

# Dynamics of interacting electrons in ac-driven dimer chains

G. Platero

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Mainz July 2017





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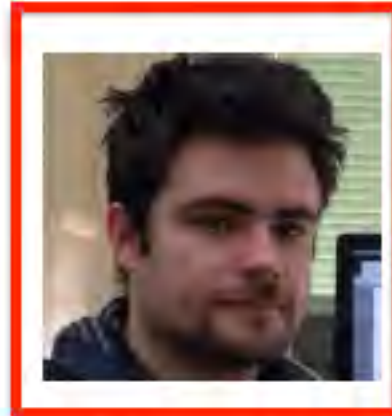


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CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS  
INSTITUTO DE CIENCIA DE MATERIALES  
DE MADRID (ICMM)



Charles Creffield



Miguel bello



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Michael Niklas  
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Sigmund Kohler  
ICMM (CSIC)

# OUTLINE

Motivation:

Quantum Transfer between distant sites at the nanoscale: Long range electron transport in quantum dot arrays

ac-driven systems

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Dynamics of interacting electrons (Doublons) in dimer chains: interplay between interaction, topology and driving.

Doublon dynamics in 2D lattices

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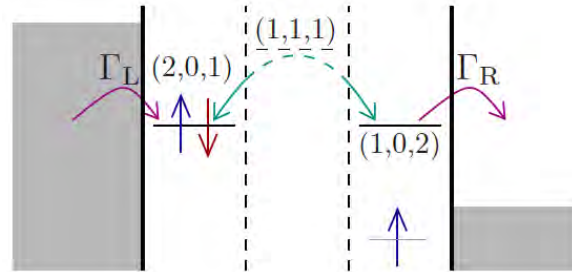
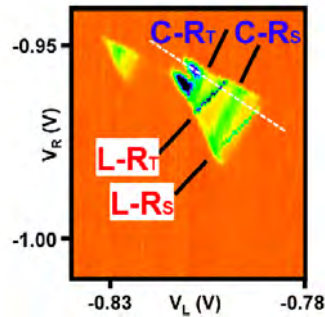
Doublon dynamics in 2D lattices

Electron transport in a dimer chain: edge state current blockade

Transport statistics: current and shot noise in ac driven dimer chains

## Long range quantum transfer in TQDs

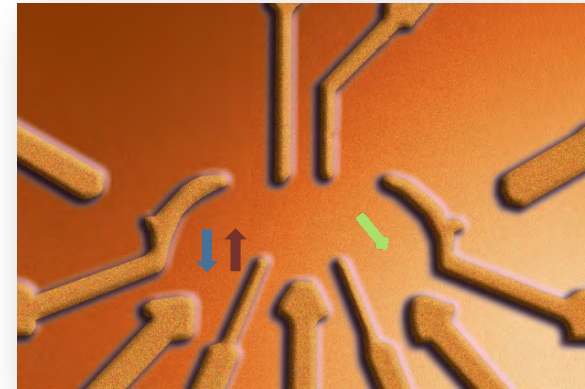
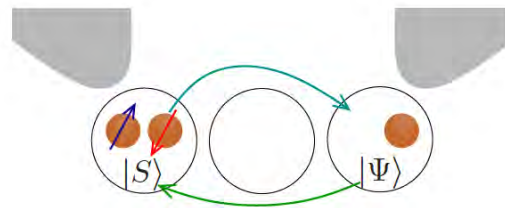
L-R resonance of (2,0,1) and (1,0,2)  
intermediate (1,1,1) far in energy



$$\Delta E \gg \tau \quad |LR_-\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow, 0, \psi\rangle - |\psi, 0, \uparrow\downarrow\rangle)$$

As an electron tunnels from one extreme to the other an arbitrary spin state  $\psi$  is transferred in the opposite direction:

$$|\psi\rangle_{l=L,R} = c_\uparrow |\uparrow\rangle_l + c_\downarrow |\downarrow\rangle_l$$

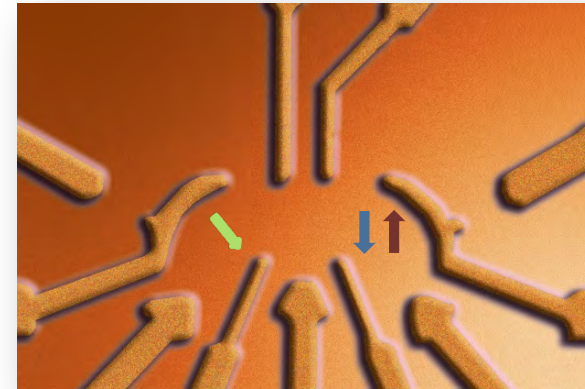
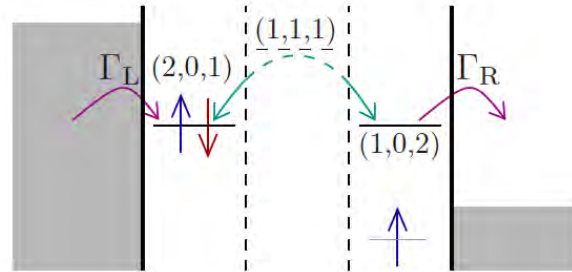
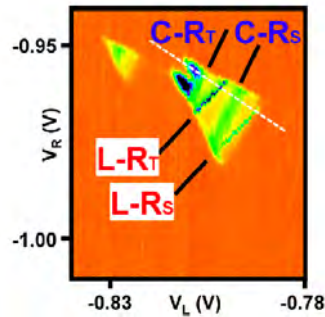


M. Busl et al., Nature Nanotech, 8, 262 (2013)

R. Sánchez et al., PRL, 112, 176803 (2014)

## Long range quantum transfer in TQDs

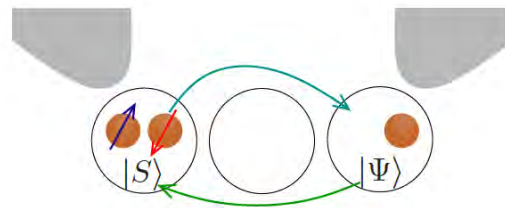
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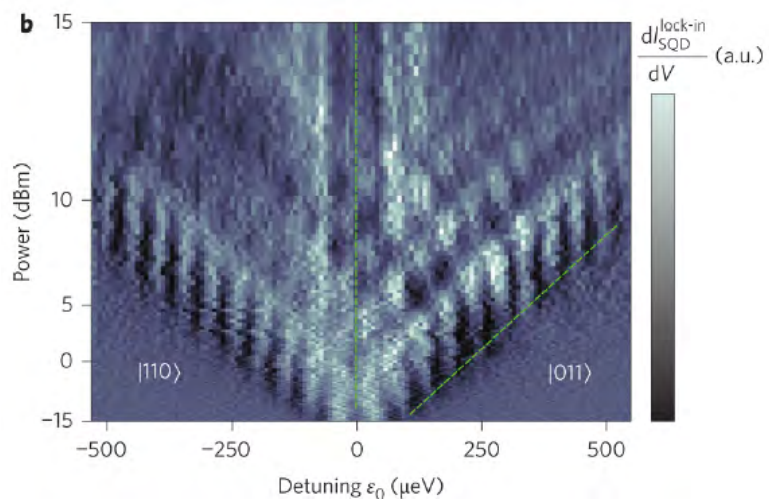
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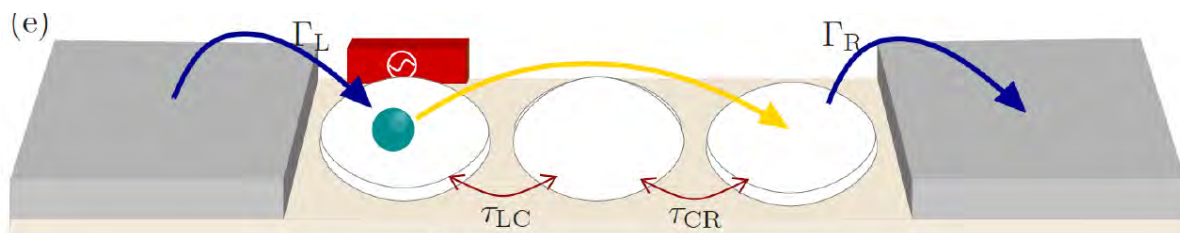


F.R. Braakman et al, Nature Nanotech, 8, 432 (2013)

9 QDs in Si/Ge heterostructure



PHYSICAL REVIEW APPLIED 6, 054013 (2016)

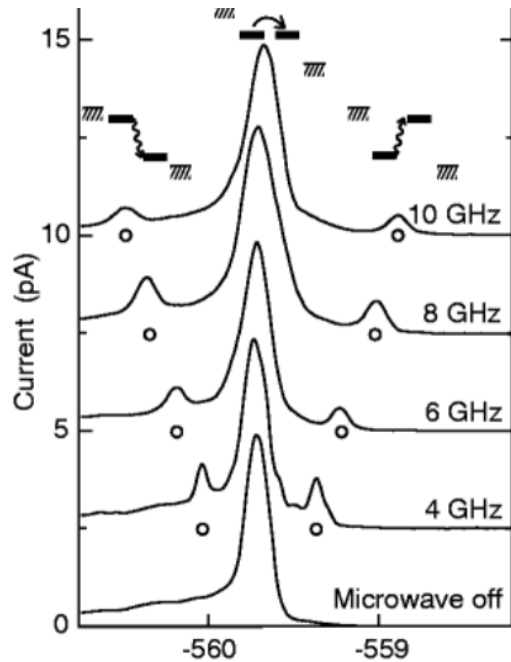


J. Petta



# Driving with periodic AC electric fields

## Photoassisted Tunneling (PAT) in quantum dots



Coherent destruction of tunnel  
P. Hänggi, PRL, 1991

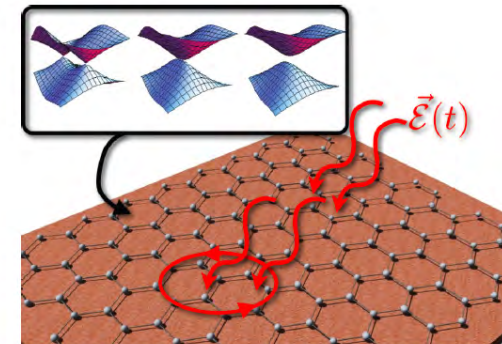
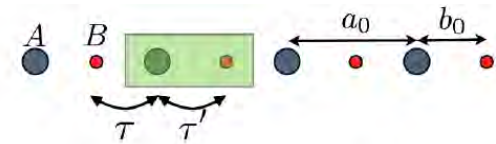


Bonds renormalization

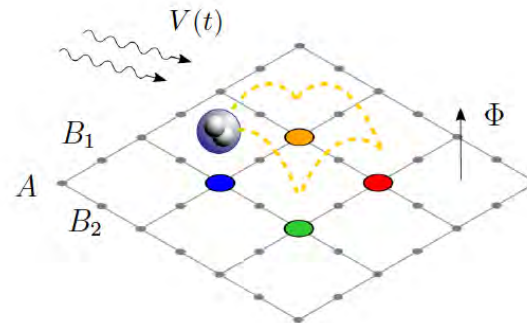


T. H. Oosterkamp  
et al., Nature 395, 873-876,  
1998

Floquet-Bloch Theory and Topology in  
Periodically Driven Lattices,  
A. Gómez-León and G. P., PRL, 110,200403  
(2013)



P. Delplace, A. Gómez-León  
and G. P., PRB, 88,245  
(2013)



M. Bello,  
C.E. Creffield,  
G.P., PRB 2017

## Floquet Theory

Analog to Bloch theory, but in time: time periodic hamiltonians  $H(t) = H(t + T)$

Solutions:  $|\Psi_\alpha(t)\rangle = e^{-i\varepsilon_\alpha t} |u_\alpha(t)\rangle$

Floquet States:  $|u_\alpha(t + T)\rangle = |u_\alpha(t)\rangle$

Quasi-energies:  $\varepsilon_\alpha \in [-\omega/2, \omega/2]$

Floquet Equation:  $\mathbf{H}(t)|u_\alpha\rangle = \varepsilon_\alpha |u_\alpha\rangle$  where  $\mathbf{H}(t) = H(t) - i\partial_t$

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## Floquet-Bloch Theory

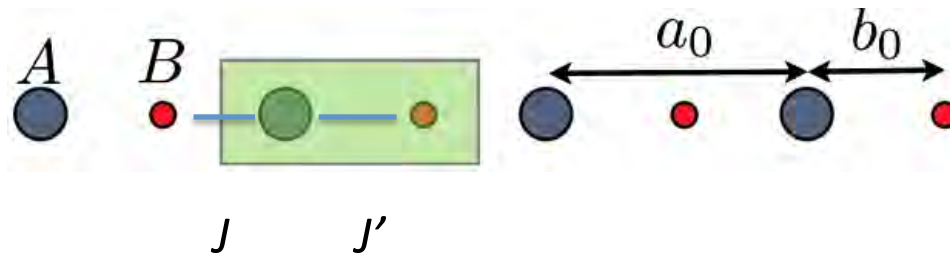
$$H(x + a_i, t + T) = H(x + a_i, t) = H(x, t + T)$$

