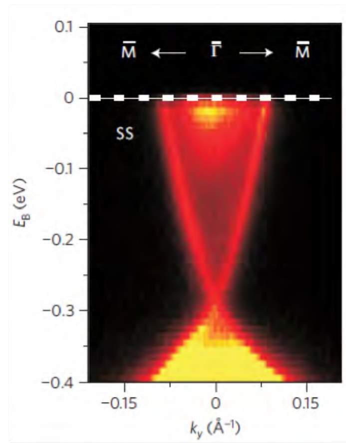
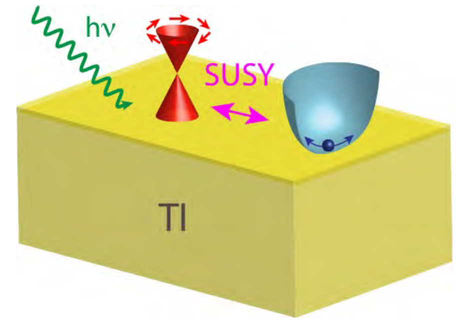
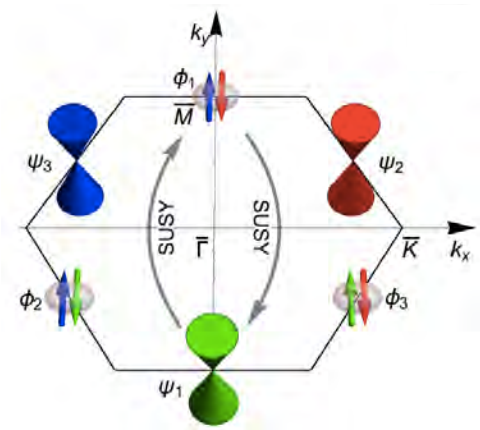
Superconducting Dirac fermions and mirror symmetry

Joseph Maciejko

University of Alberta







Spin Dynamics in the Dirac Systems

Schloss Waldthausen, Mainz

June 7, 2017

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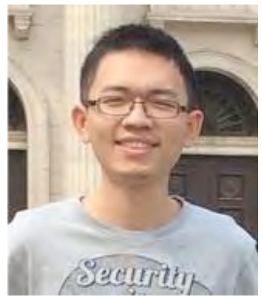
Collaborators

N. Zerf
(Heidelberg)

C.-H. Lin
(Alberta)



S.-K. Jian
(IASTU)

W. Witczak-Krempa
(Montréal)

H. Yao
(IASTU)

W. Witczak-Krempa and JM, Phys. Rev. Lett. **116**, 100402 (2016)
N. Zerf, C.-H. Lin, and JM, Phys. Rev. B **94**, 205106 (2016)
S.-K. Jian, C.-H. Lin, JM, and H. Yao, Phys. Rev. Lett. **118**, 166802 (2017)

Platforms for novel quantum criticality?

- Topological surface states = novel gapless fermionic vacua with "anomalous" character (can only exist on boundary)

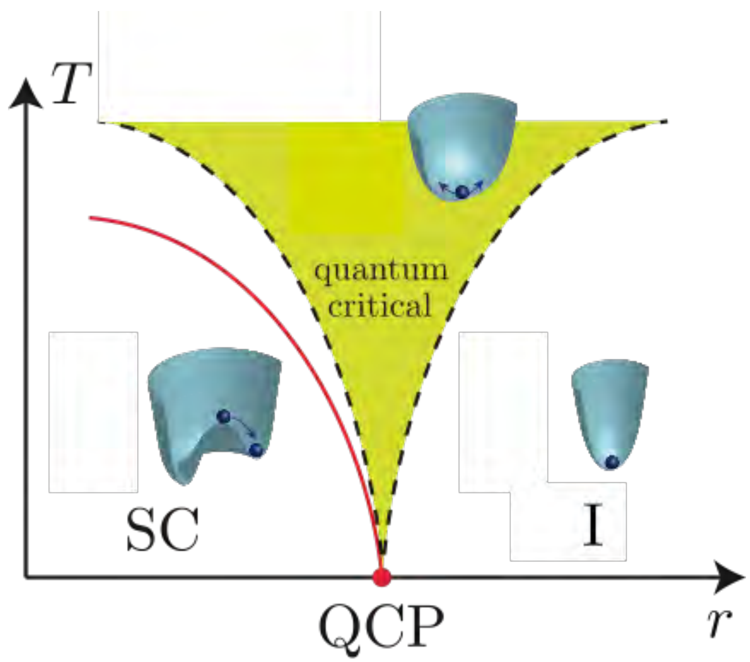
- Possibility of novel "boundary" quantum critical phenomena impossible (or hard) to realize in "bulk" systems?

- Focus on **semimetal-superconductor transition** on surface of 3D TI: odd number of 2D Dirac fermions with U(1) and T symmetries

Outline

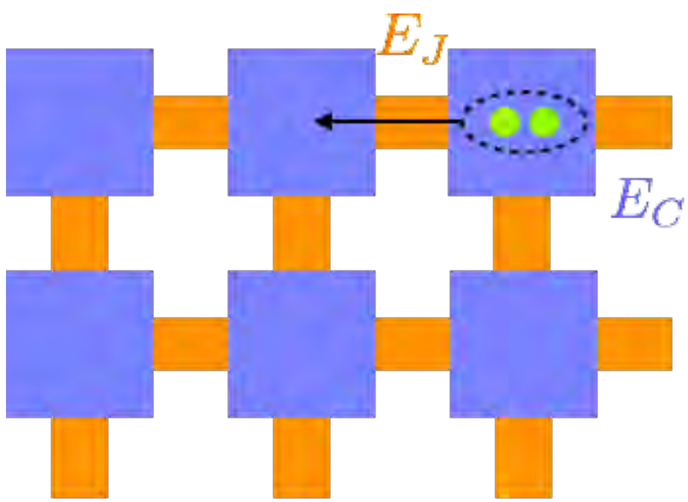
- Warm-up: boson superconductor-insulator transition (SC-I) vs Dirac fermion superconductor-semimetal transition (SC-SM)
- Superconductivity with one Dirac cone (Sb_2Te_3?)
- Superconductivity with three Dirac cones (SmB_6?)

SC-I transition of bosons



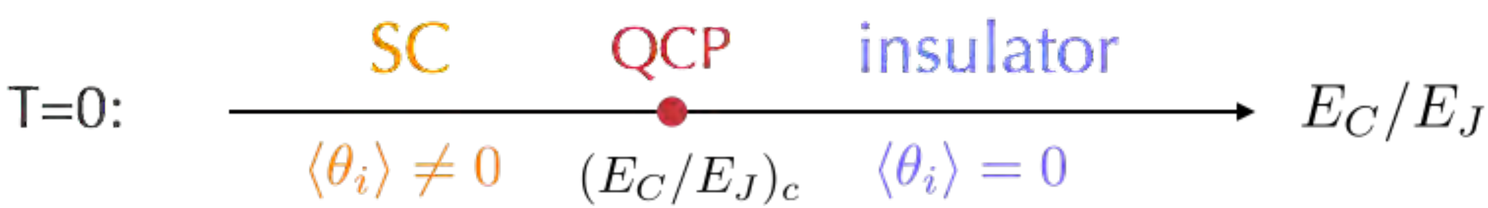
Josephson junction array

- SC islands coupled via Cooper pair tunneling
- Assume $E_C, E_J \ll \Delta$: no low-energy fermions



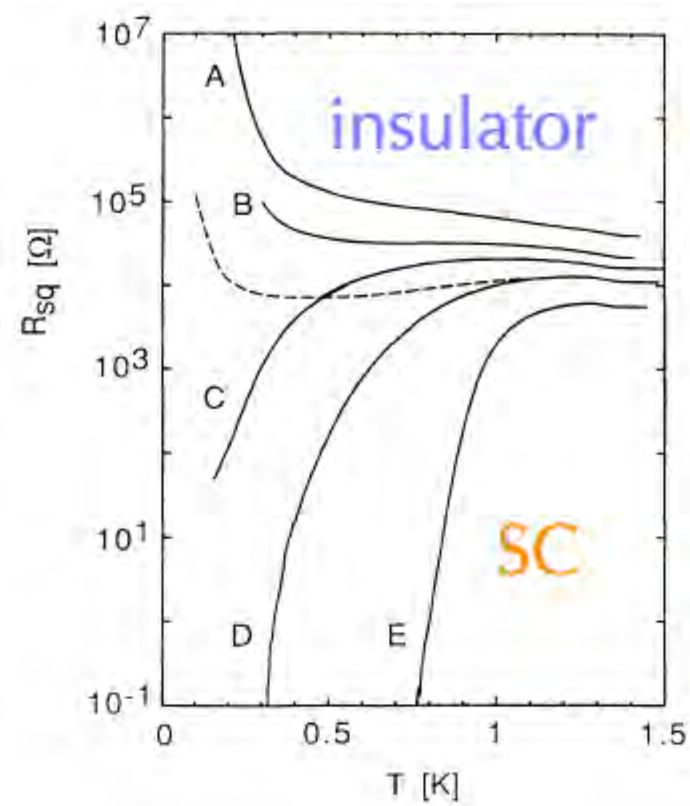
$$H = E_C \sum_i n_i^2 - E_J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

