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From the parity anomaly to a Majorana fermion - realization of the ultrarelativistic physics in topological insulators

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SPICE Workshop
"Topology Matters"
28 July 2017

Outline

1. Majorana fermions in topological insulators proximitized with superconductor:

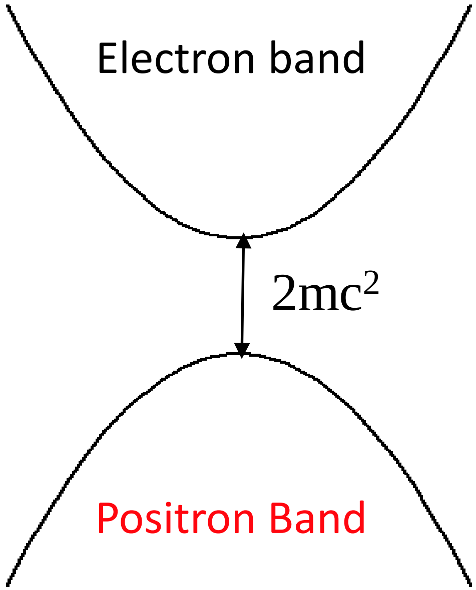
a) Hybrid topological insulator/ superconductor junctions: How does the helicity appear here?

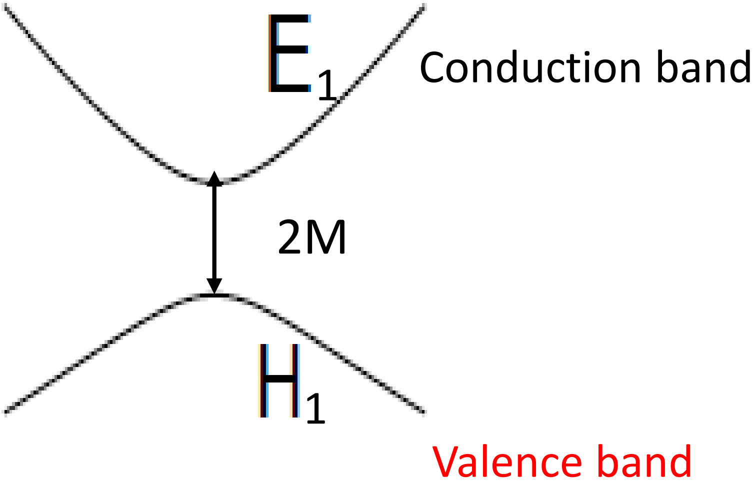
b) Is thermal conductance a way to detect topological Andreev bound states?

c) AC Josephson effect as a tool to detect 4π supercurrent. What is the regime where Shapiro steps and power emission might indicate the 4π supercurrent?

Dirac Fermions in Vacuum and in Semiconductor

1902 - 1984





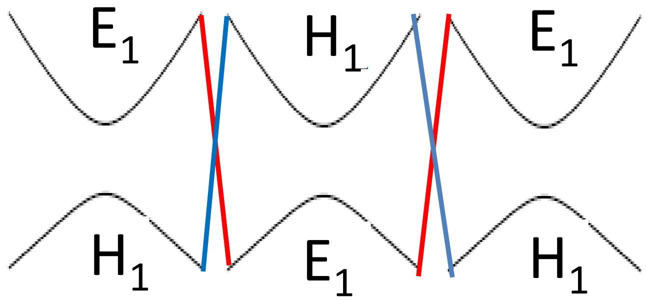
$c \Rightarrow A/\hbar \approx 5.6 * 10^{5}$ m/s

$mc^2 \Rightarrow M \approx 10$ meV

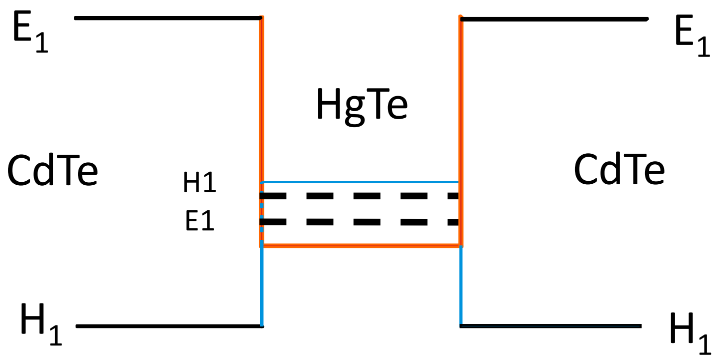
Why is that interesting? :

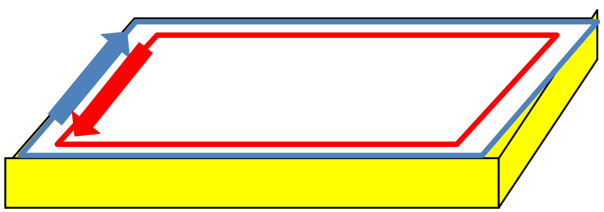
1. Testing ultra-relativistic physics in the Solid State Lab.
2. Boundary between the material where Dirac Fermion resides and vacuum.
3. Information about the topology.

Inversion of bands: key to topological insulator



Example: HgTe Quantum Well



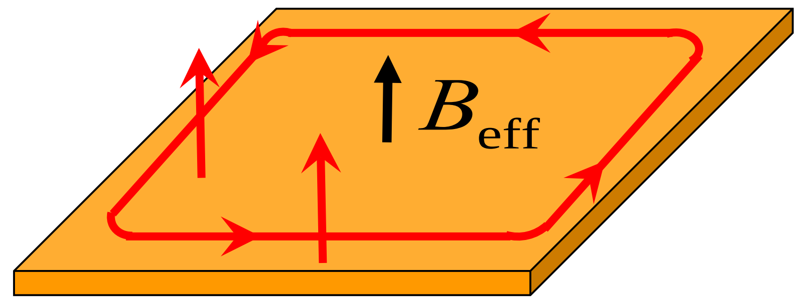


Topological insulator have a bulk gap
while the metallic (edge or surface) states
(at the boundary) cannot be removed by a deformation

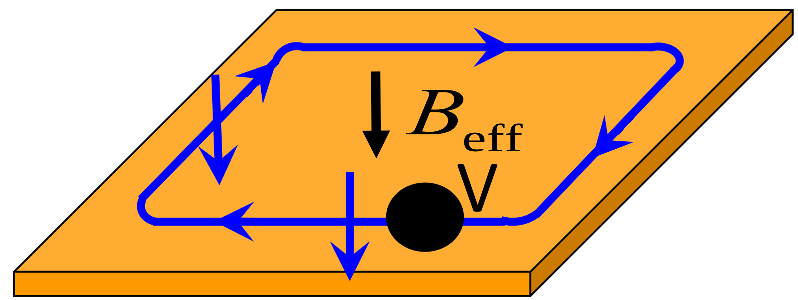
A. Bernevig, T. Hughes, S.C. Zhang Science 314, 1757 (2006).
M. König et al. Science 318, 766 (2007).

Quantum spin Hall effect as characterization of 2D topological insulator

- The QSH state can be thought of as two copies of QH states, one for each spin component, each seeing the opposite magnetic field.



- The QSH state does not break the time reversal symmetry, and can exist without any external magnetic field.



$$|\Psi>=T|\Phi> \quad [V,T]=0$$

$$<\Psi|V|\Phi>=<T\Phi|V|\Phi>=<TV\Phi|T^2|\Phi>=-<T\Phi|V|\Phi> \Rightarrow <\Psi|V|\Phi>=0$$

G. Tkachov and E. M. Hankiewicz PRL 104, 166803 (2010)
G. Tkachov and E.M. Hankiewicz Phys. Status Solidi 250, 215 (2013)

Effect is protected against elastic scattering by time reversal symmetry

Formation of the helical edge states
Spin-momentum locking: direction of spin follows momentum

