SPICE Workshop on Spin Dynamics in the Dirac System
Schloss Waldthausen, Mainz, 6 June 2017

# Exotic Phenomena in Topological Insulators and Superconductors

**Yoichi Ando**

**Physics Institute II, University of Cologne**

# Outline

- Topological Protection and Planar Hall Effect
- Topological Superconductivity in $Cu_xBi_2Se_3$
- Recent Efforts at Cologne

Funding: DFG CRC1238

CRC1238 Control and Dynamics of Quantum Materials

ERC Advanced Grant
"MajoranaTopIn"

# Collaborators

- **University of Cologne**

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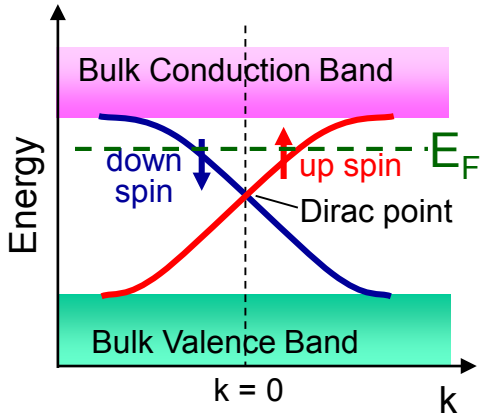
*Past members*:

K. Segawa, S. Sasaki, Z. Ren, M. Kriener, M. Novak, K. Eto

- **Theory:** Profs. Rosch, Altland (Cologne)
- **Fab.**: Prof. Matsumoto (Osaka)
- **NMR**: Prof. Zheng (Okayama)
- **Magnetocaloric**: Prof. Maeno (Kyoto)

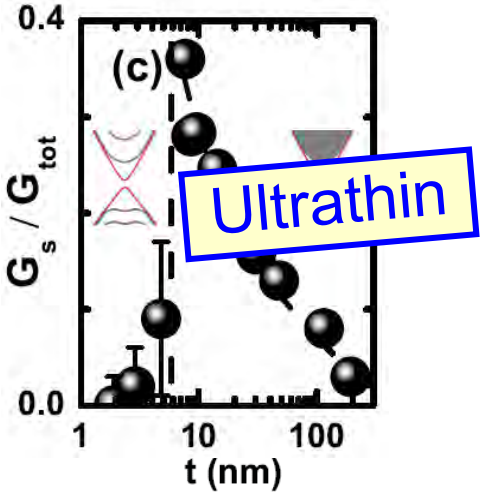
# Topological Protection and Planar Hall Effect

# Topological Protection



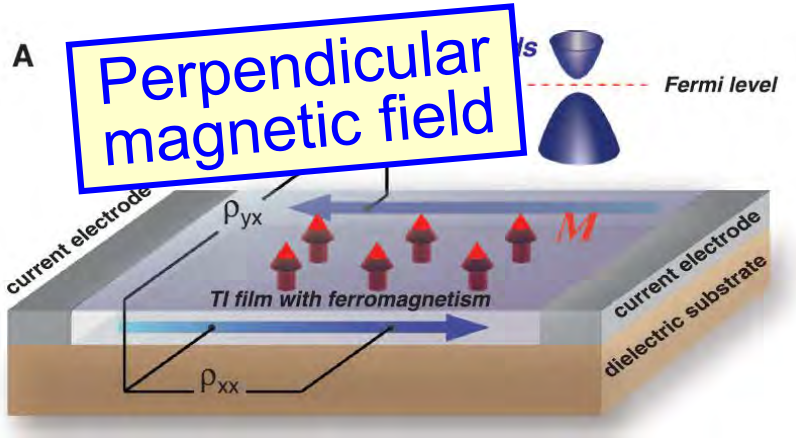

Protection is lifted by Gap Opening

## Top-Bottom Surface Hybridization




Taskin, Ando *et al.*,
PRL (2012)

## Quantum Anomalous Hall Effect



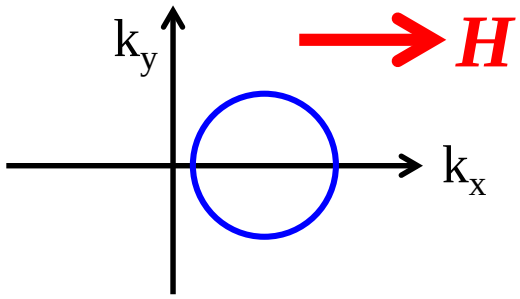

Chang, Xue *et al.*,
Science (2013)

Time-Reversal-Breaking Effect of **Parallel Magnetic Field?**

# Parallel Magnetic Field

For surface Dirac fermions, Zeeman field acts like the vector potential:

$$\boldsymbol{k}\cdot\boldsymbol{\sigma} \rightarrow (\boldsymbol{k} + g\mu_B \boldsymbol{H})\cdot\boldsymbol{\sigma}$$



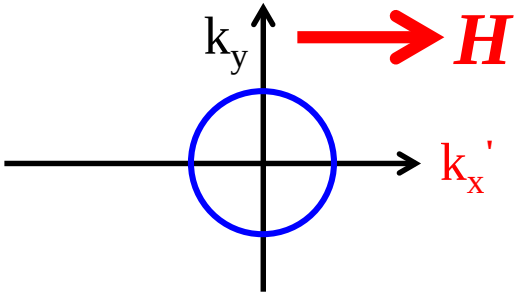

Dirac Hamiltonian

$$H = v_F \boldsymbol{k}\cdot\boldsymbol{\sigma} = v_F \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix}$$

$$\rightarrow H = v_F(\boldsymbol{k} + g\mu_B \boldsymbol{H})\cdot\boldsymbol{\sigma} = v_F \begin{pmatrix} m_z & k_x + m_x - i(k_y + m_y) \\ k_x + m_x + i(k_y + m_y) & -m_z \end{pmatrix}$$

Parallel magnetic field does not open a gap, and its effect can be gauged away by a simple shift of the frame.

➡ To first order, there is no effect on surface transport properties.



# Planar Hall Effect & In-Plane Anisotropy

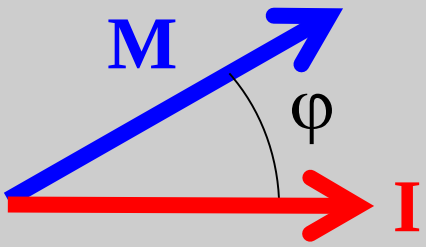
When the resistivity tensor is anisotropic,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} R_{\parallel} & 0 \\ 0 & R_{\perp} \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

$$= \begin{pmatrix} R_{\parallel}\cos^2\varphi + R_{\perp}\sin^2\varphi & (R_{\parallel} - R_{\perp})\cos\varphi\sin\varphi \\ (R_{\parallel} - R_{\perp})\cos\varphi\sin\varphi & R_{\parallel}\sin^2\varphi + R_{\perp}\cos^2\varphi \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$

AMR

PHE




**FM metal**

$R_{//} > R_{\perp}$

Angular Magnetoresistance (AMR)

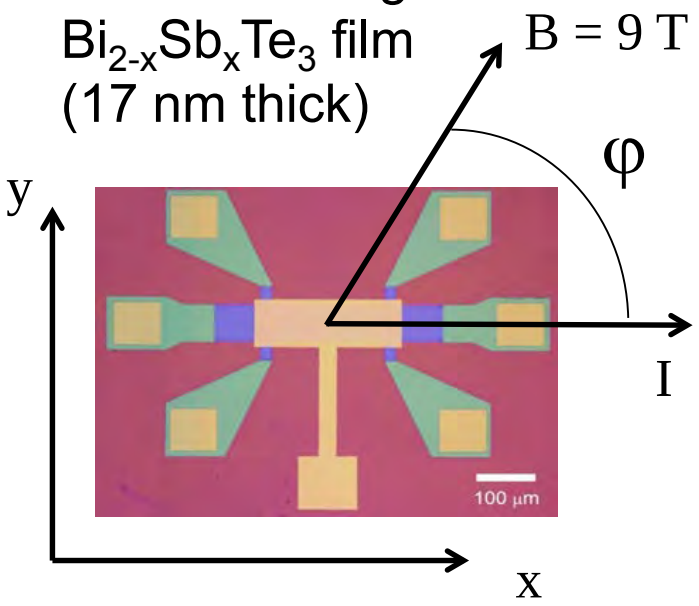
$$R_{xx} = R_{\perp} + (R_{||} - R_{\perp})\cos^2\varphi$$

Planar Hall Effect (PHE)

$$R_{yx} = (R_{||} - R_{\perp})\cos\varphi\sin\varphi$$

# Bulk-Insulating TI in Parallel Field

Bulk-insulating $Bi_{2-x}Sb_xTe_3$ film (17 nm thick)