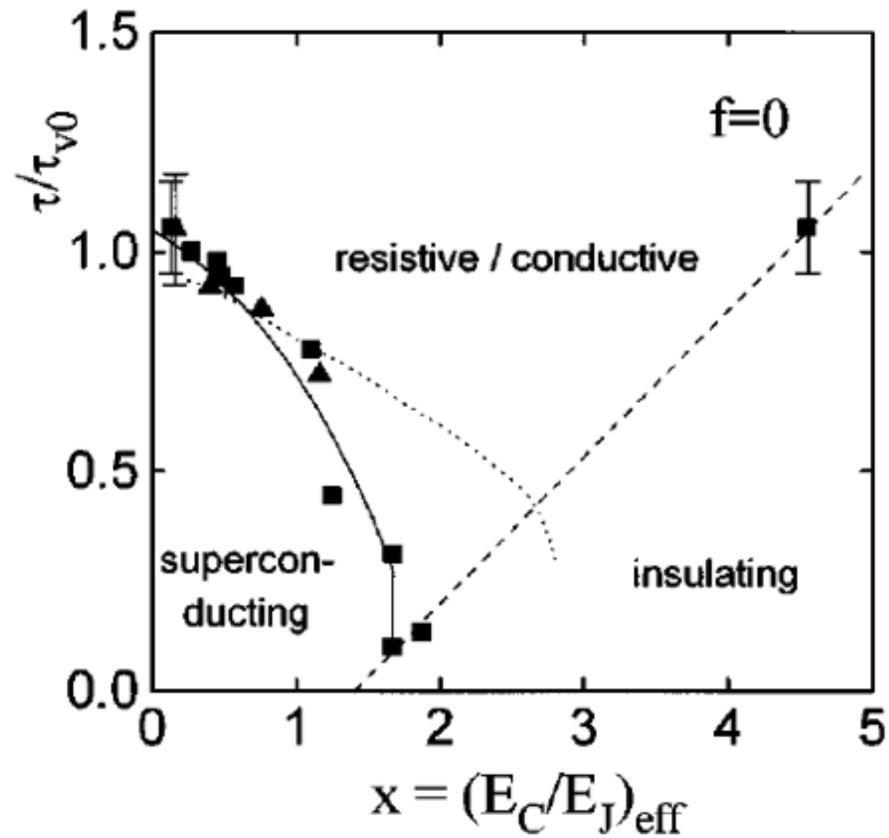
$$[n_i, \theta_j] = i\delta_{ij}$$



(Al junctions)

van der Zant, PRB '96



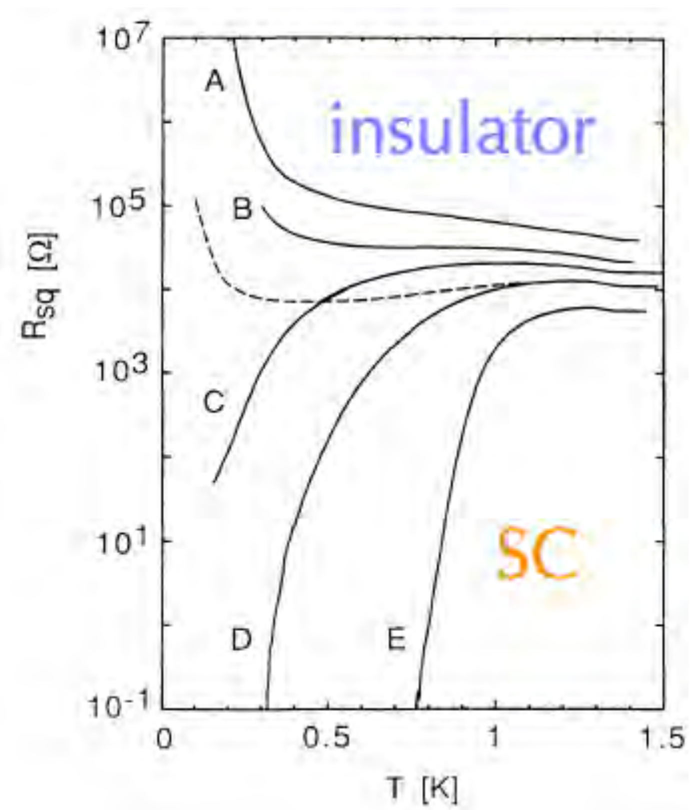


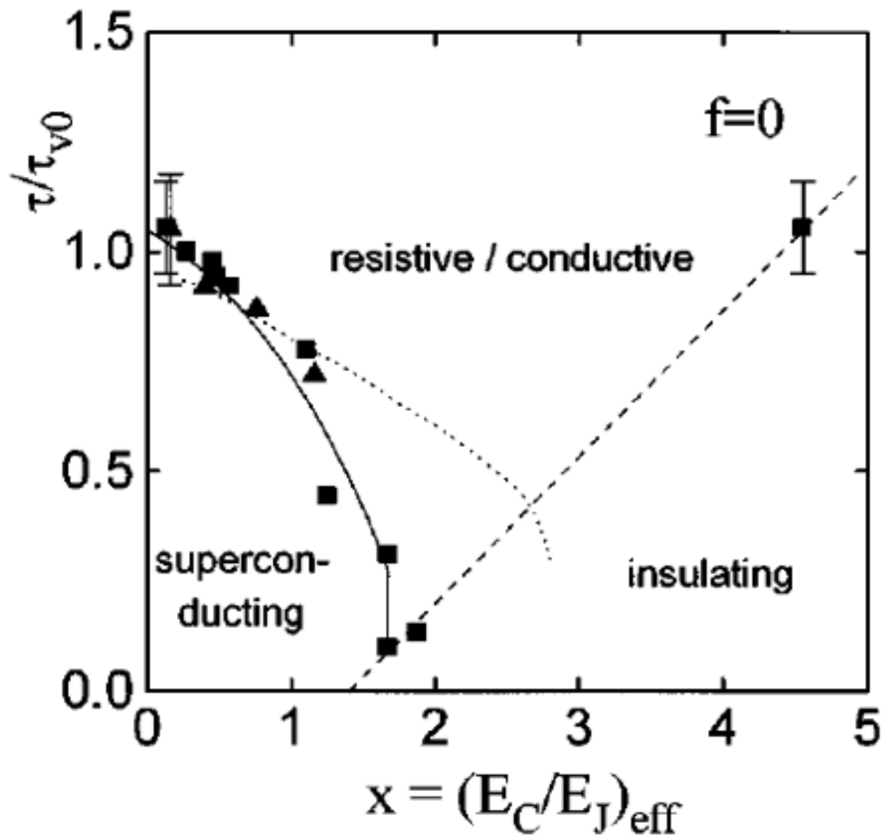
Geerligs et al., PRL '89

$$(E_C/E_J)_c \approx 1.7$$

(Al junctions)

van der Zant, PRB '96





Geerligs et al., PRL '89

$$(E_C/E_J)_c \approx 1.7$$

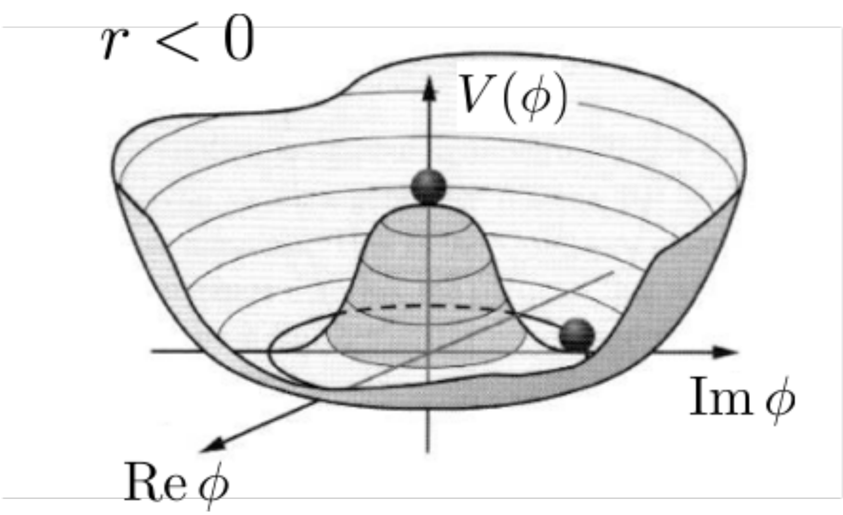
- also Bose-Hubbard model with ^{87}Rb atoms

(Spielman et al., PRL '07; Endres et al., Nature '12)

Landau-Ginzburg theory

- Coarse-grained description: order parameter = bosonic Cooper pair field $\phi(\boldsymbol{r}, \tau)$

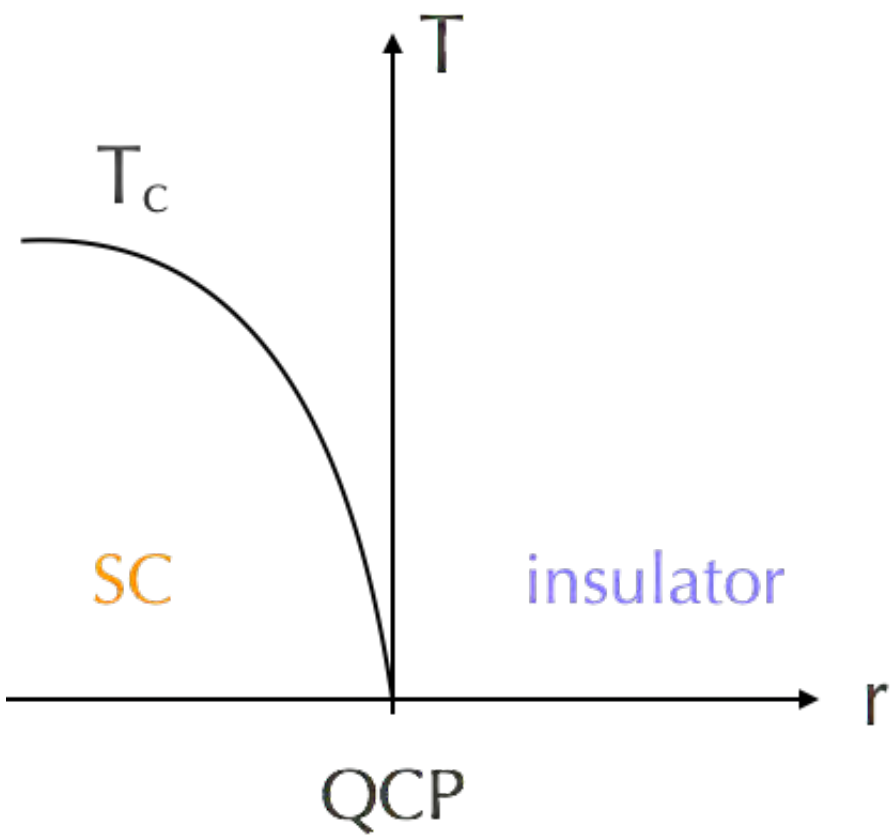
$$\mathcal{L} = |\partial_\tau \phi|^2 + c_b^2 |\nabla \phi|^2 + V(\phi)$$



$$V(\phi) = r|\phi|^2 + u|\phi|^4$$

$$r \sim \frac{E_C}{E_J} - \left(\frac{E_C}{E_J}\right)_c$$

Quantum critical point



- QCP is strongly coupled: O(2) Wilson-Fisher fixed point (3DXY)
- Emergent Lorentz invariance

