Charge-Spin Coupling in Superconducting Structures: Non-dissipative Magnetoelectric Effects

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Outline

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Direct and inverse spin Hall effect

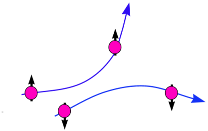
Coupling of charge and spin currents mediated by SOC

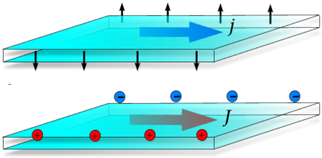
Direct SHE: “primary” charge current j_k generates spin current J_k^a

Inverse SHE: “primary” spin current J_k^a generates charge current j_k

$$J_i^a = \theta_{ik}^a j_k , \qquad j_i = \theta_{ik}^a J_k^a$$

$$\theta_{ik}^a = -\theta_{ki}^a$$





For cubic materials $\theta_{ik}^a = \theta \varepsilon_{ika}$

θ – spin Hall angle

Bulk magnetoelectric effects

Coupling between bulk spin polarization and electric current

breaking of inversion symmetry (gyrotropy) is required

Edelstein effect: Spin induced by electric field

$$S^a(\omega) = \sigma_k^a(\omega) E_k(\omega)$$

σ_k^a – Edelstein conductivity

$E_k = \mathbf{i}\omega A_k \quad \Rightarrow \quad \sigma_k^a(\omega)$ is related to the spin-current Kubo correlator $\chi_k^a(\omega)$

$$\sigma_k^a(\omega) = \frac{1}{\mathbf{i}\omega}\langle\langle \hat{S}^a; \hat{j}_k \rangle\rangle_\omega = \frac{1}{\mathbf{i}\omega}\chi_k^a(\omega)$$

Gauge invariance implies $\chi_k^a(\omega) \sim \omega$ at $\omega \to 0$ for finite $\sigma_k^a(0)$

Inverse Edelstein (spin-galvanic) effect

$$j_k = \chi_k^a(\omega)\mu_B B^a = \sigma_k^a \left[\mu_B \dot{B}^a\right]$$

Induced charge current $\propto$ rate of spin generation

Spin diffusion and spin torque in the presence of SOC

Intrinsic SOC: $\hat{H}_{\rm so} = \frac{1}{2}\Omega^a(\mathbf{p})\sigma^a, \quad \Omega^a(-\mathbf{p}) = -\Omega^a(\mathbf{p})$

Spins of *moving* electrons precess with $\mathbf{p}$-dependent angular velocity.
Precession of moving elementary spins lead to an average torque $\mathcal{T}^a$

Spin is not conserved in the presence of SOC

$$\partial_t S^a - D\nabla^2 S^a = \mathcal{T}^a$$

Gradient expansion of the spin torque:

$$\mathcal{T}^a = -\Gamma^{ab}S^b + P_k^{ab}\partial_k S^b + C_k^a \partial_k n$$

Γ^{ab} – DP spin relaxation tensor (random motion of elementary spins)

$P_k^{ab} = -P_k^{ba}$ – spin precession tensor (diffusive motion of spins)

C_k^a – “spin Hall torque” (motion of spins generated by the charge flow)

Coupled spin-charge diffusion in normal conductors

Stationary spin-charge diffusion equations

$$D\nabla^2 S^a - \Gamma^{ab} S^b + P_k^{ab}\partial_k S^b + C_k^a \partial_k n = 0$$

$$D\nabla^2 n + C_k^a \partial_k S^a = 0$$

I. Spin-charge coupling C_k^a leads to the Edelstein effect:

$$j_k = -D\partial_k n \;\Rightarrow\; S^a = \left(\hat{\Gamma}^{-1}\right)^{ab} C_k^a \partial_k n \;\Rightarrow\; \sigma_k^a = \frac{\partial n}{\partial \mu}\left(\hat{\Gamma}^{-1}\right)^{ab} C_k^a$$