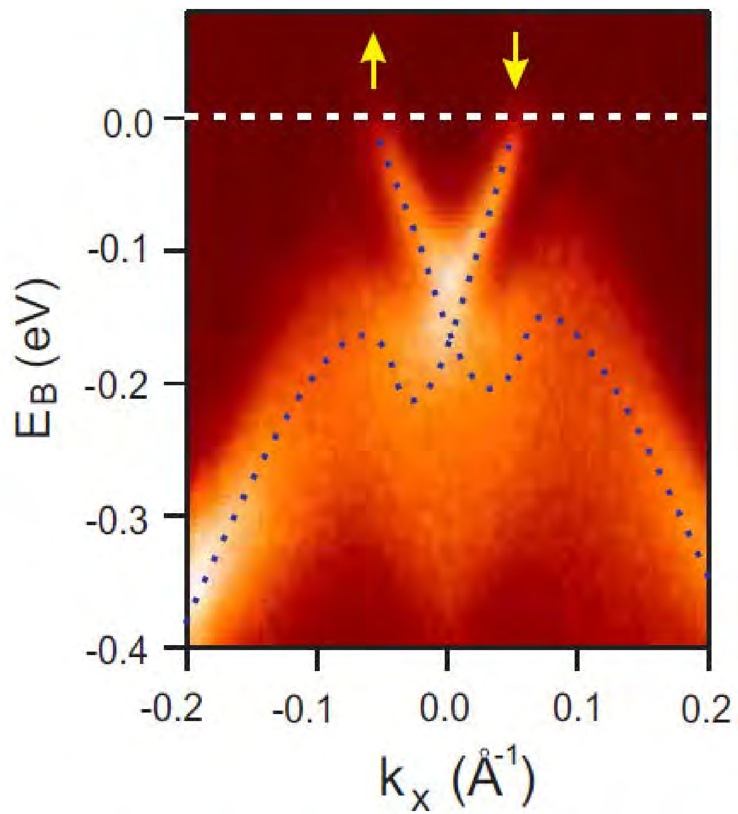
Spin-momentum locking in 3D Topological Insulators

Metallic surface state

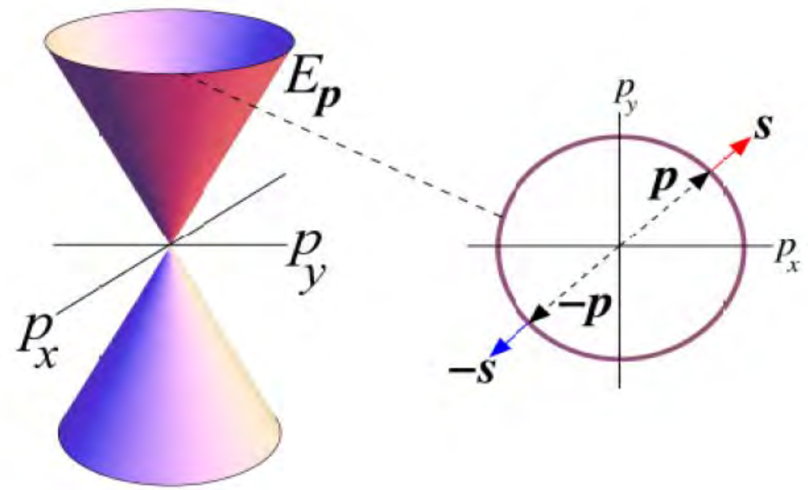
$$h_p = v\boldsymbol{\sigma} \cdot \boldsymbol{p}$$

$Bi_{1-x}Sb_x$, Bi_2Se_3, Bi_2Te_3

ARPES: direct visualization of Dirac state



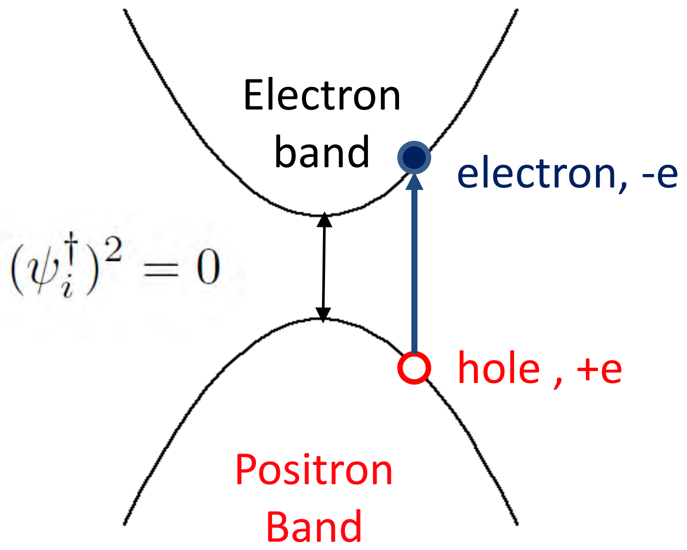
Hasan's group (Princeton) Nature 2008

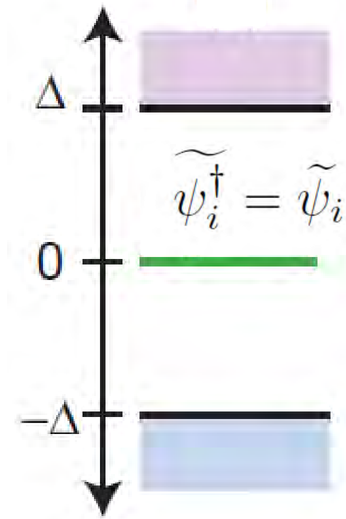


$$\langle \boldsymbol{p} | \hat{V} | -\boldsymbol{p} \rangle = 0$$

States with **p** and **−p** are orthogonal.
Lack of backscattering.

Topological Insulator vs Topological Superconductor





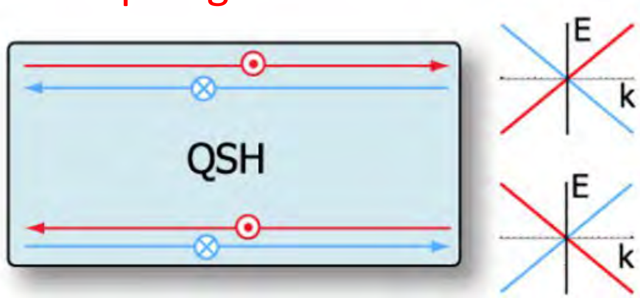
Dirac equation with complex field, charged particle with spin 1/2

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

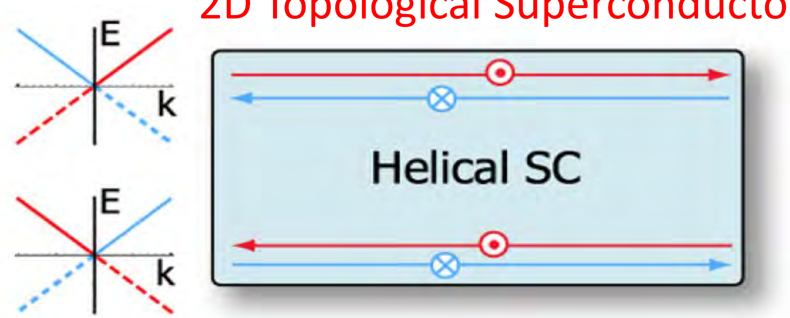
Majorana equation with real field, neutral particle with spin 1/2

$$(i\tilde{\gamma}^\mu \partial_\mu - m)\tilde{\psi} = 0$$

2D Topological Insulator



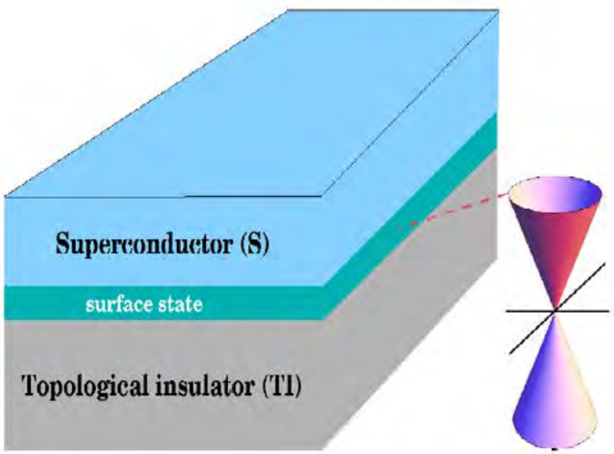
2D Topological Superconductor



X-L. Qi , T.L. Hughes, S. Raghu and S.-C. Zhang Phys. Rev. Lett. 102, 187001 (2009)

What kind of order parameter is induced by conventional superconductor in the 3D topological insulator?

