Spin torque induced by triplet supercurrent
and
Supercurrent induced noncollinear order

Rina Takashima
Kyoto Univ.

KYOTO UNIVERSITY KYOTO JAPAN FOUNDED 1897

JSPS

Topological Materials Science

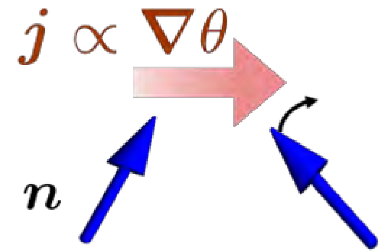
OSS Oxide Superspin Core-to-Core Programme JSPS-EPSRC

in collaboration with
T. Yokoyama (Tokyo Institute of Technology), S. Fujimoto (Osaka University)
Y. Motome, Y. Kato (University of Tokyo), Y. Yanase (Kyoto University)

Outline of this talk

1st part

Spin-torque induced by spin-triplet supercurrent

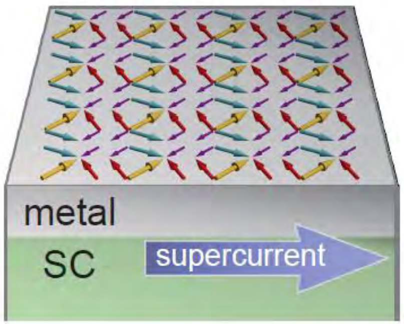


R. Takashima, S. Fujimoto, T. Yokoyama, PRB **96**, 121203 (R) (2017)

2nd part

Noncollinear magnetic order

induced by supercurrent

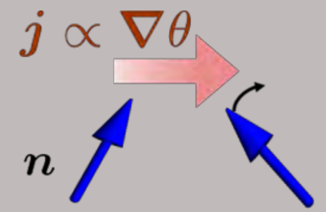


R. Takashima, Y. Kato, Y. Yanase, Y. Motome (to be submitted)

Outline

1st part

Spin-torque induced by spin-triplet supercurrent



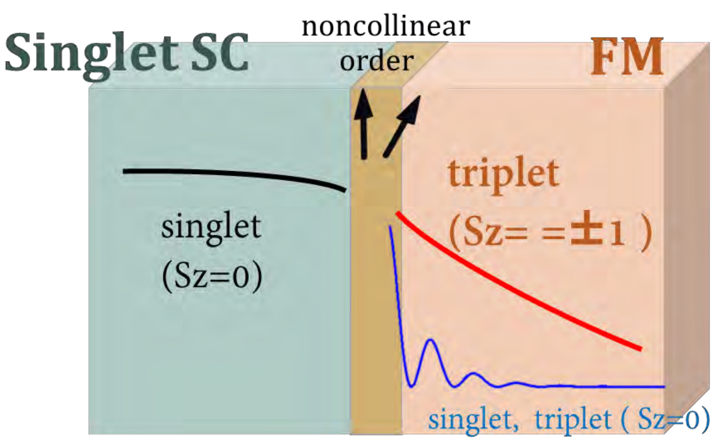
R. Takashima, S. Fujimoto, T. Yokoyama, *arXiv:* 1706.02296 (to appear in PRB(R))

1. Motivation
2. Result : general form of spin torque
3. Application: Domain wall dynamics

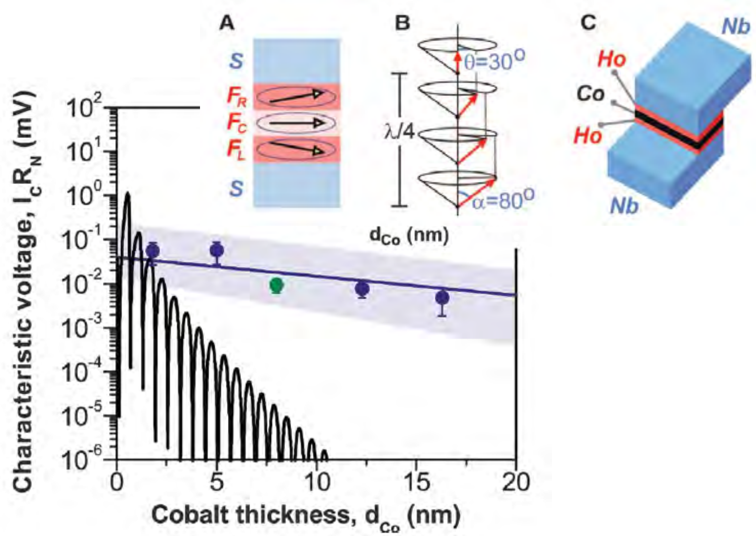
Triplet Cooper pairs

- **Spin-triplet** proximity effect inside ferromagnet

  - **triplet SC** | **FM** with Sr_2RuO_4 Anwar *et al.* Nat. commun. (2016)
  - **singlet SC** | **noncollinear magnet** | **FM** Robinson *et al*, Science (2010) Khaire et al, PRL (2010)



Singlet-Triplet Conversion



Robinson *et al*, Science (2010)

Interplay of **spin-triplet pairing** and **magnetic moment** ?

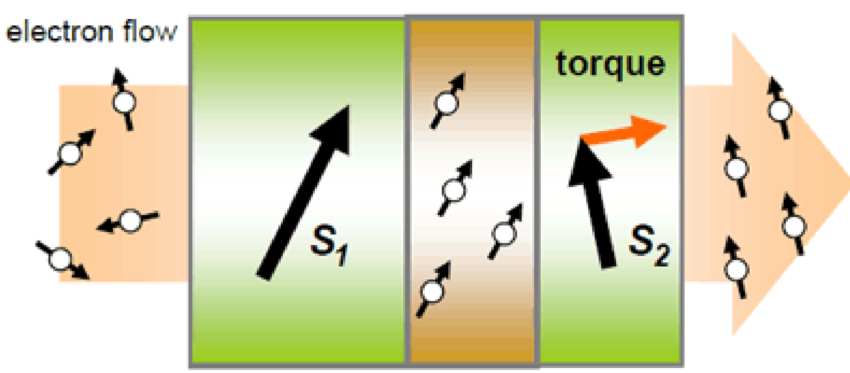
Current-induced torque in normal magnet

- Electric current in magnet exerts **spin-torque** on localized moment

(spin-transfer torque)