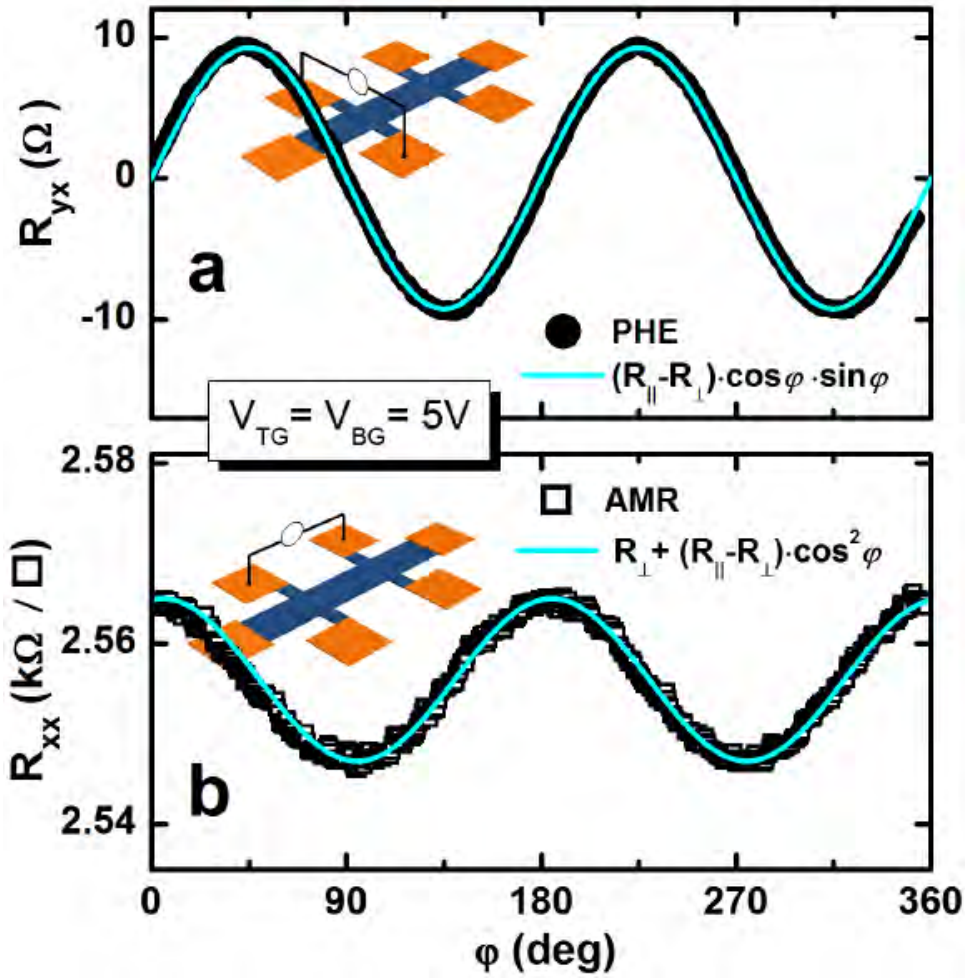

PHE

$$R_{yx} = (R_{||} - R_{\perp}) \cos\varphi \, \sin\varphi$$

AMR

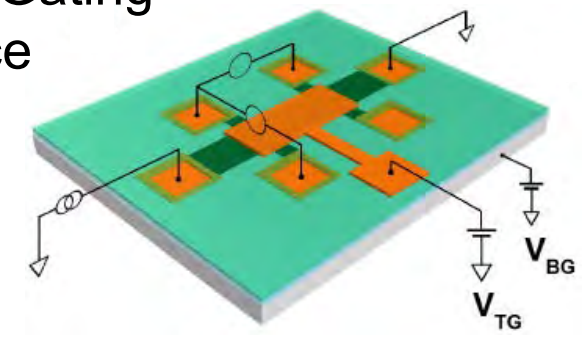
$$R_{xx} = R_{\perp} + (R_{||} - R_{\perp}) \cos^2\varphi$$

