



Fingerprints of fractionalized excitations in scattering probes of quantum spin liquids

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Spin Dynamics in the DIRAC Systems, SPICE
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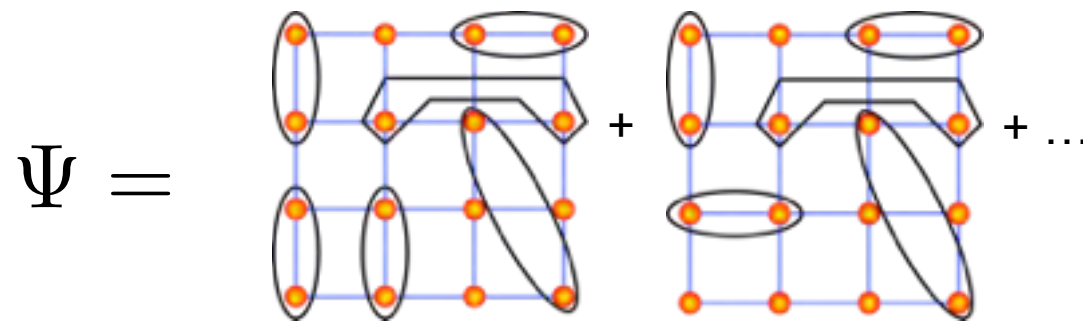


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Quantum spin liquids

QSL: State of interacting spins that breaks no rotational or translational symmetry and has only short range spin correlations.

1973: Anderson proposes the “Resonating Valence Bond” state - a prototype of the modern QSLs



Unlike states with broken symmetry, QSLs are not characterized by any local order parameter.

QSLs are characterized by topological order and long range entanglement (difficult to probe experimentally).

QSLs supports excitations with *fractional quantum numbers and statistics*.

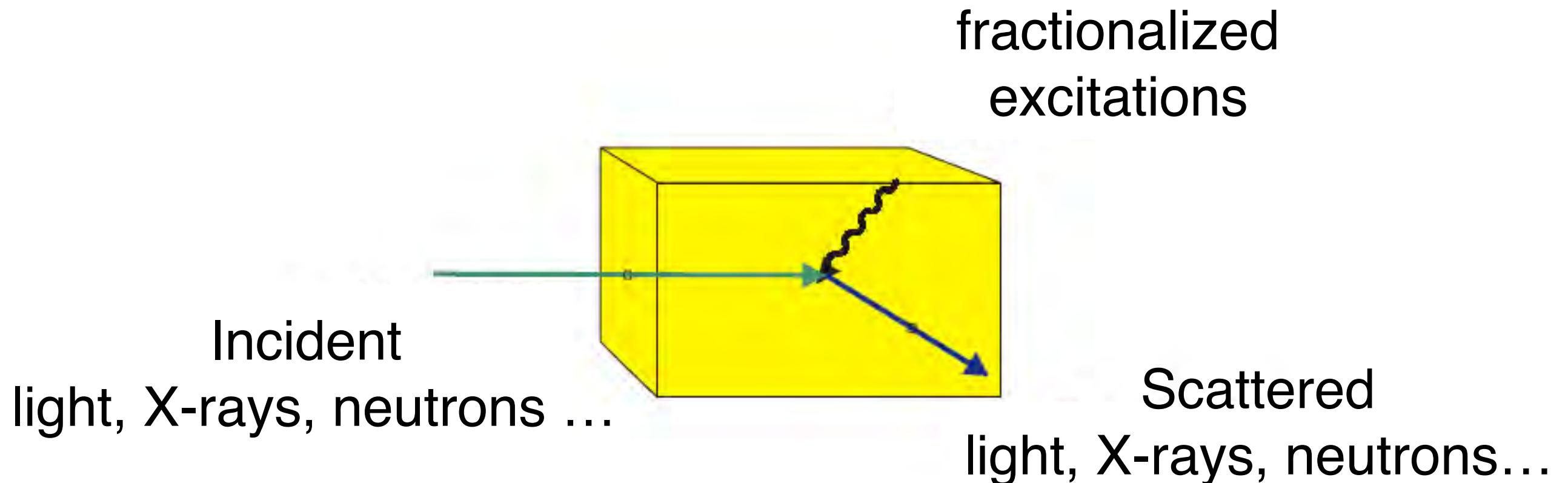
Quantum spin liquids

Main Question:

How to probe fractionalized quasiparticles in QSL and their statistics?

Take home message:

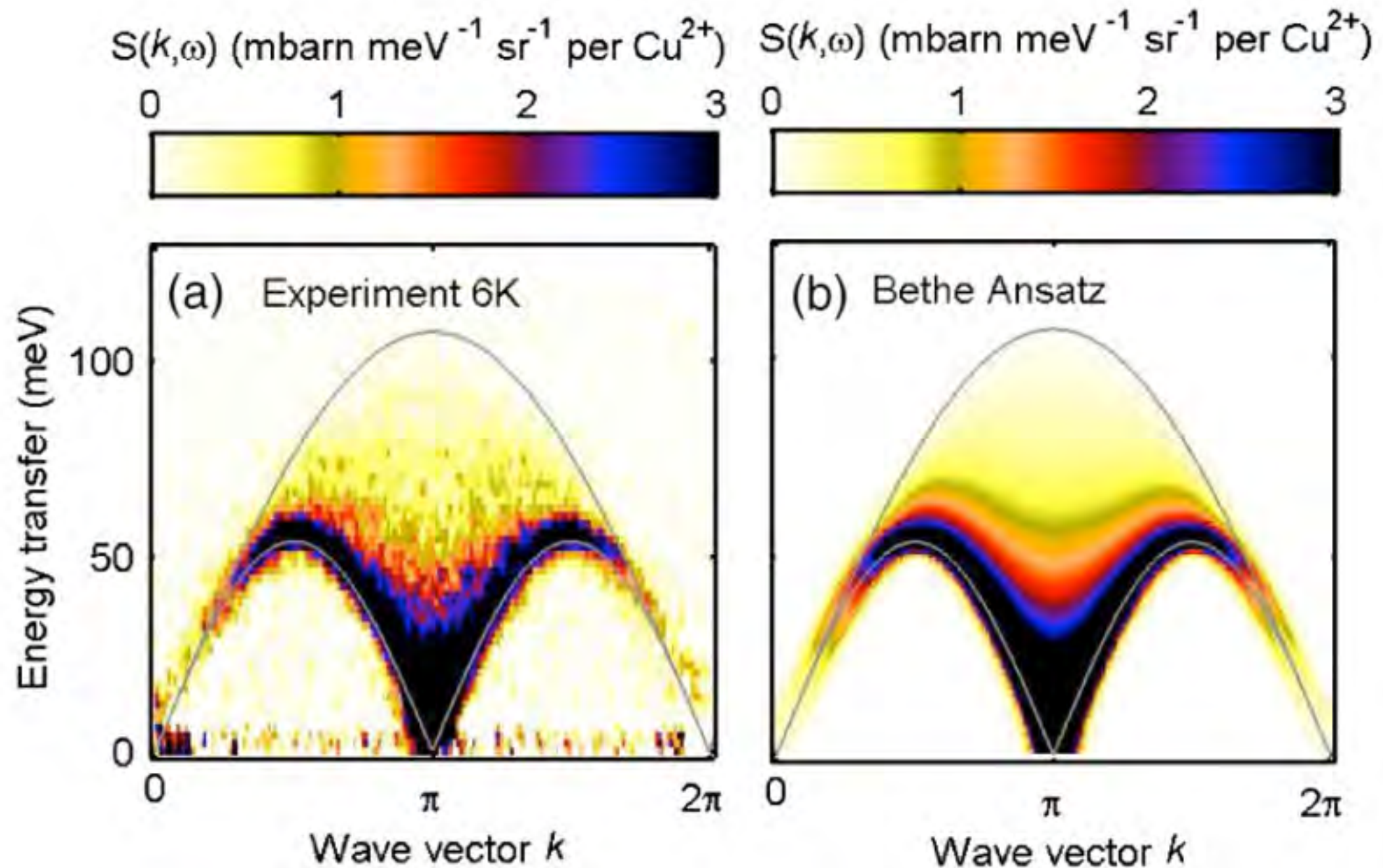
Signatures of quantum order are mainly in the excitations



Since excitations carry fractional quantum numbers relative to the local degrees of freedom, only *multiple quasiparticles* can couple to external probes: **Response from QSL is always a multi-particle continuum.**



Spinon excitations probed by neutrons: KCuF_3



The fractionalization was definitively identified by excellent **quantitative** agreement between experiments and exact calculation based on the Bethe Ansatz.

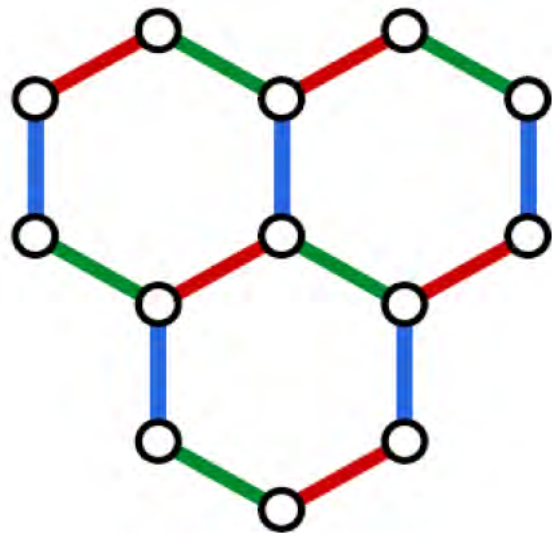
Probing continua of excitations in Kitaev spin liquids

- Spin liquid ground state and fractionalized excitations in 2D and 3D Kitaev models
- Raman response in 2D & 3D Kitaev model
- RIXS response in 2D & 3D Kitaev models
- Conclusions



Kitaev model on the honeycomb lattice

$$H = - \sum_{x\text{-bonds}} J_x \sigma_j^x \sigma_k^x - \sum_{y\text{-bonds}} J_y \sigma_j^y \sigma_k^y - \sum_{z\text{-bonds}} J_z \sigma_j^z \sigma_k^z$$



$$\begin{array}{ll} \sigma^x \sigma^x & \text{red line} \\ \sigma^y \sigma^y & \text{green line} \\ \sigma^z \sigma^z & \text{blue line} \end{array}$$

Exactly solvable 2D model

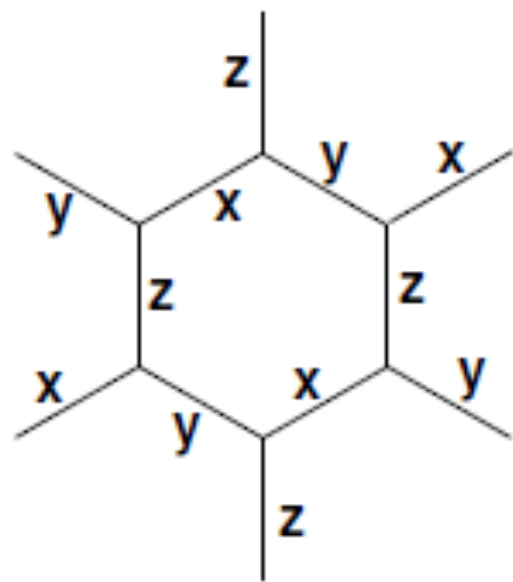
Spin liquid ground state

Fractionalized excitation

Mapping spins to Majorana fermions:

$$\sigma_i^a = i c_i c_i^a, \quad a = x, y, z$$

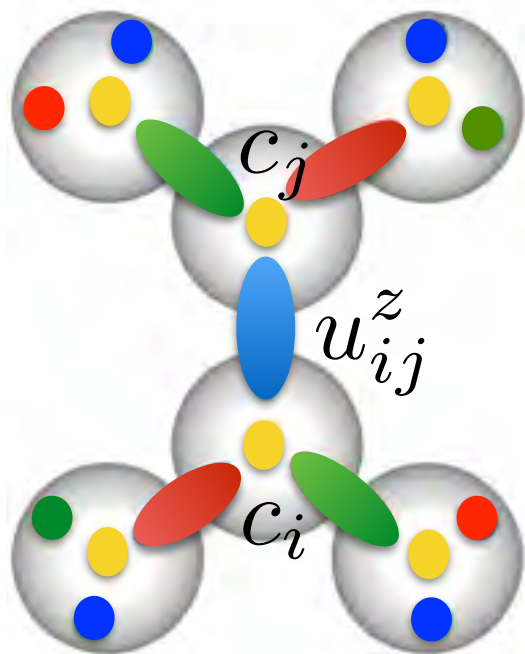
Spin fractionalization and Majorana fermions



Large number of conserved quantities,
local plaquette operators:

$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

$$[H, W_p] = 0$$



Quadratic Hamiltonian in each flux sector:

$$H = - \sum_{a=x,y,z} J_a \sum_{\langle ij \rangle_a} i c_i \hat{u}_{\langle ij \rangle_a} c_j$$

$$\hat{u}_{\langle ij \rangle_a} \equiv i c_i^a c_j^a$$

Excitations in the 2D Kitaev spin liquid

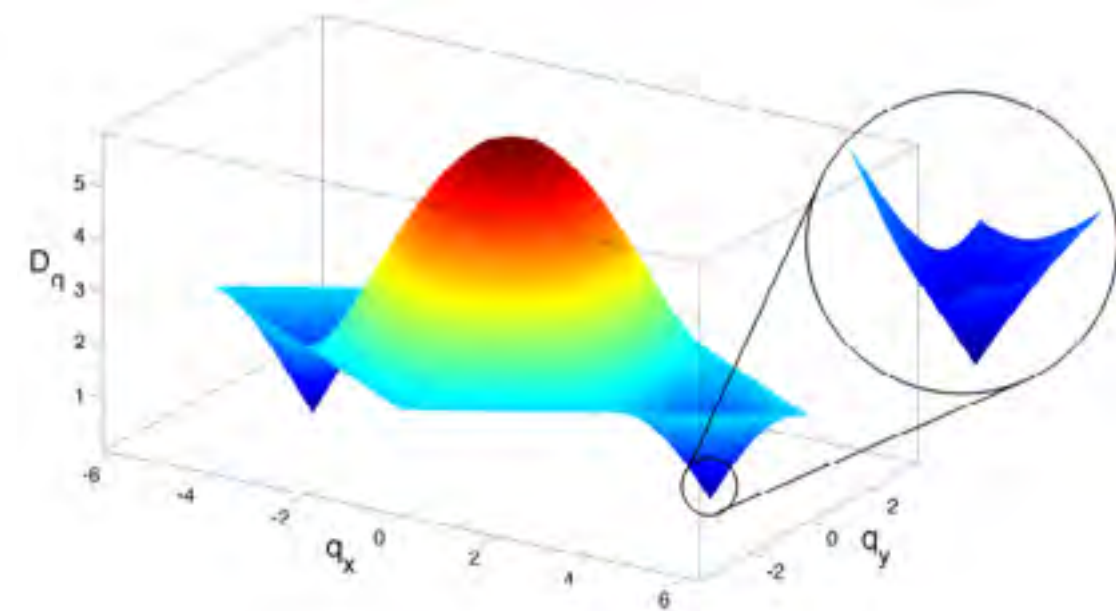
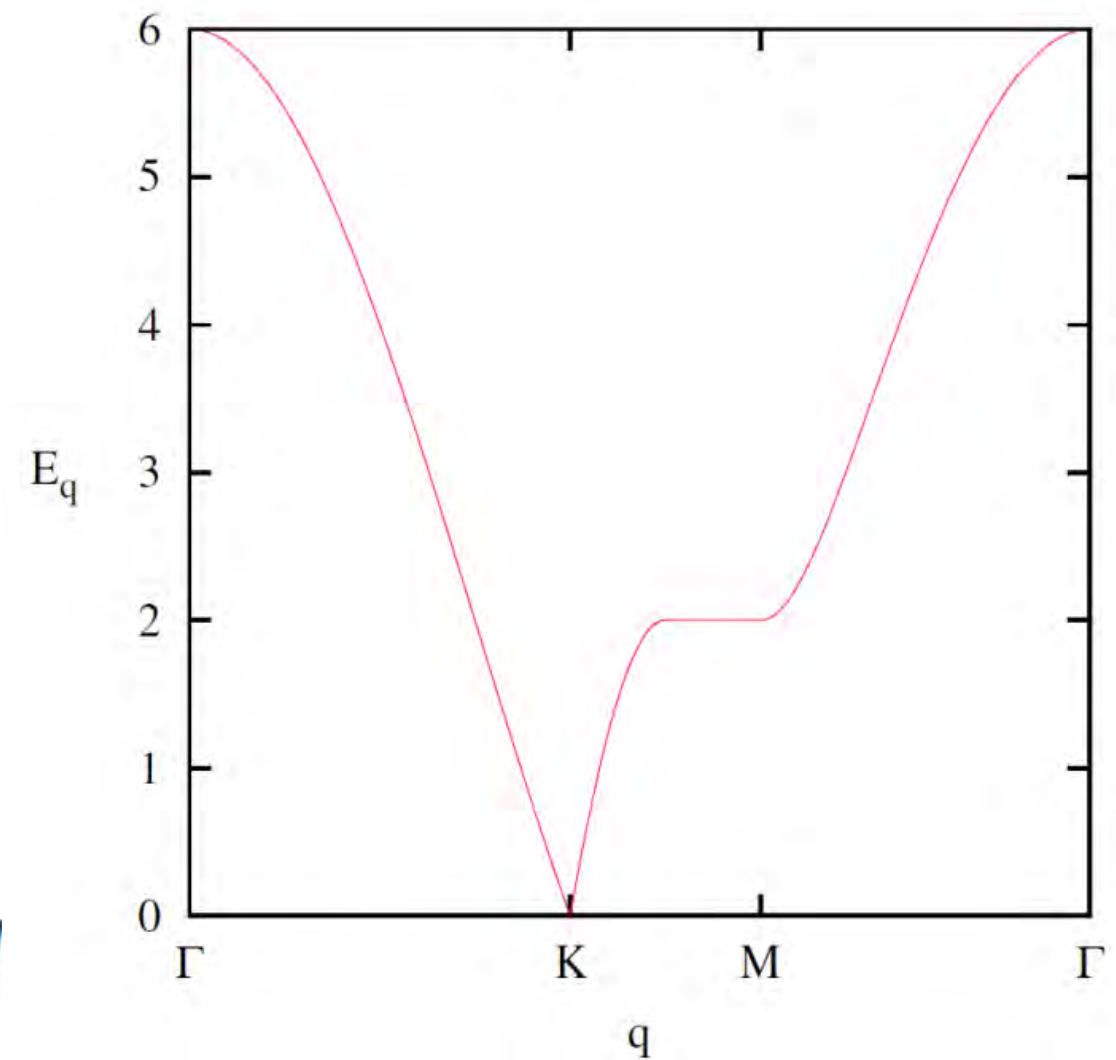
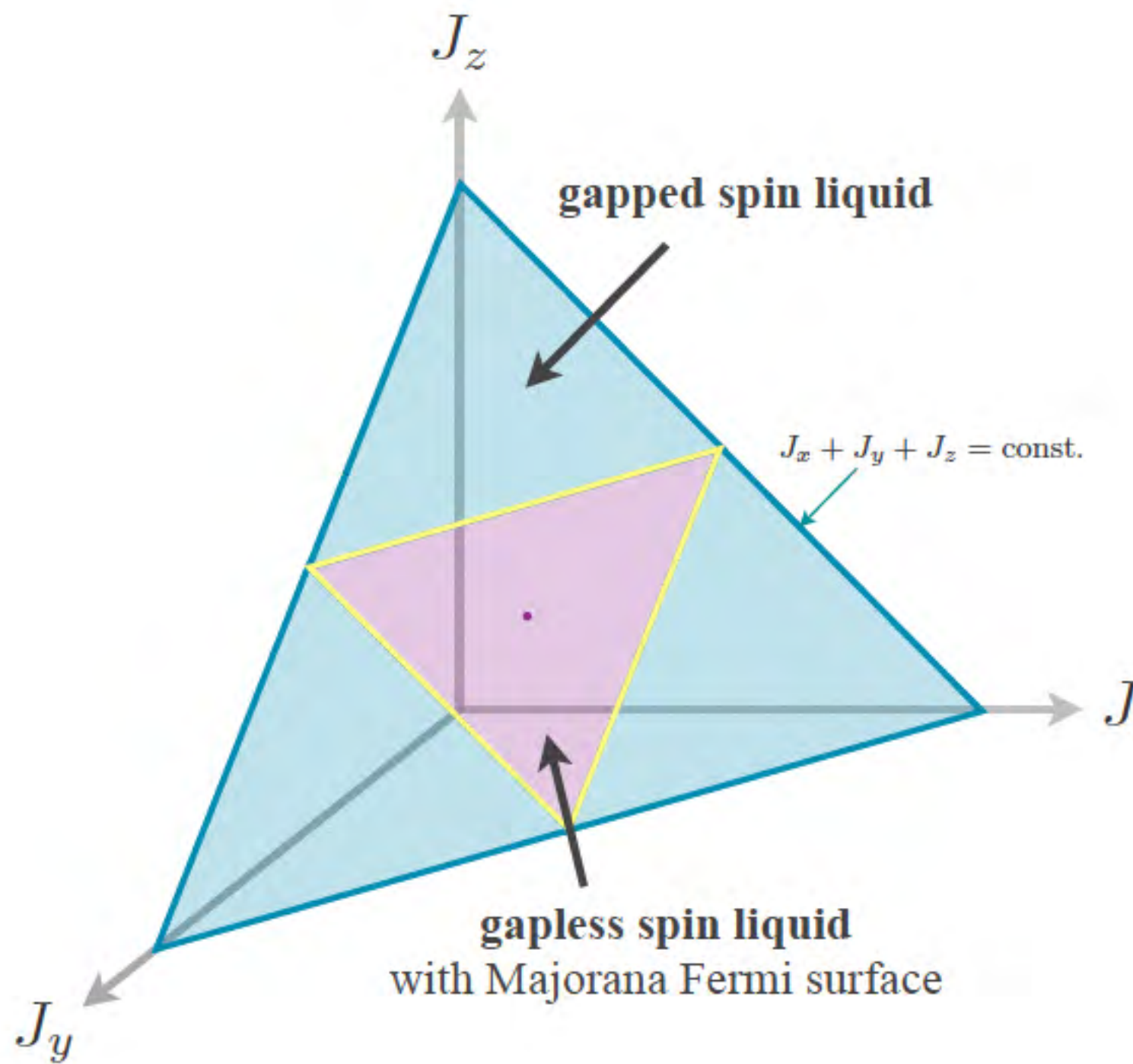
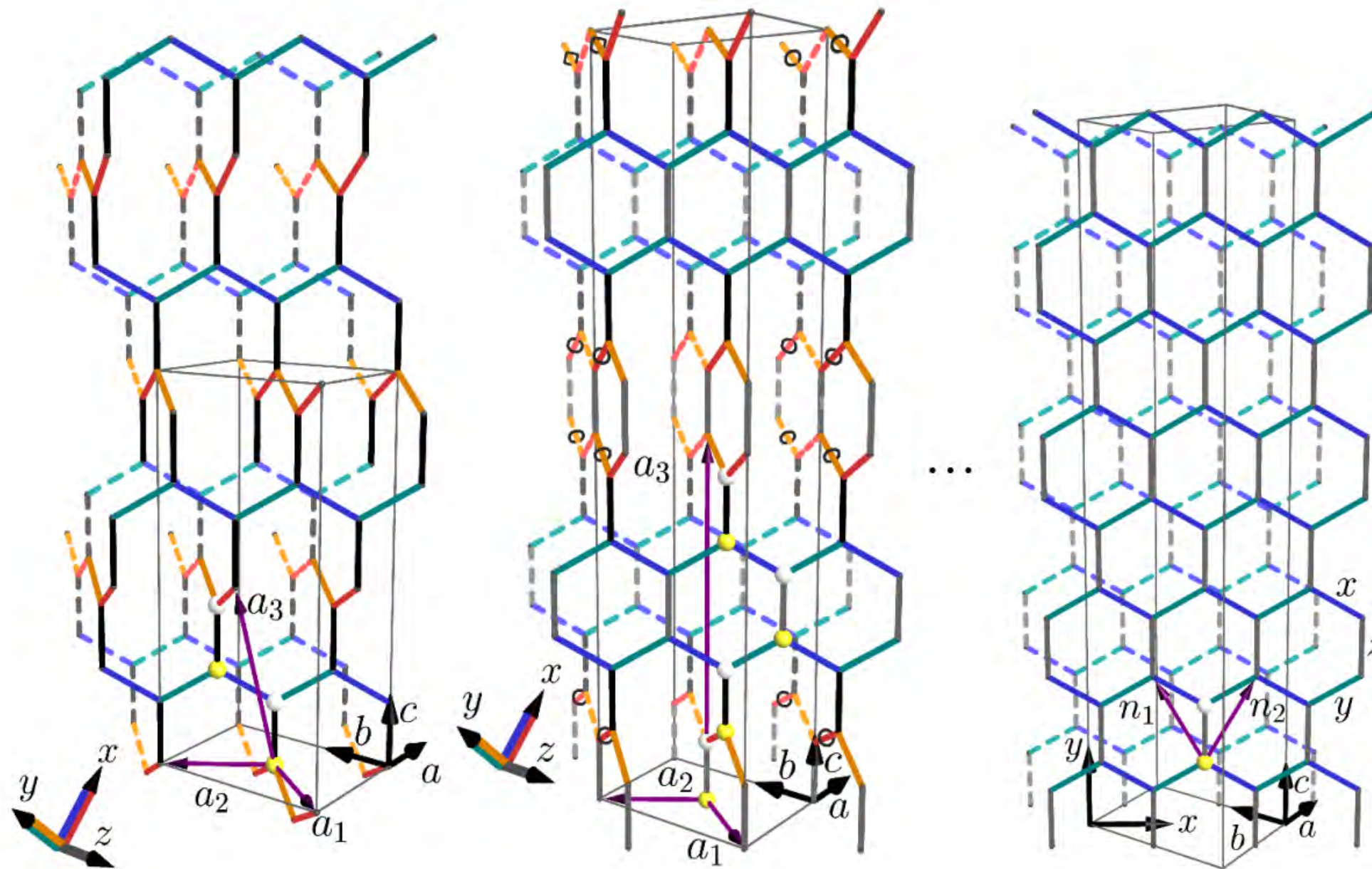