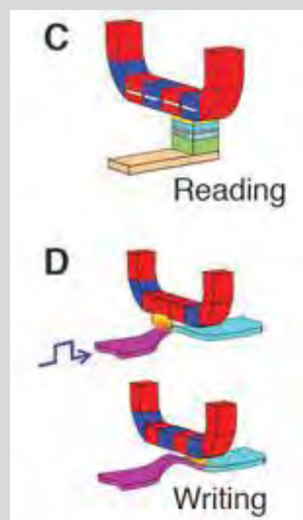
- **Manipulation of spin** $\Rightarrow$ Application in magnetic devices

Spin angular momentum is transferred

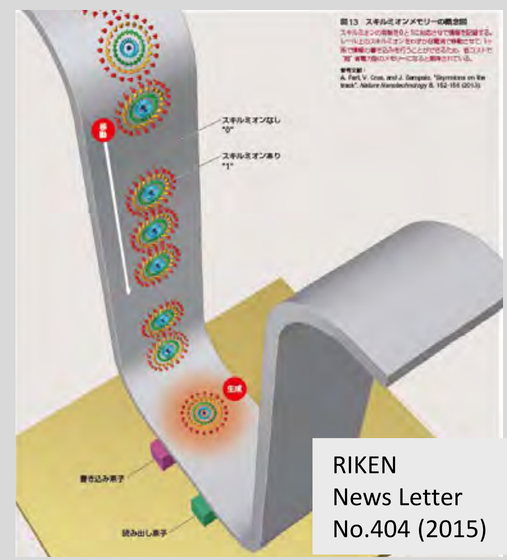


https://docs.quantumwise.com/

Racetrack memory using ***domain wall / Skyrmions***



Parkin et al Science (2008)

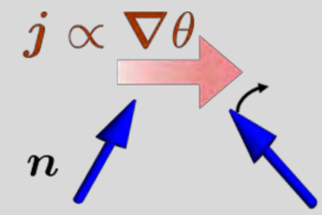


RIKEN
News Letter
No.404 (2015)

Motivation of our work

Question: How **triplet-correlation** changes **spin transfer torque?**

We study
spin-transfer torque induced by **triplet supercurrent**



c.f.) early works for spin-torque in magnetic Josephson junction:
Waintal& Brouwer PRB(2002), Y. Tserkovnyak &A. Brataas PRB (2002), etc

keypoint **:**

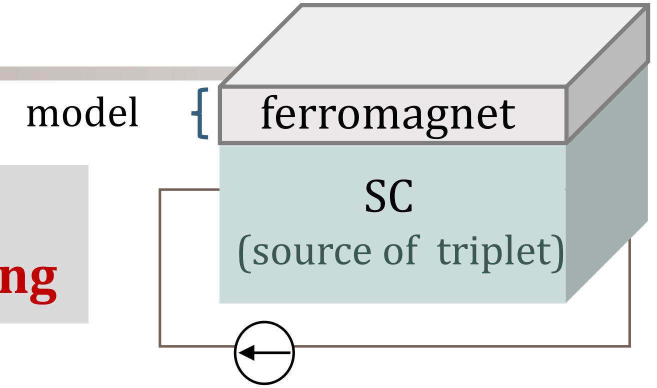
- Supercurrent-induced torque might realize **energetically efficient devices**

- Triplet order parameter (=***d*** vector) might give **new type of torque** ?

$$\chi_{\mu\nu} = \chi_1 \delta_{\mu\nu} - \chi_2 \langle \hat{d}_\mu(\boldsymbol{k}) \hat{d}_\nu(\boldsymbol{k}) \rangle_{FS}$$

(spin susceptibility characterizes spin-transfer process)

Model



metallic magnet (**s-d model**)
with proximity induced **triplet pairing**

$$H = -t \sum_{\langle i,j \rangle} c_{i\alpha}^\dagger c_{j\alpha} - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha} - J_{\rm sd} S \sum_i \boldsymbol{n}(\boldsymbol{r}_i) \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{i\alpha}^\dagger c_{i\beta}$$

$$+ \frac{\Delta_0}{2} \sum_{\langle i,j \rangle} e^{i\boldsymbol{\kappa}\cdot(\boldsymbol{r}_i + \boldsymbol{r}_j)} c_{i\alpha}^\dagger \left[(\boldsymbol{d}_{ij} \cdot \boldsymbol{\sigma}) i\sigma_y \right] c_{j\beta}^\dagger + H.c,$$

(square lattice)

$c_{i\alpha}$: conduction electron (site i, spin α)

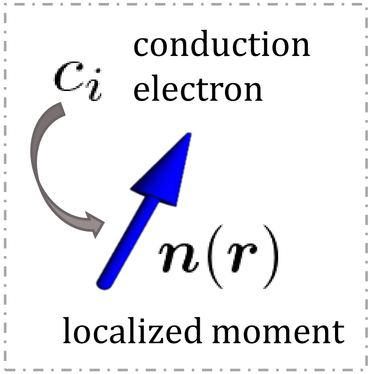
$\boldsymbol{n}_i$: localized moment (site i)

$\boldsymbol{d}_{ij}$: triplet order parameter (site i, j)

supercurrent flow is given by the spatial gradient of SC phase $e^{i\boldsymbol{\kappa}\cdot(\boldsymbol{r}_i + \boldsymbol{r}_j)}$

$$\boldsymbol{j} = -2ten_{\rm s}a^2\boldsymbol{\kappa} \quad (\kappa a \ll 1)$$

Calculation of spin torque



- local spin torque : $\tau_{\rm STT} = 2J_{\rm sd}\boldsymbol{n} \times \delta \boldsymbol{s}_i$

$\delta \boldsymbol{s}_i$ = local spin density of electrons under **supercurrent**

➡ we calculate **spin density** within **linear response**

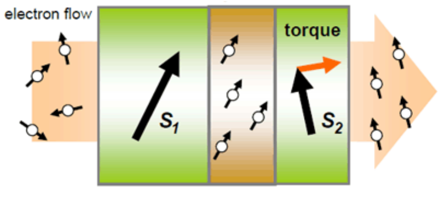
- We assume
  - Localized moment varies smoothly
  - Exchange splitting is large $J_{\rm sd}S \gg \Delta_0$