Proximity effect in terms of tunneling model



Surface state Hamiltonian (electron, hole)

Induced singlet pairing

$$\hat{H}_p^{\rm eff} = \begin{bmatrix} v\boldsymbol{\sigma}\cdot\boldsymbol{p} - \mu - i\Gamma(\epsilon) & \Delta(\epsilon) i\sigma_y e^{i\phi} \\ -\Delta(\epsilon) i\sigma_y e^{-i\phi} & v\boldsymbol{\sigma}^*\cdot\boldsymbol{p} + \mu - i\Gamma(\epsilon) \end{bmatrix}$$

Spectrum shift due to tunneling

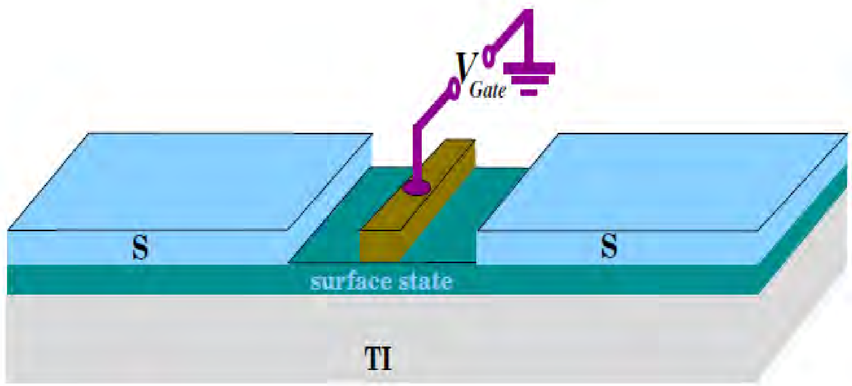
$$G_{12,\boldsymbol{p}} = \frac{1}{2}\frac{(\sigma_0 + \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) i\sigma_y\, \Delta(\epsilon)\, e^{i\phi}}{\mathcal{E}^2(\epsilon) - v^2(p - p_F)^2 - \Delta^2(\epsilon)}$$

Mixed s-wave and p-wave correlations.
Due to broken spin rotational symmetry: spin singlet can evolve into spin triplet

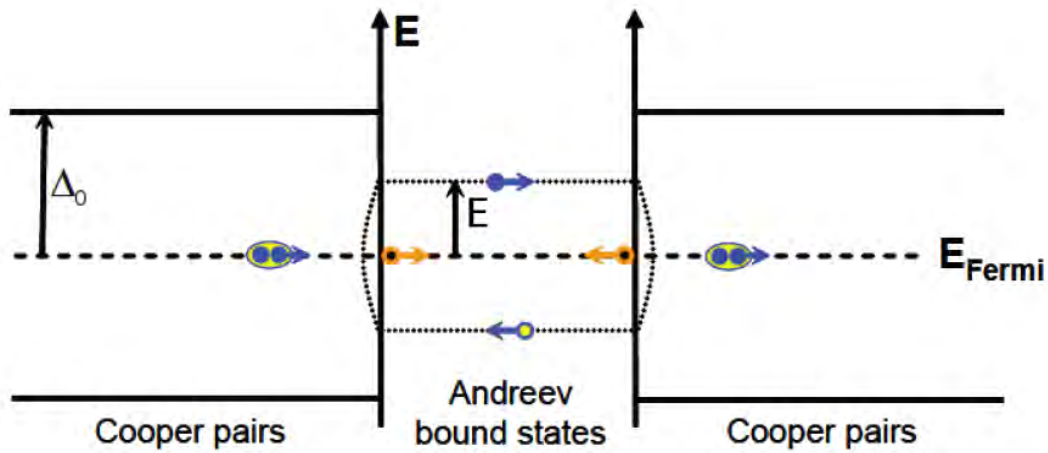
G. Tkachov and E.M. Hankiewicz Review Article: PSS B 250, 215 (2013)
G. Tkachov and E.M. Hankiewicz Phys. Rev. B ***88****, 075401 (2013)*

Generalized model based on Fu and Kane PRL 100,096407(2008) (p-wave)

Andreev Bound States



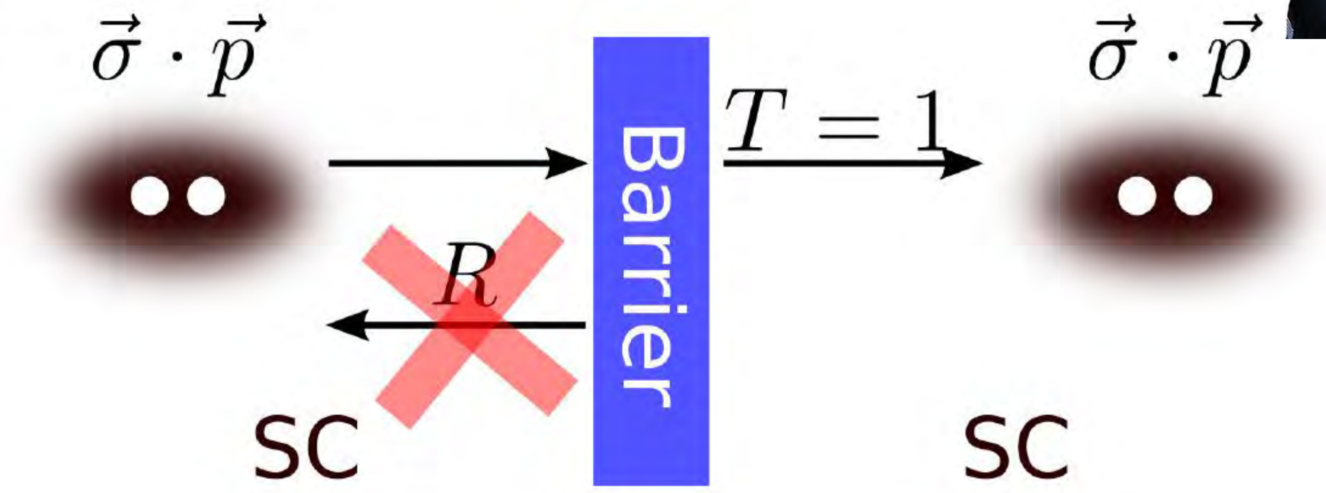
Match wave functions of left and right superconductors assuming the delta-like barrier in the middle.



Electron converts into hole, hole converts into an electron.
If many reflections, the Andreev bound state forms.

Beenakker Springer Series in Solid-State Sciences. Vol. 109,1992

Superconducting Klein tunneling

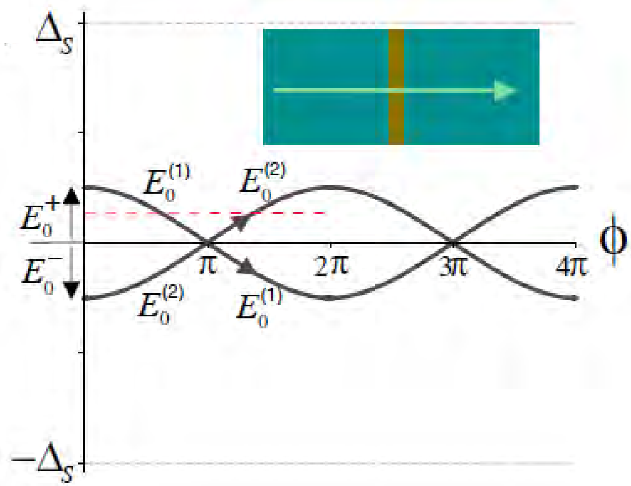


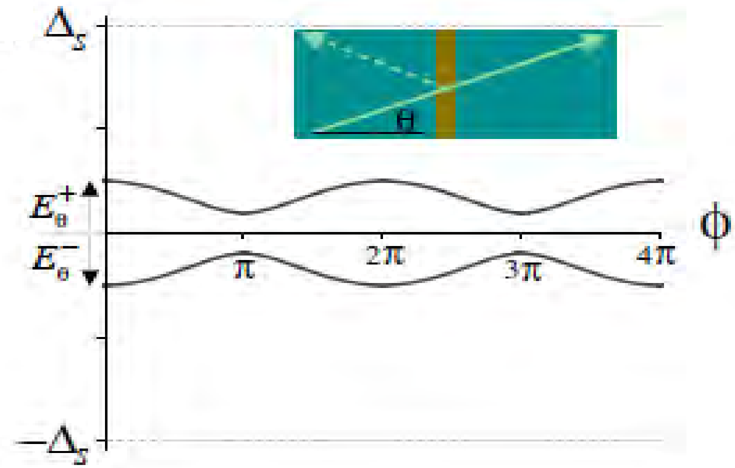
$$T_\theta = \frac{\cos^2(\theta)}{1 - \sin^2(\theta)/(1 + Z^2)}$$

Passing through the barrier of helical Cooper pairs/helical Andreev bound states independent of the height of the barrier

G. Tkachov and E.M. Hankiewicz Review Article: PSS B 250, 215 (2013)
G. Tkachov and E.M. Hankiewicz Phys. Rev. B **88**, 075401 (2013)

Topological versus trivial Andreev bound states





Topological Andreev bound states
are connected by time reversal symmetry

Trivial Andreev bound states
for oblique incidence.

