Spin Transport using Magneto-elastic Bosons

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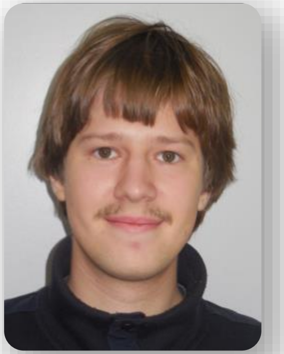
Team

University of Kaiserslautern (Germany)



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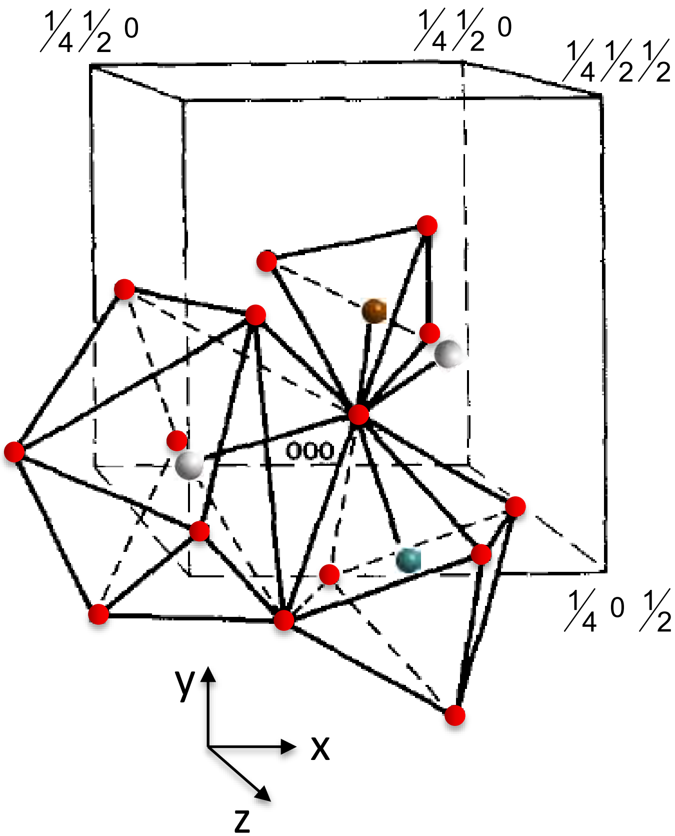
Yttrium Iron Garnet $Y_3Fe_5O_{12}$ (YIG)

"Yttrium-Iron Garnet is a marvel of nature.
Its role in the physics of magnets is analogous to that of germanium in semiconductor physics, water in hydrodynamics, and quartz in crystal acoustics."

V. Cherepanov, I. Kolokolov, and V. L'vov, *The saga of YIG: spectra, thermodynamics, interaction and relaxation of magnons in a complex magnet*, Phys. Rep. **229**, 81–144 (1993).



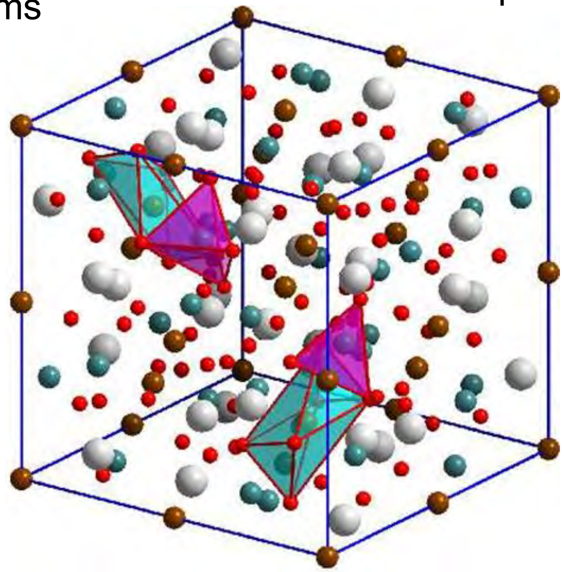
Unit cell

- 48 oxygen atoms
- 8 octahedral iron atoms (spin 5/2 up)
- 12 tetrahedral iron atoms (spin 5/2 down)
- 12 dodecahedral yttrium atoms

→ Magnetic moment of a unit cell is 10 Bohr magnetons at zero temperature

Bulk YIG crystal

Wiki



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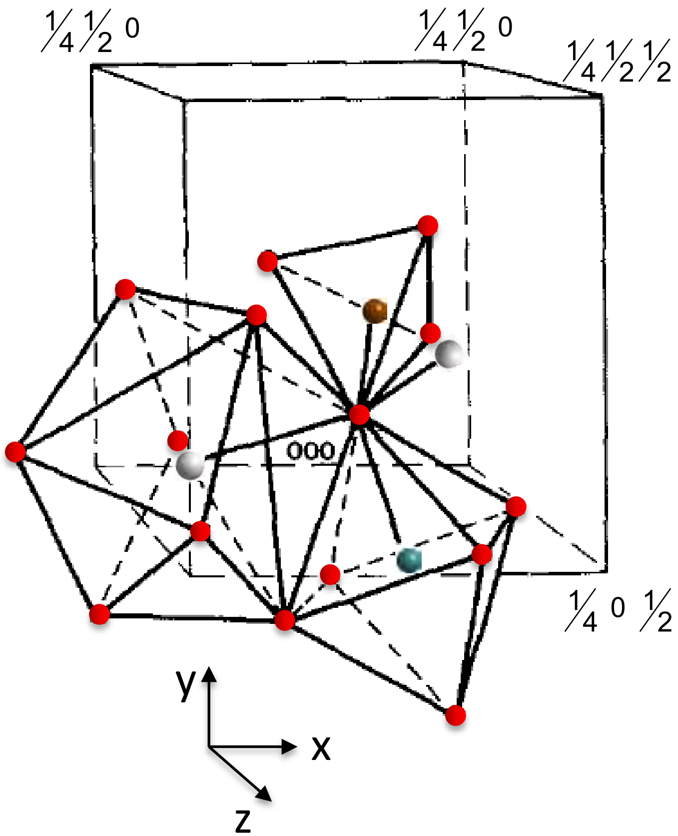

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- Room temperature ferrimagnet (T_C = 560 K)
- Longest known magnon lifetime (up to 700 ns)
- Very low phonon damping

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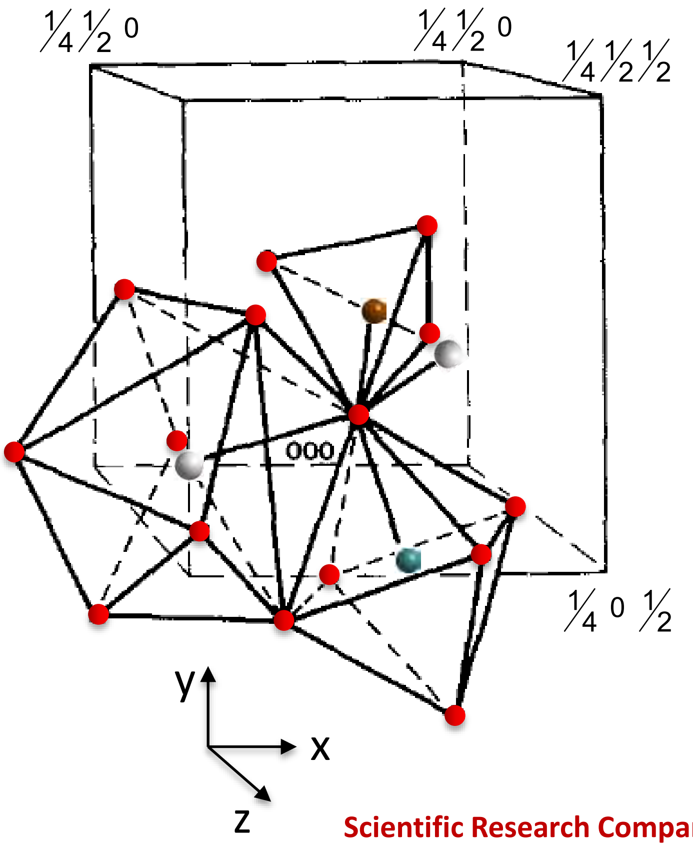

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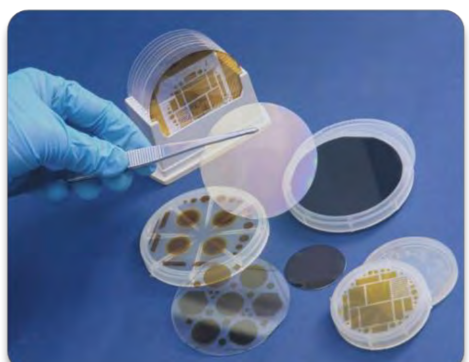
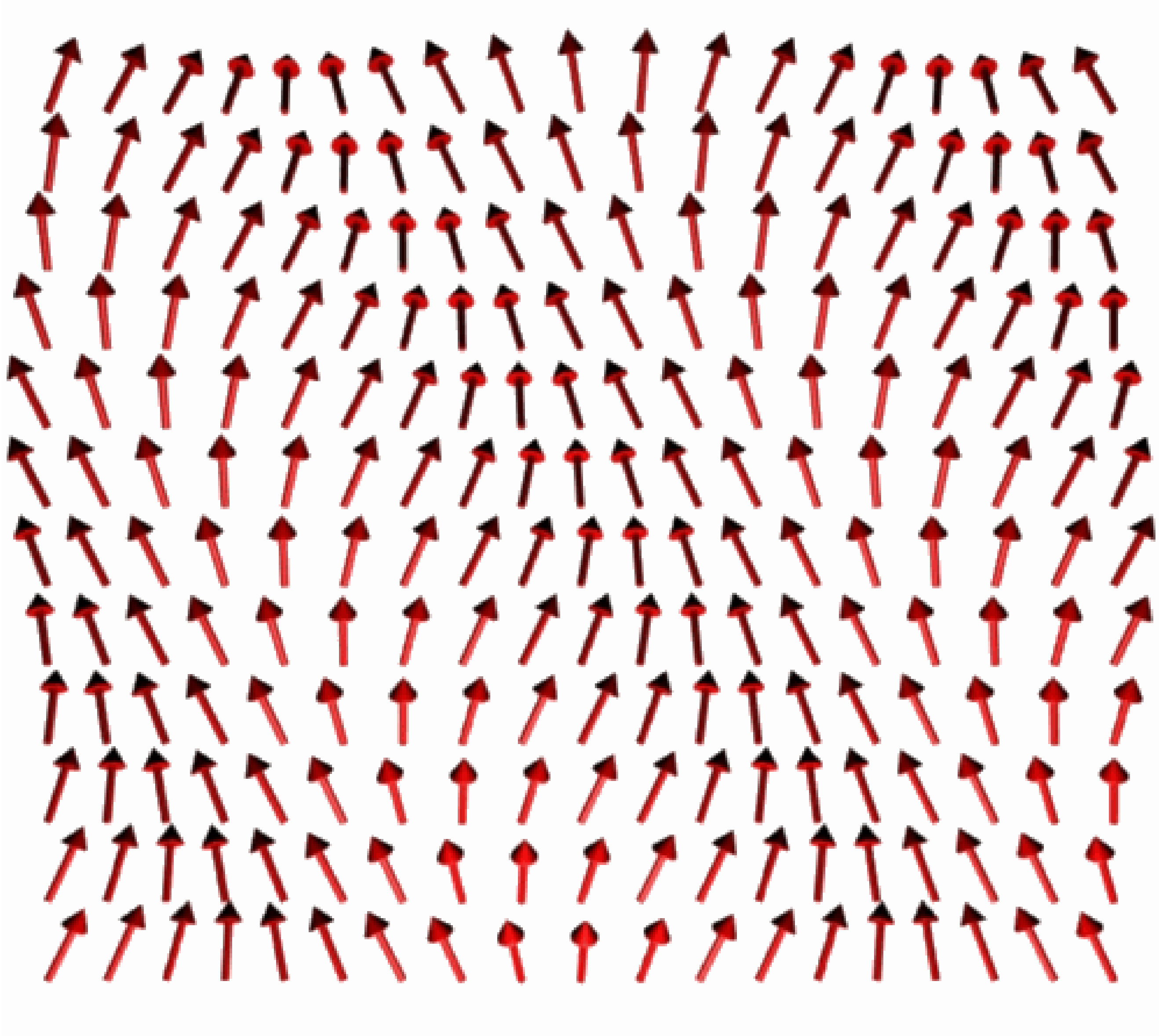
Single-crystal YIG film



Scientific Research Company "Carat", Lviv, Ukraine

- Room temperature ferrimagnet (T_C = 560 K)
- Longest known magnon lifetime (up to 700 ns)
- Very low phonon damping

Spin waves



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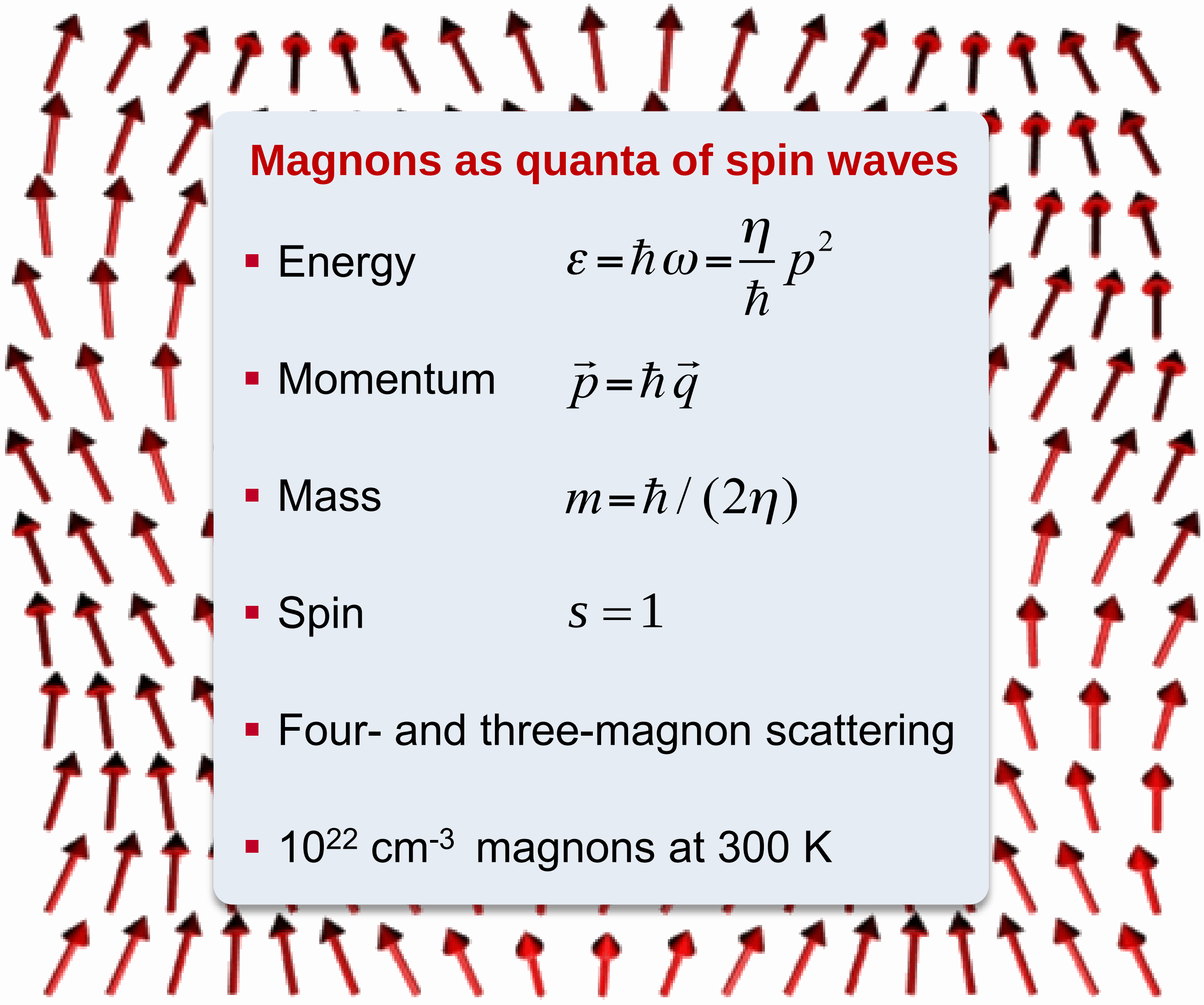
Magnon gas



