Interactions and transport in Majorana wires

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Content

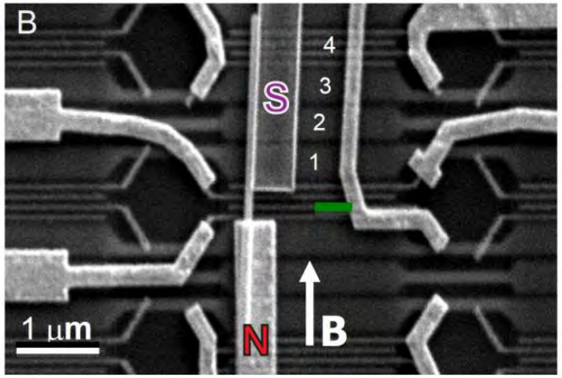
Low energy transport theory in Majorana wire junctions,
PRB 94, 155445 (2016)
Alex Zazunov, Reinhold Egger and ALY

Josephson effect in multiterminal topological junctions
(in preparation)
With R. Egger and A. Zazunov (Dusseldorf) and Miguel Alvarado (UAM)

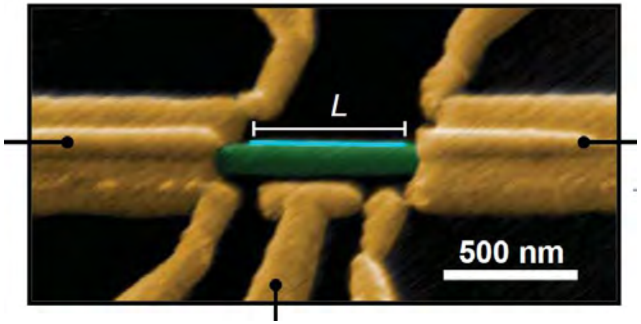
Zero-energy pinning from interactions in Majorana nanowires,
NPJ Quantum Materials 2, 13 (2017)
Fernando Dominguez, Jorge Cayao, Pablo San José, Ramón Aguado, ALY & Elsa Prada

Hybrid nanowire devices (exp)

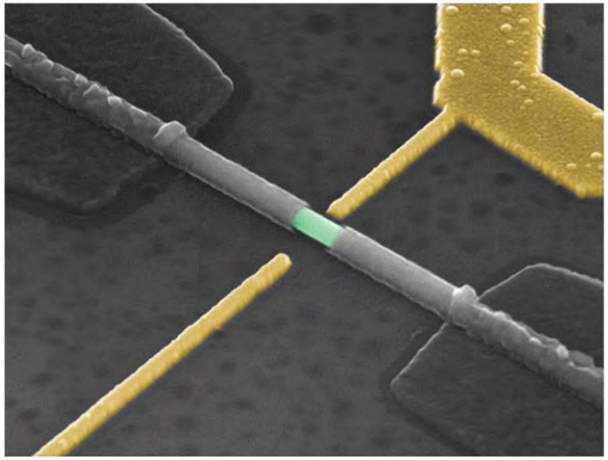
Delft



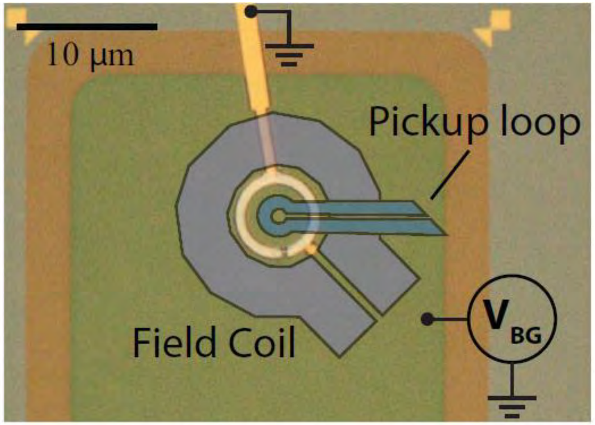
Copenhagen

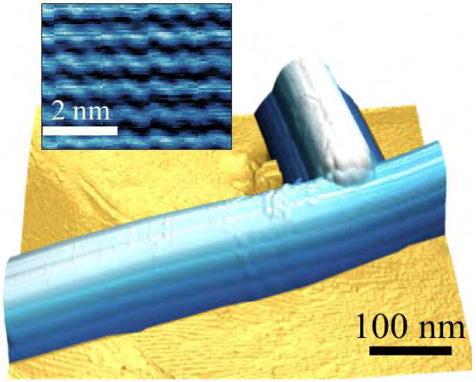


Saclay



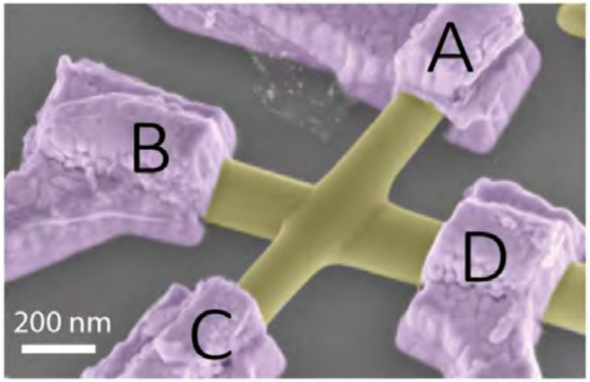
Stanford/Copenhagen





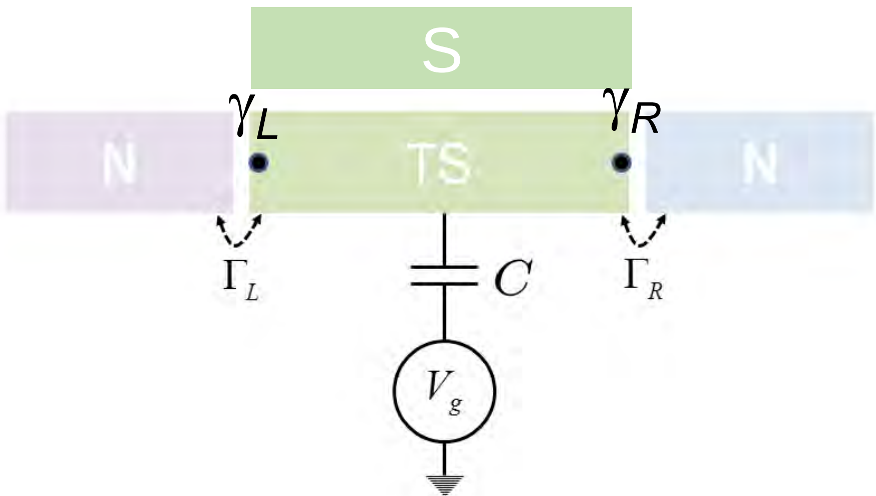
Weizmann

Delft

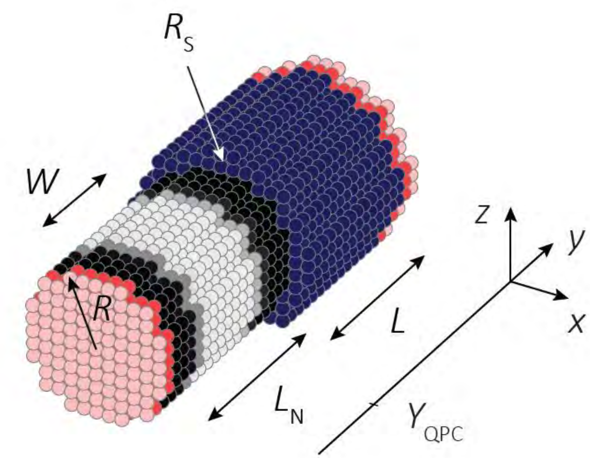


Theoretical modeling

Minimal: lowest energy states
Role of interactions
Analytical results



Extended: large discrete basis
Role of disorder
Numerical



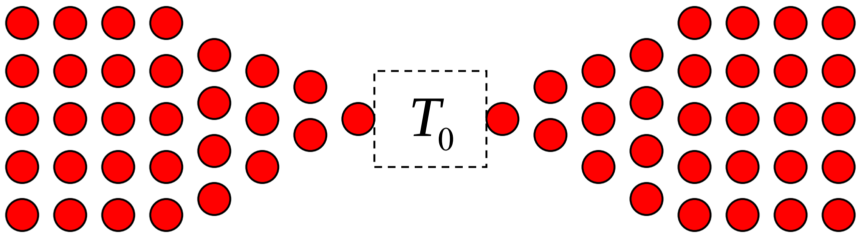
Intermediate: effective low energy theory

Transport, Subgap+continuum
possible analytical results

Hamiltonian Approach

e.g. Cuevas, Martín-Rodero & ALY, PRB (1996)

Left lead



Right lead

$$H_{contact} = H_L + H_R + \sum_{\sigma} \underbrace{T_0 (c^{+}_{L\sigma} c_{R\sigma} e^{i\phi(t)/2} + c^{+}_{R\sigma} c_{L\sigma} e^{-i\phi(t)/2})}_{H_T}$$

normal case: *transmission coefficient*

$$\tau = \frac{4\beta}{(1+\beta)^2} \qquad \beta = \left(\frac{T_0}{W}\right)^2$$

bandwidth

$$\phi(t) = \phi_0 + \frac{2eVt}{\hbar}$$

BCS superconductors:

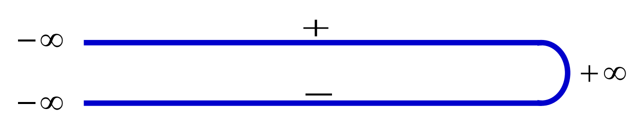
$$H_{L,R} = \sum_{k\sigma} \varepsilon_k^{L,R} c^{+}_{k\sigma} c_{k\sigma} + \sum_k \Delta_{L,R} c^{+}_{k\uparrow} c^{+}_{-k\downarrow} + h.c.$$

Keldysh + Nambu formalism →

$$\hat{\Psi}_i = \begin{pmatrix} c_{i\uparrow} \\ c^{+}_{i\downarrow} \end{pmatrix} \qquad \hat{T}_{LR}(t) = T_0 \begin{pmatrix} e^{i\phi(t)/2} & 0 \\ 0 & -e^{-i\phi(t)/2} \end{pmatrix} = \hat{T}^{*}_{RL}(t)$$

$$S_{eff}[\hat{\Psi}_L, \hat{\Psi}_R] = \int_C dt \left(\hat{\bar{\Psi}}_L, \hat{\bar{\Psi}}_R\right) \begin{pmatrix} \hat{g}_L^{-1} & \hat{T}_{LR} \\ \hat{T}_{RL} & \hat{g}_R^{-1} \end{pmatrix} \begin{pmatrix} \hat{\Psi}_L \\ \hat{\Psi}_R \end{pmatrix}$$