  ➡ we only take equal spin pairing ((anti)parallel to ***n***)

Result: supercurrent-induced torque

- Obtained torque $\boldsymbol{\tau}_{\mathrm{STT}} = \sum_{\nu=x,y} \frac{-\tilde{P}_\nu a^3}{2eS} j_\nu \left(-\partial_\nu \boldsymbol{n} + \tilde{\beta}_\nu \boldsymbol{n} \times \partial_\nu \boldsymbol{n} \right).$

j_ν : supercurrent density



https://docs.quantumwise.com/

$\frac{\partial \boldsymbol{n}}{\partial t} \sim \boldsymbol{\tau}_{\mathrm{STT}}$

$\boldsymbol{\tau}_{\mathrm{STT}} \propto -\partial_\nu \boldsymbol{n}$: direct transfer of spin from neighboring sites (~"adiabatic torque")

$\boldsymbol{\tau}_{\mathrm{STT}} \propto \boldsymbol{n} \times \partial_\nu \boldsymbol{n}$: **deviation** from direct transfer (~"β term")

$\tilde{P}_\nu$ ~ spin polarization of electrons

$\tilde{\beta}_\nu$ -originate in **order parameter .** $\tilde{\beta}_\nu \propto |\Delta_0|^2$

- depend on the direction of ***n*** ***(spatial dependence)***

---

explicit form:

$$\tilde{P}_\nu = \frac{J_{\mathrm{sd}} S}{n_e a^3} \left[\frac{1}{2} (\pi_\nu^{xx} + \pi_\nu^{yy}) + \frac{1}{|\partial_\nu \boldsymbol{n}|^2} \left(-\pi_\nu^{(1)} \left((\partial_\nu \theta)^2 - \sin^2 \theta (\partial_\nu \phi)^2 \right) + 2\pi_\nu^{(2)} \sin\theta \partial_\nu \theta \partial_\nu \phi \right) \right],$$

$$\tilde{\beta}_\nu = -\frac{J_{\mathrm{sd}} S}{n_e a^3} \frac{1}{\tilde{P}_\nu} \frac{1}{|\partial_\nu \boldsymbol{n}|^2} \left(\pi_\nu^{(2)} \left((\partial_\nu \theta)^2 + \sin^2 \theta (\partial_\nu \phi)^2 \right) + 2\pi_\nu^{(1)} \sin\theta \partial_\nu \theta \partial_\nu \phi \right),$$

$\pi_\nu^{xx}, \pi_\nu^{yy}, \pi_\nu^{(i)}$: spin-spin correlation

What causes β term?

c.f.) **Normal system**

Zhang& Li (2004) , Tatara et al. (2008), Tserkovnyak et al(2008)

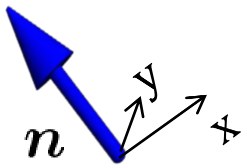
$$\boldsymbol{\tau}_{\rm nor} = \sum_{\nu=x,y} \frac{-Pa^3}{2eS} j_\nu^{\rm nor} \left(-\partial_\nu \boldsymbol{n} + \beta \boldsymbol{n} \times \partial_\nu \boldsymbol{n}\right).$$

- β is qualitatively important
- magnetic impurity scattering / mistracking $\rightarrow$ ***β* term**

With **triplet-SC correlation**

anisotropy in **spin susceptibility** $\rightarrow$ deviation from direct transfer

$$\tilde{\beta}_\mu \neq 0 \quad \Rightarrow \quad \pi^{xx} - \pi^{yy} \neq 0, \quad \pi^{xy} \neq 0$$



π^{ab}: spin-spin correlation

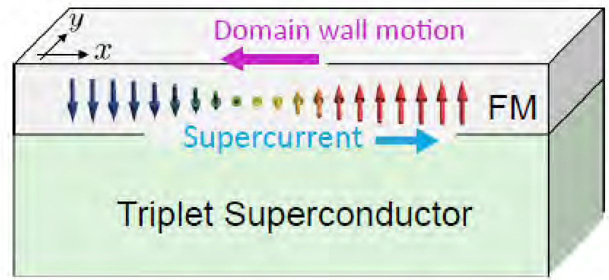
***β* term** can be controlled by **triplet order parameters (d-**vector).

(⇔in normal metals, it depends on **extrinsic scattering**)

Application: Domain wall manipulation

- Domain wall texture in ferromagnetic metal
- Assume the ***d***-vector

$$\boldsymbol{d}(\boldsymbol{k}) = (-\sin k_y, \sin k_x, \delta \sin k_x)$$



Possible origin:

spin-orbit coupling due to **structure inversion asymmetry** $\boldsymbol{g}_{\mathrm{so}}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$

➡ $\boldsymbol{d}(\boldsymbol{k}) \parallel \boldsymbol{g}_{so}(\boldsymbol{k})$ is favored

- Apply a current (=phase gradient) ➡ Domain wall moves

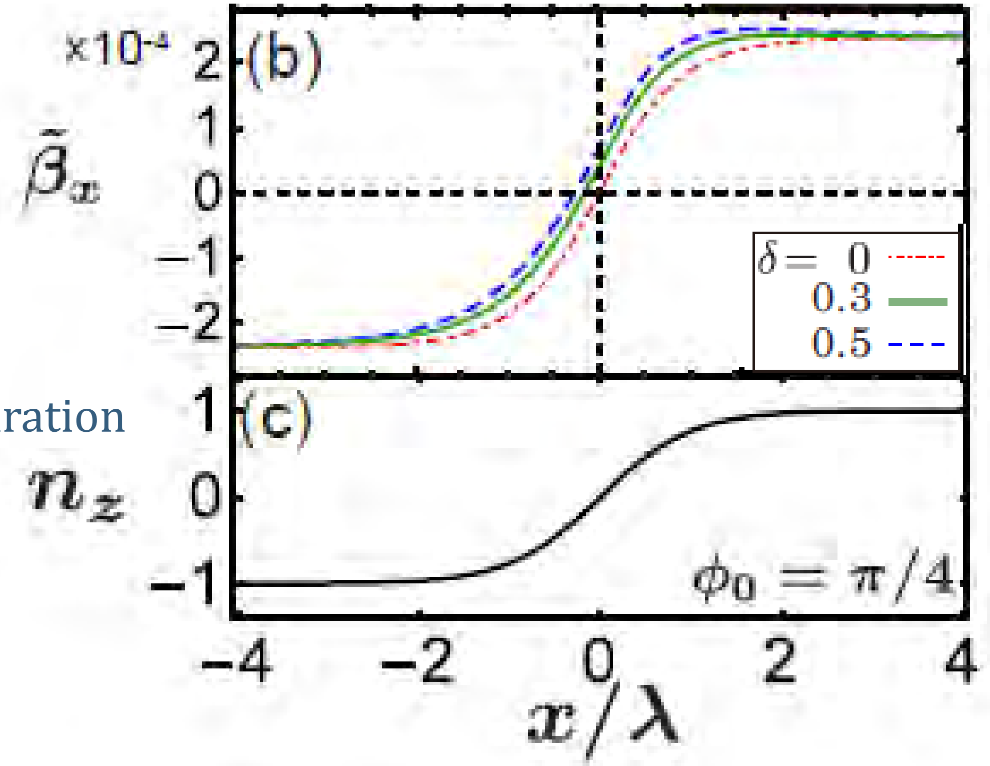
- EOM of collective coordinates (*X*: domain wall center)

