Multi-terminal Josephson junctions

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with

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SPICE-Workshop

Exotic New States in Superconducting Devices: The Age of the Interface

Mainz – September 28, 2017

UNIVERSITÉ Grenoble Alpes

DE LA RECHERCHE À L'INDUSTRIE cea GRENOBLE

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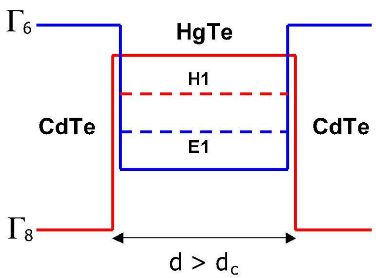
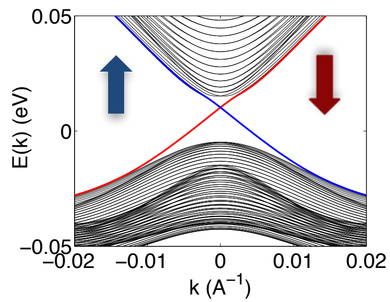
Pheliqs Quantum Physics & Engineering

Topological materials

Topological insulators / superconductors:

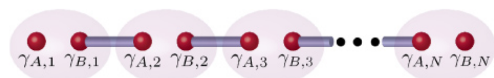
- gap in the bulk + topologically-protected surface states

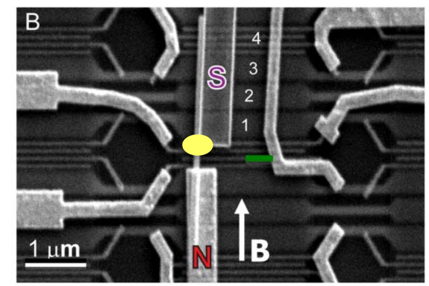


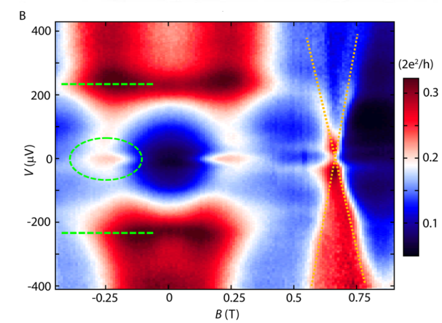


Kitaev chain
(1D spinless p-wave SC)
→ Majorana bound states

Kitaev (2001)





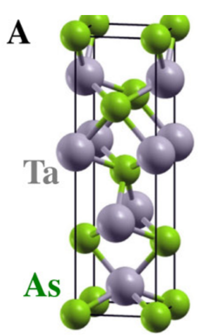


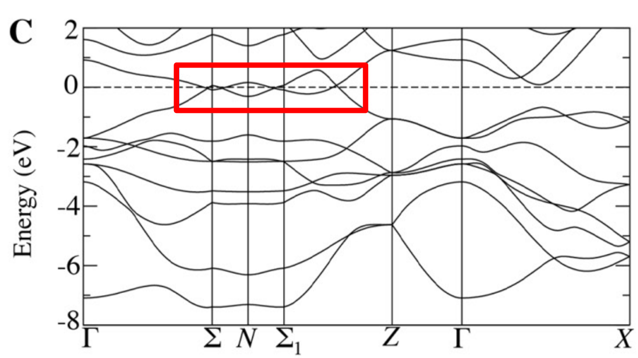
quantum spin Hall insulator
→ helical edge states

Fu & Kane (2005), Bernevig *et al.* (2006), König *et al.* (2007)

Oreg *et al.* (2010)
Lutchyn *et al.*
(2010)

Mourik *et al.* (2012)



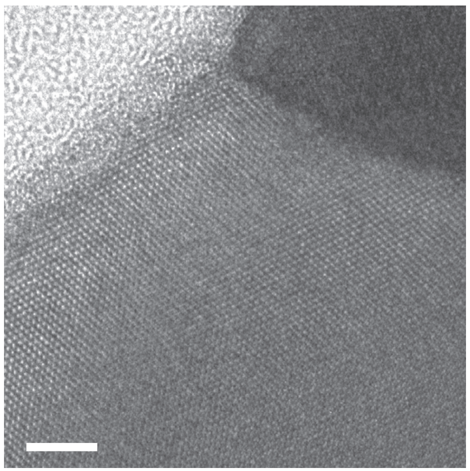


Weyl points

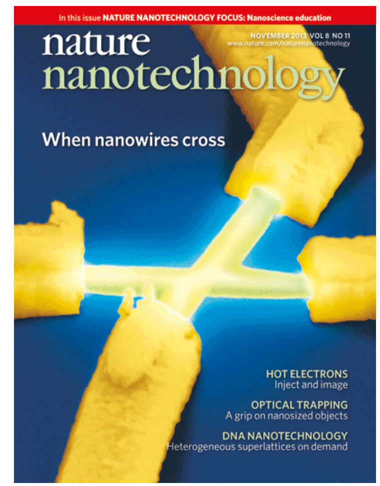
Wan *et al.* (2011)
Xu *et al.* (2015)

Main idea

materials



complicated bandstructure
in d dimensions



Plissard *et al.* (2013)

artificial material in $d = n - 1$ dimensions ?

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The analogy

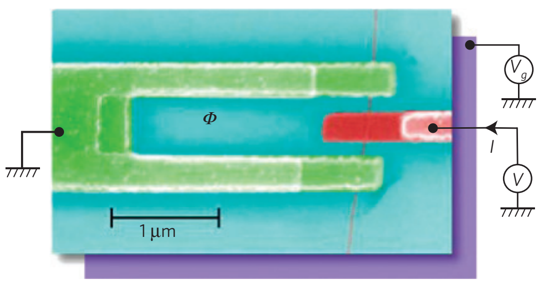
2-terminal junctions:

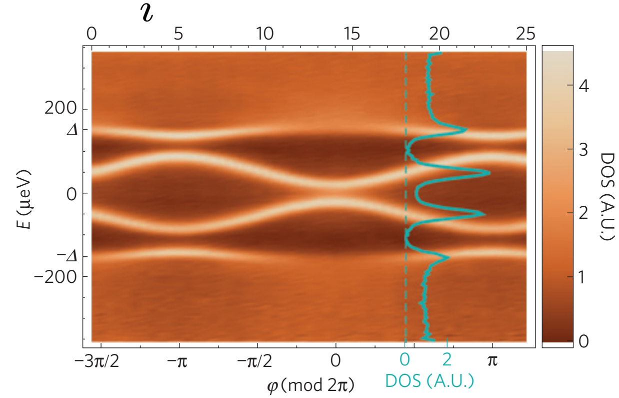
simplest case: Josephson energy

$$E = -E_J \cos\phi \;\; \rightarrow \;\; I_J = \frac{2e}{\hbar}\frac{\partial E}{\partial \phi} = I_c \sin\phi$$

in general:

Andreev bound states (ABS) $E = -\sum_i E_A^{(i)}(\phi)$ (+ continuum)





Pillet
et al.
(2013)

discrete Andreev spectrum
in a junction with few channels

The analogy

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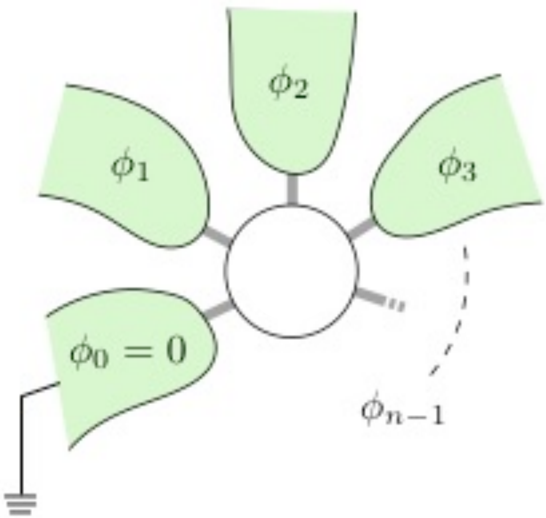


$\rightarrow$ n-terminal junctions: $\quad E_A^{(i)}(\phi_1, \phi_2, \ldots, \phi_{n-1})$

ABS energy = periodic fct of $n-1$ phase differences

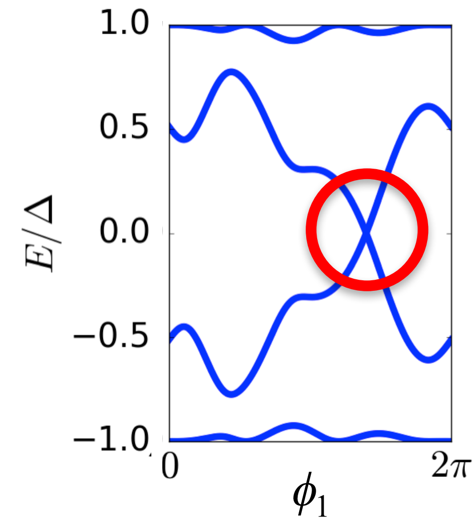
analogy:

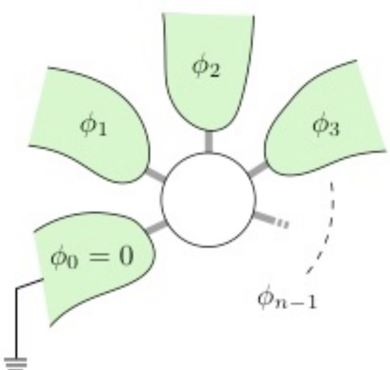
n-terminal junction	$\rightarrow$	$d = n-1$ dimensional material
Andreev spectrum	$\rightarrow$	band structure
phase differences	$\rightarrow$	quasi-momenta $k_x, k_y, k_z, \ldots$

Topology: more information in the wavefunctions than in the spectrum!

Main result

topologically-protected Weyl singularities
in the ABS spectrum of junctions with $n \geq 4$ terminals





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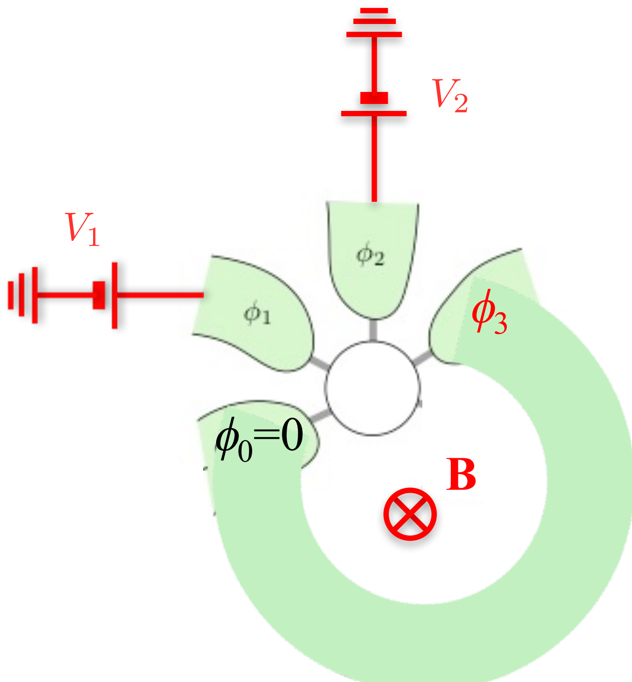
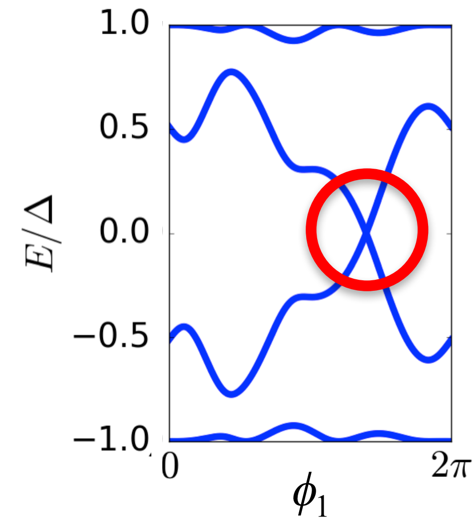
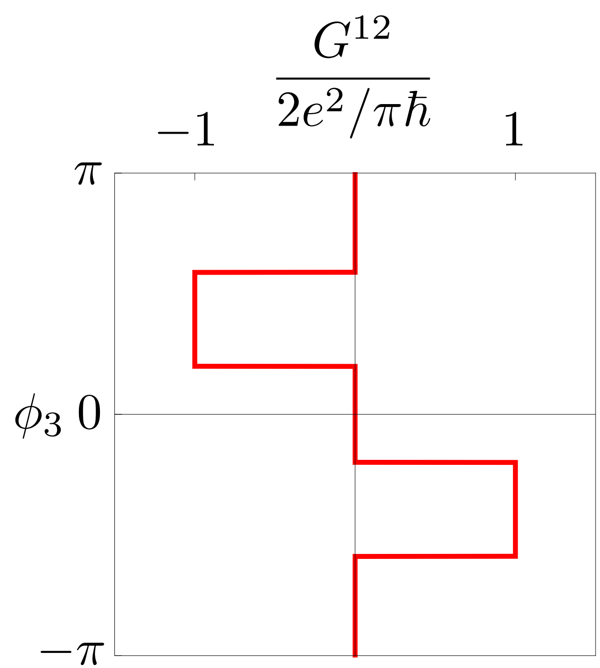