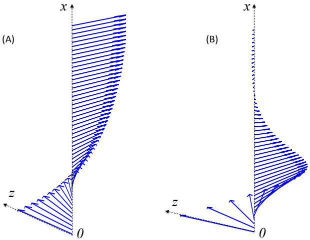
II. $P_k^{ab} \neq 0$ and/or anisotropy of Γ^{ab} generate “spin helix”:

$$D\partial_x^2 S^a + P_x^{ab}\partial_x S^b - \Gamma^{ab} S^b = 0$$

$$\mathbf{S}(x=0) = \hat{\mathbf{z}} S_0^z$$

(A) $\hat{H}_{\rm so} = p_z(\alpha\sigma^x + \beta\sigma^y)$

(B) $\hat{H}_{\rm so} = \alpha \mathbf{p}\cdot\boldsymbol{\sigma}$



What we can expect in superconducting systems

I. Non-dissipative Edelstein/spin-galvanic effect

The supercurrent $\boldsymbol{j} = n_s \boldsymbol{v}_s$ with the superfluid velocity $\boldsymbol{v}_s \sim \nabla\varphi - e\boldsymbol{A}$

$\Rightarrow$ gauge invariance does not forbid a static $\chi_k^a = \langle\langle \hat{S}^a ; \hat{j}_k \rangle\rangle_{\omega=0}$

$$S^a = \chi_k^a \partial_k \varphi, \quad j_k = e\chi_k^a h^a$$

$$F_L = h^a \chi_k^a (\partial_k \varphi - eA_k) = \mathbf{T} \cdot \mathbf{v}_s \text{ – Lifshitz invariant}$$

II. Charge-Spin conversion $\mapsto$ Singlet-Triplet conversion

- SHE $\mapsto$ accumulation of triplet condensate at the sample edges
- “Spin Helix” $\mapsto$ rotation of the triplet and LRTC in S/F structures

Outline

Green functions in superconductors

Superconductivity: Pair correlations between t-conjugated states

$$\underbrace{iG_{\alpha\beta}(1,2) = \langle T\{\psi_\alpha(1)\psi^\dagger_\beta(2)\}\rangle,}_{\text{normal Green function}} \quad \underbrace{iF_{\alpha\beta}(1,2) = \langle T\{\psi^\dagger_\alpha(1)\psi^\dagger_\beta(2)\}\rangle}_{\text{anomalous Green function}}$$

Nambu representation: Matrix in the subspace of t-conjugates states

$$\check{\mathbb{G}} = \begin{bmatrix} \hat{G} & \hat{F} \\ -\hat{\bar{F}} & \hat{\bar{G}} \end{bmatrix}, \quad \hat{\bar{F}} = \sigma^y \hat{F}^* \sigma^y$$ – time-reversal operation

Quasiclassical GF: Dynamics of quasiparticles at the Fermi surface

$$\check{\mathbb{G}}(\mathbf{r}_1,\mathbf{r}_2) \approx -i\pi N_F \langle e^{ip_F \mathbf{n}(\mathbf{r}_1-\mathbf{r}_2)} \check{\mathfrak{g}}(\mathbf{n},\mathbf{r})\rangle_{\mathbf{n}} \qquad \check{\mathfrak{g}} = \begin{bmatrix} \hat{g} & \hat{f} \\ \hat{\bar{f}} & -\hat{\bar{g}} \end{bmatrix}$$

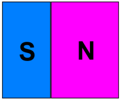
In s-wave BCS superconductor (equilibrium Matsubara formalism):

$$\check{\mathfrak{g}}_{BCS} = \tau_3 g_{BCS} + \tau_2 f_{BCS}; \; g_{BCS} = \frac{\omega}{\sqrt{\omega^2 + |\Delta|^2}} \quad f_{BCS} = \frac{|\Delta| e^{i\varphi}}{\sqrt{\omega^2 + |\Delta|^2}}$$

Diffusion of the condensate: Usadel equation

Anomalous GF $\hat{f}(\omega, \mathbf{r})$ describes the spectral condensate density

When "injected" from SC the condensate diffuses into a normal metal

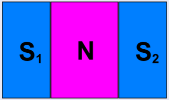


$$\left.\begin{array}{r} D\nabla^2 \hat{f} - 2|\omega|\hat{f} = 0 \\ -\partial_x \hat{f}|_S = i\gamma f_{BSC} \end{array}\right\} \Rightarrow \hat{f} = i\frac{\gamma}{\kappa_\omega} f_{BSC} e^{-\kappa_\omega x}$$

$\kappa_\omega^2 = 2|\omega|/D \;\Rightarrow$ the penetration depth $\xi_T \sim \sqrt{D/T_c}$