Keldysh contour

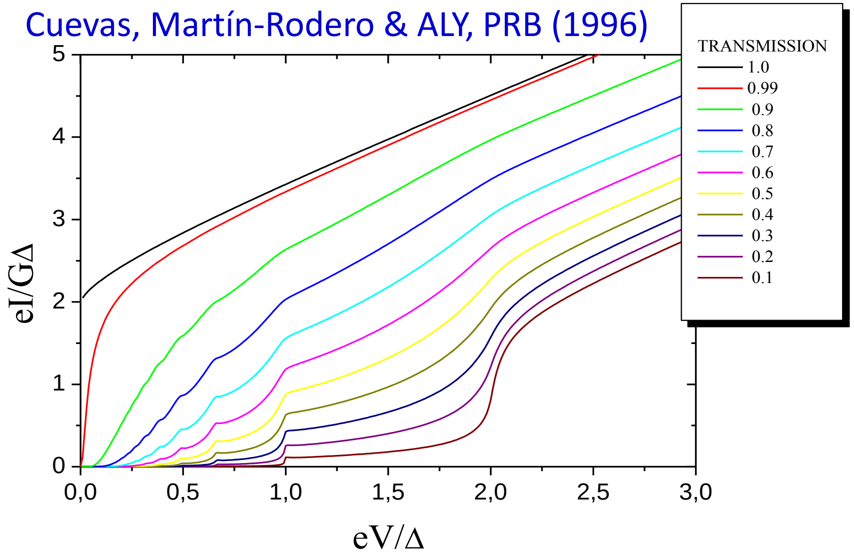


Basic ingredient: **Boundary Green functions**

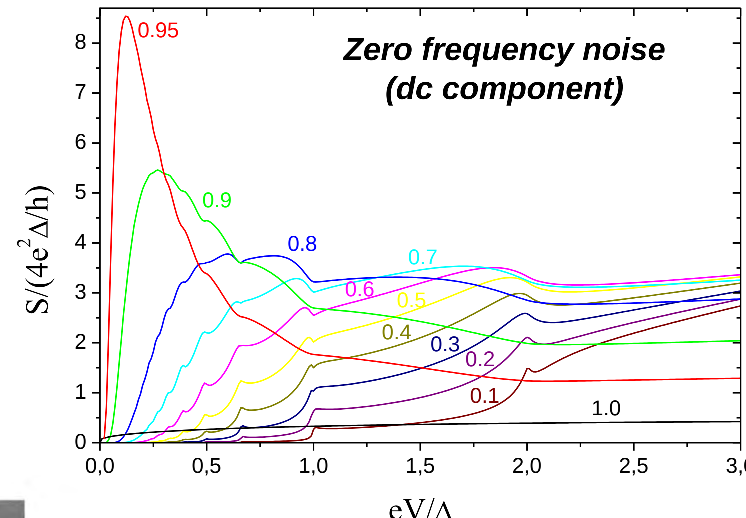
BCS case

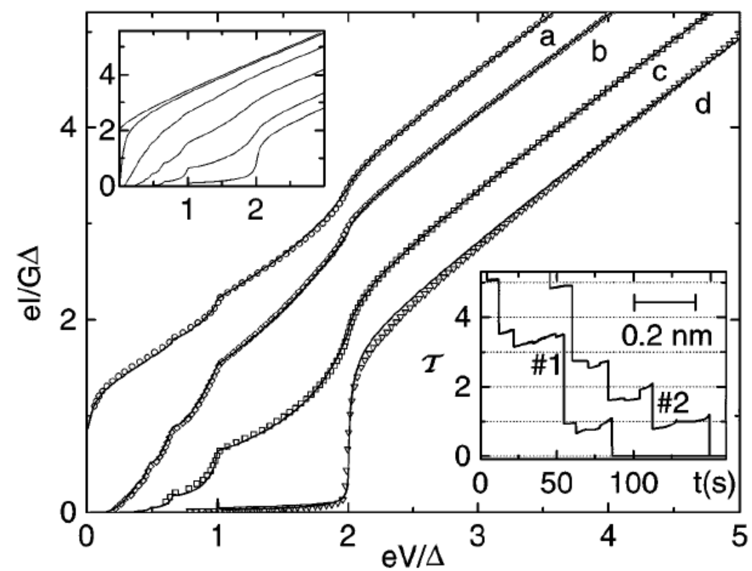
$$\hat{g}^{R,A}_{BCS} = \frac{-(\omega \pm i0^+)\hat{\sigma}_0 + \Delta\hat{\sigma}_x}{W\sqrt{\Delta^2 - (\omega \pm i0^+)^2}}$$

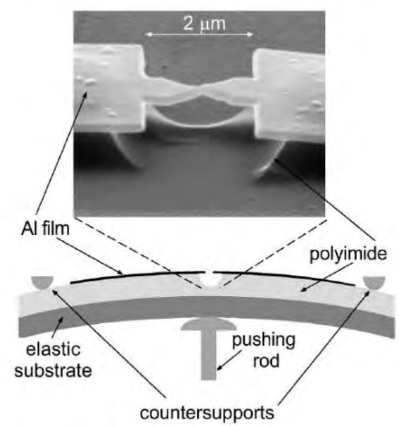
Cuevas, Martín-Rodero & ALY, PRB (1996)



Cuevas, Martín-Rodero & ALY, PRL (1999)

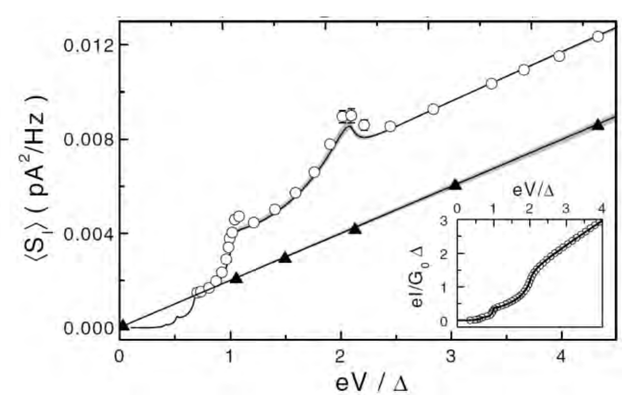






E. Scheer et al, PRL (1997)

R. Cron et al, PRL (2001)



TS case: Boundary GF for the Kitaev model

L/R chains in real space

$$H_{L/R} = \sum_{j \in L/R} t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + \text{h.c.}$$

infinte chain (k space, Nambu)

$$H_0 = \sum_k \Psi_k^\dagger \underbrace{\begin{pmatrix} t\cos k & -i\Delta \sin k \\ i\Delta \sin k & -t\cos k \end{pmatrix}}_{\mathcal{H}_k} \Psi_k$$

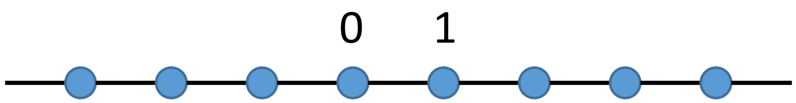
$$\Psi_k^T = \begin{pmatrix} c_k & c_{-k}^\dagger \end{pmatrix}$$

infinite chain GF in real space

$$\hat{G}_{ij}^0 = \sum_k \left[\omega - \mathcal{H}_k\right]^{-1} e^{ik|i-j|}$$

$$\hat{G}_{00}^0 = \frac{-\omega}{\sqrt{(\omega^2 - \Delta^2)(\omega^2 - t^2)}} \sigma_0$$

$$\hat{G}_{01}^0 = \frac{t(z_1^2 + 1) + \Delta(z_1^2 - 1)\sigma_x}{\sqrt{(\omega^2 - \Delta^2)(\omega^2 - t^2)}} \sigma_z$$

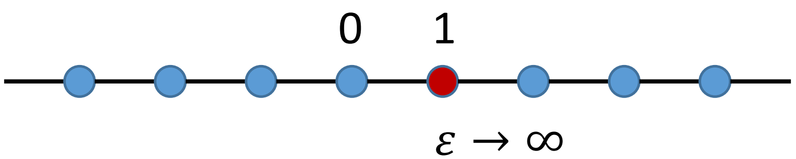


$$z_1^2 = \frac{2\omega^2 - (t^2 + \Delta^2)}{t^2 - \Delta^2} - \text{sign}\left(2\omega^2 - (t^2 + \Delta^2)\right) \sqrt{\left(\frac{2\omega^2 - (t^2 + \Delta^2)}{t^2 - \Delta^2}\right)^2 - 1}$$

Dyson equation for chain breaking

$$\hat{g}_L = \hat{G}_{00}^0 - \hat{G}_{01}^0 \left(\hat{G}_{00}^0\right)^{-1} \hat{G}_{10}^0$$

$$\hat{g}_R = \hat{G}_{00}^0 - \hat{G}_{10}^0 \left(\hat{G}_{00}^0\right)^{-1} \hat{G}_{01}^0$$



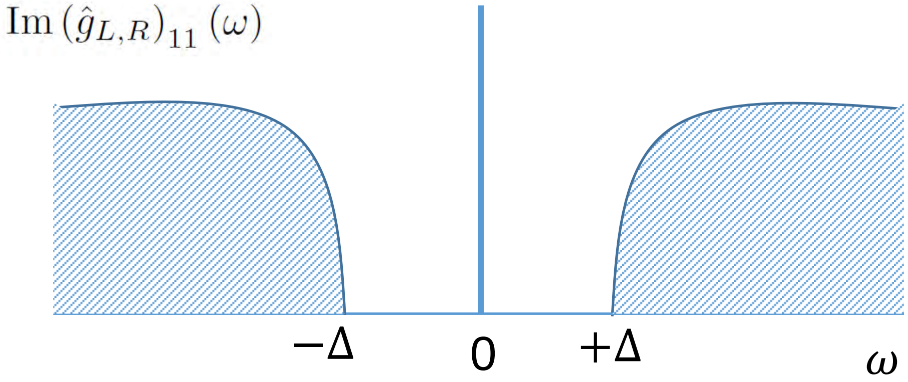
Boundary GF for the Kitaev model

Zazunov, Egger & ALY, PRB (2016)

Boundary GFs in $t >> \Delta$ *limit*

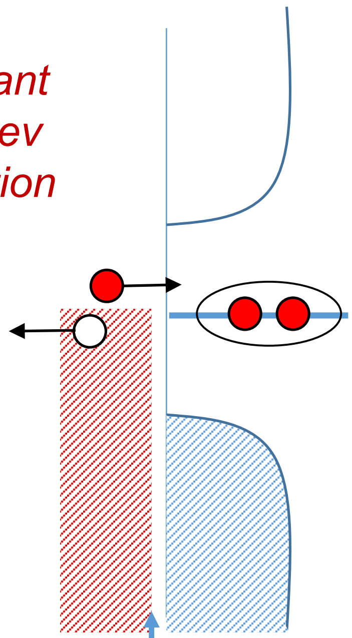
$$\hat{g}_L = \frac{2}{|t|\omega}\begin{pmatrix} \sqrt{\Delta^2-\omega^2} & \Delta \\ \Delta & \sqrt{\Delta^2-\omega^2} \end{pmatrix}$$

$$\hat{g}_R = \frac{2}{|t|\omega}\begin{pmatrix} \sqrt{\Delta^2-\omega^2} & -\Delta \\ -\Delta & \sqrt{\Delta^2-\omega^2} \end{pmatrix}$$



N-TS case: conductance and noise

Zazunov, Egger & ALY, PRB (2016)

