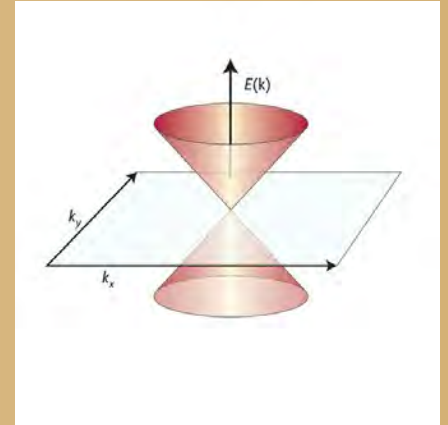


# Weyl physics in superconducting junctions

Yuli V. Nazarov  
TU Delft

Roman Riwar, Manuel Houzet,  
Julia Meyer, Tomohiro Yokoyama,  
Evgeny Repin, Yungang Chen, Johannes Reutlinger



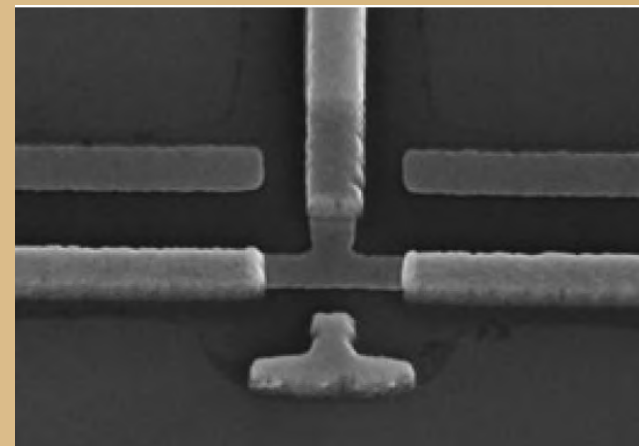
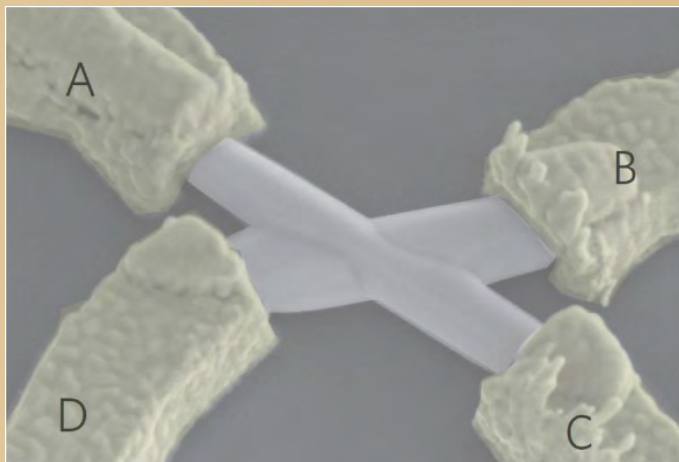
# Outline

- MultiTerminal Superconducting Junctions
- MTSJ as (topological) material
- Weyl singularities in 4-terminal junction
- From 3D to 2D –flexible dimensionality
- Transconductance quantization
- Vicinity of the singularity
  - Spin-orbit – spintronics!
  - Tricky interaction effect: from cones to pancakes