Supercurrent: $\mathbf{j} = \pi N_F D \,\mathrm{Im} \sum_\omega \mathrm{tr}\{\hat{\bar{f}} \nabla \hat{f}\}$ $\left(cf.\ \mathbf{j} \sim \mathrm{Im}\{\Psi^* \nabla \Psi\}\right)$

Josephson effect in S/N/S structures



$\mathrm{Im}\{f^* \nabla f\} \neq 0$ only if $\varphi_2 - \varphi_1 \neq 0$

$$j_x = j_c \sin(\varphi_2 - \varphi_1)\,, \quad j_c \sim e^{-L/\xi_T}$$

Singlet-triplet coupling in a ferromagnet

Spin structure of the condensate function: $\hat{F}(\omega) = \hat{F}_s(\omega) + \hat{F}_t(\omega)$

$$\hat{F} = F_s(\omega)(|\uparrow\rangle\langle\downarrow| - |\downarrow\rangle\langle\uparrow|) + F_t^z(\omega)(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|) \\ + F_t^+(\omega)|\uparrow\rangle\langle\uparrow| + F_t^-(\omega)|\downarrow\rangle\langle\downarrow|$$

Pauli principle requires $F_t^a(-\omega) = -F_t^a(\omega)$: "odd-frequency triplet"

$$F_s(\omega) \mapsto f_s(\omega), \qquad F_t^a(\omega) \mapsto \mathrm{sgn}(\omega) f_t^a(\omega)$$

The exchange field in a ferromagnet $\hat{H}_{\mathrm{ex}} = \sigma^z h^z$ mixes f_s and f_t^z

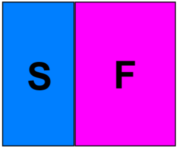
Usadel equation in the presence of an exchange/Zeeman field

$$D\nabla^2 f_s - 2|\omega| f_s + i\, 2h^a f_t^a = 0$$
$$D\nabla^2 f_t^a - 2|\omega| f_t^a + i\, 2h^a f_s = 0$$

Singlet-triplet conversion is accompanied with the phase shift of $\pi/2$ because the exchange field breaks the time reversal symmetry

Proximity effect in S/F structures

S/F structure with perfectly transparent interface



$$D\nabla^2 f_s - 2|\omega| f_s + i\,2h^a f_t^a = 0$$
$$D\nabla^2 f_t^a - 2|\omega| f_t^a + i\,2h^a f_s = 0$$
$$f_s|_S = f_{BSC}, \qquad f_t^a|_S = 0$$

Singlet condensate penetrates F-region where the exchange field converts it into the odd-frequency triplet condensate parallel to $\boldsymbol{h}$

$$f_s = f_{BCS}\cos(x/\xi_h)e^{-x/\xi_h}$$
$$f_t^z = i\, f_{BCS}\sin(x/\xi_h)e^{-x/\xi_h}$$

If $h \gg T_c$, the penetration depth $\xi_h = \sqrt{D/h} \ll \xi_T$

The phase of the generated triplet condensate is shifted by $\pi/2$

SOC in superconducting structures: General picture

Spin-charge diffusion equations in normal conductors

$$D\nabla^2 n + C_k^a \partial_k S^a = 0$$

$$D\nabla^2 S^a - \Gamma^{ab} S^b + P_k^{ab} \partial_k S^b + C_k^a \partial_k n = 0$$

SOC is time-reversal symmetric $\Rightarrow$ it acts in exactly the same way on t-conjugated states $\Rightarrow$ we expect: $n \mapsto f_s(\omega), \quad S^a \mapsto f_t^a(\omega)$

Expected form of Usadel equations in the presence SOC

$$D\nabla^2 f_s - 2|\omega| f_s + C_k^a \partial_k f_t^a + i2h^a f_t^a = 0$$

$$D\nabla^2 f_t^a - (2|\omega|\delta^{ab} + \Gamma^{ab}) f_t^b + P_k^{ab} \partial_k f_t^b + C_k^a \partial_k f_s + i2h^a f_s = 0$$

- Additional relaxation (Γ^{ab}) and spin rotation (P_k^{ab}) of the triplet: Generation of long-range triplet condensate (LRTC) in S/F
- Additional t-even (C_k^a) channel of the singlet-triplet conversion: Interference of two conversion channels $\mapsto$ anomalous current

