

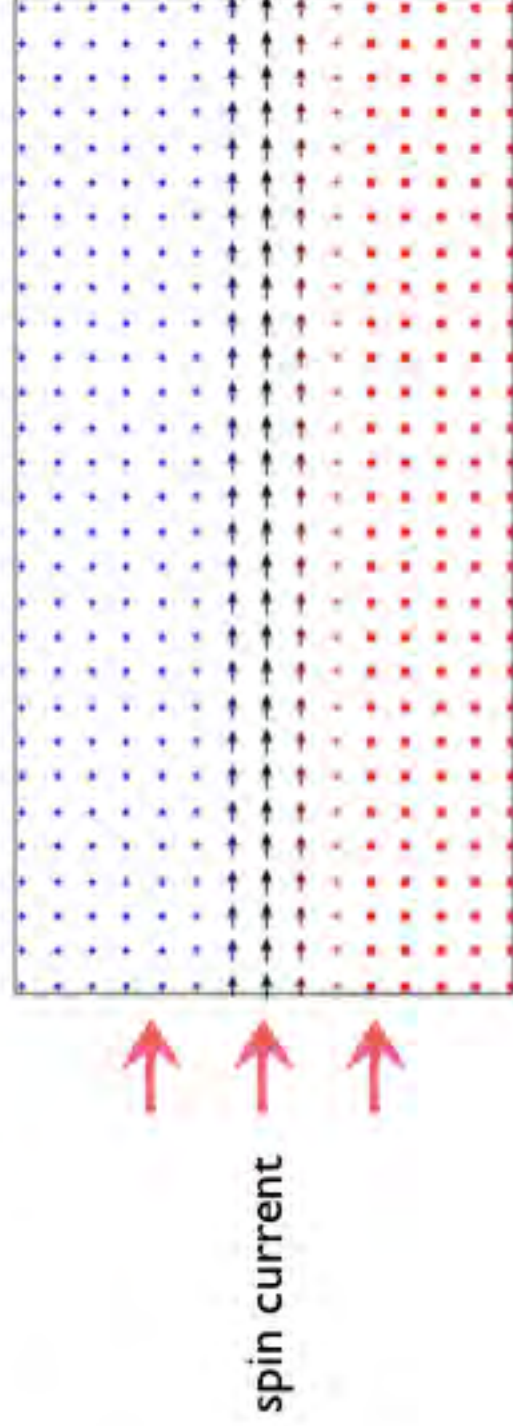


Spin Superfluid in a Magnetic Domain Wall

Se Kwon Kim

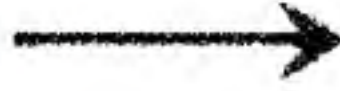
UCLA

*in collaboration with Yaroslav Tserkovnyak (UCLA)
Young Research Leaders Group Workshop, SPICE at Mainz, Germany, 2017*



How to achieve an efficient spin transport?

zero-resistance mass and charge transport
(superconductivity, quantum Hall effects, ...)



efficient spin transport

Novel types of spin transport in charge insulators



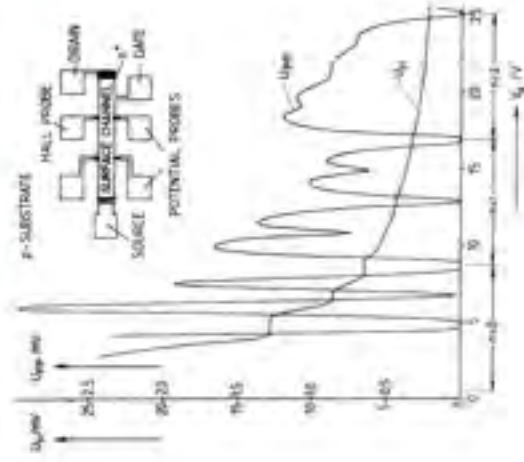
superfluid mass transport

P. Kapitza, Nature (1938)

superfluid spin transport

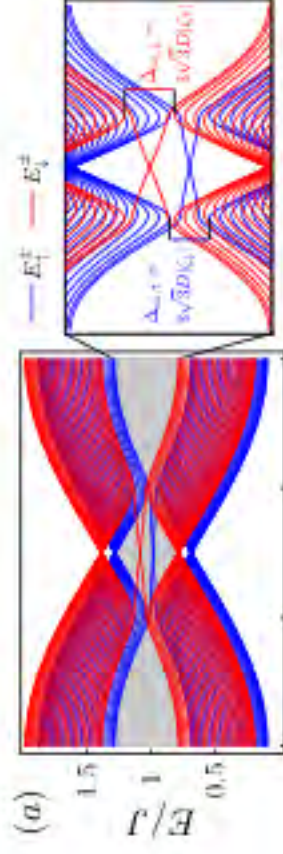
Sonin, Sov. Phys. JETP (1978)
 König, Bomsager, and MacDonald, PRL (2001)
 Chen and Signt, PRB (2014)
 Taker and Tserkovnyak, PRL (2014)
 Holmqvist, et al., PRL (2015)

the figure is taken from <https://en.wikipedia.org/wiki/Superfluidity>.



K.v. Klitzing; G. Dorda; M. Pepper PRL 1980

quantum Hall effects



Matsumoto and Murakami, PRL (2011)
 Zhang et al., PRB (2013)
 Shindou et al., PRB (2013)
 Mook et al., PRB (2014)
 Nakata et al., PRB (2017)

magnonic quantum Hall effects

Novel types of spin transport in charge insulators

NEWER TYPES OF SPIN TRANSPORT IN CHARGE INSULATORS



superfluid mass transport

P. Kapitza, Nature (1938)

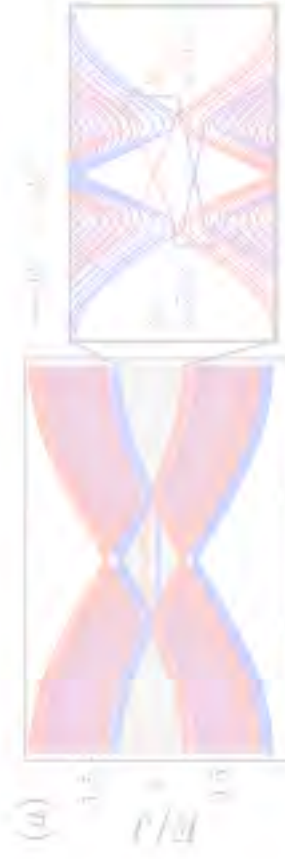
* the figure is taken from <https://en.wikipedia.org/wiki/Superfluidity>.

superfluid spin transport

- Sonin, Sov. Phys. JETP (1978)
- König, Bonsager, and MacDonald, PRL (2001)
- Chen and Sigrist, PRB (2014)
- Takei and Tserkovnyak, PRL (2014)
- Holmqvist, et al., PRL (2015)



Quantum Hall effects



magnon's quantum Hall effects

1. Mass Superfluidity

2. Spin Superfluid in Easy-plane Magnets

3. Spin Superfluid in a Domain Wall in Easy-axis Magnets

4. Phase Slips as Skyrmion Generations

5. Superfluid-induced Emergent Magnetic Field on Magnons



Mass Superfluidity

Letters to the Editor

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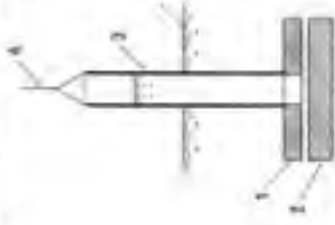
Correspondents are invited to address similar instructions to their communications.

Viscosity of Liquid Helium below the λ -Point

The abnormally high first conductivity of helium at below the λ -point, as first observed by Keesom, suggested to us the possibility of an explanation in terms of superconductor currents. This explanation would require helium II to have an abnormally low viscosity η , at present, the only viscosity anomaly we know of below the λ -point (see note in *Nature*, 1937, and above) that there is a drop in viscosity below the λ -point by a factor of 2 compared with liquid helium at normal pressures, and by a factor of 8 compared with the value just above the λ -point. In these experiments, however, no fluids were used to ensure that the motion was laminar, and not turbulent.

The question is, then, that liquid helium has a specific density ρ of about 0.125, and very different from that of an ordinary fluid, while its viscosity η is very small compared to that of a gas, making its kinematic viscosity $\nu = \eta/\rho$ extraordinarily small. Consequently when the liquid is in motion in an ordinary environment, the Reynolds number (see below) is very high, which is why the motion is laminar, especially in the medical use of the liquid, namely, the dropping of an anesthetic mixture, the hypodermic needle used to keep very low. This experiment was not fulfilled in the Toronto apparatus, and the reduced value of viscosity then refers to turbulent motion, and consequently may be higher by any amount than the real value.

The very small kinematic viscosity of liquid helium II at this point is difficult to measure. It is an attempt to get this low viscosity, the following method (see next paragraph) is used in the apparatus. The viscosity was measured by the method of Couette, where the liquid flows between the shaft and a glass tube. The shaft and glass tube were of plane ends were optically flat (within 100 Å), and were being adjustable by micrometer screws. The upper disk, 1.5 cm in diameter with a central hole of 1.0 cm, was stationary while a glass tube (B) was fixed. Lowering and raising the plunger in this liquid helium by means of the thread (A), the level of the liquid column in the



tube, the gap between them being adjustable by micrometer screws. The upper disk, 1.5 cm in diameter with a central hole of 1.0 cm, was stationary while a glass tube (B) was fixed. Lowering and raising the plunger in this liquid helium by means of the thread (A), the level of the liquid column in the

tube B could be set above or below the level of the liquid in the surrounding helium tank. The amount of flow and the pressure were obtained from the difference of the two levels, which was measured by a manometer.

