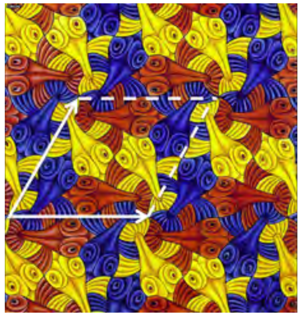
New Avenues toward Complex Pairing States

Matthias Eschrig

SEPnet & Hubbard Theory Consortium
Department of Physics,
Royal Holloway, University of London

Schloss Waldthausen, Mainz, 27. September 2017



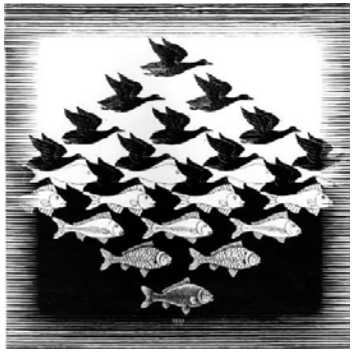
Symmetry

Coherence

Emergence

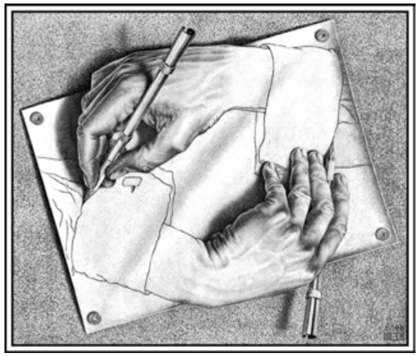
Concepts in Condensed Matter Physics

Geometry



Topology

Correlation



Basics of superconducting spintronics: combination of superconductivity and magnetism

Andreev bound states
at spin-active interfaces

singlet-triplet mixing due to interface:

Tokuyasu, Sauls, Rainer 1988

$$\uparrow_{\underline{k}}\downarrow_{-\underline{k}} = e^{i\vartheta}\, \uparrow_{k}\downarrow_{-k}$$

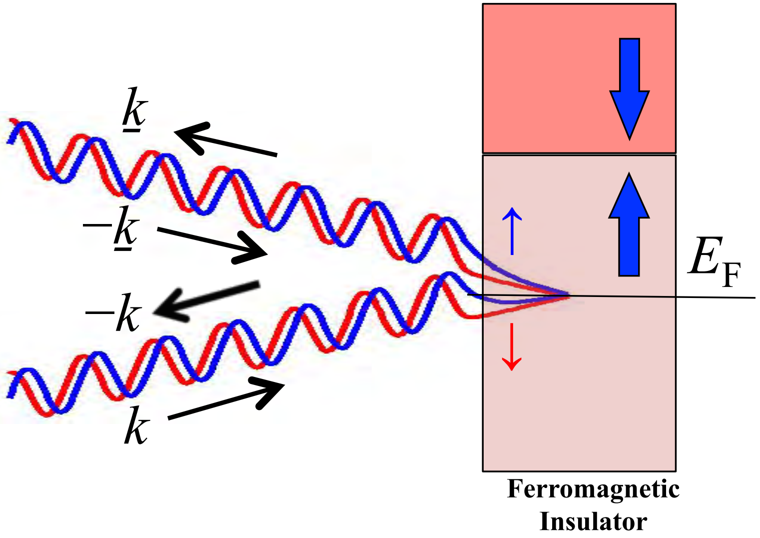
$$\downarrow_{\underline{k}}\uparrow_{-\underline{k}} = e^{-i\vartheta}\, \downarrow_{k}\uparrow_{-k}$$

$$\uparrow_{\underline{k}} = e^{i(\psi+\vartheta/2)} \uparrow_{k}$$

$$\downarrow_{\underline{k}} = e^{i(\psi-\vartheta/2)} \downarrow_{k}$$

$$\uparrow_{-k} = e^{i(\psi+\vartheta/2)} \uparrow_{-\underline{k}}$$

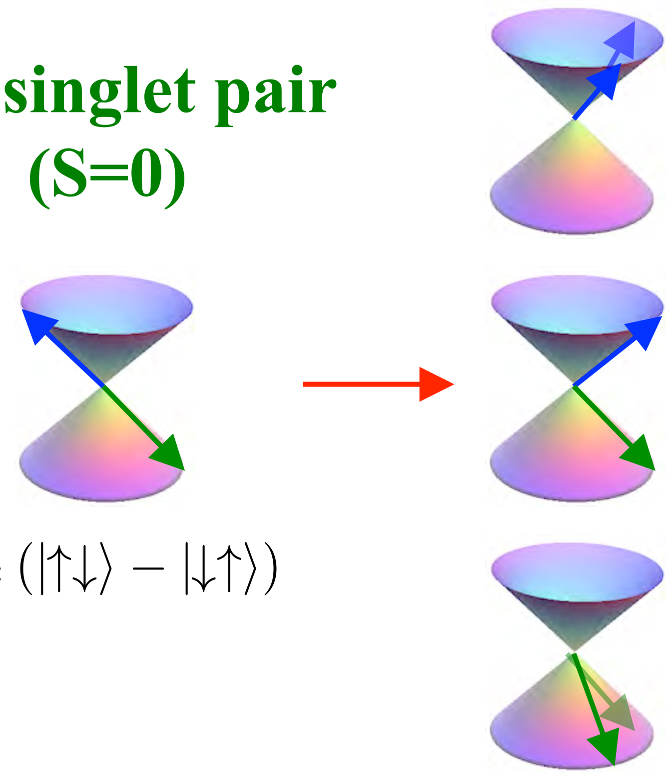
$$\downarrow_{-k} = e^{i(\psi-\vartheta/2)} \downarrow_{-\underline{k}}$$



$$(\uparrow\downarrow - \downarrow\uparrow) \rightarrow (\uparrow\downarrow e^{i\vartheta} - \downarrow\uparrow e^{-i\vartheta}) = \cos(\vartheta)(\uparrow\downarrow - \downarrow\uparrow) + i\sin(\vartheta)(\uparrow\downarrow + \downarrow\uparrow)$$

$$\boxed{\cos(\vartheta)(\uparrow\downarrow - \downarrow\uparrow) + i\sin(\vartheta)(\uparrow\downarrow + \downarrow\uparrow)}$$

Spin of the Cooper pair:



spin triplet pair
(S=1)

$S_z=+1$

$|\uparrow\uparrow\rangle$

spin singlet pair
(S=0)

$S_z=0$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

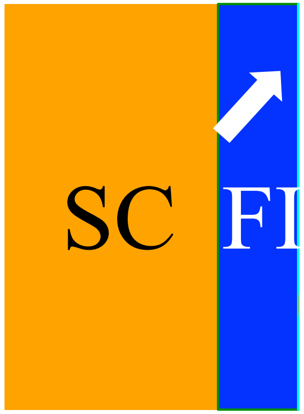
$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

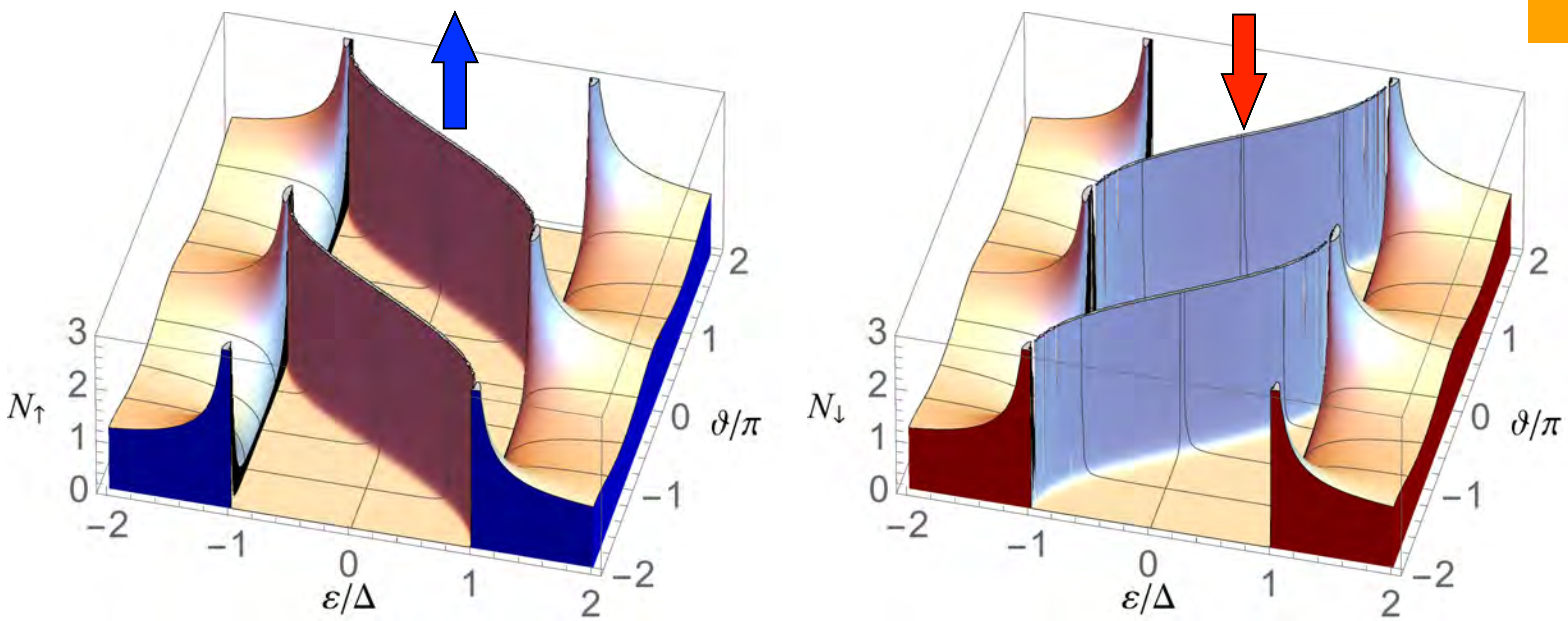
$S_z=-1$

$|\downarrow\downarrow\rangle$

Spin-polarized Andreev bound states at S/FI interfaces:

