Magnon Transport Both in Ferromagnetic and Antiferromagnetic Insulating Magnets

Kouki Nakata

University of Basel

KN, S. K. Kim (UCLA), J. Klinovaja, D. Loss (2017)
arXiv:1707.07427

See also review article [KN, Simon (Paris) & Loss, J. Phys. D (2017)] with [KN, JK & DL, PRB (2017)] on magnonic ``quantum'' Hall effect and Wiedemann-Franz law in a topological ferromagnet

@Mainz 31 (Monday) July 2017: 35 +10 min (14:00-)

Magnonic Topological Insulator in Antiferromagnet

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GOAL

Topological insulator

Kane & Mele, PRL (2005, 2005).
Bernevig & Zhang, PRL (2006).

Magnonic Topological insulator: A bosonic analog

KN, Kim, Klinovaja & Loss (2017).
arXiv:1707.07427

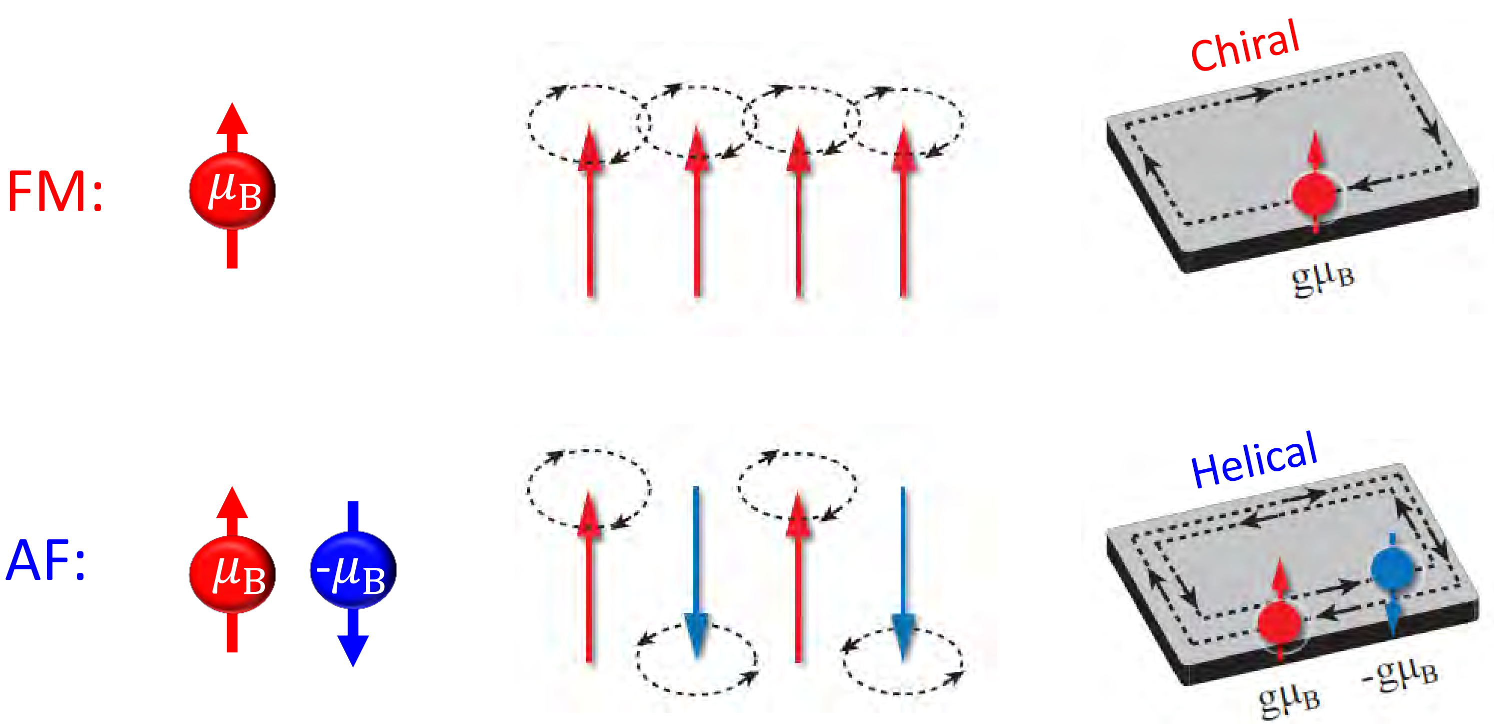
BASIC IDEA

Antiferromagnet (AF): Néel Order

AF = Independent copies of FM

Anderson, Phys. Rev. (1952)
Kubo, Phys. Rev. (1952)

$\sigma g \mu_{\mathrm{B}}$: Up- ($\sigma = 1; \uparrow$) & down- ($\sigma = -1; \downarrow$) magnons → Direction of cyclotron motion; opposite



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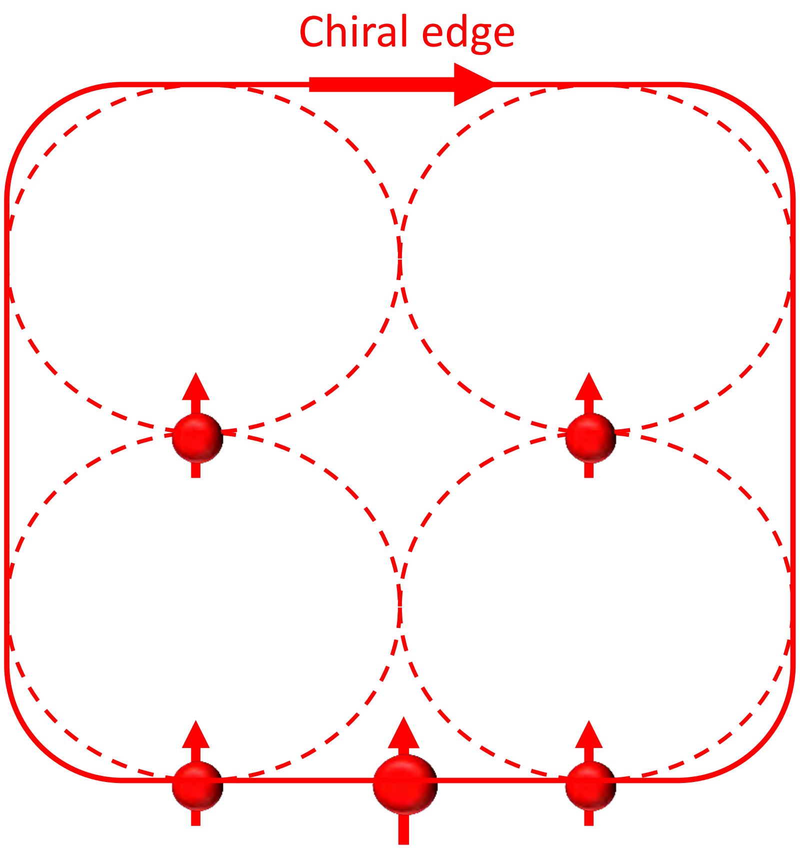
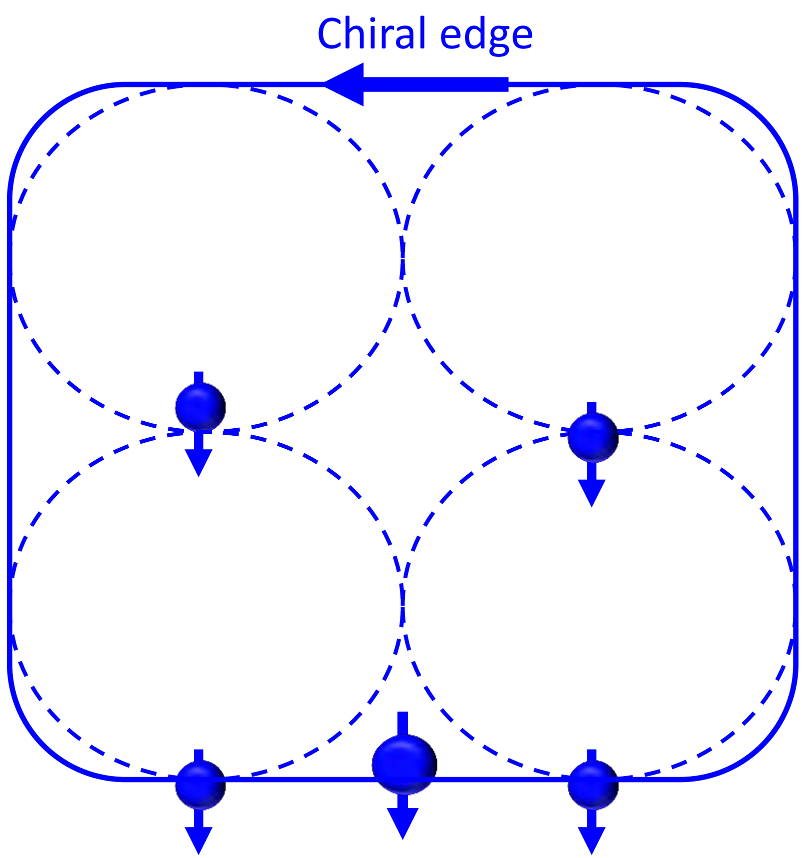
Up-magnon: $\sigma = 1; \uparrow$



Down-magnon: $\sigma = -1; \downarrow$



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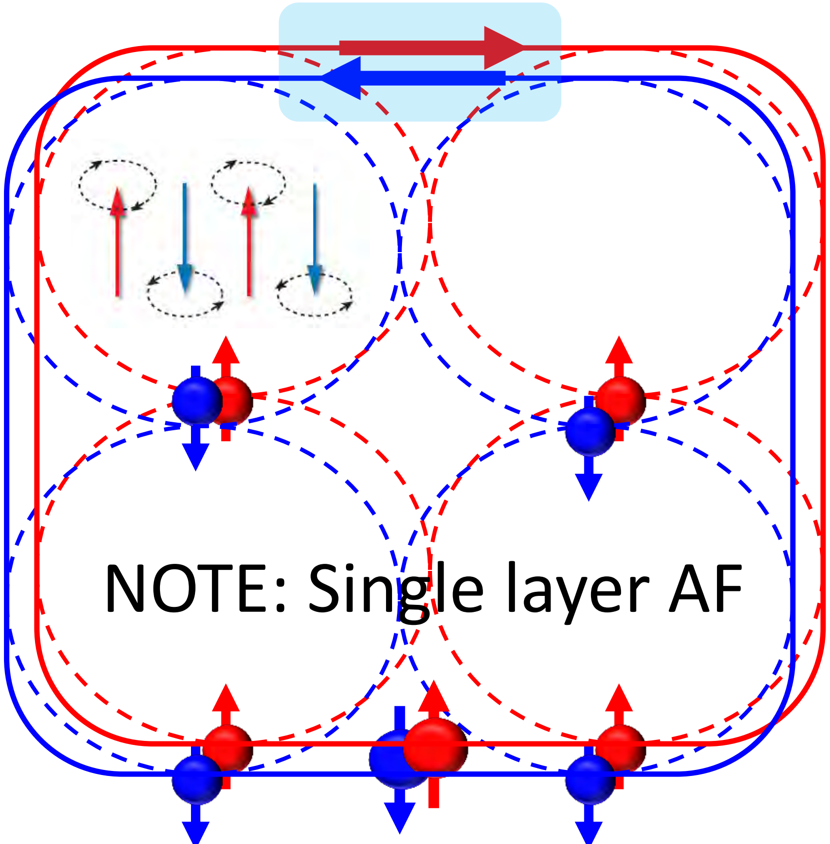
(Single layer) AF: Helical edge magnon