Solutions in the Floquet-Bloch form:  $|\Psi_{\alpha,k}(x, t)\rangle = e^{ikx - i\varepsilon_{\alpha,k}t} |u_{\alpha,k}(x, t)\rangle$

$$\mathbf{H}(k, t)|u_{\alpha,k}\rangle = \varepsilon_{\alpha,k} |u_{\alpha,k}\rangle \quad \mathbf{H}(k, t) = e^{-ikx} (H(t) - i\partial_t) e^{ikx} = H(k, t) - i\partial_t$$

Peierls substitution  $k \rightarrow K(t) = k + A(t)$

## Finite dimer chains: role of edge states in the charge dynamics



$$H = -J' \sum_{i=1, \sigma}^M c_{2i\sigma}^\dagger c_{2i-1\sigma} - J \sum_{i=1, \sigma}^{M-1} c_{2i+1\sigma}^\dagger c_{2i\sigma} + H.c.$$

Su-Schrieffer-Heeger Model (SSH)

W. P. Su, J. R. Schrieffer, and A. J. Heeger,  
Phys. Rev. Lett. 42,1698 (1979).

nature  
physics

ARTICLES

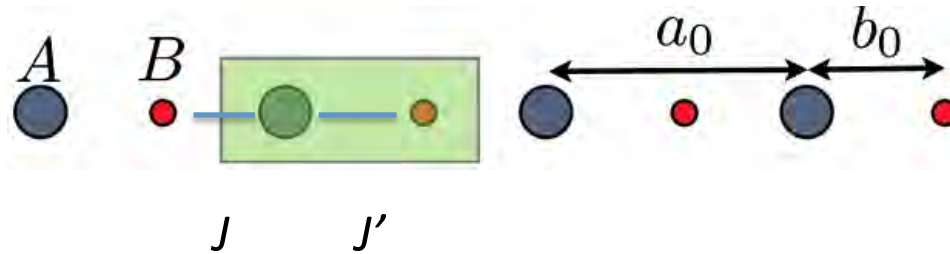
PUBLISHED ONLINE: 3 NOVEMBER 2013 | DOI: 10.1038/NPHYS2790

M. Atala et al. 2013

**Direct measurement of the Zak phase in topological Bloch bands**

Ramsey interferometry combined with coherent Bloch oscillations

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Su-Schrieffer-Heeger Model (SSH)

Topological invariant in 1 D: The Zak phase

$$Z = \oint \langle u_{\alpha, k} | i\partial_k | u_{\alpha, k} \rangle dk$$

Zak phase : the Berry's phase picked up by a particle moving across the Brillouin zone.  
It characterizes the topological properties of 1D solids.

$Z = 0$  Trivial phase

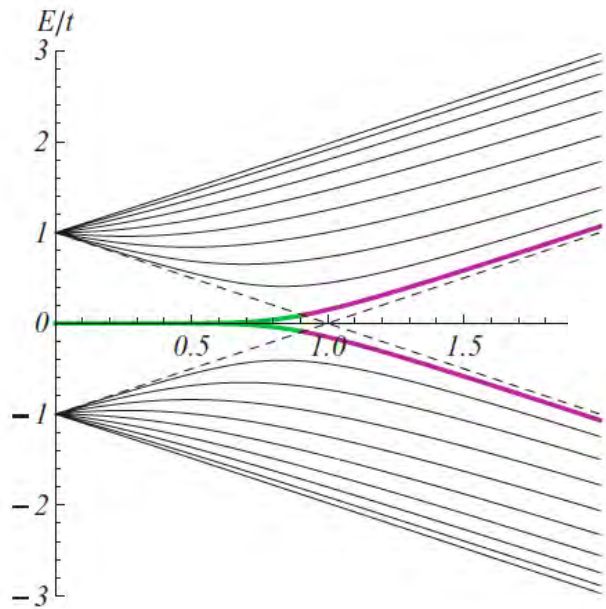
$$\frac{J'}{J} > 1$$

$Z = \pi$  Non trivial topological phase:

$$\frac{J'}{J} < 1$$

edge states

## Finite system



$$\left(\frac{J'}{J}\right)_c = 1 - \frac{1}{M+1}$$

$$N_{bulk} = 2M \quad J'/J > (J'/J)_c$$

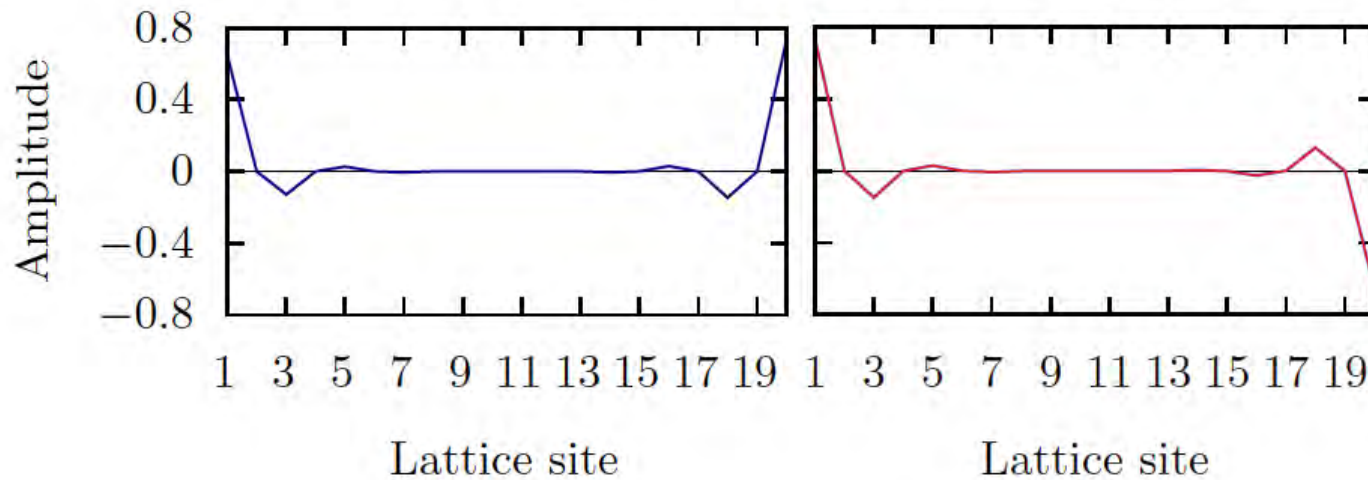
$$N_{bulk} = 2(M-1) \quad J'/J < (J'/J)_c$$

P. Delplace et al. PRB, 84, 195452

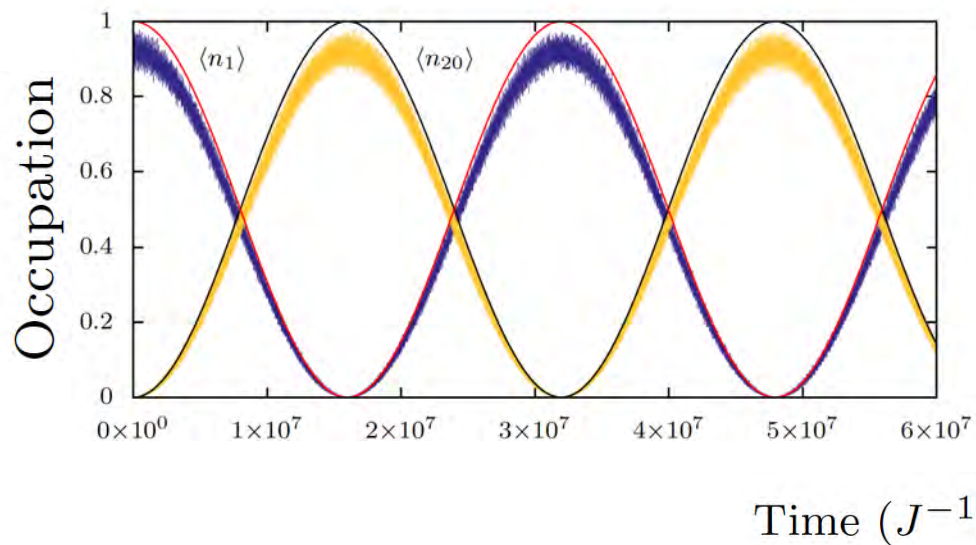
$$\lambda = \frac{J'}{J}$$

$$\frac{J'}{J}$$

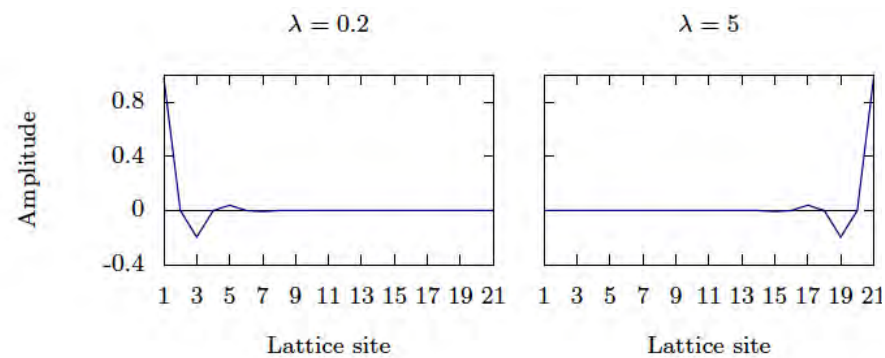
Edge states form a non-local two-level system.  
Long-range transfer



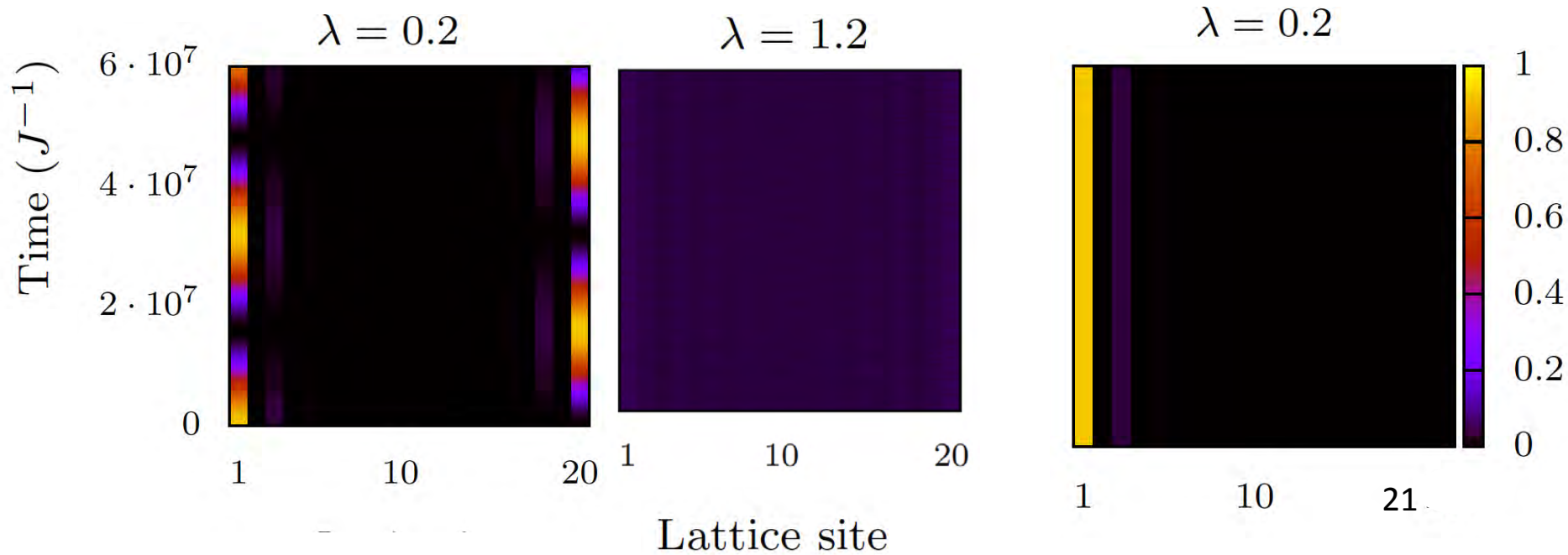
$$|1\rangle \simeq \frac{1}{\sqrt{2}}(|e_+\rangle + |e_-\rangle), \quad |N\rangle \simeq \frac{1}{\sqrt{2}}(|e_+\rangle - |e_-\rangle)$$



Odd number of sites



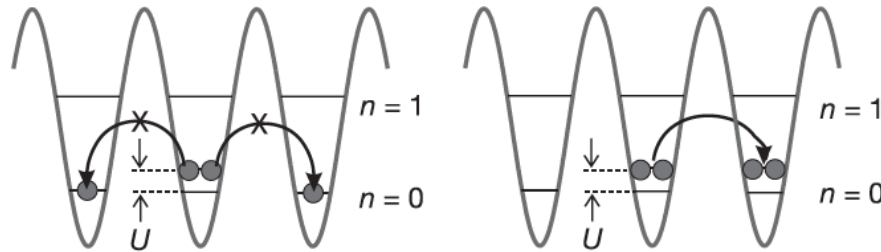
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Transfer of interacting electrons in a dimer chain