$$\partial_t X = \frac{v_c}{(1+\alpha^2)} \left(\tau(\phi_0) j_x + \alpha F(\phi_0) j_x + \sin 2\phi_0\right),$$

$$\partial_t \phi_0 = \frac{-1}{(1+\alpha^2) t_0} \left(\alpha \tau(\phi_0) j_x - F(\phi_0) j_x + \alpha \sin 2\phi_0\right),$$

(detail) Spatial dependence of β

domain wall configuration

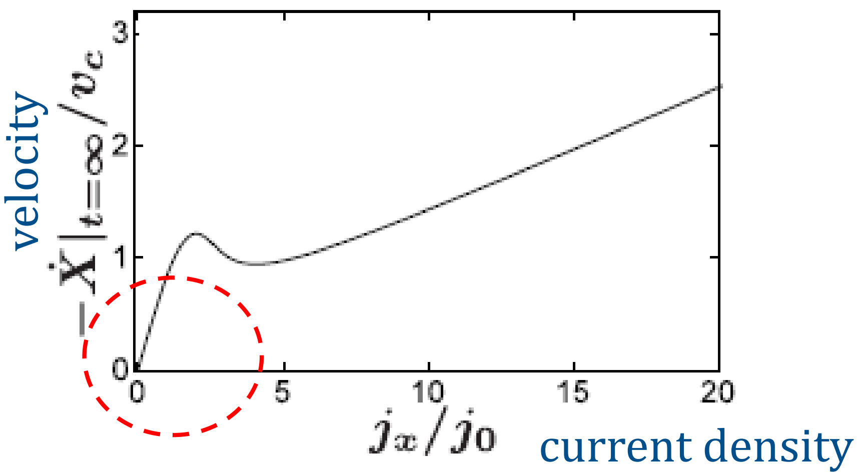


$\tilde{\beta}_\nu$ **has strong spatial dependence**

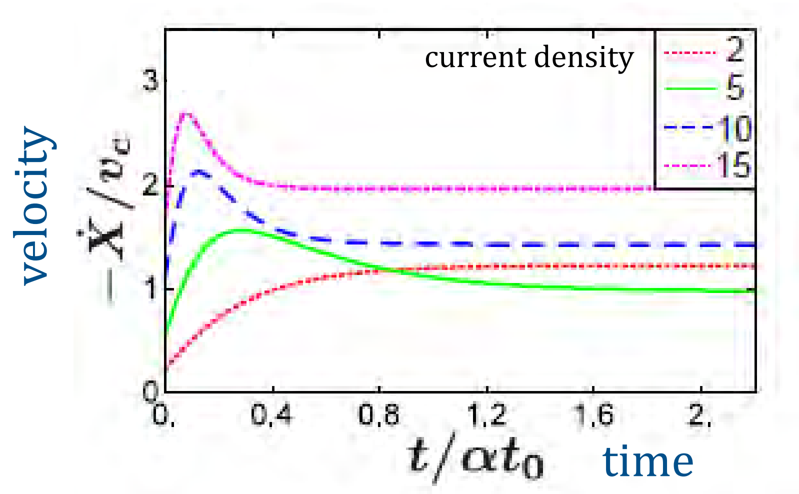
Domain wall dynamics

Under a constant supercurrent ,

Current dependence of velocity at t =∞



Time dependence of velocity



✓No threshold current density

*without extrinsic pinning

⇔ w/o β terms, threshold current exists

∗ β terms arises from d-vector

$$\tilde{\beta}_\nu \propto |\Delta_0|^2$$

✓No oscillatory motion

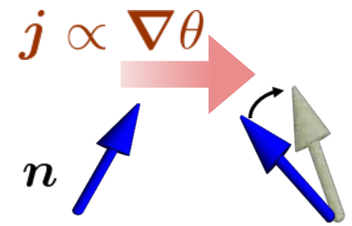
⇔Normal metal, oscillation occurs

∗ β depends on space

Summary of 1^{st} part

RT, Fujimoto, Yokoyama, PRB **96**, 121203 (R)

Spin-transfer torque by triplet supercurrent



✓ We obtain the spin-torque given by

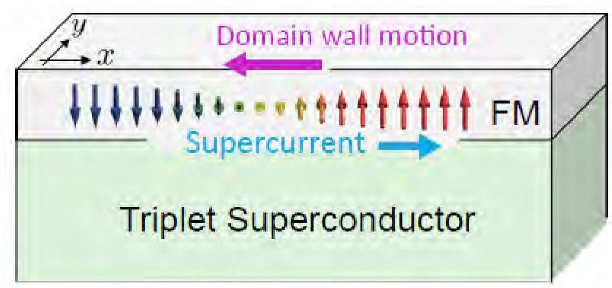
$$\boldsymbol{\tau}_{\mathrm{STT}} = \sum_{\nu=x,y} \frac{-\tilde{P}_\nu a^3}{2eS} j_\nu \left(-\partial_\nu \boldsymbol{n} + \tilde{\beta}_\nu \boldsymbol{n} \times \partial_\nu \boldsymbol{n} \right).$$

