Review: arXiv:1706.08245.

# Superconductor/ferromagnet systems out of equilibrium


Tero T. Heikkilä

Nanoscience Center/Department of Physics,
University of Jyväskylä, Finland


Collaborators:

Sebastián Bergeret and Asier Ozaeta - San Sebastian
F. Giazotto and Pauli Virtanen - SNS Pisa
M. Silaev, I. Maasilta, Faluke Aikebaier, Risto Ojajärvi, Jyväskylä


Funding by

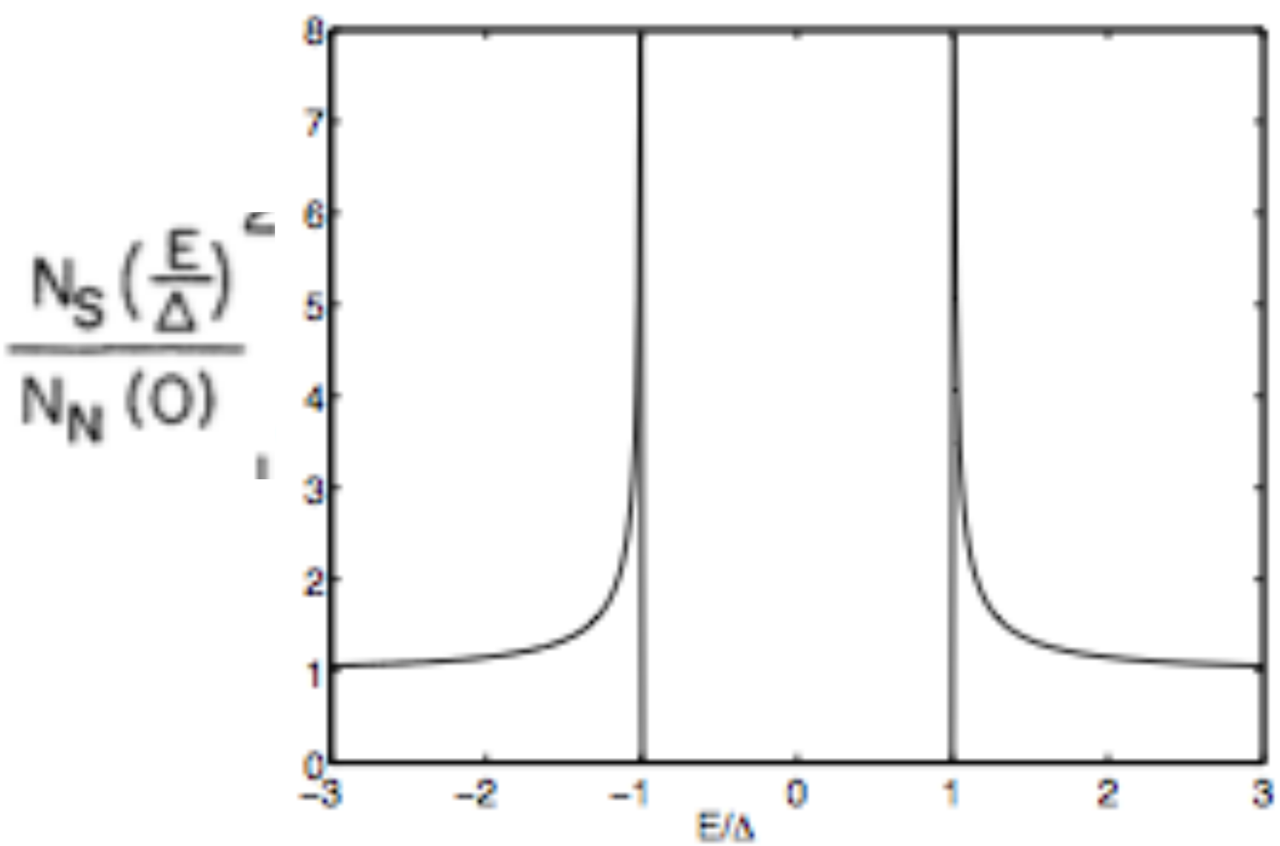
# Superconductivity and ferromagnetism

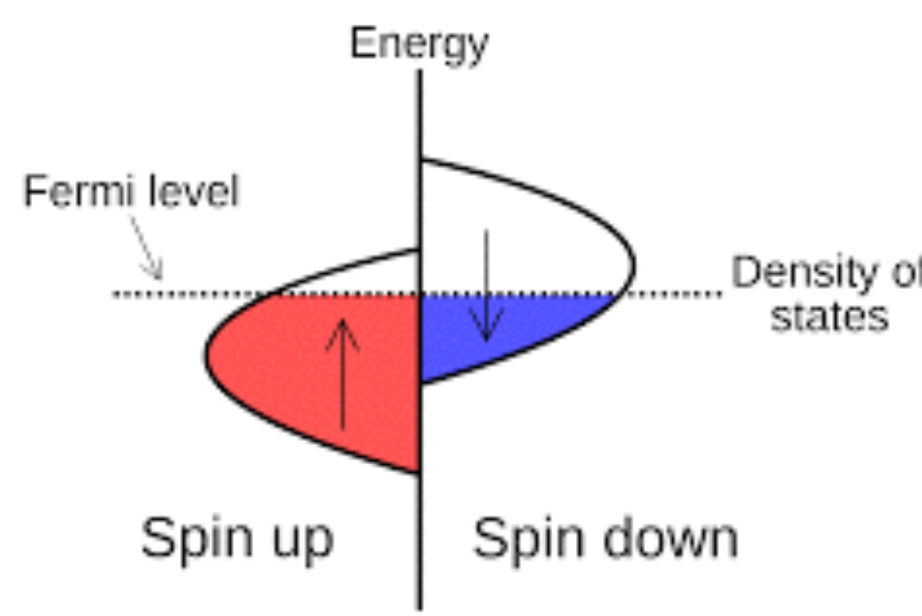
Superconductor

$\langle \psi_\uparrow \psi_\downarrow \rangle$

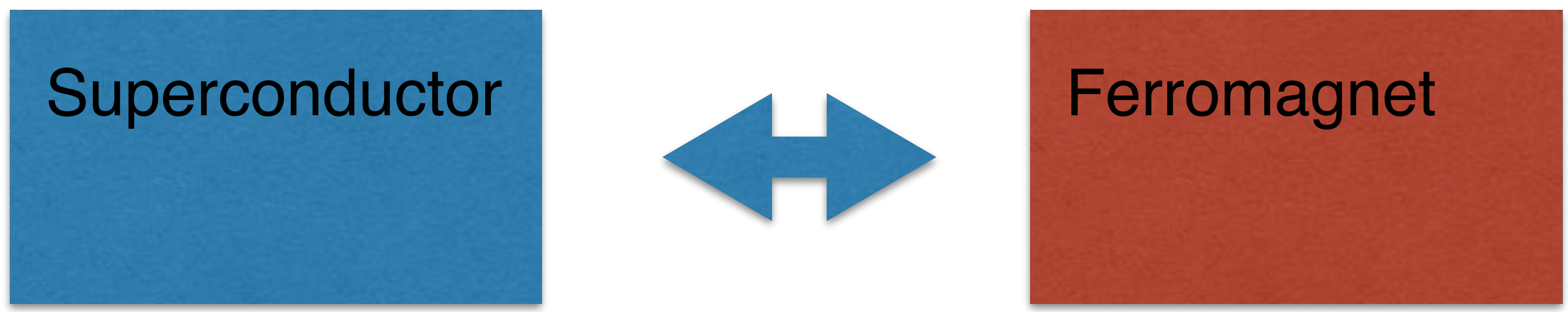
Ferromagnet

$\vec{M}$



# Coupling between ferromagnetism and superconductivity




Equilibrium: proximity effect length scales:

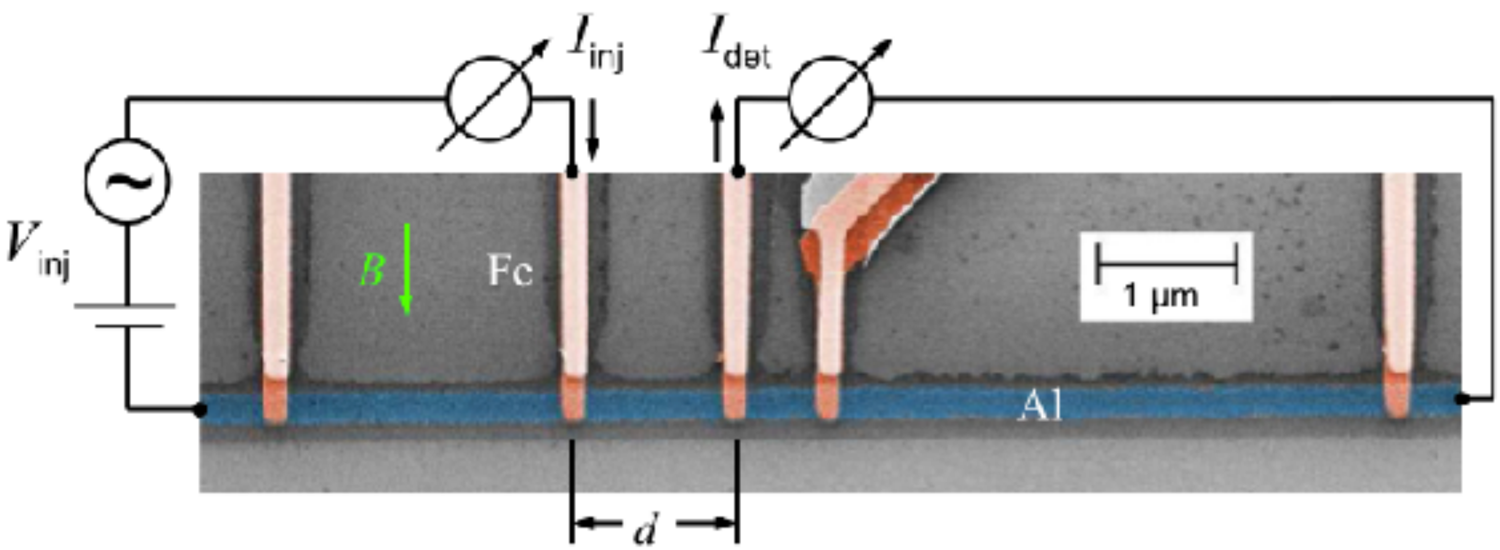
S: • $\xi_0 \sim 10 \ldots 200$ nm

F: • $\ell_m \sim$ few nm

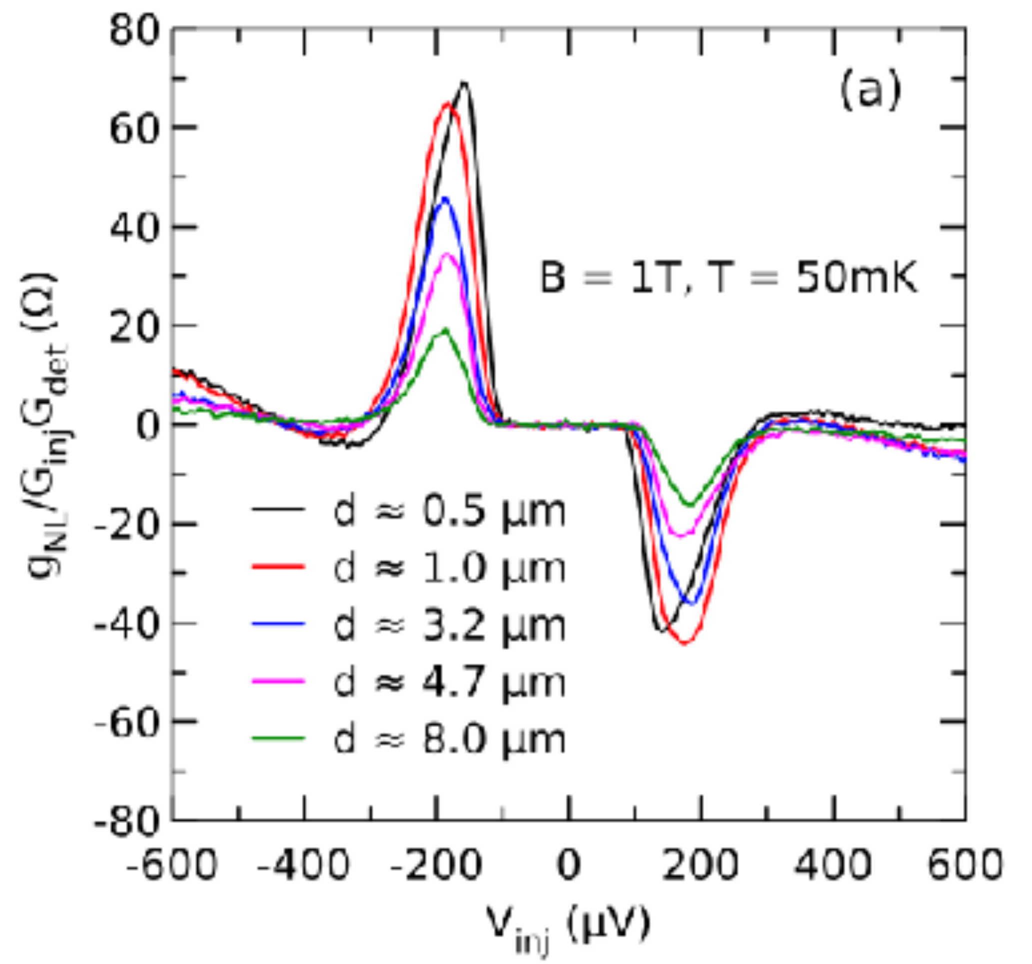
Out of equilibrium: modified charge/spin/heat transport, effect on order parameter length scales:

- $\ell_{\rm in} \sim \ell_{\rm e-ph} \sim$ several $\mu$m at low $T$
- $\ell_{\rm sf} \sim$ hundreds of nm

# Long-range spin accumulation




Hübler, Wolf, Beckmann, von Löhneysen, PRL **109,** 207001 (2012)




$$g_{NL} = \frac{dI_{\mathrm{det}}}{dV_{\mathrm{inj}}}$$

$$\text{Measured spin diffusion length} \sim 3\ \mu\text{m} \sim 10 \times \ell_{sf}^{N}$$

Also: Quay, Chevallier, Bena & Aprili, Nat. Phys. **9**, 84 (2013)
Yang, Yang, Takahashi, Maekawa & Parkin, Nature Mat. **9**, 586 (2010)

# Why was this strange?

Normal state: $\ell_{\rm spin} \sim 300$ nm

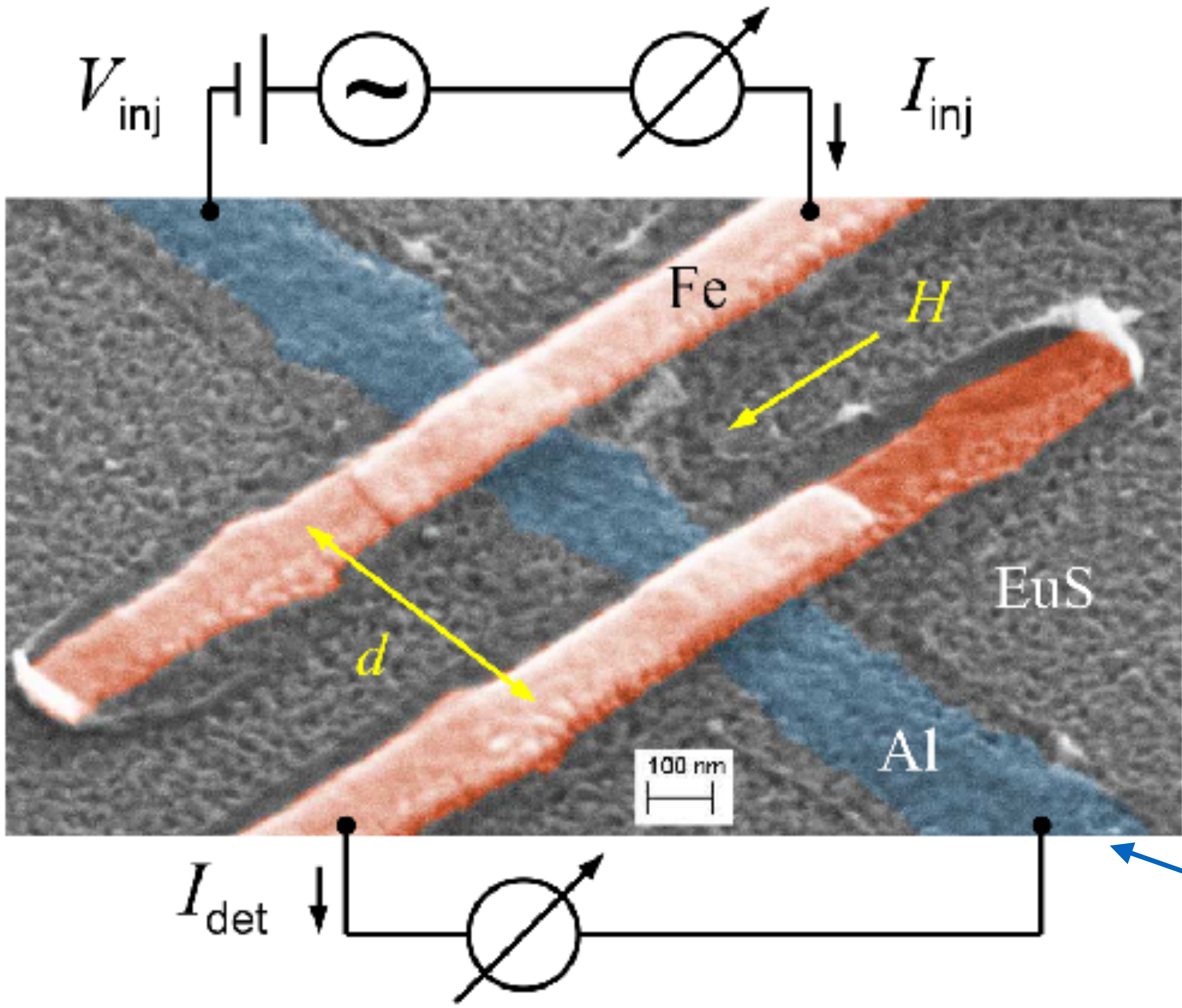
Effect of superconductivity (Morten & Belzig, 2004): depends on the type of relaxation (spin-orbit or spin-flip)

Spin-orbit: $\ell^S_{\rm spin} \approx \ell^N_{\rm spin}$

Spin-flip: $\ell^S_{\rm spin} \ll \ell^N_{\rm spin}$

In any case one should not get the observed length $\ell^S_{\rm spin} \sim 10\ell^N_{\rm spin}$