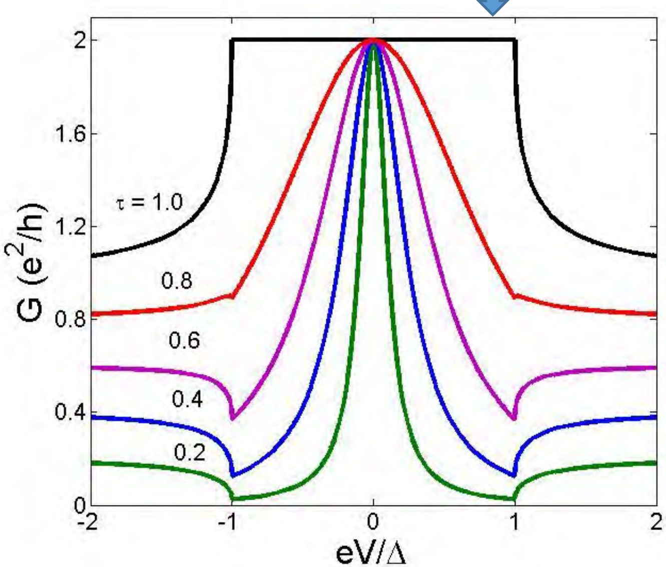


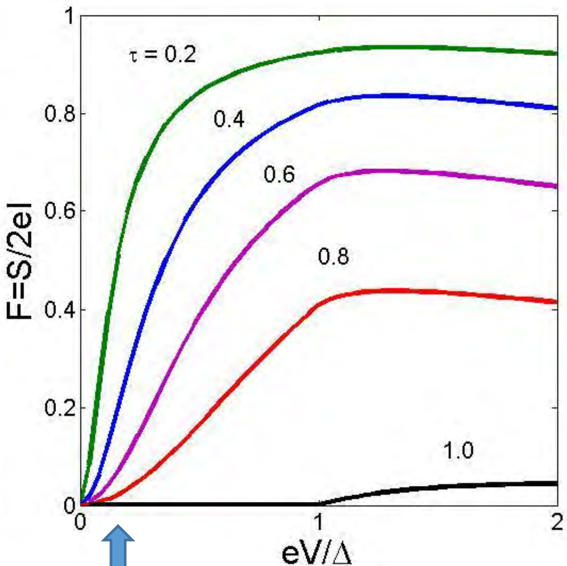
$$G(V, T=0) = \frac{2e^2}{h} J(eV)$$

zero-temperature conductance

$$J(\omega) = \begin{cases} 1/(1+\omega^2/\Gamma^2), & |\omega| < \Delta, \\ \tau \frac{\tau + (2-\tau)\sqrt{1-(\Delta/\omega)^2}}{\left[2-\tau+\tau\sqrt{1-(\Delta/\omega)^2}\right]^2}, & |\omega| \geq \Delta, \end{cases}$$

$$\Gamma = \frac{\tau\Delta}{2\sqrt{1-\tau}}$$

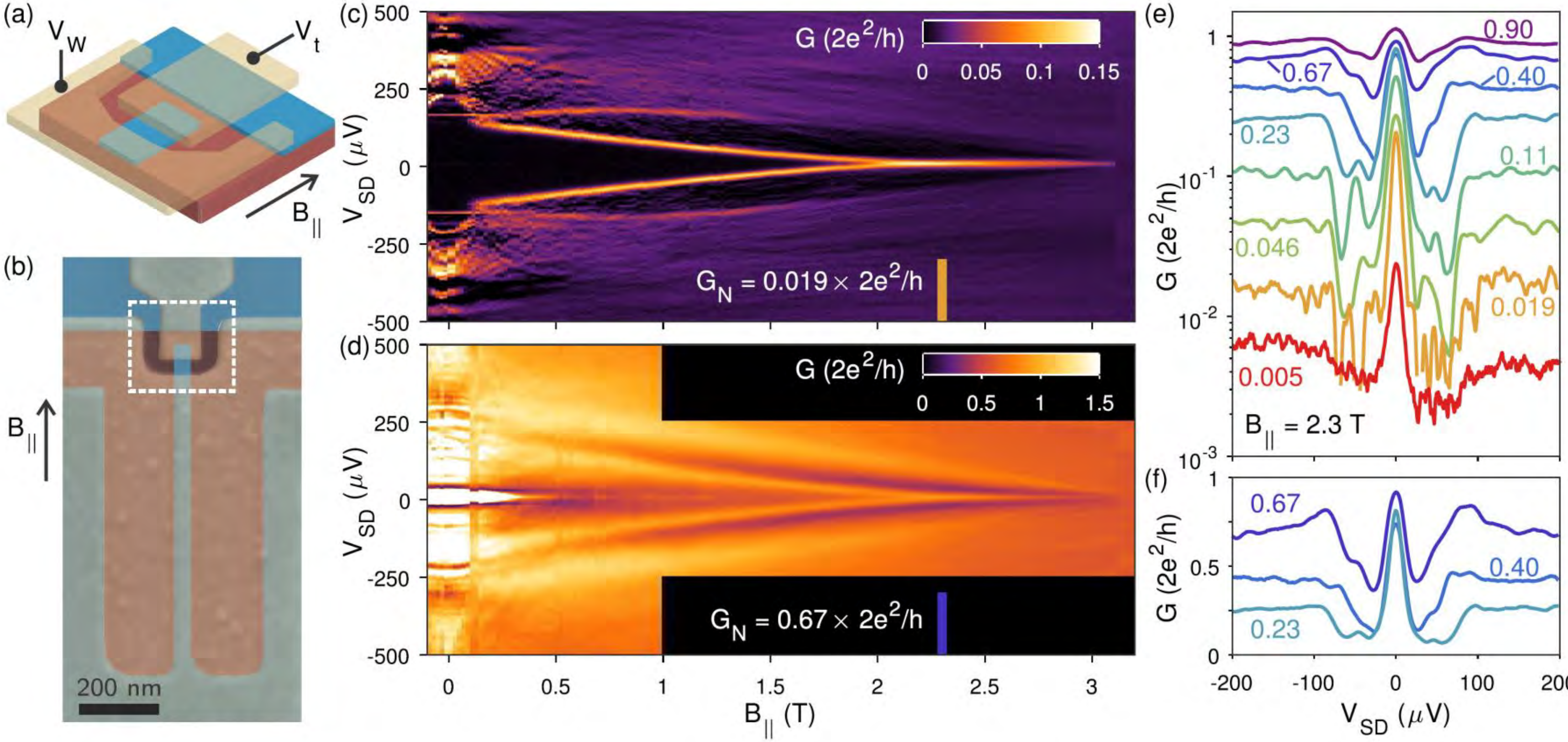




Subgap shot-noise

$$S = \frac{4e^2\Gamma}{h}\left(\tan^{-1}(eV/\Gamma) - \frac{eV/\Gamma}{1+(eV/\Gamma)^2}\right)$$

Nagging issue: $2e^2/h$ or not?