Magnons as quanta of spin waves

- Energy $\varepsilon = \hbar\omega = \frac{\eta}{\hbar} p^2$
- Momentum $\vec{p} = \hbar\vec{q}$
- Mass $m = \hbar / (2\eta)$
- Spin $s = 1$
- Four- and three-magnon scattering
- 10^{22} cm^{-3} magnons at 300 K

Magnon spectrum of in-plane magnetized YIG film

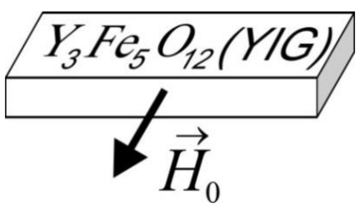
Landau-Lifshitz equation: $$\frac{\partial \vec{M}}{\partial t} = -|\gamma| \vec{M} \times \vec{H}_{\text{eff}}$$

$$\vec{H}_{\text{eff}}(\vec{r}) = \vec{H}_0 + \int_V \tilde{G}(\vec{r}, \vec{r}\,') \cdot \vec{M}(\vec{r}\,') dr'^3 + \frac{\eta}{\gamma M_S} \nabla^2 \vec{M} + \ldots$$

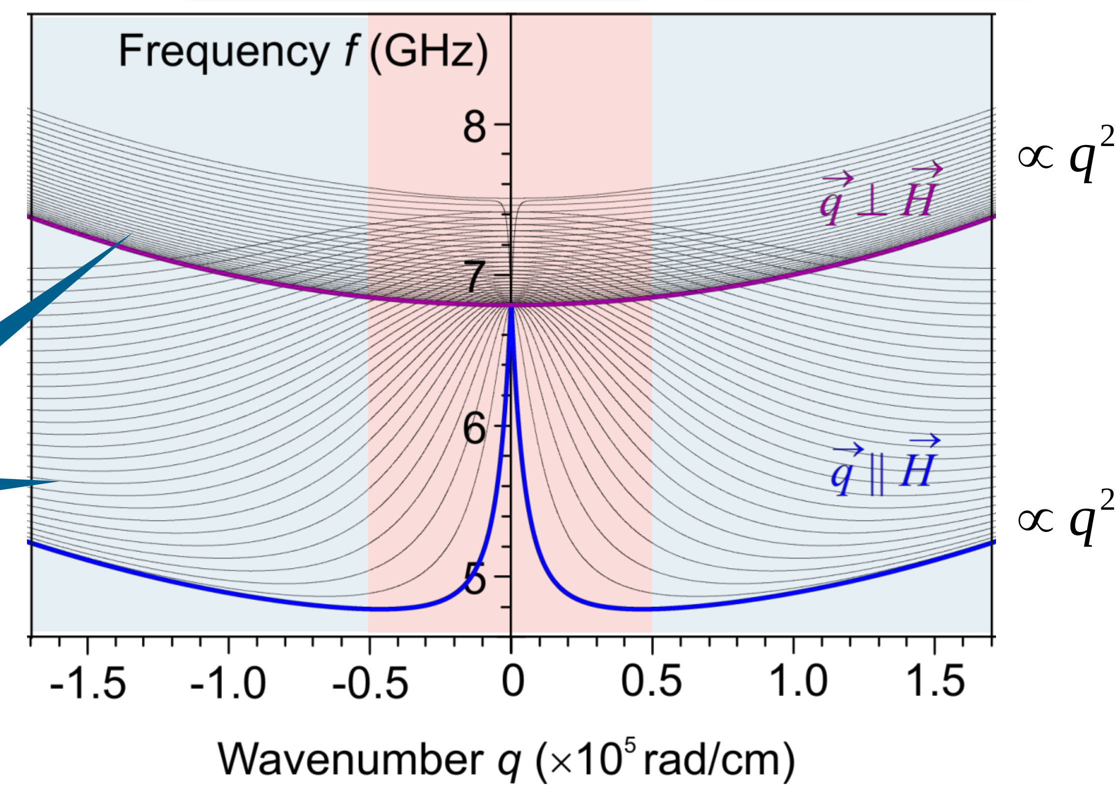
dipolar interaction

exchange interaction

$H_0 = 1710$ Oe



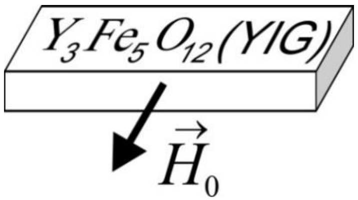
Thickness modes having a non-uniform harmonic distribution of dynamic magnetization along the film thickness



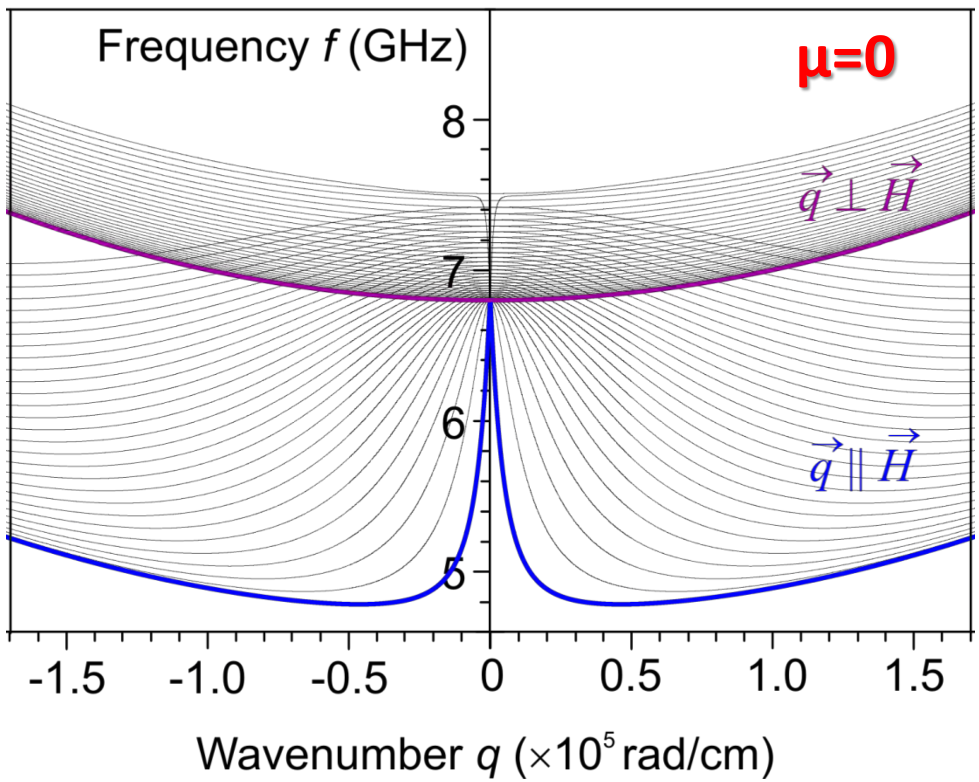
6 µm thick YIG film

Magnon distribution

$H_0 = 1710$ Oe



Magnons are bosons (s=1)
and similar to other quasi-particles
are described in thermal equilibrium
by Bose-Einstein distribution
with zero chemical potential



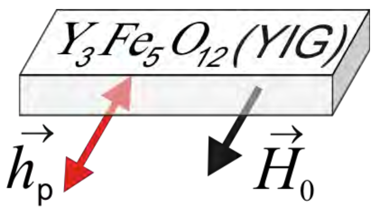
Bose-Einstein distribution

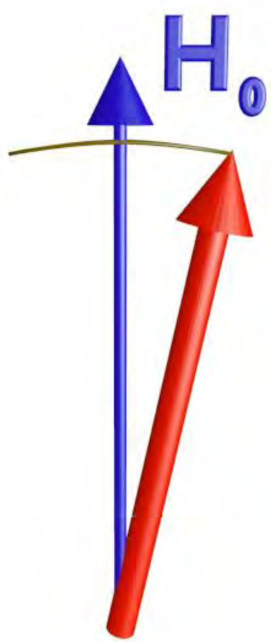
$$\rho(f) = \frac{D(f)}{\exp\left(\frac{hf - \mu}{k_B T}\right) - 1}$$

μ: chemical potential

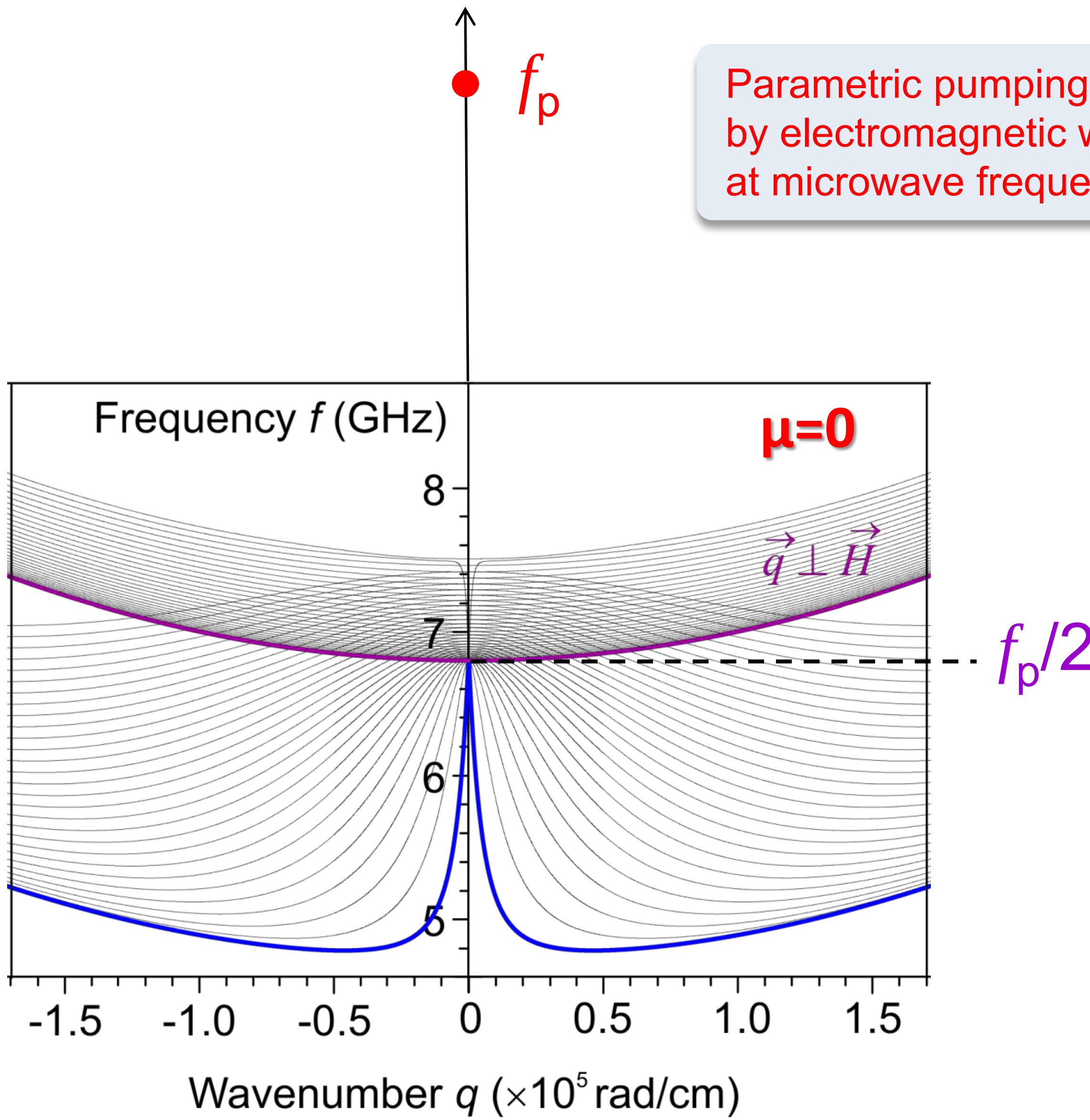
Control of magnon gas density by parametric pumping

$H_0 = 1710$ Oe





Parametric pumping
by electromagnetic wave
at microwave frequency



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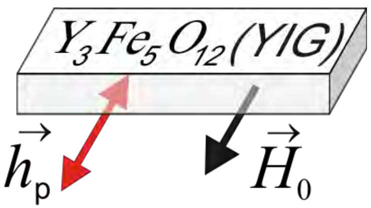
Control of magnon gas density by parametric pumping

Energy and momentum conservation laws

$$\begin{cases} \vec{q}_{\mathrm{sw}} + \vec{q}\,'_{\mathrm{sw}} = \vec{q}_{\mathrm{p}} \simeq 0 \\ f_{\mathrm{sw}} + f\,'_{\mathrm{sw}} = f_{\mathrm{p}} \end{cases}$$