Similar effects on magnetic proximity from a ferromagnetic insulator:




Wolf, Sürgers, Fischer & Beckmann, PRB **90**, 144509 (2014)

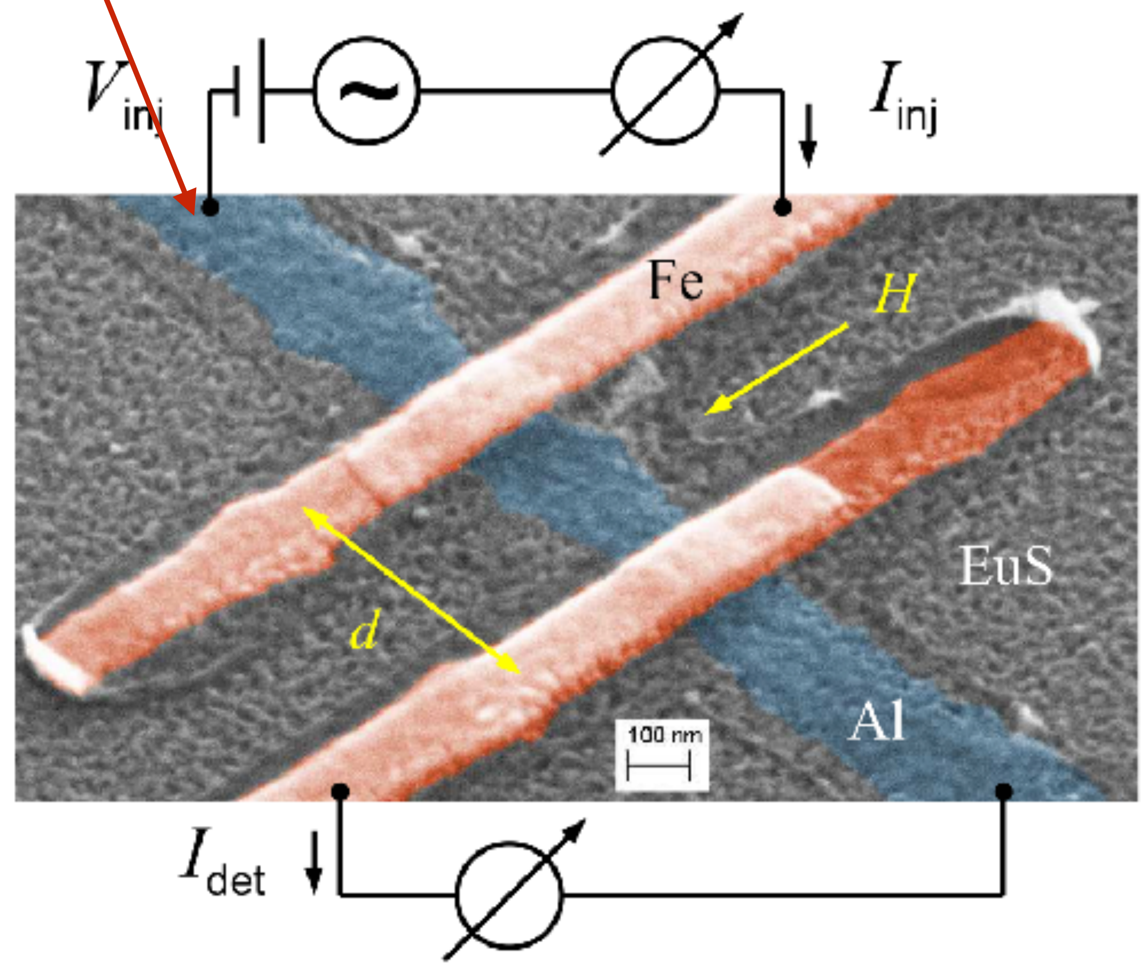
“Ferromagnetic superconductor”

Our task:

1. Describe such "ferromagnetic superconductors" S'

2. Describe the F-S' interface (tunnelling model): boundary conditions

3. Find the kinetic equations inside S' (spin injection)




Wolf, Sürgers, Fischer & Beckmann, PRB **90**, 144509 (2014)

Review: [arXiv:1706.08245](arXiv:1706.08245).

# Theory: diffusive limit

"spectrum"

$$\check{g} = \begin{pmatrix} \boxed{\hat{g}^R} & \hat{g}^K \\ 0 & \boxed{\hat{g}^A} \end{pmatrix}$$

$\hat{g}^{R/K/A}$: $4\times 4$ matrices (spin $\times$ Nambu)

Diffusive limit: Usadel equation

$$D\nabla\cdot(\check{g}\nabla\check{g}) + [i\varepsilon\tau_3 - i\boldsymbol{h}\cdot\boldsymbol{\sigma}\tau_3 - \check{\Delta} - \check{\Sigma}, \check{g}] = 0. \qquad \check{g}^2 = \check{1}$$

exchange field

pair potential $\check{\Delta} = \Delta\tau_1$

self-energy for extra scattering

$$\check{\Sigma} = \check{\Sigma}_{\text{el}} + \check{\Sigma}_{\text{inel}}$$

$$\check{\Sigma}_{\text{el}} = \check{\Sigma}_{\text{so}} + \check{\Sigma}_{\text{sf}} + \check{\Sigma}_{\text{orb}}$$

$$\check{\Sigma}_{\text{inel}} = \check{\Sigma}_{\text{qp-qp}} + \check{\Sigma}_{\text{qp-ph}}$$

$$\Delta = \frac{\lambda}{16}\int_{-\Omega_D}^{\Omega_D} d\epsilon \,\mathrm{Tr}[\tau_1 \check{g}^K(\epsilon)]$$

# Theory: diffusive limit

Equilibrium, no gradients:

$$\cancel{D\nabla(\hat{g}^R\nabla \hat{g}^R)} + [i\epsilon\tau_3 - i\mathbf{h}\cdot\sigma\tau_3 - \hat{\Delta} - \hat{\Sigma}, \hat{g}^R] = 0$$

exchange field

pair potential $\check{\Delta} = \Delta\tau_1$

self-energy for extra scattering

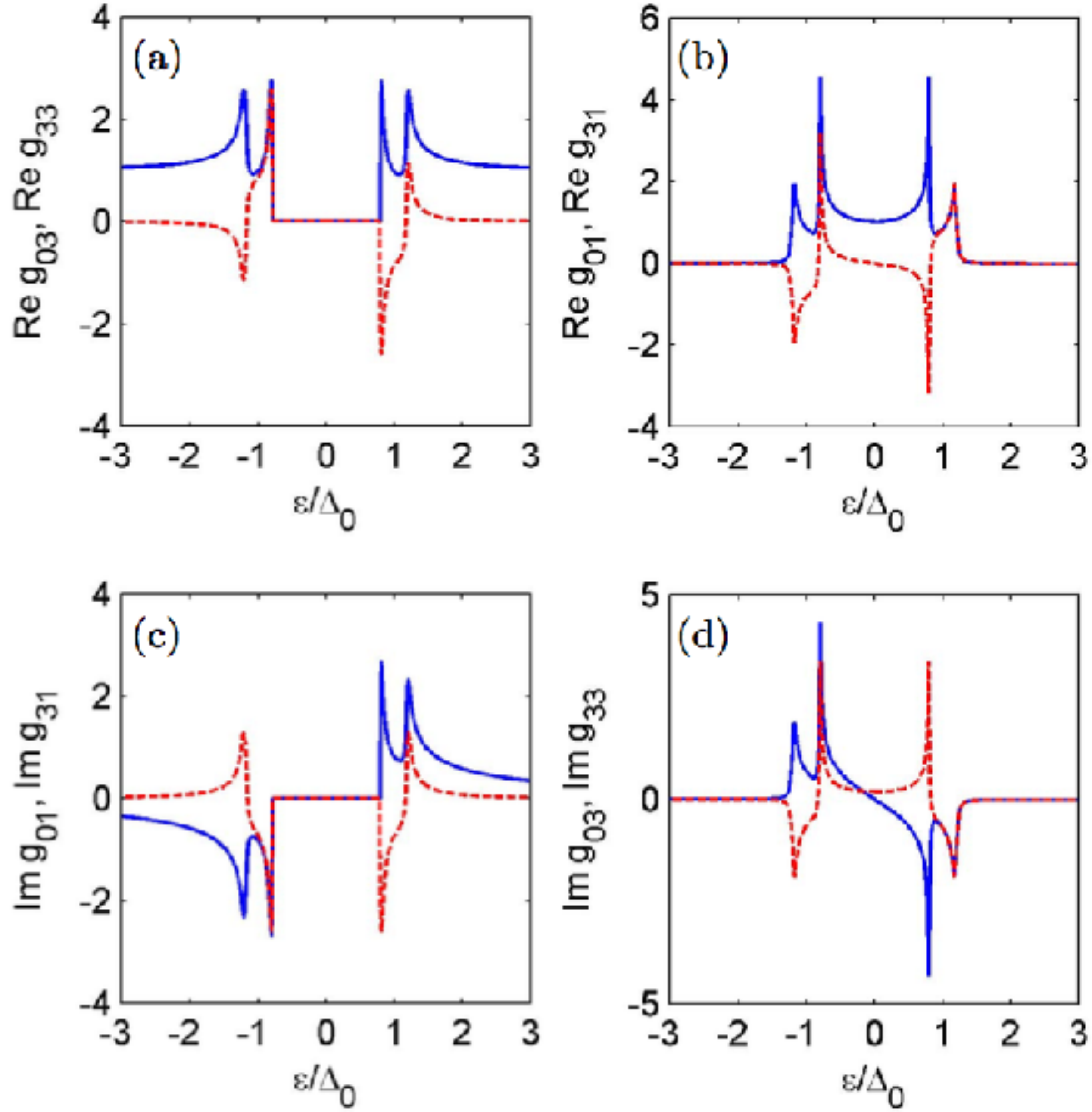
$$\check{\Sigma} = \check{\Sigma}_{\mathrm{el}} + \check{\Sigma}_{\mathrm{inel}}$$

$$\check{\Sigma}_{\mathrm{el}} = \check{\Sigma}_{\mathrm{so}} + \check{\Sigma}_{\mathrm{sf}} + \check{\Sigma}_{\mathrm{orb}}$$

$$\check{\Sigma}_{\mathrm{inel}} = \check{\Sigma}_{\mathrm{qp-qp}} + \check{\Sigma}_{\mathrm{qp-ph}}$$

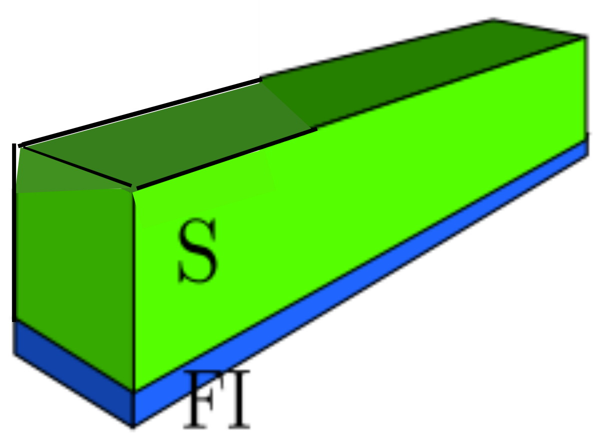
$$\Delta = \frac{\lambda}{16}\int_{-\Omega_D}^{\Omega_D} d\epsilon \mathrm{Tr}[\tau_1(\hat{g}^R - \hat{g}^A)] \tanh\left(\frac{\epsilon}{2T}\right)$$

# Theory: diffusive limit

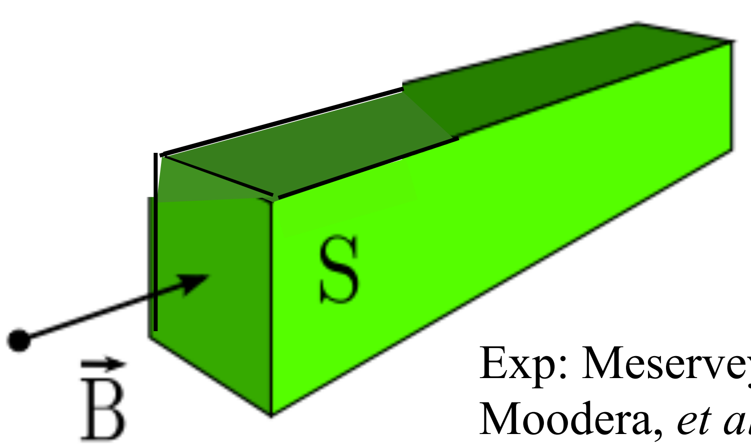


# Ferromagnetic superconductors

S-F bilayer (FI=F insulator)

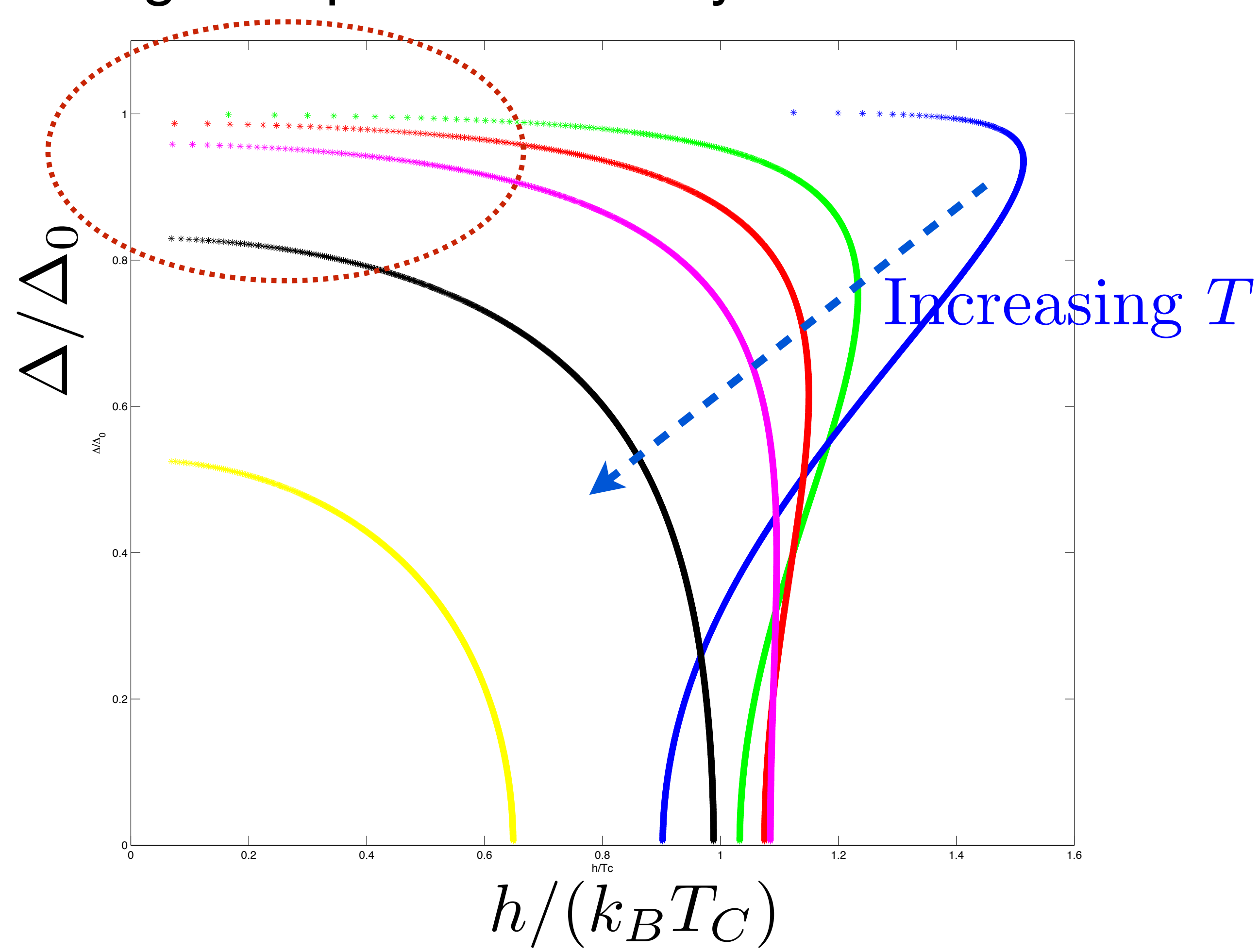



Similar effect from Zeeman field




Exp: Meservey, Tedrow (1971)
Moodera, *et al.* (1990, 2013)
Beckmann, *et al.* (2014)
and many others

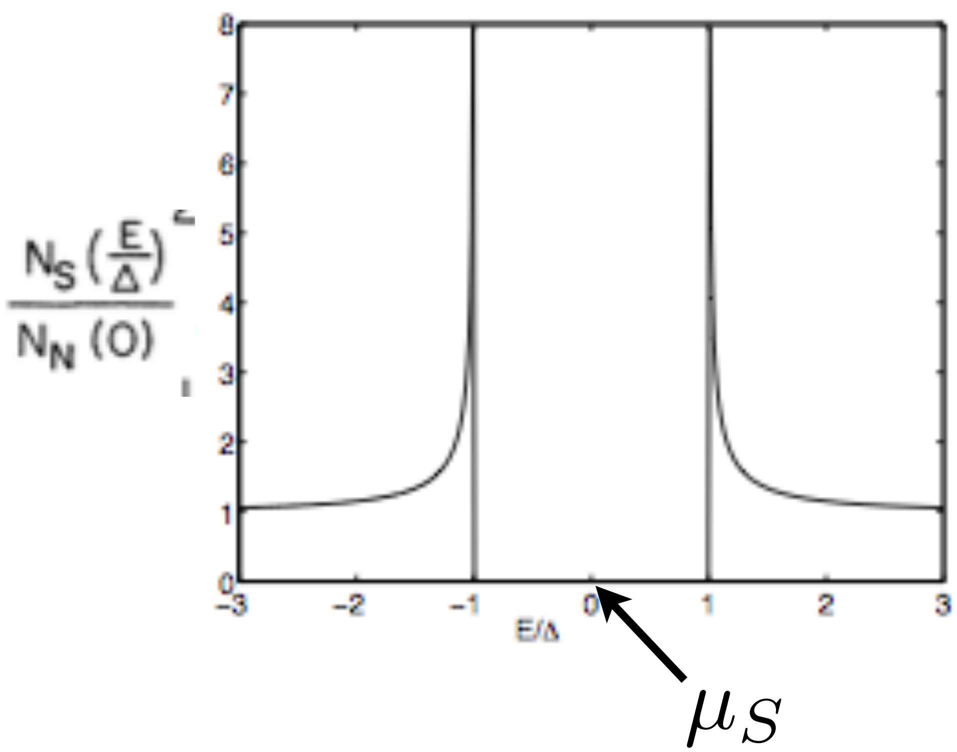
Spin-splitting field *h* in a superconductor: kills singlet superconductivity