The results of the measurements were rather striking. When there was no distance between the disks, and the plates 1 and 2 were brought into contact, the observation of optical changes, the experiment was suspended to be about half a minute, the flow of liquid above the λ -point could be only just detected over several minutes, while below the λ -point the liquid column flowed quite easily, and that level in the tube B settled down to a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1/100 times smaller than that of helium I at normal pressures.

The experiment also shows that in the case of helium II, the pressure drop across the gap is proportional to the square of the velocity of flow, which means that the flow rate here has laminar character. However, we calculate the viscosity, assuming the flow to have been laminar, to obtain a value of the order 10^{-4} dyne/cm², which is probably still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small pressure but higher than 1000 dyne/cm², a value for which turbulence might well be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present apparatus (namely, 1.5 cm gap) is already very striking, since it is now almost 10⁶ times smaller than that of hydrogen gas (presumably thought to be the fluid of least viscosity). The present limit is perhaps sufficient to compare by analogy with superconductors, that the helium here has a great critical special state which might be called a "superfluid".

P. Keesom.

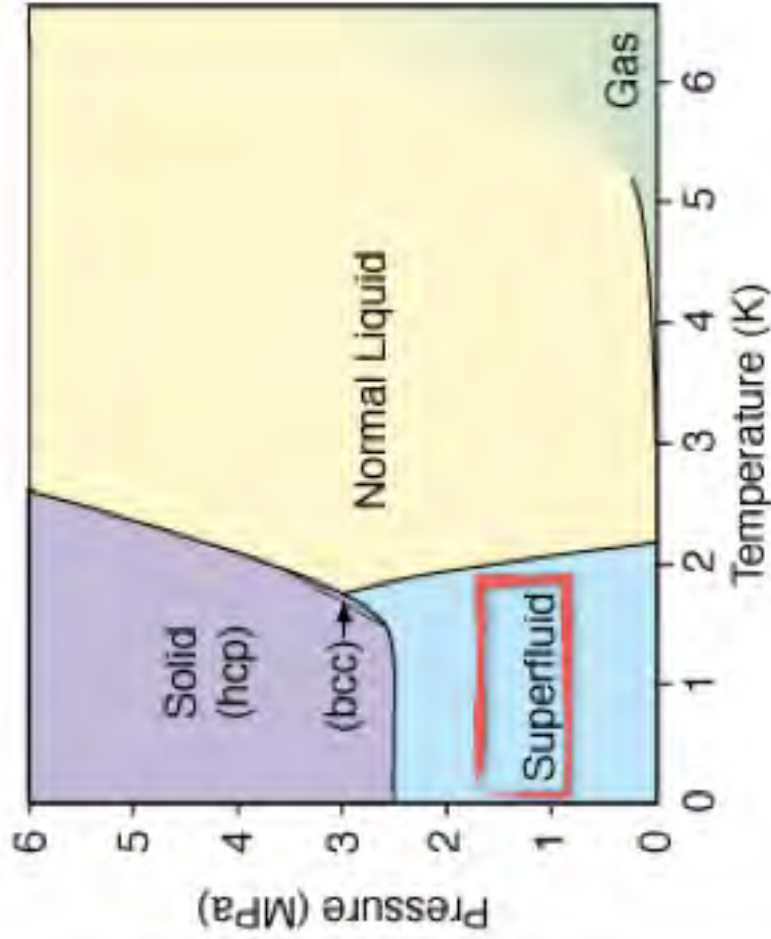
Institute for Physical Problems,

Academy of Sciences,

Moscow,

U.S.S.R.

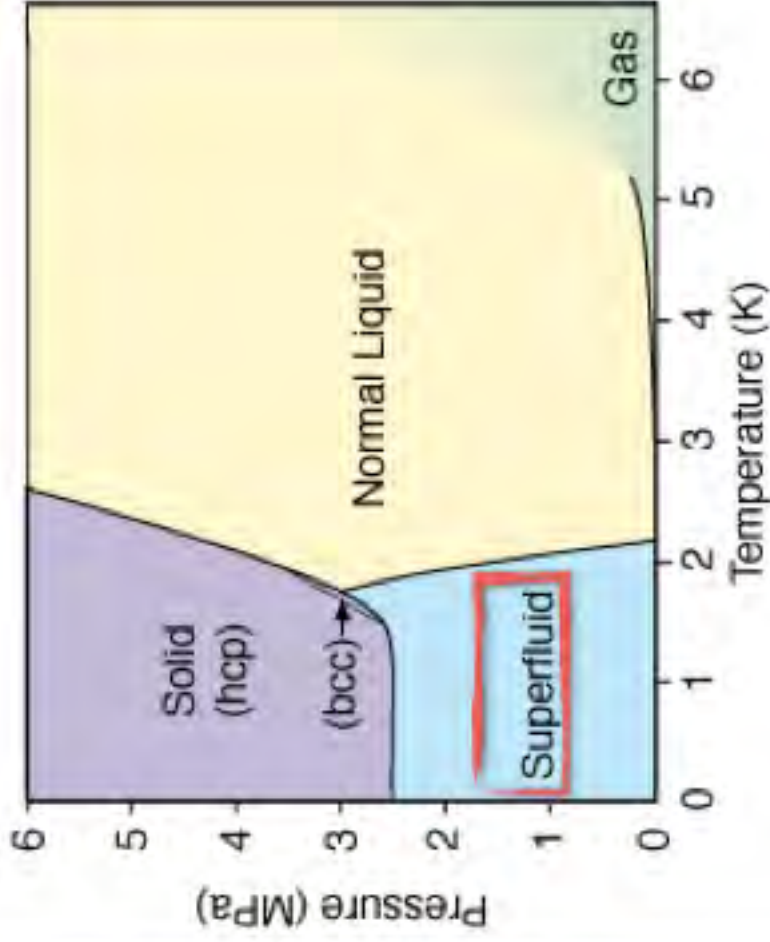
(Received January 20, 1938; in final form February 1, 1938; in revised form February 1, 1938.)



helium-4 in the superfluid phase has zero viscosity (resistance to flow in a liquid)

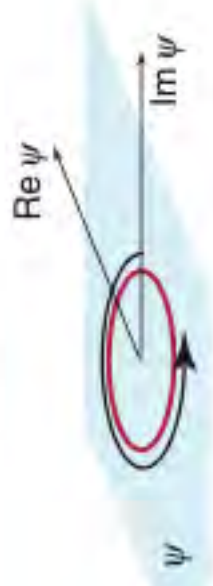
* the phase diagram figure is taken from <http://t1.ttk.fkf/research/theory/helium.html>

Mass Superfluidity



helium-4

$$\psi = \sqrt{n} e^{i\phi}$$



order parameter: macroscopic wave function

* the phase diagram figure is taken from <http://t1.tkk.fi/research/theory/helium.html>

Sonin, AP (2010)

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

n : nonequilibrium number density

ϕ : phase

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

$U(1)$ symmetry of the Hamiltonian: $\phi(\mathbf{r}, t) \mapsto \phi(\mathbf{r}, t) + \Delta\phi$

└ number conservation

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

$$\dot{\phi} = \frac{i}{\hbar} [H, \phi] = -\frac{n}{\hbar C}$$

: Josephson relation

$$\dot{n} = \frac{i}{\hbar} [H, n] = \frac{A\nabla^2\phi}{\hbar}$$

: continuity equation $(\dot{n} + \nabla \cdot \mathbf{J} = 0)$

$$\hookrightarrow \mathbf{J} = -\frac{A\nabla\phi}{\hbar}$$

: supercurrent

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

$$\ddot{\phi} + c^2 \nabla^2 \phi = 0, \quad c = \frac{1}{\hbar} \sqrt{\frac{A}{C}}$$

: sound-wave speed

$$\hookrightarrow \omega = \pm ck$$

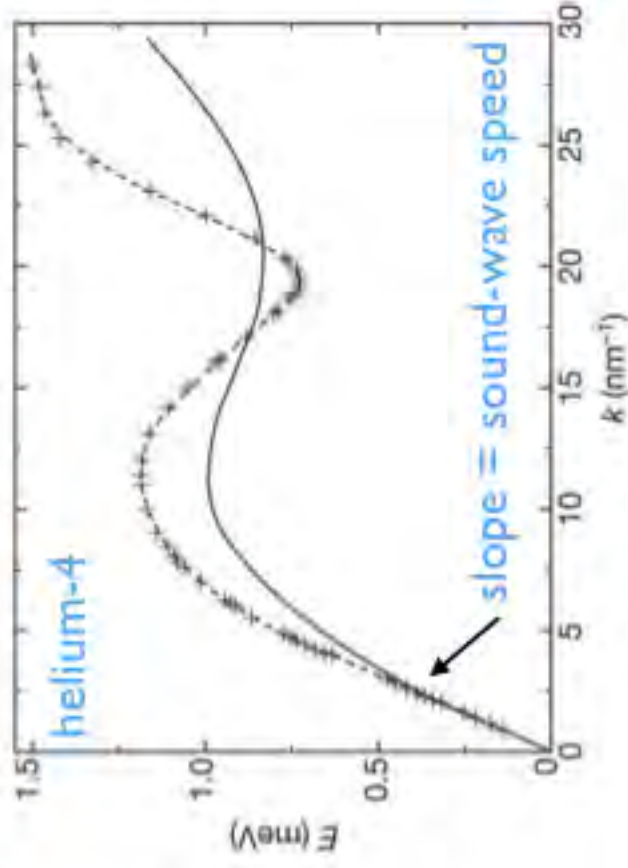
: linear dispersion relation

Superfluid Primer

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

density-phase conjugate relation: $[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$

$\omega = \pm ck$: linear dispersion relation



Landau criterion for stable supercurrent:

$$v < c$$

Godfrin et al., Nature (2012)

Sonin, AP (2010)

Essential Ingredients for Superfluidity

$$c = \sqrt{\frac{A}{m}} \quad A(\nabla\phi)^2$$

$$H = \int dV \left[\frac{\hbar^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

n : nonequilibrium number density

ϕ : phase

$$[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$$

density-phase conjugate relation:

$$\phi(\mathbf{r}, t) \mapsto \phi(\mathbf{r}, t) + \Delta\phi$$

U(1) symmetry of the Hamiltonian:



particle number is conserved.
excitations have the linear dispersion.
superfluid is supported.