Fig. from M. Hermanns et al, 2014

3D Kitaev family



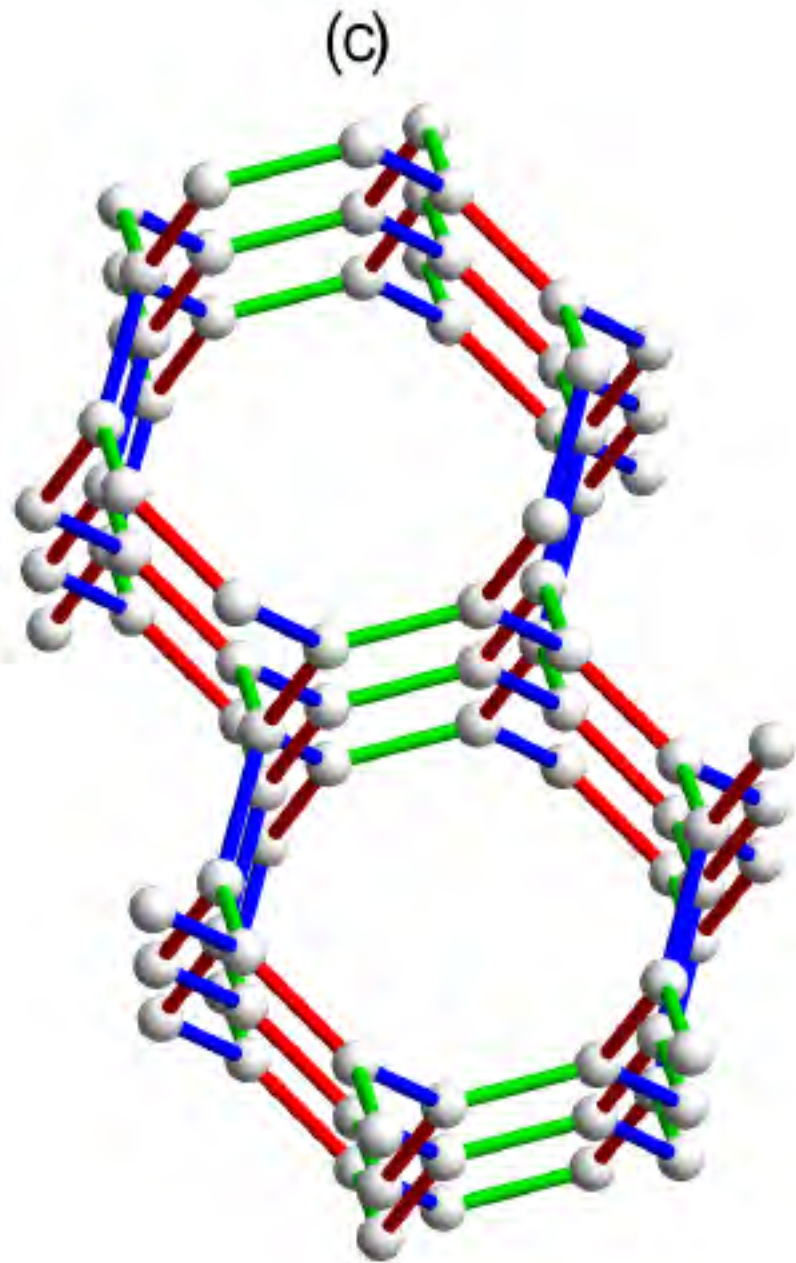
$$H_{<0>}$$

$$H_{<1>}$$

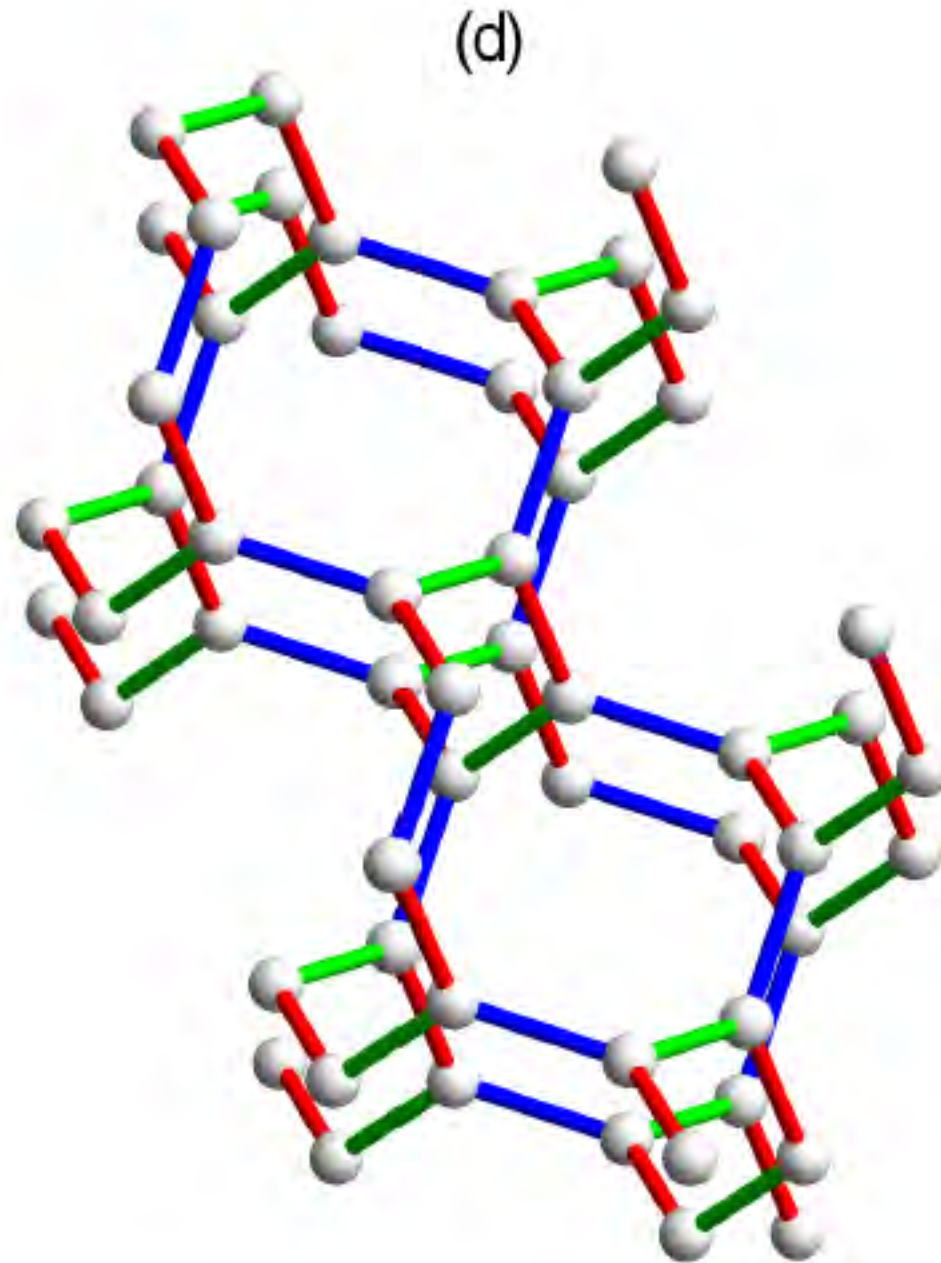
$$H_{<\infty>}$$

Hyperhoneycomb lattices

3D Kitaev family

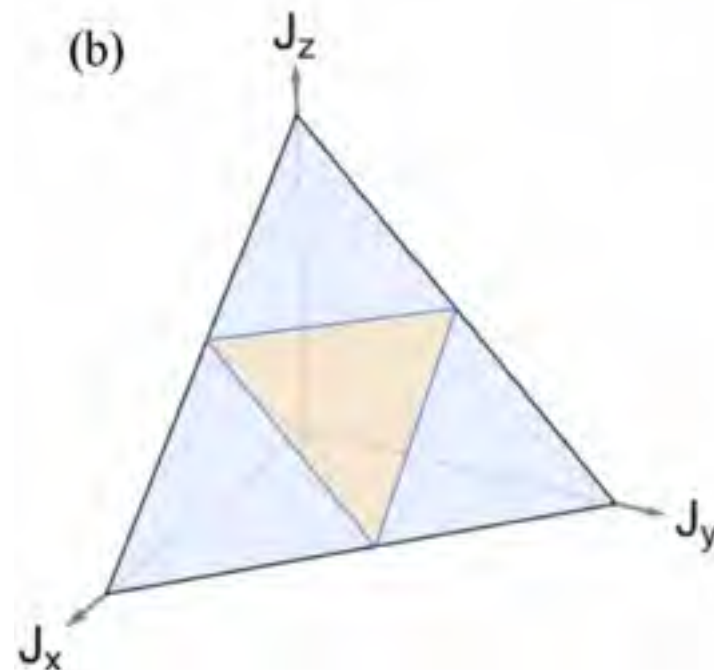
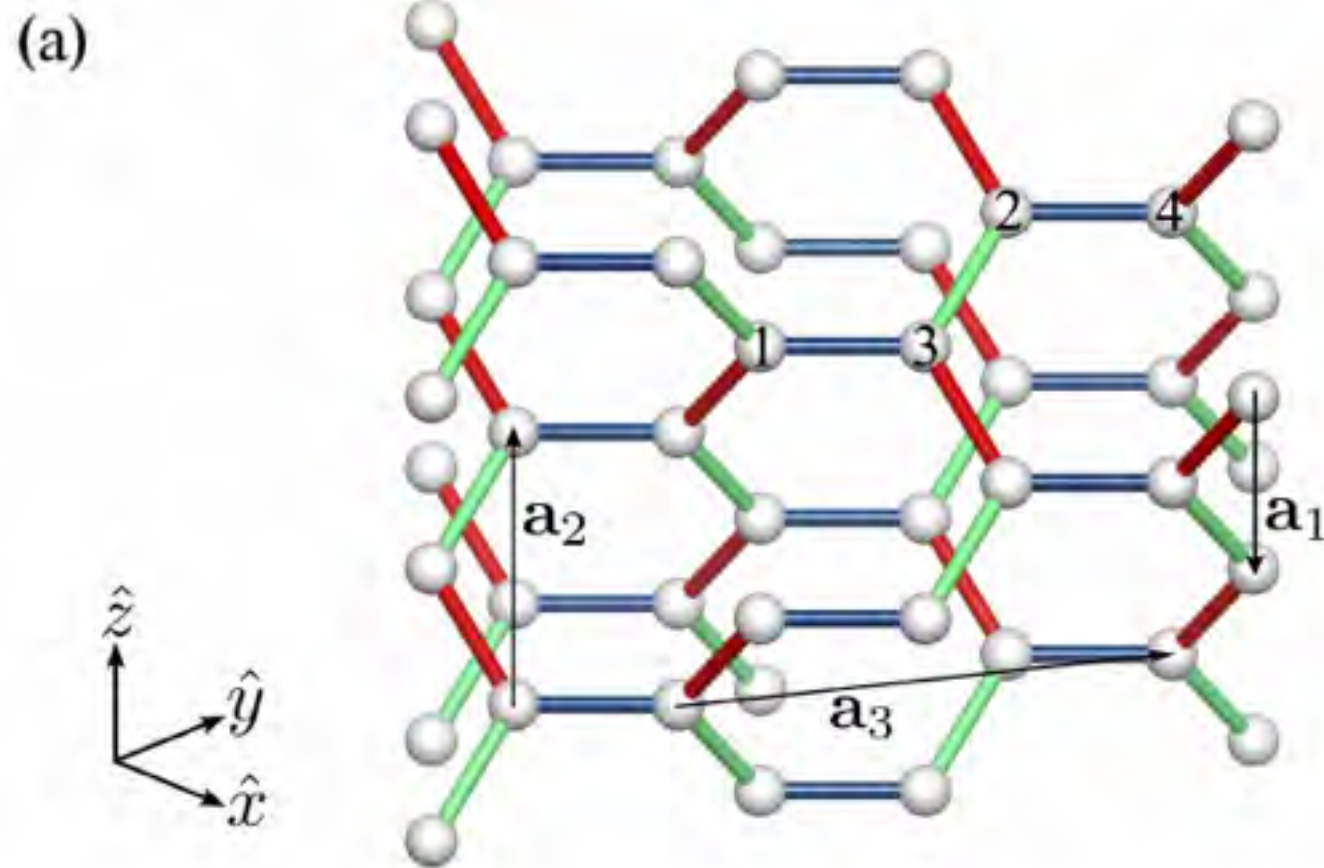