Magnetic-field-induced resistivity anisotropy

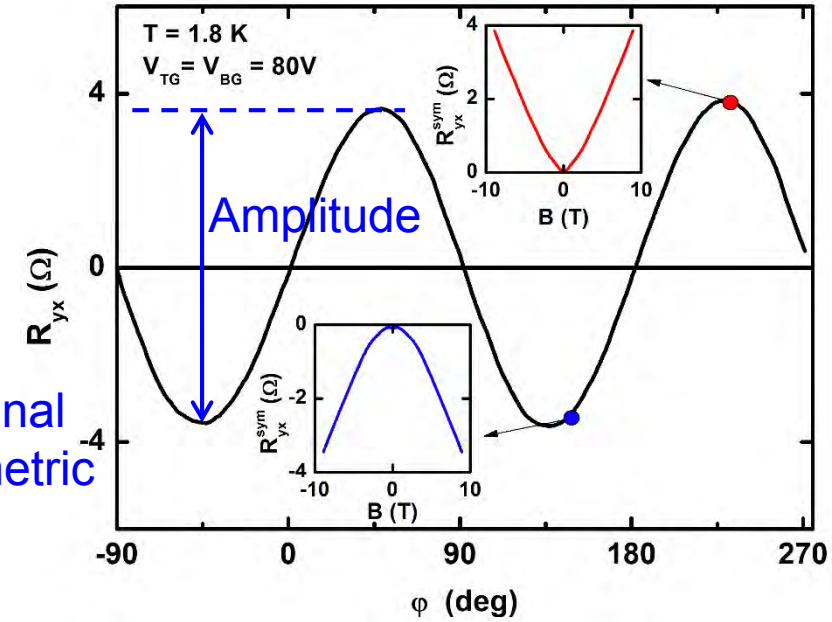


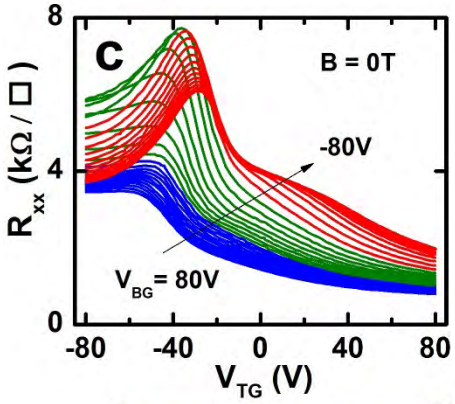
# PHE from TI surface

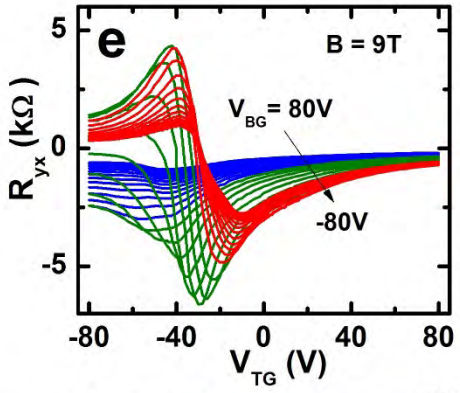
Dual-Gating Device

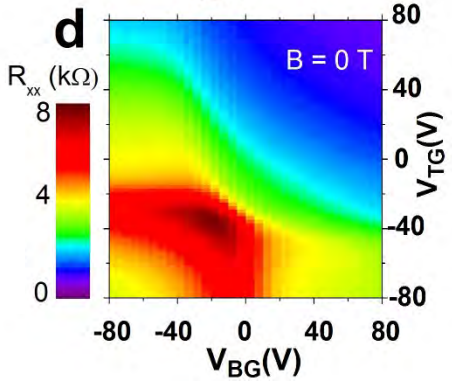


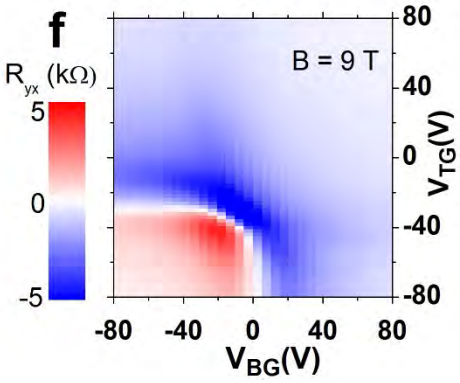

PHE signal is symmetric w.r.t. B

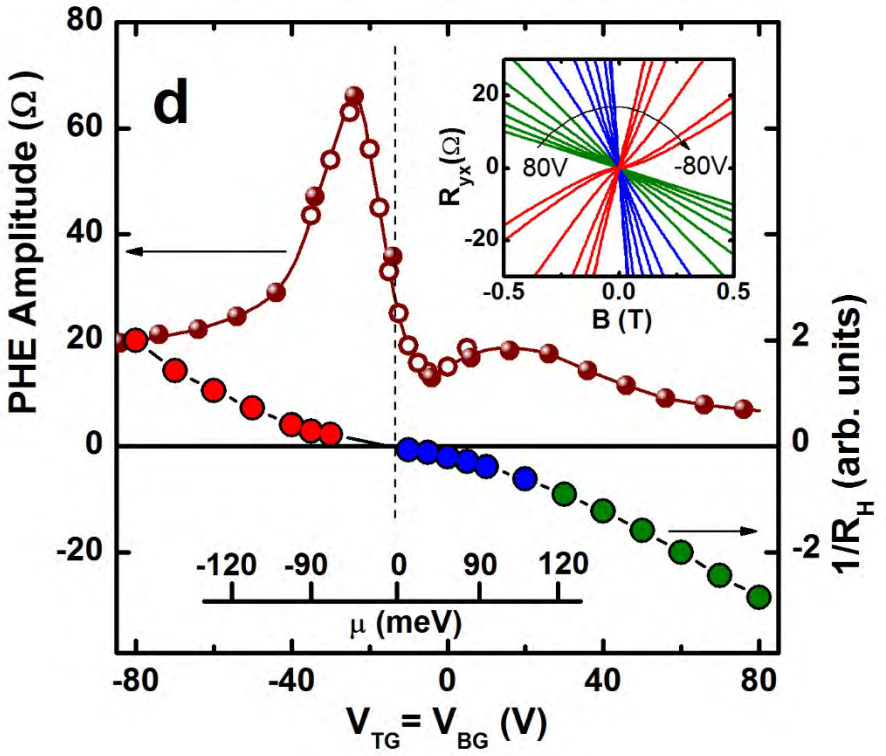


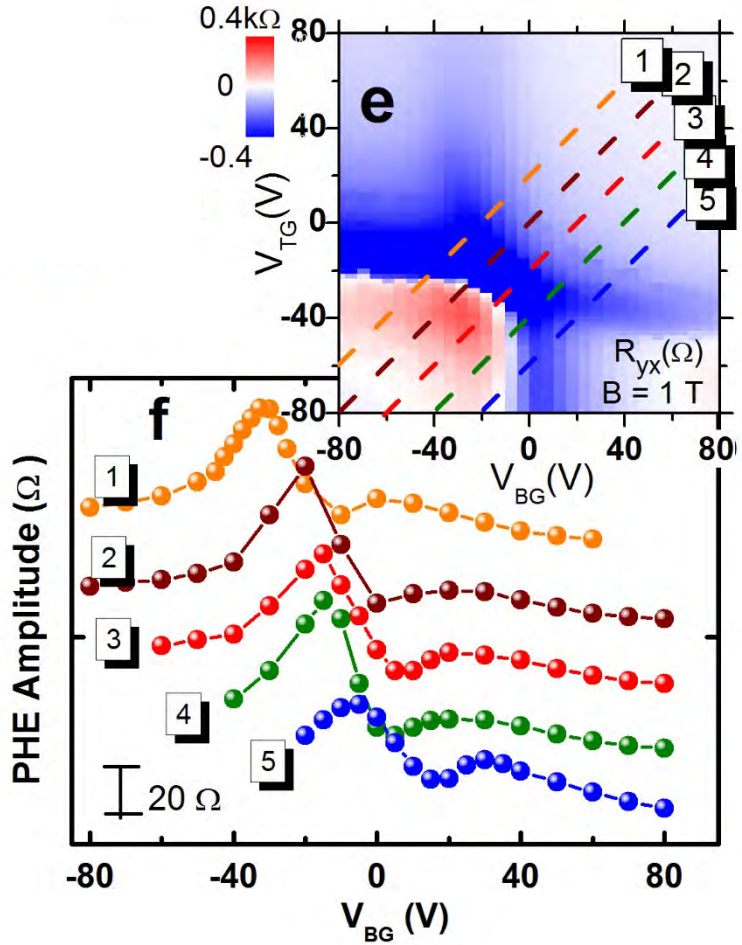



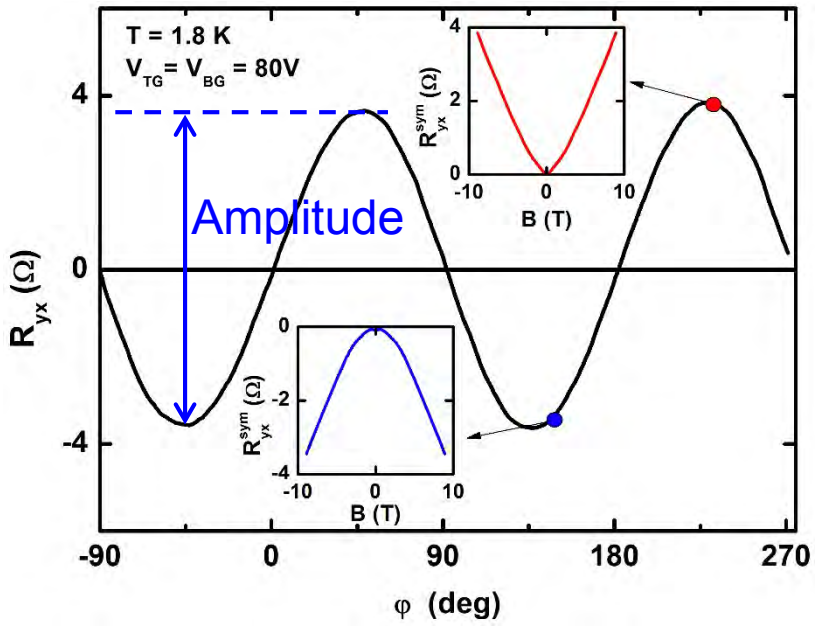



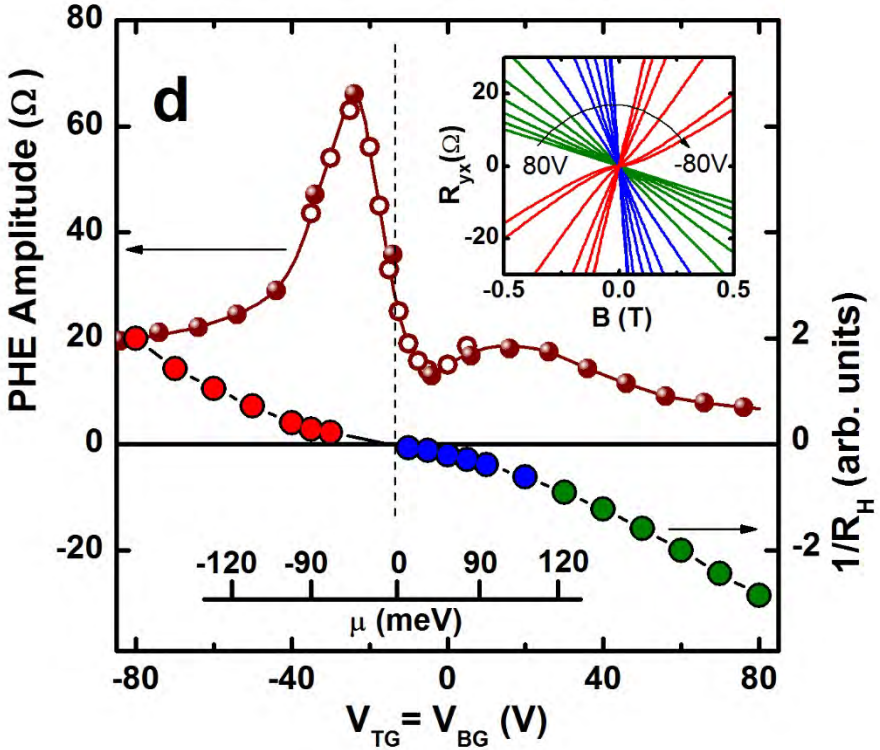



# PHE from TI surface

Double-peak structure is associated with the Dirac Point



# Theoretical Analysis of PHE

$$H = \sum_{k,\alpha,\beta} h_{\alpha\beta}(\boldsymbol{k})\psi^{\dagger}_{\alpha}(\boldsymbol{k})\psi_{\beta}(\boldsymbol{k}) + \sum_{\alpha,\beta}((\epsilon+\mu)\delta_{\alpha\beta} - \boldsymbol{B}\boldsymbol{\sigma})d^{\dagger}_{\alpha}d_{\beta} + V\sum_{k,\alpha,i} e^{-i\boldsymbol{k}\boldsymbol{R}_i}\psi^{\dagger}_{\alpha}(\boldsymbol{k})d_{\alpha} + \text{h.c.}$$

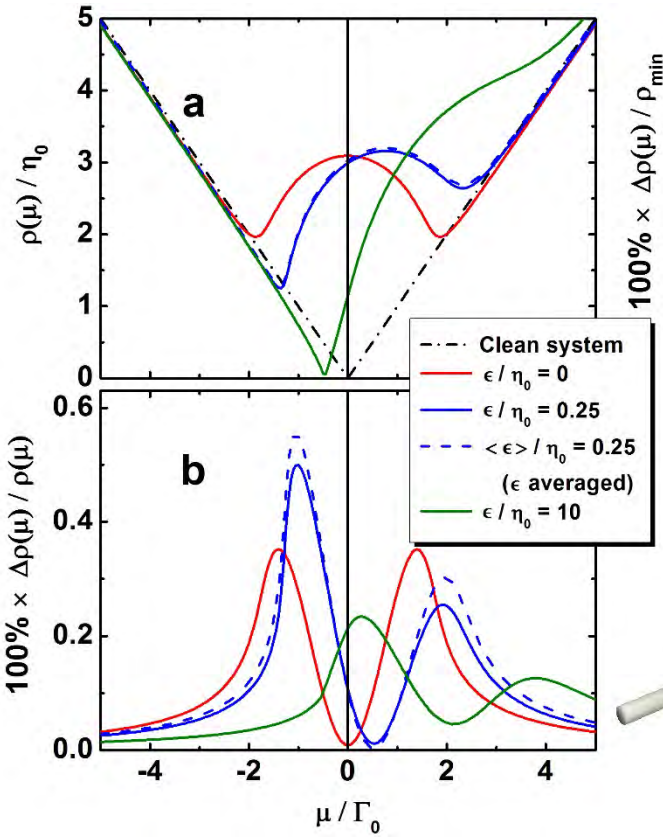
Resonance energy

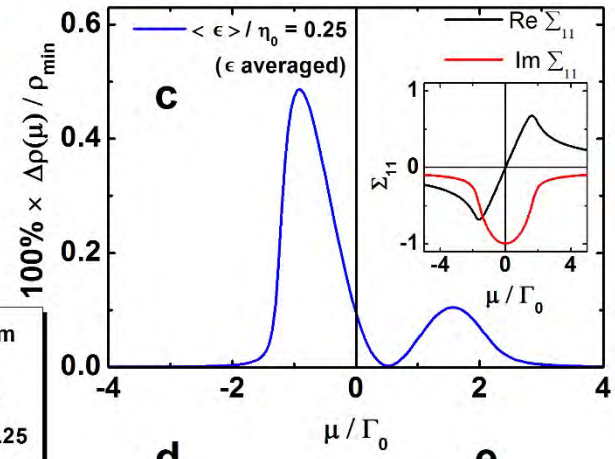
TSS

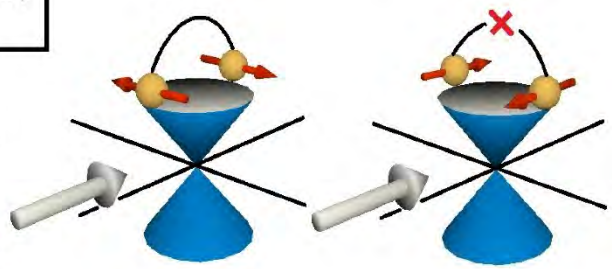
Impurity states in B

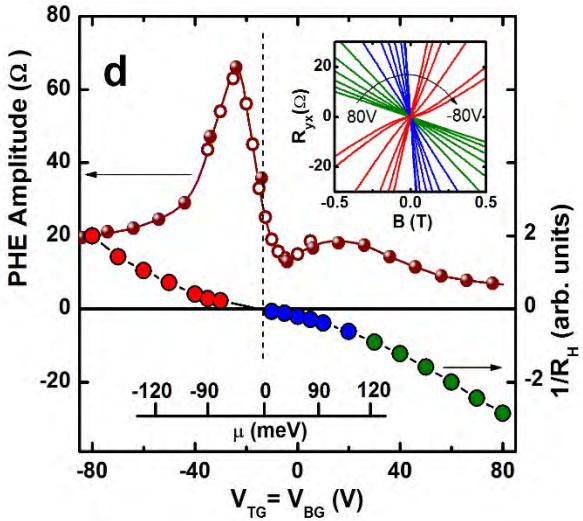
hybridization

$$\sigma^{\parallel/\perp} = \frac{2e^2v_F^2}{\pi}\int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{|\mu-\Sigma_{11}|^2 \pm (v_Fk_{\parallel}^2 - |v_Fk_{\perp}-\Sigma_{12}|^2)}{|(\mu-\Sigma_{11})^2 - (v_F^2k_{\parallel}^2 + (v_Fk_{\perp}-\Sigma_{12})^2)|^2} \approx \sigma_0\left(1 \mp \frac{c}{2}\left(\frac{\mathrm{Im}\Sigma_{12}}{\mathrm{Im}\Sigma_{11}}\right)^2\right)$$




Spin-polarization of monmagnetic impurities → Anisotropic scattering

Peak position → $\ell$ ~ 7 nm

PHE amplitude → $\ell_\phi$ ~ 70 nm

# Topological Superconductor

IOP Publishing

Reports on Progress in Physics

Rep. Prog. Phys. **80** (2017) 076501 (42pp)

https://doi.org/10.1088/1361-6633/aa6ac7

**Review**

# Topological superconductors: a review

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1. Introduction
2. Concept of topology in quantum mechanics
3. Basics of superconductivity
4. Theory of topological superconductors
5. Majorana fermions
6. Routes to topological superconductivity
7. Materials realizations
8. Properties of topological superconductors
9. Outlook

# Possible Topological Superconductors

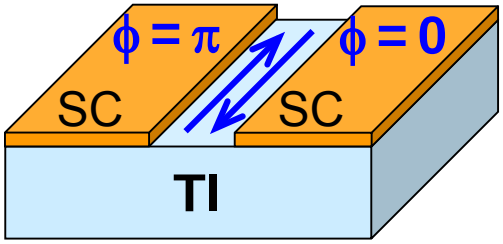
- "Periodic Table" of topological invariant

Schnyder-Ryu-Furusaki-Ludwig (2008)
Kitaev (2009)

