Spin Dynamics in the Dirac Systems
Mainz, Jun. 6, 2017

Topology and symmetry in topological semimetals: tutorial

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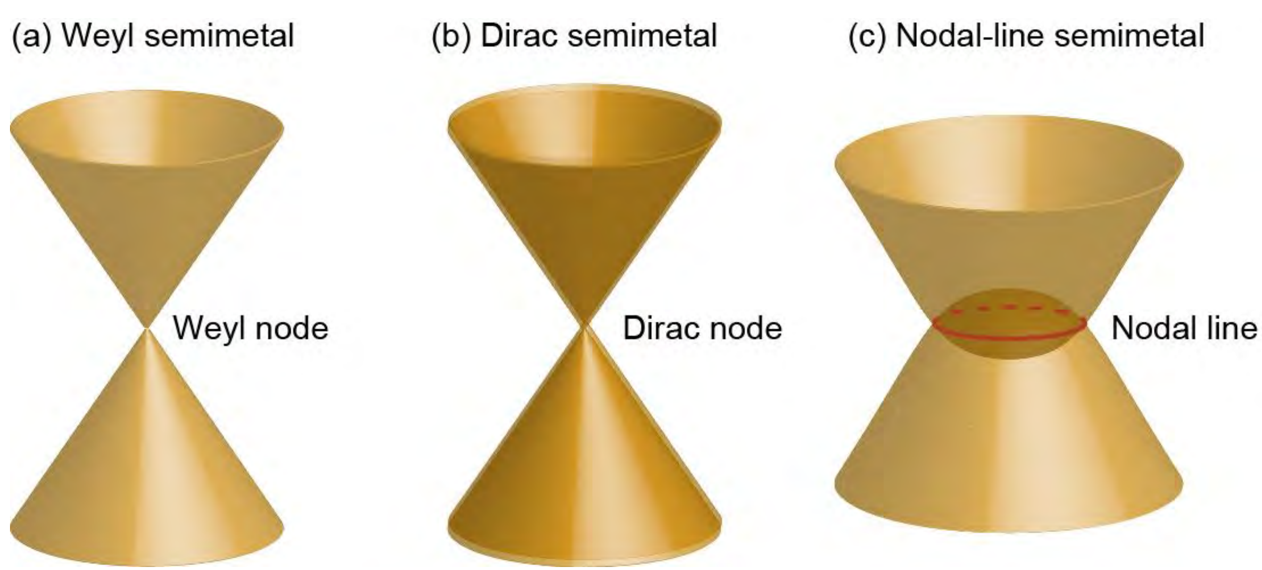


No Kramers degeneracy

Kramers degeneracy

Various topological phases in electronic systems

(1) Bulk is an insulator: "topological insulator" in a broader sense

(1-1) Integer quantum Hall system (Chern insulator)

(1-2) topological insulator

(1-3) topological crystalline insulator

…..

(2) Bulk is a metal: topological metal (topological semimetal)

(2-1) Dirac semimetal

(2-2) Weyl semimetal

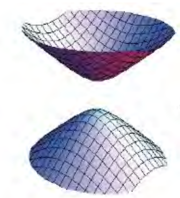
(2-3) nodal-line semimetal

……

(2) Bulk is a metal: topological metal (topological semimetal)

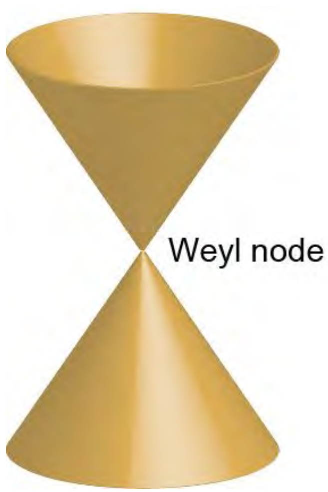
Band degeneracy occurs

- at high-symmetry points/lines
- At other general points, bands usually anticross.



→ Nevertheless, in some cases, anticrossing does not happen, and band degeneracy occurs at general k points
= topological metal (topological semimetal)

Weyl semimetal:

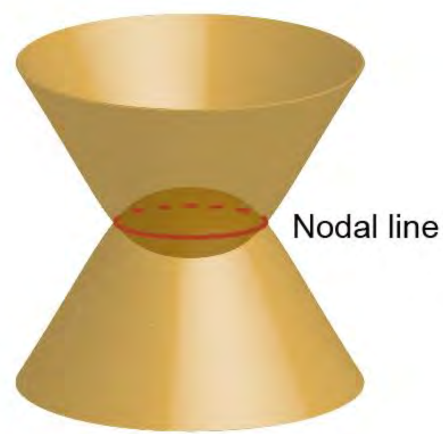


No Kramers degeneracy

Dirac semimetal:

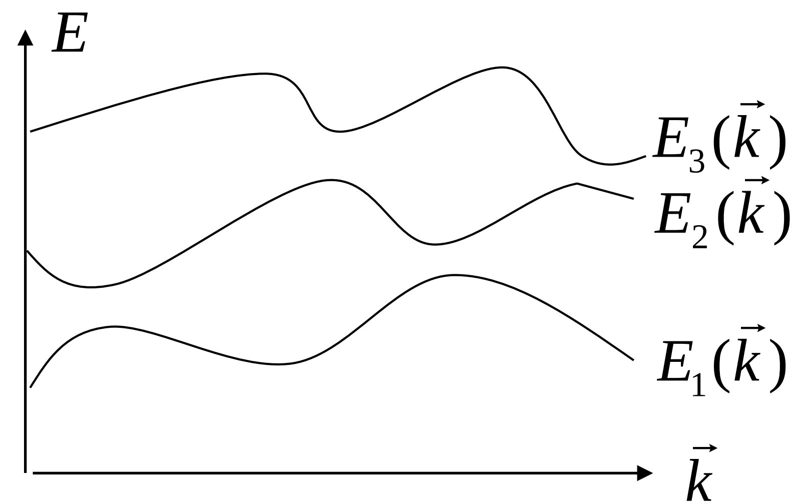
Kramers degeneracy

Nodal-line semimetal:



Degeneracies in electronic states in crystals

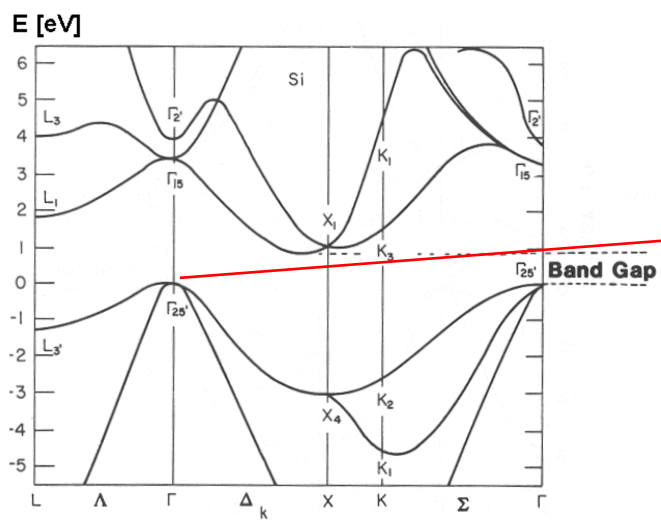
$$H(\vec{k}) \rightarrow E_1(\vec{k}), E_2(\vec{k}), E_3(\vec{k}), ...$$



- When do degeneracies appear?
- What conditions are required?

$$E_n(\vec{k}) = E_m(\vec{k})$$

Case 1: symmetry: High-dimensional irreducible representation of a little group



At high-symmetry points/lines, there might be degeneracies due to symmetry

https://www.tf.uni-kiel.de/matwis/amat/semi_en/kap_2/backbone/r2_1_5.html

Degeneracies in electronic states in crystals

Case 2: topology "accidental degeneracy"

Wigner, Herring , Volovik, Murakami, ...

Simple example: general 2*2 Hamiltonian in k space

$$H(\vec{k}) = \begin{pmatrix} a(\vec{k}) & b(\vec{k}) \\ b^*(\vec{k}) & c(\vec{k}) \end{pmatrix}$$

$a(\vec{k}), c(\vec{k})$: real

$b(\vec{k})$: complex

$$\rightarrow E(\vec{k}) = \frac{a+c}{2} \pm \sqrt{\left(\frac{a-c}{2}\right)^2 + |b|^2}$$

Degeneracy appears when
$$\begin{cases} a(\vec{k}) = c(\vec{k}) \\ \mathrm{Re}\, b(\vec{k}) = 0 \\ \mathrm{Im}\, b(\vec{k}) = 0 \end{cases}$$

In 3D k space: they may have solutions

In 2D k space: they have no solutions (unless there are additional constraints)

Dimensionality matters !

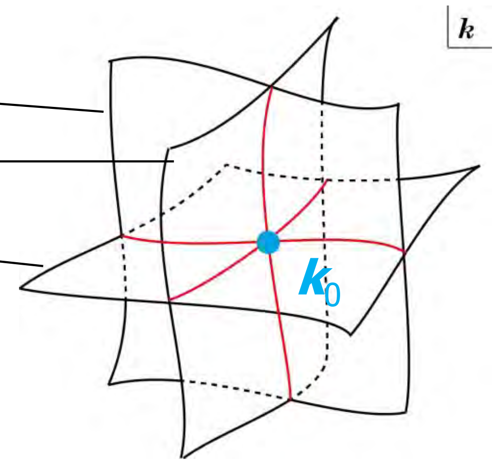
Degeneracies in electronic states in crystals

Case 2: topology

$$H(\vec{k}) = \begin{pmatrix} a(\vec{k}) & b(\vec{k}) \\ b^*(\vec{k}) & c(\vec{k}) \end{pmatrix} \quad \rightarrow E(\vec{k}) = \frac{a+c}{2} \pm \sqrt{\left(\frac{a-c}{2}\right)^2 + |b|^2}$$

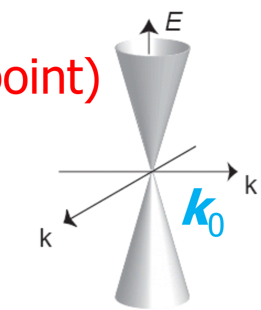
Degeneracy appears when

$$\begin{cases} a(\vec{k}) = c(\vec{k}) \\ \mathrm{Re}\, b(\vec{k}) = 0 \\ \mathrm{Im}\, b(\vec{k}) = 0 \end{cases}$$



In 3D k space: they may have solutions:
isolated points in k space: $\boldsymbol{k} = \boldsymbol{k}_0$

Apart from this degeneracy point at $\boldsymbol{k}_0$,
energy separation between two bands is linear in wavevector
= forming a Dirac cone
$\boldsymbol{k} = \boldsymbol{k}_0$: Weyl node (: not necessarily at high-symmetry point)



Note: this Weyl node cannot be removed perturbatively !
i.e. topologically stable

$$H(\vec{k}) \rightarrow H(\vec{k}) + \delta H(\vec{k})$$

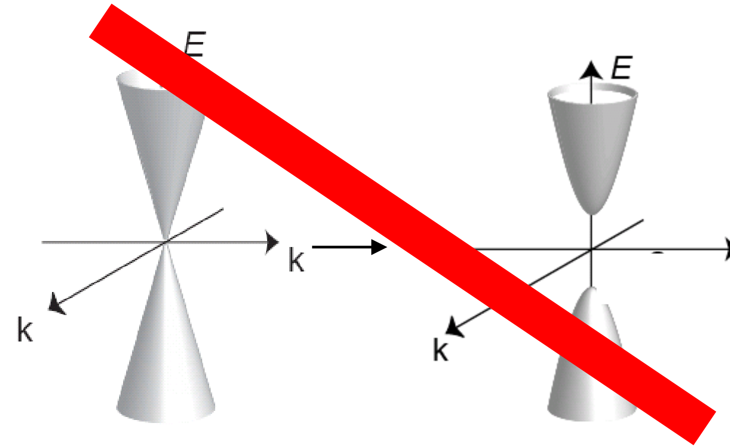
Weyl nodes in 3D is topologically stable

3D Weyl node :

$$H(\vec{k},m) = k_x\sigma_x + k_y\sigma_y + k_z\sigma_z \boxed{+ m\sigma_z}$$

$$= k_x\sigma_x + k_y\sigma_y + \ k_z + m \ \sigma_z$$

Weyl point moves but gap does not open
(0,0,0)→(0,0,-*m*)



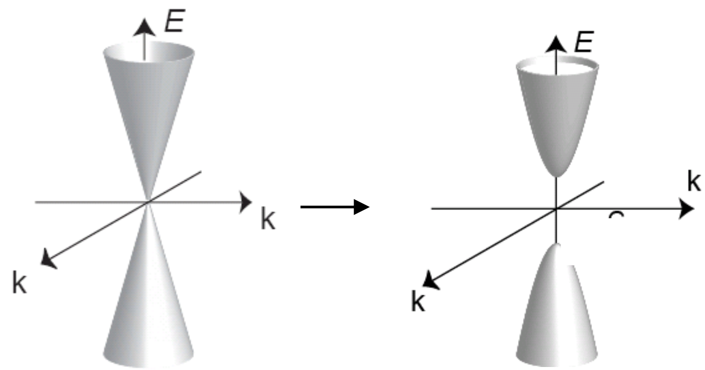
3D Weyl node is topological.

2D Weyl node :

$$H(\vec{k},m) = k_x\sigma_x + k_y\sigma_y \boxed{+ m\sigma_z}$$

parameter m opens a gap.

$m = 0$ $m \neq 0$



Apart from 2-band model:

How one can topologically characterize Weyl nodes?

