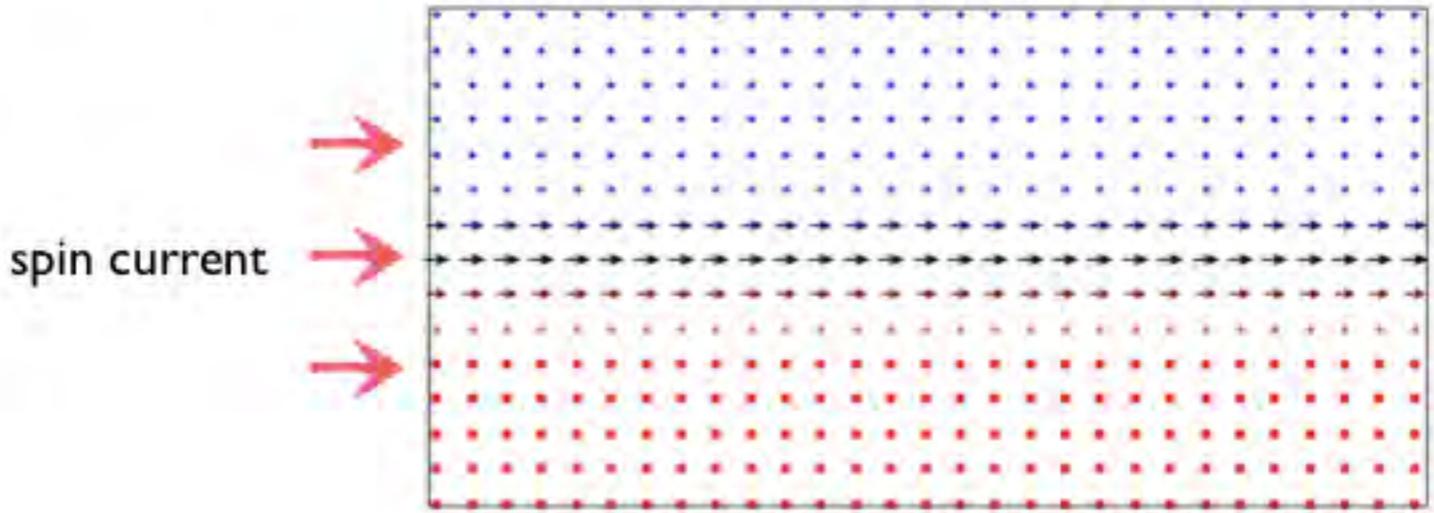
Spin Superfluid in a Magnetic Domain Wall

Se Kwon Kim
UCLA

in collaboration with Yaroslav Tserkovnyak (UCLA)
Young Research Leaders Group Workshop, SPICE at Mainz, Germany, 2017



SKK and Tserkovnyak, *PRL* 119, 047202 *(2017)*

How to achieve an efficient spin transport?

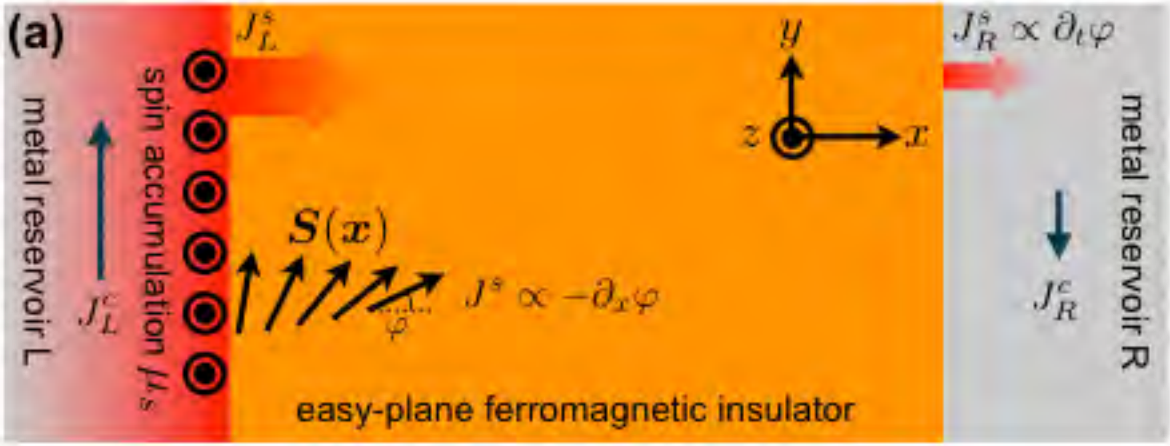
zero-resistance mass and charge transport
(superconductivity, quantum Hall effects, …)

↓

efficient spin transport

Novel types of spin transport in charge insulators

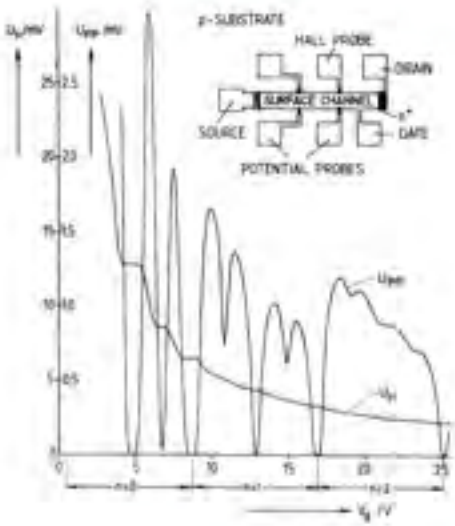
~



superfluid mass transport P. Kapitza, *Nature* (1938)

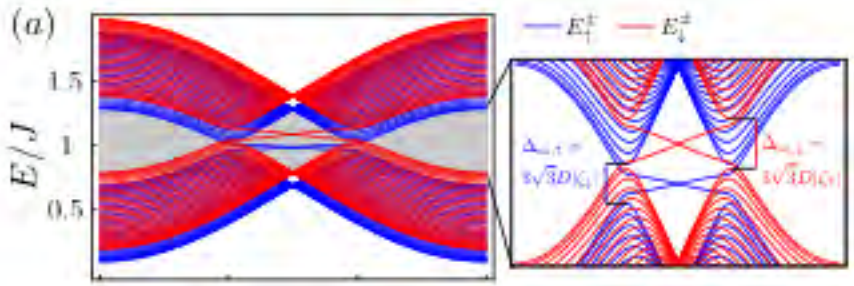
superfluid spin transport Sonin, *Sov. Phys. JETP* (1978)
König, Bønsager, and MacDonald, *PRL* (2001)
Chen and Signst, *PRB* (2014)
Takei and Tserkovnyak, *PRL* (2014)
Holmqvist, *et al.*, *PRL* (2015)
...

* the figure is taken from https://en.wikipedia.org/wiki/Superfluidity.



K. v. Klitzing; G. Dorda; M. Pepper PRL 1980

~



Matsumoto and Murakami, *PRL* (2011)
Zhang et *al.*, *PRB* (2013)
Shindou et *al.*, *PRB* (2013)
Mook et *al.*, *PRB* (2014)
Nakata et *al.* *PRB* (2017)
...

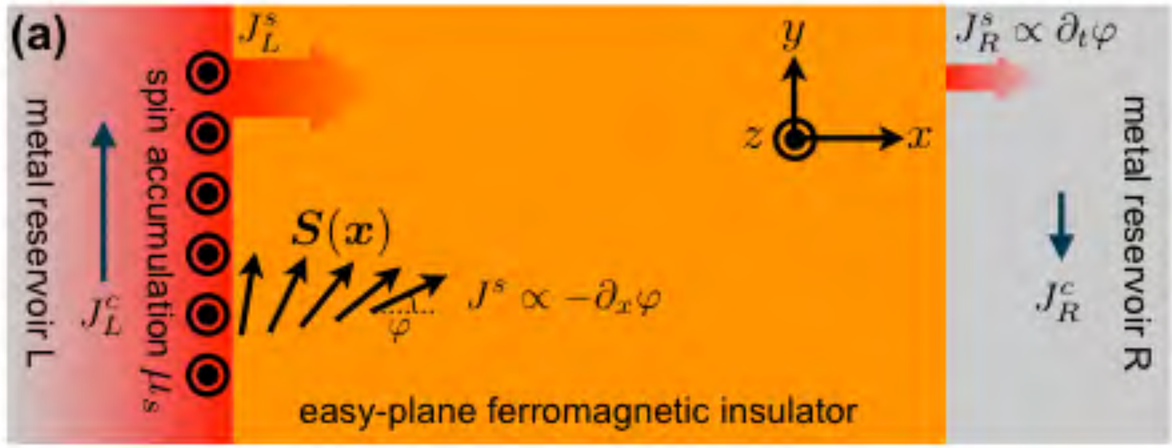
quantum Hall effects

magnonic quantum Hall effects

Novel types of spin transport in charge insulators

Novel types of spin transport in charge insulators

~



superfluid mass transport P. Kapitza, *Nature* (1938)