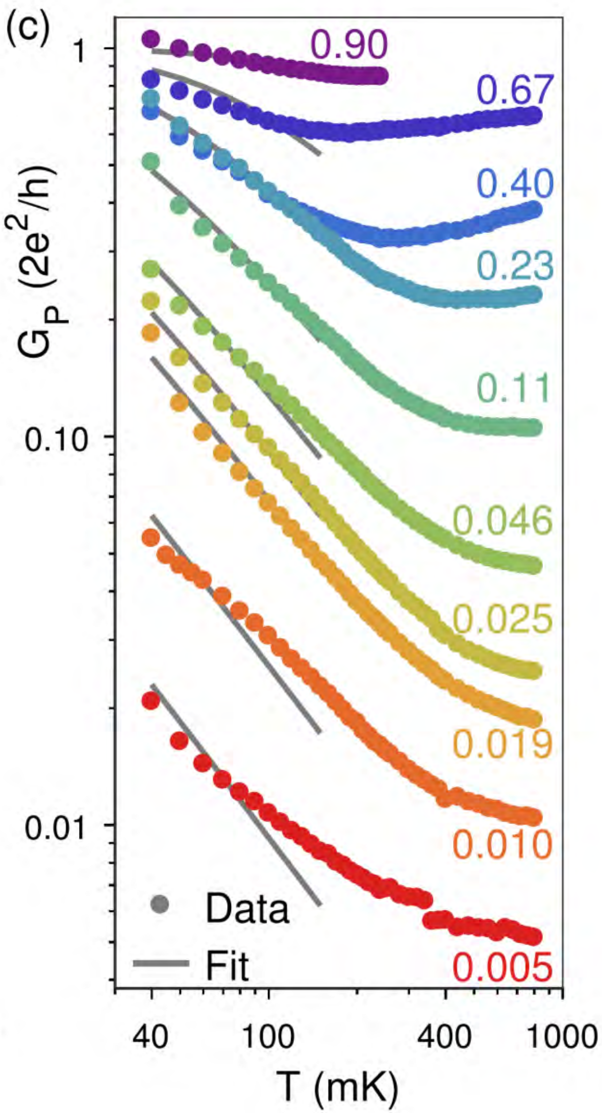


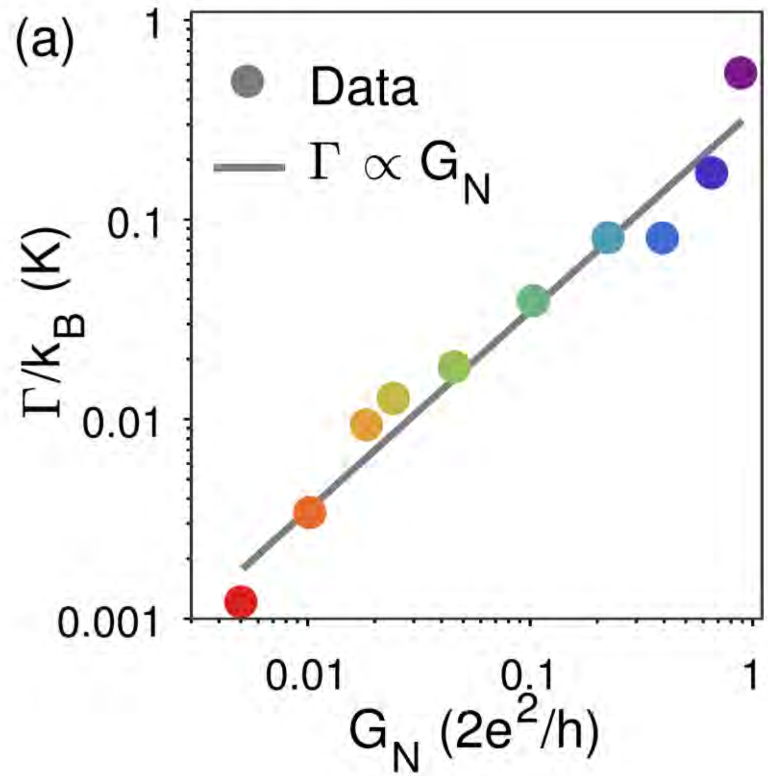
Al/InGaAs/InAs

Nichele et al. (CPH), preprint 2017

Courtesy by K. Flensberg

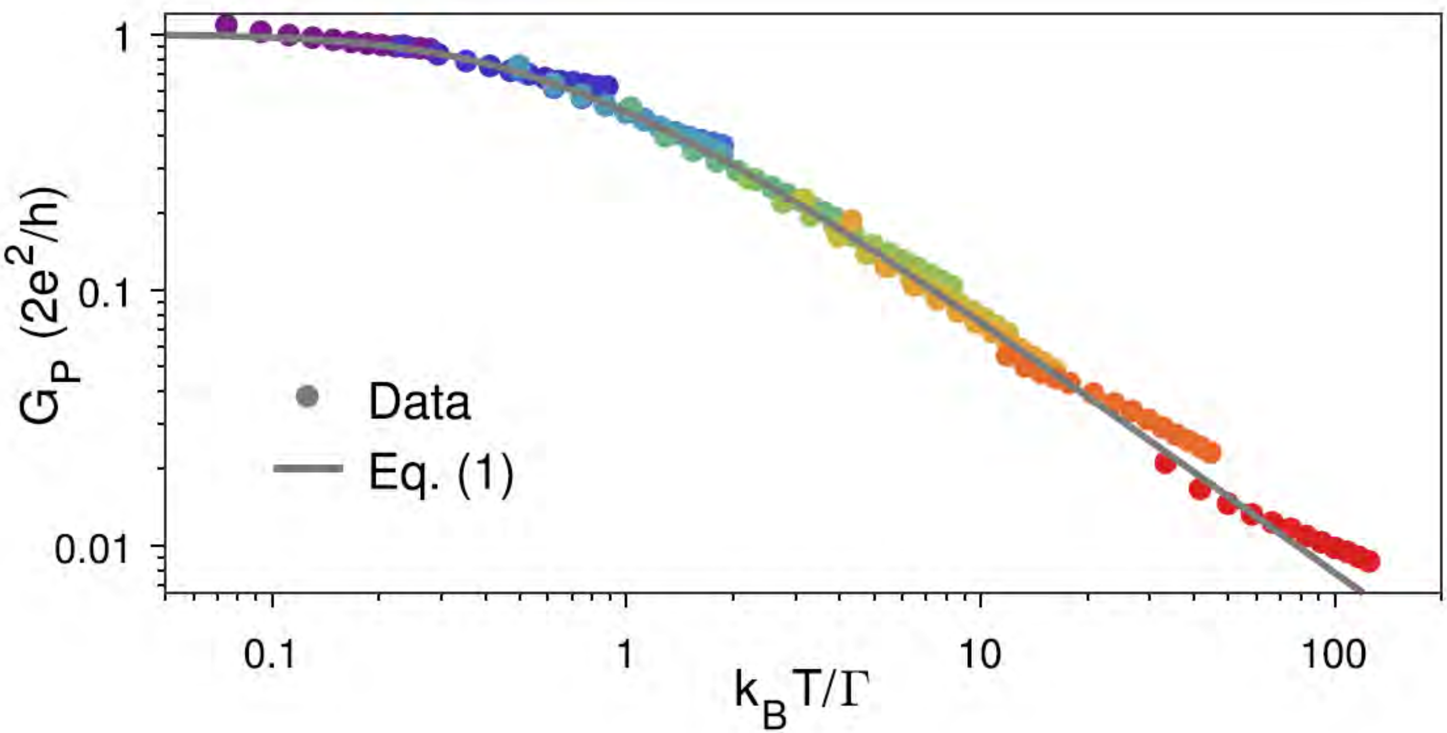
$$G_{\mathrm{P}} \approx \frac{e^2}{h}\int_{-\infty}^{\infty} d\omega \frac{2\Gamma^2}{\omega^2+\Gamma^2}\frac{1}{4k_{\mathrm{B}}T\cosh^2(\omega/(2k_{\mathrm{B}}T))}$$
$$= \frac{2e^2}{h} f(k_{\mathrm{B}}T/\Gamma),$$





$$\Gamma = \frac{\tau \Delta_{\mathrm{topo}}}{2\sqrt{1-\tau}}$$

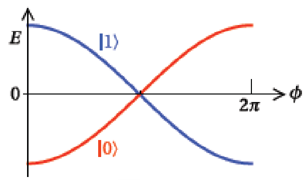
Courtesy by K. Flensberg



Courtesy by K. Flensberg

Equilibrium TS-TS case: frequency dependent noise

Zazunov, Egger & ALY, PRB (2016)

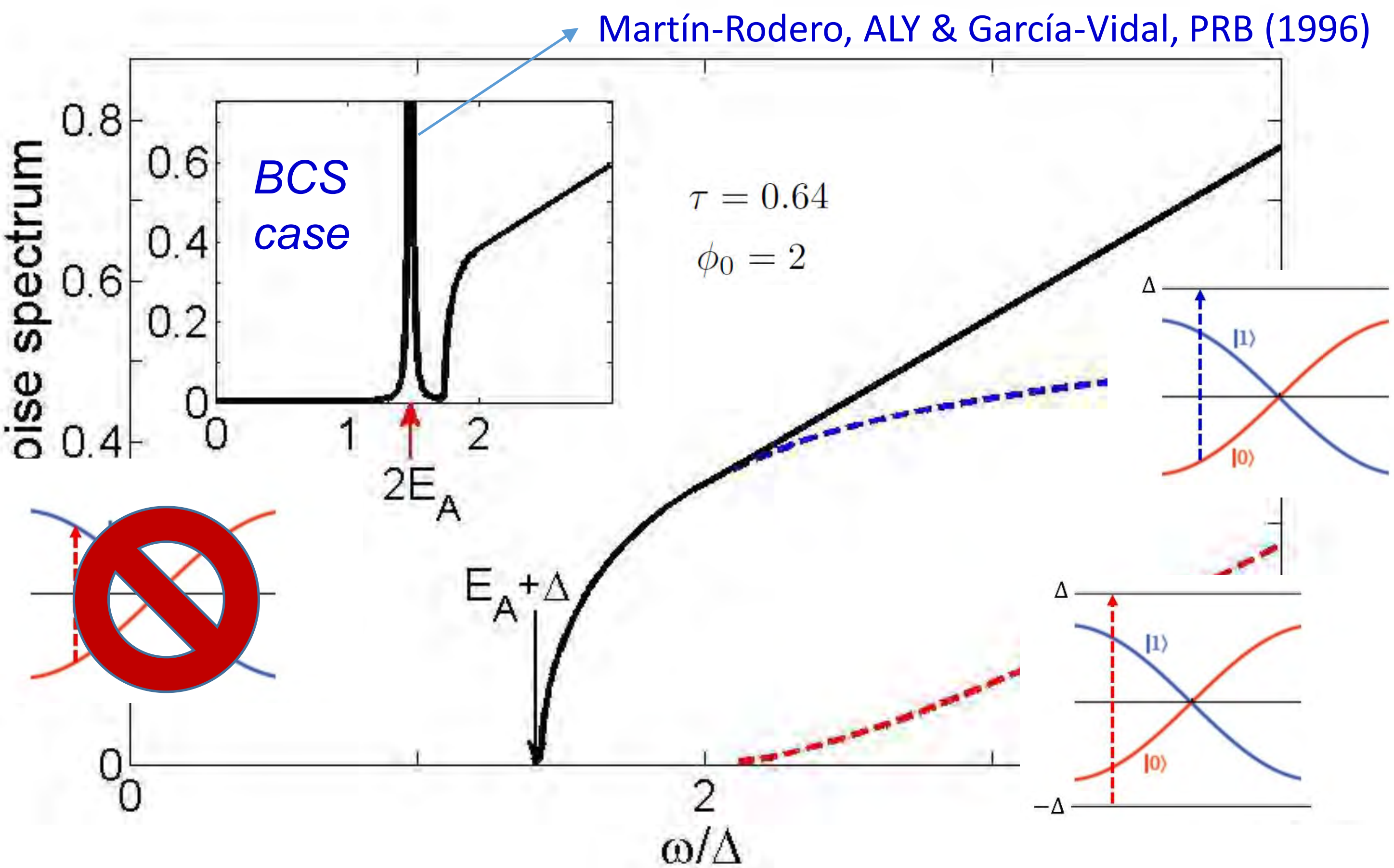


$$E_A(\phi_0) = \sqrt{\tau}\Delta\cos(\phi_0/2)$$

Andreev bound states (ABS): 4π periodicity

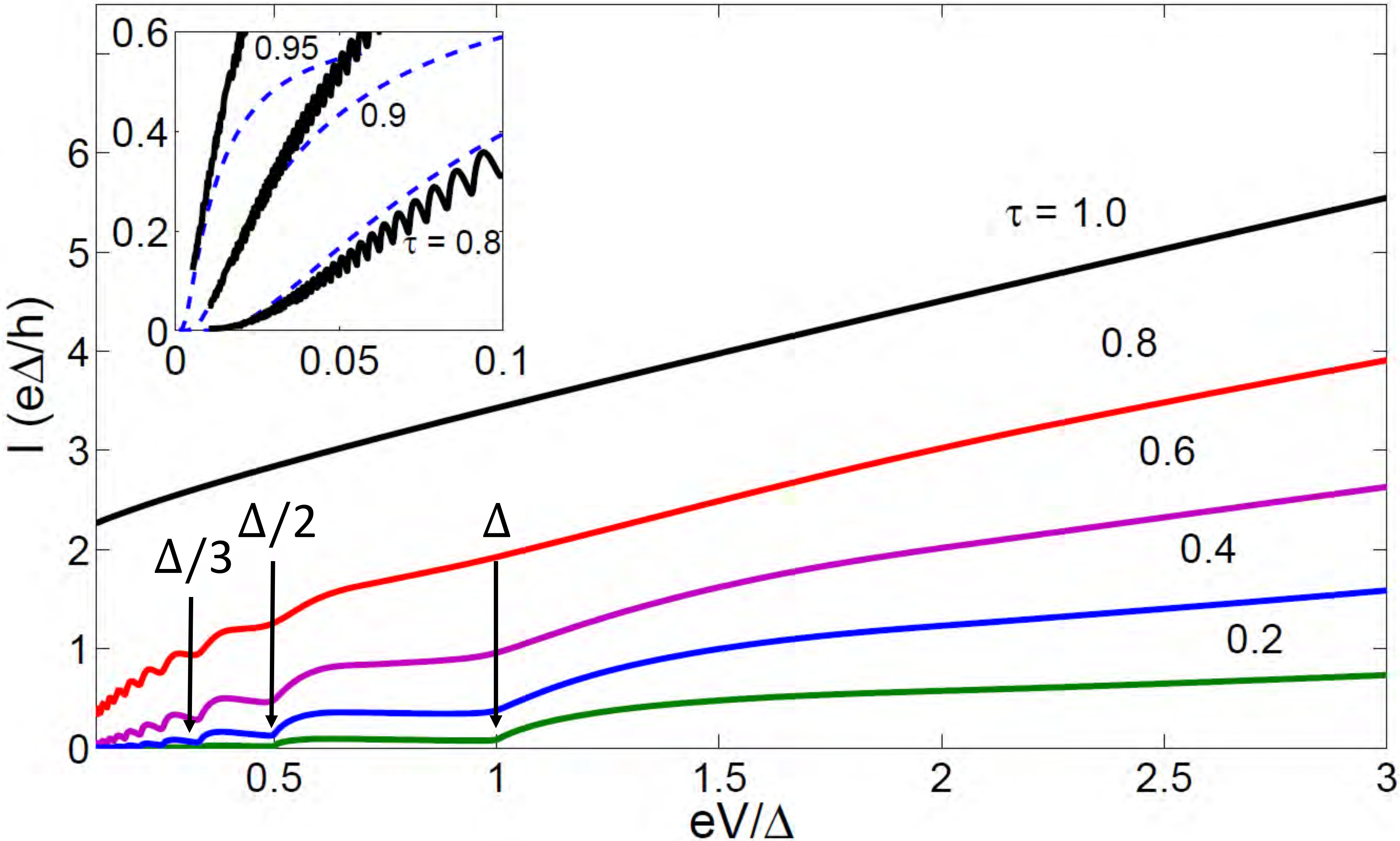
$$I(\phi_0) = \pm\frac{e\sqrt{\tau}\Delta}{2\hbar}\sin(\phi_0/2)$$