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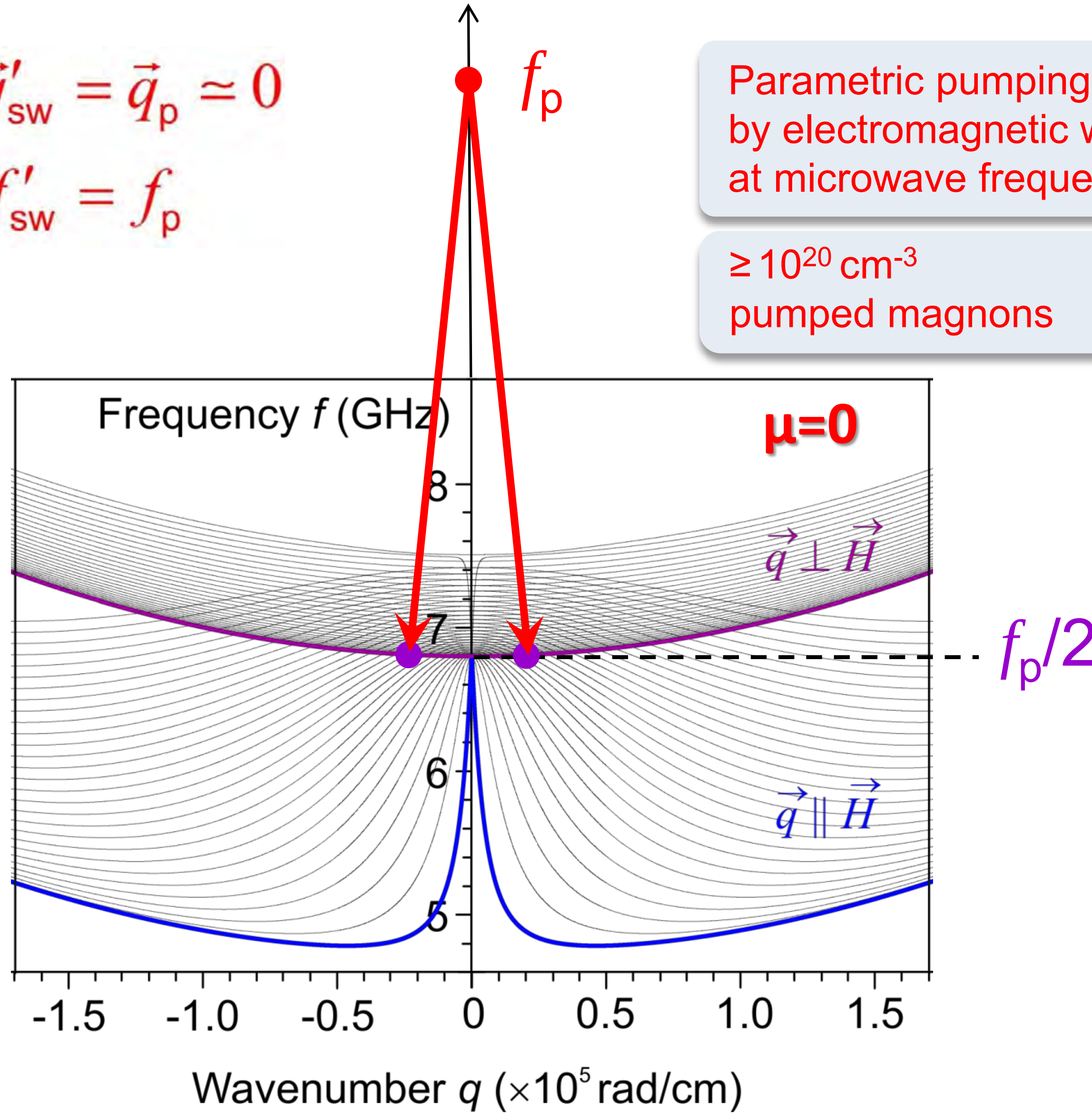
Parametric pumping by electromagnetic wave at microwave frequency

$\geq 10^{20}$ cm^{-3} pumped magnons



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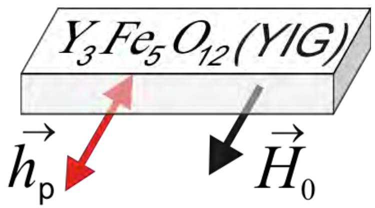

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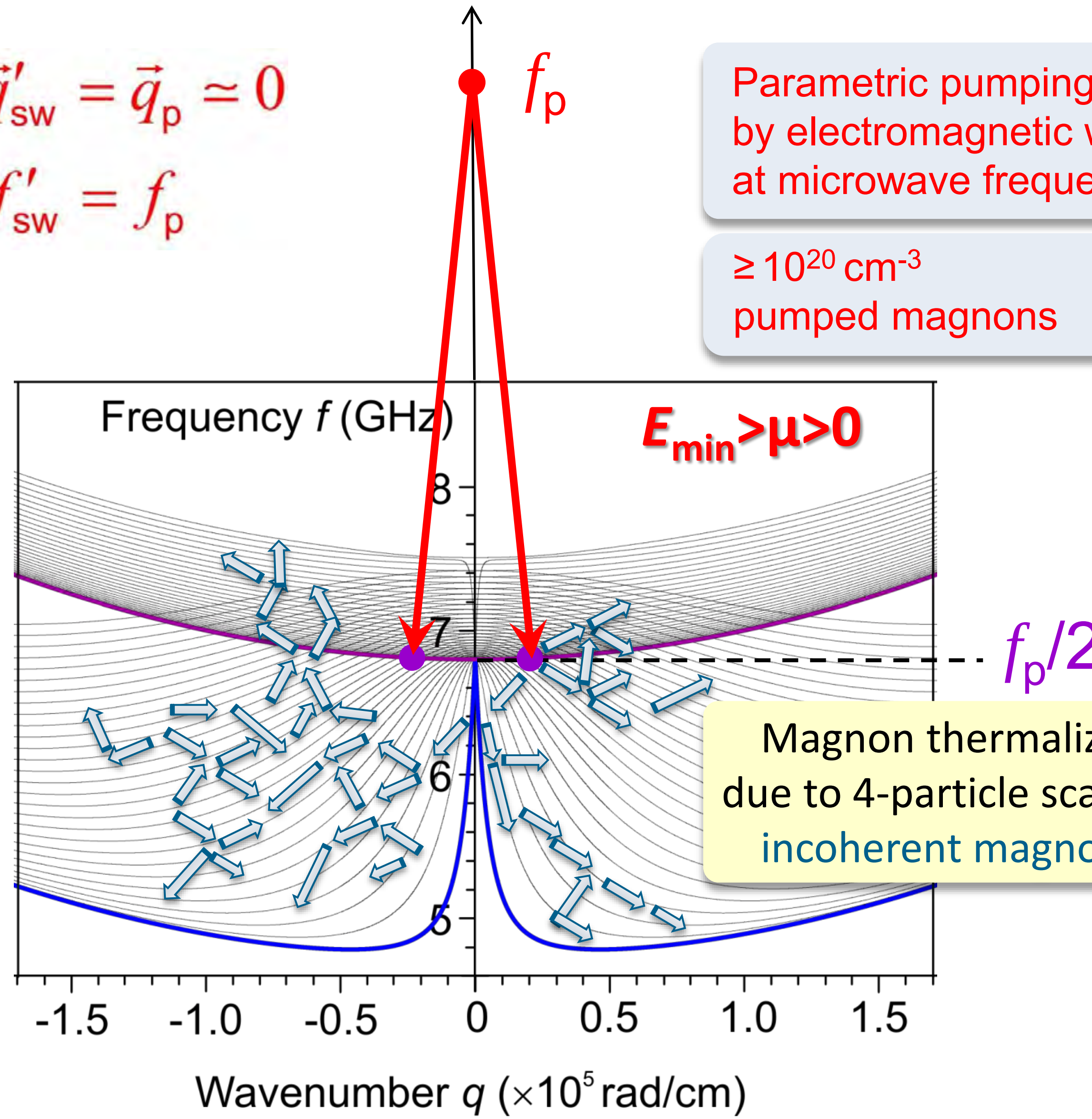


Magnon thermalization due to 4-particle scattering: incoherent magnon gas

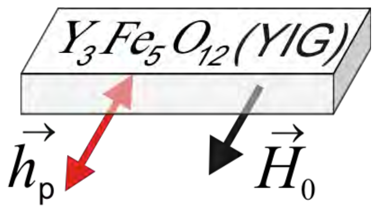
Bose-Einstein distribution

$$\rho(f) = \frac{D(f)}{\exp\left(\dfrac{hf - \mu}{k_B T}\right) - 1}$$

Bose-Einstein condensation of magnons

Energy and momentum conservation laws

$$\begin{cases} \vec{q}_{sw} + \vec{q}\,'_{sw} = \vec{q}_{p} \simeq 0 \\ f_{sw} + f'_{sw} = f_{p} \end{cases}$$



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Parametric pumping by electromagnetic wave at microwave frequency

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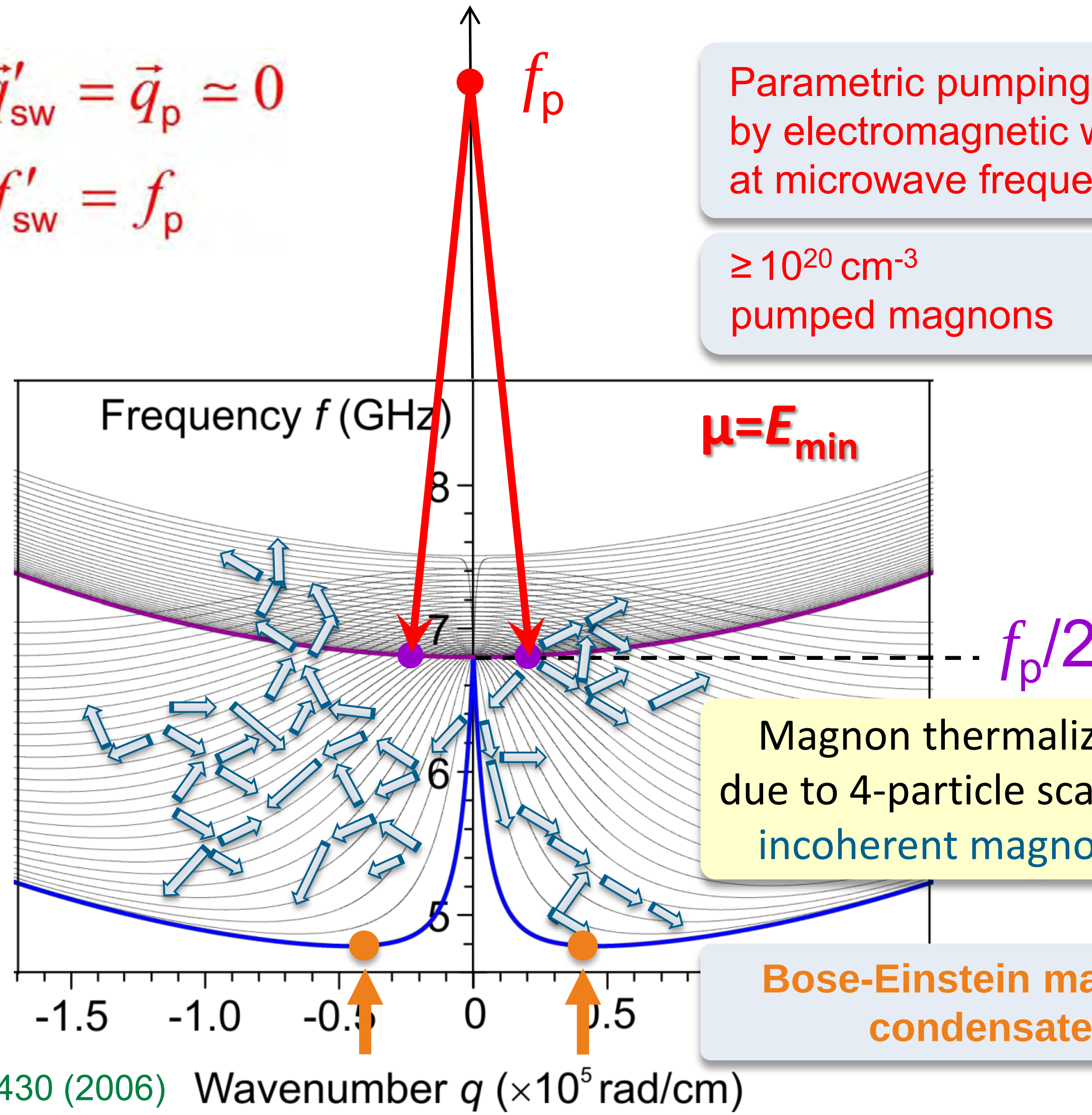
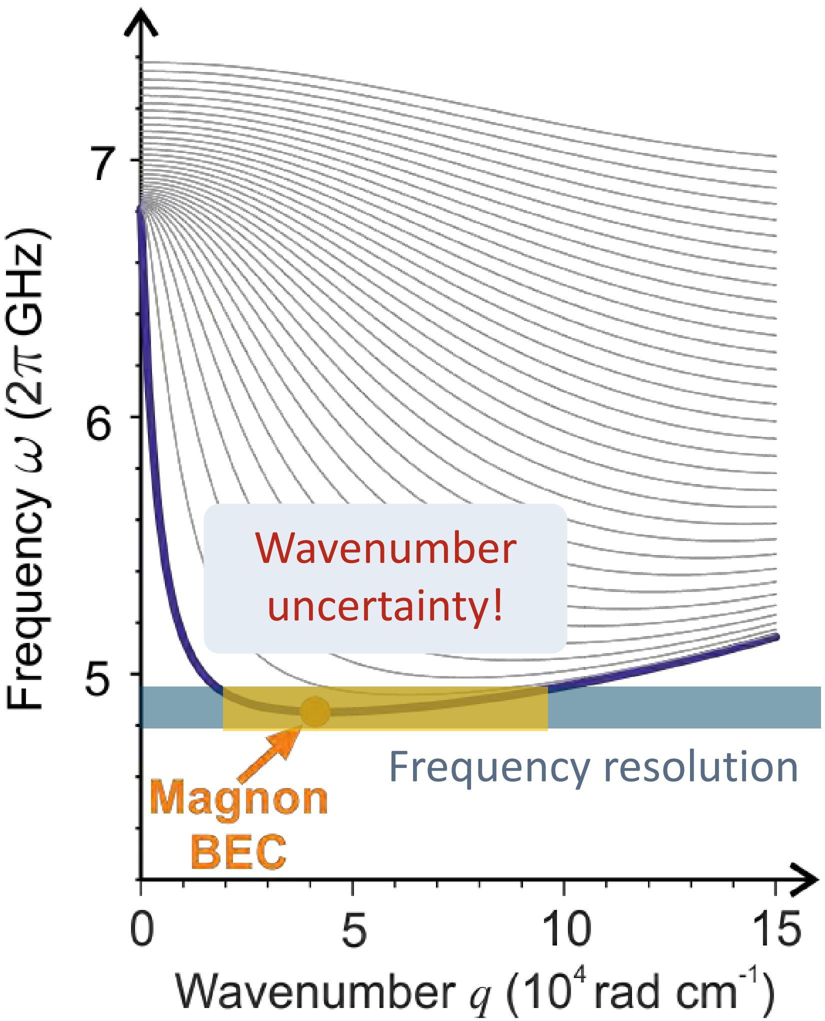


Magnon thermalization due to 4-particle scattering: incoherent magnon gas

Bose-Einstein magnon condensate

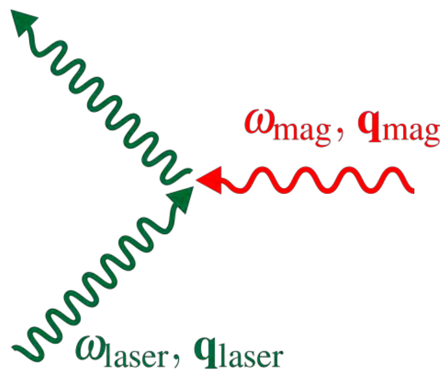
S.O. Demokritov *et al.*, Nature **443**, 430 (2006)

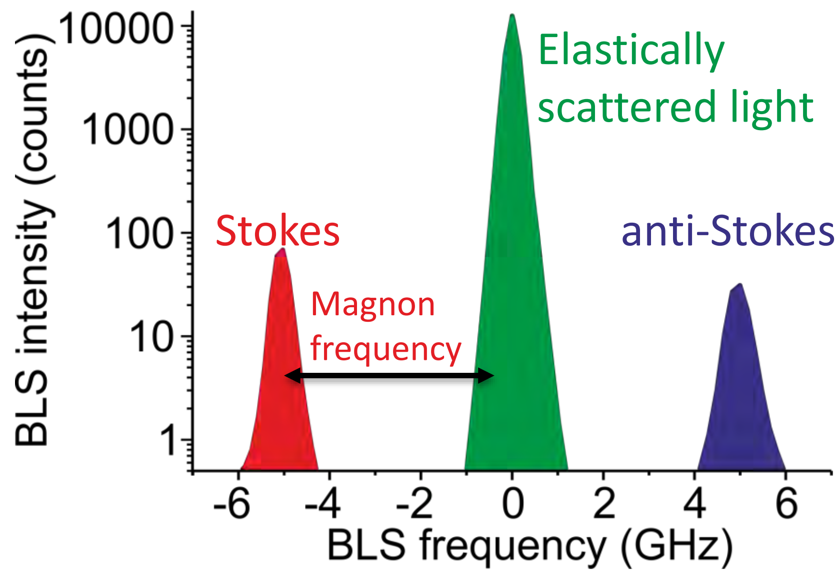
Brillouin light scattering spectroscopy



Inelastic scattering of photons on magnons:

$$\omega_{sc} = \omega_{laser} \pm \omega_{mag}$$
$$\mathbf{q}_{sc} = \mathbf{q}_{laser} \pm \mathbf{q}_{mag}$$