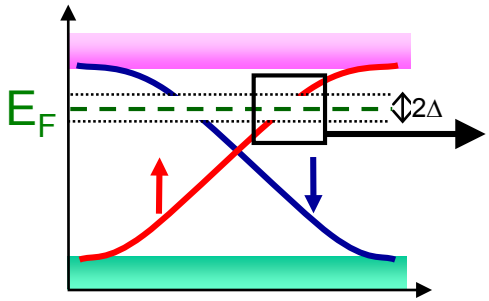
SC in TI surface
Chiral p-wave

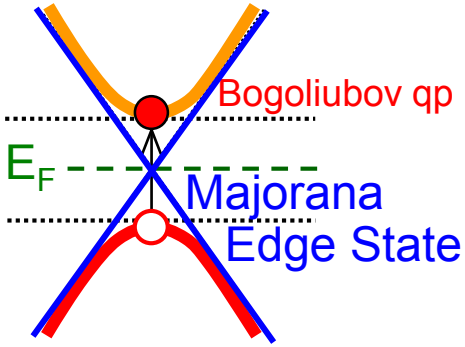
| | 1D | 2D | 3D | |
|---|---|---|---|---|
| Time-Reversal Broken (TRB) | $Z_2$ | Z | - | (D) |
| Time-Reversal Invariant (TRI) | $Z_2$ | $Z_2$ | Z | (DIII) |

- Surface State of TIs




Fu & Kane (2008)




- $Sr_2RuO_4$

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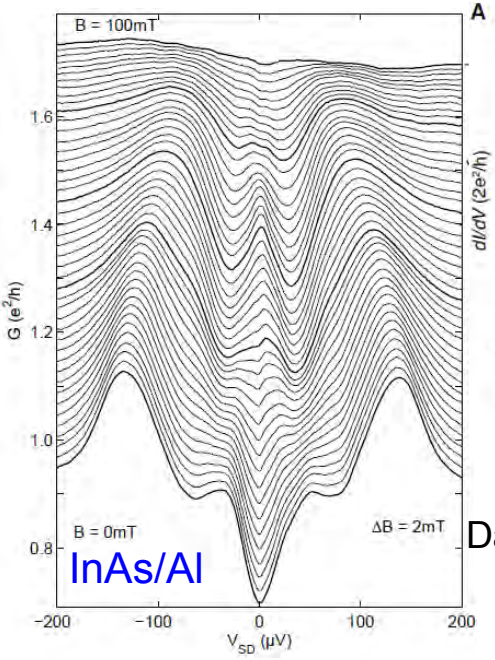
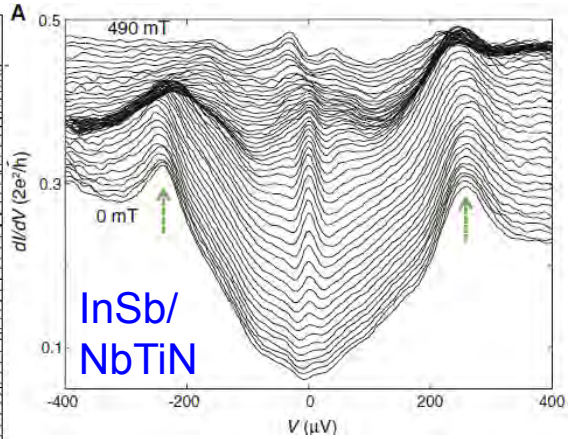


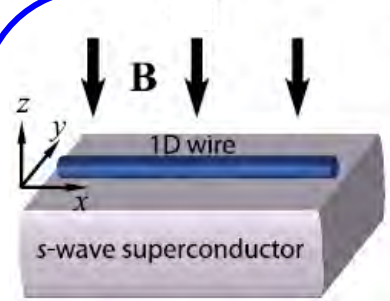



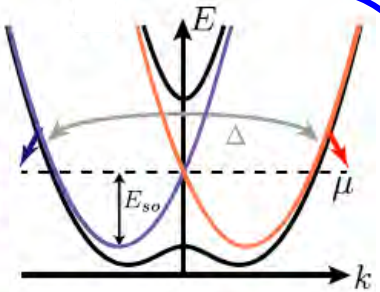

Mourik *et al.*, Science (2012)

Das *et al.*, Nature Phys. (2012)

Oreg *et al.*, PRL (2010)

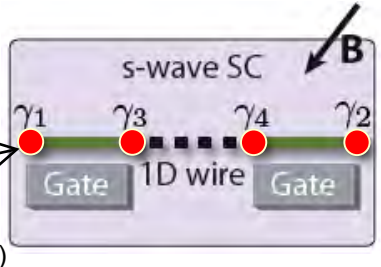
Lutchyn *et al.*, PRL (2010)




1D Nanowire of InSb or InAs

Majorana End-State




Alicea, RPP (2012)

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Kitaev (2009)

Kitaev model

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Chiral p-wave

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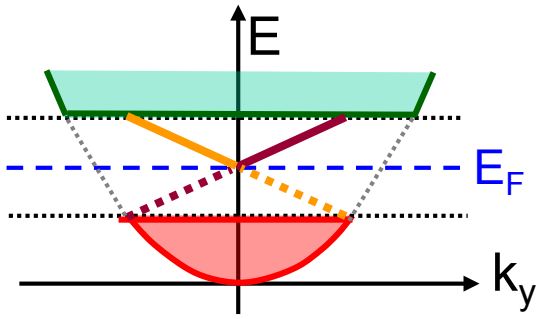
| Time-Reversal Broken (TRB) | $Z_2$ | $Z$ | - | (D) |
| Time-Reversal Invariant (TRI) | $Z_2$ | $Z_2$ | $Z$ | (DIII) |

Superfluid $^3$He-B phase

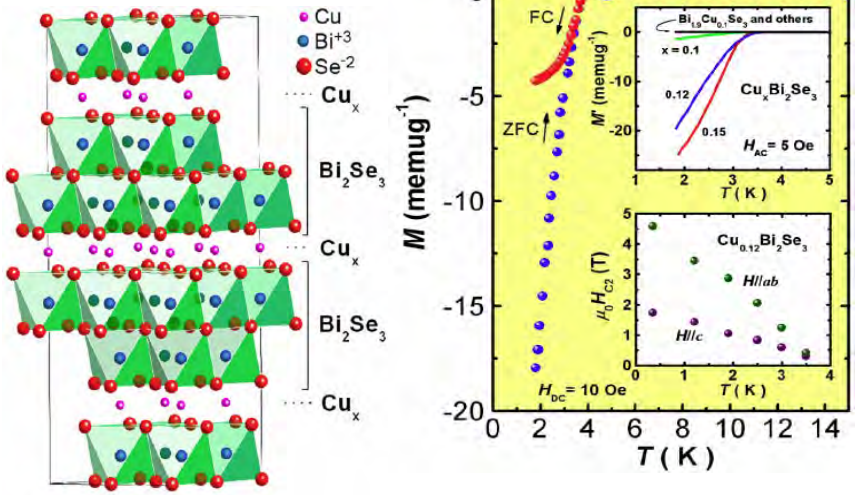
- New 3D topological state of matter

- The surface state may host
  Helical Majorana Fermions
  that are itinerant and massless



# SC in $Cu_xBi_2Se_3$

Hor et al., PRL (2010)




- Four-component Hamiltonian of $Bi_2Se_3$

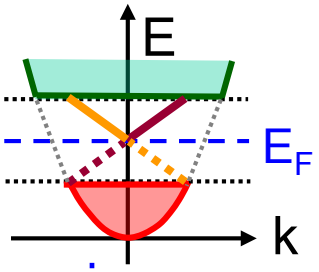
$$H_0(\mathbf{k}) = m\Gamma_0 + v(k_x\Gamma_1 + k_y\Gamma_2) + v_z k_z \Gamma_3$$

with the basis

$( |P1_z^+\uparrow\rangle, |P1_z^+\downarrow\rangle, |P2_z^-\uparrow\rangle, |P2_z^-\downarrow\rangle )$

Topological SC
State in the bulk

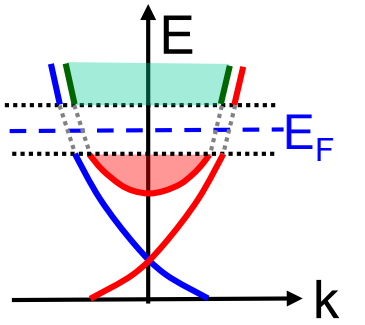
Fu & Berg, PRL (2010)

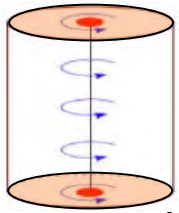



→ Helical Majorana fermions

| Types of pairing in terms of field operators | Rep. | $P$ |
|---|---|---|
| $\Delta_1$: $c^\dagger_{1\uparrow}c^\dagger_{1\downarrow} + c^\dagger_{2\uparrow}c^\dagger_{2\downarrow}$; $c^\dagger_{1\uparrow}c^\dagger_{2\downarrow} - c^\dagger_{1\downarrow}c^\dagger_{2\uparrow}$ | $A_{1g}$ | + |
| $\Delta_2$: $c^\dagger_{1\uparrow}c^\dagger_{2\downarrow} + c^\dagger_{1\downarrow}c^\dagger_{2\uparrow}$ | $A_{1u}$ | − |
| $\Delta_3$: $c^\dagger_{1\uparrow}c^\dagger_{1\downarrow} - c^\dagger_{2\uparrow}c^\dagger_{2\downarrow}$ | $A_{2u}$ | − |
| $\Delta_4$: $(ic^\dagger_{1\uparrow}c^\dagger_{2\uparrow} - ic^\dagger_{1\downarrow}c^\dagger_{2\downarrow}, c^\dagger_{1\uparrow}c^\dagger_{2\uparrow} + c^\dagger_{1\downarrow}c^\dagger_{2\downarrow})$ | $E_u$ | $(-,-)$ |

Conventional SC
State in the bulk
→ Proximity SC



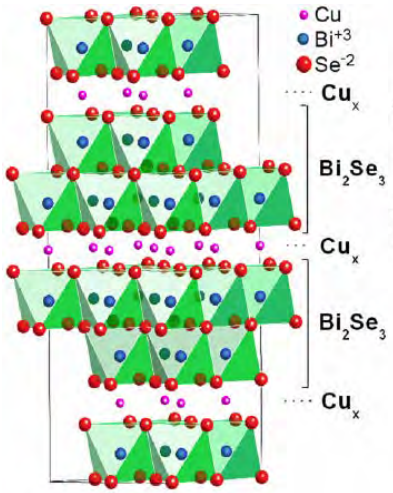



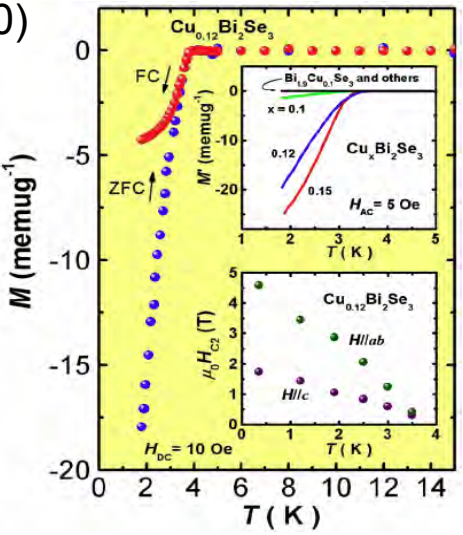
Majorana zero mode in vortices

Hosur *et al.*, PRL (2011)