zero-temperature Josephson current



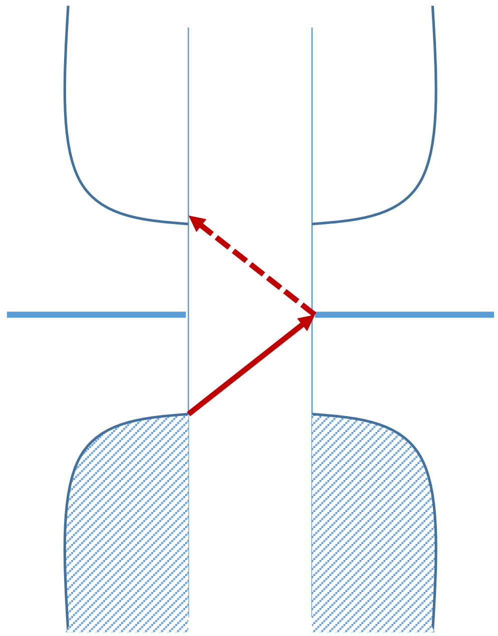
Non-equilibrium TS-TS case: MAR regime

Zazunov, Egger & ALY, PRB (2016)

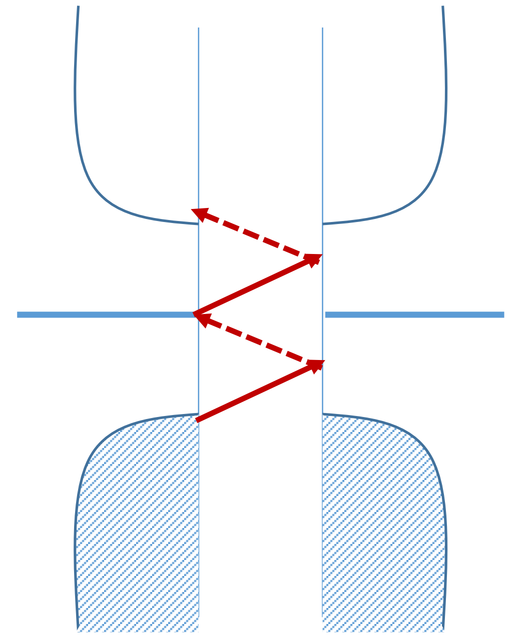


Badiane et al., PRL (2011)
San José et al., NJP (2013)

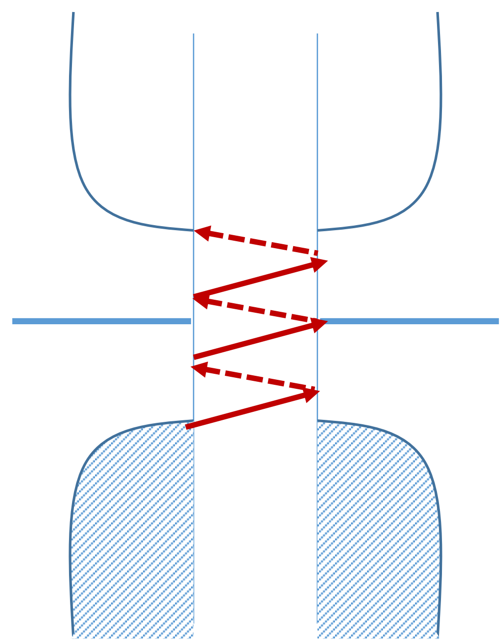
Subgap features at Δ/n instead of 2Δ/n



$V = \Delta$



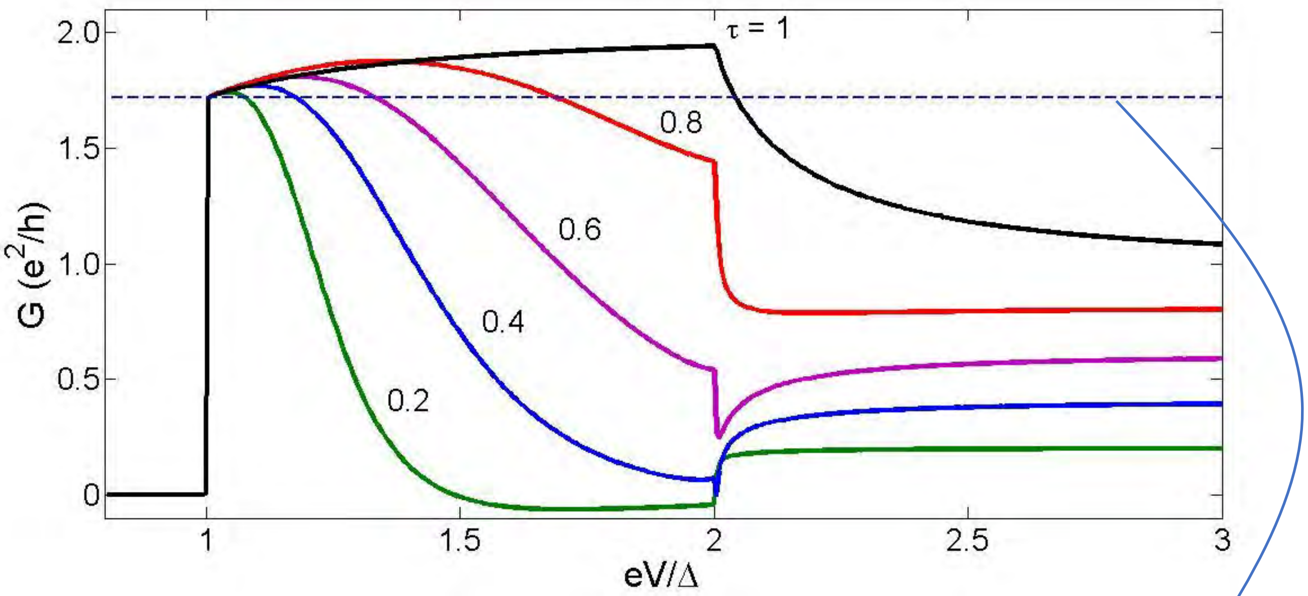
$V = \Delta/2$

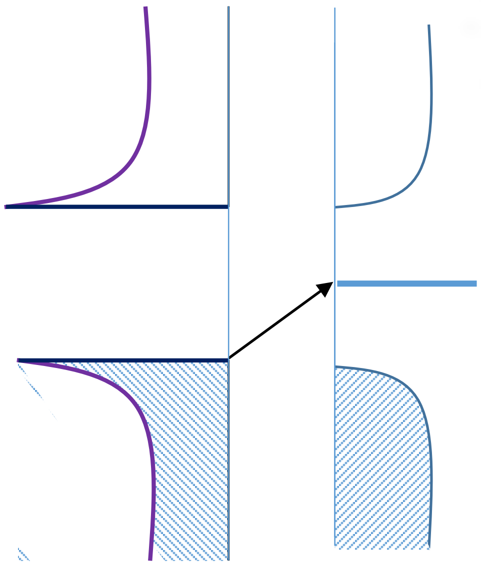


$V = \Delta/3$

S-TS case: differential conductance

Zazunov, Egger & ALY, PRB (2016)





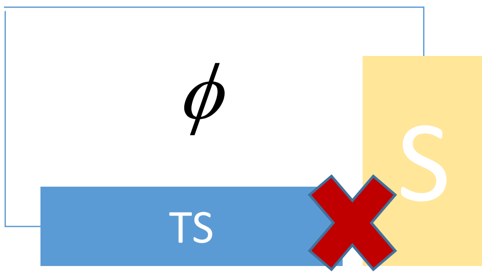
$V = \Delta$

$$G = (4 - \pi)\,\frac{2e^2}{h}$$

Peng et al., PRL (2015)

Multiterminal S-TS junctions

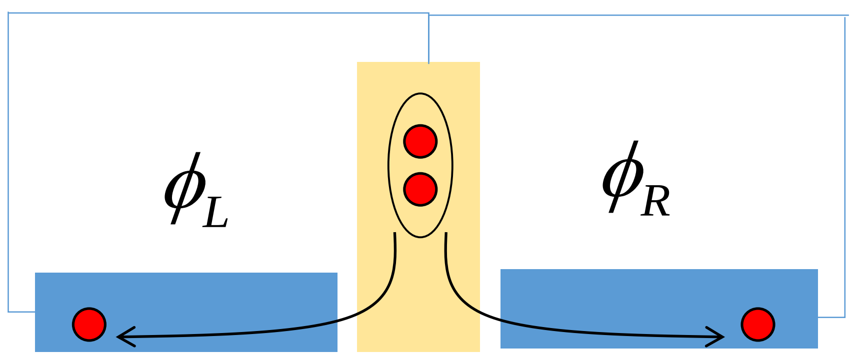
Previous work: topological states from multiterminal → Heck et al., PRB (2014)
Riwar et al., Nature Comm. (2016)



Two-terminal S-TS: Josephson blockade

Zazunov & Egger, PRB (2012)
Zazunov, Egger & ALY, PRB (2016)

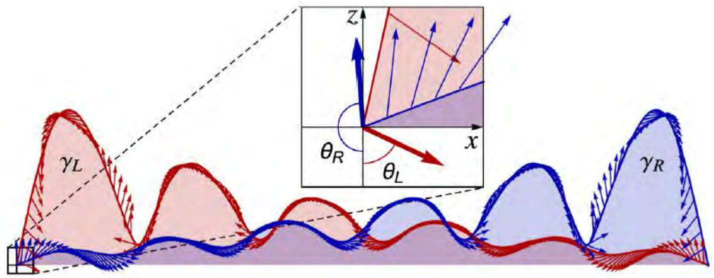
Three-terminal S-TS: lifting of Josephson blockade?