✓ a new type of $\tilde{\beta}$ term : **Interplay** of ***d***-vector and magnetic moment ***n***

triplet correlation changes spin susceptibility of electrons (~spin transfer process)

✓ domain wall manipulation

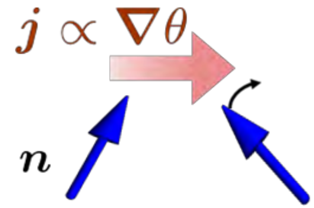
- threshold current density is lowerd
- No oscillatory motion



Outline

1st part

Spin-torque induced by spin-triplet supercurrent

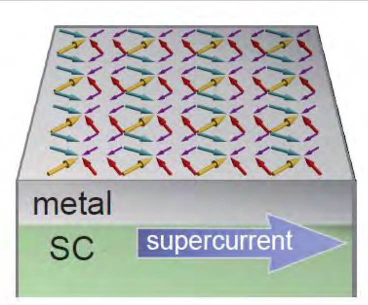


R. Takashima, S. Fujimoto, T. Yokoyama, *arXiv:* 1706.02296 (to appear in PRB(R))

2nd part

Noncollinear magnetic order

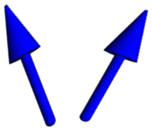
controlled by spin-singlet supercurrent



R. Takashima, Y. Kato, Y. Yanase, Y. Motome (to be submitted)

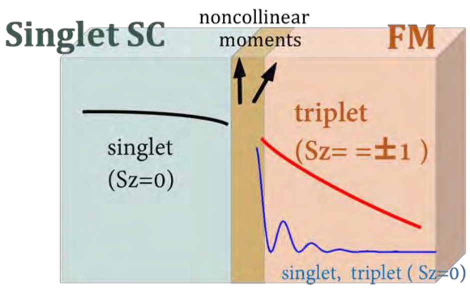
Noncollinear magnetism and SC proximity effect

Noncollinear magnetic order : Spins are not in parallel/antiparallel

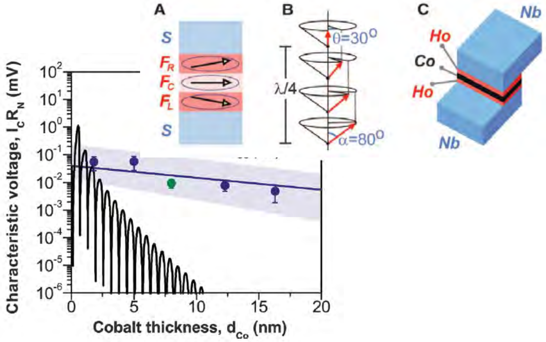


Noncollinear magnetic order is important in physics of **SC proximity effects**

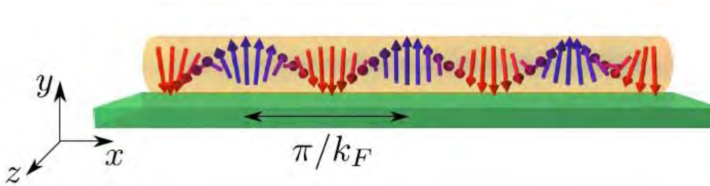
- Singlet-triplet pairing conversion — Keizer et al, Nat. Lett. (2006); Robinson *et al*, Science (2010)
- Topological superconductor **w/o spin-orbit coupling** — Klinovaja et al. (2013)



Singlet-Triplet Conversion



Robinson *et al*, Science (2010)



Klinovaja et al. (2013)

Motivation of our work

Question: Can we **switch/control** noncollinear magnetic order in the presence of SC proximity effect?

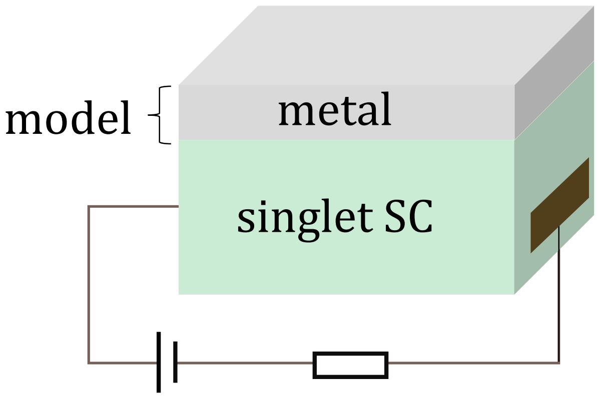
➡ can be used

- to switch /optimize **the singlet-triplet conversion**
- to externally **control topological SC** and Majorana zero modes etc

In our work:

We propose a new way to induce **noncollinear magnetic order** by a **supercurrent**

Model



2d Correlated metal attached to **s-wave SC** with a supercurrent

repulsive Hubbard interaction

$$H = \sum_{\boldsymbol{k}\sigma} \xi_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma} - \frac{2U}{3} \sum_i \boldsymbol{m}_i \cdot (c^\dagger_{i\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} c_{i\sigma'}) + \sum_i (\Delta e^{2i\boldsymbol{\kappa}\cdot\boldsymbol{r}_i} c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} + \mathrm{h.c.}) + \frac{2U}{3} \sum_i |\boldsymbol{m}_i|^2$$

- mean field of spin density

$$\boldsymbol{m}_i = \frac{1}{2} \langle c^\dagger_{i\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} c_{i\sigma'} \rangle$$