# Repulsively bound atom pairs in an optical lattice

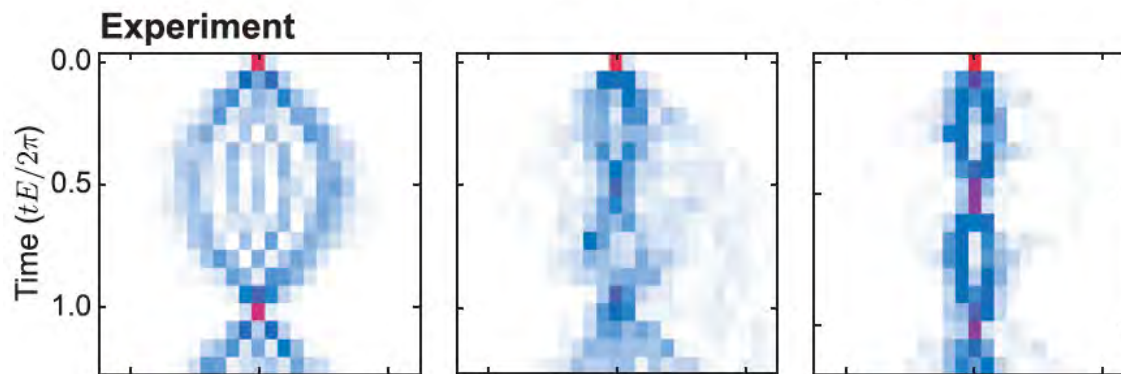
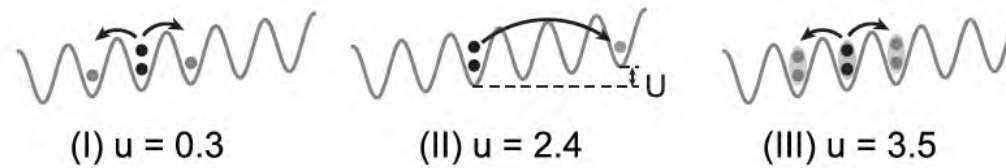


Vol 441|15 June 2006|doi:10.1038/nature04918

K. Wrinkler et al.

## Strongly correlated quantum walks in optical lattices

Philipp M. Preiss,<sup>1</sup> Ruichao Ma,<sup>1</sup> M. Eric Tai,<sup>1</sup> Alexander Lukin,<sup>1</sup> Matthew Rispoli,<sup>1</sup> Philip Zupancic,<sup>1\*</sup> Yoav Lahini,<sup>2</sup> Rajibul Islam,<sup>1</sup> Markus Greiner<sup>1†</sup>



Science Vol. 347, Issue 6227, pp. 1229-1233 (2015)

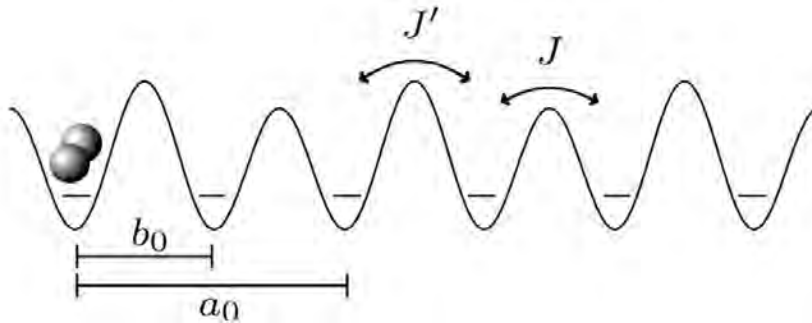
## SSH-Hubbard model

$$H = -J' \sum_{i=1, \sigma}^M c_{2i\sigma}^\dagger c_{2i-1\sigma} - J \sum_{i=1, \sigma}^{M-1} c_{2i+1\sigma}^\dagger c_{2i\sigma} + H.c. + U \sum_{i=1}^{2M} n_{i\uparrow} n_{i\downarrow}$$

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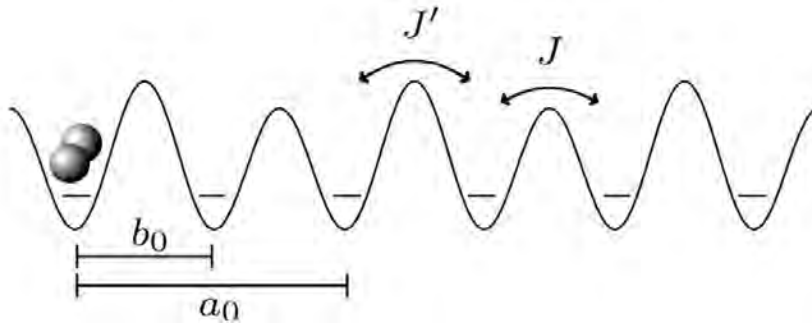
- ▶ Strongly interacting regime:  $U > 4J$



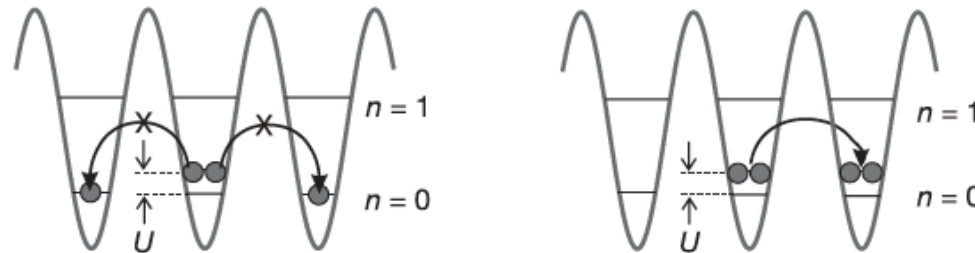
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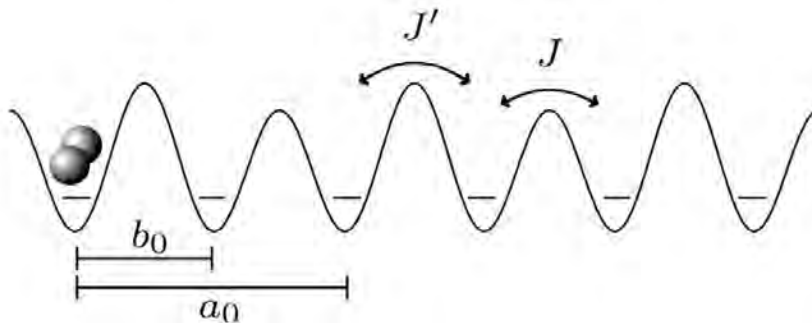
- ▶ Doublons cannot decay due to energy conservation



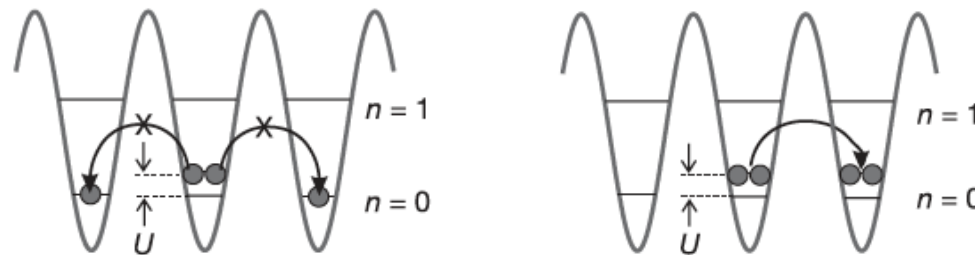
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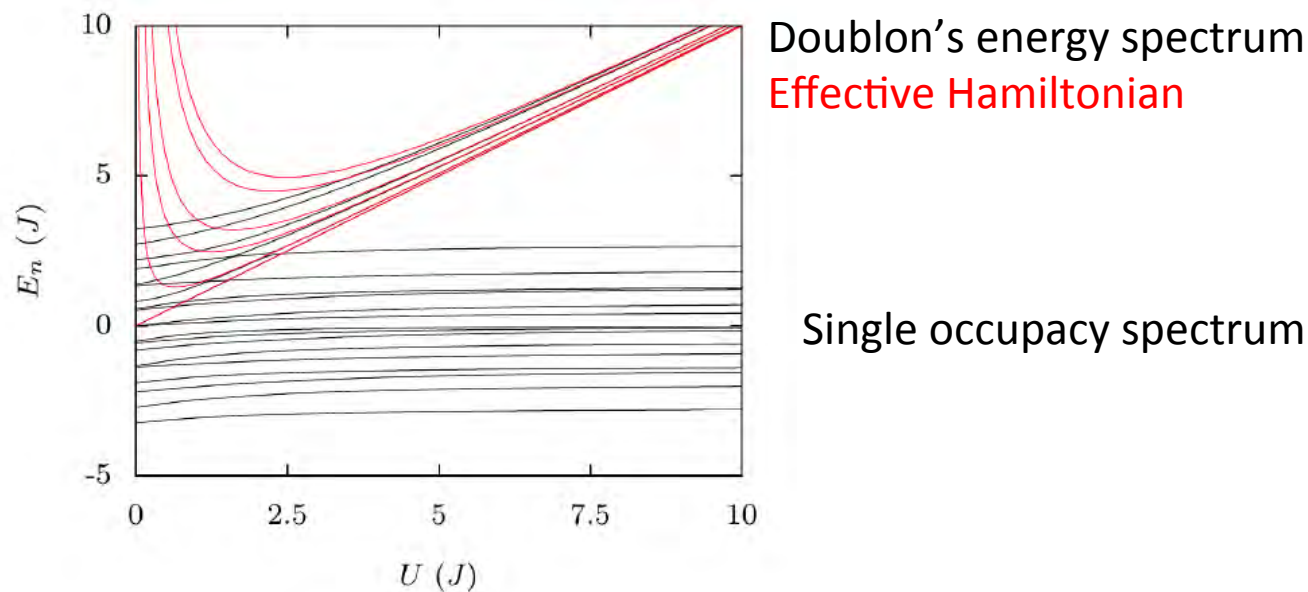
- ▶ Effective Hamiltonian for doublons?

- ▶ Unitary transformation that block-diagonalizes the Hamiltonian, perturbatively in powers of  $J/U$  up to first order.

$$H_{\text{eff}} = J'_{\text{eff}} \sum_{i=1}^M d_{2i}^\dagger d_{2i-1} + J_{\text{eff}} \sum_{i=1}^{M-1} d_{2i+1}^\dagger d_{2i} + H.c. + \sum_{i=1}^{2M} \mu_i n_i^d$$

$$d_i^\dagger = c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger \qquad n_i^d = d_i^\dagger d_i$$

F. Hofmann et al., PRB, 85,205127 (2012), A. H. MacDonald et al., PRB, 37, 9753 (1988)



M. Bello et al. , Scientific Rep. 6, 22562 (2016).

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dimer tight-binding
effective chemical potential

$$J'_{\text{eff}} = 2J'^2/U \quad J_{\text{eff}} = 2J^2/U$$

$$\mu_i = U + \sum_{\langle i,j \rangle} J_{ij}^{\text{eff}} \quad \text{Depends on number of neighbours}$$

For a finite lattice:

$$\mu_i = \begin{cases} \mu_{\text{bulk}} = J'_{\text{eff}} + J_{\text{eff}} + U & \text{if } 1 < i < 2M \\ \mu_{\text{edge}} = J'_{\text{eff}} + U & \text{if } i \in \{1, 2M\} \end{cases}$$

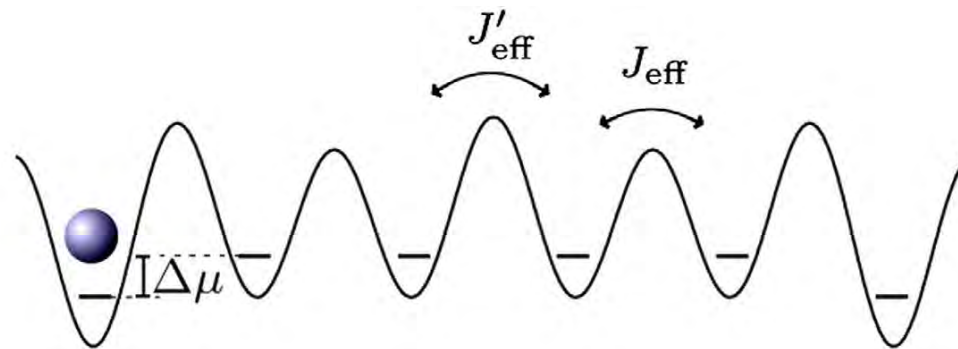
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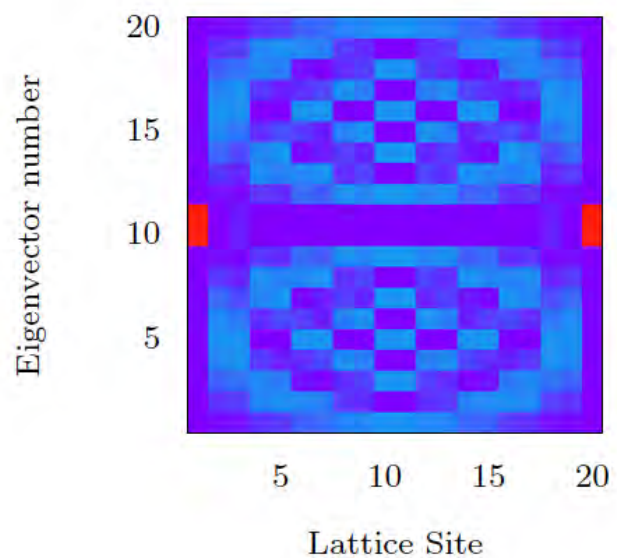
$$\mu_i = U + \sum_{\langle i,j \rangle} J_{ij}^{\text{eff}} \quad \text{Depends on number of neighbours}$$