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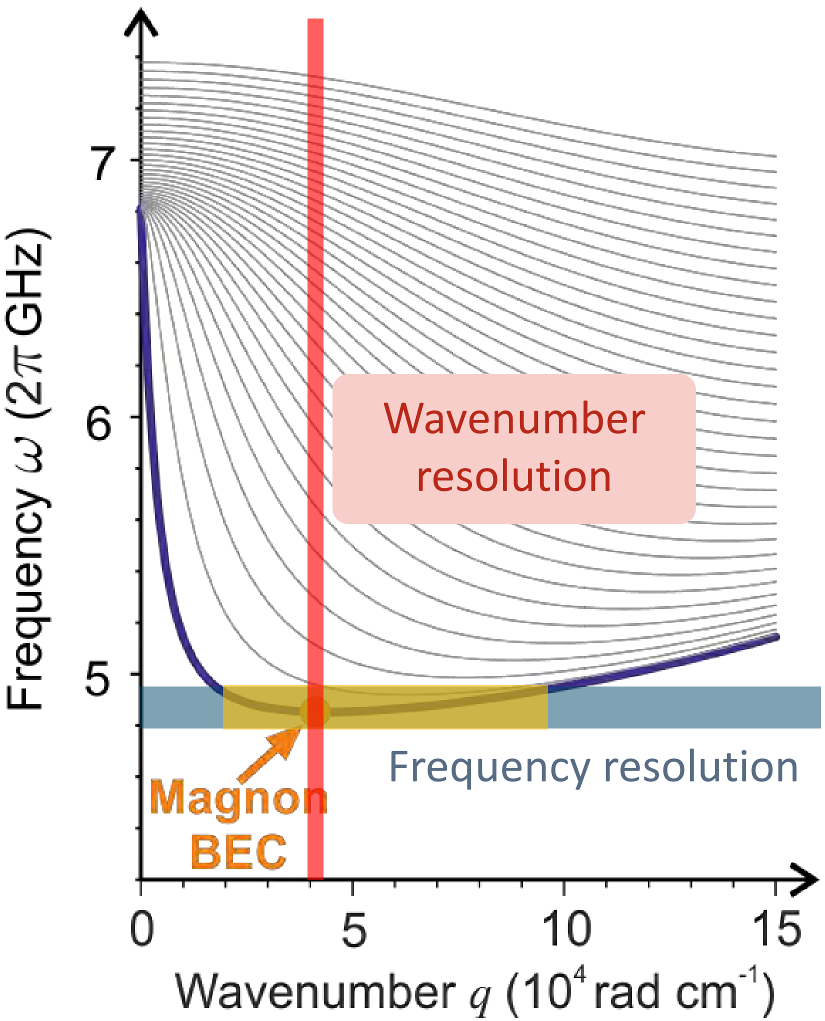
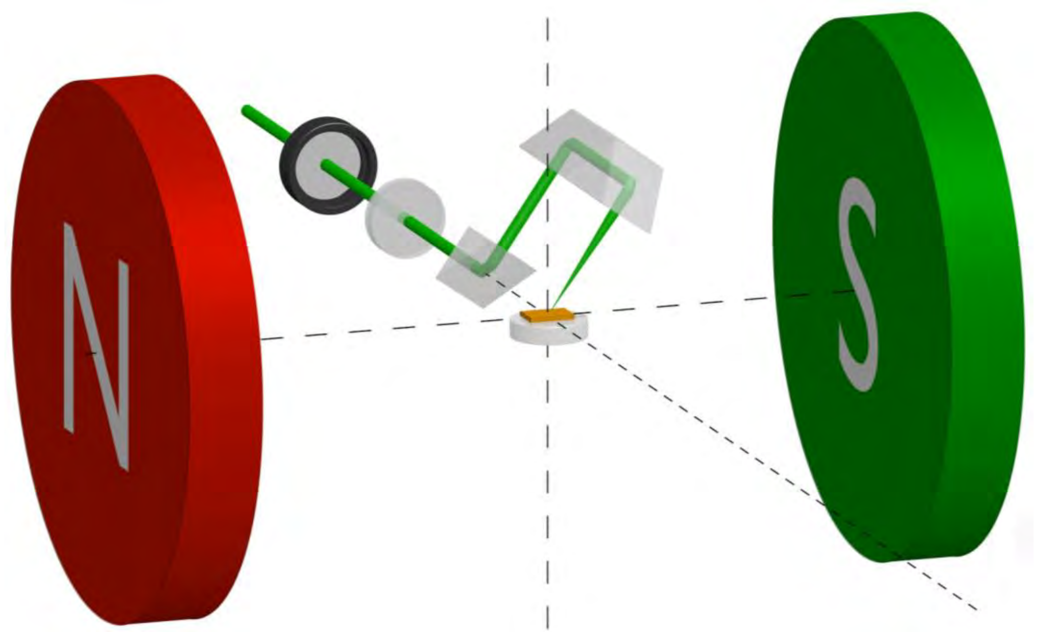
Wavenumber resolution



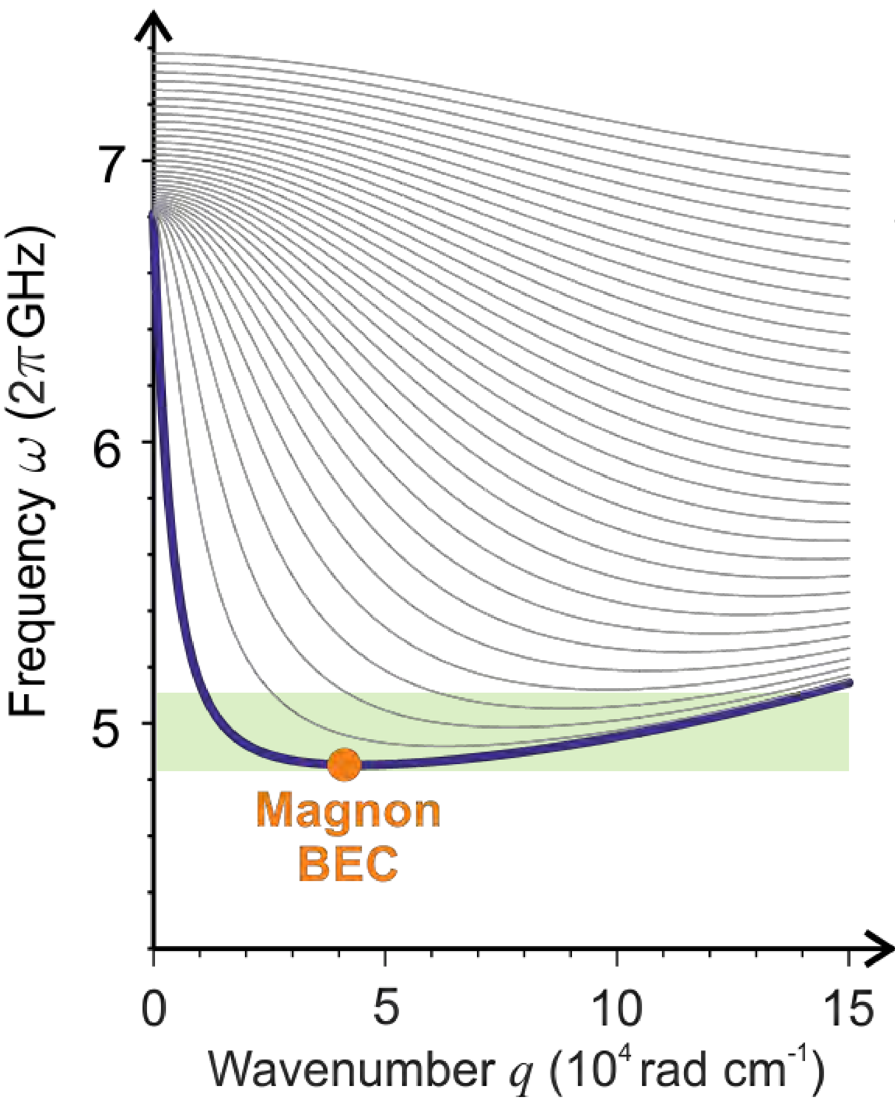


$$q_{magnon} = 2q_{Laser} \sin\left(\Theta_{\parallel}\right)$$

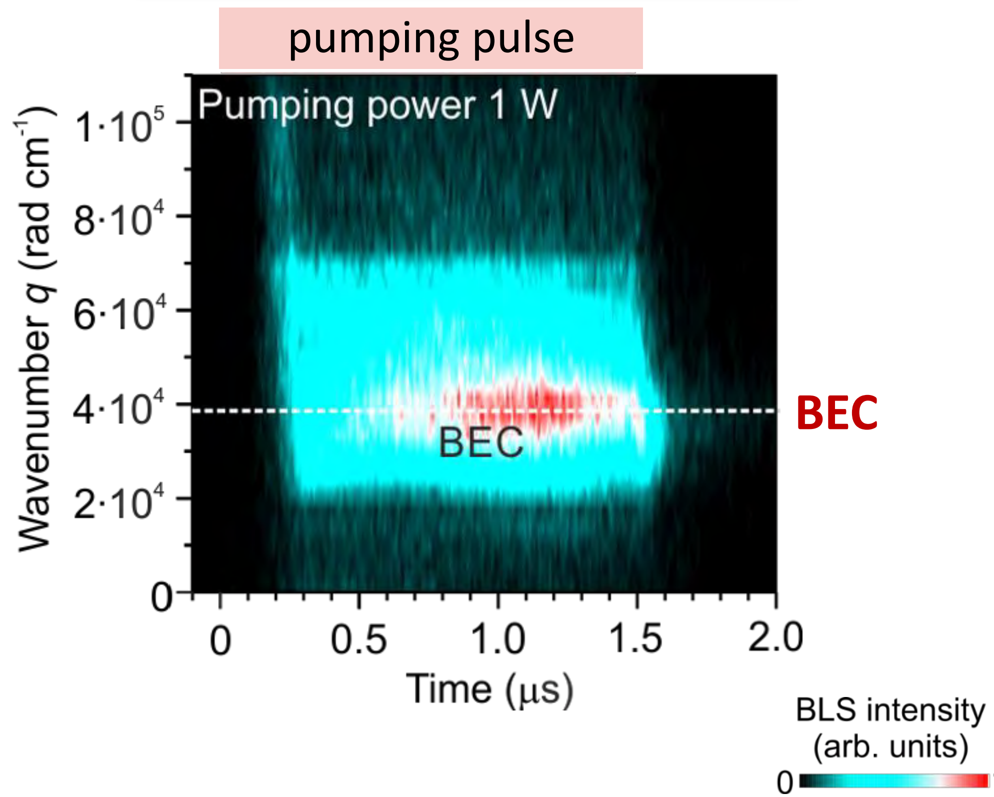
Max wavenumber	2.36×10^5 rad/cm
Wavenumber resolution	0.02×10^5 rad/cm

D.A. Bozhko, PhD thesis (2017)

Bose-Einstein magnon condensate

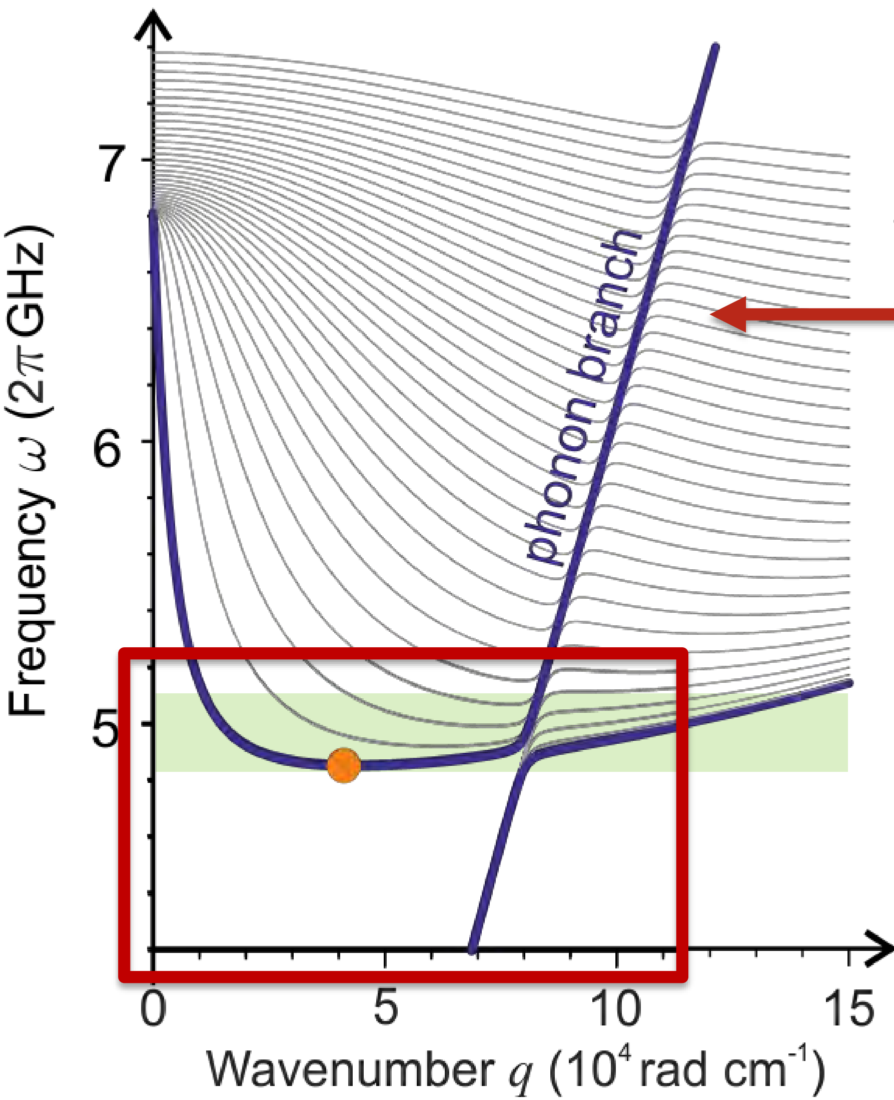


Narrow pumping area **50 µm**

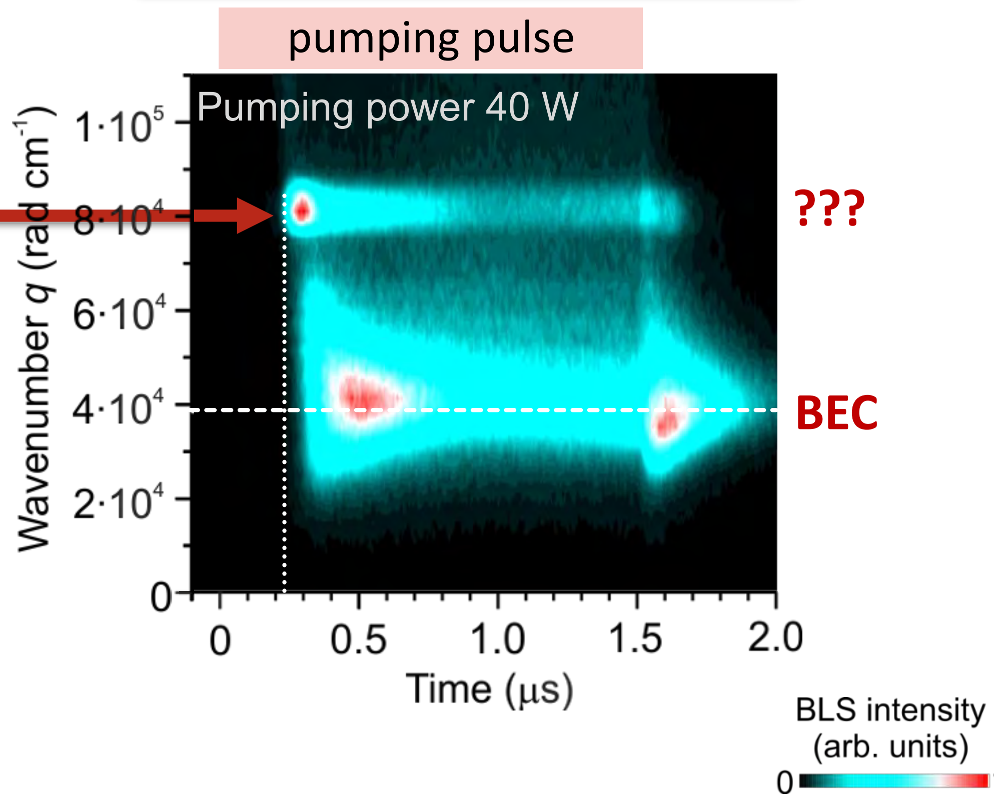


A.A. Serga *et al*., Nat. Commun. **5**, 3452 (2014)