- singlet supercurrent $\boldsymbol{j} \propto \boldsymbol{\kappa}$

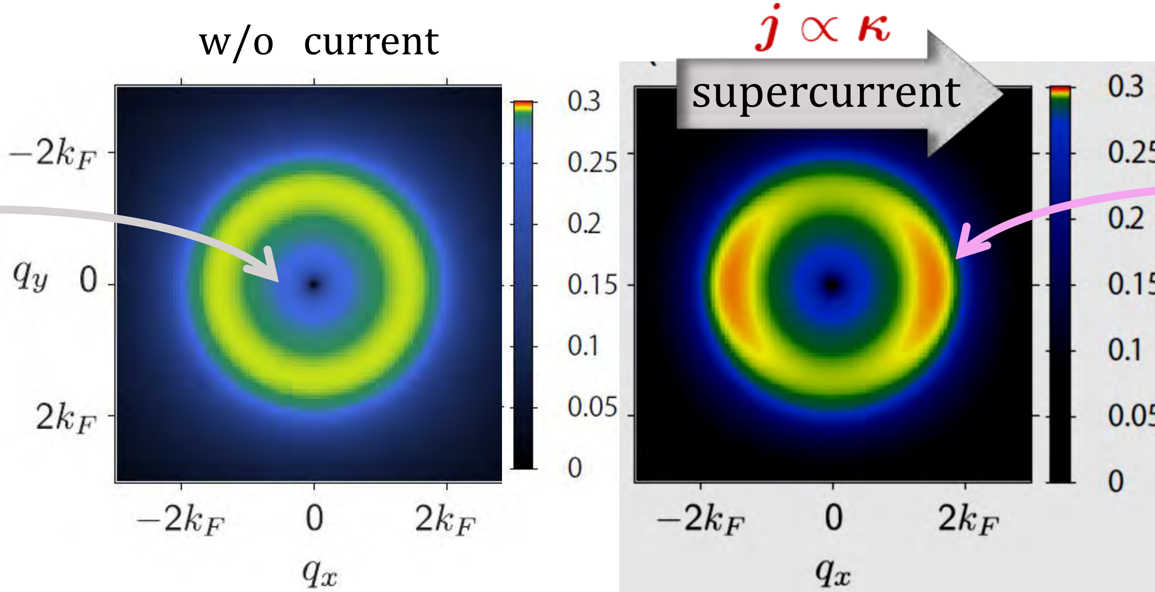
(spatial gradient of SC phase)

Magnetic instability

- bare spin susceptibility $\chi(\boldsymbol{q})$ in the continuum model : $\xi_{\boldsymbol{k}} = \frac{k^2}{2m} - \epsilon_F$

suppression by singlet gap $q \sim 0$

Anderson&Suhl (1959)



increase

$\kappa/k_F = 0.022$

$\Delta/\epsilon_F = 0.05$

$$\chi(\boldsymbol{q}) - \chi_{\boldsymbol{\kappa}=\boldsymbol{0}}(q) = \frac{a^2|\boldsymbol{\kappa}|^2}{\epsilon_F} f\left(\frac{q}{k_F}, \frac{|\Delta|}{\epsilon_F}\right) + \frac{a^2(\boldsymbol{\kappa}\cdot\hat{\boldsymbol{q}})^2}{\epsilon_F} g\left(\frac{q}{k_F}, \frac{|\Delta|}{\epsilon_F}\right) + O\left((\kappa/k_F)^4\right)$$

much smaller than g

>0 and peak at $q/k_F \sim 2$

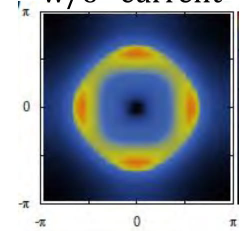
$$g(x,y) = \frac{x^2}{\pi^2}\int_0^\infty \int_0^{2\pi} \tilde{k}d\tilde{k}d\theta \frac{\sqrt{(\tilde{\xi}_1^2+y^2)(\tilde{\xi}_2^2+y^2)} - \tilde{\xi}_1\tilde{\xi}_2 - y^2}{\sqrt{(\tilde{\xi}_1^2+y^2)(\tilde{\xi}_2^2+y^2)}\left(\sqrt{(\tilde{\xi}_1^2+y^2)} + \sqrt{(\tilde{\xi}_2^2+y^2)}\right)^3},$$

Supercurrent leads to **magnetic instability**

Magnetic order in lattice system

- square lattice model : $\xi_{\boldsymbol{k}} = -2t(\cos(k_x a) + \cos(k_y a)) - \mu$

Instability : $\boldsymbol{m}_{\boldsymbol{q}=(\pm Q,0)}$, $\boldsymbol{m}_{\boldsymbol{q}=(0,\pm Q)}$ $(Q \sim 2\pi/3a)$ $\mu/t = -2.96$

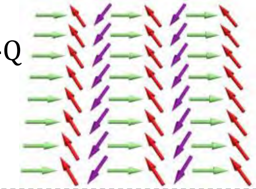


- Variational ansatz

$(M_0, Q$: variational parameter$)$

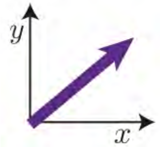
$$\boldsymbol{m}_i^{\text{single}} = M_0 \begin{pmatrix} \cos(Qx_i) \\ \sin(Qx_i) \\ 0 \end{pmatrix} \qquad \boldsymbol{m}_i^{\text{double}} = M_0 \begin{pmatrix} \cos(Qx_i) \\ \cos(Qy_i) \\ 0 \end{pmatrix},$$