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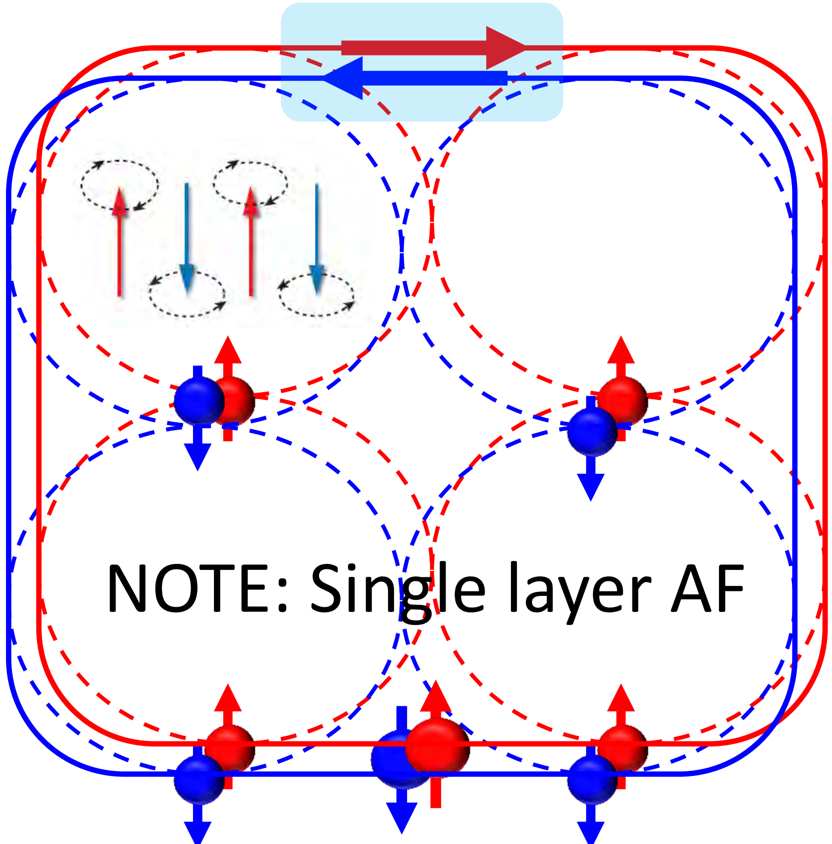
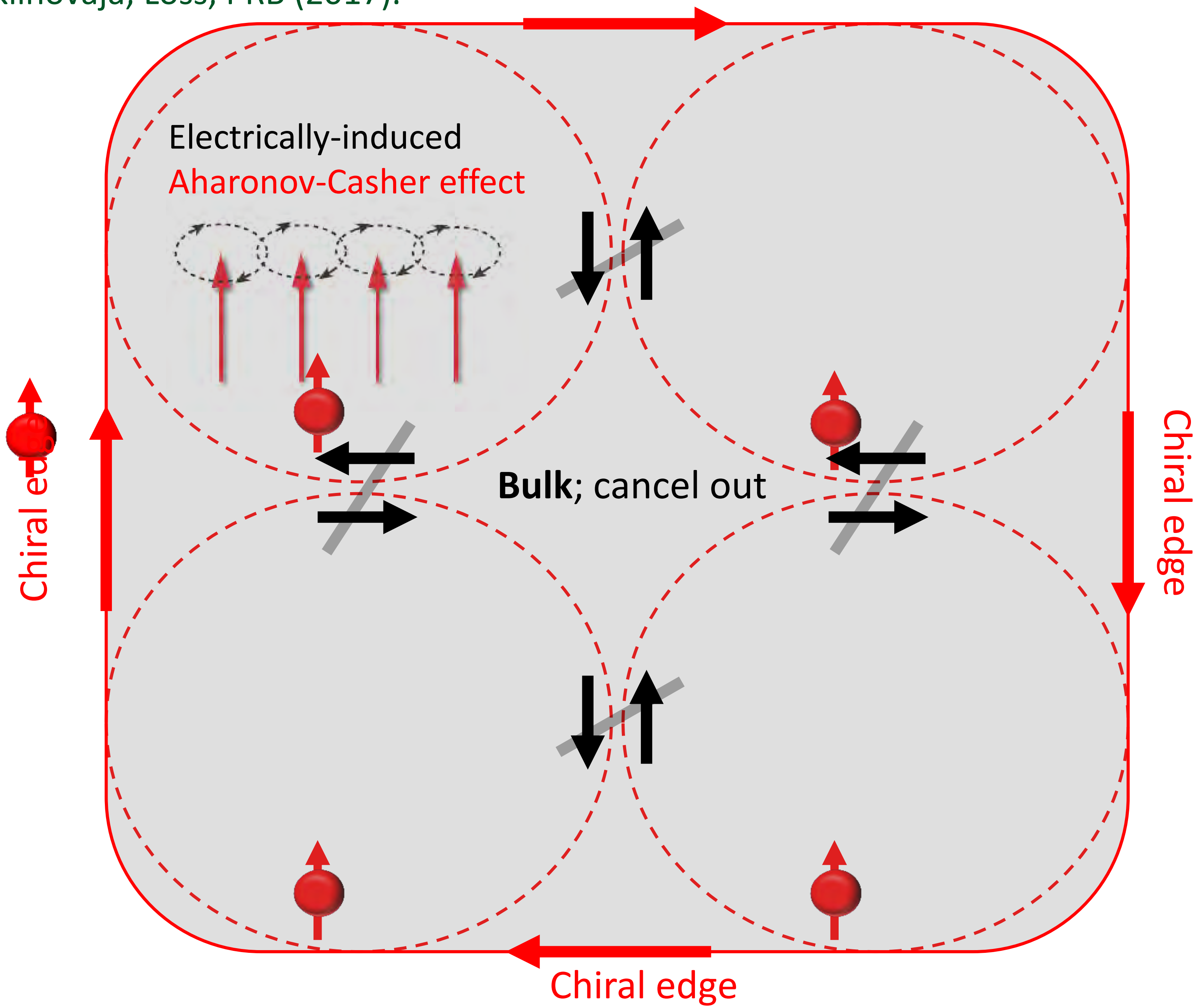
Q. How to realize such a cyclotron motion of each magnon ?
A. Aharonov-Casher (AC) effect on magnons in electric field

(Single layer) AF: Helical edge magnon



``Quantum'' Hall effect of magnon in ferromagnet (FM): Chiral edge by AC effect

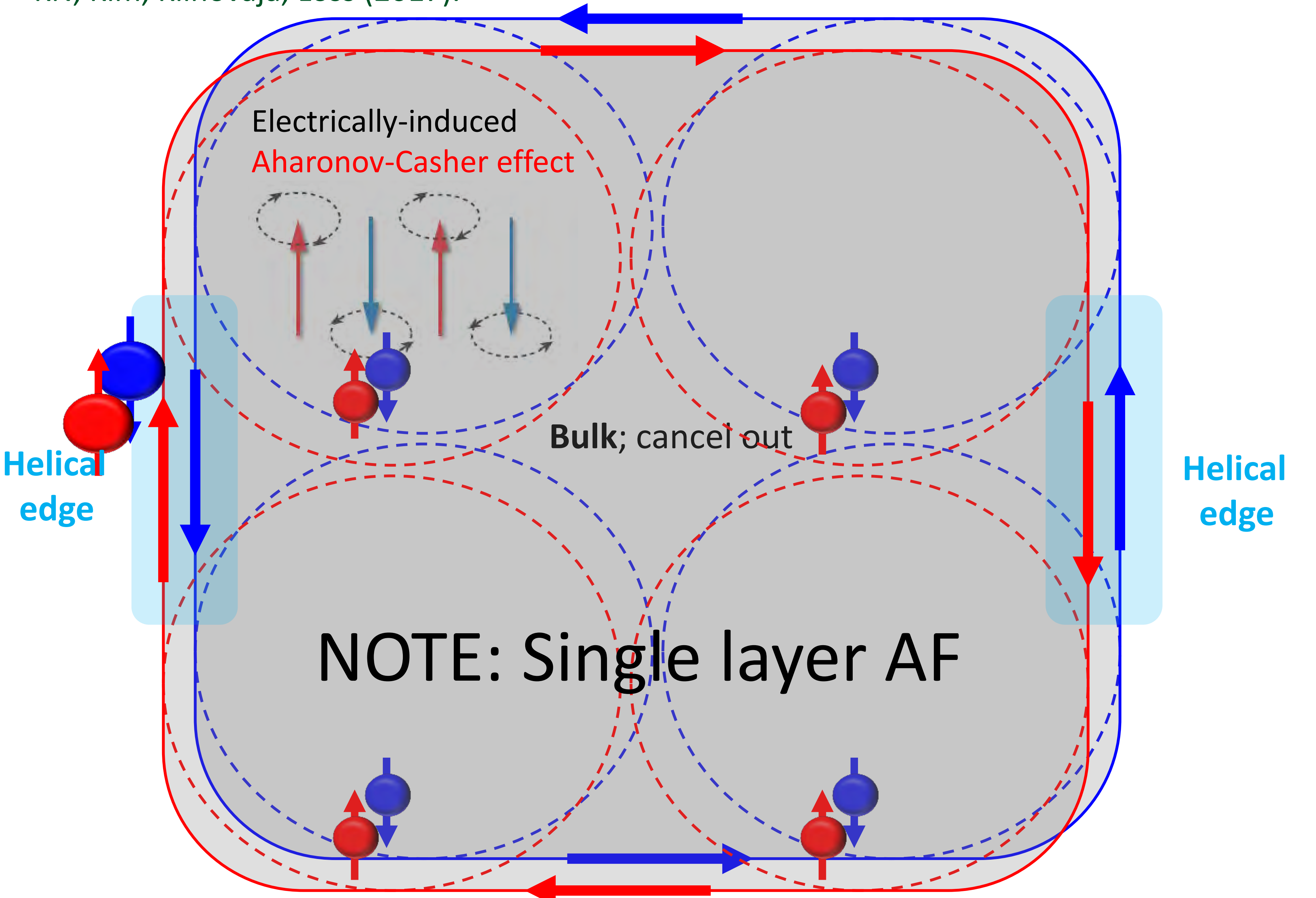
KN, Klinovaja, Loss, PRB (2017).



``Quantum'' spin Hall effect of magnons in AF (single layer): Helical edge by AC effect

KN, Kim, Klinovaja, Loss (2017).

= Magnonic topological insulator (TI)



PURPOSE OF TALK

To share the ``qualitative'' understanding of such mechanism in topological AFs:

See [KN, Kim, Klinovaja, Loss, arXiv:1707.07427] for details

Q. How to realize such a 2-dim (single layer) topological AF ???

A. Aharonov-Casher effect on magnons in electric field

Magnonic topological insulator (TI) in AF

1) Helical edge magnon state

2) Topological invariant:

Topological Hall effect of bulk magnons

(*e.g.*, spin, thermal, Nernst & Ettinghausen effects)

3) Bosonic Wiedemann-Franz (WF) law:

Universal thermomagnetic properties

OUTLINE OF TALK

I. Topological FM

KN, Klinovaja & Loss, PRB (2017)

Chiral edge

II. Topological AF

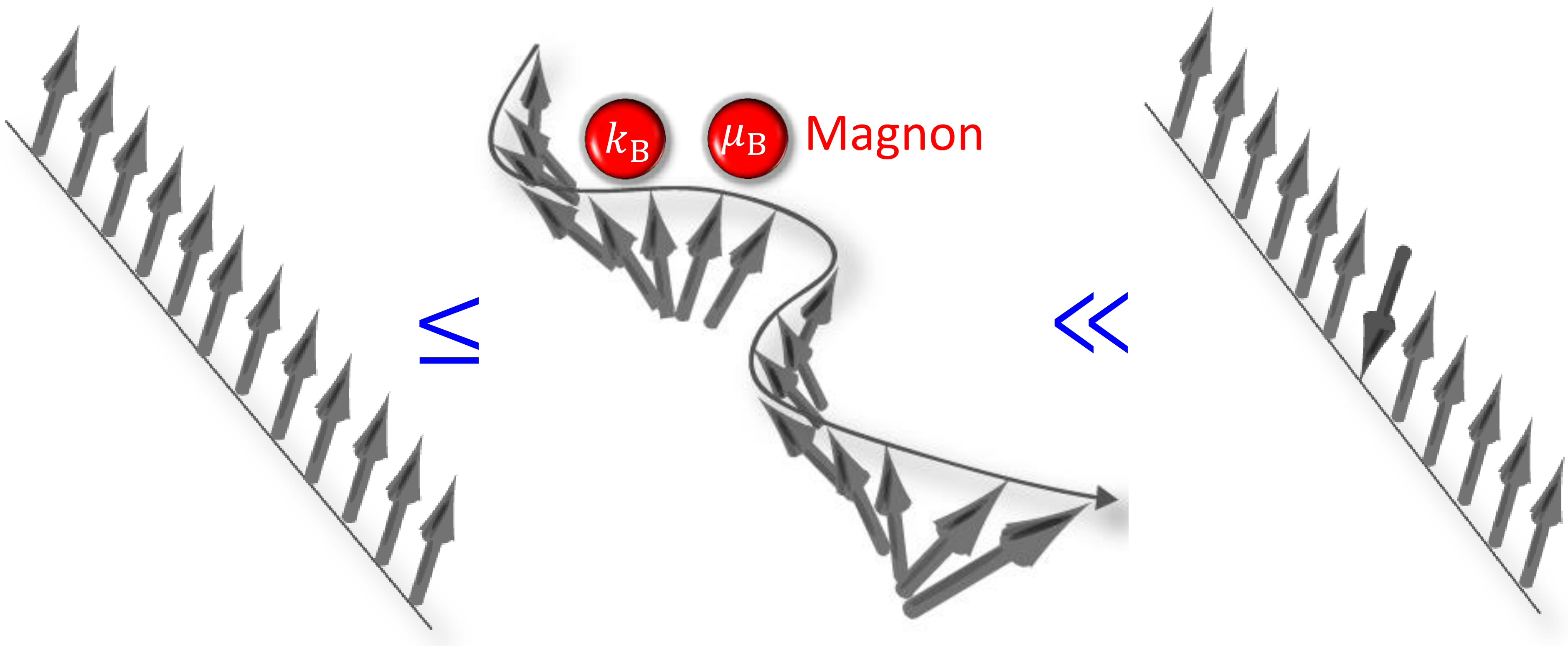
KN, Kim, Klinovaja & Loss (2017)

Helical edge

Let's start !!