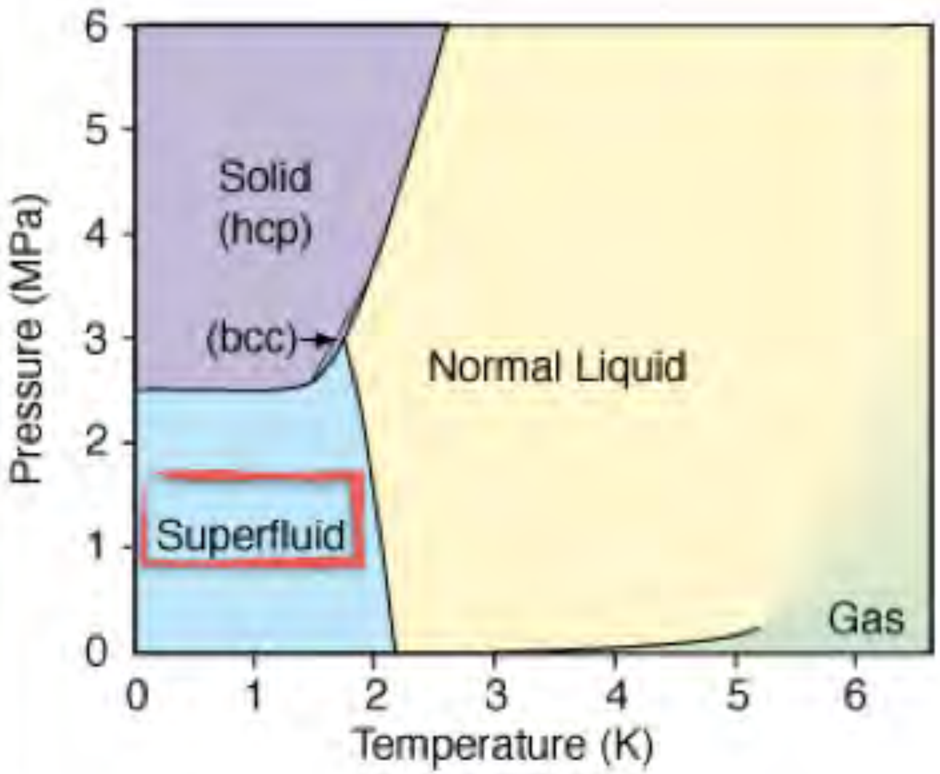
* the figure is taken from https://en.wikipedia.org/wiki/Superfluidity.

superfluid spin transport Sonin, *Sov. Phys. JETP* (1978)
König, Bønsager, and MacDonald, *PRL* (2001)
Chen and Sigrist, *PRB* (2014)
Takei and Tserkovnyak, *PRL* (2014)
Holmqvist, *et al*, *PRL* (2015)
...

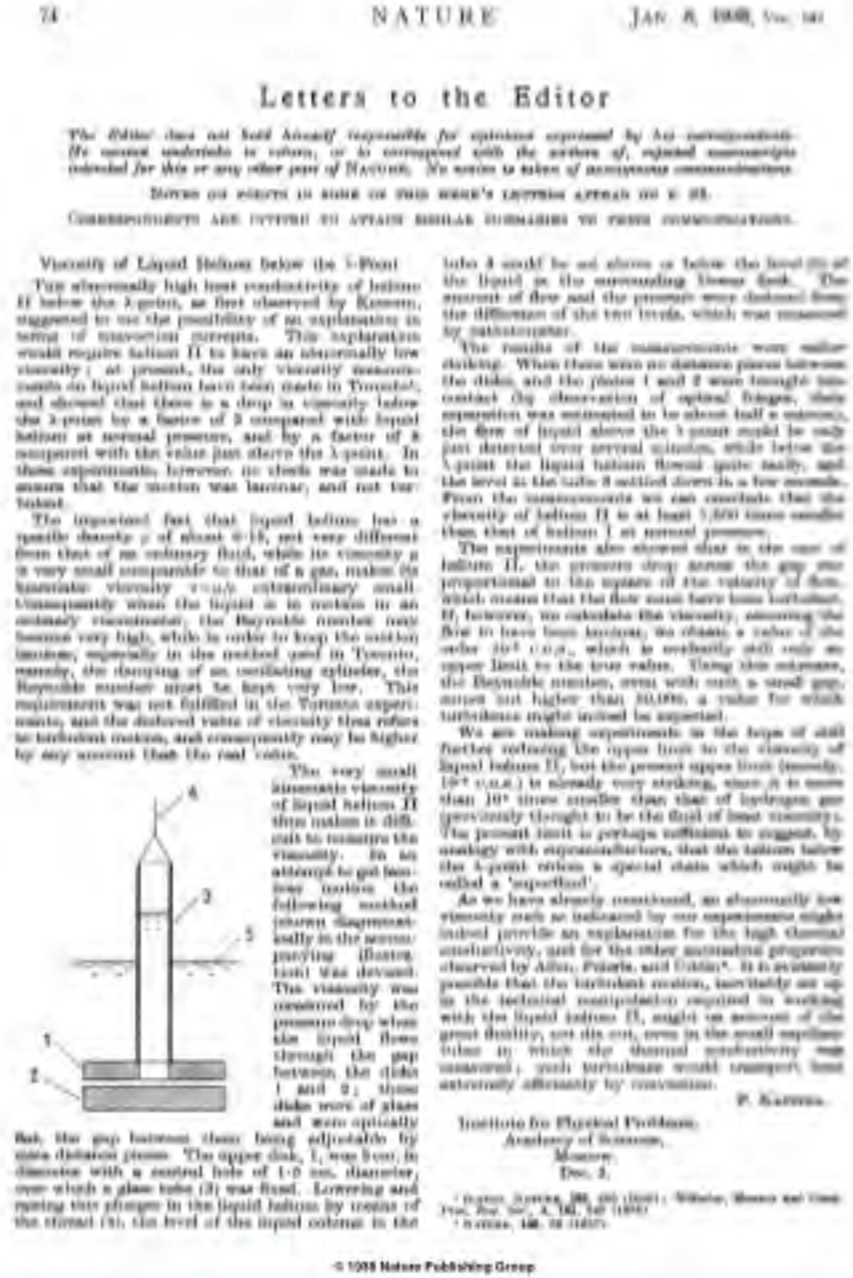
~

Table of Contents

1. Mass Superfluidity
2. Spin Superfluid in Easy-plane Magnets
3. Spin Superfluid in a Domain Wall in Easy-axis Magnets
4. Phase Slips as Skyrmion Generations
5. Superfluid-induced Emergent Magnetic Field on Magnons



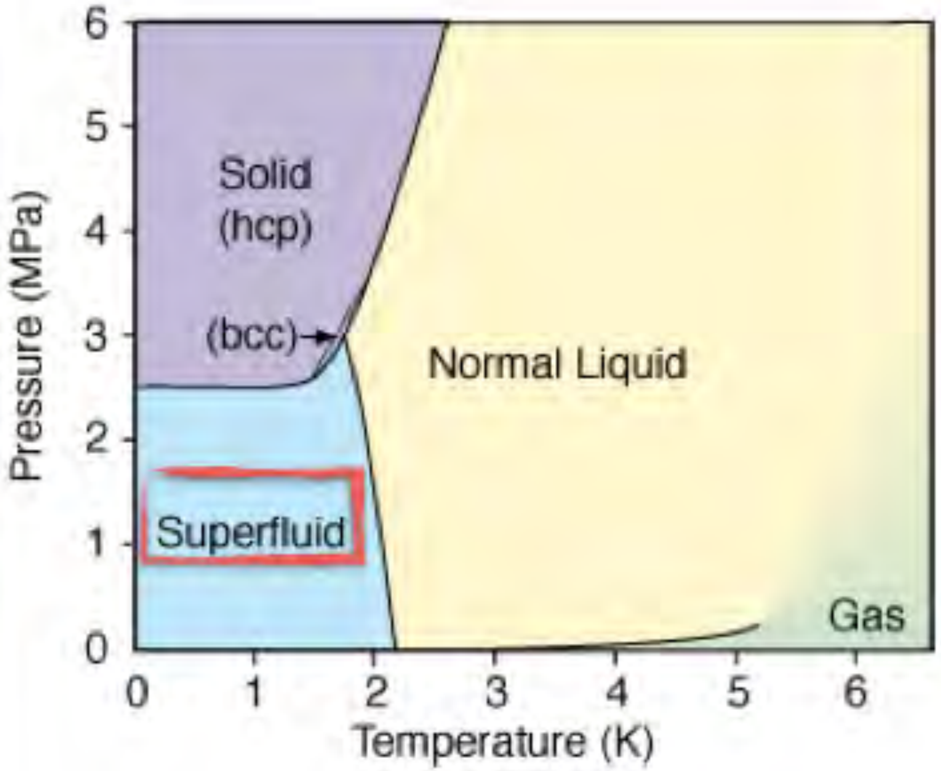
helium-4 in the superfluid phase has
zero viscosity (resistance to flow in a liquid)



* the phase diagram figure is taken from http://ltl.tkk.fi/research/theory/helium.html

P. Kapitza, *Nature* (1938)

Mass Superfluidity



helium-4

$$\psi = \sqrt{n} e^{i\phi}$$

order parameter: macroscopic wave function

* the phase diagram figure is taken from http://ltl.tkk.fi/research/theory/helium.html

Sonin, *AP* (2010)

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\boldsymbol{\nabla}\phi)^2}{2} \right]$$

n : nonequilibrium number density

ϕ : phase

density-phase conjugate relation: $[n(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\delta(\mathbf{r}-\mathbf{r}')$

U(1) symmetry of the Hamiltonian: $\phi(\mathbf{r},t) \mapsto \phi(\mathbf{r},t) + \Delta\phi$

→ number conservation

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\boldsymbol{\nabla}\phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\delta(\mathbf{r}-\mathbf{r}')$

$$\dot{\phi} = \frac{i}{\hbar}[H,\phi] = -\frac{n}{\hbar C}$$: Josephson relation

$$\dot{n} = \frac{i}{\hbar}[H,\rho] = \frac{A\nabla^2\phi}{\hbar}$$: continuity equation $(\dot{n} + \boldsymbol{\nabla}\cdot\mathbf{J} = 0)$

$\rightarrow$ $$\mathbf{J} = -\frac{A\boldsymbol{\nabla}\phi}{\hbar}$$: supercurrent