Outline

Microscopic model of intrinsic SOC: SU(2) gauge field

One-particle Hamiltonian with intrinsic SOC: $\Omega^a(\hat{\mathbf{p}}) = \frac{1}{2m}\{\mathcal{A}^a_k, \hat{p}_k\}$

$$H_0 = \frac{\mathbf{p}^2}{2m} - \frac{1}{2}\Omega^a(\mathbf{p})\sigma^a - h^a\sigma^a + V_{\text{imp}} \mapsto \frac{1}{2m}(p_k - \hat{\mathcal{A}}_k)^2 - \hat{\mathcal{A}}_0 + V_{\text{imp}}$$

SOC $\hat{\mathcal{A}}_k = \frac{1}{2}\mathcal{A}^a_k\sigma^a$, and the exchange field $\hat{\mathcal{A}}_0 = \frac{1}{2}\mathcal{A}^a_0\sigma^a \equiv h^a\sigma^a$ enter as space- and time-components of an effective SU(2) gauge filed

2D Rashba-Dresselhaus SOC: $\mathcal{A}^y_x = -\mathcal{A}^x_y = \alpha, \quad \mathcal{A}^x_x = -\mathcal{A}^y_y = \beta$

$\hat{H}$ is form-invariant under a local SU(2) rotation: $U = \exp\left[\frac{i}{2}\theta^a(\mathbf{r})\sigma^a\right]$

$$H \mapsto UHU^{-1}, \quad \mathcal{A}_\mu \mapsto U\mathcal{A}_\mu U^{-1} - i(\partial_\mu U)U^{-1}$$

Gauge symmetry $\Rightarrow$ SOC can enter only via covariant combinations!

SU(2) Covariant Usadel equation

- Covariant derivative: $\tilde{\nabla}_k \hat{O} = \partial_k \hat{O} - i[\hat{\mathcal{A}}_k, \hat{O}]$
- Gauge field strength: $\hat{\mathcal{F}}_{\mu\nu} = \frac{1}{2}\mathcal{F}^a_{\mu\nu}\sigma^a = \partial_\mu \hat{\mathcal{A}}_\nu - \partial_\nu \hat{\mathcal{A}}_\mu - i[\hat{\mathcal{A}}_\mu, \hat{\mathcal{A}}_\nu]$

"Electric" field: $\mathcal{E}^a_k = \mathcal{F}^a_{0k}$, "Magnetic" filed: $\mathcal{B}^a_i = \varepsilon_{ijk}\mathcal{F}^a_{jk}$

Make Usadel equation covariant by $\partial_k \cdot \mapsto \tilde{\nabla}_k \cdot = \partial_k \cdot - i[\hat{\mathcal{A}}_k, \cdot]$

$$\hat{f} = f_s + f^a_t \sigma^a \qquad \begin{cases} D\tilde{\nabla}^2 \hat{f} - 2|\omega|\hat{f} + i\{\hat{h}, \hat{f}\} = 0 \\ \nu_k \tilde{\nabla}_k \hat{f}|_S = i\gamma f_{BSC} \end{cases}$$

Covariant Laplacian: $$\tilde{\nabla}^2 \hat{f} = \nabla^2 \hat{f} - \underbrace{2i[\hat{\mathcal{A}}_k, \partial_k \hat{f}]}_{\text{rotation}} - \underbrace{[\hat{\mathcal{A}}_j, [\hat{\mathcal{A}}_j, \hat{f}]]}_{\text{DP relaxaton}}$$

Spin torque: $P^{ab}_k = 2D\varepsilon^{abc}\mathcal{A}^c_k, \;\; \Gamma^{ab} = D\left(\mathcal{A}^c_k \mathcal{A}^c_k \delta^{ab} - \mathcal{A}^a_k \mathcal{A}^b_k\right)$

$C^a_k = 0 \;\rightarrow$ SOC does not induce singlet-triplet coupling at this level

Generation of Long-range Triplet Condensate (LTRC)

Simple example of LRTC: "3D Rashba" SOC $\mathcal{A}_k^a = \alpha \delta_k^a$ and $\boldsymbol{h} = \hat{\boldsymbol{z}} h^z$