QCP: optical conductivity

- Universal quantum critical conductivity in d=2 (Damle, Sachdev, PRB '97):

$$\sigma(\omega, T) = \frac{e^2}{\hbar}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

- T=0 optical conductivity is frequency-independent: **universal constant**

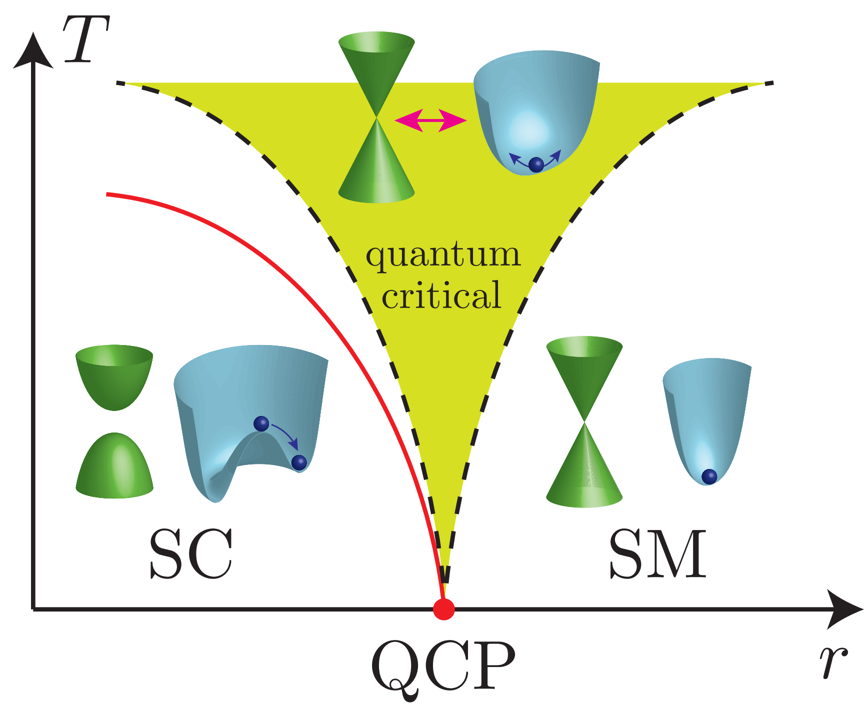
$$\sigma(\omega, 0) = \frac{e^2}{\hbar}\Sigma(\infty) = \frac{e^2}{\hbar}\sigma_\infty$$

- For boson SC-I transition: no exact result, long history – response function of a strongly correlated system with no quasiparticles! (Fisher, Grinstein, Girvin, PRL '90; Fazio, Zappalà, PRB '96; Šmakov, Sørensen, PRL '05…)

- QMC + holography + conformal bootstrap (Katz et al., PRB '14; Gazit et al., PRL '14; Witczak-Krempa et al., Nat. Phys. '14; Kos et al., JHEP '15):

$$\sigma_\infty \simeq 0.226$$

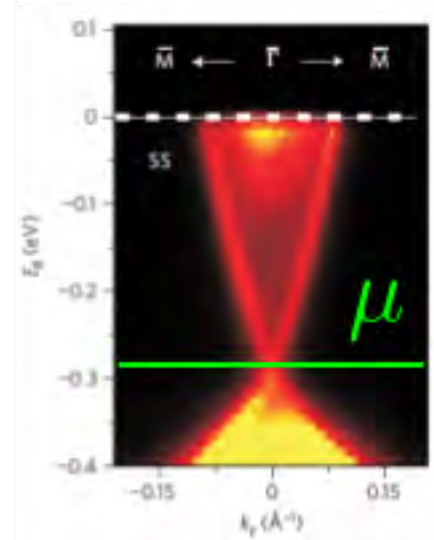
SC-SM transition of Dirac fermions



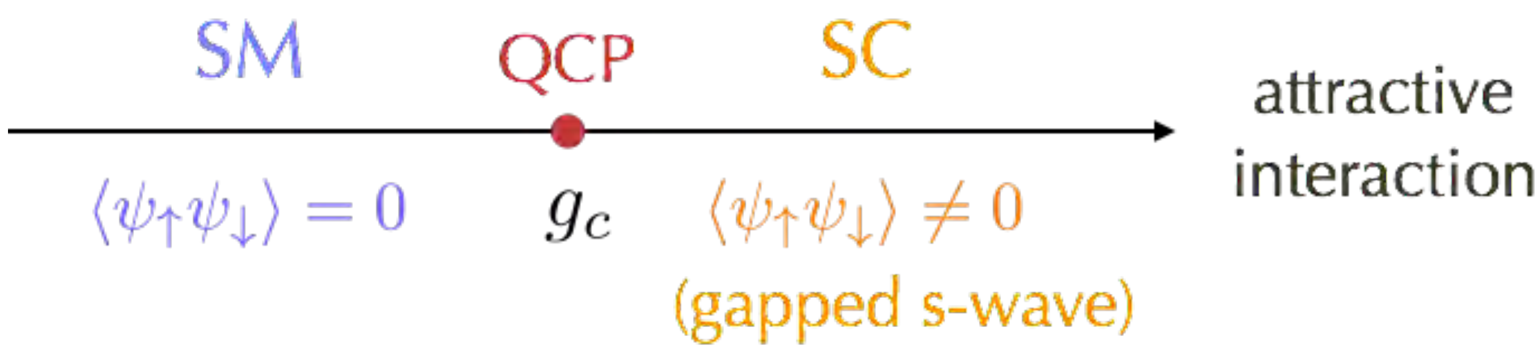
SM-SC transition of 2D Dirac fermions

- Pairing instability of **single** 2D Dirac fermion: 3D TI surface

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$

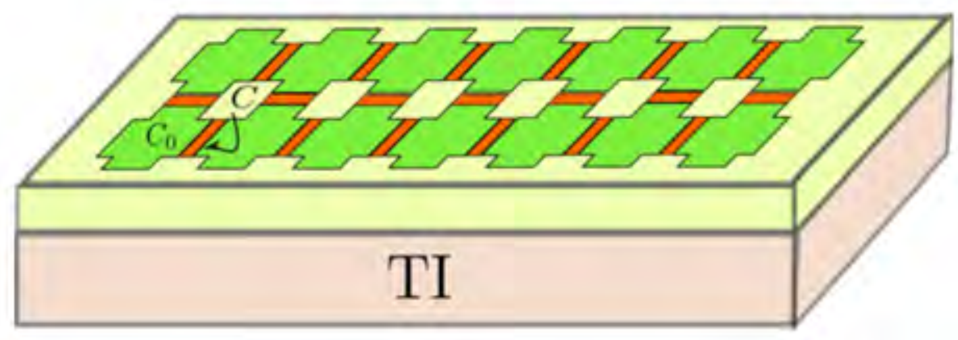


- Consider chemical potential at Dirac point: vanishing DOS implies QCP (Roy, Juričić, Herbut, PRB ‘13; Nandkishore, JM, Huse, Sondhi, PRB ’13)



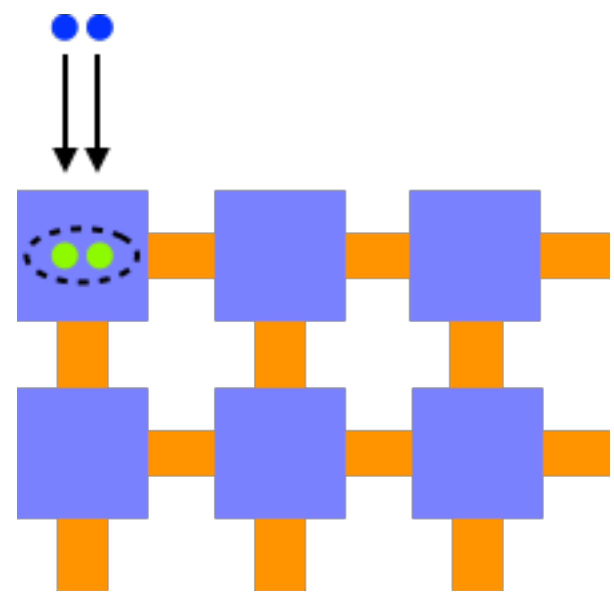
Route #1: Josephson engineering

- JJA on surface of TI



Ponte and Lee, NJP '14

- Pairs of Dirac electrons tunnel to SC island and vice-versa



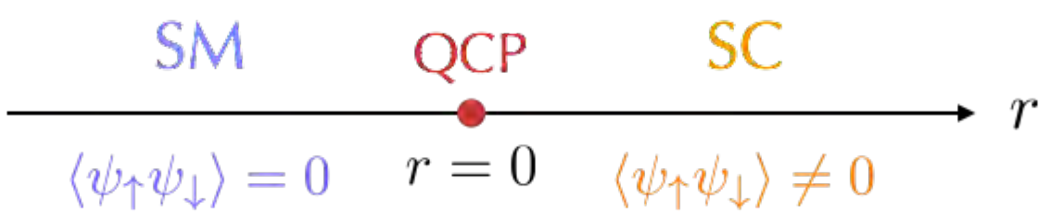
Landau-Ginzburg theory

- Low-energy theory has bosons **and** fermions (pair-breaking effects)

$$\mathcal{L} = i\bar{\psi}(\gamma_0\psi_0 + c_f\gamma_i\partial_i)\psi \qquad \text{Dirac}$$

$$+|\partial_0\phi|^2 + c_b^2|\partial_i\phi|^2 + r|\phi|^2 + \lambda^2|\phi|^4 \qquad \text{JJA}$$

$$+2h(\phi^*\psi_\uparrow\psi_\downarrow + \text{c.c.}) \qquad \text{Dirac-JJA tunneling}$$



Route #2: Intrinsic SC?