Berry curvature $$\vec{B}_n(\vec{k}) = i \left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

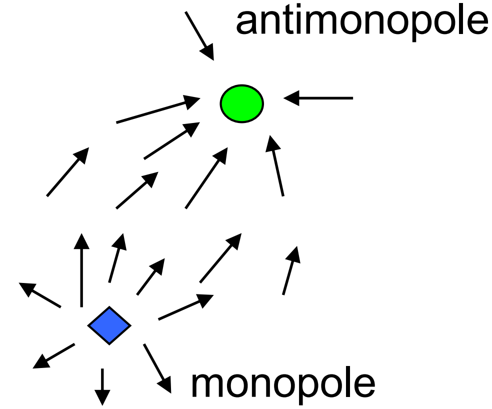
Berry curvature in k space

☐ Berry curvature

$$\vec{B}_n(\vec{k}) = i\left\langle \frac{\partial u_n}{\partial \vec{k}} \right| \times \left| \frac{\partial u_n}{\partial \vec{k}} \right\rangle$$

$u_{n\vec{k}}$: periodic part of the Bloch wf.

$$\psi_{n\vec{k}}(\vec{x}) = u_{n\vec{k}}(\vec{x})e^{i\vec{k}\cdot\vec{x}}$$ (n : band index)



: "magnetic field in k-space"

☐ Monopole density

$$\rho_n\left(\vec{k}\right) = \frac{1}{2\pi}\nabla_{\vec{k}} \cdot \vec{B}_n\left(\vec{k}\right)$$

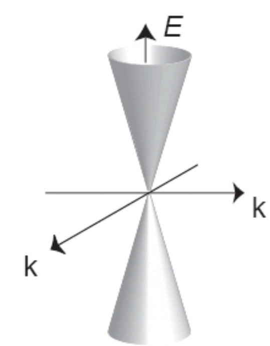
⇒

Quantization of monopole charge

$$\rho_n\left(\vec{k}\right) = \sum_i q_l \delta\left(\vec{k} - \vec{k}_l\right)$$

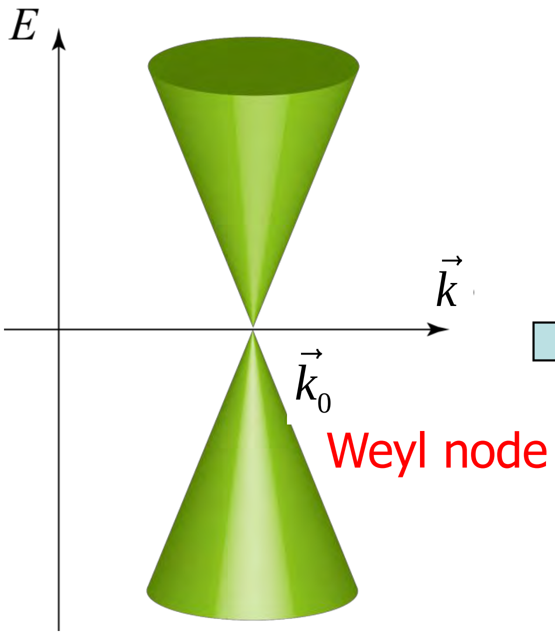
q_l : integer

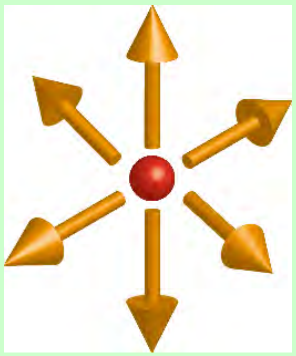
Position of Weyl node



3D Weyl nodes = monopole or antimonopole for Berry curvature

$$\begin{cases} B_n(\vec{k}) = i\left\langle \dfrac{\partial u_{nk}}{\partial k} \right| \times \left| \dfrac{\partial u_{nk}}{\partial k} \right\rangle & \text{: Berry curvature} \\ \rho_n(\vec{k}) = \dfrac{1}{2\pi} \nabla_{\vec{k}} \cdot \vec{B}_n(\vec{k}) & \text{: monopole density} \end{cases}$$

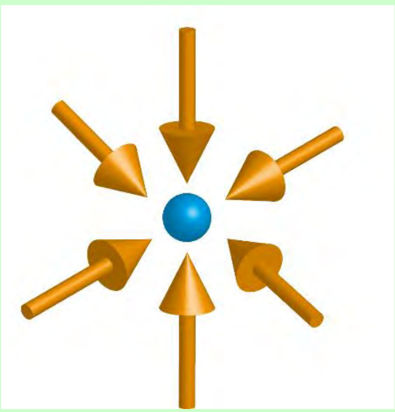




Monopole at $\vec{k} = \vec{k}_0$

$Q = +1$

or



Antimonopole at $\vec{k} = \vec{k}_0$

$Q = -1$

- Weyl nodes are either monopole or antimonopole
- Quantization of monopole charge

  → Weyl nodes can be created only as a monopole-antimonopole pair

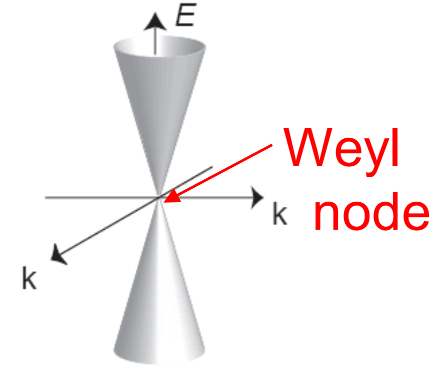
C. Herring, Phys. Rev. 52, 365 (1937).
G. E. Volovik, The Universe in a Helium Droplet (2007).
S. Murakami, New J. Phys. 9, 356 (2007).

Weyl semimetal and Fermi arc

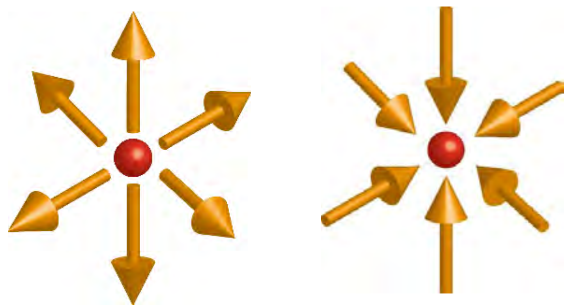
Weyl semimetal

Murakami, New J Phys. (2007)
Wan et al., Phys. Rev. B (2011)

= Bulk 3D Dirac cones **without** degeneracy at or around the Fermi energy



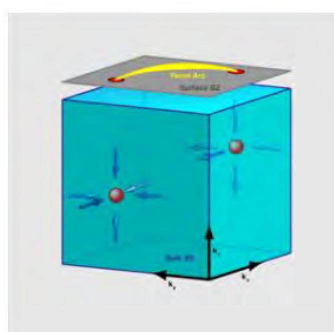
- Weyl nodes are either monopole or antimonopole for Berry curvature



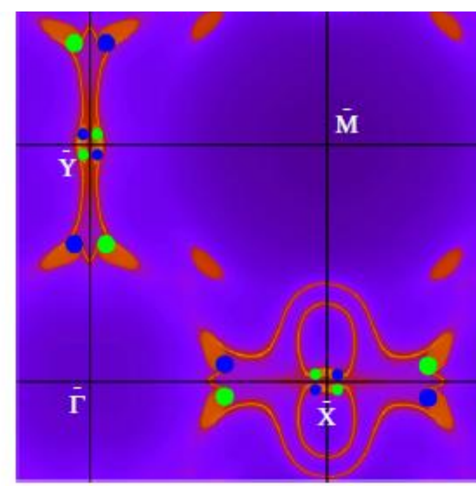
$$B_n(\vec{k}) = i\left\langle \frac{\partial u_{nk}}{\partial k}\right| \times \left| \frac{\partial u_{nk}}{\partial k}\right\rangle$$

$$\rho_n(\vec{k}) = \frac{1}{2\pi}\nabla_{\vec{k}}\cdot \vec{B}_n(\vec{k})$$

- Surface Fermi arc
  – connecting between Weyl nodes



TaAs surface Fermi arc
(Xu et al.,Lv et al., Yang et al. '15)

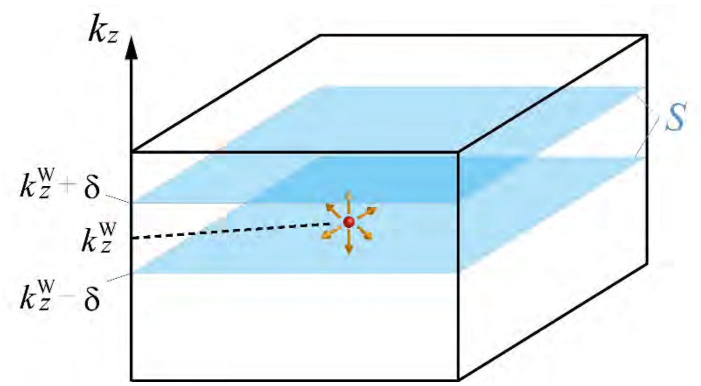


3D system with Weyl node

Even in 3D, one can consider the plane, k_z=const., to be a 2D system and consider its Chern number.

$$Ch_{(n)}(k_z) = \frac{1}{2\pi} \int_{k_z = const} \vec{B}\left(\vec{k}\right) \cdot \vec{n}\, dS$$

This Chern number depends on k_z



When a monopole (Weyl node with Q=1) is at k_z^W the Chern number jumps by +1

E.g. $\begin{cases} Ch_{(n)}(k_z^W - \delta) = 0 \\ Ch_{(n)}(k_z^W + \delta) = 1 \end{cases}$, chiral surface states exist not at $k_z < k_z^W$

but at $k_z > k_z^W$

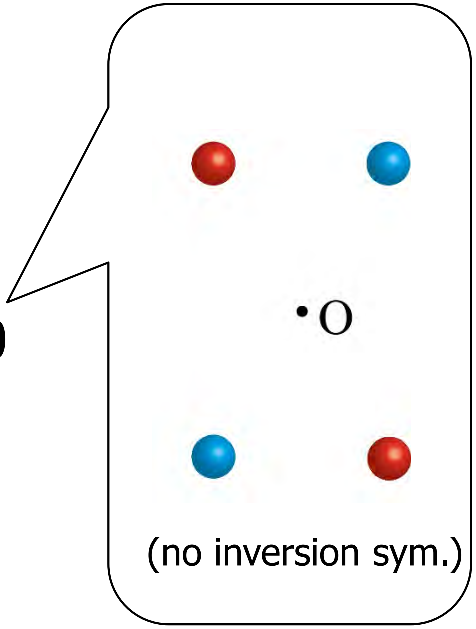
→ Surface states starting from the Weyl node projection = Fermi arc

Wan et al. (2011)

Symmetry and monopole density

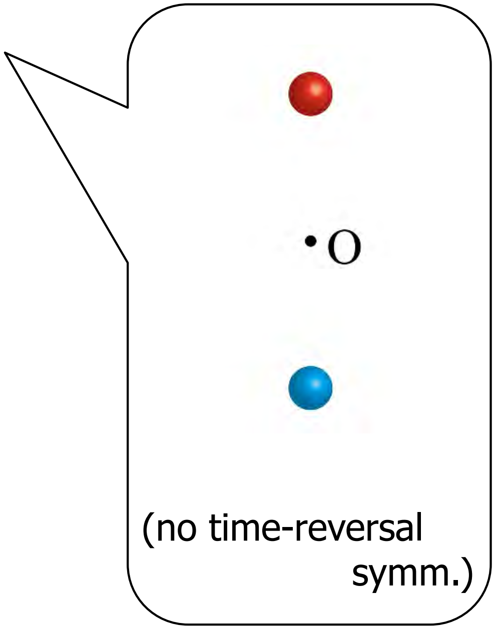
- Time-reversal symmetry

  → Monopoles distribute symmetrically w.r.t. k=0



- Inversion symmetry

  → Monopoles distribute antisymmetrically with respect to k=0



- Total monopole charge in the whole BZ vanishes.

- Inversion + Time-reversal
  → monopoles do not exist
  → Weyl semimetal cannot be realized
  (Dirac semimetal is possible)

Topological metals with bulk Dirac cones

Dirac semimetals =

bulk 3D Dirac cone with Kramers degeneracy

Both inversion- or time-reversal symmetry are required

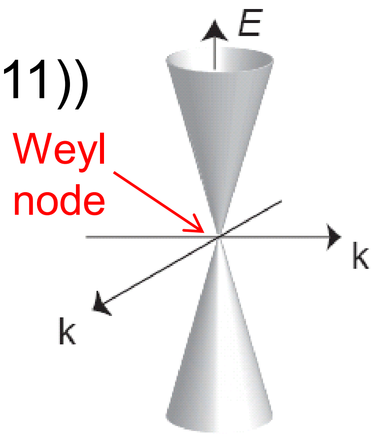
- β-BiO_2 (Young et al., (2011))
- A_3Bi (A = Na, K, Rb) (Wang et al., (2011))
- Cd_3As_2 (Wang et al., (2012))
- NI/TI multilayer (Burkov, Balents)

Weyl semimetals=

bulk 3D Dirac cone without Kramers degeneracy

Either inversion- or time-reversal symmetry should be broken

- pyrochlore iridates
  (Wan et al., PRB (2011), Yang et al., PRB(2011))
- NI/TI multilayer (Burkov, Balents)
- TaAs
- YbMnBi2 …



Effective model : insulator – Weyl semiemtal

Okugawa, Murakami, Phys. Rev. B 89, 235315 (2014)

$$H = \gamma(k_x^2 - m)\sigma_x + v(k_y\sigma_y + k_z\sigma_z)$$

Bulk dispersion

$$E = \pm\sqrt{\gamma^2(k_x^2 - m)^2 + v^2k_y^2 + v^2k_z^2}$$

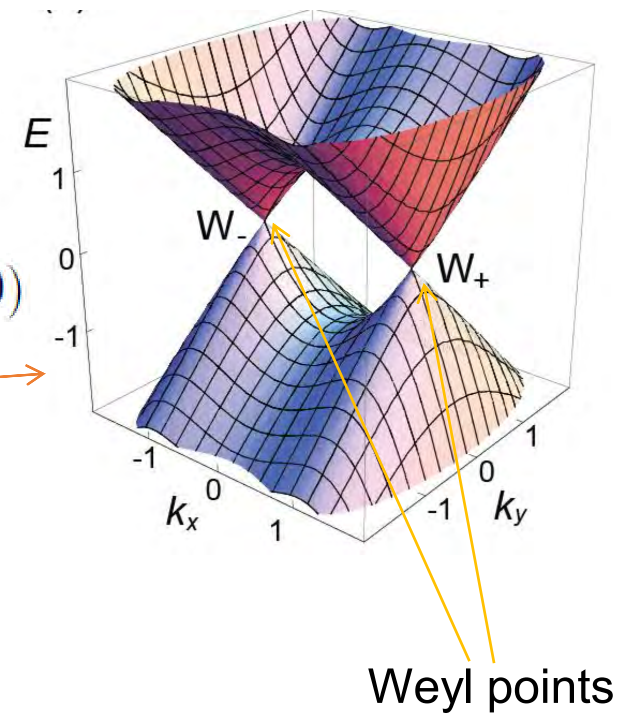
m<0: bulk gap = $2\gamma|m|$

= topological or normal insulator

m>0: bulk is gapless

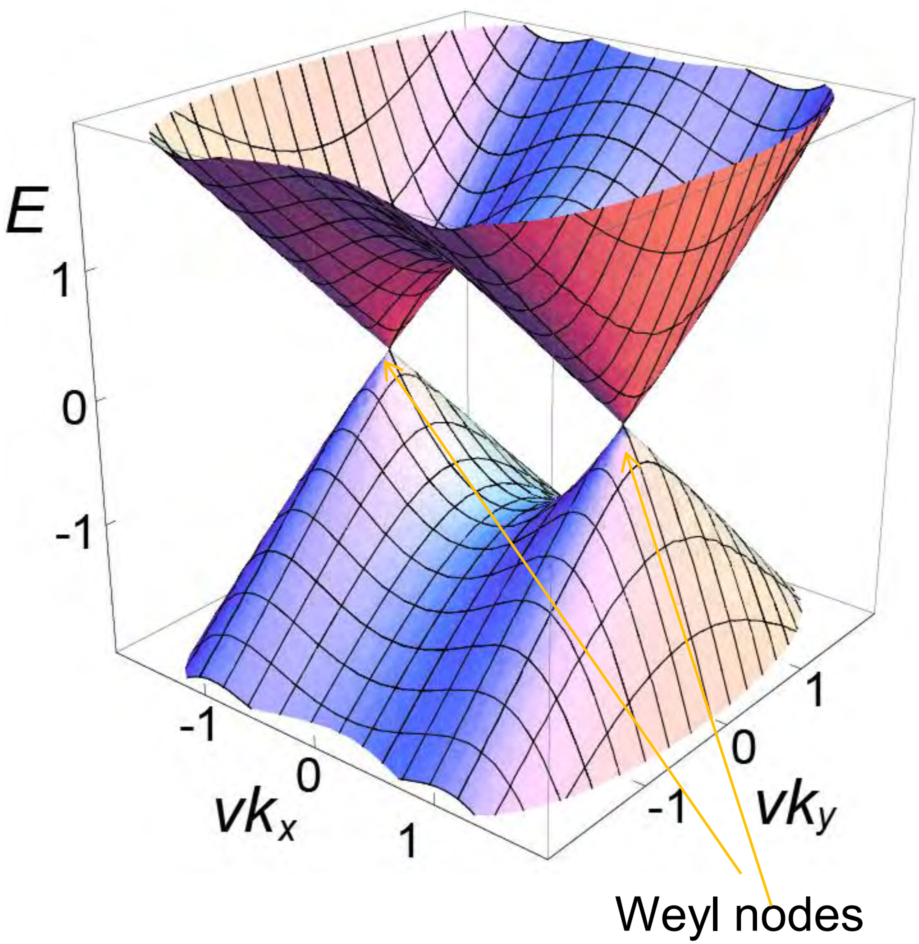
gap closed at $\mathrm{W}_{\pm}$: $\boldsymbol{k} = (\pm\sqrt{m}, 0, 0)$