Hyperhexagon

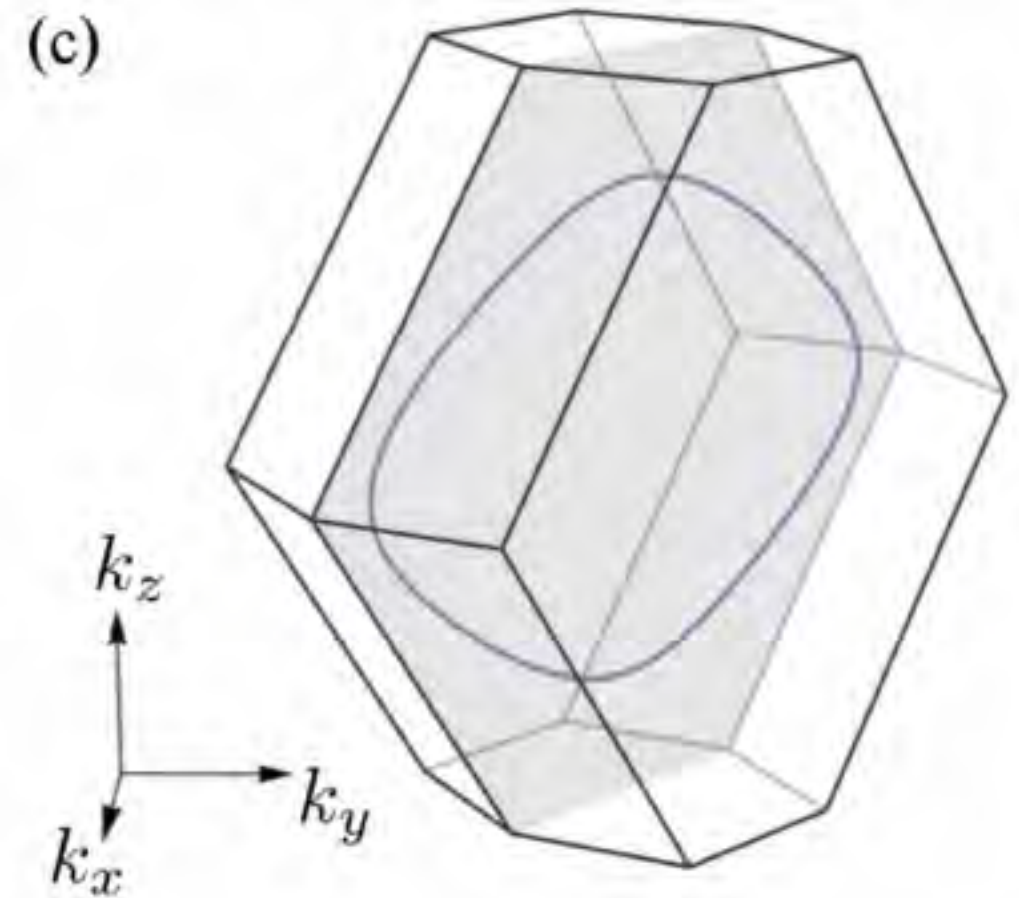


Hypercuboctagon

Hyperhoneycomb lattice

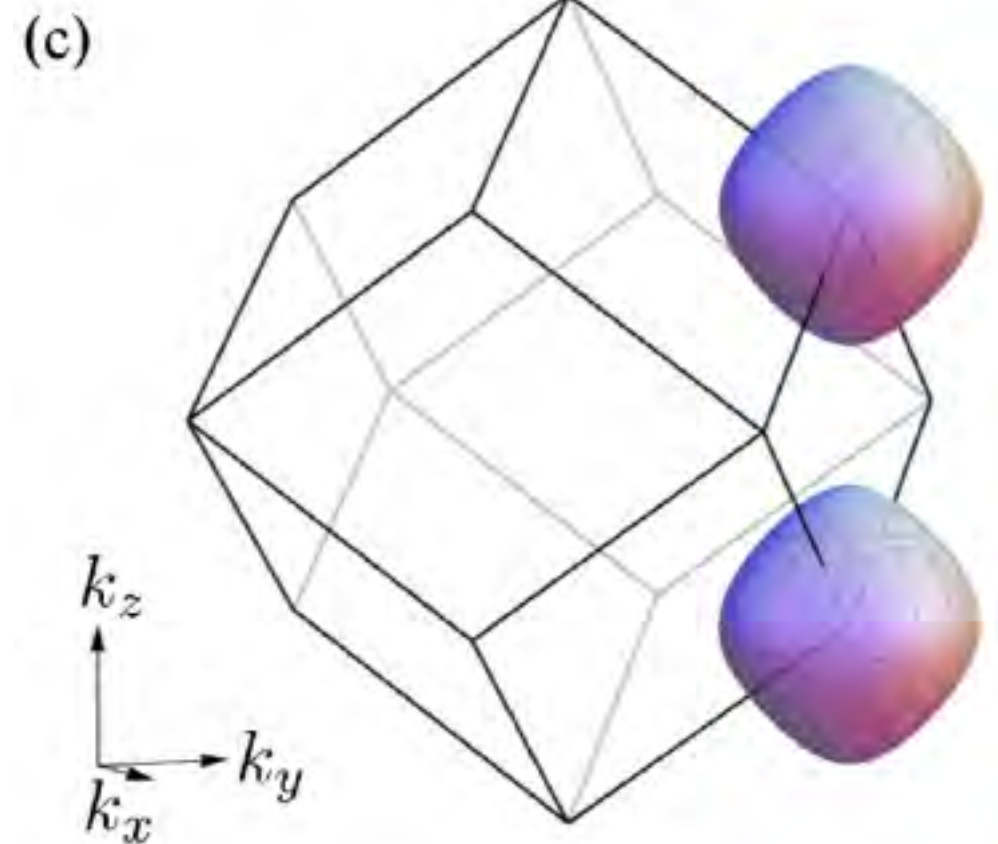
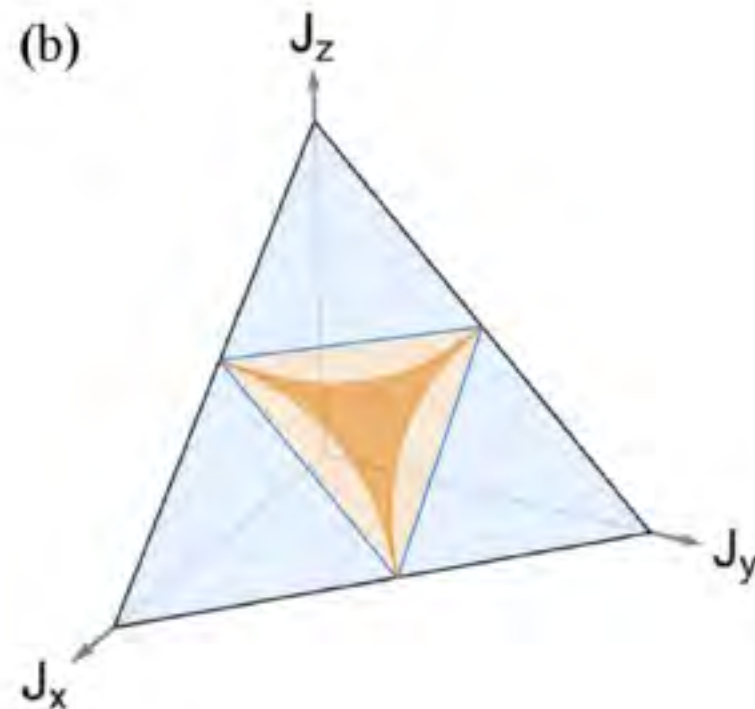
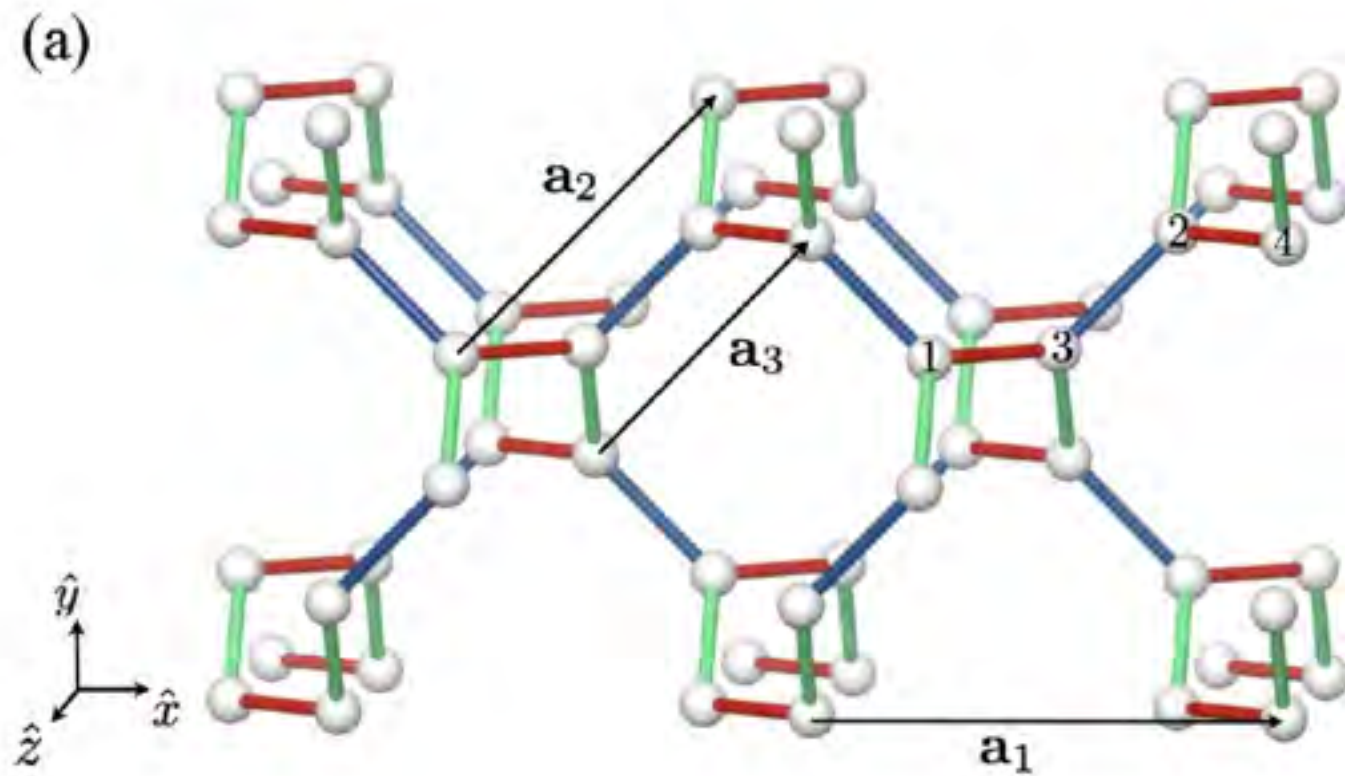


closed line of Dirac nodes

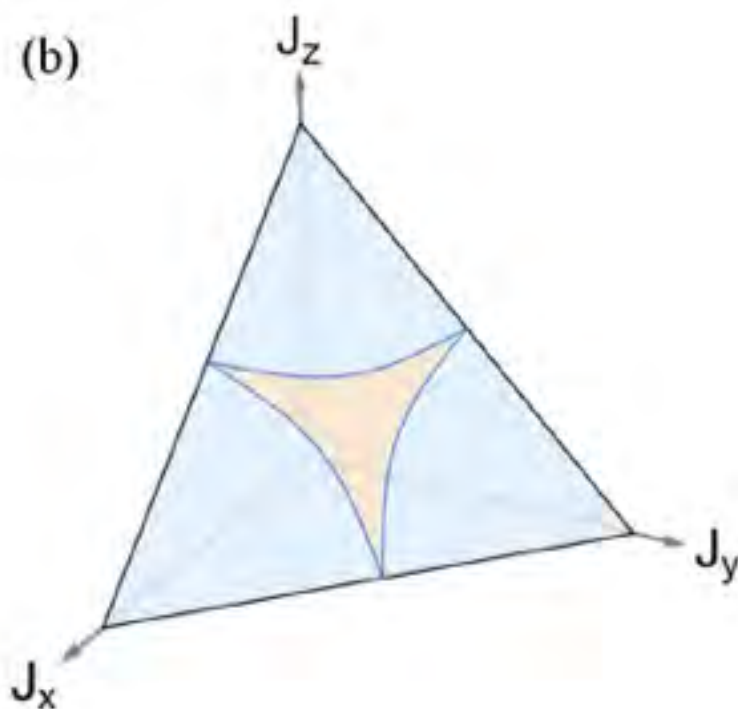
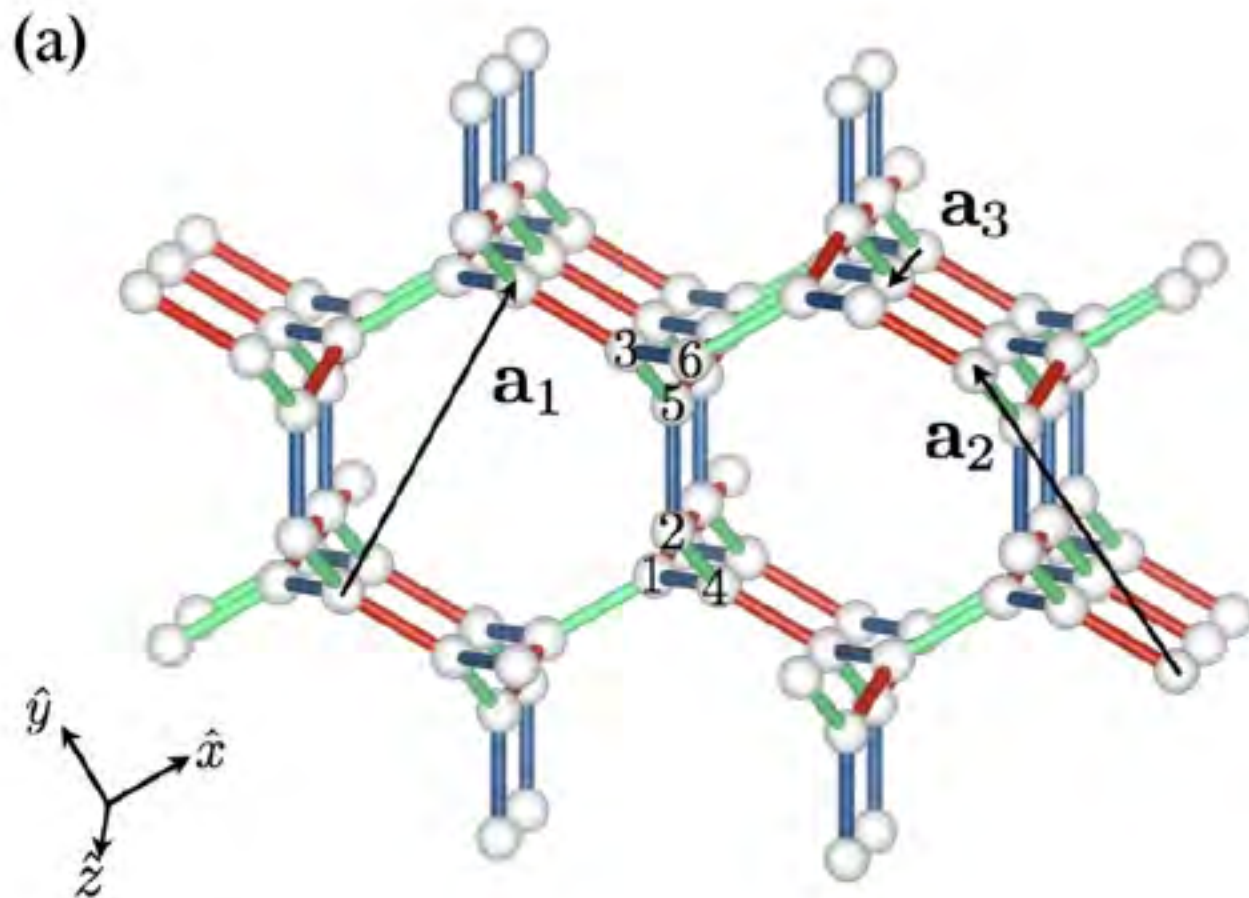


Hyperoctagon lattice Majorana metal

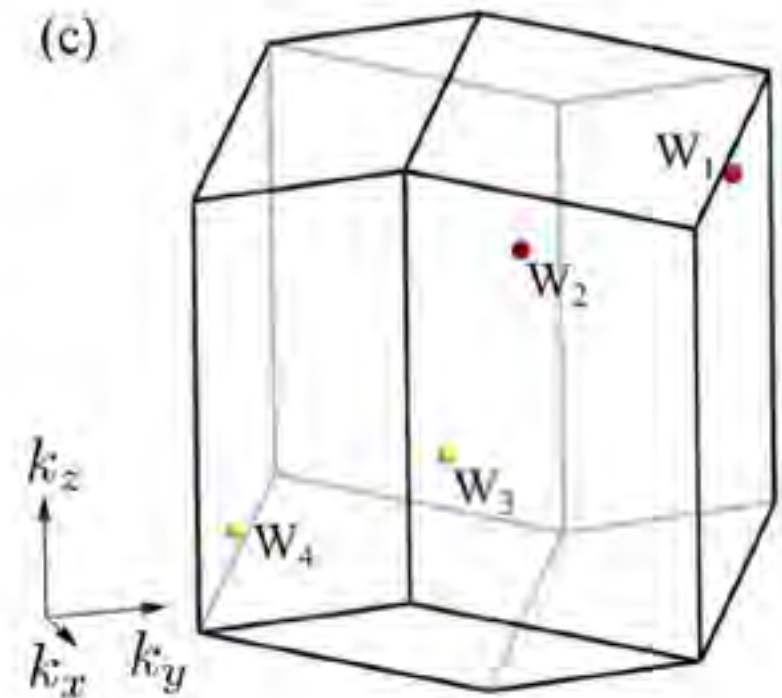
Fermi surfaces



Hyperhexagon lattice



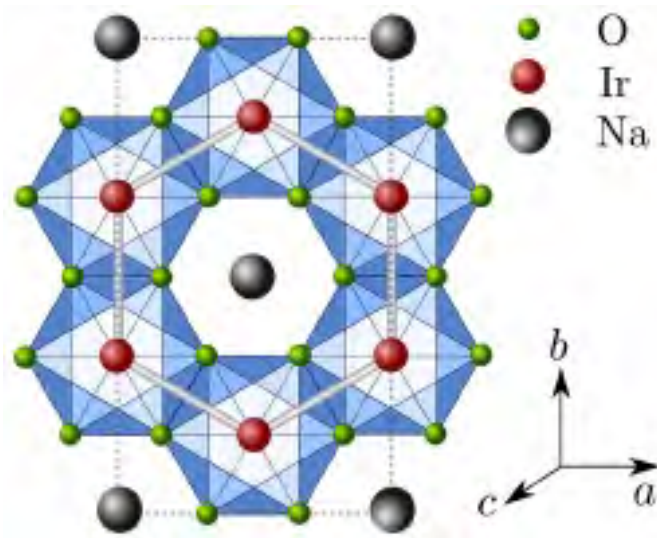
gapless Weyl points



$$\hat{H}_{2 \times 2} = \mathbf{v}_0 \cdot \mathbf{q} \mathbb{1} + \sum_{j=1}^3 \mathbf{v}_j \cdot \mathbf{q} \sigma_j$$

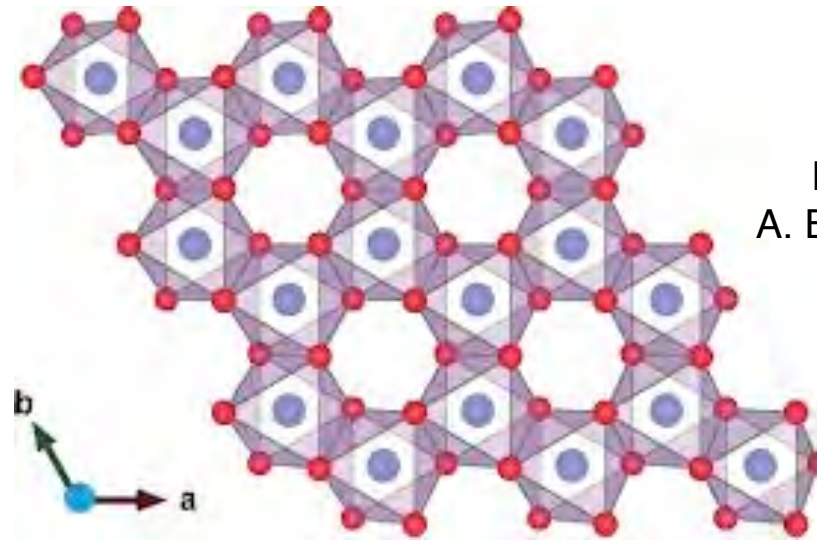
Experimental realizations

Na_2IrO_3
 $\alpha\text{-Li}_2\text{IrO}_3$



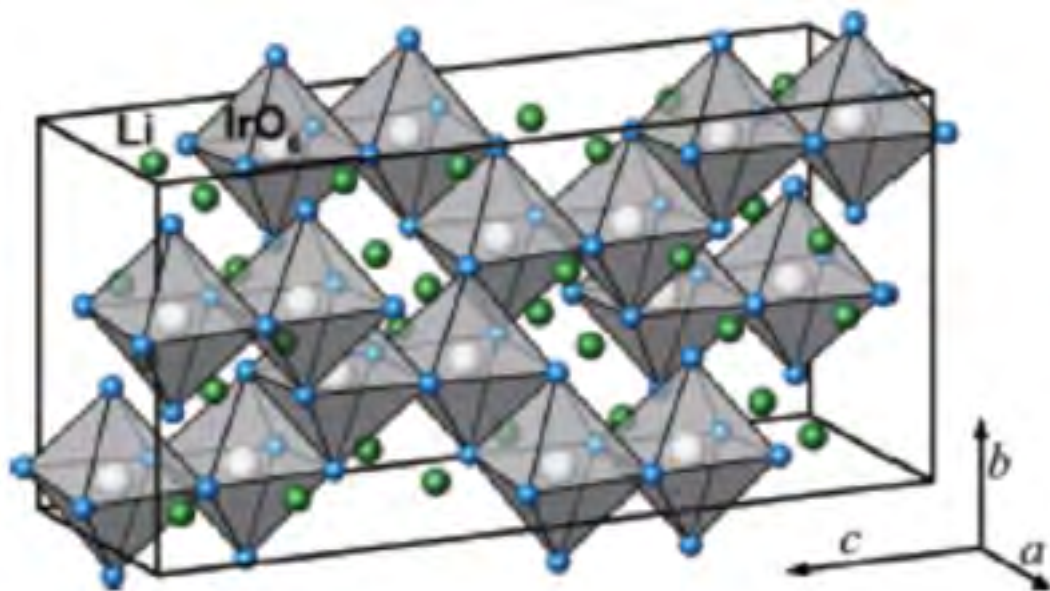
Y.Singh, P. Gegenwart,
PRL 2010, 2011

$\alpha\text{-RuCl}_3$



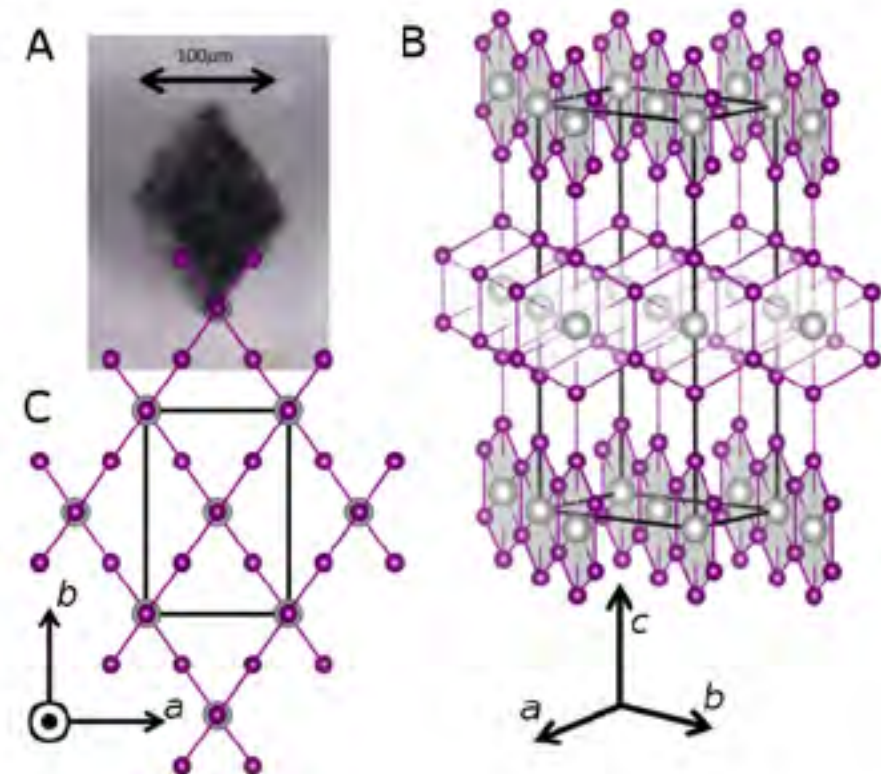
K. Plumb et al, Phys. Rev. B (2014)
A. Banerjee et al, Nature Materials (2016)