This forbids the presence of edge states for doublons

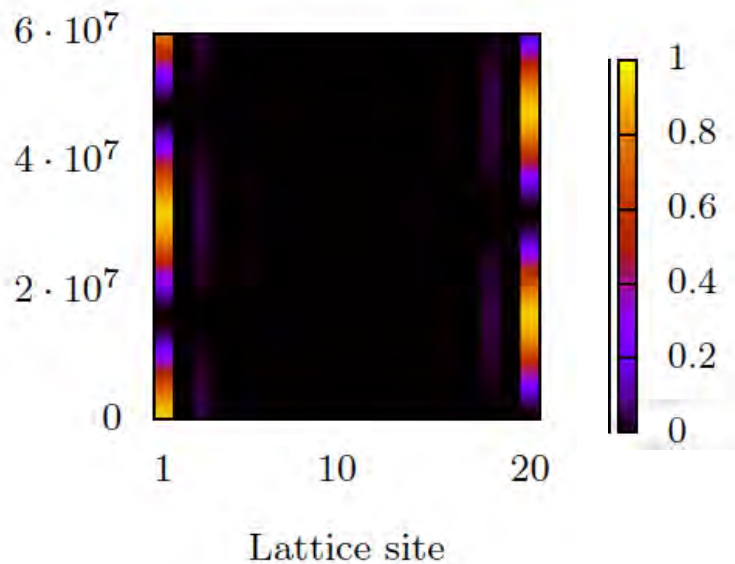




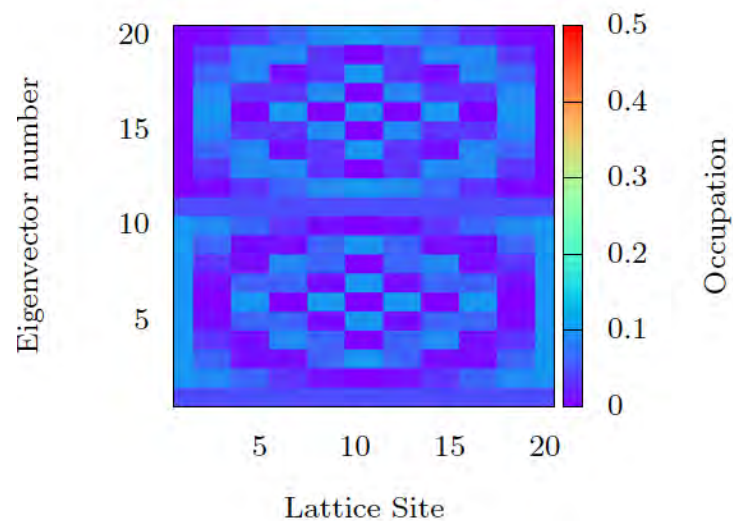
Non-interacting particles



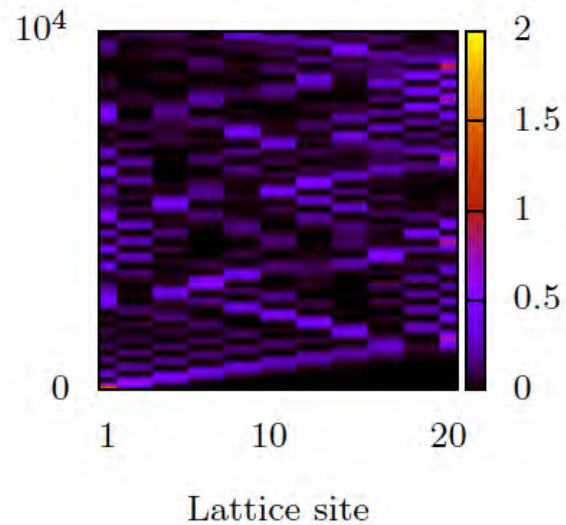
$$\lambda = 0.2$$



Effective Model for doublons



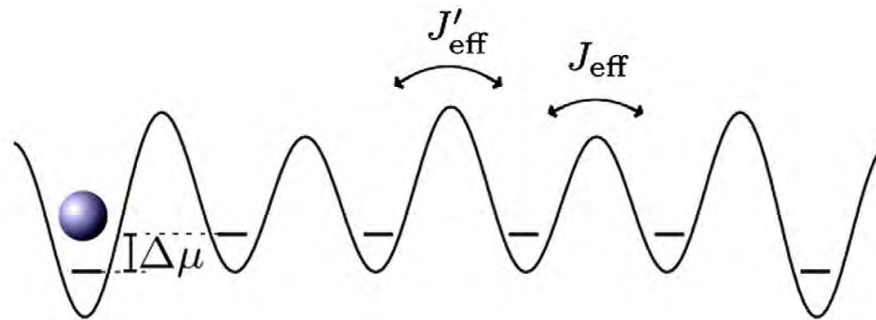
$$U = 10J$$



## Conclusions

- ▶ Long-range transfer of particles can be produced in a dimer chain thanks to the presence of edge states

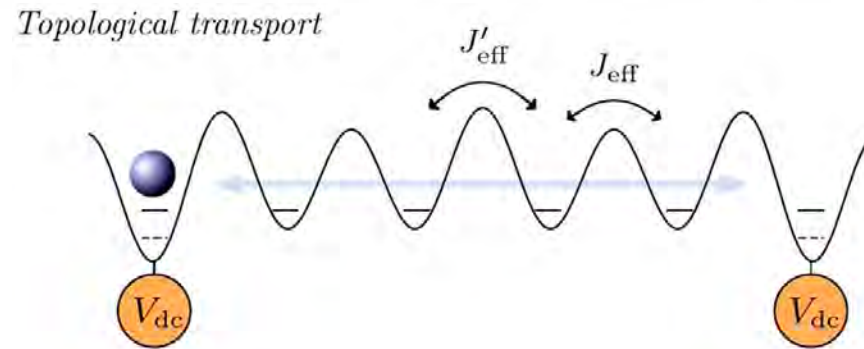
Doublons edge states do not occur naturally in the dimer chain.



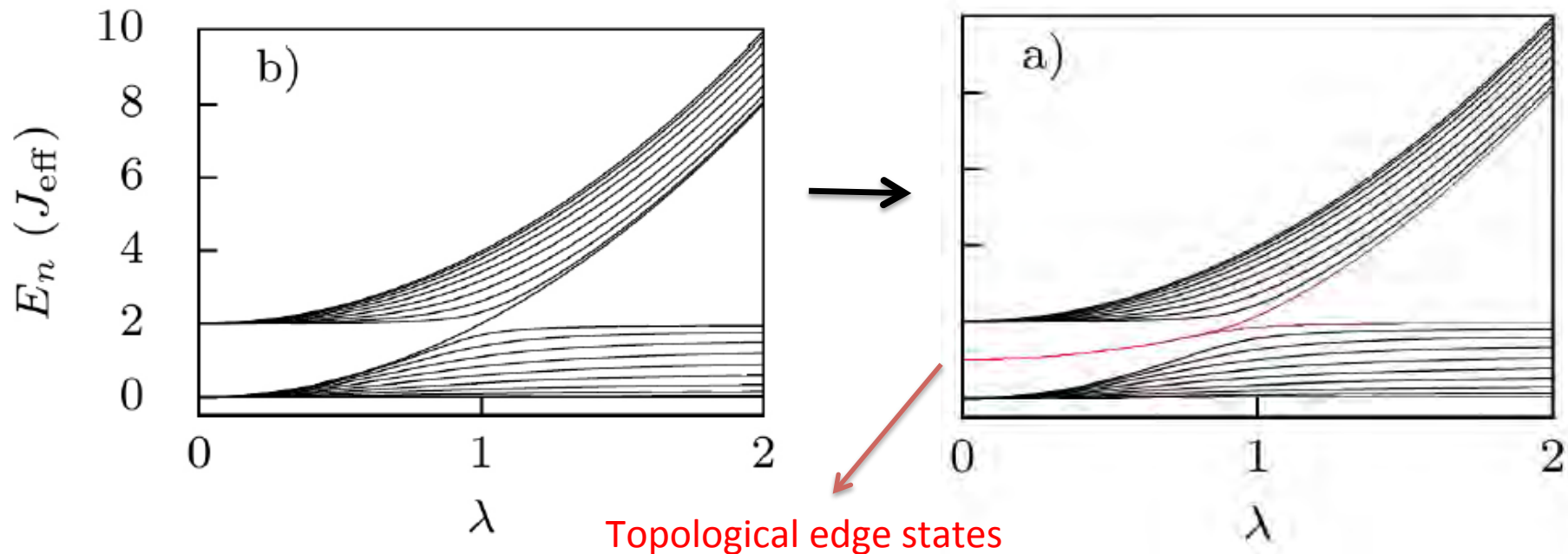
However they can be induced by different means:

1)

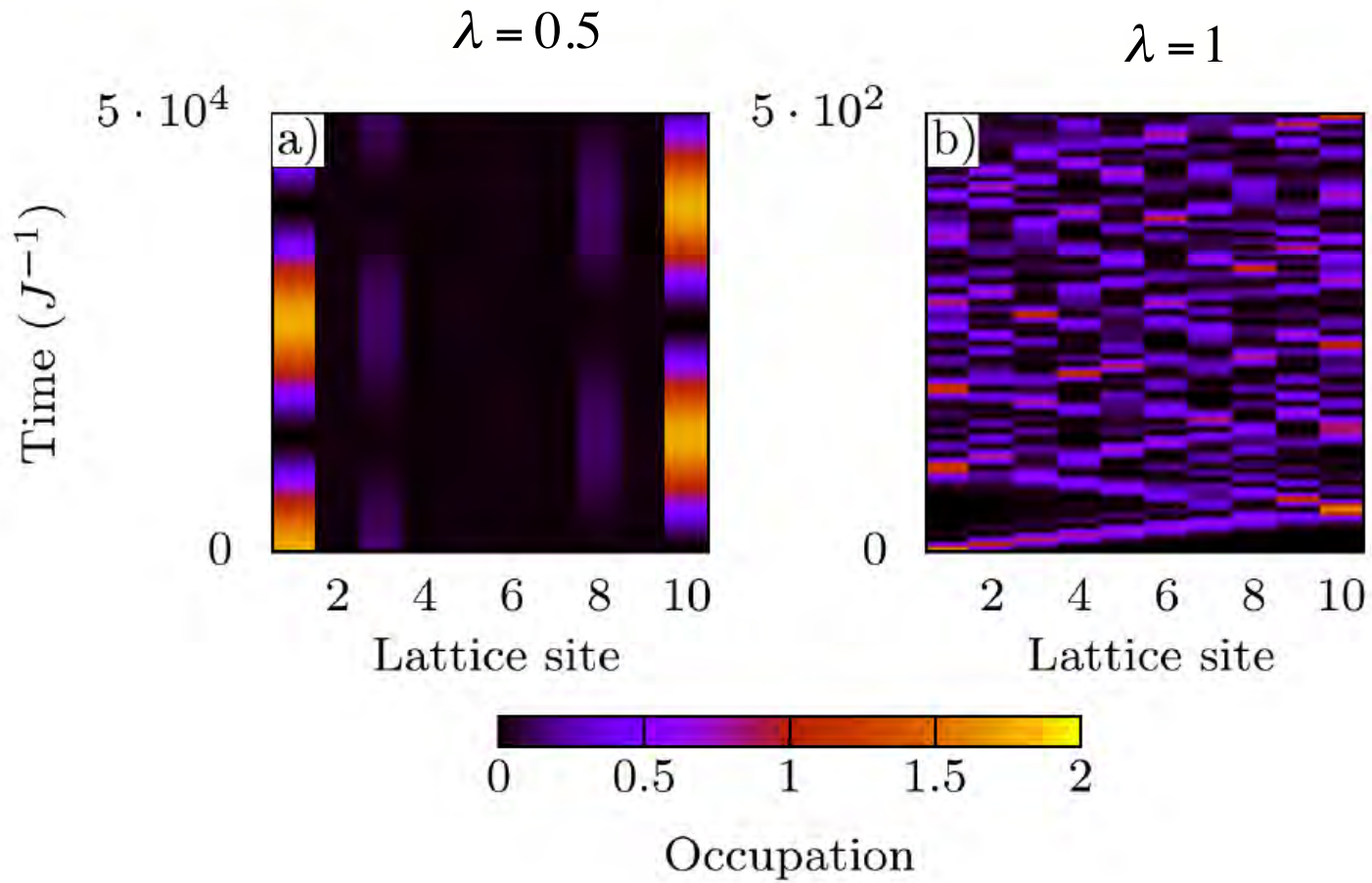
If we add a gate potential at the ends of the chain to compensate for the chemical potential difference, we recover the SSH model for doublons