# Multi-terminal superconducting junctions

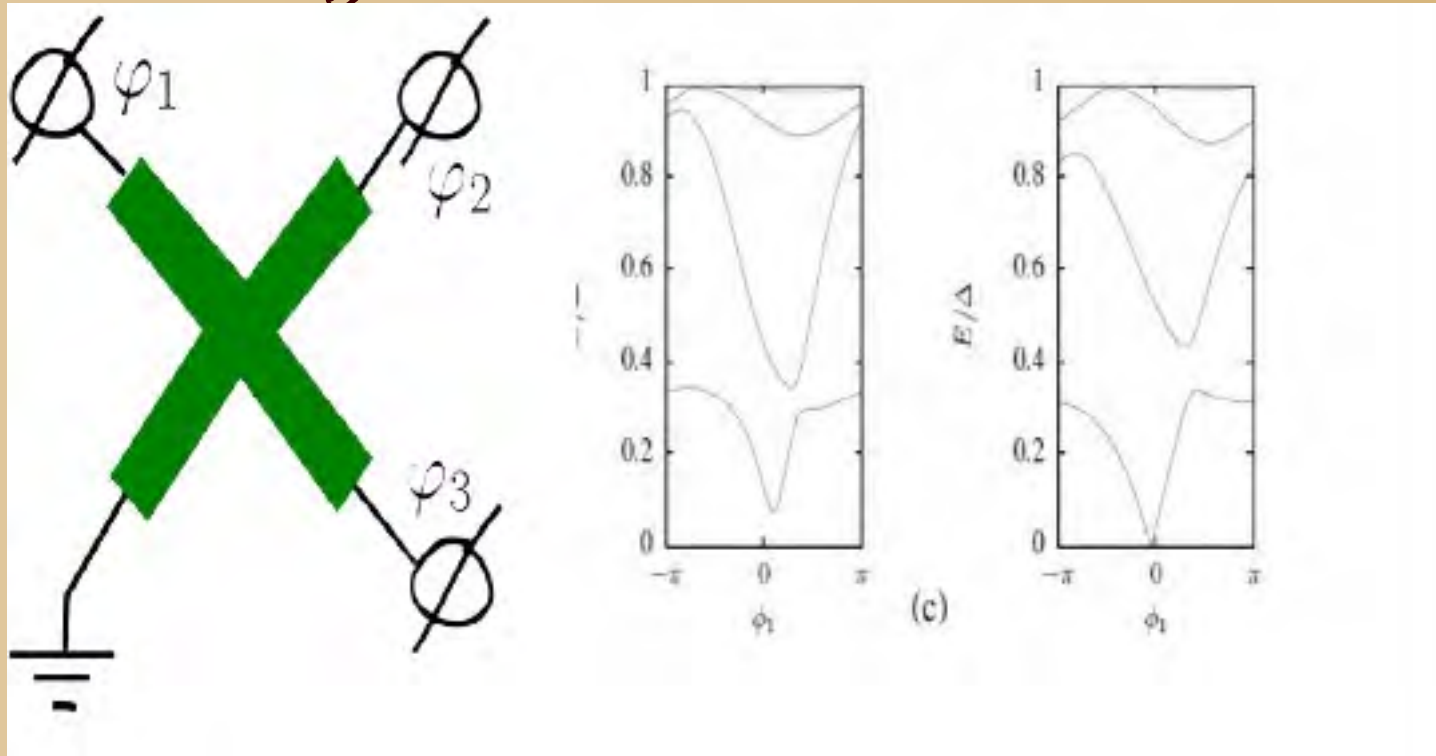
- Josephson junction( 2 terminals)  $E = -E_J \cos \varphi$
- More transparent – Andreev bound states
- $E = -\frac{1}{2} \sum_p E_p$     $E_p = \Delta \sqrt{1 - T_p \sin(\varphi/2)^2}$
- More terminals – more superconducting phases
- same Andreev states

$$E_p(\phi_1, \dots, \phi_{n-1})$$



# The analogy: junction – material

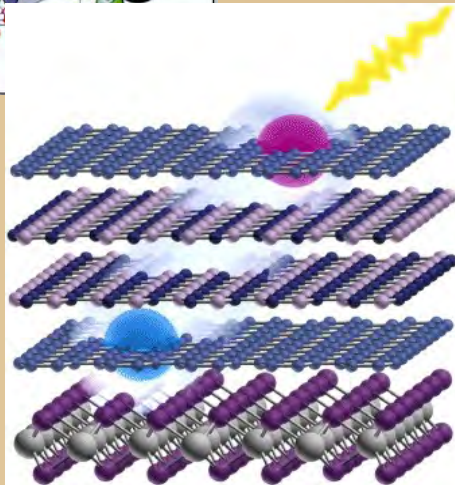
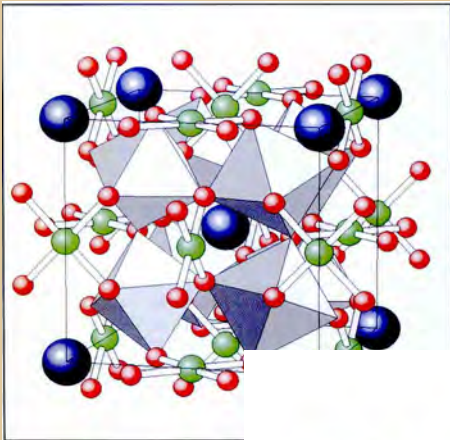
$$Q_x, Q_y, Q_z \leftrightarrow \varphi_1, \varphi_2, \varphi_3$$



- Incomplete
  - Gap edge, continuous spectrum
  - Filling: all quasimomenta, phases - one

