Two-Terminal Conductance via Majorana Fermions

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Phys. Rev. B **93**, 235431 (2016), arXiv 1701.0084, arXiv 1706.04726

Outline

- Engineering and probing Majorana states in nanowires
- Theory of two-terminal conductance of a wire across the topological transition
- Interpretation of existing experiments
- Electron correlations beyond perturbations in tunneling

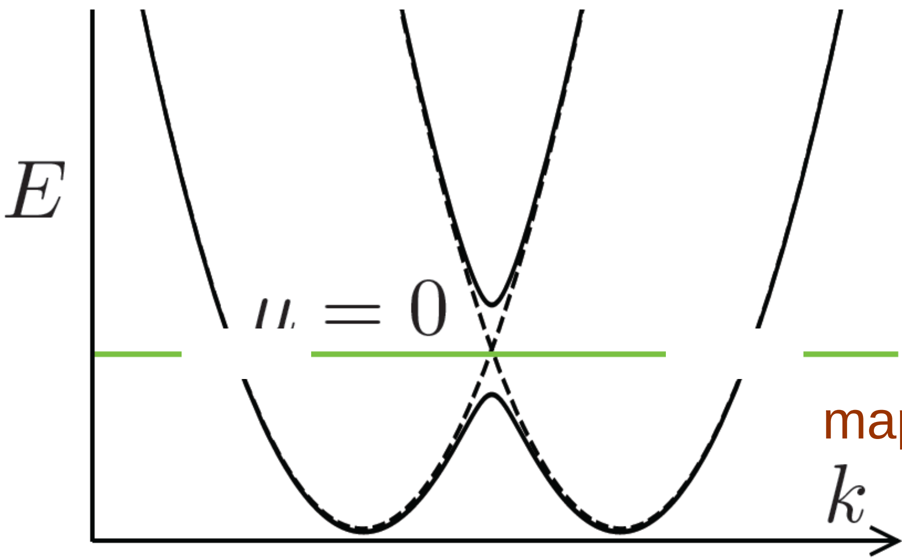
“Engineered” Majorana states: SO wire

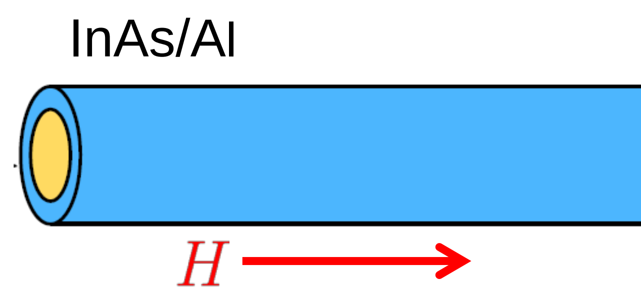
A quantum wire with SO interaction $\mathcal{H} = \frac{1}{2m}(p - p_0\sigma_x)^2 \psi^\dagger_{p\sigma}\psi_{p\sigma}$

placed in a uniform magnetic field $\mathcal{H} = -B\sigma_y \psi^\dagger_{p\sigma}\psi_{p\sigma}\,, \quad B = g\mu_B H$

in proximity with s-wave superconductor $\mathcal{H} = \Delta_s \psi^\dagger_{p\sigma}\psi^\dagger_{\overline{p}\,\overline{\sigma}} + \mathrm{hc}$

$\Delta_s < B$



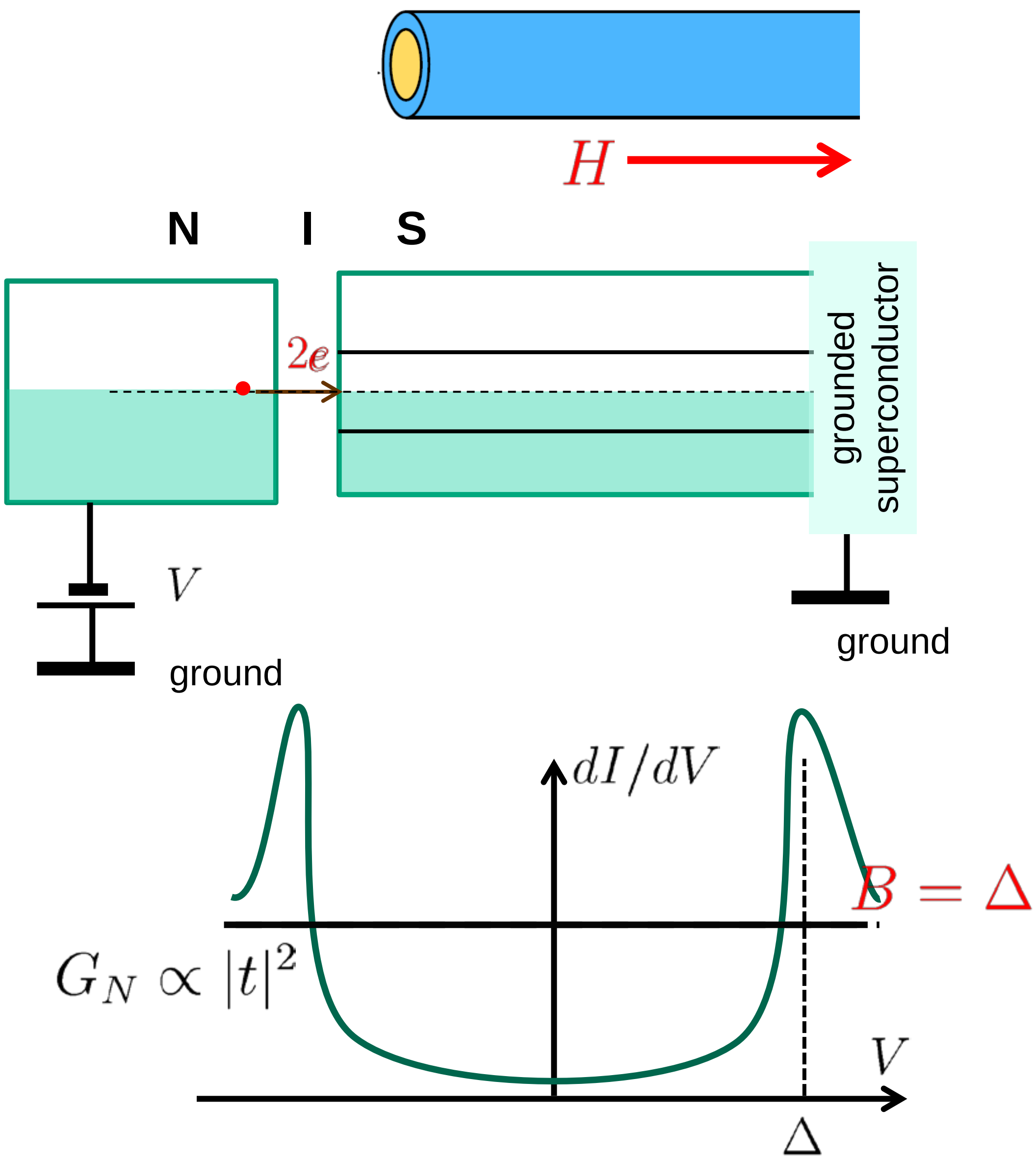


maps onto 1D p-wave superconductor

R.M. Lutchyn, J.D. Sau, and S. Das Sarma, Phys. Rev. Lett. **105**, 077001 (2010).

Y. Oreg, G. Refael, and F. von Oppen, Phys. Rev. Lett. **105**, 177002 (2010).

Andreev reflection: from trivial to topological state



$$G(V,T) = \frac{dI}{dV}$$

$$|\text{even}\rangle \underset{e}{\rightarrow} |\text{odd}\rangle \underset{e}{\rightarrow} |\text{even}\rangle$$

$G(eV < \Delta, T = 0)$:

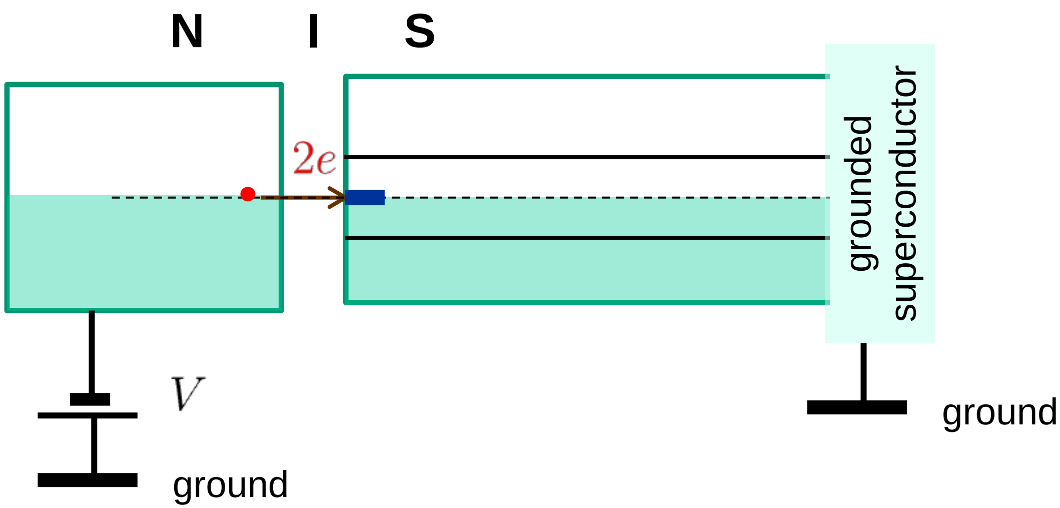
Andreev reflection from s-wave superconductor,

$$G_A \propto |t|^4 \propto G_N^2$$

(two-electron tunneling)

Majorana resonance in conduction: single junction

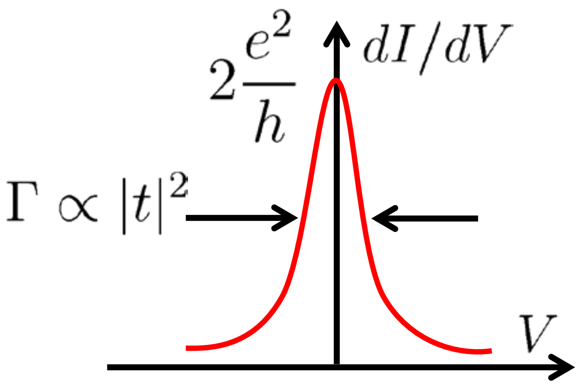
Single-junction conductance, **N-I-S** setting (Law, Lee, Ng, PRL 2009)



$$|\text{even}\rangle \xrightarrow[e]{} |\text{odd}\rangle \xrightarrow[e]{} |\text{even}\rangle$$

Resonant Andreev reflection: zero-bias peak in dI/dV

$$G_{\mathrm{M}}^{\mathrm{max}}(T=0) = 2\frac{e^2}{h}$$

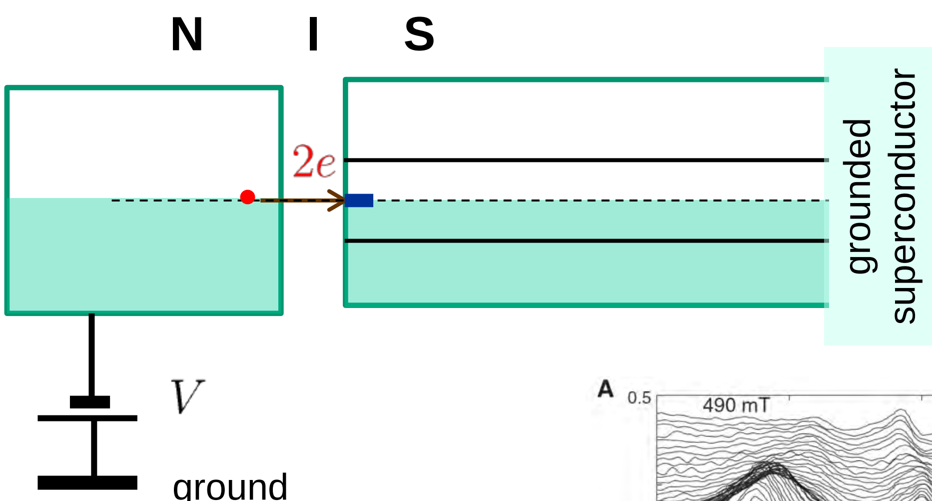


The peak width at *T=0* (but not the peak height) depends on the tunneling amplitude $|t|$

$$G_{\mathrm{M}}^{\mathrm{max}}(T=0) \gg G_N \propto |t|^2$$

Majorana resonance in conduction: single junction

Single-junction conductance, **N-I-S** setting (Law, Lee, Ng, PRL 2009)