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\boldsymbol{\nabla}\phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\delta(\mathbf{r}-\mathbf{r}')$

$$\ddot{\phi} + c^2\nabla^2\phi = 0\,, \quad c = \frac{1}{\hbar}\sqrt{\frac{A}{C}}$$: sound-wave speed

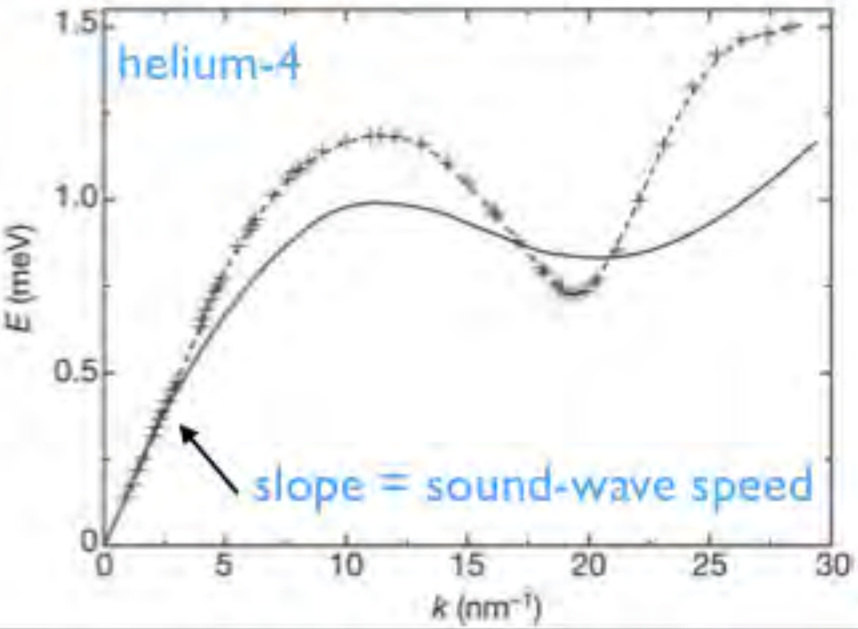
$\hookrightarrow \omega = \pm ck$: linear dispersion relation

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla \phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

$\omega = \pm ck$: linear dispersion relation



Landau criterion for stable supercurrent:

$$v < c$$

Godfrin et al., *Nature* (2012)

Sonin, *AP* (2010)

Essential Ingredients for Superfluidity

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla \phi)^2}{2} \right]$$

n : nonequilibrium number density

ϕ : phase

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

U(1) symmetry of the Hamiltonian: $\phi(\mathbf{r}, t) \mapsto \phi(\mathbf{r}, t) + \Delta\phi$

particle number is conserved.
excitations have the linear dispersion.
superfluid is supported.

Spin Superfluid in Easy-plane Magnets

$$H = \int dV \left[K n_z^2 \quad A(\nabla \mathbf{n})^2 \right]$$

$$H = \int dV \left[\frac{\quad}{2} + \frac{\quad}{2} \right]$$

easy-plane anisotropy: $K > 0$

The ground states are continuously degenerate with U(1) topology.

$$H \approx \int dV \left[\frac{K n_z^2}{2} + \frac{A(\nabla \phi)^2}{2} \right]$$

Sonin, *Sov. Phys. JETP* (1978)
König, Bønsager, and MacDonald, *PRL* (2001)
Chen and Sigrist, *PRB* (2014)
Takei and Tserkovnyak, *PRL* (2014)
Holmqvist, *et al.*, *PRL* (2015)

n_z : nonequilibrium spin density (polarized in the z direction)

ϕ : in-plane angle

Spin Superfluid in Easy-plane Magnets

$$\int \left[K n_z^2 \quad A(\nabla \phi)^2 \right]$$

$$H \approx \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla\phi)^2}{2} \right]$$

spin (angular momentum) is the generator of spin rotations

density-phase conjugate relation: $[sn_z(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')$

s : spin (scalar) density

U(1) symmetry of the Hamiltonian: $\phi(\mathbf{r},t) \mapsto \phi(\mathbf{r},t) + \Delta\phi$

→ spin conservation

Spin Superfluid in Easy-plane Magnets

$$H \approx \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla\phi)^2}{2} \right]$$

$$H \approx \int dV \left[\frac{\quad}{2} + \frac{\quad}{2} \right]$$

density-phase conjugate relation: $[sn_z(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')$

$$\dot{\phi} = \frac{i}{\hbar}[H,\phi] = -\frac{Kn_z}{s}$$: Josephson relation

$$\dot{n}_z = \frac{i}{\hbar}[H,n_z] = \frac{A\nabla^2\phi}{s}$$: continuity equation $(s\dot{n}_z + \boldsymbol{\nabla}\cdot\mathbf{J}_s = 0)$

↳ $\mathbf{J}_s = -A\boldsymbol{\nabla}\phi$: spin supercurrent

Spin Superfluid in Easy-plane Magnets

$$H \approx \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\boldsymbol{\nabla}\phi)^2}{2} \right]$$

$$\int \left[\ \ 2 \qquad 2 \ \ \right]$$

density-phase conjugate relation: $[sn_z(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')$

$$\ddot{\phi} + c_s^2\nabla^2\phi = 0\,, \quad c = \frac{AK}{s}$$: spin-wave speed

↳ $\omega = \pm c_s k$: linear dispersion relation

Landau criterion for stable supercurrent: $v < c_s$

Comparison

mass superfluid	spin superfluid in easy-plane magnets
$\int \left[n^2 \quad A(\boldsymbol{\nabla}\phi)^2 \right]$	$\int \left[Kn^2 \quad A(\boldsymbol{\nabla}\phi)^2 \right]$

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2}\right]$$

$$[n(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\delta(\mathbf{r}-\mathbf{r}')$$

$$\ddot{\phi} + c^2\nabla^2\phi = 0\,, \quad c = \frac{1}{\hbar}\sqrt{\frac{A}{C}}$$

sound-wave speed

$$H \approx \int dV \left[\frac{K n_z^2}{2} + \frac{A(\nabla\phi)^2}{2}\right]$$

$$[s n_z(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')$$

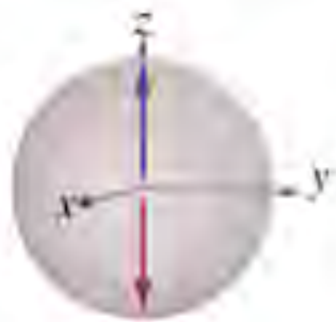
$$\ddot{\phi} + c_s^2\nabla^2\phi = 0\,, \quad c = \frac{AK}{s}$$

spin-wave speed

Easy-axis Magnets?

$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\boldsymbol{\nabla}\mathbf{n})^2}{2}\right]$$

easy-axis anisotropy: $K < 0$

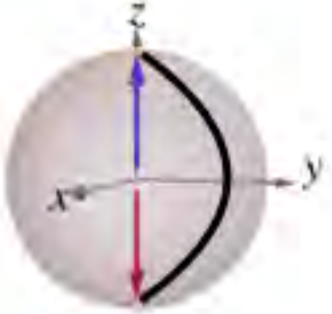


Ground states are discrete.
Excitations are gapped.
U(1) symmetry is not spontaneous broken.
No spin superfluidity.

Easy-axis Magnets?

$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\nabla \mathbf{n})^2}{2}\right]$$

easy-axis anisotropy: $K < 0$



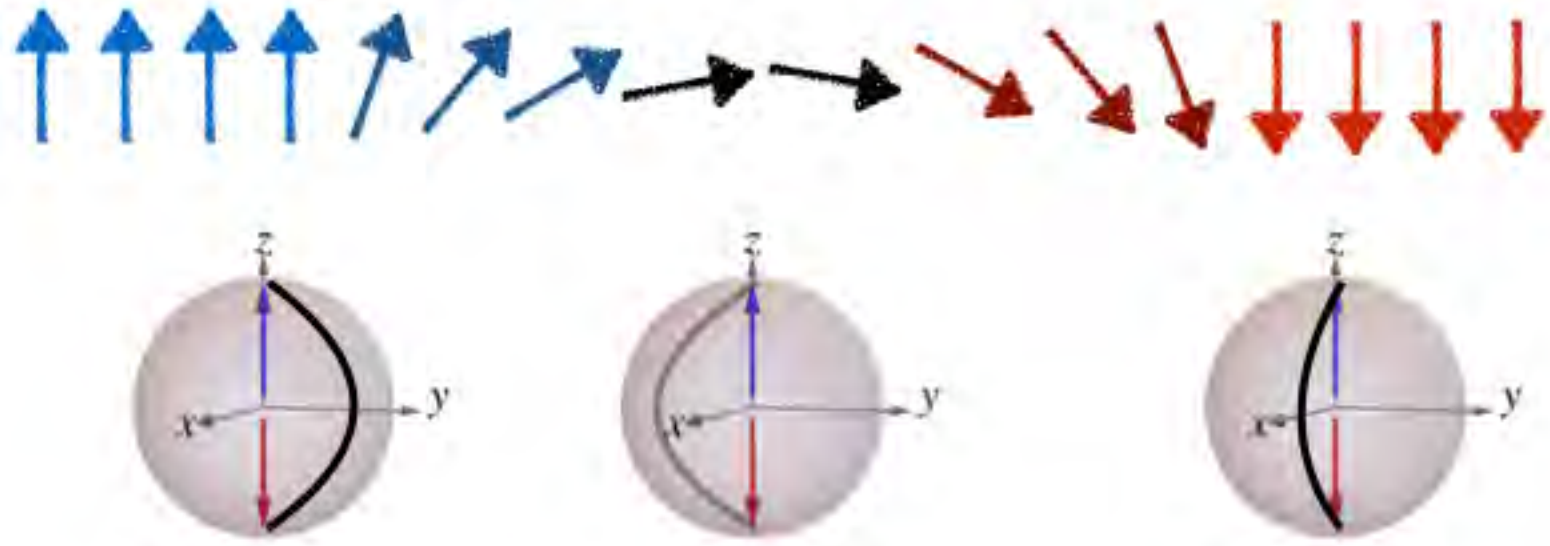
No spin superfluidity,
but we have a domain wall.