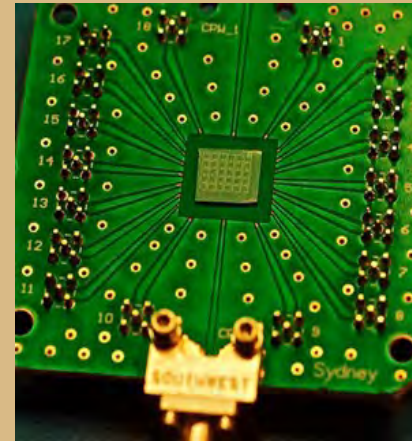
# Breakthrough to higher dimensions

material



material :  
say, 5 d

device

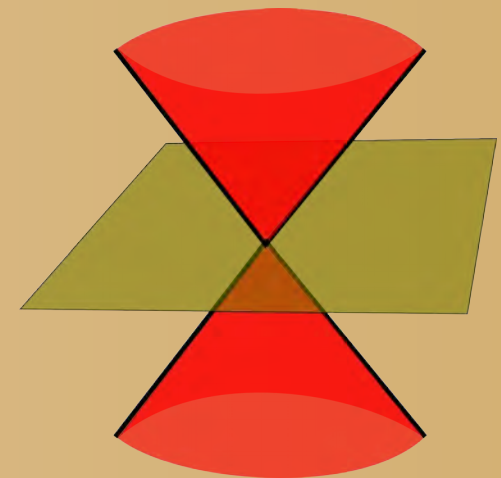
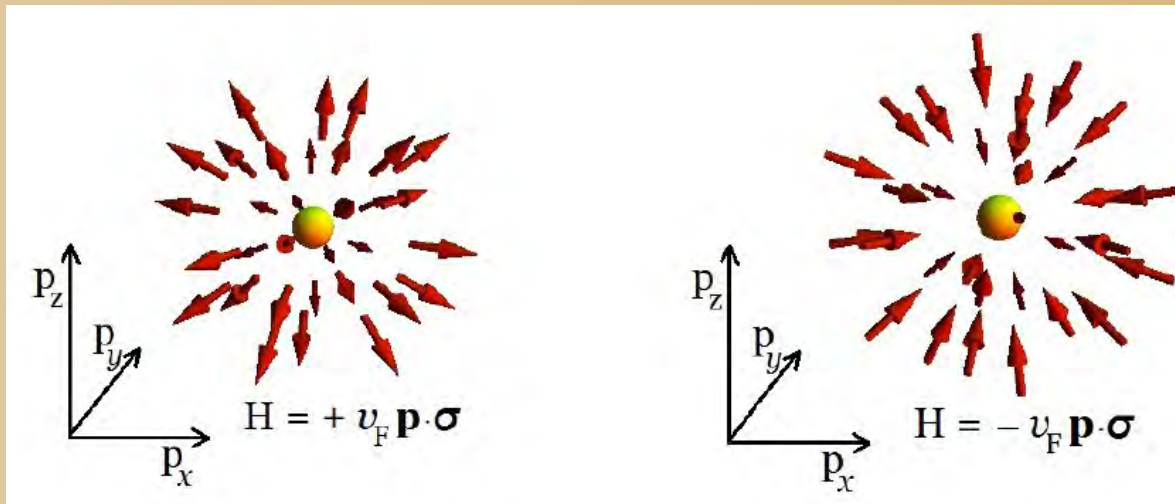


Nanodevice  
(multi-  
terminal  
sup.jun)



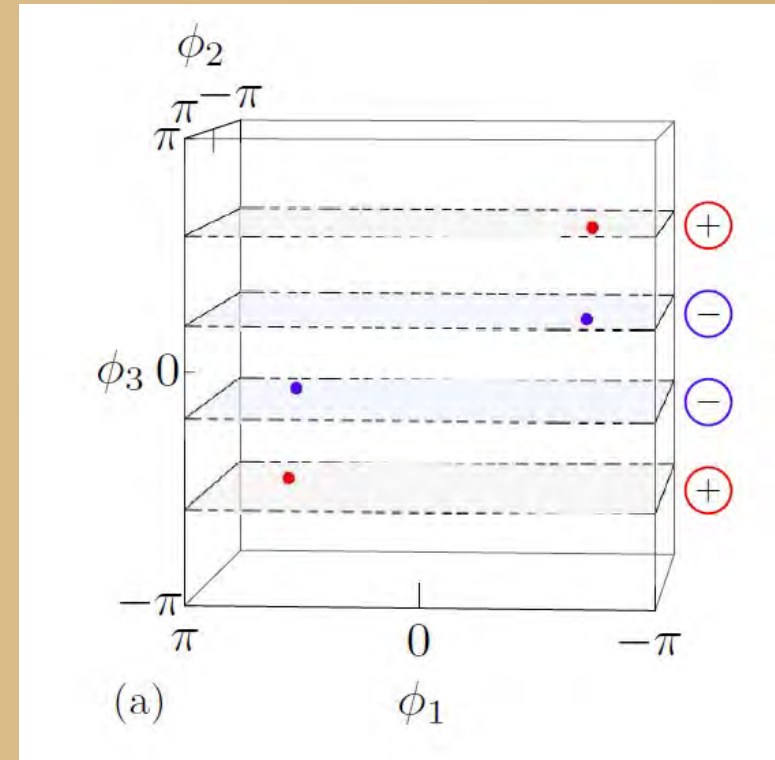
# Weyl singularity

- Massless particle with conical spectrum
- A missed paragraph in Landau and Lifshitz: Levels or bands do cross in 3-D parametric space
- In the vicinity of the crossing point:  $\hat{H} = v_F \vec{\tau} \cdot \vec{p}$
- Occurrence in materials – **recently proved**
- **A monopole of the Berry curvature field -topstable**