$$\Psi_0^{(2)}(x) = \tau_0 i \sigma_y \Psi_0^{(1)*}(x)$$

Topological Andreev bound states are
the eigenstates of spin helicity

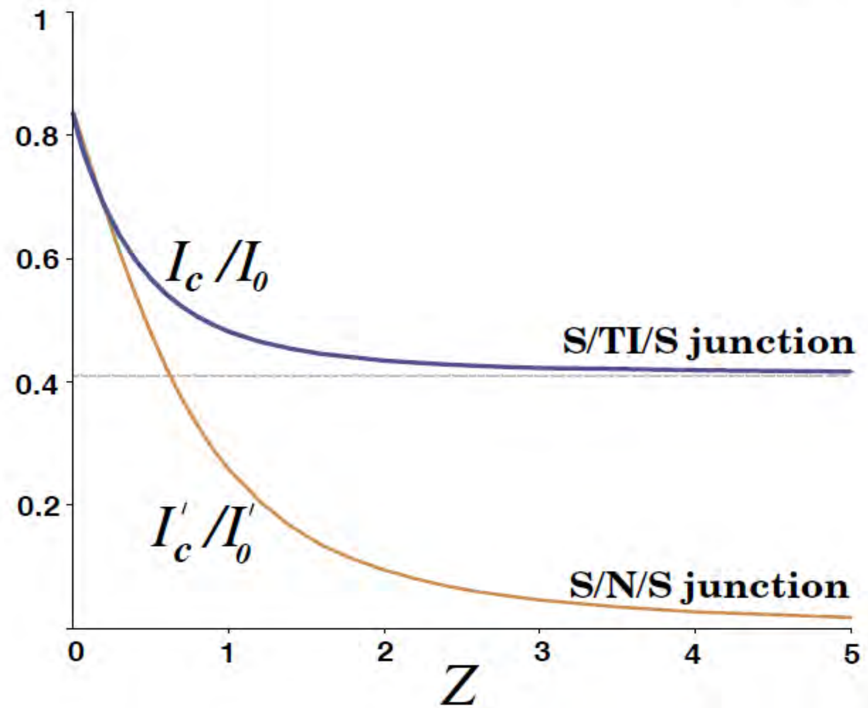
How to see helical Andreev bound state?

S/TI/S junction

$$\varepsilon = \pm\Delta\cos\phi/2$$

S/N/S junction

$$\varepsilon = \pm\Delta\sqrt{1 - D\sin^2\phi/2}$$

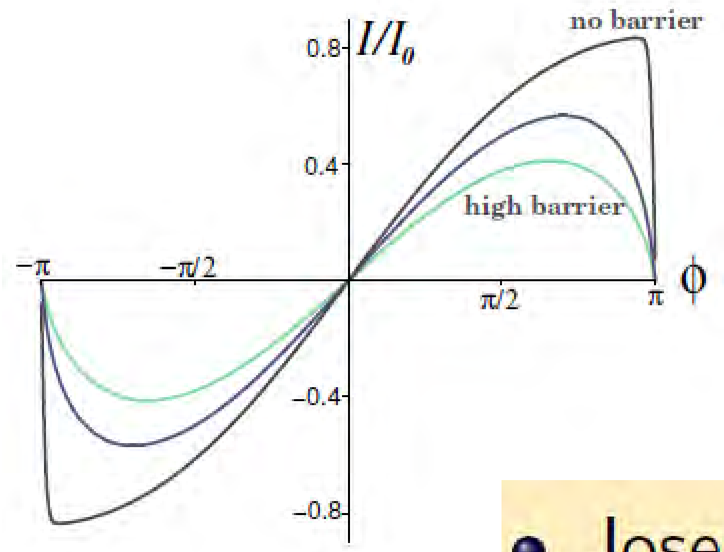


Our recipe: Tune the barrier at the normal part by a gate in the Josephson junction: only topological component stays.

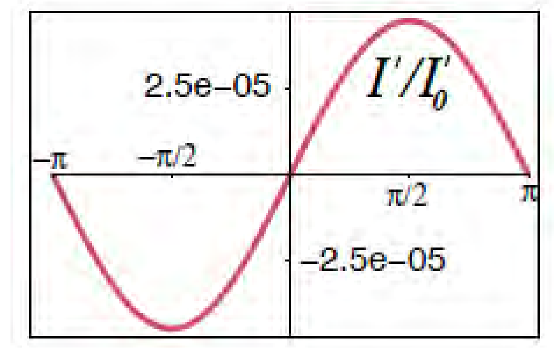
G. Tkachov and E.M. Hankiewicz
Phys. Rev. B **88**, 075401 (2013)

Non-sinusoidal current phase relation as a signature of helical Andreev bound state

S/TI/S



S/N/S



- Josephson current $I \propto \frac{\partial \varepsilon}{\partial \phi}$

We predict higher signal and non-sinusoidal current phase relation for the helical Andreev bound state.

G. Tkachov and E.M. Hankiewicz Phys. Rev. B ***88****, 075401 (2013)*

Sochnikov, L. Maier, C. Watson, J. R. Kirtley, C. Gould, G. Tkachov, E. M. Hankiewicz, C. Brüne, H. Buhmann, L.W. Molenkamp, and K. A. Moler
PRL ***114****, 066801 (2015)*

Outline

1. Majorana fermions in topological insulators proximitized with superconductor:

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How does the helicity appear here?

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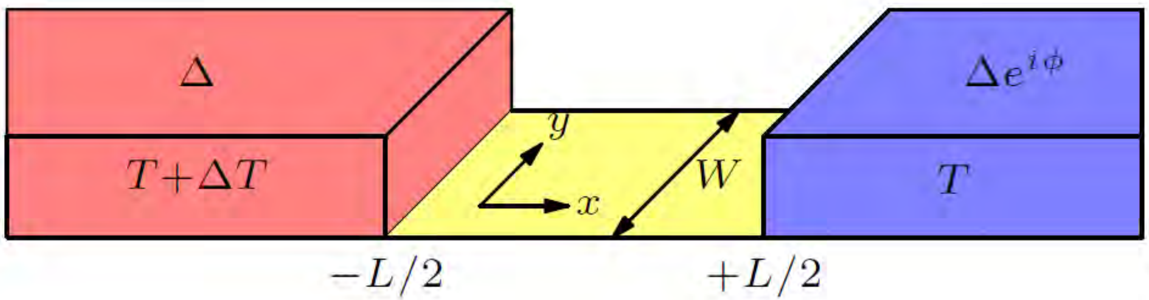
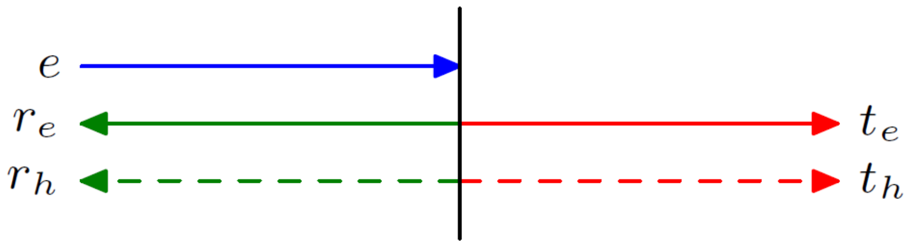
Heat conductance
to detect the helical Andreev bound states