= Weyl semimetal

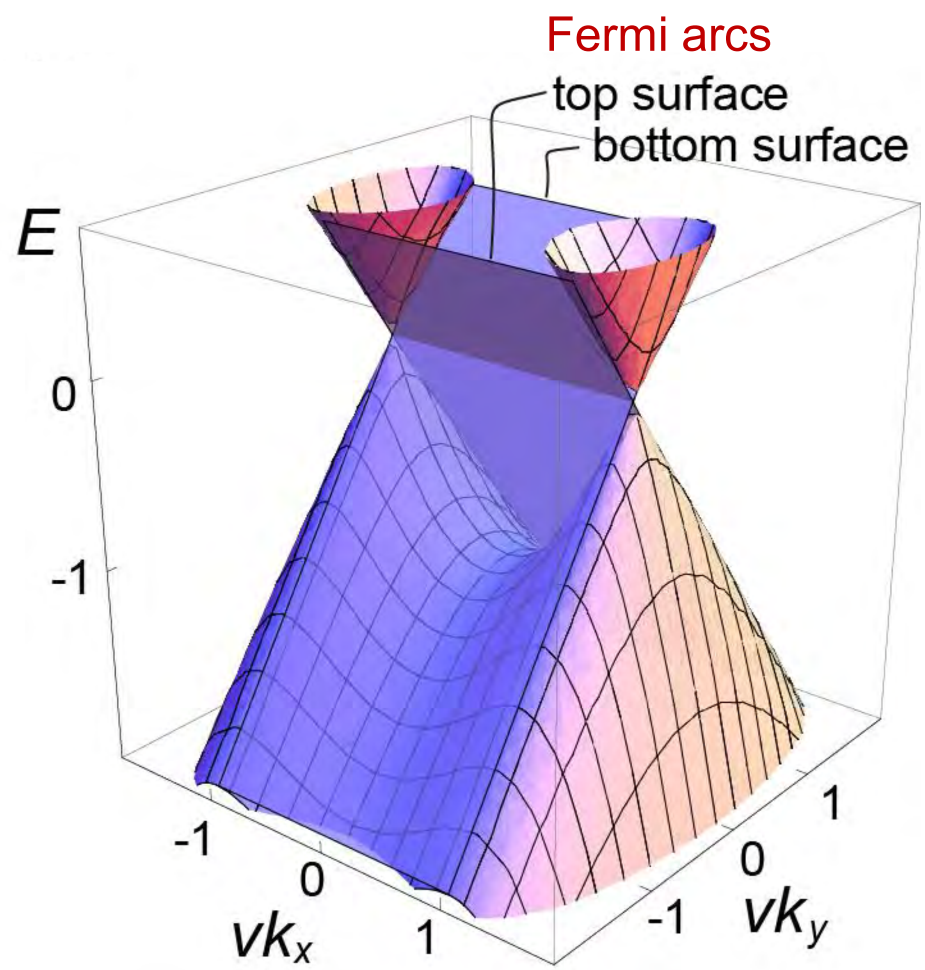


Surface Fermi arc : effective model calc.

Bulk band structure



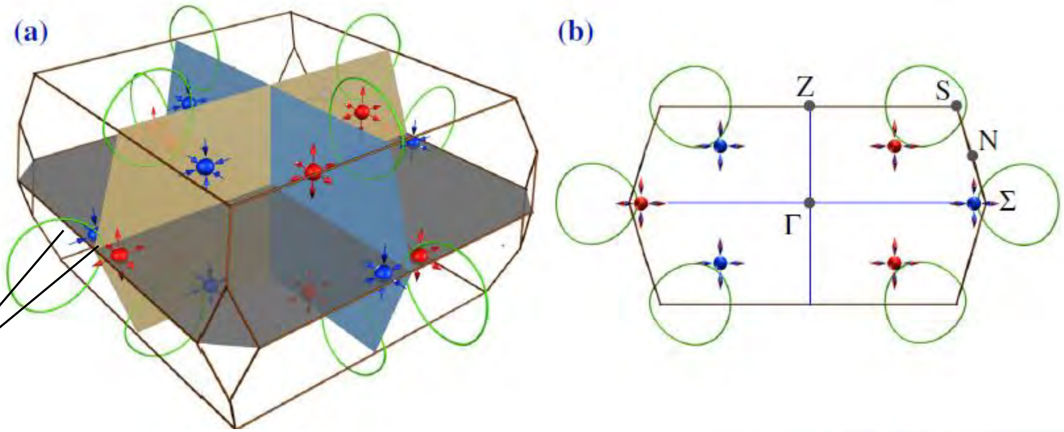
Bulk + surface



Okugawa, Murakami, Phys. Rev. B 89
235315 (2014)

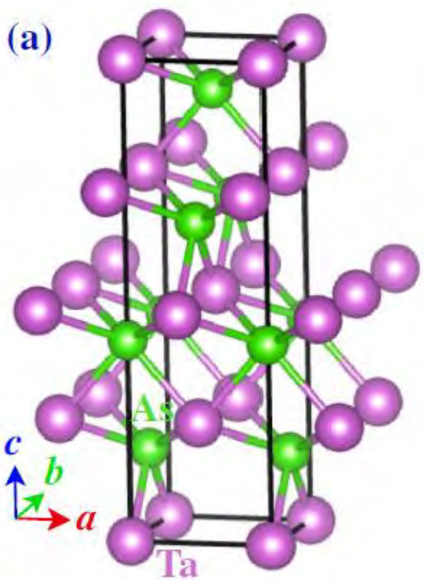
Weyl semimetal TaAs

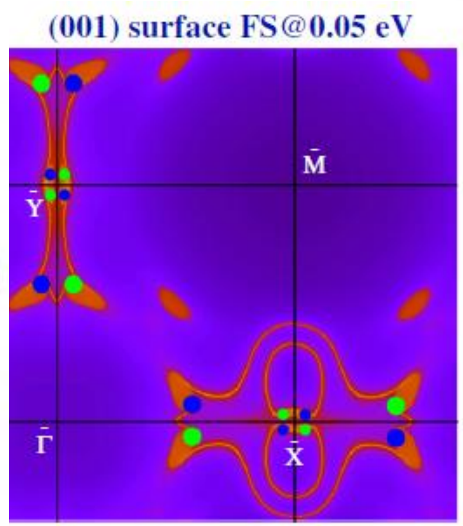
Lv et al., Xu et al. (2015):



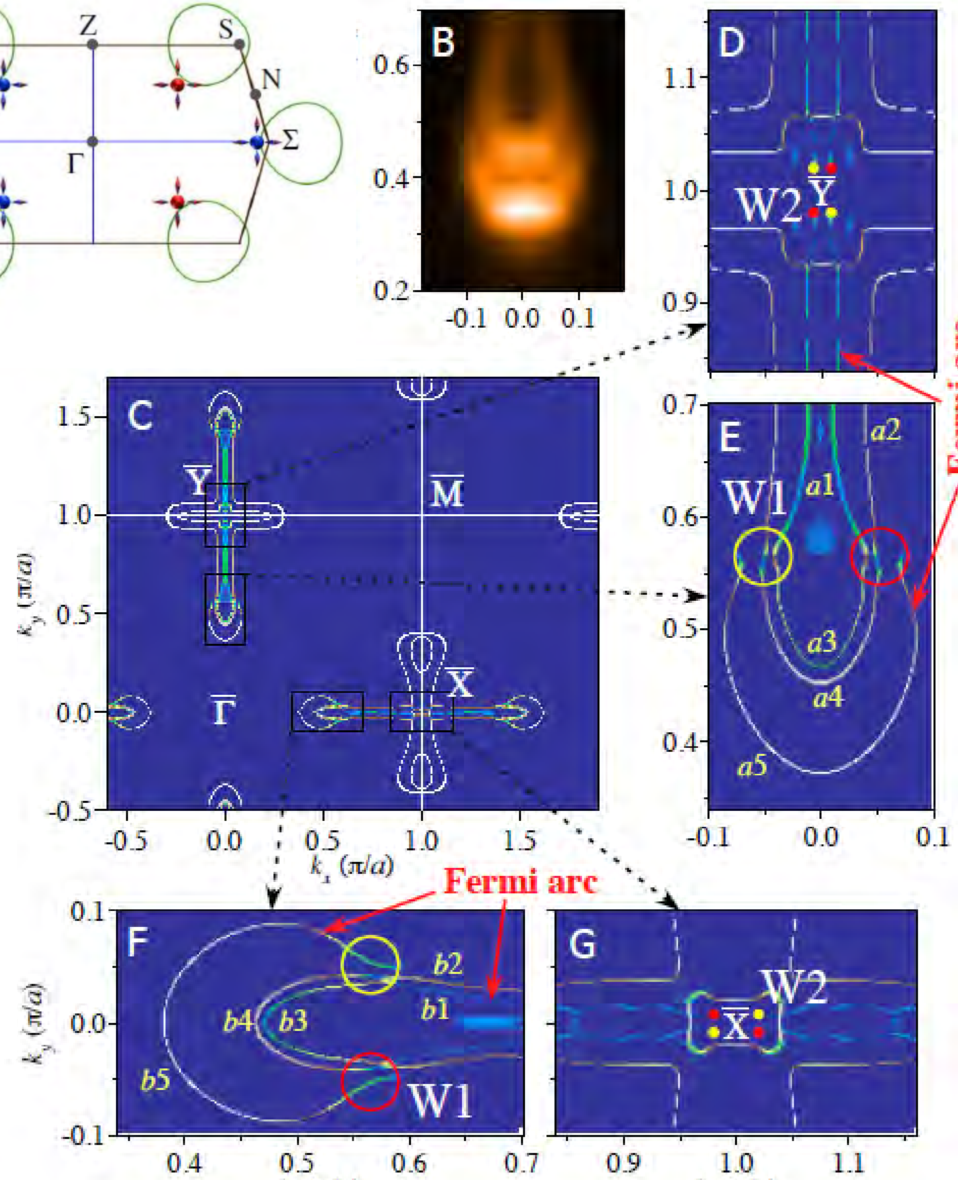
Weyl node

Weng et al., PRX (2015): theory





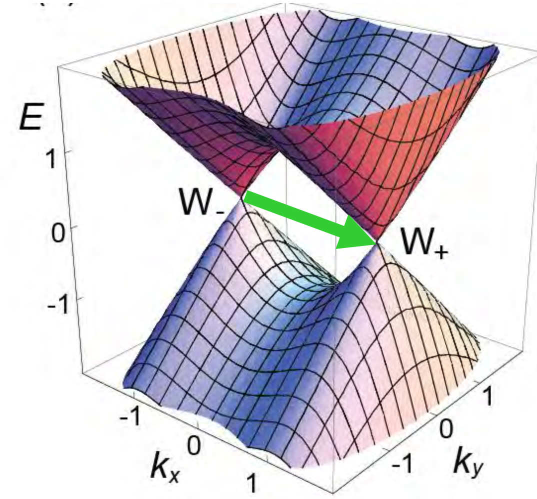
Fermi arc

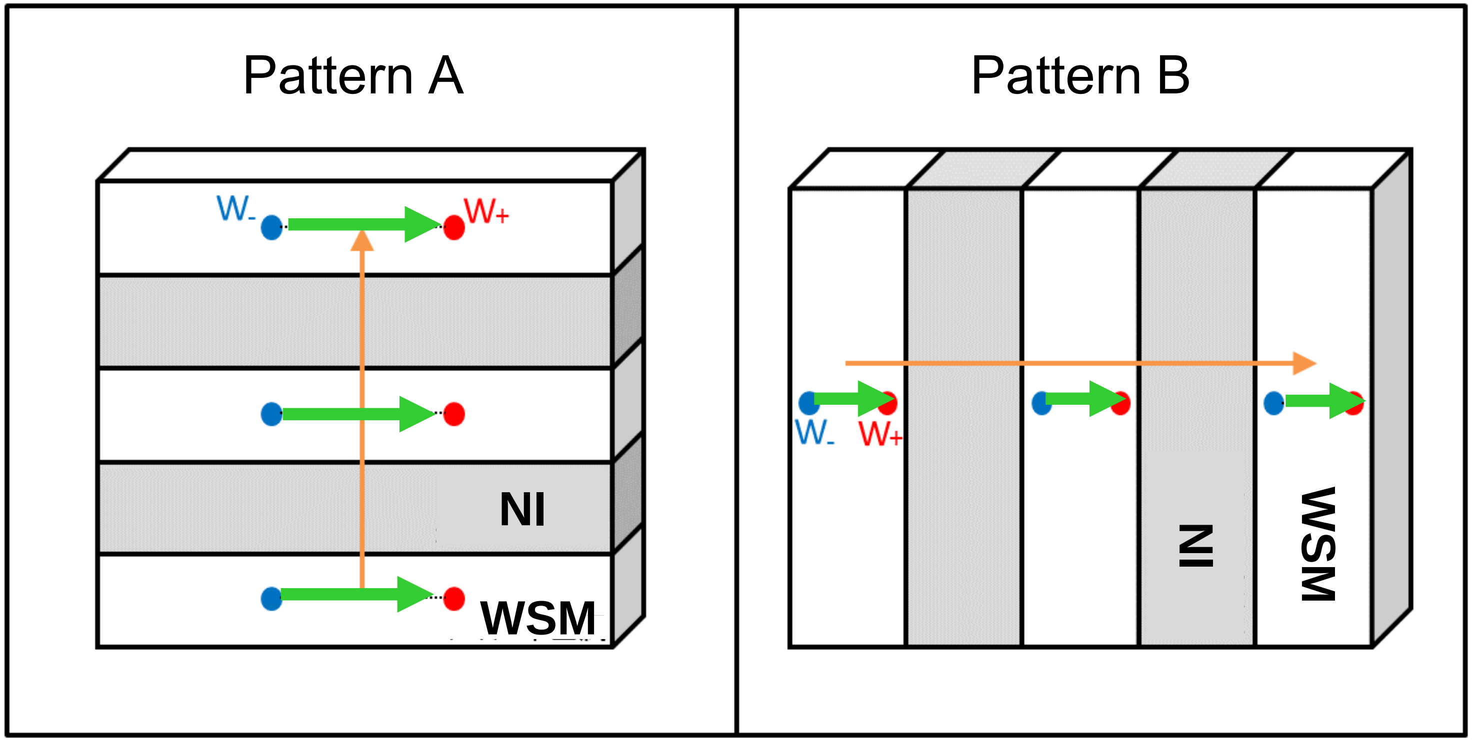


Multilayers of a Weyl semimetal and a normal insulator

K. Yokomizo and S. Murakami

Phys. Rev. B **95**, 155101 (2017)