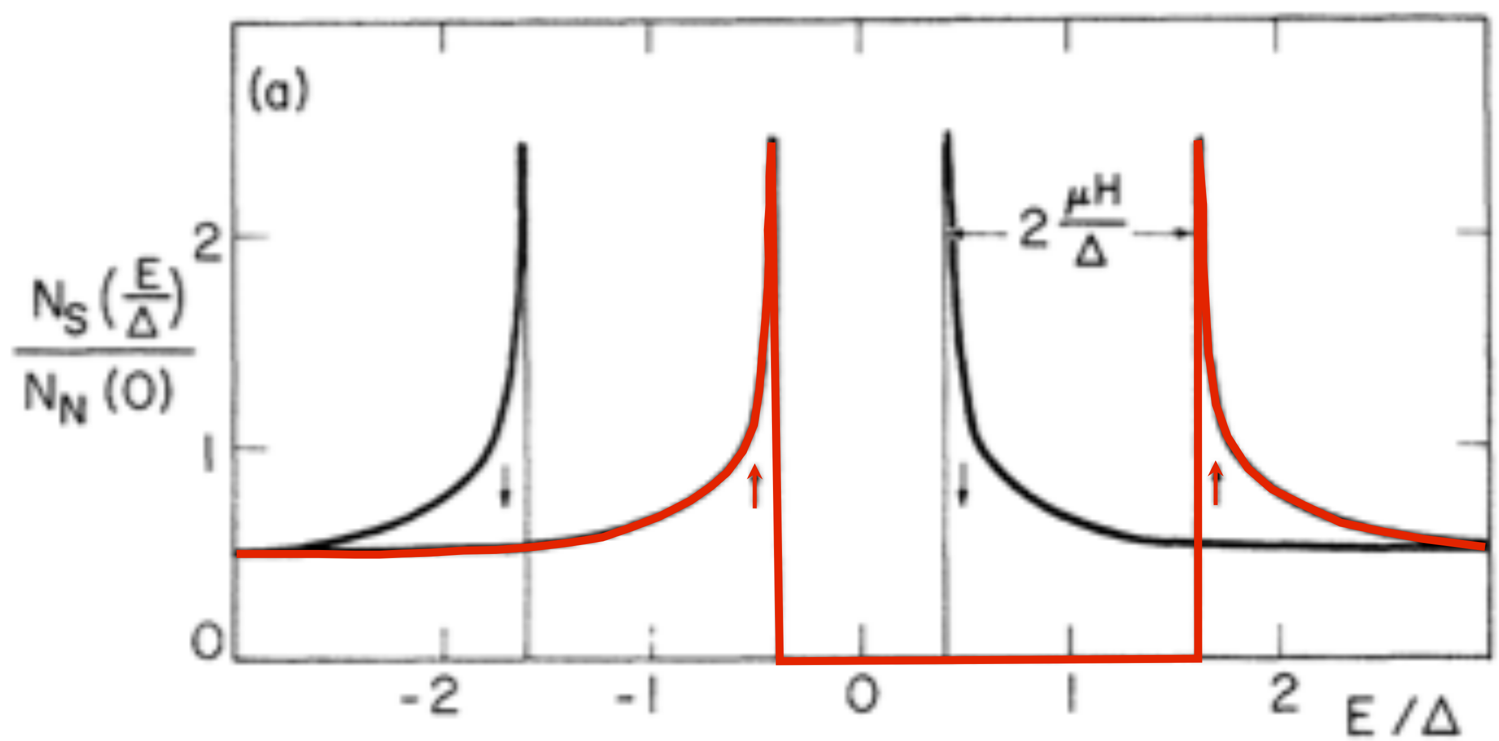

Chandrasekhar (1962), Clogston (1962), Fulde & Ferrell (1964), Larkin & Ovchinnikov (1965), Maki & Tsuneto (1965)
Alexander, *et al.* (1985), Catelani, *et al.*, (2008), and many others

# Spin-split density of states

BCS density of states




With exchange field $h$:

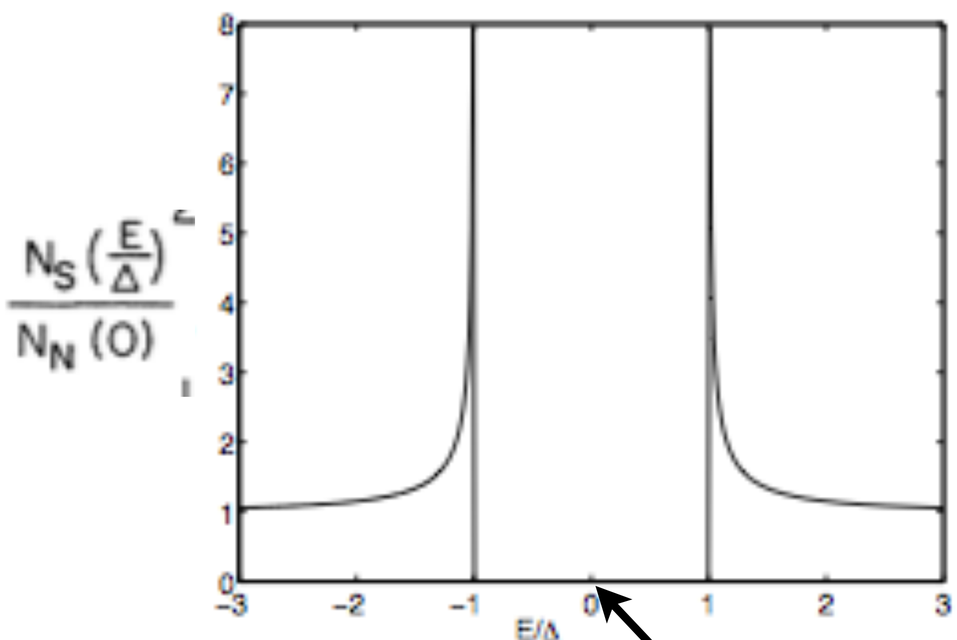



Meservey & Tedrow, PRL **27**, 919 (1971)

(ALSO IN J. MOODERA'S TALK)

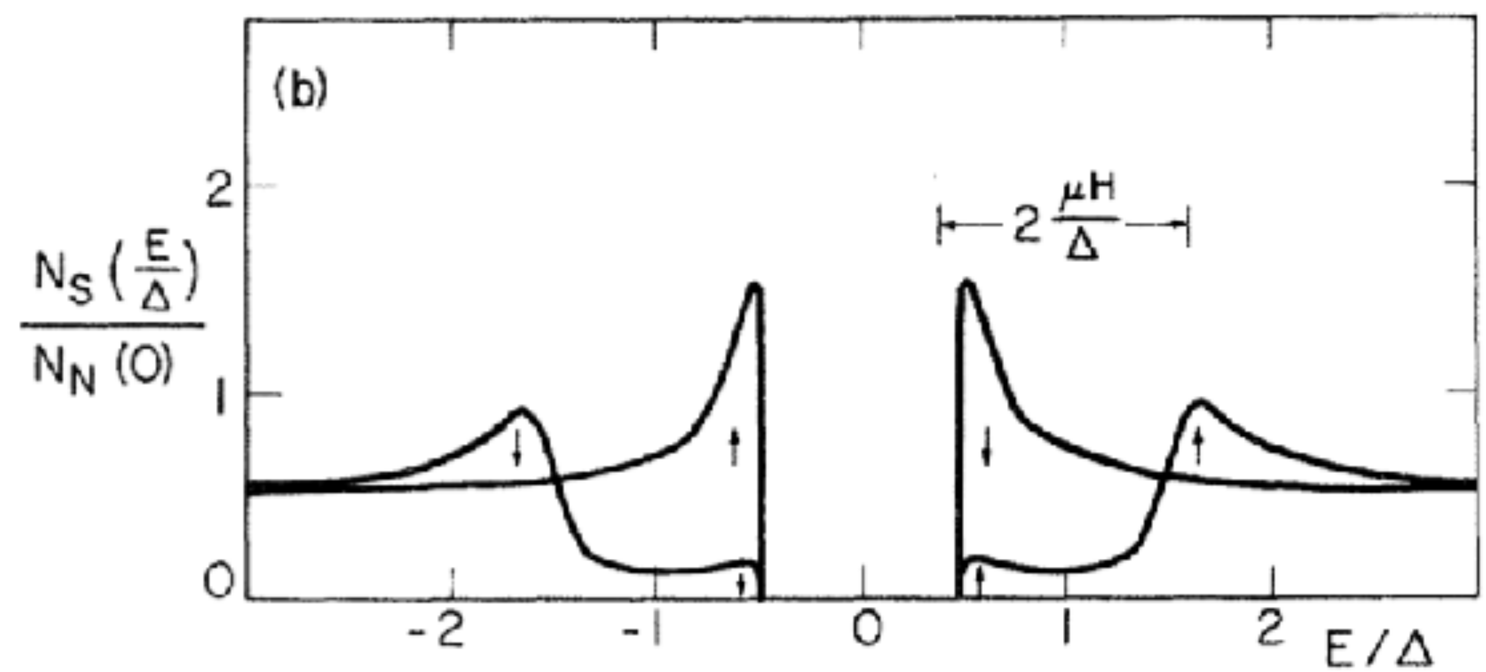
# Superconductor and an exchange field

BCS density of states




$\mu_S$

With exchange field $h$
and spin-orbit scattering




Tedrow & Meservey, PRL **27**, 919 (1971)

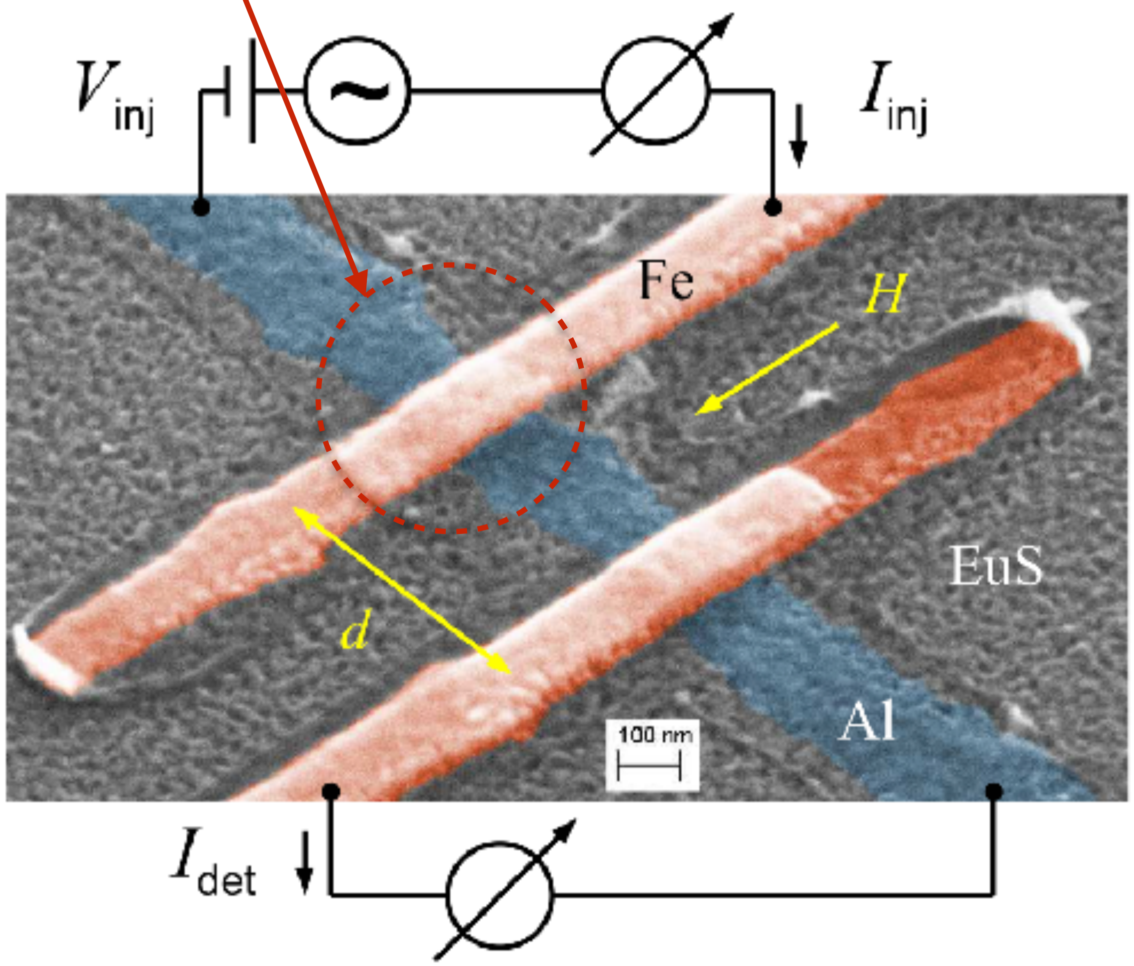
Our task:

1. Describe such "ferromagnetic superconductors" S' ✔

2. Describe the F-S' interface (tunnelling model): boundary conditions

3. Find the kinetic equations inside S' (spin injection)

CONCENTRATE ON THE NONEQUILIBRIUM PART!




Wolf, Sürgers, Fischer & Beckmann, PRB **90**, 144509 (2014)

Review: [arXiv:1706.08245](arXiv:1706.08245).

DNNFU

# Theory: Keldysh part

$$\check{g} = \begin{pmatrix} \hat{g}^R & \boxed{\hat{g}^K} \\ 0 & \hat{g}^A \end{pmatrix} \quad \text{and } \check{g}^2 = \check{1}$$

Satisfied by $\quad \hat{g}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A$

Nambu ($\tau$) - spin ($\sigma$) space:

$$\hat{f} = f_L \hat{1} + f_T \tau_3 + \sum_j (f_{Tj} \sigma_j + f_{Lj} \sigma_j \tau_3)$$

Interpretation: energy/charge/spin/spin energy (later in the talk)

# Theory: boundary conditions

Here: tunnel barrier with (normal state) spin polarisation $P = \frac{G_\uparrow - G_\downarrow}{G_\uparrow + G_\downarrow}$

Tunneling matrix $\hat{\Gamma} = t\tau_3 + u\sigma_3$ (interface with $\vec{M}||\hat{u}_z$)

B.c.: equate the *matrix currents* on both sides with

"Left" "Right"

$$\check{j}_k \equiv \check{g}\nabla_k\check{g} = -\frac{1}{R_\Box \sigma_N}\left[\hat{\Gamma}\check{g}_N\hat{\Gamma}^\dagger, \check{g}\right]$$

N-state interface resistance/area

N-state wire conductivity

Bergeret, Verso, Volkov, PRB **86**, 214516 (2012)

Completely general interface: M. Eschrig, A. Cottet, W. Belzig & J. Linder, NJP **17**, 083037 (2015)

DNNFU

# Keldysh part

$$\check{j}_k = \begin{pmatrix} \hat{j}_k^R & \boxed{\hat{j}_k^K} \\ 0 & \hat{j}_k^A \end{pmatrix}$$

Still a Nambu x spin matrix! For case of collinear magnetizations: N or F - insulator - spin-split superconductor:

Spectral

Charge current

Energy current

Spin current

Spin energy current

$$\begin{pmatrix} j_c \\ j_e \\ j_s \\ j_{se} \end{pmatrix} = \kappa \begin{pmatrix} N_+ & PN_- & PN_+ & N_- \\ PN_- & N_+ & N_- & PN_+ \\ PN_+ & N_- & N_+ & PN_- \\ N_- & PN_+ & PN_- & N_+ \end{pmatrix} \begin{pmatrix} \tilde{f}_T \\ \tilde{f}_L \\ \tilde{f}_{T3} \\ \tilde{f}_{L3} \end{pmatrix}$$

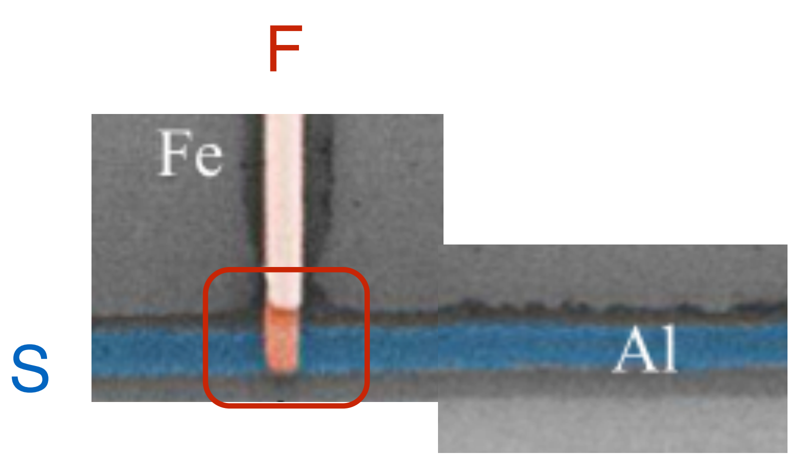
$$N_+(\varepsilon) = \frac{1}{8}\mathrm{Tr}[\tau_3(\hat{g}^R - \hat{g}^A)] \qquad N_-(\epsilon) = \frac{1}{8}\mathrm{Tr}[\tau_3\sigma_3(\hat{g}^R - \hat{g}^A)]$$

Energy integral: in a few minutes…

$$\kappa = 1/(R_\square e^2)$$

# Huge thermoelectric effects in FIS junctions

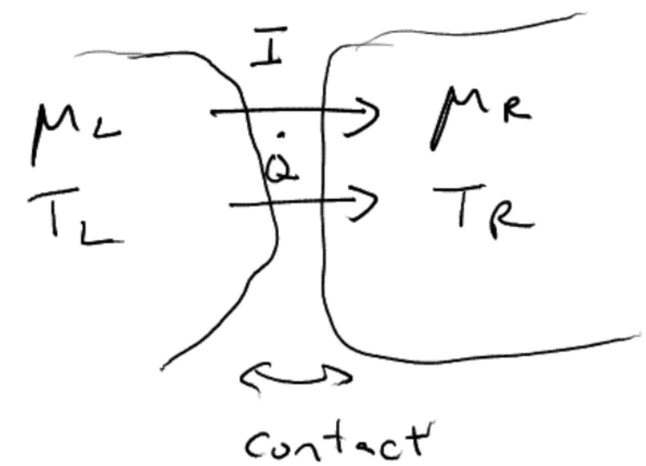
Details: Ozaeta, Virtanen, Bergeret & TTH, PRL **112**, 203603 (2014)
see also Machon, Eschrig, Belzig, PRL **110**, 047002 (2013), NJP **16**, 073002 (2014); Kalenkov & Zaikin, PRB **90**, 134502 (2014), PRB **91**, 064504 (2015)



# Thermoelectric effects

Linear response charge and heat currents across an interface:

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$$




$eV = \mu_L - \mu_R$

$\Delta T = T_L - T_R$

Max. efficiency of thermoel. conversion

$$\max \eta = \eta_{\mathrm{Carnot}} \frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+1}$$

Thermoelectric figure of merit

$$ZT = \frac{\alpha^2}{G_{\mathrm{th}}GT - \alpha^2}$$

# Linear response

Current density, linearised Boltzmann:

$$j = e \int dE \nu(E) D(E) \nabla f(E) = \sigma \nabla \mu / e + \alpha' \nabla T$$

Symmetric

$$\sigma = e^2 \int dE \nu(E) D(E) \partial_\mu f(E) = e^2 \int dE \frac{\nu(E) D(E)}{4 k_B T \cosh^2\left(\frac{E}{2 k_B T}\right)}$$

$$\alpha' = e \int dE \nu(E) D(E) \partial_T f(E) = e \int dE \frac{E \nu(E) D(E)}{4 k_B T \cosh^2\left(\frac{E}{2 k_B T}\right)}$$

Antisymmetric: requires E-dependence and e-h asymmetry!