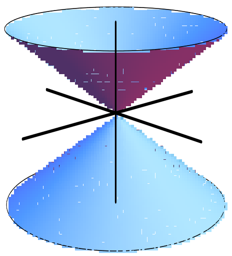
manifestations:
quantized transconductance
between 2 voltage-biased terminals

Outline

- Weyl singularities
- Andreev bound state (ABS) spectrum of multi-terminal junctions
- Quantized transconductance
- Beyond the adiabatic regime
- Conclusion

Weyl singularities

- topologically protected zero-energy states



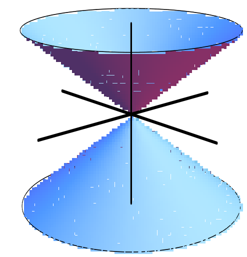
3D Weyl Hamiltonian: $$H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j$$

where σ_i 2 x 2 Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Weyl singularities

- topologically protected zero-energy states



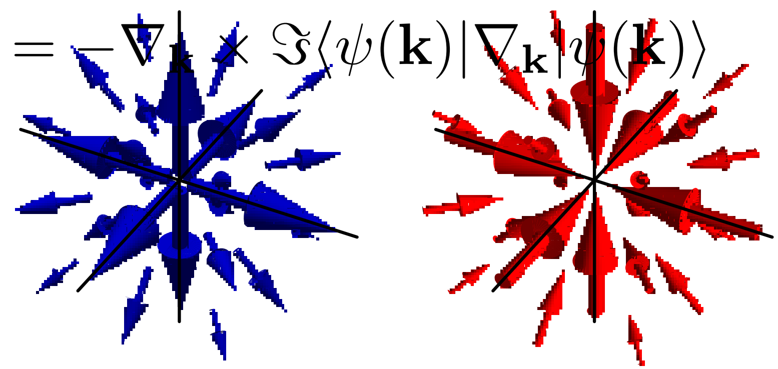
3D Weyl Hamiltonian: $H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j$

Weyl points carry a topological charge:

- Weyl points are monopoles of Berry curvature

$$\mathbf{B}(\mathbf{k}) = -\nabla_{\mathbf{k}} \times \Im \langle \psi(\mathbf{k}) | \nabla_{\mathbf{k}} | \psi(\mathbf{k}) \rangle$$

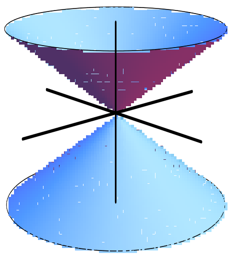
$$\chi = \frac{1}{2\pi} \oint d\mathbf{S}(\mathbf{k}) \cdot \mathbf{B}(\mathbf{k})$$
$$= \operatorname{sign} \det[\{v_{ij}\}] = \pm 1$$



Weyl semimetals have been discovered recently (TaAs …)

Weyl singularities

- topologically protected zero-energy states



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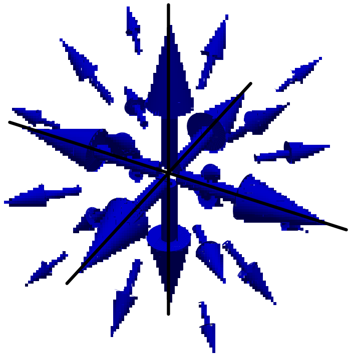
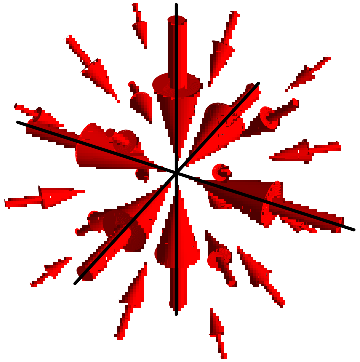
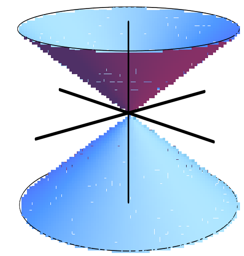




vs Chern number: $C = \frac{1}{2\pi} \oint_{\text{2D BZ}} (dk)\, B_z(\mathbf{k}) \quad \in \mathbb{Z}$

Weyl singularities

- topologically protected zero-energy states



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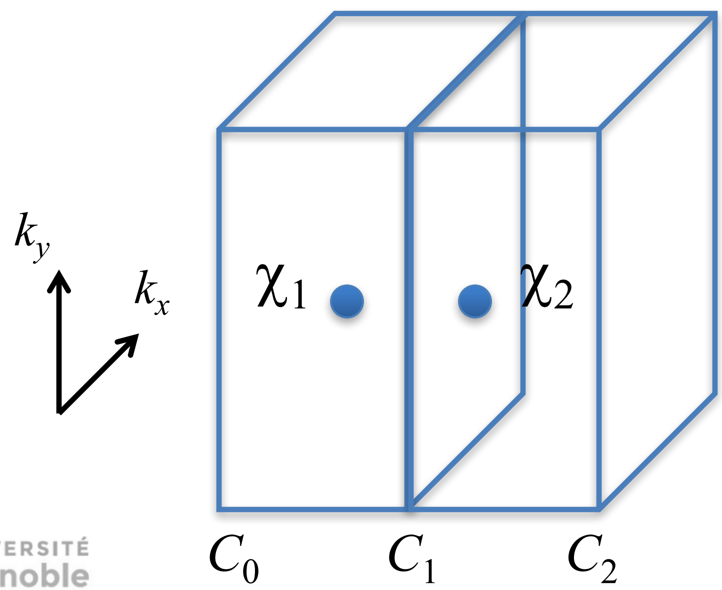


$$C = \frac{1}{2\pi} \oint_{\mathrm{2D\ BZ}} (dk)\, B_z(\mathbf{k})$$

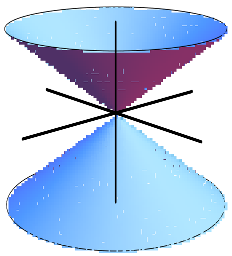
$\rightarrow$ Chern number changes when crossing a Weyl Point:

$$C_1 = C_0 + \chi_1$$

$$C_2 = C_1 + \chi_2 = C_0 + \chi_1 + \chi_2$$

Weyl singularities

- topologically protected zero-energy states



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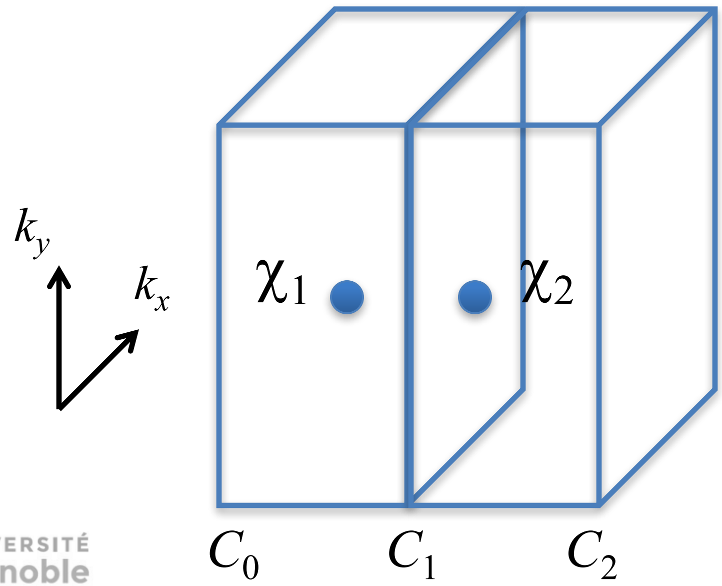

$$C = \frac{1}{2\pi} \oint_{\text{2D BZ}} (dk)\, B_z(\mathbf{k})$$

non-zero Chern number

$\rightarrow$ edge states

$\rightarrow$ quantum Hall effect

Thouless *et al.* (1982)

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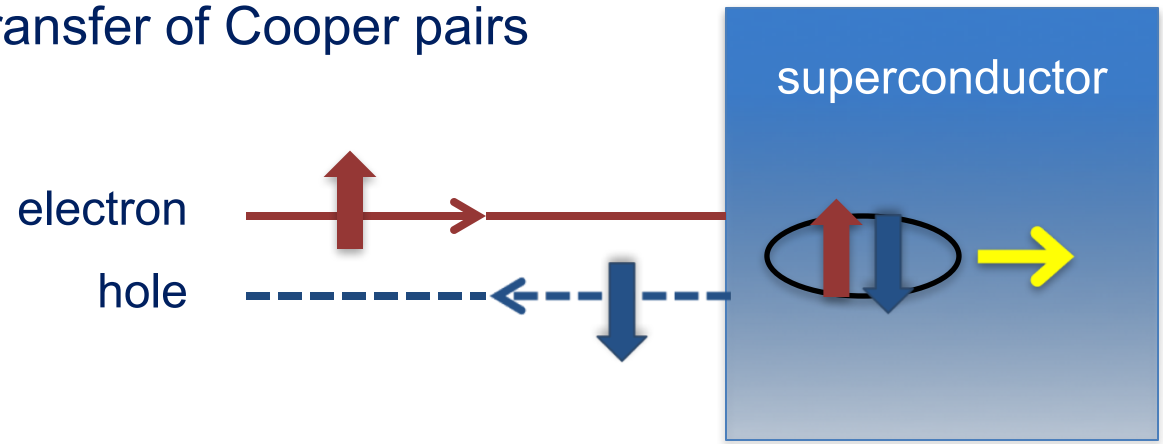
Andreev reflection

charge transfer between a "non-superconductor" and a superconductor at $\varepsilon < \Delta$:

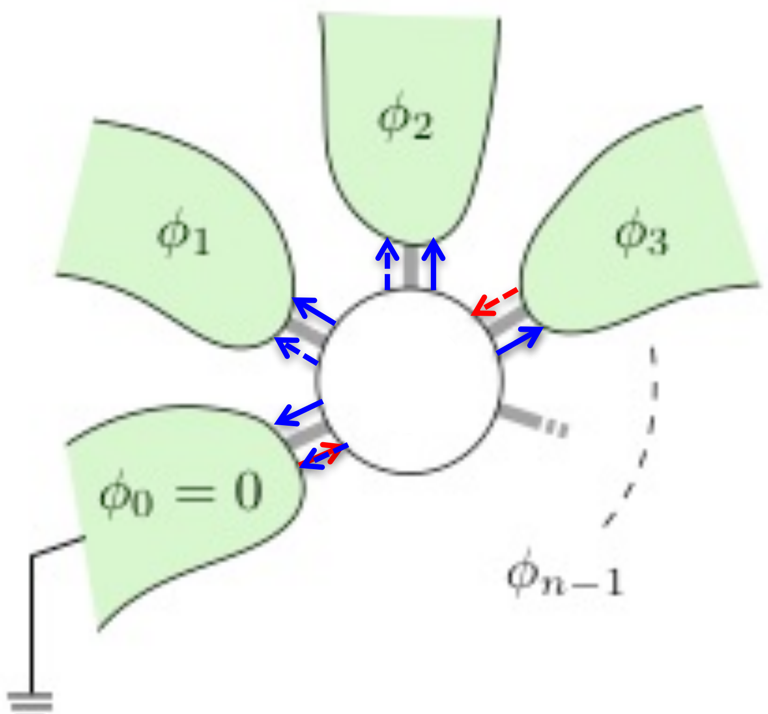
- no quasi-particle states
- only transfer of Cooper pairs