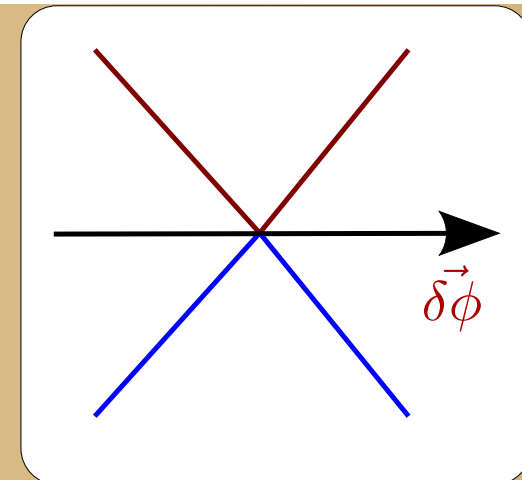


# Weyl singularity in 4-terminal superjunctions

- At zero energy – important
- Three fields – three parameters – three phases – one-goal game?
- Beenakker formula – from scatt.matrix
- Come in pairs of 4
- (+ - charges and inversion (time-reversal) symmetry)



$$\hat{H} = I\vec{\tau} \cdot \delta\vec{\phi}$$

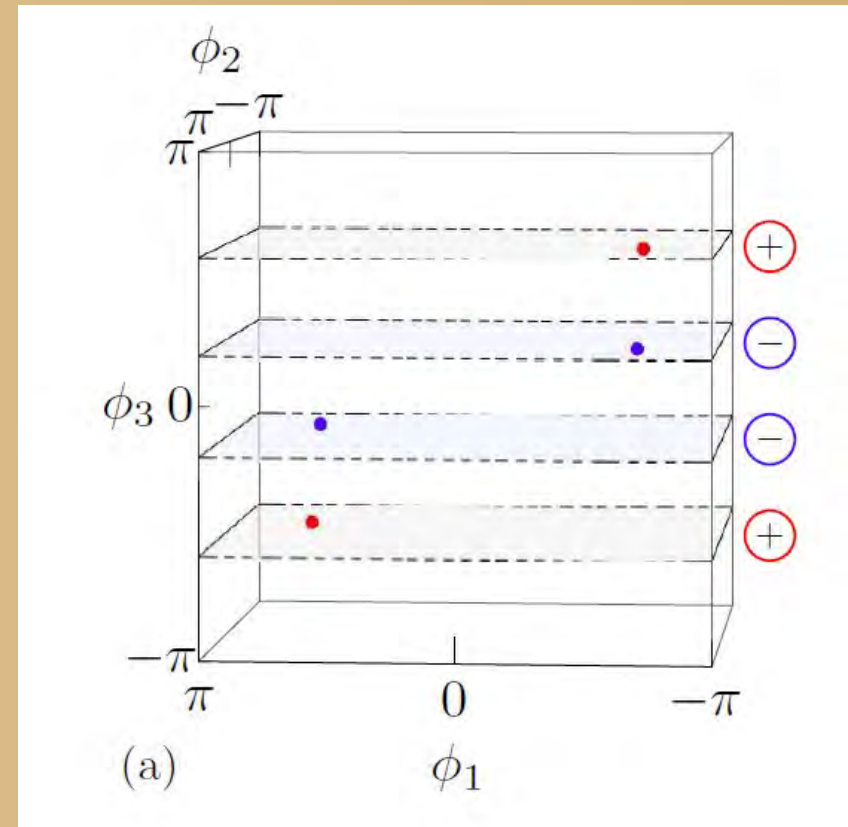


# From 3D to 2D

- Flexibility of dimensions
- 2D bandstructure – a moving plane

- $\varphi_3$  – control parameter
- Plane passes a W.S =
- Change of Chern number
- Topologically non-trivial  
2D material