### Topological Direct Transfer



# Topological Direct Transfer

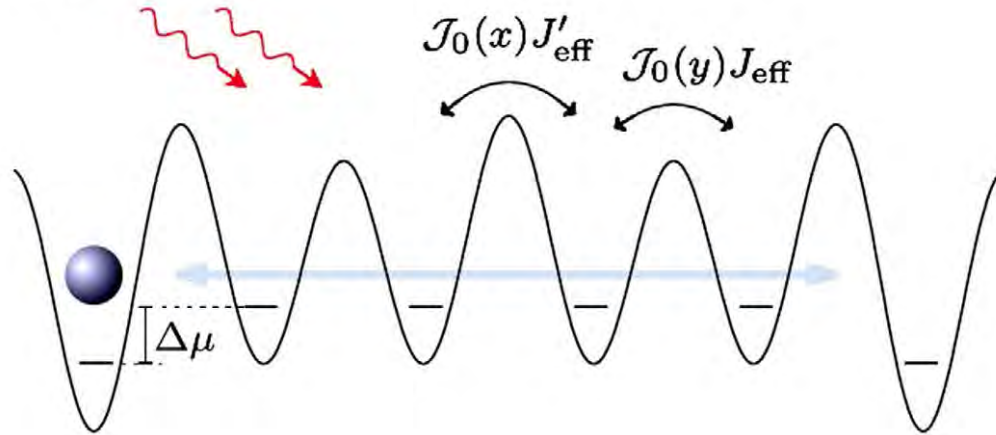


$U = 16J$ ; a)  $\lambda = 0.5$ , b)  $\lambda = 1$

2)

With AC driving: Shockley transfer:

*Shockley transport*



# Floquet theory and HFE

N. Goldman et al., PRX, 4, 031027  
M. Bukov et al., Adv. Phys. 64,139

$$H(t) = H_{hopp} + H_U + E \cos(\omega t) \sum_{i=1}^{2M} x_i (n_{i\uparrow} + n_{i\downarrow})$$

Effective Hamiltonian, regime  $U \gg \omega > J, J'$

Only the hopping parameters become renormalized by the ac field:

$$J'_{eff} \longrightarrow \mathcal{J}_0\left(\frac{2Eb_0}{\omega}\right) J'_{eff}$$

$$J_{eff} \longrightarrow \mathcal{J}_0\left(\frac{2E(a_0 - b_0)}{\omega}\right) J_{eff}$$



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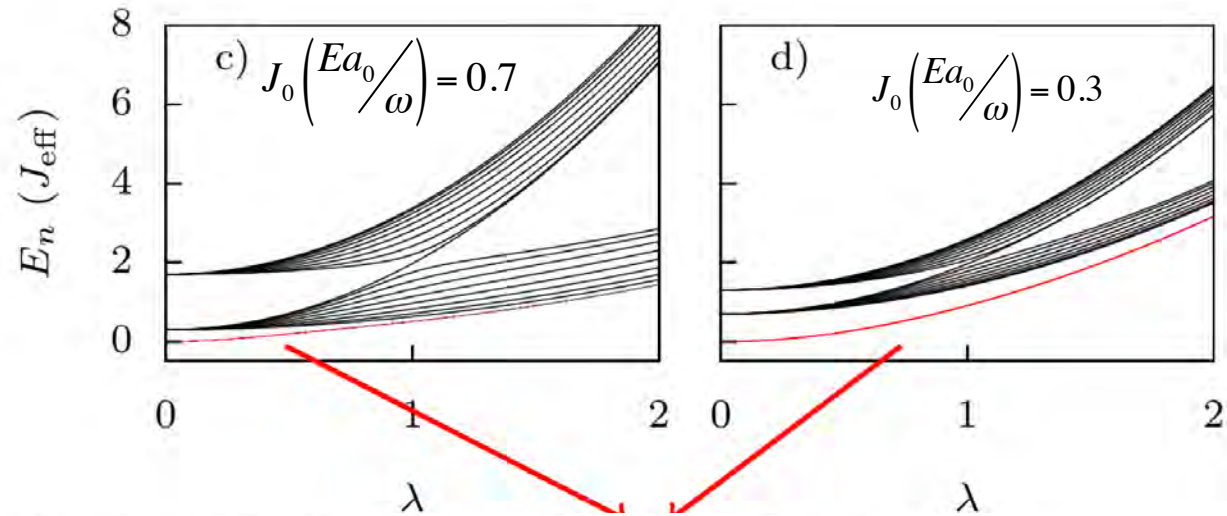
$$J_{eff} \longrightarrow \mathcal{J}_0\left(\frac{2E(a_0 - b_0)}{\omega}\right) J_{eff}$$



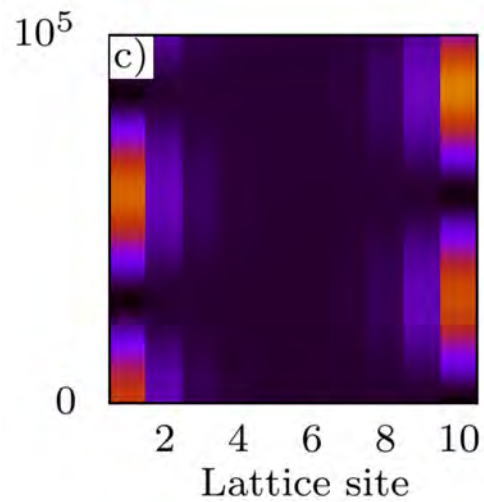
$$H_{eff} = \mathcal{J}_0(x) J'_{eff} \sum_{i=1}^M d_{2i}^\dagger d_{2i-1} + \mathcal{J}_0(y) J_{eff} \sum_{i=1}^{M-1} d_{2i+1}^\dagger d_{2i} + h.c. + \sum_{i=1}^{2M} \mu_i n_i^d \quad \begin{aligned} x &= \frac{2Eb_0}{\omega} \\ y &= \frac{2E(a_0 - b_0)}{\omega} \end{aligned}$$

The ac field allows to tune the ratio between the hoppings and  $\Delta\mu$

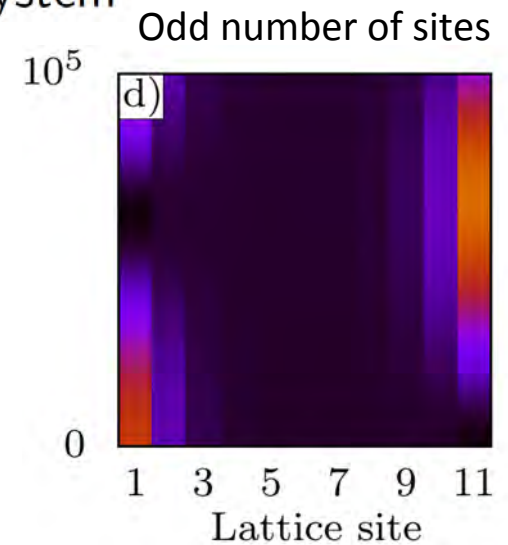
# Inducing Shockley-like edge states



These edge states also form a non-local two level system



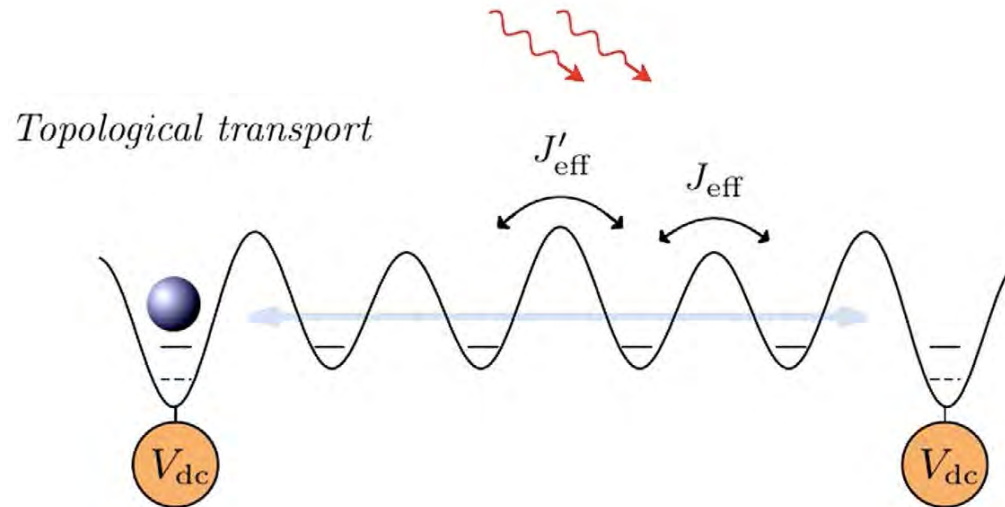
Shockley Direct Transfer





3)

Combination of both: AC induced topological transfer



AC fields + gate potentials

SSH model for doublons with tunable hoppings:

$$H_{\text{eff}} = \mathcal{J}_0(x) J'_{\text{eff}} \sum_{i=1}^M d_{2i}^\dagger d_{2i-1} + \mathcal{J}_0(y) J_{\text{eff}} \sum_{i=1}^{M-1} d_{2i+1}^\dagger d_{2i} + H.c.$$

$$x = \frac{2Eb_0}{\omega}$$

$$y = \frac{2E(a_0 - b_0)}{\omega}$$

A. Gómez-León & G. Platero PRL **110**, 200403 (2013)

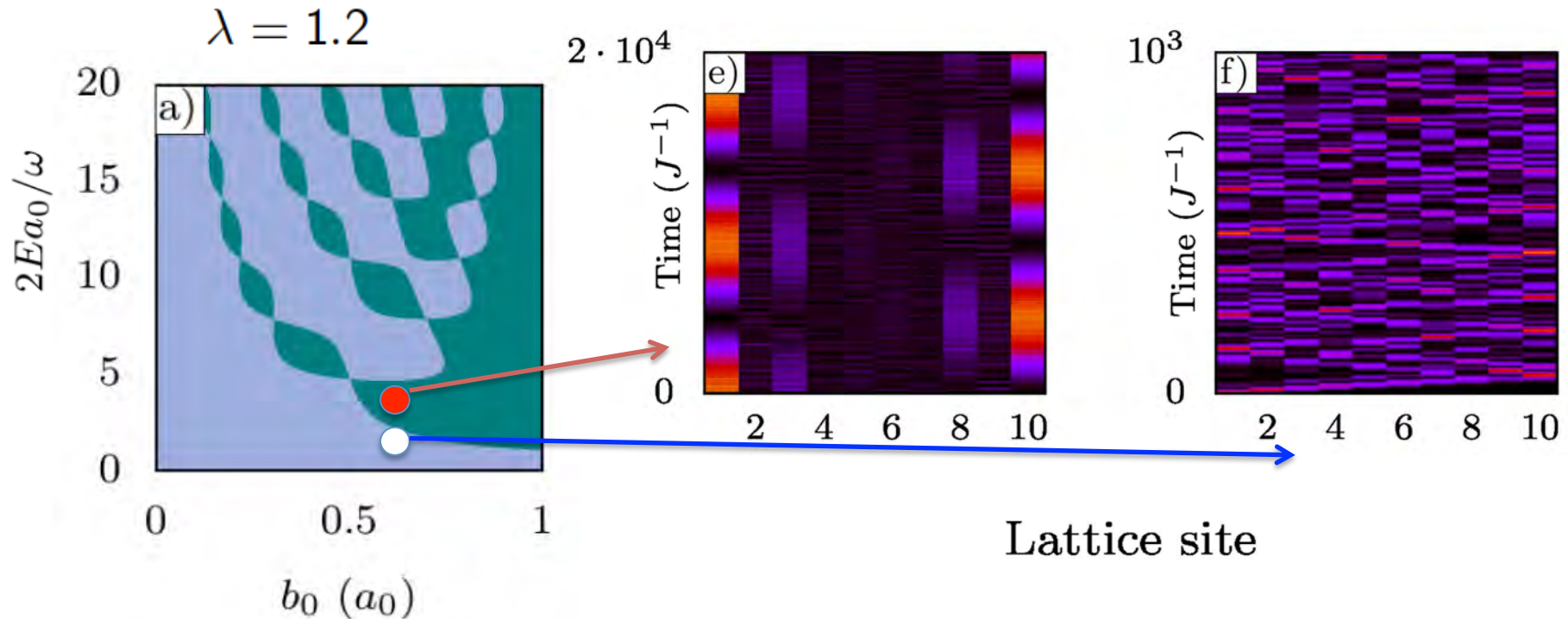
# AC fields + gate potentials

SSH model for doublons with tunable hoppings:

⇒ Control over the topology of the system:

$$Z=\pi \quad \left| \frac{\lambda^2 \mathcal{J}_0\left(\frac{2E}{\omega} b_0\right)}{\mathcal{J}_0\left(\frac{2E}{\omega} (a_0 - b_0)\right)} \right| < 1 \quad \lambda = \frac{J'}{J}$$