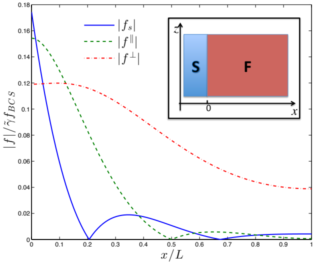
$$D\tilde{\nabla}^2 \hat{f} - 2|\omega|\hat{f} + i\{\hat{h}, \hat{f}\} = 0\,; \quad \hat{f} = f_s + f_t^{||}\sigma^z + f_t^{\perp}\sigma^y$$

$$D\partial_x^2 f_s - 2|\omega| f_s - 2ih^z f_t^{||} = 0$$
$$D\partial_x^2 f_t^{||} + 2D\alpha\partial_x f_t^{\perp} - 2\left(D\alpha^2 + |\omega|\right) f_t^{||} - 2ih^z f_s = 0$$
$$D\partial_x^2 f_t^{\perp} - 2D\alpha\partial_x f_t^{||} - 2\left(D\alpha^2 + |\omega|\right) f_t^{\perp} = 0$$

General condition for LRTC

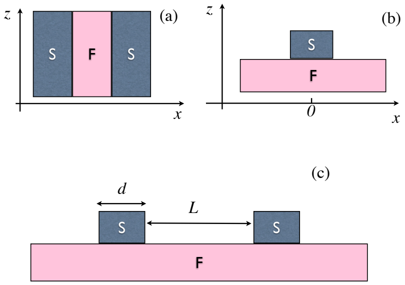
The component $\hat{\mathcal{A}}_k$ along inhomogeneity should not commute with $\hat{h}$, $[\hat{\mathcal{A}}_0, \hat{\mathcal{A}}_k] \neq 0$

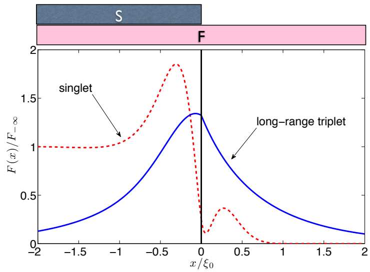
LRTC is generated by the SU(2) electric field $\mathcal{F}_{0k}^a = \mathcal{E}_k^a$ in the direction of diffusion!



LRTC: "vertical" vs "lateral" S/F/S structures

SOC related to geometric constraints and/or heterointerfaces generates LRTC only in lateral S/F/S structures





Long-range Josephson effect in a lateral S/F/S structure

$$I_c = \frac{\mathcal{S}\sigma_F}{e}\mathrm{tr}(\hat{\mathcal{F}}^{\perp}_{x0})^2 T\sum_{\omega}\kappa_\omega C^2(\omega_n)e^{-\kappa_\omega L} \sim \mathrm{tr}(\hat{\mathcal{E}}^{\perp}_{x})^2 e^{-L/\xi_T}$$

Beyond the leading order: Singlet-Triplet coupling

Include the effect of SU(2) magnetic field $\hat{\mathfrak{F}}_{jk} = -i[\hat{\mathcal{A}}_j, \hat{\mathcal{A}}_k]$

Linearized covariant Eilenberger equation for anomalous GF $\hat{f}(\mathbf{n})$

$$v_F n_k \tilde{\nabla}_k \hat{f}(\mathbf{n}) + \Big\{\omega - \underbrace{i\hat{\mathcal{A}}_0 - \frac{\hat{\mathfrak{F}}_{jk}}{2m} n_j \partial_{n_k}}_{\text{singlet-triplet coupling}}, \hat{f}(\mathbf{n})\Big\} = \frac{\mathrm{sgn}(\omega)}{\tau}\left[\hat{f}(\mathbf{n}) - \langle \hat{f} \rangle\right]$$

SU(2) “Lorentz force”$\to$ spin-dependent deflection of trajectories

$\to$ conversion of moving charge into moving spin $\to$ SHE

In combination with SOC-induced precession of moving spins $\to$

- Spin-galvanic and Edelstein effects in normal metals
- Additional t-even channel of singlet-triplet conversion in SC