# SC in $Cu_xBi_2Se_3$

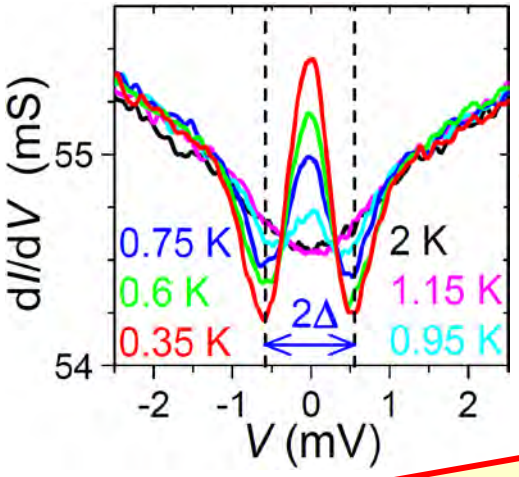
Hor et al., PRL (2010)

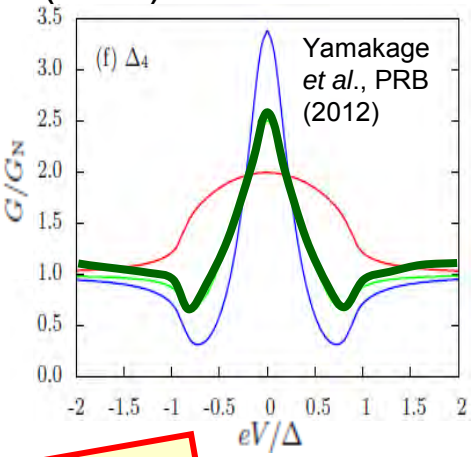



Point-contact spectroscopy

Sasaki, Ando *et al.*, PRL (2011)

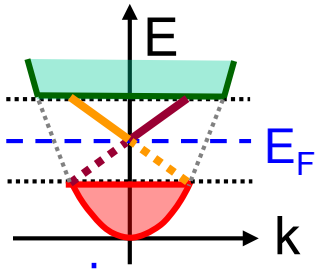



**ZBCP due to helical Majorana fermions?**

Topological SC
State in the bulk

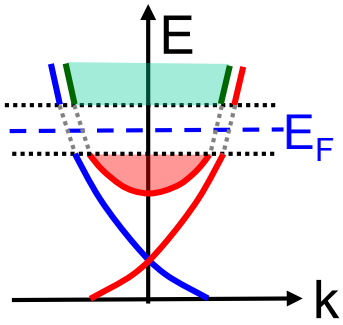
Fu & Berg, PRL (2010)

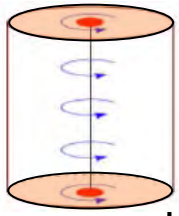



→ Helical Majorana fermions

| Types of p… terms of field operators | Rep. | $P$ |
|---|---|---|
| $\Delta_1$: $c^\dagger_{1\uparrow}c^\dagger_{1\downarrow}+c^\dagger_{2\uparrow}c^\dagger_{2\downarrow}$; $c^\dagger_{1\uparrow}c^\dagger_{2\downarrow}-c^\dagger_{1\downarrow}c^\dagger_{2\uparrow}$ | $A_{1g}$ | + |
| $\Delta_2$: $c^\dagger_{1\uparrow}c^\dagger_{2\downarrow}+c^\dagger_{1\downarrow}c^\dagger_{2\uparrow}$ | $A_{1u}$ | − |
| $\Delta_3$: $c^\dagger_{1\uparrow}c^\dagger_{1\downarrow}-c^\dagger_{2\uparrow}c^\dagger_{2\downarrow}$ | $A_{2u}$ | − |
| $\Delta_4$: $(ic^\dagger_{1\uparrow}c^\dagger_{2\uparrow}-ic^\dagger_{1\downarrow}c^\dagger_{2\downarrow}, c^\dagger_{1\uparrow}c^\dagger_{2\uparrow}+c^\dagger_{1\downarrow}c^\dagger_{2\downarrow})$ | $E_u$ | $(-,-)$ |

Conventional SC
State in the bulk
→ Proximity SC



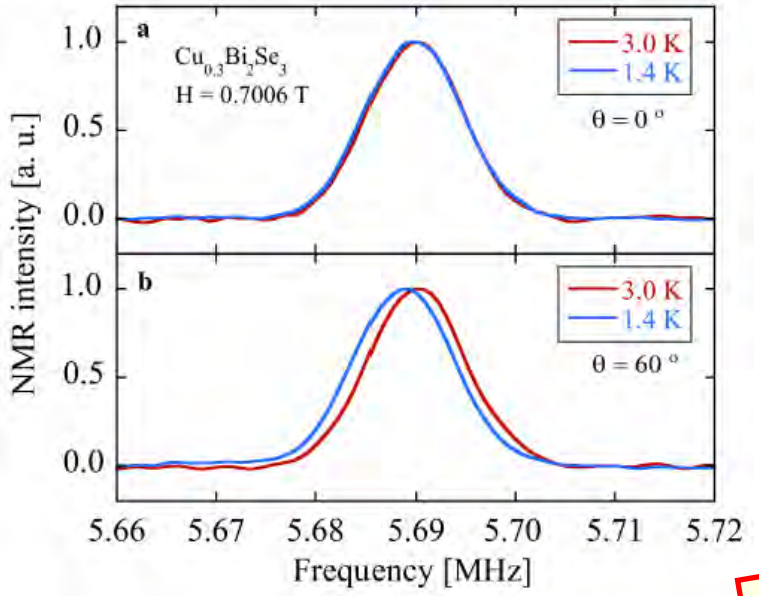



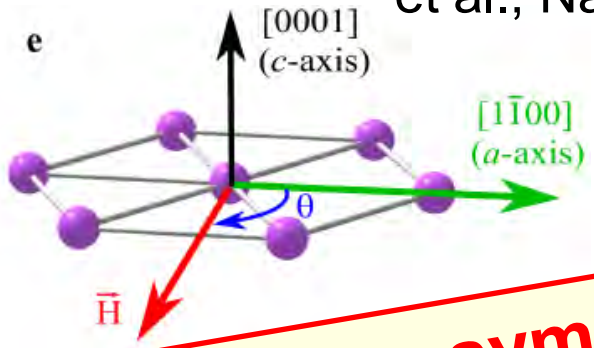
Majorana zero mode in vortices

Hosur *et al.*, PRL (2011)

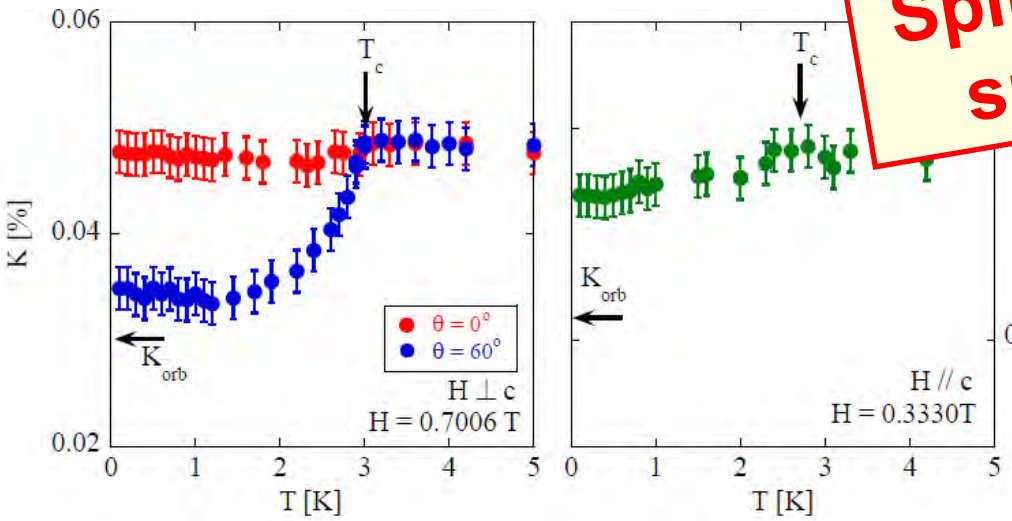
# NMR Experiment on $Cu_xBi_2Se_3$

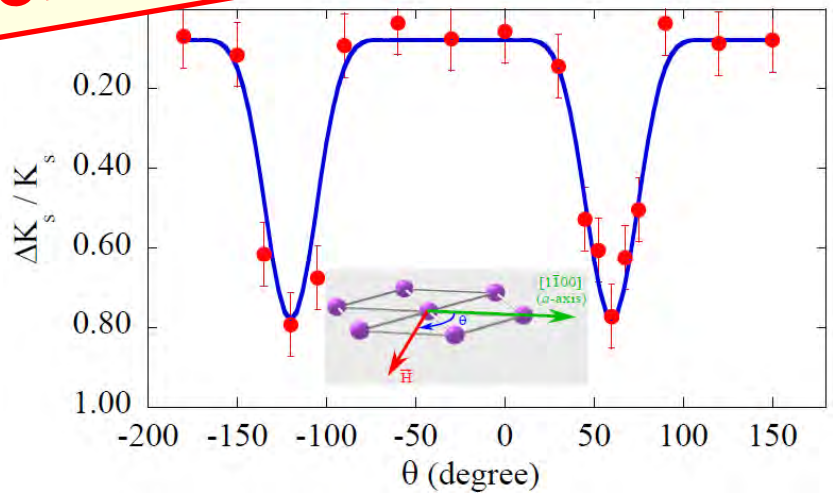
Matano, Ando, Zheng et al., Nat. Phys. (2016)