Easy-axis Magnets?

$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\boldsymbol{\nabla}\mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$

easy-axis anisotropy: $K < 0$

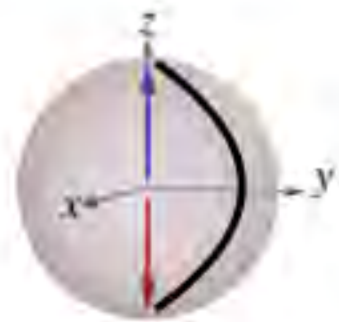


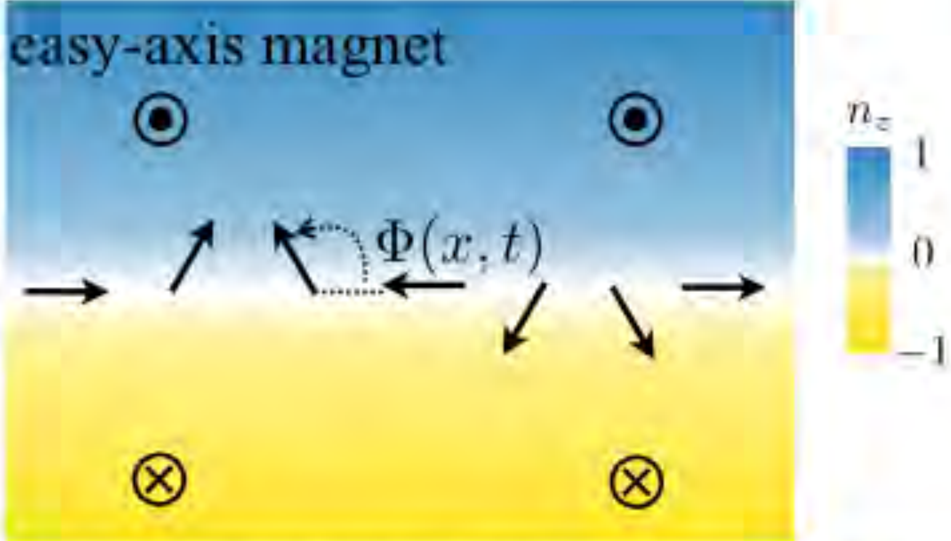
At the domain-wall center, spins lie in the xy plane with an **arbitrary** in-plane angle.

Domain Wall

$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\nabla \mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$

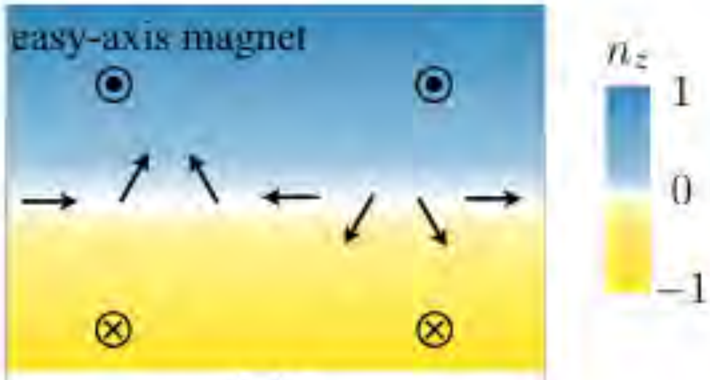




Domain walls are continuously degenerate with U(1) topology.
Excitations are gapless.
U(1) symmetry is spontaneous broken.
Possible existence of spin superfluidity.

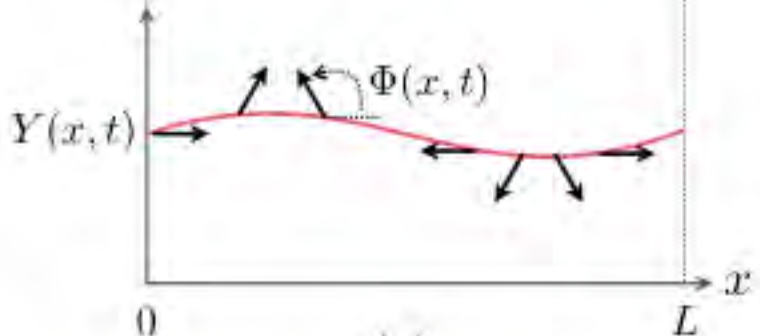
SKK and Tserkovnyak, *PRL* 119, 047202 *(2017)*

Spin Superfluid inside a Domain Wall



(a)

$$H = \int dx (\kappa Y^2 + \eta \Phi'^2)/2$$



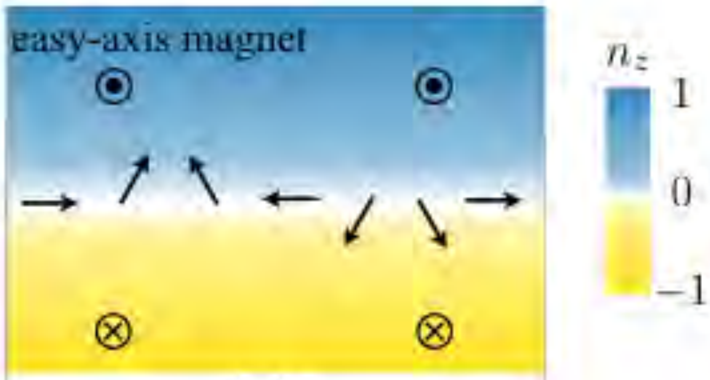
(c)

$$\int dx\, Y(x,t) \sim S_z(t)$$

density-phase conjugate relation: $[sn_z(\mathbf{r},t), \phi(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')$

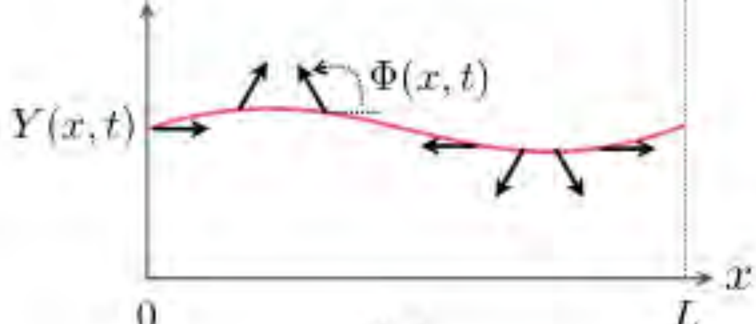
position-phase conjugate relation: $[Y(x,t), \Phi(x',t)] = \frac{i\hbar}{2s}\delta(x-x')$

Spin Superfluid inside a Domain Wall



(a)

$$H = \int dx(\kappa Y^2 + \eta\Phi'^2)/2$$



(c)

$$[Y(x,t), \Phi(x',t)] = \frac{i\hbar}{2s}\delta(x-x')$$

$$2s\dot{\Phi} = -\kappa Y \ ,$$:Josephson relation

$$-2s\dot{Y} = \eta\Phi'' \ .$$: continuity equation

$$I_s = -\eta\Phi'$$: spin supercurrent

Comparison

mass superfluid

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\boldsymbol{\nabla}\phi)^2}{2}\right]$$

spin superfluid in a domain wall

$$H = \int dx(\kappa Y^2 + \eta\Phi'^2)/2$$

$$[n(\mathbf{r},t),\phi(\mathbf{r}',t)] = i\delta(\mathbf{r}-\mathbf{r}')$$

$$\ddot{\phi} + c^2\nabla^2\phi = 0\,, \quad c = \frac{1}{\hbar}\sqrt{\frac{A}{C}}$$