Spin Superfluid in Easy-plane Magnets

$$H = \int dV [Kn_z^2 + A(\nabla\mathbf{n})^2]$$

$$H = \int dV \left[\frac{v^2}{2} + \frac{K}{2} \right]$$

easy-plane anisotropy: $K > 0$

The ground states are continuously degenerate with $U(1)$ topology.

$$H \approx \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla\phi)^2}{2} \right]$$

n_z : nonequilibrium spin density (polarized in the z direction)

ϕ : in-plane angle

Spin Superfluid in Easy-plane Magnets

$$f = \left[Kn_z^2 + A(\nabla\phi)^2 \right]$$



Somin, Sov. Phys. JETP (1978)

König, Bönsager, and MacDonald, PRL (2001)

Chen and Sigrist, PRB (2014)

Takel and Tserkovnyak, PRL (2014)

Holmqvist, et al., PRL (2015)

$$H \approx \int dV \left[\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 \right]$$

spin (angular momentum) is the generator of spin rotations



density-phase conjugate relation: $[S n_z(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\hbar \delta(\mathbf{r} - \mathbf{r}')$

S : spin (scalar) density

$U(1)$ symmetry of the Hamiltonian: $\phi(\mathbf{r}, t) \mapsto \phi(\mathbf{r}, t) + \Delta\phi$



spin conservation

Spin Superfluid in Easy-plane Magnets

$$H \approx \int dV \left[K n_z^2 + A(\nabla \phi)^2 \right]$$

$$\mu \approx \int dV \left[\frac{1}{2} \rho v^2 + \frac{1}{2} \rho \phi^2 \right]$$

density-phase conjugate relation: $[s n_z(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i \hbar \delta(\mathbf{r} - \mathbf{r}')$

$$\dot{\phi} = \frac{i}{\hbar} [H, \phi] = -\frac{K n_z}{s}$$

∴ Josephson relation

$$\dot{n}_z = \frac{i}{\hbar} [H, n_z] = \frac{A \nabla^2 \phi}{s}$$

∴ continuity equation ($s \dot{n}_z + \nabla \cdot \mathbf{J}_s = 0$)

$$\hookrightarrow \mathbf{J}_s = -A \nabla \phi$$

∴ spin supercurrent

Spin Superfluid in Easy-plane Magnets

$$H \approx \int dV \left[\frac{K n_z^2}{2} + \frac{A (\nabla \phi)^2}{2} \right]$$

density-phase conjugate relation: $[sn_z(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\hbar\delta(\mathbf{r} - \mathbf{r}')$

$$\ddot{\phi} + c_s^2 \nabla^2 \phi = 0, \quad c = \frac{AK}{s} \quad \text{: spin-wave speed}$$

$$\hookrightarrow \omega = \pm c_s k \quad \text{: linear dispersion relation}$$

Landau criterion for stable supercurrent: $v < c_s$

Comparison

mass superfluid

spin superfluid in easy-plane magnets

$$c \sim [n^2 A(\nabla\phi)^2]$$

$$c \sim [Kn^2 A(\nabla\phi)^2]$$

$$H = \int dV \left[\frac{n_0^2}{2C} + \frac{z\mathbf{n}(\mathbf{v}\cdot\boldsymbol{\phi})}{2} \right]$$

$$[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$$

$$\ddot{\phi} + c^2 \nabla^2 \phi = 0, \quad c = \frac{1}{\hbar} \sqrt{\frac{A}{C}}$$

sound-wave speed

$$H \approx \int dV \left[\frac{\mathbf{n}(\mathbf{v}\cdot\boldsymbol{\phi})}{2} + \frac{z\mathbf{n}(\mathbf{v}\cdot\boldsymbol{\phi})}{2} \right]$$

$$[sn_z(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\hbar\delta(\mathbf{r} - \mathbf{r}')$$

$$\ddot{\phi} + c_s^2 \nabla^2 \phi = 0, \quad c = \frac{AK}{s}$$

spin-wave speed

Easy-axis Magnets?

$$H = \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla\mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$



Ground states are discrete.

Excitations are gapped.

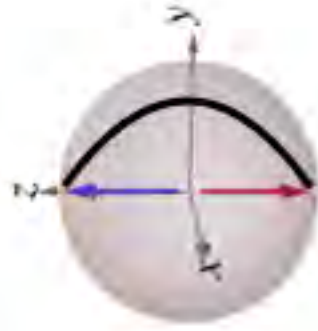
$U(1)$ symmetry is not spontaneously broken.

No spin superfluidity.

Easy-axis Magnets?

$$H = \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla\mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$



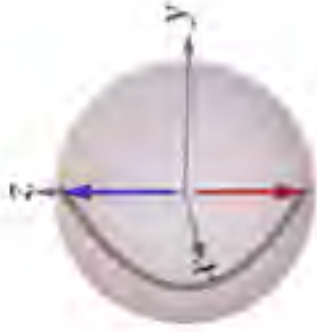
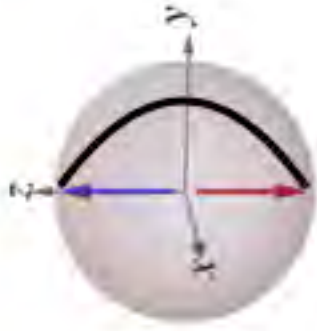
No spin superfluidity,
but we have a domain wall.



Easy-axis Magnets?

$$H = \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla \mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$



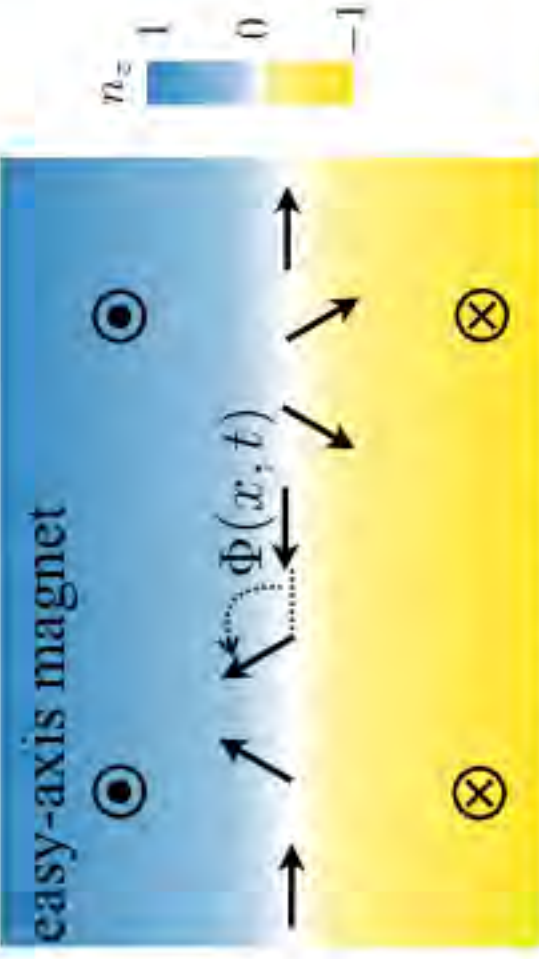
At the domain-wall center, spins lie in the xy plane with an **arbitrary** in-plane angle.

Domain Wall

$$H = \int dV \left[\frac{Kn_z^2}{2} + \frac{A(\nabla \mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$





Domain walls are continuously degenerate with $U(1)$ topology.

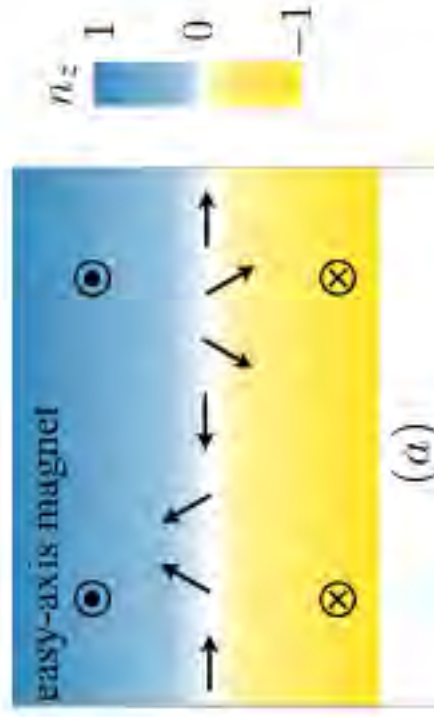
Excitations are gapless.