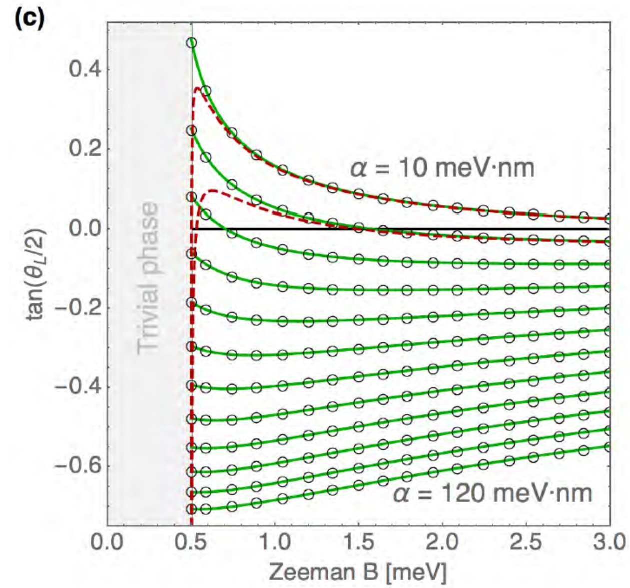


Multiterminal S-TS junctions: role of MBS spin structure

MBS spin-structure in single wire:

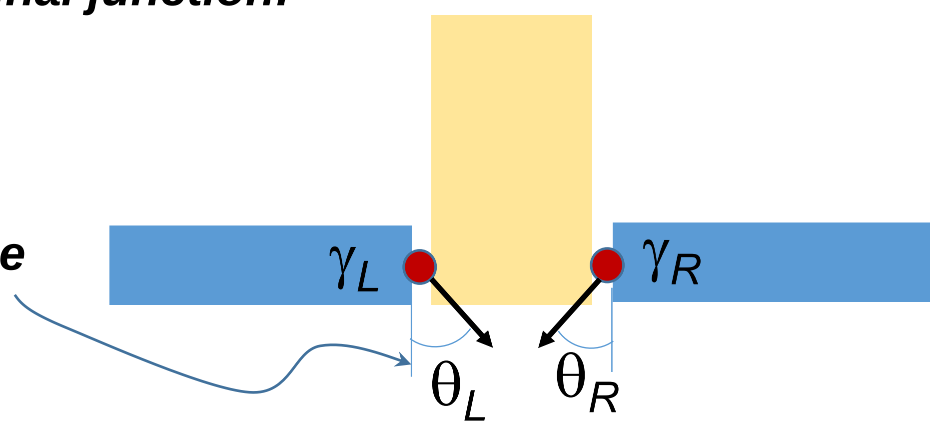
Sticlet, Bena & Simon, PRL (2012)
Prada, Aguado & San-José, arXiv 1702.02525





MBS spin-structure in multiterminal junction:

Spin canting angle



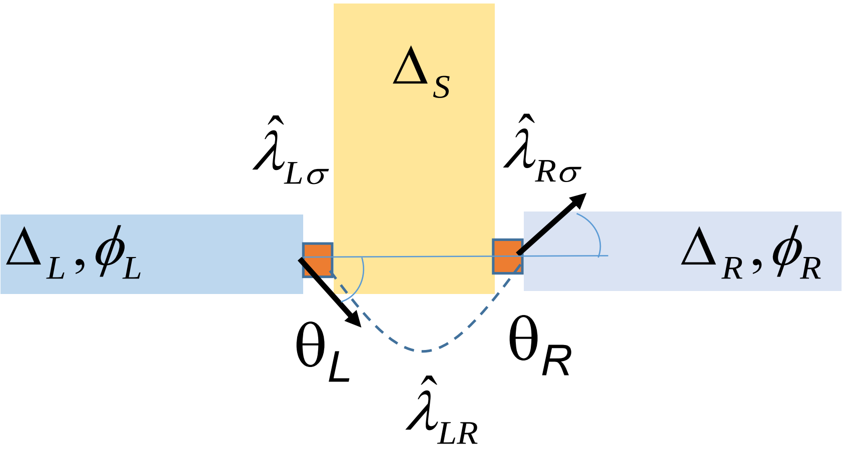
Multiterminal S-TS junctions: modeling

$$H_T = \sum_{\mu\equiv L,R;\sigma} \hat{\psi}^\dagger_{s\sigma}\hat{\lambda}_{\mu\sigma}\hat{\psi}_\mu + \hat{\psi}^\dagger_L \hat{\lambda}_{LR}\hat{\psi}_R \;+\; \text{h.c.}$$

$$\hat{\lambda}_{LR} = \lambda_{LR}\tau_z e^{i\tau_z(\phi_L-\phi_R)/2}$$

$$\hat{\lambda}_{\mu\uparrow} = \lambda_\mu \begin{pmatrix} e^{i\phi_\mu/2}\cos\frac{\theta_\mu}{2} & 0 \\ 0 & -e^{-i\phi_\mu/2}\sin\frac{\theta_\mu}{2} \end{pmatrix}$$

$$\hat{\lambda}_{\mu\downarrow} = \lambda_\mu \begin{pmatrix} e^{i\phi_\mu/2}\sin\frac{\theta_\mu}{2} & 0 \\ 0 & -e^{-i\phi_\mu/2}\cos\frac{\theta_\mu}{2} \end{pmatrix}$$



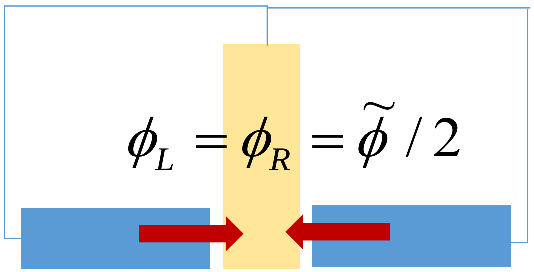
$$\hat{G}^{-1} = \hat{g}^{-1} - \hat{\Sigma} \qquad \hat{g} = \begin{pmatrix} g_L & 0 \\ 0 & g_R \end{pmatrix} \qquad \hat{\Sigma} = \begin{pmatrix} \Sigma_{LL} & \Sigma_{LR} \\ \Sigma_{RL} & \Sigma_{RR} \end{pmatrix}$$

$$I_j = \frac{e}{h}\int d\omega\, n_F(\omega)\text{Re Tr}\left[\sigma_z \left\{\hat{\Sigma}^A, \hat{G}^A\right\}_{jj}\right]$$

Multiterminal S-TS junctions: CPR results

$$\lambda_L = \lambda_R = \lambda \qquad \Delta_L = \Delta_R = \Delta \qquad \lambda_{LR} = 0 \qquad \theta = \theta_L - \theta_R$$

"parallel" case

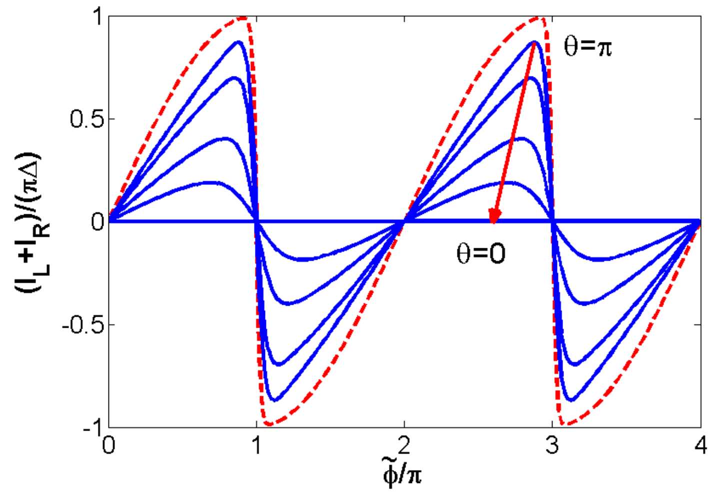


$\Delta_s \to \infty$ *limit*

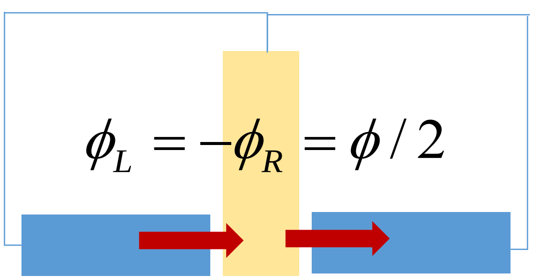
$$\epsilon_A = \sqrt{\tau}\Delta\cos(\tilde{\phi}/2)$$

$$\tau = 4\Lambda_\theta^2/(1+\Lambda_\theta^2)^2$$

$$\Lambda_\theta = \lambda^2 \sin(\theta/2)$$



"serial" case



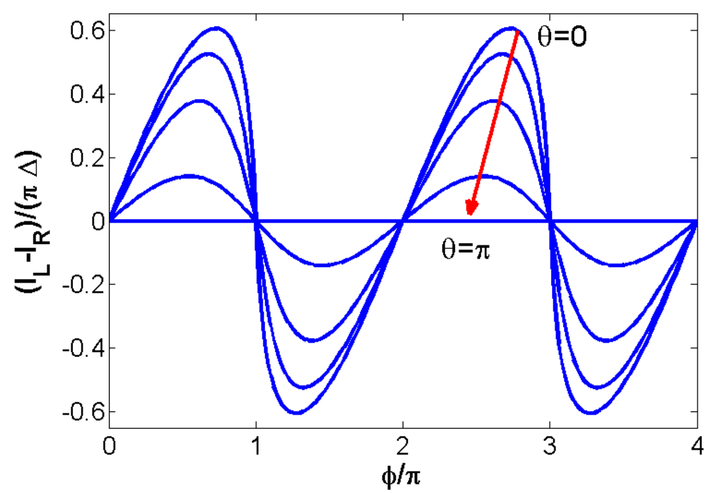
$\Delta_s \to 0$ *limit*

$$\epsilon_A(\phi) = \tilde{\Delta}\sqrt{1-\tau\sin^2(\phi/2)}$$

$$\tau = \cos^2(\theta/2)$$

$$\tilde{\Delta} = \Delta/\sqrt{1+x^2}$$

$$x = \frac{1+\lambda^4\sin^2(\theta/2)}{2\lambda^2}$$



Boundary GF for the spinful wire model

infinte wire (k space, Nambu)

$$H_0 = \sum_k \Psi_k^\dagger \underbrace{\left((\epsilon(k) - \mu)\sigma_0\tau_z + \alpha\sin(k)\sigma_z\tau_z + V_x\sigma_x\tau_0 + \Delta\sigma_0\tau_x\right)}_{\mathcal{H}_k} \Psi_k$$

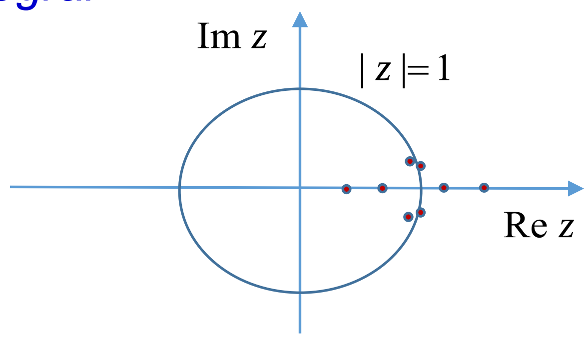
$$\epsilon(k) = -2t\left(\cos(k) - 1\right)$$

$$\Psi_k^T = \left(c_{k\uparrow}\ ,\ c_{k\downarrow}\ ,\ c_{-k\downarrow}^\dagger\ ,\ -c_{-k\uparrow}^\dagger\right)$$