Andreev reflection
(two-electron tunneling):
zero-bias peak in dI/dV

$$G_{\mathrm{M}}^{\mathrm{max}}(T=0) = 2\frac{e^2}{h}$$

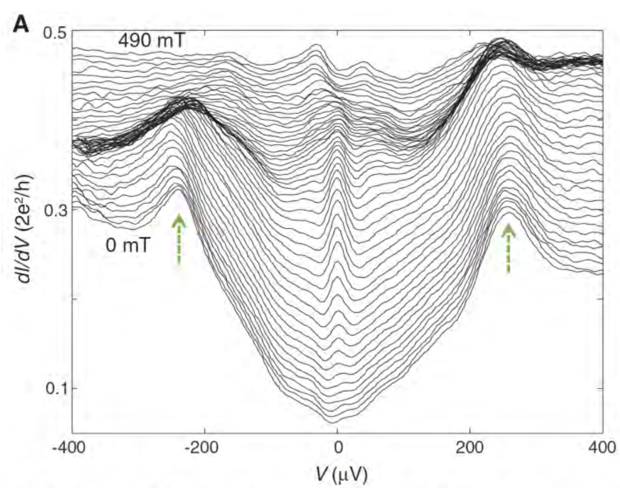


Fig. 2. Magnetic field–dependent spectroscopy. **(A)** dI/dV versus V at 70 mK taken at different B fields (from 0 to 490 mT in 10-mT steps; traces are offset for clarity, except for the lowest trace at $B = 0$). Data are from device 1.

Zero-bias conductance peak

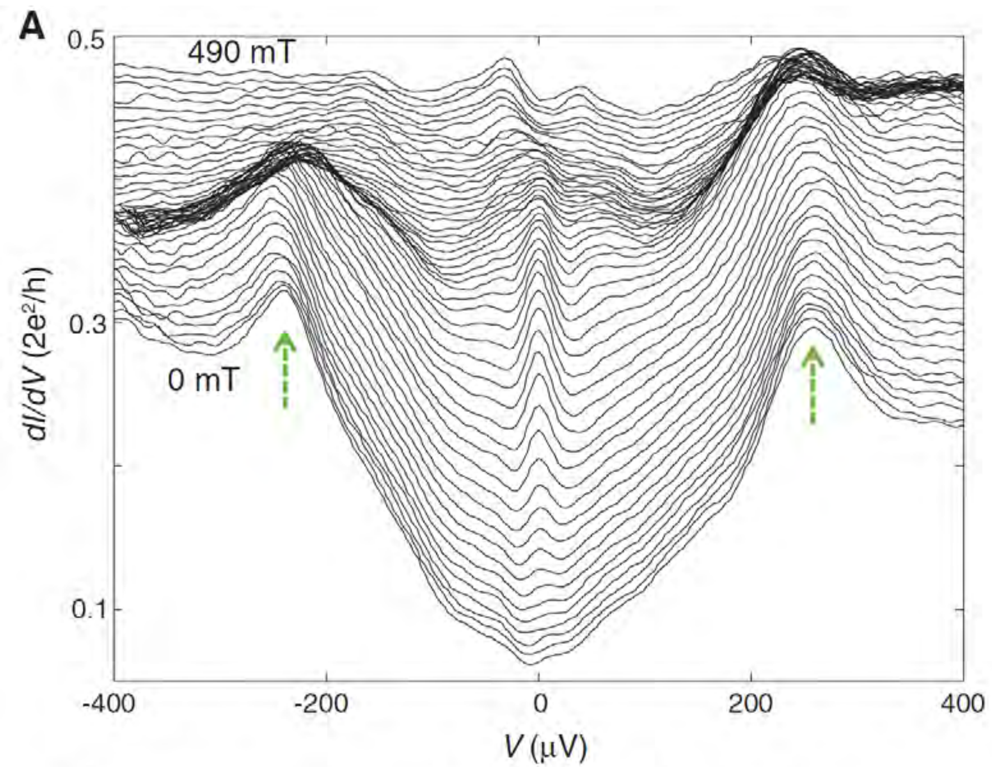
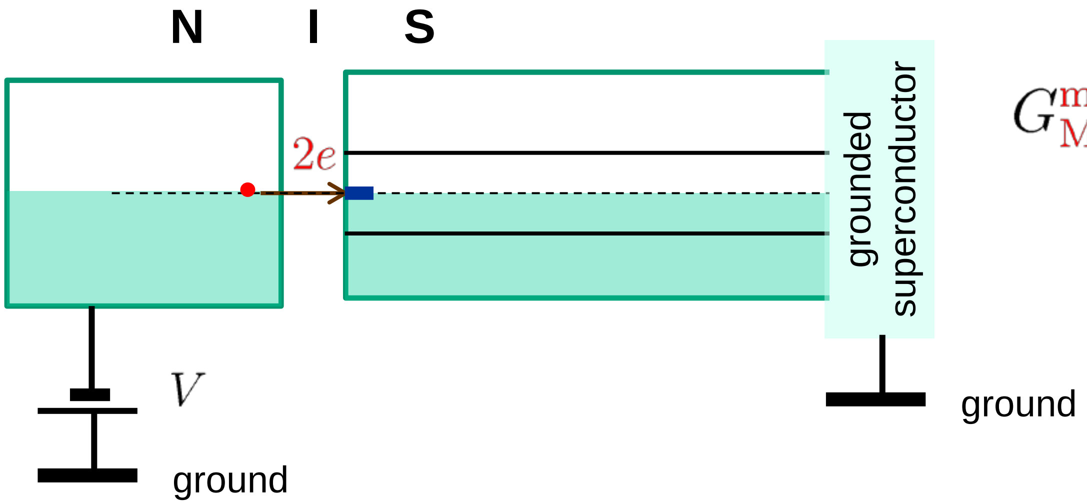


Fig. 2. Magnetic field–dependent spectroscopy. (**A**) *dI*/*dV* versus *V* at 70 mK taken at different *B* fields (from 0 to 490 mT in 10-mT steps; traces are offset for clarity, except for the lowest trace at $B = 0$). Data are from device 1.

Mourik, Zuo, S.M. Frolov, Plissard, Kouwenhoven, Science (2012)

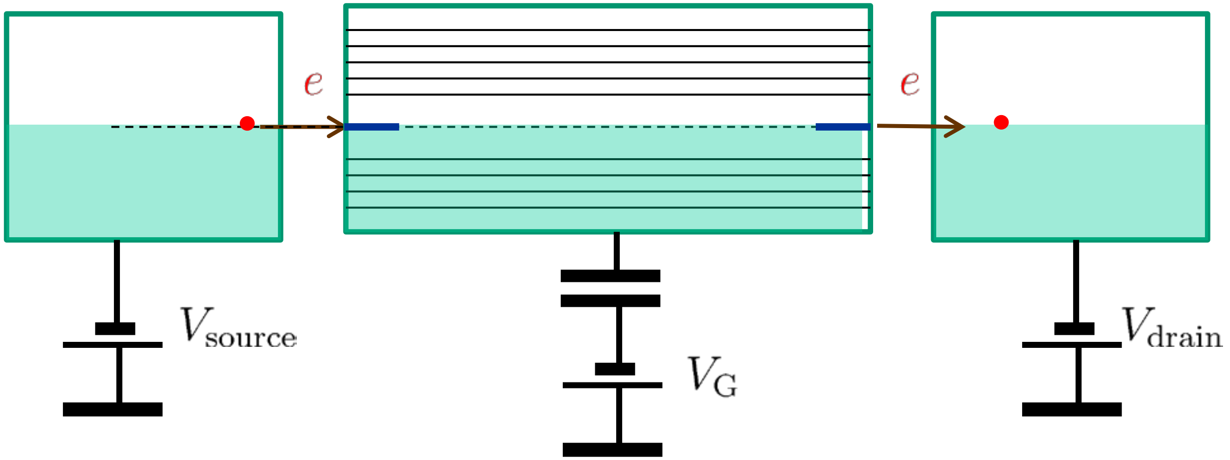
Transport through a wire segment: more knobs to turn



$$G_{\mathrm{M}}^{\max}(T = 0) = 2\frac{e^2}{h}$$

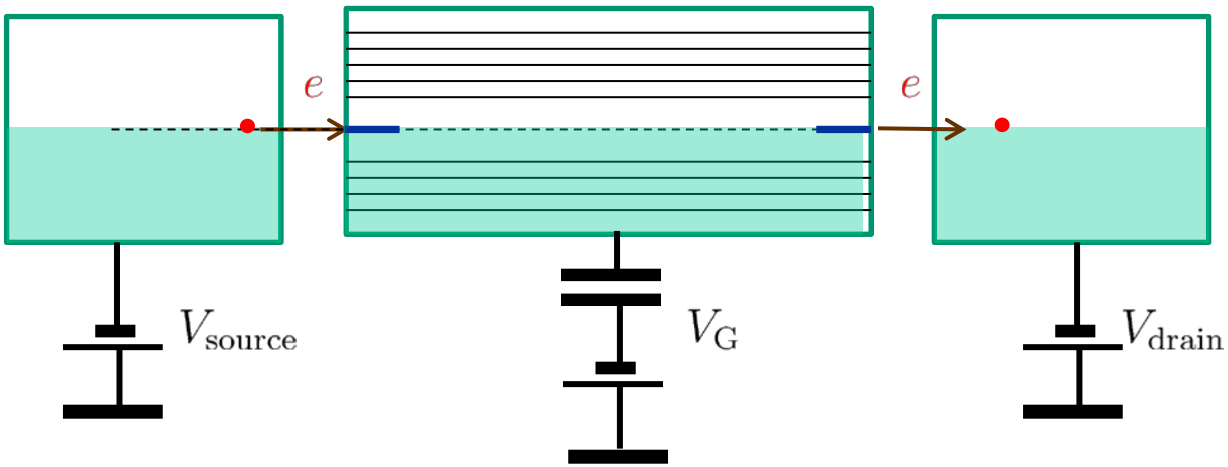
Majoranas allow for resonant one-electron tunneling

("teleportation" - L Fu, PRL 2010)