M. Bello, C. E. Creffield, and G. Platero, Scientific Rep. 6, 22562 (2016).



# AC fields + gate potentials

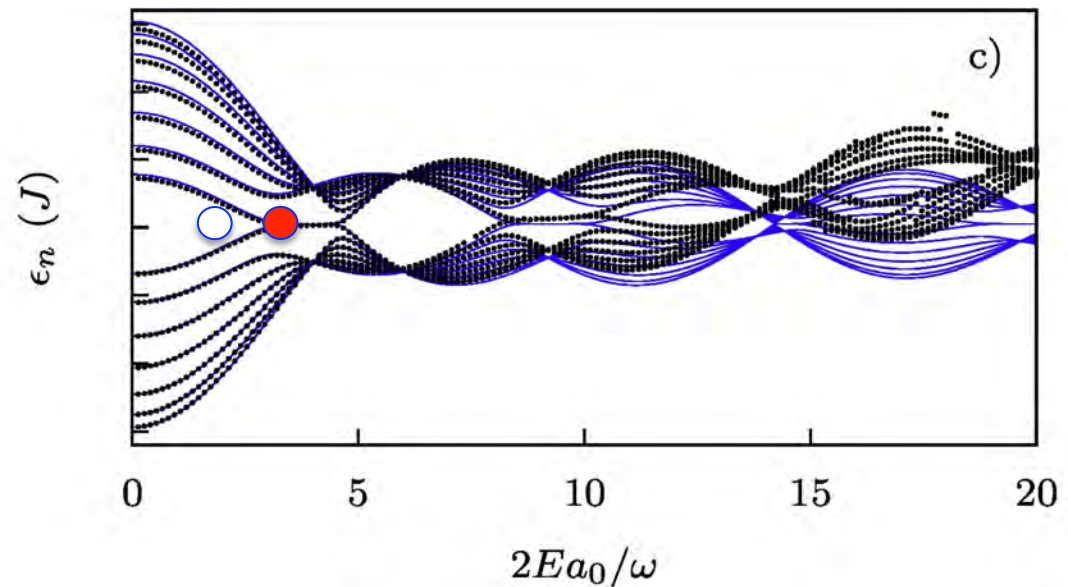
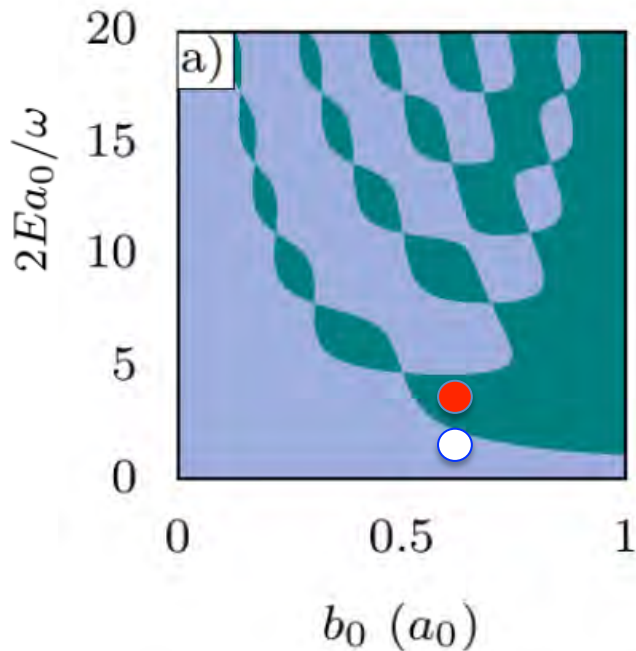
SSH model for doublons with tunable hoppings:

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$$Z=\pi \quad \left| \frac{\lambda^2 \mathcal{J}_0\left(\frac{2E}{\omega} b_0\right)}{\mathcal{J}_0\left(\frac{2E}{\omega} (a_0 - b_0)\right)} \right| < 1 \quad \lambda = \frac{J'}{J}$$

M. Bello, C. E. Creffield, and G. Platero, Scientific Rep. 6, 22562 (2016).

$\lambda = 1.2$



$U = 16J, b_0 = 0.6a_0$  and  $\omega = 2J$ .

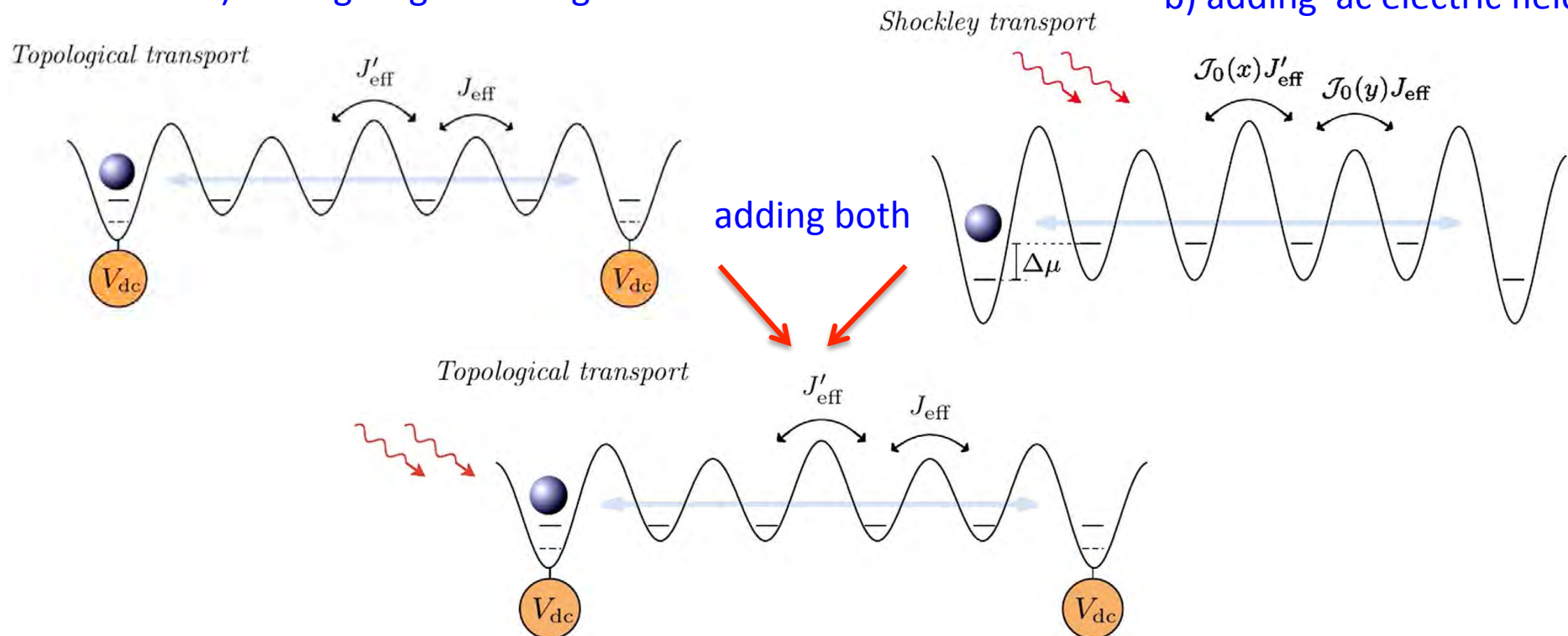
# Conclusions

- ▶ Long-range transfer of particles can be produced in a dimer chain thanks to the presence of edge states

Doublons edge states do not occur naturally in the dimer chain, however they can be induced by different means:

a) adding dc gate voltages

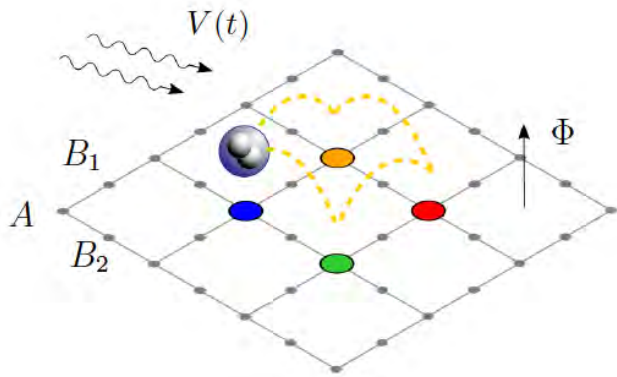
b) adding ac electric fields



# Doublons in 2D lattices driven with ac electric fields and static magnetic fields

$$H(t) = -J \sum_{\langle i,j \rangle, \sigma} e^{i\phi_{ji}} c_{j\sigma}^\dagger c_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_i V_i(t) (n_{i\uparrow} + n_{i\downarrow})$$

Circular polarization  $V_i(t) = x_i E \cos(\omega t) + y_i E \sin(\omega t)$

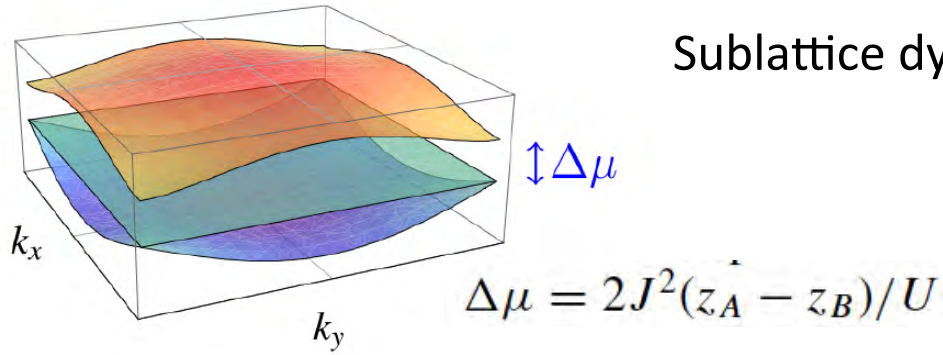


$$H_{\text{eff}} = \sum_{\langle i,j \rangle} J_{ij}^{\text{eff}} d_i^\dagger d_j + \sum_i \mu_i^{\text{eff}} d_i^\dagger d_i$$

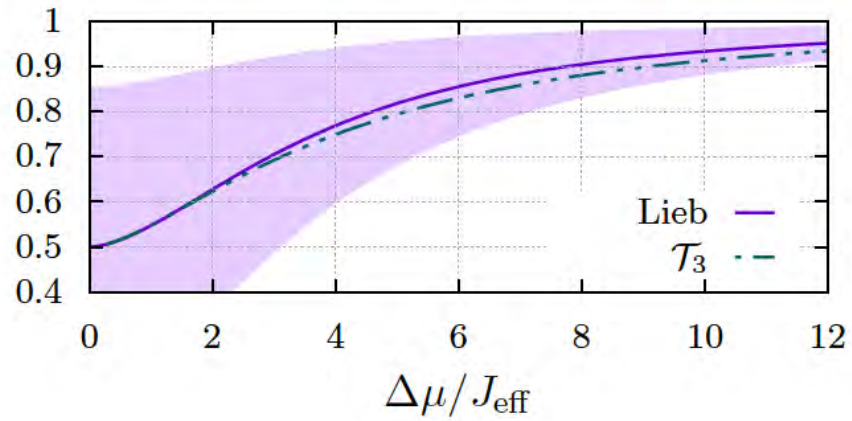
$$J_{ij}^{\text{eff}} = \frac{2J^2 e^{i2\phi_{ij}}}{U} \mathcal{J}_0\left(\frac{2E\delta}{\omega}\right) \quad \mu_i^{\text{eff}} = \frac{2J^2 z_i}{U}$$

# Sublattice dynamics

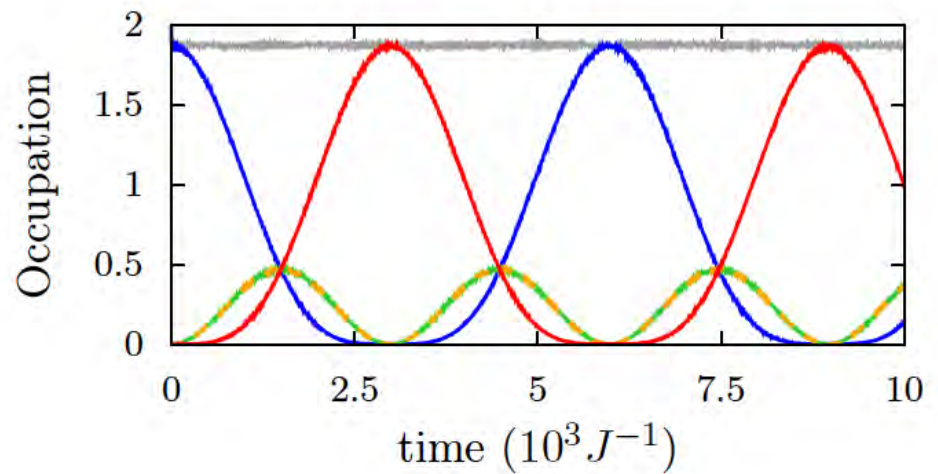
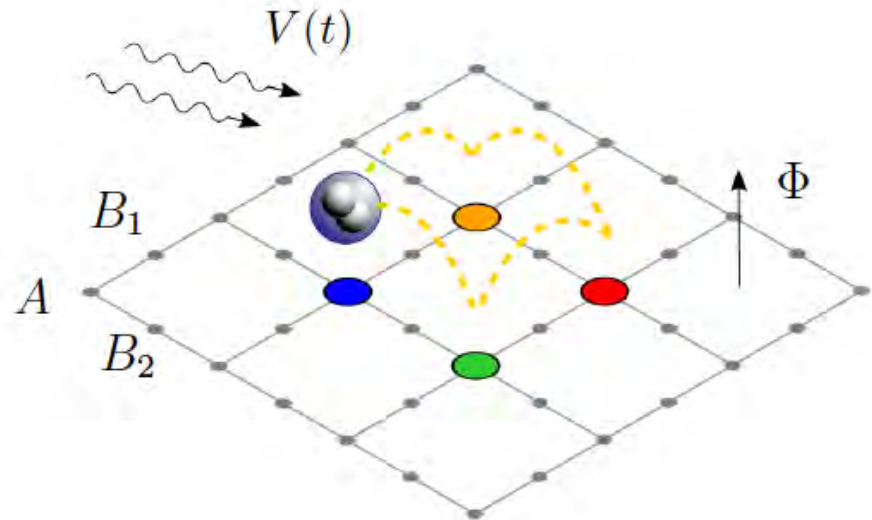
$$\Phi = 0$$