Beyond the leading order: The diffusive limit $\tau \to 0$

$$\hat{f}(\mathbf{n}) \approx \hat{f} + n_k \hat{f}_k, \qquad \hat{f} = \langle \hat{f}(\mathbf{n}) \rangle \gg \hat{f}_k$$

$$\frac{1}{3} v_F \tilde{\nabla}_k \hat{f}_k + 2\omega \hat{f} - i\{\hat{\mathcal{A}}_0, \hat{f}\} = 0$$

$$\hat{f}_k \approx -\tau v_F \Big(\underbrace{\mathrm{sgn}(\omega) \tilde{\nabla}_k \hat{f}}_{\text{“covariant diffusion”}} - \underbrace{\frac{\tau}{2m} \{\hat{\mathfrak{F}}_{kj}, \tilde{\nabla}_j \hat{f}\}}_{\text{“spin Hall” term}} \Big)$$

Generalized Usadel equation and physical observables

$$D\tilde{\nabla}^2 \hat{f} - 2|\omega|\hat{f} + \mathrm{sgn}(\omega)\Big(i\{\hat{\mathcal{A}}_0, \hat{f}\} + \frac{\tau D}{2m}\{\tilde{\nabla}_k \hat{\mathfrak{F}}_{kj}, \tilde{\nabla}_j \hat{f}\}\Big) = 0$$

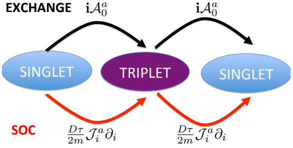
$$j_k = \pi N_0 D \,\mathrm{Im} \sum_\omega \mathrm{tr}\Big[\hat{\bar{f}} \hat{f}_k\Big] \mathrm{sgn}(\omega)$$

$$S^a = \pi N_0 \,\mathrm{Im} \sum_\omega \mathrm{tr}\Big[\sigma^a \hat{\bar{f}} \hat{f}\Big] \mathrm{sgn}(\omega)$$

Non-dissipative magnetoelectric effects

$$\hat{f} = f_s + \mathrm{sgn}(\omega)\sigma^a f_t^a$$

$$D\nabla^2 f_s - 2|\omega| f_s + \left(i\mathcal{A}_0^a + \frac{\tau D}{2m}\mathfrak{J}_k^a \partial_k\right) f_t^a = 0$$
$$D(\tilde{\nabla}^2 \hat{f}_t)^a - 2|\omega| f_t^a + \left(i\mathcal{A}_0^a + \frac{\tau D}{2m}\mathfrak{J}_k^a \partial_k\right) f_s = 0$$



$\hat{\mathfrak{J}}_k = \tilde{\nabla}_j \hat{\mathfrak{F}}_{jk} \mapsto [\hat{\mathcal{A}}_j, [\hat{\mathcal{A}}_j, \hat{\mathcal{A}}_k]]$: pseudotensor $\sim$ equilibrium spin current

Edelstein and spin-galvanic effects in bulk superconductors

$$S^a = \chi_k^a \partial_k \varphi, \qquad j_k = \chi_k^a \mathcal{A}_k^a$$

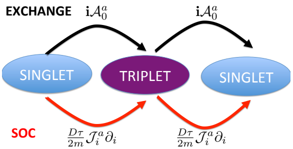
$$\chi_k^a = \pi N_0 \frac{\tau D}{m} \sum_\omega \frac{\Delta^2}{\omega^2} \left[(\hat{\Gamma} + 2|\omega|)^{-1}\right]^{ab} \mathfrak{J}_k^b \sim \frac{\alpha^3}{\alpha^2 + \xi_T^{-2}}$$

Non-dissipative magnetoelectric effects: φ_0-junctions

$$I(\varphi) = I_c \sin(\varphi + \varphi_0), \qquad \varphi_0 \neq \{0, \pi\} \text{ – anomalous phase}$$

Current through the right interface at $x = x_R$

$$I_x \sim \sum_{\omega} \mathrm{Im}\left[f_s^*(x_R) f_{BSC}^R\right] \sim \sin(\varphi + \varphi_0)$$