M. Fogelström, PRB 62, 11812 (2000)

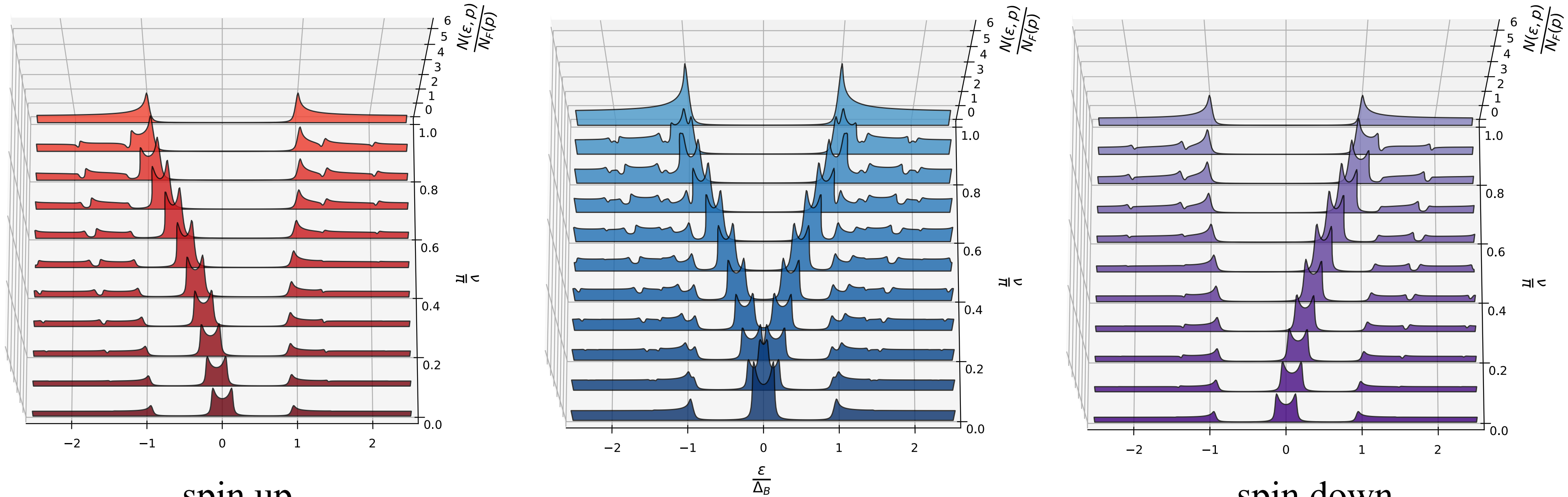




Slab of Superfluid 3He B with spin-active interfaces:

$$\Delta = [\sigma_X \Delta_{Xx} p_x + \sigma_Y \Delta_{Yy} p_y + \sigma_Z \Delta_{Zz} p_z](i\sigma_Y)$$

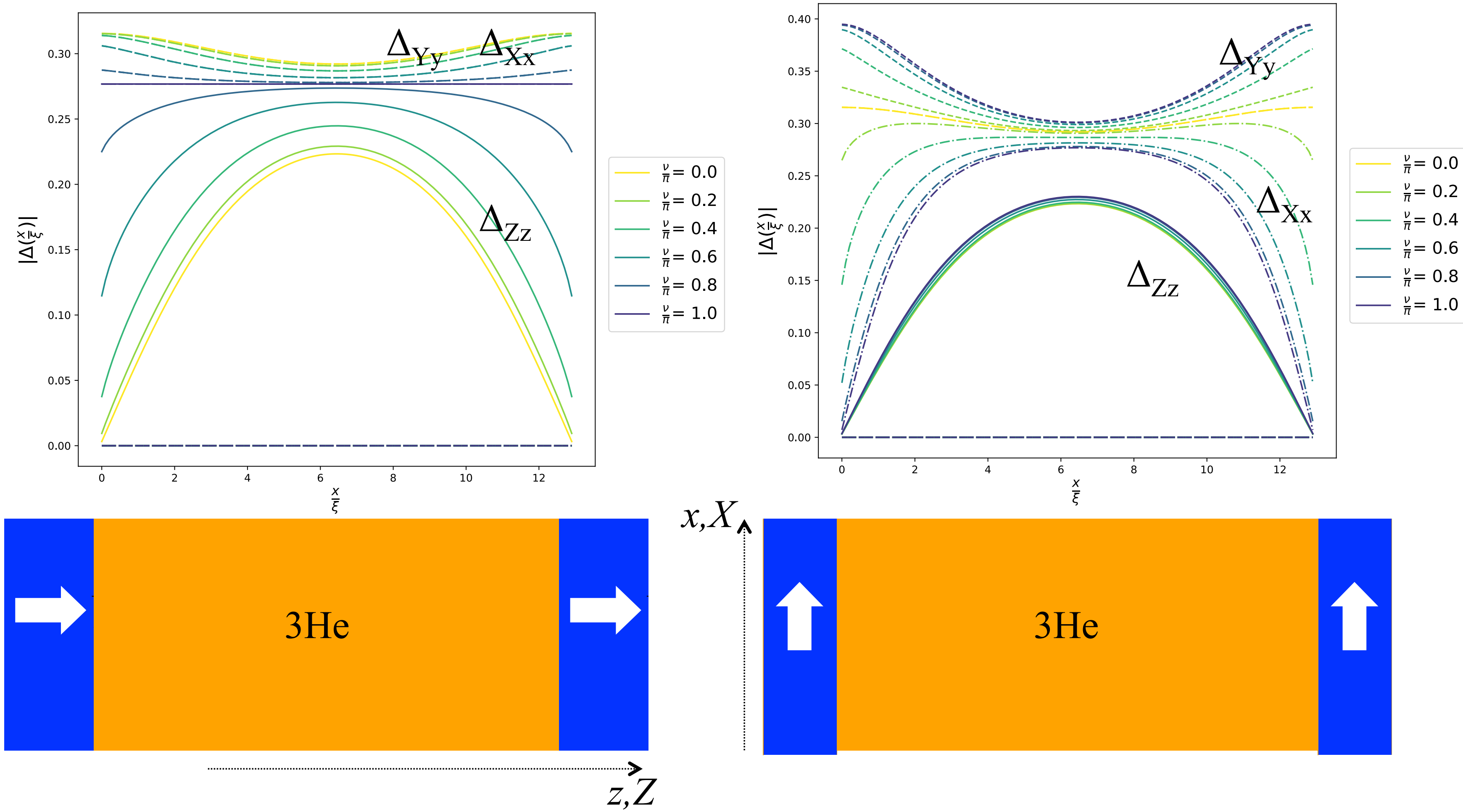
LDOS at interface for normal impact trajectory



spin up

spin down

Slab of Superfluid 3He B with spin-active interfaces:



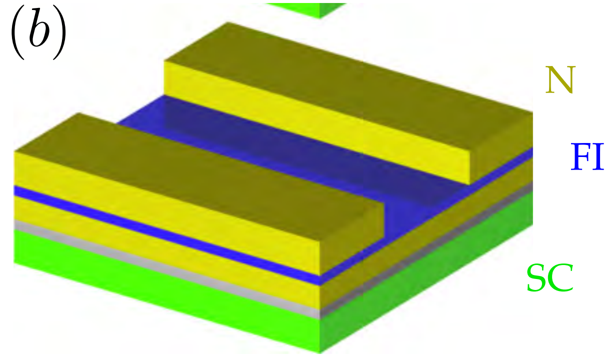
$$\Delta = [\sigma_X \Delta_{Xx} p_x + \sigma_Y \Delta_{Yy} p_y + \sigma_Z \Delta_{Zz} p_z](i\sigma_Y)$$

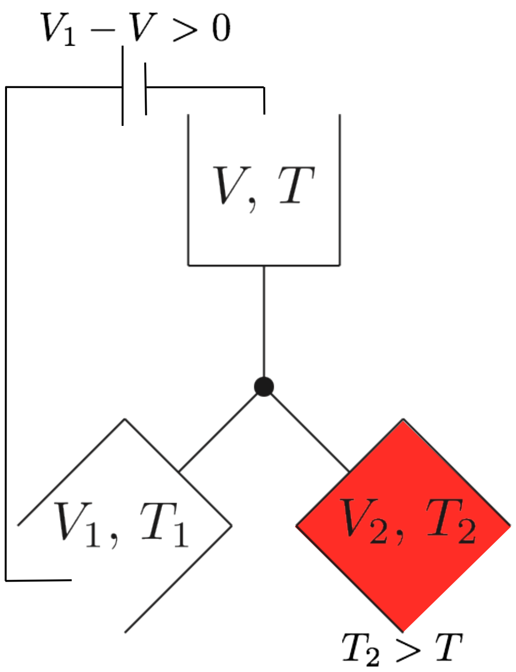
Large Thermoelectric effects

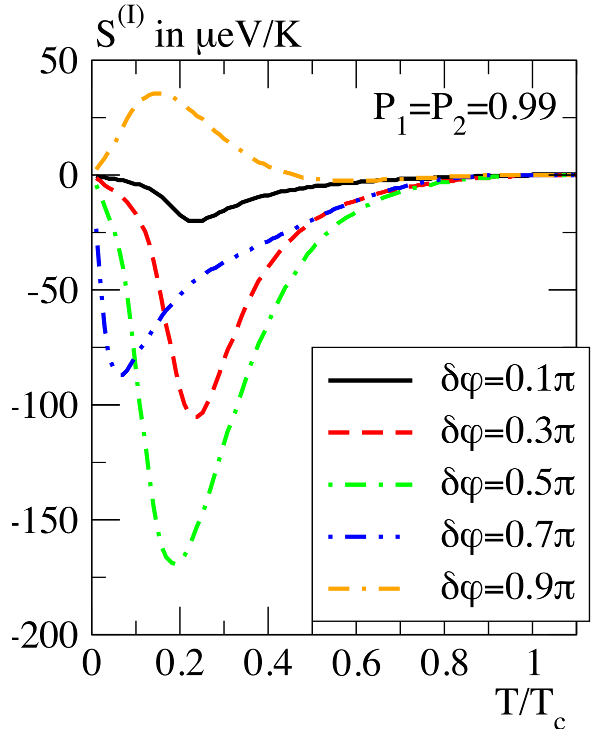
PRL **110**, 047002 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Nonlocal Thermoelectric Effects and Nonlocal Onsager relations in a Three-Terminal Proximity-Coupled Superconductor-Ferromagnet Device

P. Machon,[1] M. Eschrig,[2] and W. Belzig[1]
[1]*Department of Physics, University of Konstanz, D-78457 Konstanz, Germany*
[2]*SEPnet and Hubbard Theory Consortium, Department of Physics, Royal Holloway, University of London, Egham, Surrey TW20 0EX, United Kingdom*