Magnon Carries μ_B & k_B

Low-energy collective excitation in insulating FM & AF



QUESTION

Q. Can magnon μ_{B} (boson) transport be similar to electron e (fermion) transport ?

Yes !! KN, Simon & Loss, J. Phys. D (2017): Review article

Electron e Fermion	Magnon μ_{B} Boson
Wiedemann-Franz (WF) law Franz & Wiedemann, Annalen der Physik (1853)	Magnonic Wiedemann-Franz law KN, Simon & Loss, PRB (2015)
Josephson effect Josephson, Phys. Lett. (1962)	Magnonic Josephson effect KN, Hoogdalem, Simon & Loss, PRB (2014) KN, Simon & Loss, PRB (2015)
Integer quantum Hall effect (IQHE) Klitzing *et al.*, PRL (1980) TKNN, PRL (1982) / Kohmoto, Ann. Phys. (1985)	Magnonic ``quantum'' Hall effect: Chiral edge KN, Klinovaja & Loss, PRB (2017) Topological ferromagnet (FM)
Topological insulator (TI) Kane & Mele, PRL (2005, 2005). Bernevig & Zhang, PRL (2006). Quantum spin Hall effect (QSHE)	Magnonic topological insulator: Helical edge KN, Kim, Klinovaja & Loss (2017) arXiv:1707.07427 Topological antiferromagnet (AF): Magnonic quantum spin Hall effect

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Aharonov-Casher (AC) phase:

Aharonov & Casher, PRL (1984)

Observation for magnons:

Zhang *et al*. (Yale), PRL (2014)

Electric-field coupling to magnons

Magnon μ_B Boson
Magnonic Wiedemann-Franz law KN, Simon & Loss, PRB (2015)
Magnonic Josephson effect KN, Hoogdalem, Simon & Loss, PRB (2014) KN, Simon & Loss, PRB (2015)
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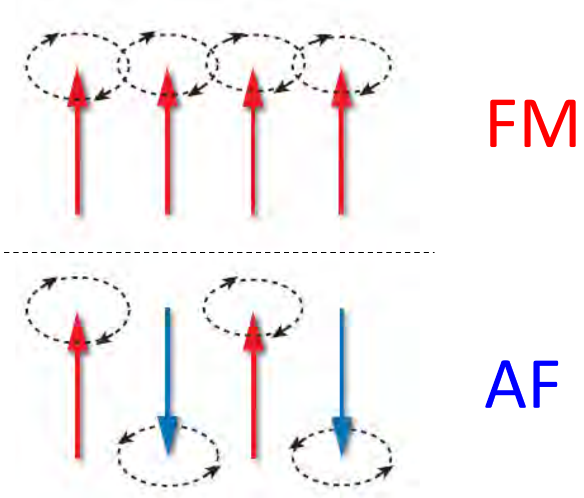

Magnon μ_B Boson
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Topological FM

KN, Klinovaja & Loss, PRB (2017)

STRATEGY

Magnonic classical Hall effect in Aharonov-Casher phase: No topological edge

Meier & Loss, PRL (2003)

↓ Topological invariant: Chern (TKNN) integer

TKNN, PRL (1982)

Kohmoto, Ann. Phys. (1985)

Magnonic ``quantum'' Hall effect in topological FM: Chiral edge

KN, Klinovaja & Loss, PRB (2017)

↓ $\mathbb{Z}_2$ topological number

Hasan & Kane, RMP (2010) *etc.*

Magnonic ``quantum'' spin Hall effect in topological AF: Helical edge

KN, Kim, Klinovaja & Loss (2017)

Geometric Phases

Aharonov-Bohm (AB) phase	Aharonov-Casher (AC) phase
Aharonov & Bohm, Phys. Rev. (1959)	Aharonov & Casher, PRL (1984) Mignani, J. Phys. A (1991). Meier & Loss, PRL (2003).
(Electrically) charged particle : e	Magnon = Magnetic dipole: $\vec{\mu} = g\mu_{\mathrm{B}}\boldsymbol{e}_z$
Magnetic field coupling with electrons: $\theta_{\mathrm{AB}} = \frac{e}{\hbar c}\oint d\boldsymbol{l}\cdot\boldsymbol{A}$	Electric field coupling with magnons: $\theta_{\mathrm{AC}} = \frac{g\mu_{\mathrm{B}}}{\hbar c^2}\oint d\boldsymbol{l}\cdot(\boldsymbol{E}\times\boldsymbol{e}_z) = \frac{g\mu_{\mathrm{B}}}{\hbar c}\oint d\boldsymbol{l}\cdot\boldsymbol{A}_{\mathrm{m}}$
Magnetic vector potential $\boldsymbol{A}$ $\boldsymbol{\nabla}\times\boldsymbol{A} = \boldsymbol{B}$	Electric vector potential: $\boldsymbol{A}_{\mathrm{m}} \equiv \boldsymbol{E}\times\boldsymbol{e}_z/c$ $\boldsymbol{\nabla}\times\boldsymbol{A}_{\mathrm{m}} = \varepsilon\boldsymbol{e}_z/c$ for $E = \varepsilon(-x, 0, 0)$

Observation of AC effect on magnons:
Zhang *et al*. (Yale), PRL (2014)
Electric-field coupling to magnons

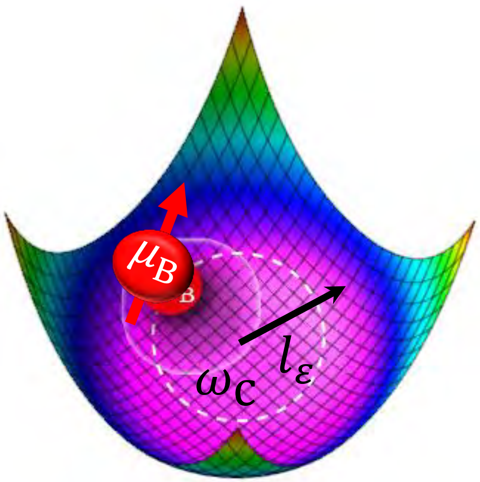
Hoogdalem *et al*., PRB (2013) [Katsura *et al*., PRL (2005)]:
DM int. → An analogue of artificial gauge field
→ AC effect

FM: Landau Level in AC Effect

KN, Klinovaja & Loss, PRB (2017)

External electric field gradient $\mathcal{E}$

KN, Klinovaja & Loss, PRB (2017)



AC effect-induced topological FM:

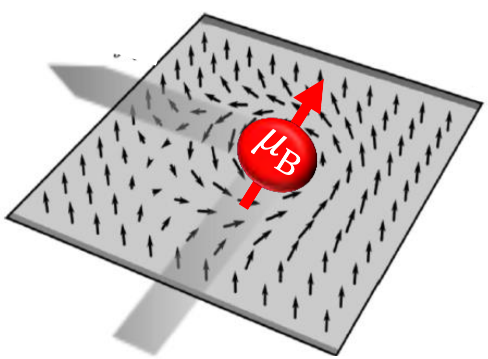
$$\nabla \times \mathbf{A}_{\mathrm{m}} = \frac{\mathcal{E}}{c}\mathbf{e}_z \quad \begin{cases} \mathbf{E}(\mathbf{r}) = \mathcal{E}(-x,0,0) \\ \mathbf{A}_{\mathrm{m}}(\mathbf{r}) = \frac{1}{c}\mathbf{E}\times\mathbf{e}_{\mathbf{z}} \\ \mathbf{A}_{\mathrm{m}}(\mathbf{r}) = (\mathcal{E}/c)(0,x,0) \end{cases}$$

Landau energy level: $E_n = \hbar\omega_c\left(n+\frac{1}{2}\right) \quad \omega_c = \frac{g\mu_{\mathrm{B}}\mathcal{E}}{mc^2}$

Cyclotron motion: $l_{\mathcal{E}} \equiv \sqrt{\hbar c^2/g\mu_{\mathrm{B}}\mathcal{E}}$

Cf. Skyrmion lattice induced by DM int.

Hoogdalem, Tserkovnyak & Loss, PRB (2013)



DM int. → Vector potential analogous to $\boldsymbol{A}_{\mathrm{m}}$

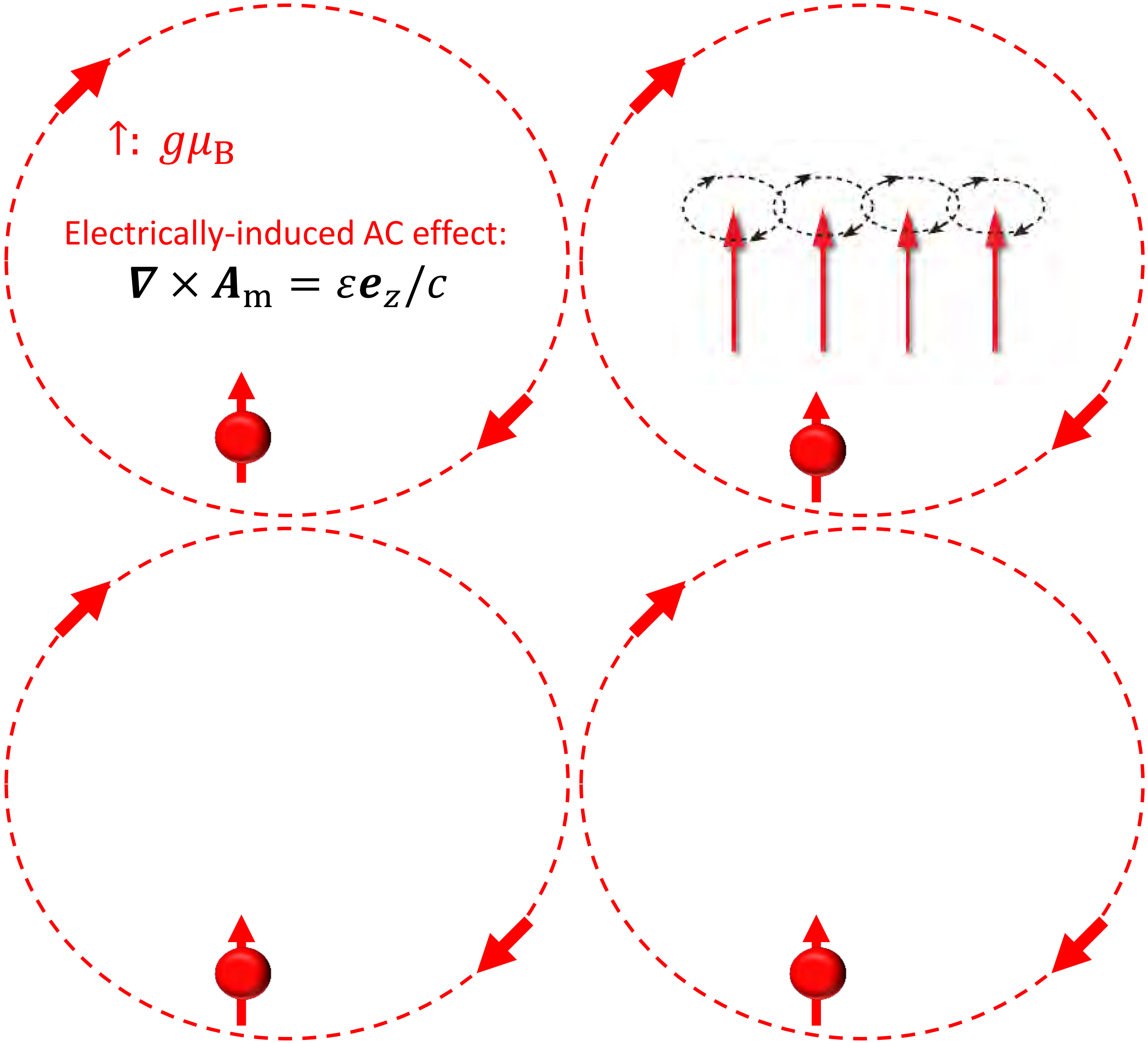
Average fictitious field (textured magnetization)

Landau gap: $\Delta E_n = 2.5$ meV $= 18$ K

Within experimental reach: Nagaosa & Tokura, Nat. Nano. (2013)

``Quantum'' Hall effect of up-magnon in FM: Chiral edge by cyclotron motion

KN, Klinovaja & Loss, PRB (2017)