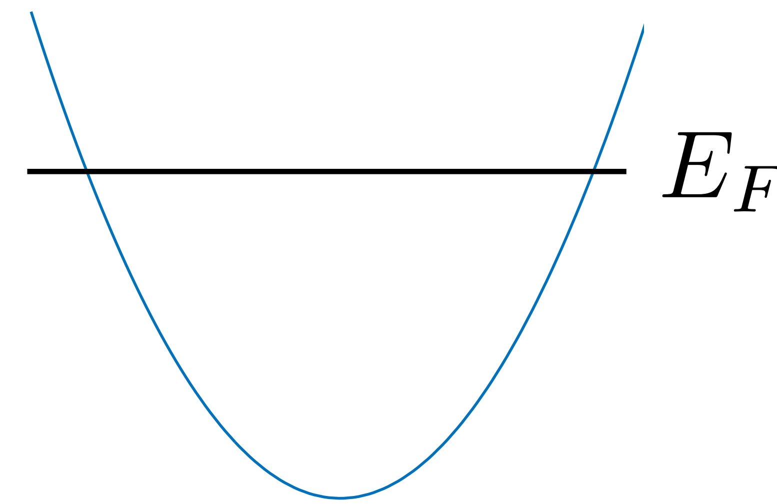
Sommerfeld expansion: Drude conductivity and Mott relation

$$\sigma = e^2 D(E_F) \nu(E_F) \qquad \alpha = e \mathcal{L} T \frac{\sigma'}{\sigma}, \quad \sigma' = e^2 (D'(E_F) \nu(E_F) + D(E_F) \nu'(E_F))$$

# Thermoelectricity in metals

Estimate: simple model with quadratic dispersion + Mott relation

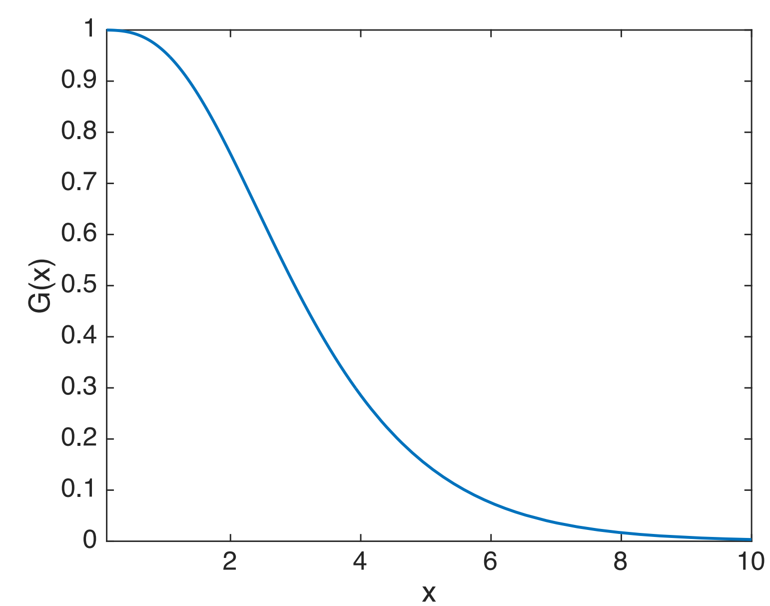
$$\alpha'_N = \frac{\pi^2}{6}\frac{T}{E_F} \ll 1$$




The size of thermoelectric coefficient in bulk superconductors:

$$\alpha = \alpha_N G(\Delta/T), \quad G(x) = \frac{3}{2\pi^2}\int_x^\infty \frac{y^2 dy}{\cosh^2(y/2)}$$

Small Small



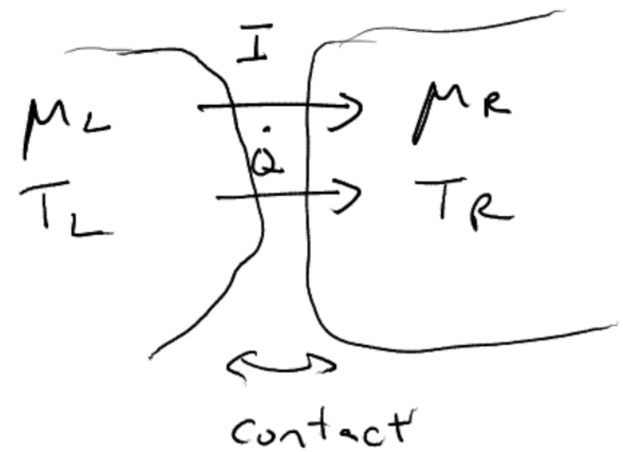

Gal'perin, Gurevich & Kozub
Sov. Phys. JETP (1974)

(Therefore, strongest thermoelectrics are semiconductors, with record-high ZT~3)

# Interfaces

Linear response charge and heat currents across an interface:

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$$




$eV = \mu_L - \mu_R$

$\Delta T = T_L - T_R$

Tunneling limit:

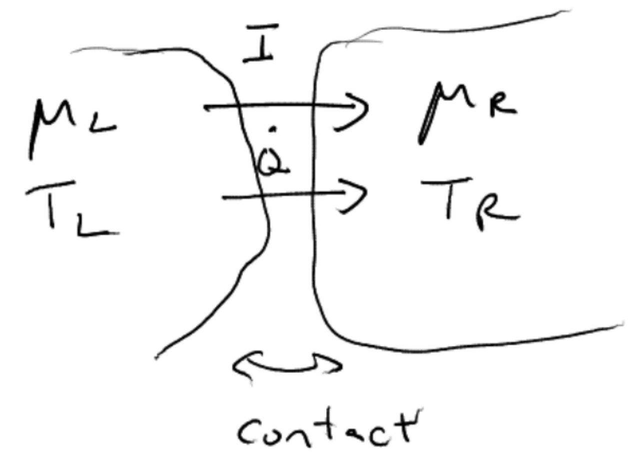
$I = \sum_\sigma \frac{1}{2eR_\sigma} \int_{-\infty}^{\infty} dE N_{L\sigma}(E) N_{R\sigma}(E) [f_0(E;\mu_L,T_L) - f_0(E;\mu_R,T_R)]$

$\dot{Q}_L = \sum_\sigma \frac{1}{2e^2R_\sigma} \int_{-\infty}^{\infty} dE (E-\mu_L) N_{L\sigma}(E) N_{R\sigma}(E) [f_0(E;\mu_L,T_L) - f_0(E;\mu_R,T_R)]$

# Thermoelectric effects

Linear response charge and heat currents across an interface:

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T/T \end{pmatrix}$$




$eV = \mu_L - \mu_R$

$\Delta T = T_L - T_R$

antisymmetric

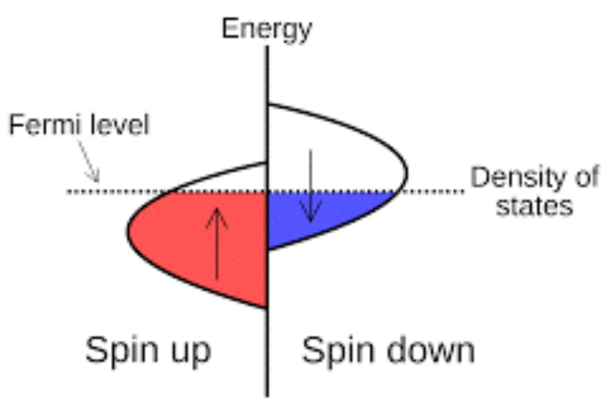
Linear response thermoelectric coefficient:

$$\alpha = \sum_\sigma \frac{1}{2eR_\sigma} \int_{-\infty}^{\infty} dE \, \frac{E N_{L\sigma}(E) N_{R\sigma}(E)}{4k_B T \cosh^2\left(\frac{E}{2k_B T}\right)}$$

Requirement for large thermoelectric effects:

- Strong energy dependence (density of states, scattering time)
- Electron-hole asymmetry

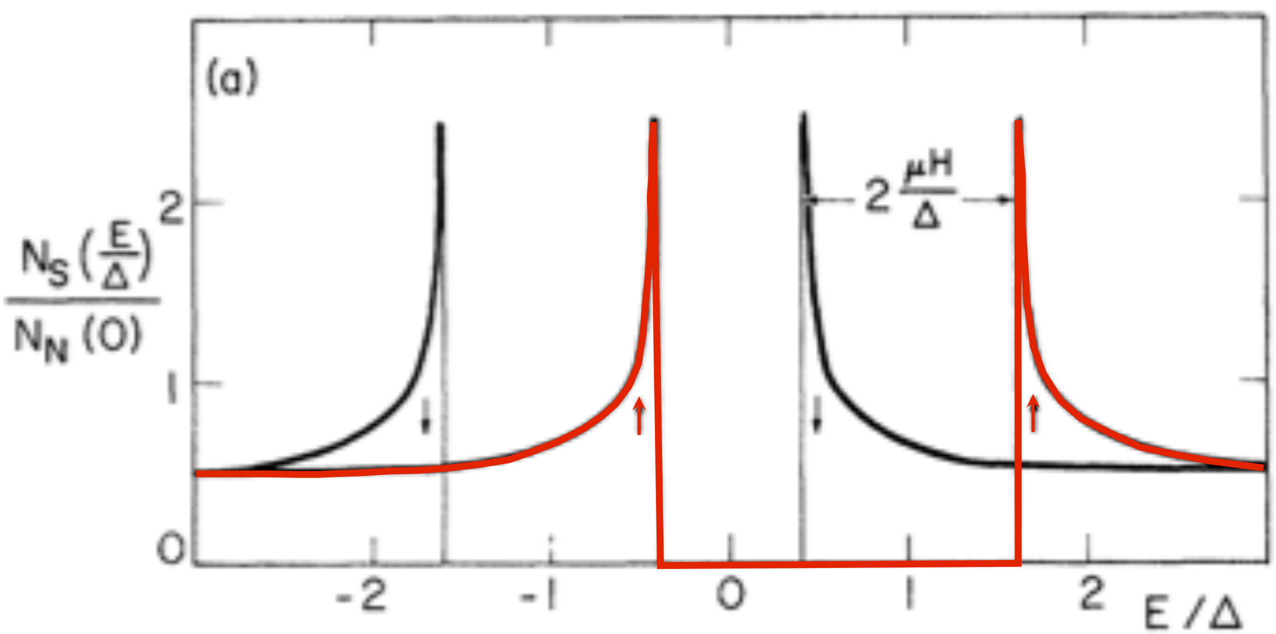
# Super-ferro: ingredients




Ferromagnet:
spin-dependent
Fermi level

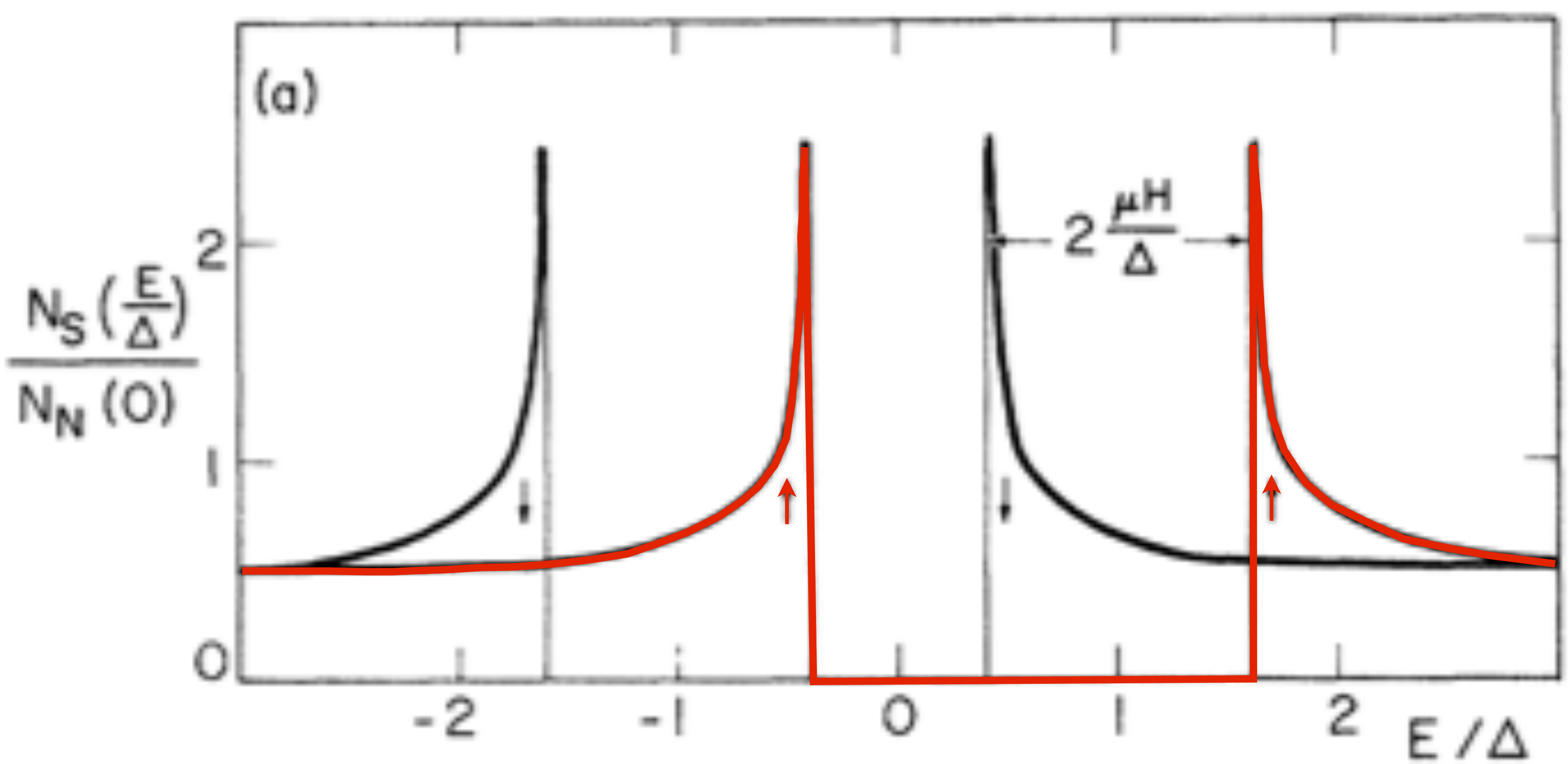
$\longrightarrow$ spin-dependent
contact resistance

Polarization: $P = \frac{R_\uparrow - R_\downarrow}{R_\uparrow + R_\downarrow} \in [-1, 1]$




superconductor
with spin splitting $h$

# Large thermoelectric effect:




1. Large e-h asymmetry *per spin*

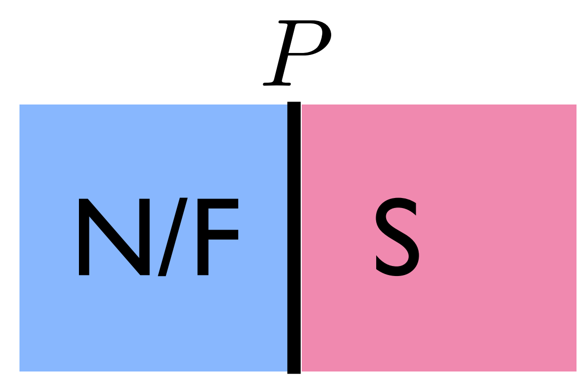
2. For $P \neq 0$, different spin contributions weighed differently

# Spin-splitting field + polarization

Linear response: $\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & P\alpha \\ P\alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$ $\begin{pmatrix} I_S \\ \dot{Q}_S \end{pmatrix} = \begin{pmatrix} PG & \alpha \\ \alpha & PG_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$

Polarization!

$$\alpha = \frac{1}{2eR_T} \int_{-\infty}^{\infty} dE \frac{E[N_\uparrow(E) - N_\downarrow(E)]}{4k_B T \cosh^2\left(\frac{E}{2k_B T}\right)}$$




Requirement for large thermoelectric effects:

- Strong energy dependence (density of states, scattering time) ✔
- Electron-hole asymmetry ✔

# Keldysh part

$$\check{j}_k = \begin{pmatrix} \hat{j}_k^R & \boxed{\hat{j}_k^K} \\ 0 & \hat{j}_k^A \end{pmatrix}$$

Still a Nambu x spin matrix! For case of collinear magnetizations:
N or F - insulator - spin-split superconductor:

Spectral

Charge current
Energy current
Spin current
Spin energy current

$$\begin{pmatrix} j_c \\ j_e \\ j_s \\ j_{se} \end{pmatrix} = \kappa \begin{pmatrix} N_+ & PN_- & PN_+ & N_- \\ PN_- & N_+ & N_- & PN_+ \\ PN_+ & N_- & N_+ & PN_- \\ N_- & PN_+ & PN_- & N_+ \end{pmatrix} \begin{pmatrix} \tilde{f}_T \\ \tilde{f}_L \\ \tilde{f}_{T3} \\ \tilde{f}_{L3} \end{pmatrix}$$

$$N_+(\varepsilon) = \frac{1}{8}\mathrm{Tr}[\tau_3(\hat{g}^R - \hat{g}^A)] \qquad N_-(\epsilon) = \frac{1}{8}\mathrm{Tr}[\tau_3\sigma_3(\hat{g}^R - \hat{g}^A)]$$