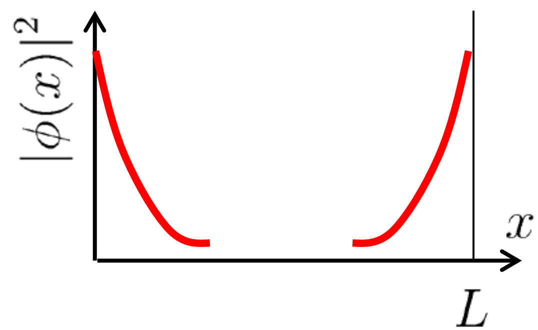


Transport through a wire segment

Resonant one-electron tunneling (“teleportation” - L Fu, PRL 2010)



$$E(n, V_G) = \frac{e^2}{2C}(n - C_g V_g)^2$$



$$\nu(x, \varepsilon \to 0) \propto |\langle \mathrm{odd}|\psi^\dagger(x)|\mathrm{even}\rangle|^2 \delta(\varepsilon) \sim |\phi(x)|^2 \delta(\varepsilon)$$

$$|n\rangle \xrightarrow[e]{} |n+1\rangle \xrightarrow[e]{} |n\rangle$$

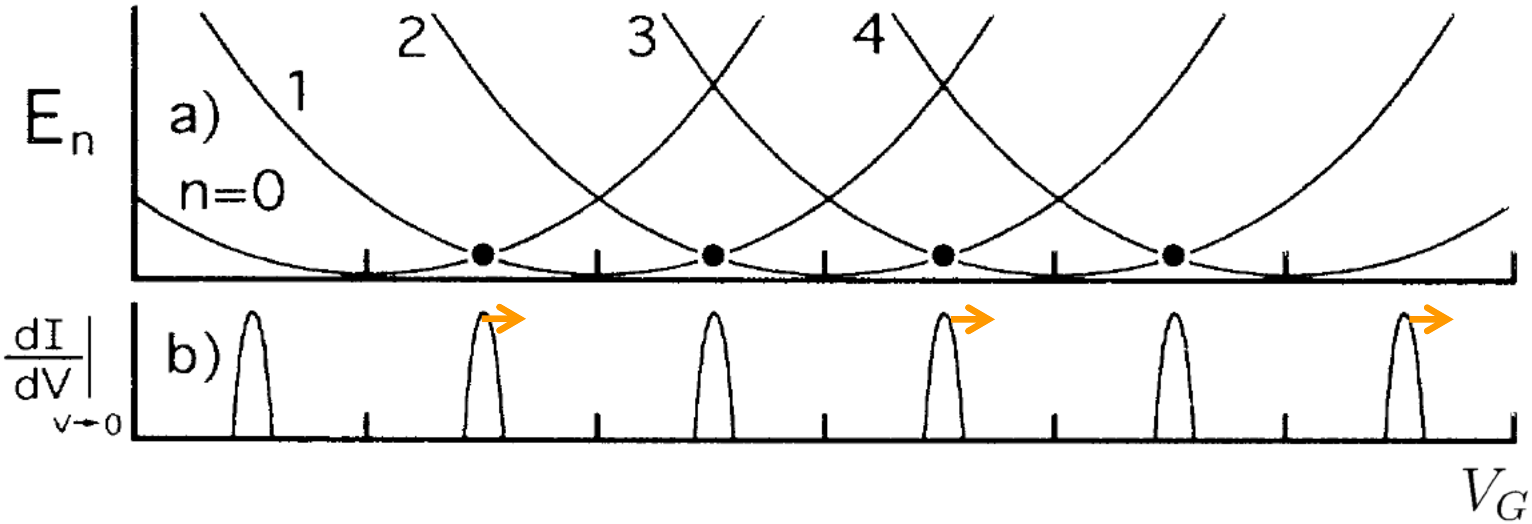
Linear conductance $G(V_G)$ reaches maxima at $E(n, V_G) = E(n+1, V_G)$

This condition defines a periodic set of V_G (for non-overlapping Majoranas)

Coulomb blockade peaks

Conductance reaches maxima at $E(n, V_G) = E(n+1, V_G)$

This condition defines a periodic set of V_G (for non-overlapping Majoranas)

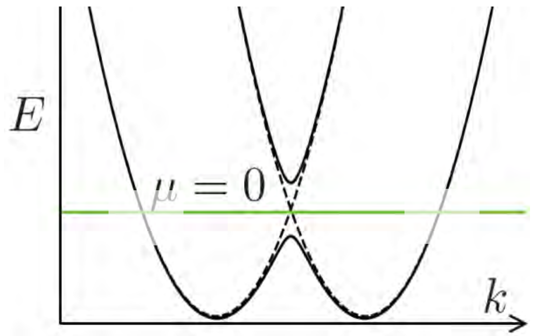


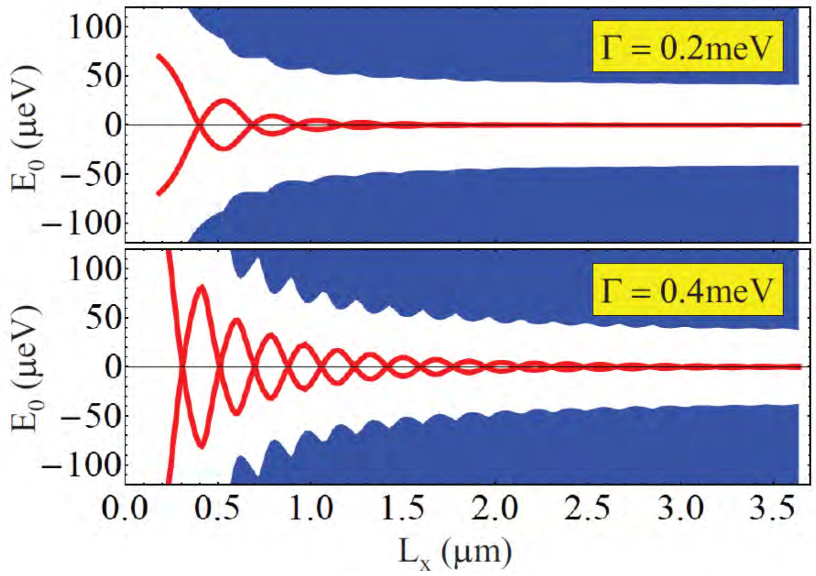
Hybridization between the end states: period doubling (even set shifts wrt odd set)

Majoranas in a finite-length wire segment

Hybridization between the end states: period doubling
(even set shifts wrt odd set)

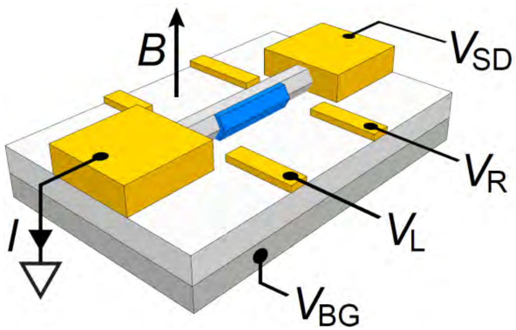
$$E_{\text{odd}} - E_{\text{even}} = f_{\text{osc}}(L, H) \cdot \exp\{-L/\xi(H)\}$$

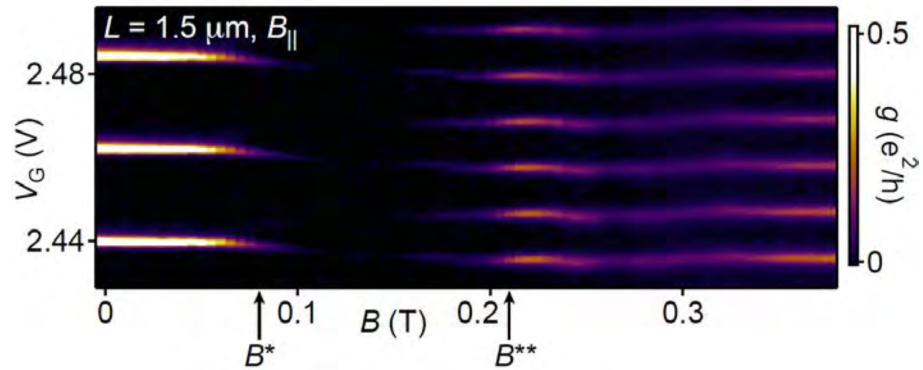




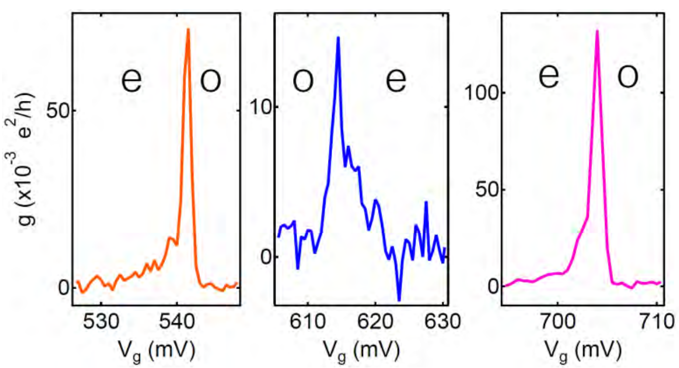
T. Stanescu, R. Lutchyn, S. Das Sarma, PRB 2013

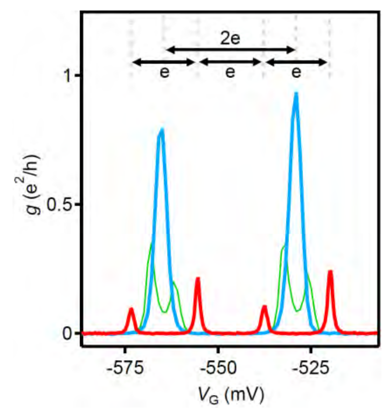
Specific motivation for this work: two-terminal measurements (Qdev Lab, Copenhagen)





Skewed peaks



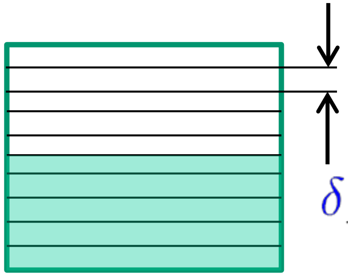


Short wires (L~250nm)
[Higginbotham *et al*,
Nat. Phys., **11**, 1017 (2015)]

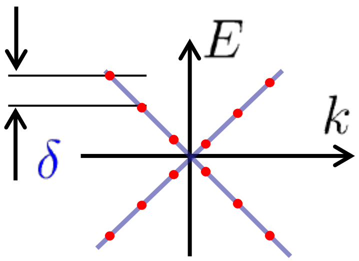
Long wires (L~1μm)
[Albrecht *et al*,
Nature **531**, 206 (2016)]

Quantitative theory of two-terminal conductance

Energy scales - 1

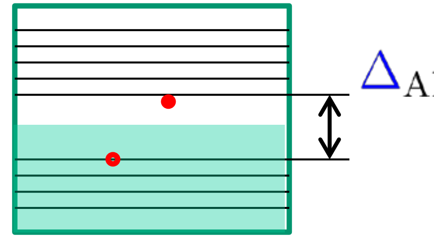


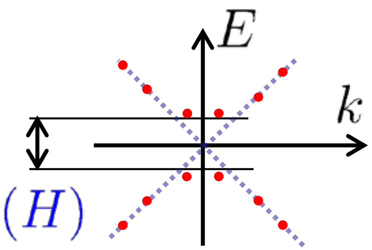
$$\delta_{\rm Al} = (V \cdot \nu_{\rm Al})^{-1}$$



$$\boxed{\delta > \delta_{\rm Al}}$$

wire



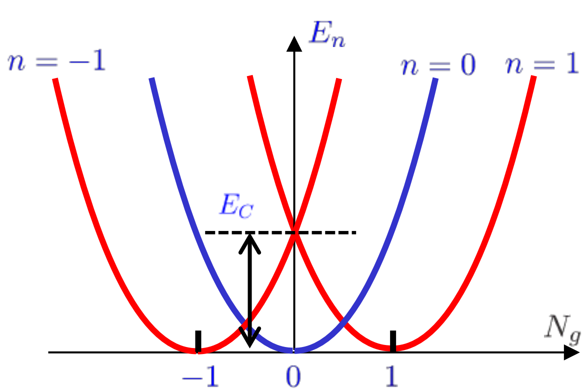


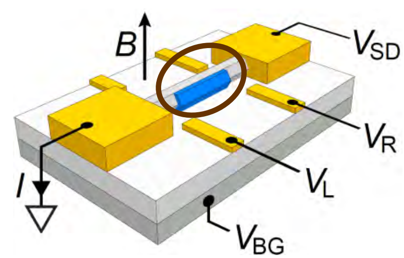
$$\Delta = \Delta(H)$$

$$\boxed{\Delta(H) < \Delta_{\rm Al}}$$

Quasiparticle “poisoning” temperature: $T_{\rm p} = \dfrac{\Delta}{\ln(\Delta/\delta)}$