- Tunable by  $\varphi_3$





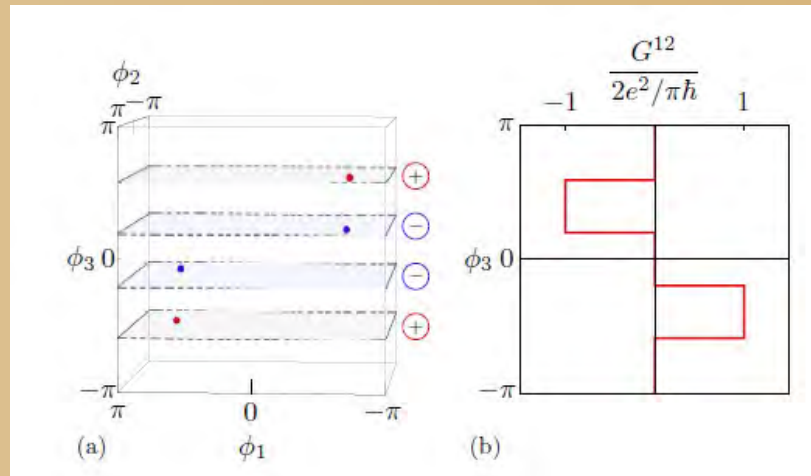
# Transconductance quantization

- Berry curvature and ad.transport
- Sensitive to the local Berry curvature
- Apply (incommensurate) voltages
- Phases are swept over BZ
- Sup.current vanishes
- What remains?

$$I_{\alpha}(t) = \frac{2e}{\hbar} \frac{\partial E}{\partial \phi_{\alpha}} - 2e \dot{\phi}_{\beta} B^{\alpha\beta}$$

Leading order

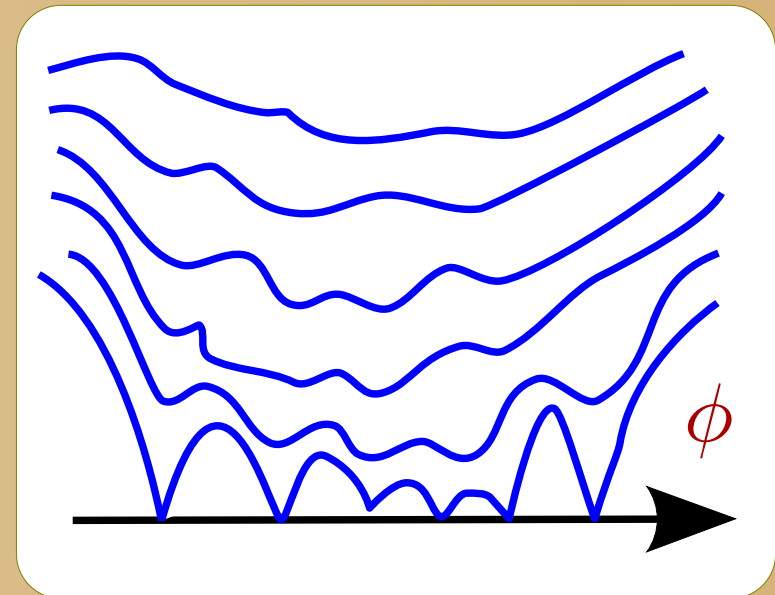
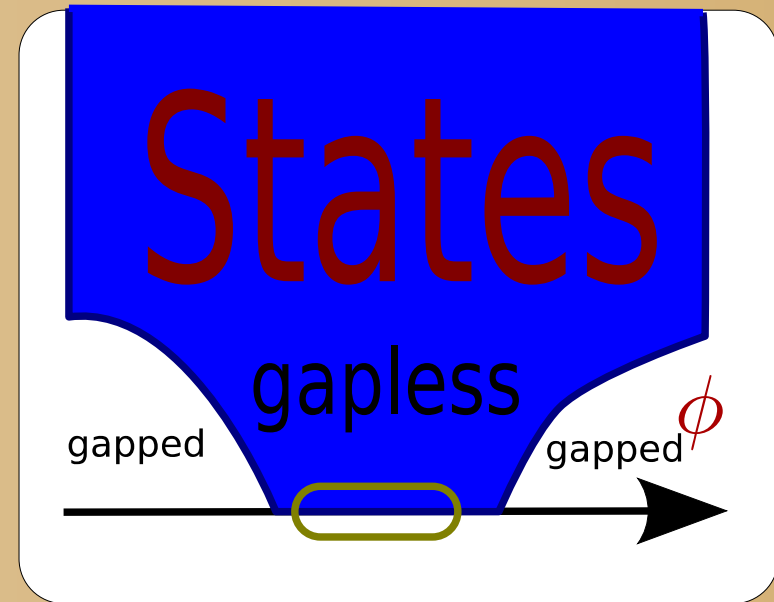
First correction



$$I_1 = G_{12} V_2; \quad I_2 = -G_{12} V_1; \quad G_{12} \equiv (2e^2 / \pi \hbar) C$$