Infinite wire: Real space GF as contour integral

$$\hat{G}^0(k,\omega) = \left[\omega - \mathcal{H}_k\right]^{-1} \qquad z = e^{ik}$$

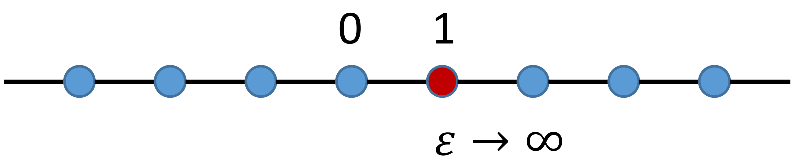
$$\hat{G}^0_{lm}(\omega) = \oint_{|z|=1} \frac{dz}{iz}\hat{G}^0(z,\omega)z^{(l-m)}$$



Dyson equation for chain breaking

$$\hat{g}_L = \hat{G}^0_{00} - \hat{G}^0_{01}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{10}$$

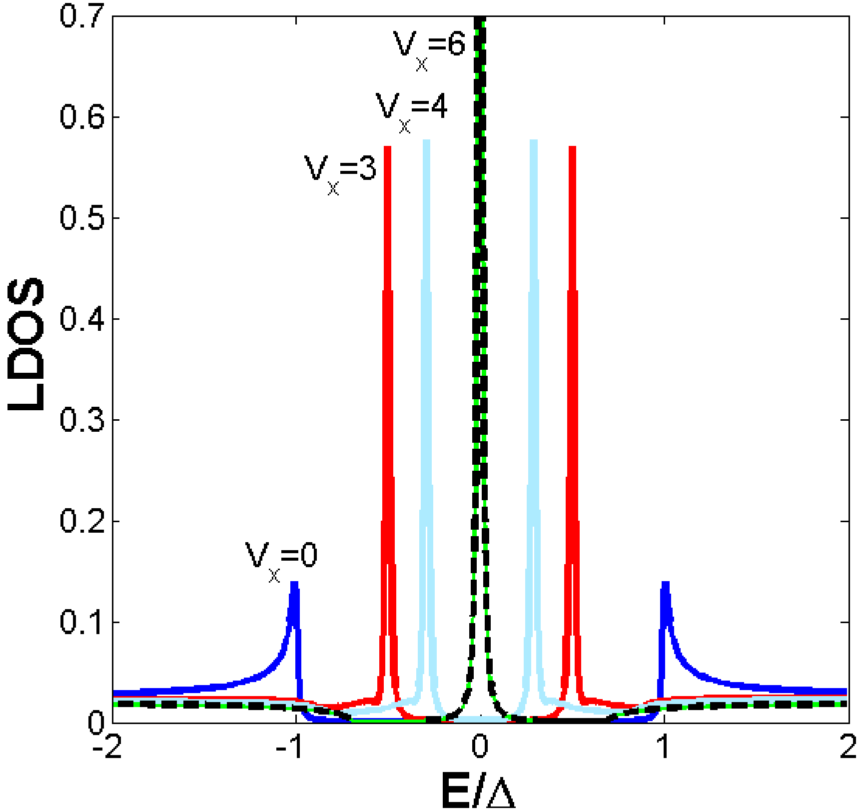
$$\hat{g}_R = \hat{G}^0_{00} - \hat{G}^0_{10}\left(\hat{G}^0_{00}\right)^{-1}\hat{G}^0_{01}$$



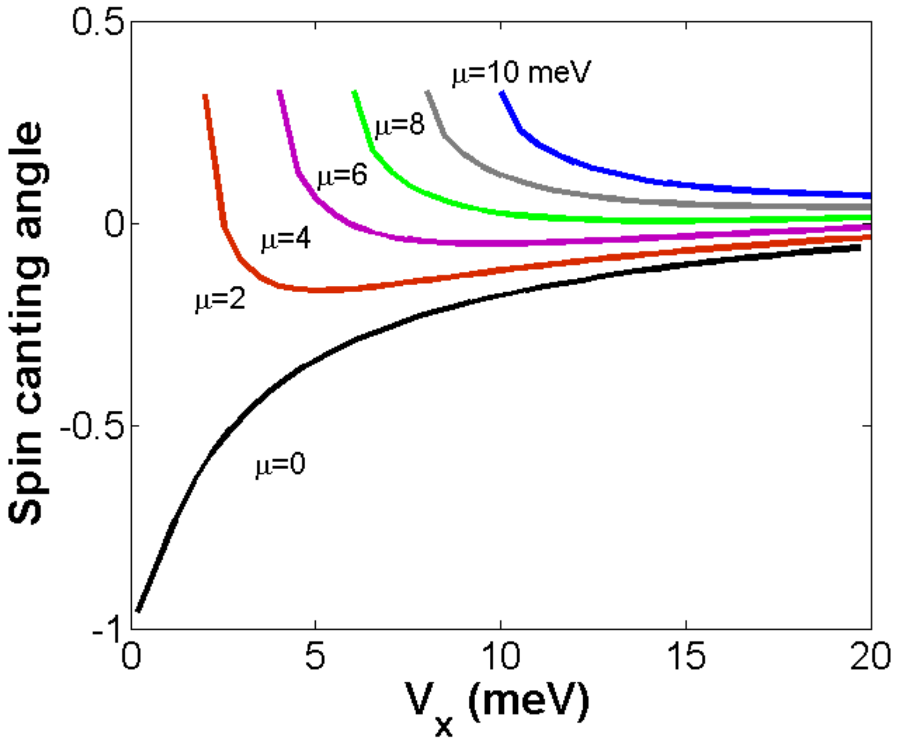
Boundary GF: LDOS and spin canting angle

$$\mu = 5meV$$

$$V_c = \sqrt{\mu^2 + \Delta^2} \approx 5\,\text{meV}$$



$$S_{\alpha,j}(\omega) = \frac{1}{2\pi i}\text{Tr}\left[(1+\tau_z)\,\sigma_\alpha\left(G_j^A(\omega) - G_j^R(\omega)\right)\right]$$



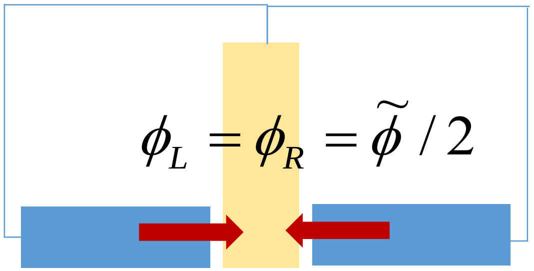
Parameters suitable for InAs/Al

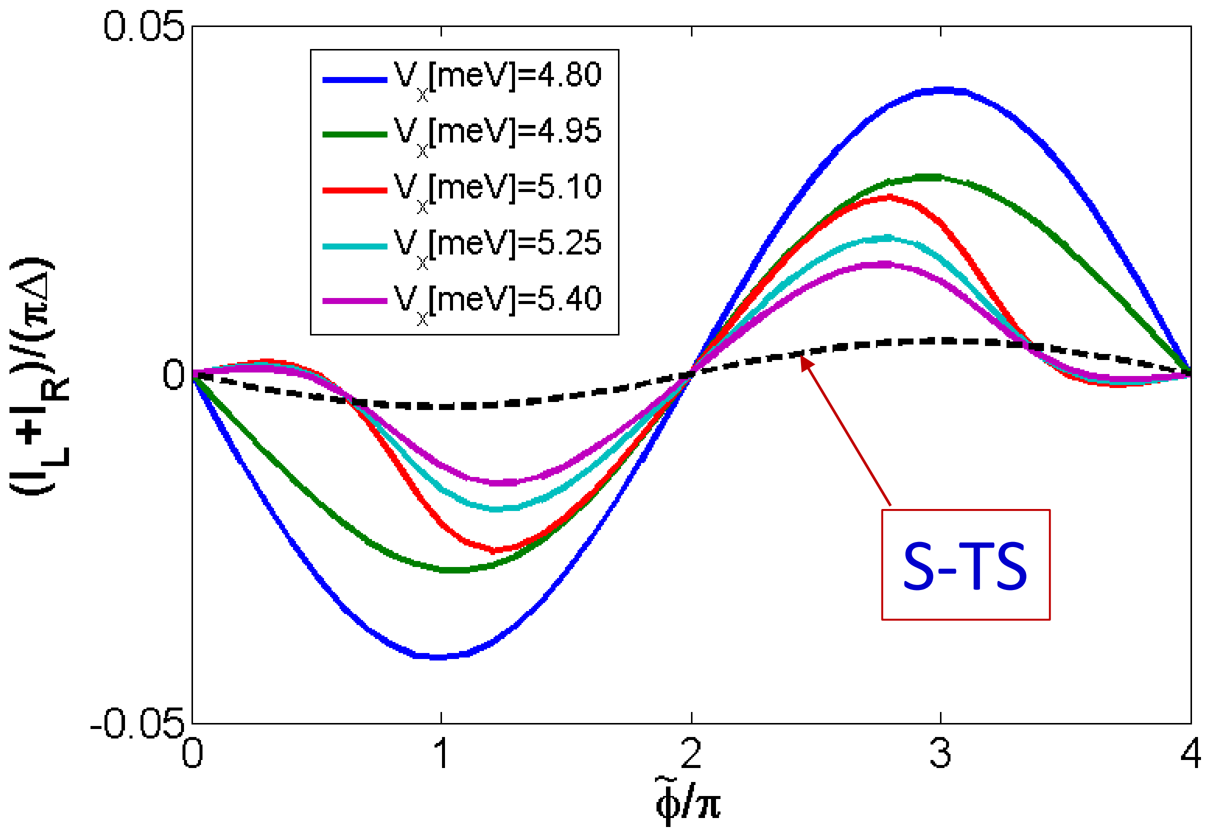
$$t = 20\,\text{meV}; \alpha = 4\,\text{meV}; \Delta = 0.2\,\text{meV}$$