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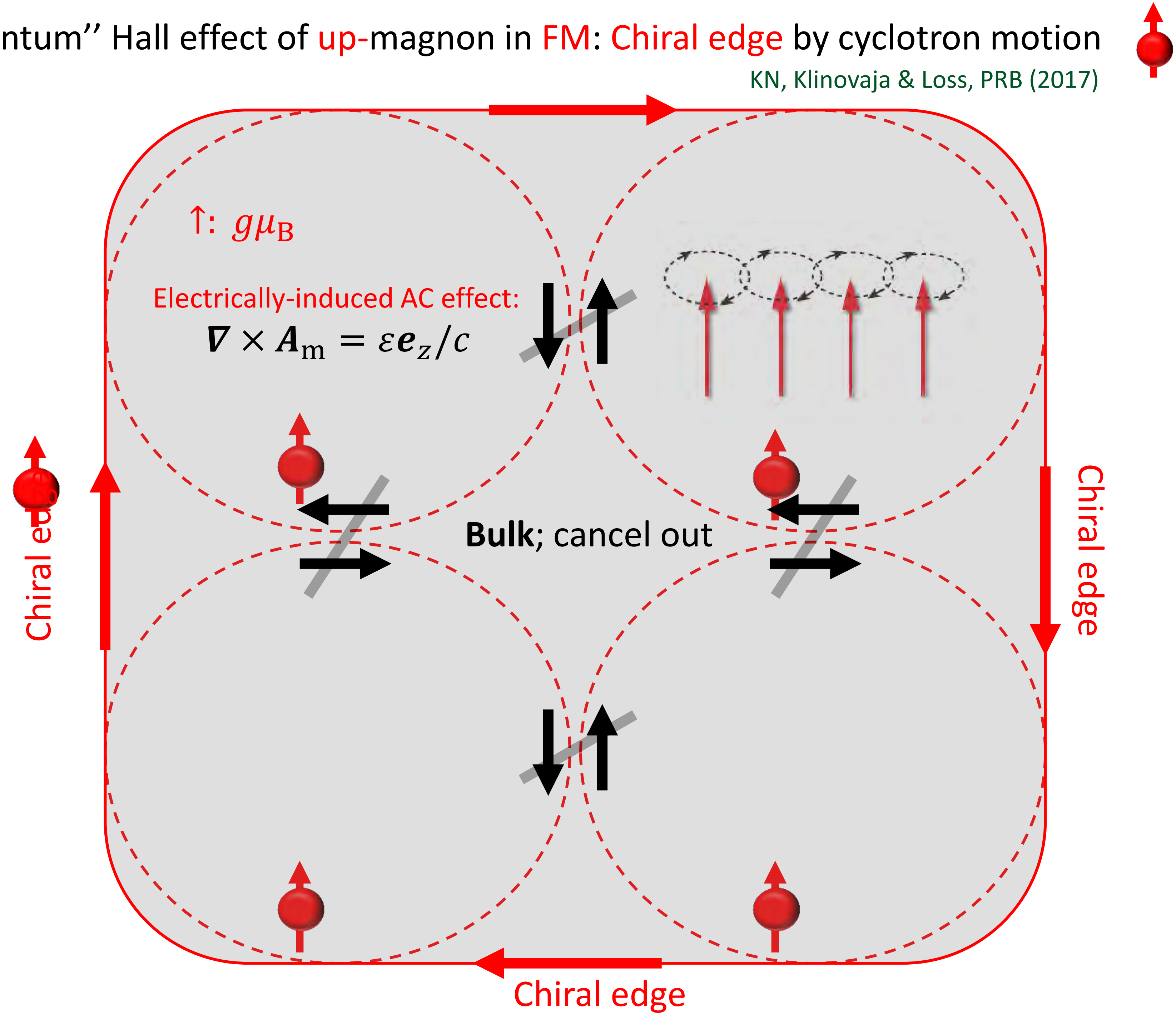


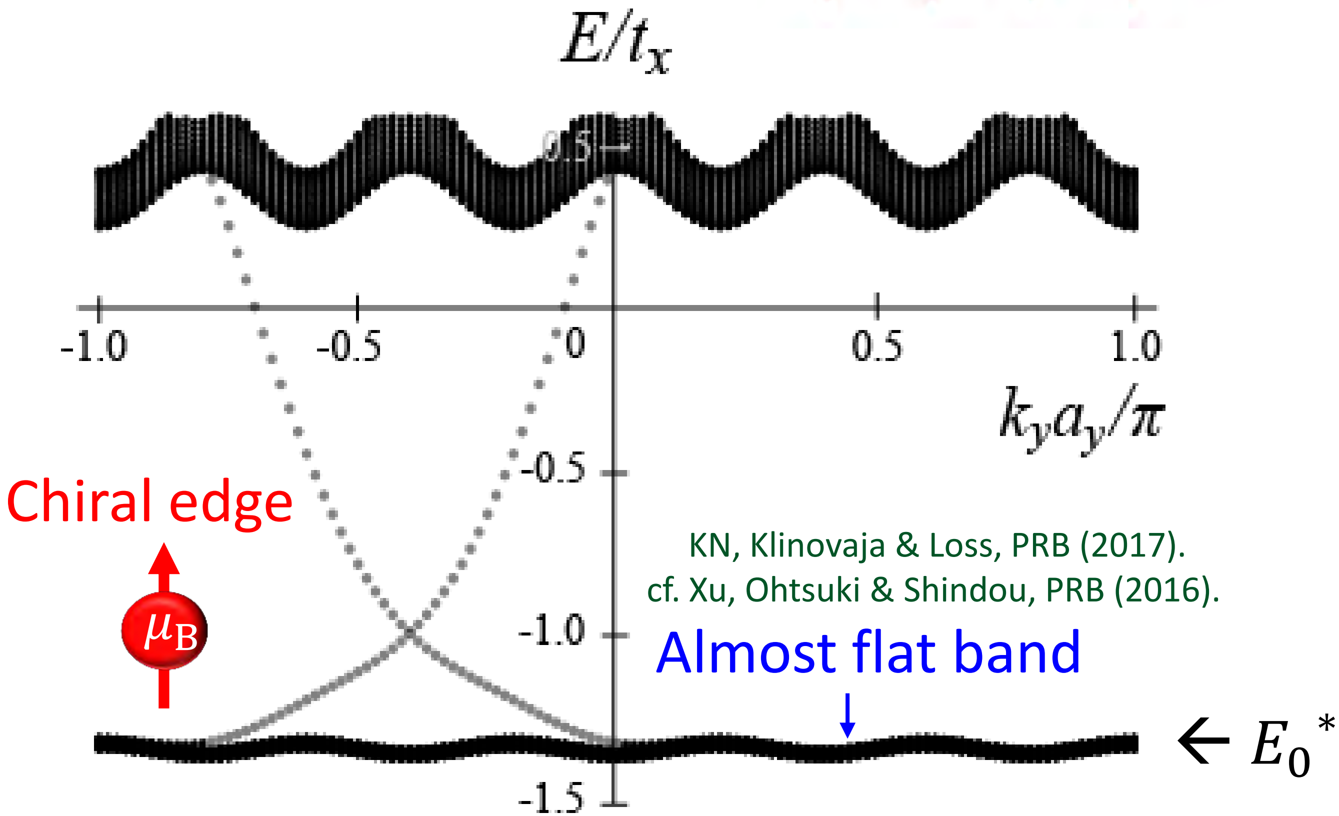
Chiral Edge Magnon States: Topological FM

KN, Klinovaja & Loss, PRB (2017)

$$\mathcal{H}_{\rm m} = -\sum_{\langle ij \rangle} t_{ij} e^{i\theta_{ij}} (a_i a_j^\dagger + \text{H.c.}) \quad \theta_{ij} = (g\mu_B/\hbar c^2) \int_{\mathbf{x}_i}^{\mathbf{x}_j} d\mathbf{r} \cdot (\mathbf{E} \times \mathbf{e}_z)$$

Electrically-induced AC effect: $\boldsymbol{A}_{\rm m} \equiv \boldsymbol{E} \times \boldsymbol{e}_z / c$

$$\boldsymbol{\nabla} \times \boldsymbol{A}_{\rm m} = \varepsilon \boldsymbol{e}_z / c$$

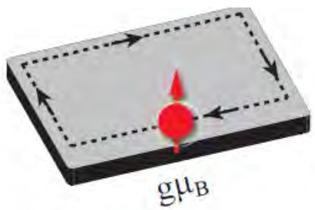


KN, Klinovaja & Loss, PRB (2017).
cf. Xu, Ohtsuki & Shindou, PRB (2016).

Magnonic WF Law in Topological FM: Bulk

KN, Klinovaja & Loss, PRB (2017)

Chiral edge:



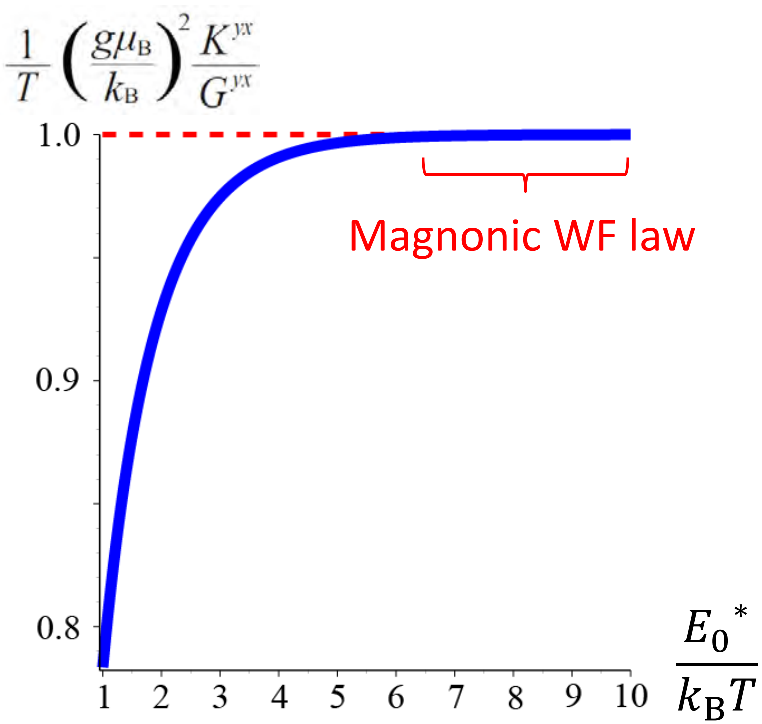
Chern integer:

$$\mathcal{N}_{0\uparrow} = +1$$

Hall coefficients: $L_{ij}^{yx} \propto N_{0\uparrow}$ in almost flat band

$$\begin{pmatrix} \langle j_y \rangle \\ \langle j_y^Q \rangle \end{pmatrix} = \begin{pmatrix} L_{11}^{yx} & L_{12}^{yx} \\ L_{21}^{yx} & L_{22}^{yx} \end{pmatrix} \begin{pmatrix} -\partial_y B \\ -\partial_y T/T \end{pmatrix}$$

cf. Matsumoto & Murakami, PRL (2011)



Magnonic WF law in `quantum' Hall system :

Universal at low temperature ($k_B T \ll E_0{}^*$)

$$\frac{K^{yx}}{G^{yx}} = \frac{1}{T}\frac{L_{22}^{yx} - \frac{(L_{12}^{yx})^2}{L_{11}^{yx}}}{L_{11}^{yx}} \fallingdotseq \left(\frac{k_B}{g\mu_B}\right)^2 T \propto T$$

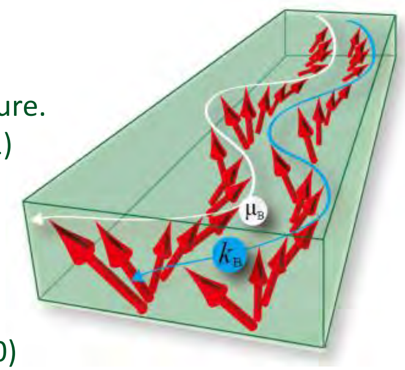
Counter-current by magnetization gradient

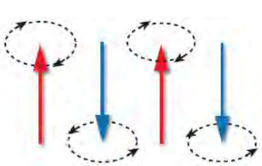
NOTE: $K^{yx} \neq L_{22}^{yx}/T$ for boson

Observation of Magnonic Edge State & WF Law

Within experimental reach

2006	Quasi-equilibrium magnon-BEC	Demokritov *et al.*, Nature
	Inverse spin-Hall effect	Saitoh *et al.*, APL
2008	Spin-Seebeck effect	Uchida *et al.* (`08, `10, `11), Nature. Theory: Adachi *et al.*, PRB (2011)
2010	Spin-wave spin current	Kajiwara *et al.*, Nature
	Magnon thermal Hall effect: Magnonic thermal conductivity	Onose *et al.*, Science Theory: Katsura *et al.*, PRL (2010) Matsumoto & Murakami, PRL (2011) Shindou *et al.*, PRB (`13): Chiral edge magnon mode
2014	Aharonov-Casher effect on magnon	Yale-group, PRL: Observation. Aharonov & Casher, PRL (1984)
2016	Snell's law for spin-waves	Tanabe *et al.* APE (2014). Stigloher *et al.*, PRL (2016).
	Magnonic spin conductivity	Cornelissen *et al.*, PRB (2016).
	Magnon planar Hall effect	Liu *et al.*, PRB (2017).





