SPICE – Topology Matters

Moiré Patterns and Chiral-Boundary Mode Networks in Bilayer Graphene

Allan MacDonald and Dmitry Efimkin

Dmitry Efimkin

Ashley DaSilva Rafi Bistritzer
Emanuel Tutuc Brian LeRoy

Valley Hall Effect
in
Bilayer Graphene

Joaquin M. Luttinger

1923–1997

BIOGRAPHICAL *Memoirs*

A Biographical Memoir by
Walter Kohn

While visiting UC Berkeley in 1954, Quin heard about some surprising Hall effect measurements. In ferromagnets, the size of this effect was three orders of magnitude larger than in non-magnetic metals; it exhibited a previously unheard-of temperature dependence; and it depended strongly on the unmagnetized resistance. Quin and Robert Karplus, who had recently moved to Berkeley from Harvard, argued that many, if not all, of these properties could be qualitatively accounted for by making reasonable assumptions about the effects of the external magnetic field and spin-orbit coupling.

$$-\int_{\Omega} \frac{\partial w_{n\mathbf{k}}{}^{*}(\mathbf{r})}{\partial k_b} \frac{\partial w_{n\mathbf{k}}(\mathbf{r})}{\partial k_a} d^3x$$

Karplus and Luttinger - Phys. Rev. (1954)

KL vs. TKNN

$$-\int_{\Omega} \frac{\partial w_{nk}{}^{*}(\mathbf{r})}{\partial k_b} \frac{\partial w_{nk}(\mathbf{r})}{\partial k_a} d^3x$$

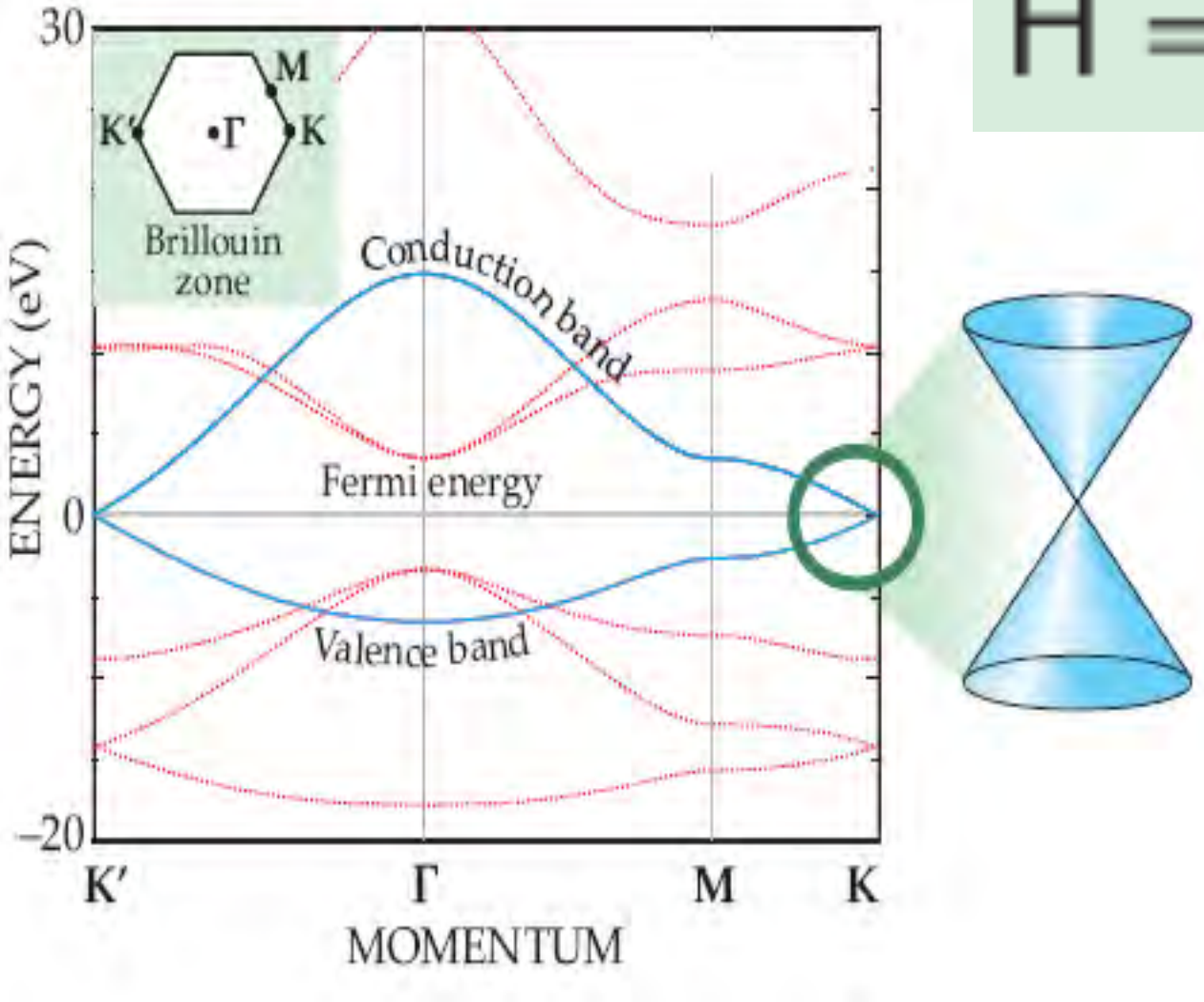
KL (1954)

$$\sigma_{\mathrm{H}} = \frac{ie^2}{2\pi h} \sum \int d^2k \int d^2r \left(\frac{\partial u^*}{\partial k_1} \frac{\partial u}{\partial k_2} - \frac{\partial u^*}{\partial k_2} \frac{\partial u}{\partial k_1} \right)$$

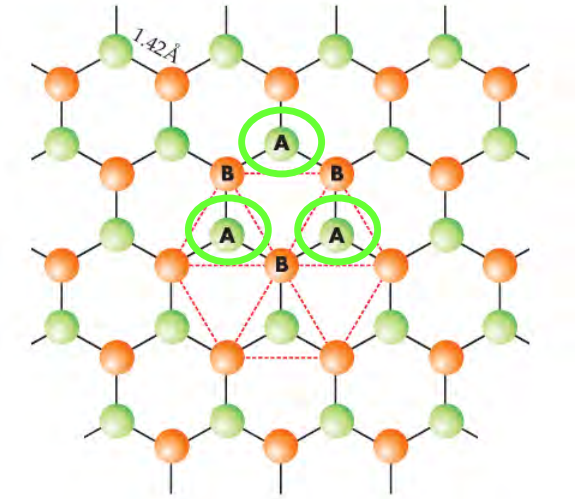
Thouless et al. (TKNN) – PRL (1982)

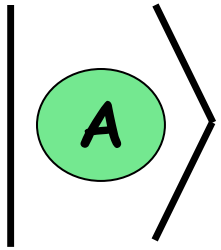
2D Massless Dirac Equation

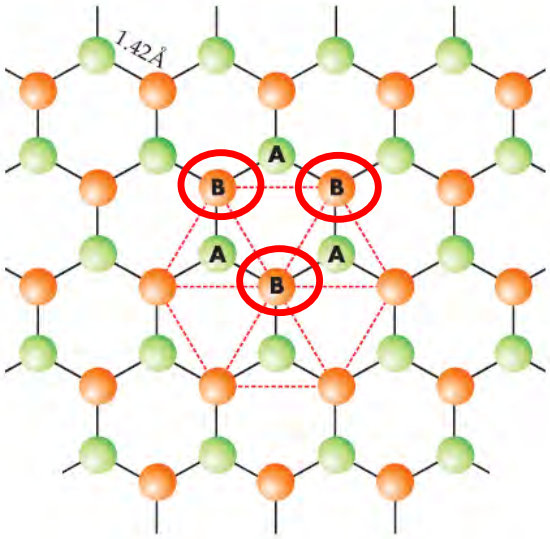
$$H = -\hbar v \mathbf{k} \cdot \boldsymbol{\tau}$$