$U(1)$ symmetry is spontaneous broken.

Possible existence of spin superfluidity.

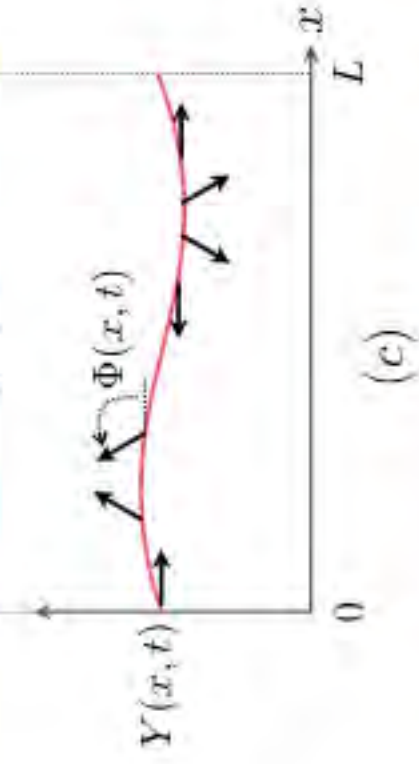
SKK and Tserkovnyak, PRL 119, 047202 (2017)

Spin Superfluid inside a Domain Wall



$$H = \int dx (\kappa Y^2 + \eta \Phi'^2) / 2$$

$$\int dx Y(x, t) \sim S_z(t)$$

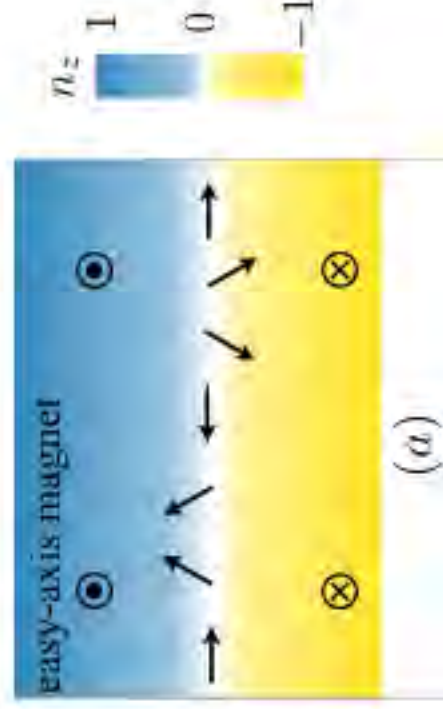


density-phase conjugate relation: $[sn_z(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\hbar\delta(\mathbf{r} - \mathbf{r}')$

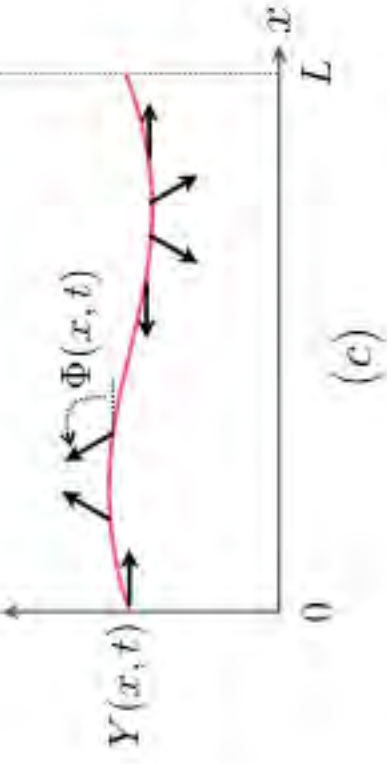


position-phase conjugate relation: $[Y(x, t), \Phi(x', t)] = \frac{i\hbar}{2s}\delta(x - x')$

Spin Superfluid inside a Domain Wall



$$H = \int dx (\kappa Y'^2 + \eta \Phi'^2) / 2$$



$$[Y(x, t), \Phi(x', t)] = \frac{i\hbar}{2s} \delta(x - x')$$

$$2s\dot{\Phi} = -\kappa Y, \quad \text{:Josephson relation}$$

$$-2s\dot{Y} = \eta\Phi'' \quad \text{:continuity equation}$$

$$\hookrightarrow I_s = -\eta\Phi' \quad \text{:spin supercurrent}$$

Comparison

mass superfluid

$$H = \int dV \left[\frac{n^2}{2C} + \frac{A(\nabla\phi)^2}{2} \right]$$

spin superfluid in a domain wall

$$H = \int dx (\kappa Y^2 + \eta\Phi'^2)/2$$

$$[n(\mathbf{r}, t), \phi(\mathbf{r}', t)] = i\delta(\mathbf{r} - \mathbf{r}')$$

$$\ddot{\phi} + c^2 \nabla^2 \phi = 0, \quad c = \frac{1}{\hbar} \sqrt{\frac{A}{C}}$$

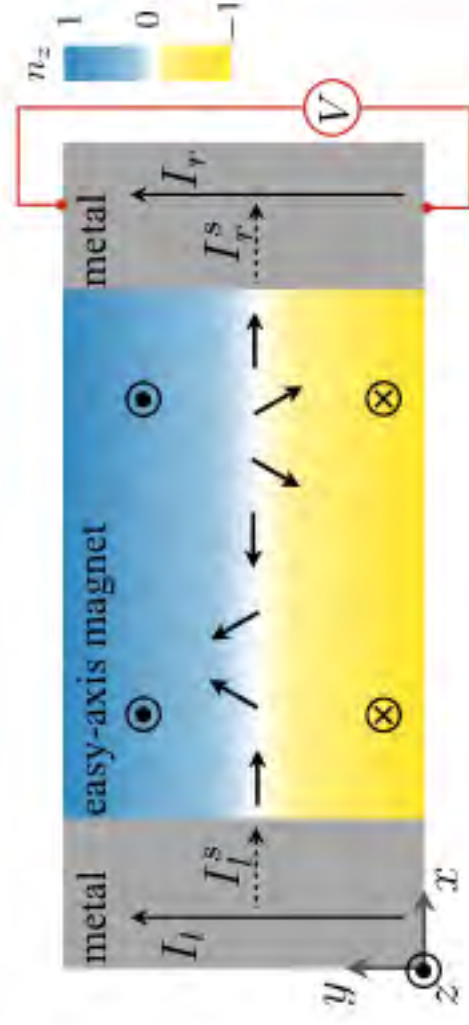
sound-wave speed

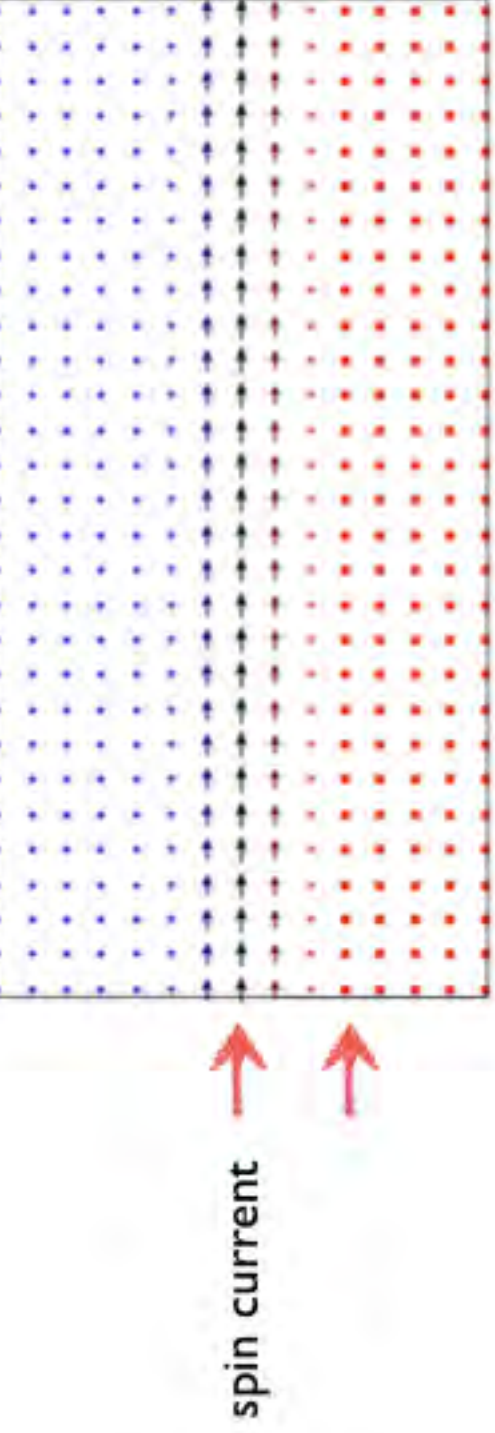
$$[Y(x, t), \Phi(x', t)] = \frac{i\hbar}{2s} \delta(x - x')$$

$$\ddot{\Phi} + c_s^2 \nabla^2 \Phi = 0, \quad c_s = \frac{\sqrt{\kappa\eta}}{2s}$$