Spin Seebeck effect in AF	Seki *et al.*, (2015). Theory: Ohnuma *et al.*, PRB (2013)
Magnonic spin Nernst effect in AF	Shiomi *et al.*, arXiv:1706.03978. cf. Theory [Cheng *et al.*, PRL (2016)] & [Zyuzin *et al.*, PRL (2016)]

Remark:

Thermal (Hall) Conductance for Boson

KN, Simon & Loss, PRB (2015)
KN, Klinovaja & Loss, PRB (2017)
KN, Simon & Loss, J. Phys. D (2017): Review article

Thermal Conductivity $K \approx L^{22}$ for Fermions

Textbook by Ashcroft & Mermin

$$\begin{array}{r} \text{Charge} \\ \text{Heat} \end{array} \begin{pmatrix} \mathbf{J}_e \\ \mathbf{J}_Q \end{pmatrix} = \begin{pmatrix} L^{11} & L^{12} \\ L^{21} & \underbrace{L^{22}}_{K} \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ -\nabla T \end{pmatrix}$$

WF law: Electron = Fermion

*Lifshitz & Pitaevskii (Vol. 10)

$$\frac{K}{\sigma} \approx \frac{L^{22} + \mathcal{O}((k_B T/\epsilon_F)^2)^*}{L^{11}} \overset{!}{=} \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T$$

Thermal Conductivity $K \neq L^{22}$ for Magnons

KN, Simon & Loss, PRB (2015)

$$\begin{array}{r} \text{Magnon} \\ \text{Heat} \end{array} \begin{pmatrix} \mathbf{I}_m \\ \mathbf{I}_Q \end{pmatrix} = \underbrace{\begin{pmatrix} L^{11} & L^{12} \\ L^{21} & L^{22} \end{pmatrix}}_{K} \begin{pmatrix} \nabla B \\ -\nabla T \end{pmatrix}$$

WF law: Magnon = Boson

$$\frac{K}{G} \equiv \frac{L^{22} - L^{21}L^{12}/L^{11}}{L^{11}} \overset{!}{=} \left(\frac{k_B}{g\mu_B}\right)^2 T$$

Textbook by Ashcroft & Mermin Eq. (13.56): *K is measured under conditions of no quasi-particle current*

Thermal conductivity K: $\mathbf{I}_Q \equiv -K \cdot \nabla T$ with $\mathbf{I}_m \overset{!}{=} 0$

$$\mathbf{I}_m = L^{11}\nabla B - L^{12}\nabla T \overset{!}{=} 0 \;\rightarrow\; \nabla B^* = \frac{L^{12}}{L^{11}}\nabla T$$

$$\mathbf{I}_Q = L^{21}\nabla B^* - L^{22}\nabla T = -\underbrace{(L^{22} - L^{21}L^{12}/L^{11})}_{K}\nabla T$$

Magnetization gradient:

Johnson & Silsbee (1987)
Basso *et al.* (2016)

Topological AF

KN, Kim, Klinovaja, Loss (2017)

STRATEGY

Magnonic classical Hall effect in Aharonov-Casher phase: No topological edge

Meier & Loss, PRL (2003)

↓ Topological invariant: Chern (TKNN) integer

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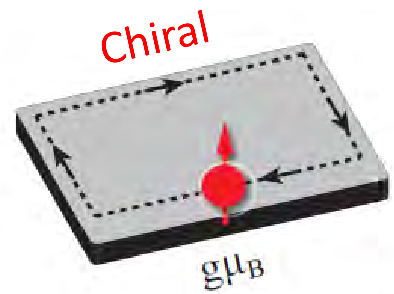
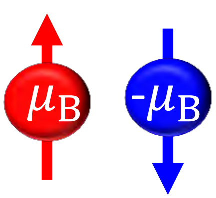
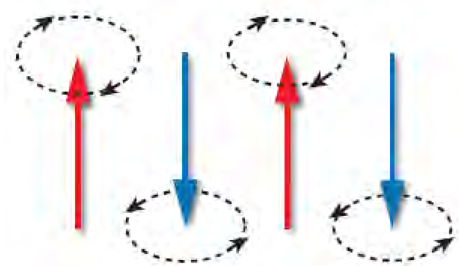
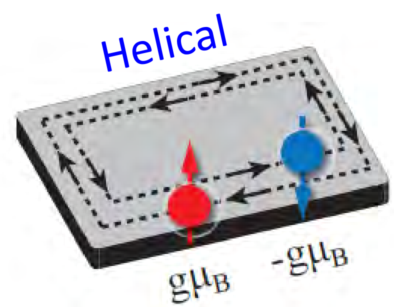
FM:



AF:







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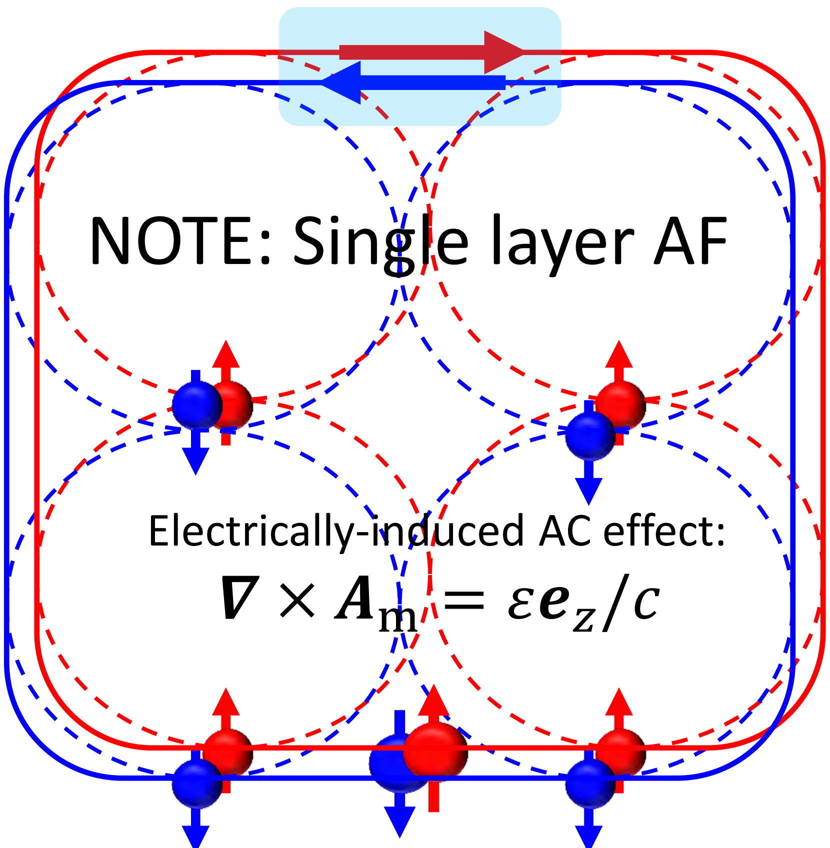
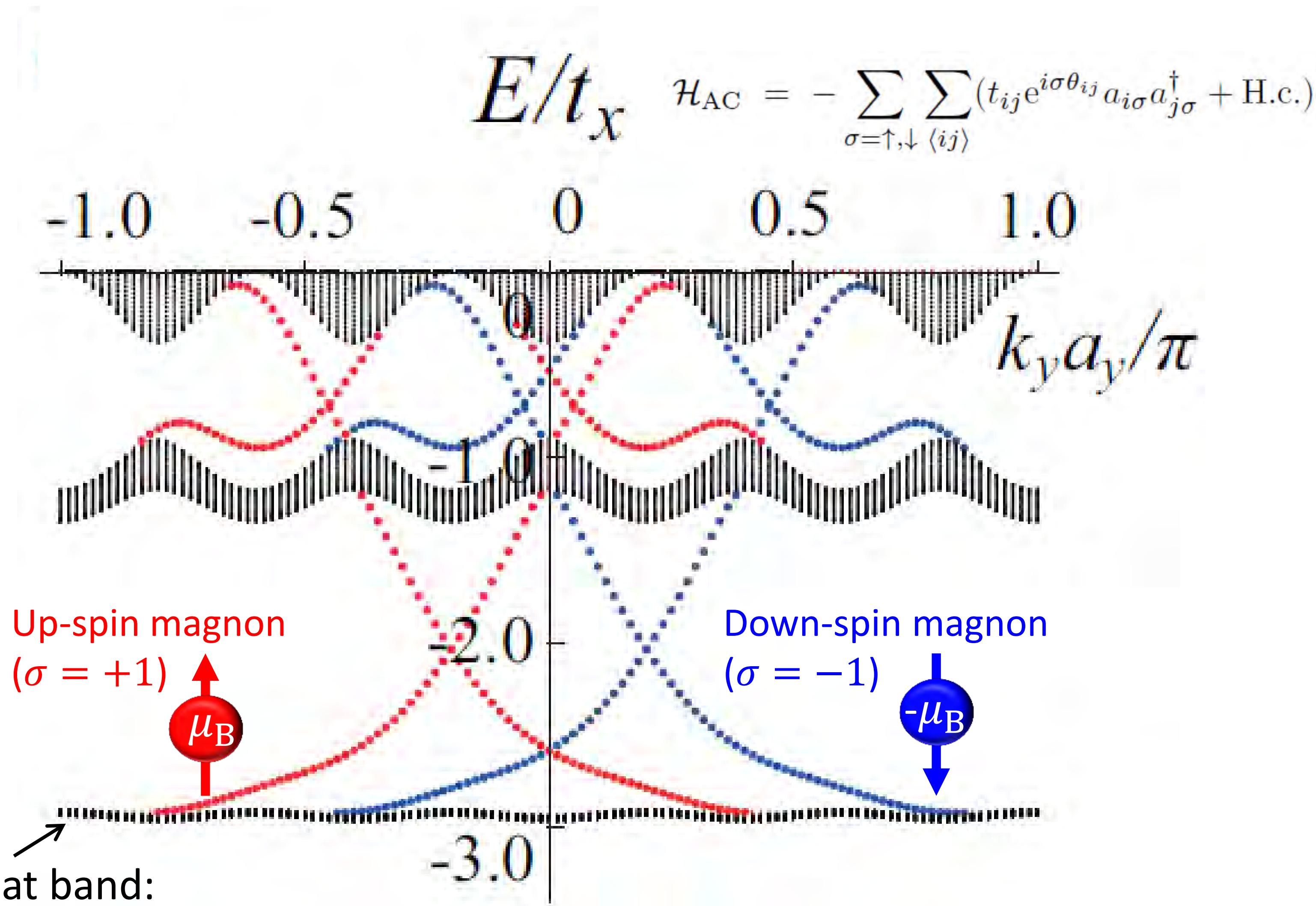
(Single layer) AF: Helical edge magnon



Helical Edge Magnon States: Topological AF

KN, Kim, Klinovaja & Loss (2017) arXiv:1707.07427



Almost flat band:

cf. KN, Klinovaja & Loss, PRB (2017)

Magnonic TI: AF in AC effect

$\boldsymbol{\nabla} \times \boldsymbol{A}_{\mathrm{m}} = \varepsilon \boldsymbol{e}_z / c$

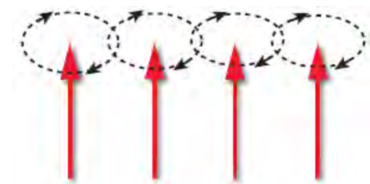
Up- & down-magnons $\sigma g \mu_{\mathrm{B}}$ along the opposite direction → $N_{0\sigma} = \pm 1 = \sigma$

FM

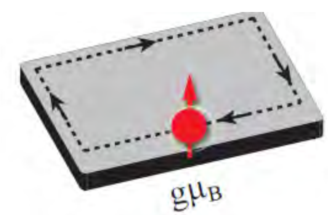
TKNN integer: Chiral edge

KN, Klinovaja & Loss, PRB (2017)

$$\mathcal{N}_{0\uparrow} = +1$$



Chiral edge:

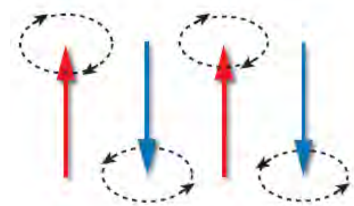


AF

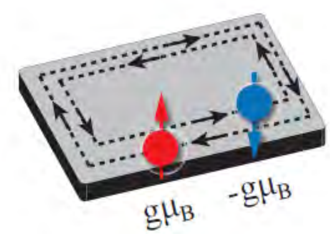
Magnonic TI: Helical edge

KN, Kim, Klinovaja & Loss (2017) $\sigma = \pm 1 = \uparrow, \downarrow$

$$\mathcal{N}_{0\sigma} = \sigma$$



Helical edge:

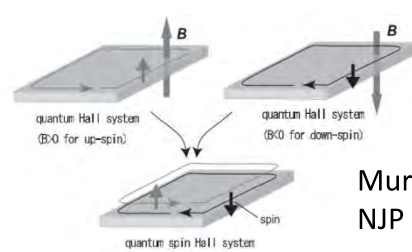


Total Chern number: $\sum_{\sigma} \mathcal{N}_{0\sigma} = \mathcal{N}_{0\uparrow} + \mathcal{N}_{0\downarrow} = 0$

$\mathbb{Z}_2$ topological invariant: $\mathcal{Z}_0 \equiv \frac{(\mathcal{N}_{0\uparrow} - \mathcal{N}_{0\downarrow})}{2} = 1$