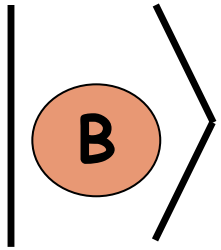


Pseudospins - Graphene

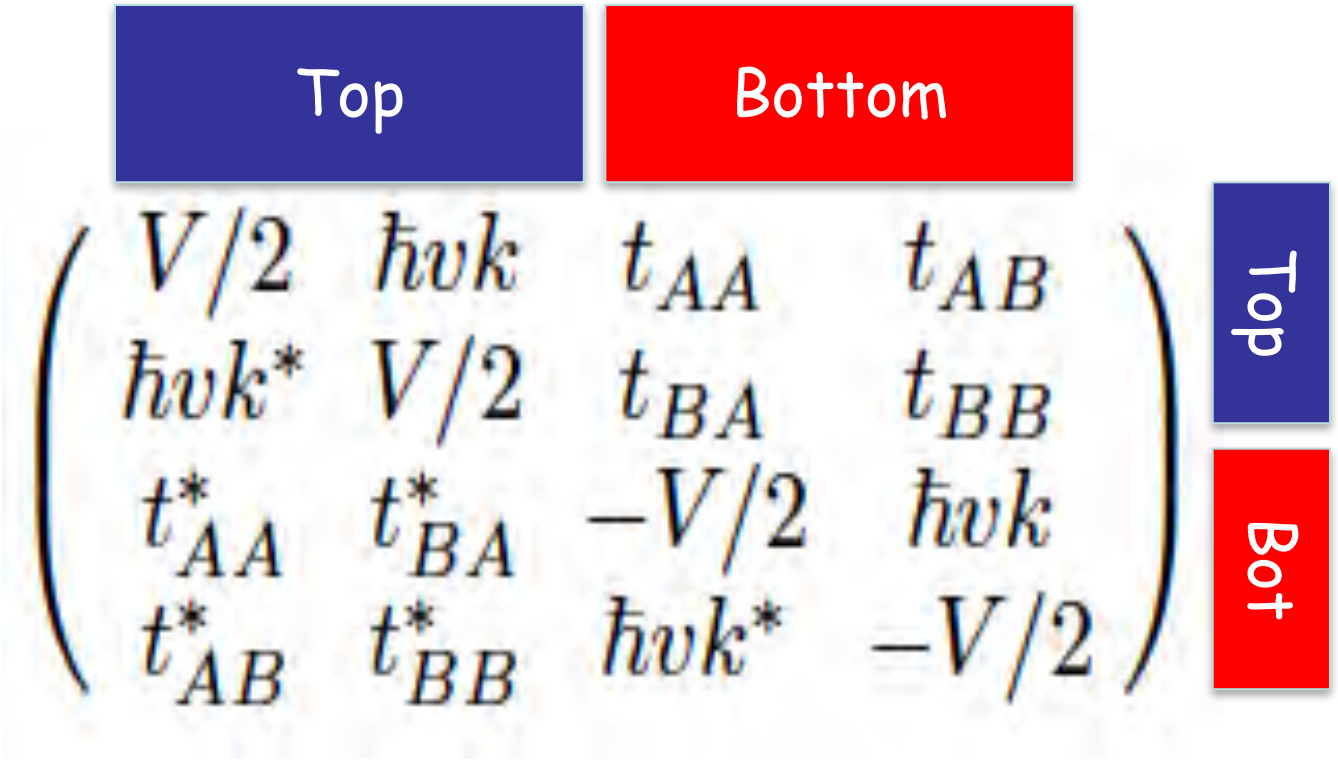








Graphene Bilayers



$k=k_x+ i\, k_y = |k|\, e^{i\theta}$

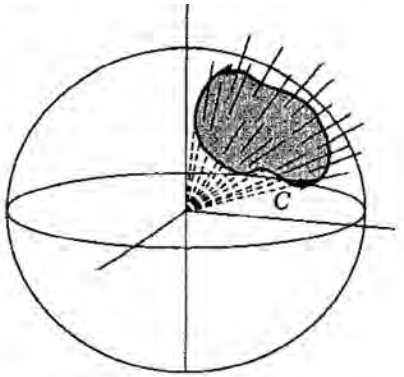
AB Stacking

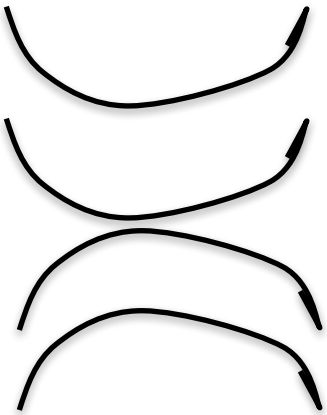
Top | Bottom

$$\begin{pmatrix} V/2 & \hbar v k & 0 & 0 \\ \hbar v k^* & V/2 & \gamma_1 & 0 \\ 0 & \gamma_1 & -V/2 & \hbar v k \\ 0 & 0 & \hbar v k^* & -V/2 \end{pmatrix}$$

Top

Bot



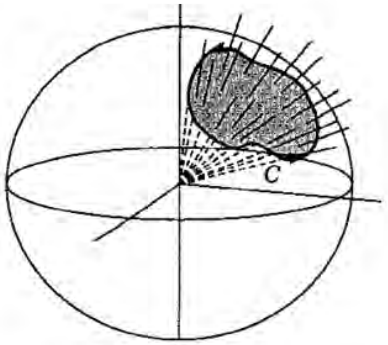


Top | Bot

$$\begin{pmatrix} V/2 & (\hbar v k)^2/\gamma_1 \\ (\hbar v k^*)^2/\gamma_1 & -V/2 \end{pmatrix}$$

$\sigma_{VH} = -\,\mathrm{sgn}(V)\, e^2/h$

BA Stacking



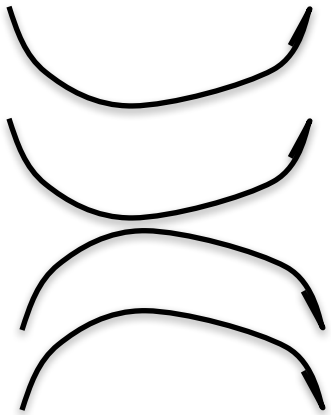
Top	Bottom

$$\begin{pmatrix} V/2 & \hbar v k & 0 & \gamma_1 \\ \hbar v k^* & V/2 & 0 & 0 \\ 0 & 0 & -V/2 & \hbar v k \\ \gamma_1 & 0 & \hbar v k^* & -V/2 \end{pmatrix}$$

Top

Bot

Chiral Boundary Modes
Zhang et al. PNAS (2013)
Vaezi et al. PRX (2013)

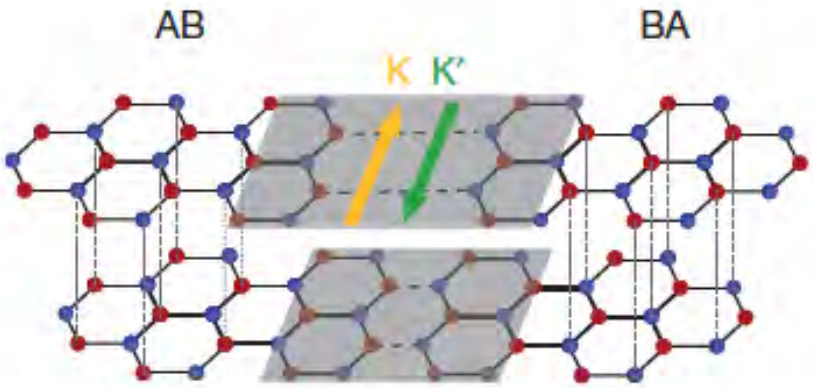


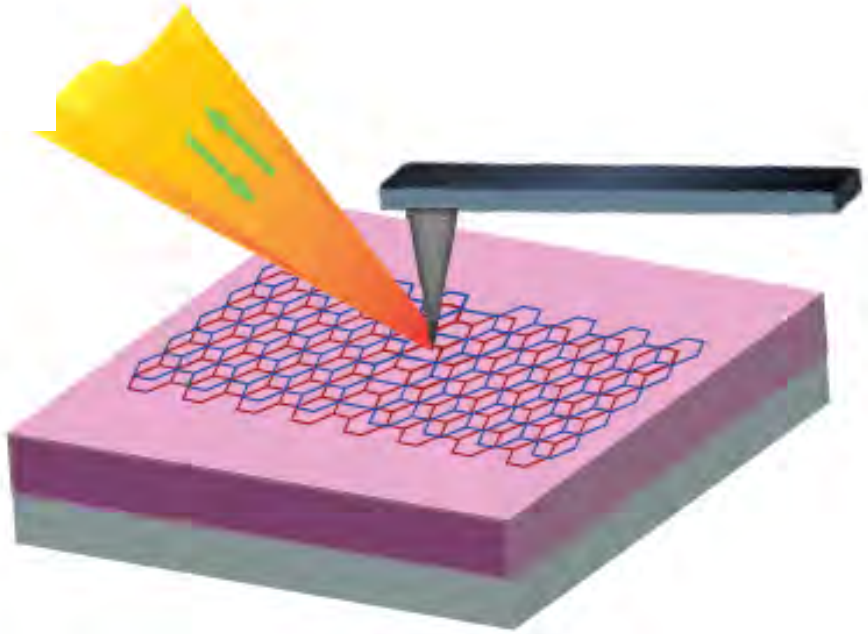
Top	Bot

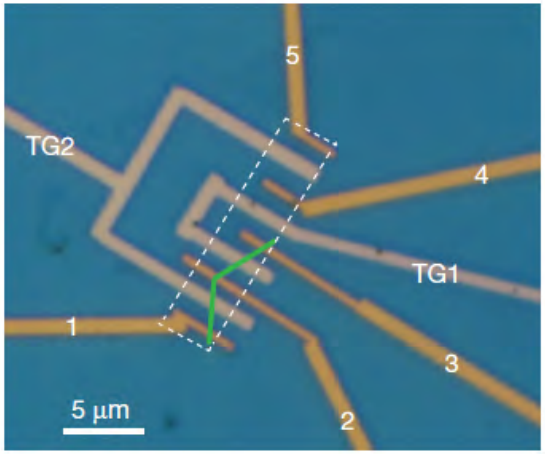
$$\begin{pmatrix} V/2 & (\hbar v k)^2/\gamma_1 \\ (\hbar v k^*)^2/\gamma_1 & -V/2 \end{pmatrix}$$

$$\sigma_{VH} = - \, \mathrm{sgn}(V) \, e^2/h$$

Domain Wall Transport







$\xi \approx 0.5\ \mu m$

F. Wang et al.
Nature (2015)

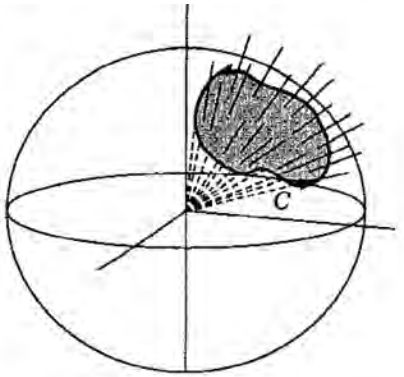
AA Stacking

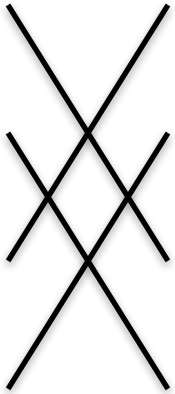
Top | Bottom

$$\begin{pmatrix} V/2 & \hbar v k & \gamma_1 & 0 \\ \hbar v k^* & V/2 & 0 & \gamma_1 \\ \gamma_1 & 0 & -V/2 & \hbar v k \\ 0 & \gamma_1 & \hbar v k^* & -V/2 \end{pmatrix}$$