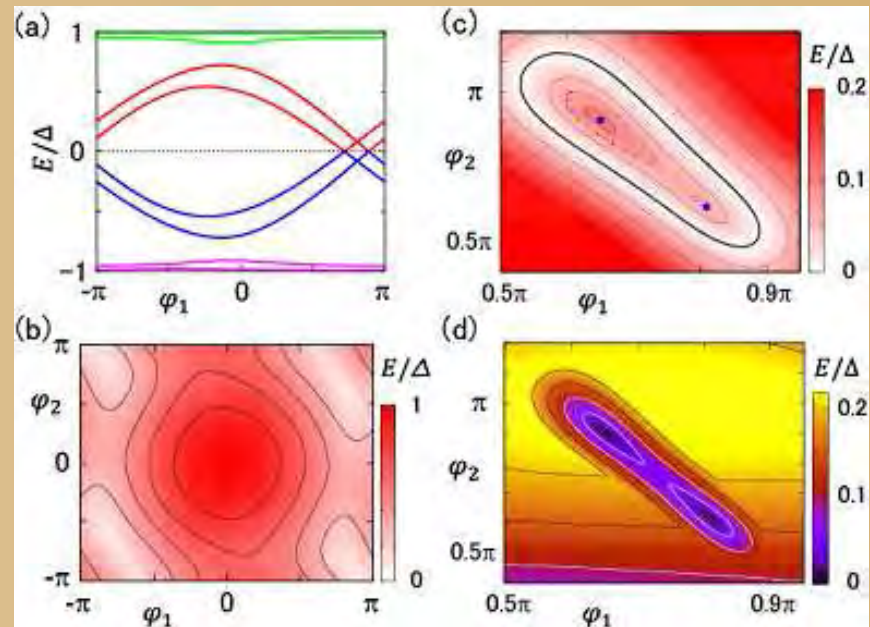
# Semiclassical structures

- Many channels
- Quasi-continuous spectrum
- Gapped and Gapless phases
- Gapless – a vault of Weyl singularities



# In the vicinity

- Spin-orbit and spintronics
- interaction

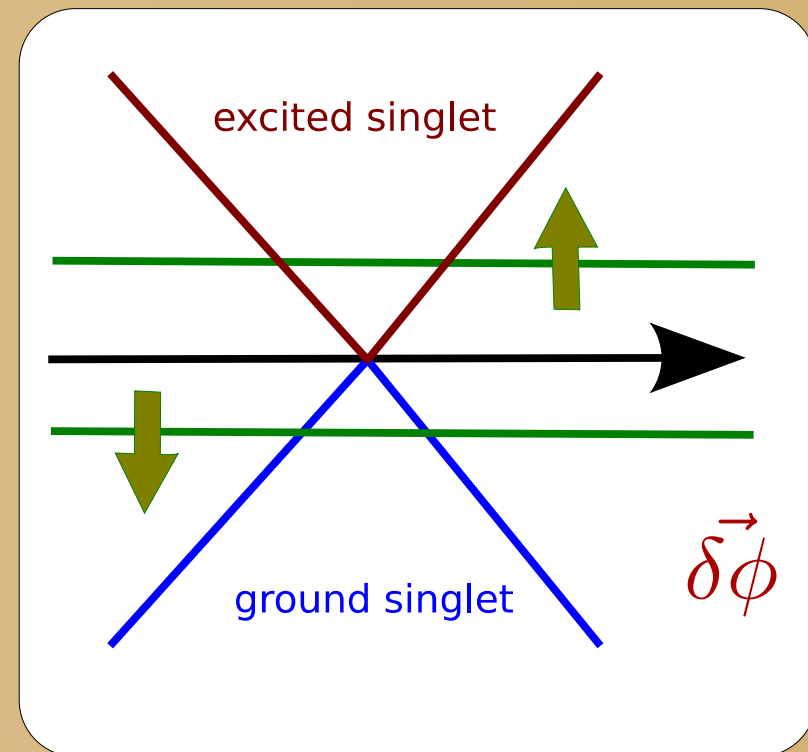
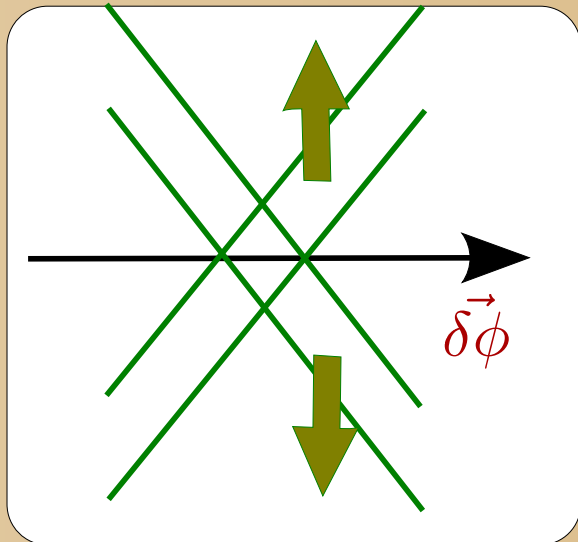