$\beta\text{-Li}_2\text{IrO}_3$



T. Takayama et al, PRL (2015)

$\gamma\text{-Li}_2\text{IrO}_3$

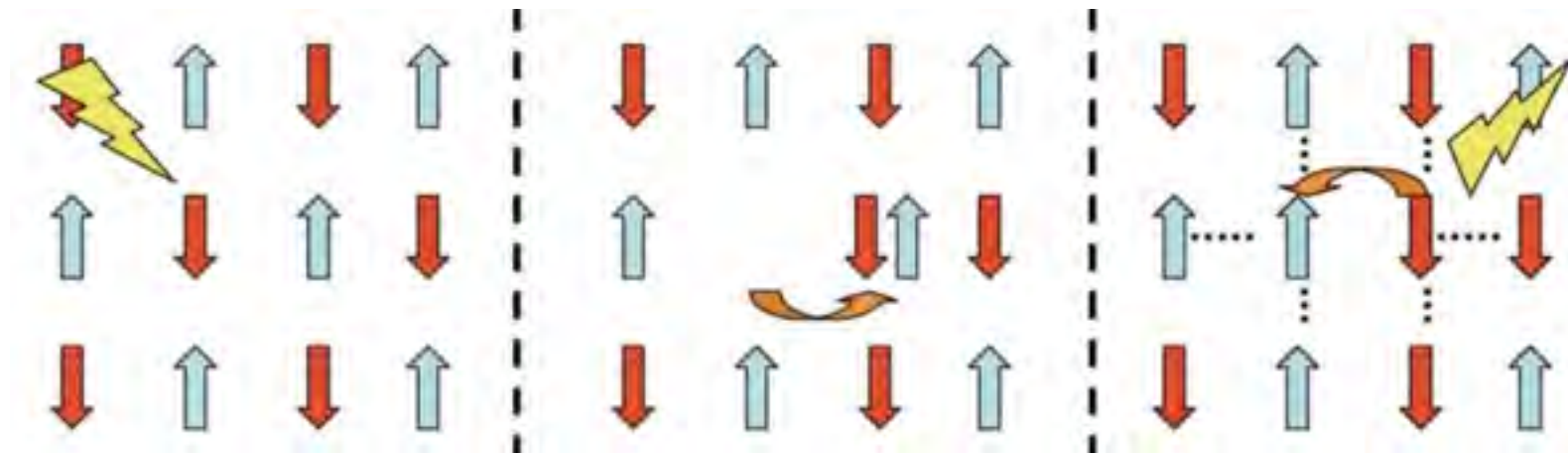


Modic, Nature Communications 5, 4203 (2014)

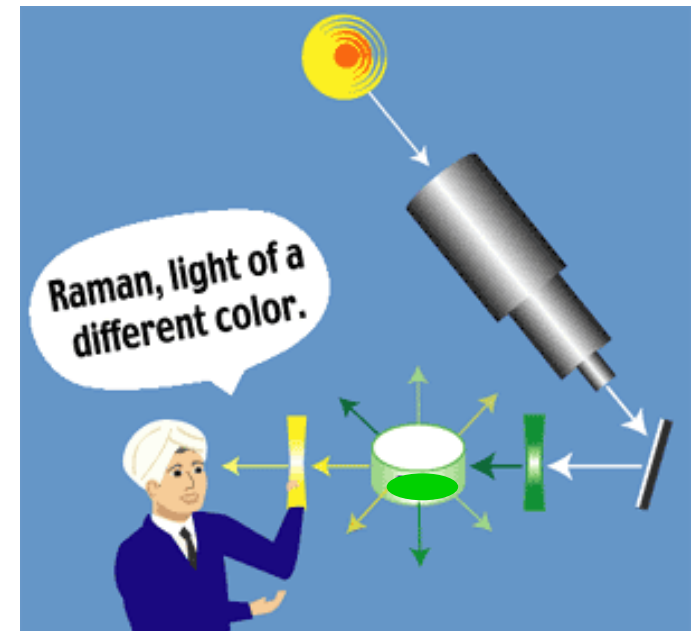
Spectroscopy of Kitaev Spin Liquids

Raman scattering in a nutshell

Photon-in photon-out process



Photon induced spin exchange

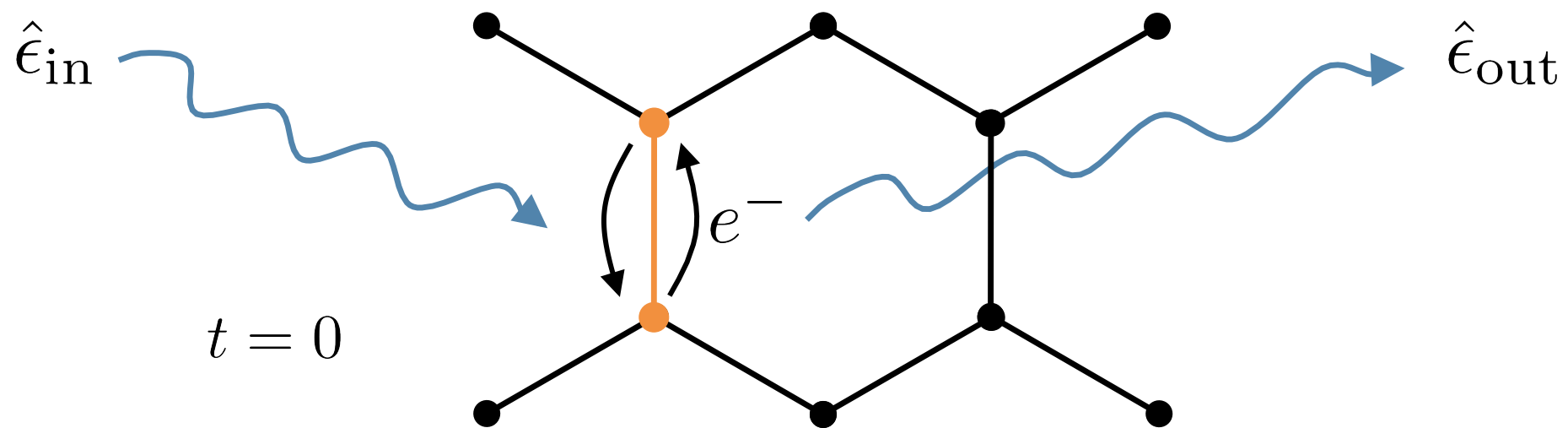


Two-magnon scattering in Mott insulators:
Loudon-Fleuri scattering vertex

$$R = \sum_{i, \pm \delta_\mu} (\hat{\epsilon}_{\text{in}} \cdot \delta_\mu) (\hat{\epsilon}_{\text{out}} \cdot \delta_\mu) J_\mu \mathbf{S}_i \cdot \mathbf{S}_{i \pm \delta_\mu}$$

Raman Scattering in Kitaev model

$$I_{\text{Ram}}(\omega) \propto \int dt e^{i\omega t} \langle R e^{iHt} R \rangle$$



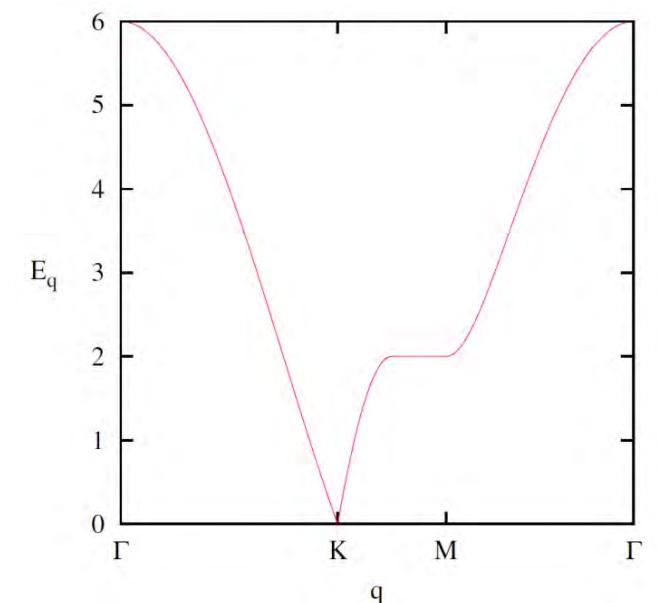
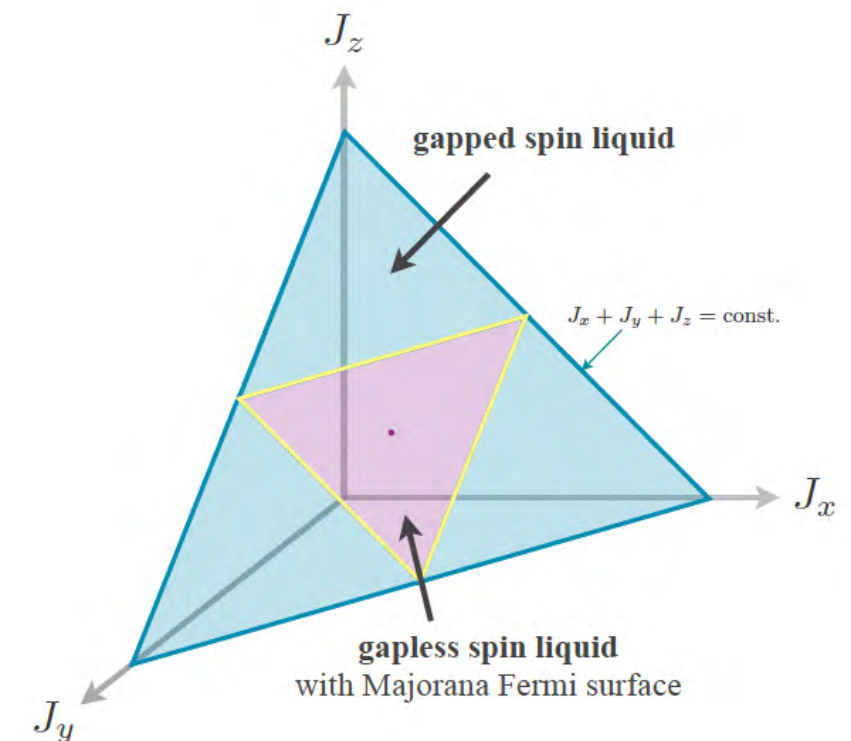
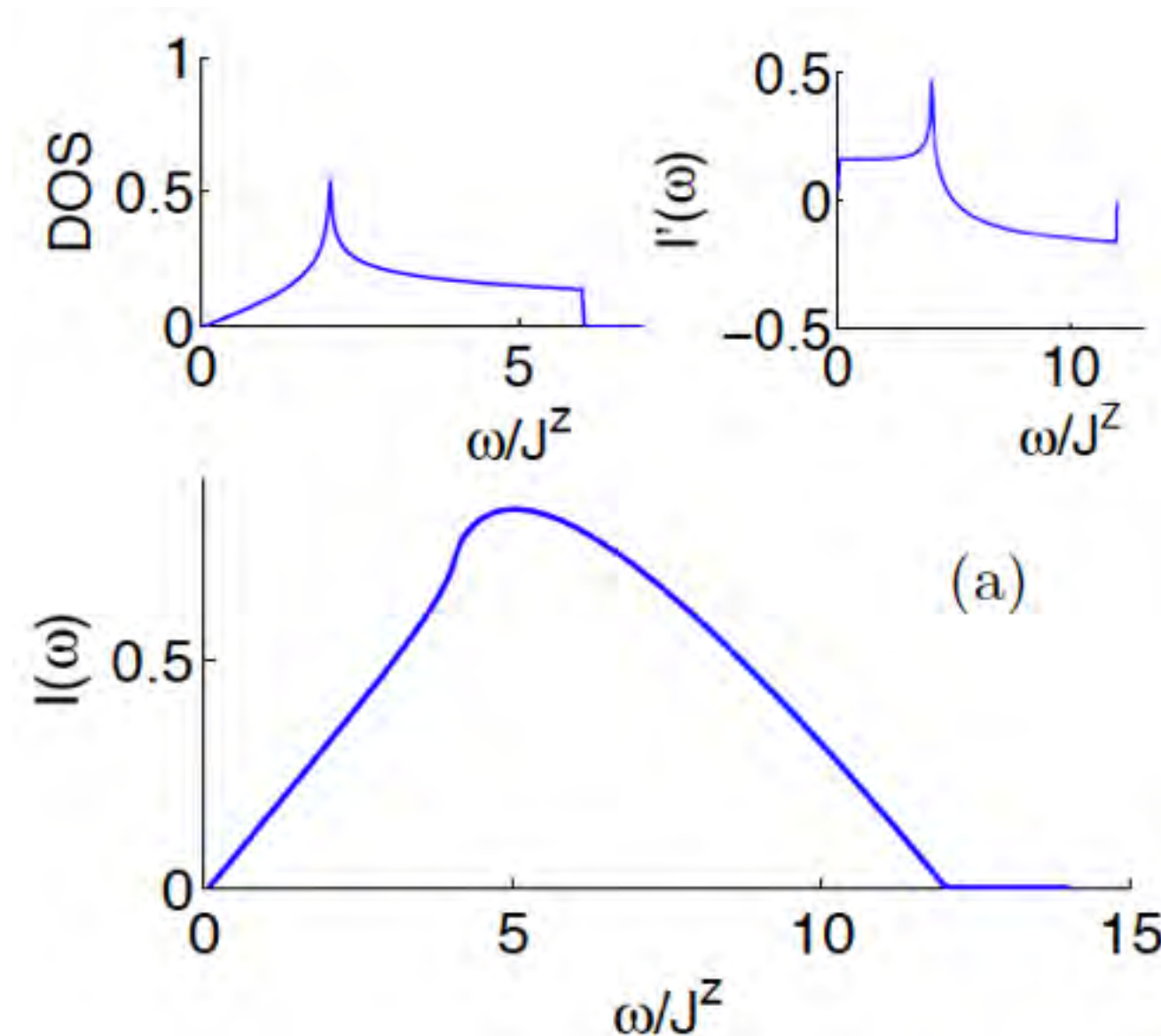
$$R = \sum_{\langle ij \rangle^\alpha} (\epsilon_{\text{in}} \cdot \mathbf{d}^\alpha) (\epsilon_{\text{out}} \cdot \mathbf{d}^\alpha) J^\alpha \sigma_i^\alpha \sigma_j^\alpha$$

$$= i \sum_{\langle ij \rangle^\alpha} (\epsilon_{\text{in}} \cdot \mathbf{d}^\alpha) (\epsilon_{\text{out}} \cdot \mathbf{d}^\alpha) J^\alpha u_{\langle ij \rangle^\alpha} c_i c_j$$

Raman vertex: diagonal in fluxes but **creates two Majorana fermions**

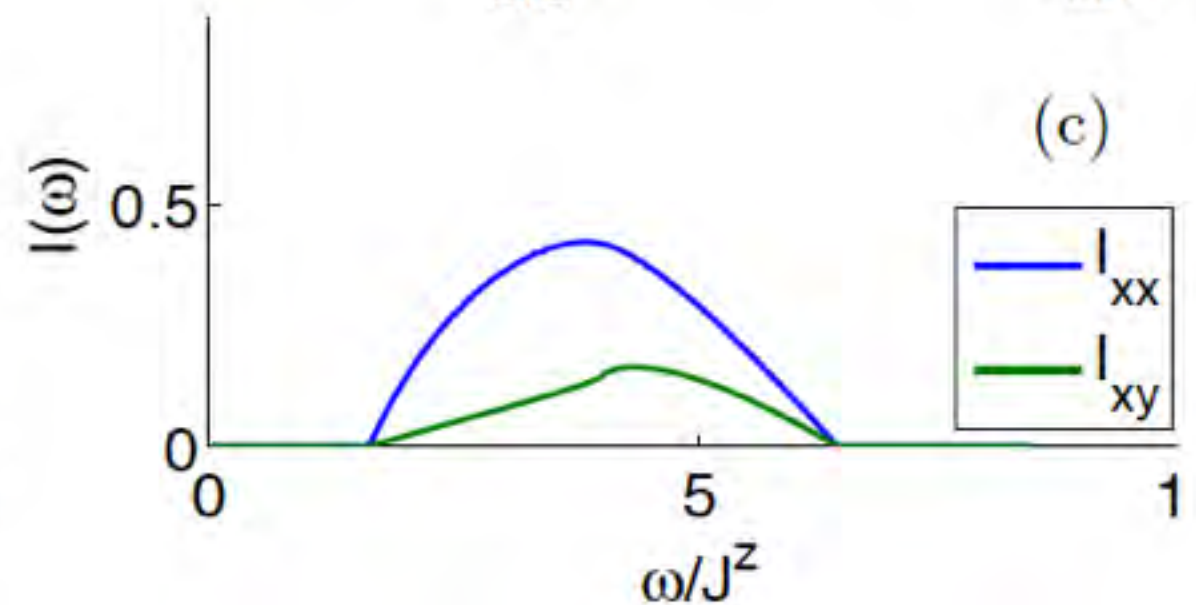
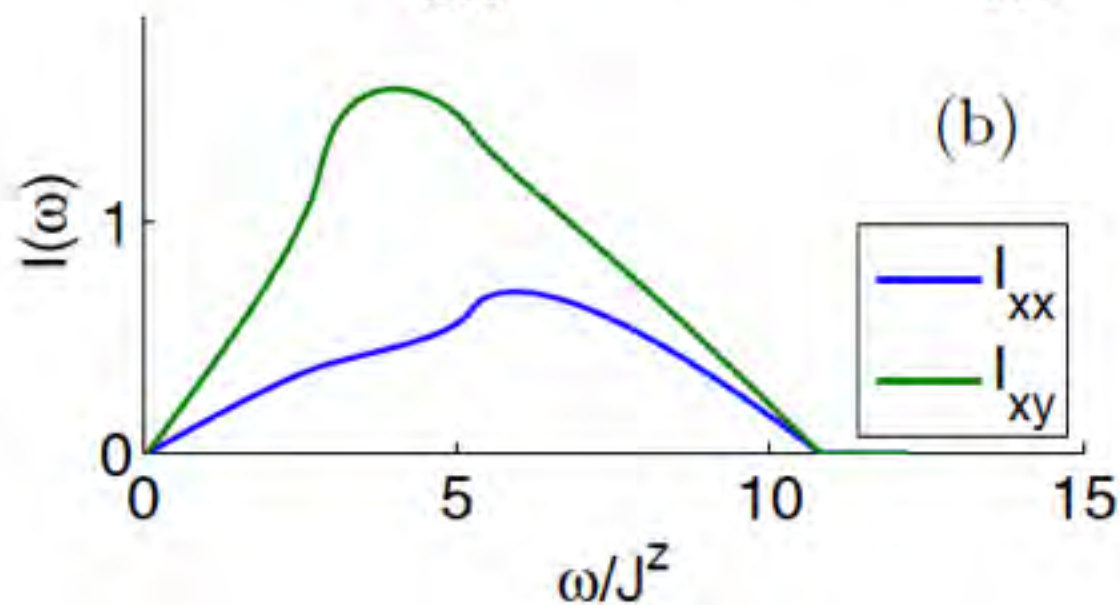
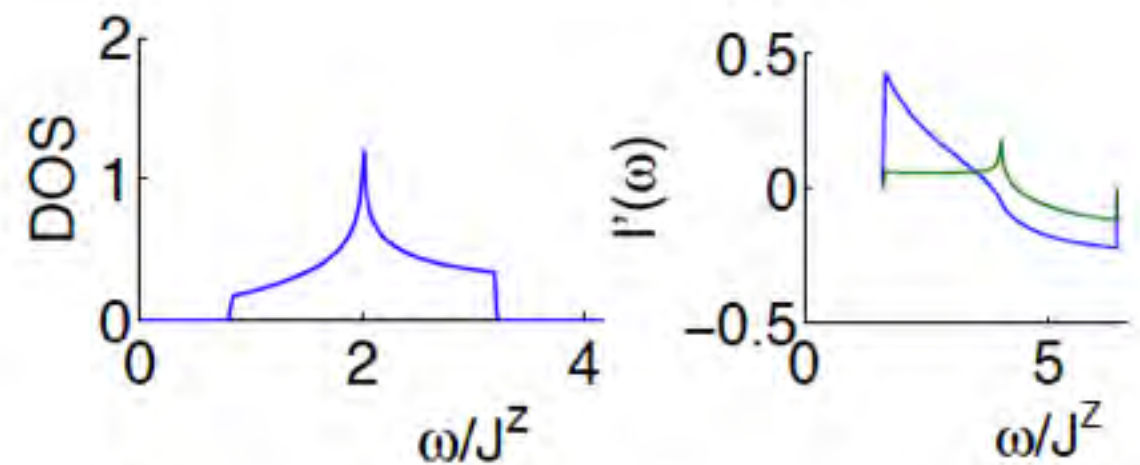
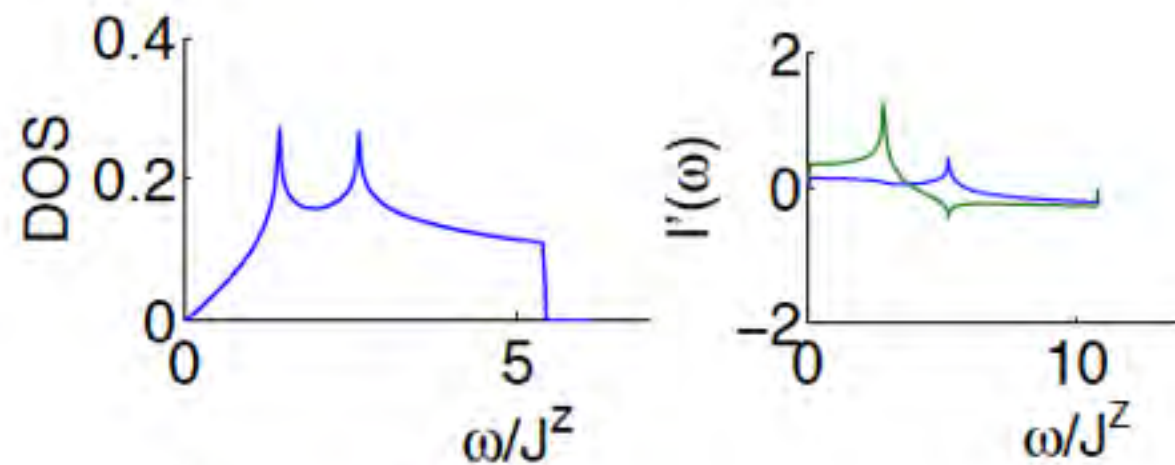
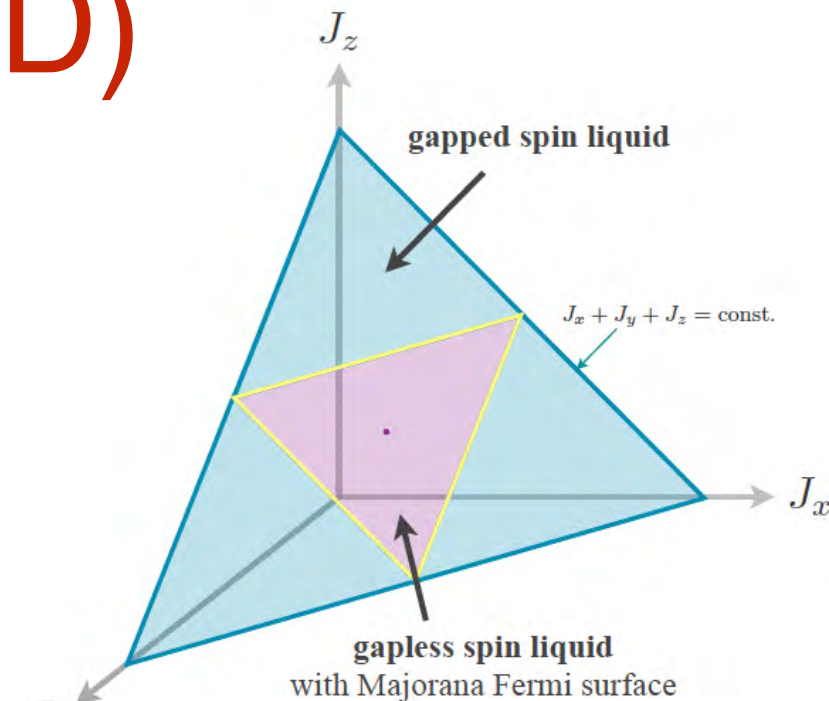
Raman Scattering results (2D)