Top

Bot

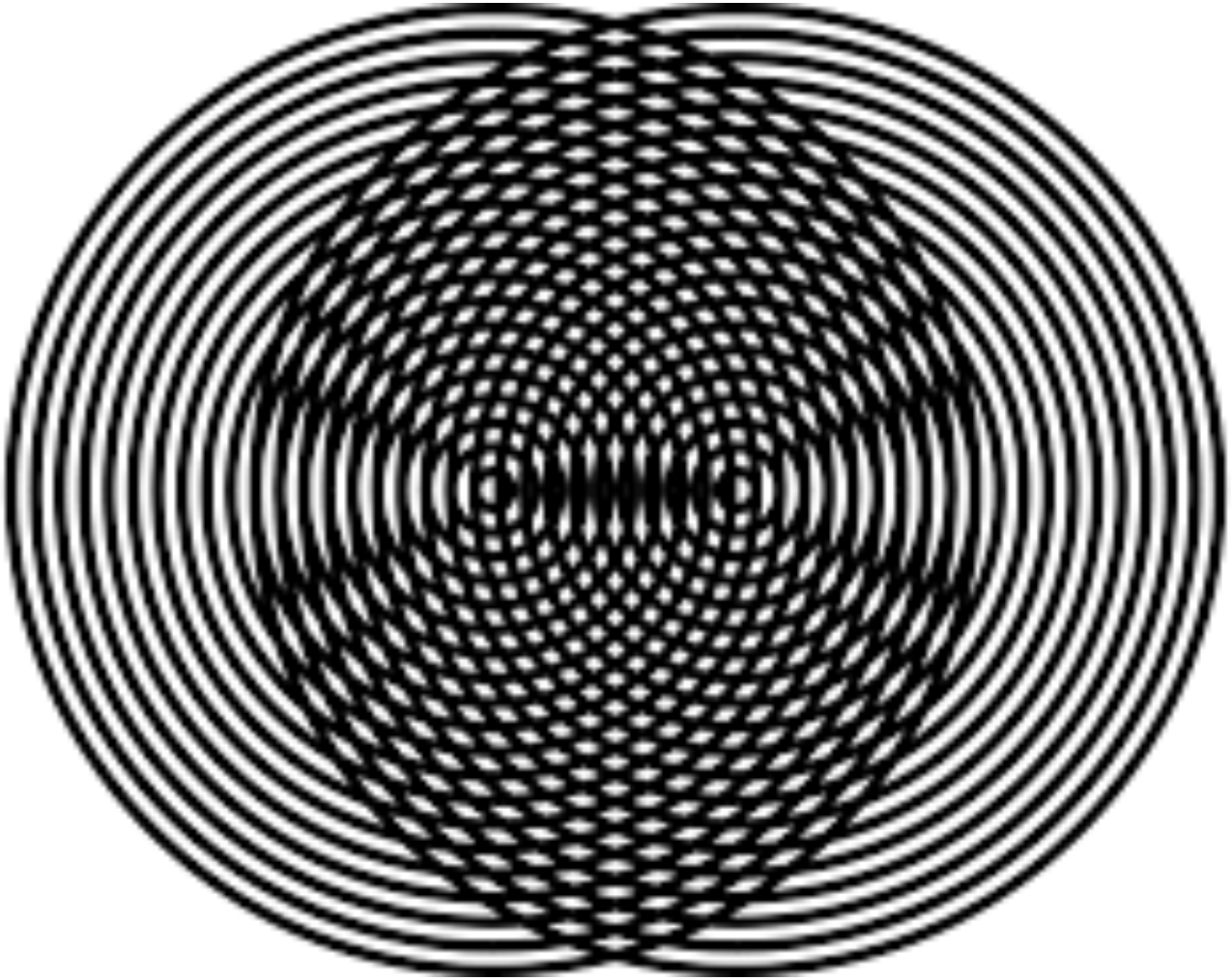


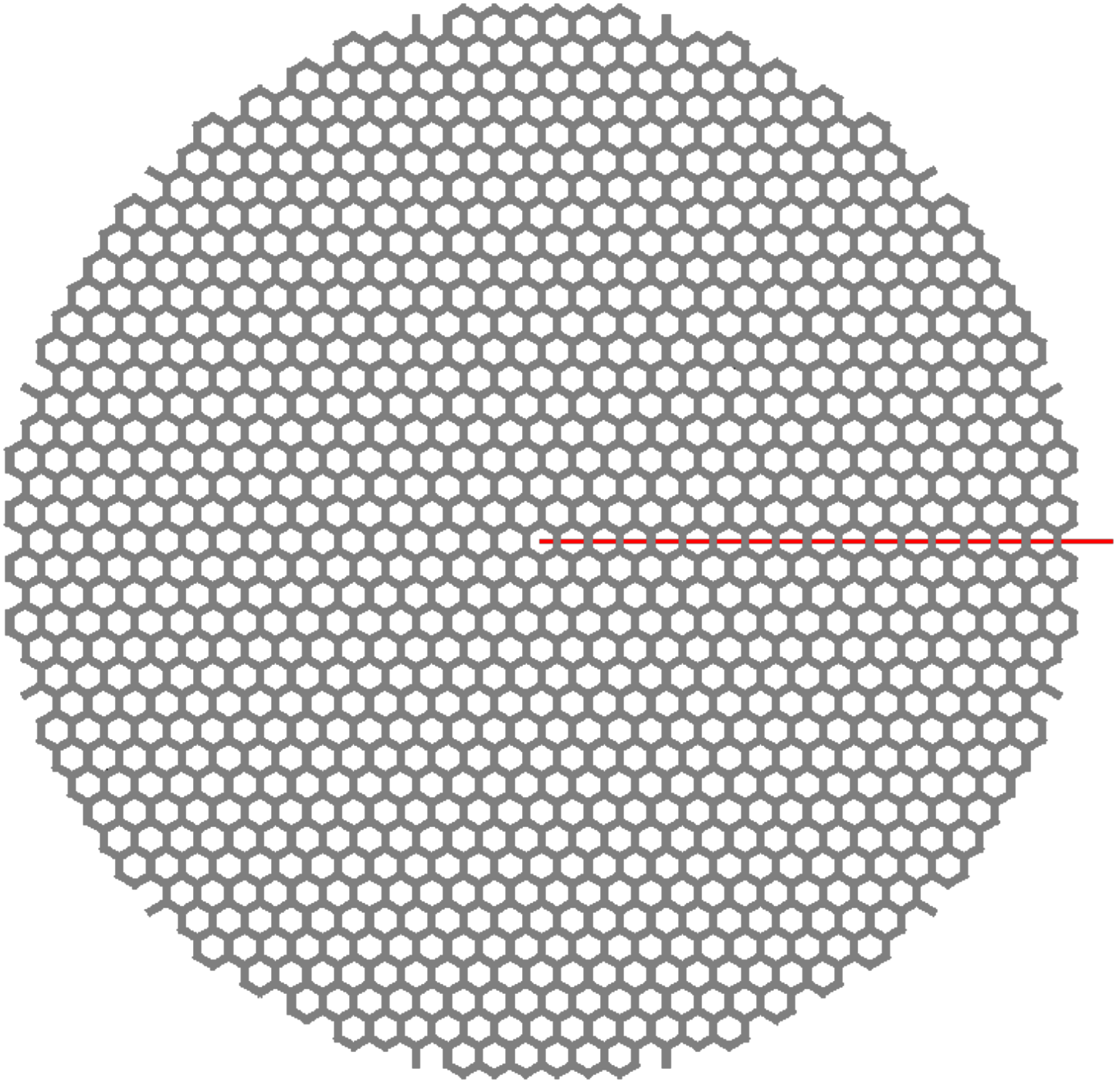


$$\begin{pmatrix} 0 & \hbar v k \\ \hbar v k^* & 0 \end{pmatrix} \pm \sqrt{(V/2)^2 + \gamma_1^2}$$