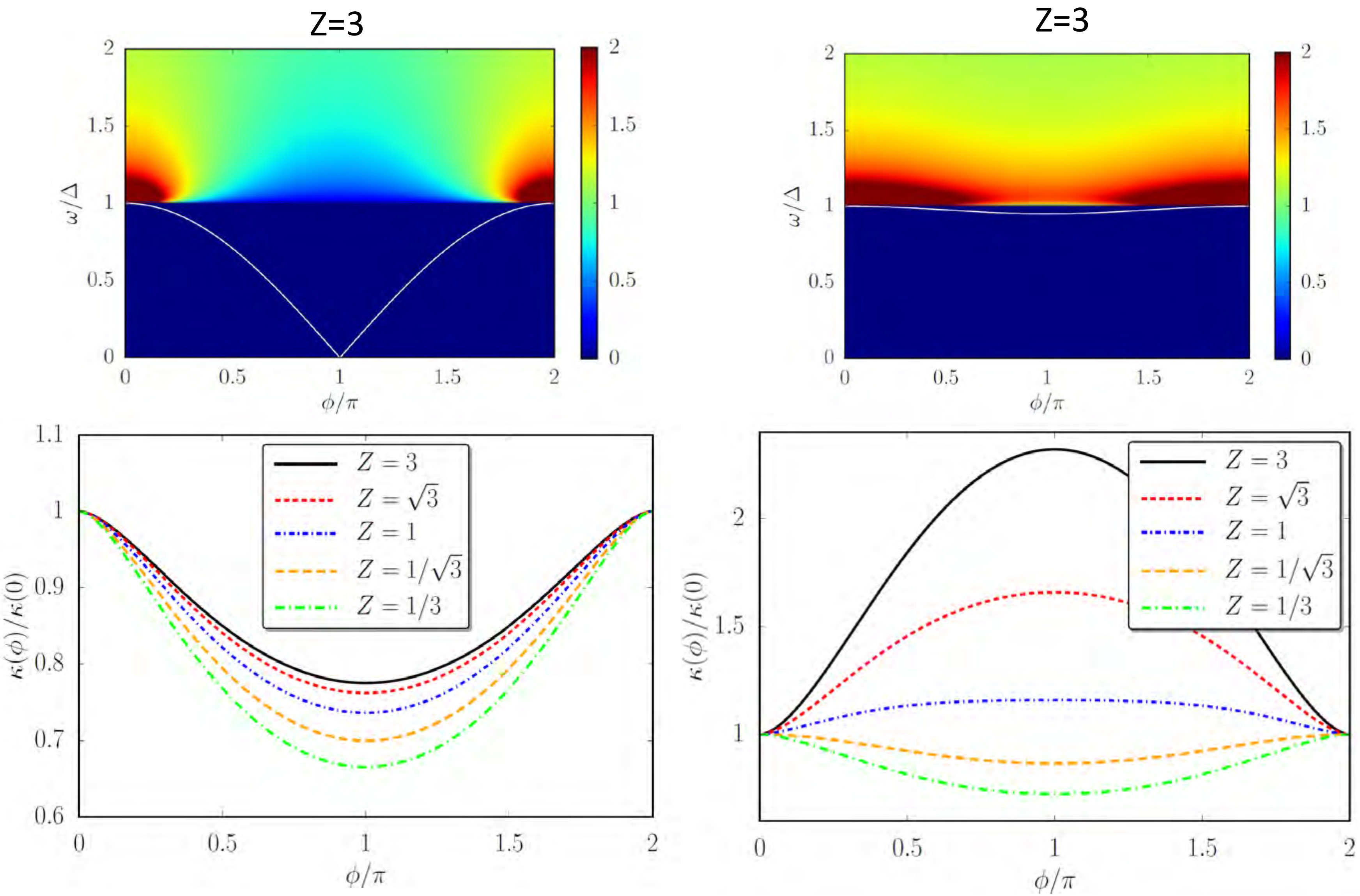
- Transmission probability $\mathcal{T}_e(\omega,\phi) = |t_e|^2 + |t_h|^2$
- Thermal conductance

$$\kappa(\phi) = \frac{2}{h}\int_{\Delta}^{\infty} d\omega\, \omega \mathcal{T}_e(\omega,\phi)\frac{df}{dT}$$

Heat current carried by quasiparticles above the gap
Complementary to Josephson current due to Andreev bound states inside the gap
No issues with quasiparticle poisoning

B. Sothmann and E. M. Hankiewicz PRB 94, 081407(R) (2016)

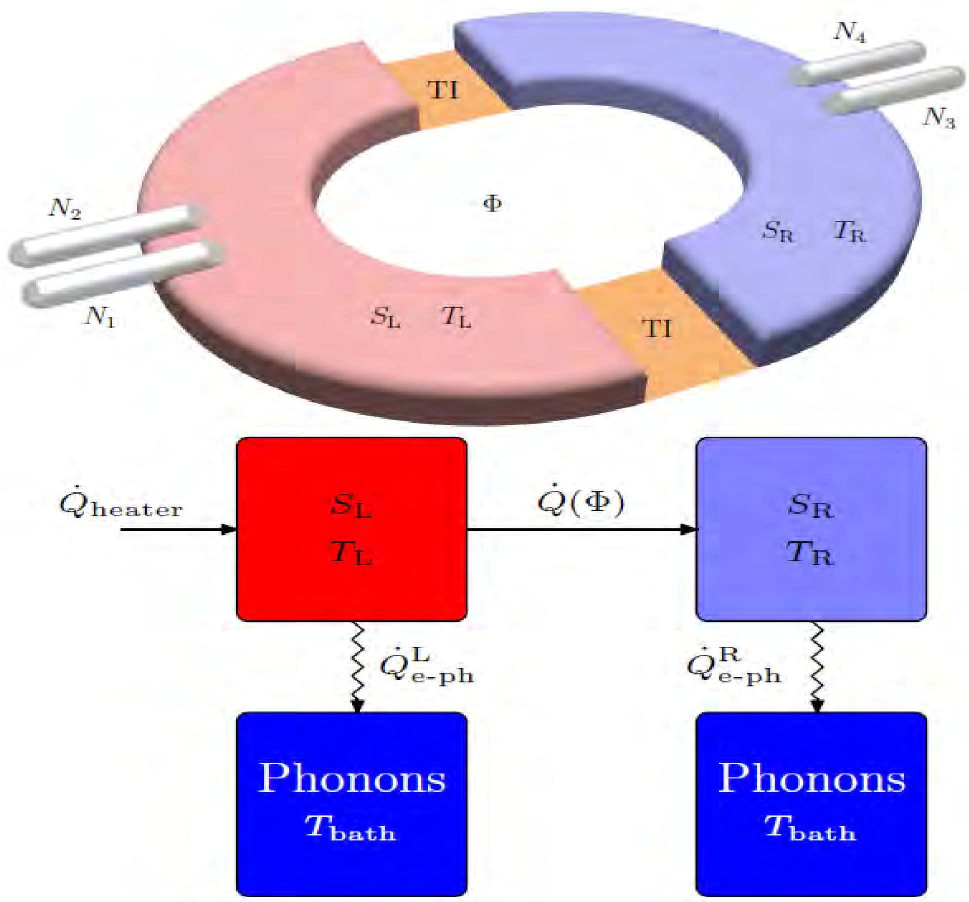
Andreev bound states in S/TI/S and S/N/S junctions



Helical Andreev bound states dominate thermo-transport even for many channels.
Smoked gun test for helical Andreev bound states.

B. Sothmann and E. M. Hankiewicz PRB 94, 081407(R) (2016)

Suggestion for an experiment



SQUID with normal metal electrodes connected to the superconductors via tunnel barrier

Similar to the experiment by Giazotto et al. Nature 2012

$$\dot{Q}(\Phi) = 2\kappa(\phi)(T_L - T_R)$$

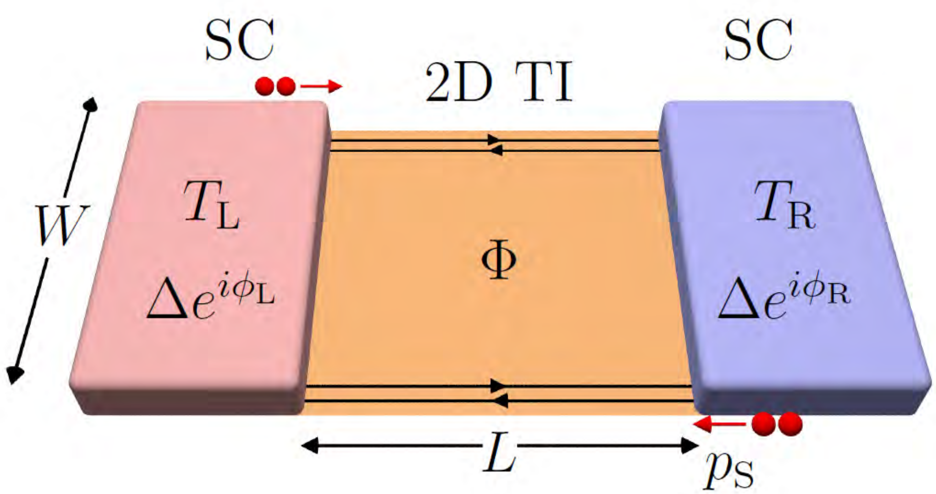
$$T_L = 500\,\mathrm{mK}$$

$$T_{bath} = 100\,\mathrm{mK}$$

We predict measurable change of the drain temperature T_R around 20 -30mK between maximum and minimum of phase.