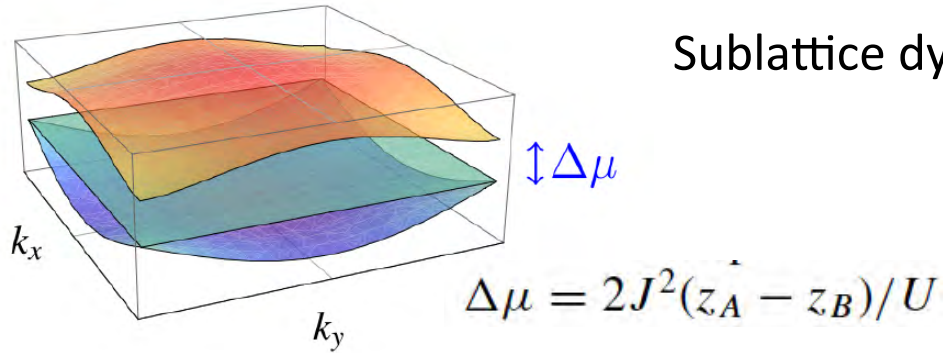
The ac field modifies the relative weight of the Bloch states on each sublattice



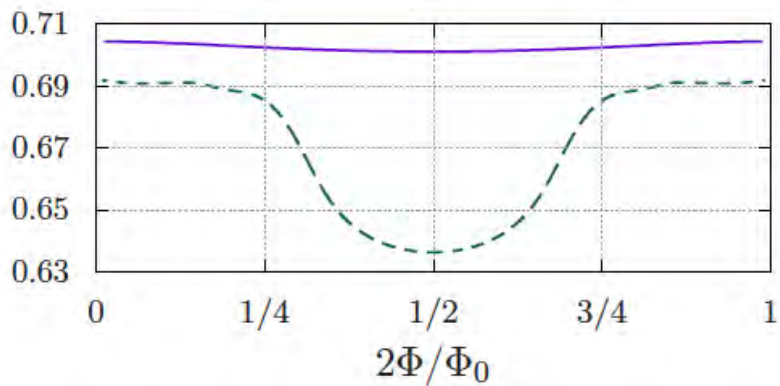
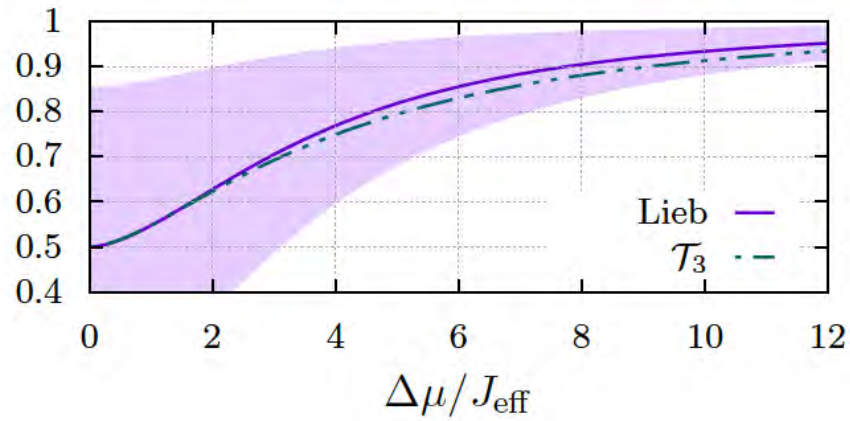
Time average probability for the doublon to remain in sublattice A



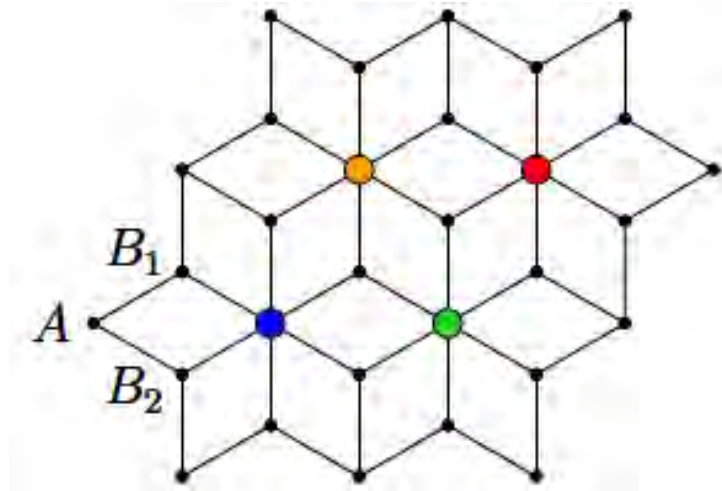
# Sublattice dynamics



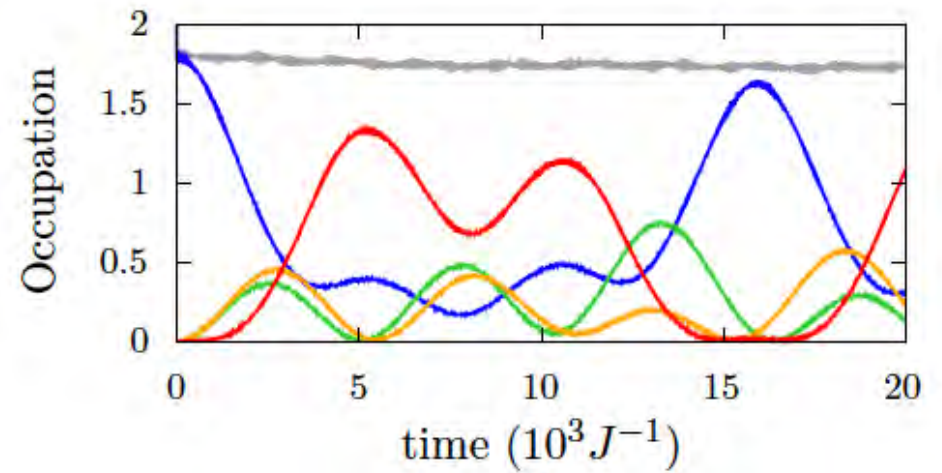
The ac field modifies the relative weight of the Bloch states on each sublattice



$\Phi = 0$

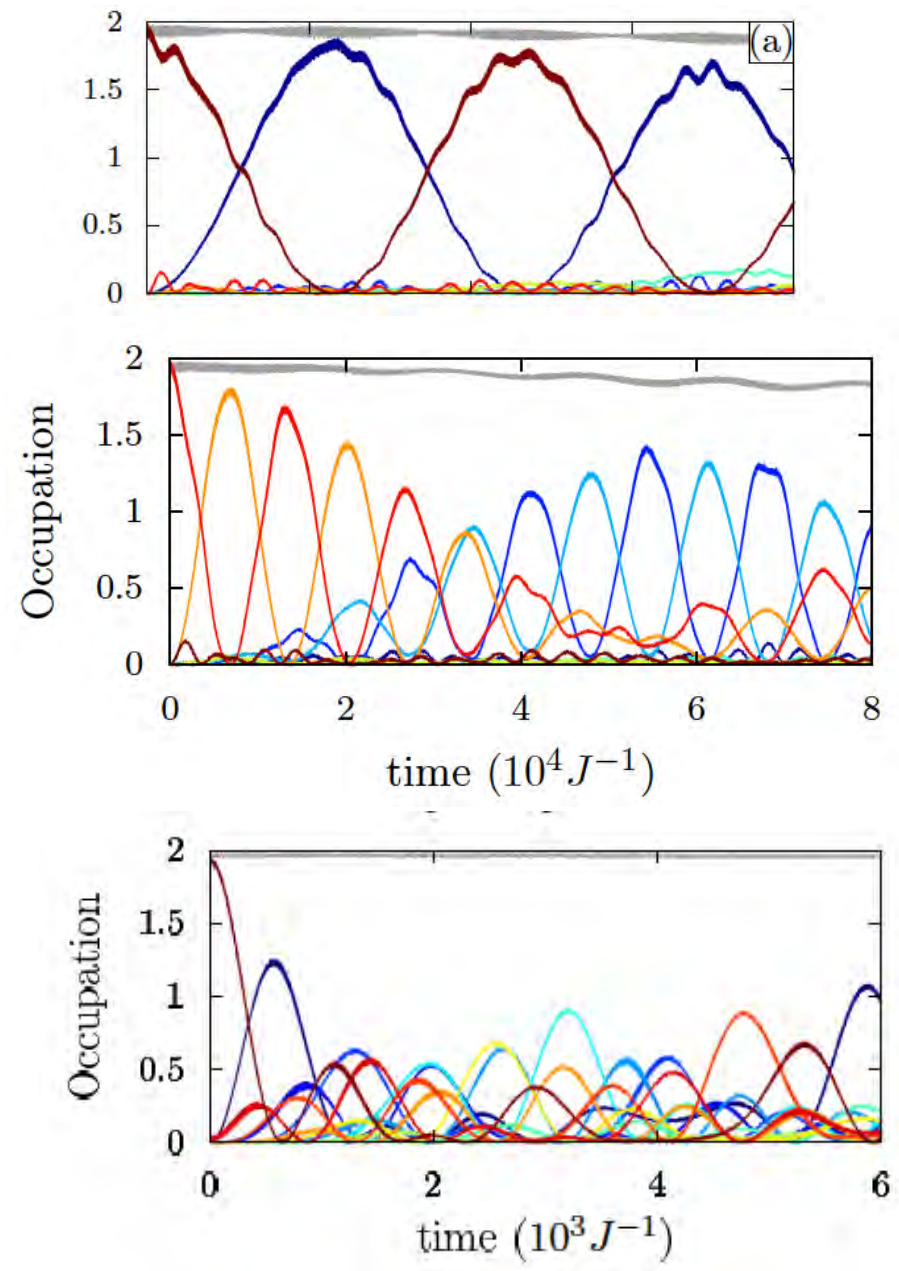
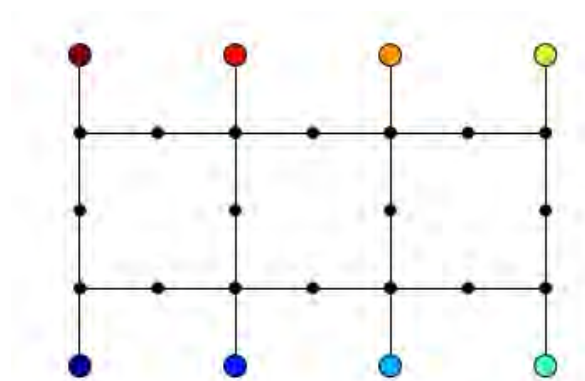


Vidal et al., PRL 1998 Aharonov-Bohm Cages



# Edge dynamics and long range transfer of doublons

$$\Phi = 0$$

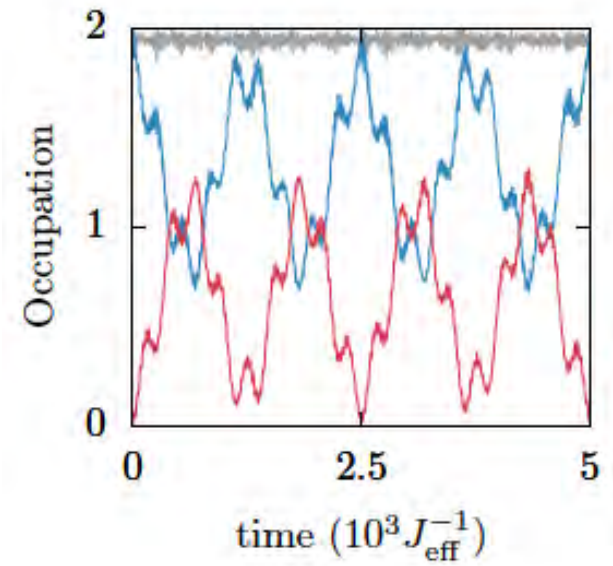
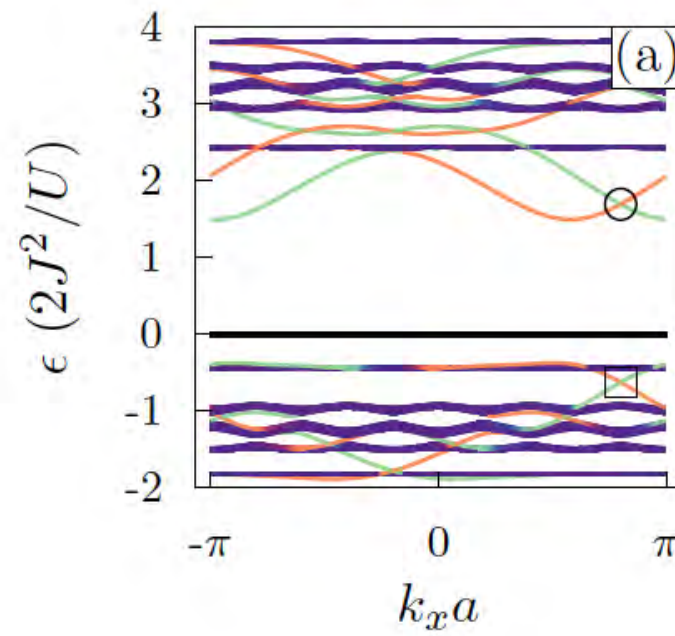