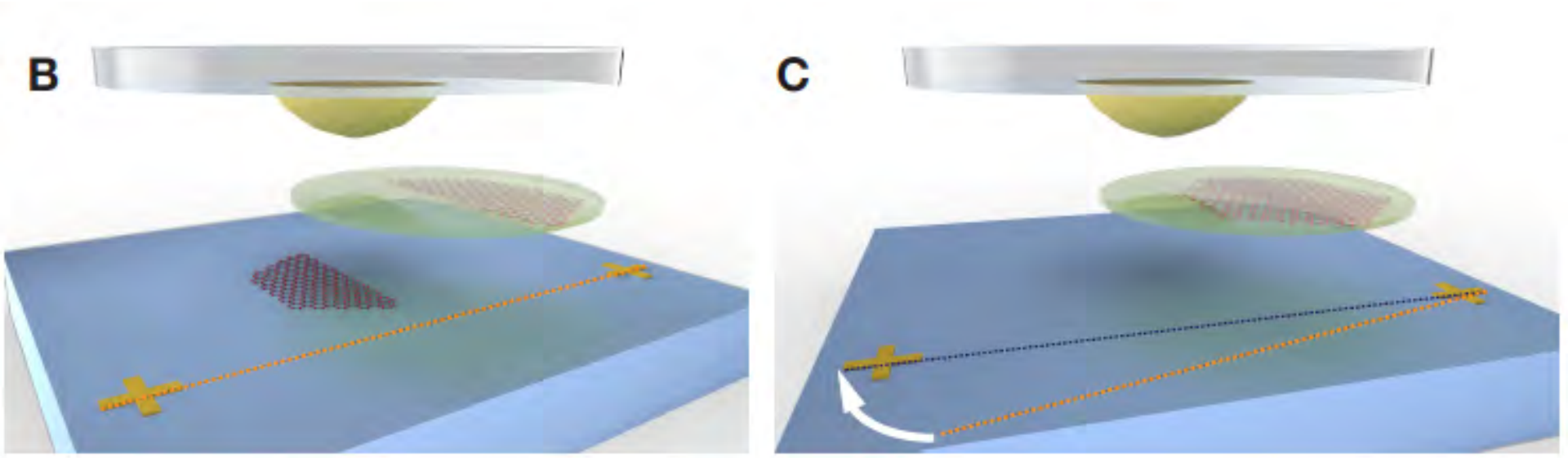
Moiré Patterns
in
Bilayer Graphene

Moiré Patterns



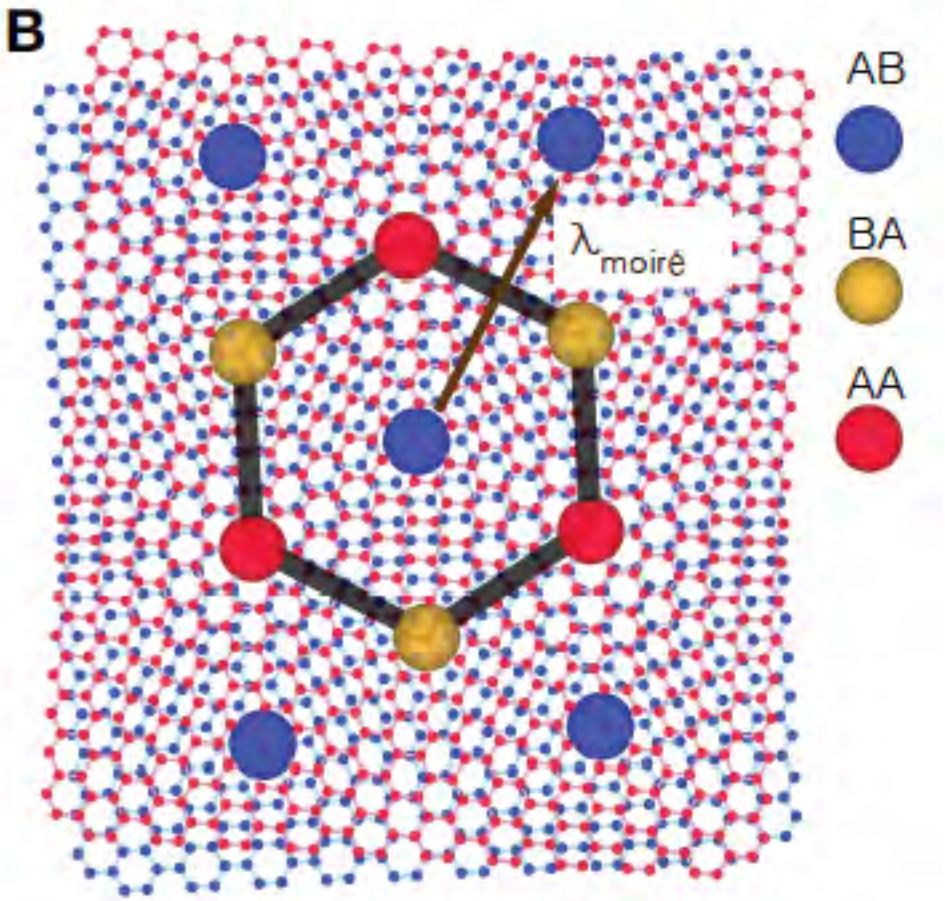


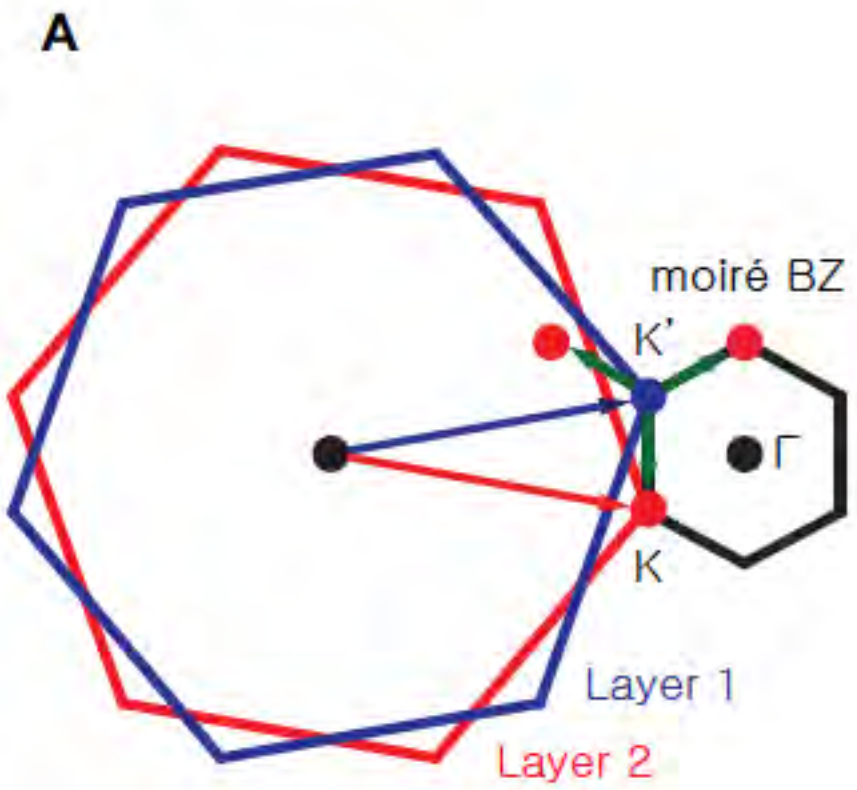
Long Period Moiré



Emanuel Tutuc
PNAS (2017)

Graphene on Graphene Moiré





Tutuc et al. PNAS (2017)

Moiré Band Model

$$T^{\alpha\beta}(\mathbf{r}) = w \sum_{j=1}^{3} \exp(-i\mathbf{q}_j \cdot \mathbf{r})\ T_j^{\alpha\beta}$$

φ = 2π/3

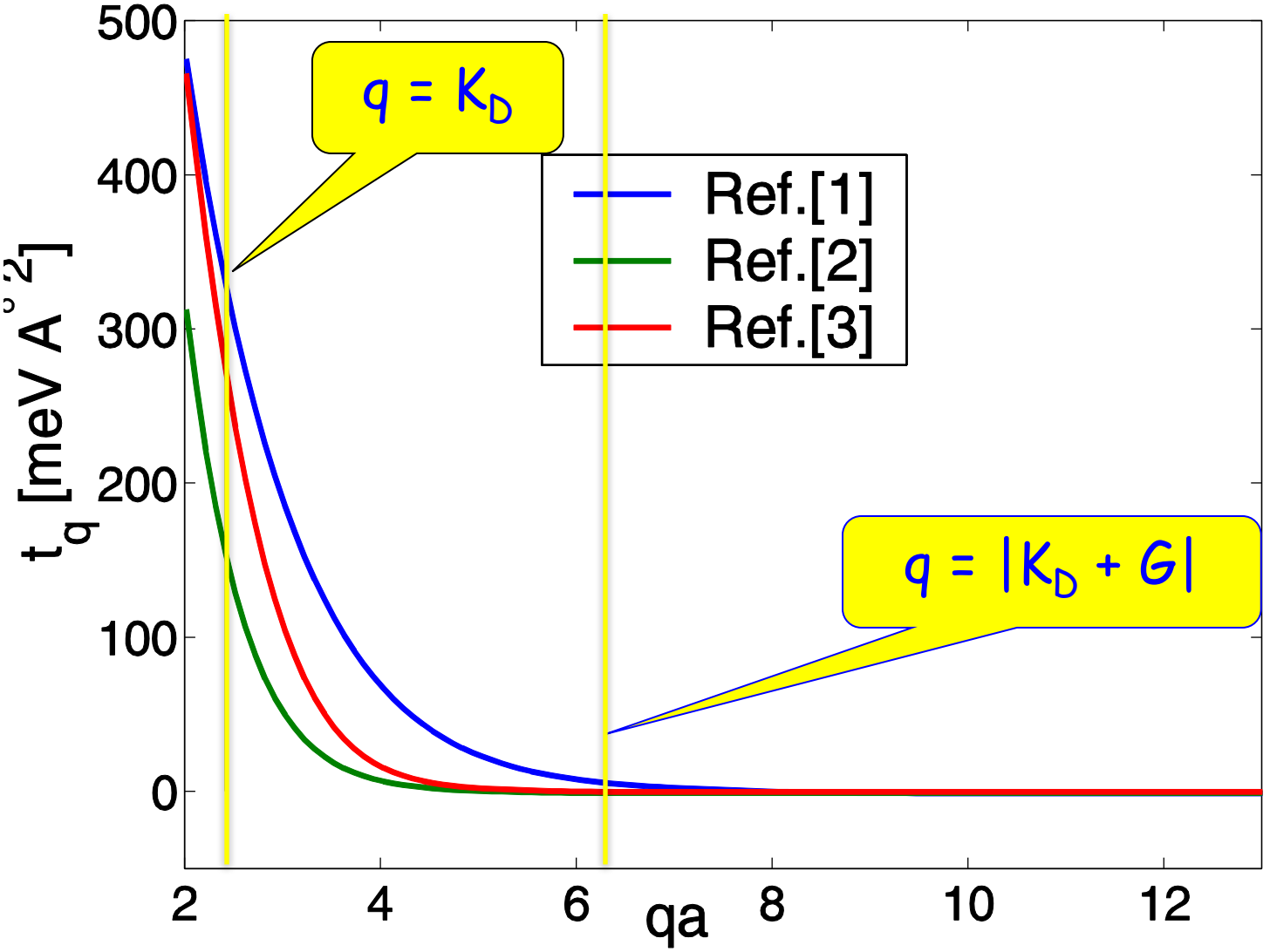
$$T_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$T_2 = e^{-i\mathcal{G}^{(2)\prime}\cdot\mathbf{d}} \begin{pmatrix} e^{-i\phi} & 1 \\ e^{i\phi} & e^{-i\phi} \end{pmatrix},$$

$$T_3 = e^{-i\mathcal{G}^{(3)\prime}\cdot\mathbf{d}} \begin{pmatrix} e^{i\phi} & 1 \\ e^{-i\phi} & e^{i\phi} \end{pmatrix},$$