isotropic point:
polarization independent



Raman Scattering results (2D)

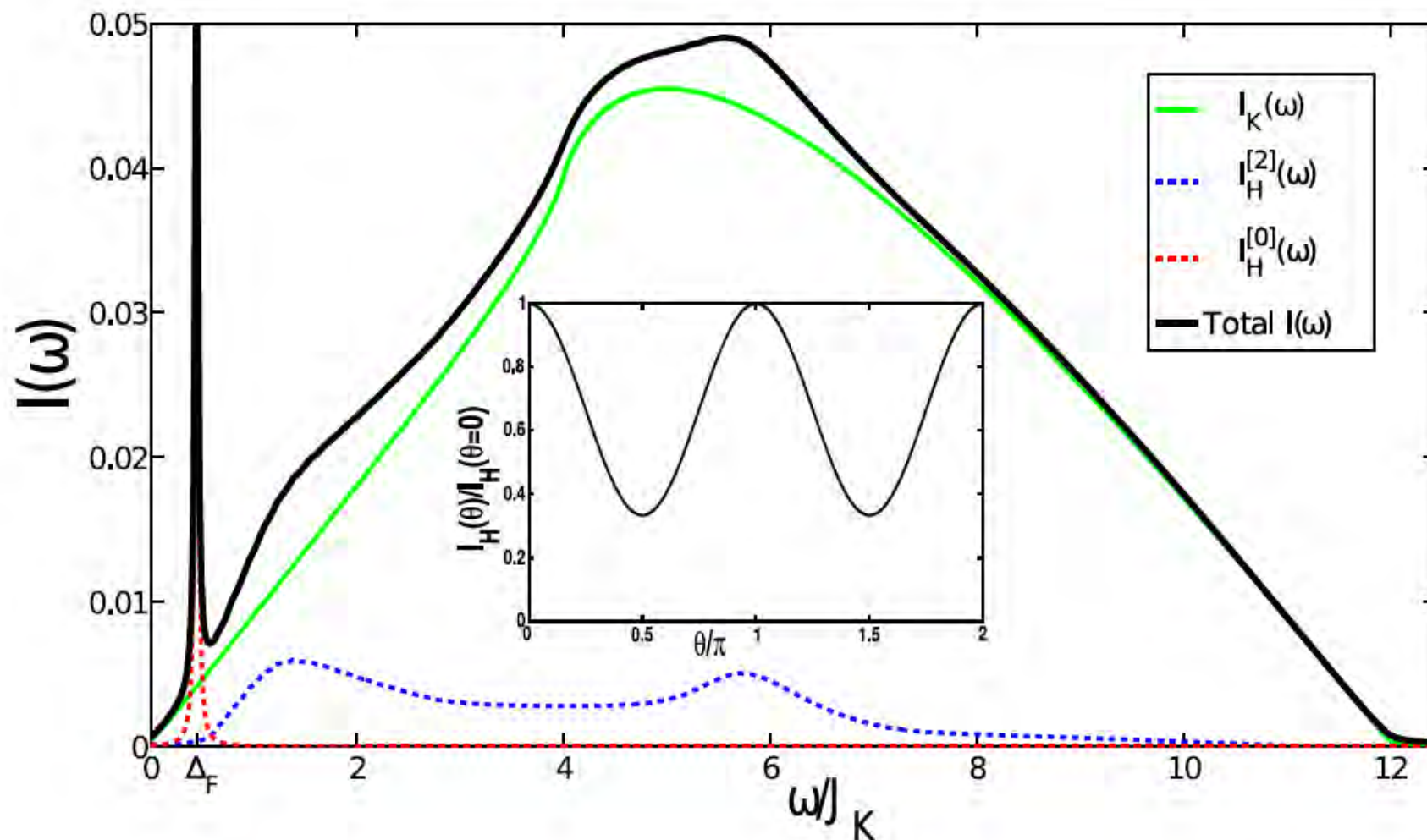
anisotropic point:
polarization dependence



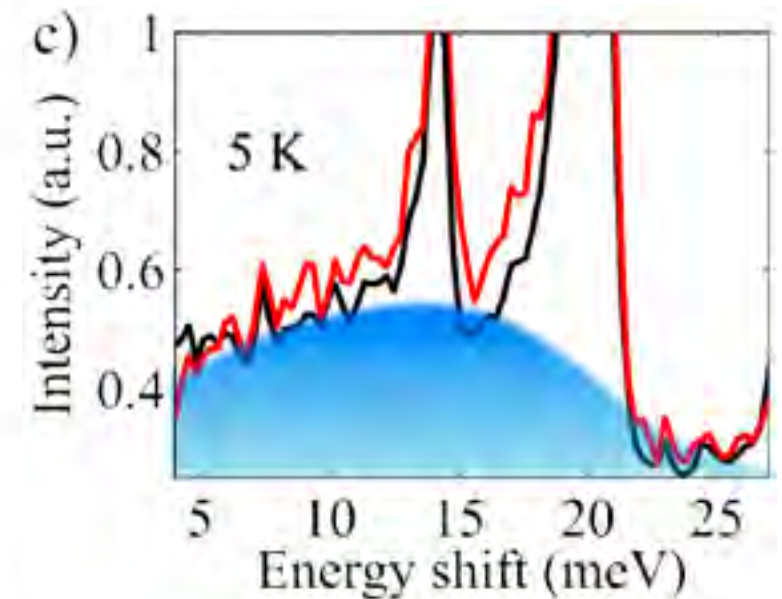
Raman Scattering results (2D)

$$I(\omega) = I_K(\omega) + I_H(\omega)$$

computed perturbatively



L. Sandilands, Y.J. Kim, K.S. Burch
Phys. Rev. Lett. 114 (2015)

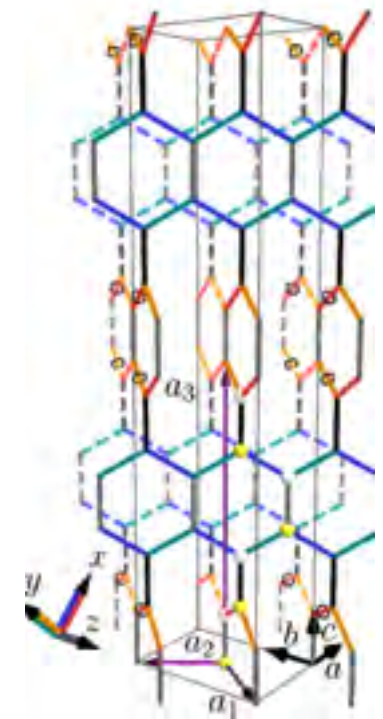
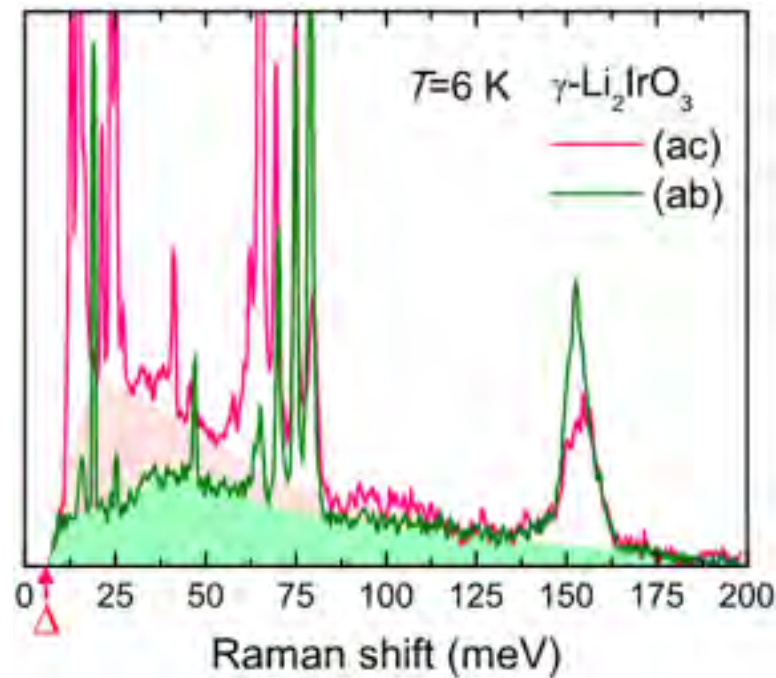
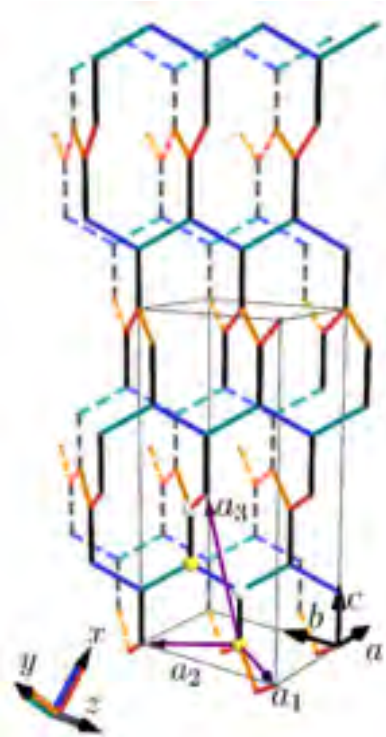


Big 'hump' with fine features of the Majorana DOS. **Salient signatures of fractionalization are visible!**
(comparison gives $J_K \sim 8 \text{ meV}$)

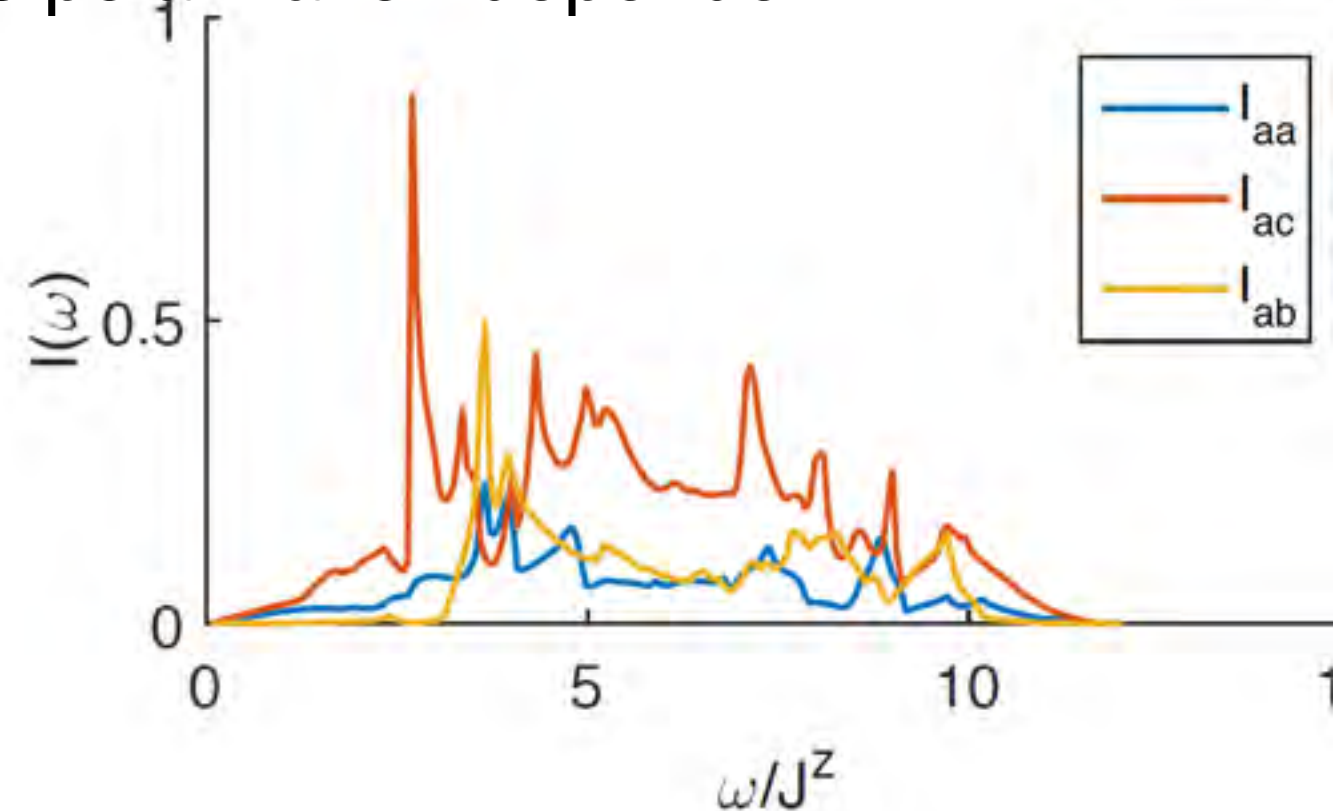
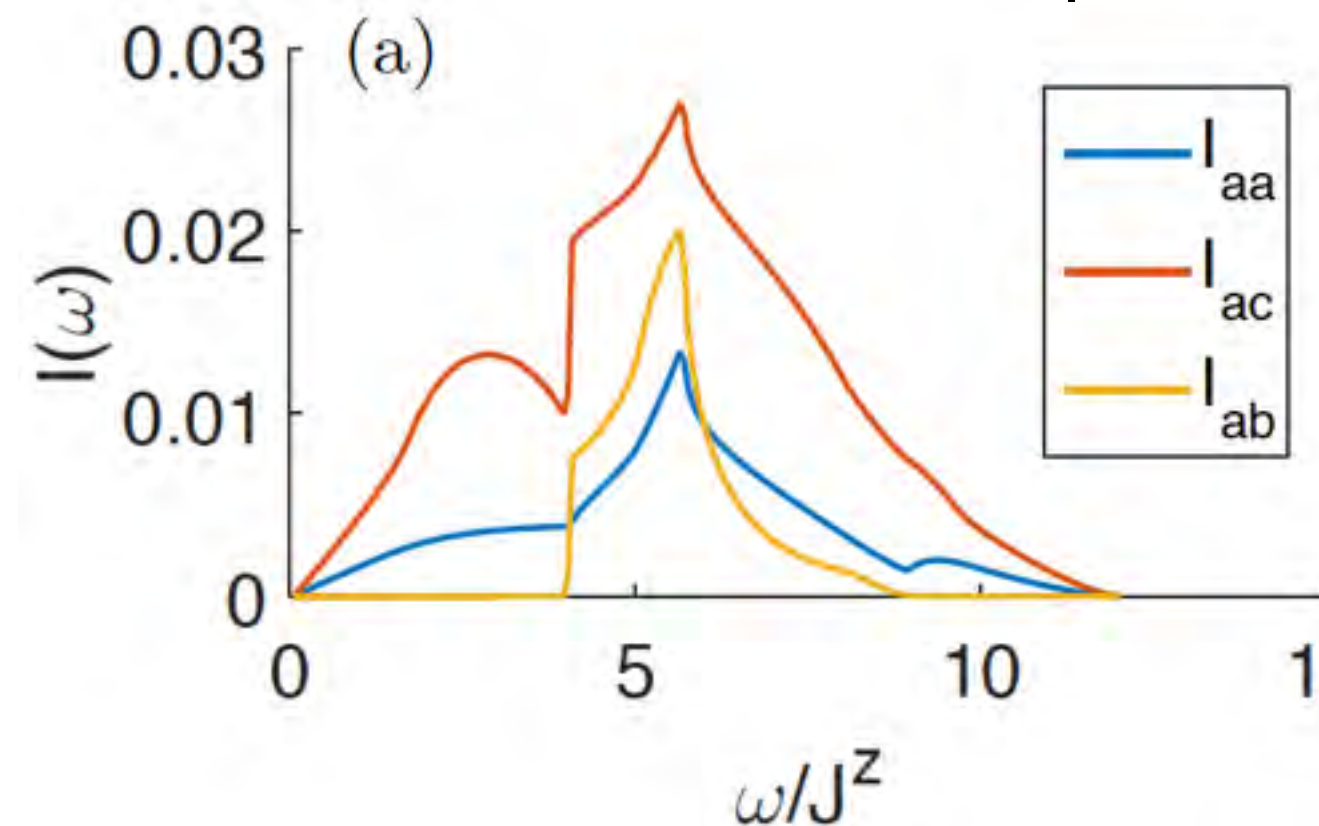
The Heisenberg contribution depends on the photon polarizations, δ -function peak at the four flux gap.

Raman Scattering results (3D)

Glamazda, Lemmens, Do, Choi, Choi, Nature Comm. 7 (2016)

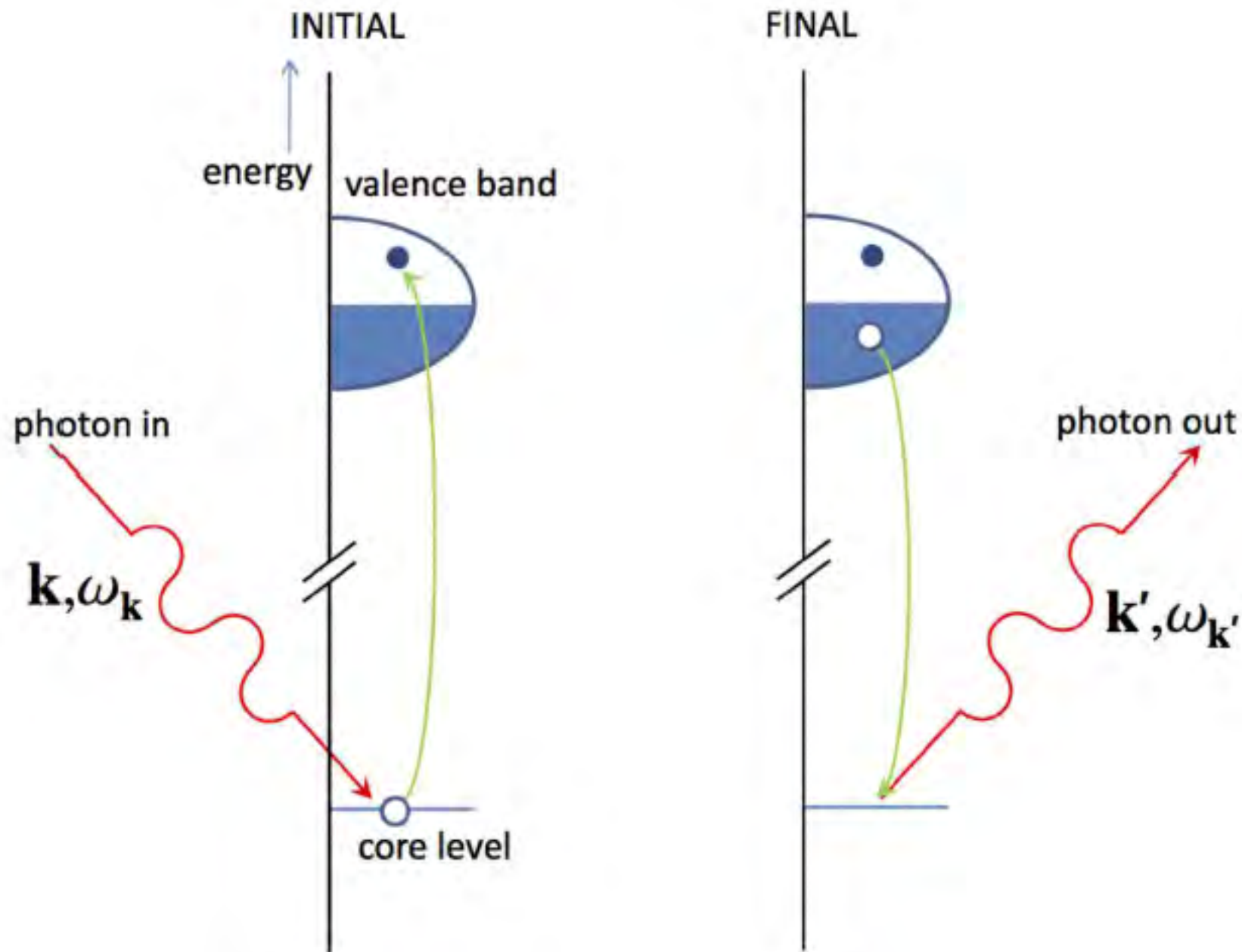


The Raman response is polarization dependent!