Energy integral: in a few minutes…

$$\kappa = 1/(R_\square e^2)$$

DNNFU

# Theory: General structure

$$\begin{pmatrix} I \\ \dot{Q} \\ I_s \\ \dot{Q}_s \end{pmatrix} = \begin{pmatrix} G & P\alpha & PG & \alpha \\ P\alpha & G_{\mathrm{th}}T & \alpha & PG_{\mathrm{th}}T \\ PG & \alpha & G & P\alpha \\ \alpha & PG_{\mathrm{th}}T & P\alpha & G_{\mathrm{th}}T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T/T \\ V_s/2 \\ -\Delta T_s/2T \end{pmatrix}$$

$$G = e^2\kappa \int_{-\infty}^{\infty} N_+ \frac{\partial f_{\mathrm{eq}}}{\partial \varepsilon} d\varepsilon$$

$$G_{\mathrm{th}} = \frac{\kappa}{T} \int_{-\infty}^{\infty} N_+ \varepsilon^2 \frac{\partial f_{\mathrm{eq}}}{\partial \varepsilon} d\varepsilon$$

$$\alpha = -e\kappa \int_{-\infty}^{\infty} N_- \varepsilon \frac{\partial f_{\mathrm{eq}}}{\partial \varepsilon} d\varepsilon,$$

# Thermally activated transport

Behavior for $k_B T \lesssim \Delta - h$

$$G \approx G_T \sqrt{2\pi\tilde{\Delta}} \cosh(\tilde{h}) e^{-\tilde{\Delta}},$$

$$G_{\text{th}} \approx \frac{k_B G_T \Delta}{e^2} \sqrt{\frac{\pi}{2\tilde{\Delta}}} e^{-\tilde{\Delta}} \left[ e^{\tilde{h}} (\tilde{\Delta} - \tilde{h})^2 + e^{-\tilde{h}} (\tilde{\Delta} + \tilde{h})^2 \right],$$

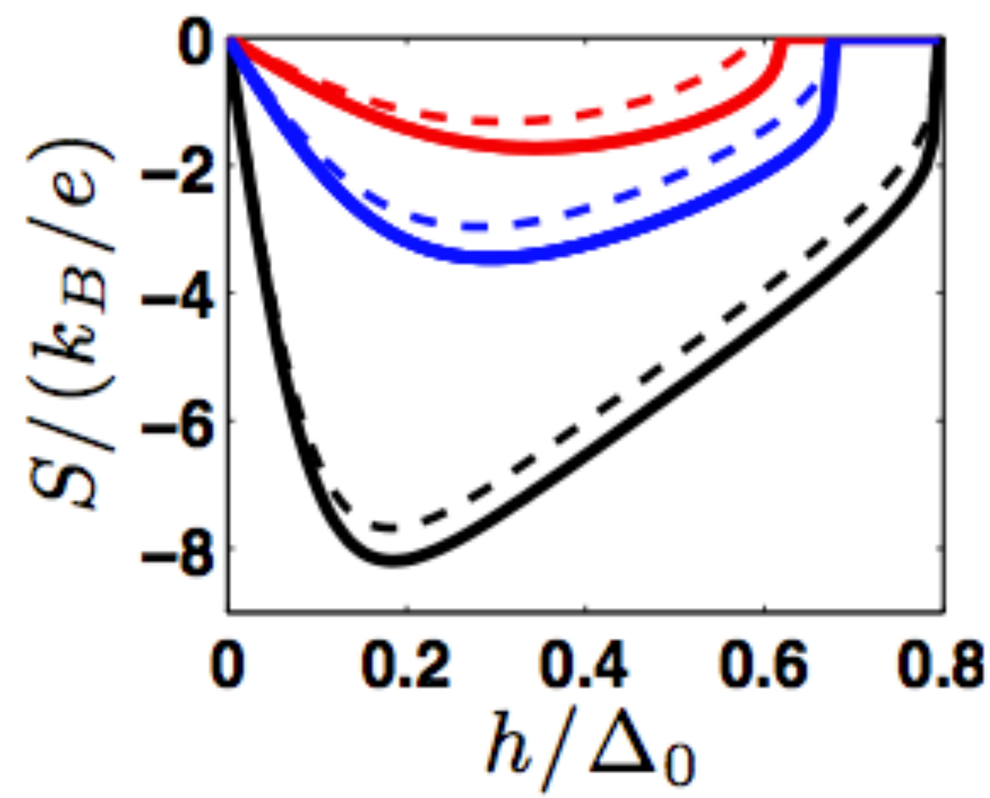
$$\alpha \approx \frac{G_T}{e} \sqrt{2\pi\tilde{\Delta}} e^{-\tilde{\Delta}} \left[ \Delta \sinh(\tilde{h}) - h \cosh(\tilde{h}) \right]$$

Thermopower $S = \left(\frac{V}{\Delta T}\right)_{I=0} = -\frac{P\alpha}{GT}$

$$\tilde{h} = \frac{h}{k_B T}; \tilde{\Delta} = \frac{\Delta}{k_B T}$$

# FI(FS) Thermopower

Thermopower $S = \left(\frac{V}{\Delta T}\right)_{I=0} = -\frac{P\alpha}{GT}$




$T = 0.3\Delta$

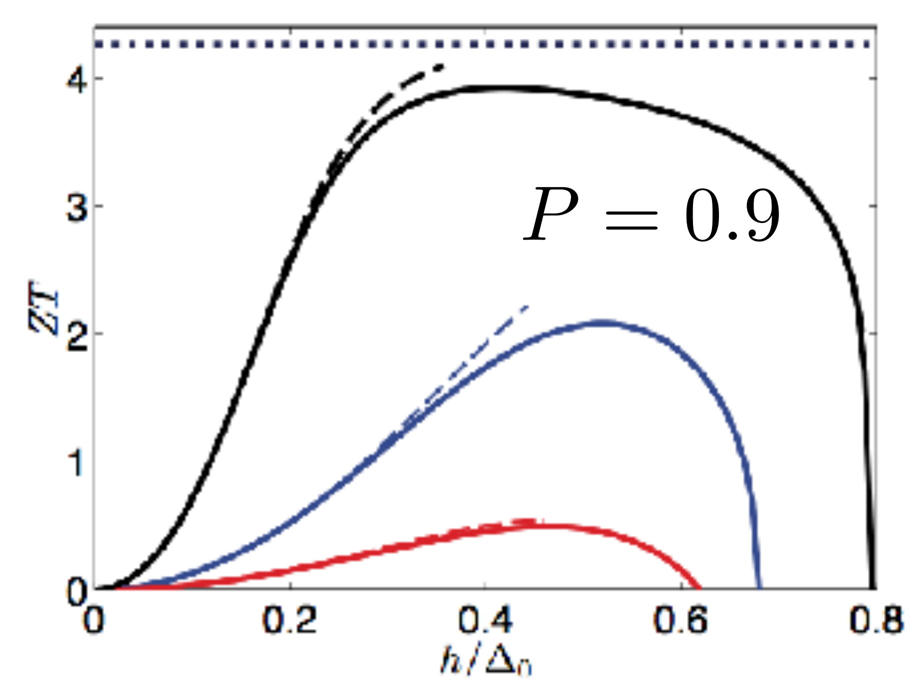
$T = 0.2\Delta$

$T = 0.1\Delta$

$$S_{\max} \approx -\frac{k_B}{e} P \frac{\Delta}{k_B T}$$

Figure of merit (ideal case):

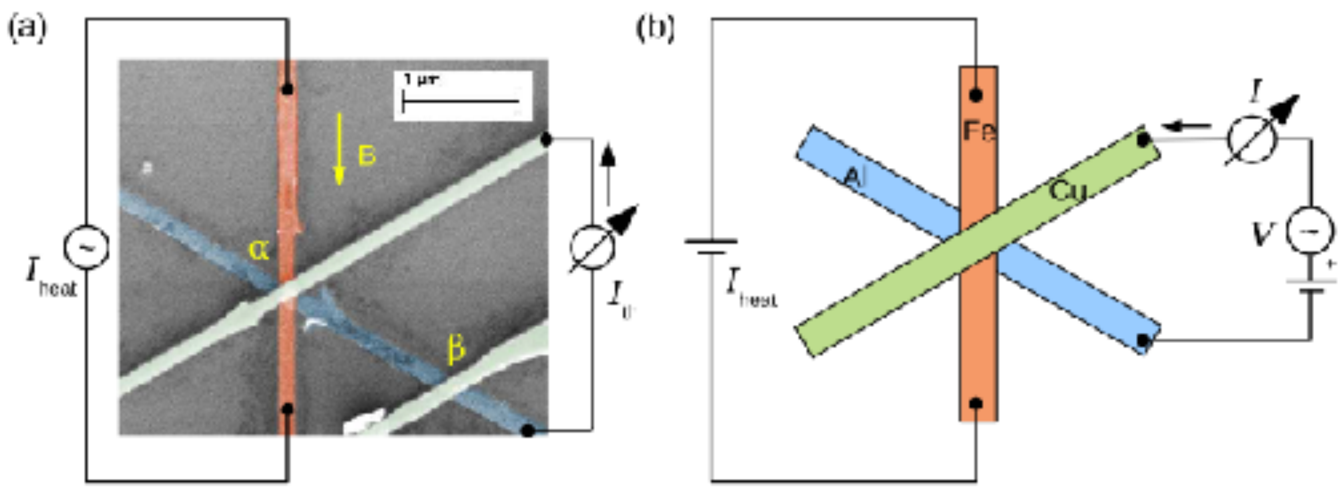
$$ZT_{\max} \overset{k_B T \ll \Delta - h}{\approx} \frac{P^2}{1-P^2}$$

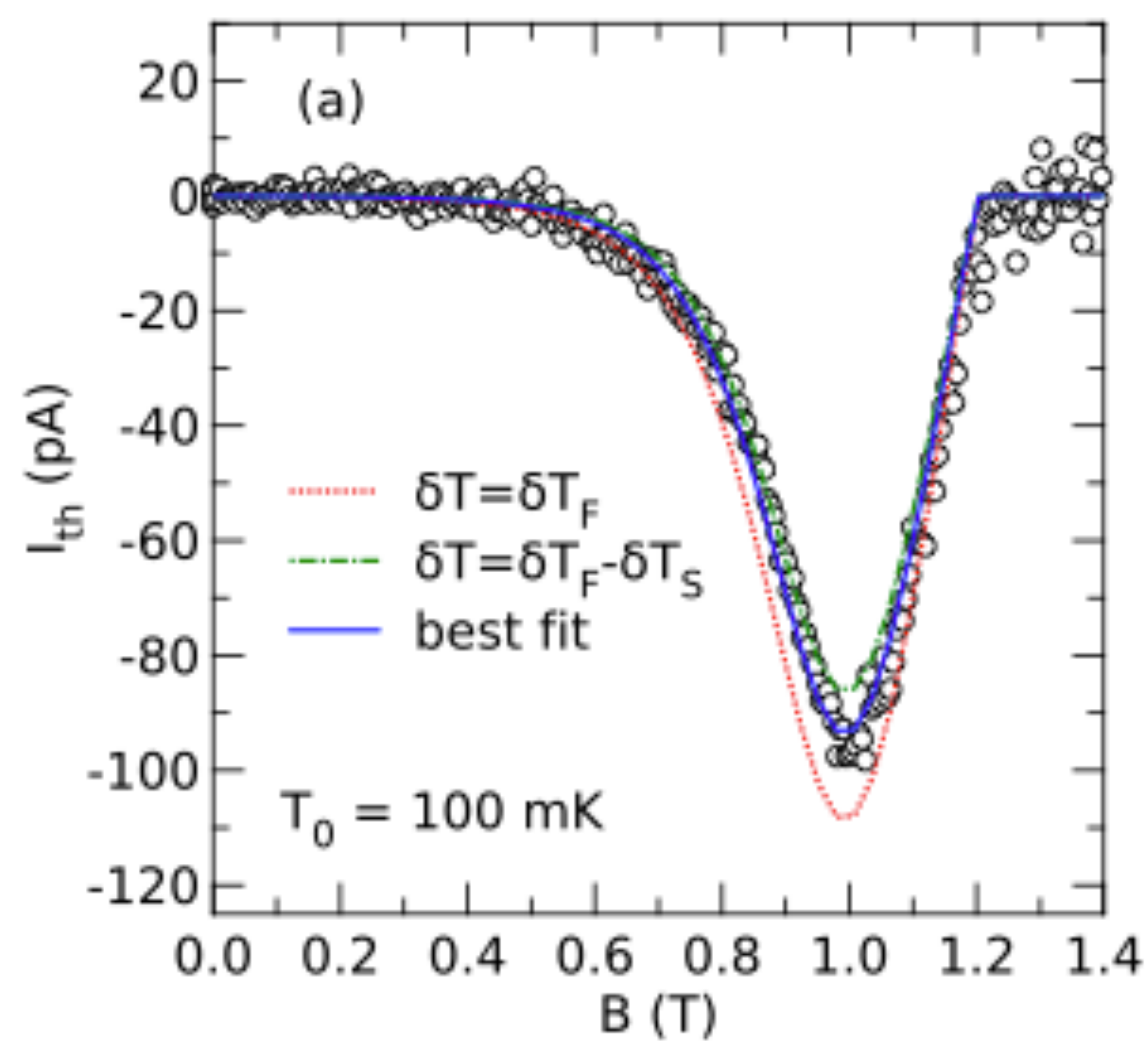



Efficiency (max power): $\eta = \eta_{CA} ZT/(ZT+2)$,
$\eta_{CA} = 1 - \sqrt{T_{\rm cold}/T_{\rm hot}}$

# Observation of Thermoelectric Currents in High-Field Superconductor-Ferromagnet Tunnel Junctions

S. Kolenda, M. J. Wolf,* and D. Beckmann†
*Institute of Nanotechnology, Karlsruhe Institute of Technology, Karlsruhe, Germany*