Condensation scenarios and phonons



Wide pumping area **500 µm**

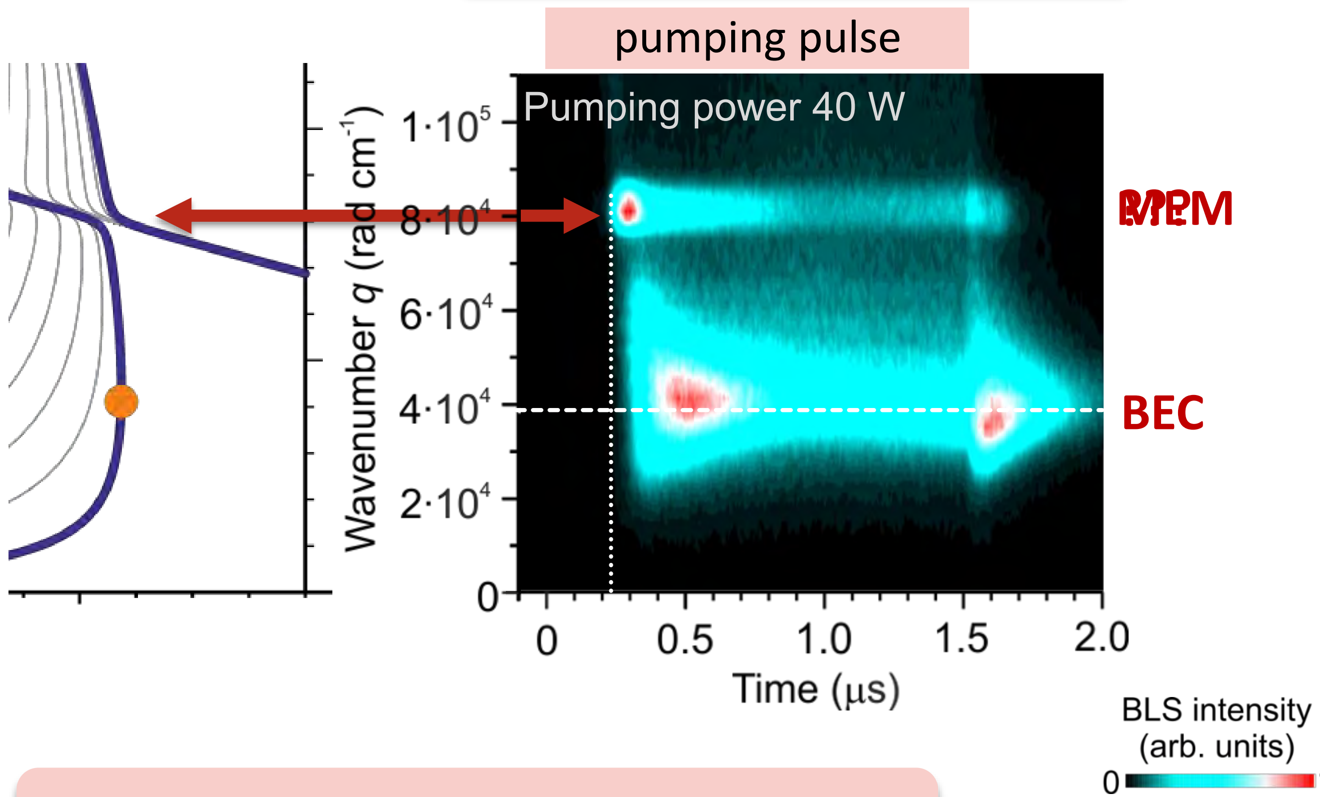


D.A. Bozhko *et al*., Phys. Rev. Lett. **118**, 237201 (2017)

Condensation scenarios and phonons

Wide pumping area **500 µm**

Due to it’s high group velocity magneto-elastic mode suffers smaller leakage losses, and thus is much more intensive, in the case of a wide pumping area



Why and how quasi-particles accumulate in the magneto-elastic mode (MEM)?

Intercoupling of BEC and MEM in a parametrically populated magnon gas

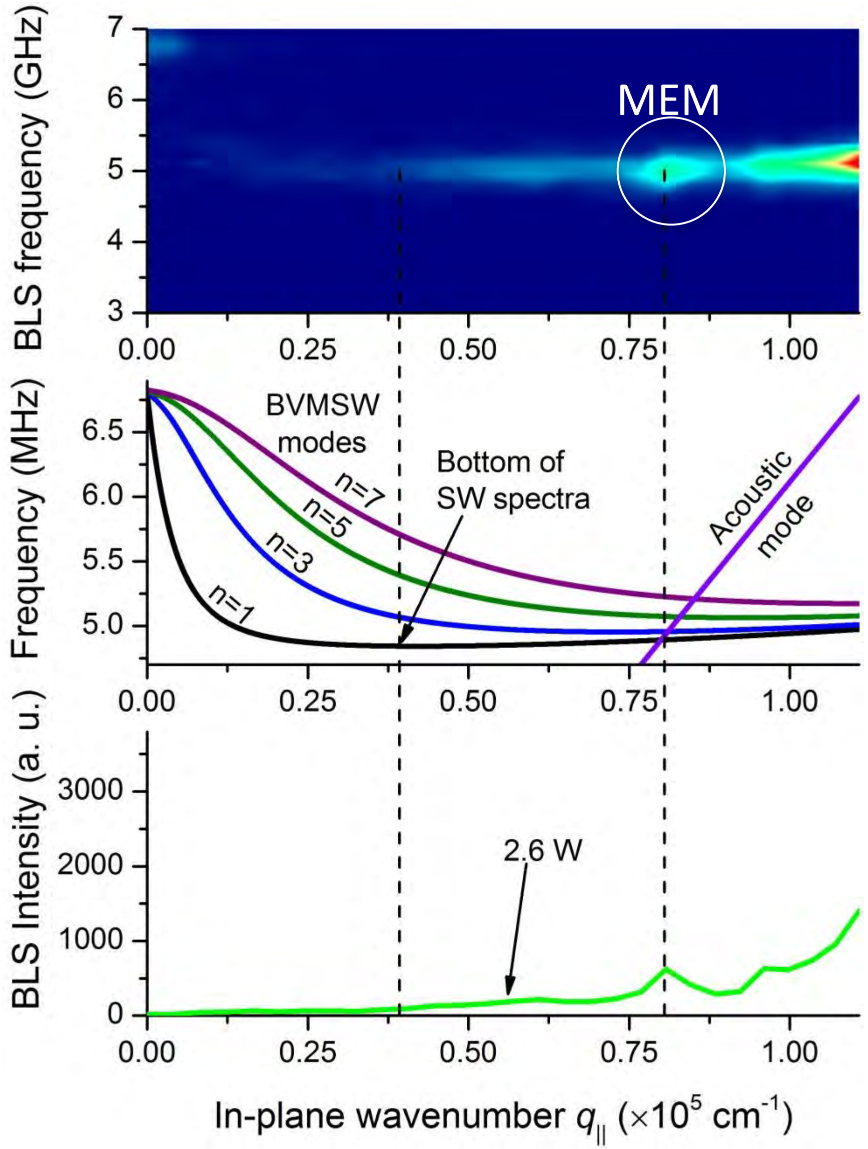
Magnon spectrum population at different pumping powers

$H_0 = 1710$ Oe

$\frac{f_p}{2} = 6810$ MHz

Calculated magnon spectrum

Population of the low energy states at different pumping powers



MEM peak appears below the threshold of magnon BEC formation

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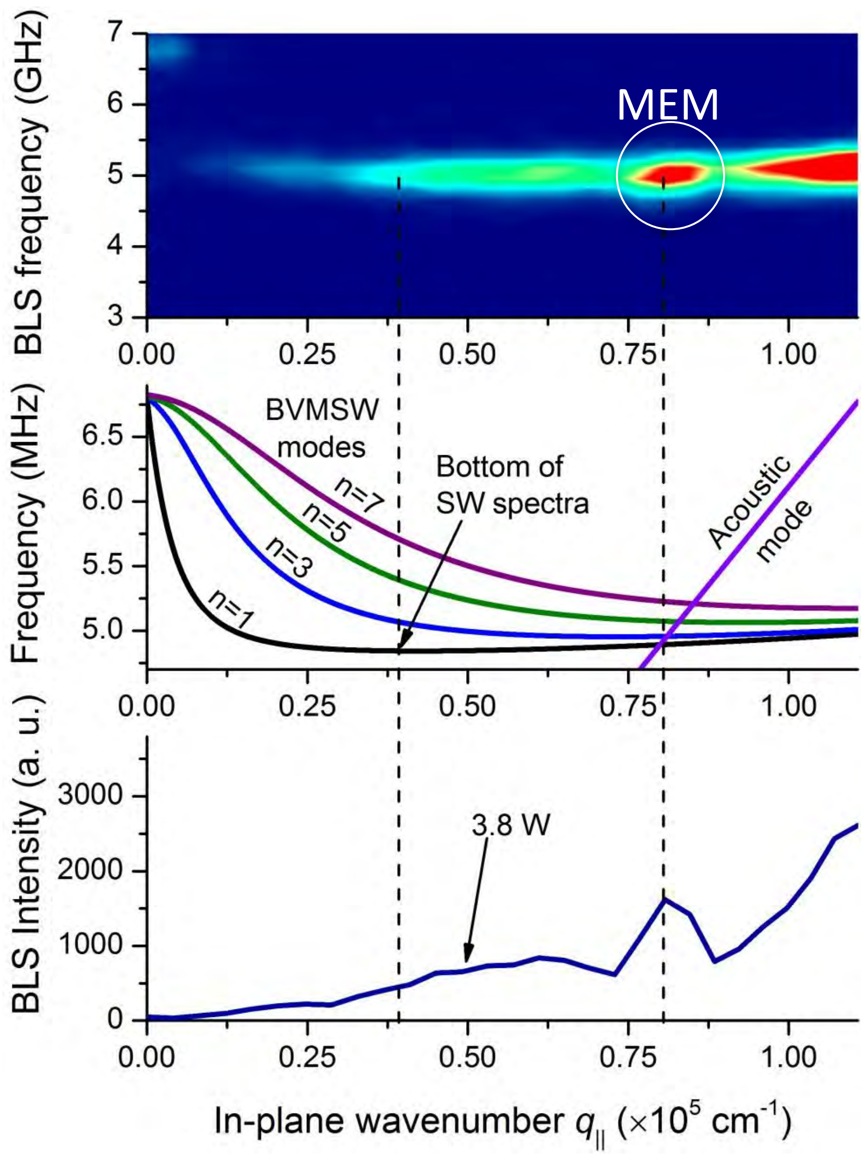

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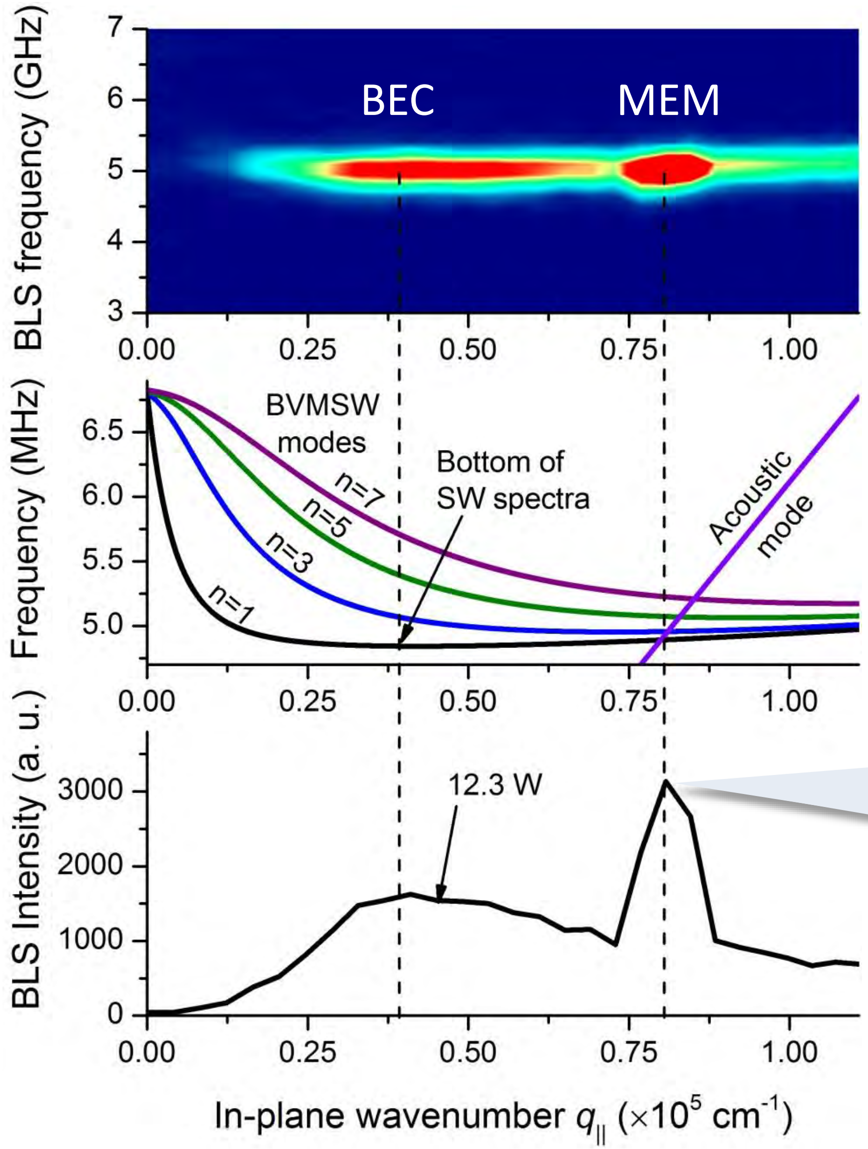