Bistritzer & AHM - PNAS (2011)

q-space Interlayer Tunneling

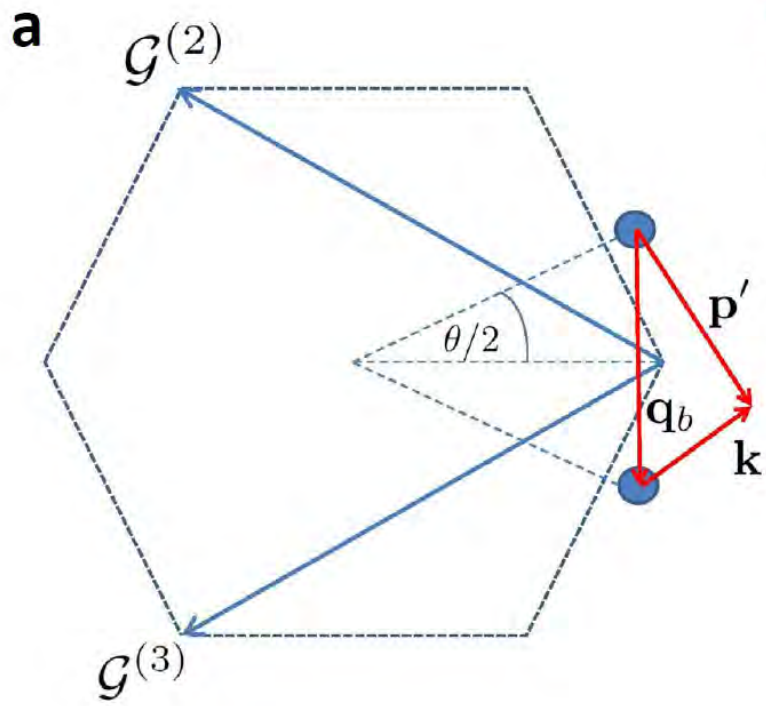


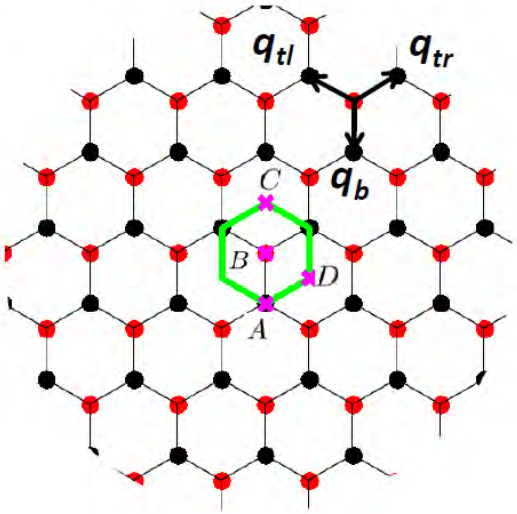
Interlayer Tunneling
-
Momentum Space

$$T^{\alpha\beta}_{\mathbf{k}\mathbf{p}'} = \sum_{\mathbf{G_1}\mathbf{G_2}} \frac{t_{\bar{\mathbf{k}}+\mathbf{G_1}}}{\Omega} e^{i[\mathbf{G_1}\tau_\alpha - \mathbf{G_2}(\tau_\beta - \tau) - \mathbf{G}'_2\mathbf{d}]} \delta_{\bar{\mathbf{k}}+\mathbf{G_1},\bar{\mathbf{p}}'+\mathbf{G}'_2}$$

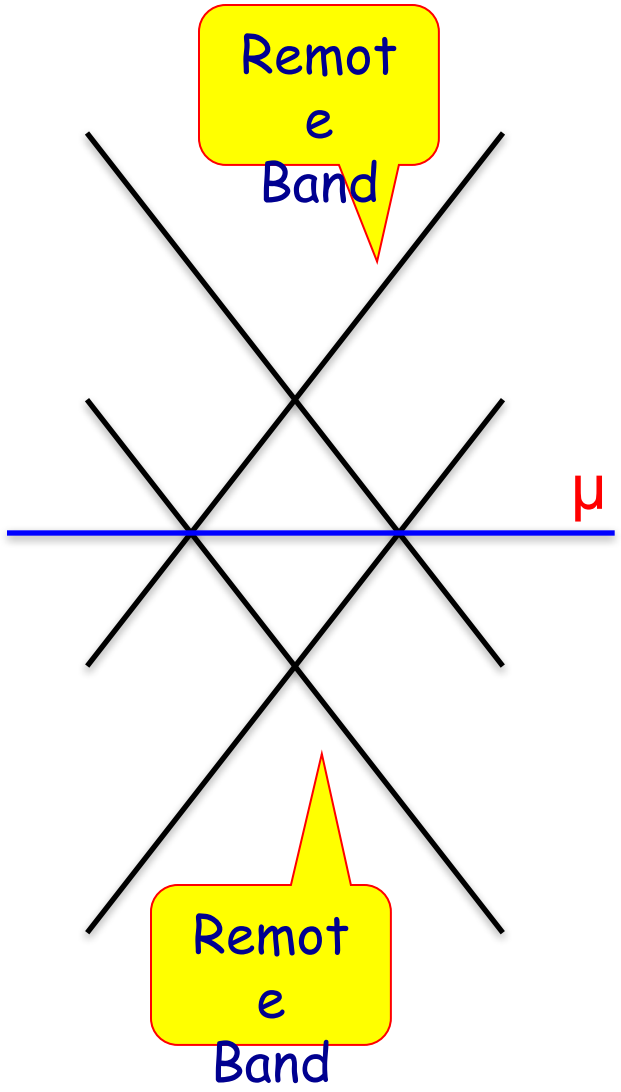
t_q decreases very rapidly with q !