Only fit parameter: $P \approx 0.08$

# What to do with it?

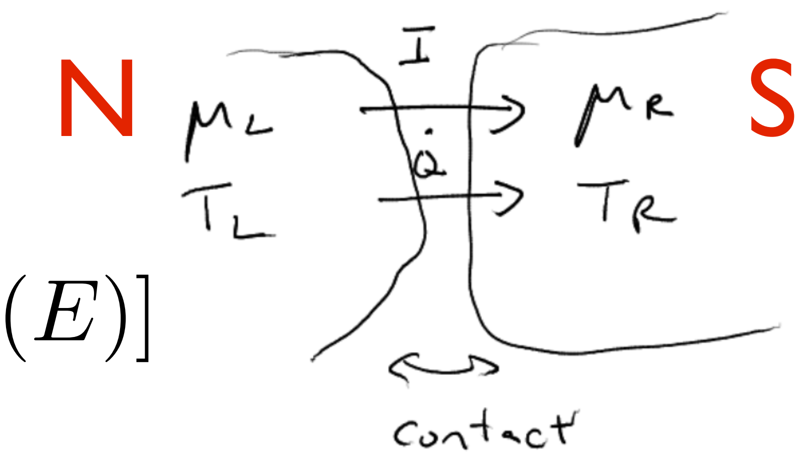
Power conversion

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$$

Cooling

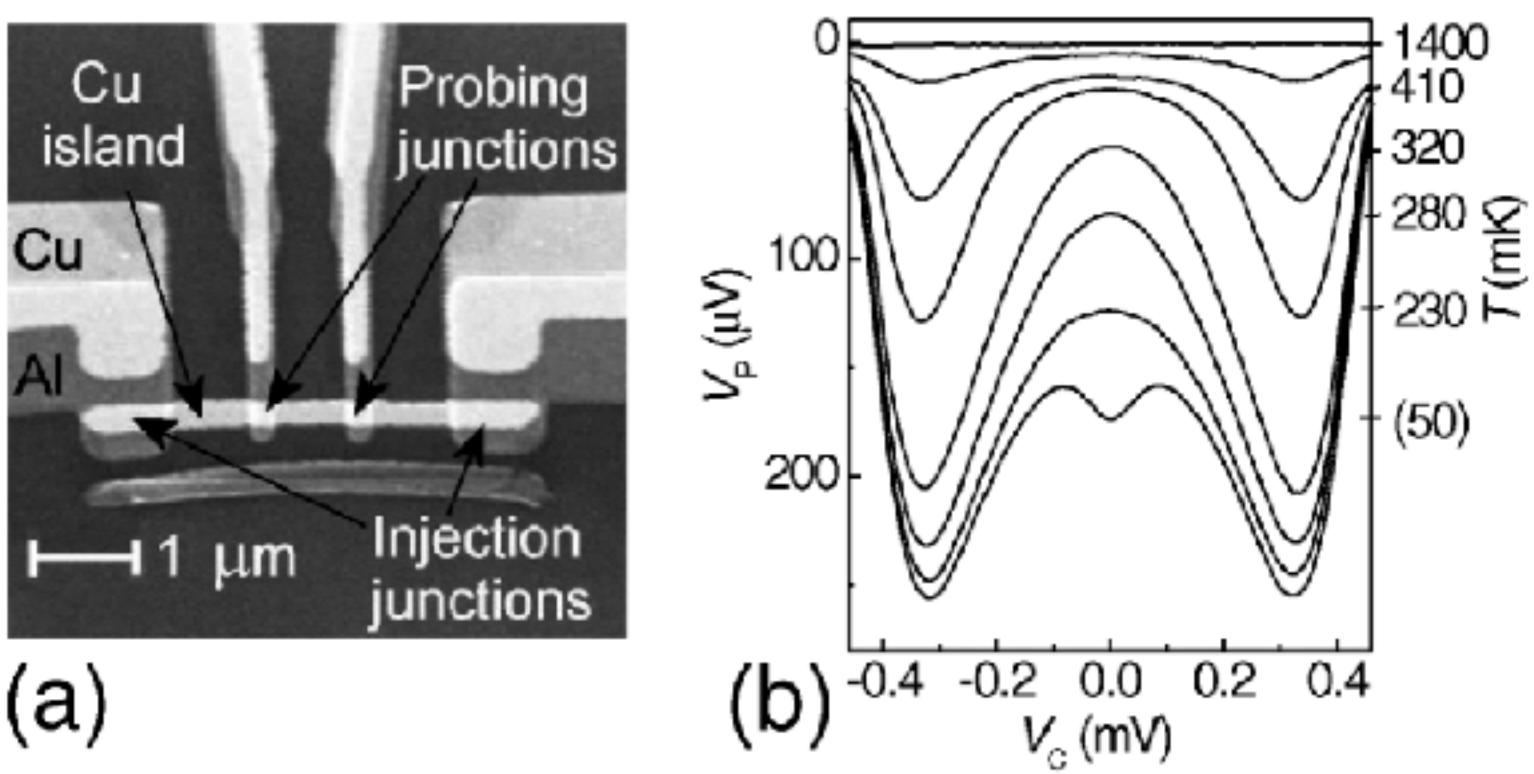
# Electronic cooling

Cooling even when $h = 0$



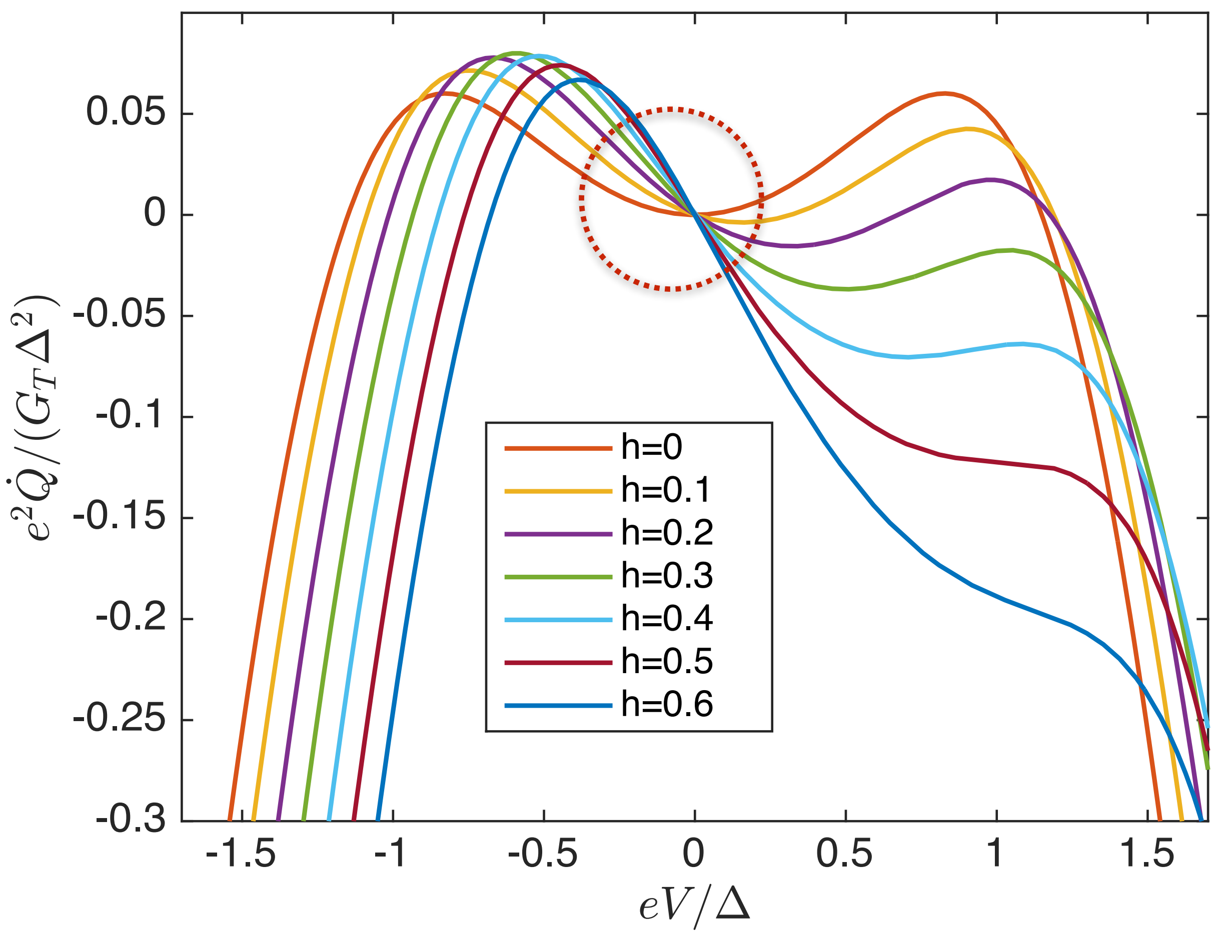

$$\dot{Q}_N = \frac{G_T}{e}\int_{-\infty}^{\infty} dE(E-\mu_N)N_S(E)[f_N(E)-f_S(E)]$$

Heat balance: $\dot{Q}(V;T_I,T_{\rm bath}) = P_{\rm e-ph}(T_I,T_{\rm ph})$

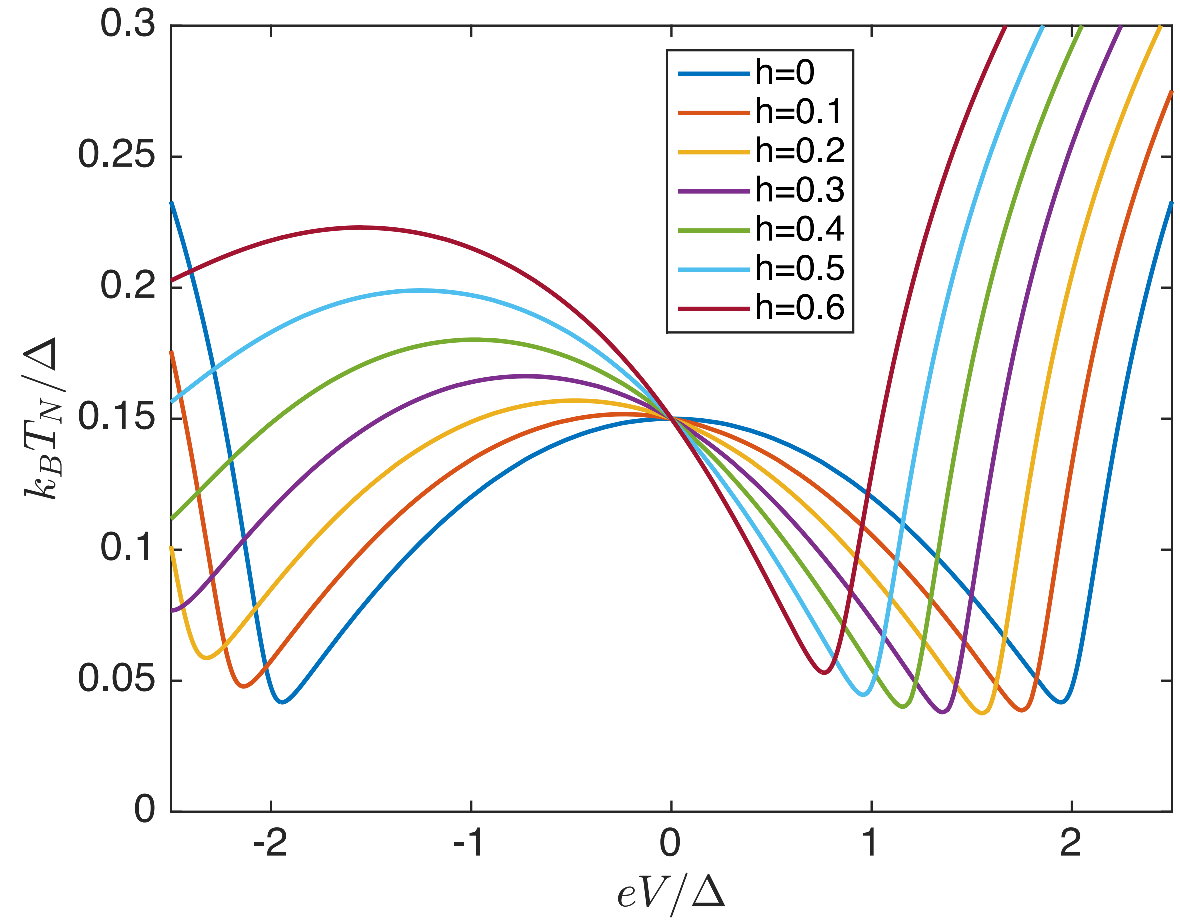



Pekola, TTH, *et al.*, PRL (2004)

# With spin splitting




Cooling power

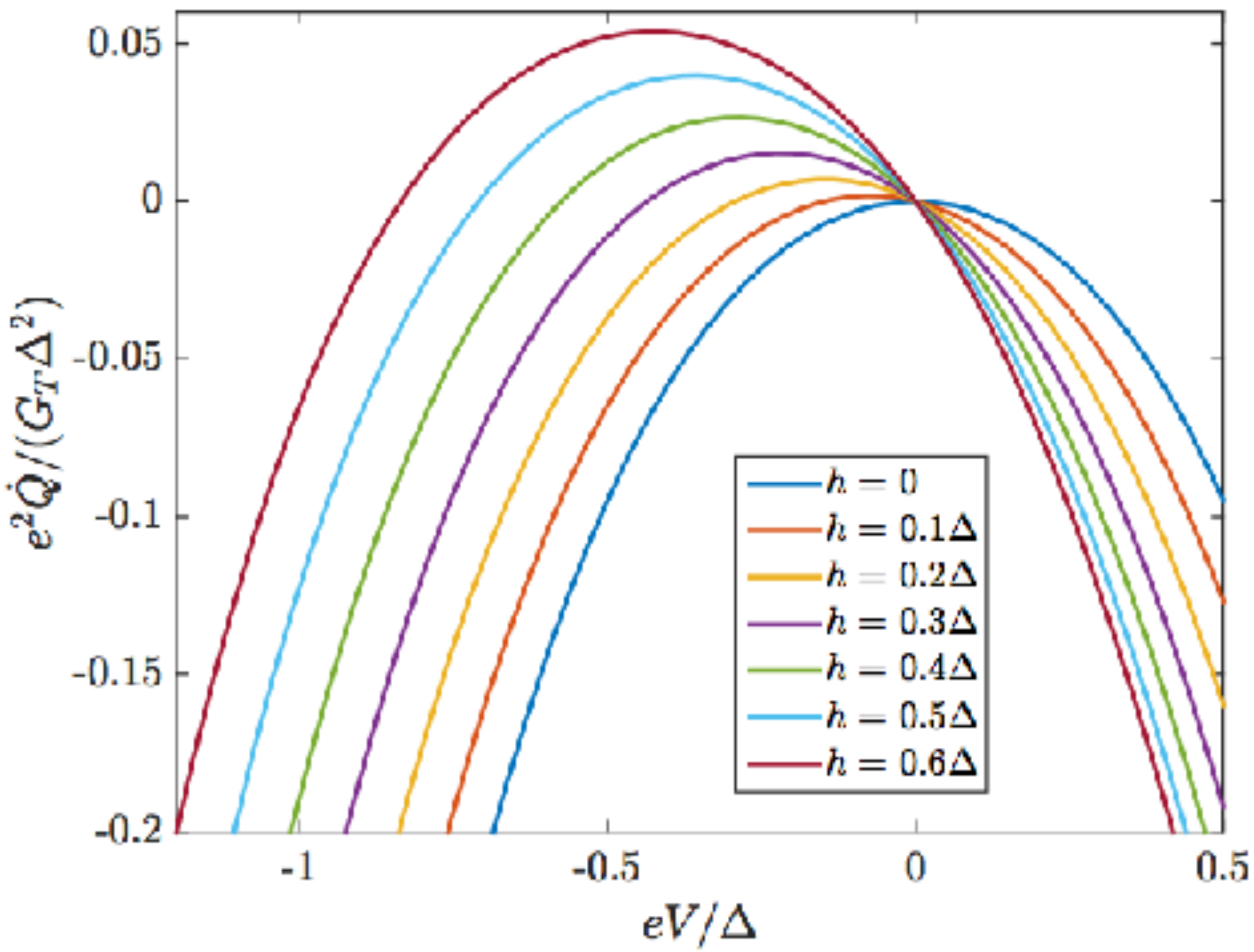



Electron temperature

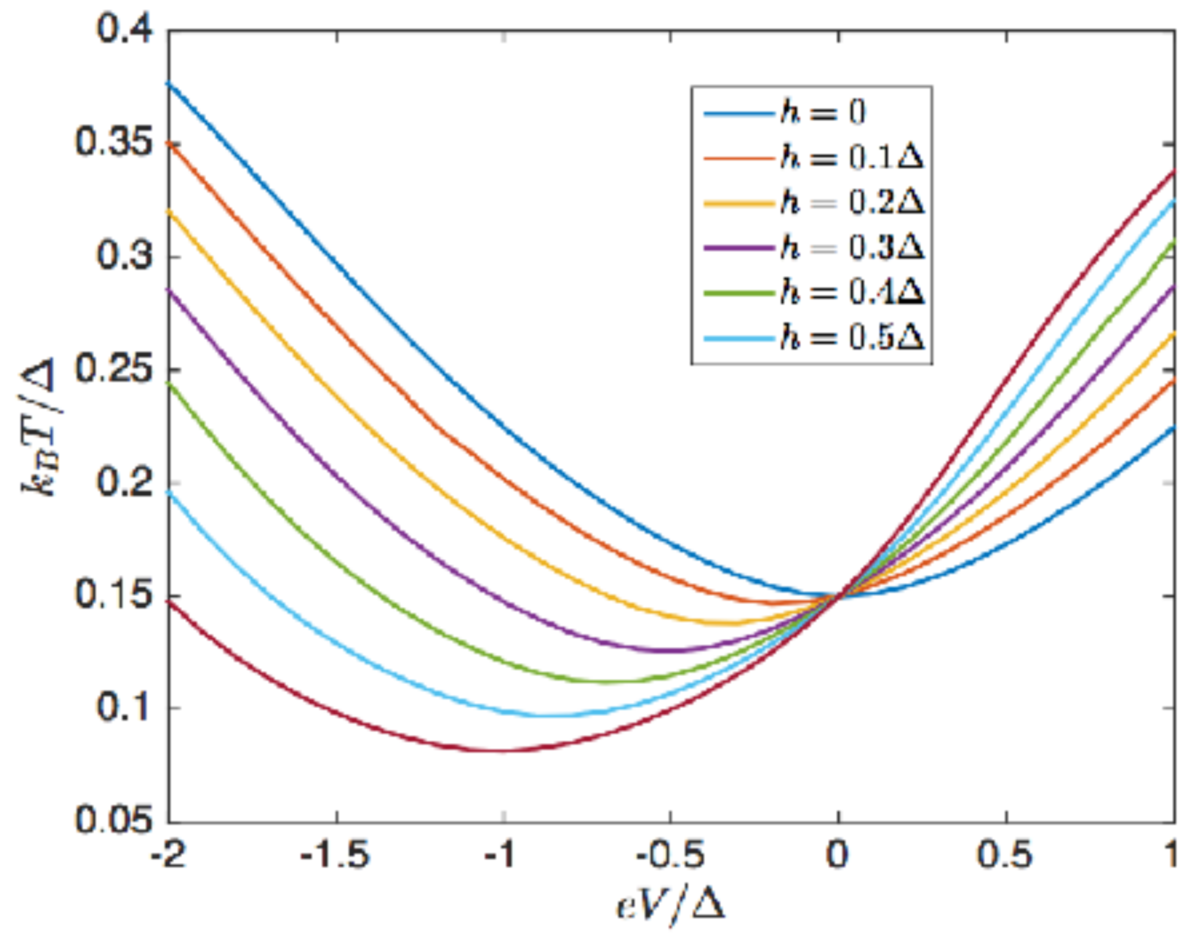
S'-FI-N-FI-S'