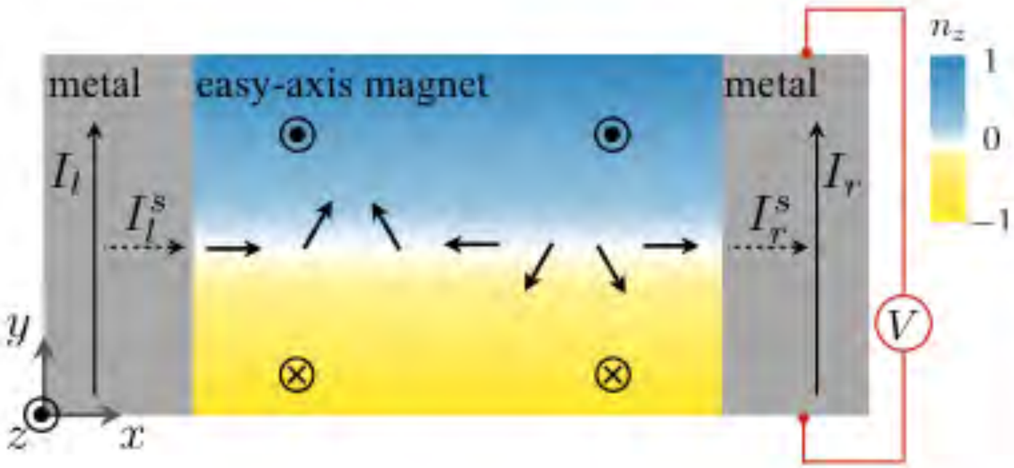
sound-wave speed

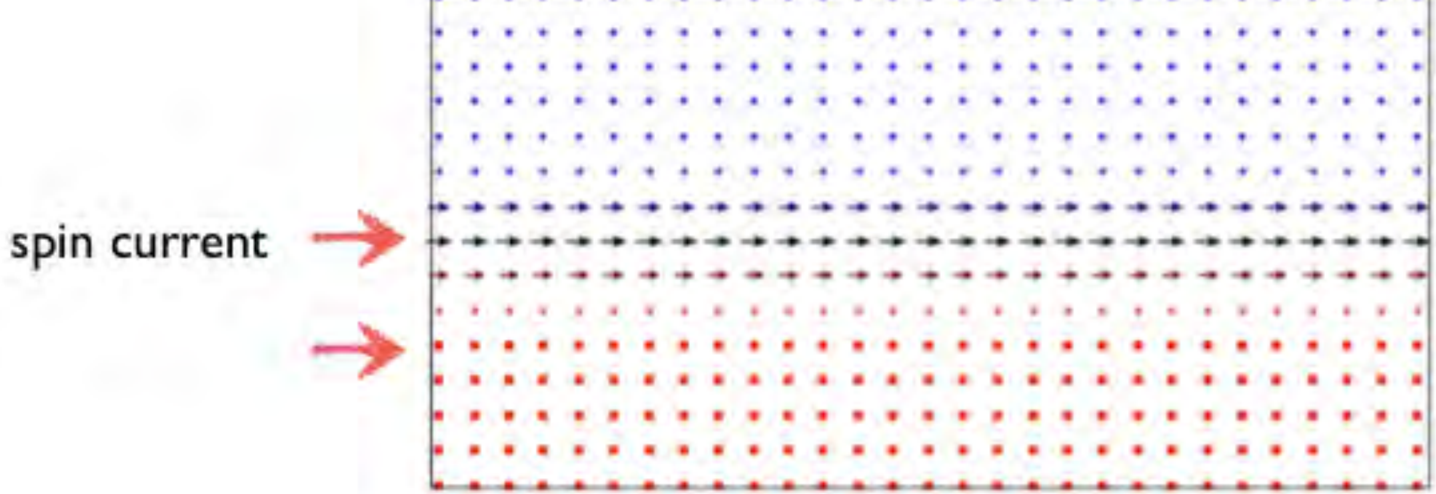
$$[Y(x,t),\Phi(x',t)] = \frac{i\hbar}{2s}\delta(x-x')$$

$$\ddot{\Phi} + c_s^2\nabla^2\Phi = 0\,, \quad c_s = \frac{\sqrt{\kappa\eta}}{2s}$$

spin-wave speed

Micromagnetic Simulations

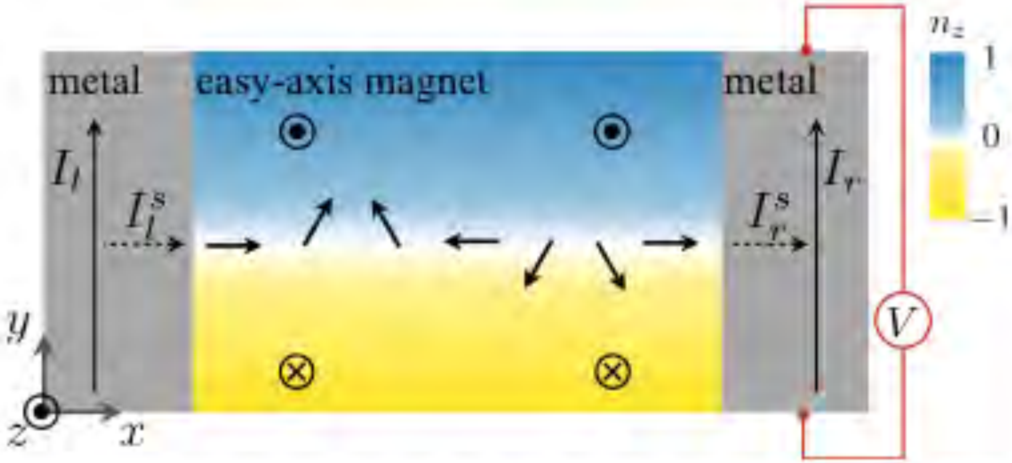


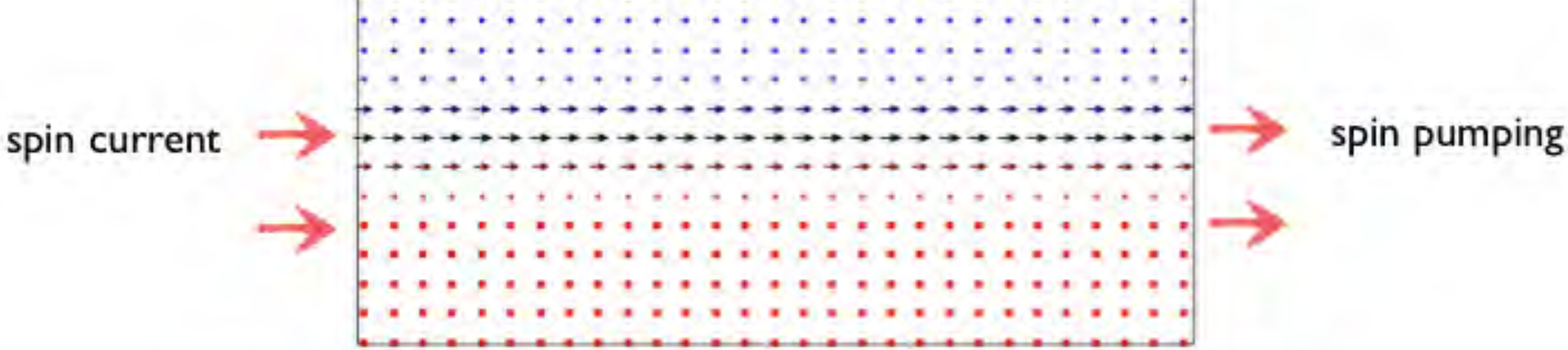


spin supercurrent: $I_s = -\eta\Phi'$

domain-wall precession: $\dot{\Phi}(x,t) \equiv \Omega$

Experimental Probe



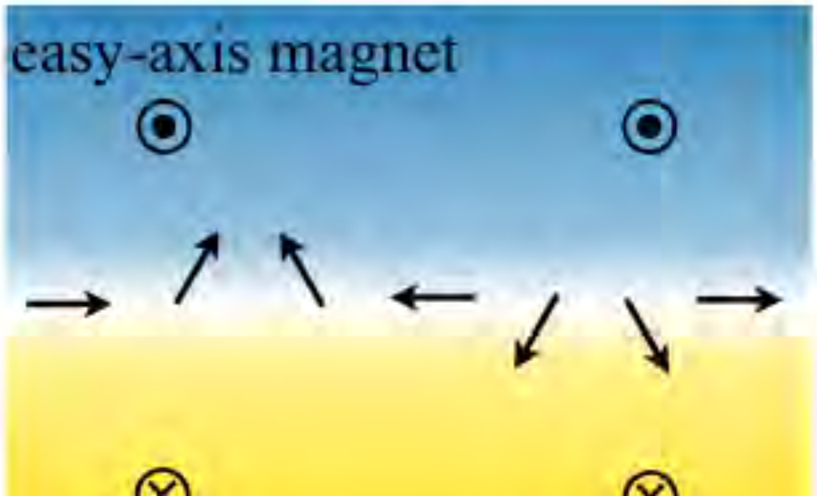


Superfluid spin transport can be probed experimentally
by measuring the inverse spin Hall voltage in the adjacent metal.

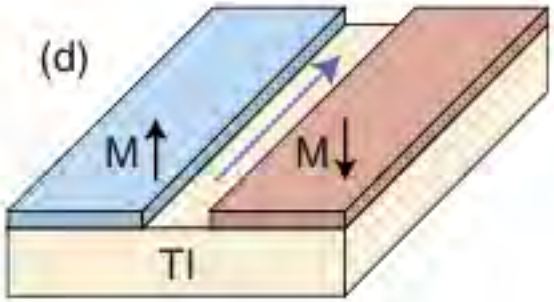
$$J_l = 10^{10}\ \mathrm{A/m^2} \to |\Delta V| = \frac{5\mu\mathrm{V}}{1 + L/1\mu\mathrm{m}}$$

SKK and Tserkovnyak, *PRL* 119, 047202 *(2017)*

Analogy to an Edge Mode between Topological Insulators



(d)



By interpreting the magnon **chirality** as a **topological invariant**, the spin superfluid along the domain wall can be interpreted as a gapless edge mode between two topological distinct insulators.

Hasan and Kane, RMP (2010)

Domain Wall as a Spin Superfluid Track

Areal view of Mainz, penetrated by the Rhine river.
The Rhine is among the most important arteries of industrial transport in the world.

* the figure is taken from https://en.wikipedia.org/wiki/Mainz https://www.britannica.com/place/Rhine-River

Domain Wall as a Spin Superfluid Track

A domain wall can serve as a **versatile** track for spin superfluids, because it can be moved easily by several means such as heat flux or microwave.