Hasan & Kane, RMP (2010) *etc.*

cf. An electronic TI by AB phase



Murakami, NJP (2007)

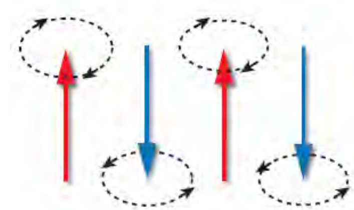
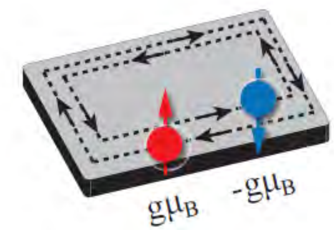
Magnonic TI: AF in AC effect

cf. KN, Klinovaja & Loss, PRB (2017)

Spin — Nernst*: Ensured by $\mathbb{Z}_2$ invariant

$$\begin{array}{l}\text{Spin:}\\ \text{Heat:}\end{array}\begin{pmatrix}\langle \mathcal{J}_y\rangle \\ \langle \mathcal{J}_y^Q\rangle\end{pmatrix} = \begin{pmatrix} L'_{11}(\mathcal{N}_{0\uparrow}+\mathcal{N}_{0\downarrow}) & L'_{12}(\mathcal{N}_{0\uparrow}-\mathcal{N}_{0\downarrow}) \\ L'_{21}(\mathcal{N}_{0\uparrow}-\mathcal{N}_{0\downarrow}) & L'_{22}(\mathcal{N}_{0\uparrow}+\mathcal{N}_{0\downarrow})\end{pmatrix}\begin{pmatrix}\partial_x B \\ -\frac{\partial_x T}{T}\end{pmatrix}$$

Ettinghausen — Heat

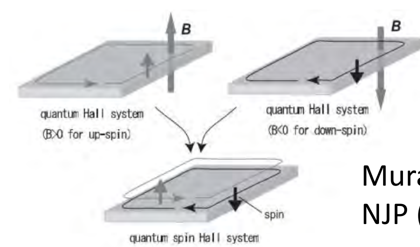
AF	**Magnonic TI:** Helical edge KN, Kim, Klinovaja & Loss (2017) $\sigma = \pm 1 = \uparrow, \downarrow$ $$\mathcal{N}_{0\sigma} = \sigma$$	 Helical edge: 

Total Chern number: $\sum_\sigma \mathcal{N}_{0\sigma} = \mathcal{N}_{0\uparrow} + \mathcal{N}_{0\downarrow} = 0$

$\mathbb{Z}_2$ topological invariant: $\mathcal{Z}_0 \equiv \frac{(\mathcal{N}_{0\uparrow} - \mathcal{N}_{0\downarrow})}{2} = 1$

Hasan & Kane, RMP (2010) *etc.*

cf. An electronic TI by AB phase



Murakami, NJP (2007)

NOTE. AF magnon Nernst*: Report of observation in a AF [Shiomi, Takashima & Saitoh, arXiv:1706.03978].
cf. Theory [Cheng *et al.*, PRL (2016)] & [Zyuzin *et al.*, PRL (2016)]

LAST QUESTION

Is magnonic Wiedemann-Franz law satisfied in insulating AFs ?

FM: [KN, Simon & Loss, PRB (2015)] & [KN, Klinovaja & Loss, PRB (2017)]

``No'' in topological AF (magnonic TI):

$G_{\mathrm{AF}}^{yx}=0, \quad K_{\mathrm{AF}}^{yx}=0$: Topological invariant $\sum_{\sigma}\mathcal{N}_{0\sigma}=\mathcal{N}_{0\uparrow}+\mathcal{N}_{0\downarrow}=0$

$\because G_{\mathrm{AF}}^{yx} \propto \mathcal{N}_{0\uparrow}+\mathcal{N}_{0\downarrow}=0 \quad K_{\mathrm{AF}}^{yx} \propto \mathcal{N}_{0\uparrow}+\mathcal{N}_{0\downarrow}=0$

``Yes'' in non-topological AF:

WF law for bulk magnons AF: $\frac{K_{\mathrm{AF}}}{G_{\mathrm{AF}}} \rightleftarrows \frac{5}{2}\left(\frac{k_{\mathrm{B}}}{g\mu_{\mathrm{B}}}\right)^2 T$ FM: $\frac{K_{\sigma}}{G_{\sigma}} \rightleftarrows \frac{5}{2}\left(\frac{k_{\mathrm{B}}}{g\mu_{\mathrm{B}}}\right)^2 T$

∵ AF = Independent copies of FM

GOAL

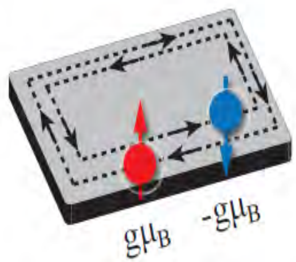
Topological insulator

Kane & Mele, PRL (2005, 2005).
Bernevig & Zhang, PRL (2006).

Magnonic Topological insulator: A bosonic analog

KN, Kim, Klinovaja & Loss (2017).
arXiv:1707.07427

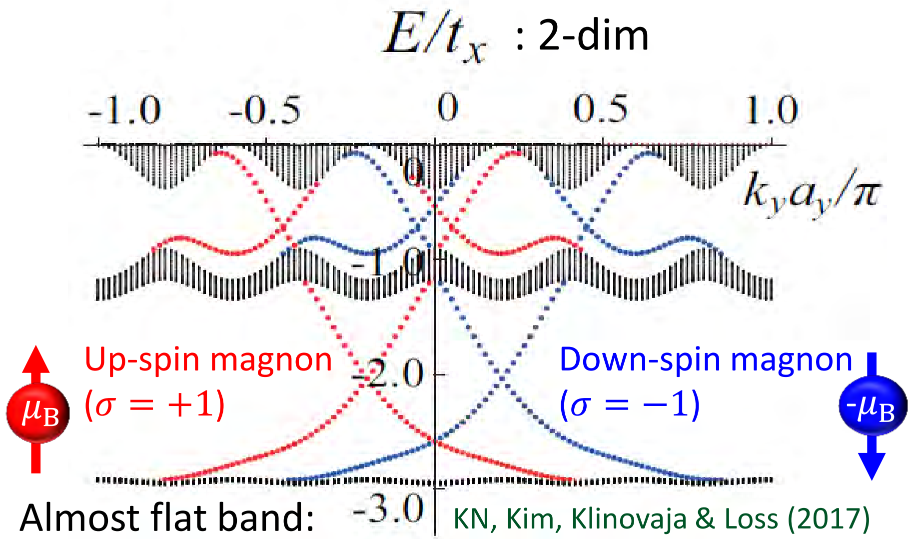
SUMMARY



Magnonic Topological Insulator in Antiferromagnet:

Helical edge magnon state in Aharonov-Casher effect

KN (Basel), S. K. Kim (UCLA), J. Klinovaja, D. Loss. arXiv:1707.07427



KN, Klinovaja & Loss, PRB (2017)

Chern integer: $\mathcal{N}_{0\sigma} = \sigma$

$$\sum_\sigma \mathcal{N}_{0\sigma} = \mathcal{N}_{0\uparrow} + \mathcal{N}_{0\downarrow} = 0$$

$\mathbb{Z}_2$ topological invariant:

$$\mathcal{Z}_0 \equiv \frac{(\mathcal{N}_{0\uparrow} - \mathcal{N}_{0\downarrow})}{2} = 1$$

Spin — Nernst*: Ensured by $\mathbb{Z}_2$ invariant

$$\begin{matrix}\text{Spin:}\\ \text{Heat:}\end{matrix}\begin{pmatrix}\langle \mathcal{J}_y \rangle \\ \langle \mathcal{J}_y^Q \rangle\end{pmatrix} = \begin{pmatrix} L'_{11}(\mathcal{N}_{0\uparrow} + \mathcal{N}_{0\downarrow}) & L'_{12}(\mathcal{N}_{0\uparrow} - \mathcal{N}_{0\downarrow}) \\ L'_{21}(\mathcal{N}_{0\uparrow} - \mathcal{N}_{0\downarrow}) & L'_{22}(\mathcal{N}_{0\uparrow} + \mathcal{N}_{0\downarrow}) \end{pmatrix} \begin{pmatrix} \partial_x B \\ -\frac{\partial_x T}{T} \end{pmatrix}$$

Ettinghausen — Heat

Magnonic WF law:

KN, Simon & Loss, PRB (2015)

No, in topological AF.

Yes, in trivial AF.

*Report of observation in a AF [Shiomi, Takashima & Saitoh, arXiv:1706.03978]

APPENDIX

NOTE: *pptx-file for animation is available in the link:*
https://www.dropbox.com/s/9be58ryqrdhsyu5/AFmagnonicZ2TI_Animation_KoukiNakata.pptx?dl=0

Onsager Coefficients: Bulk Magnon Transport

KN, Kim, Klinovaja, Loss (2017)

Topologically trivial AF

Hamiltonian: $\mathcal{H} = \sum_{\sigma=\uparrow,\downarrow} \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}\sigma} a^\dagger_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}$

$$\begin{pmatrix} \langle j_{x\sigma} \rangle \\ \langle j^Q_{x\sigma} \rangle \end{pmatrix} = \begin{pmatrix} \overset{G_\sigma}{L_{11\sigma}} & L_{12\sigma} \\ L_{21\sigma} & L_{22\sigma} \end{pmatrix} \begin{pmatrix} \partial_x B \\ -\partial_x T/T \end{pmatrix}$$

Particle current: $j^{\mathrm{P}}_{x\sigma} = j_{x\sigma}/(\sigma g \mu_{\mathrm{B}})$

Thermal conductivity: $K_\sigma = \frac{1}{T}\left(L_{22\sigma} - \frac{L_{12\sigma}L_{21\sigma}}{L_{11\sigma}}\right)$

WF law for bulk magnons:

AF: $\frac{K_{\mathrm{AF}}}{G_{\mathrm{AF}}} \overset{\rightarrow}{=} \frac{5}{2}\left(\frac{k_{\mathrm{B}}}{g\mu_{\mathrm{B}}}\right)^2 T$ FM: $\frac{K_\sigma}{G_\sigma} \overset{\rightarrow}{=} \frac{5}{2}\left(\frac{k_{\mathrm{B}}}{g\mu_{\mathrm{B}}}\right)^2 T$

Topological AF

Hamiltonian: $\mathcal{H}_{\mathrm{AC}} = -\sum_{\sigma=\uparrow,\downarrow} \sum_{\langle ij \rangle} (t_{ij} e^{i\sigma\theta_{ij}} a_{i\sigma} a^\dagger_{j\sigma} + \mathrm{H.c.})$

i.e., $\mathcal{H}_{\mathrm{m}\sigma} = \frac{1}{2m}\left(\mathbf{p} + \sigma \frac{g\mu_{\mathrm{B}}}{c}\mathbf{A}_{\mathrm{m}}\right)^2$

$$\begin{pmatrix} \langle j_{y\sigma} \rangle \\ \langle j^Q_{y\sigma} \rangle \end{pmatrix} = \begin{pmatrix} \overset{G_\sigma{}^{yx}}{L^{yx}_{11\sigma}} & L^{yx}_{12\sigma} \\ L^{yx}_{21\sigma} & L^{yx}_{22\sigma} \end{pmatrix} \begin{pmatrix} \partial_x B \\ -\partial_x T/T \end{pmatrix}$$

Particle Hall current: $j^{\mathrm{P}}_{y\sigma} = j_{y\sigma}/(\sigma g \mu_{\mathrm{B}})$

$$\langle j^{\mathrm{P}}_{y\sigma} \rangle = \sigma \mathcal{N}_{0\sigma} \frac{L'_{11}}{g\mu_{\mathrm{B}}} \partial_x B - \mathcal{N}_{0\sigma} \frac{L'_{12}}{g\mu_{\mathrm{B}}} \frac{\partial_x T}{T}$$

Spin & thermal Hall conductances:

$$G^{yx}_{\mathrm{AF}} = 0, \quad K^{yx}_{\mathrm{AF}} = 0 \quad K^{yx}_\sigma = \left(L^{yx}_{22\sigma} - \frac{L^{yx}_{21\sigma}L^{yx}_{12\sigma}}{L^{yx}_{11\sigma}}\right)/T$$

Antiferromagnet	Topologically trivial bulk: Sec. II	Topological bulk: Sec. III B
Magnetic field gradient:	Helical magnon transport. Spin: $\sum_\sigma L_{11\sigma} \neq 0$. Heat: $\sum_\sigma L_{21\sigma} = 0$.	Magnon Hall transport. Spin: $\sum_\sigma L^{yx}_{11\sigma} = 0$. Heat: $\sum_\sigma L^{yx}_{21\sigma} \neq 0$.
Thermal gradient:	Magnon transport. Spin: $\sum_\sigma L_{12\sigma} = 0$. Heat: $\sum_\sigma L_{22\sigma} \neq 0$.	Helical magnon Hall transport. Spin: $\sum_\sigma L^{yx}_{12\sigma} \neq 0$. Heat: $\sum_\sigma L^{yx}_{22\sigma} = 0$.