# SO = spin splitting

- Since there is no time reversibility,

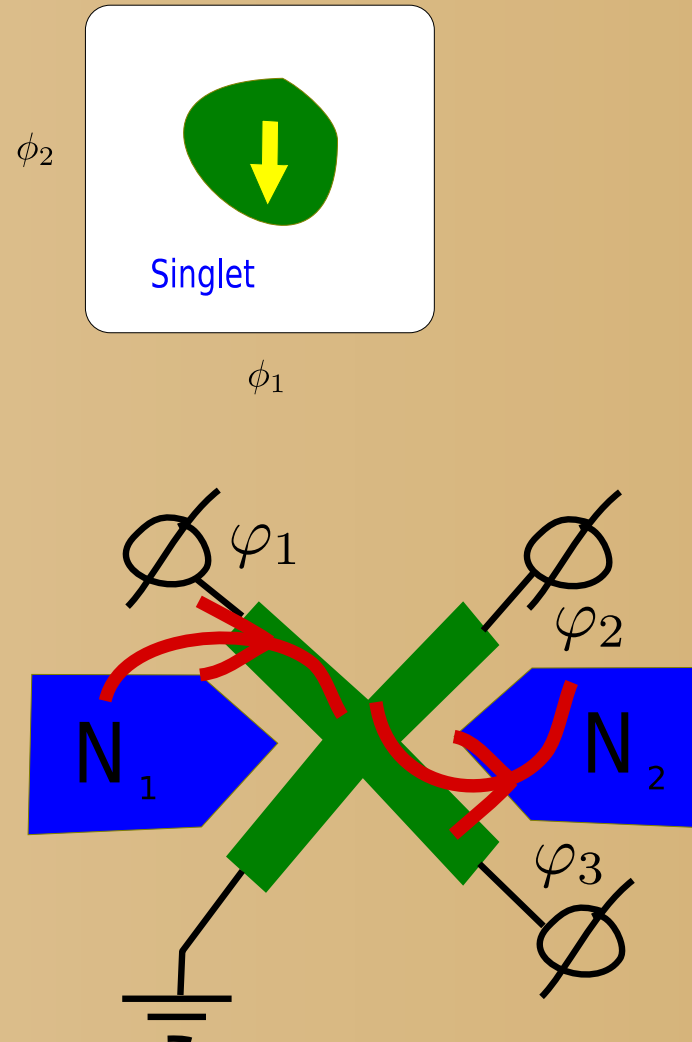
$$\hat{H} = I\vec{\tau} \cdot \vec{\delta\phi} + \vec{\sigma} \cdot \vec{B}$$

- Spin-split cone or flat spinful states
- Weyl singularity departs(?) from zero energy



# Spintronics

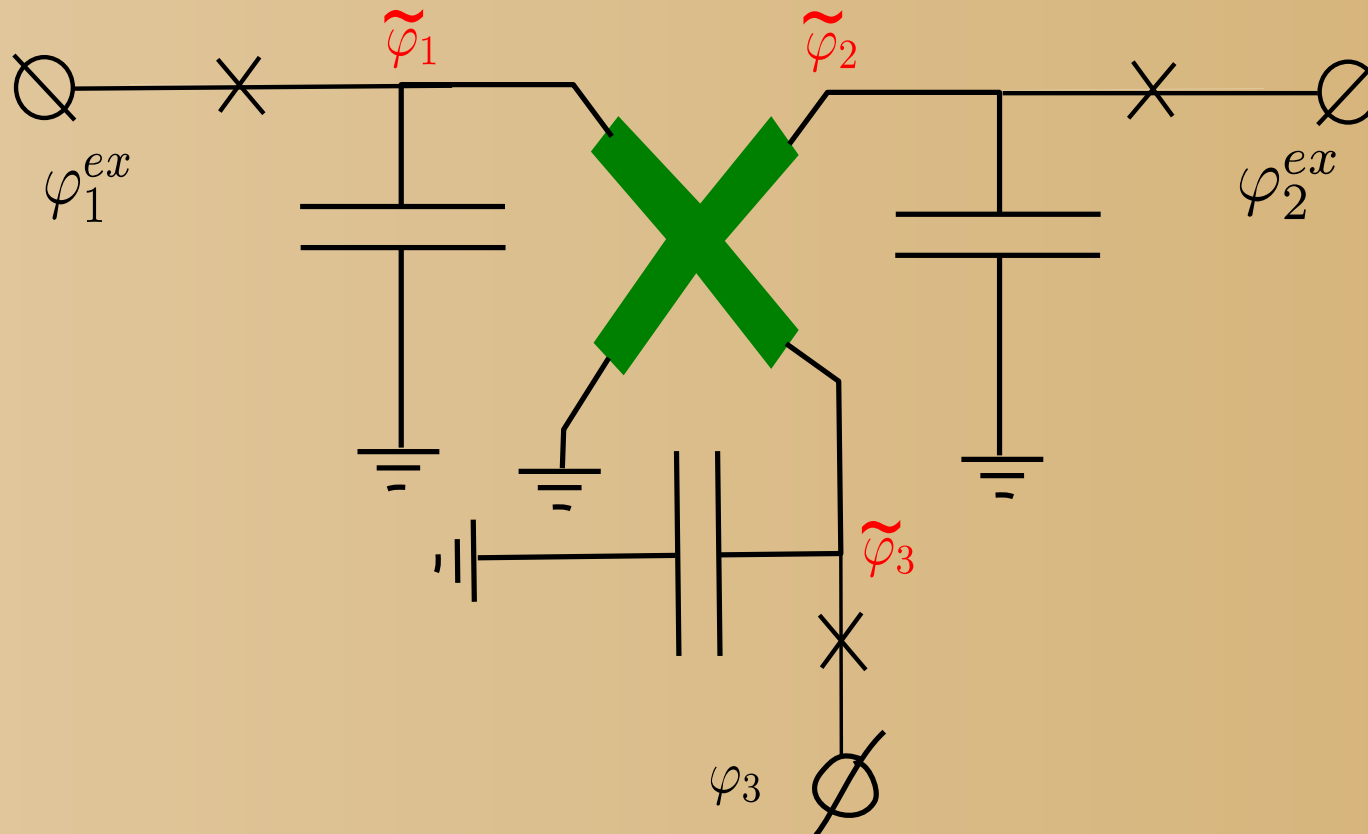
- Magnetic state in the vicinity
- Normal lead is needed for equilibration
- With 2: spin filter driven by tiny phase differences