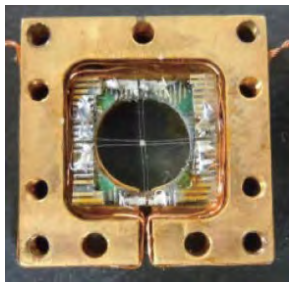



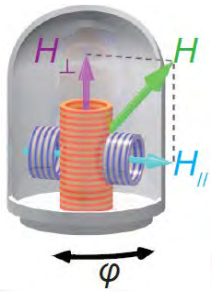

**Spin-rotation symmetry is spontaneously broken**

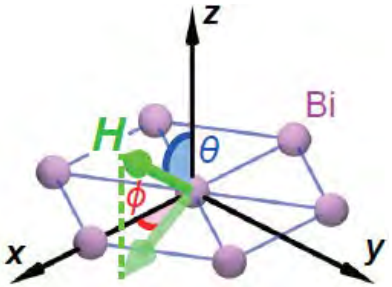


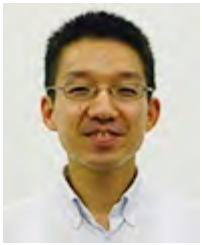



# Specific-Heat of $Cu_xBi_2Se_3$

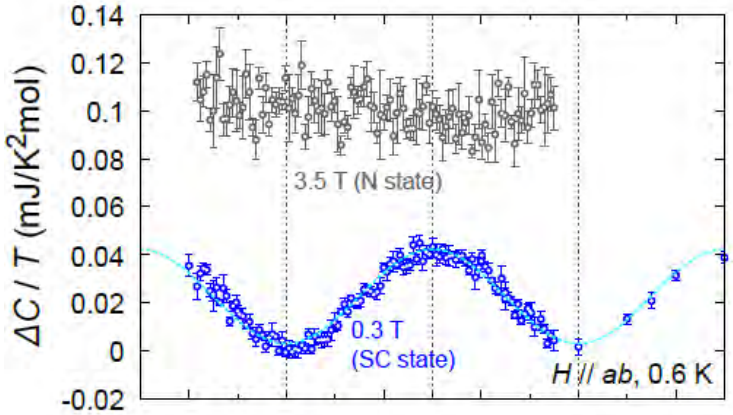


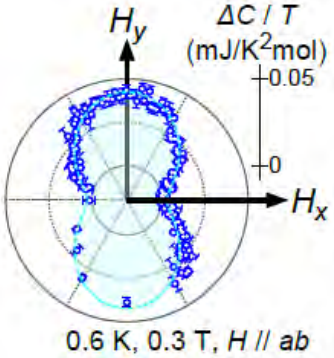


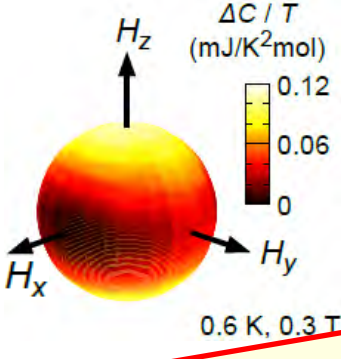



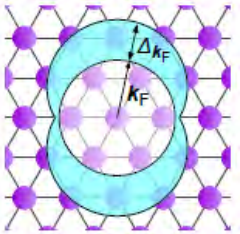



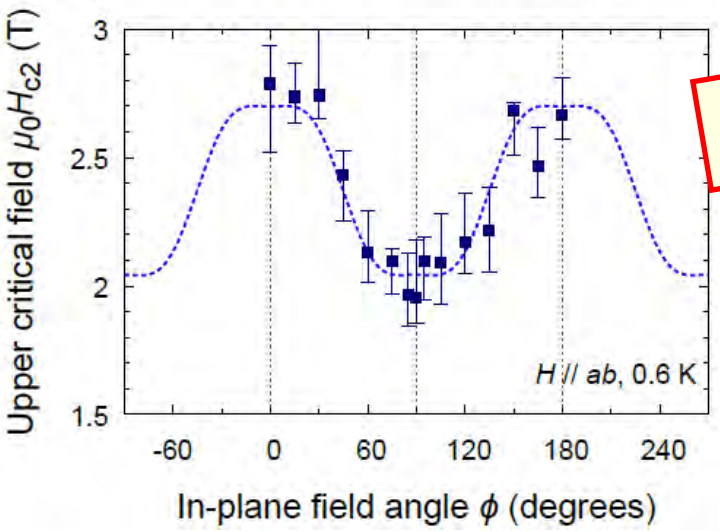
Yonezawa, Ando, Maeno et al., Nat. Phys. (2016)



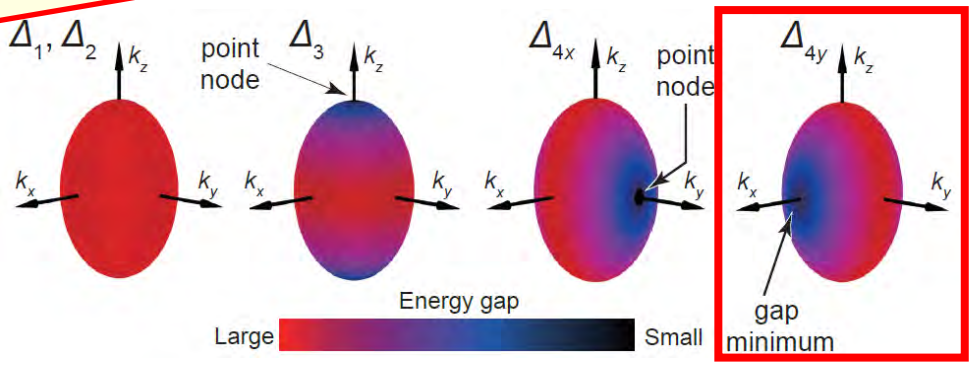



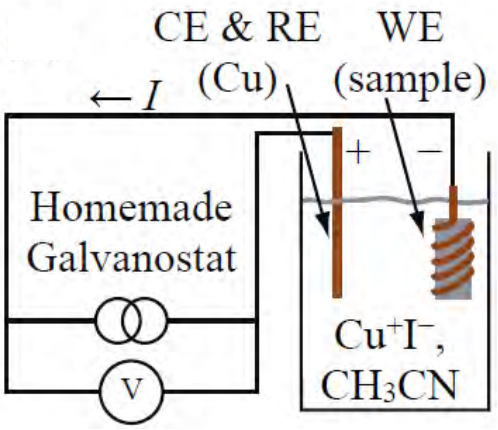



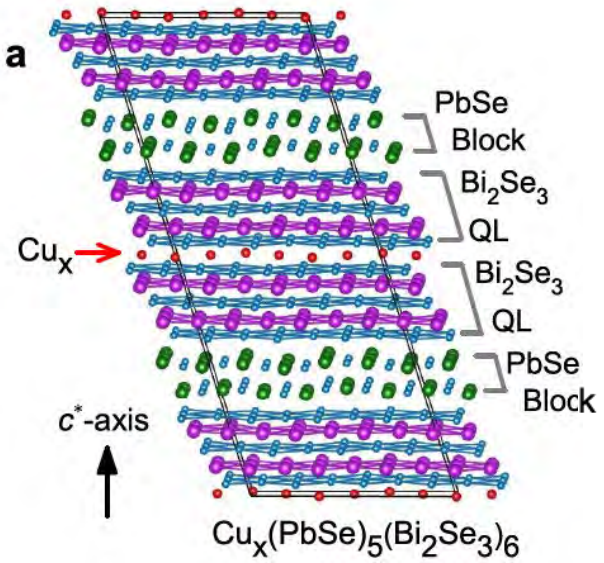


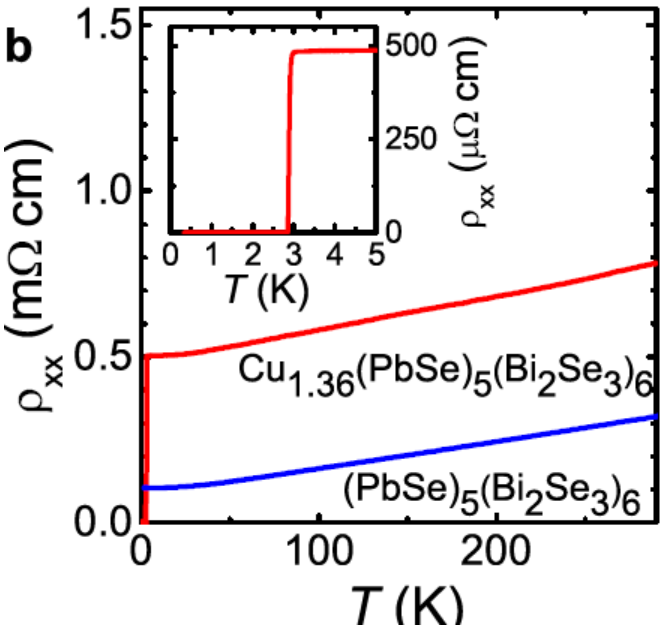

**Nematic superconductivity**



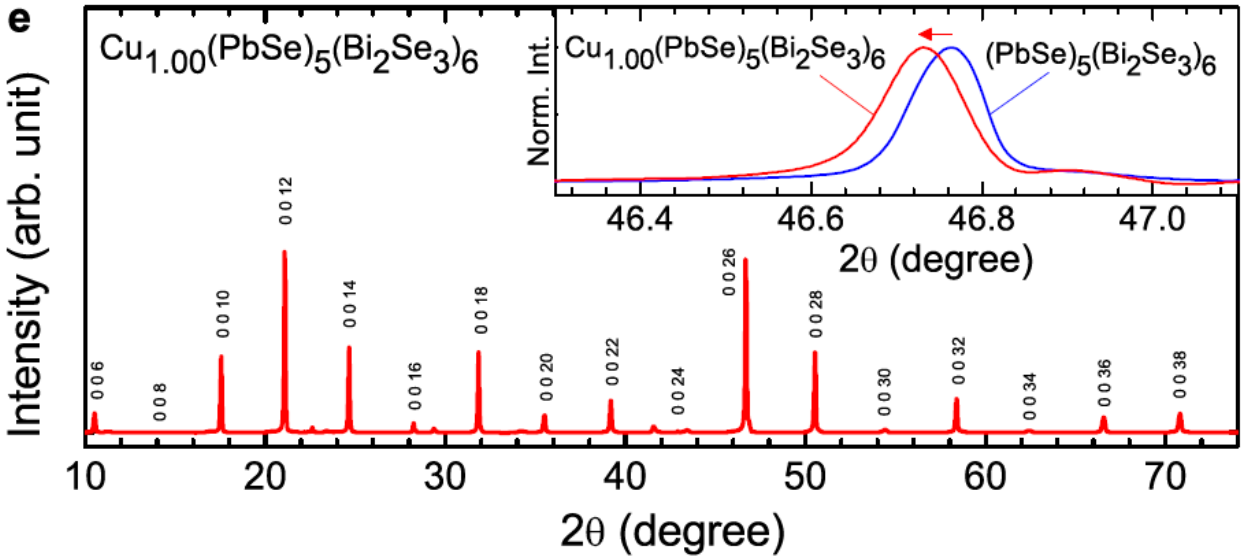
# Cu-intercalated $(PbSe)_5(Bi_2Se_3)_6$