- Anisotropic (2D) diamagnetic screening: surface SC

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ARTICLE

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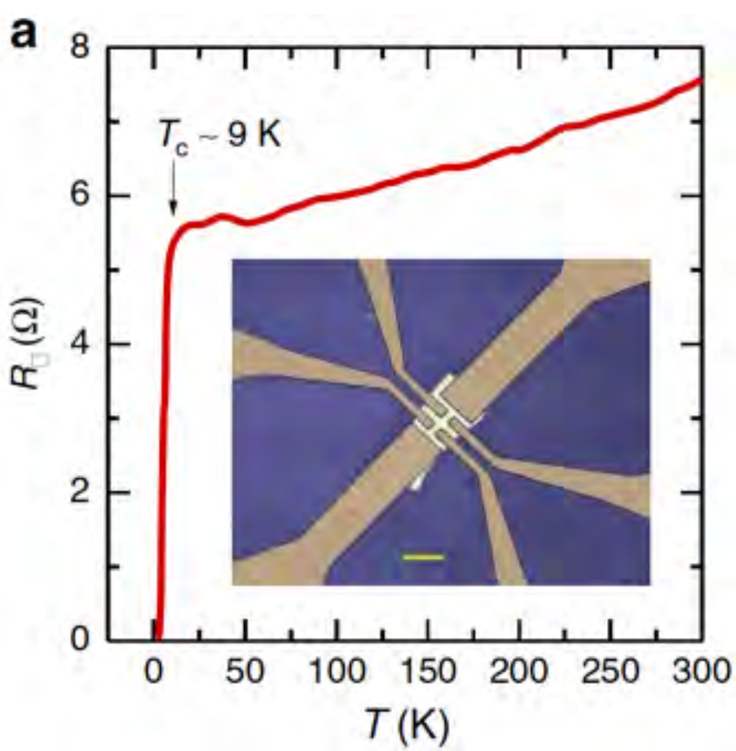
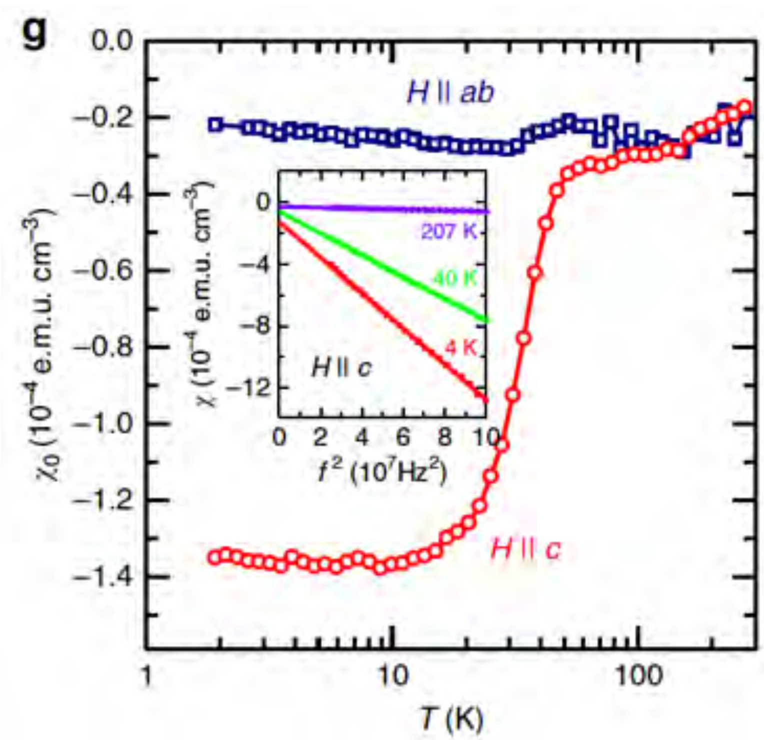
Emergent surface superconductivity in the topological insulator Sb_2Te_3

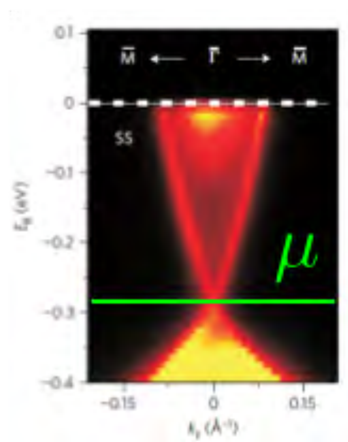
Lukas Zhao[1], Haiming Deng[1], Inna Korzhovska[1], Milan Begliarbekov[1], Zhiyi Chen[1], Erick Andrade[2], Ethan Rosenthal[2], Abhay Pasupathy[2], Vadim Oganesyan[3,4] & Lia Krusin-Elbaum[1,4]

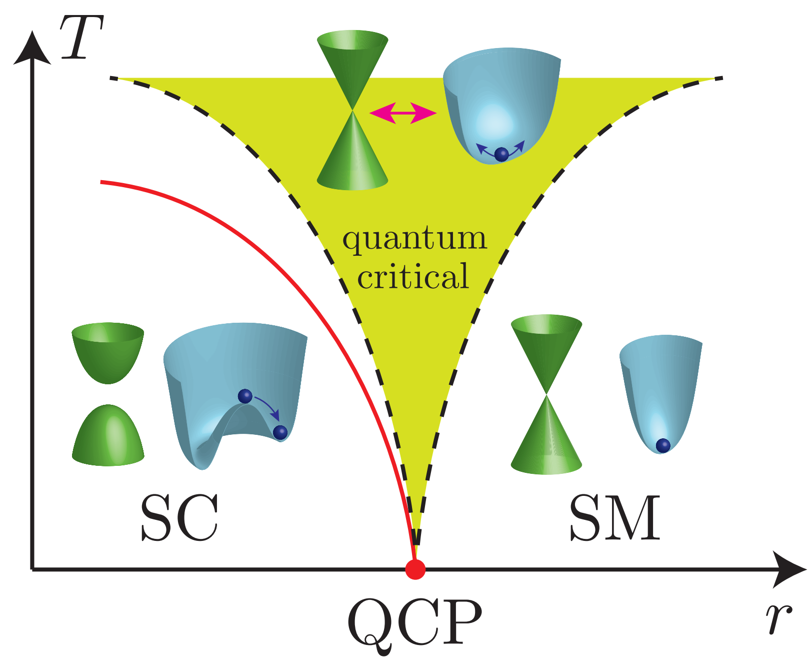




Semimetal-superconductor QCP

- QCP has an emergent (2+1)D **supersymmetry**: N=2 Wess-Zumino model (Grover, Sheng, Vishwanath, Science '14; Ponte, Lee, NJP '14)





$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + |\partial_\mu\phi|^2 + r|\phi|^2 + h^2|\phi|^4 + h(\phi^*\psi^T i\sigma^y\psi + \text{h.c.})$$

SUSY QCP: critical exponents

- Strongly coupled QCP: anomalous dimensions exactly known from SUSY (Aharony et al., NPB ‘97)

$$\eta_\phi = \eta_\psi = \frac{1}{3}$$

- Correlation length exponent: $\xi \sim (g - g_c)^{-\nu}$

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2) \approx 0.75$$

1-loop RG (Thomas, ‘05; Lee, PRB ’07)

$$\nu = \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{24} + \left(\frac{\zeta(3)}{6} - \frac{1}{144}\right)\epsilon^3 + \mathcal{O}(\epsilon^4) \approx 0.985$$

3-loop RG (Zerf, Lin, JM, PRB ’16)

$\nu \approx 0.9174$ Padé extrapolation of 3-loop result (Fei et al., PTEP ‘16)

$\nu \approx 0.9173$ conformal bootstrap (Bobev et al., PRL ‘15)

QCP: optical conductivity

$$\sigma(\omega, 0) = \frac{e^2}{\hbar}\Sigma(\infty) = \frac{e^2}{\hbar}\sigma_\infty$$

PRL **101**, 196405 (2008) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2008

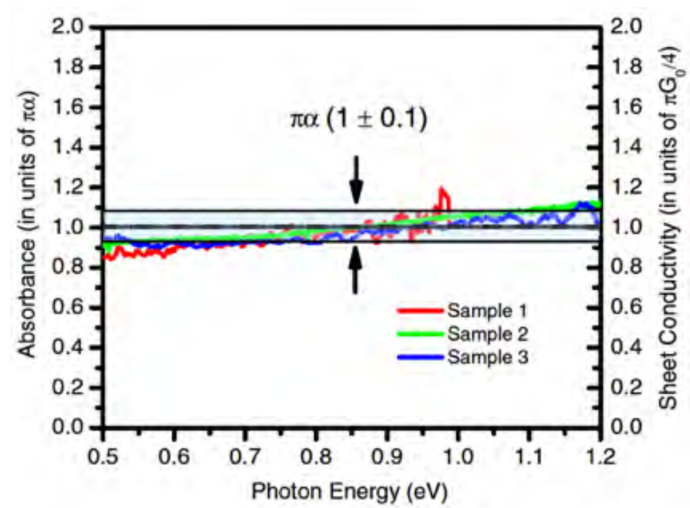
Measurement of the Optical Conductivity of Graphene

Kin Fai Mak,[1] Matthew Y. Sfeir,[2] Yang Wu,[1] Chun Hung Lui,[1] James A. Misewich,[2] and Tony F. Heinz[1,*]
[1]*Departments of Physics and Electrical Engineering, Columbia University, 538 West 120th Street, New York, New York 10027, USA*
[2]*Brookhaven National Laboratory, Upton, New York 11973, USA*
(Received 28 June 2008; published 7 November 2008)

Optical reflectivity and transmission measurements over photon energies between 0.2 and 1.2 eV were performed on single-crystal graphene samples on a SiO_2 substrate. For photon energies above 0.5 eV, graphene yielded a spectrally flat optical absorbance of $(2.3 \pm 0.2)\%$. This result is in agreement with a constant absorbance of $\pi\alpha$, or a sheet conductivity of $\pi e^2/2h$, predicted within a model of noninteracting massless Dirac fermions. This simple result breaks down at lower photon energies, where both spectral

- e.g., graphene = free Dirac CFT:

$$\frac{\hbar\omega}{k_B T} \sim \frac{1\ \text{eV}}{300\ \text{K}} \sim 39 = \infty$$



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$$\sigma_\infty = 1/4 = 4 \times 1/16$$