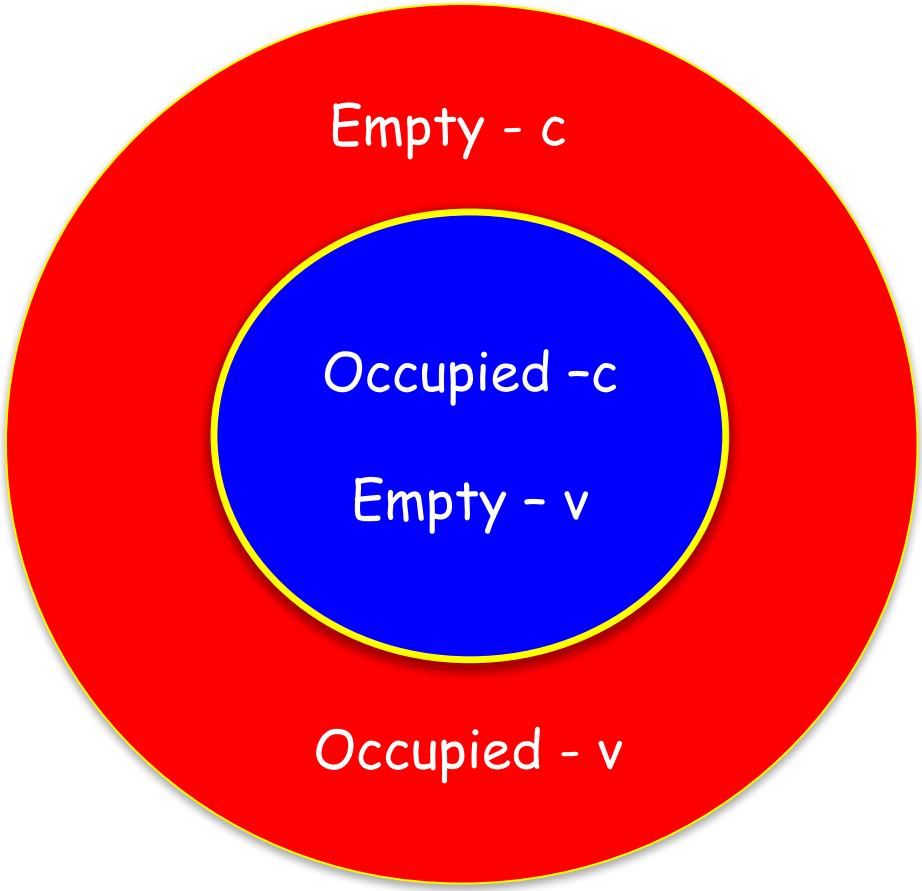
Continuum Model Momentum Boosts





Biased Bilayer Graphene



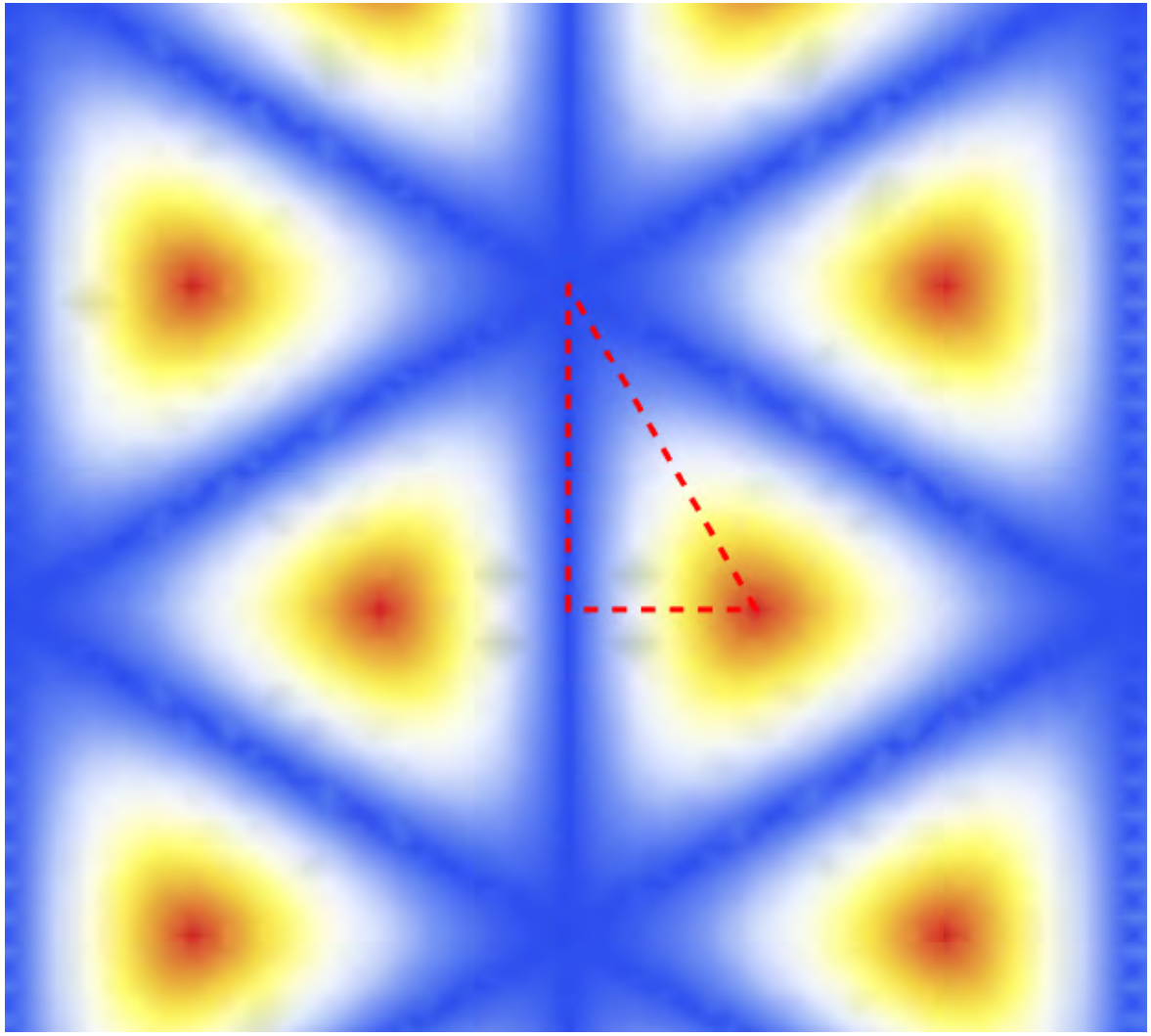


Tunneling Matrix Elements

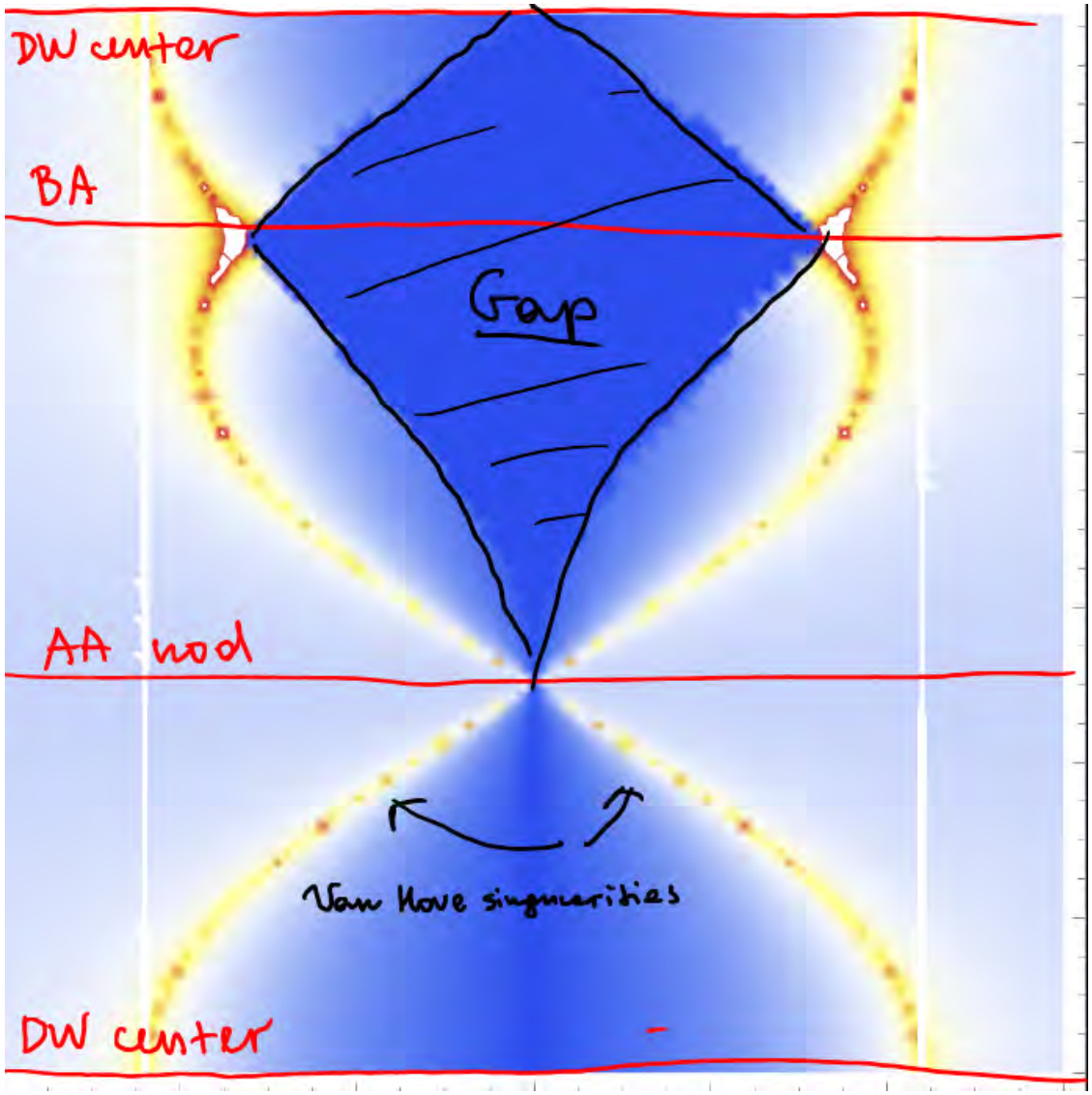
$$T = \begin{pmatrix} T_{oA} & T_{AB} \\ T_{BA} & T_{oB} \end{pmatrix} \rightarrow$$

$$t_o = \frac{1}{2}\left\{ T_{BA} e^{-i\varphi p} - T_{AB} e^{i\varphi p} \right\}$$

Local Gap

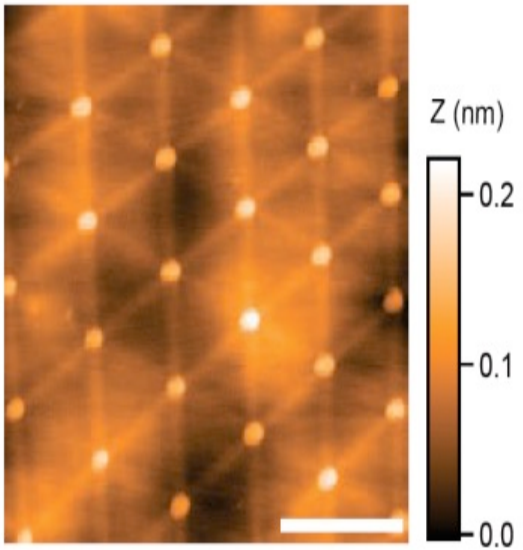


Local Approximation for DOS

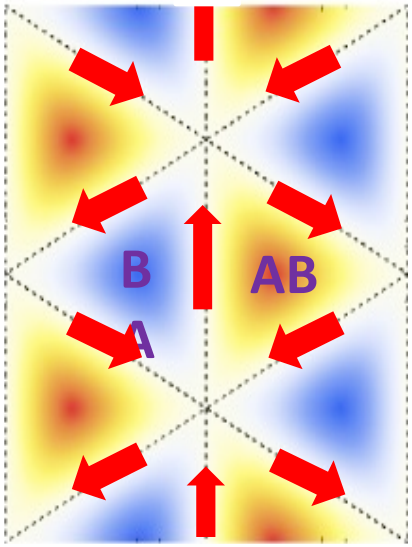


Helical network model

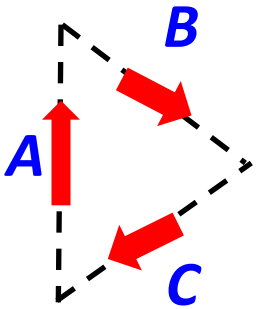
STM by LeRoy



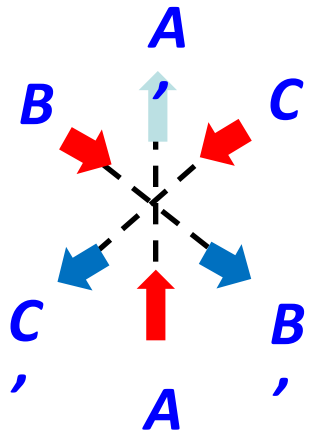
Dirac mass



Unit cell



Scattering node



Scattering matrix parametrization

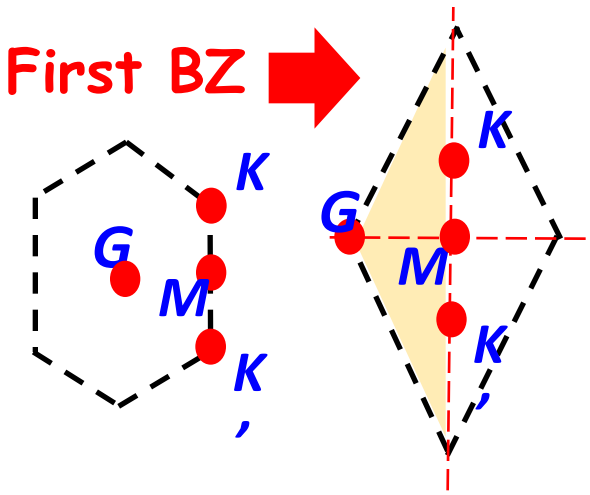
$$\bar{T} = \begin{pmatrix} \cos\theta e^{i\chi} & \frac{\sin\theta}{\sqrt{2}} & \frac{\sin\theta}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & -\frac{1+\cos\theta e^{-i\chi}}{2} & \frac{1-\cos\theta e^{-i\chi}}{2} \\ \frac{\sin\theta}{\sqrt{2}} & \frac{1-\cos\theta e^{-i\chi}}{2} & -\frac{1+\cos\theta e^{-i\chi}}{2} \end{pmatrix}$$