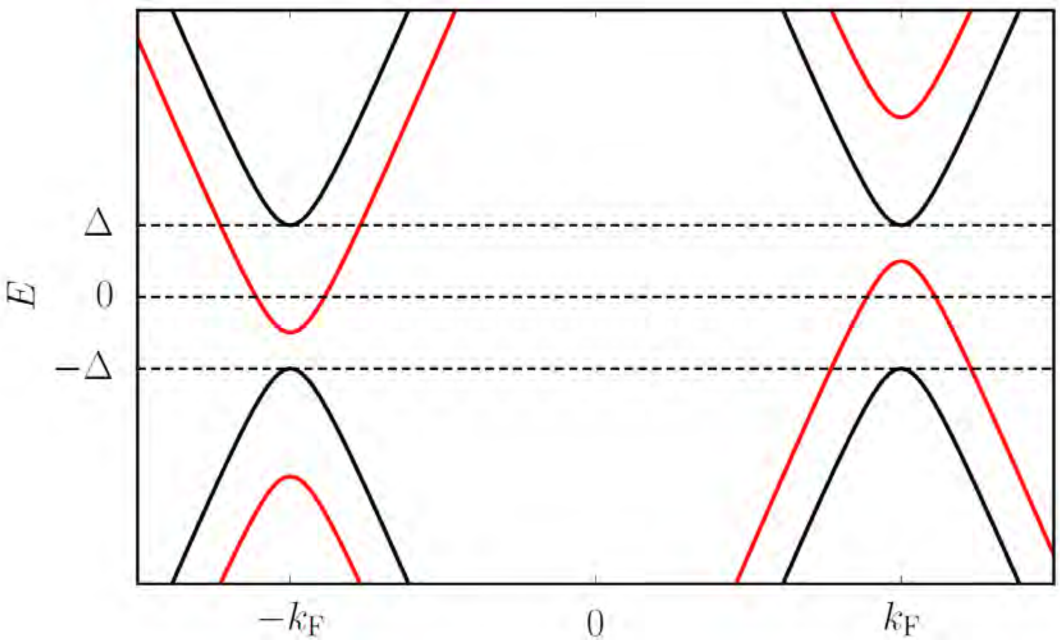
B. Sothmann and E. M. Hankiewicz PRB 94, 081407(R) (2016)

Thermal switch based on 2D topological insulators



$$H_{\mathrm{BdG}} = \begin{pmatrix} h(x) & i\sigma_y \Delta(x) \\ -i\sigma_y \Delta^*(x) & -h^*(x) \end{pmatrix}$$

$$h(x) = v_F \sigma_x \left(-i\hbar\partial_x \pm \frac{p_S}{2}\right) - \mu.$$



The superconducting gap closes for a small flux allowing to switch thermal transport.

B. Sothmann, F. Giazotto and E. M. Hankiewicz *New J. Phys. 19, 023056 (2017)*

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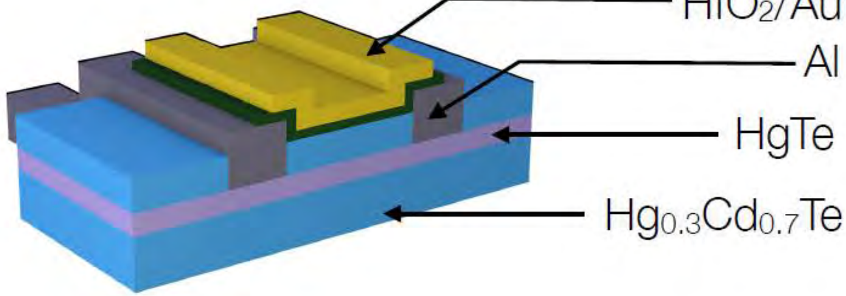
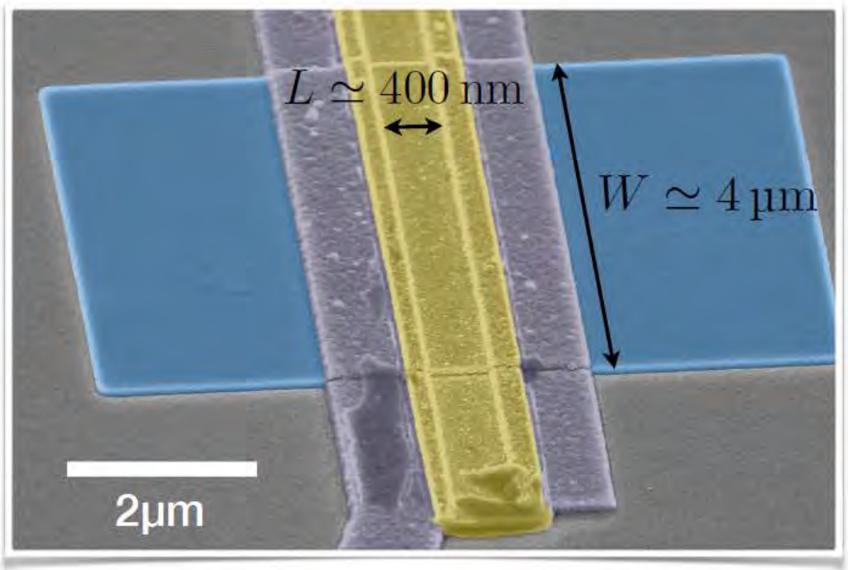
Josephson junction on quantum spin Hall insulator





Josephson junctions

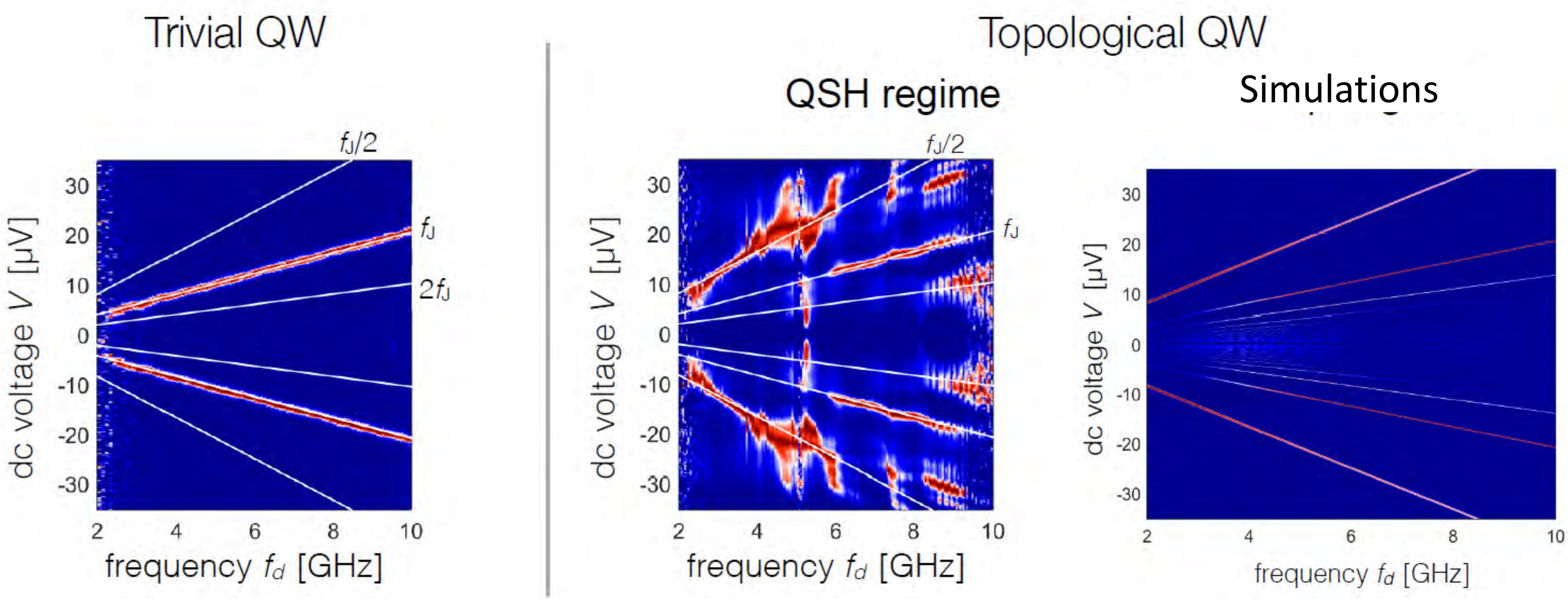
- μ≈3 10^5 $cm^2V^{-1}s^{-1}$
- Al contacts (in situ)
- HfO_2 /Au gate
- no overlap of edge states
- ballistic / intermediate