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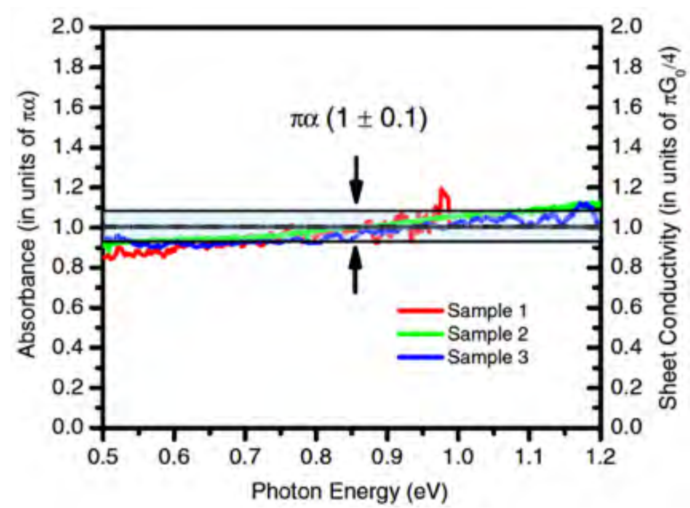

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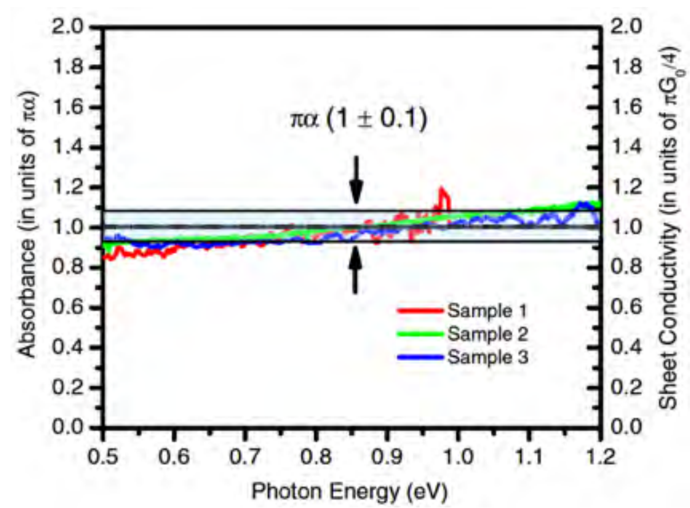
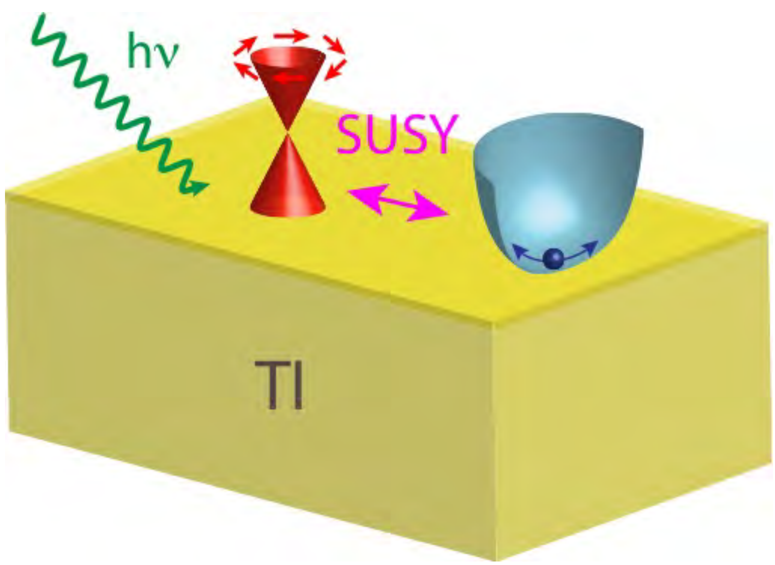


QCP = strongly interacting Dirac fermions + Cooper pairs!

SUSY QCP: optical conductivity

- Optical conductivity at the strongly correlated Dirac SM-SC QCP can be calculated exactly using SUSY:

$$\sigma(\omega, 0) = \frac{5(16\pi - 9\sqrt{3})}{243\pi}\frac{e^2}{\hbar} \approx 0.2271\frac{e^2}{\hbar}$$



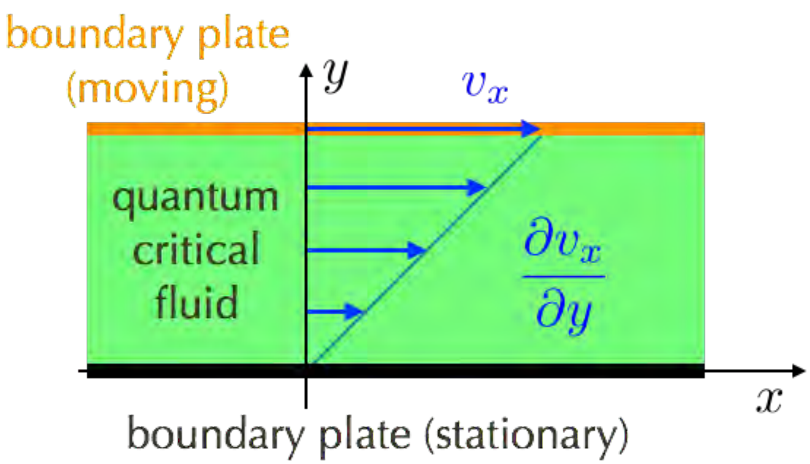
- Reason: 2-point function of stress tensor can be computed from partition function of N=2 WZ model on “squashed” S^3 (Closset et al., JHEP ’13; Nishioka, Yonekura, JHEP ‘13)

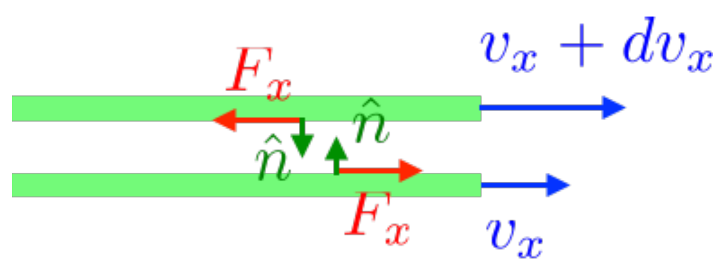
- U(1) current and stress tensor belong to the same SUSY multiplet

Witczak-Krempa and JM, PRL ’16

SUSY QCP: shear viscosity

- Optical conductivity and dynamical shear viscosity are related by SUSY:





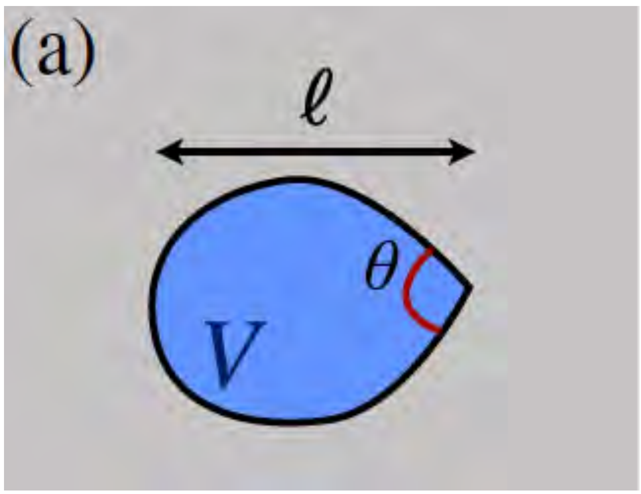
shear stress: $T_{xy} = \frac{F_x}{L} = \eta \frac{\partial v_x}{\partial y}$

$$\eta(\omega, 0) = \frac{\hbar\omega^2}{1944}(16\pi - 9\sqrt{3}) \approx 0.0178\hbar\omega^2$$

Witczak-Krempa and JM, PRL '16

SUSY QCP: entanglement entropy

- 2-point function of stress tensor also determines corner entanglement entropy



$$S = B\ell/\delta - a(\theta)\ln(\ell/\delta) + \ldots$$

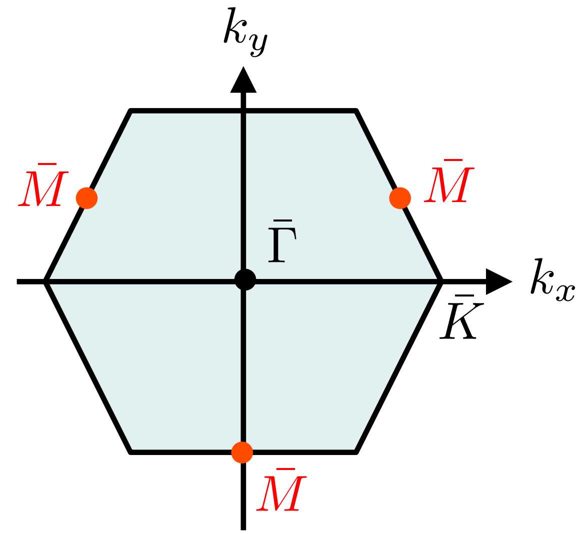
$$a(\theta) \simeq \lambda(\pi - \theta)^2$$