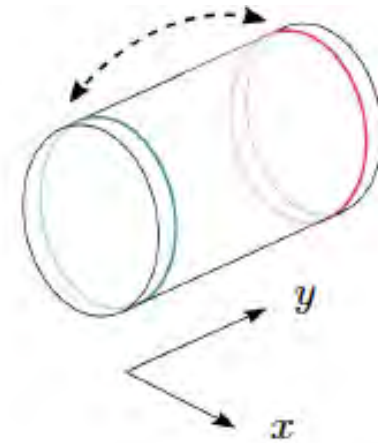
$$\Phi \neq 0$$

$$\Phi/\Phi_0 = 1/10$$

$$N_y = 100$$

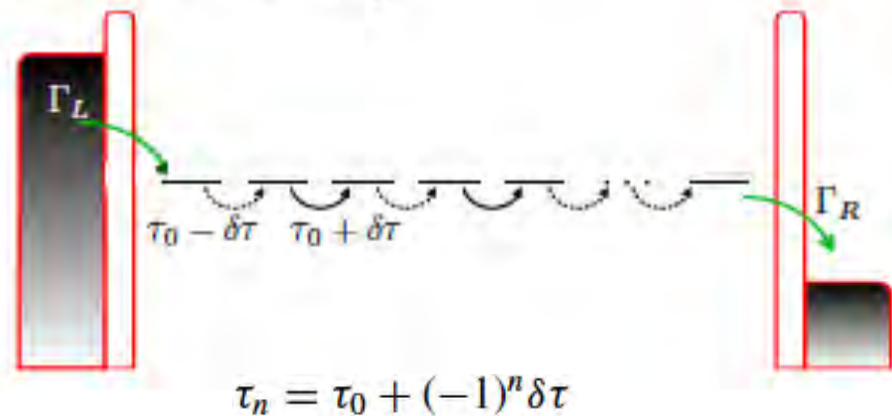


M. Bello, C.E. Creffield and G. Platero, PRB, 95, 094303 (2017)



## Quantum transport in a dimer chain

- ▶ Transport from source to drain (voltage bias  $V$ )
- ▶ Low temperatures
- ▶  $|\tau_0 \pm \delta\tau| \ll eV \ll U_d \rightarrow$   
only single-electron states



- ▶ Master equation

$$\dot{\rho} = -\frac{i}{\hbar} [H_{SSH}, \rho] + \Gamma_L \mathcal{D}(c_1^\dagger) \rho + \Gamma_R \mathcal{D}(c_N) \rho \Rightarrow$$

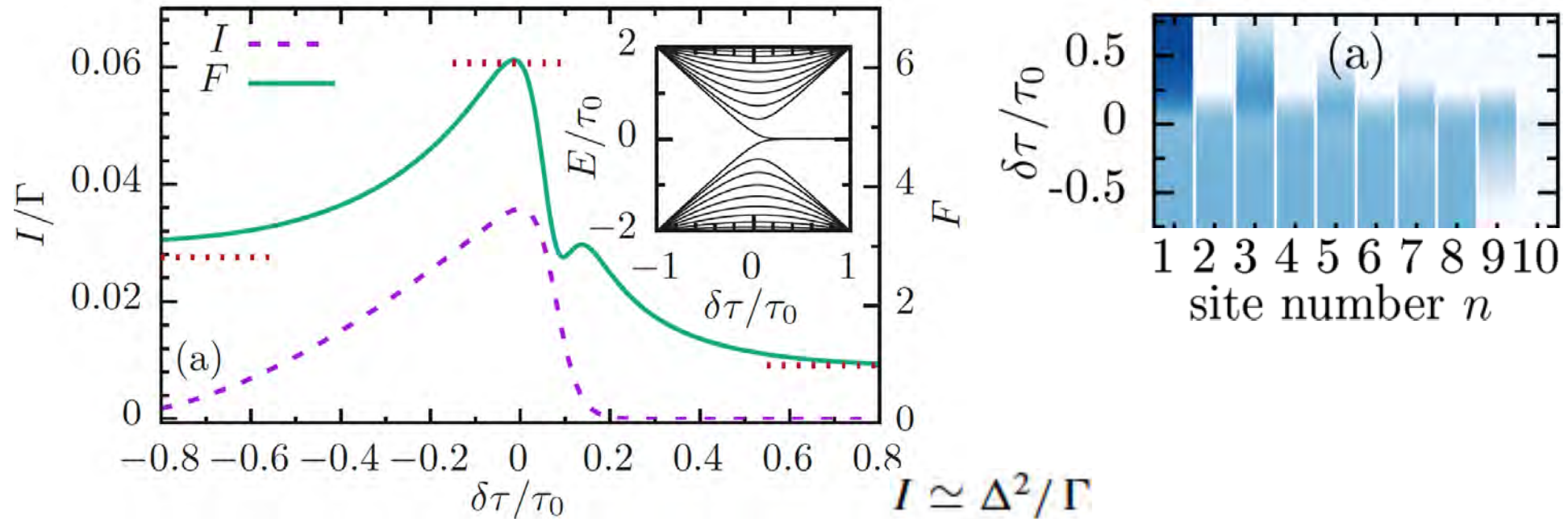
- ▶ steady state
- ▶ current

Lindblad operator:  $\mathcal{D}(x)\rho = (2x\rho x^\dagger - x^\dagger x\rho - \rho x^\dagger x) / 2$

- ▶  $\delta\tau < 0$  no edge states
- ▶  $\delta\tau > 0$  two edge states

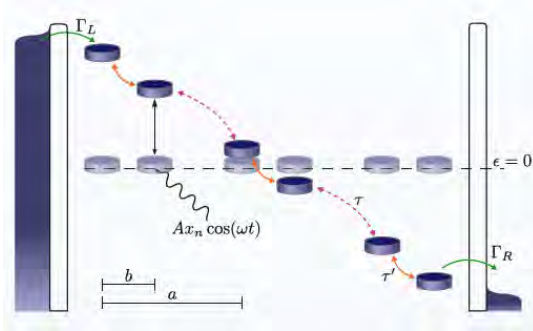
$$\Delta \approx \tau_0 \exp(-N\delta\tau/\tau_0)$$

N=20 atoms



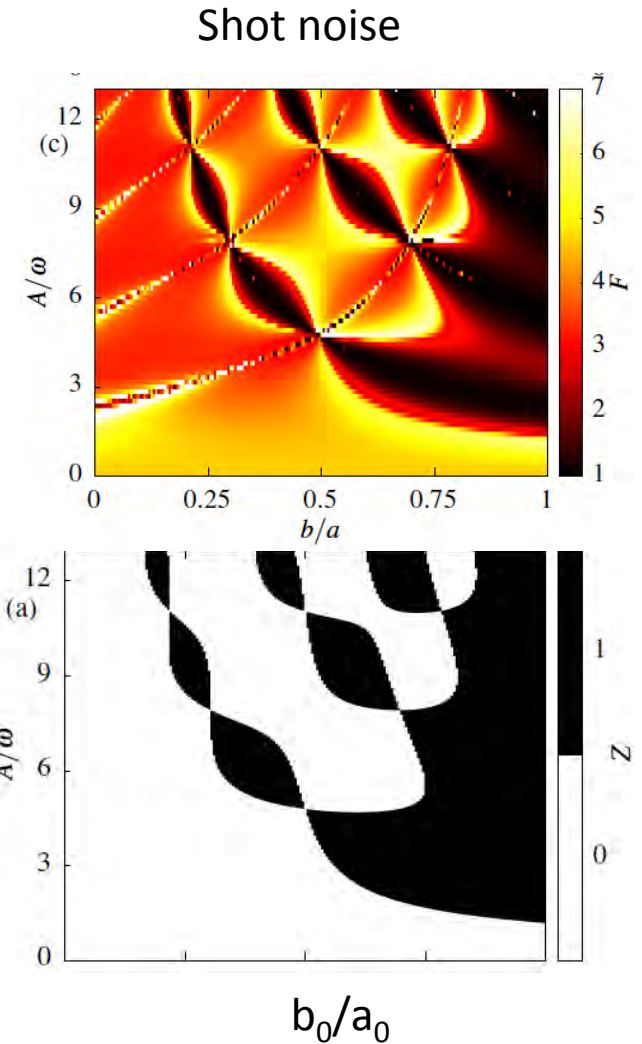
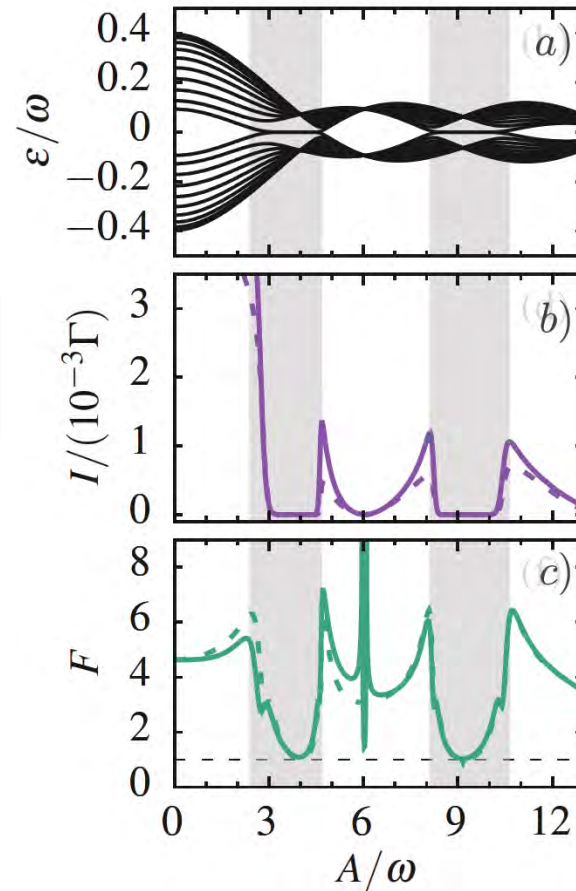
- ▶ Electron blocked at the source edge state
- ▶ Coulomb interaction avoids a second electron to enter
- ▶ edge state at the source + Coulomb interaction  $\Rightarrow$  edge-state blockade

# Dimer chain: AC driven transport to characterize the topology



$$H(t) = H_{\text{SSH}} + A \sum_{n=1}^N x_n c_n^\dagger c_n \cos(\omega t)$$

While the shape of the current suppression would be sufficient to identify edge-state blockade, the Fano factor exhibits clearer fingerprints of the topological phase diagram.



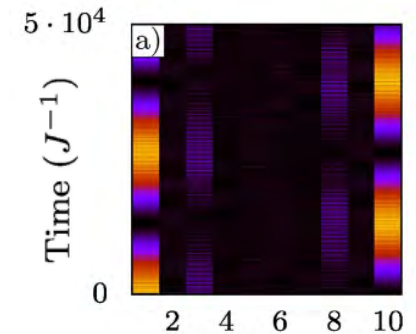
M. Niklas, M. Benito, S. Kohler and G. Platero, Nanotechnology, 2017

Zak phase

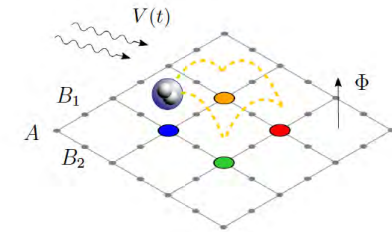
# Summary

Long range transfer of particles in a dimer chain mediated by edge states

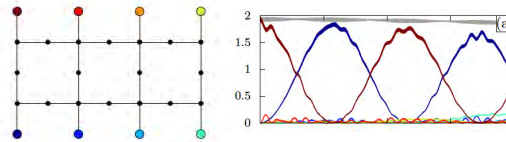
Doublons edge states in the dimer chain induced by dc voltages and ac electric fields: Long range transfer of doublons



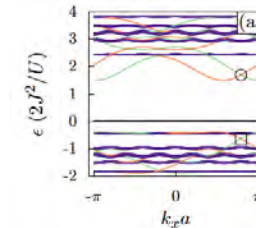
Sublattice Doublon dynamics in 2D lattices controlled by ac electric fields



long range Doublon transfer



coexistence of Schockley and topological edge states



Current edge state blockade in a dimer chain:  
ac transport: fingerprints of topology in the noise