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Formation of the magnon BEC is accompanied by **saturation** of the MEM peak

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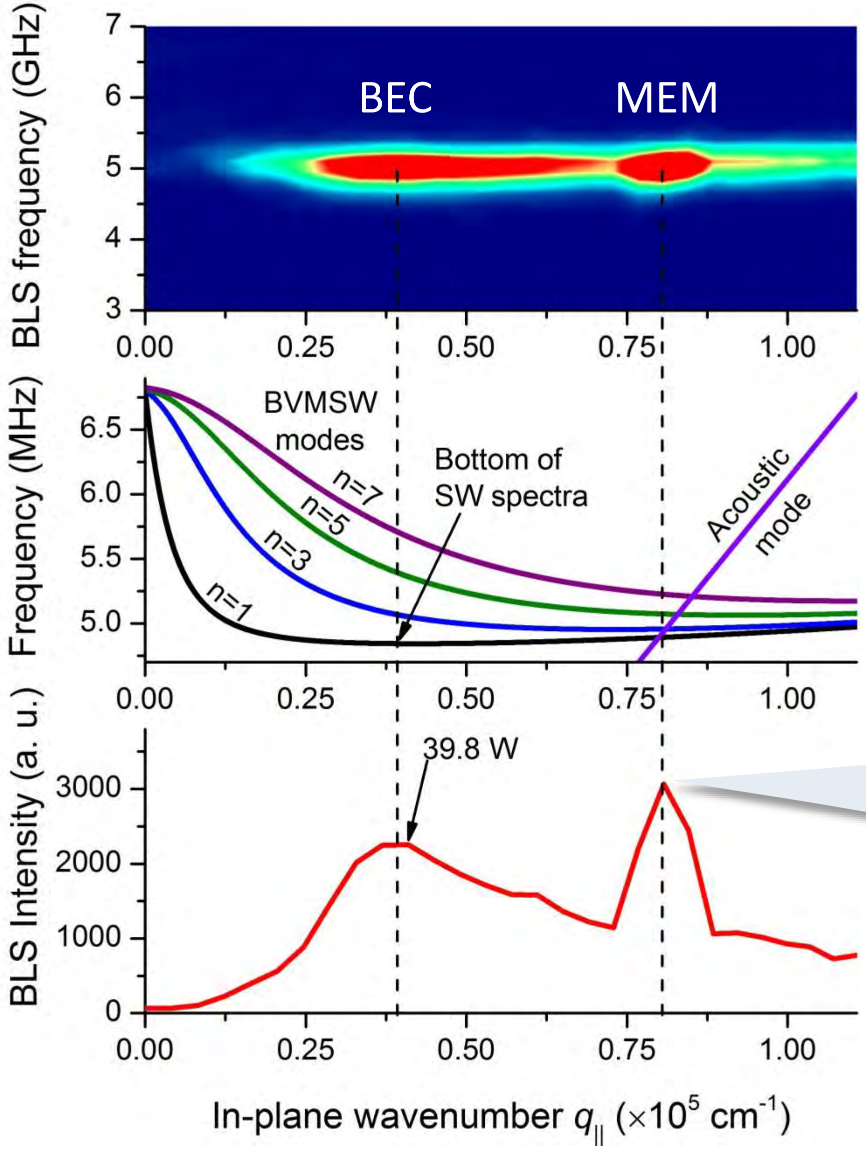

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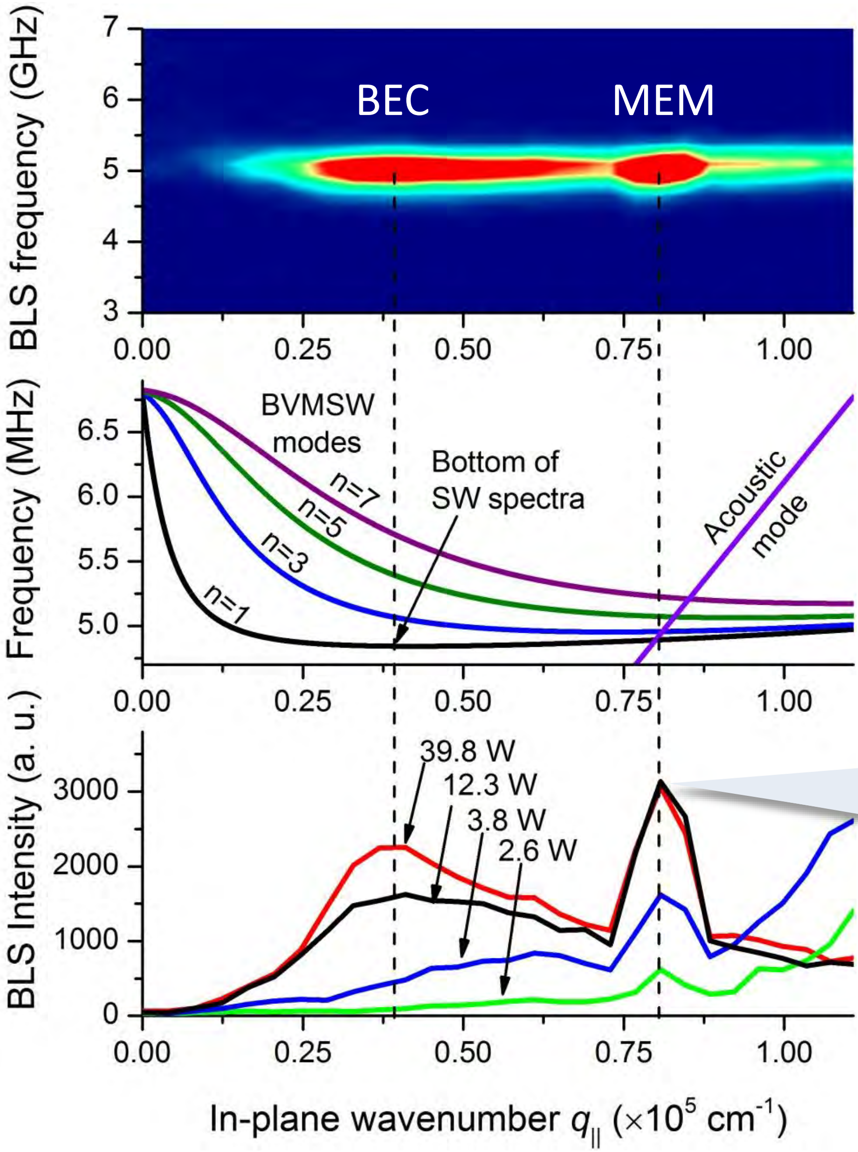

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Magnon bottleneck and accumulation of magnon-phonon hybrid particles

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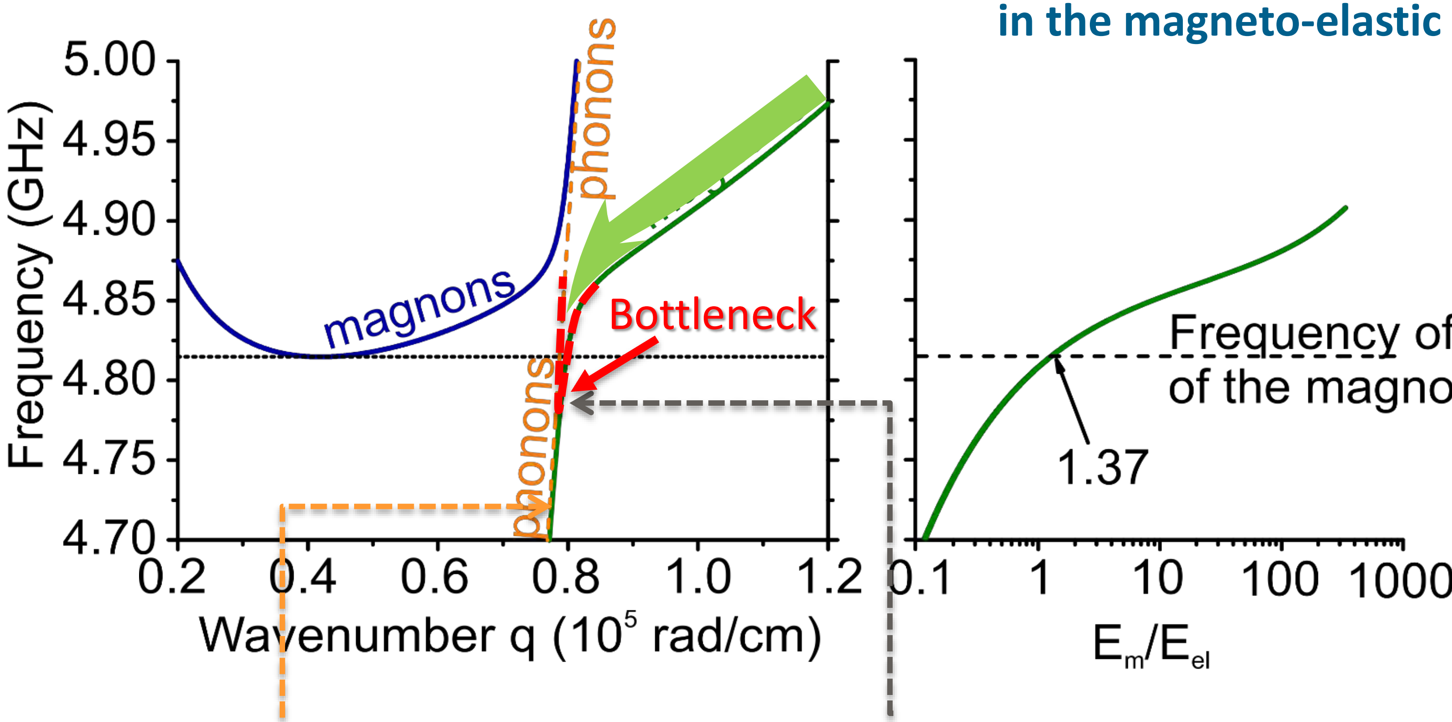
Magnon-phonon hybridization area

Ratio of magnetic E_m and elastic E_{el} energies in the magneto-elastic magnon mode



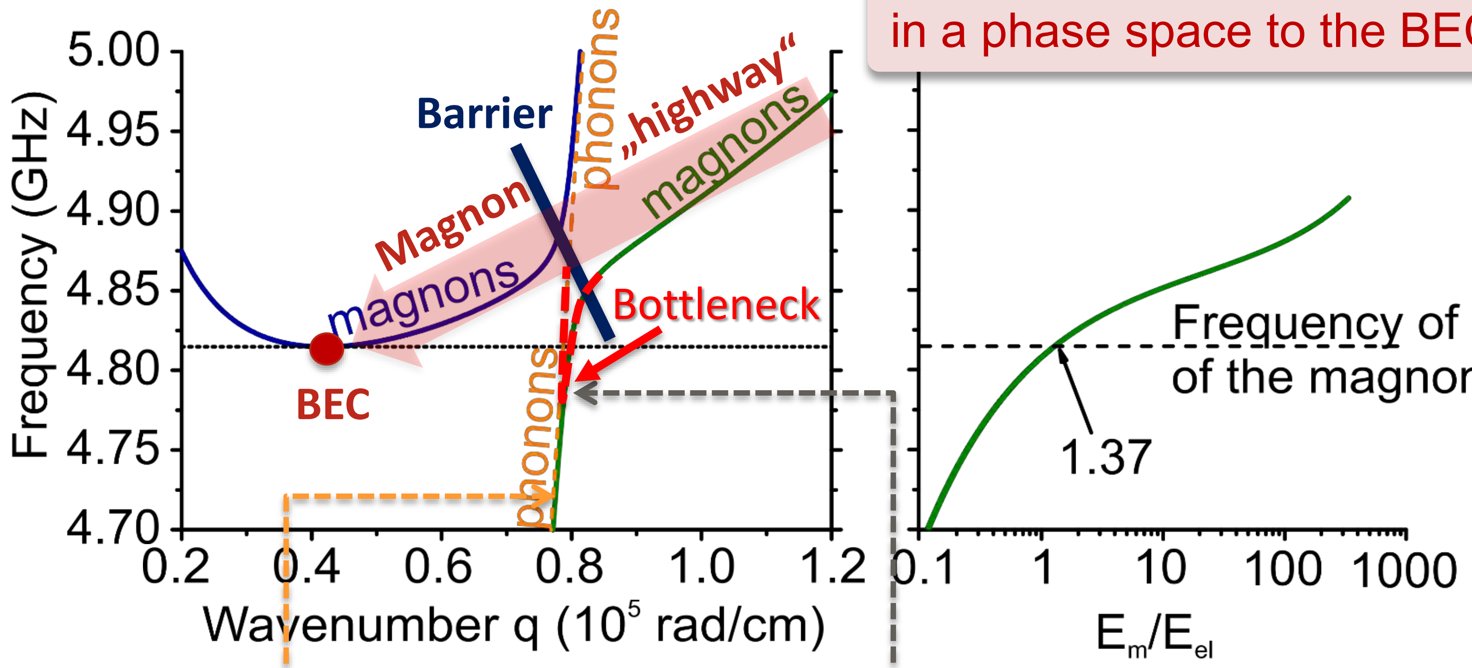
Pure phonon states - no non-linear scattering and thus no connection with upper magnon states

Accumulation of the hybridized magneto-elastic bosons at the bottom of the magnon spectrum

Magnon bottleneck and accumulation of magnon-phonon hybrid particles

Magnon-phonon hybridization area

Magnon “highway” - current of magnons in a phase space to the BEC state



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Accumulation of the hybridized magneto-elastic bosons at the bottom of the magnon spectrum

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Hamiltonian approach to magnon-phonon hybridization

Hamiltonian equation of motion:

$$i\frac{\partial a_q}{\partial t} = \frac{\partial \mathcal{H}}{\partial a_q^*}$$

$$i\frac{\partial b_q}{\partial t} = \frac{\partial \mathcal{H}}{\partial b_q^*}$$

Magnon – phonon hybridization Hamiltonian

$$\mathcal{H} = \mathcal{H}_2 + \mathcal{H}_4$$

$$\mathcal{H}_2 = \sum_q \left[\omega_q^{\mathrm{m}} a_q a_q^* + \omega_q^{\mathrm{p}} b_q b_q^* + \frac{\Delta}{2}\left(a_q b_q^* + a_q^* b_q\right) \right],$$

magnons phonons hybridization

ω_q^{m} - magnon dispersion law

ω_q^{p} - phonon dispersion law

Δ - coupling amplitude

$$\mathcal{H}_4 = \frac{1}{4} \sum_{q_1+q_2=q_3+q_4} T_{12,34} a_1^* a_2^* a_3 a_4$$