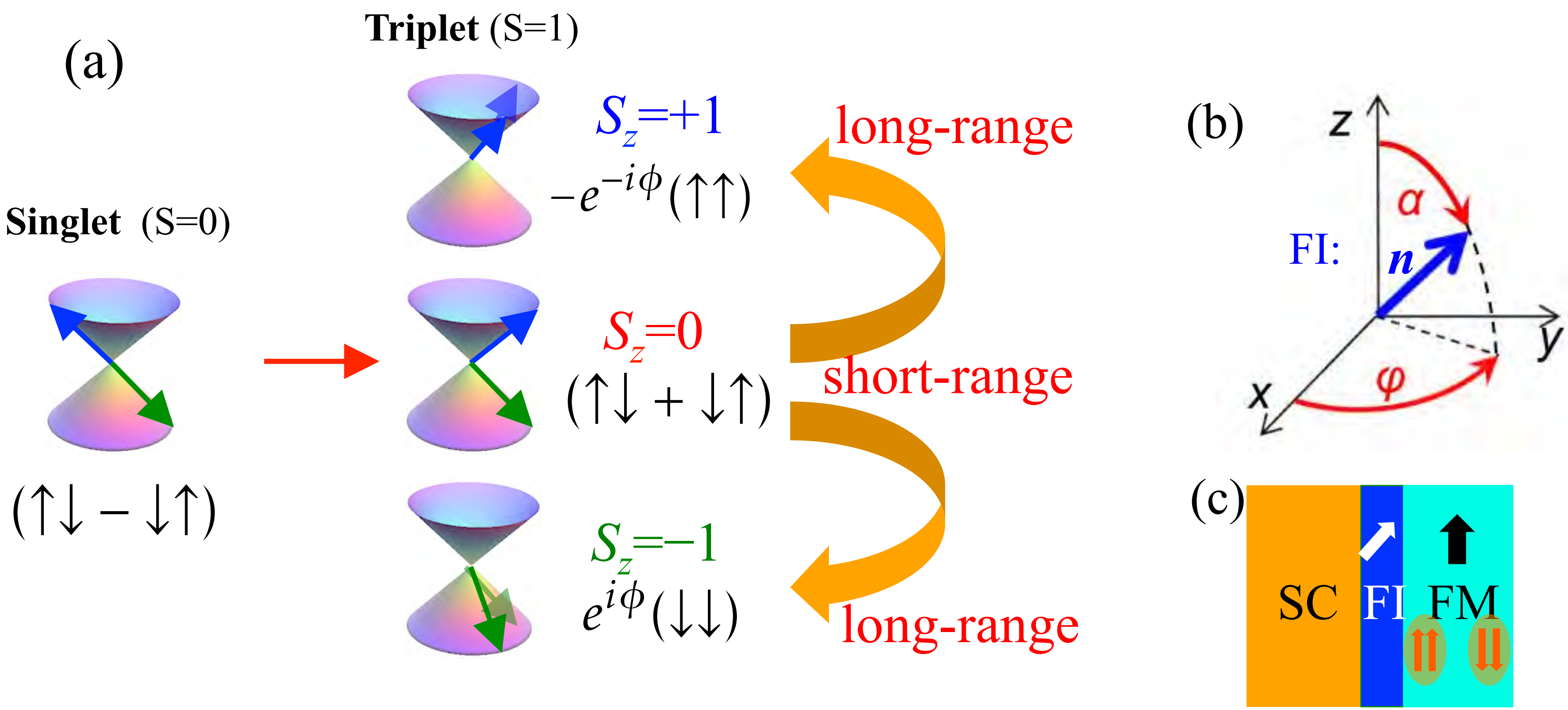




values of 100 μV / K easily reachable

Long-range spin triplet supercurrents

Spin of the Cooper pair:

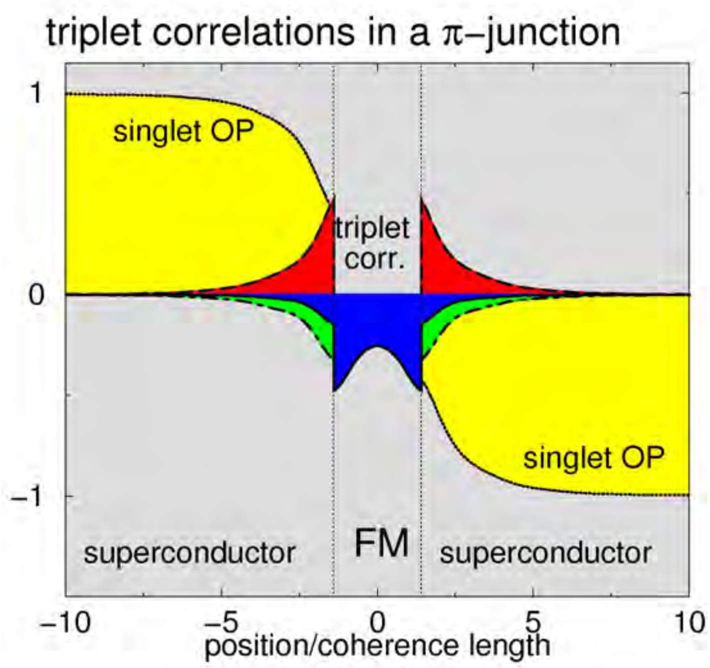


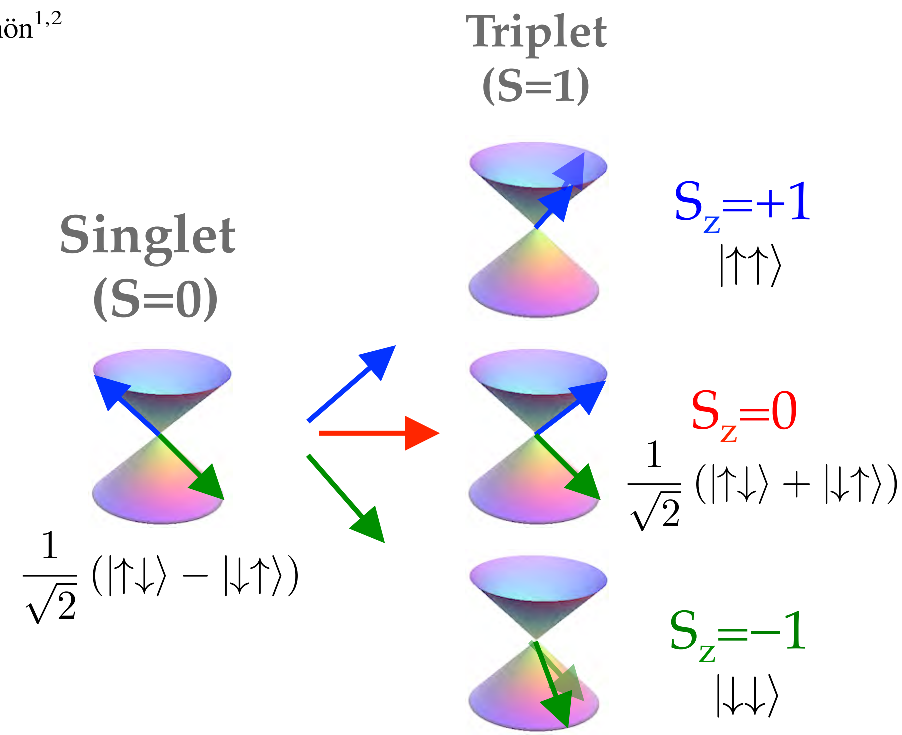
Induced triplet supercondutivity

VOLUME 90, NUMBER 13 PHYSICAL REVIEW LETTERS week ending 4 APRIL 2003

Theory of Half-Metal/Superconductor Heterostructures

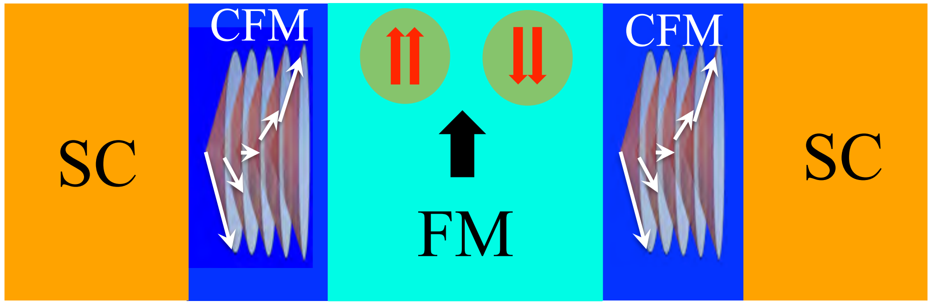
M. Eschrig,[1] J. Kopu,[1] J. C. Cuevas,[1] and Gerd Schön[1,2]



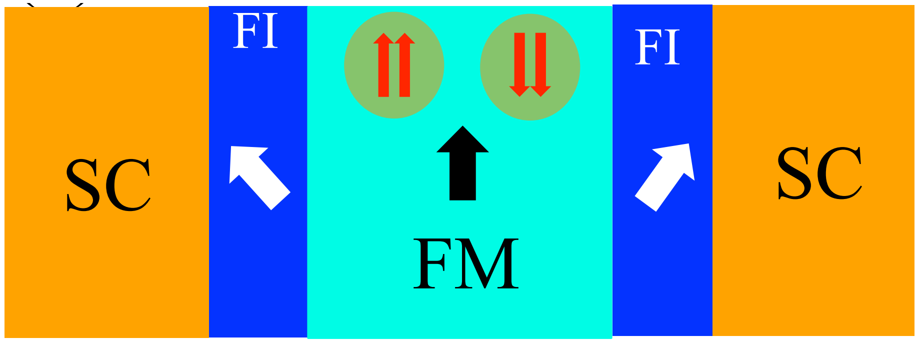


superconductivity with broken spin rotational symmetry
triplet pairs coupled to singlet condensate

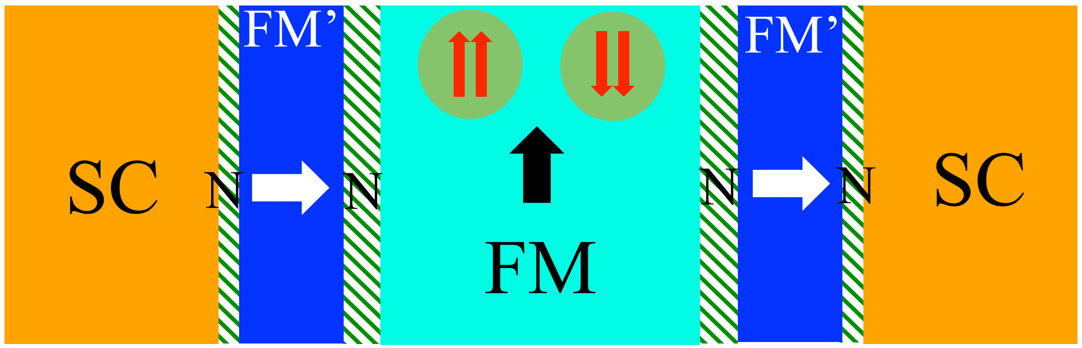
Various geometries appropriate for the generation of long-range spin-triplet supercurrents