spin-wave speed

Micromagnetic Simulations

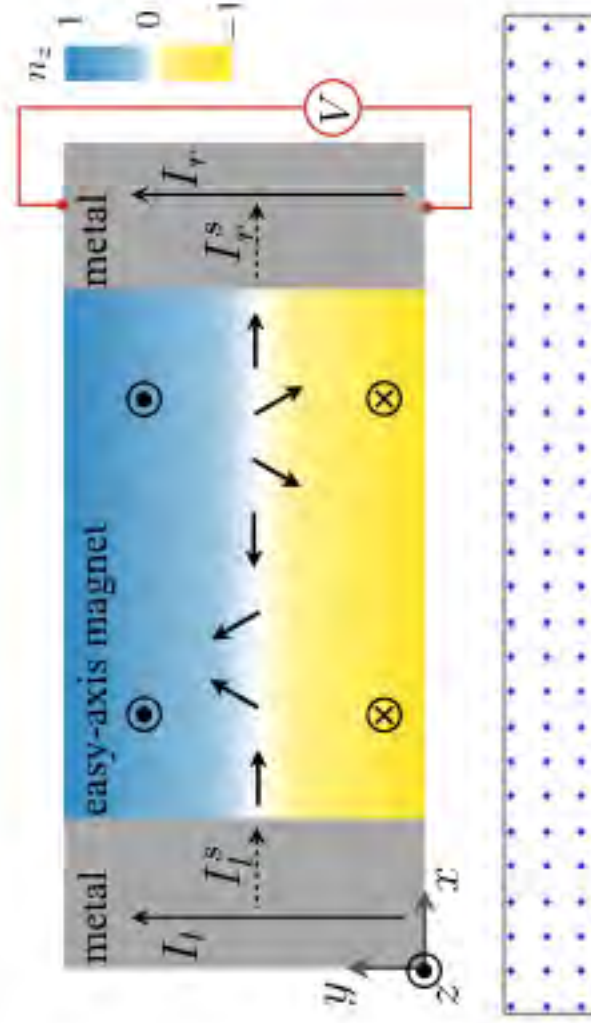


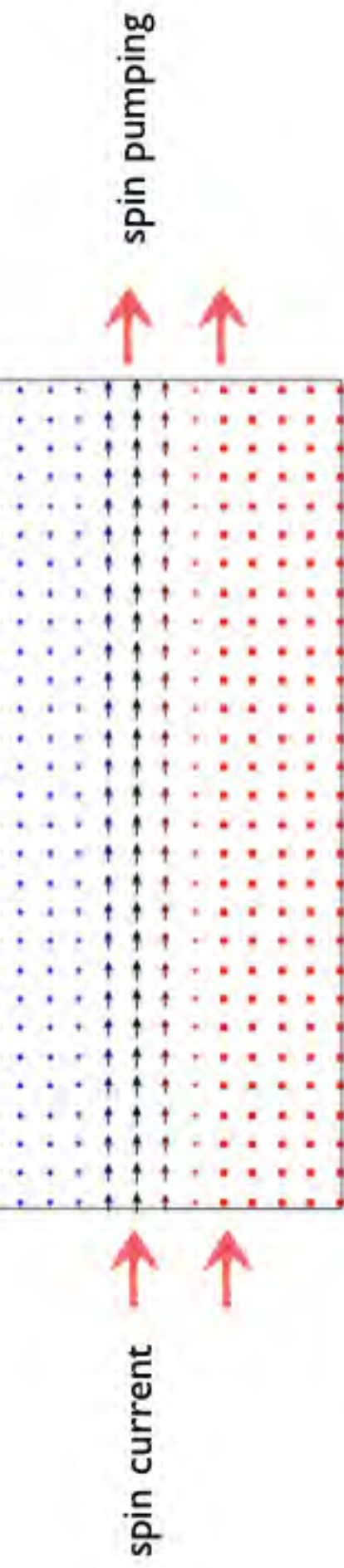


spin supercurrent: $I_s = -\eta\Phi'$

domain-wall precession: $\dot{\Phi}(x, t) \equiv \Omega$

Experimental Probe



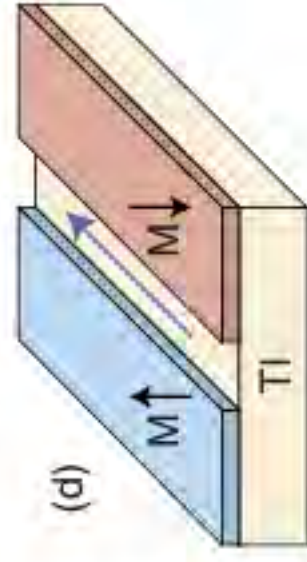
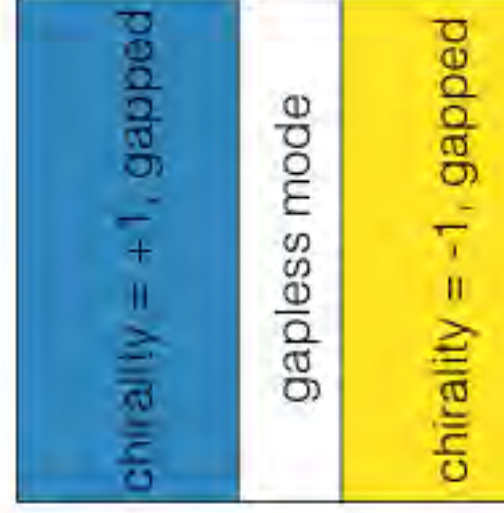
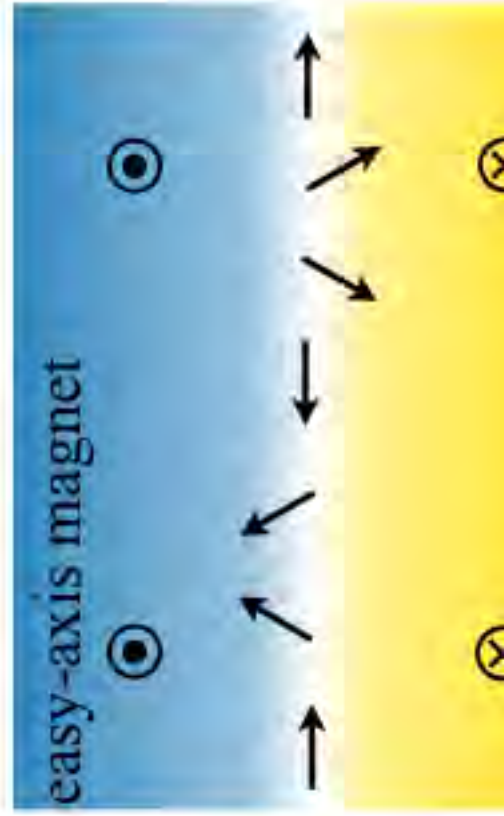


Superfluid spin transport can be probed experimentally by measuring the inverse spin Hall voltage in the adjacent metal.

$$J_I = 10^{10} \text{ A/m}^2 \rightarrow |\Delta V| = \frac{5\mu\text{V}}{1 + L/1\mu\text{m}}$$

SKK and Tserkovnyak, PRL 119, 047202 (2017)

Analogy to an Edge Mode between Topological Insulators



By interpreting the magnon **chirality** as a **topological invariant**, the spin superfluid along the domain wall can be interpreted as a gapless edge mode between two topological distinct insulators.

Hasan and Kane, RMP (2010)

Domain Wall as a Spin Superfluid Track





Aerial view of Mainz, penetrated by the Rhine river.
The Rhine is among the most important arteries of industrial transport in the world.

* the figure is taken from <https://en.wikipedia.org/wiki/Mainz>

<https://www.britannica.com/place/Rhine-River>

Domain Wall as a Spin Superfluid Track





A domain wall can serve as a **versatile** track for spin superfluids, because it can be moved easily by several means such as heat flux or microwave.

* the figure is taken from <https://en.wikipedia.org/wiki/Mainz>

Phase Slips in Mass and Charge Superfluidity



the order parameter:

$$\psi = \sqrt{n}e^{i\phi}$$

metastable states

phase winding number: n

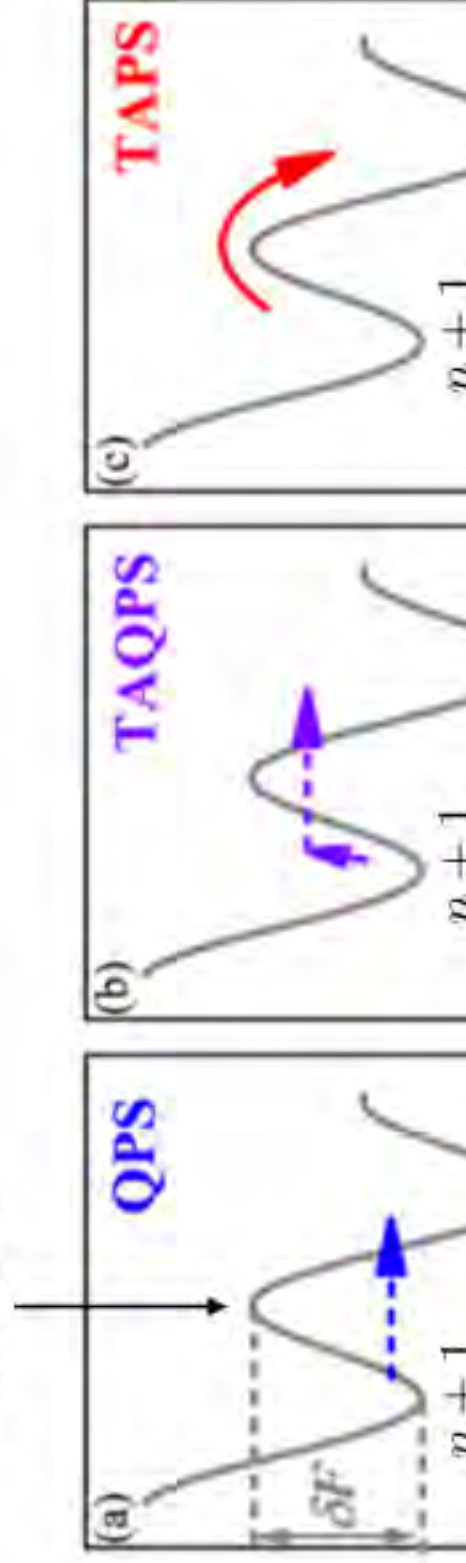
supercurrent: $\propto n$

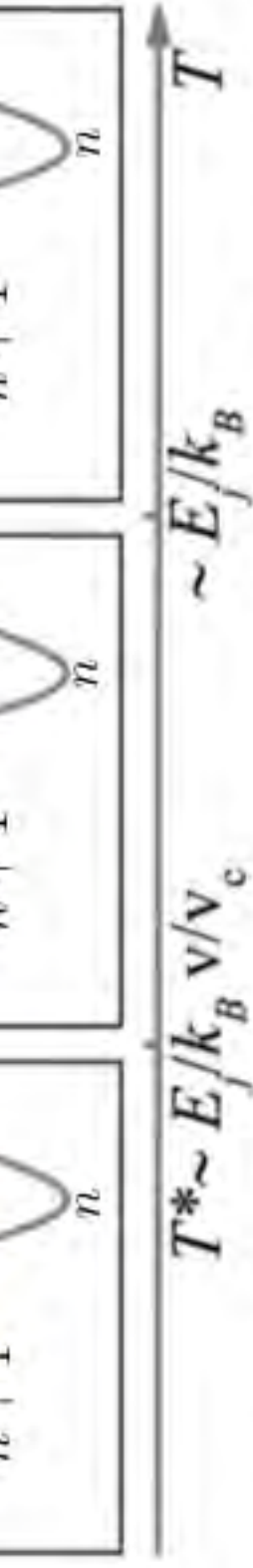
energy: $\propto n^2$

Phase Slips in Mass and Charge Superfluidity

phase slips = change of the winding number n

saddle point

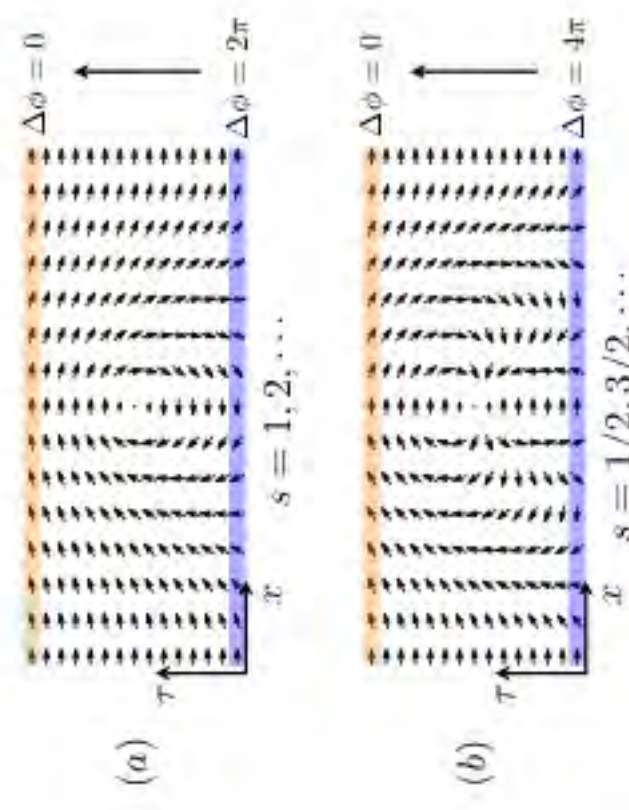
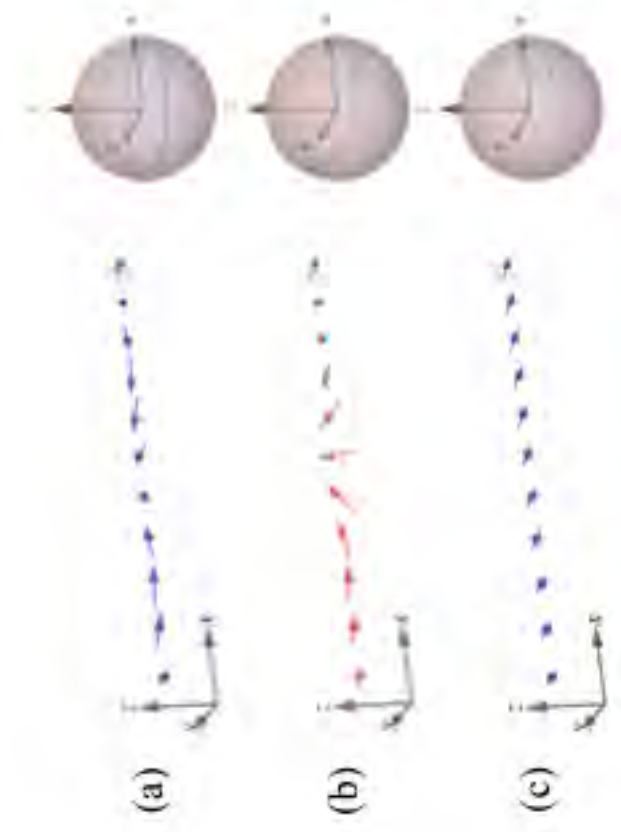




Phase-slips are activated (a) via quantum tunnelling events (QPS) for $T \ll T^*$, (b) via quantum tunnelling events assisted by the temperature (TAQPS) for $T \sim T^*$ and (c) via thermal fluctuations (TAPS) for $T \gg \Delta F/k_B$. In a lattice $T^* \approx \Delta F/k_B \approx v/v_c$ and $\Delta F \approx \xi_c$. See text.

Ianzzi et al., Sci. Rep. (2016)

Phase Slips of Spin Superfluid in Easy-plane Magnets



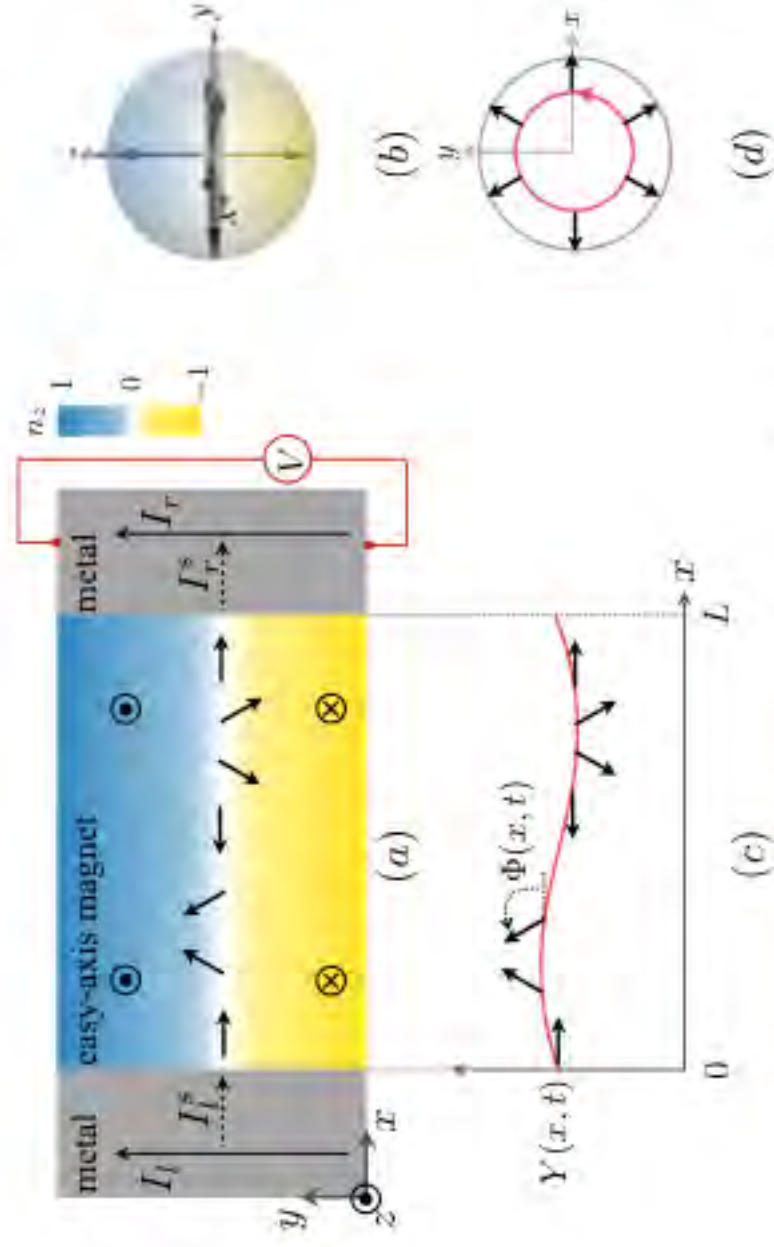
thermally-activated phase slips

SKK, Takei and Tserkovnyak, PRB (2016)

quantum phase slips

SKK and Tserkovnyak, PRL (2016)