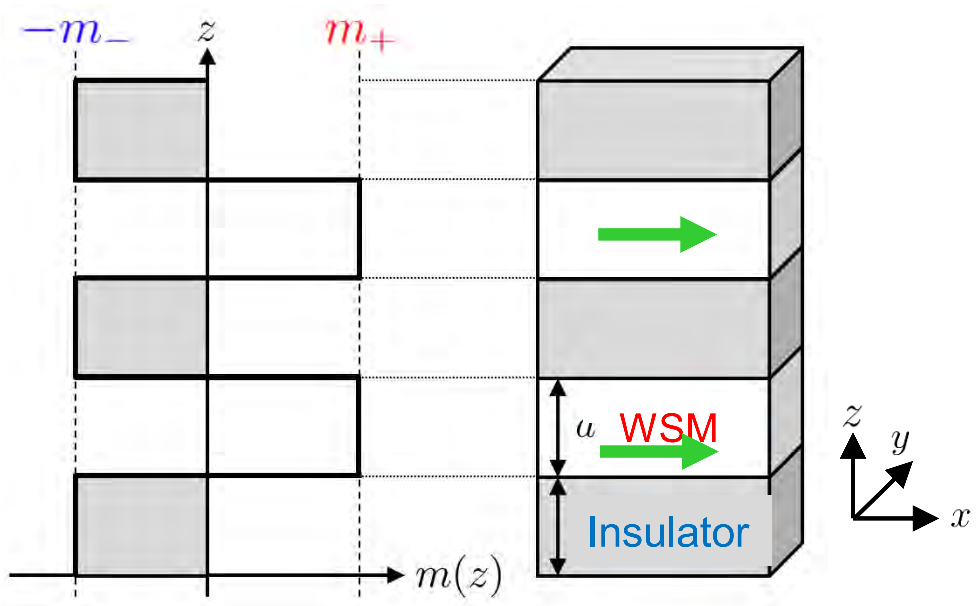




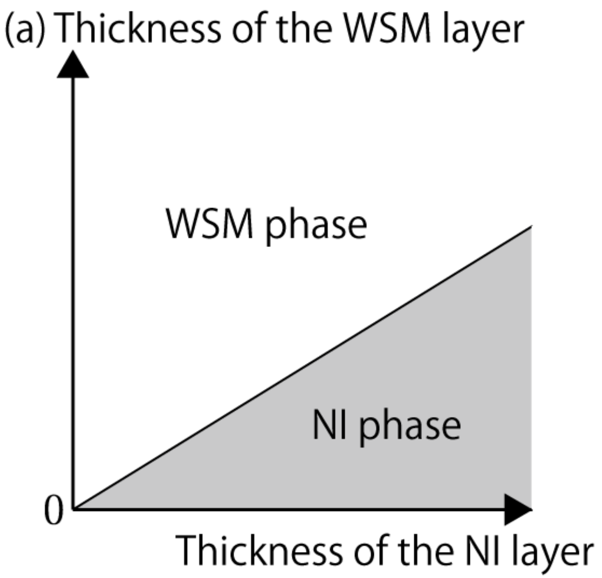
Multilayer (Pattern A)

$$H = \gamma\left(k_x^2 - m(z)\right)\sigma_x + v\left(k_y\sigma_y - i\partial_z\sigma_z\right)$$

Spatial modulation



Phase diagram of the multilayer



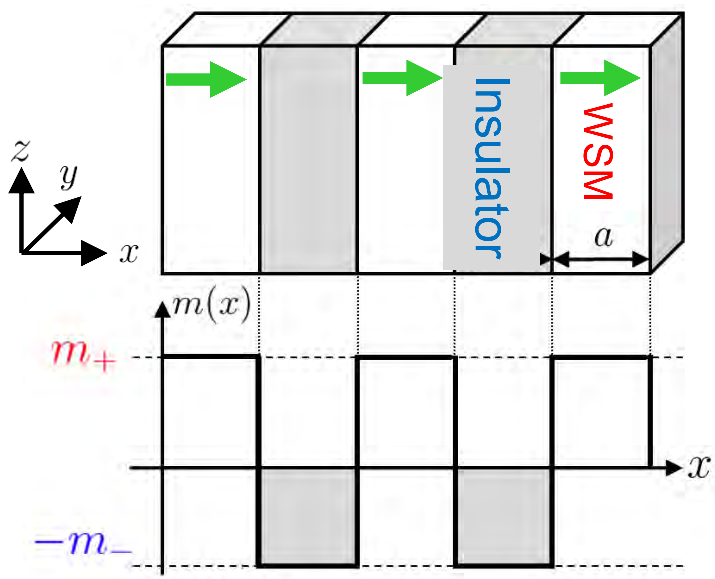
The multilayer becomes the WSM phase
by increasing the thickness of the WSM layer

Multilayer (Pattern B)

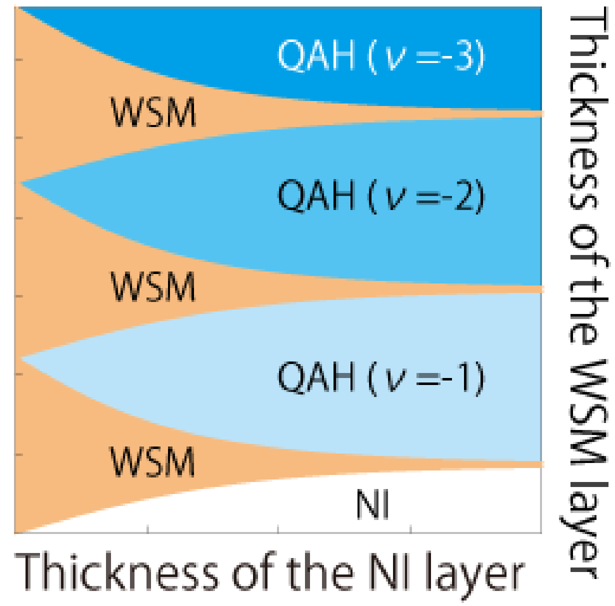
Hamiltonian

$$H = \gamma \left(-\partial_x^2 - \underline{m(x)}\right) \sigma_x + v \left(k_y \sigma_y + k_z \sigma_z\right)$$

Spatial modulation



Phase diagram of the multilayer

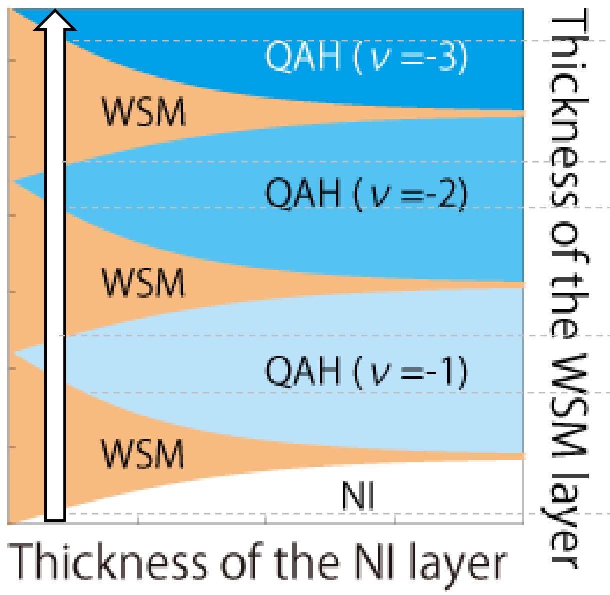


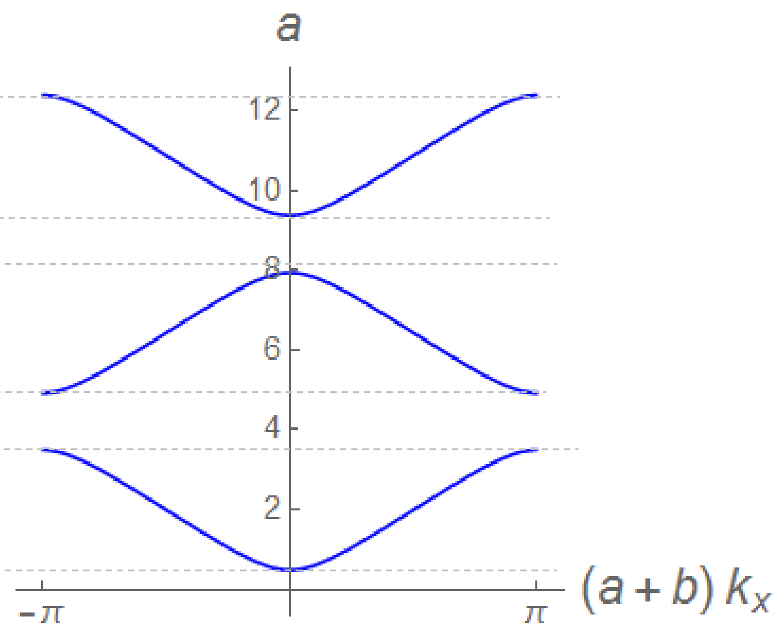
(1) WSM phases periodically emerge

(2) Quantum anomalous Hall (QAH) phases which have different Chern number periodically emerge

Trajectory of the Weyl nodes in multilayer (Pattern B)

thickness of the WSM layer: increase
thickness of the NI layer: const





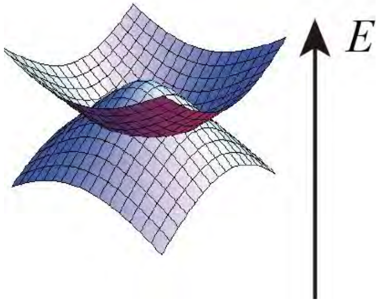
Trajectory of the Weyl nodes

K. Yokomizo and S. Murakami

Phys. Rev. B **95**, 155101 (2017)

Nodal-line semimetal

Nodal-line semimetal: bulk gap closes along a loop in k space



2 typical mechanisms

(i) Mirror symmetric system:

Mirror eigenvalues are different between the valence and the conduction bands

- spinless (SOC=0) $M = \pm 1$
- spinful (nonzero SOC) $M = \pm i$

→ No anticrossing between the two bands (← prohibited hybridization)

→ Nodal line on a mirror plane

Dirac line node

- Carbon allotropes
- Cu3PdN
- Ca3P2
- LaN
- CaAgX (X=P,As)

Weyl line node

- HgCr2Se4
- TlTaSe2

2 typical mechanisms

(ii) Spinless (SOC=0) & time-reversal sym. & inversion symm.

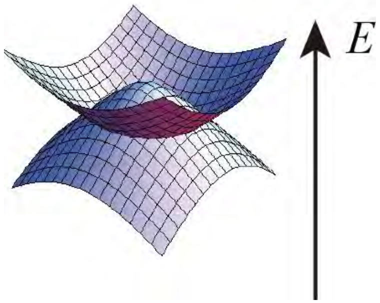
topological nodal line at generic position in k space

(Example)

$$H(\vec{k}) = \begin{pmatrix} a(\vec{k}) & b(\vec{k}) \\ b^*(\vec{k}) & c(\vec{k}) \end{pmatrix}$$

$a(\vec{k}), c(\vec{k})$: real

$b(\vec{k})$: complex

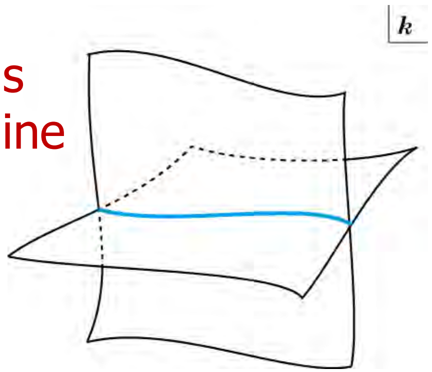


With the above 3 conditions → Hamiltonian is a real matrix.

$b(\vec{k})$: real

Degeneracy appears when $\begin{cases} a(\vec{k}) = c(\vec{k}) \\ \mathrm{Re}\, b(\vec{k}) = 0 \end{cases}$

2 conditions → nodal line



Topological characterization:

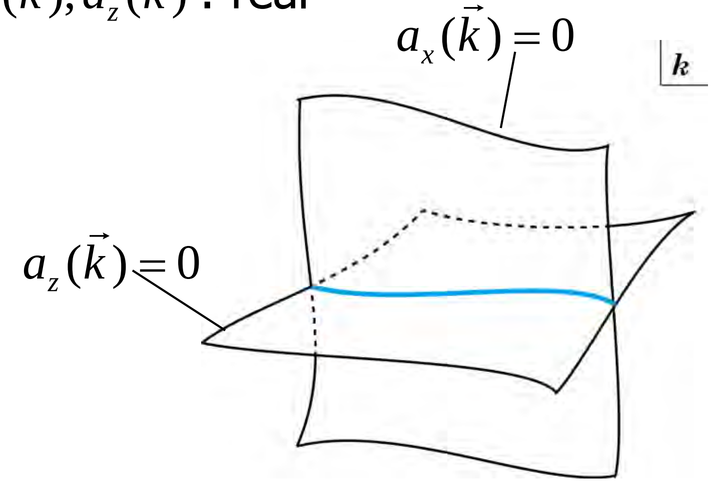
π Berry phase around the nodal line

（Example）

Rewrite the matrix in terms of Pauli matrices. Omit the trace part.

$$H(\vec{k}) = \begin{pmatrix} a_z(\vec{k}) & a_x(\vec{k}) \\ a_x(\vec{k}) & -a_z(\vec{k}) \end{pmatrix}$$

$a_x(\vec{k}), a_z(\vec{k})$: real



Phase of $z(\vec{k}) \equiv a_x(\vec{k}) + i a_z(\vec{k})$ winds by 2π around the nodal line.

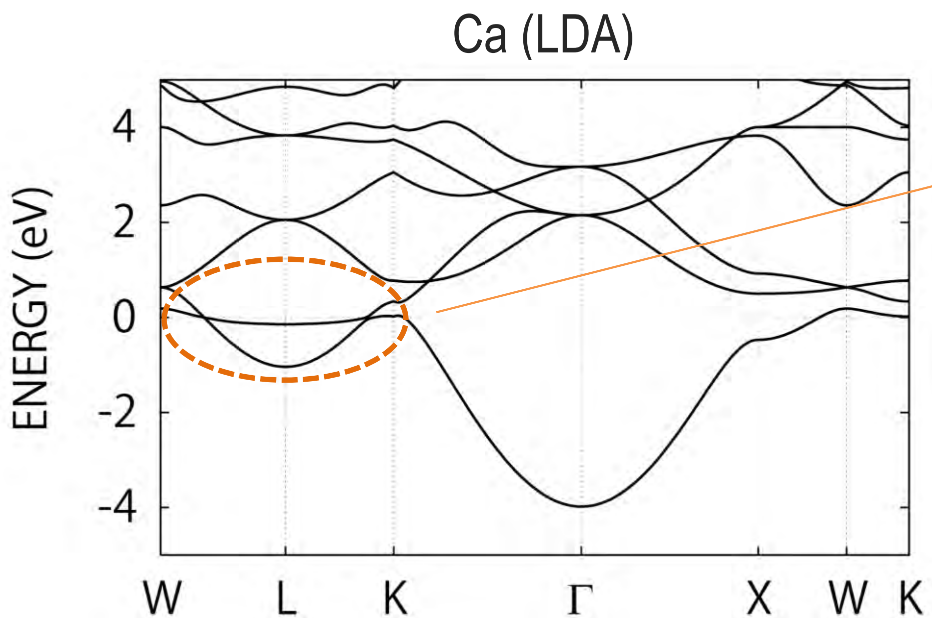
→ Nodal line is topological

In general systems, the nodal line is characterized by π Berry phase.