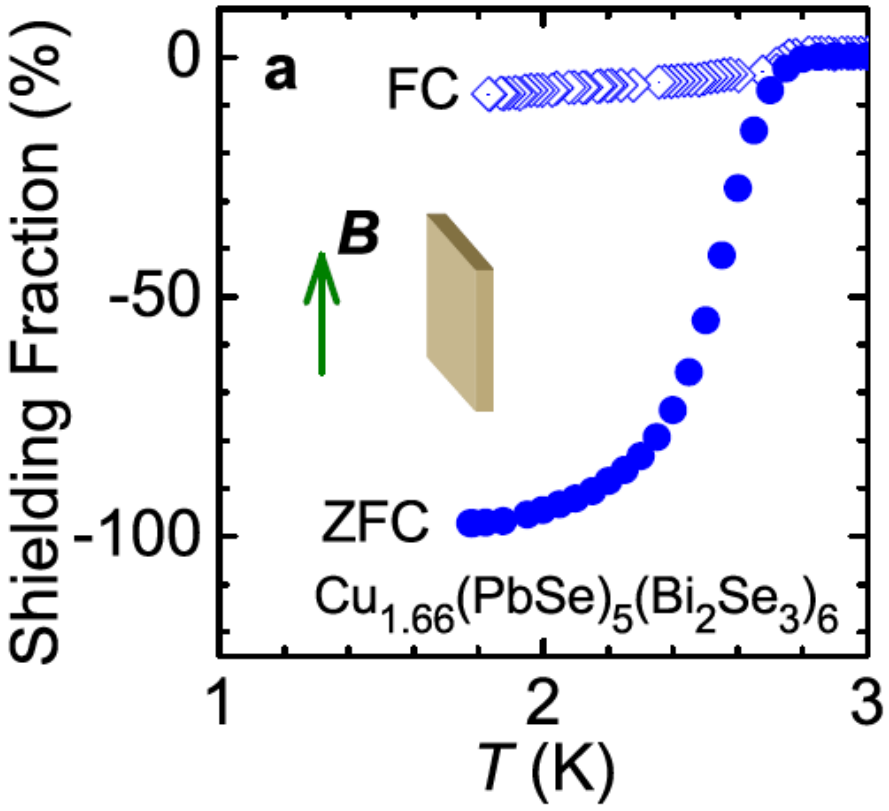



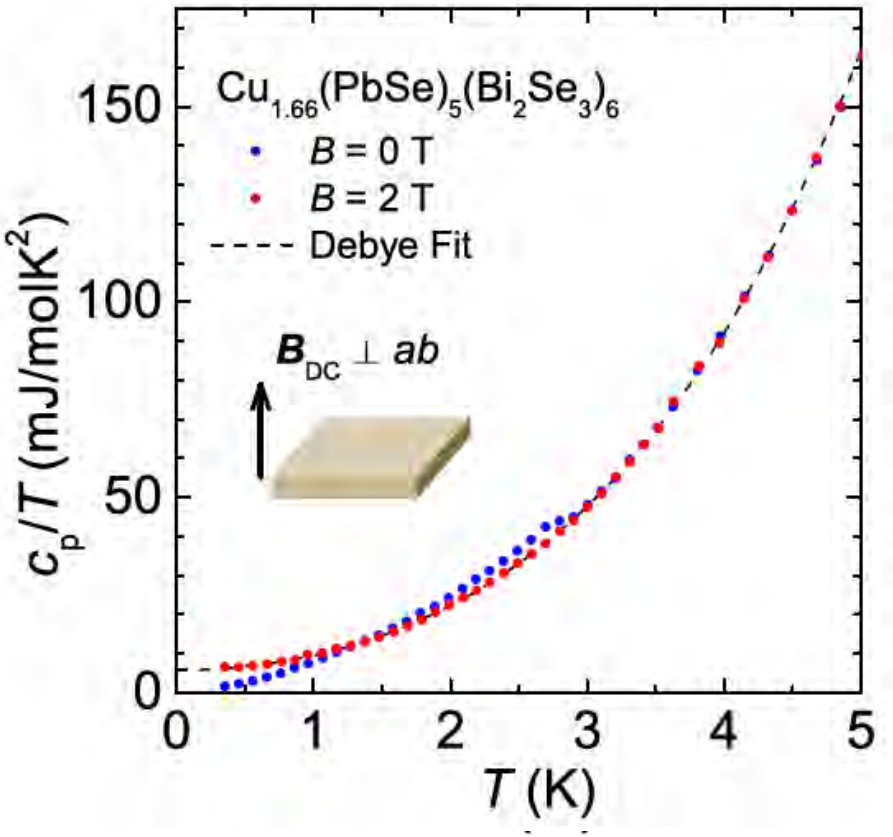




Sasaki, Segawa, Ando, PRB (2014)



# Nearly 100% Volume Fraction

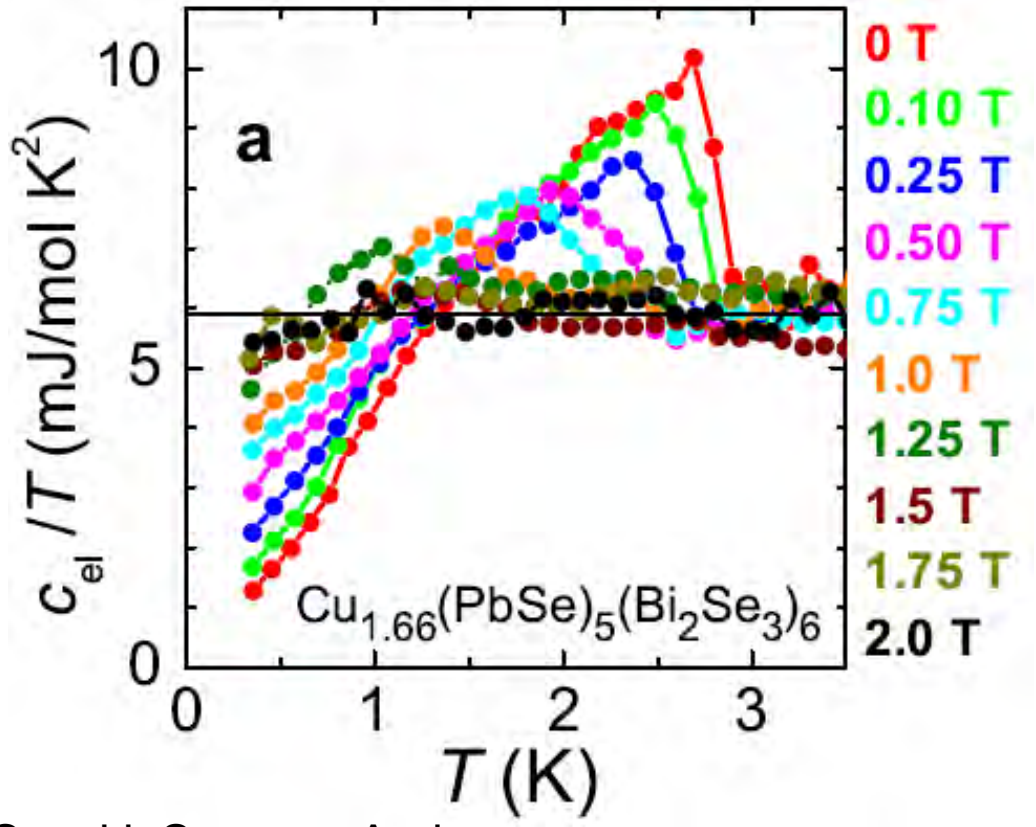


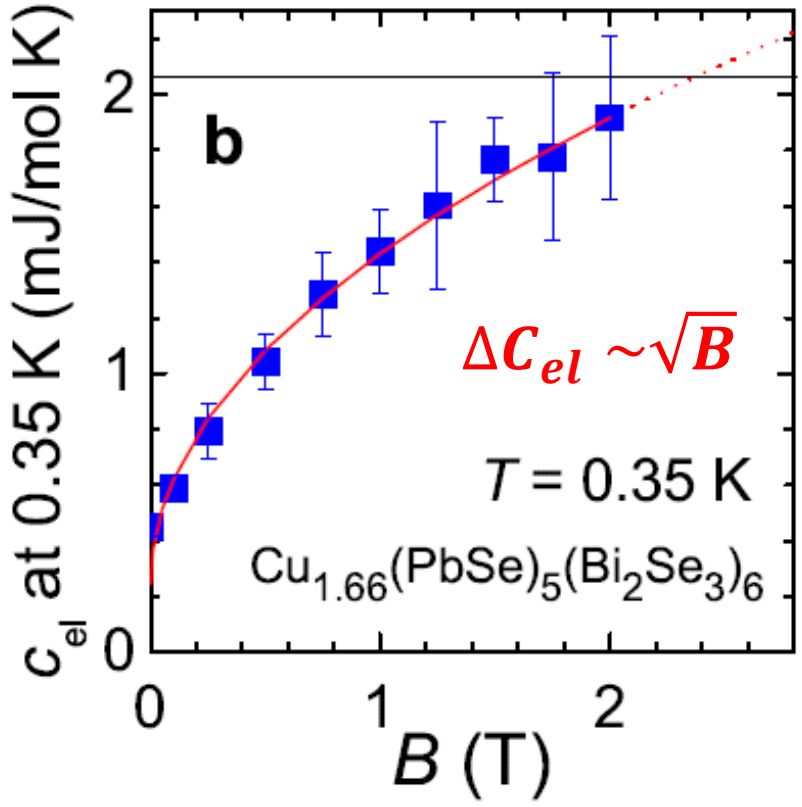




Sasaki, Segawa, Ando
PRB (2014)

Specific-heat behavior is very different from BCS, suggesting a gap with line nodes

# Magnetic-Field Dependence of $C_{el}$



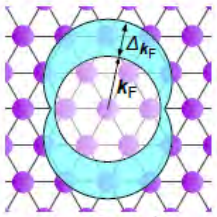




Sasaki, Segawa, Ando
PRB (2014)

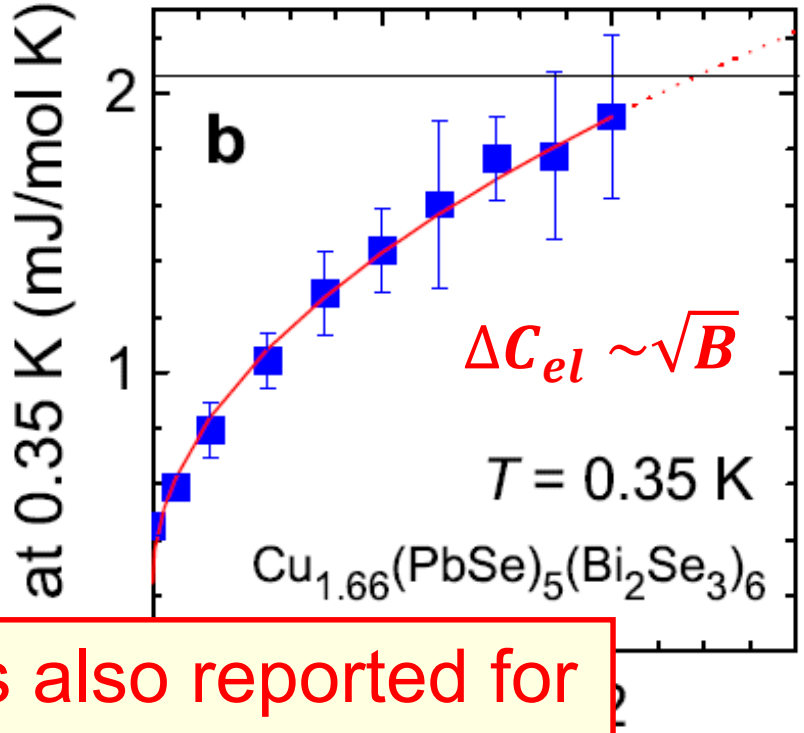
At low T, $C_{el}(B)$ behaves as $\sim\sqrt{B}$ ,
pointing to the existence of **line nodes**.