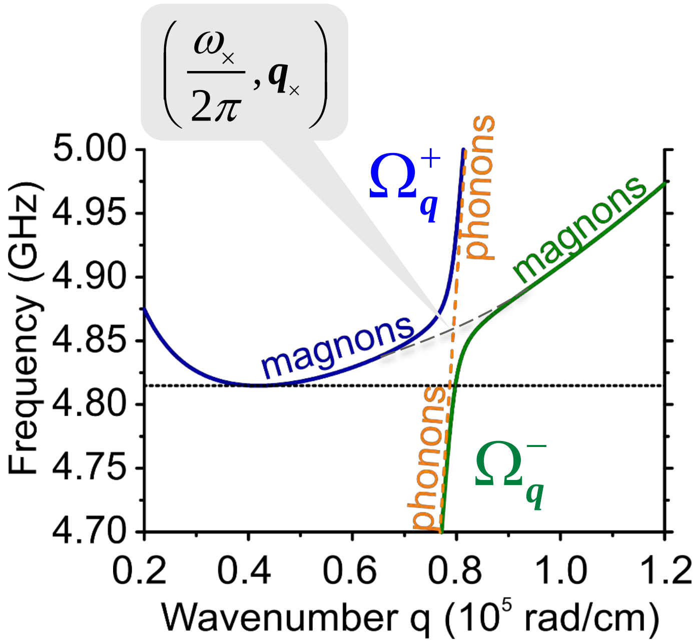
$T_{12,34}$ - interaction amplitudes

Interaction Hamiltonian of 2↔2 magnon scattering

Magnon-phonon hybridization

Transition to hybridized MEM modes $c_q^{\pm}$ using linear canonical Bogolyubov transformation (rotation by the angle φ_q in the (a_q, b_q) plane)

$$\begin{cases} a_q = \cos(\varphi_q) c_q^- + \sin(\varphi_q) c_q^+ \\ b_q = -\sin(\varphi_q) c_q^- + \cos(\varphi_q) c_q^+ \end{cases} \qquad \cos(\varphi_q) = \frac{1}{\sqrt{2}}\left[1 + \frac{o_q}{\sqrt{1+o_q^2}}\right]^2, \quad o_q = \frac{\omega_q^{\mathrm{p}} - \omega_q^{\mathrm{m}}}{\Delta}, \quad \Delta = \Omega_{q_\times}^+ - \Omega_{q_\times}^-$$

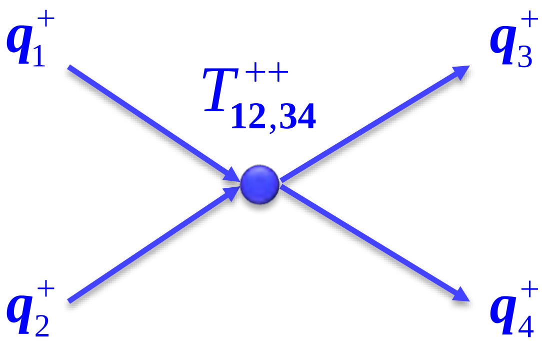


Diagonal quadratic Hamiltonian for the upper and lower MEM modes

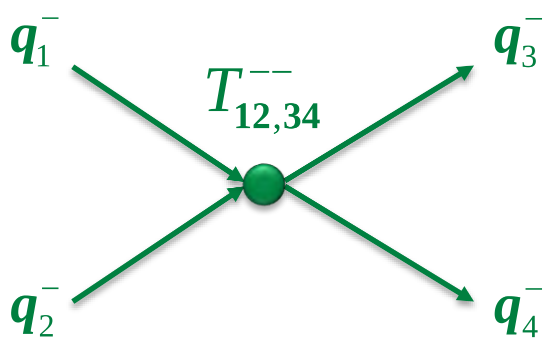
$$\tilde{\mathcal{H}}_2 = \sum_q \left[\Omega_q^+ c_q^+ c_q^{+*} + \Omega_q^- c_q^- c_q^{-*}\right]$$

$$\Omega_q^{\pm} = \frac{1}{2}\left\{\omega_q^{\mathrm{m}} + \omega_q^{\mathrm{p}} \pm \sqrt{\left[\omega_q^{\mathrm{m}} - \omega_q^{\mathrm{p}}\right]^2 + \Delta^2}\right\}$$

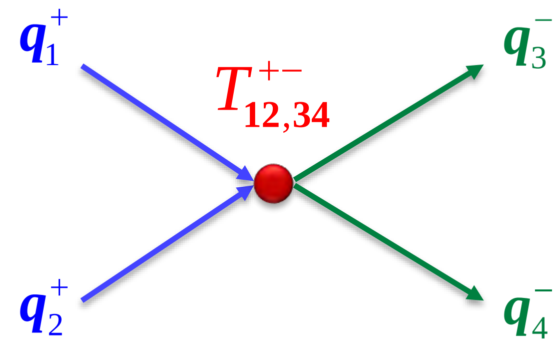
Interaction amplitudes $T_{12,34}$ of the upper and lower MEMs



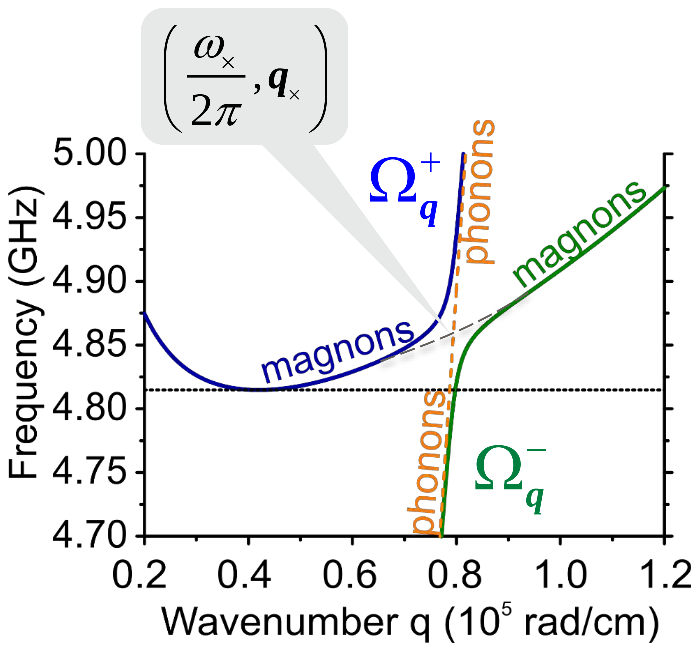
Upper-Upper interaction



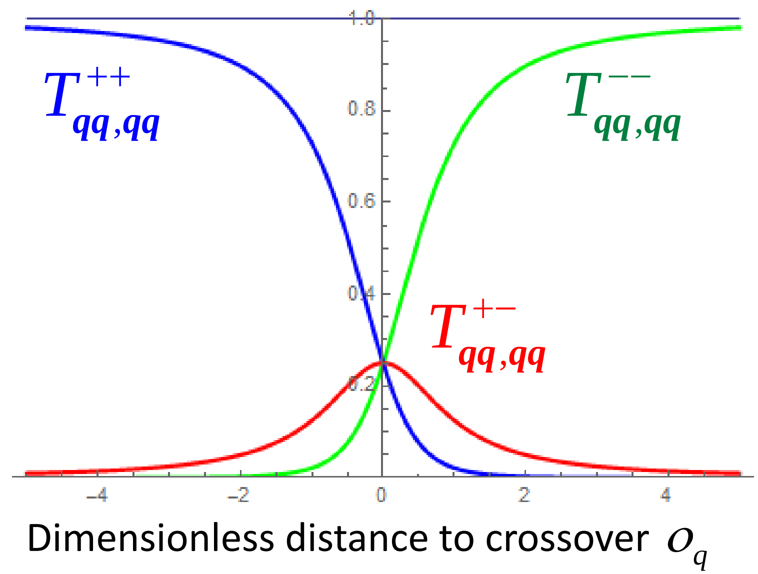
Lower-Lower interaction



Cross Upper-Lower MEMs interaction



Wavenumber dependence of interaction amplitudes



Statistical description

$$\frac{\partial \mathcal{N}_q^-}{\partial t} = \frac{d\mu_q}{dq} - F_q^{-+}$$

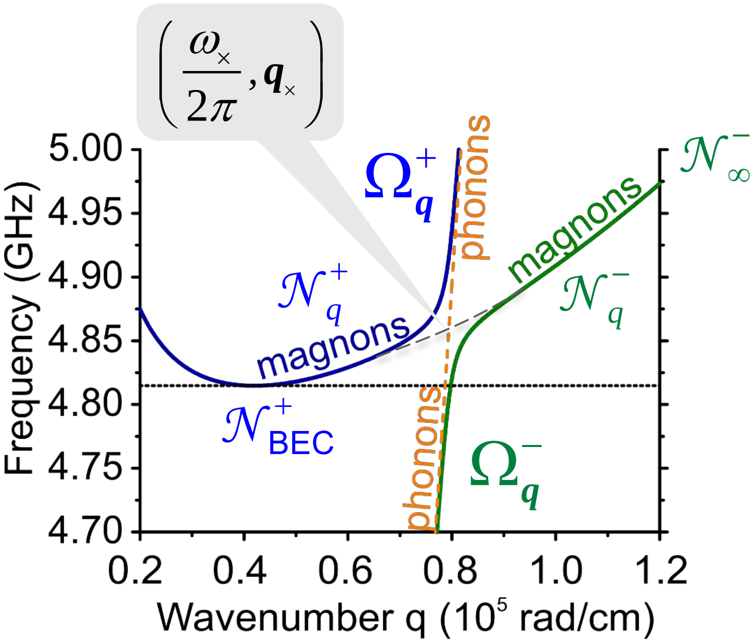
- balance equation for the lower MEM mode occupation numbers $\mathcal{N}^-$

$$\mu_q \propto \left|T_q^{--}\right|^2 \left(\mathcal{N}_q^-\right)^3$$

- flux of $\mathcal{N}^-$ towards the hybridization region

$$F_q^{-+} \propto \left|T_q^{-+}\right|^2 \left(\mathcal{N}_q^-\right)^2 \mathcal{N}_q^+$$

- transition rate $\mathcal{N}^- \rightarrow \mathcal{N}^+$ in the hybridization region



The dimensionless lower-MEM and upper-MEM densities:

$$\mathcal{N}_q^- = \frac{N_q^-}{N_{o_q \simeq +5}^-}, \quad \mathcal{N}_q^- = \frac{N_q^-}{N_{o_q \simeq -5}^-}$$

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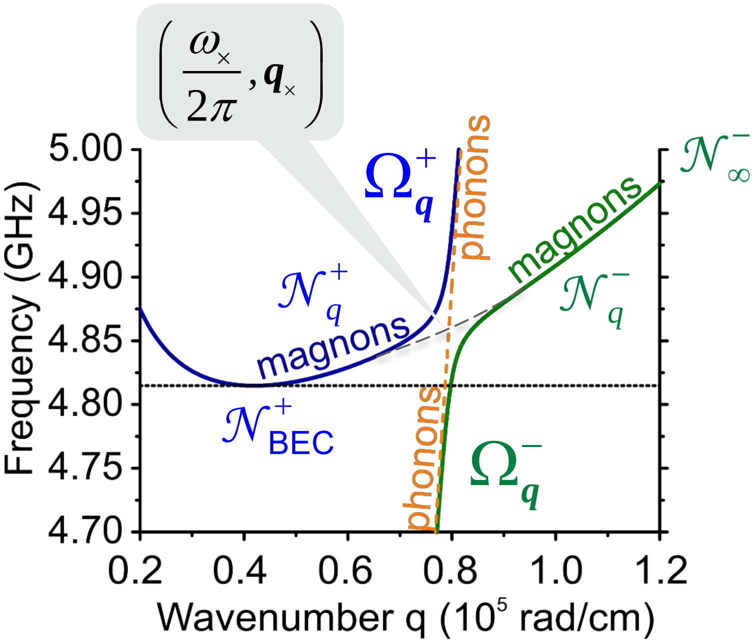


Taking into account the equations for T_q

$$\frac{d}{dq}\left[\cos^8\varphi_{\boldsymbol{q}}\left(\mathcal{N}_q^-\right)^3\right] = 3a\left(\mathcal{N}_q^-\right)^2\cos^4\varphi_{\boldsymbol{q}}\sin^4\varphi_{\boldsymbol{q}}$$

$$\cos\left(\varphi_{\boldsymbol{q}}\right) = \frac{1}{\sqrt{2}}\left[1 + \frac{o_q}{\sqrt{1+o_q^2}}\right]^2, \quad o_q = \frac{\omega_q^{\text{p}} - \omega_q^{\text{m}}}{\Delta}$$

$$\mathcal{N}_q^- = \frac{N_q^-}{N_{o_q \simeq +5}^-}, \quad \mathcal{N}_q^- = \frac{N_q^-}{N_{o_q \simeq -5}^-}, \quad a \simeq \frac{N_{o_q \simeq -5}^+}{N_{o_q \simeq +5}^-} \simeq \frac{\mathcal{N}_{\text{BEC}}^+}{\mathcal{N}_\infty^-}$$

Bottleneck accumulation

$$\frac{\partial \mathcal{N}_q^-}{\partial t} = \frac{d\mu_q}{dq} - F_q^{-+}$$

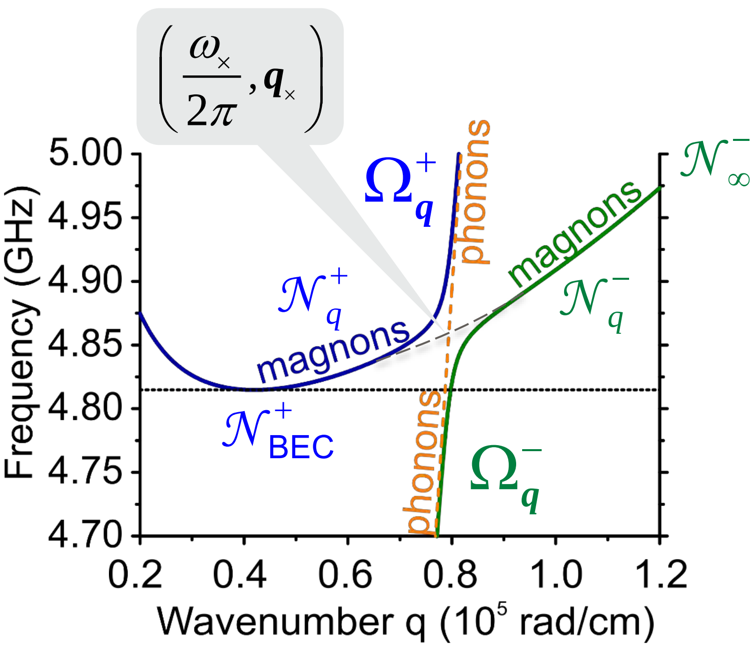
$$\mu_q \propto \left|T_q^{--}\right|^2 \left(\mathcal{N}_q^-\right)^3$$

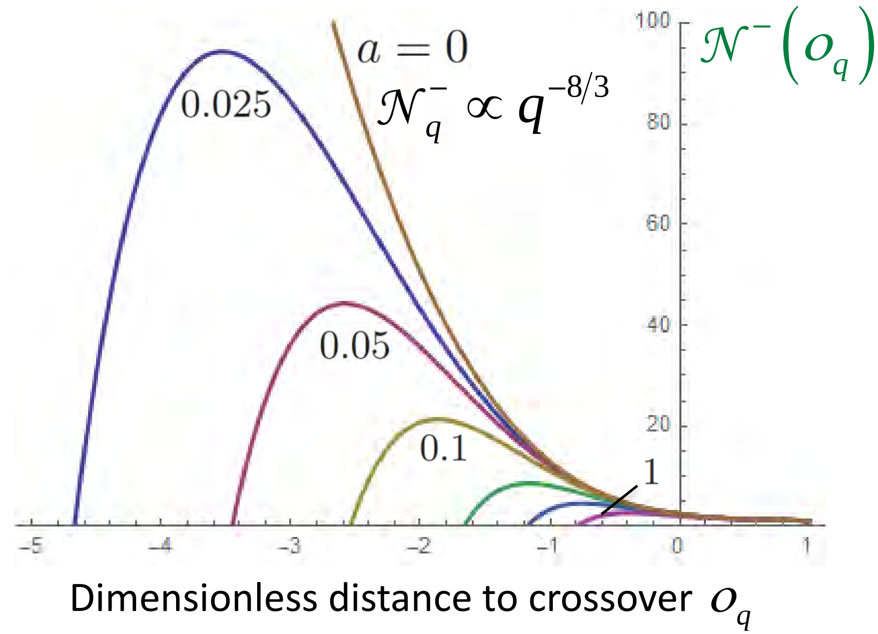
$$F_q^{-+} \propto \left|T_q^{-+}\right|^2 \left(\mathcal{N}_q^-\right)^2 \mathcal{N}_q^+$$