RIXS in a nutshell

In L-edge experiments, the scattering typically, though not always, happens through a direct RIXS process.

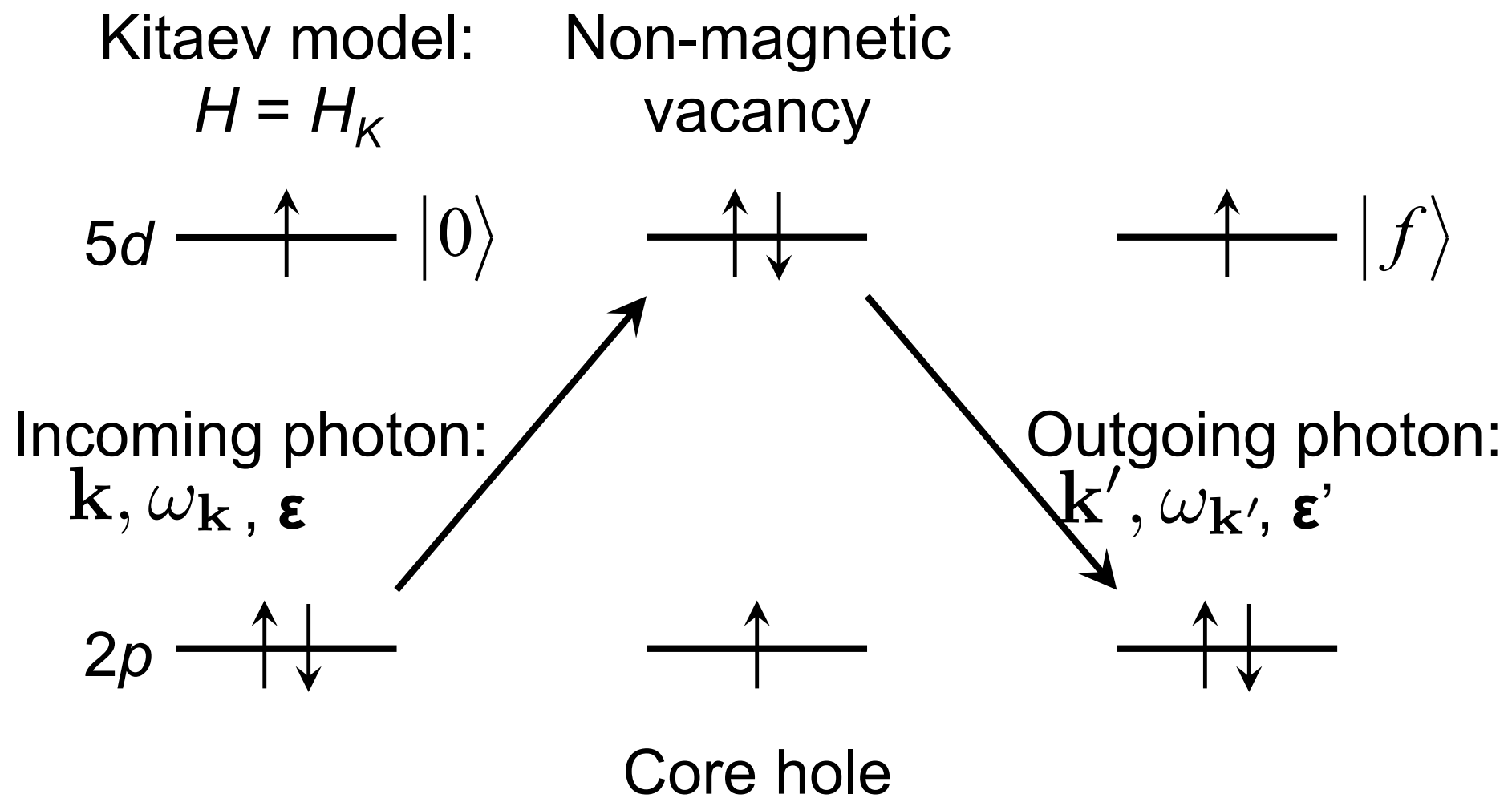


$$\mathbf{q} = \mathbf{k}' - \mathbf{k}$$

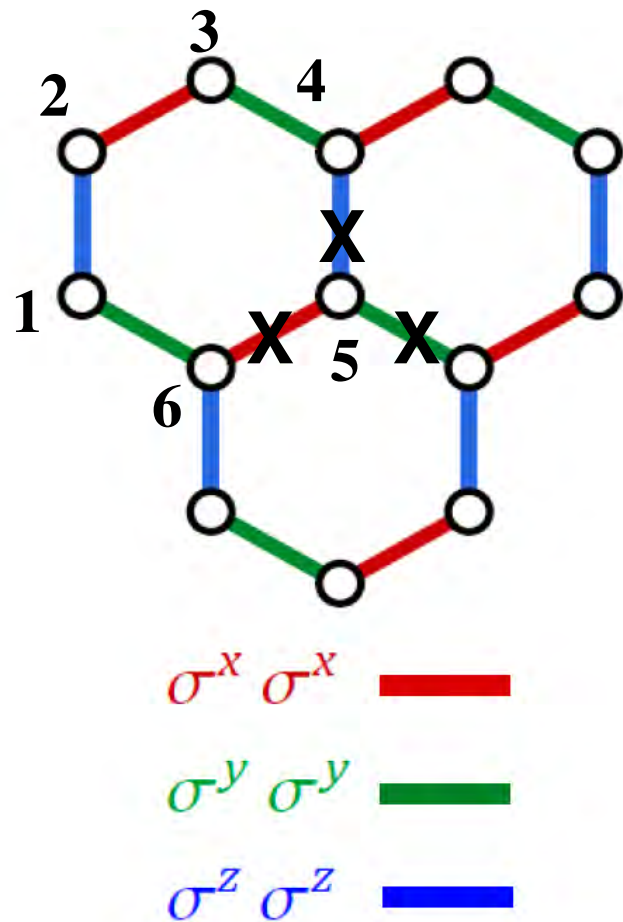
$$\omega = \omega_{\mathbf{k}'} - \omega_{\mathbf{k}}$$

RIXS from Ir⁴⁺

(Na,Li)₂IrO₃ with Ir⁴⁺ in 5d⁵ configuration [*L*₃ edge]:



Intermediate state with a vacancy



The Kitaev model with a single vacancy at site \mathbf{r}
 = the original Kitaev model with switched off
 couplings around site \mathbf{r} (**exactly solvable**)

We demand that the vacancy is always in the
 spin-up state.

$$d_{\mathbf{r},\downarrow}^\dagger \rightarrow \frac{1}{2} (1 + \sigma_{\mathbf{r}}^z), \quad d_{\mathbf{r},\uparrow}^\dagger \rightarrow \frac{1}{2} \sigma_{\mathbf{r}}^x (1 - \sigma_{\mathbf{r}}^z)$$

$$d_{\mathbf{r},\downarrow}^\dagger |\uparrow\rangle = |\uparrow\rangle$$

$$d_{\mathbf{r},\uparrow}^\dagger |\uparrow\rangle = 0$$

$$d_{\mathbf{r},\downarrow}^\dagger |\downarrow\rangle = 0$$

$$d_{\mathbf{r},\uparrow}^\dagger |\downarrow\rangle = |\uparrow\rangle$$

RIXS amplitude for the Kitaev model

$$I(\omega, \mathbf{q}) = \sum_m \left| \sum_{\alpha, \beta} T_{\alpha\beta} A_{\alpha\beta}(m, \mathbf{q}) \right|^2 \delta(\omega - E_m)$$

$$A_{\alpha\beta}(m, \mathbf{q}) = \sum_{\mathbf{r}, \tilde{n}_{\mathbf{r}}} \frac{\langle m | d_{\mathbf{r}, \alpha} | \tilde{n}_{\mathbf{r}} \rangle \langle \tilde{n}_{\mathbf{r}} | d_{\mathbf{r}, \beta}^\dagger | 0 \rangle}{\Omega - E_{\tilde{n}} + i\Gamma} e^{i\mathbf{q} \cdot \mathbf{r}}$$

$$\mathbf{q} \equiv \mathbf{k} - \mathbf{k}'$$

Kramers–Heisenberg formula

The four fundamental RIXS channels are introduced by decomposing the polarization tensor into

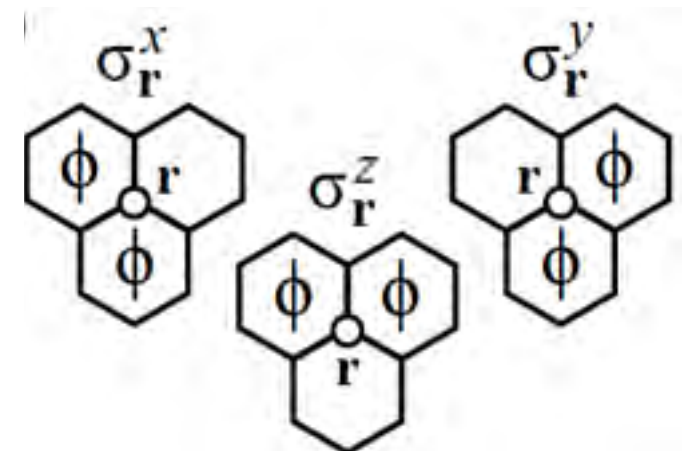
(a) Spin-conserving (SC) channel with

$$T_{\alpha\beta} \propto \sigma_{\alpha\beta}^0$$

create two flux excitations

(b) three non spin-conserving (NSC) channels with

$$T_{\alpha\beta} \propto \sigma_{\alpha\beta}^{x,y,z}$$



Fast collision approximation

$(\text{Na,Li})_2\text{IrO}_3$ and $\alpha\text{-RuCl}_3$: $\Gamma / J_{x,y,z} \gg 1$

$$t \sim 1 / \Gamma \ll 1 / J_{x,y,z} \rightarrow$$

The lowest order RIXS amplitude is then

$$\begin{aligned} A_\eta(m, \mathbf{q}) &\propto \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} \langle m | \sigma_{\mathbf{r}}^\eta \left[1 - \frac{i\tilde{H}(\mathbf{r})}{\Gamma} \right] | 0 \rangle \\ &= \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} \langle m | \sigma_{\mathbf{r}}^\eta \left[1 - \frac{i}{\Gamma} \sum_{\kappa} J_\kappa \sigma_{\mathbf{r}}^\kappa \sigma_{\kappa(\mathbf{r})}^\kappa \right] | 0 \rangle \end{aligned}$$

NSC channels recover INS amplitudes for infinite Γ

Flux creation: Finite gap, little dispersion

Results: SC channel 2D Kitaev model

$$A_0(m, \mathbf{q}) \propto \sum_{\mathbf{r}} e^{i\mathbf{q} \cdot \mathbf{r}} \langle m | \left[1 - \frac{i}{\Gamma} \sum_{\kappa=x,y,z} J_{\kappa} \sigma_{\mathbf{r}}^{\kappa} \sigma_{\kappa(\mathbf{r})}^{\kappa} \right] | 0 \rangle$$

X

Elastic response

Inelastic response


$|m\rangle \neq |0\rangle$

no flux and two fermion excitations

$$\omega = \varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{q}-\mathbf{k}}$$

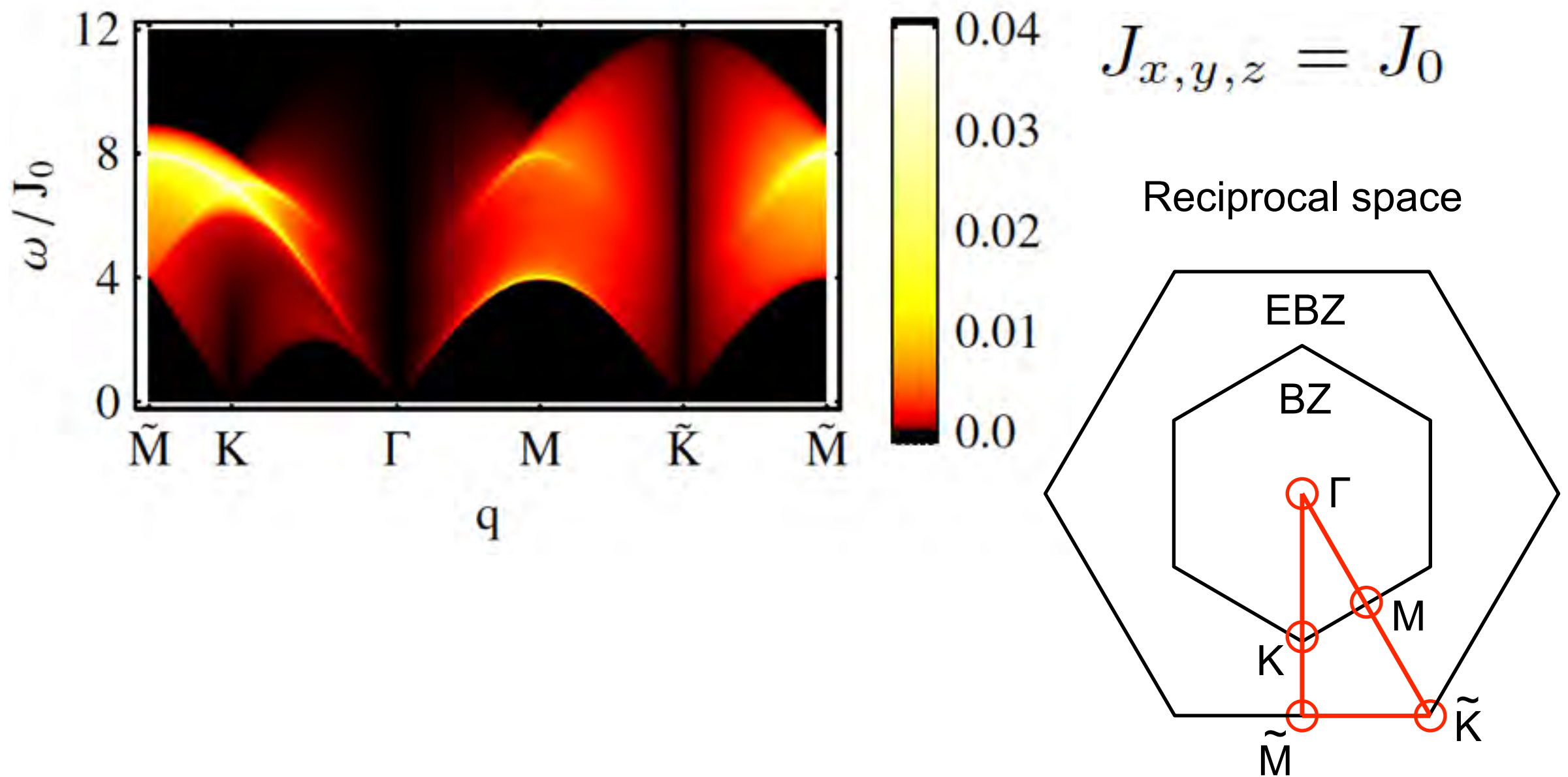
$I_0(\omega, \mathbf{q})$ as a histogram of $|A_0(m, \mathbf{q})|^2$

$$I_0(\omega, \mathbf{q}) \propto \int_{\text{BZ}} d^2\mathbf{k} \, \delta(\omega - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}-\mathbf{k}}) [\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}-\mathbf{k}}]^2 \left| 1 - e^{i\varphi_{\mathbf{k}}} e^{i\varphi_{\mathbf{q}-\mathbf{k}}} \right|^2$$

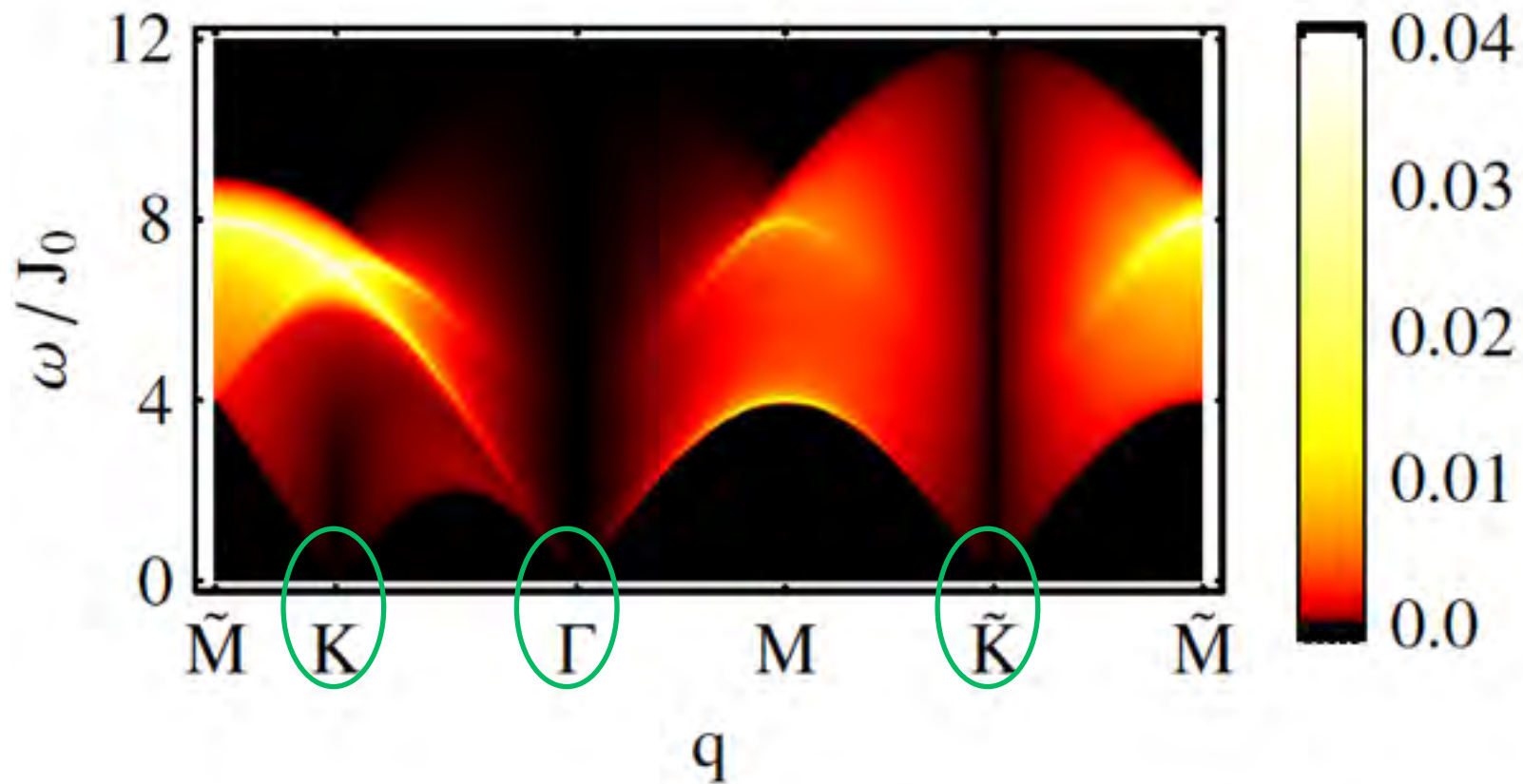
A  B

interference between the two sublattices

RIXS response in SC channel

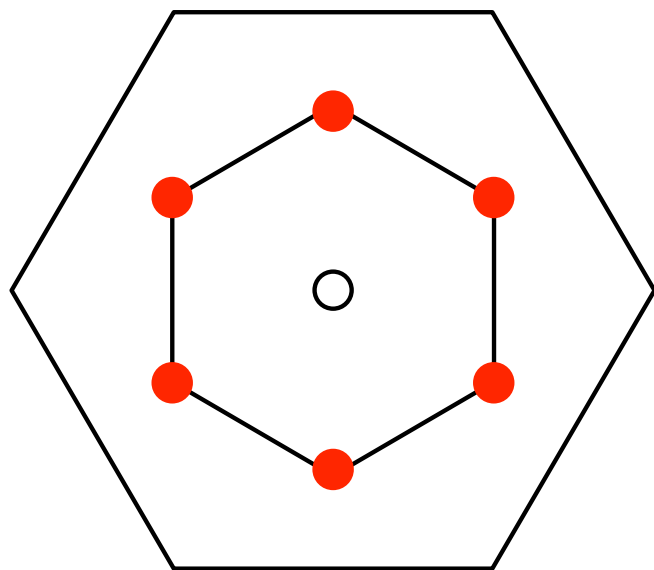


RIXS response in SC channel



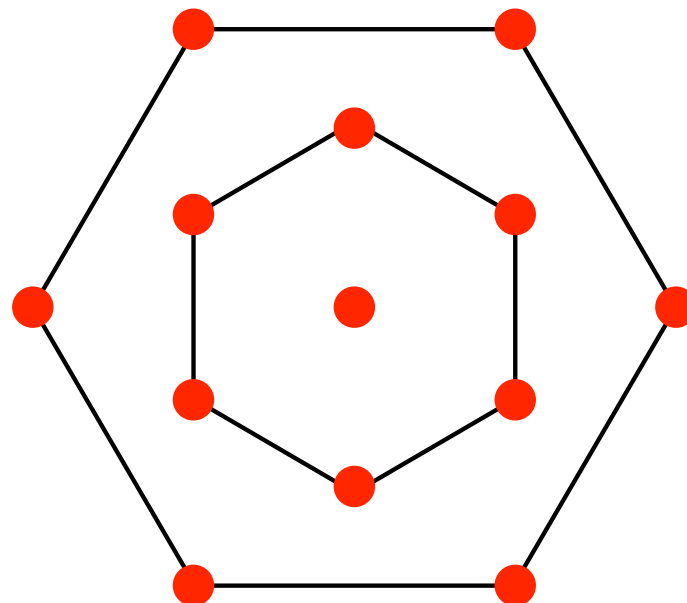
Gapless response at a finite number of discrete points