* the figure is taken from https://en.wikipedia.org/wiki/Mainz

Phase Slips in Mass and Charge Superfluidity

the order parameter: $\psi = \sqrt{n} e^{i\phi}$

metastable states

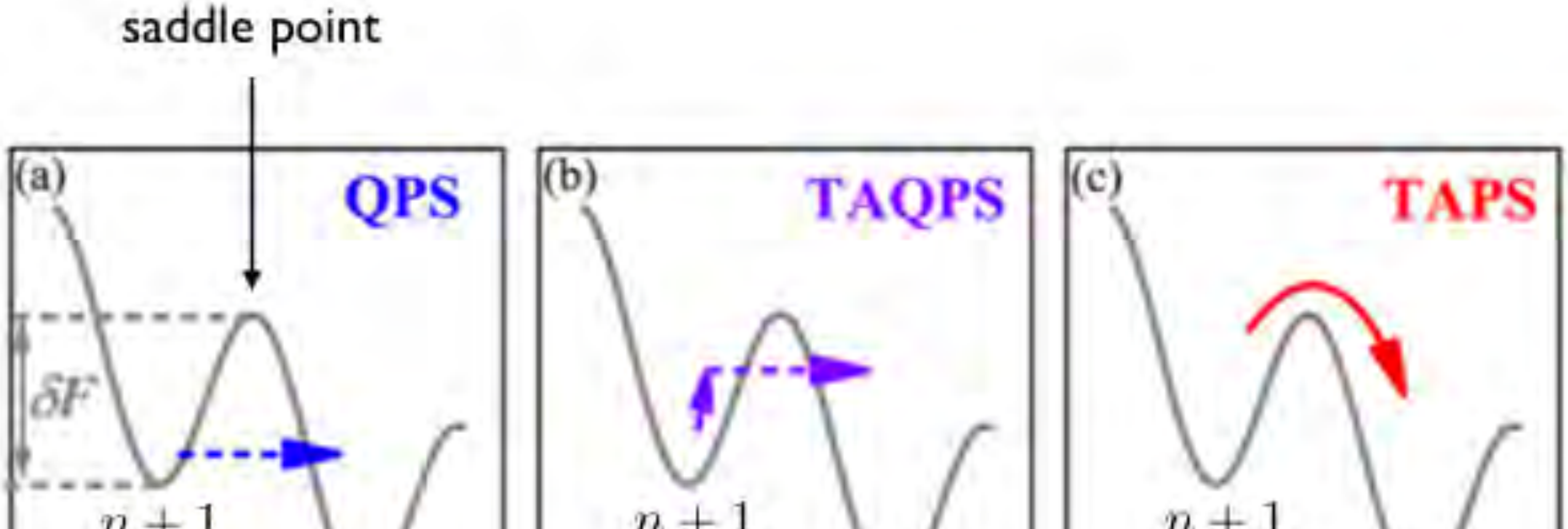
phase winding number: n

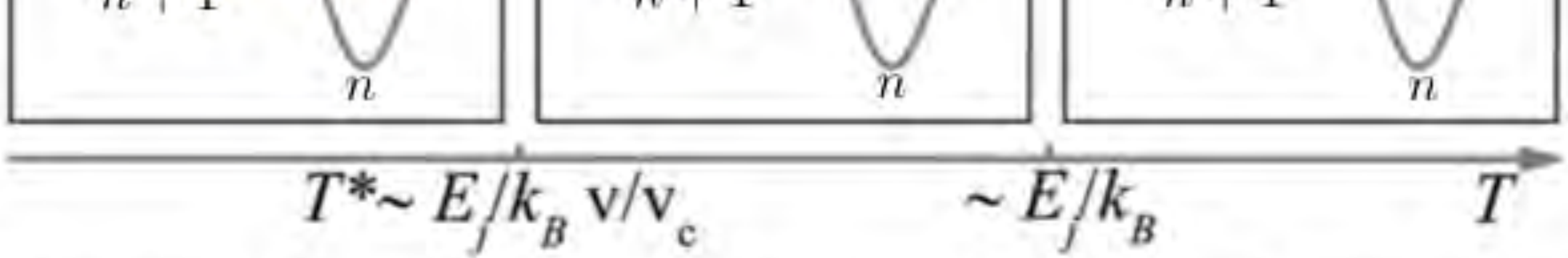
supercurrent: $\propto n$

energy: $\propto n^2$

Phase Slips in Mass and Charge Superfluidity

phase slips = change of the winding number **n**

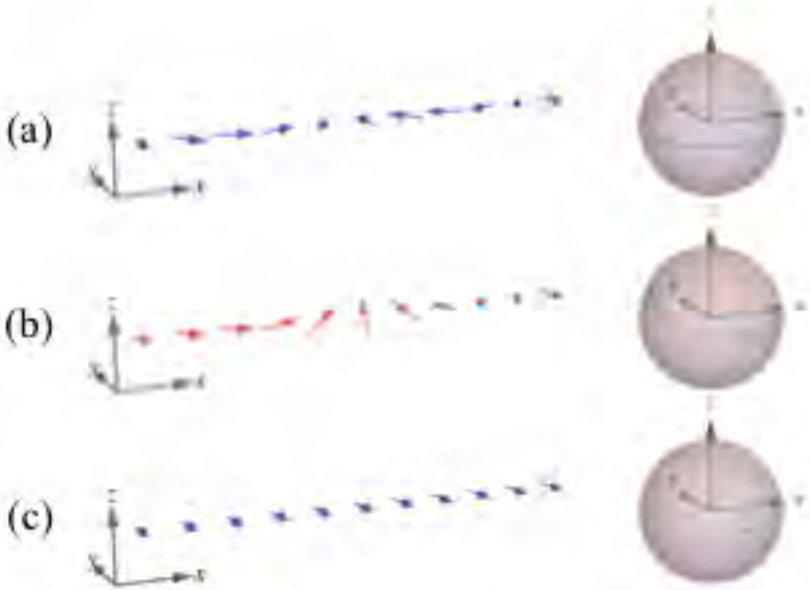


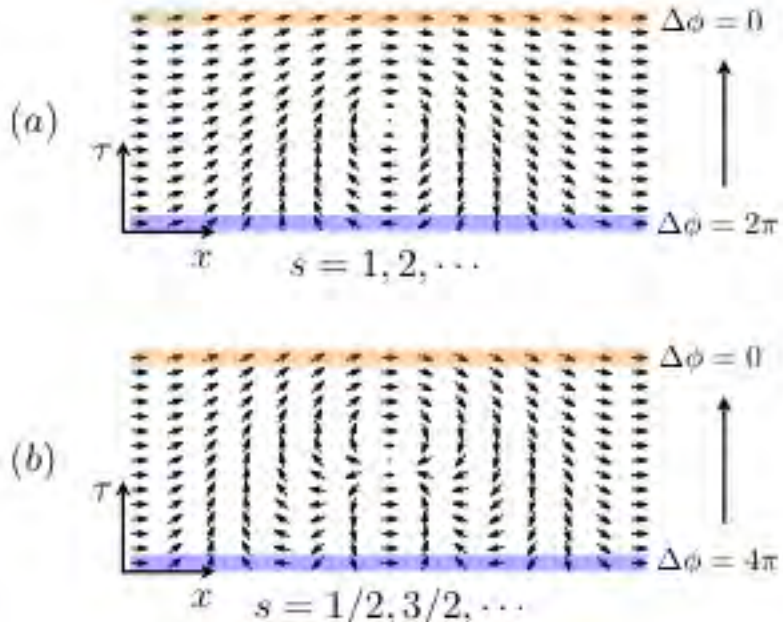


Phase-slips are activated (a) via quantum tunneling events (QPS) for $T \lesssim T^*$, (b) via quantum tunnelling events assisted by the temperature (TAQPS) for $T^* \lesssim T \lesssim \delta F/k_B$ and (c) via thermal fluctuations (TAPS) for $T \gg \delta F/k_B$. In a lattice $T^* \approx E_J/k_B \times v/v_c$ and $\delta F \approx E_J$, see text.

Ianzi et al. Sci. Rep. (2016)

Phase Slips of Spin Superfluid in Easy-plane Magnets





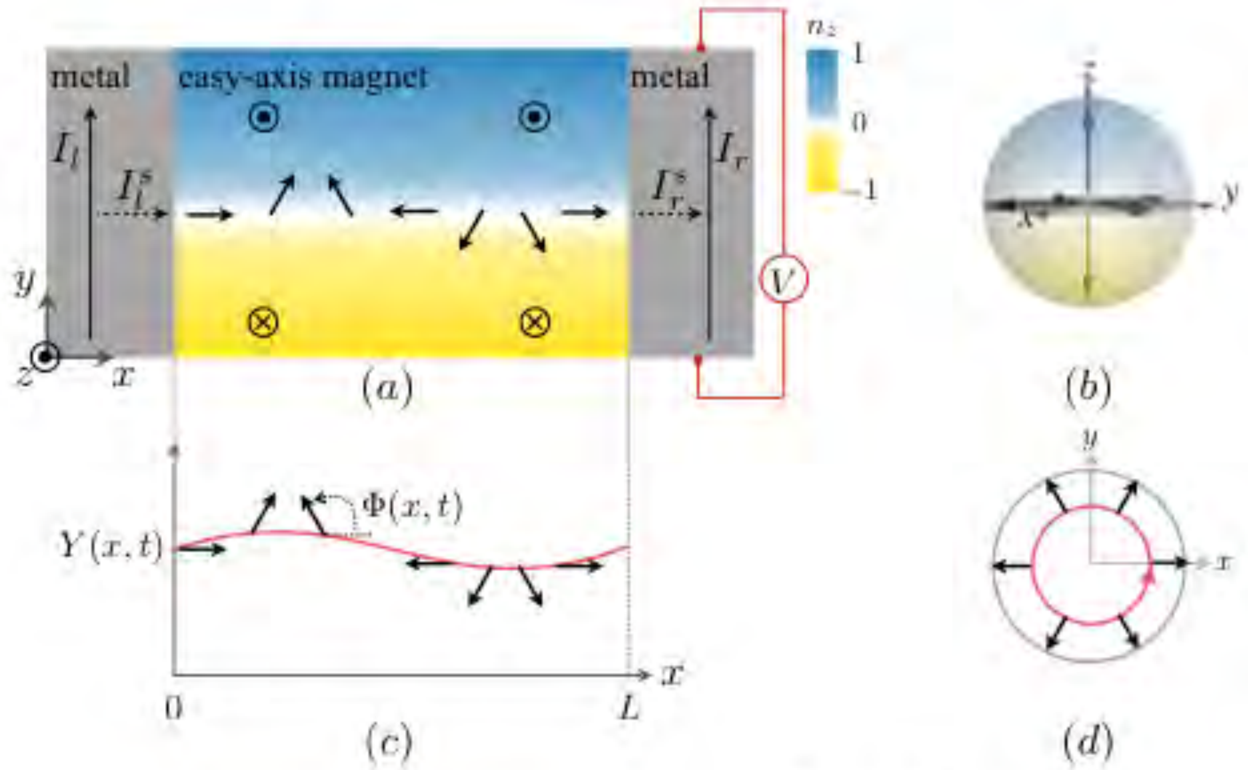
thermally-activated phase slips

SKK, Takei and Tserkovnyak, *PRB* (2016)

quantum phase slips

SKK and Tserkovnyak, *PRL* (2016)