# Interaction? Interaction!

- Deviations and fluctuations of the phases

$$\hat{H} = \sum_k E_k^J \frac{(\varphi_k - \tilde{\varphi}_k)^2}{2} + E_k^C \frac{\tilde{q}_k^2}{2}$$

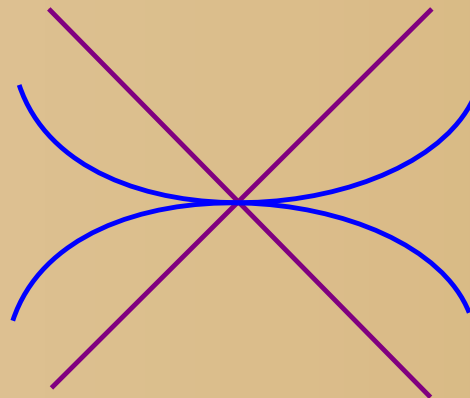
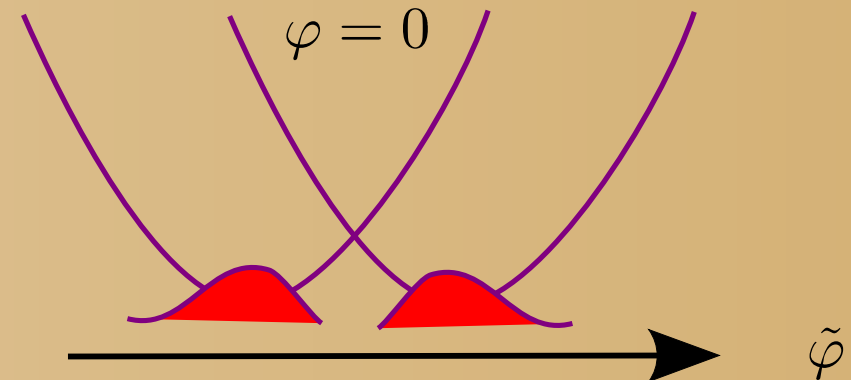
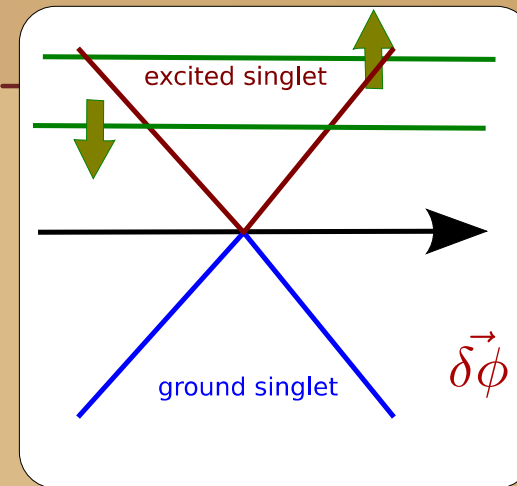




# What does the interaction do?

- Conical point survives :)
- Even states are favoured
- Small quantum fluctuations
  - Strongly anisotropic cone
  - 2D region of almost degenerate levels:

**pancake**



# Conclusions

- Weyl physics – in MSJ
  - Higher (flexible) dimensions
  - Superconducting QHE
- In the vicinity of the singularity
  - Spintronics
  - pancake