Dirac points



K points

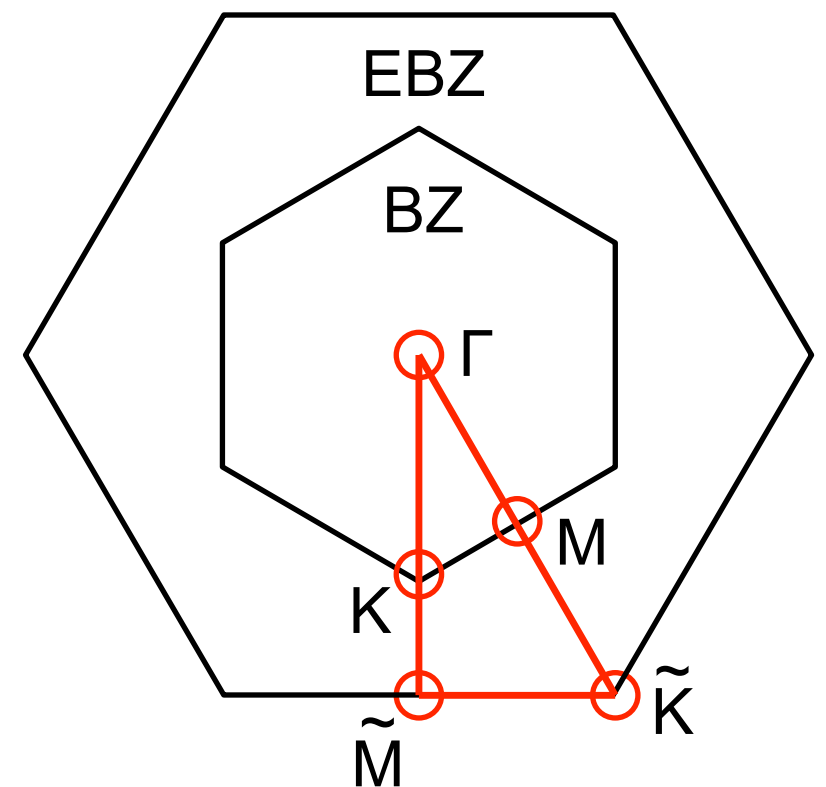
Gapless points



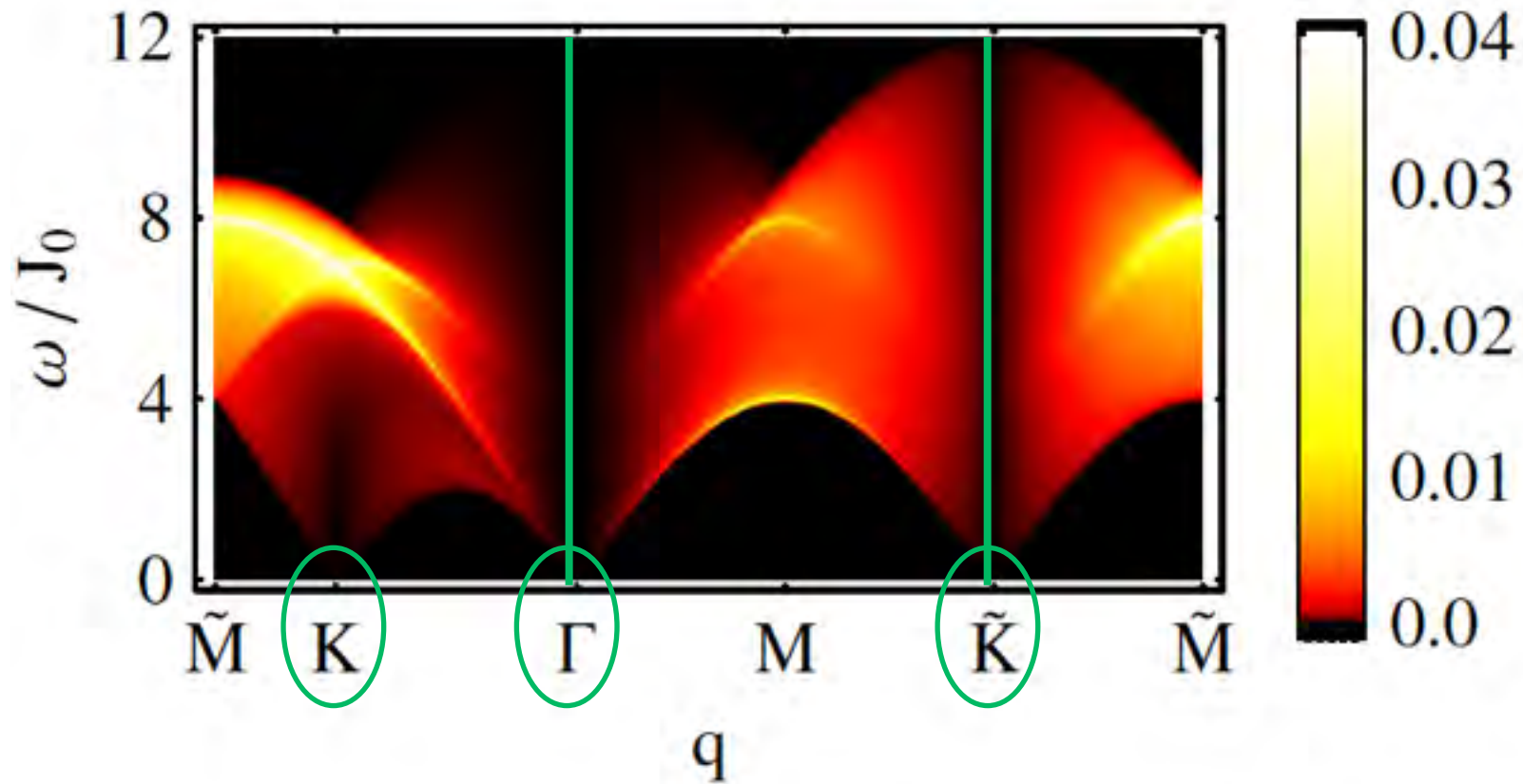
Γ , K, \tilde{K} points

$$J_{x,y,z} = J_0$$

Reciprocal space



RIXS response in SC channel

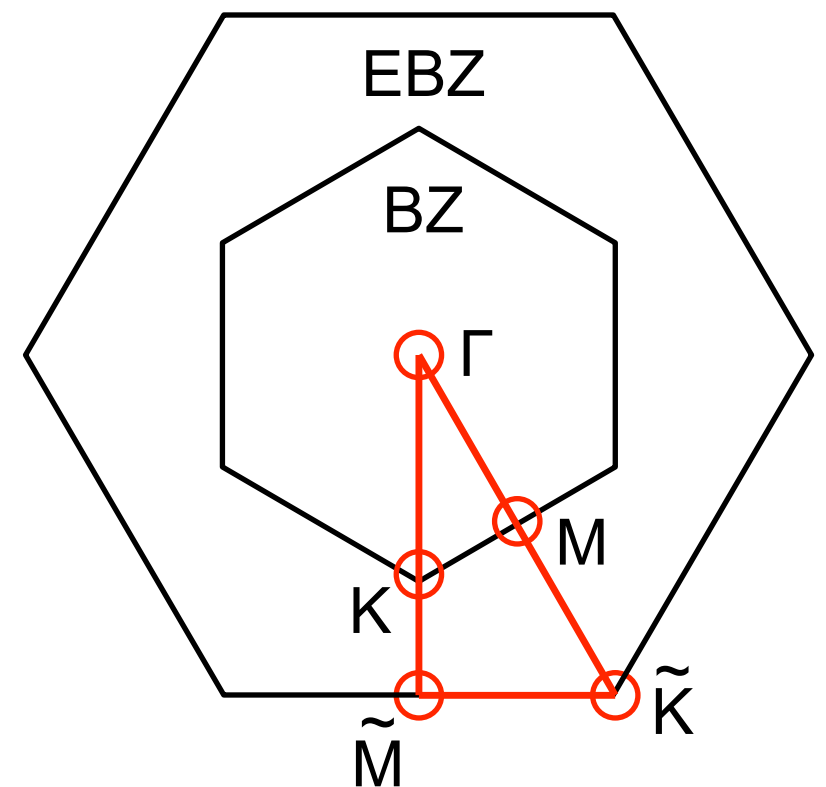


the response actually vanishes at the Γ and \tilde{K} points due to the factor

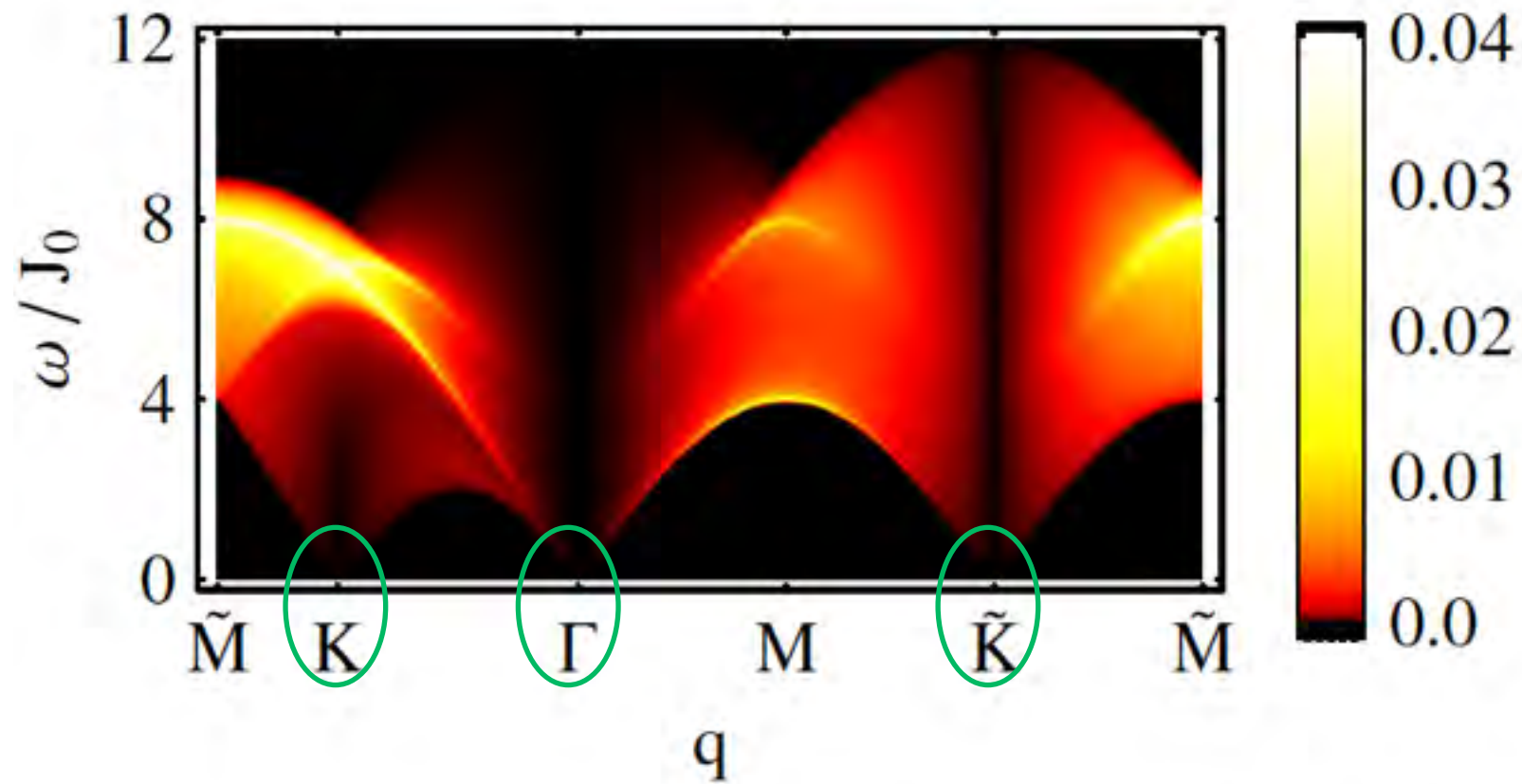
$$\tilde{I}_0(\mathbf{k}, \mathbf{q}) \propto [\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}-\mathbf{k}}]^2$$

$$J_{x,y,z} = J_0$$

Reciprocal space

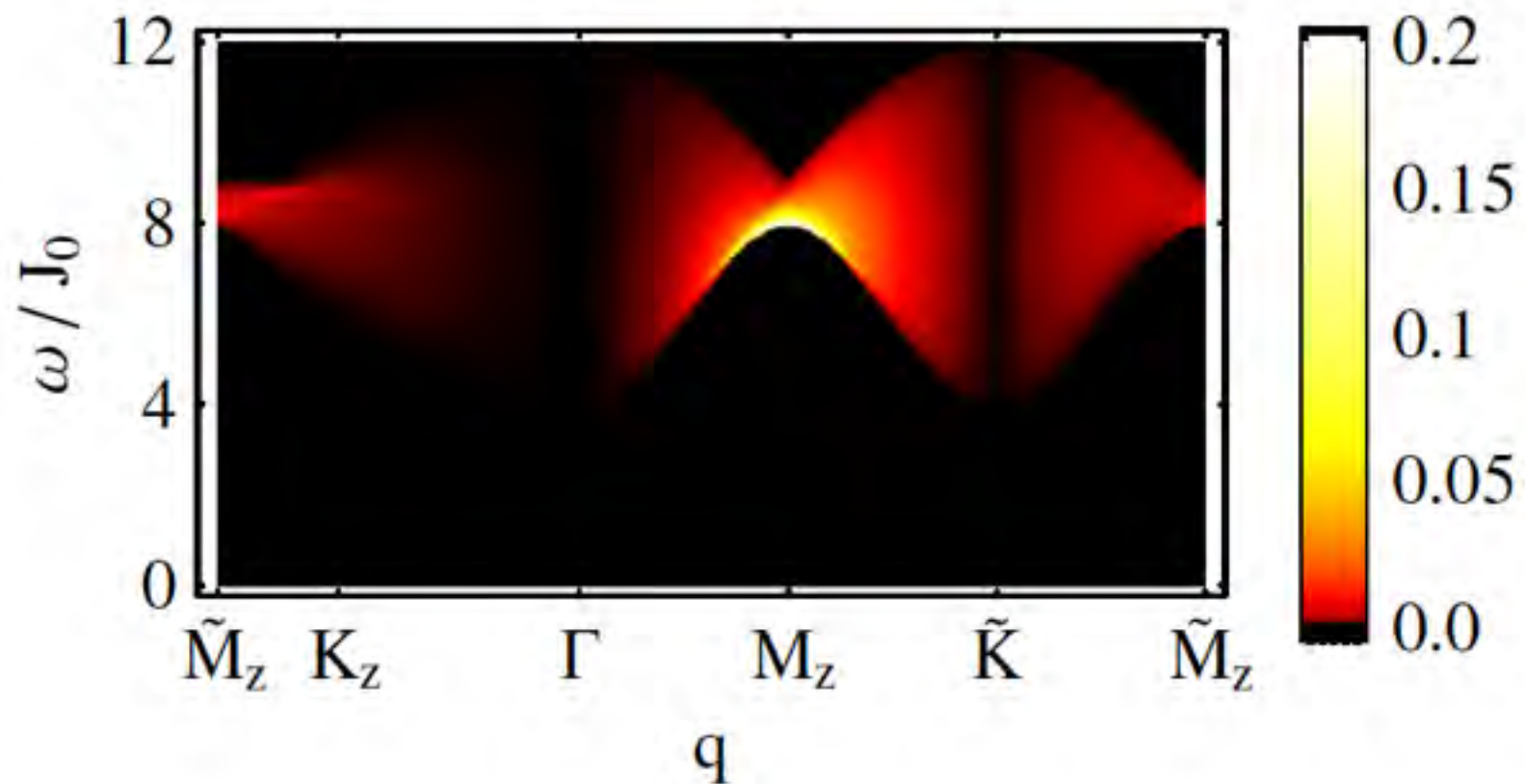
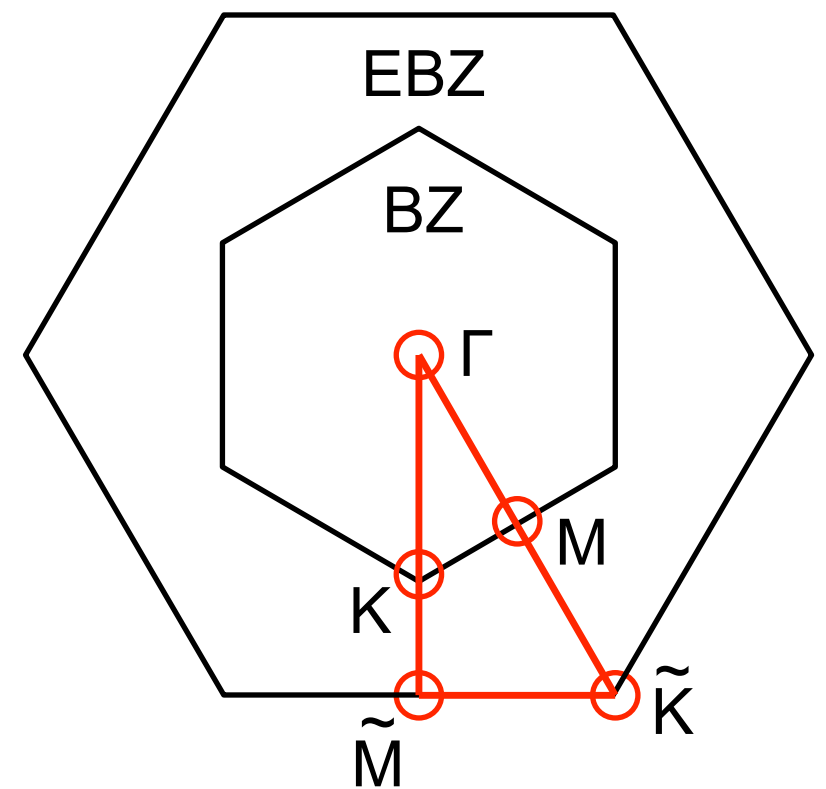


RIXS response in SC channel



$$J_{x,y,z} = J_0$$

Reciprocal space



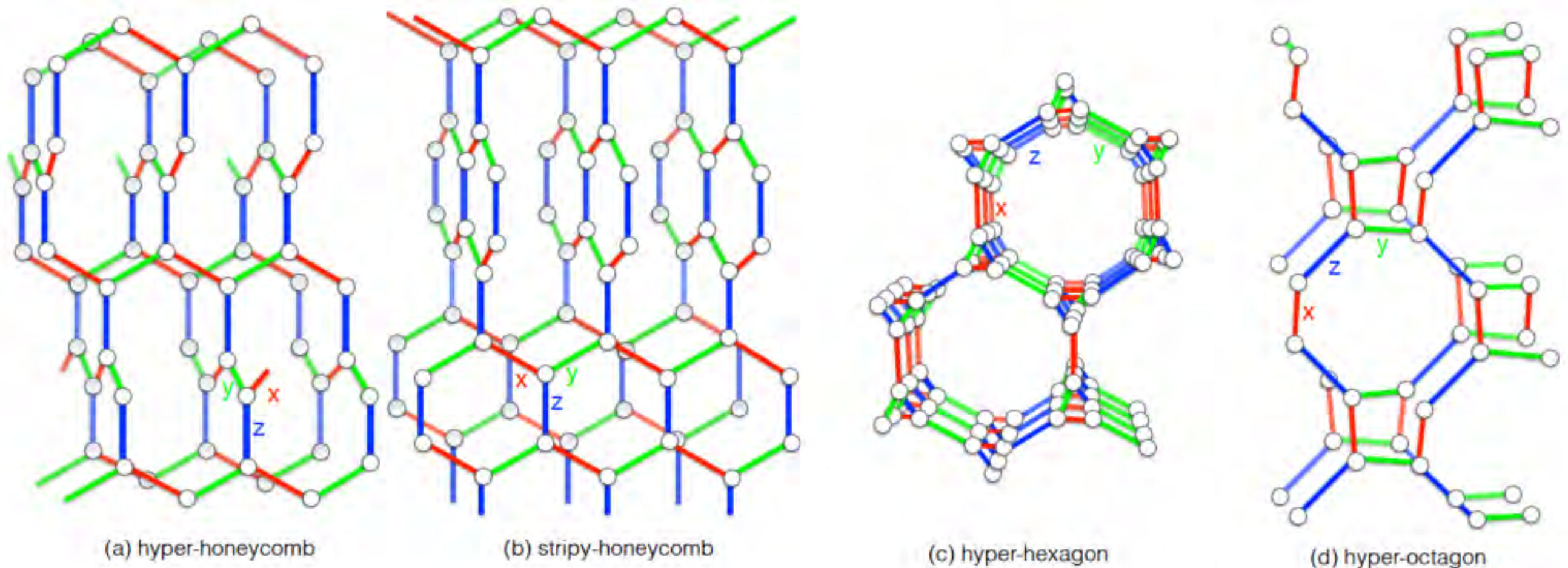
$$J_{x,y} = J_0/2$$

$$J_z = 2J_0$$

Results: SC channel in 3D Kitaev models

$$I_0(\omega, \mathbf{q}) \propto \sum_{\mathbf{k}, \mu, \mu'} |(\mathcal{A}_{\mathbf{q}, \mathbf{k}})_{\mu\mu'}|^2 \delta(\omega - \varepsilon_{\mathbf{k}, \mu} - \varepsilon_{\mathbf{q}-\mathbf{k}, \mu'})$$

For each model, the low-energy(gapless) response is determined by the nodal structure of the fermions.