U(1) Winding Number and Skyrmion Charge

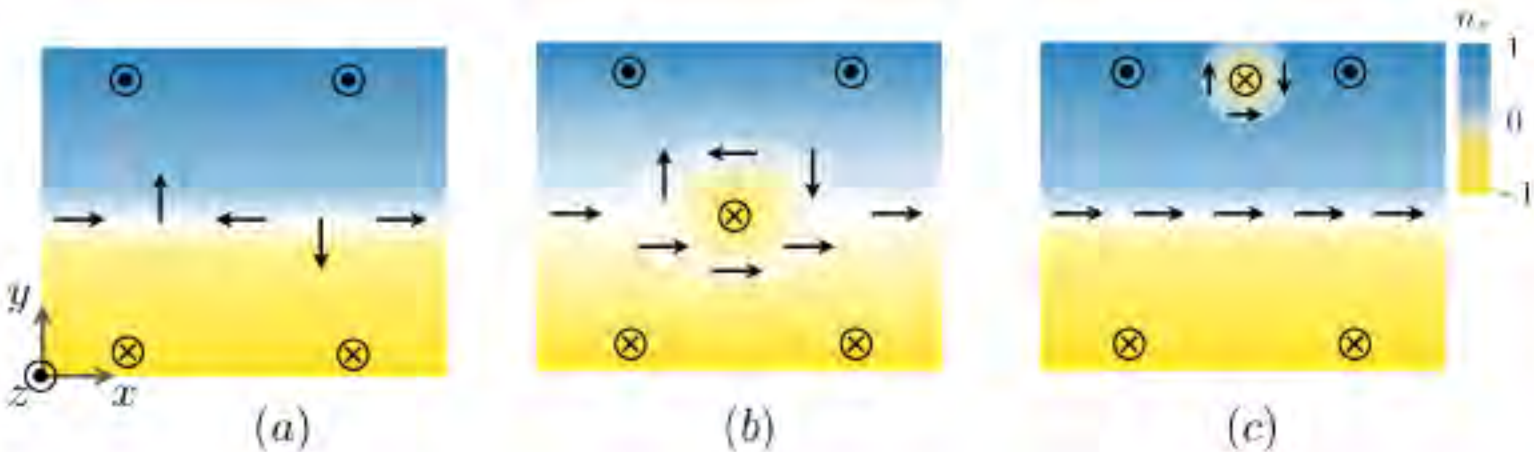


U(1) winding number: $$w = \frac{1}{2\pi} \int dx\, \partial_x \Phi$$

Skyrmion charge: $$Q = \frac{1}{4\pi} \int dxdy\, \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$$

$\omega = Q$ in metastable states (with periodic boundary conditions)

Phase Slips for Spin Superfluid inside a Domain Wall

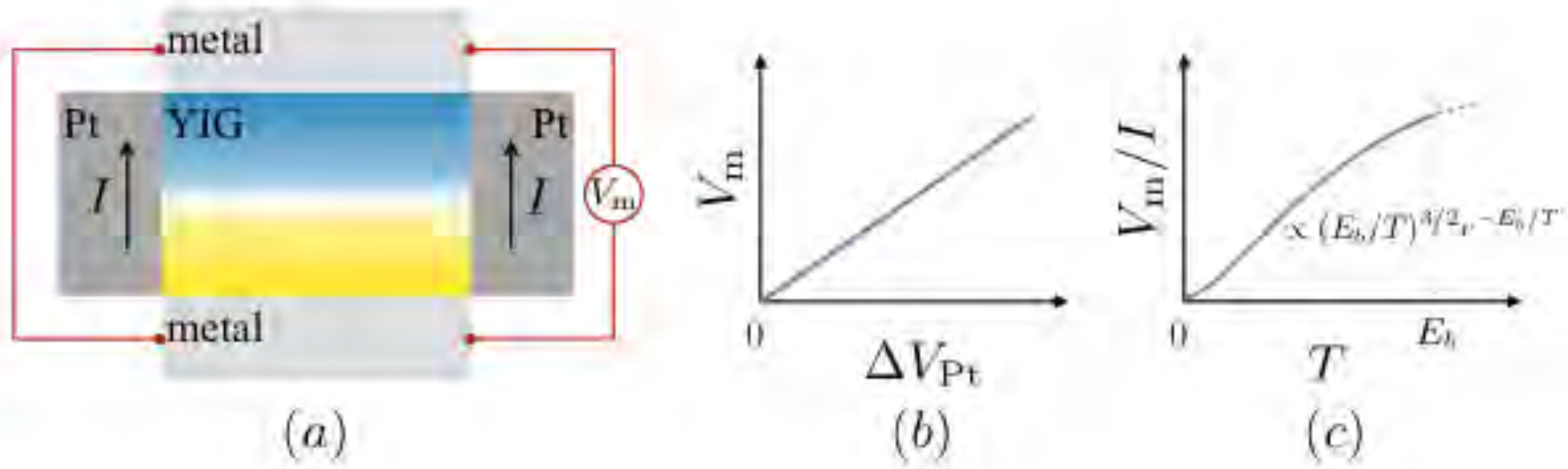


$\omega = 1$ $\omega = 0$

$Q = 1$ (localized in the domain wall) $Q = 1$ (isolated skyrmion)

Phase slips, which change the U(1) winding number, generate Skyrmions.

Phase Slips for Spin Superfluid inside a Domain Wall

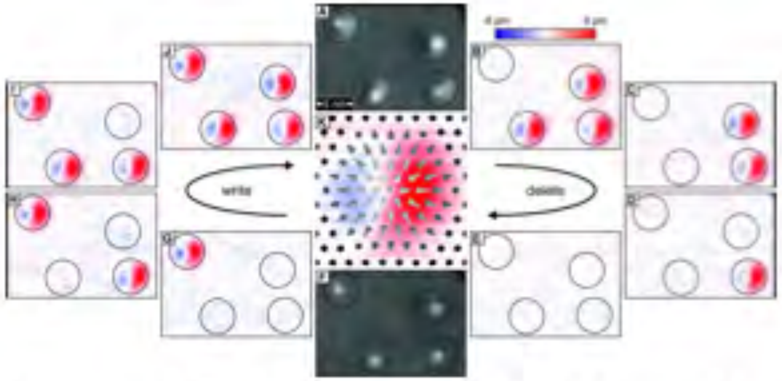


Skyrmion creation by phase slips can be probed experimentally by measuring the skyrmion-induced electromotive voltage (V_m) in the adjacent metal.

$$J_l = J_r = 10^{10}\ \mathrm{A/m^2} \rightarrow |V_m| \sim 7\ \mathrm{nV\ at}\ T = 300\mathrm{K}$$

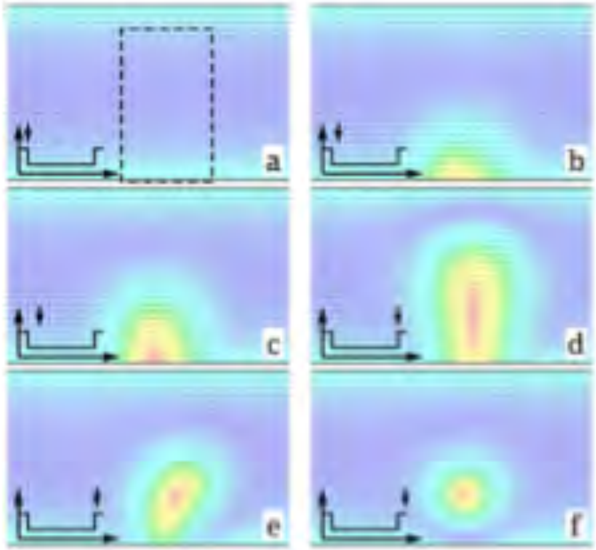
Ochoa, SKK, and Tserkovnyak *PRB* (2016)
SKK and Tserkovnyak *PRL* 119, 047202 (2017)

Wanted: Efficient Creation of Skyrmions



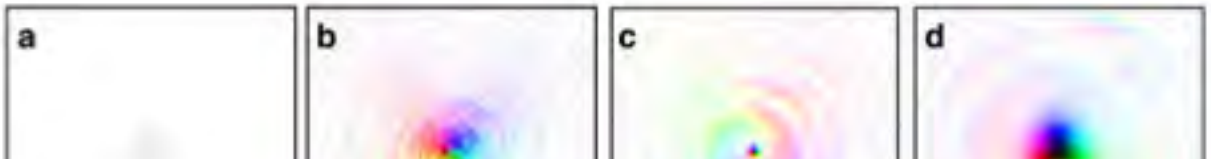
Writing and Deleting Single Magnetic Skyrmions

Niklas Romming, Christian Hanneken, Matthias Menzel, Jessica E. Bickel*, Boris Wolter, Kirsten von Bergmann†, André Kubetzka†, Roland Wiesendanger *Science* (2013)