$$L \ll l \qquad L \lesssim \xi$$

l = 2.4μm
ξ = 600nm
Δ_{ind} = 80μeV

E. Bocquillon et al. Nat. Nanotechnology 12, 137 (2017)

Emission spectra for topologically trivial and nontrivial quantum wells



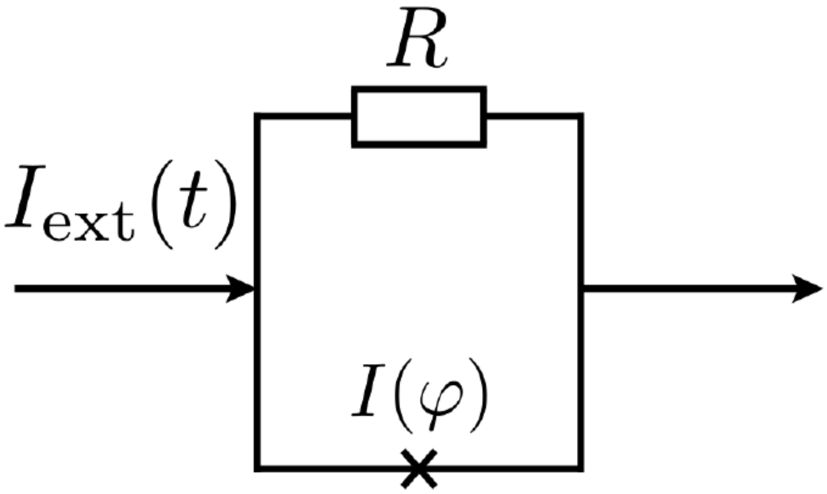
No 4π modes in trivial quantum wells.
4π modes/fractional frequency in non-trivial quantum wells but only for low frequencies!

R. Deacon … E.M. Hankiewicz … and L. Molenkamp PRX **7**, 021011 (2017)
F. Dominguez … and E.M. Hankiewicz, Phys. Rev. B 95, 195430 (2017) -

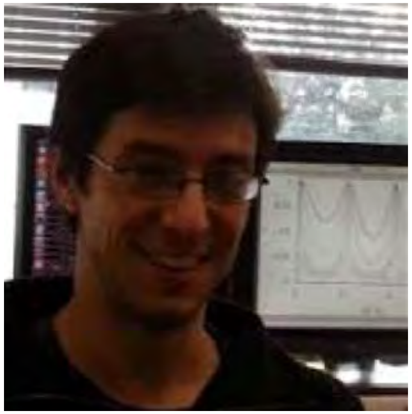
Resistively shunted junction

$$I_{\text{ext}}(t) = \frac{\hbar}{2eR}\frac{d\varphi}{dt} + I(\varphi) \qquad I_{\text{ext}}(t) = I_0 + I_{\text{ac}}\sin(\omega_{\text{ac}}t)$$

$$I(\varphi) = I_{2\pi}\sin(\varphi) + I_{4\pi}\sin(\varphi/2)$$



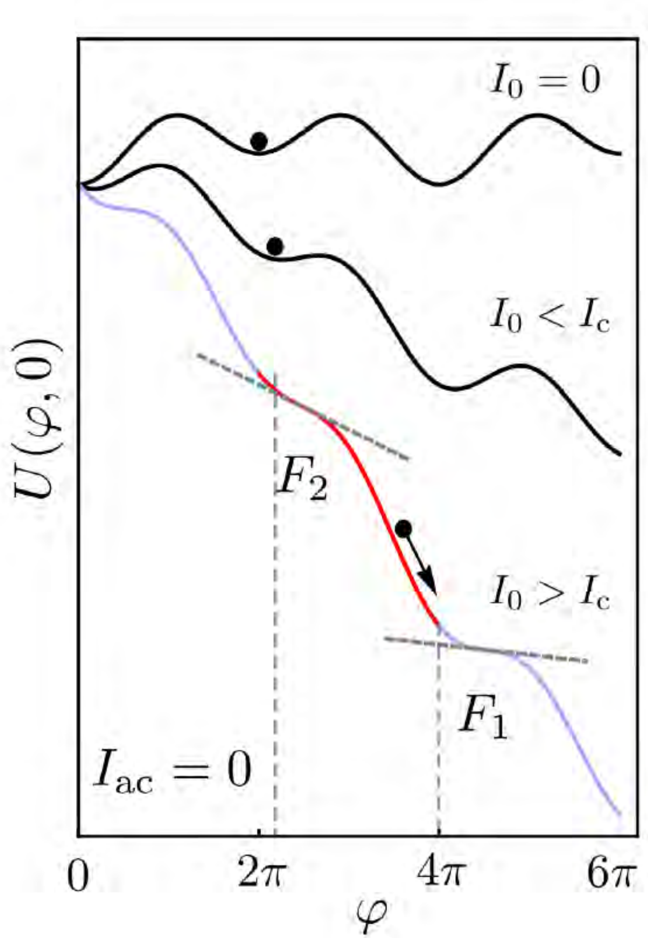
Washboard potential



Potential of the sliding ball

$$m\ddot{\varphi} = -\frac{\hbar}{2eR}\dot{\varphi} - \partial_\varphi U(\varphi)$$

$$U(\varphi) = -I_{2\pi}\cos(\varphi) - I_{4\pi}\cos(\varphi/2)) - I_0\varphi$$



Time spent by ball at different slopes

$$T_1 = \frac{\hbar}{2eR}\int_0^{2\pi} \frac{d\varphi}{I_0 - I_{2\pi}\sin(\varphi) - I_{4\pi}\sin(\varphi/2)}$$

$$T_2 = \frac{\hbar}{2eR}\int_{2\pi}^{4\pi} \frac{d\varphi}{I_0 - I_{2\pi}\sin(\varphi) - I_{4\pi}\sin(\varphi/2)}$$

In the limit: $I_{2\pi} >> I_{4\pi}$ $I_C \sim I_{2\pi}$

$$I_0 \sim I_c$$

$$T_1 \propto 1/F_1 = 1/(I_0 - I_c)$$

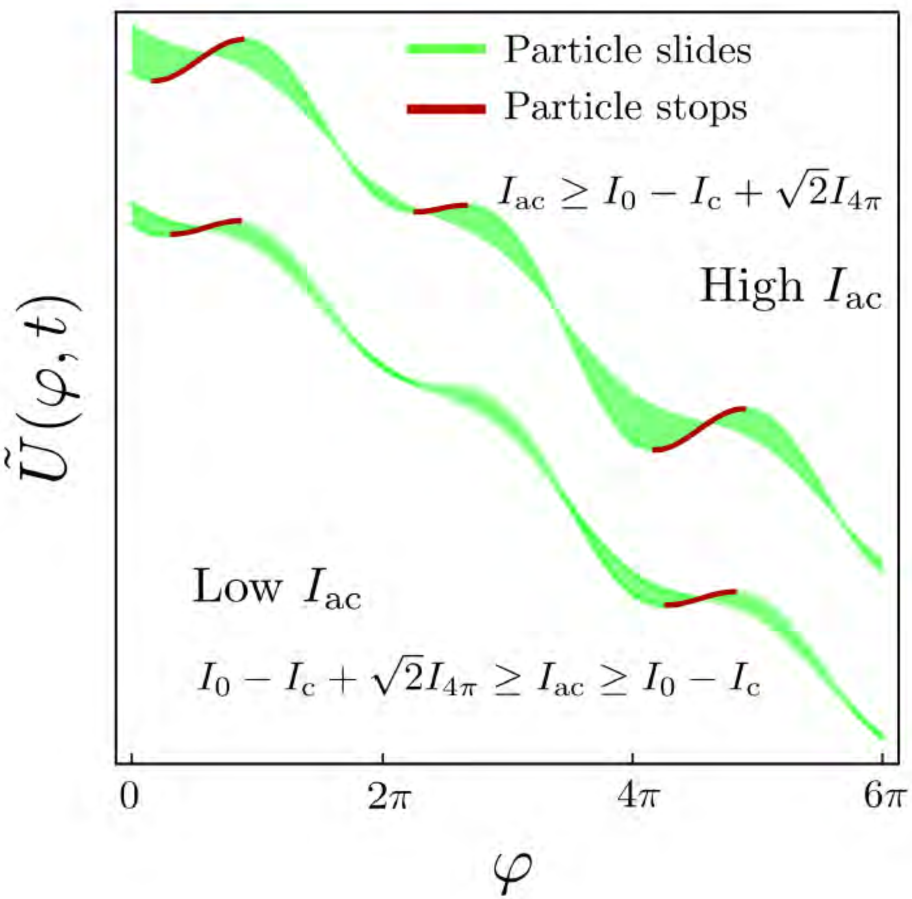
$$T_2 \propto 1/F_2 \approx 1/(I_0 - I_c + \sqrt{2}I_{4\pi})$$

$$T_1/T_2 \gg 1$$

The mass stops only every second step/
4π periodicity of the supercurrent

F. Dominguez … and E.M. Hankiewicz, Phys. Rev. B 95, 195430 (2017) -

Why is the small dc current/ small driving frequency causing 4pi supercurrent?