Bergeret et al 2001



Eschrig et al 2003, 2007



Houzet et al 2007

Phase batteries

Phase batteries due to geometric phases:

Prediction:

geometric phase φ_2-φ_1
modifies Josephson phase

$$
\begin{aligned}
I_{\uparrow\uparrow} &= I_{1,1}\sin(2\Delta\chi) - I_{1,0}\sin(\Delta\chi - \Delta\phi) \\
I_{\downarrow\downarrow} &= I_{1,1}\sin(2\Delta\chi) - I_{0,1}\sin(\Delta\chi + \Delta\phi).
\end{aligned}
$$

couple to FM spin via FMR to induce precession of spins →Δφ(t)

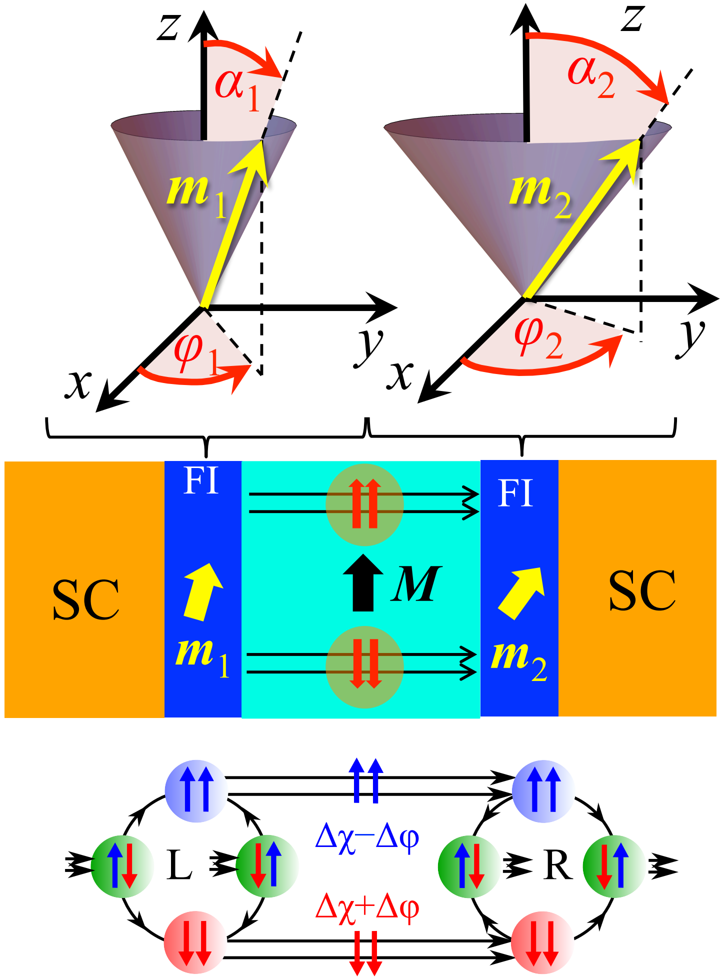
IOP Publishing
Reports on Progress in Physics
Rep. Prog. Phys. **78** (2015) 104501 (50pp)
doi:10.1088/0034-4885/78/10/104501

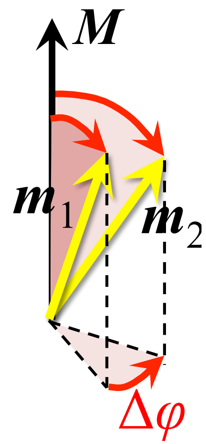
Report on Progress

Spin-polarized supercurrents for spintronics: a review of current progress

Matthias Eschrig

non-coplanar spin arrangement





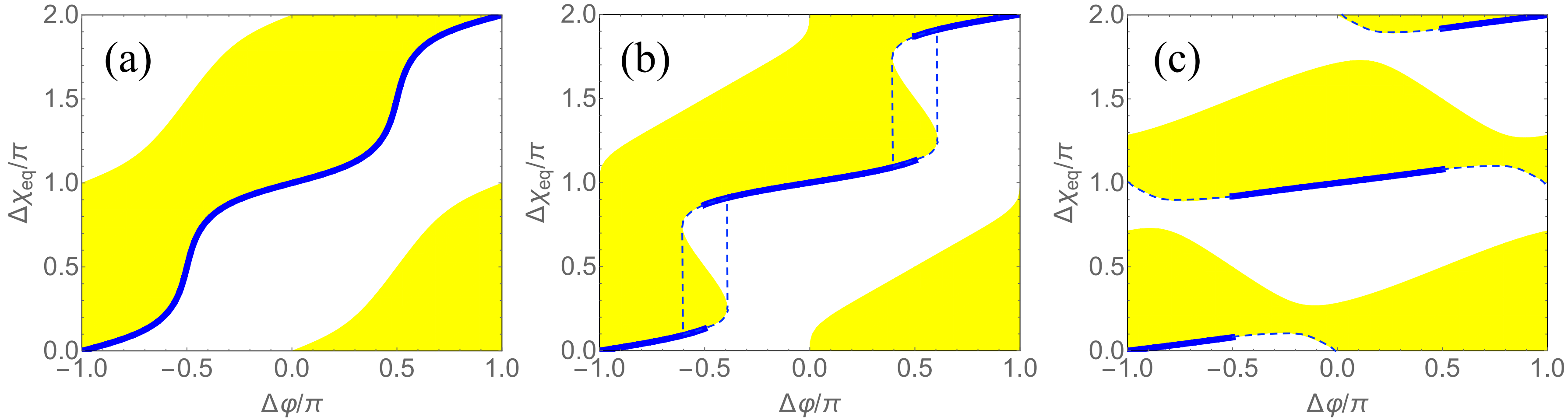
Crossed pair transmission

Phase batteries due to geometric phases:

<u>Prediction:</u>
geometric phase φ_2-φ_1
modifies Josephson phase

$$I_{\uparrow\uparrow} = I_{1,1}\sin(2\Delta\chi) - I_{1,0}\sin(\Delta\chi - \Delta\phi)$$
$$I_{\downarrow\downarrow} = I_{1,1}\sin(2\Delta\chi) - I_{0,1}\sin(\Delta\chi + \Delta\phi)$$

$I_{1,0} = 0.7, I_{0,1} = 0.3$



$I_{1,1} = 0.05$ $I_{1,1} = 0.25$ $I_{1,1} = 0.4$

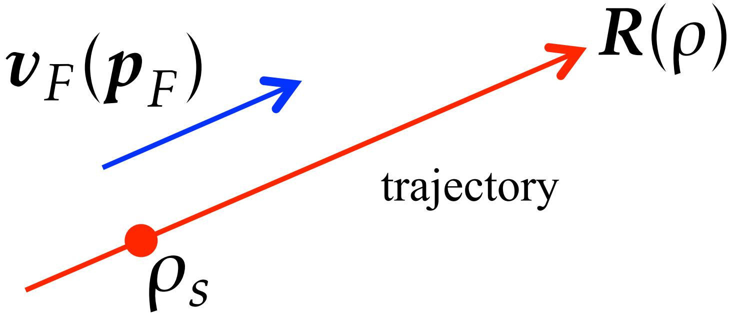
Full lines: equilibrium phase with lowest free energy for given $\Delta\varphi$

M.E. to be published in "Handbook on Spin Transport and Magnetism"

Topology in superconductivity:

zero-energy Andreev bound states in time-reversal symmetric superconductors

Andreev equations:



$$\boldsymbol{R}(\rho) = \boldsymbol{R}_s + \hbar \boldsymbol{v}_F(\boldsymbol{p}_F)(\rho - \rho_s)$$

$$\boxed{\begin{pmatrix} -i\sigma_0\partial_\rho & \Delta \\ -\tilde{\Delta} & i\sigma_0\partial_\rho \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \varepsilon \begin{pmatrix} u \\ v \end{pmatrix}}$$

$$\Delta = [\Delta_0 + \boldsymbol{\Delta}\cdot\boldsymbol{\sigma}]i\sigma_y$$

time-reversal symmetry: $\Delta_0^* = \Delta_0$, $\boldsymbol{\Delta}^* = \boldsymbol{\Delta}$

internal discrete symmetries allow a canonical transformation to purely off-diagonal Hamiltonian:

$$\mathcal{W} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -\sigma_y \\ -\sigma_y & -1 \end{pmatrix} \qquad \mathcal{W}^\dagger\mathcal{W} = 1 \qquad \begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \mathcal{W}\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} u - \sigma_y v \\ -\sigma_y u - v \end{pmatrix}$$

$$\boxed{\begin{pmatrix} 0 & i\sigma_y\partial_\rho - \Delta \\ i\sigma_y\partial_\rho - \Delta^\dagger & 0 \end{pmatrix}\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \varepsilon \begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix}}$$

Andreev equations:

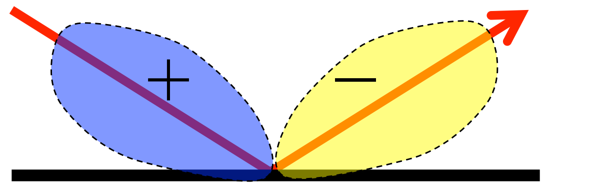
$$\begin{pmatrix} 0 & i\sigma_y\partial_\rho - \Delta \\ i\sigma_y\partial_\rho - \Delta^\dagger & 0 \end{pmatrix}\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \varepsilon \begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix}$$

$$\varepsilon = 0 \qquad \boxed{\underline{u} = \underline{u}_s e^{-\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'} \qquad \underline{v} = \underline{v}_s e^{\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}}$$

$\underline{v}$ stays finite for both $\rho \to \infty$ and $\rho \to -\infty$

$\underline{u}$ diverges for both $\rho \to \infty$ and $\rho \to -\infty$

$\Delta_0(-\infty) > 0$ $\qquad$ $\Delta_0(\infty) < 0$



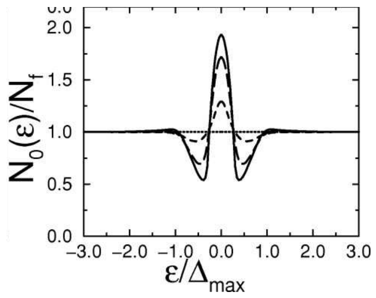
$$\underline{u}_s = 0 \text{ and } \underline{u}(\rho) \equiv 0$$

$$\boxed{u(\rho) \equiv \sigma_y v(\rho)}$$

$$\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \mathcal{W}\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} u - \sigma_y v \\ -\sigma_y u - v \end{pmatrix}$$

$$\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \begin{pmatrix} 0 \\ \underline{v}_s \end{pmatrix} e^{\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}$$

$$\boxed{\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sigma_0 \\ \sigma_y \end{pmatrix} u_s e^{\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}}$$