Solution of the kinetic equation

$$\mathcal{N}_q^- = \frac{1}{(\cos\varphi_q)^{8/3}}\left[1 - a\int_q^\infty \frac{(\sin\varphi_p)^4\, dp}{(\cos\varphi_p)^{4/3}}\right]$$

$$a \simeq \frac{\mathcal{N}_{\mathrm{BEC}}^+}{\mathcal{N}_\infty^-}$$ ← relative population of the BEC state





Bottleneck accumulation

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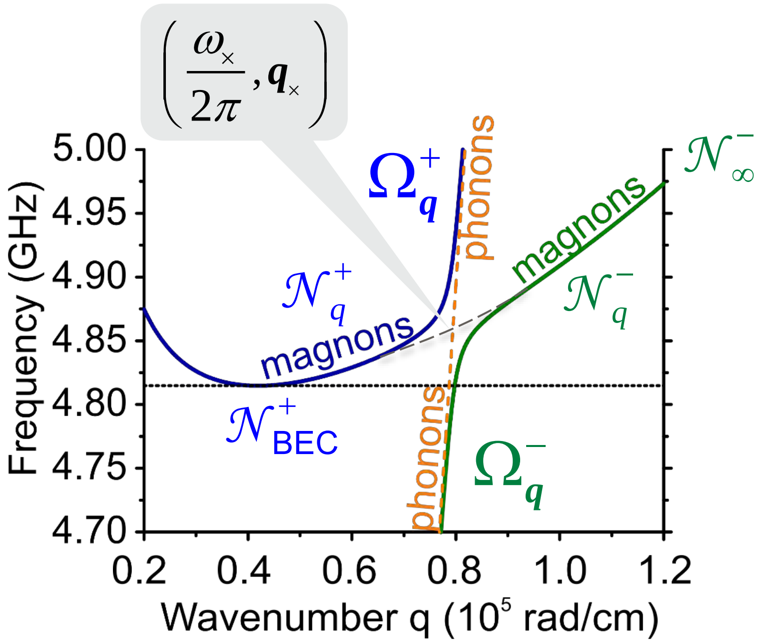
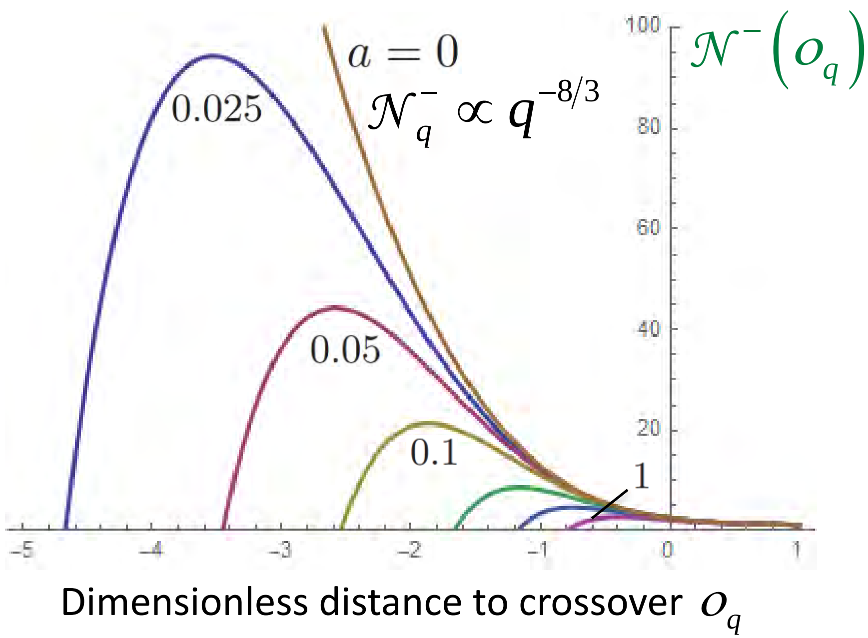
Increase in the magnon BEC population $\mathcal{N}_{\text{BEC}}^+$ decreases bottleneck effect and explains the MEM saturation phenomenon

$$a \simeq \frac{\mathcal{N}_{\text{BEC}}^+}{\mathcal{N}_\infty^-}$$

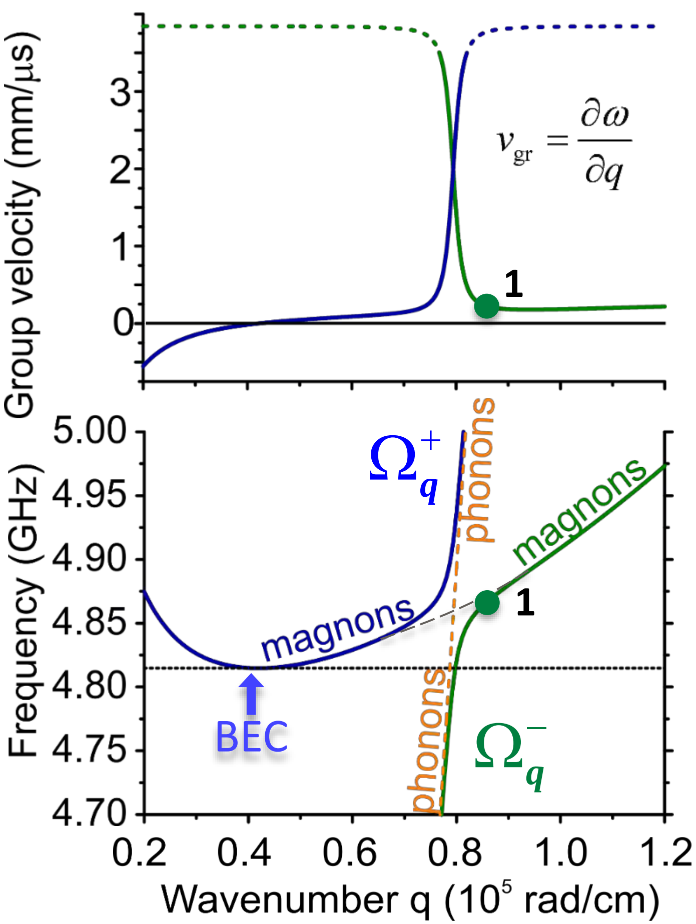
D.A. Bozhko *et al*., Phys. Rev. Lett. **118**, 237201 (2017)



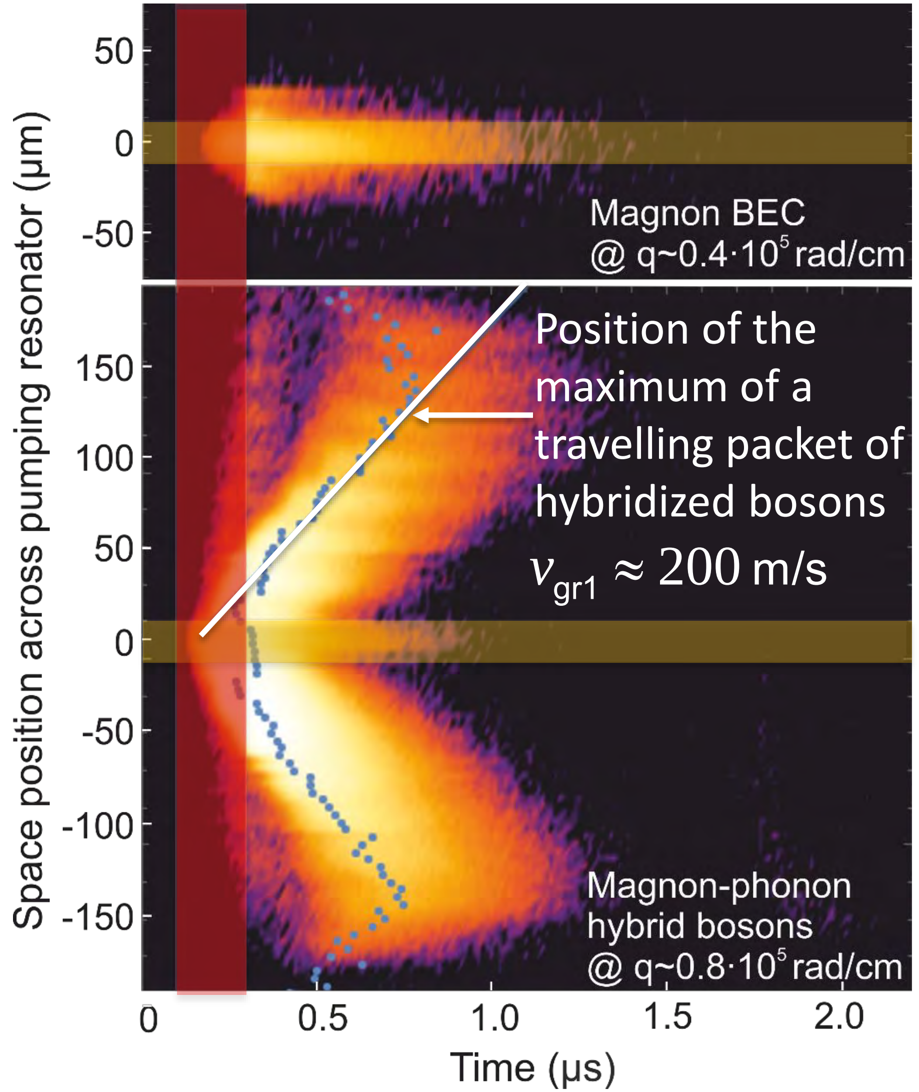


Transport measurements

Group velocities of magnon-phonon bosons in hybridization area



Experimental time-space diagram for BEC and MEM



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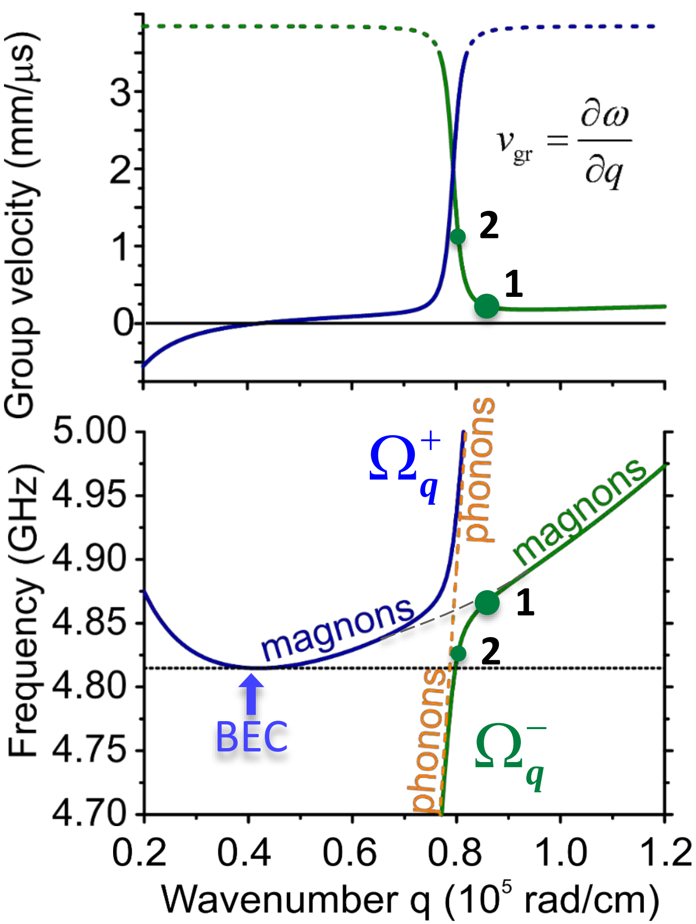

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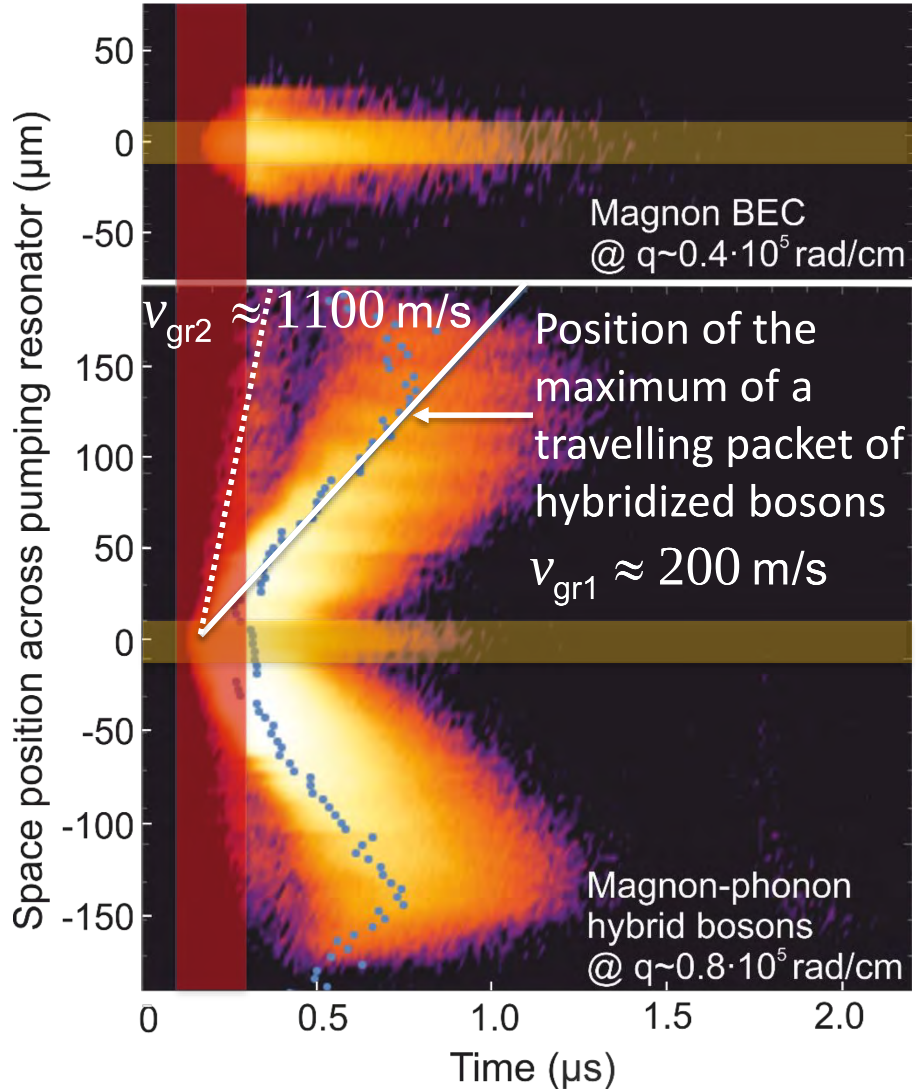

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Summary

- Observed effects evidence the bottleneck accumulation of hybridized magneto-elastic bosons at the bottom of the magnon spectrum
- Developed minimal model qualitatively describes the observed phenomena
- Transport measurements give an information about spectral positions of accumulated quasi-particles
- Accumulated hybridized bosons with non-zero group velocity can be used for spin transport
- Bottleneck accumulation can occur in any multicomponent gas-mixture of interacting quasiparticles with different scattering amplitudes