Charging energy



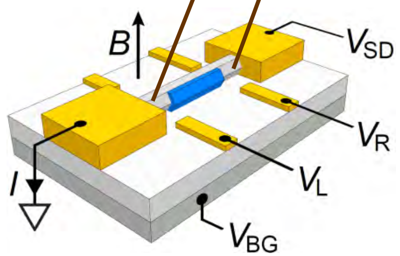


$$E_C = 2e^2/C$$

Long wire segments: $E_C < \Delta(H=0)$

Energy scales - 2

$$\boxed{g_L, g_R \lesssim 1}$$



Conductance of single-channel junctions, in units of e^2/h

Energy scale for quantum fluctuations of charge ("charge-Kondo")

$$T_{\mathrm{K}} \lesssim E_C \exp\left\{-\pi^2/(g_L + g_R)\right\}$$

[Ion Garate, PRB **84**, 085121(2011)]

Assumptions and aims of the theory

$$\xi_M \simeq \frac{v_F}{|\Delta(H)|}$$

Majoranas resolved only in long wires, $L \gg \xi_M$

$$\delta \approx \frac{\pi v_F}{L}$$

$$|\Delta(H)| < \Delta(H=0)$$

$$\boxed{\delta \ll \Delta(H=0)}$$

weak-tunneling regime $\quad \frac{T_{\mathrm{K}}}{T} \to 0$

level spacing small, temperature finite $\quad \frac{\delta^2}{T\Delta} \ll 1$

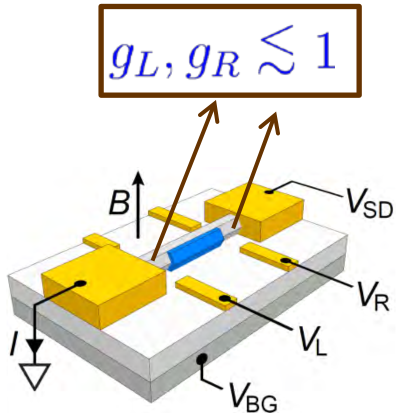
probability of quasiparticle poisoning low: $\quad \frac{T}{T_{\mathrm{p}}} \lesssim 1$

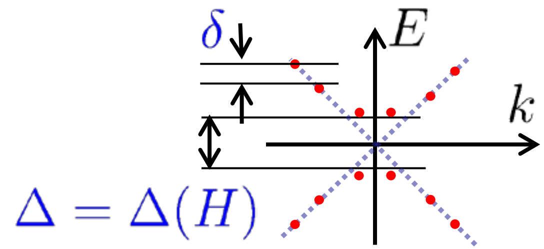
$$T_{\mathrm{p}} = \frac{\Delta}{\ln(\Delta/\delta)}$$

We aim at evaluation of the two-terminal conductance $G(V_g, H)$ as a function of gate voltage V_g at a set of fixed values of H, in the leading orders of small parameters: g_L , g_R , and $\delta/\sqrt{T\Delta}$;

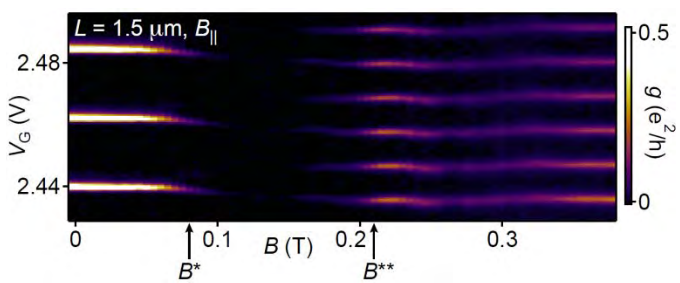
Coulomb blockade peaks: heights, widths, shapes

Conclusions (abbreviated)





To the order $g_L \cdot g_R$, and $(\delta/\sqrt{T\Delta})^0$, conductance $G(V_g, H)$ is finite only in the regimes $\Delta(H) > E_C$ [Cooper pair transport] and $\Delta(H) < 0$ [resonant tunneling via Majorana states].



Some history: experiments and theory for $\Delta(H) > E_C$

VOLUME 70, NUMBER 12 PHYSICAL REVIEW LETTERS 22 MARCH 1993

Even-Odd Asymmetry of a Superconductor Revealed by the Coulomb Blockade of Andreev Reflection

T. M. Eiles,[a] John M. Martinis, and Michel H. Devoret[b]
National Institute of Standards and Technology, Boulder, Colorado 80303
(Received 20 October 1992)

We have measured at low temperatures the current through a submicrometer superconducting island connected to two normal metal leads by ultrasmall tunnel junctions. As the bias voltage is lowered well below twice the superconducting energy gap, the current changes from being e periodic with gate charge to $2e$ periodic. This behavior is clear evidence that there is a difference in the total energy between the

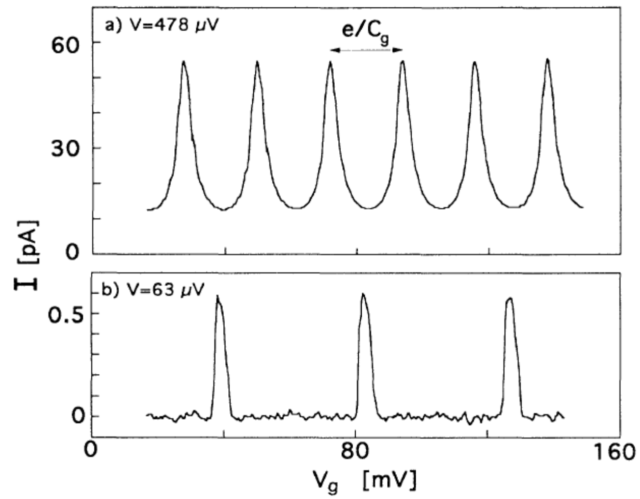


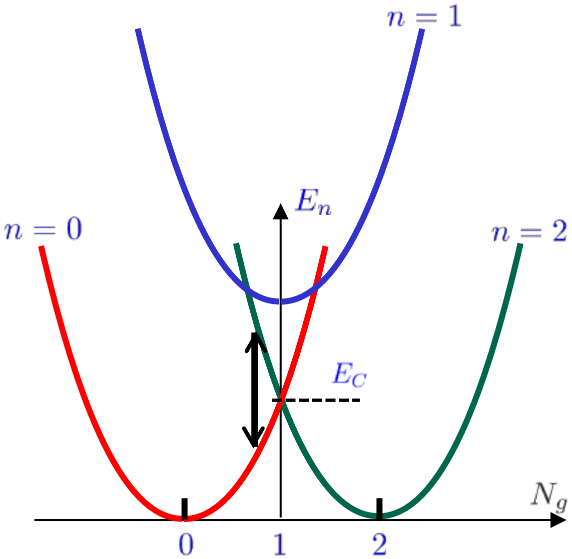
FIG. 3. Current through the electrometer vs V_g for bias voltage (a) $V \approx 2\Delta/e$ and (b) $V \approx \Delta/4e$. Arrow in (a) shows gate voltage corresponding to e periodicity.

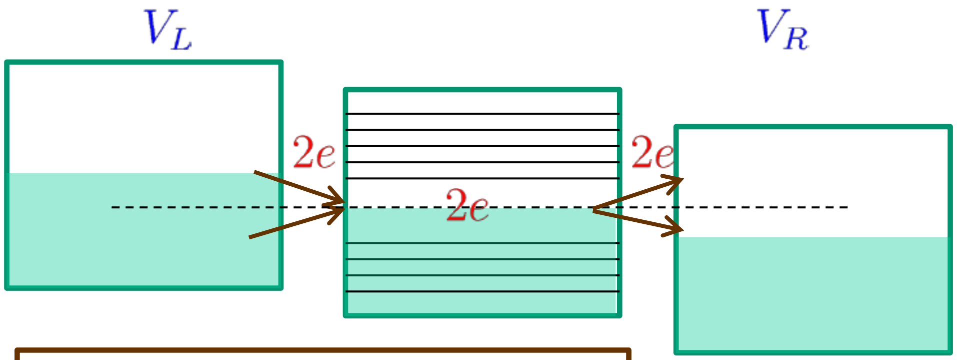
VOLUME 70, NUMBER 26 PHYSICAL REVIEW LETTERS 28 JUNE 1993

Coulomb Blockade of Two-Electron Tunneling

F. W. J. Hekking,[1] L. I. Glazman,[2] K. A. Matveev,[2] and R. I. Shekhter[3]
[1] *Institut für Theoretische Festkörperphysik, Universität Karlsruhe, Postfach 6980, 7500 Karlsruhe, Federal Republic of Germany*
[2] *Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455*

Sequential tunneling of pairs, $\Delta(H) > E_C$





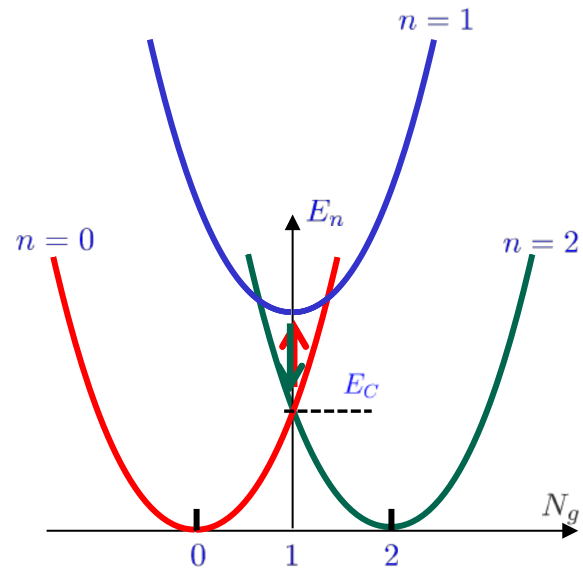
$$\boxed{\Gamma_i(\epsilon_i) = \frac{2\pi}{\hbar}\gamma_i \frac{\epsilon_i}{\exp(\epsilon_i/k_BT) - 1}}, \quad i = L, R$$

$$\epsilon_i = 2E_C\eta - 2eV_i\,, \; \eta = N_g - N_g^*$$

charge degeneracy points: $N_g^* = 2(l+1)$

The *2e* tunneling occurs in two *e*-steps, via an intermediate state with a minimal energy $\Delta - E_C$

$$\boxed{\gamma_i = \frac{g_i^2}{(2\pi^2)^2}.}$$



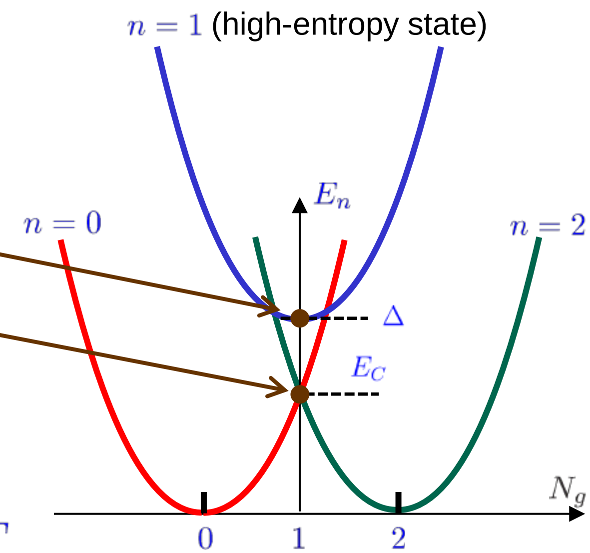
Conductance peaks at $\Delta(H) > E_C$

The Andreev conductance near the charge degeneracy points at low temperatures:

$$G_{2e}(T,\eta) = \frac{e^2}{2h}\frac{g_L^2 g_R^2}{g_L^2+g_R^2}\frac{2\eta E_C/T}{\sinh(2\eta E_C/T)}$$

$$\eta = N_g - N_g^*$$

Peak height is *T*-independent, peak width ~*T*



Occupation of this state would block 2e-tunneling

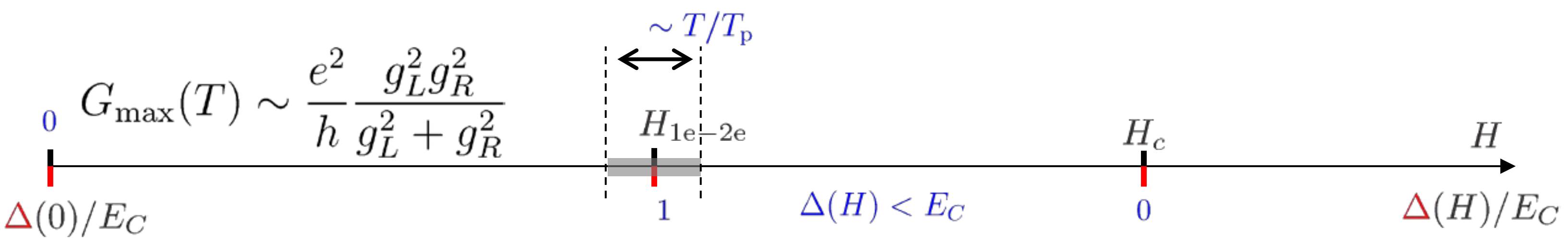
degenerate state

Free energy difference:

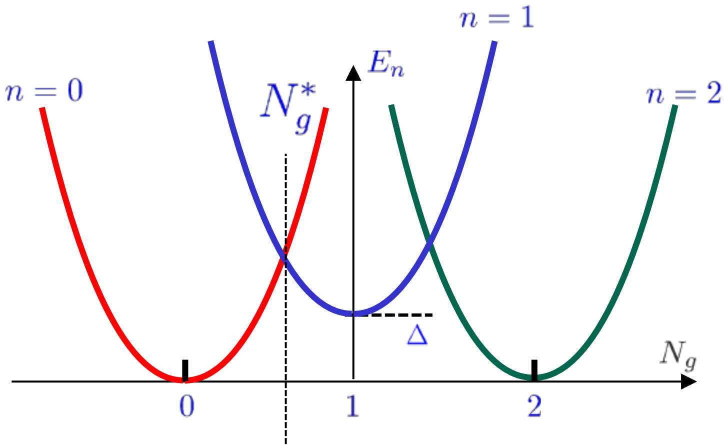
$$\Delta F = (\Delta - E_C) - T\ln\sqrt{\frac{T\Delta}{\delta^2}}$$

The Andreev conductance is “poisoned” at $\dfrac{\Delta(H)-E_C}{E_C} \lesssim \dfrac{T}{T_{\rm p}}$

$$G_{\max}(T) \sim \frac{e^2}{h}\frac{g_L^2 g_R^2}{g_L^2+g_R^2}$$



Single-electron sequential tunneling, $\Delta(H) < E_C$



charge degeneracy point, $\eta = 0$

Any one
may receive an electron

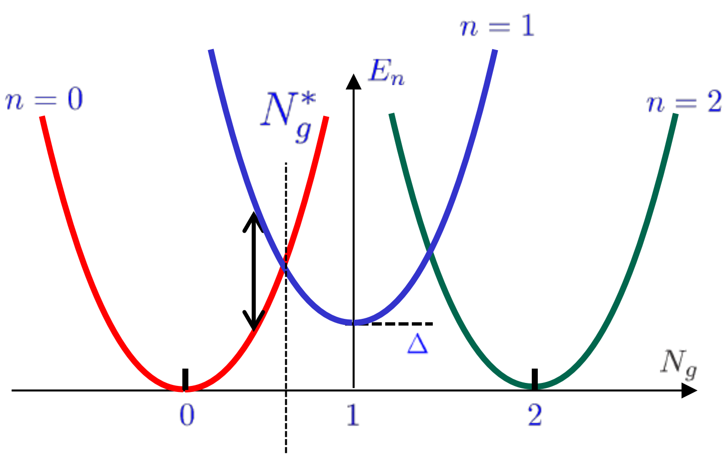
only one state may
release an electron,
without breaking pairs

$$\text{tunnel-out rate} \propto |\psi(r_{\text{junction}})|^2 \propto \frac{1}{L} \propto \delta$$

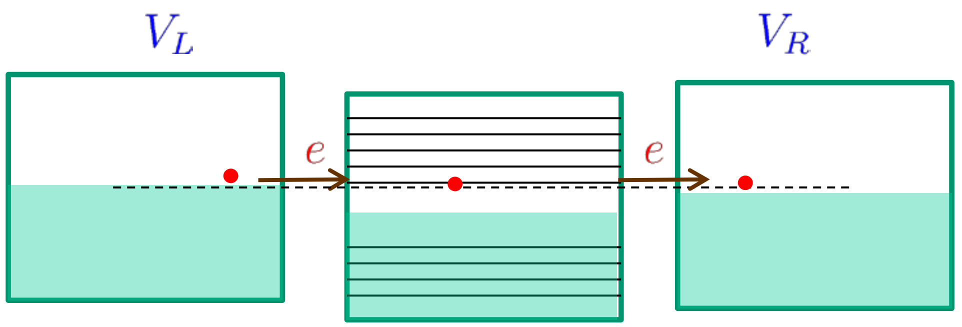
$$G_{\max}(T) \sim \frac{e^2}{h} \frac{g_L g_R}{g_L + g_R} \frac{\delta}{T}$$

vanish in the limit $\delta \to 0$

Single-electron sequential tunneling, $\Delta(H) < E_C$



charge degeneracy point, $\eta = 0$



activated sequential tunneling, activation energy $\epsilon = E_C|\eta|$

$\eta = N_g - N_g^*$

Resonance at $\eta = 0$ is "poisoned"; at finite *T*, conductance peak is suppressed and shifted to $\eta < 0$

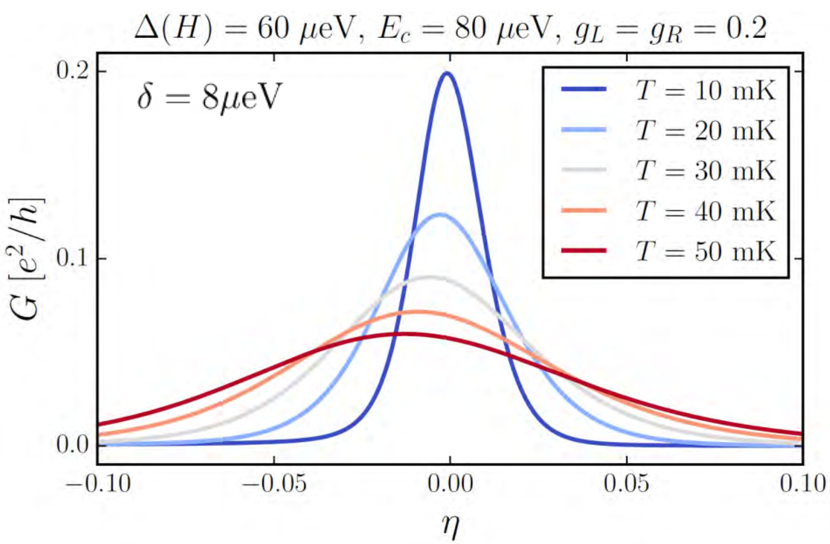
$$\boxed{G_e(T,\eta) = \frac{e^2}{h}\frac{g_L g_R}{g_L + g_R}\frac{\delta}{T}}\frac{1}{(\delta/2\sqrt{T\Delta})\exp(-2\eta E_C/T)+1}\int \frac{dw}{2\sqrt{2w}}\frac{1}{\exp(2\eta E_C/T)+e^w}$$

$$G_{\max}(T) \sim \frac{e^2}{h}\frac{g_L g_R}{g_L + g_R}\frac{\delta}{T}$$

Broad maximum, $\Delta\eta \sim |\eta_{\text{peak}}|$, at $\eta_{\text{peak}} \simeq -\frac{\Delta}{8E_C}\frac{T}{T_{\text{p}}}$

vanish in the limit $\delta \to 0$

Conductance peaks at $\Delta(H) < E_C$



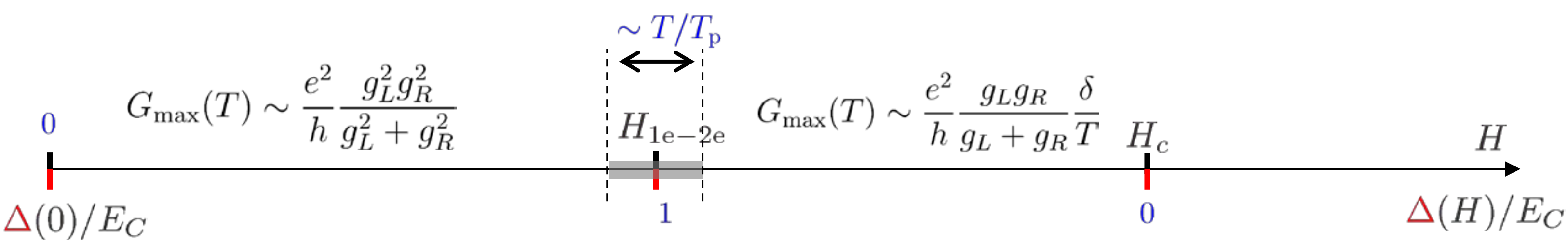
$$\eta_{\text{peak}} \simeq -\frac{\Delta}{8T_{\text{p}}}\frac{T}{E_C}$$

$$\Delta\eta \sim |\eta_{\text{peak}}|$$

$$G_{\max}(T) \sim \frac{e^2}{h}\frac{g_L g_R}{g_L + g_R}\frac{\delta}{T}$$

tails: activation, $|\eta| \gg \eta_{\text{peak}}$

$$G_{\text{e}}(T,\eta) \propto \exp(-E_C|\eta|/T)$$

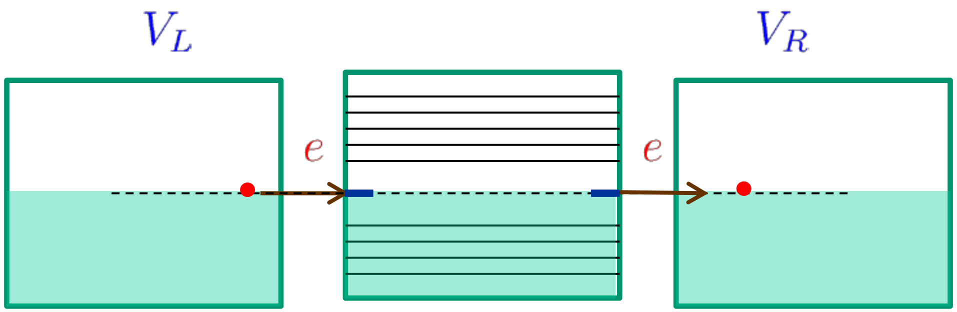


$\Delta(H) > E_C$: sequential 2e-tunneling, peaks are symmetric in η

$\Delta(H) < E_C$: sequential e-tunneling, peaks symmetry is preserved within ~1%