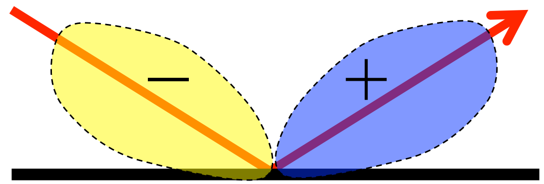
Andreev equations:

$$\begin{pmatrix} 0 & i\sigma_y\partial_\rho - \Delta \\ i\sigma_y\partial_\rho - \Delta^\dagger & 0 \end{pmatrix}\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \varepsilon\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix}$$

$\varepsilon = 0$

$$\boxed{\underline{u} = \underline{u}_s e^{-\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'} \qquad \underline{v} = \underline{v}_s e^{\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}}$$

$\Delta_0(-\infty) < 0$ $\Delta_0(\infty) > 0$



$\underline{u}$ stays finite for both $\rho \to \infty$ and $\rho \to -\infty$.
$\underline{v}$ diverges for both $\rho \to \infty$ and $\rho \to -\infty$

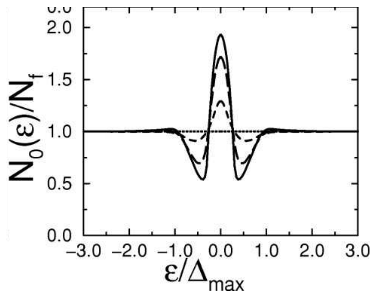
$$\underline{v}_s = 0 \text{ and } \underline{v}(\rho) \equiv 0$$

$$\boxed{u(\rho) \equiv -\sigma_y v(\rho)}$$

$$\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \begin{pmatrix} \underline{u}_s \\ 0 \end{pmatrix} e^{-\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}$$

$$\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \mathcal{W}\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} u - \sigma_y v \\ -\sigma_y u - v \end{pmatrix}$$

$$\boxed{\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \sigma_0 \\ -\sigma_y \end{pmatrix} u_s e^{-\int_{\rho_s}^{\rho}\Delta_0(\rho')d\rho'}}$$

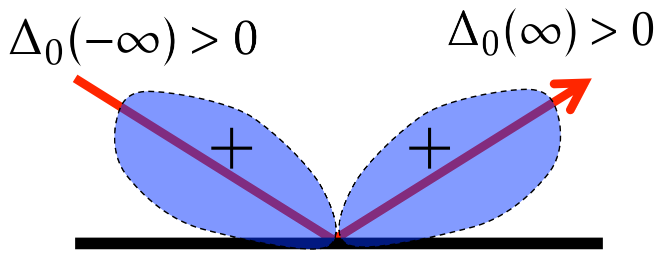


Andreev equations:

$$\begin{pmatrix} 0 & i\sigma_y\partial_\rho - \Delta \\ i\sigma_y\partial_\rho - \Delta^\dagger & 0 \end{pmatrix}\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \varepsilon \begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix}$$

$\varepsilon = 0$

$$\boxed{\underline{u} = \underline{u}_s e^{-\int_{\rho_s}^{\rho} \Delta_0(\rho')\mathrm{d}\rho'} \qquad \underline{v} = \underline{v}_s e^{\int_{\rho_s}^{\rho} \Delta_0(\rho')\mathrm{d}\rho'}}$$



$\underline{u}$ diverges for $\rho \to -\infty$

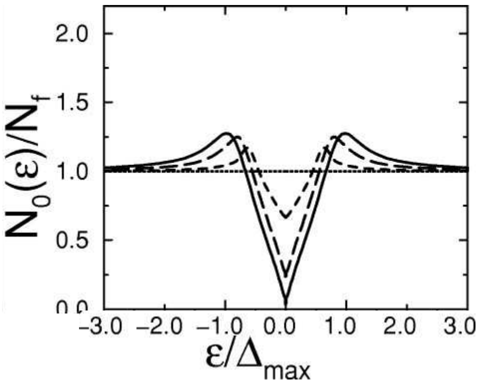
$\underline{v}$ diverges for $\rho \to \infty$

$\underline{u}_s = 0$ and $\underline{u}(\rho) \equiv 0$

$\underline{v}_s = 0$ and $\underline{v}(\rho) \equiv 0$

$$\boxed{u(\rho) \equiv v(\rho) \equiv 0}$$

$$\begin{pmatrix} \underline{u} \\ \underline{v} \end{pmatrix} = \mathcal{W}\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} u - \sigma_y v \\ -\sigma_y u - v \end{pmatrix}$$



$$\boxed{\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

No eigenvector for $\varepsilon = 0$.

No bound state for $\varepsilon = 0$.

Atiyah-Patodi-Singer theorem:

If for a trajectory described by the Andreev Hamiltonion with time-reversal symmetry the condition

$$\Delta_0(-\infty)\Delta_0(\infty) < 0$$

holds, then there will be a topological zero-energy bound state in the eigenvalue spectrum.

Similar statement holds for triplet order parameter with spin conserved in one direction.

See also M. Stone, Ann. Phys. **155**, 56 (1984)

Bulk-surface correspondence:

Topology can be inferred from bulk Hamiltonian

$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{k}} \left(c^{\dagger}_{\boldsymbol{k}\uparrow}, c^{\dagger}_{\boldsymbol{k}\downarrow}, c_{-\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow}\right) \hat{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c_{\boldsymbol{k}\downarrow} \\ c^{\dagger}_{-\boldsymbol{k}\uparrow} \\ c^{\dagger}_{-\boldsymbol{k}\downarrow} \end{pmatrix} \qquad \hat{H}(\boldsymbol{k}) = \begin{pmatrix} \hat{\mathcal{E}}(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k}) \\ -\hat{\Delta}(-\boldsymbol{k})^* & -\hat{\mathcal{E}}(-\boldsymbol{k})^* \end{pmatrix}$$

Due to time-reversal and particle-hole symmetry one can perform the following transformations:

$$\hat{\mathcal{W}}\hat{H}(\boldsymbol{k})\hat{\mathcal{W}}^{\dagger} = \begin{pmatrix} \hat{0} & \hat{D}(\boldsymbol{k}) \\ \hat{D}^{\dagger}(\boldsymbol{k}) & \hat{0} \end{pmatrix} \qquad \hat{D}(\boldsymbol{k}) = \hat{L}^{\dagger}_{\boldsymbol{k}}\hat{\Sigma}(\boldsymbol{k})\hat{R}_{\boldsymbol{k}} \qquad \hat{q}(\boldsymbol{k}) = \hat{L}^{\dagger}_{\boldsymbol{k}}\hat{R}_{\boldsymbol{k}}$$

1D winding number along contractable loop within Brillouin zone (not $\mathcal{T}$-invariant, class AIII):

$$N_{\mathcal{L}} = \oint_{\mathcal{L}} \frac{\mathrm{d}l}{2\pi i} \mathrm{Tr}\left[q^{-1}\nabla_l q\right]$$

d-wave:

$$\Delta(\boldsymbol{k}) = \Delta_0[\cos(k_x) - \cos(k_y)]i\sigma_y \qquad |N_{\mathcal{L}}| = 2$$

Symmetry and topology in non-centrosymmetric superconductors:

strong spin-orbit coupling

spin-orbit interaction: $$\hat{H}_{SO} = \frac{\hbar e}{4m^2c^2}\hat{\boldsymbol{\sigma}}\left[\boldsymbol{\nabla}\Phi(\boldsymbol{r}) \times \hat{\boldsymbol{p}}\right]$$

In Bloch basis:

$$\hat{H} = \sum_{\boldsymbol{k}}\sum_{s,s'}\sum_{n,n'}\left[\varepsilon_n(\boldsymbol{k})\delta_{nn'}\delta_{ss'} + \boldsymbol{L}_{nn'}(\boldsymbol{k})\boldsymbol{\sigma}_{ss'}\right] c^{\dagger}_{\boldsymbol{k}ns}c_{\boldsymbol{k}n's'}$$

$$\boldsymbol{L}_{nn'}(\boldsymbol{k}) = \frac{\hbar e}{4m^2c^2}\frac{1}{\Omega}\int_{\Omega} d^3\boldsymbol{r}\; \boldsymbol{\nabla}\Phi(\boldsymbol{r}) \times \left[\phi_{\boldsymbol{k}n}(\boldsymbol{r})^*(-i\hbar\boldsymbol{\nabla})\phi_{\boldsymbol{k}n'}(\boldsymbol{r})\right]$$

Time-reversal symmetry+inversion symmetry: $$\boldsymbol{L}_{nn'}(\boldsymbol{k}) = -\boldsymbol{L}_{n'n}(\boldsymbol{k})$$

In crystals with a center of inversion the band-diagonal elements of the spin-orbit coupling vanish by symmetry: $\boldsymbol{L}_{nn}(\boldsymbol{k})=0$.