Andreev 1964

ABS spectrum of multi-terminal junctions

generalization of the scattering problem:



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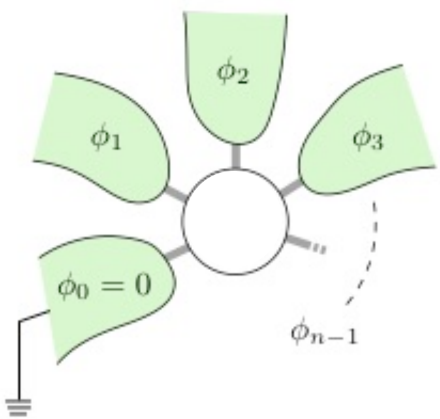
ABS spectrum of multi-terminal junctions

- ABS spectrum

  determined by $|\psi\rangle = S_N S_A |\psi\rangle$

  Beenakker (1991)



- normal scattering in the contact:

  scattering matrix S_N

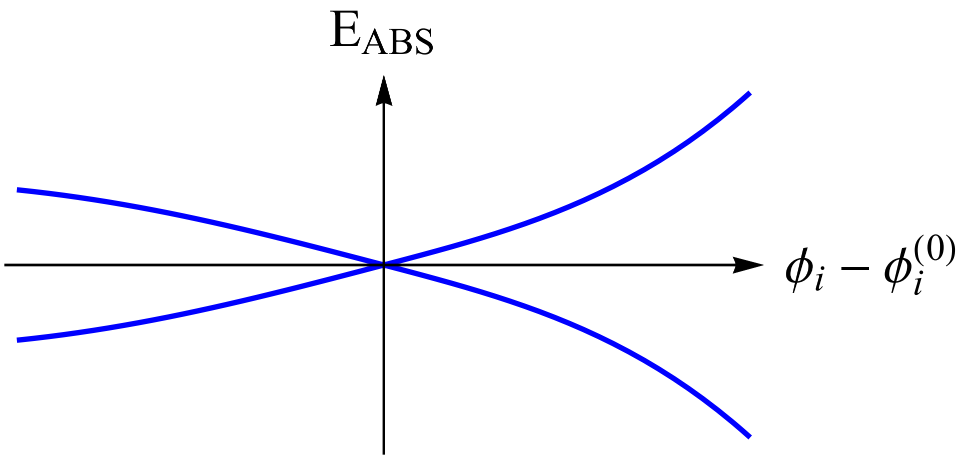
- Andreev reflection:

  scattering matrix $S_A(\phi_1, \ldots, \phi_{n\text{-}1}; E)$

- particle-hole symmetry: states come in pairs at energies $\S E$
- Weyl singularities: **doubly degenerate zero-energy states** at $\mathbf{\Phi}^{(0)}$

Weyl-Hamiltonian

- Weyl singularities: doubly degenerate zero-energy states at $\Phi^{(0)}$



- in the vicinity of the zero-energy solution at $\Phi^{(0)}$:

  effective low-energy Weyl Hamiltonian

  in the subspace of the 2 orthogonal eigenstates:

  $H_W = \sum_{\alpha,\, i} M_{\alpha i}\, \delta\phi_\alpha\, \tau_i$ where τ_i 2x2 Pauli matrices

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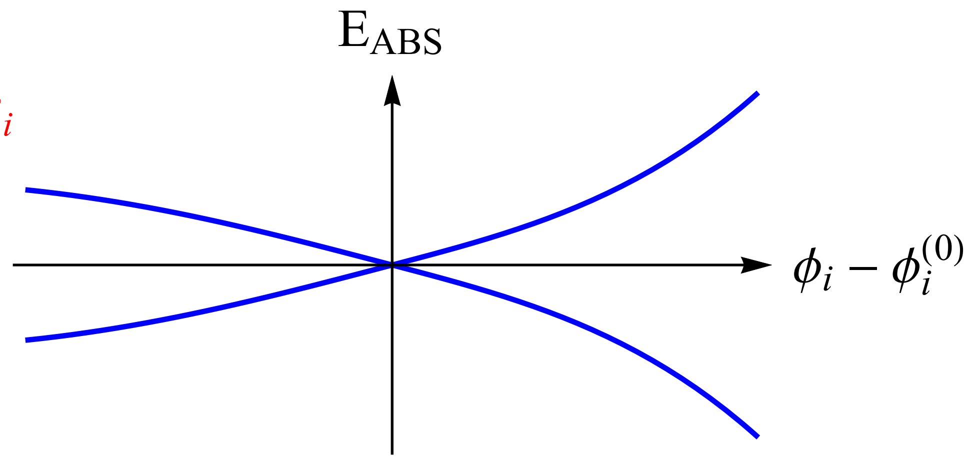
Weyl-Hamiltonian

- Weyl singularities: doubly degenerate zero-energy states at $\Phi^{(0)}$

$$H_W = \sum_{\alpha,\, i} M_{\alpha i}\, \delta\phi_\alpha\, \tau_i$$



- topological charge of the Weyl point in a 3D subspace:

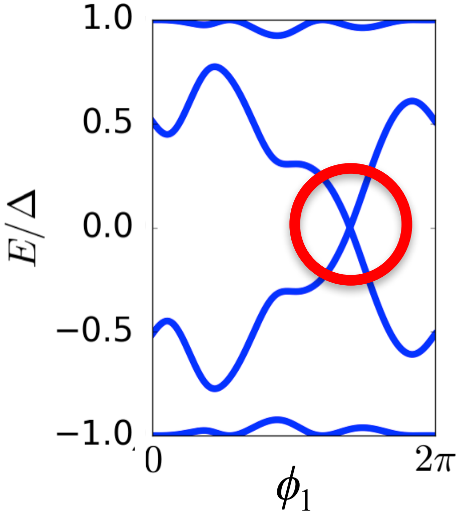
$$\chi = \text{sign det}\,[\{M_{\alpha i}\}]$$

- total topological charge = 0
- time-reversal symmetry: Weyl point at $\Phi^{(0)}$

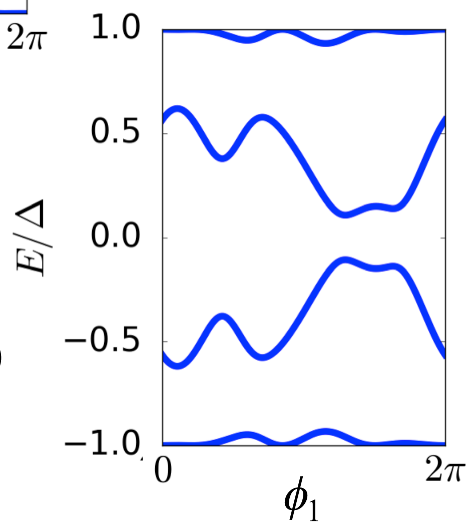
  $\rightarrow$ Weyl point with the same topological charge at $-\Phi^{(0)}$
- Weyl points come in multiples of 4