Cyclotron Motion Along the Opposite Direction

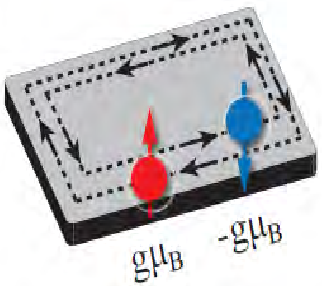


Fig. 1 (b)

KN, Kim, Klinovaja, Loss (2017)

Indeed, they form the same Landau levels[43] with the principal quantum number $n_\sigma \in \mathbb{N}_0$,

$$E_{n_\sigma} = \hbar\omega_c\left(n_\sigma + \frac{1}{2}\right) + \Delta \quad \text{for} \quad n_\sigma \in \mathbb{N}_0, \tag{15}$$

and the two magnons $\sigma g\mu_B \mathbf{e}_z$ perform cyclotron motions with the same frequency[43]

$$\omega_c = \frac{g\mu_B \mathcal{E}}{mc^2} \tag{16}$$

and same electric length[43] $l_\mathcal{E}$, defined by

$$l_\mathcal{E} \equiv \sqrt{\hbar c^2/g\mu_B \mathcal{E}}, \tag{17}$$

but along opposite direction, cf. Fig. 1 (b),

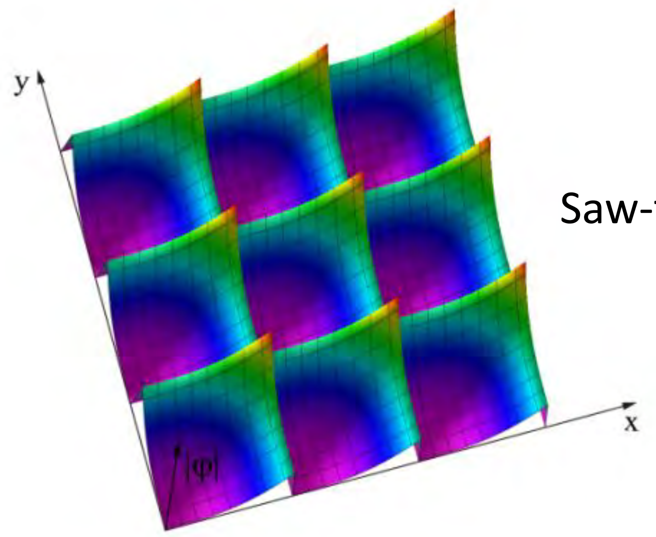
$$\frac{d}{dt}(\mathcal{R}_{x\sigma} + i\mathcal{R}_{y\sigma}) = i\sigma\omega_c(\mathcal{R}_{x\sigma} + i\mathcal{R}_{y\sigma}), \tag{18}$$

where $\mathbf{R}_{\mathcal{E}\sigma} = (\mathcal{R}_{x\sigma}, \mathcal{R}_{y\sigma})$ is the relative coordinate[82]. The factor σ in Eq. (18) is rooted in the magnetic dipole moment $\sigma g\mu_B \mathbf{e}_z$ of a magnon. The source of cyclotron motion is the electric field gradient $\mathcal{E}$ [Eqs. (14) and (16)], which is common to the both modes.

where $\mathbf{R}_{\mathcal{E}\sigma} = (\mathcal{R}_{x\sigma}, \mathcal{R}_{y\sigma}) \equiv (-l_\mathcal{E}^2 \Pi_{y\sigma}/\hbar, l_\mathcal{E}^2 \Pi_{x\sigma}/\hbar)$ and $\mathbf{\Pi}_\sigma \equiv \mathbf{p} + \sigma g\mu_B \mathbf{A}_\mathrm{m}/c$

Q. Helical edge magnon state:

Still exist in `periodic' electric vector potential $\boldsymbol{A}_{\mathbf{m}}$?



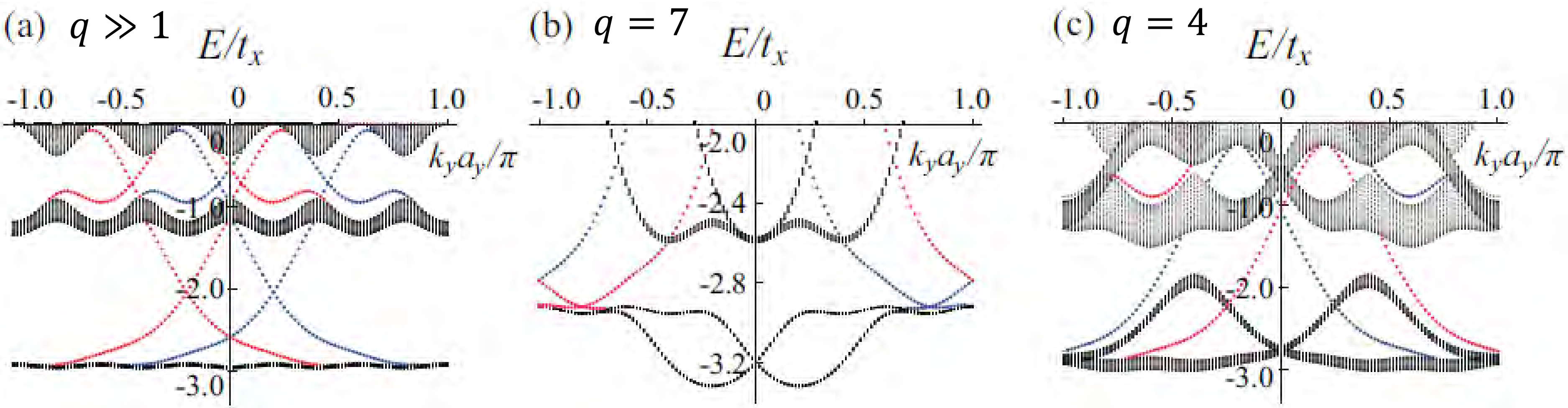
Saw-tooth shape

ANSWER: **YES.**

KN, S. K. Kim, J. Klinovaja, D. Loss (2017)

Helical Edge Magnon States in Periodic potential A_m

KN, Kim, Klinovaja, Loss (2017)



AC phase: $\theta_n = n\theta_0 \quad \theta_0 \equiv (g\mu_B/\hbar c^2)\mathcal{E} a_x a_y$

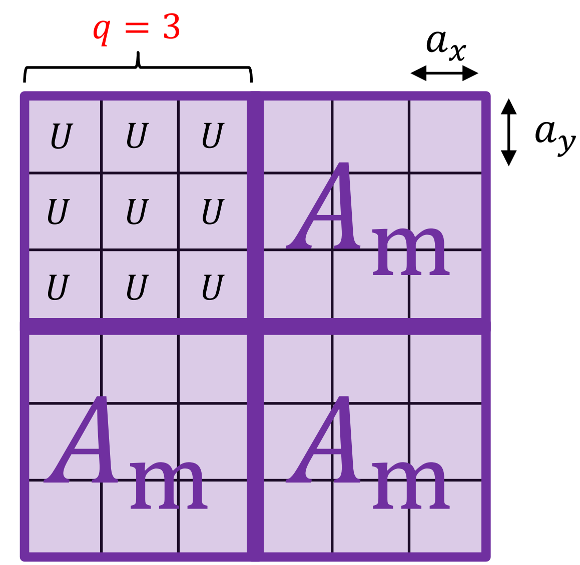
Landau gauge: $\mathbf{A}'_{\mathrm{m}} = (\mathcal{E}/c)(0, x, 0)$ → Spectrum: $E = E(k_y)$

$$\mathcal{H}_{\mathrm{AC}} = -\sum_{\sigma=\uparrow,\downarrow}\sum_{\langle ij\rangle}(t_{ij}\mathrm{e}^{i\sigma\theta_{ij}} a_{i\sigma}a^{\dagger}_{j\sigma} + \mathrm{H.c.})$$

$$\mathcal{H}_{\mathrm{AC}} = \sum_{k_y} H_{k_y}$$

$$H_{k_y} = -t_x\sum_{n,\sigma}(a^{\dagger}_{k_y,n+1,\sigma}a_{k_y,n,\sigma} + \mathrm{H.c.}) - 2t_y\sum_{n,\sigma}[\cos(k_y a_y + \sigma\theta_n)]a^{\dagger}_{k_y,n,\sigma}a_{k_y,n,\sigma}$$

Periodicity: $R_q = q a_x \quad \mathbf{A}'_{\mathrm{m}} \to \mathbf{A}'_{\mathrm{mq}} = (\mathcal{E}R_q/c)(0, \{x/R_q\}, 0)$



An Intuitive Criterion for Topological Edges

Key: Magnons experience geometric (AC) phase globally & macroscopically (*)

→ Otherwise no topological edge modes (#)

35

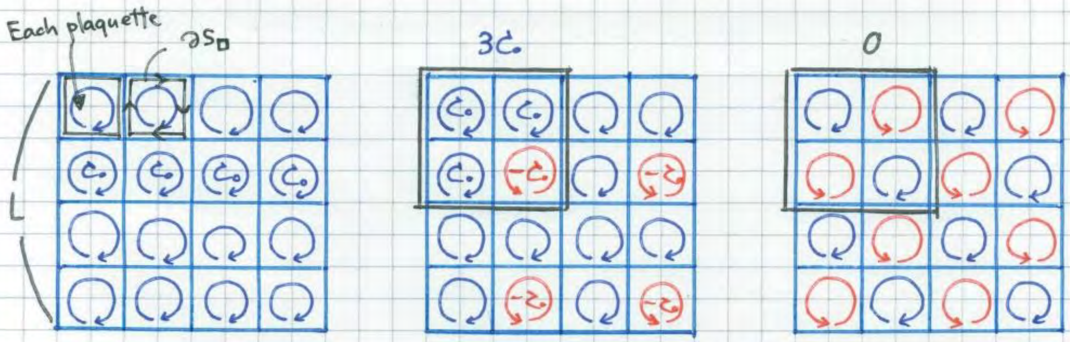
▶ Conclusion: theoretical proposal for a criterion to generate topological edge

$\nabla \times A_m = c_0 e_z$ — Source of cyclotron motion

↳ globally or macroscopically constant so that (in the sense)

$$\lim_{L\to\infty} \frac{\sum_{\square} \int dS_{\square} \cdot \nabla\times A_m}{L} = \lim_{L\to\infty}\frac{\sum_{\square} \oint_{\partial S_{\square}} dl\cdot A_m}{L} \neq 0$$

···(*)



→ Topological edge → Topological edge → No topological edge

※ Frequency of cyclotron motion $\omega_c \propto \varepsilon \propto |\nabla\times A_m|$

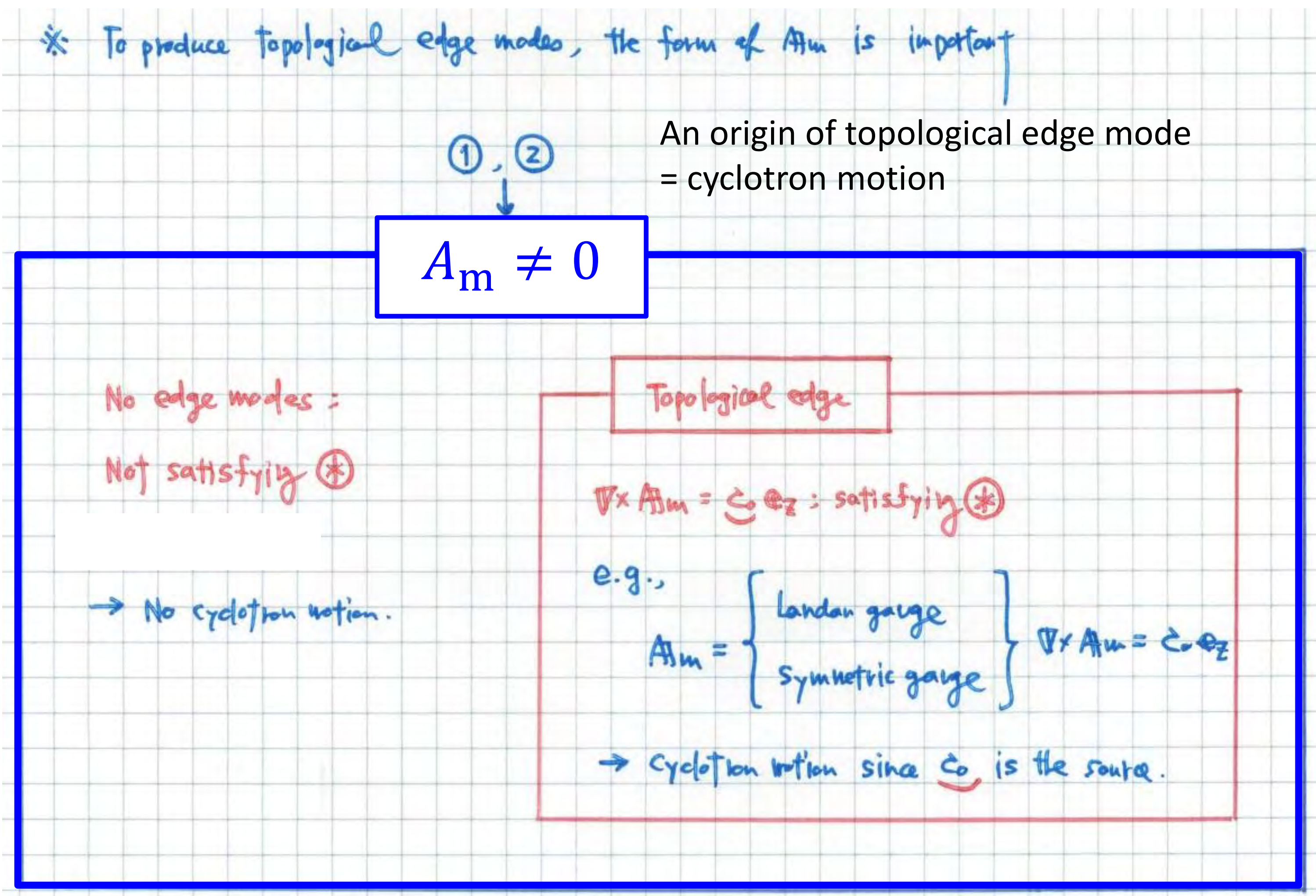
i.e., $\nabla\times A_m = \frac{\varepsilon}{c} e_z$ in Landau or symmetric gauge.