Only time-reversal symmetry $$\boldsymbol{L}_{nn'}(\boldsymbol{k}) = -\boldsymbol{L}_{n'n}(-\boldsymbol{k})$$

In non-centrosymmetric crystals, band-diagonal matrix elements can be non-zero and indeed large (30-300meV): $\boldsymbol{L}_{nn}(\boldsymbol{k})=\alpha\,\mathbf{g}(\boldsymbol{k})$

Vector field of spin-orbit vectors in non-centrosymmetric metals:

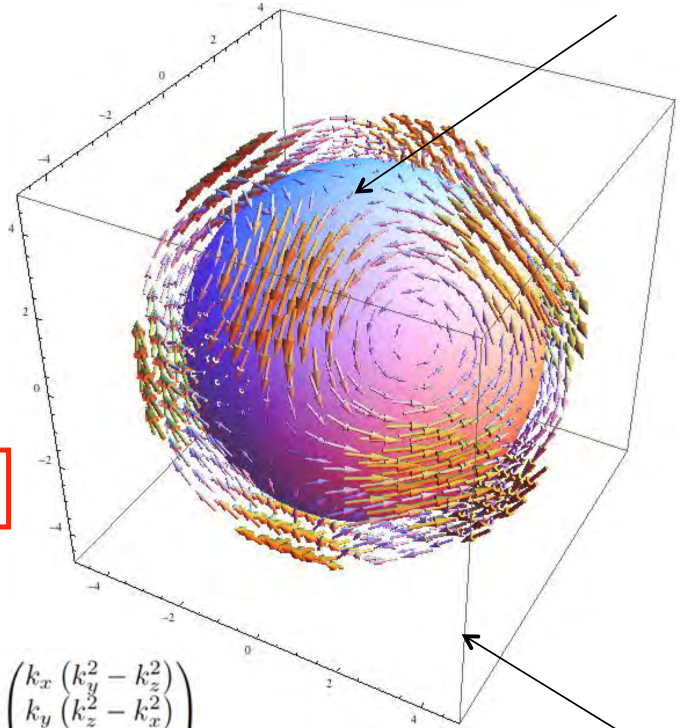
$$\hat{H} = \sum_{\boldsymbol{k}} \sum_{s,s'} \left[\varepsilon(\boldsymbol{k})\delta_{ss'} + \alpha \mathbf{g}(\boldsymbol{k})\boldsymbol{\sigma}_{ss'}\right] c^{\dagger}_{\boldsymbol{k}s} c_{\boldsymbol{k}s'}$$

single band effective Hamiltonian $\quad \mathbf{g}_{-\mathbf{k}} = -\mathbf{g}_{\mathbf{k}}$

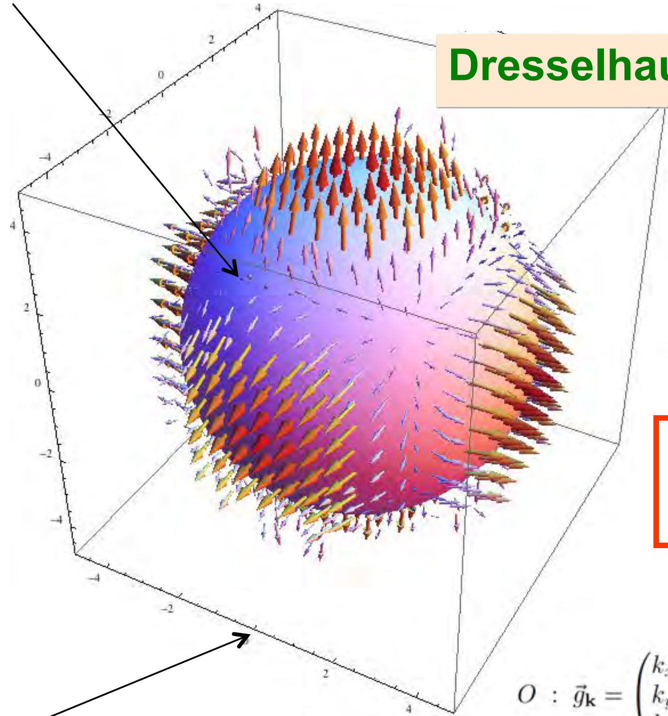
$$\epsilon_{\mathbf{k}}^{\mathrm{CUB}} = t_1 \left[\cos(k_x) + \cos(k_y) + \cos(k_z)\right]$$

Fermi surface

$$\epsilon_{\mathbf{k}}^{\mathrm{BCC}} = 8t_1 \cos(k_x/2)\cos(k_y/2)\cos(k_z/2)$$



Y_2C_3



Dresselhaus coupling

Li_2Pd_3B

Li_2Pt_3B

Brillouin zone

$$T_d \;:\; \vec{g}_{\mathbf{k}} = 2\begin{pmatrix} k_x\left(k_y^2 - k_z^2\right) \\ k_y\left(k_z^2 - k_x^2\right) \\ k_z\left(k_x^2 - k_y^2\right) \end{pmatrix}$$

cubic point group

$$O \;:\; \vec{g}_{\mathbf{k}} = \begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} + g_2 \begin{pmatrix} k_x\left(k_y^2 + k_z^2\right) \\ k_y\left(k_x^2 + k_z^2\right) \\ k_z\left(k_x^2 + k_y^2\right) \end{pmatrix}$$

full tetrahedral point group $\quad g_2 = -1.5$

Vector field of spin-orbit vectors in non-centrosymmetric metals:

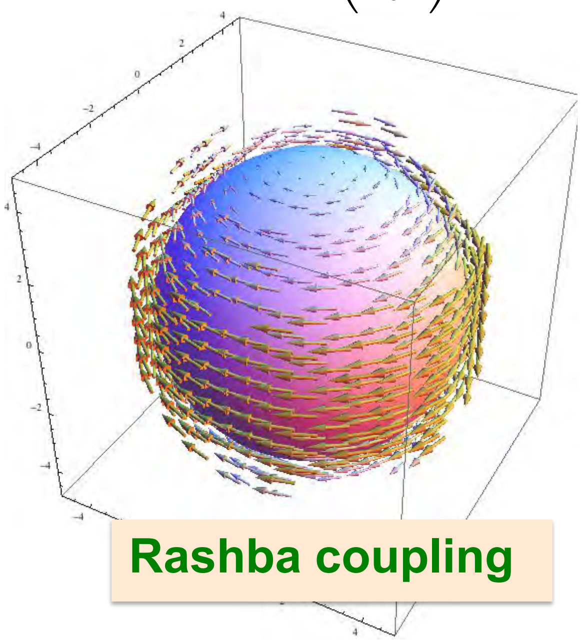
$$\hat{H} = \sum_{\boldsymbol{k}} \sum_{s,s'} \left[\varepsilon(\boldsymbol{k}) \delta_{ss'} + \alpha \mathbf{g}(\boldsymbol{k}) \boldsymbol{\sigma}_{ss'} \right] c^{\dagger}_{\boldsymbol{k}s} c_{\boldsymbol{k}s'}$$

single band effective Hamiltonian

$$\mathbf{g}_{-\mathbf{k}} = -\mathbf{g}_{\mathbf{k}}$$

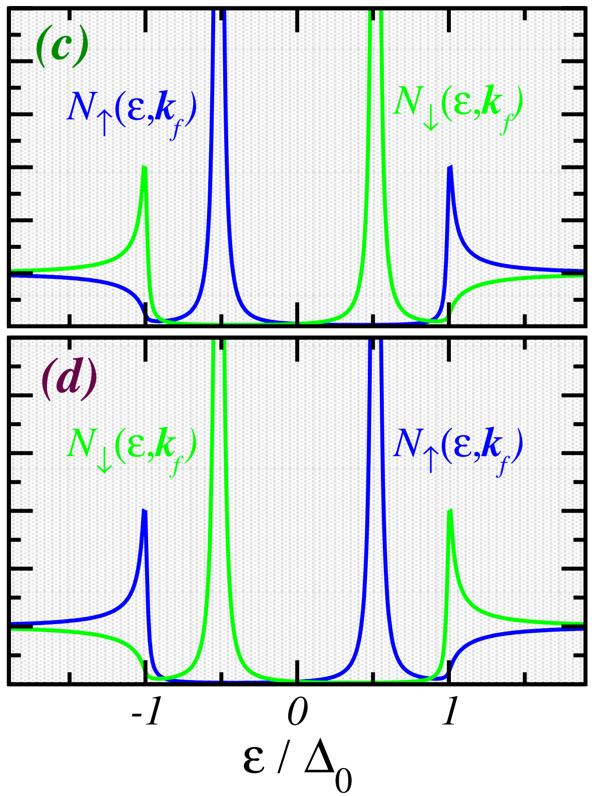
tetragonal point group

$$C_{4v} \ : \boldsymbol{g}(\boldsymbol{k}) = \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}$$



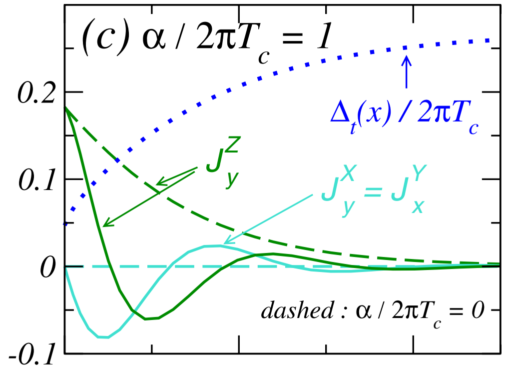
Rashba coupling

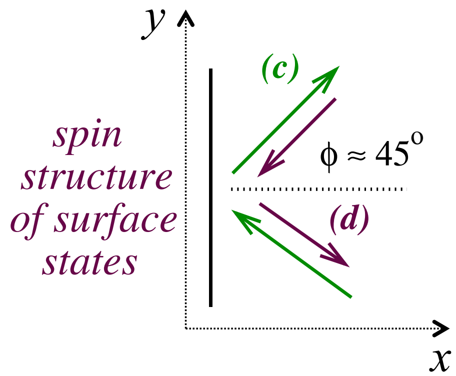
spin-polarized Andreev states



surface spin currents

Supercond.: $J_i^{spin} / (v_f N_f T_c)$





Vorontsov, Vehter, M.E., Phys. Rev. Lett. **101**, 127003 (2008)

Superconducting order parameter in non-centrosymmetric materials:

PHYSICAL REVIEW B 95, 024513 (2017)

Theory of surface spectroscopy for noncentrosymmetric superconductors

Niclas Wennerdal and Matthias Eschrig

Transport equation:

$$i\mathbf{v}_F \cdot \nabla_{\mathbf{R}} \hat{g}^{\mathrm{R,A,M}} + [z\hat{\tau}_3 - \hat{\Delta} - \hat{v}_{\mathrm{SO}}, \hat{g}]^{\mathrm{R,A,M}} = \hat{0}$$

Order parameter: singlet-triplet mixture

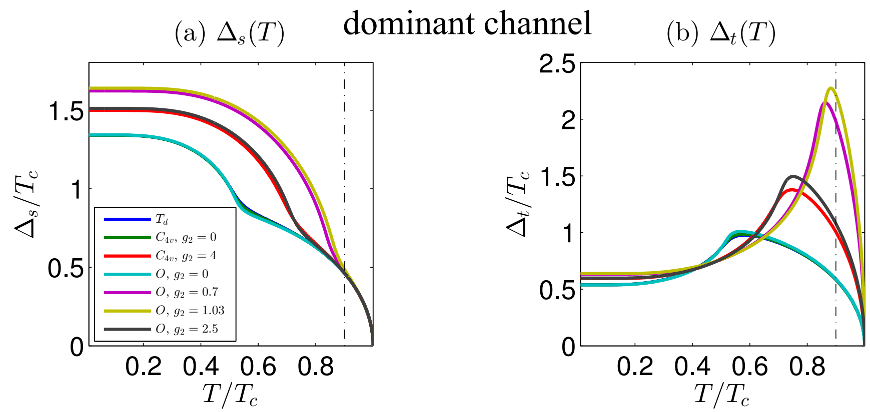
$$\boxed{\Delta(\boldsymbol{k}) = \mathcal{Y}_{\boldsymbol{k}}(\Delta_s + \Delta_t \boldsymbol{g}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma}) i\sigma_y}$$