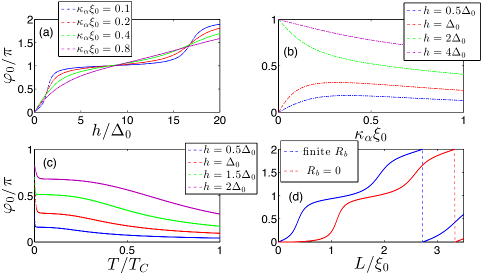


Example: Rashba SOC

$$\mathcal{A}_x^y = -\mathcal{A}_y^x = \alpha, \quad \mathcal{A}_0^y = h$$

(a)-(b) $L = \xi_0$, $T = 0.1T_c$

(c) $L = \xi_0$, $\kappa_\alpha \xi_0 = 0.2$



Outline

Extrinsic SOC in superconductors: The starting point

Spin-dependent scattering by a random potential of impurities:

$$\hat{W}(\mathbf{r}) = V(\mathbf{r}) + \hat{V}_{so}(\mathbf{r}), \quad \hat{V}_{so} = -i\lambda^2 \left(\nabla V(\mathbf{r}) \times \nabla\right) \boldsymbol{\sigma}$$

Kinetic equation for 8×8 matrix Nambu-Keldysh GF $\check{G}(\mathbf{p}, \mathbf{r}; t, t')$

$$\tau_3 \partial_t \check{G} + \partial_{t'} \check{G} \tau_3 + \frac{p_k}{m} \partial_k \check{G} + i \left[\mathbf{h}\boldsymbol{\sigma}\tau_3 + \check{\Delta}, \check{G}\right] = \mathcal{I}[\check{G}]$$

$$\mathcal{I}[\check{G}] = -i\left[\check{\Sigma}, \check{G}\right] + \frac{1}{2}\left\{\nabla_{\mathbf{r}}\check{\Sigma}, \nabla_{\mathbf{p}}\check{G}\right\} - \frac{1}{2}\left\{\nabla_{\mathbf{p}}\check{\Sigma}, \nabla_{\mathbf{r}}\check{G}\right\}$$

Contributions to the 2nd Born self-energy $\check{\Sigma} = \langle \hat{W}(\mathbf{r}) \check{G}(\mathbf{r}, \mathbf{r}') \hat{W}(\mathbf{r}') \rangle$

- $\langle V \check{G} V \rangle$ – momentum relaxation $\to \tau$
- $\langle \hat{V}_{so} \check{G} \hat{V}_{so} \rangle$ – spin relaxation (Elliot-Yafet) $\to \tau_{so}$
- $\langle V \check{G} \hat{V}_{so} \rangle$ – “side jump” SHE and spin current swapping $\to \theta, \ \kappa$

Beyond 2nd Born $\sim \langle V \check{G} V \check{G} \hat{V}_{so} \rangle$ – “skew scattering” SHE

Usadel equation: SHE and spin current swapping

Linearized Usadel equation in equilibrium $\hat{f}(\mathbf{n}) \approx \hat{f} + n_k \hat{f}_k$

$$\frac{1}{3} v_F \partial_k \hat{f}_k + 2\omega \hat{f} + i\{\mathbf{h}\boldsymbol{\sigma}, \hat{f}\} = -\mathrm{sgn}(\omega)\frac{3\hat{f} - \sigma^a \hat{f}\sigma^a}{4\tau_{so}}$$

$$\hat{f}_k = -\tau v_F \Big(\underbrace{\mathrm{sgn}(\omega)\partial_k \hat{f}}_{\text{diffusion}} - \underbrace{\frac{\theta}{2}\varepsilon_{kja}\{\sigma^a, \partial_j \hat{f}\}}_{\text{“spin Hall” term}} + i \underbrace{\mathrm{sgn}(\omega)\frac{\kappa}{2}\varepsilon_{kja}[\sigma^a, \partial_j \hat{f}]}_{\text{“swapping” term}} \Big)$$

Spin Hall and swapping contributions to $\hat{f}_k$ are purely transverse $\Rightarrow$

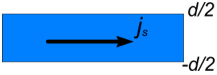
$$D\nabla^2 \hat{f} - 2|\omega|\hat{f} - i\,\mathrm{sgn}(\omega)\{\mathbf{h}\boldsymbol{\sigma}, \hat{f}\} = \frac{3\hat{f} - \sigma^a \hat{f}\sigma^a}{4\tau_{so}}$$

At S/N boundary:
$$\begin{cases} \nu_k \left(\partial_k f_s - \theta\varepsilon_{kja}\partial_j f_t^a\right) = i\gamma f_{BCS} \\ \nu_k \left(\partial_k f_t^a - \theta\varepsilon_{kja}\partial_j f_s - \kappa[\partial_a f_t^k - \delta_{ka}\partial_j f_t^j]\right) = 0 \end{cases}$$

Singlet-triplet coupling is induced at the interface: $C_k^a = \theta\varepsilon_{kja}\nu_j\delta(S)$!