single-Q



double-Q

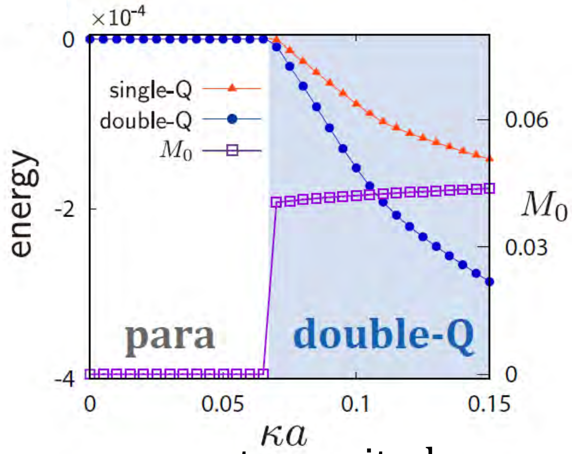
$\boldsymbol{j} \propto \kappa(1,1)$



T=0K

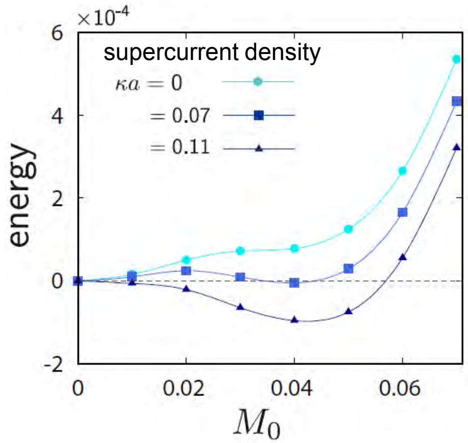
fixed U

double-Q order is stabilized



current magnitude

1st order transition behavior

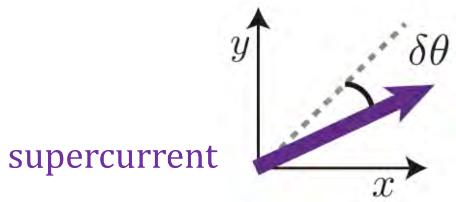


for double-Q

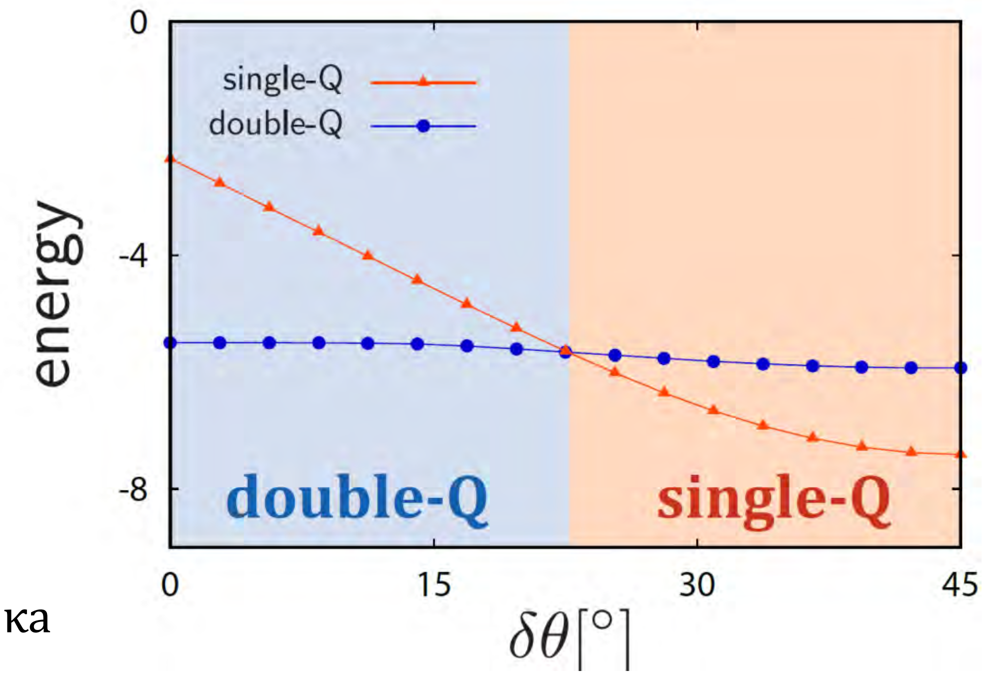
Supercurrent induces first-order transition to double-Q state

Switch to single-Q magnetic order

We can **switch** magnetic state by the **direction of supercurrent**

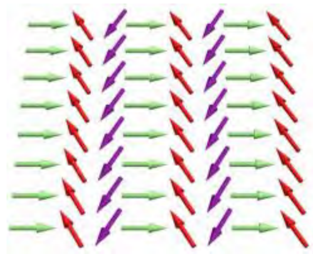


$$\boldsymbol{j} \propto \kappa(\cos(45^\circ - \delta\theta), \sin(45^\circ - \delta\theta), 0)$$



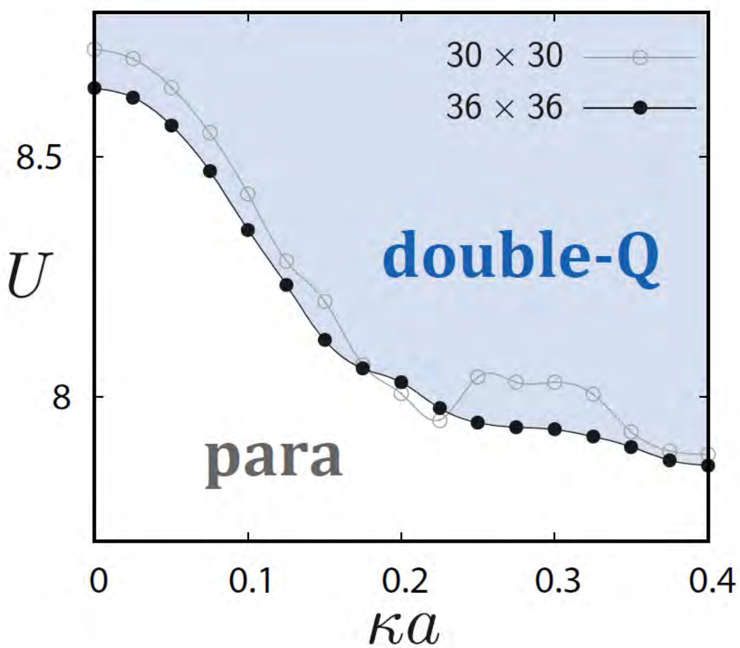
fixed U, κa

current angle



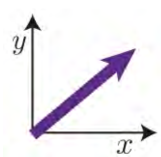
Phase diagram (T=0K)