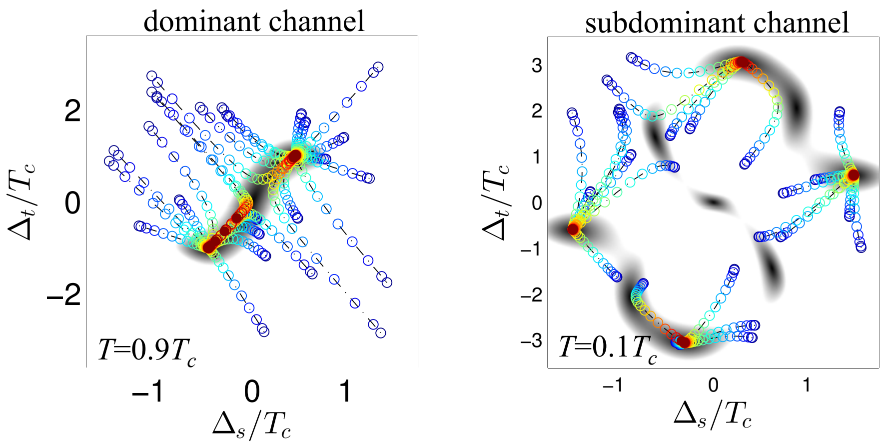
Self-consistency equation:

$$\boxed{\begin{pmatrix} \Delta_s \\ \Delta_t \end{pmatrix} = T \sum_{\epsilon_n}^{|\epsilon_n|<\epsilon_c} N_F V \left\langle A_{\mathbf{k}} \begin{pmatrix} \mathfrak{f}_+ \\ \mathfrak{f}_- \end{pmatrix} \right\rangle}$$

$$A_{\boldsymbol{k}} = \frac{1}{2} \mathcal{Y}^*_{\boldsymbol{k}} \begin{pmatrix} v_s - v_m|\boldsymbol{g}_{\boldsymbol{k}}| & v_s + v_m|\boldsymbol{g}_{\boldsymbol{k}}| \\ v_t|\boldsymbol{g}_{\boldsymbol{k}}| - v_m & -v_t|\boldsymbol{g}_{\boldsymbol{k}}| - v_m \end{pmatrix}$$

$$v_s^2 + v_t^2 + v_m^2 = 1$$





Topological invariants in non-centrosymmetric superconductors:

$$\hat{H} = \frac{1}{2}\sum_{\boldsymbol{k}} \left(c^\dagger_{\boldsymbol{k}\uparrow}, c^\dagger_{\boldsymbol{k}\downarrow}, c_{-\boldsymbol{k}\uparrow}, c_{-\boldsymbol{k}\downarrow}\right) \hat{H}(\boldsymbol{k}) \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c_{\boldsymbol{k}\downarrow} \\ c^\dagger_{-\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

$$\hat{H}(\boldsymbol{k}) = \begin{pmatrix} \hat{\mathcal{E}}(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k}) \\ -\hat{\Delta}(-\boldsymbol{k})^* & -\hat{\mathcal{E}}(-\boldsymbol{k})^* \end{pmatrix}$$

Due to time-reversal and particle-hole symmetry one can perform the following transformations:

$$\hat{\mathcal{W}}\hat{H}(\boldsymbol{k})\hat{\mathcal{W}}^\dagger = \begin{pmatrix} \hat{0} & \hat{D}(\boldsymbol{k}) \\ \hat{D}^\dagger(\boldsymbol{k}) & \hat{0} \end{pmatrix} \qquad \hat{D}(\boldsymbol{k}) = \hat{L}^\dagger_{\boldsymbol{k}}\hat{\Sigma}(\boldsymbol{k})\hat{R}_{\boldsymbol{k}} \qquad \hat{q}(\boldsymbol{k}) = \hat{L}^\dagger_{\boldsymbol{k}}\hat{R}_{\boldsymbol{k}}$$

3D winding number for fully gapped systems (class DIII):

$$\boxed{\nu = \int_{\mathrm{BZ}} \frac{\mathrm{d}^3\mathbf{k}}{24\pi^2}\varepsilon^{abc}\mathrm{Tr}\left[(q^{-1}\partial_a q)(q^{-1}\partial_b q)(q^{-1}\partial_c q)\right]}$$

topological superconductivity

(e.g. Sato *et al.* 2011, Schnyder *et al.* 2012)

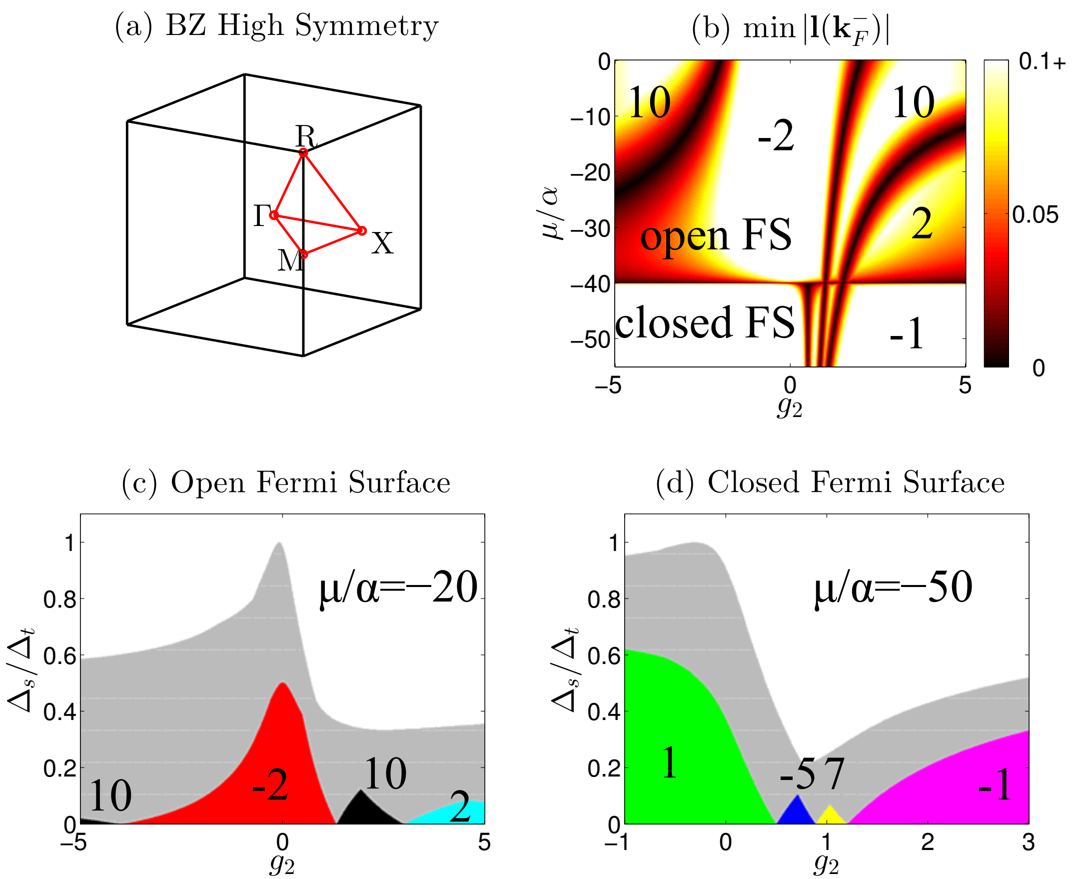
1D winding number along contractable loop (not $\mathcal{T}$-invariant, class AIII):

$$\boxed{N_{\mathcal{L}} = \oint_{\mathcal{L}} \frac{\mathrm{d}l}{2\pi i}\mathrm{Tr}\left[q^{-1}\nabla_l q\right]}$$

1D winding number along non-contractable loop of Brillouin zone torus $\mathbf{T}^3 = S_1\times S_1\times S_1$ in direction of surface normal $\mathbf{n}=(lmn)$:

$$\boxed{N_{(lmn)}(\mathbf{k}_\parallel) = \int \frac{\mathrm{d}k_\perp}{2\pi i}\mathrm{Tr}\left[q^{-1}\nabla_\perp q\right]}$$

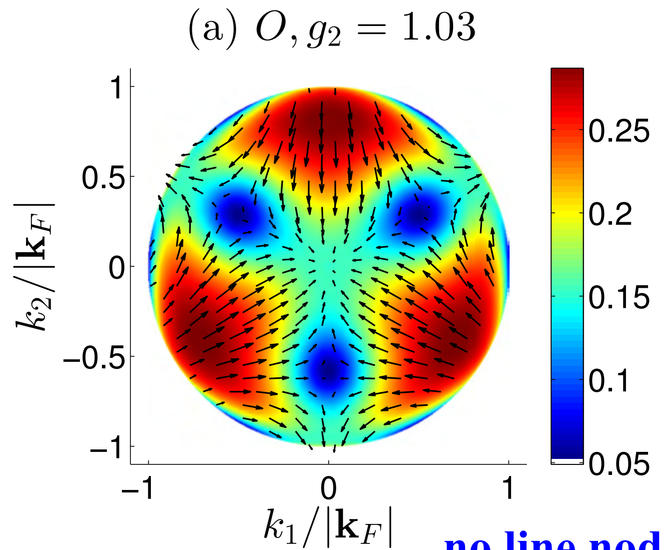
Topological phases in non-centrosymmetric superconductors



$$\hat{\mathcal{H}}_{\mathbf{k}} = \sum_{\mathbf{k}\alpha\beta} c^{\dagger}_{\mathbf{k}\alpha} \left(\xi_{\mathbf{k}}\sigma_0 + \alpha \mathbf{l}_{\mathbf{k}} \cdot \boldsymbol{\sigma}\right)_{\alpha\beta} c_{\mathbf{k}\beta}$$

Cubic point group:

$$\mathbf{l}_{\mathbf{k}} = \begin{pmatrix} \sin(k_x)\left[1 - g_2\left(\cos(k_y) + \cos(k_z)\right)\right] \\ \sin(k_y)\left[1 - g_2\left(\cos(k_z) + \cos(k_x)\right)\right] \\ \sin(k_z)\left[1 - g_2\left(\cos(k_x) + \cos(k_y)\right)\right] \end{pmatrix}$$



no line nodes!

$$\nu = \int_{\mathrm{BZ}} \frac{d^3\mathbf{k}}{24\pi^2} \varepsilon^{abc} \mathrm{Tr}\left[(q^{-1}\partial_a q)(q^{-1}\partial_b q)(q^{-1}\partial_c q)\right]$$

N. Wennerdal, M. Eschrig, PRB 2017

Surface band structure:

$$\boxed{H(\mathbf{k}_{\parallel}, k_{\perp}, \mathbf{R}) \to H(\mathbf{k}_{\parallel}, \rho, \mathbf{R})}$$

$k_{\perp}$ in the direction of the surface normal $\mathbf{n}$

Fourier transform yields a number of terms describing hopping beween layers

$$H(\mathbf{k}_{\parallel}, \rho, \mathbf{R}) = \sum_j H_j(\mathbf{k}_{\parallel}, \mathbf{R} + \frac{1}{2}\rho\mathbf{n})\, \delta(j - \rho/\rho_0)$$

with j layer index, and ρ_0 denotes lattice periodicity in direction of surface normal.