$P = 1$, antiparallel magnetizations

# Cooling of the superconductor




Cooling power




Electron temperature

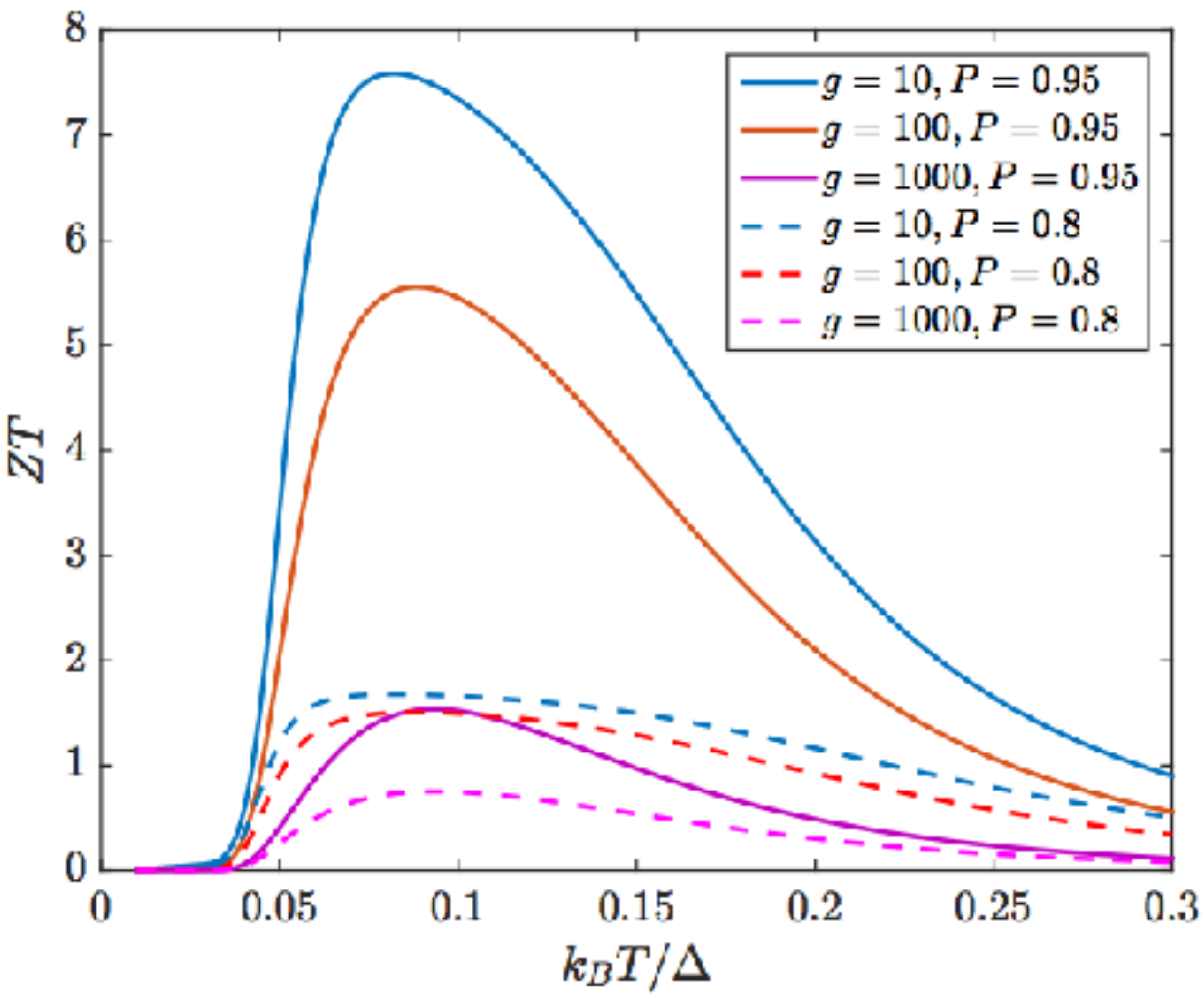
N-FI-S'-FI-N

$P=1$, antiparallel magnetizations

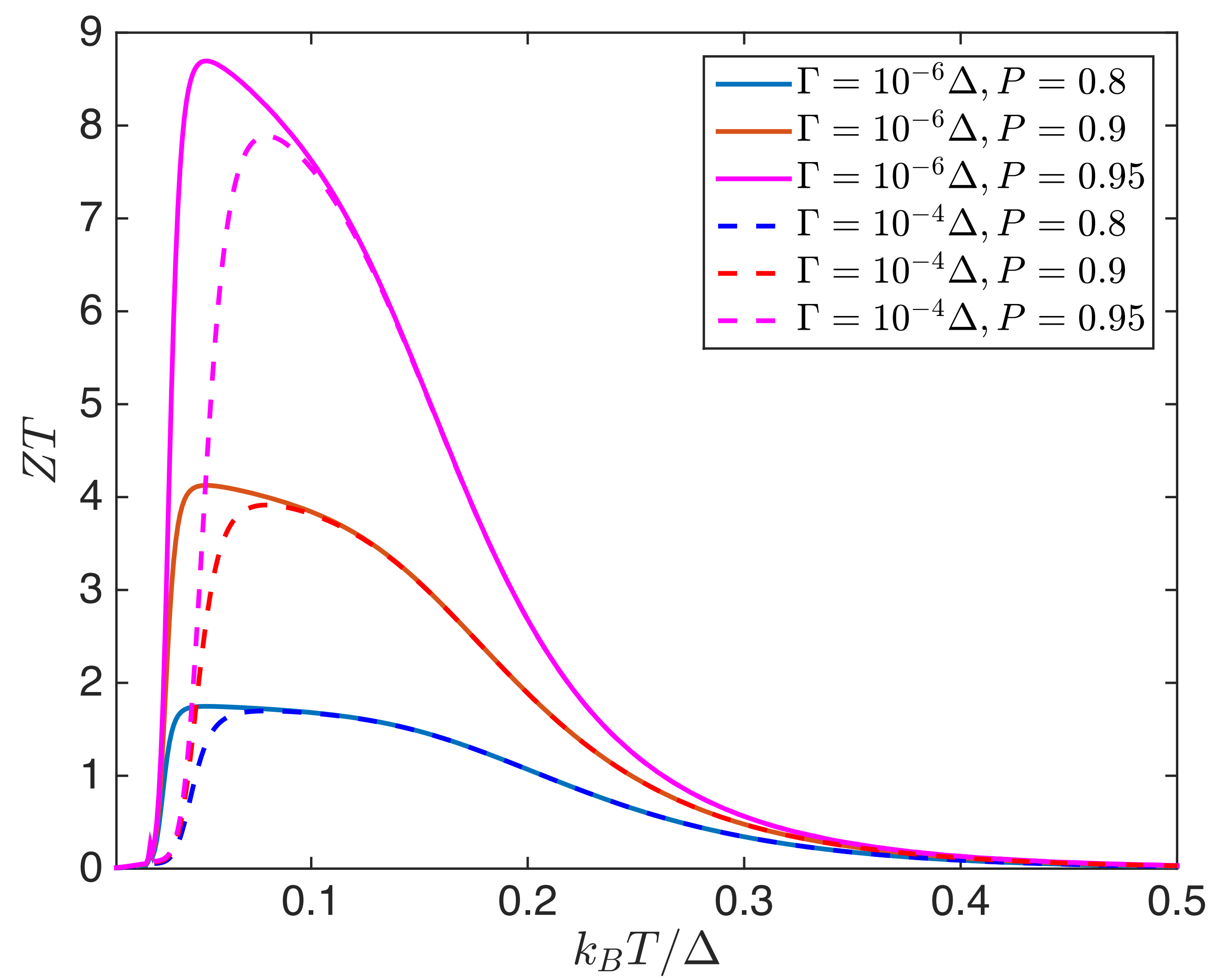
# S-F heat engine: efficiency

$h = 0.5\Delta$

FNF island (antiparallel magnetizations)



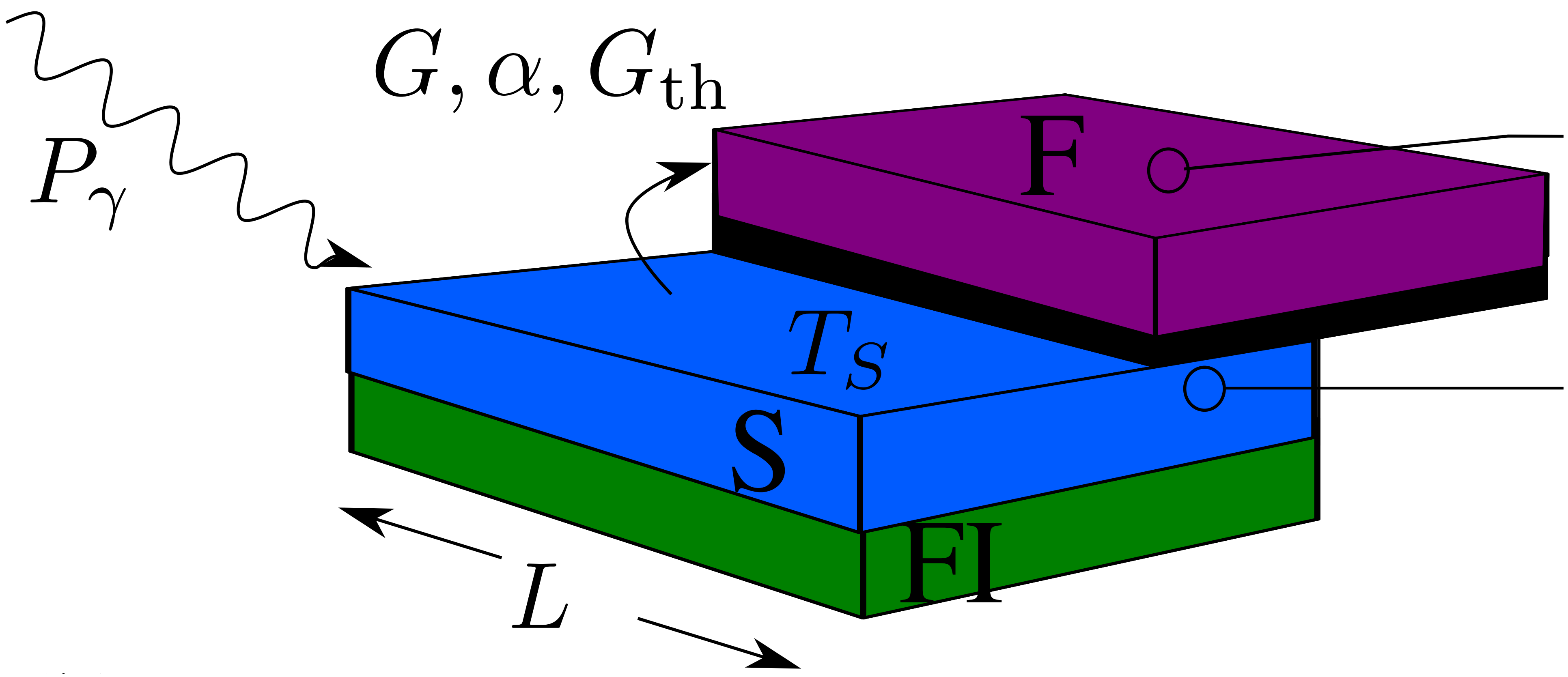

S island (F electrodes)




$g \sim$ relative strength of spurious heat conduction

$\Gamma \sim$ "Dynes" parameter, quality of the BCS gap

# Thermoelectric detector




TTH, Ojajärvi, Bergeret, Giazotto, Maasilta, submitted today…

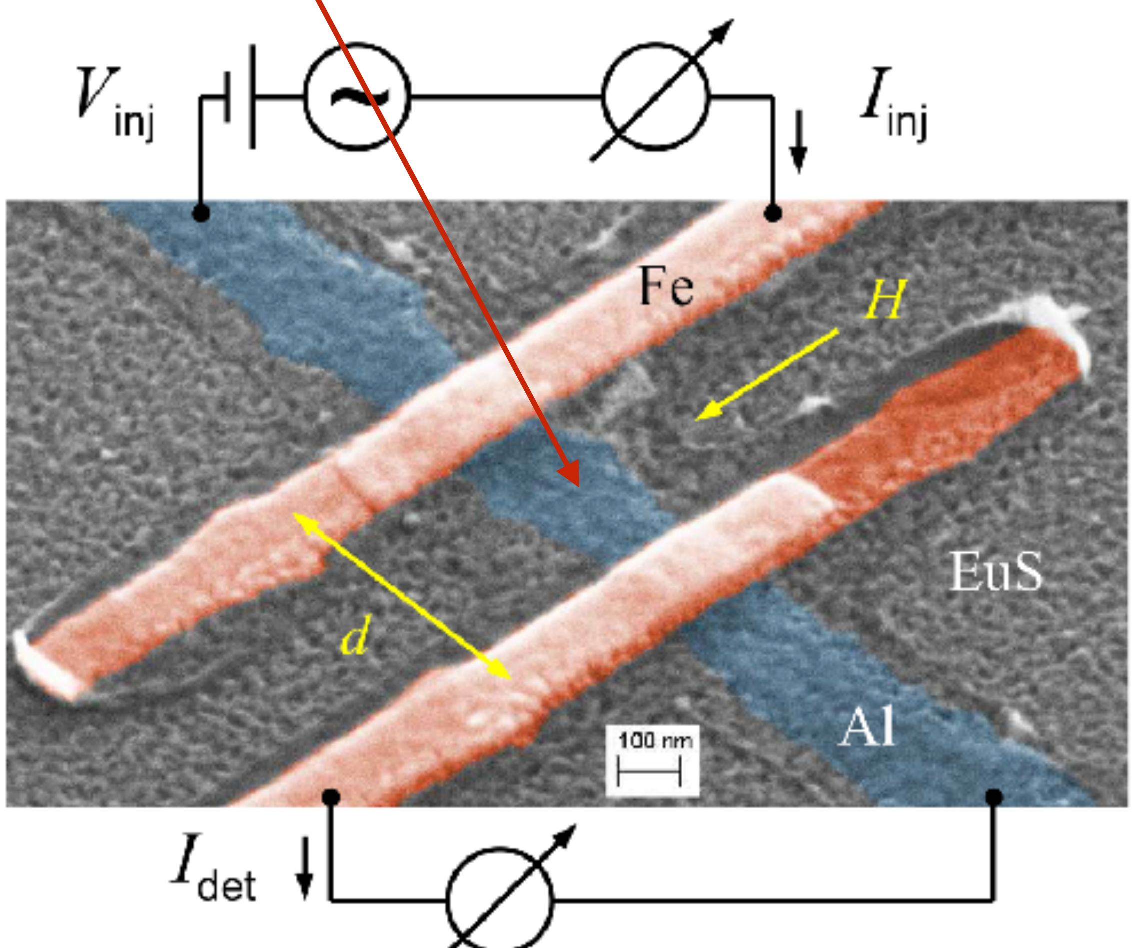
MORE DETAILS: R. OJAJÄRVI'S POSTER

Our task:

1. Describe such "ferromagnetic superconductors" S' ✔

2. Describe the F-S' interface (tunnelling model) ✔

3. Find the kinetic equations inside S' (spin injection)




Wolf, Sürgers, Fischer & Beckmann, PRB **90**, 144509 (2014)

Review: [arXiv:1706.08245](arXiv:1706.08245).

# Spin injection into a spin-split superconductor

Details: Silaev, Virtanen, Bergeret & TTH, PRL **114**, 167002 (2015)
see also Bobkov & Bobkova, Pis'ma Zh. Eksp. Teor. Fiz. **101**, 124 (2015), arXiv: 1511.07388, Krishtop, Houzet, Meyer, Phys. Rev. B **91**, 121407(R) (2015)

# Theory: Keldysh part

$$\check{g} = \begin{pmatrix} \hat{g}^R & \boxed{\hat{g}^K} \\ 0 & \hat{g}^A \end{pmatrix} \quad \text{and } \check{g}^2 = \check{1}$$

Satisfied by $\hat{g}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A$

Nambu ($\tau$) - spin ($\sigma$) space:

$$\hat{f} = f_L \hat{1} + f_T \tau_3 + \mathbf{f}_{TS} \cdot \sigma + \mathbf{f}_{LS} \cdot \sigma \tau_3$$

Energy Charge Spin Spin energy

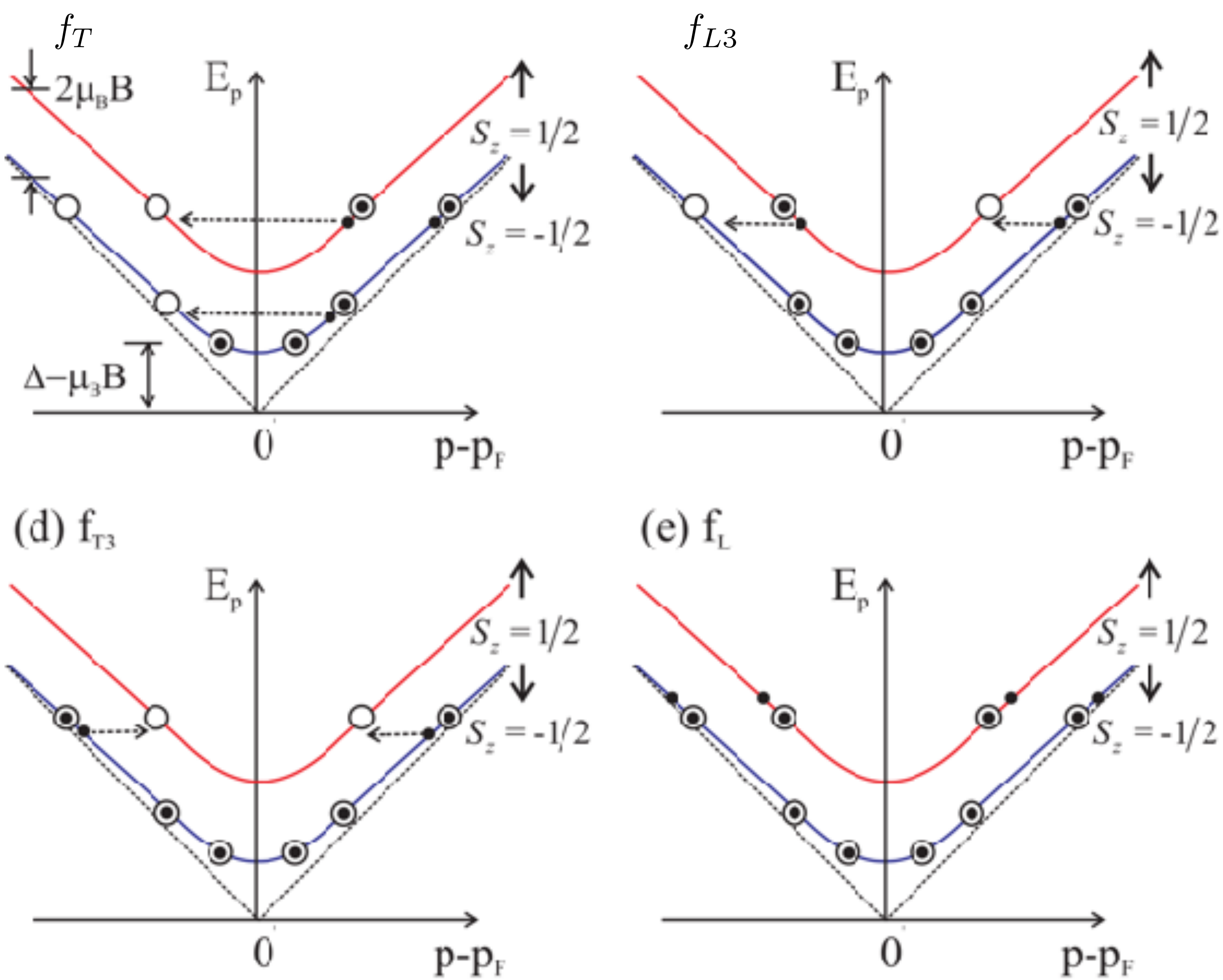
# Modes of nonequilibrium

Charge

$\mu_{\rm qp} \neq \mu_S$

Spin energy

$T_\uparrow^* \neq T_\downarrow^*$




Spin

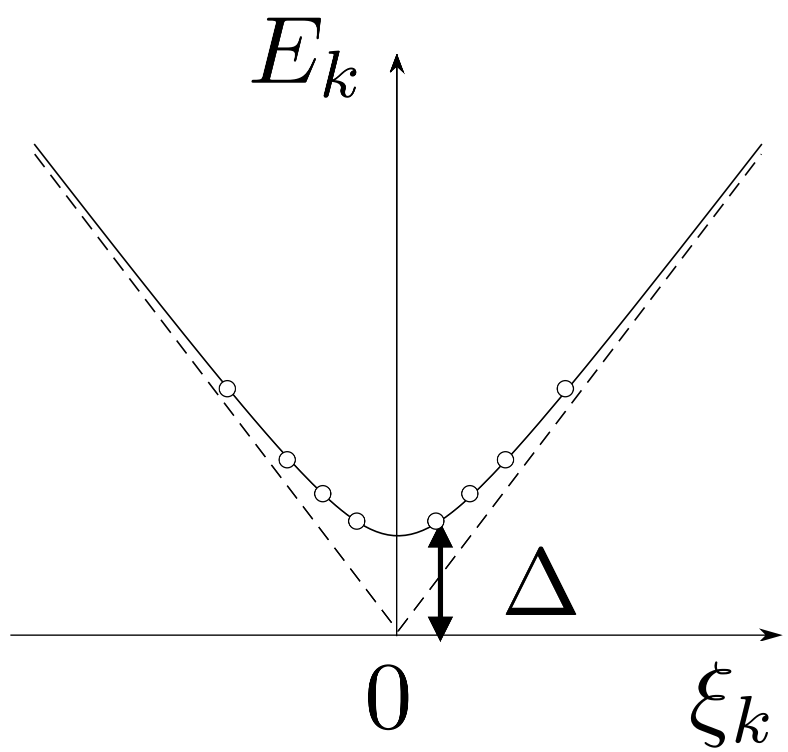
$\mu_\uparrow \neq \mu_\downarrow$

Energy

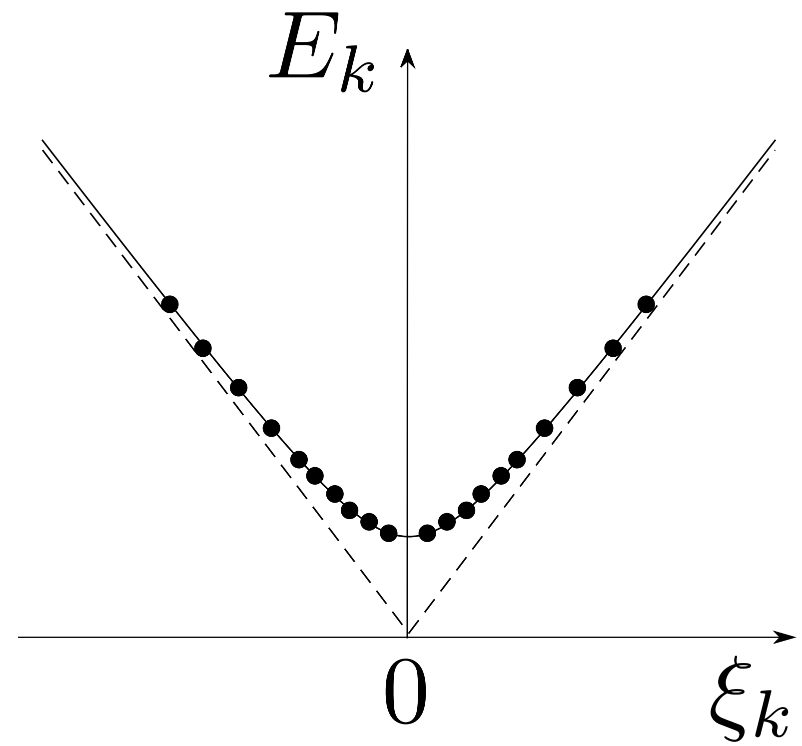
$T^* \neq T$

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)