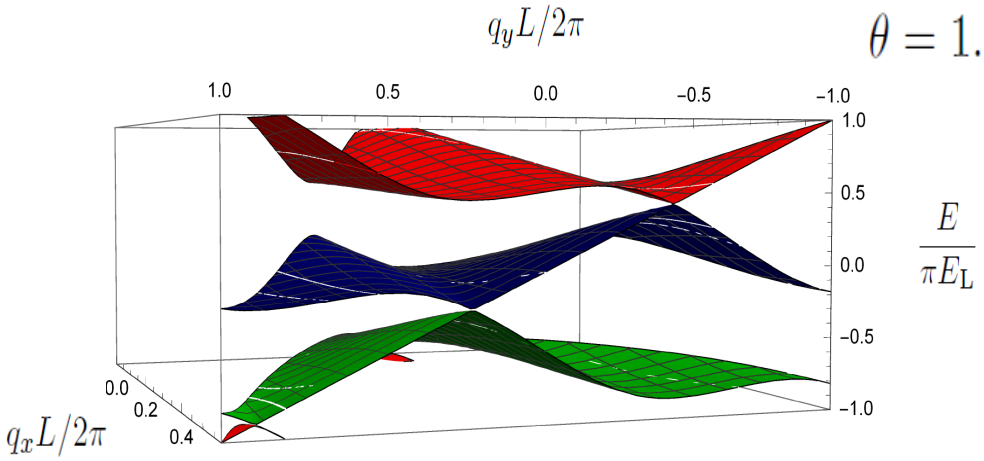
$$\chi = \arccos\left[\frac{3\cos^2(\theta) - 1}{2\cos(\theta)}\right]$$