Resonant tunneling via Majorana states $(H > H_c)$

[Quantitative transport theory of "teleportation", L. Fu, PRL **104**, 056402 (2010)]



$$\mathcal{H}_L = t_L \sum_{pn} \varphi_n^*(r_L) c_p^\dagger \psi_n + \text{h.c.}$$

$$\downarrow$$

$$\tilde{\mathcal{H}}_L = t_L \sum_p \phi_L(r_L)(c_p^\dagger \hat{N}^- - c_p \hat{N}^+)\gamma_L + \ldots$$

Partial level widths $\Gamma_{L,R} \propto |t_{L,R}|^2 |\phi_{L,R}|^2$

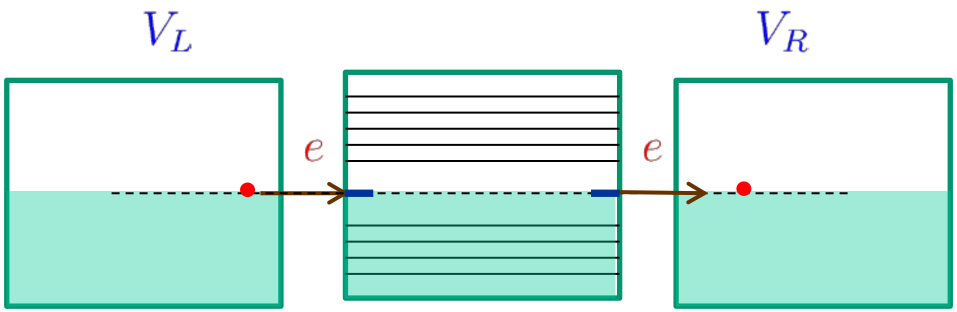
$$|\phi_s(r_s)|^2 \propto 1/\xi\,, \quad s = R, L \qquad\qquad \Gamma_{L,R} = \frac{1}{4\pi} g_{L,R} \cdot |\Delta(H)|$$

$$G_{\mathrm{M}}(T=0,\eta) = \frac{e^2}{h} \cdot \frac{4 g_L g_R \Delta^2}{(8\pi E_c \eta)^2 + (g_L + g_R)^2 \Delta^2}\,, \qquad T \ll (g_L + g_R)|\Delta|/4\pi$$

$$\eta = N_g - N_g^*\,, \quad N_g^* = \text{half} - \text{integer}$$

Resonant tunneling via Majorana states $(H > H_c)$

[Quantitative transport theory of "teleportation", L. Fu, PRL **104**, 056402 (2010)]



$$\mathcal{H}_L = t_L \sum_{pn} \varphi_n^*(r_L) c_p^\dagger \psi_n + \text{h.c.}$$

$$\downarrow$$

$$\tilde{\mathcal{H}}_L = t_L \sum_p \phi_L(r_L)(c_p^\dagger \hat{N}^- - c_p \hat{N}^+)\gamma_L + \ldots$$

Partial level widths $\Gamma_{L,R} \propto |t_{L,R}|^2 |\phi_{L,R}|^2$

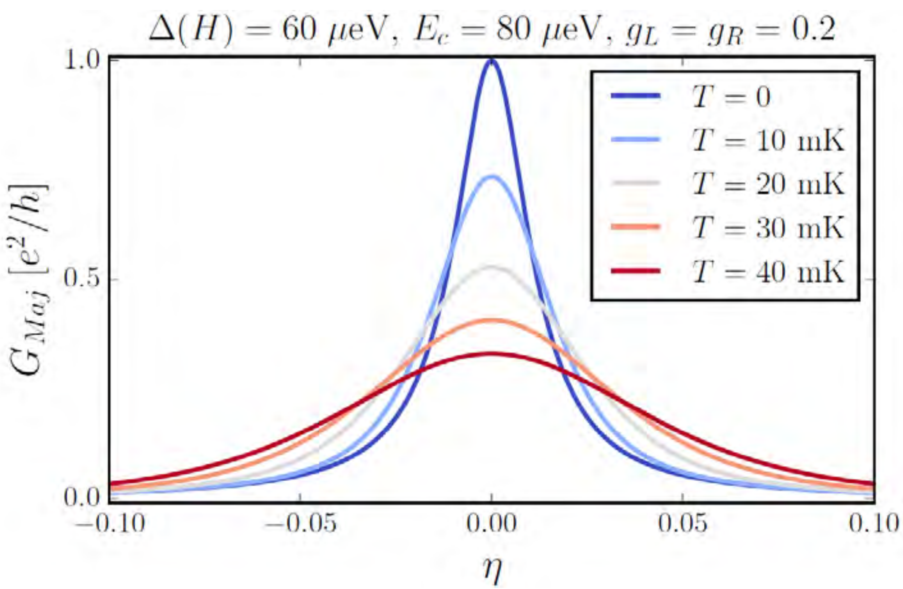
$$|\phi_s(r_s)|^2 \propto Z_0/\xi\,, \quad s = R, L \qquad\qquad \Gamma_{L,R} = \frac{1}{4\pi} g_{L,R} \cdot |\Delta(H)|$$

$$G_{\mathrm{M}}(T=0,\eta) = \frac{e^2}{h} \cdot \frac{4 g_L g_R \Delta^2}{(8\pi E_c \eta)^2 + (g_L + g_R)^2 \Delta^2}\,, \qquad T \ll (g_L + g_R)|\Delta|/4\pi$$

$$G_{\mathrm{M}}(T,\eta) = \frac{e^2}{h} \cdot \frac{g_L g_R}{g_L + g_R} \frac{|\Delta(H)|}{4T} \frac{1}{\cosh^2(\eta E_c/T)}\,, \qquad T \gg (g_L + g_R)|\Delta|/4\pi$$

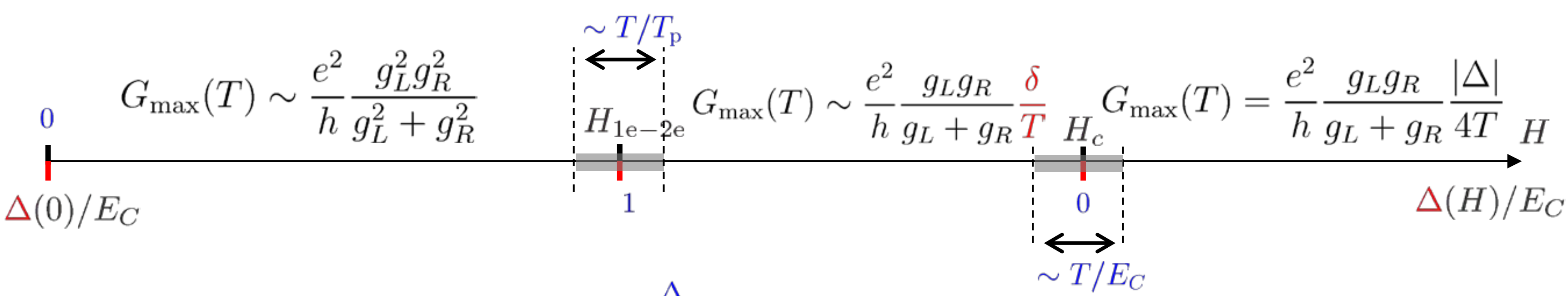
Conductance peaks are symmetric in η

Resonant tunneling via Majorana states $(H > H_c)$



$$T \gg (g_L + g_R)|\Delta|/4\pi$$

$$G_{\max}(T) = \frac{e^2}{h} \cdot \frac{g_L g_R}{g_L + g_R} \frac{|\Delta|}{4T}$$

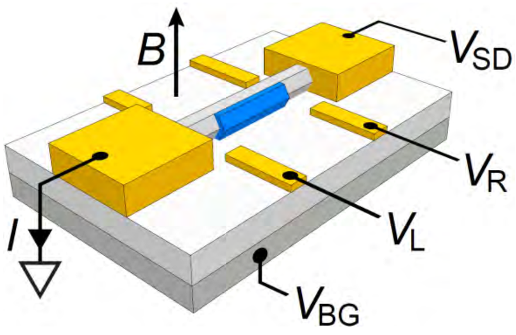


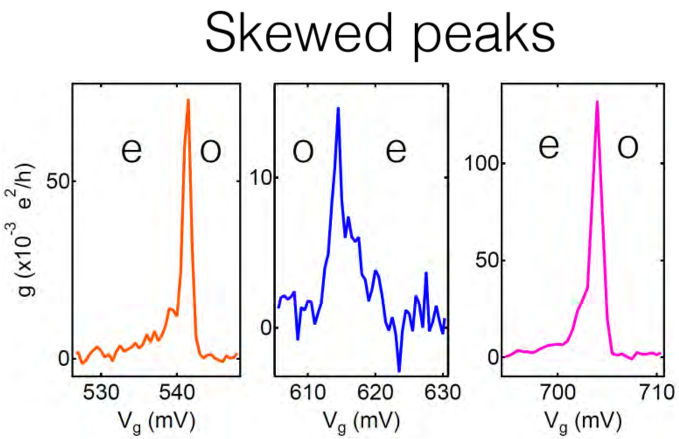
$$T_{\rm p} = \frac{\Delta}{\ln(\Delta/\delta)}$$

Outline

- Conductance in the weak-tunneling limit:
  - Conduction by Cooper pairs (large induced gap)
  - Single-electron transport (smaller gap): conductance peaks - magnitude and shape
  - Resonant tunneling via Majorana states
- **Getting back to the results of experiment**
- Beyond perturbation theory in tunneling

Peak asymmetry in short (L=250 nm) wires





Short wires (L~250nm)
[Higginbotham *et al*,
Nat. Phys., **11**, 1017 (2015)]

$$H = 0$$

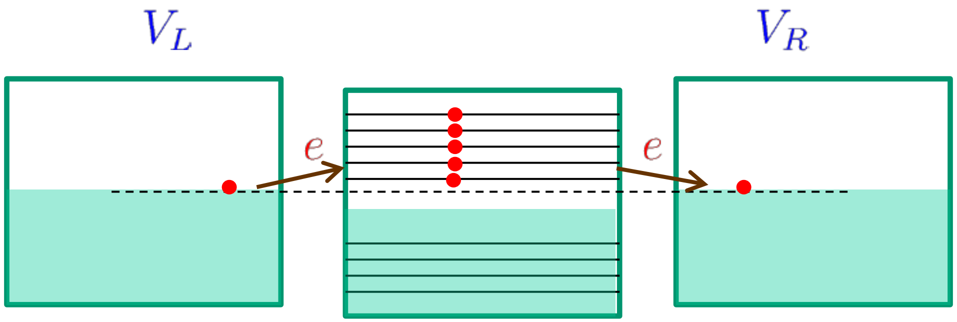
Attempt to explain by *T=0* theory

Peak asymmetry due to co-tunneling $(\Delta(H) < E_C)$

Higher-order tunneling starting from even-charge state, $\eta < 0$

$T = 0$

$\eta = N_g - N_g^*$

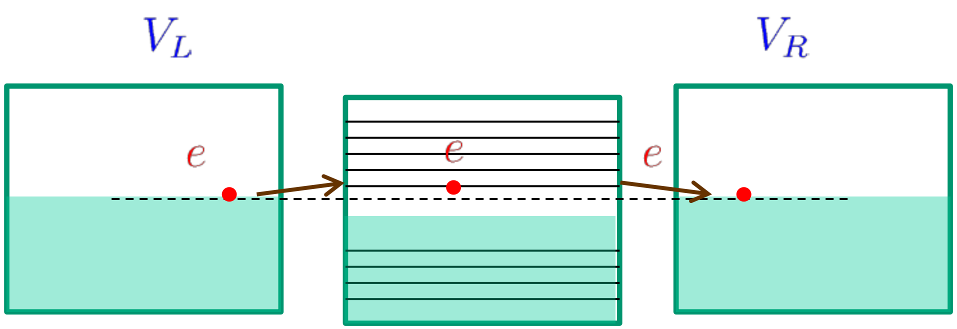


$$G_e \simeq \frac{e^2}{h}\frac{g_L g_R}{4\pi^2}\frac{\delta}{\Delta}\left(\frac{\Delta}{2E_c}\right)^{3/2}\frac{1}{|\eta|^{3/2}}$$