# Modes of nonequilibrium in superconductors

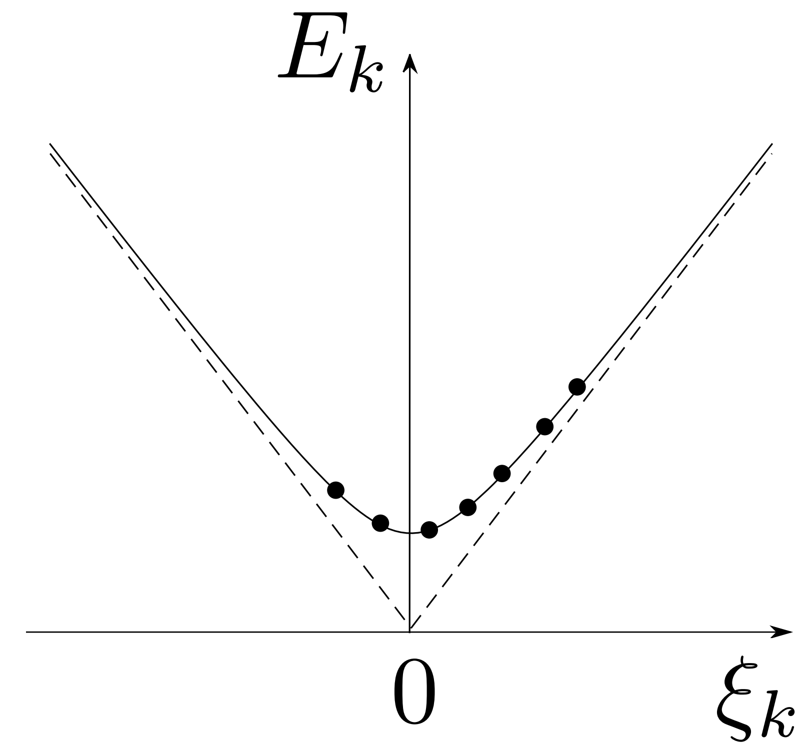



Equilibrium




Even mode

$T^* > T$




Odd mode
“Branch imbalance”
“Charge imbalance”

$$Q^* > 0$$

$$\mu_{\mathrm{QP}} > \mu_S (= 0)$$

A. Schmid and G. Schön, J. Low. Temp. Phys. **20**, 207 (1975)

# Kinetics

We start from Usadel equation

$$\check{\Lambda} = i\varepsilon\tau_3 - i(\boldsymbol{h}\cdot\boldsymbol{S})\tau_3 - \check{\Delta},$$

$$D\nabla\cdot(\check{g}\nabla\check{g}) + [\check{\Lambda} - \check{\Sigma}_{so} - \check{\Sigma}_{sf} - \check{\Sigma}_{orb}, \check{g}] = 0.$$

Spin-orbit

Spin flips

Orbital effect of a field

Charge and spin energy current:

Coupling to "condensate"

$$\boldsymbol{j_c} = \mathcal{D}_T\nabla f_T + \mathcal{D}_{L3}\nabla f_{L3} \qquad \boldsymbol{\nabla}\cdot\boldsymbol{j_c} = R_T f_T + R_{L3} f_{L3}$$

$$\boldsymbol{j_{se}} = \mathcal{D}_T\nabla f_{L3} + \mathcal{D}_{L3}\nabla f_T, \qquad \boldsymbol{\nabla}\cdot\boldsymbol{j_{se}} = (R_T + S_{L3}) f_{L3} + R_{L3} f_T$$

Spin heat relaxation

Energy and spin current:

$$\boldsymbol{j_e} = \mathcal{D}_L\nabla f_L + \mathcal{D}_{T3}\nabla f_{T3} \qquad \boldsymbol{\nabla}\cdot\boldsymbol{j_e} = 0$$

$$\boldsymbol{j_s} = \mathcal{D}_L\nabla f_{T3} + \mathcal{D}_{T3}\nabla f_L \qquad \boldsymbol{\nabla}\cdot\boldsymbol{j_s} = S_{T3} f_{T3}$$

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)

Spin relaxation

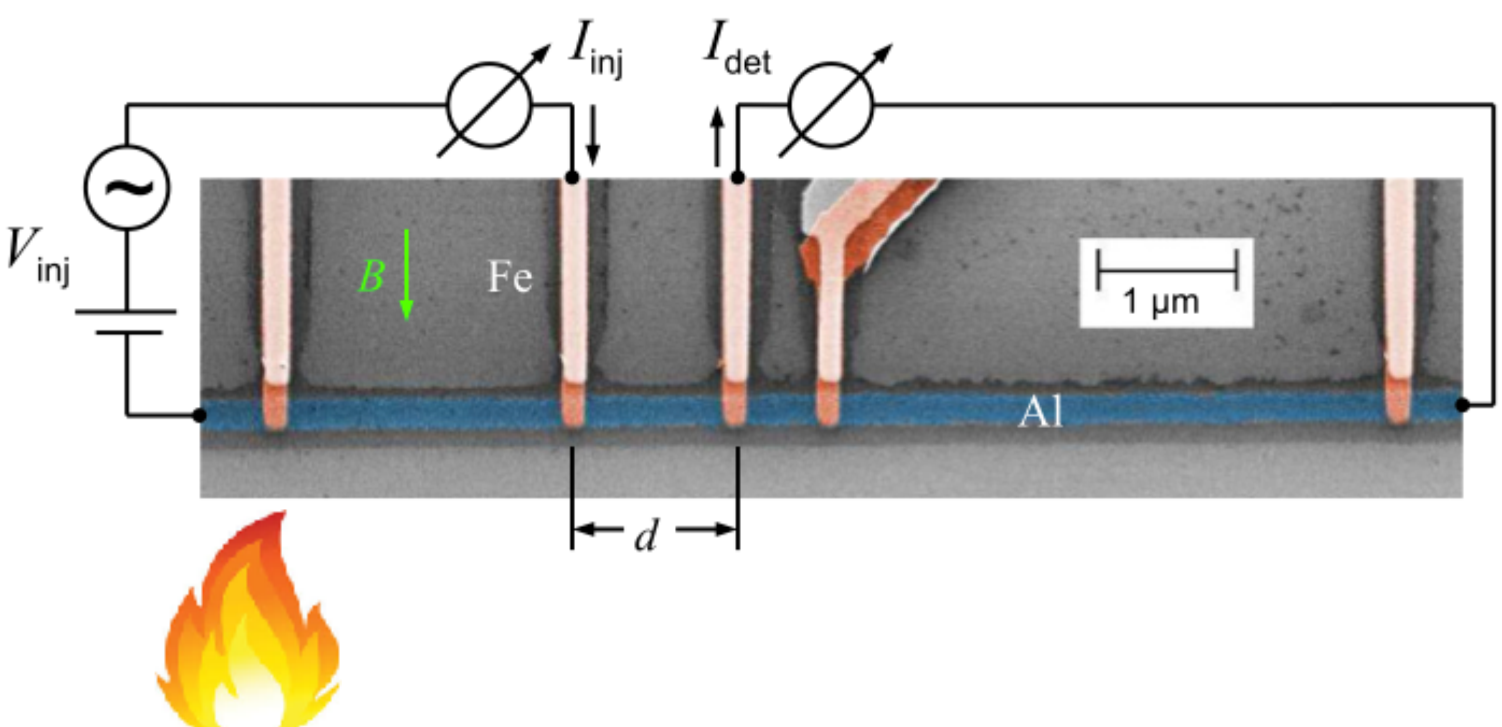
# Modes of nonequilibrium

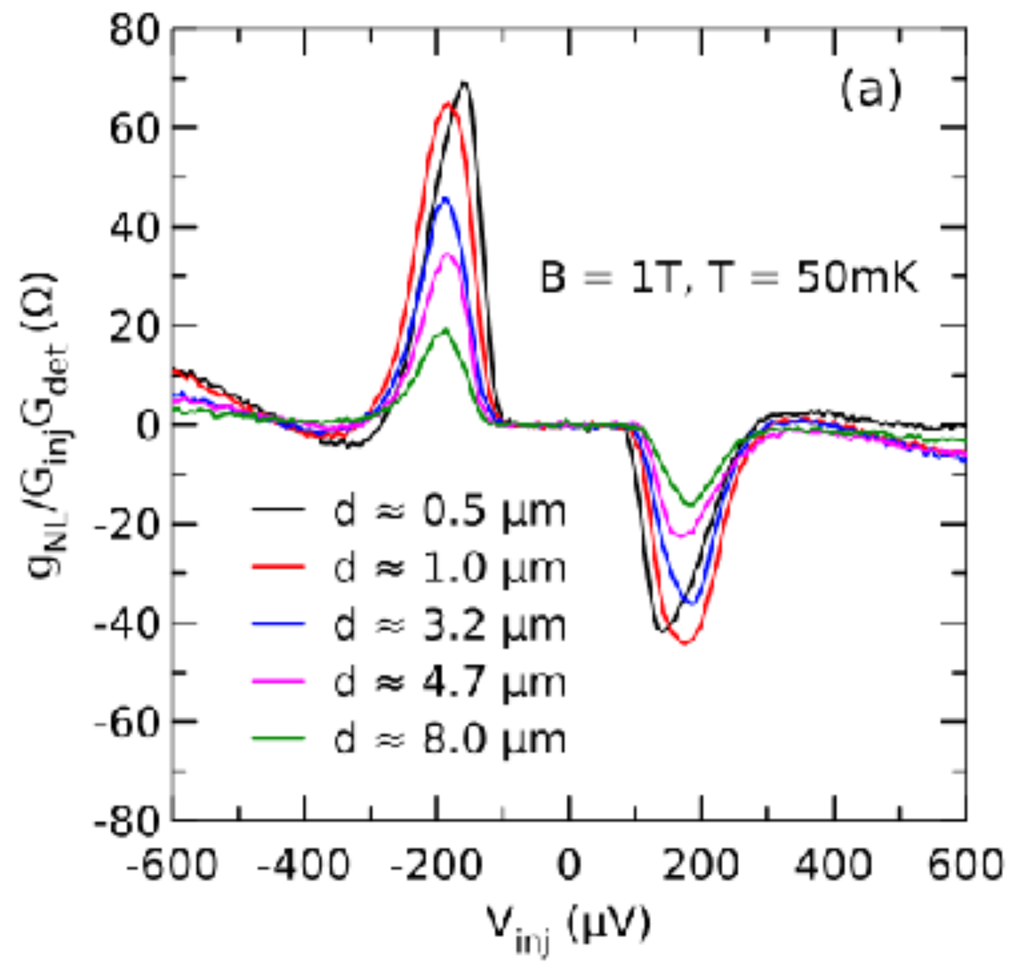
$$
\begin{tikzcd}
\text{Charge} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow] & \text{Spin energy} \arrow[d, leftrightarrow, "\text{Interface with nonzero spin polarization}"] \\
\text{Spin} \arrow[r, leftrightarrow] & \text{Energy}
\end{tikzcd}
$$

“Ferromagnetic superconductor” $N_\uparrow \neq N_\downarrow$

EXTENSION: F. AIKEBAIER’S POSTER

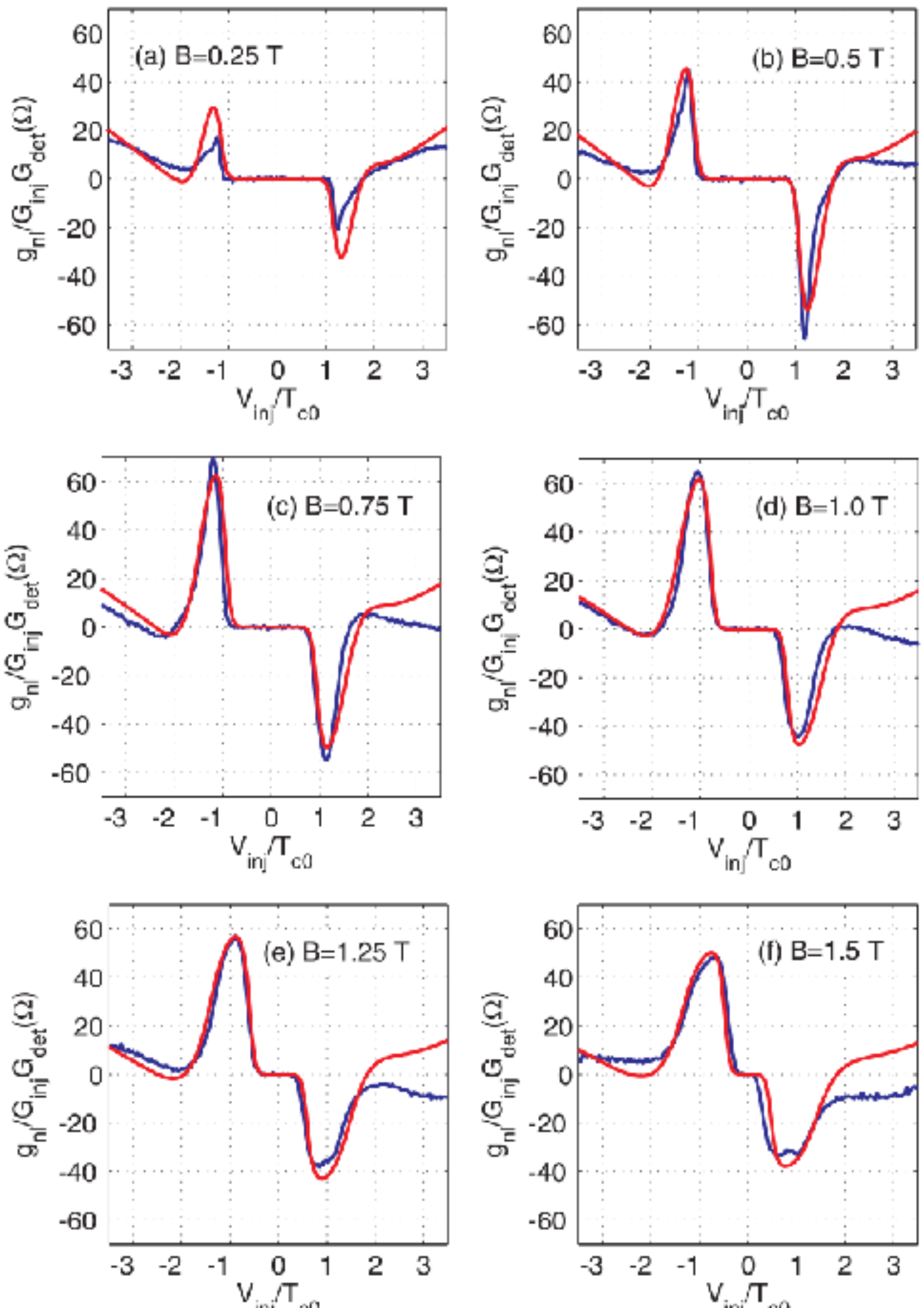
# Overall picture of the long-range effect




i) Injection heats the electrons
ii) Detector detects thermoelectrically

$$I_{\text{det}} \propto \Delta T_e(V_{\text{inj}}),\ \Delta T_e(V_{\text{inj}}) \approx \Delta T_e(-V_{\text{inj}})$$
$$\Rightarrow g_{\text{NL}}(V_{\text{inj}}) = \frac{dI_{\text{det}}}{dV_{\text{inj}}} \approx -g_{\text{NL}}(-V_{\text{inj}})$$




Fit to the experiments:
(theory=red, exp=blue)

Essential length scale: inelastic scattering length $5-10\ \mu\mathrm{m} \gg \ell_{\mathrm{sf}}$

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)

# Modes of nonequilibrium

| $\mathbf{j}$ | $\rho$ | Conservation law |
| --- | --- | --- |
| Charge current | Potential | Charge |
| Energy current | Temperature | Energy |
| Spin current | Magnetization | Angular momentum |
| Spin energy current | ? | Energy + angular momentum |

$$\partial_t \rho = -\nabla \cdot \mathbf{j}$$

# Conclusions

- Superconductivity + magnetism: rich set of noneq phenomena where some properties come only from the combination
- Other related effects: giant spin Seebeck effect, modified spin Hanle effect, rectification of ac signals, …