Helical states with dispersion

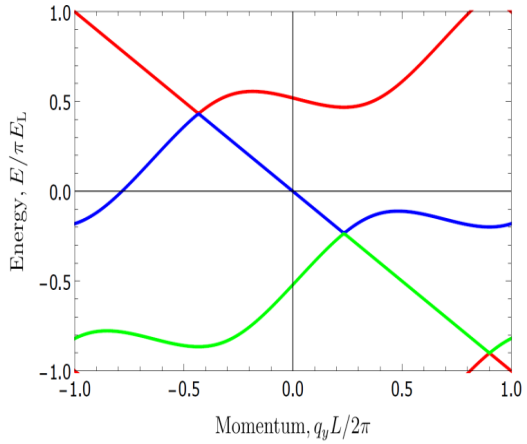
$$\epsilon = v_{||} q$$

Spectrum of the network

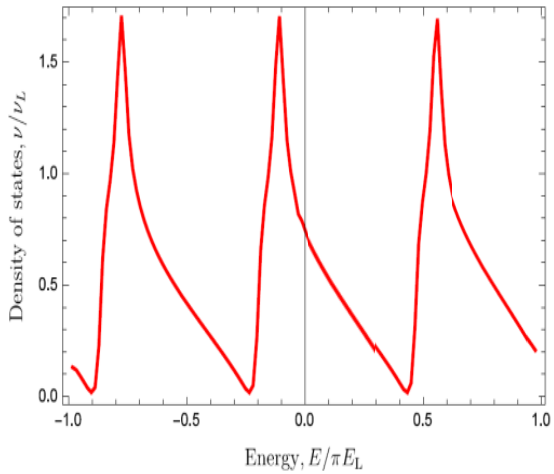




Section along KK' line

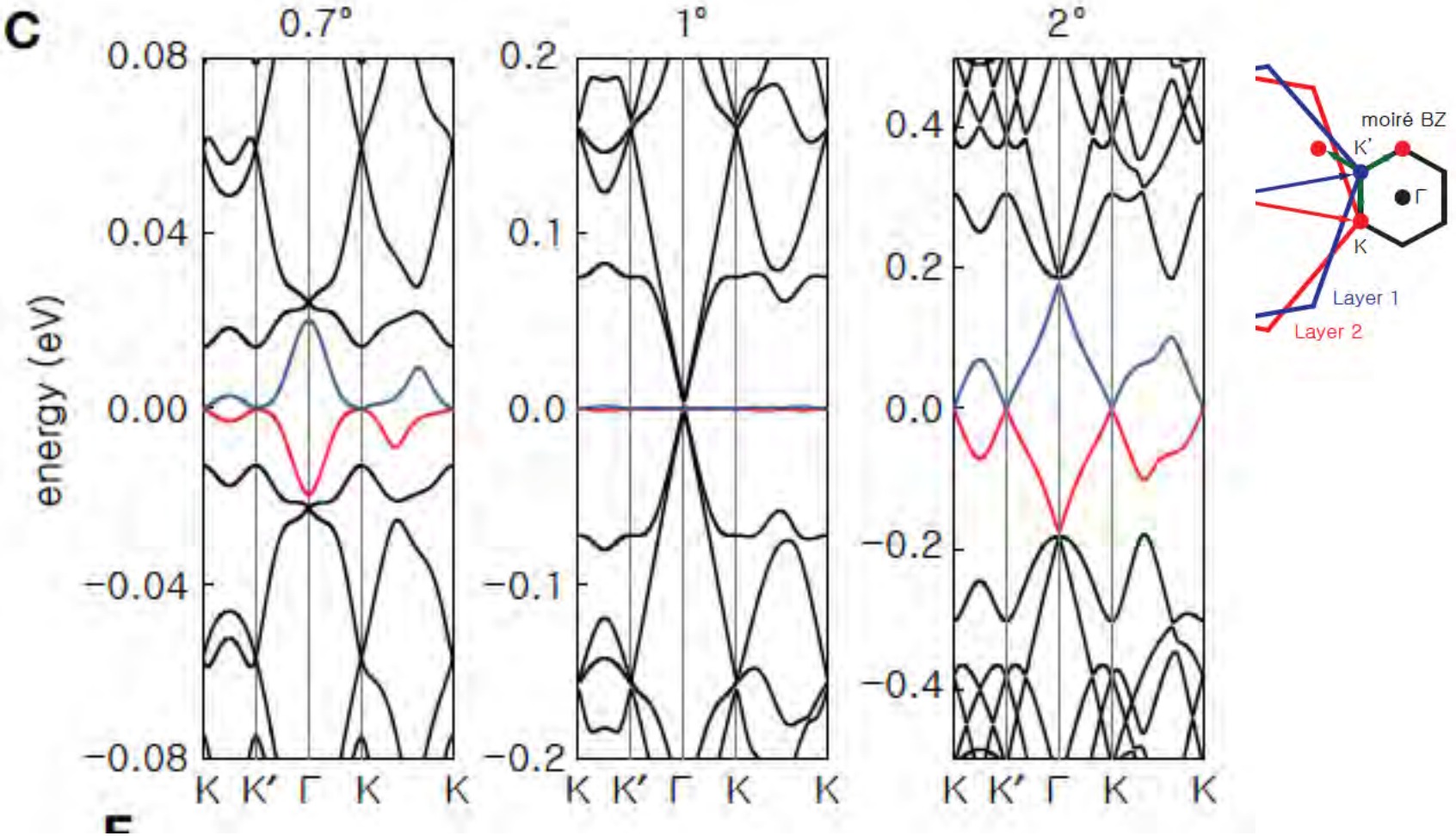


Density of states

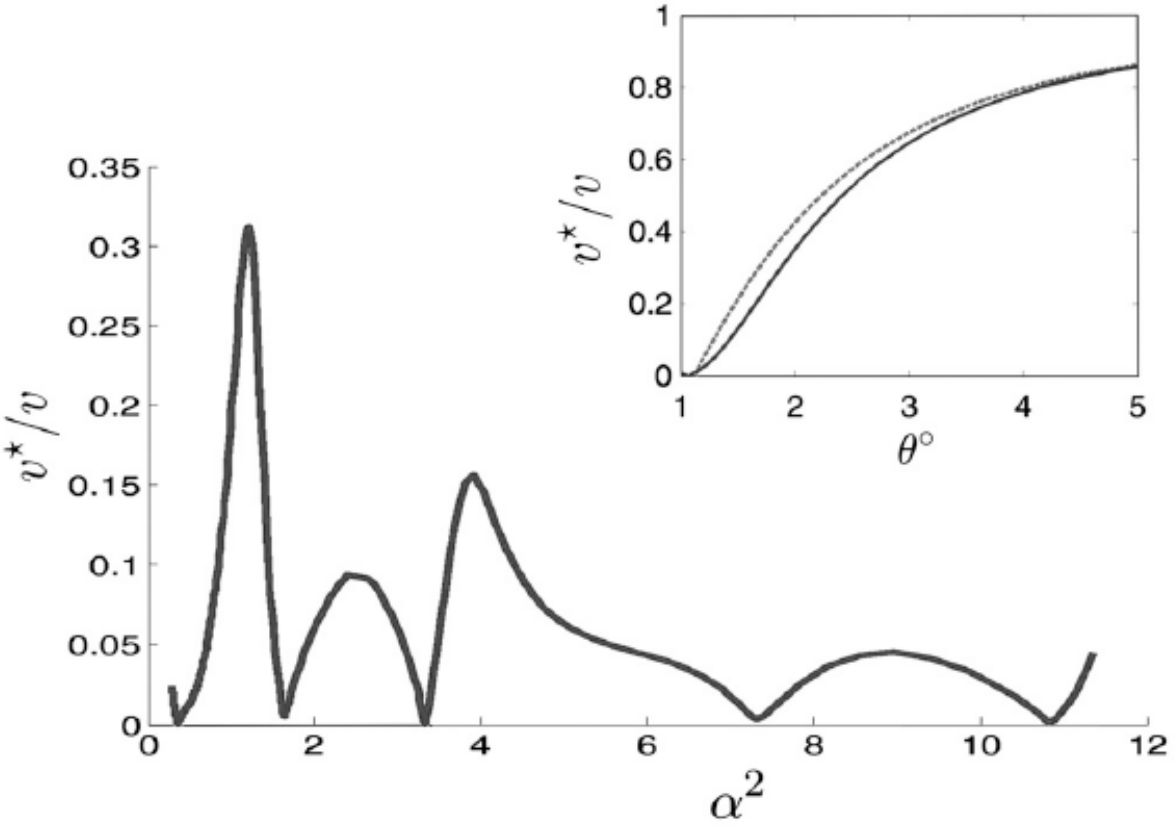


Graphene on Graphene

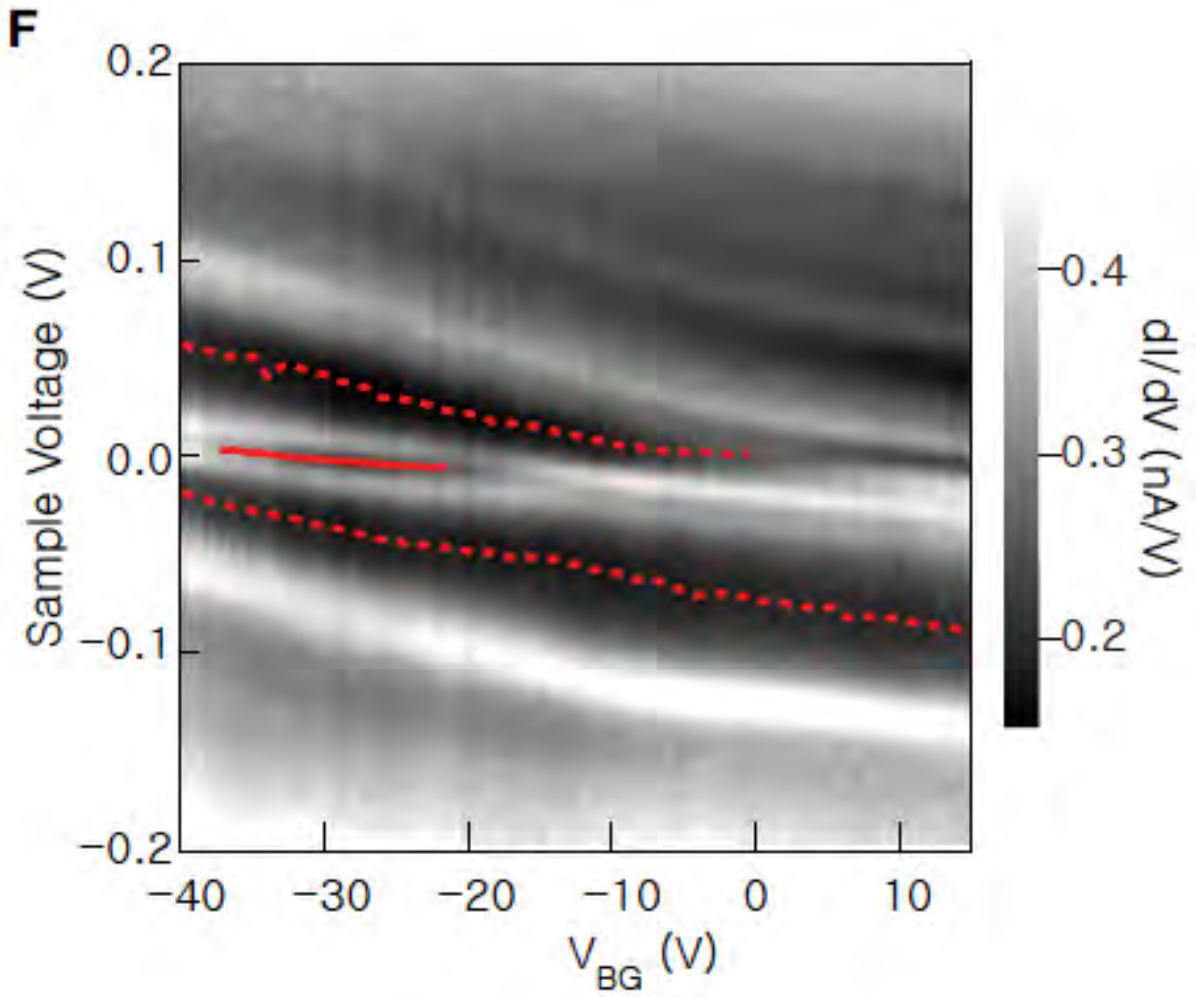
$$T^{\alpha\beta}(\mathbf{r}) = w \sum^{3} \exp(-i\mathbf{q}_j \cdot \mathbf{r}) T_j^{\alpha\beta}$$



Zero Displacement Field Magic Angle Mystery

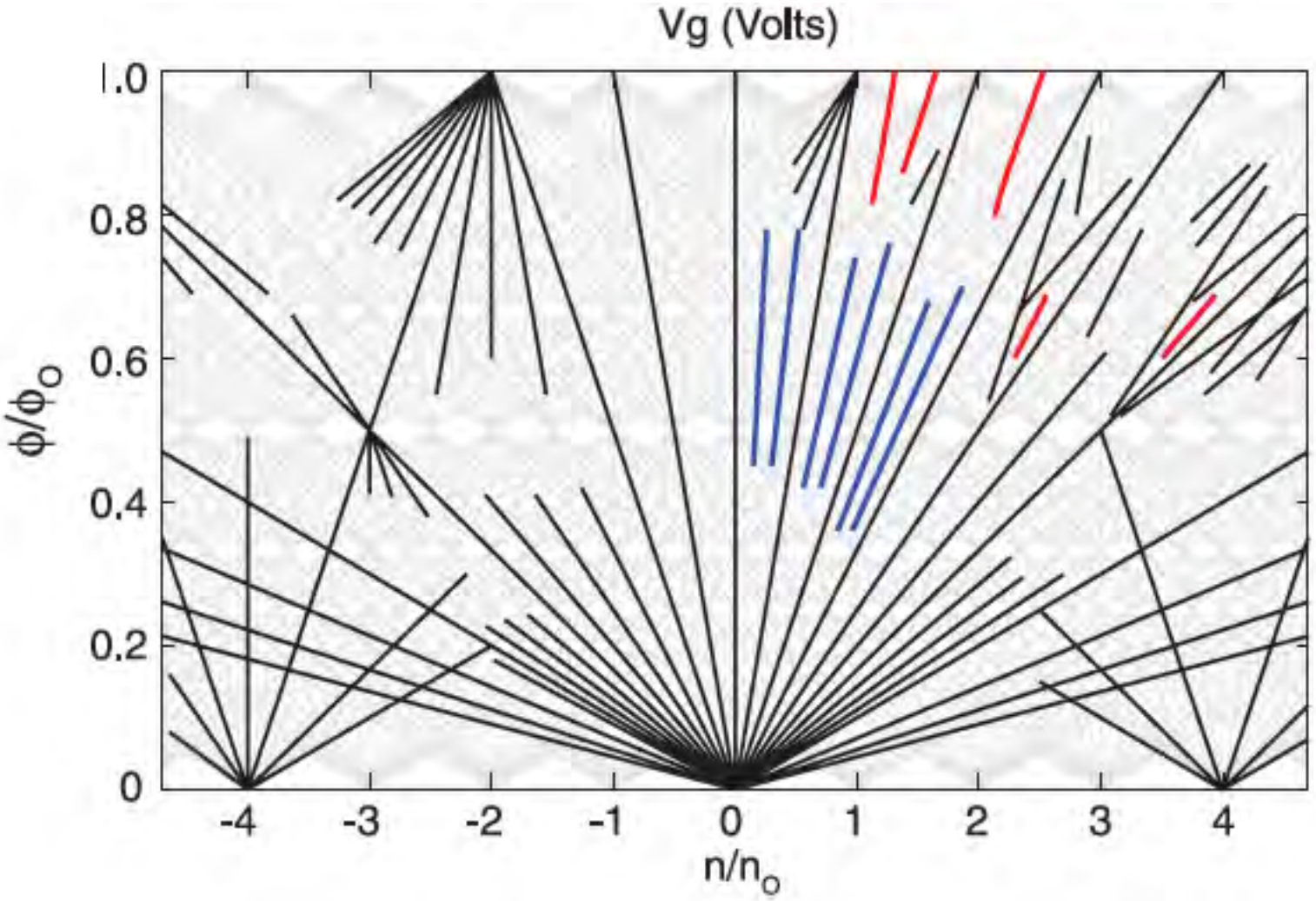


Long Period Moiré



Brian Le Roy

Butterfly Spectrum- Graphene on hBN



Cory Dean Lab – Columbia – Science (2015)

Valley Hall Effect in Bilayer Graphene

Plus Moiré Pattern

Plus Displacement Field

=

Controllable Chiral Network