ABS spectrum

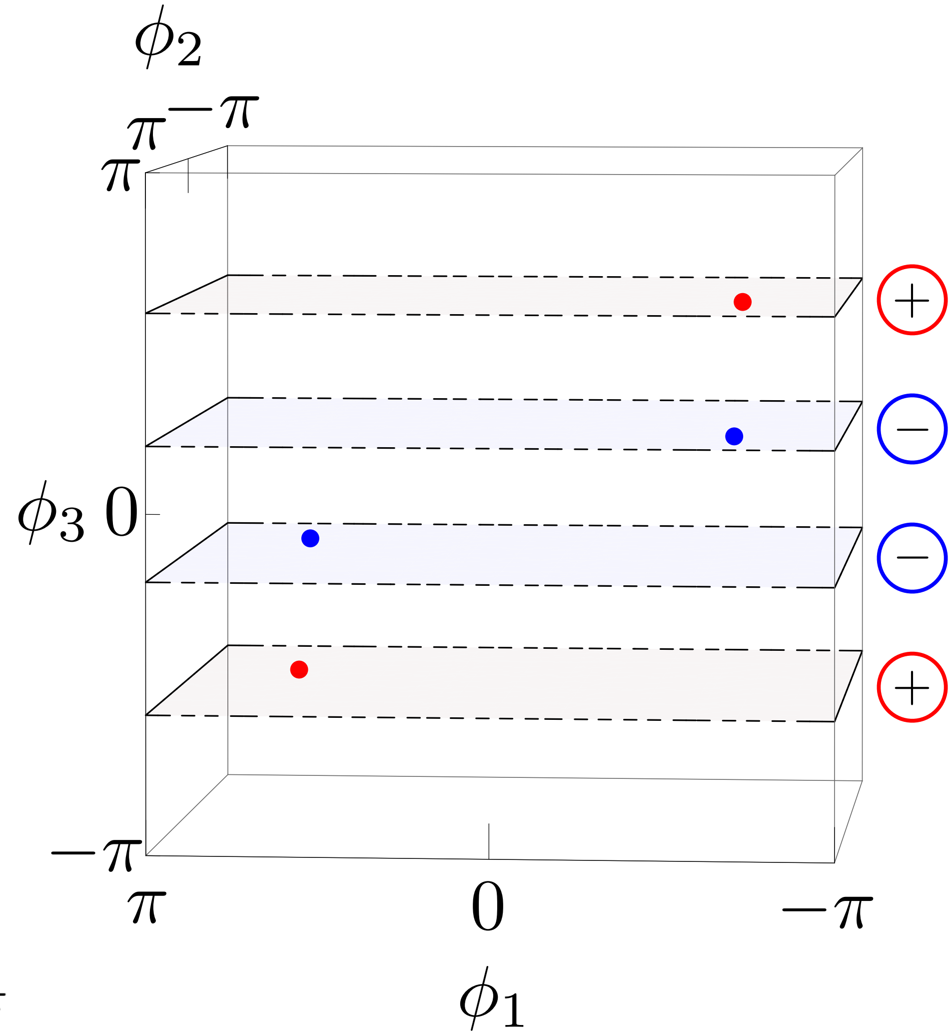
example: 4-terminal junction



$(\phi_2,\phi_3) = (\phi_2^{(0)},\phi_3^{(0)})$

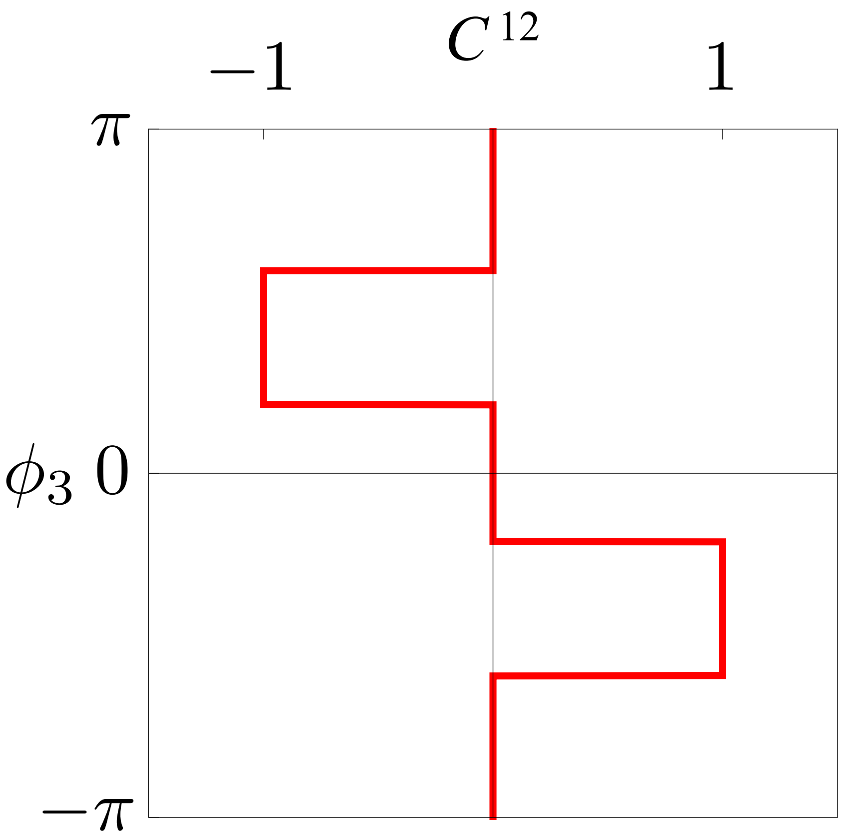


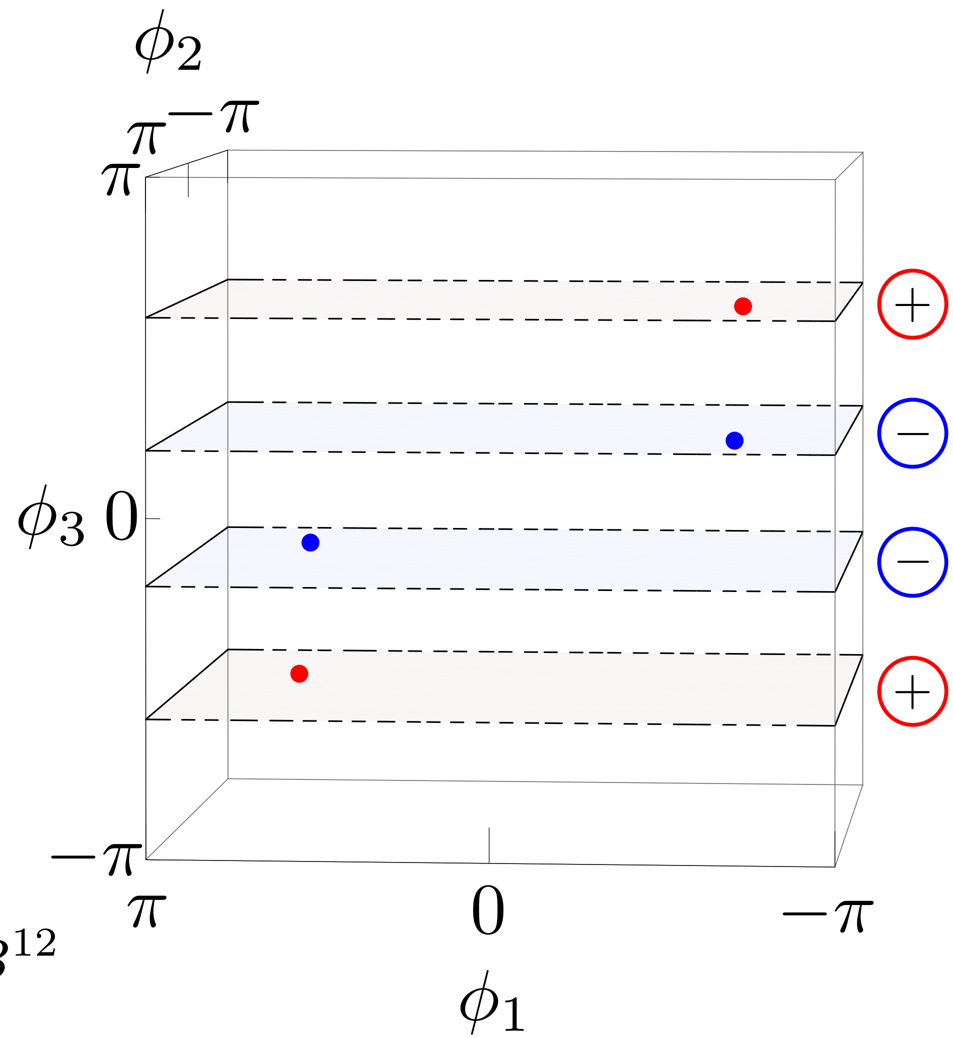
$(\phi_2,\phi_3) \neq (\phi_2^{(0)},\phi_3^{(0)})$



ABS spectrum

Chern number:

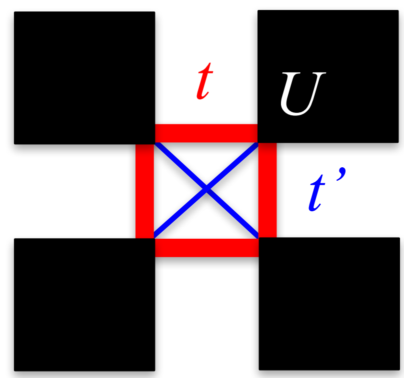




where $C^{12} = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} d\phi_1\, d\phi_2\; B^{12}$

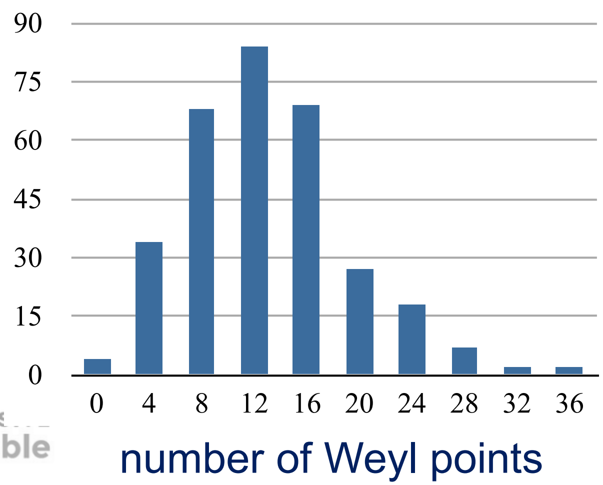
4-terminal junctions: Occurence of Weyl points

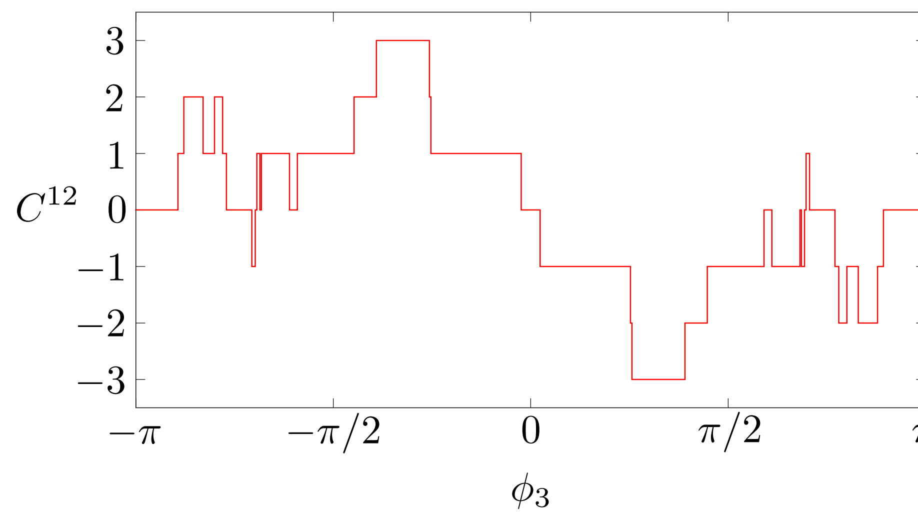
- 4 single-channel terminals:
  - ~ 5% of random scattering matrices possess Weyl points
  - simple toy models X



- 4 multi-channel terminals:

  example: $N_\alpha = 12, 11, 10, 9$





Consequences of Weyl singularities: The current

- current operator: $\hat{I}_\alpha = 2e \frac{\partial \hat{H}}{\partial \phi_\alpha}$

- use instantaneous eigenbasis $E_{A\nu}(t) \left|\psi_\nu(t)\right\rangle = \hat{H}(t) \left|\psi_\nu(t)\right\rangle$
  to compute expectation value for time-dependent phases:

contribution of ABS $$I_{\alpha\nu}(t) = \frac{2e}{\hbar} \frac{\partial E_{A\nu}(t)}{\partial \phi_\alpha} - 4e \sum_\beta \dot{\phi}_\beta \Im \langle \frac{\partial \psi_\nu}{\partial \phi_\alpha} | \frac{\partial \psi_\nu}{\partial \phi_\beta} \rangle$$

adiabatic supercurrent $I_{\alpha\nu}{}^0(t)$

first correction: $\delta I_{\alpha\nu}(t) = -2e \sum_\beta \dot{\phi}_\beta B_\nu^{\alpha\beta}$

with $B_\nu^{\alpha\beta} = 2\,\Im \left\langle \frac{\partial \psi_\nu}{\partial \phi_\alpha} \middle| \frac{\partial \psi_\nu}{\partial \phi_\beta} \right\rangle$ Berry curvature

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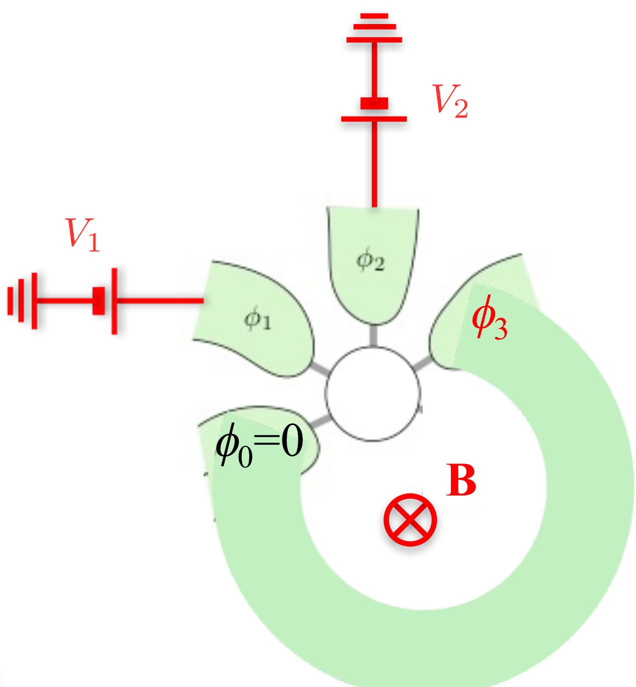
Quantized transconductance

- total current:

$$I_\alpha(t) = \sum_{k,\sigma} I_{\alpha k}(t)\left(n_{k\sigma} - \frac{1}{2}\right) = I_\alpha^0(t) - 2e\sum_{k,\sigma,\beta} \dot{\phi}_\beta B_k^{\alpha\beta}\left(n_{k\sigma} - \frac{1}{2}\right)$$

- consider 2 voltage-biased leads: $\phi_\alpha = 2eV_\alpha t$



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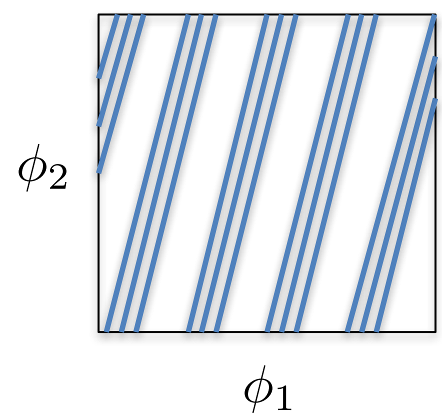
$\rightarrow$ phase sweeps 2D “Brillouin zone”

($V_{\alpha,\beta}$ ¿ Δ incommensurate)



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- consider 2 voltage-biased leads: $\phi_\alpha = 2eV_\alpha t$