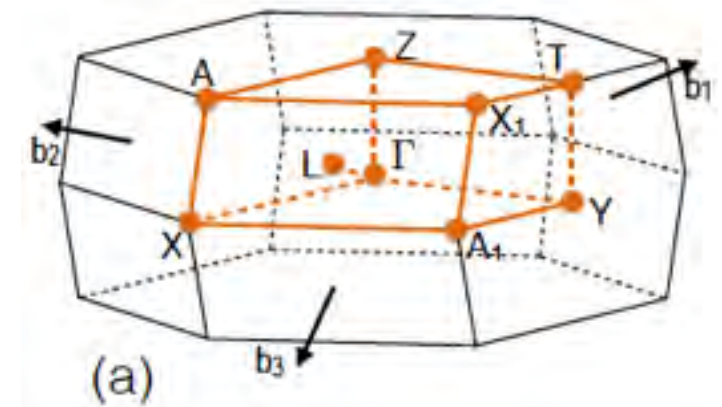
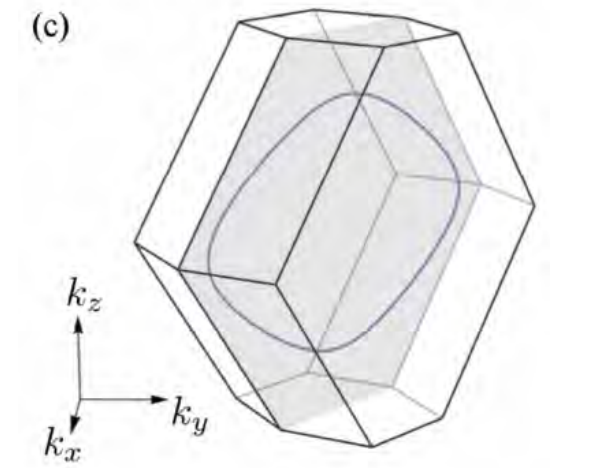
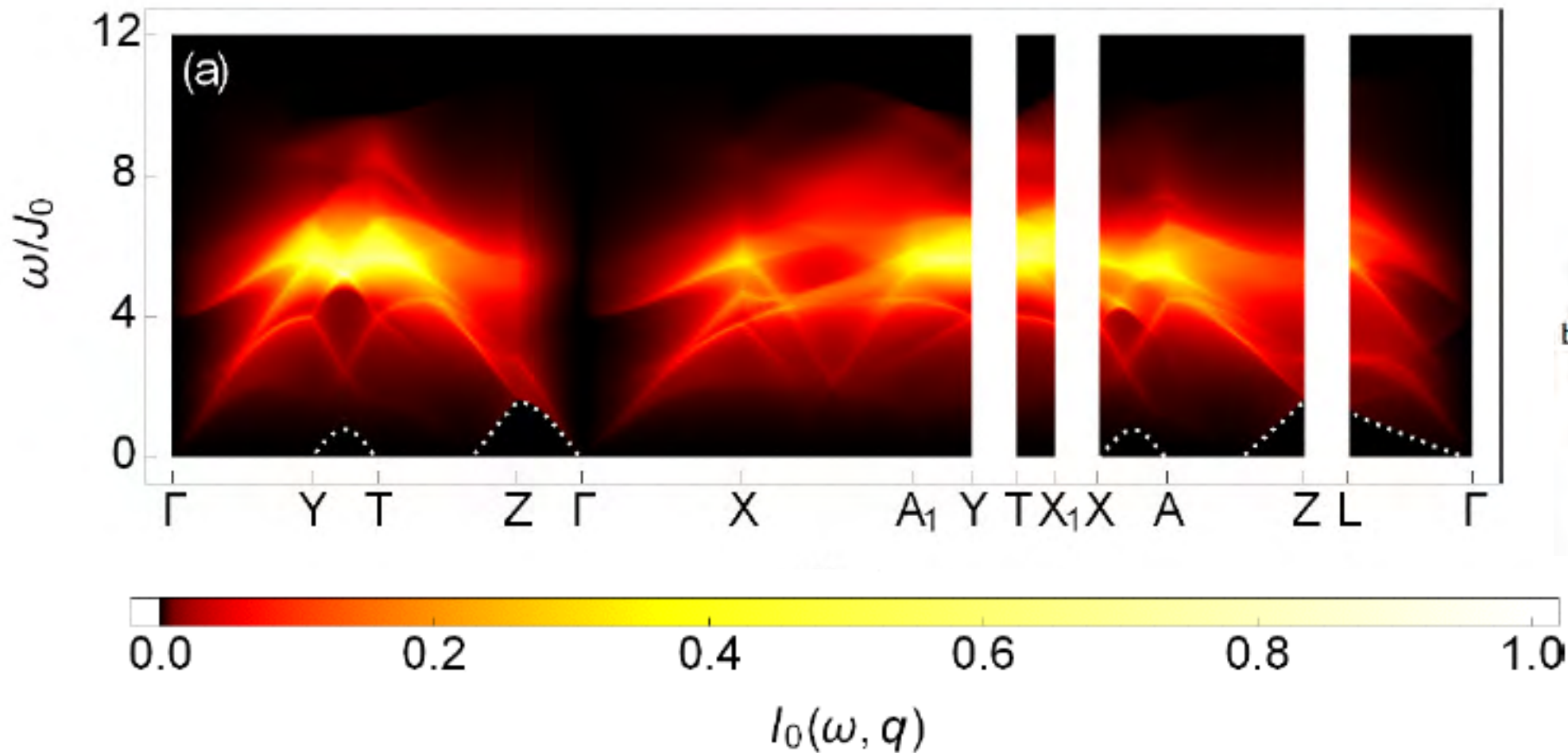


closed line of Dirac nodes

gapless Weyl points

Fermi surfaces

Hyperhoneycomb lattice

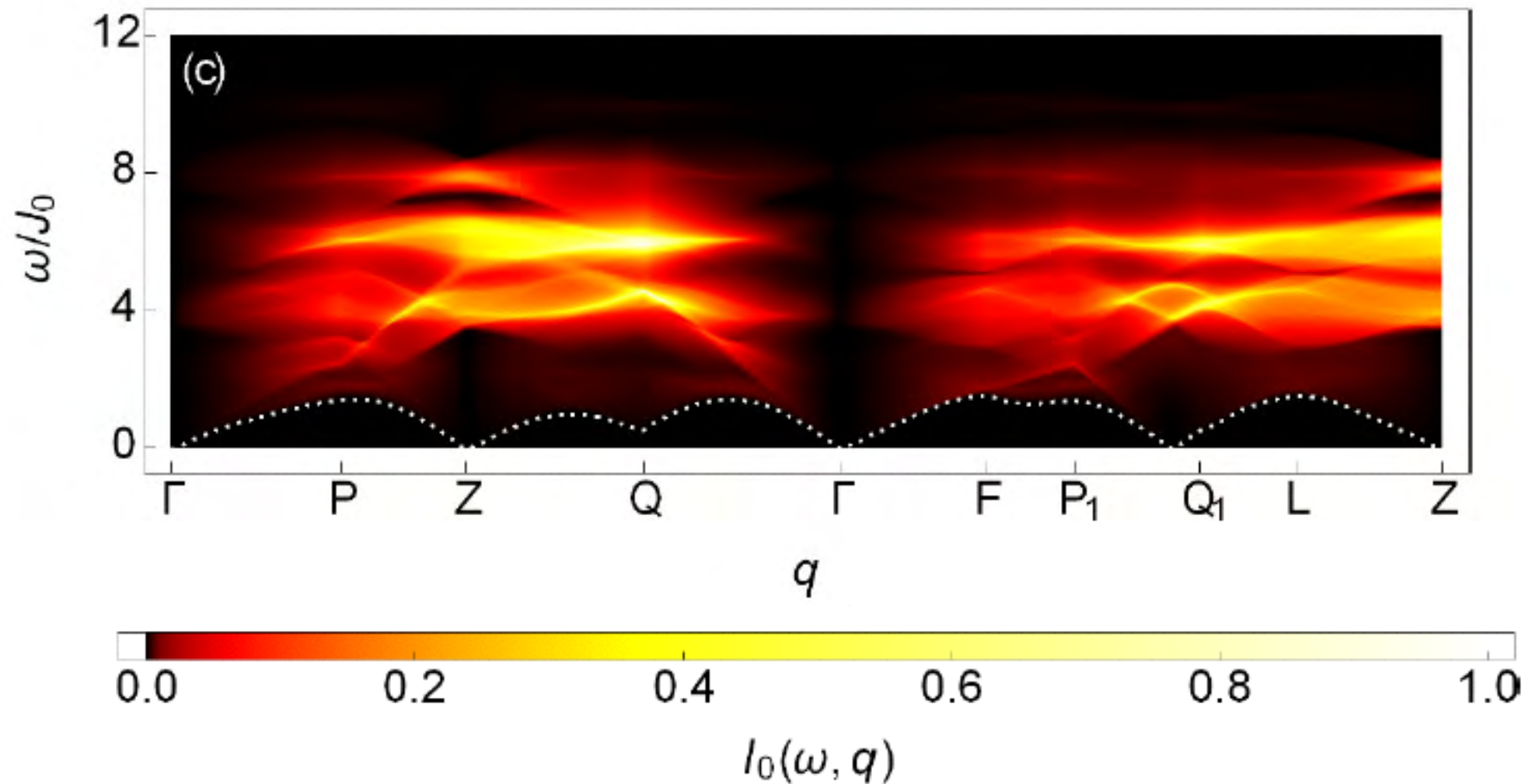


The Majorana fermions are gapless along a nodal line within the Γ -X-Y plane.



The response is thus gapless in most of the Γ -X-Y plane and also in most of the Z-A-T plane. However, it is still gapped at a generic point of the BZ.

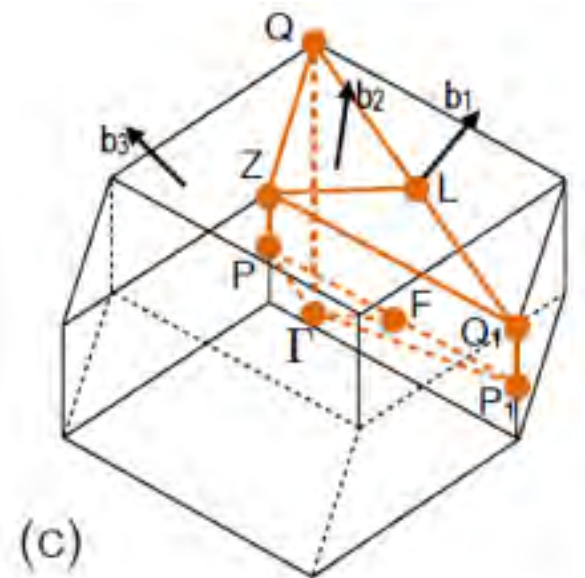
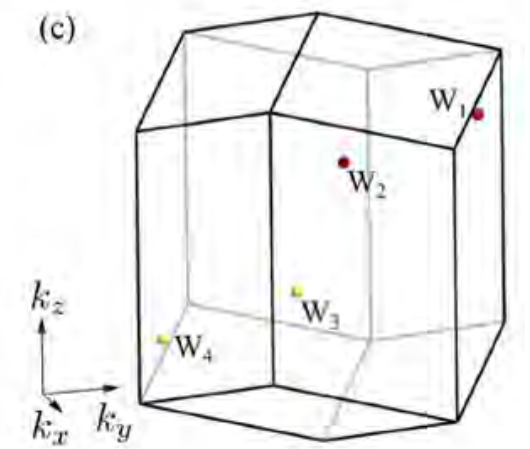
Hyperhexagon lattice



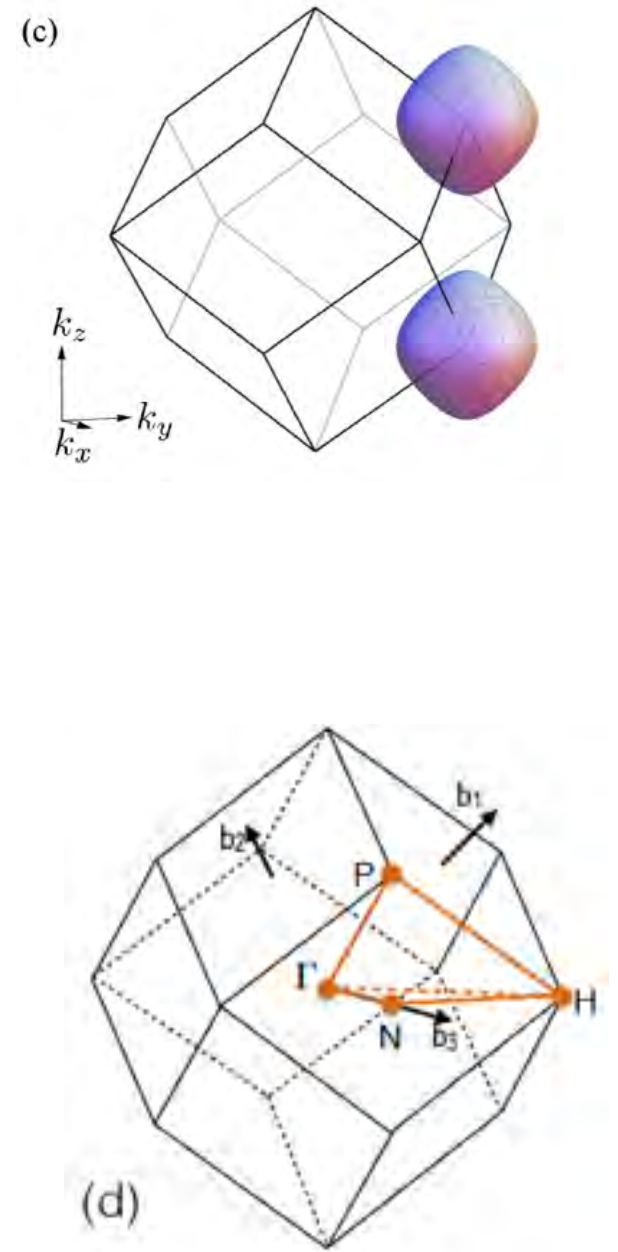
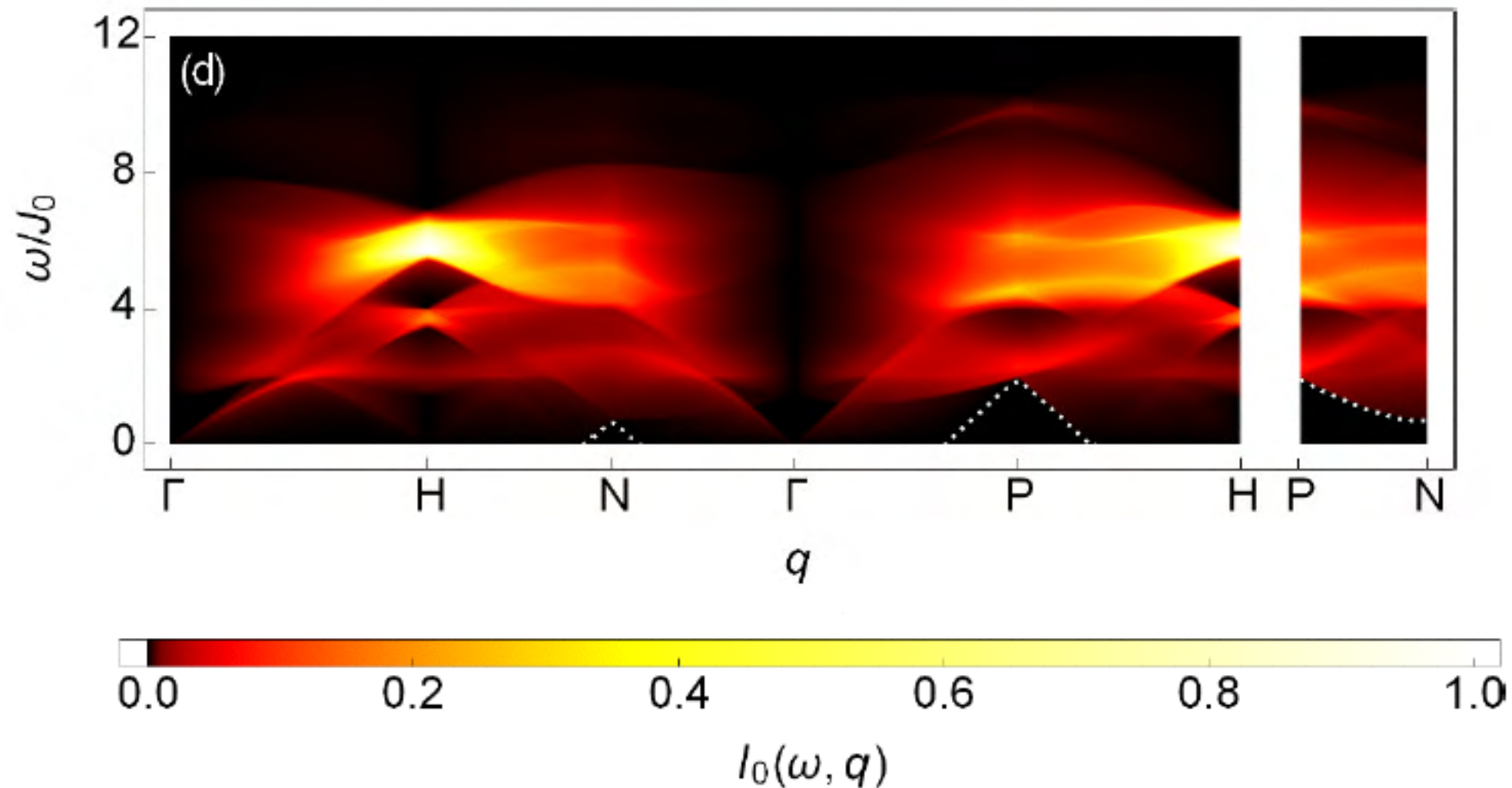
The fermions are gapless at Weyl points.



The response is thus only gapless at particular points of the BZ.



Hyperoctagon lattice



The Majorana fermions are gapless on a Fermi surface.



The response is thus gapless in most of the BZ.

Generic Kitaev spin liquids

Time-reversal-symmetric perturbations w.r.t. H_K

[Song, You, Balents, PRL 2016]

High-energy response is robust against perturbations, even beyond the phase transition into an ordered phase.

Low-energy response of a generic KSL can be completely different from that of a pure Kitaev model.

NSC channels (i.e., INS response): Gap disappears

SC channel: Two-fermion response

$$I_0(\omega, \mathbf{q}) \propto \int_{\text{BZ}} d^2\mathbf{k} \, \delta(\omega - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}-\mathbf{k}}) \times [\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}-\mathbf{k}}]^2 \left| 1 - e^{i\varphi_{\mathbf{k}}} e^{i\varphi_{\mathbf{q}-\mathbf{k}}} \right|^2$$

Robust

Robust

Survives only for perturbations with
three-fold rotation symmetry

Thank you