If driving current stops the ball only in one of the minima

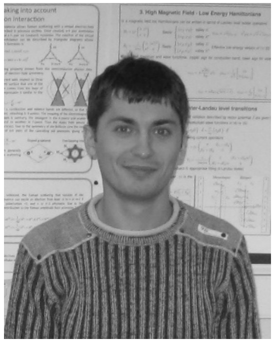
⇒ 4π contribution is visible independent how small it is

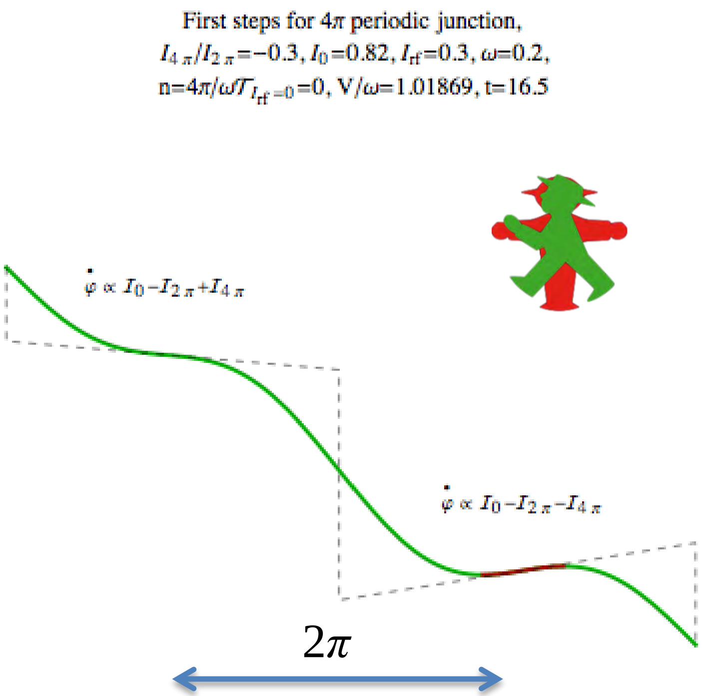
$$|F_2| \gtrsim I_{ac} \sin(\omega_{ac} t) \gtrsim |F_1|$$

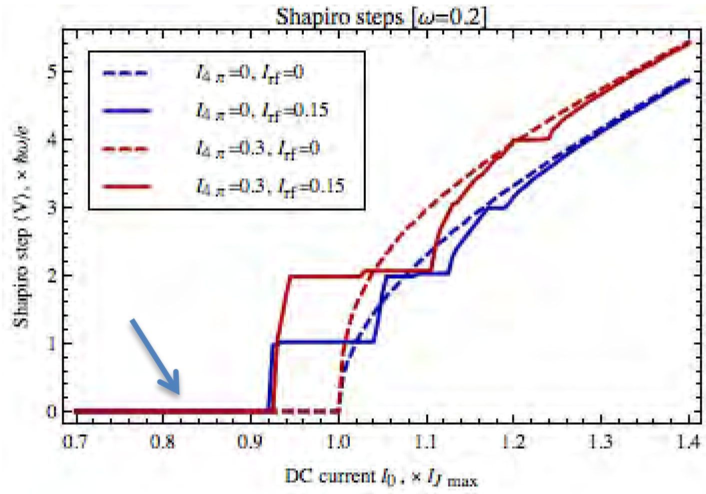
4π Shapiro steps !

F. Dominguez … and E.M. Hankiewicz, Phys. Rev. B 95, 195430 (2017) -

Shapiro step dynamics

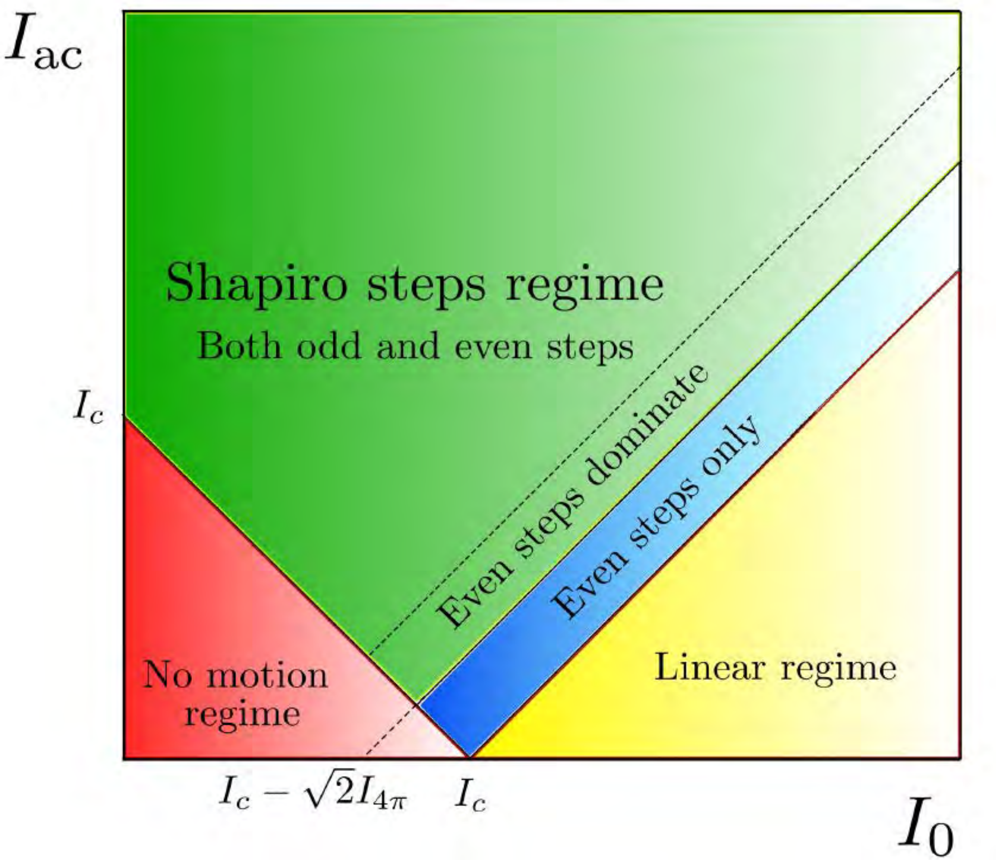






F. Dominguez … and E.M. Hankiewicz, Phys. Rev. B 95, 195430 (2017)

When does even Shapiro steps appear?



Formation of even Shapiro steps : $I_0 - I_{ac} \sim I_c - I_{4\pi}$
Small I_0 , small I_{ac} in comparison with I_c

$$\omega_0 = n\omega_{ac} = \frac{2eV}{\hbar}$$

4π component only for low bias current i.e. also for low ac frequency

F. Dominguez … and E.M. Hankiewicz, arXiv:1701.07389

Acknowledgements to my collaborators

Stanford group theory: Xiao-Liang Qi, Shou-Cheng Zhang

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Experimental group in RIKEN Japan: R. Deacon, S. Tarucha

T.M. Klapwijk: Delft

Theory in Würzburg: R. Thomale, W. Hanke, B. Trauzettel

Acknowledgements to my group

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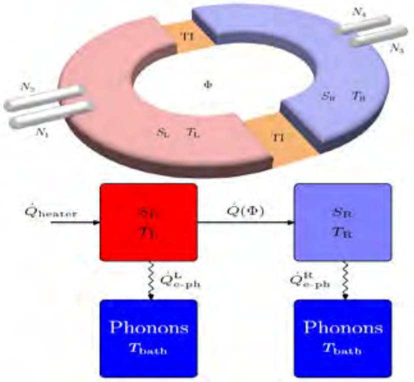
http://www.physik.uni-wuerzburg.de/fileadmin/11030030/Ewelina/index.html

Summary



1. Superconductor/3D Topological Insulator/Superconductor junction

a) We predicted existence of helical Andreev bound states, in these Josephson junctions, which have been recently detected through the non-sinusoidal current-phase relation.

b) Thermal conductance even more sensitive to helical Andreev bound states

G. Tkachov and E.M. Hankiewicz Phys. Rev. B ***88****, 075401 (2013).*
B. Sothmann and E. M. Hankiewicz PRB 94, 081407(R) (2016)

I.Sochnikov, L.Maier, C.Watson, J.R.Kirtley, C.Gould, G. Tkachov,
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2. AC Josephson effect

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