Casini, Huerta, Leitao, NPB ‘09

$$\lambda = \frac{16\pi - 9\sqrt{3}}{972\pi} \simeq 0.011356$$

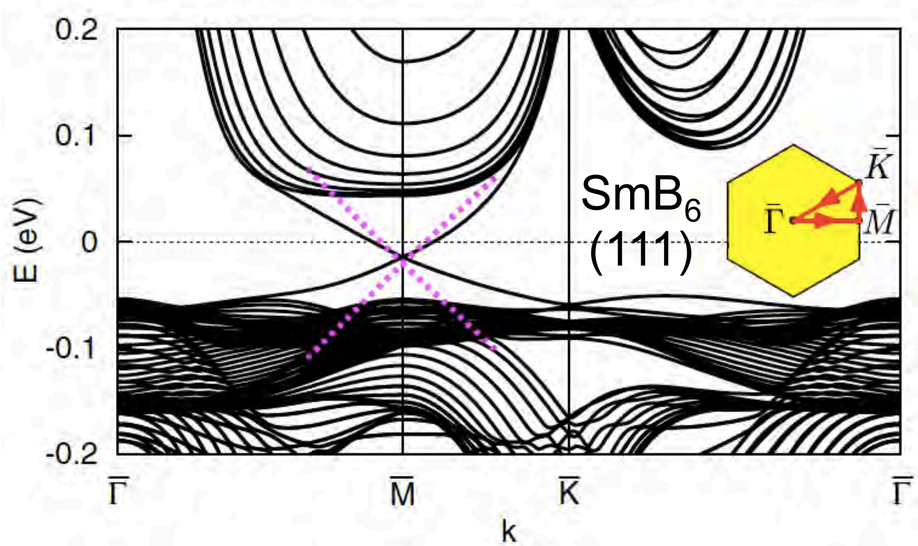
Witczak-Krempa and JM, PRL ’16

From one to three

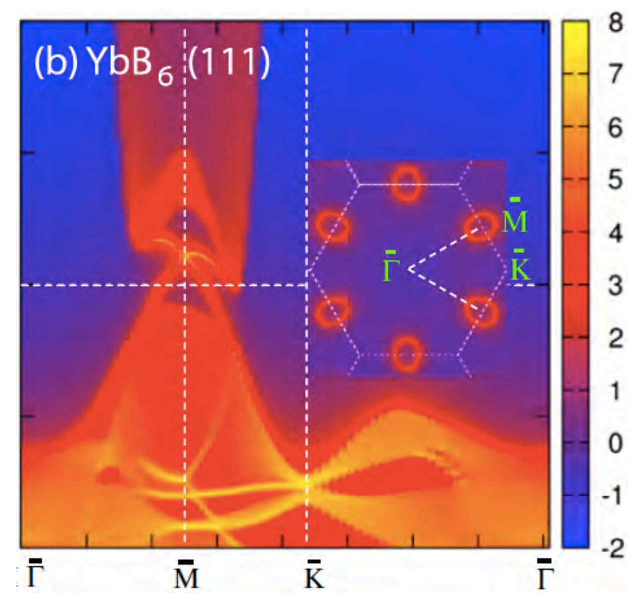


- 3D TI surface has odd # of Dirac cones: consider system with 3 cones
- (111) surface of cubic crystal has C_{3v} symmetry
- Four TRI points in surface BZ: $\bar{\Gamma}$, and three $\bar{M}$ points related by C_3 rotations

- (111) surface of SmB_6 (Ye, Allen, Sun, arXiv '13; Baruselli, Vojta, PRB '14) and YbB_6 (Weng et al., PRL '14) should host 3 **degenerate** Dirac cones at $\bar{M}$ points

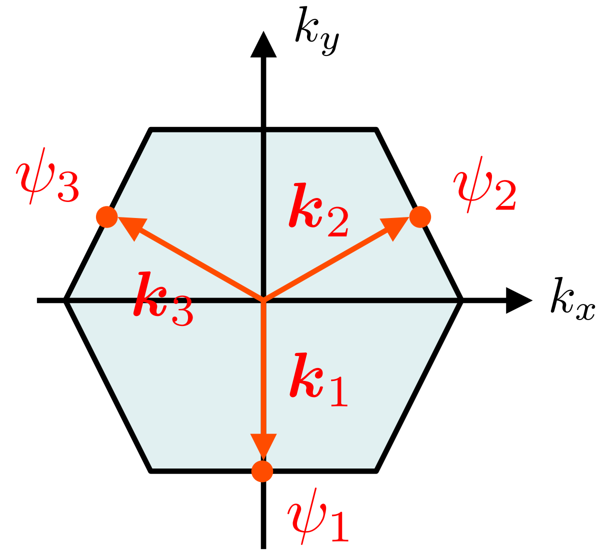


Baruselli and Vojta, PRB '16



Weng et al., PRL '14

Pairing: intra- vs intervalley

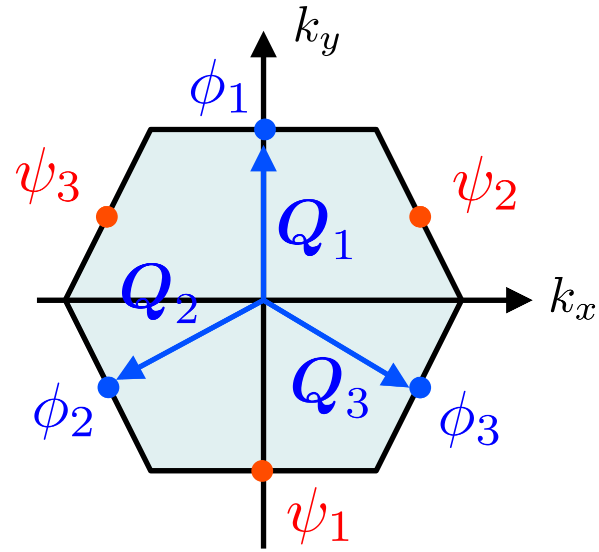


- Consider pairing instabilities for chemical potential at the Dirac point
- 2 possibilities: intravalley or intervalley pairing

- Intravalley pairing ($\langle \psi_1^T i\sigma^y \psi_1 \rangle \neq 0$, etc.) has **Q** = 0 crystal momentum: uniform SC

S.-K. Jian, C.-H. Lin, JM, H. Yao, PRL '17

Pairing: intra- vs intervalley



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- 2 possibilities: intravalley or intervalley pairing

- Intravalley pairing ($\langle \psi_1^T i\sigma^y \psi_1 \rangle \neq 0$, etc.) has **Q** = 0 crystal momentum: uniform SC

- **Intervalley** pairing ($\langle \psi_1^T i\sigma^y \psi_2 \rangle \neq 0$, etc.) has **Q**$_1$, **Q**$_2$, **Q**$_3$ ≠ 0 momentum: **pair-density-wave (PDW)**

$$\boldsymbol{Q}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_3 \quad \text{(\& cyclic permutations)}$$

$$\phi_i \sim \langle \psi_j^T i\sigma^y \psi_k \rangle, \; ijk = 123, 231, 312$$

S.-K. Jian, C.-H. Lin, JM, H. Yao, PRL ‘17

Landau theory for PDW instability

- By symmetry (C_{3v} x U(1) x TRS x translation), Landau theory for PDW instability must have the form

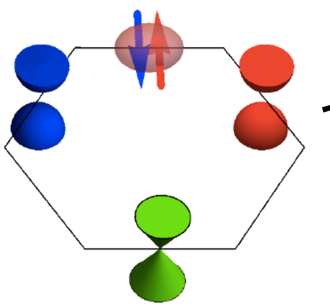
$$V = r\sum_i |\phi_i|^2 + u_1 \sum_{i<j} |\phi_i|^2|\phi_j|^2 + u_1' \sum_{i<j}(\phi_i^{*2}\phi_j^2 + \text{h.c.})$$
$$+u_2\left(\sum_i |\phi_i|^2\right)^2$$

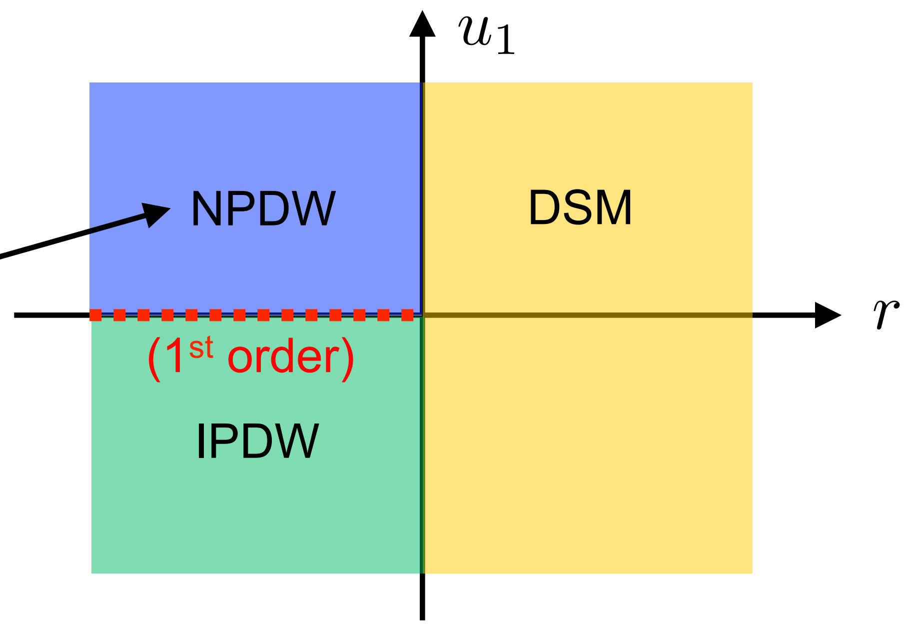
- In ordered phases, relative phase modes (Leggett modes) are gapped: can ignore at the mean-field level ($u_1 + u_1' \rightarrow u_1$)

$$V = r\sum_i |\phi_i|^2 + u_1 \sum_{i<j} |\phi_i|^2|\phi_j|^2 + u_2\left(\sum_i |\phi_i|^2\right)^2$$

Mean-field phase diagram (I)

- For $u_2 > 0$:





Dirac semimetal (DSM): $\langle \phi_i \rangle = 0$, 3 gapless Dirac cones

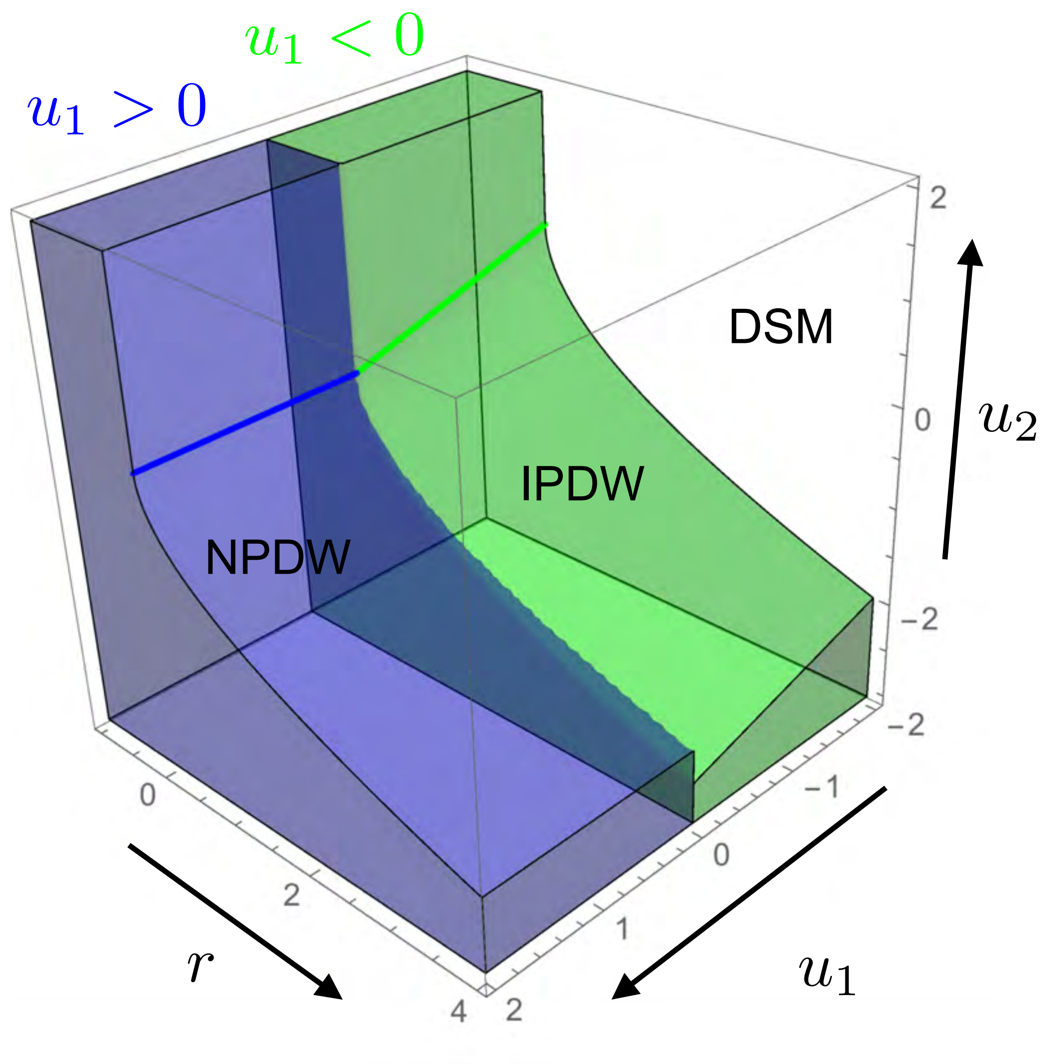
Isotropic PDW (IPDW): $\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_3 \rangle \neq 0$, 3 gapped Dirac cones

Nematic PDW (NPDW): $\langle \phi_1 \rangle \neq 0, \ \langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$ & cyclic permutations,

2 gapped & 1 gapless Dirac cones: breaks C_3

Mean-field phase diagram (II)

- For $u_2 < 0$: must add sixth-order term $\sim w\left(\sum_i |\phi_i|^2\right)^3$ to stabilize the ground state energy



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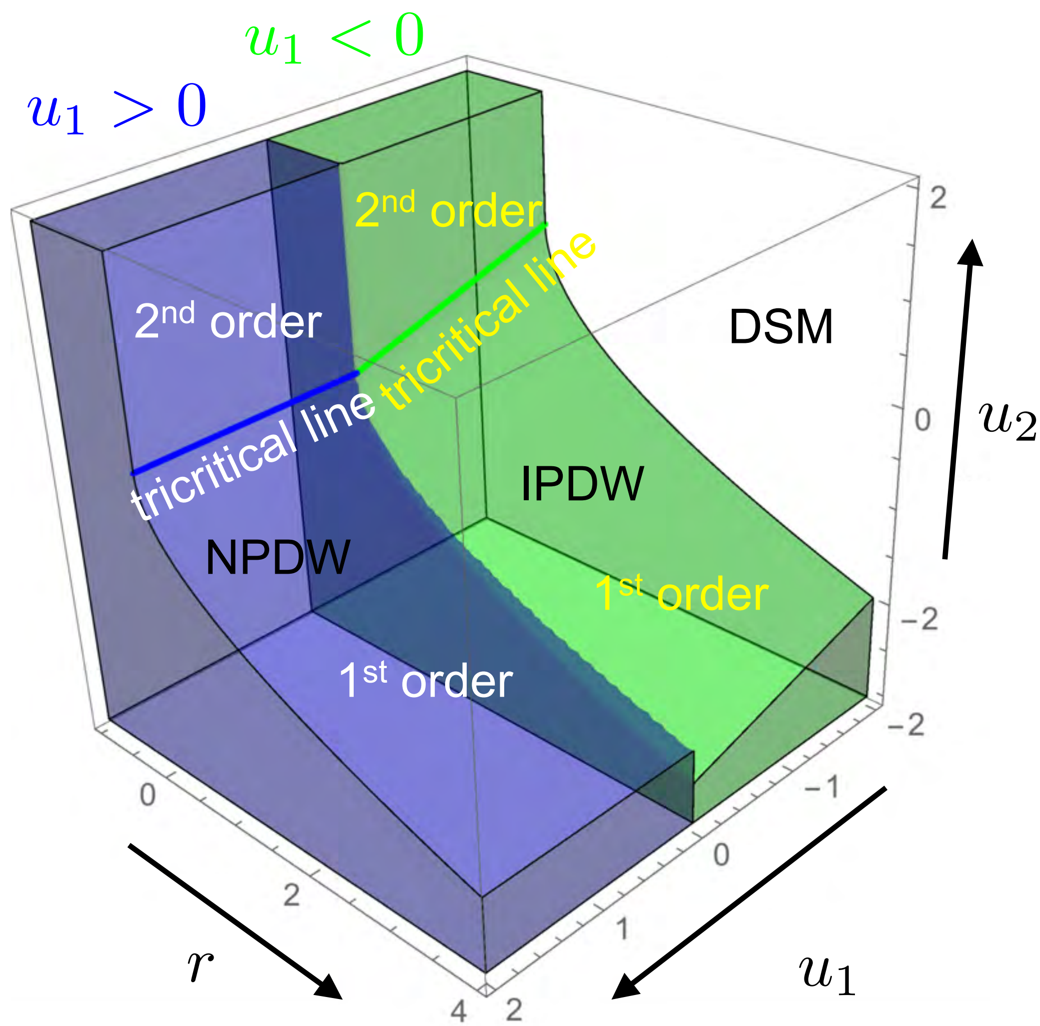
IPDW ($u_1 < 0$):

$$r = 0,\ u_2 = -u_1/3$$

NPDW ($u_1 > 0$):

$$r = 0,\ u_2 = 0$$



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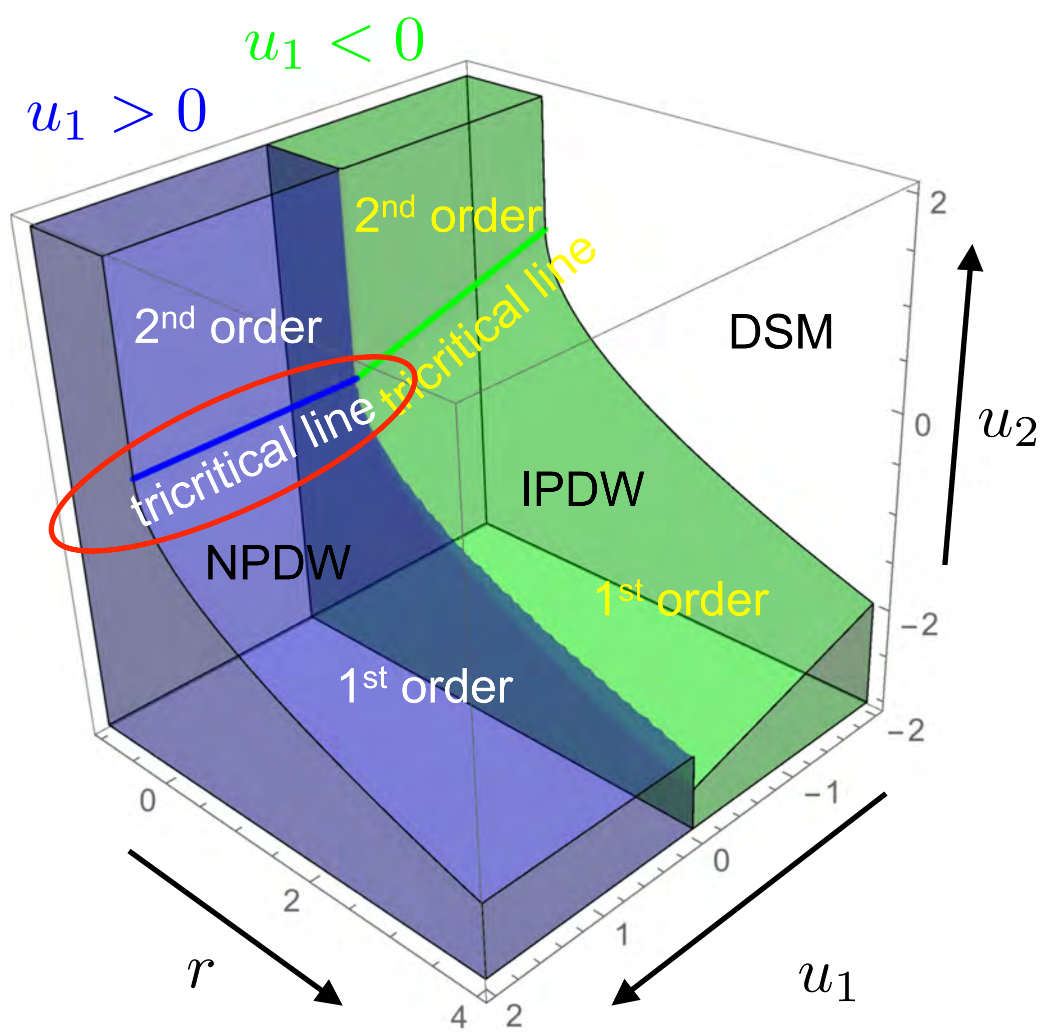


NPDW-DSM tricritical line (I)