Critical U decreases as current increases

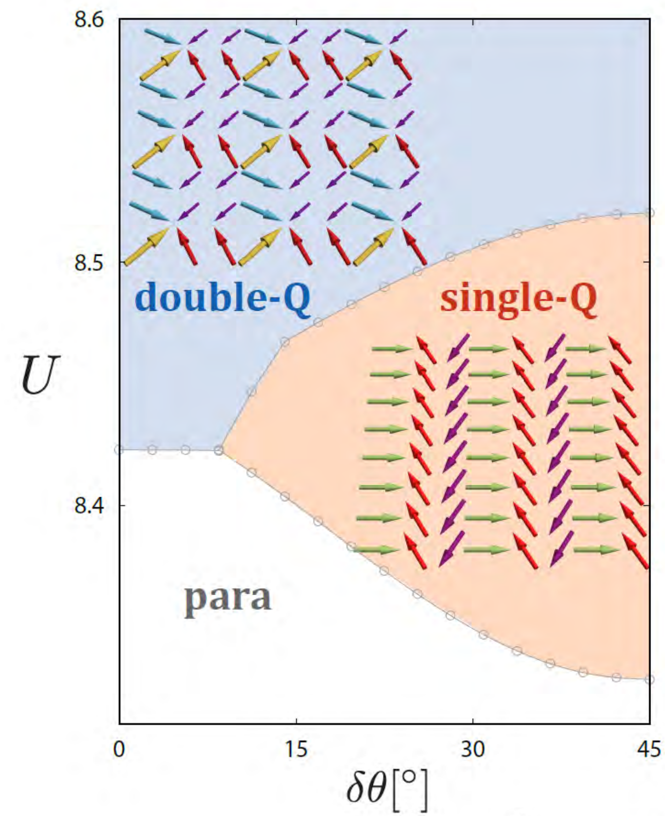


magnitude of current

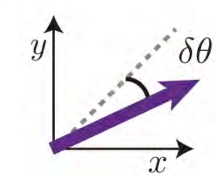
$\boldsymbol{j} \propto \kappa(1,1)$



“switch” of magnetic states



current angle

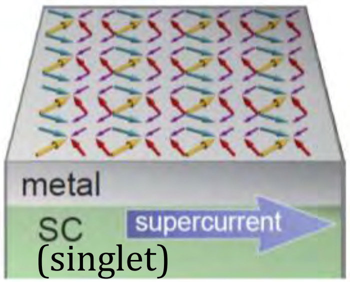


Summary of 2^{nd} part

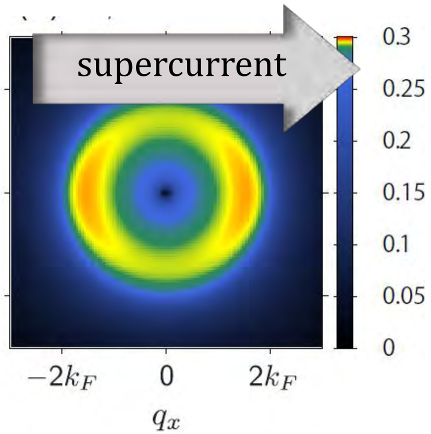
We propose a new way to control **noncollinear order** by supercurrent

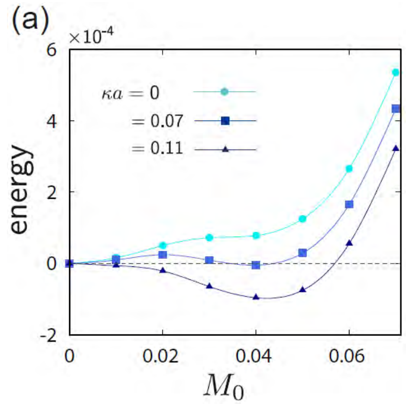
✓Supercurrent induces
1st order phase transition to double-Q state

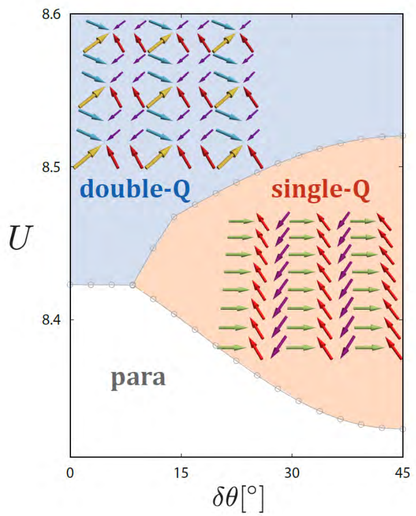
✓Switch magnetic states by **current direction**



$j \propto \kappa$





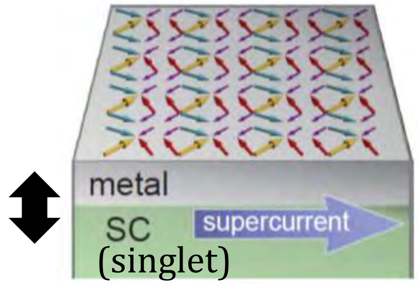


Remark

1) First-order transition→ metastable state of magnetic order w/o supercurrent
2) Different lattices/pairing ➡ a wide range of magnetic states, e.g. skyrmion
3)Rashba Spin-orbit coupling

Rashba spin orbit coupling

- Rashba SOC at the interface



$$H_{so} = \alpha \sum_{\boldsymbol{k}} \boldsymbol{g}(\boldsymbol{k}) \cdot (c^{\dagger}_{\boldsymbol{k}\sigma_1} \boldsymbol{\sigma} c_{\boldsymbol{k}\sigma_2}),$$

- Energy functional

$\boldsymbol{j} \propto \boldsymbol{\kappa}$

$$E[\{\boldsymbol{m}\}] = \frac{2UN}{3} \sum_{\boldsymbol{q}} \left(1 - \frac{2U}{3} \chi^{\mu\nu}(\boldsymbol{q})\right) m^{\mu}_{-\boldsymbol{q}} m^{\nu}_{\boldsymbol{q}} + F \sum_{i} (\hat{\boldsymbol{z}} \times \boldsymbol{\kappa}) \cdot \boldsymbol{m_i},$$

① spin-spiral plane is locked

②Inverse-Edelstein effect

➡ **in-plane magnetic field**

Realized magnetic states would be modulated

cf) w/o SOC

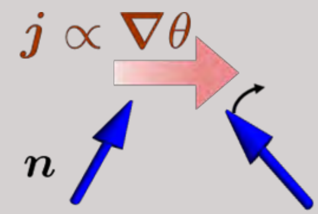
$$E[\{\boldsymbol{m}\}] = \frac{2UN}{3} \sum_{\boldsymbol{q}} \left(1 - \frac{2U}{3} \chi(\boldsymbol{q})\right) |\boldsymbol{m_q}|^2,$$

Conclusion

1st part

Background experiments on triplet-proximity effect in magnet

Model **metallic magnet + triplet pairing potential**



Spin-triplet supercurrent give a new type of **spin-transfer-torque**

RT, Fujimoto, Yokoyama, PRB **96**, 121203 (R)

2nd part

Background Rich physics arise from interplay of noncollinear order and SC

Model **2d correlated metal + singlet pairing potential**

Supercurrent induce double-Q/single-Q magnetic order

R. Takashima, Y. Kato, Y. Yanase, Y. Motome (*to be submitted*)