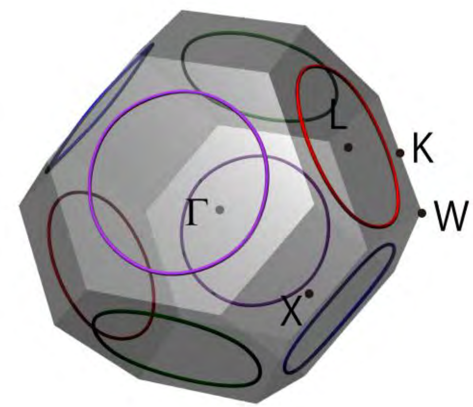
$$\phi = -i \oint_C d\vec{k} \cdot \left\langle u_n(\vec{k}) \middle| \frac{\partial}{\partial \vec{k}} \middle| u_n(\vec{k}) \right\rangle = \pi$$

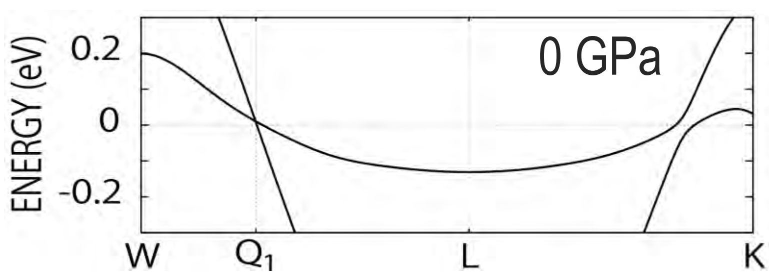
Ca have nodal lines near E_F

Hirayama, Okugawa, Miyake, Murakami
Nat. Commun.8, 14022 (2017)

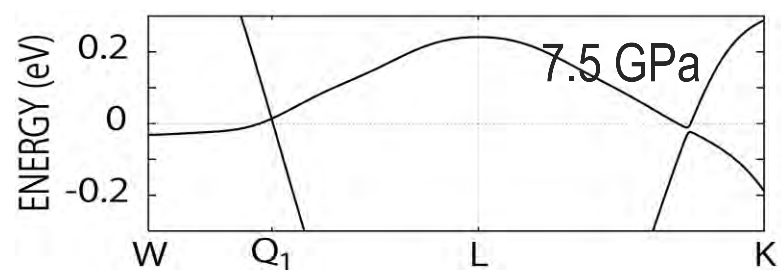


Nodal line around Fermi energy





Not semimetal at 0 GPa



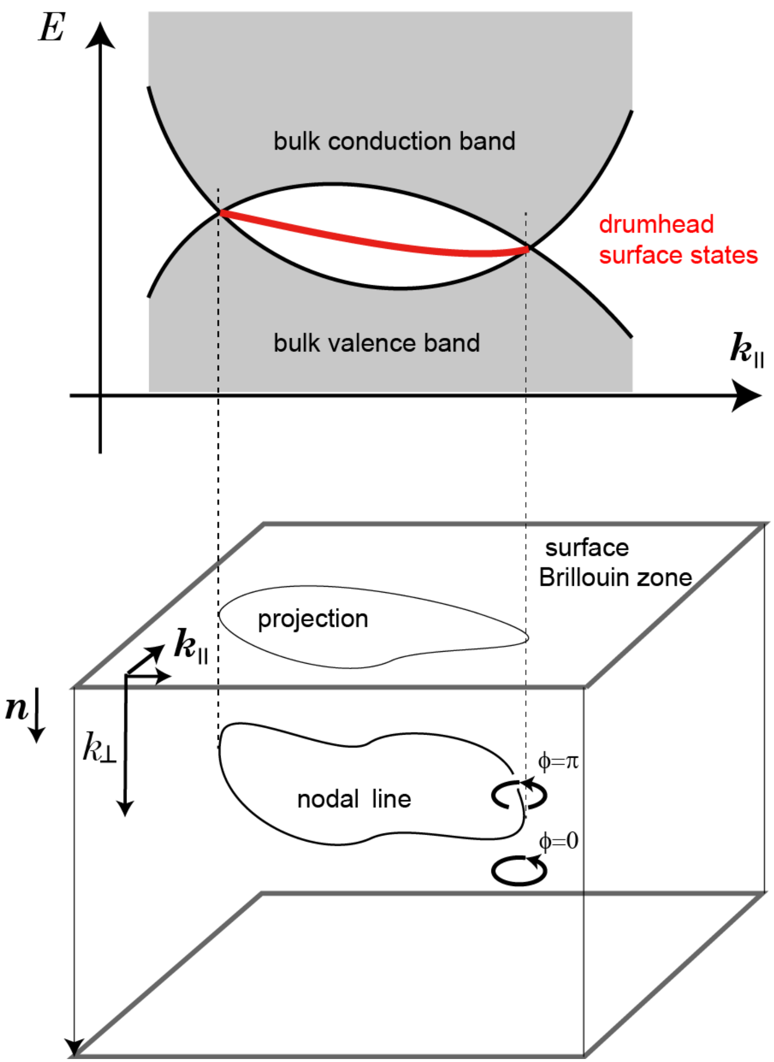
Nodal line semimetal at 7.5GPa

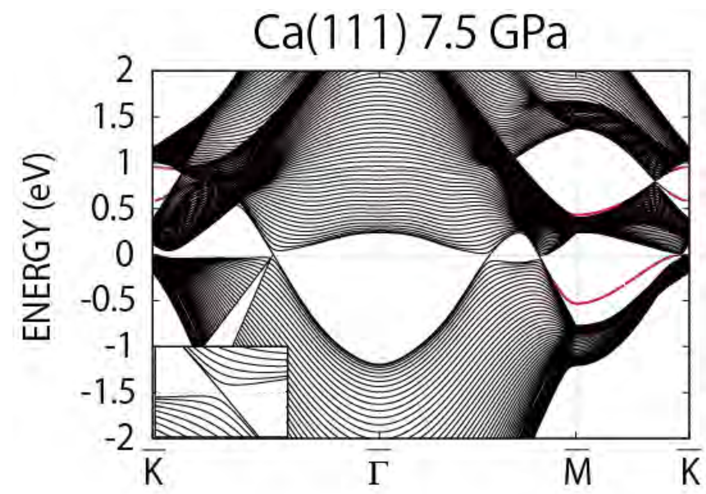
(cf.) previous works
Vasvari, Animalu, Heine, Phys. Rev. 154, 535 (1967).
Vasvari, Heine,Phil. Mag. 15, 731–738 (1967).

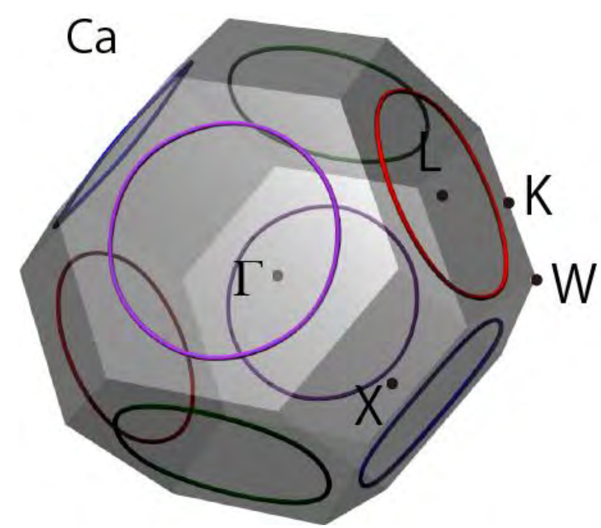
nodal-line semimetal : drumhead surface states

Surface state often appears within the region surrounded by the nodal line

- Similar to the flat-band edge states in graphene zigzag ribbon.







Nodal-line and Zak phase

Zak phase (Berry phase) for a given $\vec{k}_{\parallel}$ (= surface wavevector)

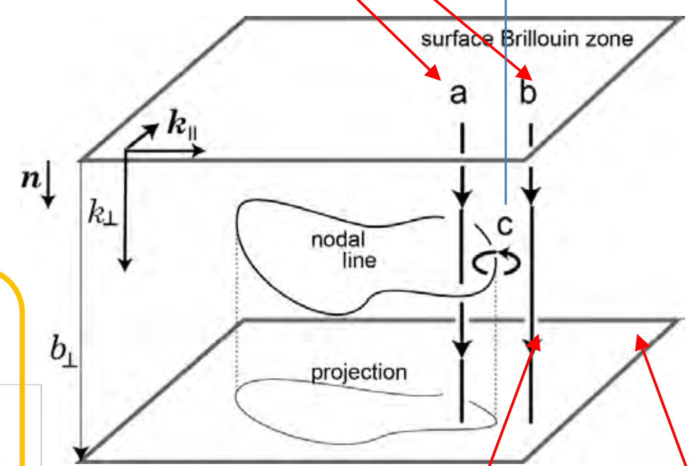
$$\theta(k_{\parallel}) = -\mathrm{i}\sum_{n}^{occ.}\int_{0}^{2\pi/a_{\perp}} dk_{\perp} \left\langle u_n(k) \middle| \frac{\partial}{\partial k_{\perp}} \middle| u_n(k) \right\rangle$$

$\theta(\vec{k}_{\parallel}) = 0 \text{ or } \pi$

← Inversion+ time-reversal symmetries

Zak phase $\theta(\vec{k}_{\parallel})$ jumps by π at the nodal line

Berry phase = π



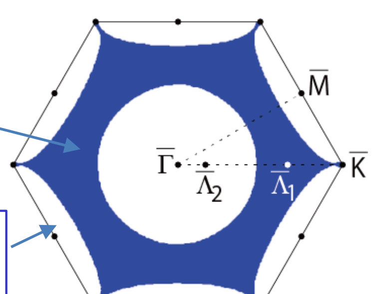
$\theta(\mathrm{a}) = \pi$

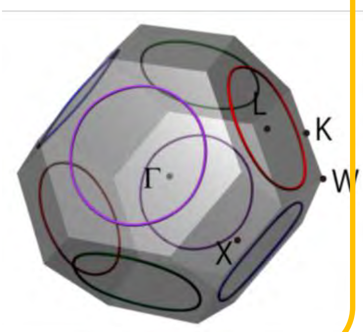
$\theta(\mathrm{b}) = 0$

(111) Surface Brillouin zone in Ca

Zak phase $\theta(\vec{k}_{\parallel}) = \pi$

Zak phase $\theta(\vec{k}_{\parallel}) = 0$





Zak phase and charge polarization

In 1D system:

Polarization σ = Zak phase $\times \frac{e}{2\pi}$

$$\sigma(\vec{k}_{\parallel}) = \frac{e}{2\pi}\theta(\vec{k}_{\parallel}) \pmod e$$

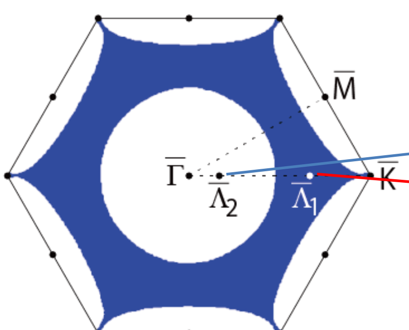
"modern theory of polarization"

(Vanderbilt, King-Smith, PRB,1993)

Total polarization for 3D system (=surface polarization charge density)

$$\sigma = \int \frac{d^2\vec{k}_{\parallel}}{(2\pi)^2}\sigma(\vec{k}_{\parallel})$$

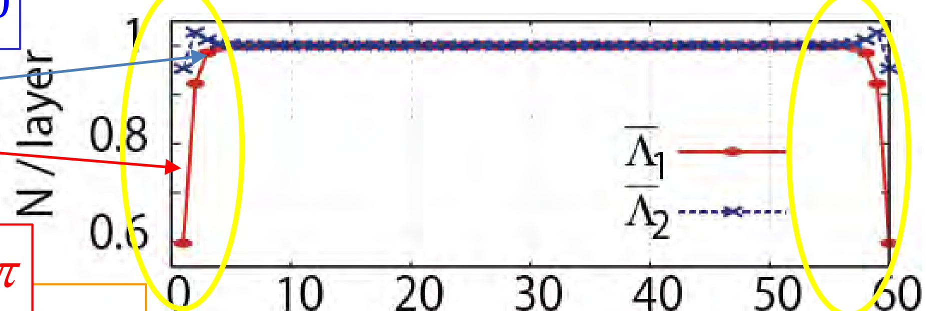
(Note: only for insulators)



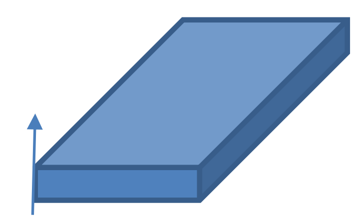
$\theta(\vec{k}_{\parallel}) = 0$

$\theta(\vec{k}_{\parallel}) = \pi$

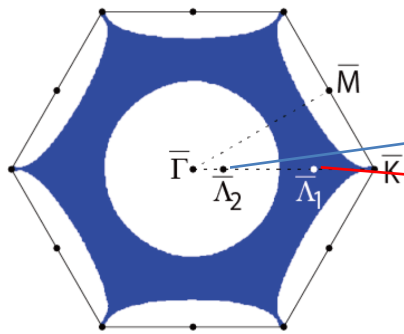
Charge profile in a slab along thickness direction

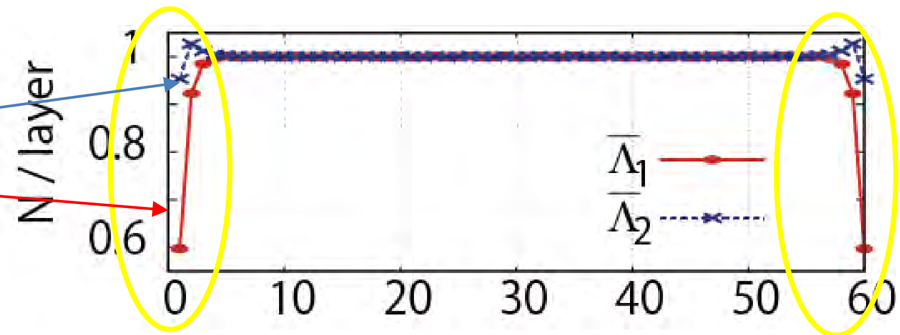


Charge is depleted by e/2

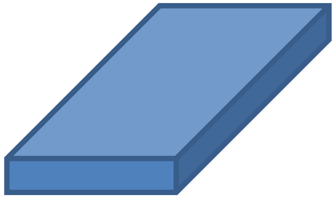


Surface charge





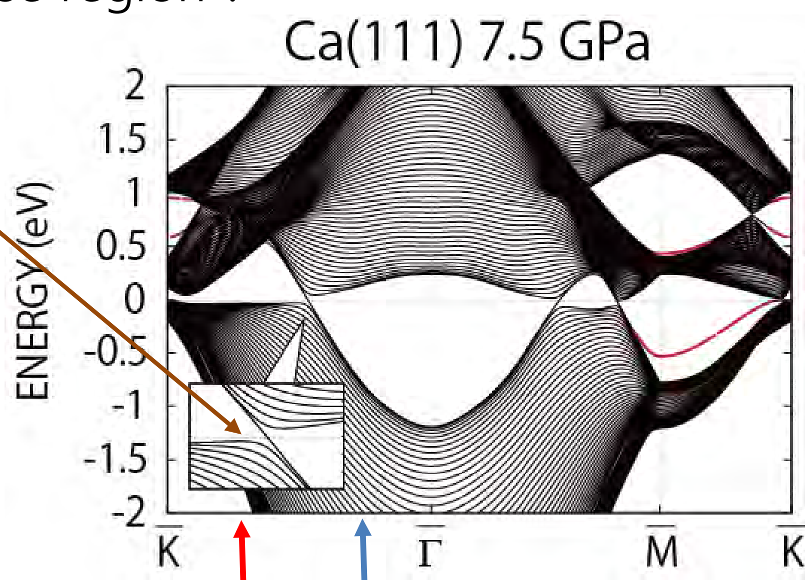
Remarks:



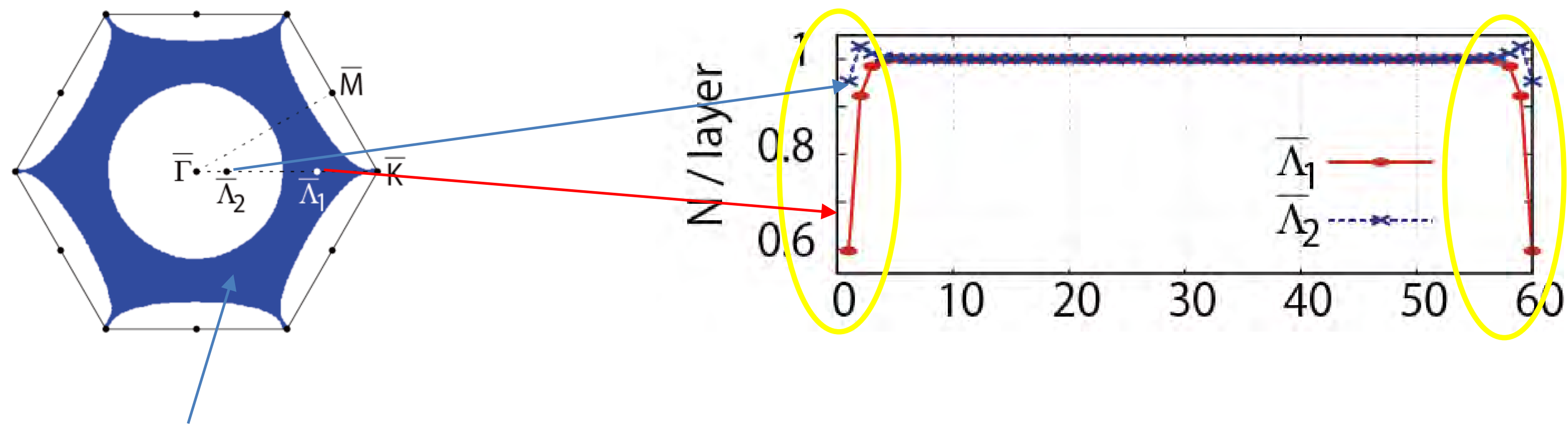
- It is not ferroelectric ← centrosymmetric fcc
- Where is the missing charge at π Zak phase region ?

The number of bulk occupied bands change at the nodal lines

Chemical potential will slightly change to accommodate missing charge.



Nodal-line and Berry phase



Area=0.485*BZ

→ surface charge density = $0.485 \cdot \frac{e}{2}$ per surface unit cell

Huge surface polarization charge

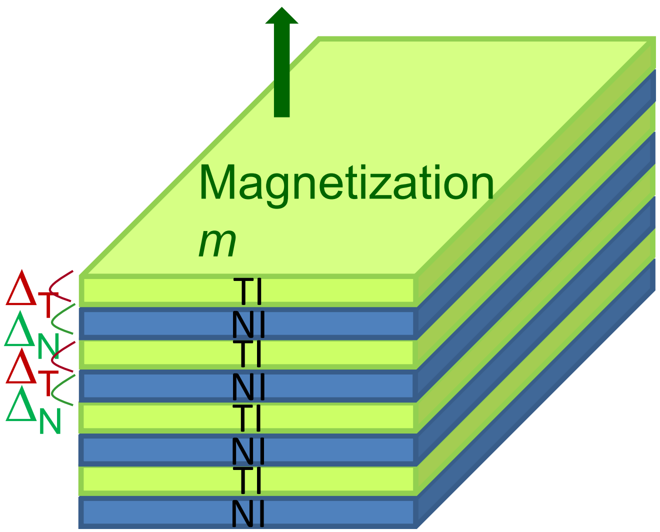
→ In metals this charge is screened by carriers and lattice

→ Charge imbalance & lattice relaxation at the surface

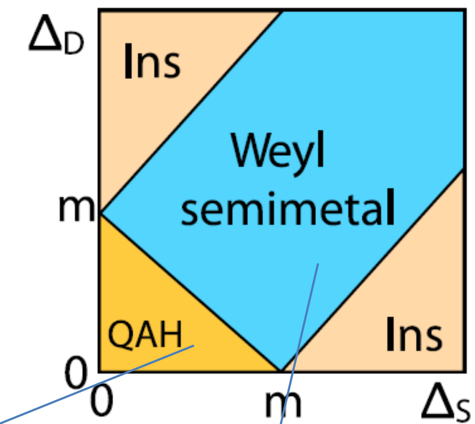
Topological metals often appears between various topological insulator phases

Topological insulator multilayer without time-reversal symmetry

Burkov, Balents, PRL 107, 127205 (2011)



m>0



<u>Quantum anomalous Hall :</u>

Ch=1 for all k_z

＝Chiral surface state for all k_z

<u>Weyl semimetal :</u>

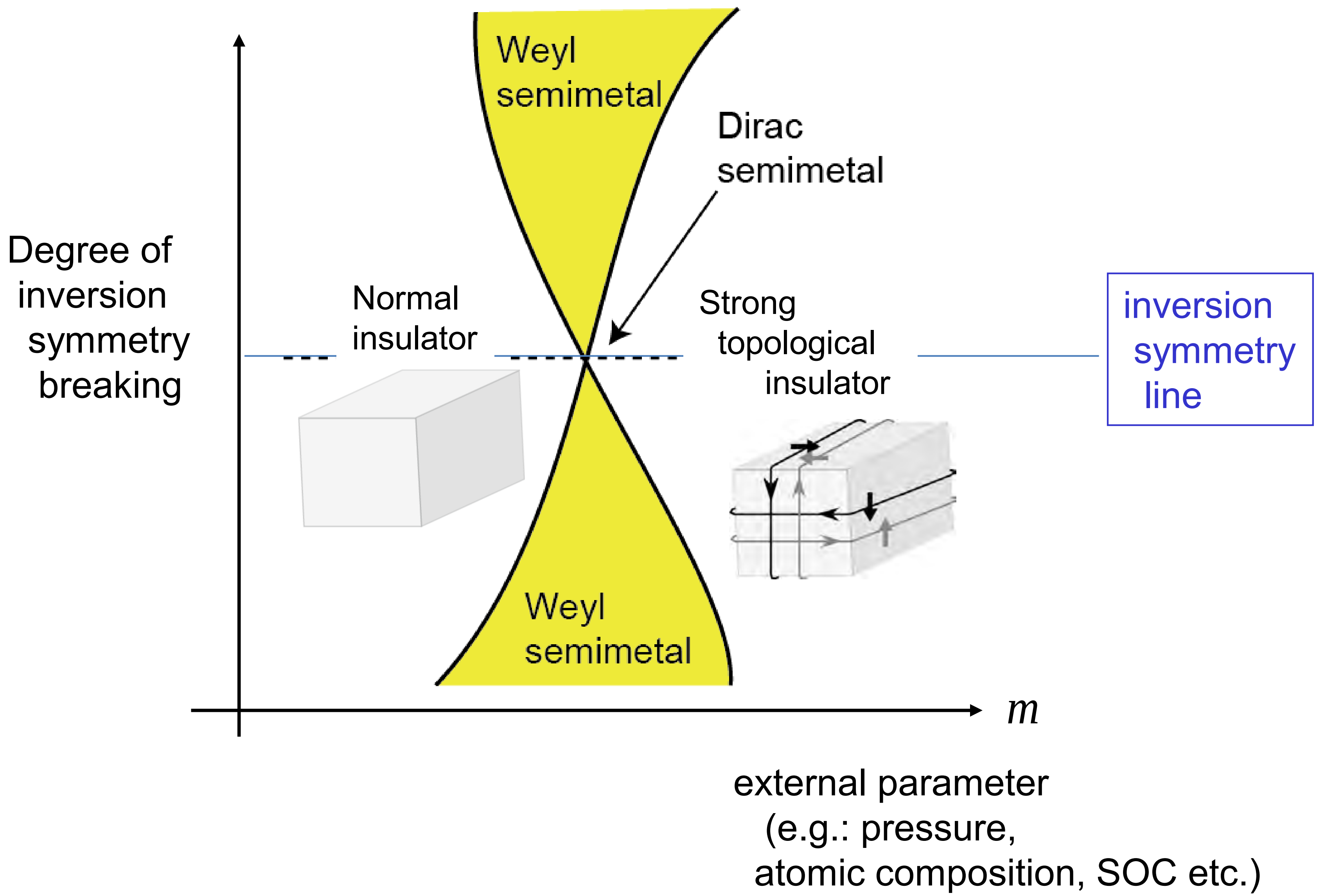
Ch=1 for k_z between the two Weyl nodes. = Fermi arc

Ch=0 otherwise

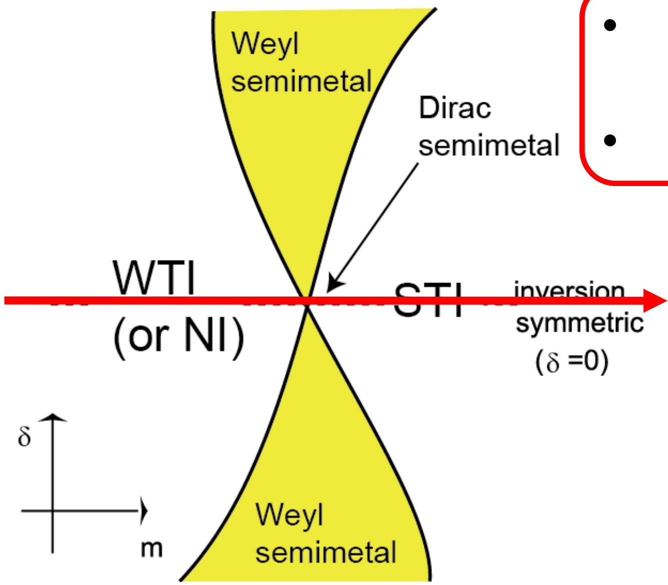
- By changing parameters, the Weyl nodes move in k-space.
- When they meet, they are annihilated in pair and the system becomes an insulator in the bulk (i.e. either QAH phase or an insulator phase).

NI-TI universal phase diagram in 3D

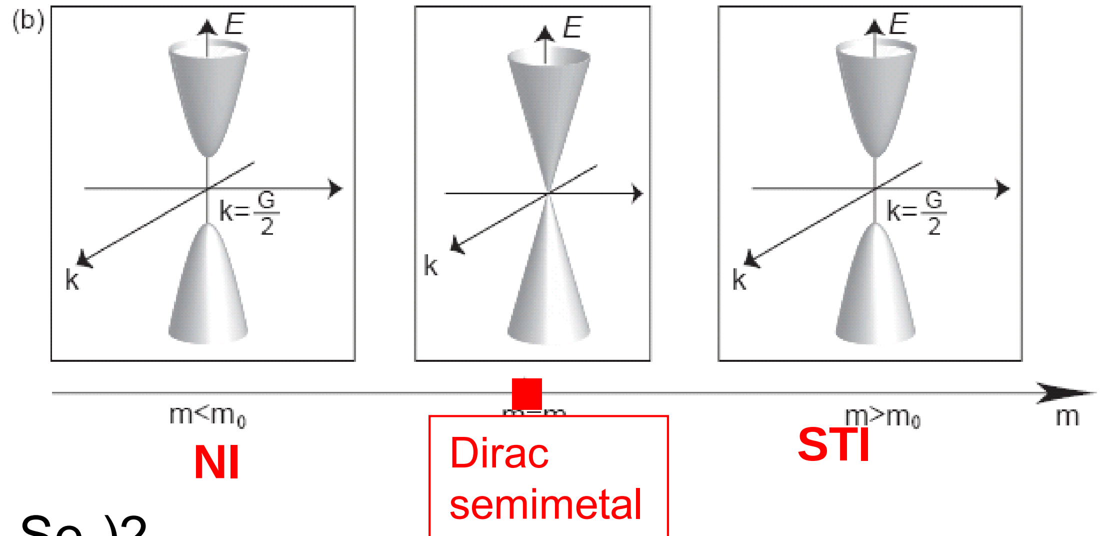
SM, New J. Phys. ('07).
SM. Kuga, PRB ('08)
SM, Physica E43, 748 ('11)



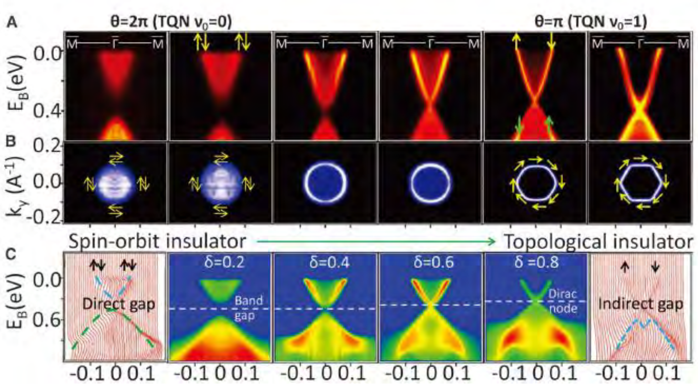
Systems with inversion symmetry



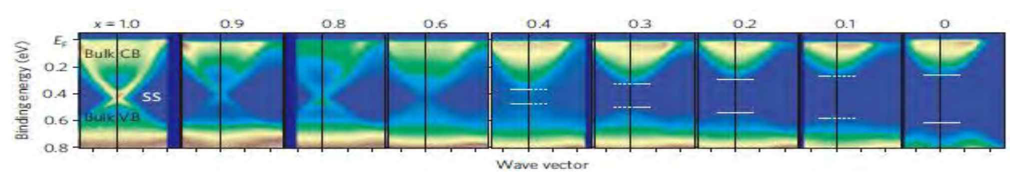
- Gap closes at TRIM
  inversion of bands with opposite parities.
- Insulator-to-insulator transition



e.g. $TlBi(S_{1-x}Se_x)2$

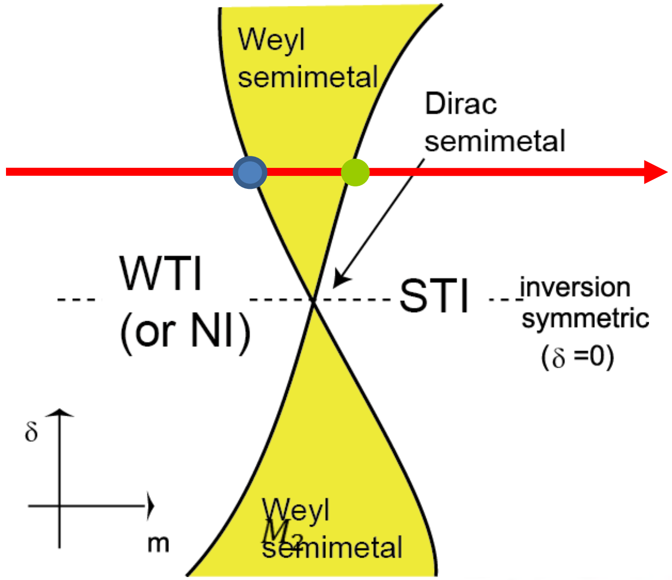


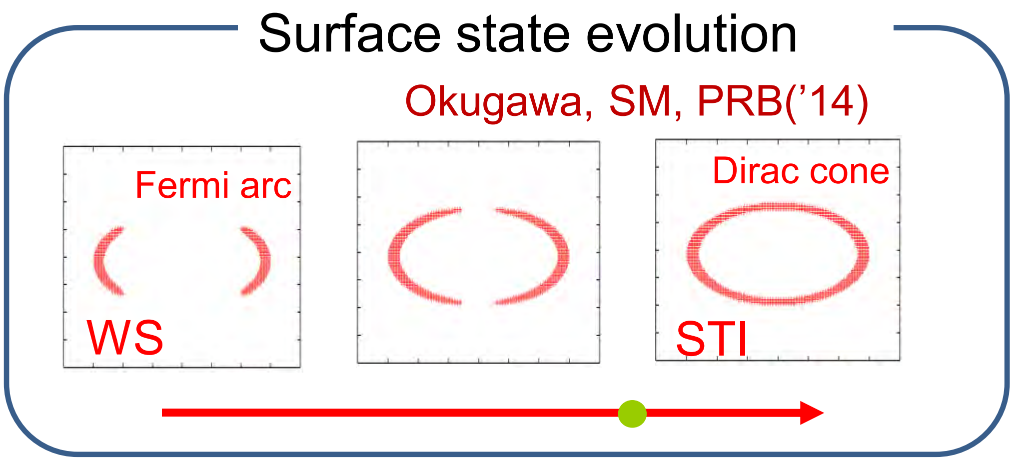
Xu et al., Science.332, 560 (‘11)