After discretization of center of mass variable $\mathbf{R}$ in steps of ρ_0:

$$\boxed{\sum_{j=-j(l)}^{j(l)} H_j\left(\mathbf{k}_{\parallel}, \mathbf{n}\rho_0(l + \frac{1}{2}j)\right) \psi_j(\mathbf{k}_{\parallel}) = E_l(\mathbf{k}_{\parallel})\psi_l(\mathbf{k}_{\parallel}),}$$

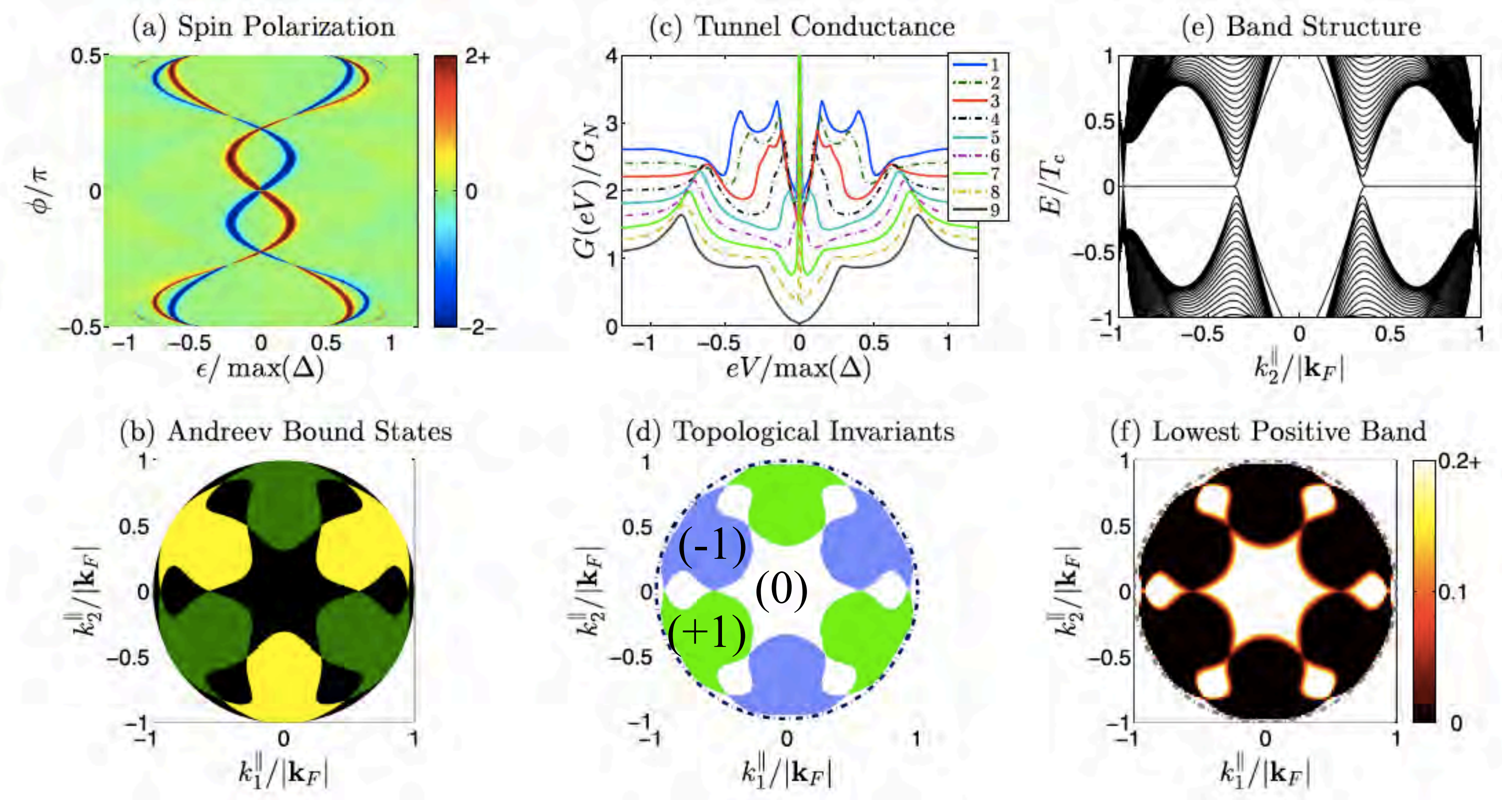
$$l = 0, 1, \ldots, L-1$$

$$j(l) = \min\{j_c, l\}$$

Only a finite number of terms due to tight binding approximation.

Nontrivial topology gives rise to dispersionless (flat-band) zero-energy Andreev bound states.

Topological surface states in non-centrosymmetric superconductors



topological winding number N_{111}

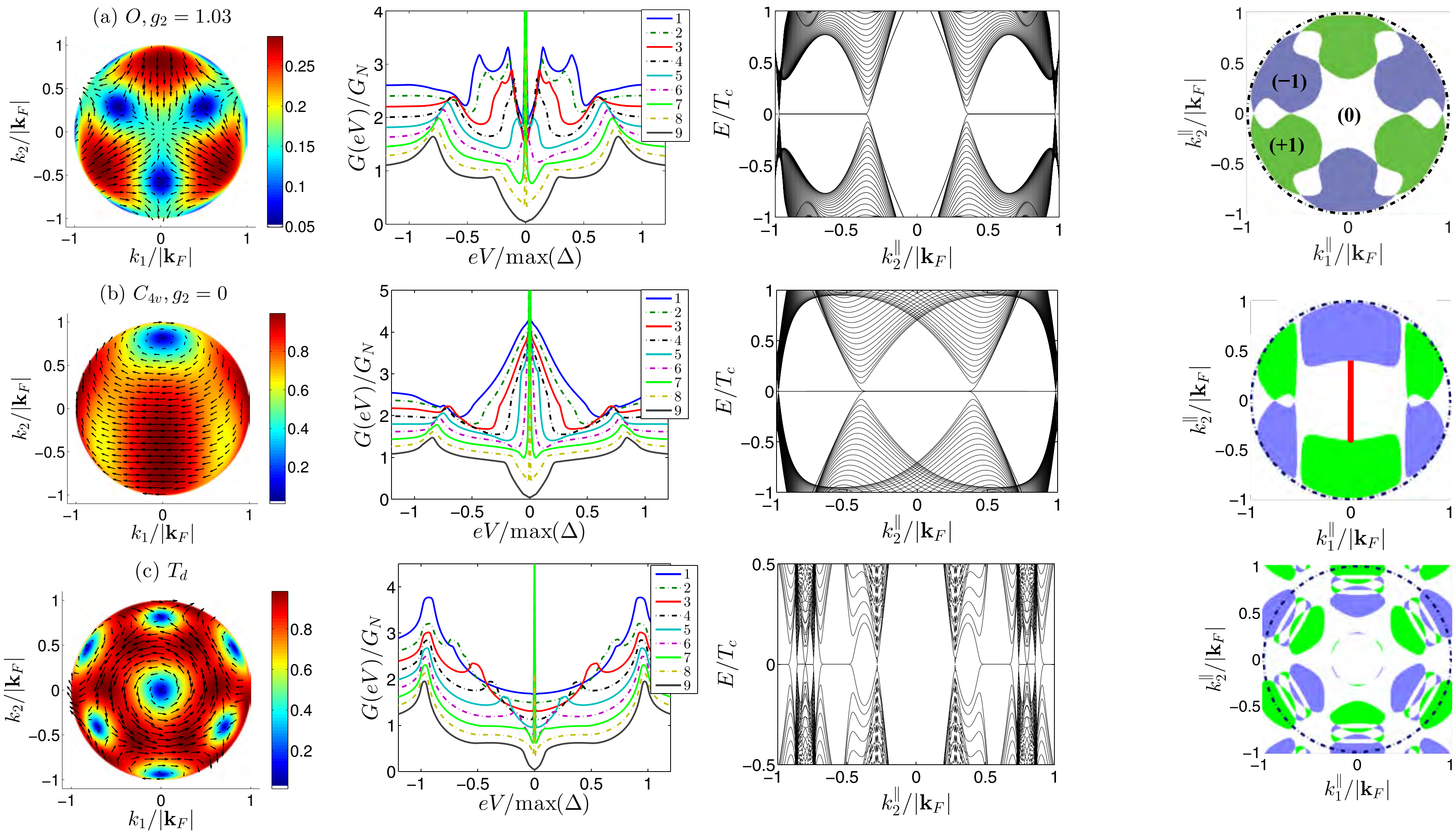
PHYSICAL REVIEW B 95, 024513 (2017)

Theory of surface spectroscopy for noncentrosymmetric superconductors

Niclas Wennerdal and Matthias Eschrig

$$N_{(lmn)}(\mathbf{k}_{\parallel}) = \int \frac{\mathrm{d}k_{\perp}}{2\pi i} \mathrm{Tr}\left[q^{-1}\nabla_{\perp} q\right]$$

Topological surface states in non-centrosymmetric superconductors



Topologically protected zero bias states for the 111 orientation are present for all three symmetries

Note the two different aspects of topology in condensed matter physics:

Topology of ground state connects **bulk** properties
with **interface** properties

Topological excitations give rise to dissipation

Summary

Superfluids and superconductors show the rich interplay between spontaneous symmetry breaking and emergent symmetry

The topology of the ground state and of the excitations is closely related to the symmetry of the order parameter in topological materials

Combination of spin and Cooper pairs opens new horizons for a quickly developing field "superspintronics", contributing to solution of energy efficiency problem for data centers and supercomputers

Condensed matter systems as test ground for particle physics: e.g. equivalents of Higgs bosons, Majorana fermions, Chiral fermions may emerge at low energies