[to the first order in δ ,
divergent at $\eta \to 0^-$]

# of almost-resonant states: $\sim E_C\eta/\delta$

Higher-order tunneling starting from odd-charge state, $\eta > 0$



$$G_e \simeq \frac{e^2}{h}\frac{g_L g_R}{4\pi^2}\frac{\delta}{\Delta}\cdot 0.57$$

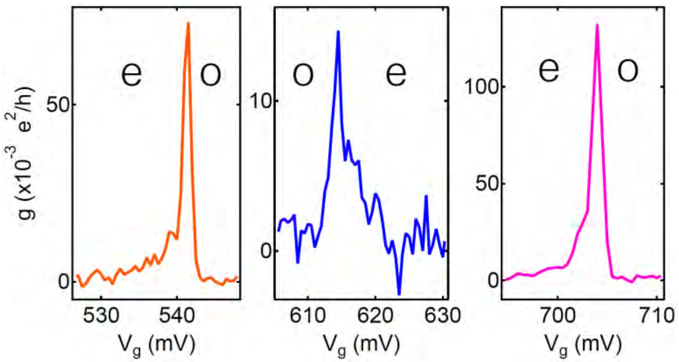
[small; same order as Averin-Nazarov
PRL **69**, 1993, (1992)]

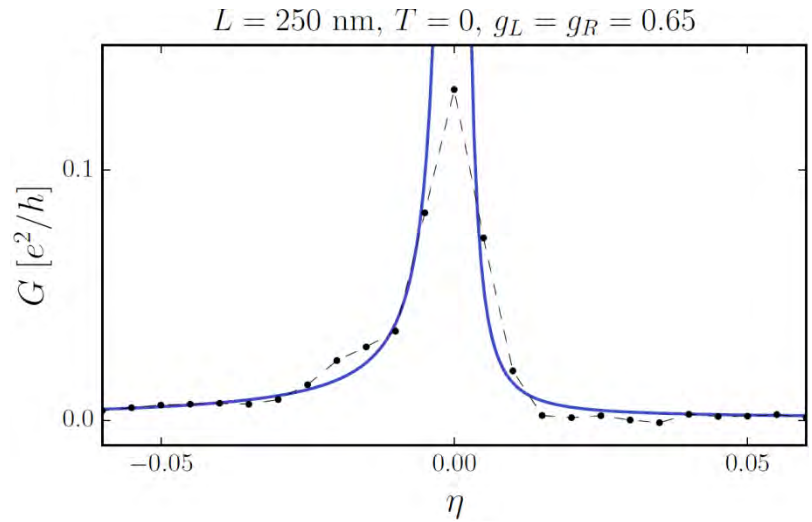
# of almost-resonant states: **one**

The even-charge side of the conductance peak is **fat** (odd side is thinner)

Peak asymmetry in short (L=250 nm) wires

Attempt to fit with *T=0* theory

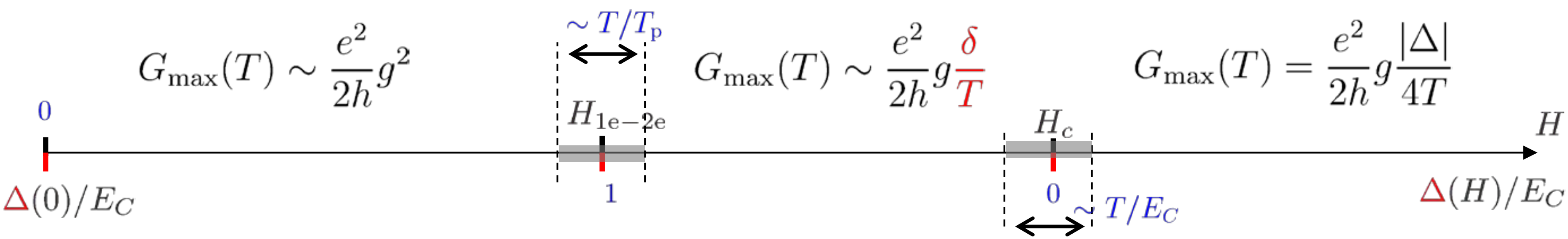


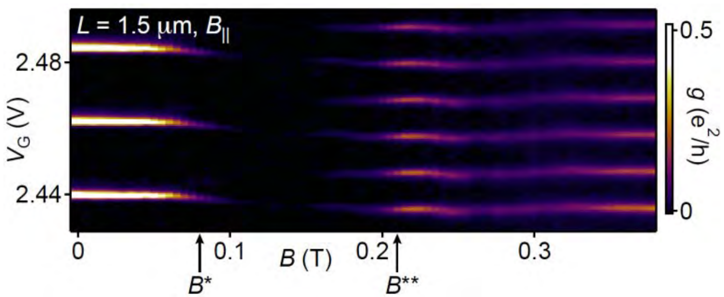


$$G_{\rm el} = \frac{e^2}{h}\,\frac{g_L g_R}{4\pi^2} \times \begin{cases} \dfrac{\delta}{\Delta}\displaystyle\int_1^\infty \frac{ds\ s}{\sqrt{s^2-1}}\,\frac{1-1/2s^2}{(s-1-2\eta E_c/\Delta)^2} + \mathcal{O}(\delta^4/\Delta^2 E_c^2) & \eta < 0 \\ \dfrac{\delta}{\Delta}\displaystyle\int_1^\infty \frac{ds\ s}{\sqrt{s^2-1}}\,\frac{1-1/2s^2}{(s+1+2\eta E_c/\Delta)^2} + \frac{\delta^2}{E_c^2}\,\frac{1}{64\eta^2} + \mathcal{O}(\delta^4/\Delta^2 E_c^2) & \eta > 0 \end{cases}$$

Non-monotonic conductance in long wires?

$$g_L \approx g_R = g$$



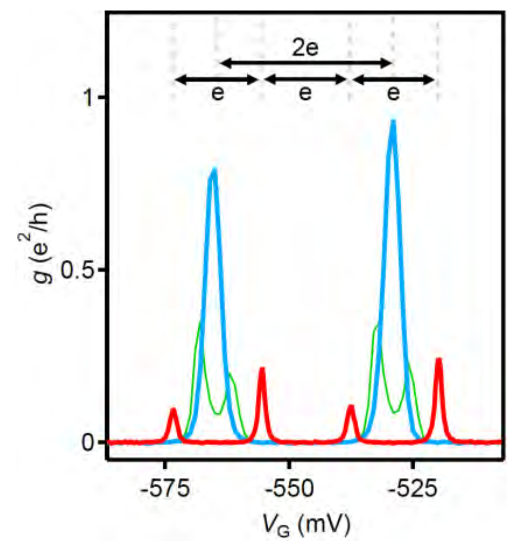


For $G_{\max}(H)$ to be non-monotonic:

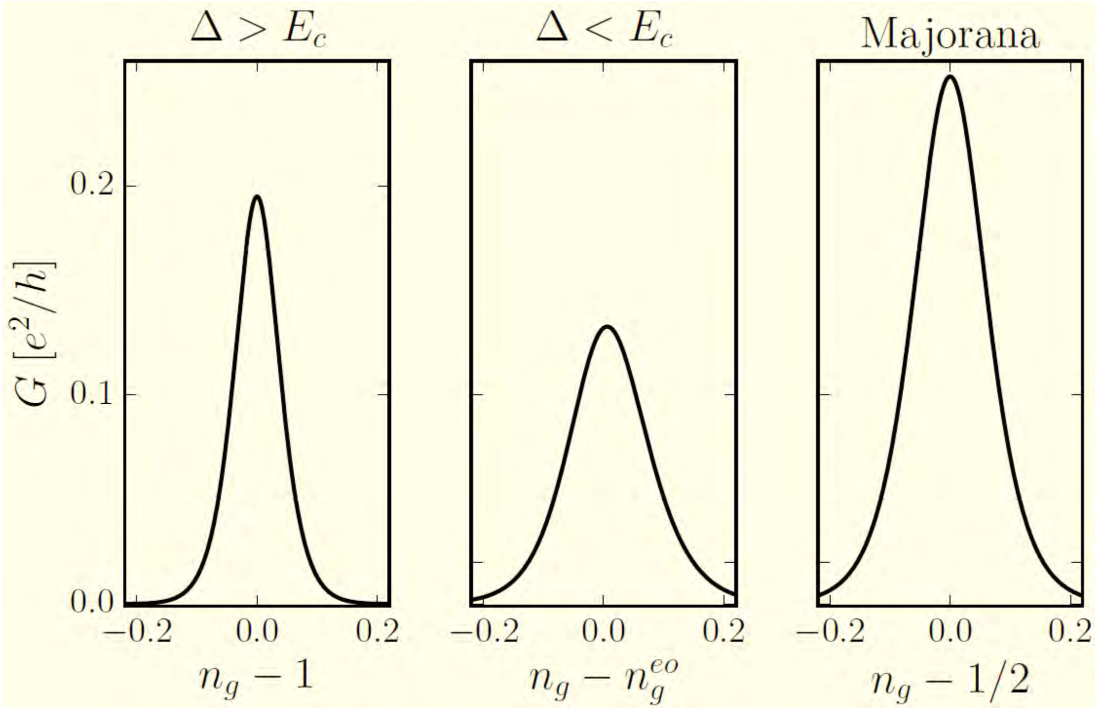
$$H = H_{1\text{e}-2\text{e}} : \ \frac{\delta}{T} < g$$

$$H = H_c : \ |\Delta(H)| > 4\delta \qquad (L \gtrsim 4\xi_M) \ \text{OK}$$

For a single-channel wire, Andreev peaks should be lower and thinner than those for Majoranas, as $g < 1$ and typically $\Delta(H)/T > 4$

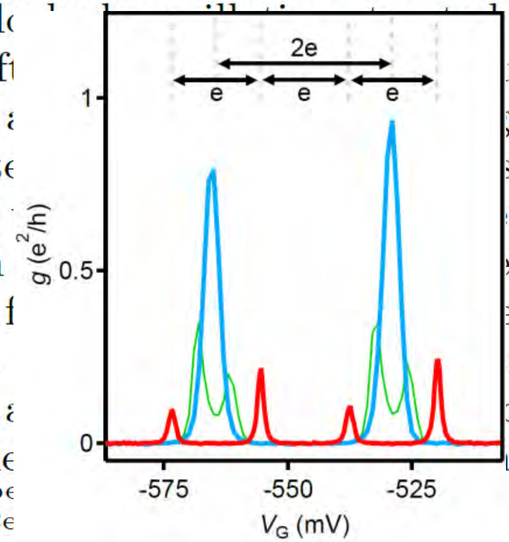


Conductance peaks in a single-channel 1 μm wire



conductance peaks in the three
different regimes of Coulomb bl… in
this work: Andreev regime (left … un-
neling regime (middle panel), … ght
panel). We have used paramete… sti-
mated for the device in Ref. [18] … ak
tunneling regime considered in … eV,
$\Delta_0 = 110\,\mu$eV, $\Delta(B) = 40\,\mu$eV f… els,
$E_c = 54\,\mu$eV, $T = 50$ mK, L … m,
$g_l = g_r = 0.58$. The value of g_l … ox-
imately match the height of the … nce
observed in the experiment (G_{2e}^{pe}

Multiple channels in a wire (?)