Edge instabilities and skyrmion creation in magnetic layers

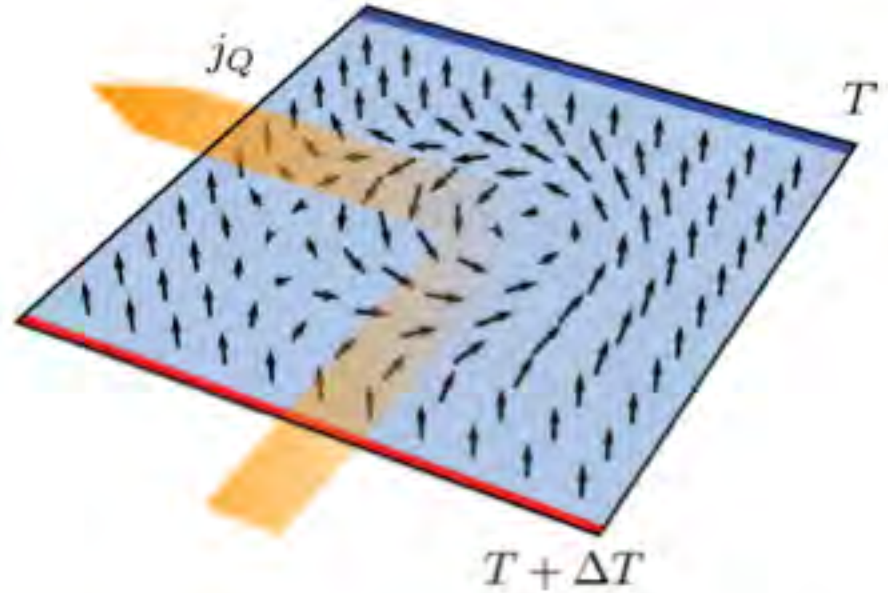
Jan Müller, Achim Rosch and Markus Garst *New J. Phys.* (2016)



Creation of skyrmions and antiskyrmions by local heating

Wataru Koshibae[1] & Naoto Nagaosa[1,2] Nat. Commun. (2014)

Skyrmion-induced Emergent Magnetic Field on Magnons



effective Lorentz force on magnons: $\mathbf{F} = \mathbf{v} \times b\hat{\mathbf{z}}\,,$

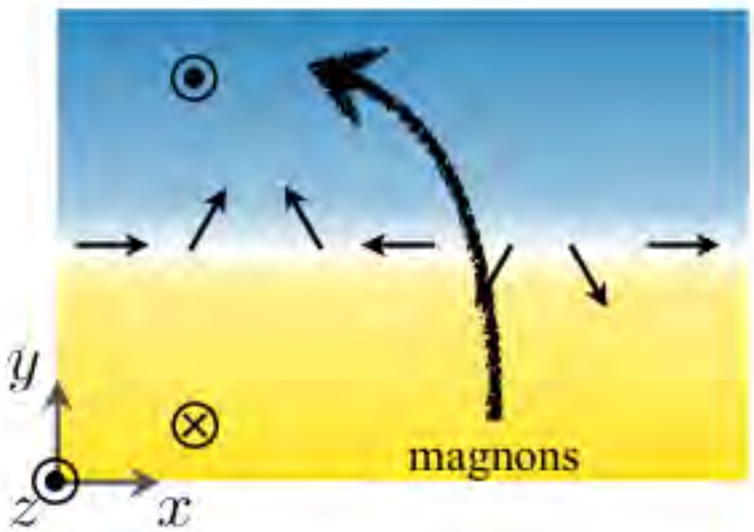
emergent magnetic field on magnons: $b = -\hbar\mathbf{n}\cdot\partial\mathbf{n}\times\partial\mathbf{n}$

emergent magnetic field on magnons: $b = -\hbar \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

cf. Skyrmion charge: $Q = \frac{1}{4\pi} \int dxdy\, \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

K. A. van Hoogdalem, Y. Tserkovnyak, and D. Loss, *PRB* (2013)

Superfluid-induced Emergent Magnetic Field on Magnons

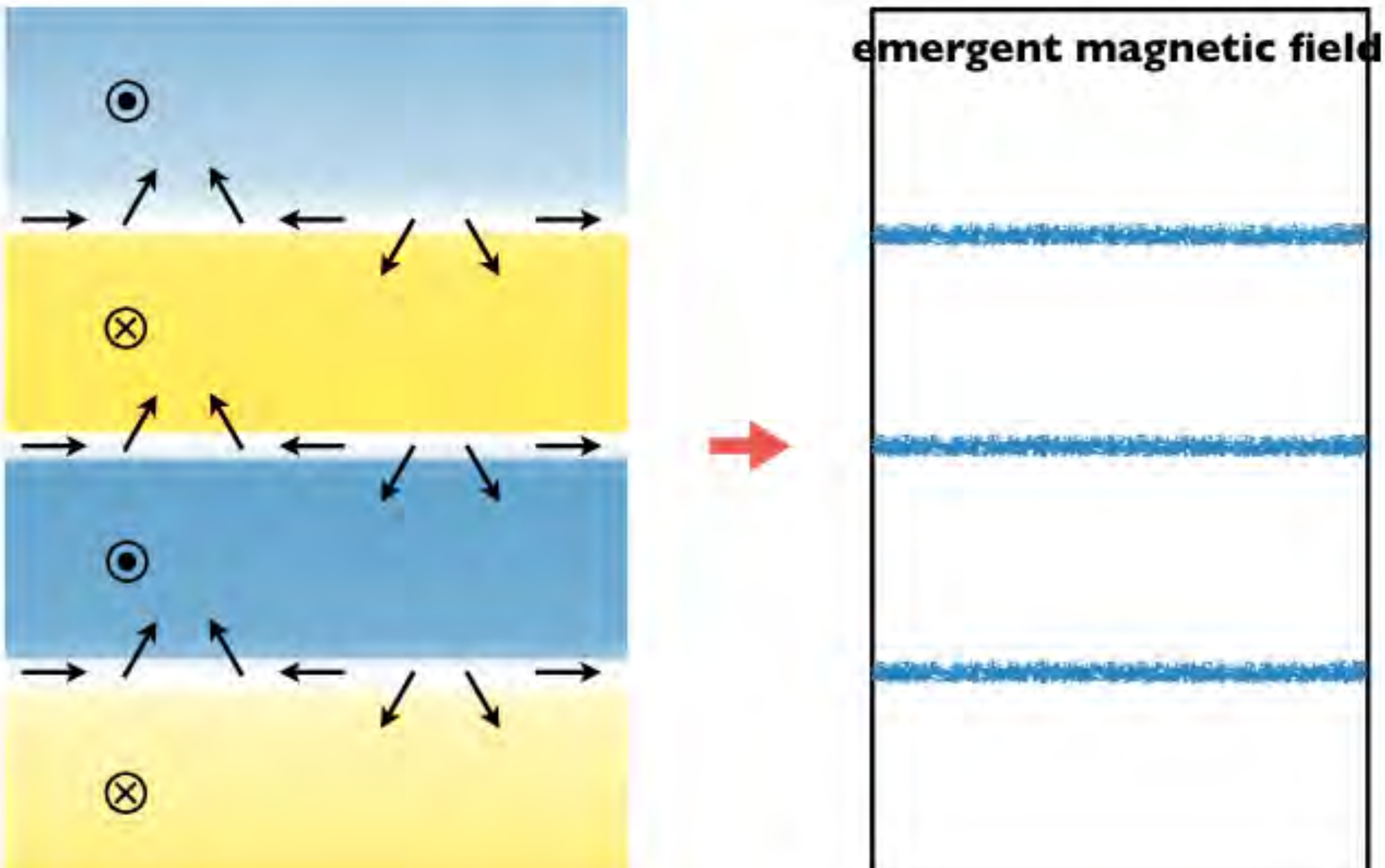


effective Lorentz force on magnons: $\mathbf{F} = \mathbf{v} \times b\hat{\mathbf{z}}\,,$

emergent magnetic field on magnons: $b = -\hbar \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

The emergent magnetic field can be **controlled** by varying the spin supercurrent.

Superfluid-induced Magnonic Crystal



An **array of domain walls** carrying the spin supercurrent forms a **magnonic crystal** with a **periodic emergent magnetic field**.

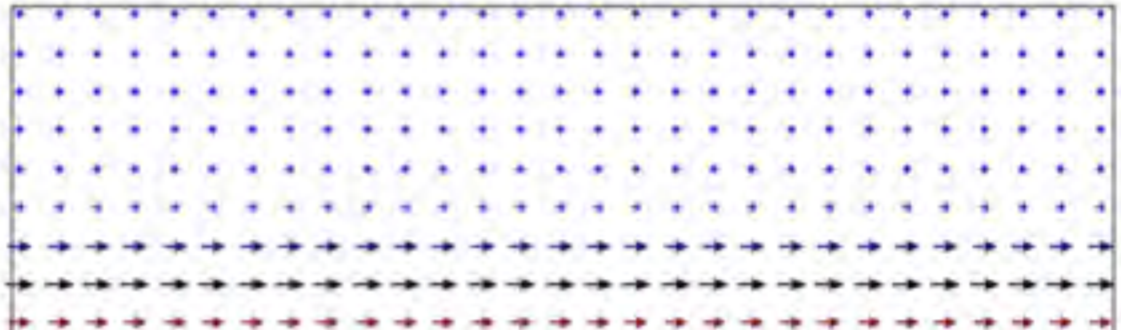
Summary

$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\boldsymbol{\nabla}\mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$

A simple easy-axis magnet is a venue for an topological orchestra of

domain wall + spin superfluid + skyrmion
+ (controllable) emergent electromagnetic field



SKK and Tserkovnyak, *PRL* 119, 047202 *(2017)*