U(1) Winding Number and Skyrmion Charge

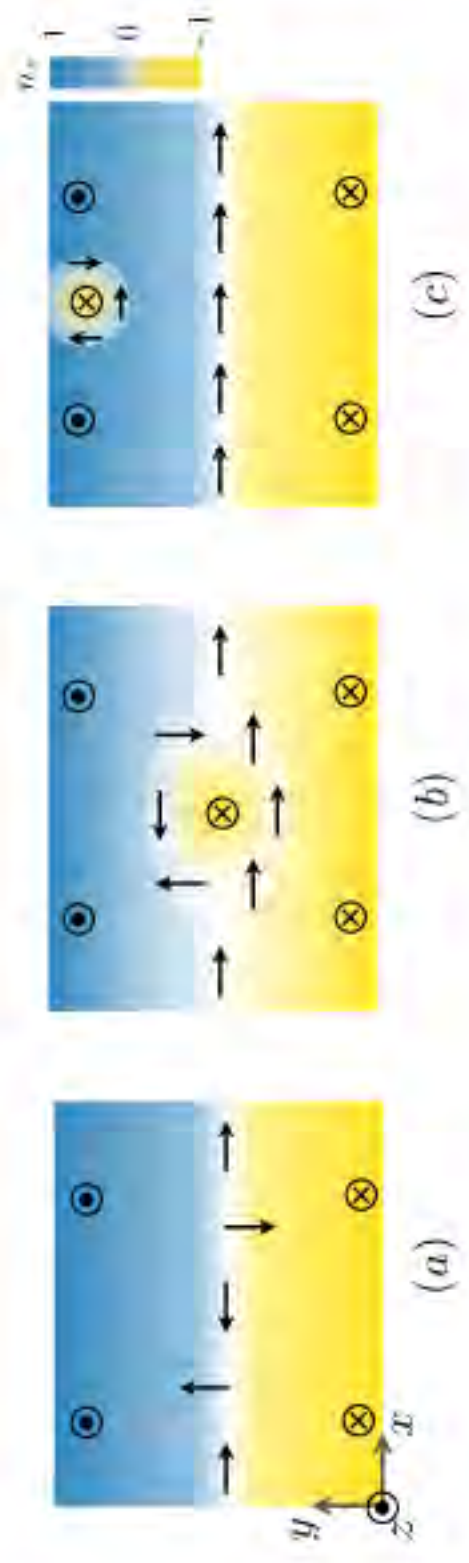


U(1) winding number: $w = \frac{1}{2\pi} \int dx \partial_x \Phi$

Skyrmion charge: $Q = \frac{1}{4\pi} \int dxdy \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

$w = Q$ in metastable states (with periodic boundary conditions)

Phase Slips for Spin Superfluid inside a Domain Wall



$w = 1$

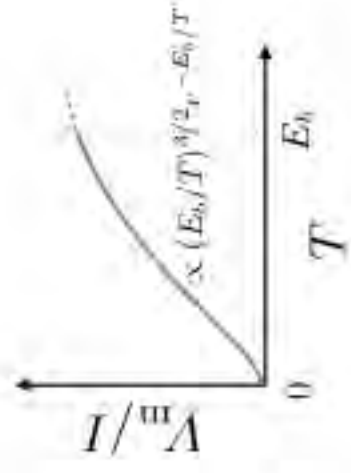
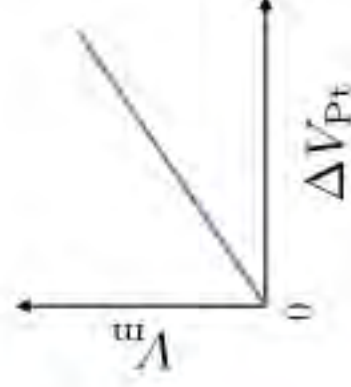
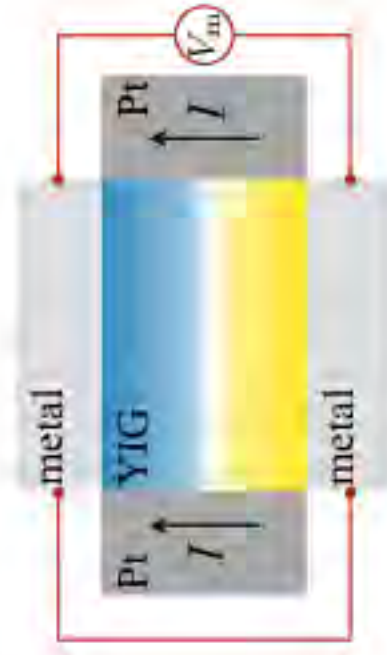
$w = 0$

$Q = 1$ (localized in the domain wall)

$Q = 1$ (isolated skyrmion)

Phase slips, which change the $U(I)$ winding number, generate Skyrmions.

Phase Slips for Spin Superfluid inside a Domain Wall



(a)

(b)

(c)

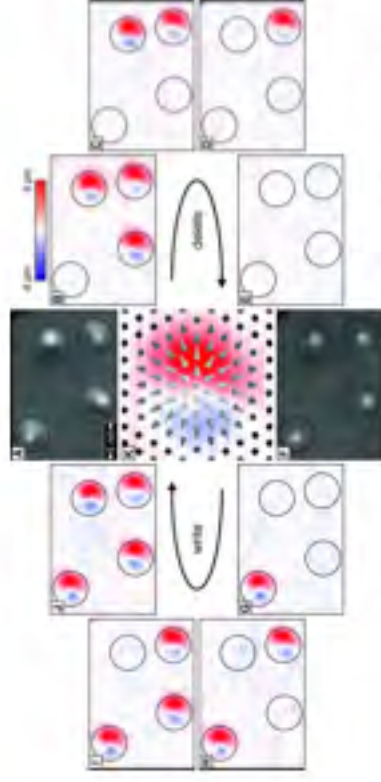
Skymion creation by phase slips can be probed experimentally by measuring the skymion-induced electromotive voltage (V_m) in the adjacent metal.

$$J_l = J_r = 10^{10} \text{ A/m}^2 \rightarrow |V_m| \sim 7 \text{ nV at } T = 300\text{K}$$

Ochoa, SKK, and Tserkovnyak *PRB* (2016)

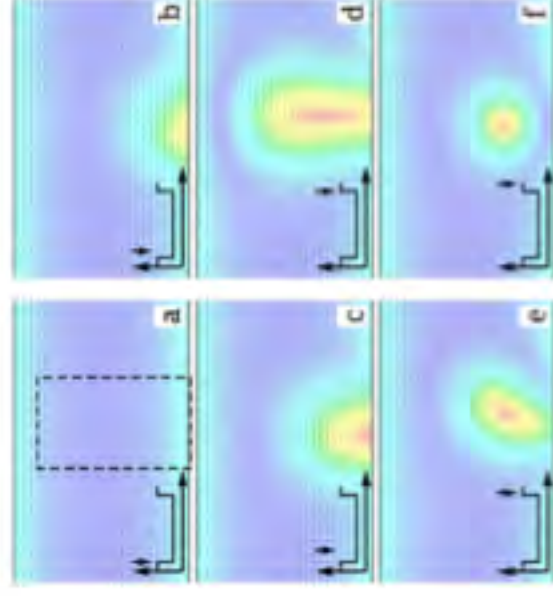
SKK and Tserkovnyak *PRL* 119, 047202 (2017)

Wanted: Efficient Creation of Skymions



Writing and Deleting Single Magnetic Skymions

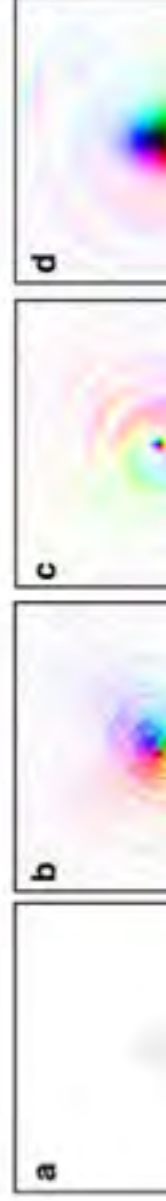
Niklas Romming, Christian Hanneken, Matthias Menzel, Jessica E. Bickel*, Boris Wolter, Kirsten von Bergmann*, André Kubetzka*, Roland Wiesendanger *Science* (2013)



Edge instabilities and skymion creation in magnetic layers

Jan Müller, Achim Rosch and Markus Garst

New J. Phys. (2016)

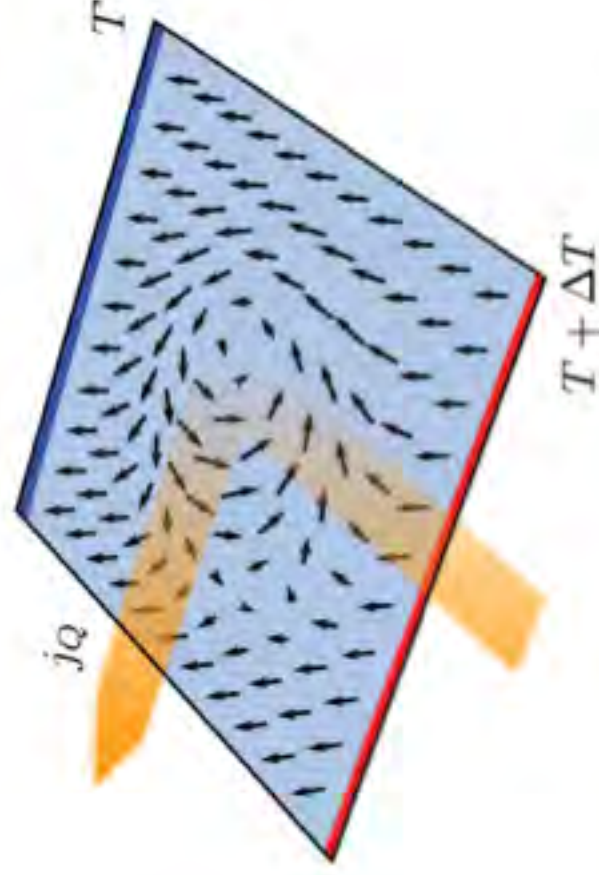




Creation of skyrmions and antiskyrmions by local heating

Wataru Koshibae¹ & Naoto Nagaosa^{1,2} *Nat. Commun.* (2014)