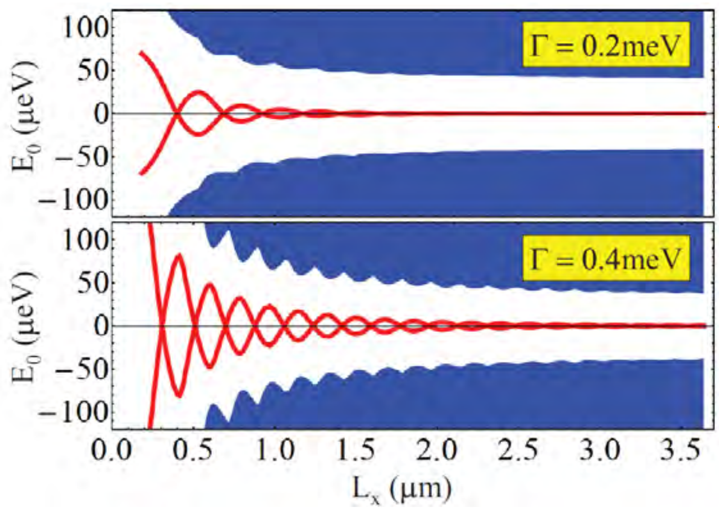


Majoranas in long wires?

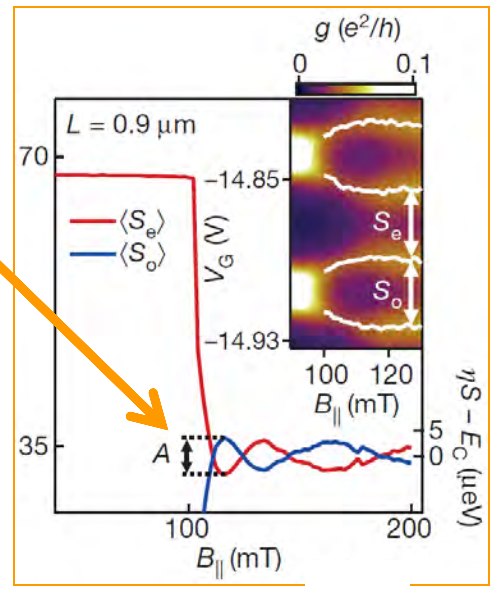
Albrecht *et al*, Nature **531**, 206 (2016)

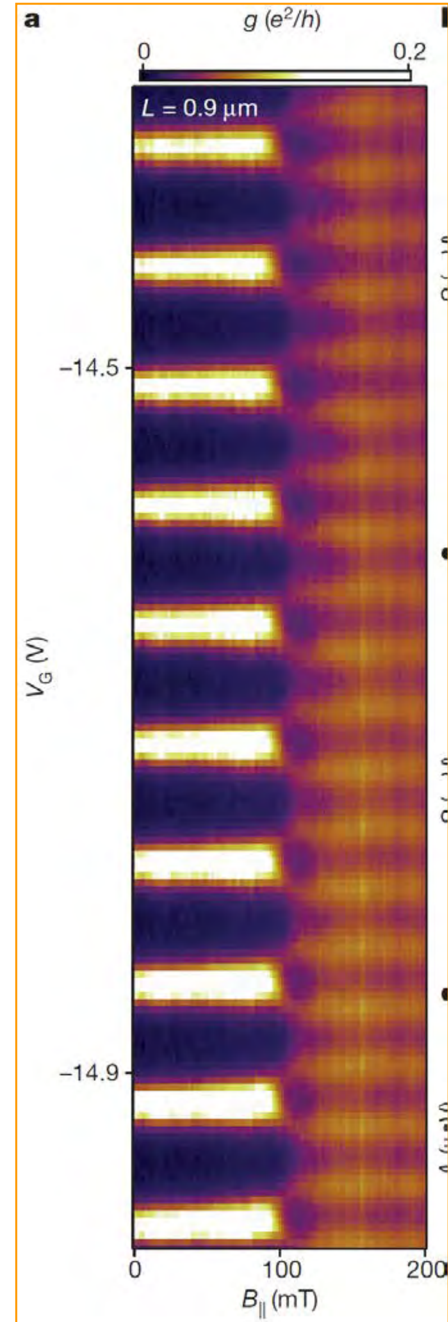
“Exponential protection of zero modes in Majorana islands”

$$E_{\text{odd}} - E_{\text{even}} = f_{\text{osc}}(L, H) \cdot \exp\{-L/\xi(H)\}$$



T. Stanescu, R. Lutchyn, S. Das Sarma, PRB 2013





Regretfully, the amplitude (A) of “wobble” is much smaller than the peak width Γ;

on a brighter side, the peaks are symmetric (Lorentzian shape), allowing to average over many peaks to extract A.

Beyond perturbation theory in tunneling

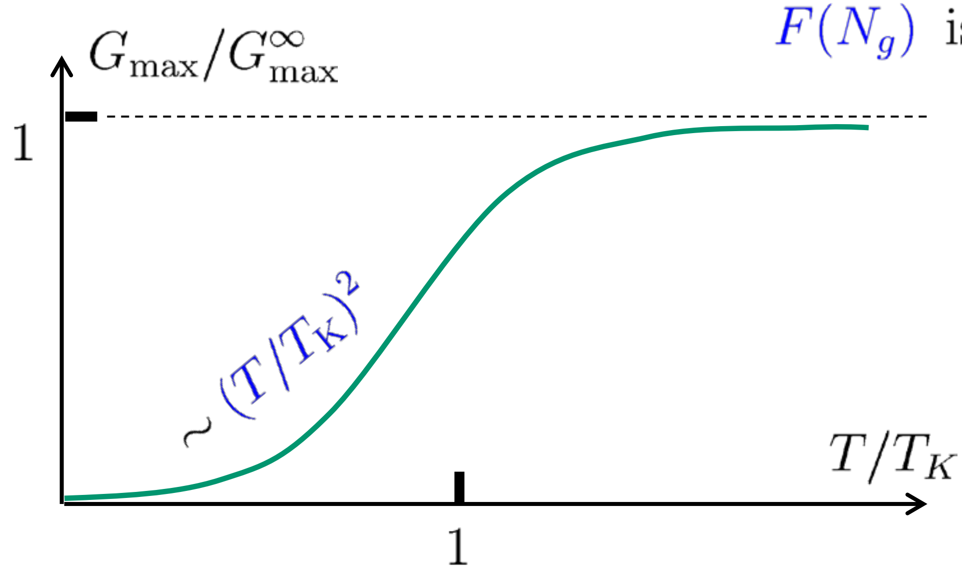
Energy scale for many-body ("charge-Kondo") effects: $T_{\rm K} \lesssim E_C \exp\left\{-\pi^2/(g_L+g_R)\right\}$

In the limit $\delta \to 0$, universal behavior of conductance at $T/T_{\rm K} \ll 1$

In s-wave state: transfer of electron *pairs* between two normal Fermi liquids (R,L); maps onto two-channel Kondo problem, arXiv 1706.04726

$$\mathcal{H}_{\rm pair-tunn} = t\sum_{\sigma_1\sigma_2}\psi^\dagger_{R,\sigma_1}\psi^\dagger_{R,\bar\sigma_1}\psi_{L,\sigma_2}\psi_{L,\bar\sigma_2} \qquad G(T,N_g) = F(N_g)\cdot T^2$$

$F(N_g)$ is max at $N_g = \text{odd}$ (generic $g_L \neq g_R$)



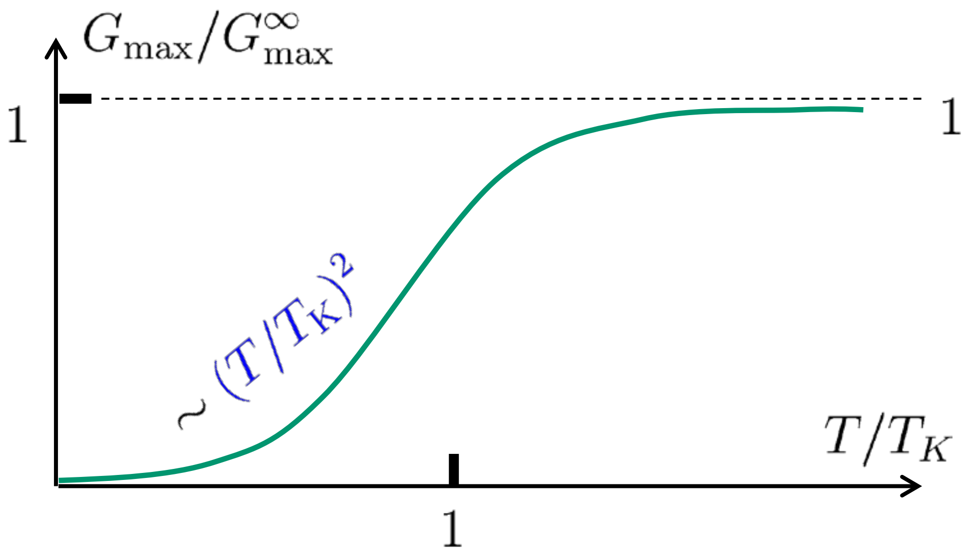
$$G^\infty_{\rm max} = G_{\rm max}(T \gg T_K) \sim \frac{e^2}{h}\frac{g_L^2 g_R^2}{g_L^2+g_R^2}$$

$$G_{\rm max}(T) \sim G^\infty_{\rm max}\cdot(T/T_K)^2$$

Non-perturbative Andreev *vs.* Majorana

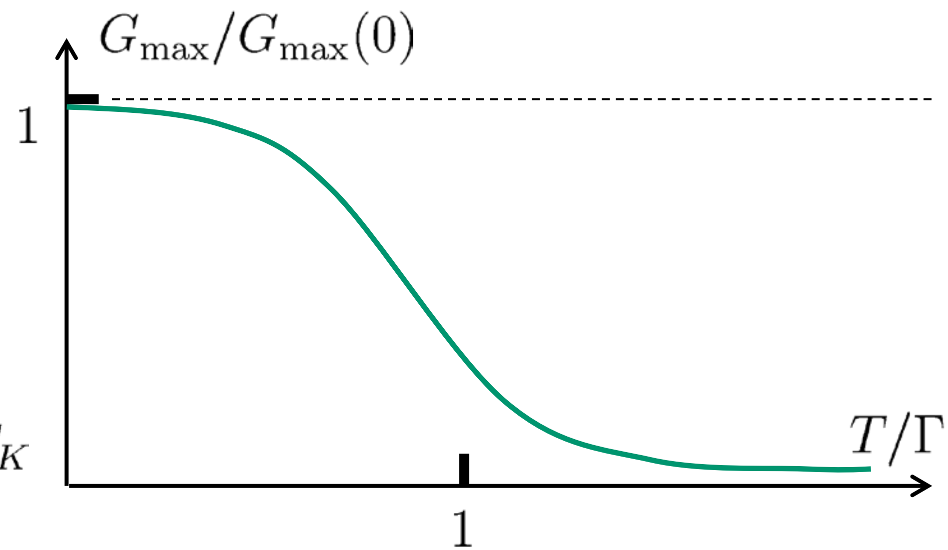
Energy scale for many-body ("charge-Kondo") effects: $T_{\mathrm{K}} \lesssim E_C \exp\left\{-\pi^2/(g_L+g_R)\right\}$

$H < H_c$:



Two-electron (inelastic) tunneling

$H > H_c$:



Single-electron resonant tunneling

(generic $g_L \neq g_R$)

$g_L \to 1$ or $g_R \to 1$ lead to $T_{\mathrm{K}} \sim E_C$

Conclusions

Quantitative predictions for peak conductance, explanation of the non-monotonic variation of peak conductance with magnetic field, B

Coulomb blockade peak asymmetry explained by resonant elastic co-tunneling

Quantitative theory for two-terminal conductance via a Majorana resonance

There is still lack of understanding, why the observed (Majorana?) “peaks” are so dim