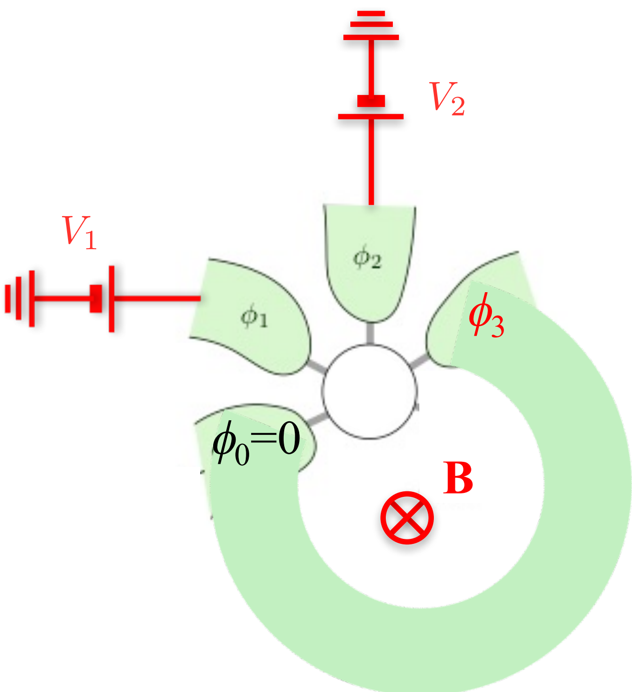
→ phase sweeps 2D "Brillouin zone"
→ time-averaged current in the ground state ($n_{k\sigma} = 0$):

$$\overline{I}_\alpha = G^{\alpha\beta}V_\beta \quad \text{with} \quad G^{\alpha\beta} = -\frac{2e^2}{\pi\hbar}C^{\alpha\beta}$$

where $C^{\alpha\beta} = -\frac{1}{2\pi}\sum_k \int_{-\pi}^{\pi} d\phi_\alpha\, d\phi_\beta\, B_k^{\alpha\beta}$ integer

= Chern number

Multiterminal junctions as topological matter

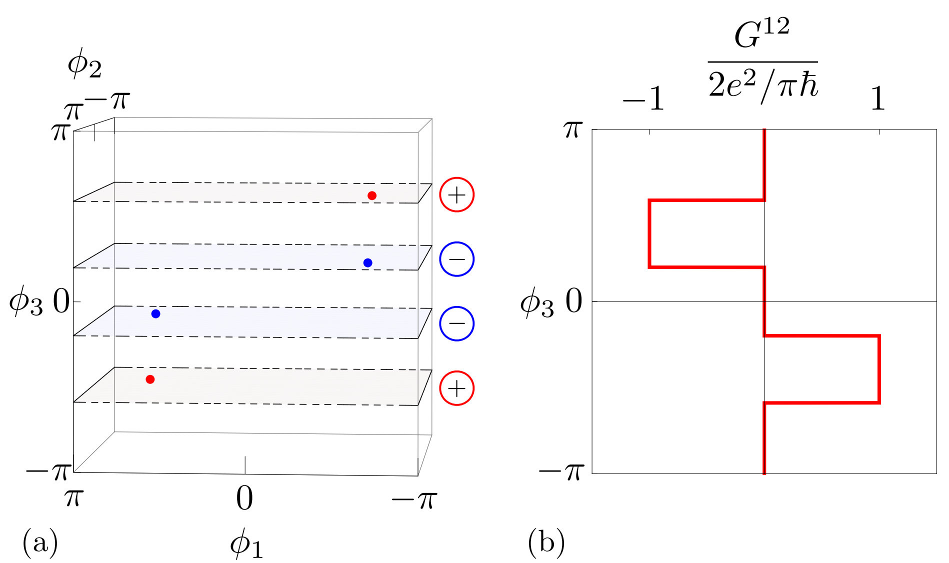


experimental manifestation:
quantized transconductance

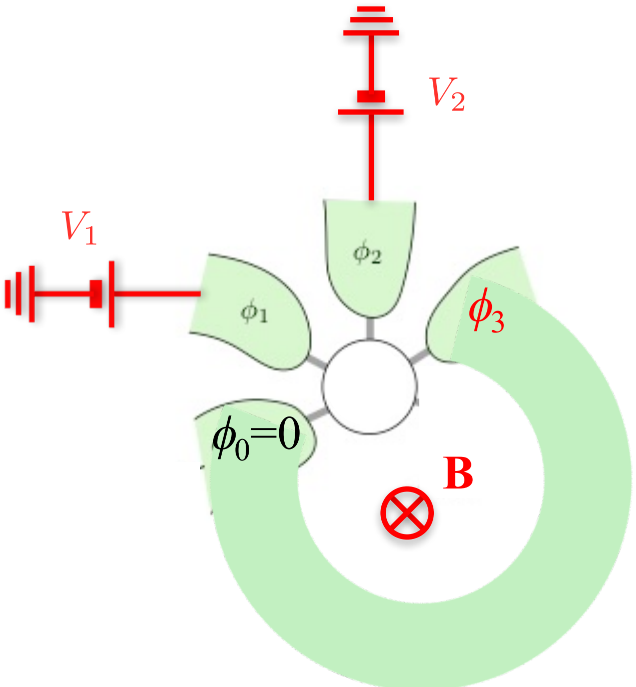
$$\bar{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = \frac{4e^2}{h} C^{\alpha\beta}$$

Chern number

MAIN RESULT



Multiterminal junctions as topological matter



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Chern number

$$\bar{I}_\alpha = -\frac{4e^2}{h} V_\beta \sum_k C_k^{\alpha\beta} (n_{k\uparrow} + n_{k\downarrow} - 1)$$

ground state: $n_{k\sigma} = 0$

$\rightarrow$ poisoning ? (Landau-Zener …)

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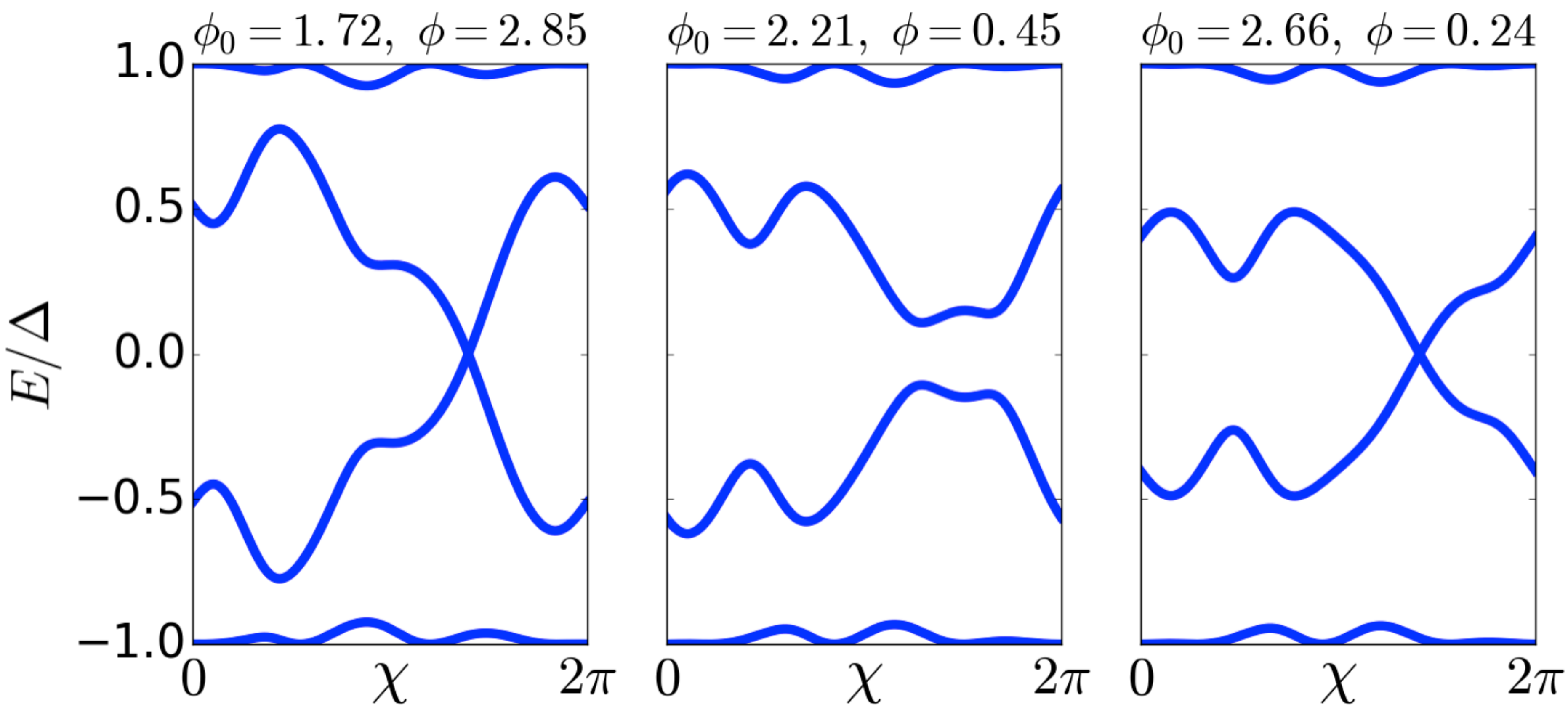
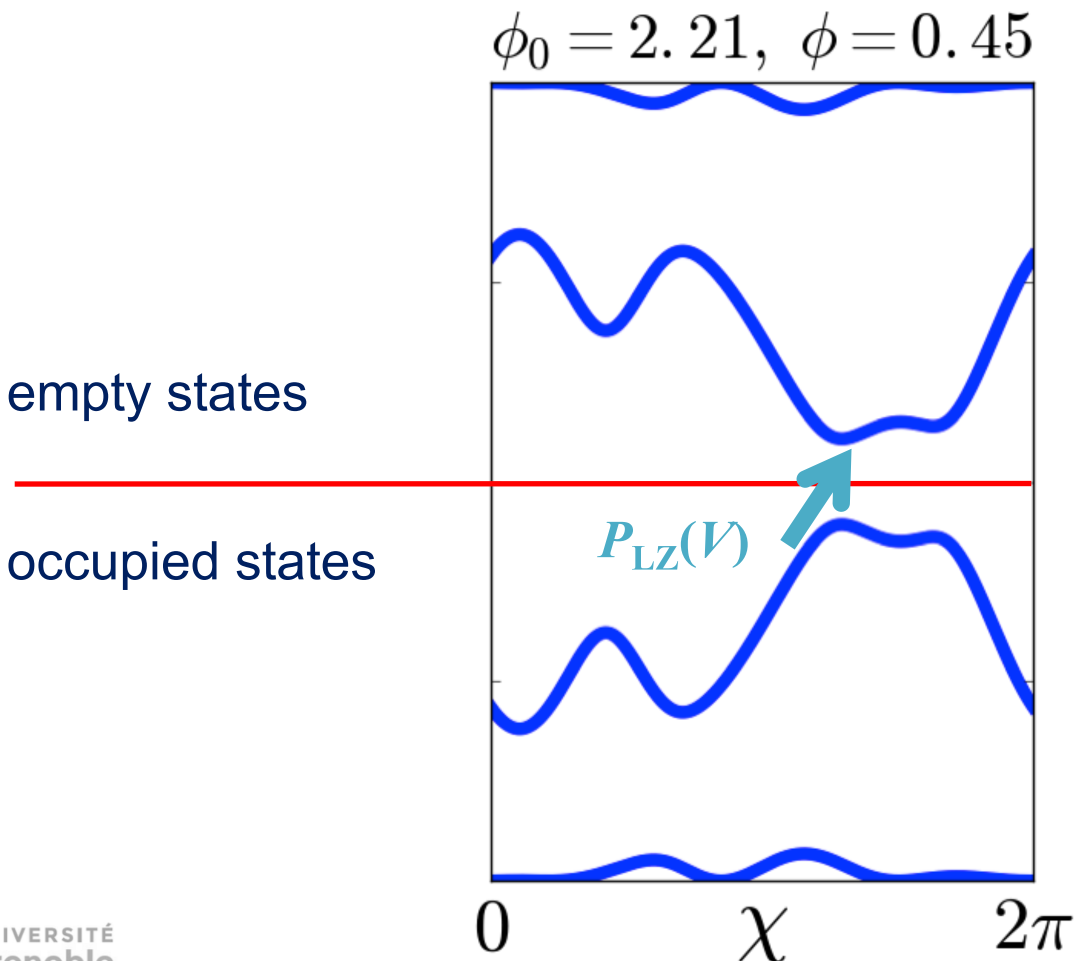
Beyond the adiabatic regime

- Landau-Zener processes:



Beyond the adiabatic regime

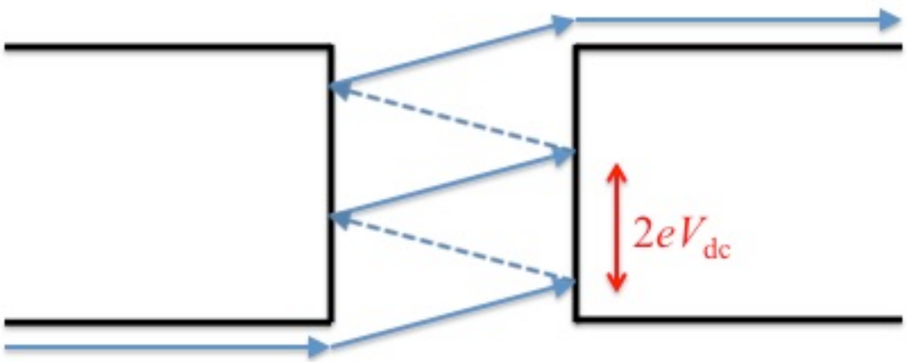
- Landau-Zener processes:



inelastic relaxation necessary to quickly recover equilibrium occupations

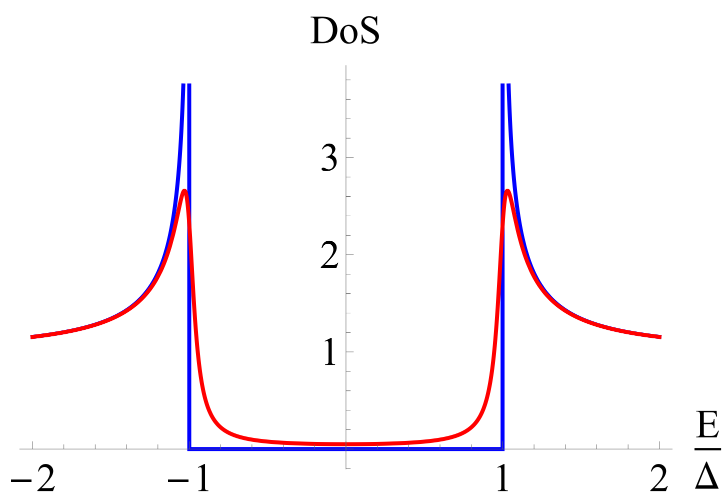
Beyond the adiabatic regime

- multiple Andreev reflections



$\rightarrow$ compute the currents using (Floquet) scattering theory

- account for inelastic relaxation with a Dynes parameter Γ in the leads

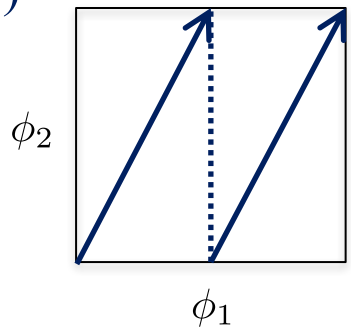


Beyond the adiabatic regime

- multiple Andreev reflections

$\rightarrow$ compute the currents using (Floquet) scattering theory

specific scattering matrix
with Weyl points at $\pm(1.7, -1.9, -2.8, 0)$ and $\pm(2.7, -1.8, 1.0, 0)$

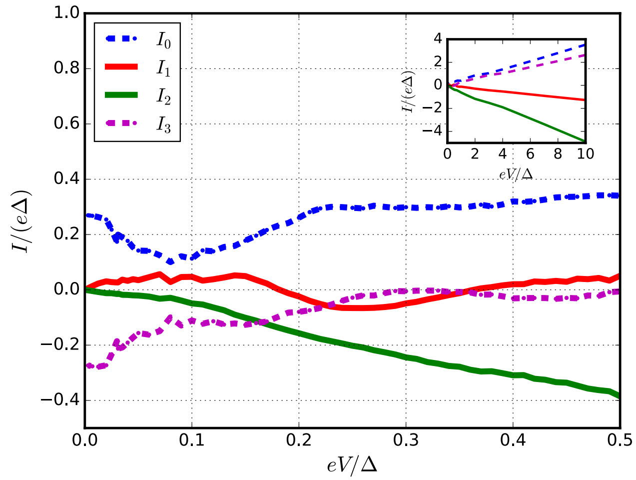


- choose $\quad \phi_1 = 2en_1Vt + \chi$
  $\phi_2 = 2en_2Vt$
- commensurate voltages $\rightarrow$ average over χ
- obtain conductances from 2 sets of voltages: $(n_1, n_2) = (1, 3)$ and $(2, 3)$

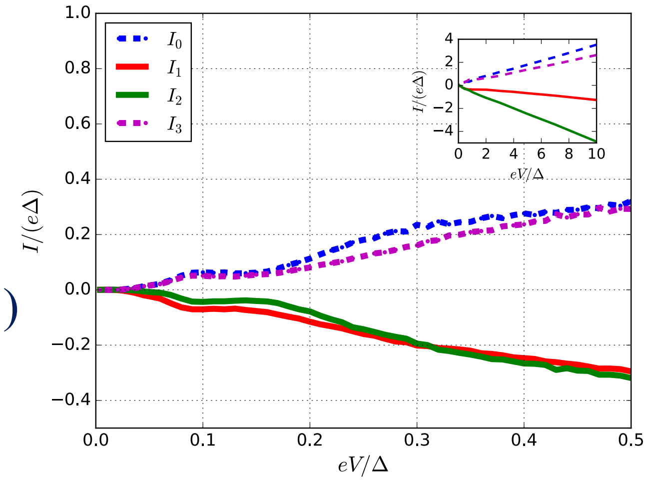
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

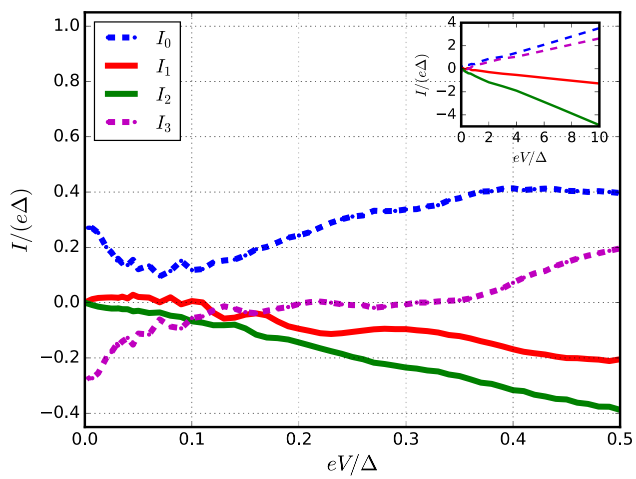
Beyond the adiabatic regime

currents as a fct of V at fixed ϕ_0 ($\Gamma = 0.002\Delta$):

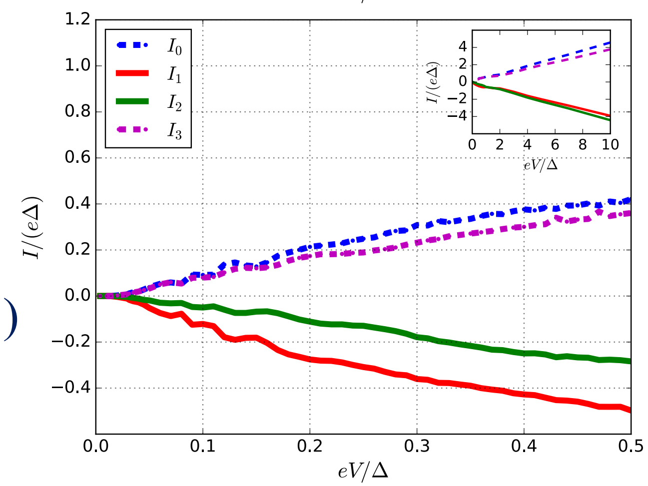


$(n_1, n_2) = (1, 3)$





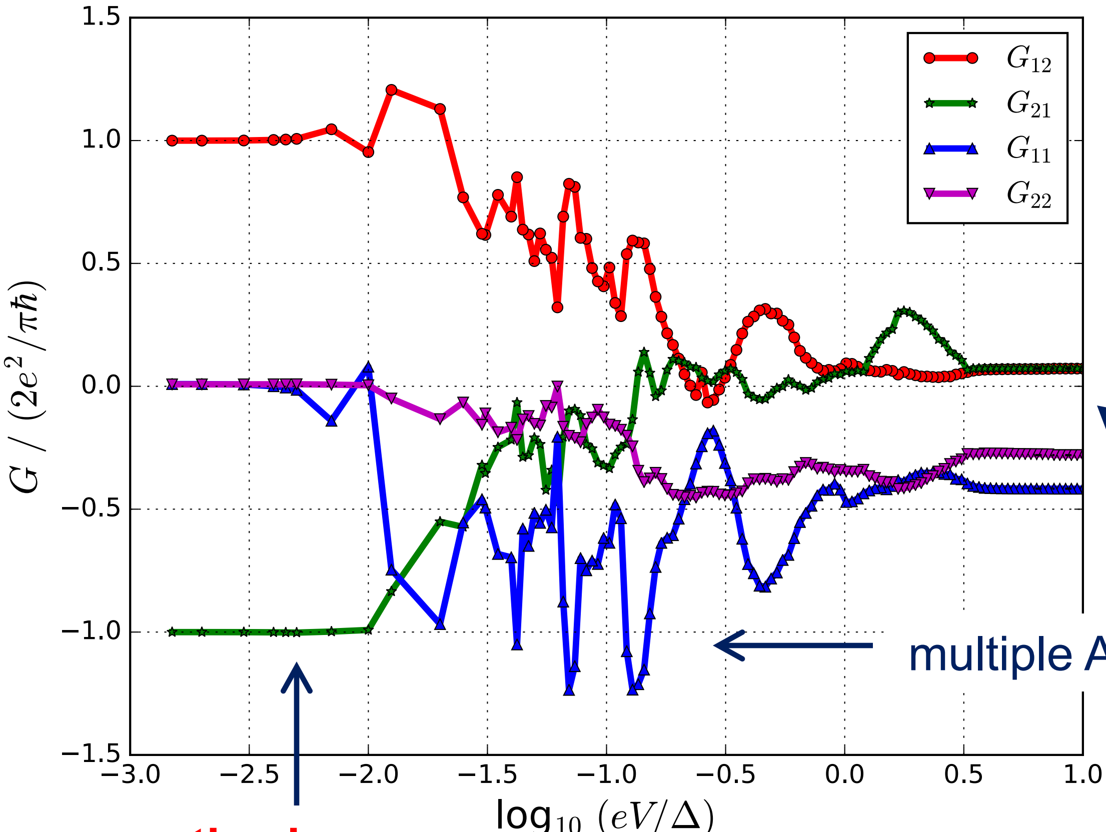
$(n_1, n_2) = (2, 3)$



$\phi_0 = 2.21$ (topological) **$\phi_0 = 0$ (trivial)**

Beyond the adiabatic regime

conductances as a fct of V at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):



normal state conductances ($G_{12} = G_{21}$)