Skyrmion-induced Emergent Magnetic Field on Magnons



effective Lorentz force on magnons:

$$\mathbf{F} = \mathbf{v} \times b\hat{\mathbf{z}},$$

emergent magnetic field on magnons:

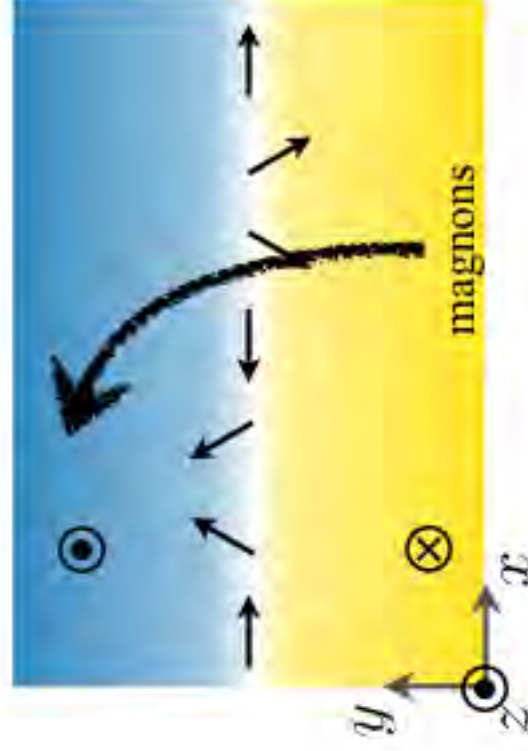
$$b = \frac{\hbar \omega}{2\pi} \frac{\partial \mathbf{n}}{\partial \mathbf{k}} \times \partial \mathbf{n}$$

emergent magnetic field on magnons: $\mathbf{U} = -\hbar \mathbf{M} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

cf. Skyrmion charge: $Q = \frac{1}{4\pi} \int dx dy \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

K.A. van Hoogdalem, Y.Tserkovnyak, and D. Loss, PRB (2013)

Superfluid-induced Emergent Magnetic Field on Magnons

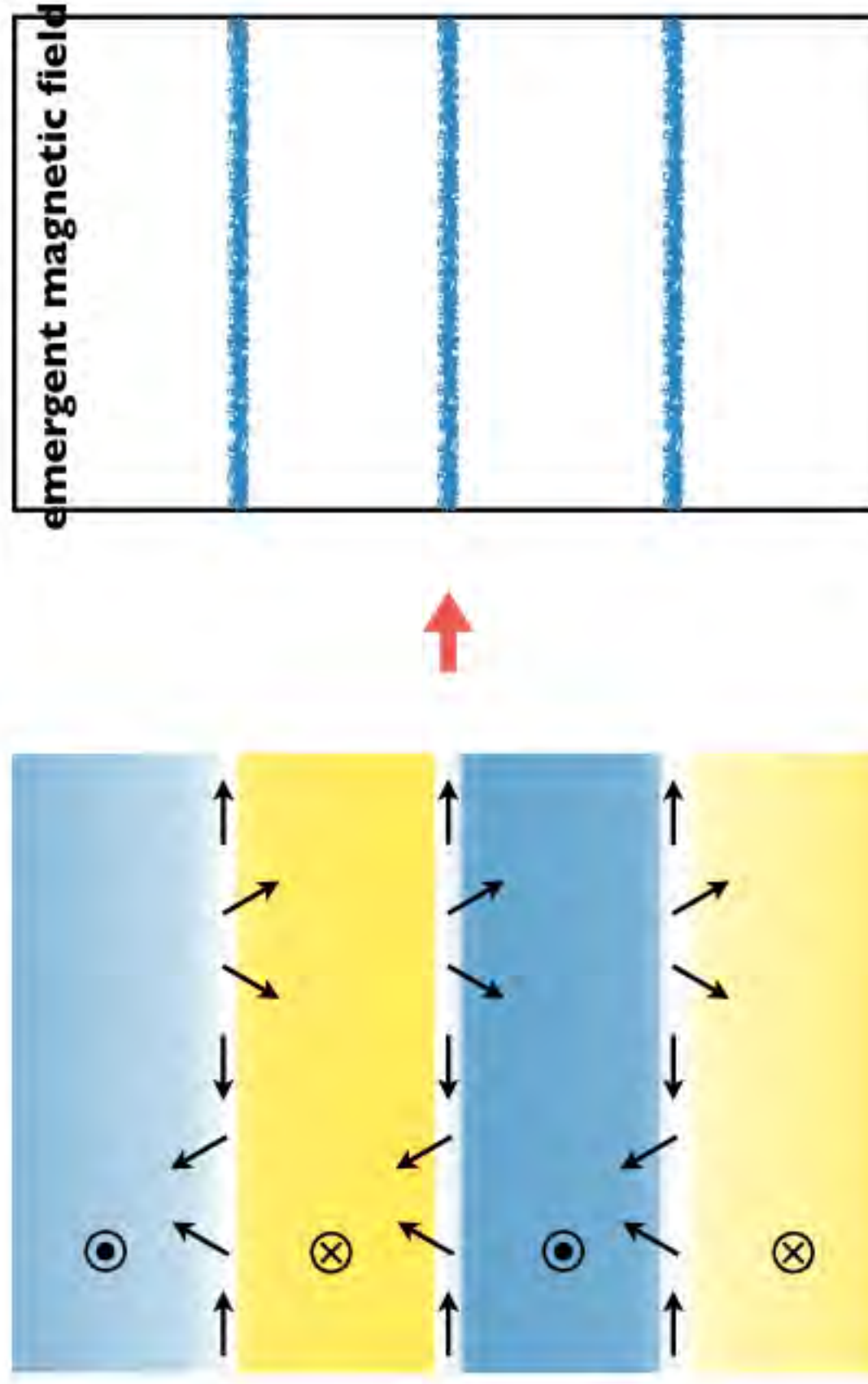


effective Lorentz force on magnons: $\mathbf{F} = \mathbf{v} \times b \hat{\mathbf{z}}$,

emergent magnetic field on magnons: $b = -\hbar \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n}$

The emergent magnetic field can be **controlled** by varying the spin supercurrent.

Superfluid-induced Magnonic Crystal



An **array of domain walls** carrying the spin supercurrent forms a **magnonic crystal** with a **periodic emergent magnetic field**.



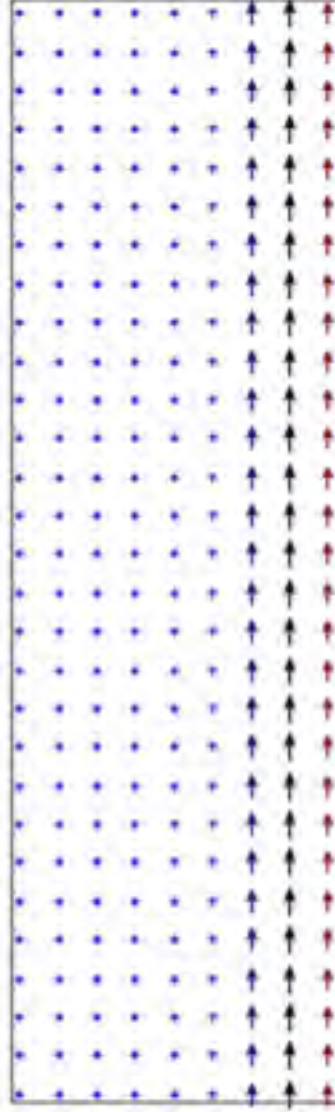
Summary

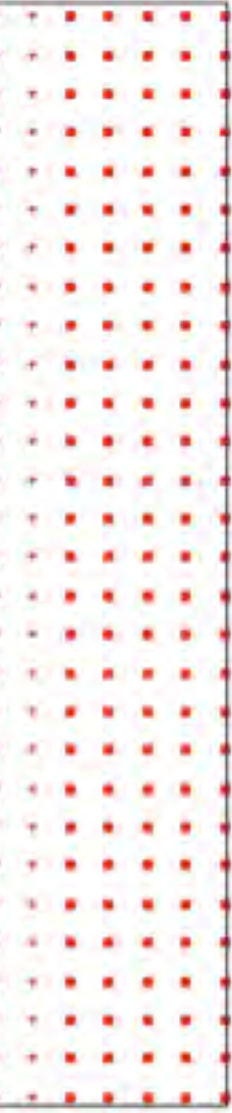
$$H = \int dV \left[\frac{K n_z^2}{2} + \frac{A(\nabla \mathbf{n})^2}{2} \right]$$

easy-axis anisotropy: $K < 0$

A simple easy-axis magnet is a venue for an topological orchestra of

- domain wall + spin superfluid + skyrmion
- + (controllable) emergent electromagnetic field





SKK and Tserkovnyak, PRL 119, 047202 (2017)