- Low-energy effective theory of the NPDW-DSM tricritical line:

$$\mathcal{L} = \sum_i \bar{\psi}_i(\partial_\tau + h_i^f)\psi_i + \sum_i \phi_i^*(-\partial_\tau^2 + h_i^b)\phi_i$$

kinetic energy of Dirac fermion/Cooper pair

$$+r\sum_i |\phi_i|^2 + u_1 \sum_{i<j} |\phi_i|^2|\phi_j|^2 + u_1' \sum_{i<j}(\phi_i^{*2}\phi_j^2 + \text{h.c.})$$

$$+u_2\left(\sum_i |\phi_i|^2\right)^2$$

"classical" Landau energy

$$+g[(\phi_1^*\psi_2^T i\sigma^y\psi_3 + \text{c.p.}) + \text{h.c.}]$$

pair breaking

- Determine critical properties using Wilson and Fisher's ε-expansion (one-loop)

NPDW-DSM tricritical line (II)

- At low energies, fermion & boson velocities become **isotropic** and **equal** to each other: **emergent Lorentz invariance**
- Unstable fixed point with two relevant directions, r and u_2: NPDW-DSM tricritical line
- Fixed point couplings:

$$g^2 = u_1 = \frac{2\epsilon}{3\pi}, \; r = u_1' = u_2 = 0$$

- Fixed point Lagrangian:

$$\mathcal{L} = \sum_i i\bar{\psi}_i \gamma_\mu \partial_\mu \psi_i + \sum_i |\partial_\mu \phi_i|^2$$
$$+g^2(|\phi_1|^2|\phi_2|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_3|^2|\phi_1|^2)$$
$$+g(\phi_1^* \psi_2 \psi_3 + \phi_2^* \psi_3 \psi_1 + \phi_3^* \psi_1 \psi_2 + \text{h.c.})$$

Emergent SUSY

- The fixed point Lagrangian on the NPDW-DSM tricritical line is a SUSY field theory known as the **XYZ model** (Aharony et al., NPB '97)

"fermionic" or "superspace" coordinates

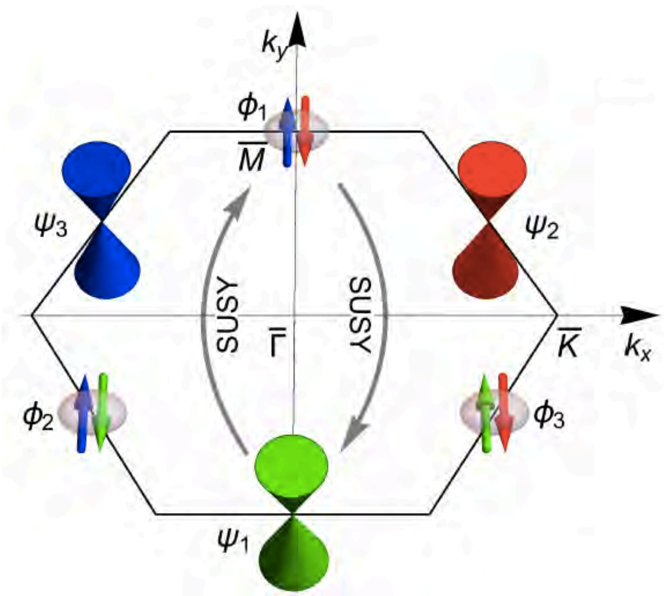
$$\mathcal{L} = \sum_{i=1}^{3} \int d^2\bar{\theta} d^2\theta \, \Phi_i^\dagger \Phi_i + g \left(\int d^2\theta \, \Phi_1 \Phi_2 \Phi_3 + \text{h.c.} \right)$$

SUSY kinetic term

"superpotential": SUSY interaction term

- Cooper pair ϕ_i and Dirac fermion ψ_i are superpartners, e.g., "components" of a single "superfield" Φ_i:

$$\Phi_i = \phi_i + \sqrt{2}\theta\psi_i + \ldots$$

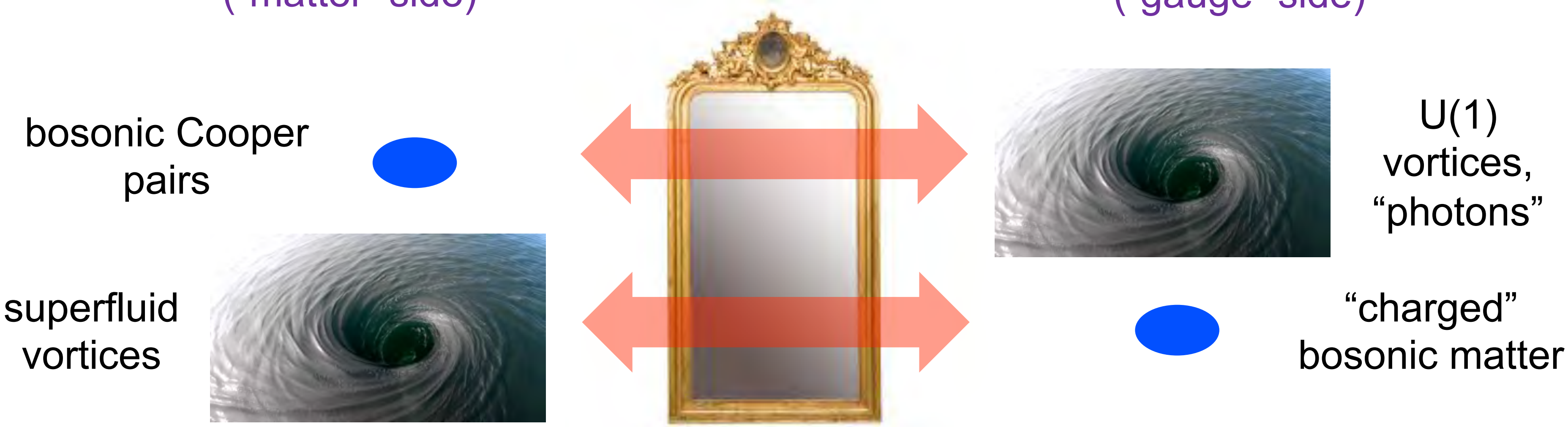


Mirror symmetry and SQED$_3$

- The XYZ model is interesting because it has an equivalent or “dual” description in terms of **N=2 supersymmetric quantum electrodynamics** in 2+1 dimensions (N=2 SQED$_3$) with a single “flavor” of matter fields
- This duality is known as **mirror symmetry** (Aharony et al., NPB ‘97) and can be understood as a SUSY version of **particle-vortex** duality (Dasgupta, Halperin, PRL ‘81)

quantum XY model
(“matter” side)

Abelian Higgs model
(“gauge” side)



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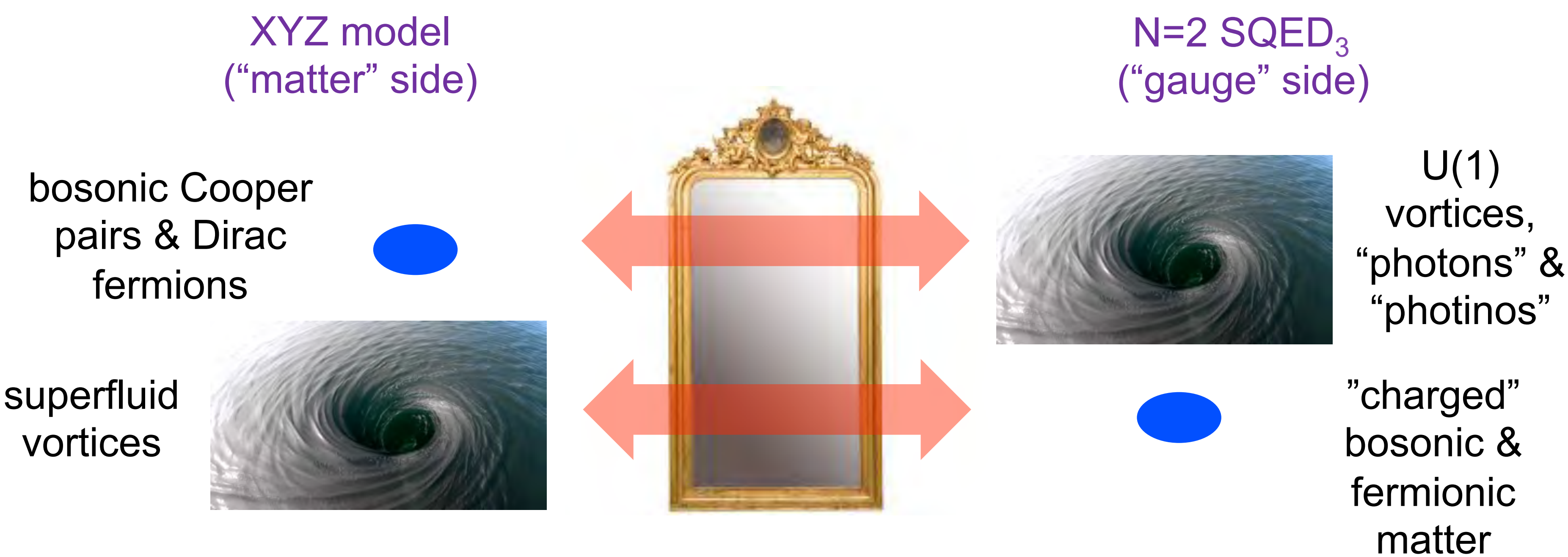

Summary

- At charge neutrality (Dirac point), the surface of 3D topological insulators can exhibit a **semimetal-superconductor quantum critical point** where gapless Dirac fermions and Cooper pairs interact strongly

- For a surface with one (three) Dirac cone(s), the QCP displays **emergent N=2 SUSY** of the Wess-Zumino (XYZ/$SQED_3$) type. Possible realization in Sb_2Te_3 (SmB_6) or other TI compounds?

- SUSY allows one to determine **exactly** certain response properties (optical conductivity, dynamical shear viscosity) of the QCP, despite strong correlations

- Possible realization of **mirror symmetry** in condensed matter

W. Witczak-Krempa and JM, Phys. Rev. Lett. **116**, 100402 (2016)
N. Zerf, C.-H. Lin, and JM, Phys. Rev. B **94**, 205106 (2016)
S.-K. Jian, C.-H. Lin, JM, and H. Yao, Phys. Rev. Lett. **118**, 166802 (2017)