multiple Andreev reflections

quantized transconductances

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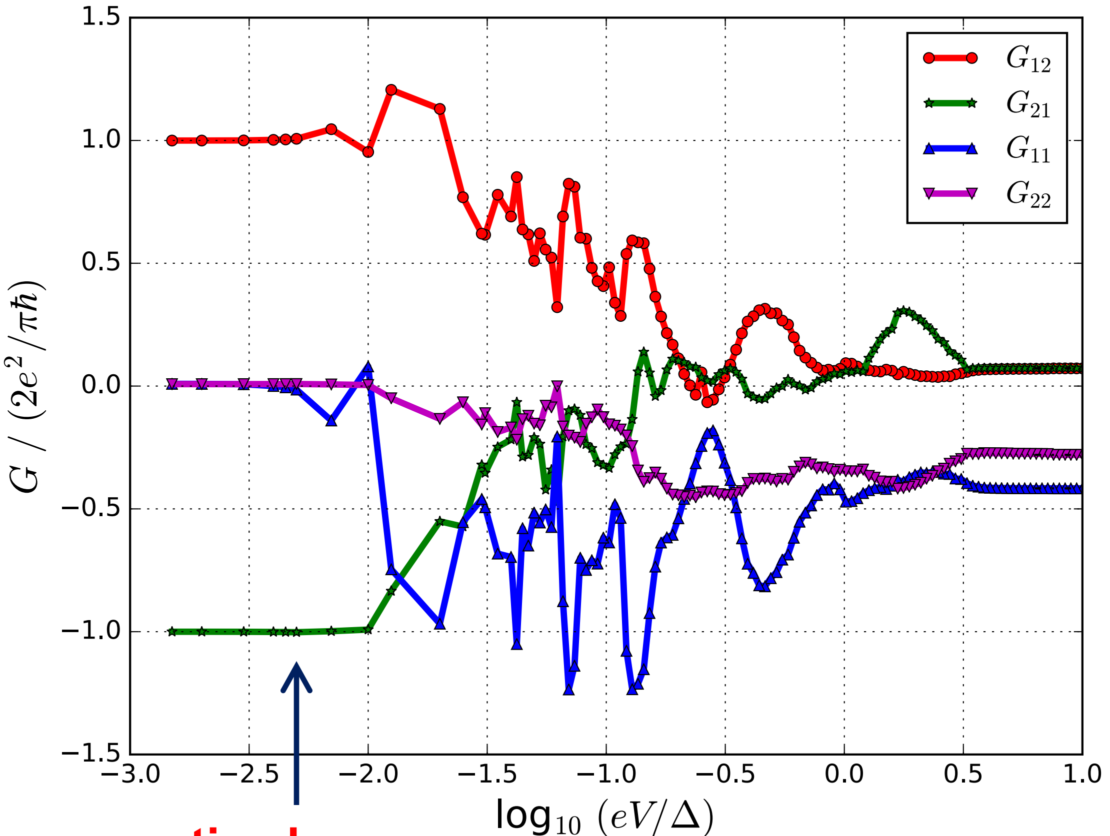
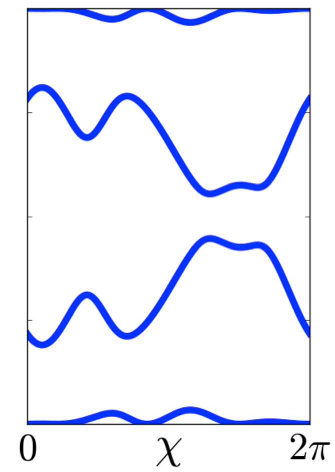
Beyond the adiabatic regime

conductances as a fct of V at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):





quantized transconductances

quantization requires fixed parity:

$$1/\tau_{LZ} = eVe^{-E_A/eV} < \Gamma$$

$$\rightarrow eV < eV_\star \sim \frac{E_A}{\log(E_A/\Gamma)}$$

where

$$E_A = \min_{\phi_1,\phi_2} E_A(\phi_0, \phi_1, \phi_2)$$

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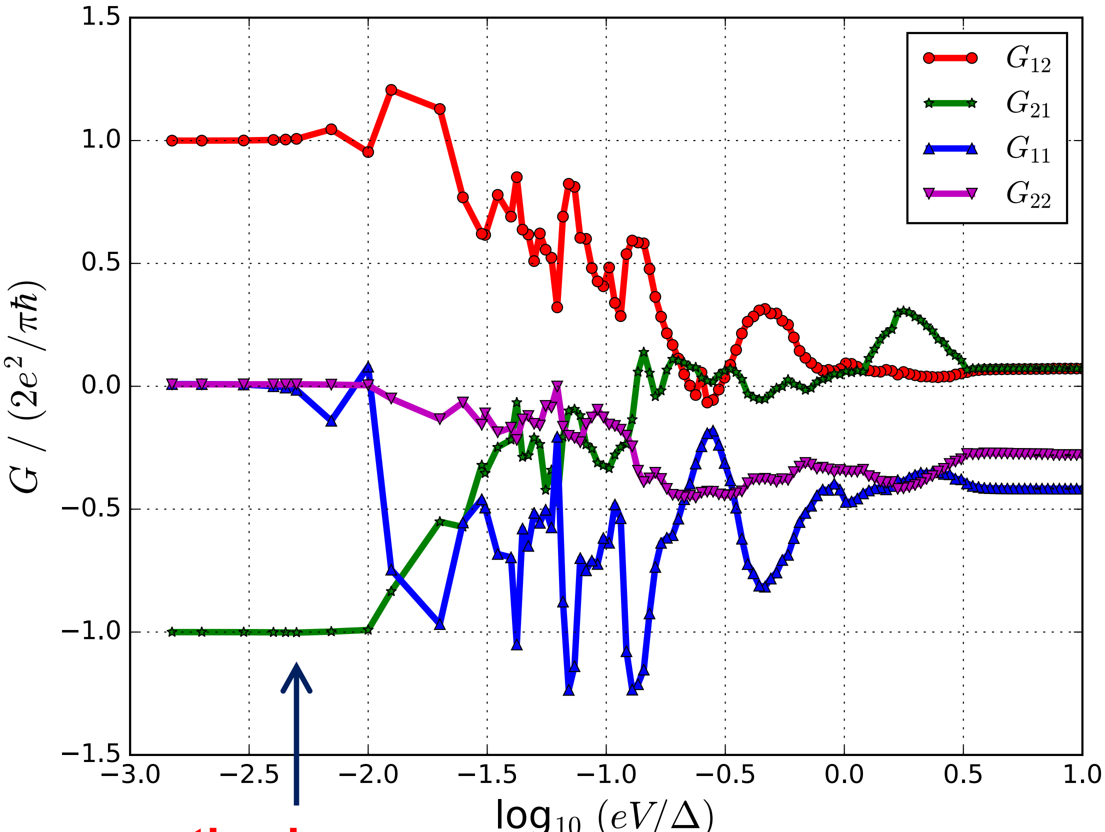
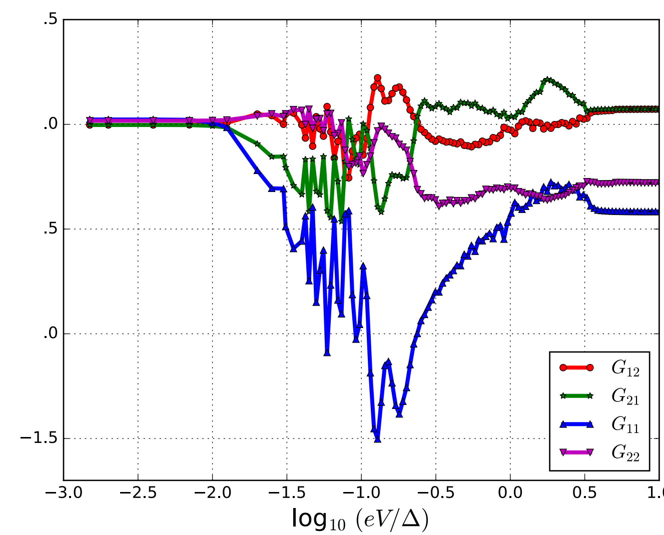
Beyond the adiabatic regime

conductances as a fct of V at fixed $\phi_0 = 2.21$ ($\Gamma = 0.002\Delta$):



quantized transconductances

for comparison: $\phi_0 = 0$



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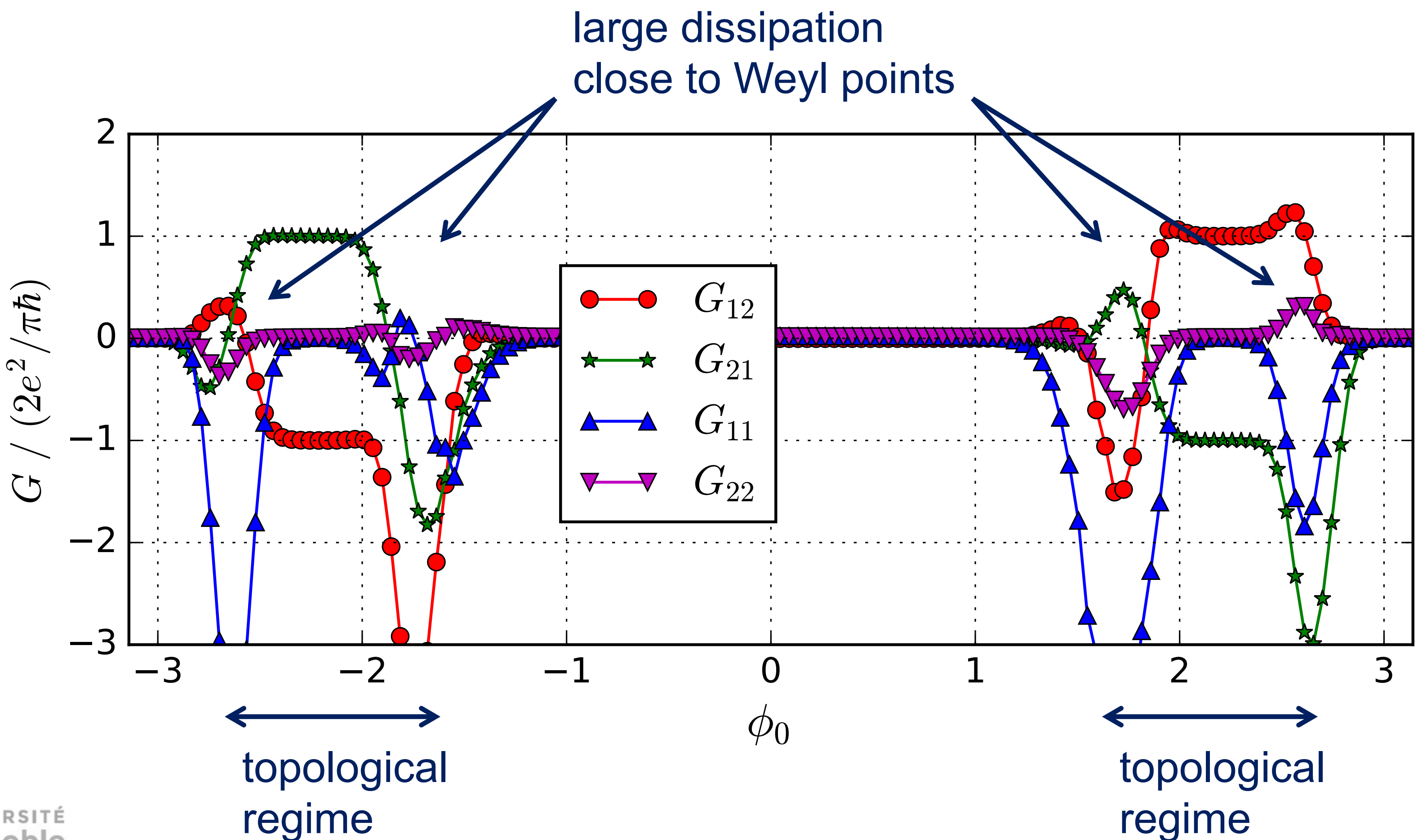
Beyond the adiabatic regime

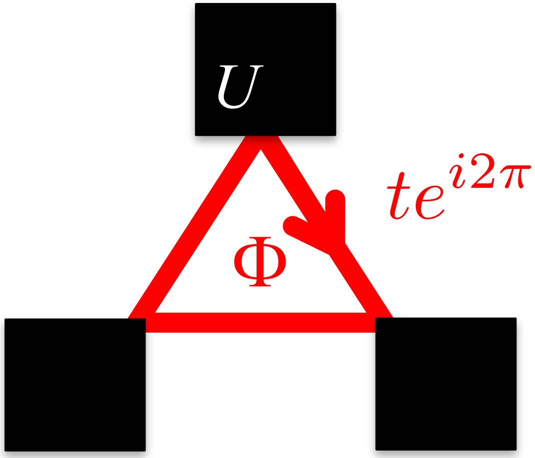
conductances as a fct of ϕ_0 at fixed $V = 0.0003\Delta/e$:



3-terminal junctions

- only 2 independent phases
- add magnetic flux through the junctions area

  $\rightarrow$ break time-reversal symmetry

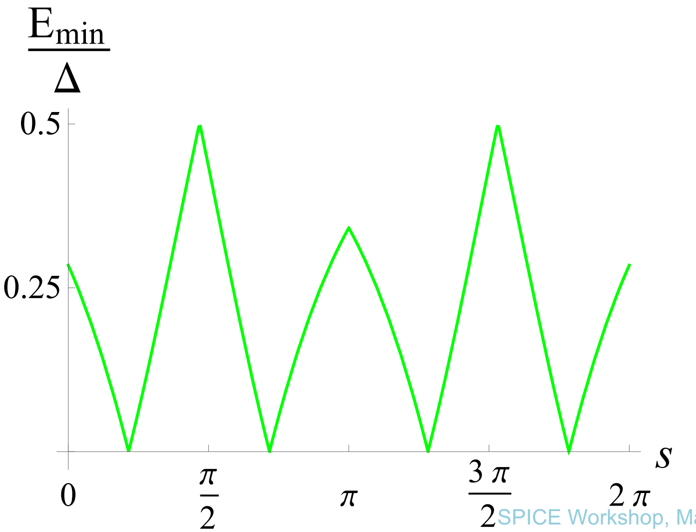


$$te^{i2\pi\Phi/(3\Phi_0)} \equiv e^{is/3}$$

example: $U = 0.1, t = 1$

minimal gap as a function of s

$\rightarrow$ 4 Weyl points



3-terminal junctions

- only 2 independent phases
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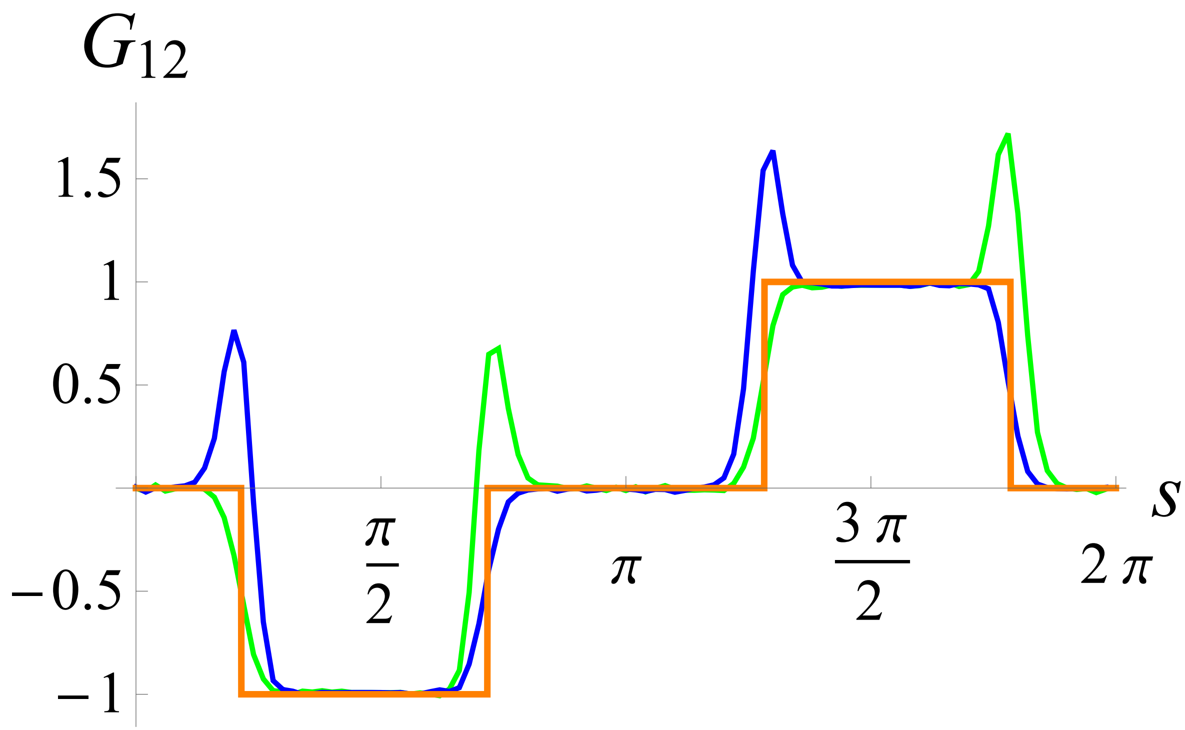
  → break time-reversal symmetry

preliminary results:

$V = 0.01\Delta, \Gamma = 0.01\Delta$

(using
only 1 voltage $V_1 = \S V$
& averaging over ϕ_2)



3-terminal junctions

- only 2 independent phases
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preliminary results: C_{12}^{ABS} = Chern number of the Andreev bound state

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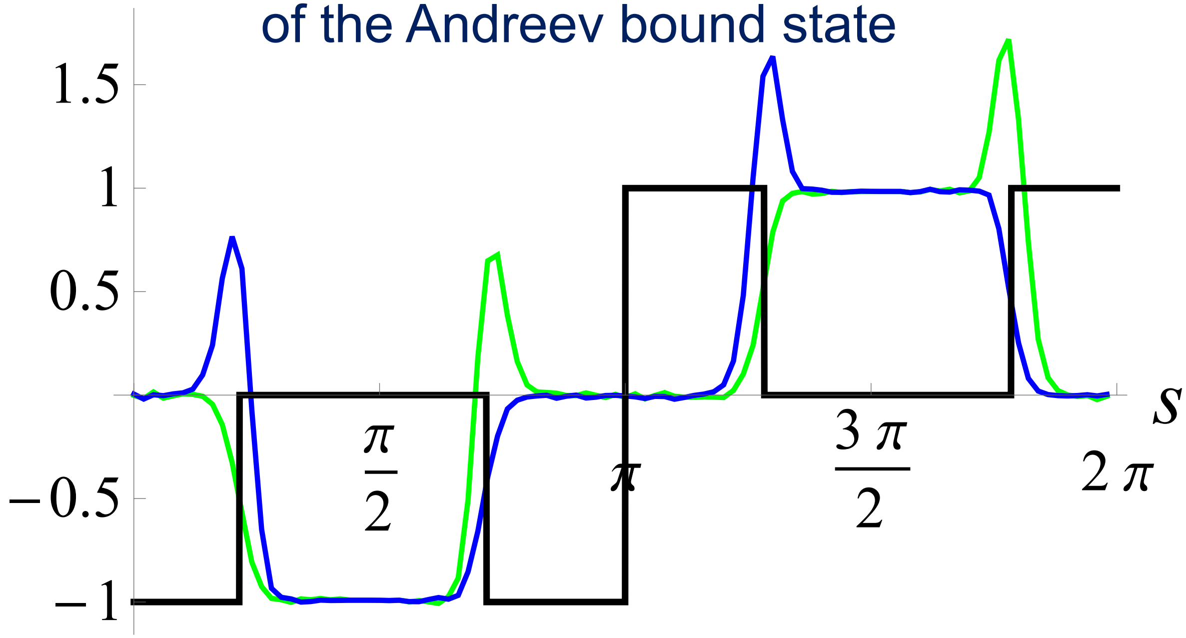


$\rightarrow$ continuum contributes to the topological properties of the junction!

Conclusion

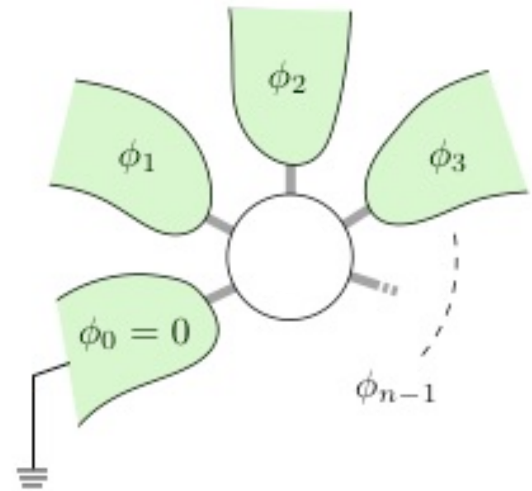
- Weyl singularities in ABS spectrum
  of multi-terminal Josephson junctions
  without any fine-tuning

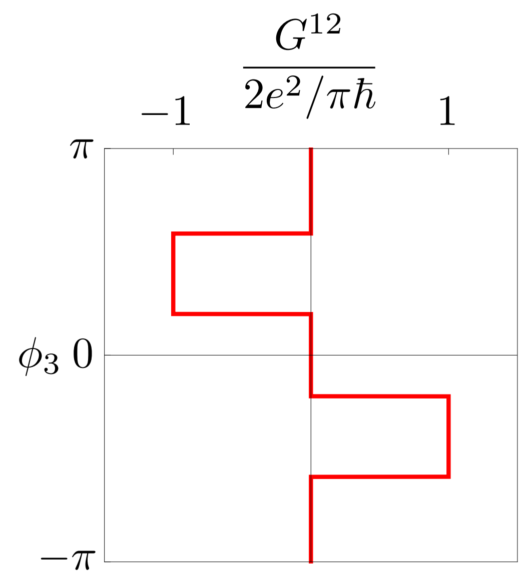
- superconducting phases = quasi-momenta

- transconductance
  between 2 voltage-biased terminals
  probes Chern number

$$\overline{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = -\frac{2e^2}{\pi\hbar} C^{\alpha\beta}$$

multi-terminal Josephson junction
= topological material





R.-P. Riwar *et al*., Nat. Commun. **7**, 11167 (2016);
E. Eriksson *et al*., PRB **95**, 075417 (2017);
JSM & M. Houzet, PRL ... (2017)

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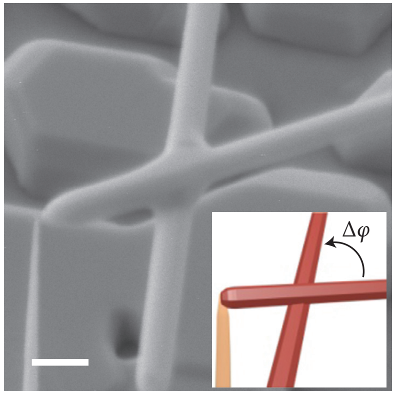
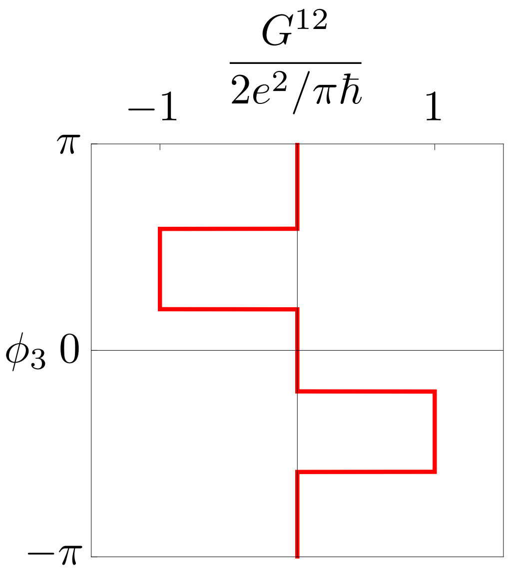


InSb nanocrosses ?
Plissard *et al.* (2013)



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Outlook

- specific realizations ?
- higher-dimensional “materials” ?
- more complex topologies ?
- edges ?

Thank you!

Conclusion

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  of multi-terminal Josephson junctions
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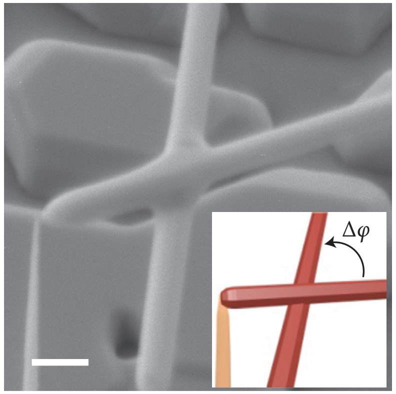

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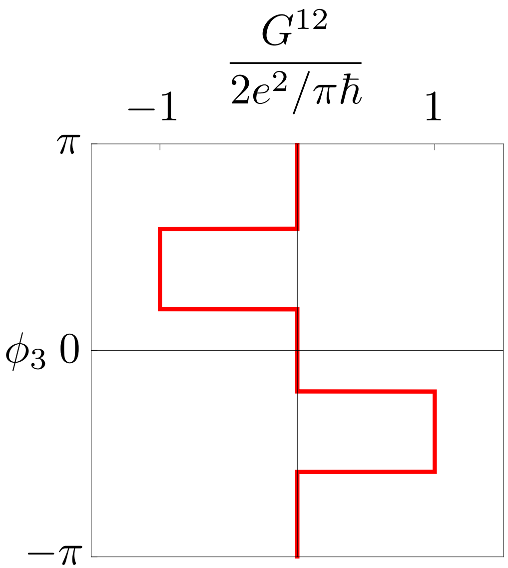

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