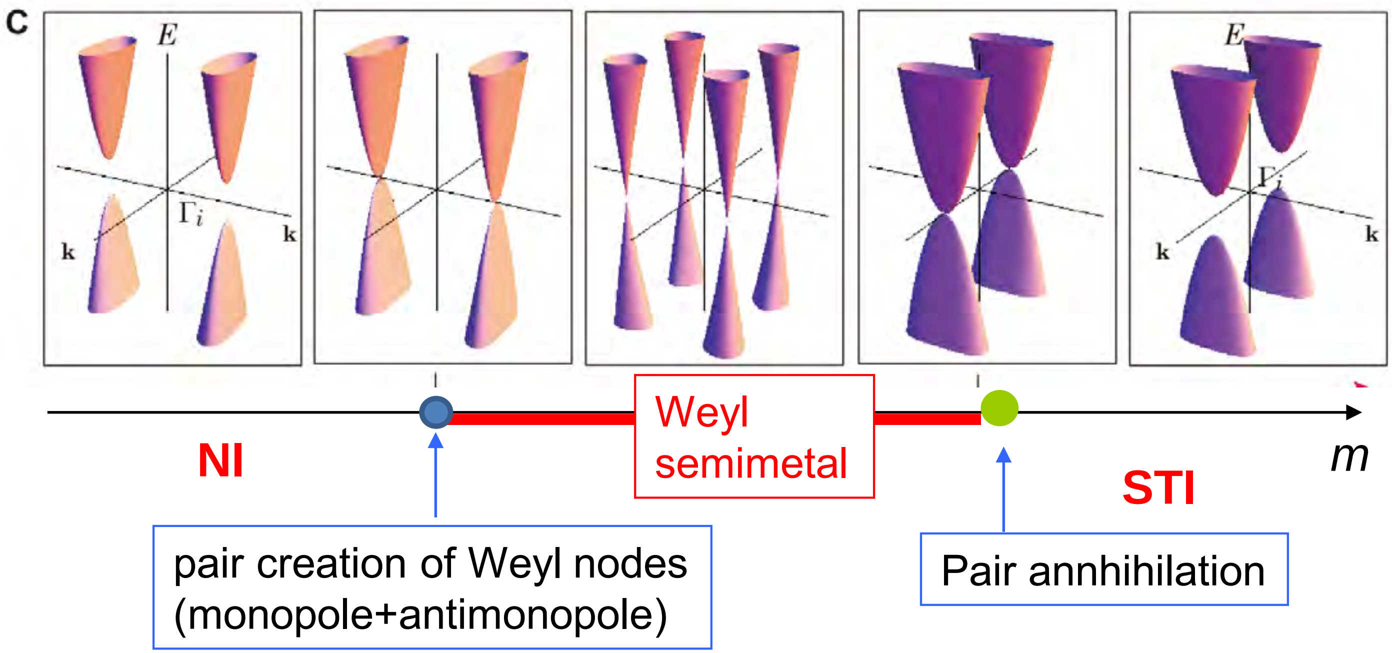


Sato et al., Nature Phys.7, 840 (‘11)

Systems without inversion symmetry





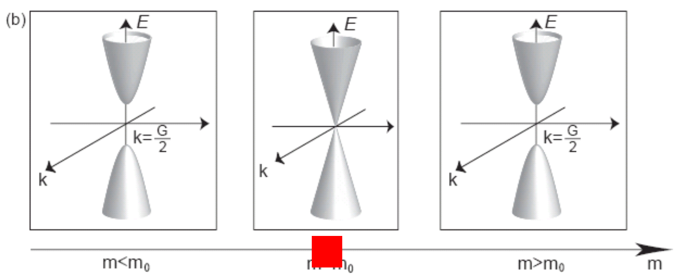


SM, New J. Phys. ('07).
SM. Kuga, PRB ('08)
SM, Physica E('11)
Okugawa, SM, PRB('14)

Z_2 topological number ν

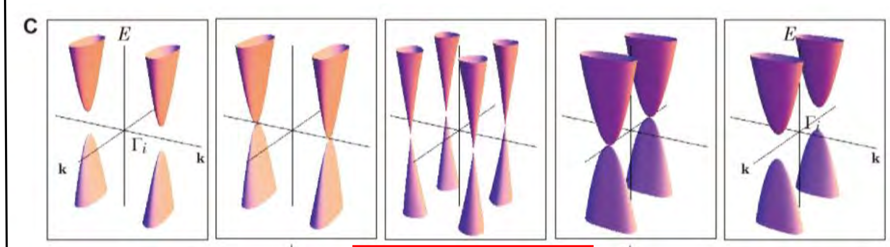
ν=0: normal insulator (NI)
ν=1: topological insulator (TI)

(A) systems with inversion symmetry	(B) systems without inversion symmetry
$$(-1)^{\nu}=\prod_i\prod_{m=1}^{N}\xi_{2m}(\Gamma_i)$$ Parity eigenvalue +1 or -1. Γ_i : TRIM. Fu,Kane, PRB(2007)	$$(-1)^{\nu}=\prod_i\frac{\sqrt{\det[w(\Gamma_i)]}}{\mathrm{Pf}[w(\Gamma_i)]}$$ $$w_{mn}(\vec{k})=\langle u_{-k,m}\vert\Theta\vert u_{k,n}\rangle$$ Fu,Kane, PRB(2006)



NI STI

Gap closes at TRIM
→parity eigenvalues are exchanged

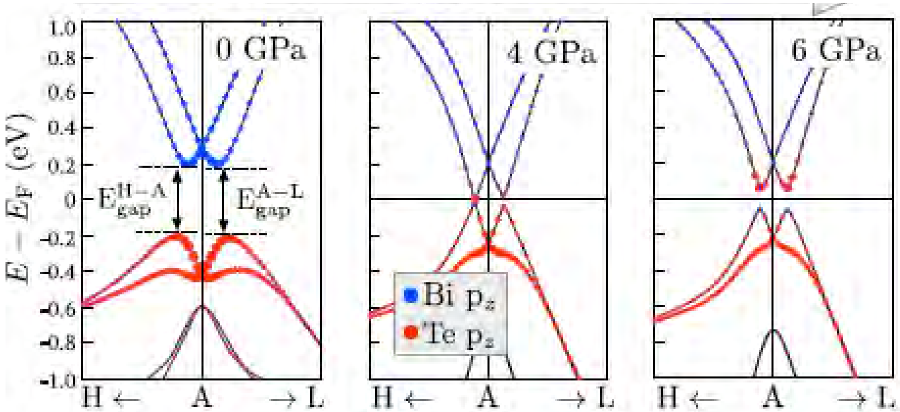


NI Weyl semimetal STI

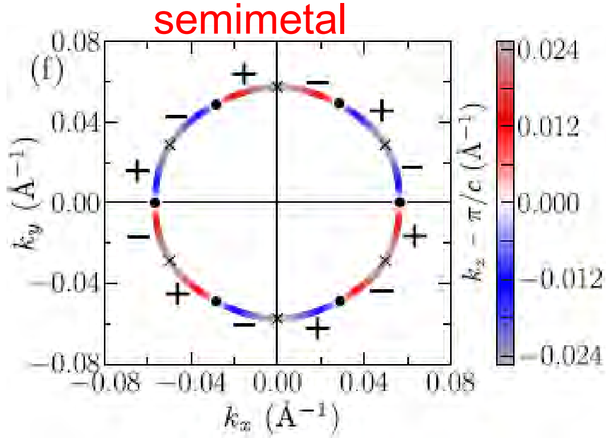
Gap closes at generic points
→This gap closing should be a pair creation of Weyl nodes.

BiTeI under pressure

Liu, Vanderbilt, PRB (2015)
Rusinov et al., NJP (2016)
See also: Bahramy et al., Nat. Commun. (2012)



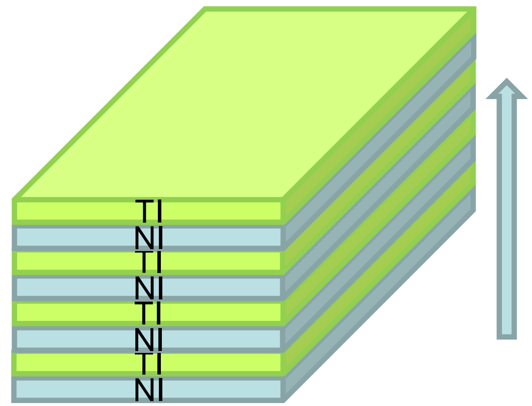
NI
Weyl semimetal
TI
p



Trajectory of Weyl nodes

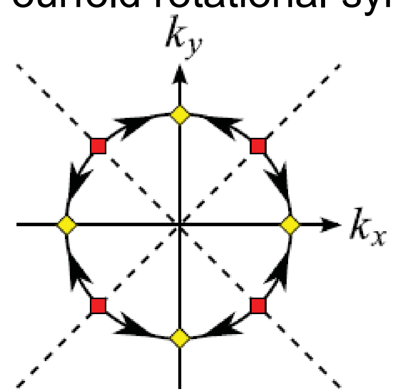
TI//NI superlattice without inversion sym.

Halasz, Balents, PRB (2012)



NI	Weyl semimetal	TI

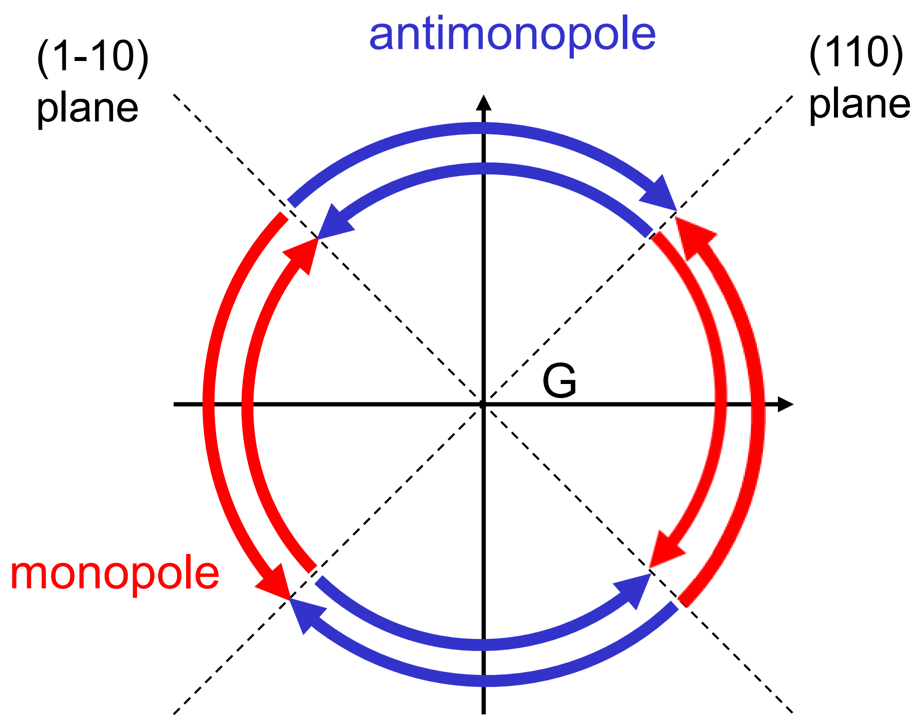
e.g.. Fourfold rotational symmetry



Trajectory of Weyl nodes

$HgTe_xS_{1-x}$ under [001] strain

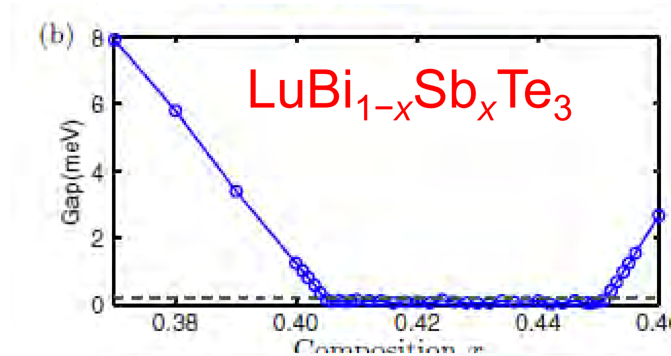
Rauch, Achilles, Henk, Mertig,,
Phys. Rev. Lett. 114, 236805 (2015).

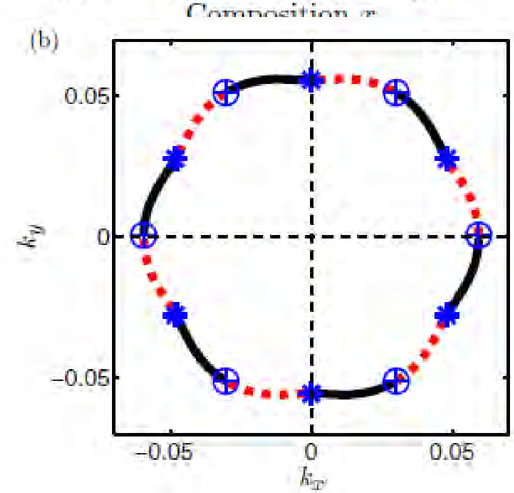


Trajectories of Weyl nodes in kz=0 plane

$LaBi_{1-x}Sb_xTe_3$, $LuBi_{1-x}Sb_xTe_{3s}$

Liu, Vanderbilt,
Phys. Rev. B **90**, 155316 (2014).





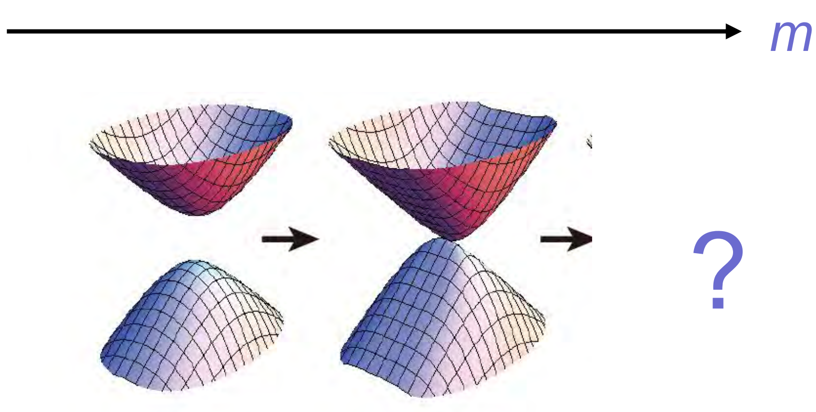
Trajectories of Weyl nodes
(within kx-ky plane)

Problem:

Start from any band insulator without inversion symmetry
(spinful + time-reversal symm.)