Multiterminal S-TS junctions: CPR across topo transition

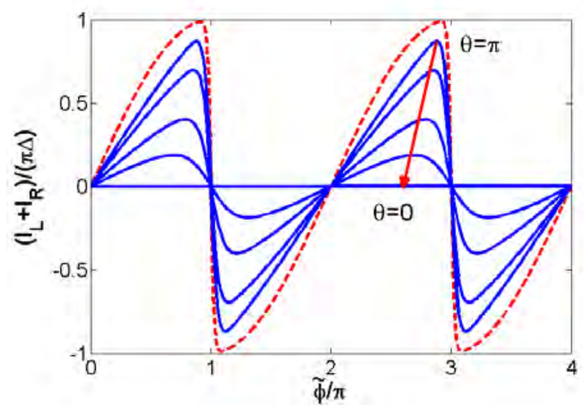
$\mu = 5meV$

"parallel" case





Kitaev limit

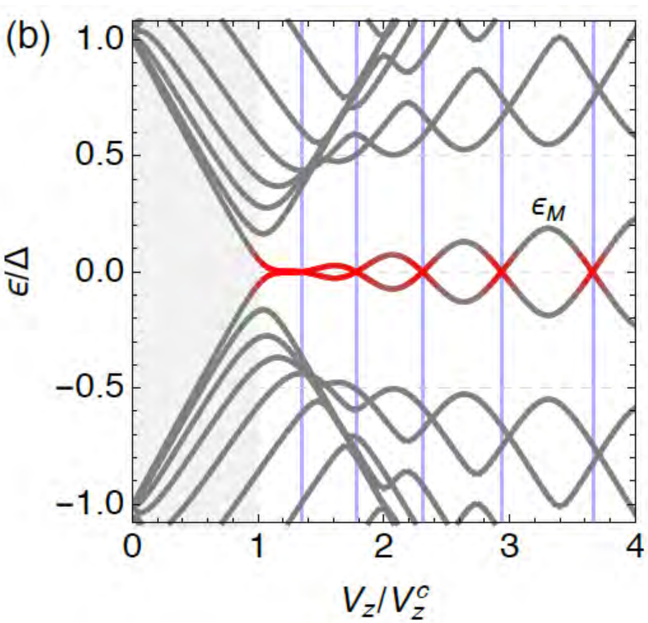


Zero-energy pinning from interactions

Dominguez, Cayao, San-José, Aguado, ALY & Prada, NPJ QM (2017)

Apparent absence of MBS hibridization in finite wires

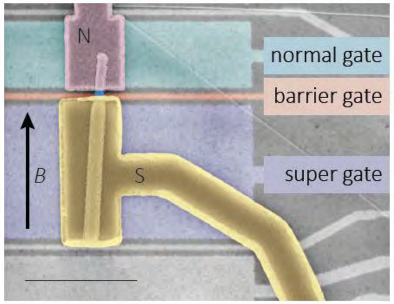
Theory (non-interacting)

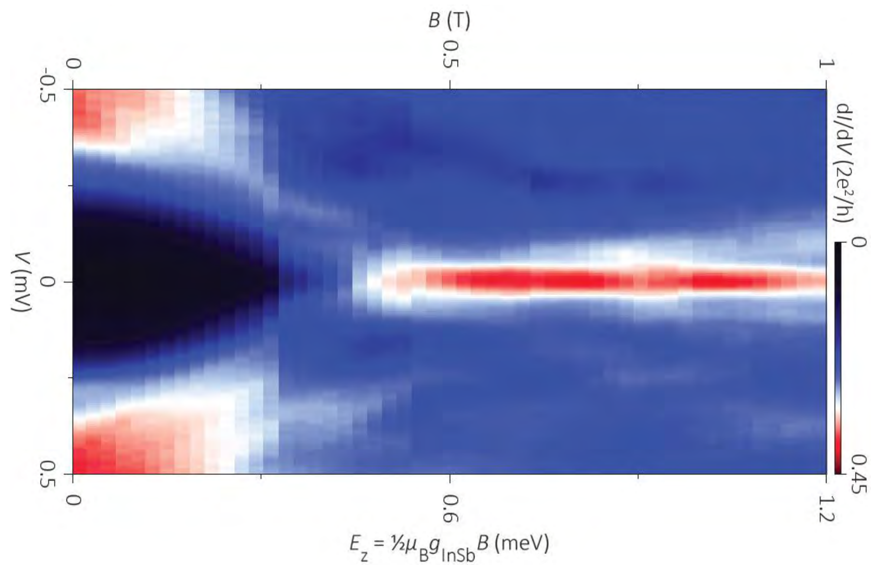


$$L = 1\,\mu\text{m};\ \alpha = 20\,\text{meVnm};$$

$$\Delta = 0.5\,\text{meV};\ m^* = 0.015 m_e$$

Exp: InSb/NbTiN

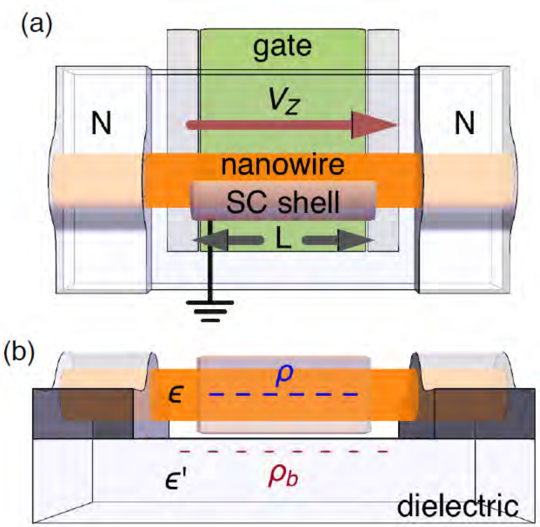




Zhang et al. arXiv 1603.04069

Zero-energy pinning from interactions

Dominguez, Cayao, San-José, Aguado, ALY & Prada, NPJ QM (2017)



electrostatic potential

$$H_{wire} = \int dx \Psi^\dagger(x) \left[\left(-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial x^2} - \mu + \phi(x) \right) \tau_z + V_Z\sigma_x + i\sigma_z\tau_z\alpha\frac{\partial}{\partial x} + \Delta\tau_x \right] \Psi(x)$$

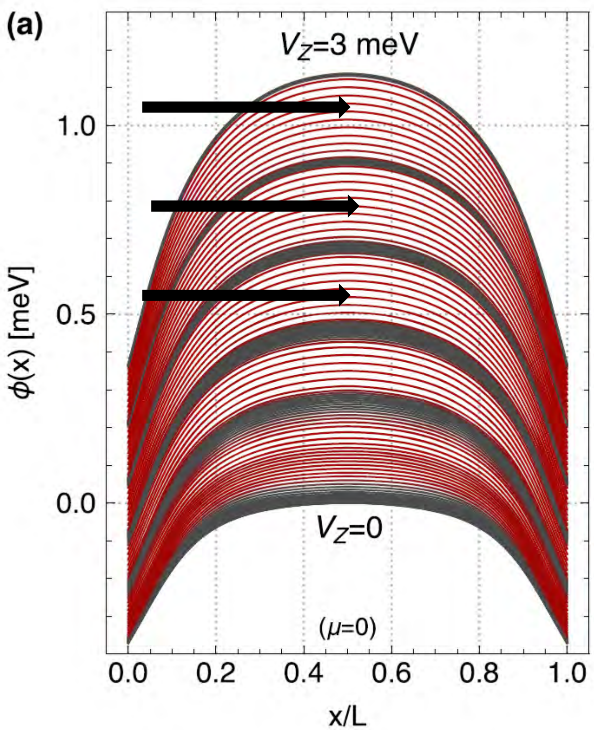
$$\Psi^T(x) = \left(\psi_\uparrow(x),\ \psi_\downarrow(x),\ \psi^\dagger_\downarrow(x),\ -\psi^\dagger_\uparrow(x) \right)$$

Poisson equation

$$-\nabla \cdot [\epsilon(r)\nabla\phi(r)] = 4\pi\rho(r)$$

Zero-energy pinning from interactions

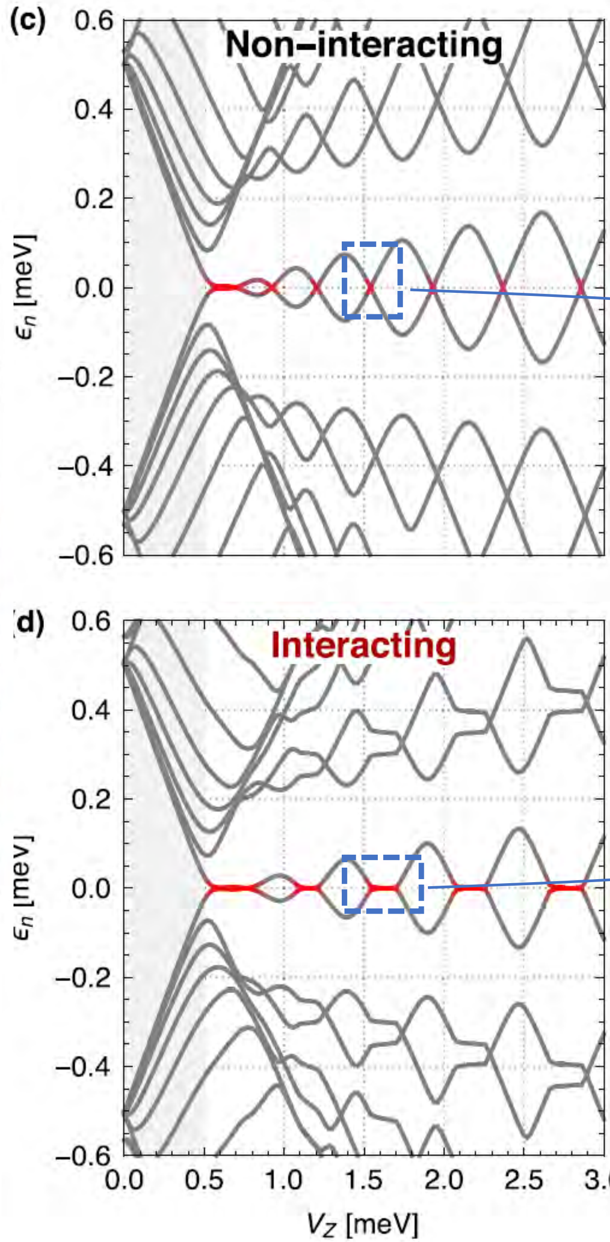
Self-consistent potential

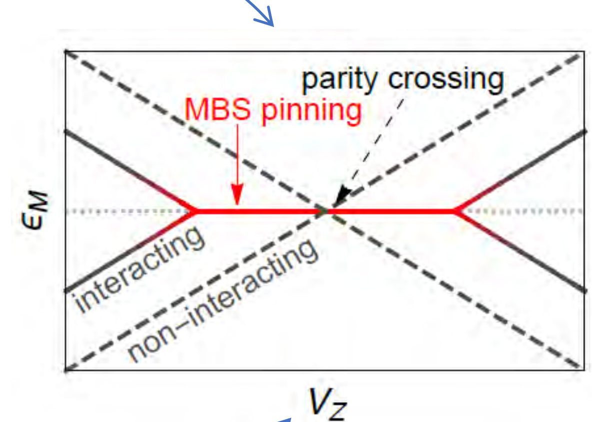


R=50 nm; L=1 μm

parameters for
a InSb/Nb wire

Energy levels





Dominguez et al., NPJ QM (2017)

Conclusions

Transport in hybrid TS junctions:

Zazunov, Egger and ALY, PRB 2016

- General GF formalism
- Unified description of MBS+continuum
- Analytical results (N-TS,TS-TS,S-TS,etc)

Josephson in multiterminal TS junctions:

(in preparation)

- Kitaev limit: role of MBS spin angle
- Boundary GF spinful model: CPR across topological transition

Interactions: Mechanism of Zero-energy pinning

Dominguez, Cayao, San-Jose, Aguado, ALY & Prada, NPJ QM 2017

Thank you!