Interface spin accumulation and extrinsic SHE in SC

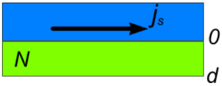
I. Non-dissipative SHE in a superconducting film



$$\left.\begin{aligned} f_s(x) &\approx i f_{BCS} e^{i\varphi(x)} \\ \partial_z f_t^y|_{\pm d/2} &= -\theta \partial_x \varphi f_{BCS} \end{aligned}\right\} \Rightarrow S^y(z) = \theta \sum_\omega \frac{j_{s,x}(\omega)}{Dk} \frac{\sinh kz}{\cosh kd/2}$$

$Dk^2 = 2|\omega| + 1/\tau_{so},$ zero net spin polarization $\langle S \rangle = 0$

II. Supercurrent-induced interface spin acumulation in S/N bilayer

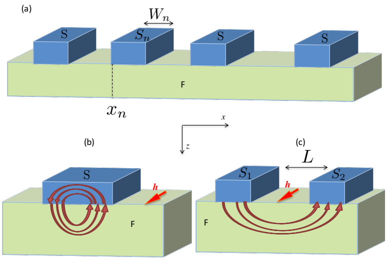


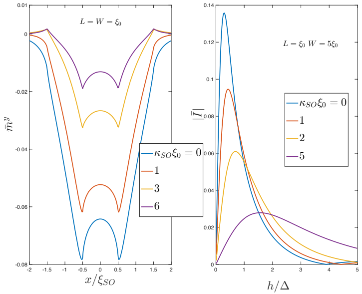
$$S^y(z) = \gamma^2 \theta \sum_\omega \frac{j_{s,x}(\omega) \cosh k_\omega (z-d)}{D k_\omega^2 k \sinh^2 k_\omega d \sinh kd} \left[\cosh kz - \cosh k_\omega d \cosh k(z-d)\right]$$

structure with broken inversion symmetry $\Rightarrow \langle S \rangle \neq 0$

Inverse effect: Spontaneous supercurrent at S/F interface

Anomalous Josephson effect in lateral structures





Supercurrent through nth S-terminal at $x_n < x < x_n + W_n$

$$j_z^{(n)}(x) = h\theta \sum_{\omega} \sum_{l=1}^{M} \frac{f_{BCS}^{(n)} f_{BCS}^{(l)}}{R_{bn} R_{bl}} \left[s(|x - x_l|) - s(|x - x_l - W_l|) \right]$$

Total anomalous current: $I = \int_{x_1}^{x_1 + W_1} dx j_z^{(1)}(x) \sim h\theta \sim \sigma_{AH}$

Outline

Summary

1. The presented theory of superconductors with SOC makes a connection between "classical spin-orbitronics" effects and phenomena mediated by SOC in superconducting structures
   - Spin helix in normal systems $\leftrightarrow$ LRTC in S/F structures
   - SHE and EE $\leftrightarrow$ Supercurrent-induced spin/triplet accumulation
   - Spin-galvanic effect (Inverse EE) $\leftrightarrow$ Josephson φ_0-junction
2. Singlet-triplet coupling and related non-dissipative magnetoelectic effects are determined by the normal state spin Hall angle and the spin-charge coupling pseudotensor C_k^a
3. On the practical side we identified lateral S/F/S structures and materials with large anomalous Hall conductivity as most promising for realization of φ_0-junctions

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- F. S. Bergeret and IVT, Phys. Rev. Lett. **110**, 117003 (2013)
- F. S. Bergeret and IVT, Phys. Rev. B **84**, 134517 (2014)
- F. S. Bergeret and IVT, EPL **110**, 57005 (2015)
- F. Konschelle, F. S. Bergeret and IVT, Phys. Rev. B **92**, 125443 (2015)
- F. S. Bergeret and IVT, Phys. Rev. B **94**, 180502 (2016)
- IVT, Phys. Rev. B **96**, 060502(R) (2017)

Thank you for your attention!