Remember AB effect; $\nabla\times A_e = B e_z$ — Source of cyclotron motion.

An Intuitive Criterion for Topological Edges

Key: Magnons experience geometric (AC) phase globally & macroscopically **(∗)**

→ Otherwise no topological edge modes (#)



Magnonic ``Quantum'' Hall Effect & WF Law

KN, Klinovaja & Loss, Phys. Rev. B 95, 125429 (2017)

See also [KN, Simon & Loss, Phys. Rev. B 92, 134425 (2015)]
for magnonic WF law in insulating FM

Chiral Edge Magnon State: Isotropy

Tight-binding model: $\mathcal{H}_{\rm m} = \sum_{k_y} H_{k_y}$ $\quad t_{ij} = J_{ij}S$

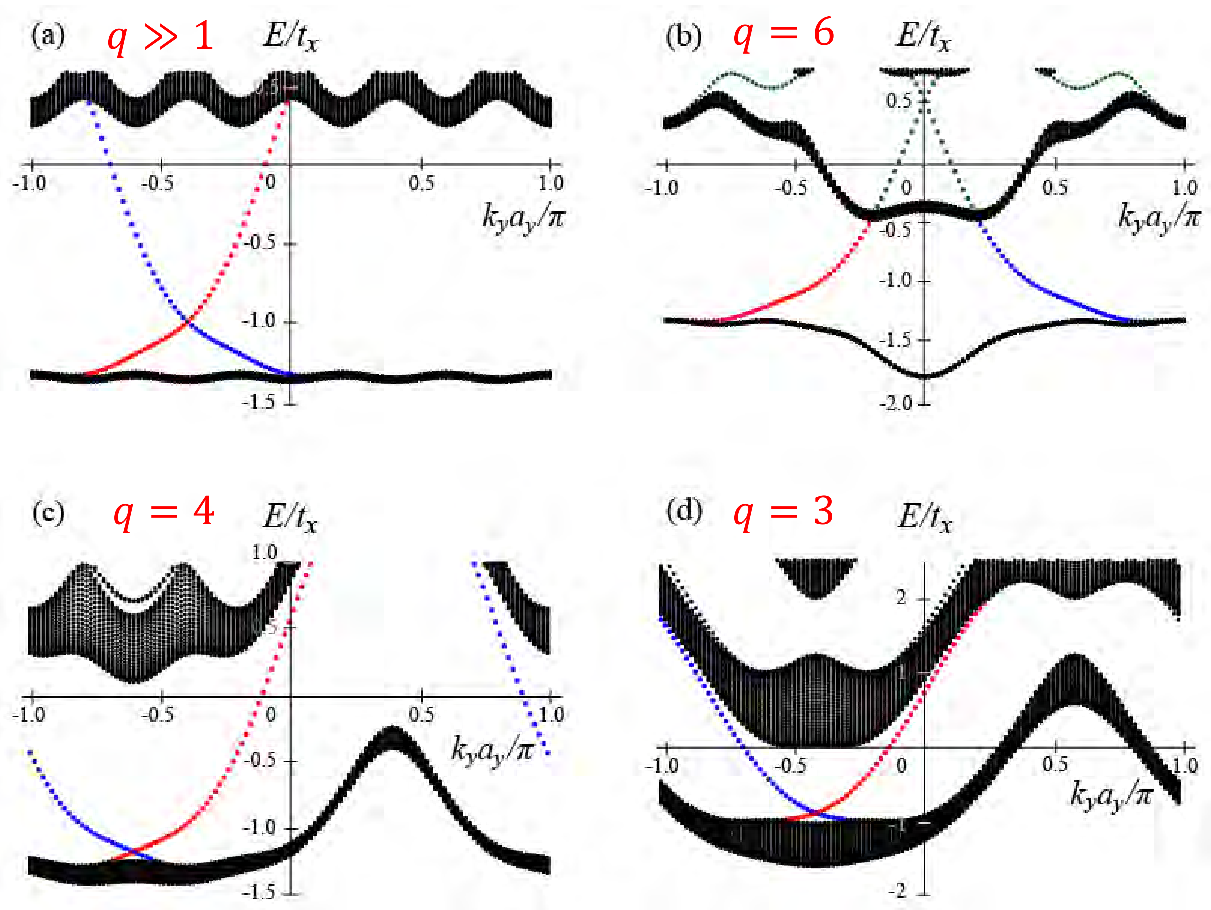
$$H_{k_y} = -t_x \sum_n (a^\dagger_{k_y,n+1} a_{k_y,n} + \text{H.c.}) - 2t_y \sum_n [\cos(k_y a_y + \theta_n)] a^\dagger_{k_y,n} a_{k_y,n}$$

AC phase: $\theta_n = n\theta_0$ $\quad \theta_0 \equiv (g\mu_{\rm B}/\hbar c^2)\mathcal{E} a_x a_y$

Landau gauge: $\mathbf{A}'_{\rm m} = (\mathcal{E}/c)(0,x,0)$ → Spectrum: $E = E(k_y)$

Periodicity: $R_q = q a_x$ $\quad \mathbf{A}'_{\rm m} \to \mathbf{A}'_{\rm mq} = (\mathcal{E}R_q/c)(0,\{x/R_q\},0)$

cf., Spin Hamiltonian: $t_{ij} = J_{ij}S$ $\quad \mathcal{H}_{\rm m} = -\sum_{\langle ij\rangle} t_{ij} e^{i\theta_{ij}}(a_i a_j^\dagger + \text{H.c.})$



Isotropic case: $J_x = J_y$

$\theta_0 = 2\pi/5 < \pi$

(a)-(d): Chiral edge states

(a)-(b): NOT flat bulk gap

(c)-(d): Bulk gap ``closed''
→ Gapless

NOTE) Weak disorder:
Edge mode will not couple to bulk
~ Weyl systems
cf., Weyl magnon in AF [Li *et al.*, Nat. Comm.(2016)]

Magnonic Wiedemann-Franz Law

KN, Simon, and Loss: Phys. Rev. B **92**, 134425 (2015)

See also [KN, Simon & Loss J. Phys. D. 50, 114004 (2017)] for review article

Universal Thermomagnetic Properties

Magnonic Wiedemann-Franz Law:

KN, Simon, and Loss, Phys. Rev. B **92**, 134425 (2015)

Ratios of L^{ij}, WF law, Seebeck, and Peltier coefficients are material independent

e vs μ_B	**Electron (metal)**	**Magnon (FI)**
	Franz and Wiedemann, Annalen der Physik **165**, 497 (1853)	KN, Simon, and Loss, Phys. Rev. B **92**, 134425 (2015)
Statistics	Fermion	Boson
WF law (Low temp.)	$\frac{K}{\sigma} \approx \frac{L^{22} + \mathcal{O}((k_B T/\epsilon_F)^2)}{L^{11}} \overset{\rightarrow}{=} \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T$ (Free electron at low temp.)	$\frac{K}{G} \equiv \frac{L^{22} - L^{21}L^{12}/L^{11}}{L^{11}} \overset{\rightarrow}{=} \left(\frac{k_B}{g\mu_B}\right)^2 T$ Low temp. : $\hbar/(2\tau) \ll k_B T \ll g\mu_B B$
Lorenz number	$\mathcal{L} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$	$\mathcal{L}_m = \left(\frac{k_B}{g\mu_B}\right)^2$
Seebeck & Peltier	$\mathcal{S} \equiv L^{12}/L^{11}$ $\quad \Pi \equiv L^{21}/L^{11}$	$\mathcal{S} \overset{\rightarrow}{=} \frac{B}{T}$ $\quad \Pi \overset{\rightarrow}{=} B$
Onsager-Thomson relation	$L^{21} = TL^{12}$ $\quad \Pi = T\mathcal{S}$	$L^{21} = TL^{12}$ $\quad \Pi = T\mathcal{S}$

OTHERS

Electron e: Fermion	Magnon μ_{B}: Boson Review article: KN, Simon & Loss. J. Phys. D: 50, 114004 (2017). arXiv:1610.08901
Wiedemann-Franz (WF) law Franz and Wiedemann, Annalen der Physik (1853)	**Magnonic Wiedemann-Franz law** KN, Simon & Loss, Phys. Rev. B 92, 134425 (2015). arXiv:1507.03807 Slide:https://www.dropbox.com/s/5n40xudfu51ibj3/MagnonicWFlaw_KoukiNakata.pdf?dl=0
Josephson effect Josephson, Phys. Lett. (1962)	**Magnonic Josephson effect** KN, Hoogdalem, Simon & Loss, Phys. Rev. B 90, 144419 (2014). arXiv:1406.7004 KN, Simon & Loss, Phys. Rev. B 92, 014422 (2015). arXiv:1502.03865 Slide:https://www.dropbox.com/s/704l39hto8pzr79/MagnonBECtransport_KoukiNakata.pdf?dl=0
Integer quantum Hall effect Klitzing *et al.*, PRL (1980) TKNN, PRL (1982) / Kohmoto, Ann. Phys. (1985)	**Magnonic `quantum' Hall effect & the WF law**: Chiral edge KN, Klinovaja & Loss, Phys. Rev. B 95, 125429 (2017). arXiv:1611.09752 Topological ferromagnet (FM) Slide:https://www.dropbox.com/s/tmryrxua10v5vgm/MagnonicQHE_KoukiNakata.pdf?dl=0
Topological insulator (TI) Kane and Mele, PRL (2005, 2005). Bernevig and Zhang, PRL (2006). Quantum spin Hall effect (QSHE)	**Magnonic topological insulator**: Helical edge KN, Kim, Klinovaja, Loss (2017). arXiv:1707.07427 Topological antiferromagnet (AF): Magnonic quantum spin Hall effects Slide: https://www.dropbox.com/s/delxgvyumuiuh99/AFmagnonicZ2TI_KoukiNakata.pdf?dl=0

Topological Phenomena

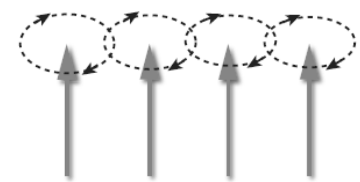
Electron: Quantum Hall Effects & Topological Insulator

1982: Thouless, Kohmoto, Nightingale, and Nijs (TKNN), PRL
1982: Halperin, PRB (1982). Hatsugai, PRL (1993)
1985: Kohmoto, Ann. Phys.
1985: Niu, Thouless, and Wu, PRB [Xiao, Chang, and Niu, RMP (2010)]

Picture from Google search

2005: Kane and Mele, PRL (2005, 2005): [Hasan and Kane, RMP (2010)]
2006: Bernevig and Zhang, PRL (2006): [Qi and Zhang, RMP (2011)]

FM insulators: Magnonic Hall Effects + …



1995: Haldane & Arovas, PRB
2009: Fujimoto, PRL

Phase twist & Berry curvature in magnonic system

2010: Onose *et al*., Science
2010: Katsura *et al*., PRL
2011: Matsumoto & Murakami, PRL & PRB

Observation of the magnon Hall effect & the theories

2013: Shindou *et al*., PRB *etc*. (2013, 2013, 2014, 2016)
2013: Zhang *et al*., PRB
2014: Mook *et al*., PRB (2014, 2014, 2015)

Magnonic Chern insulators: Chiral edge

NOTE: See [Haldane and Arovas, PRB (1995)] & [Xu, Ohtsuki, and Shindou, PRB (2016)] for disordered quantum Hall systems, and [Matsumoto & Murakami, PRL & PRB (2011)], [Shindou *et al*., PRB (2013, 2014)], & their review [Murakami & Okamoto, JPSJ (2017)} for chiral edge states in dipolar int. and the bulk-edge correspondence.