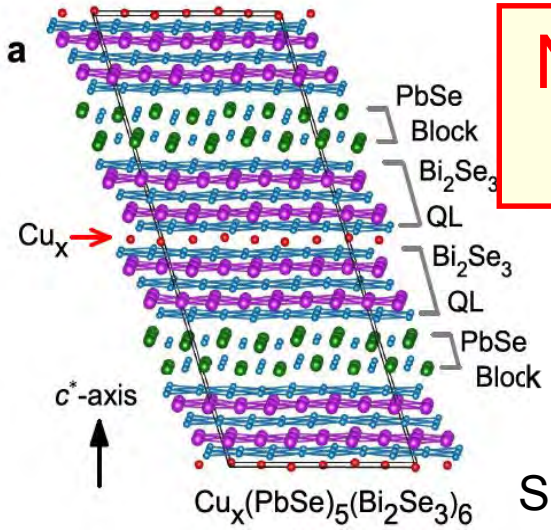
# Magnetic-Field Dependence of $C_{el}$

- Nodal Gap
  → Unconventional SC
- The nodes are most likely due to the $\Delta_4$ gap




- Same pairing as in $Cu_xBi_2Se_3$




Nematic SC is also reported for $Sr_xBi_2Se_3$ and $Nb_xBi_2Se_3$

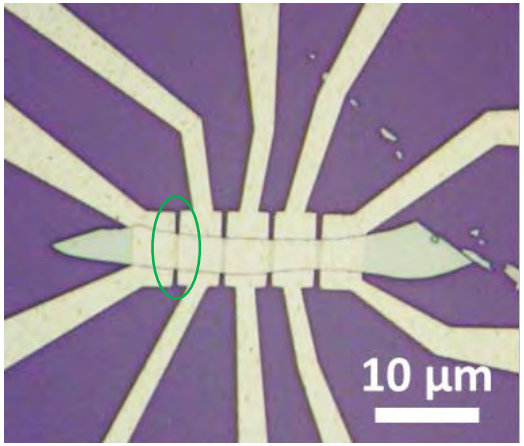
At low T, $C_{el}(B)$ behaves as $\sim\sqrt{B}$ , pointing to the existence of **line nodes**.

Sasaki, Segawa, Ando, PRB (2014)

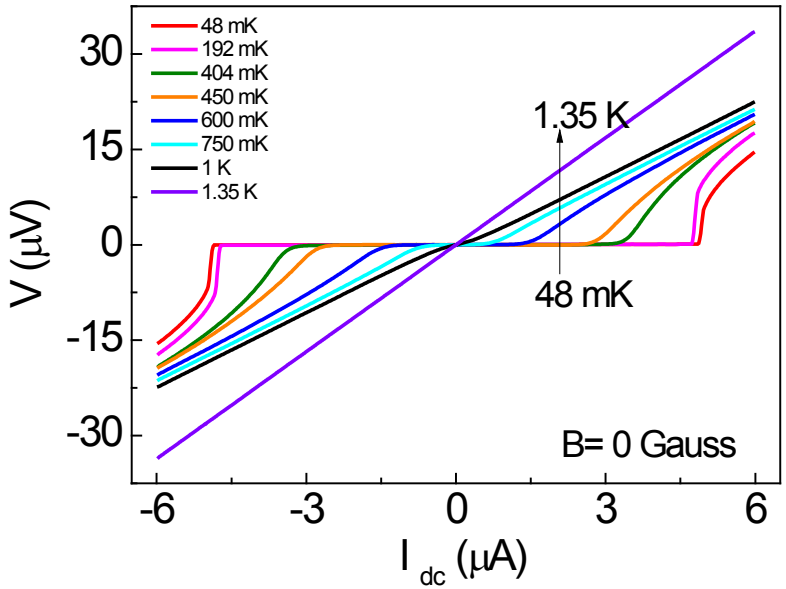
# Recent Efforts

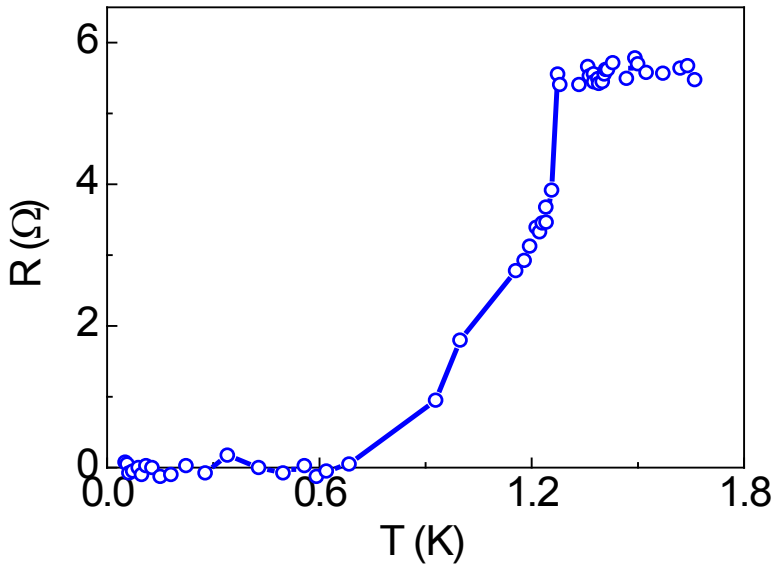
# Topological Josephson Junction

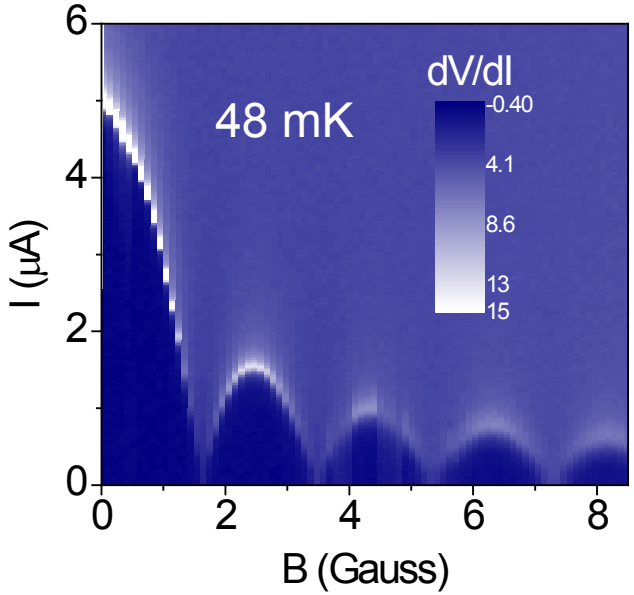
$Bi_2Se_3$ + Pt/Al/Au




Gap 600 nm



PHYSICAL REVIEW B **95**, 165424 (2017)

# Majorana qubits in a topological insulator nanoribbon architecture

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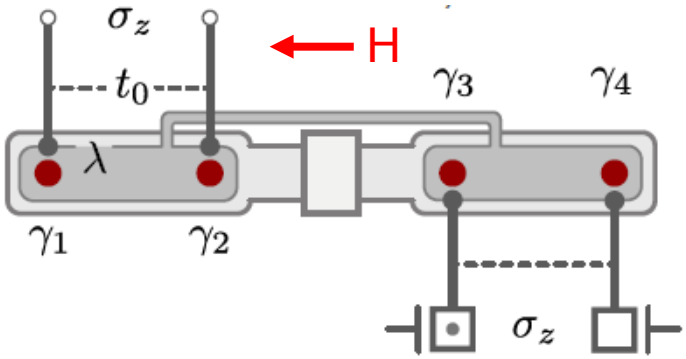
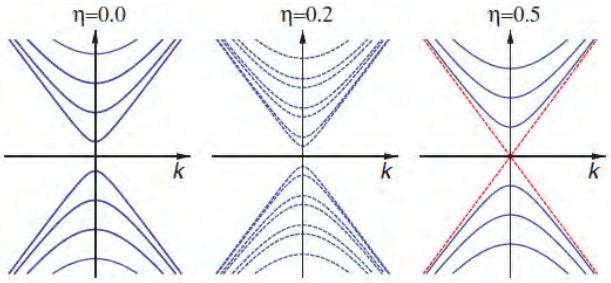
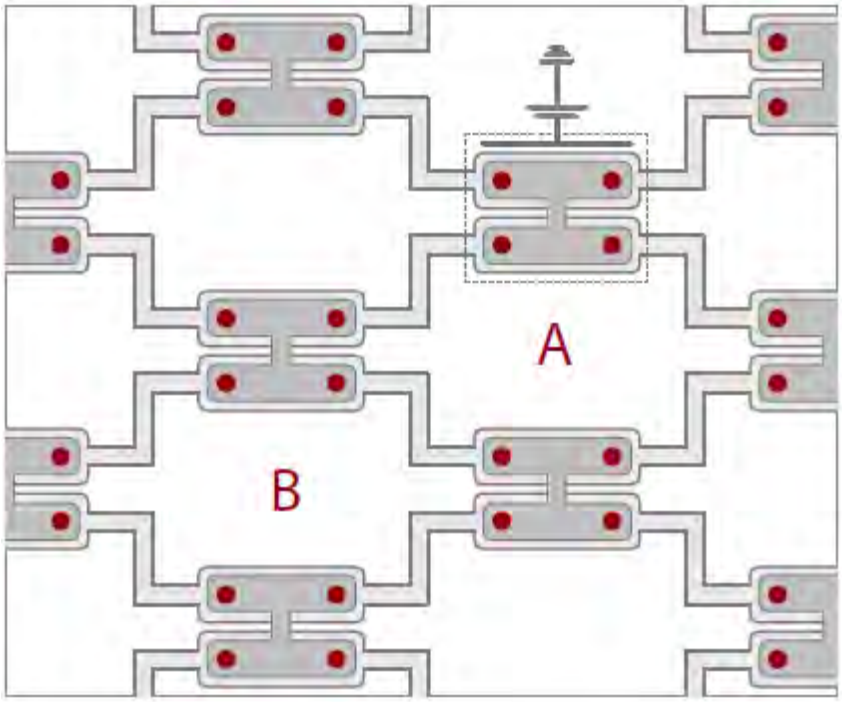
(Received 9 February 2017; published 17 April 2017)




Nanowire-based Majorana box qubit for surface code processor




Cook & Franz, PRB (2011)




"Surface Code" is a realistic architecture to realize practical quantum computations by built-in error corrections

# Summary

- Planar Hall Effect probes the anisotropic lifting of the topological protection from backscattering
- Superconducting doped $Bi_2Se_3$ realizes a Topological Nematic SC State

# Students and Postdocs wanted!

European Research Council

Established by the European Commission