→ suppose a gap closes by changing a parameter *m*

What phase appears next?

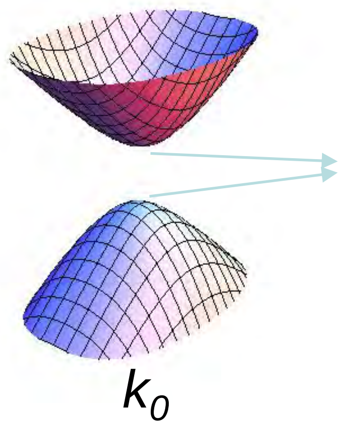


Classification
by space groups & *k*-points.

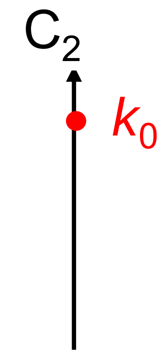
138 space groups
without inversion symm.

Murakami, Hirayama, Okugawa, Miyake,
Sci. Adv. 3, e1602680 (2017)

(Example #1): C_2 symmetry (i.e. k : invariant under C_2)



C_2 eigenvalue = +i or -i



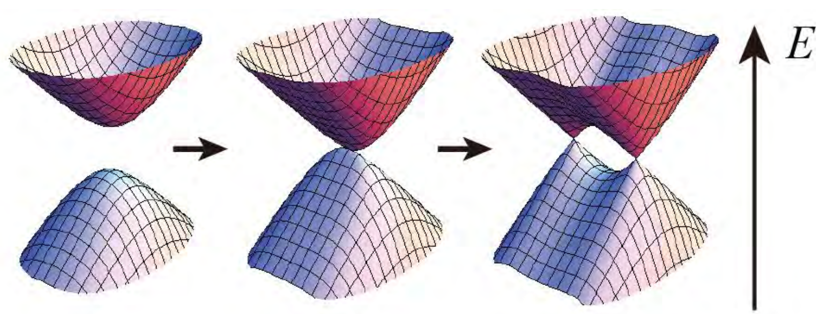
(i) <u>Same signs of C_2</u>

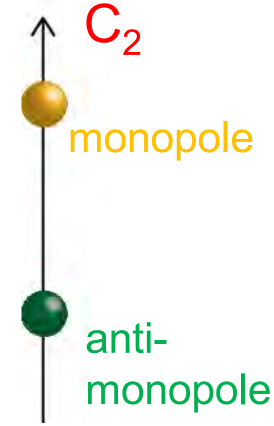
gap cannot close at k – level repulsion

(ii) <u>Different signs of C_2</u>

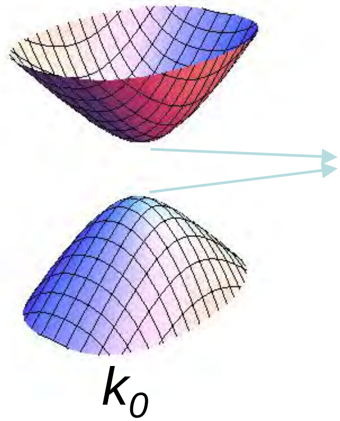
Weyl semimetal

Weyl nodes along C_2 line





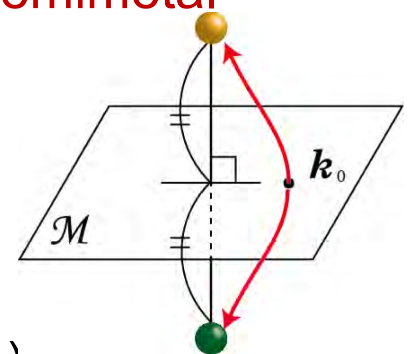
(Example #2): mirror symmetry (i.e. k : invariant under M)



M eigenvalue = +i or -i

(i) <u>Same signs of *M*</u>

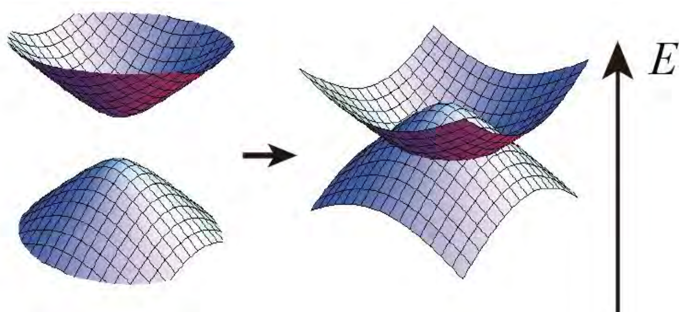
gap closes at k on the mirror plane → Weyl semimetal

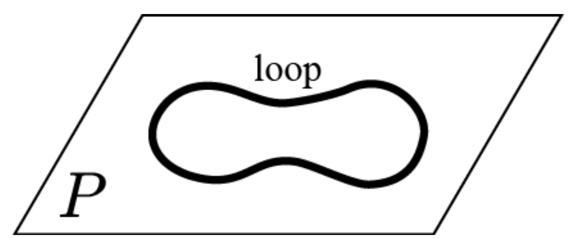


(ii) <u>Different signs of *M*</u>

nodal-line semimetal

(gap closing along a loop on a mirror plane)

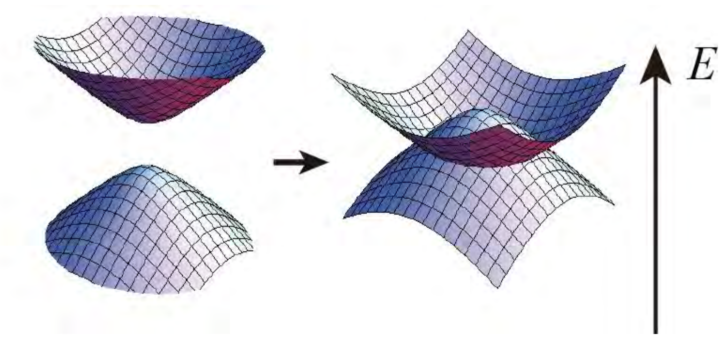


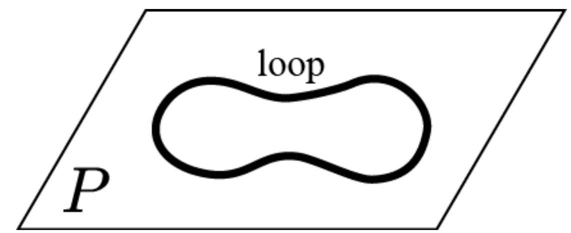


Semiconductors without inversion symmetry
→ Gap-closing always leads to topological semimetals

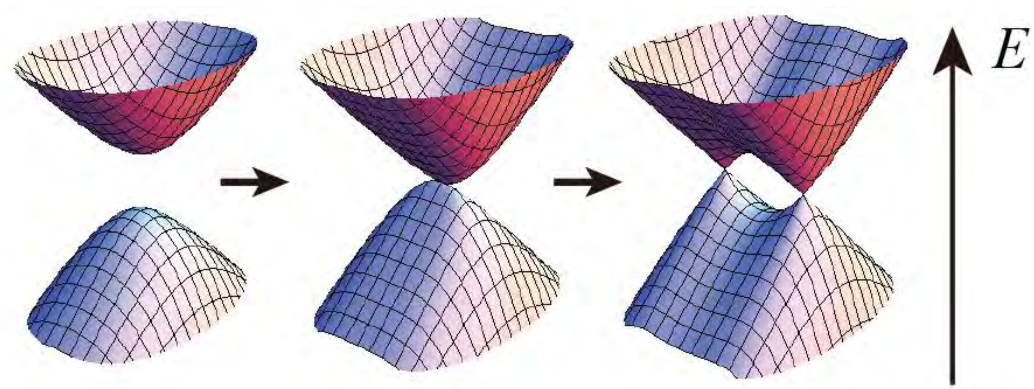
Murakami, Hirayama, Okugawa, Miyake, Sci. Adv. 3, e1602680 (2017)

(a) Nodal-line semimetal (← mirror plane)





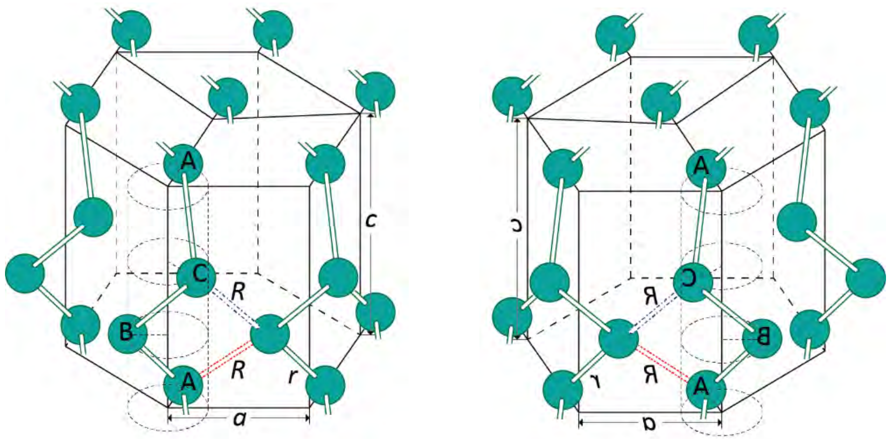
(b) Weyl semimetal



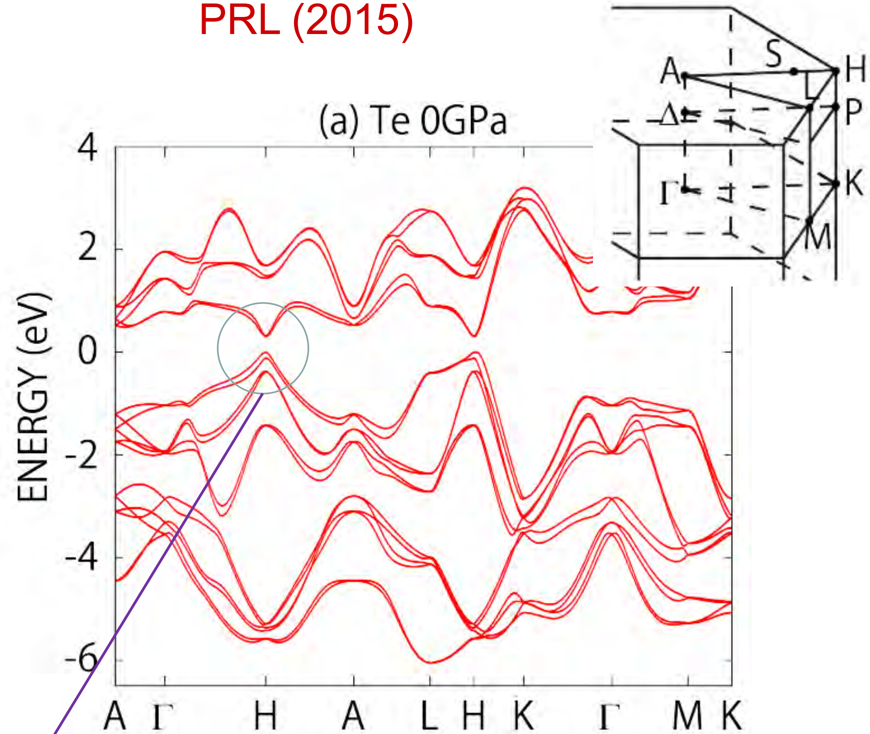
Only two possibilities. No insulator-to-insulator transition happens.
(in contrast to inversion symmetric systems)

Te : lattice with helical chains

M. Hirayama, R. Okugawa, S. Ishibashi,
S. Murakami, T. Miyake,
PRL (2015)



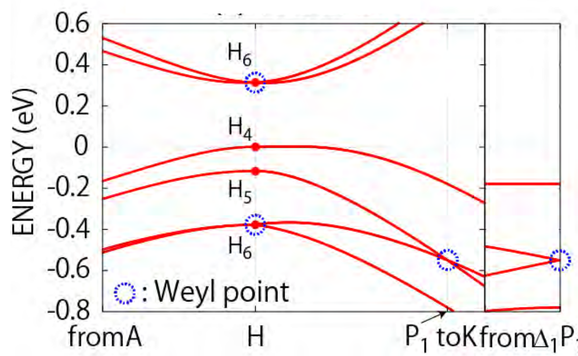
$P3_121$ $P3_221$



- Chiral lattice with helical chains
- No inversion symmetry
- No mirror symmetry

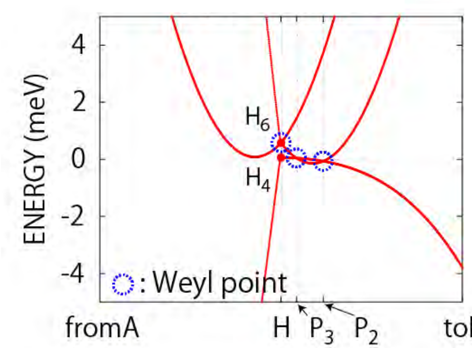
→ Allow Weyl nodes

0 GPa



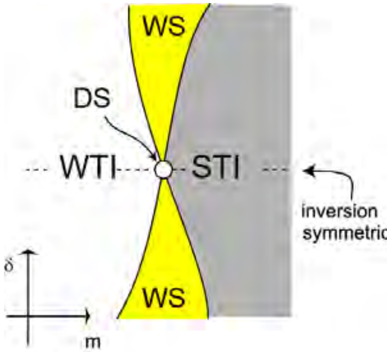
Insulator: gap=0.3eV

2.1GPa



Weyl semimetal

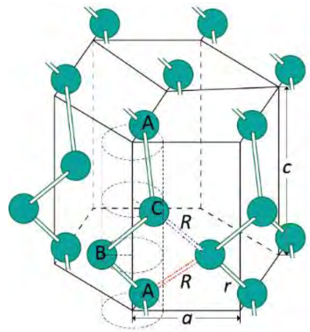
Conclusions

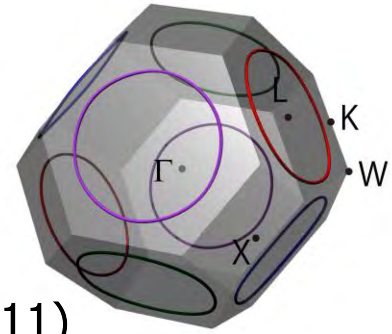


- Weyl semimetals (in inversion asymmetric systems)
  - Appear in TI-NI phase transition

    e.g. Tellurium: Weyl semimetal at high pressure

Murakami, NJP 9, 356 (2007)
Murakami, Kuga, PRB78, 165313 (2008)
Okugawa, Murakami, PRB 89, 235315 (2014)
Hirayama et al., PRL 114, 206401 (2015)
Murakami, Hirayama, Okugawa, Miyake,
Sci. Adv. 3, e1602680 (2017)





- Nodal lines in alkaline earth metals Ca, Sr, Yb
  - nodal lines if spin-orbit couping is neglected
  - large "polarization" for $k_{//}$ inside the nodal line
  - surface Rashba SOC is enhanced e.g. Bi/Sr(111), Bi/Ag(111)

Hirayama et al., Nat. Commun.8, 14022 (2017)