Topological superconductivity and Majorana bound states in chains of magnetic adatoms

Felix von Oppen

Freie Universität Berlin

w/ Yang Peng, Falko Pientka, Yuval Vinkler-Aviv, Leonid Glazman, Michael Ruby, Benjamin Heinrich, Katharina Franke

Literature

Les Houches lecture notes:

F. von Oppen, Y. Peng and F. Pientka
Topological superconducting phases in one dimension
Topological Aspects of Condensed Matter Physics:
Lecture Notes of the Les Houches Summer School
(2017)

see, eg, group website under publications

The failing @nytimes

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lays Hard to … elebrities and … | BITS A World of Surveillance Doesn't Always Help to Catch a Thief | How a Putin Fan Overseas Pushed Pro-Trump Propaganda to Americans | ITINERARIES Communal TV Screens in a Binge-Watching Age? Won't Do, Airlines | BITS Daily Report: No Special Perks on Snapchat for Celebrities | TECH TIP Give the Windows 10 Start Menu a Classic Look | OP-ED Goog… Cars,…

TECHNOLOGY

Microsoft Spends Big to Build a Computer Out of Science Fiction

By JOHN MARKOFF NOV. 20, 2016

Todd Holmdahl will direct Microsoft's quantum computing efforts. Ian C. Bates for The New York Times

SAN FRANCISCO — Microsoft is putting its considerable financial and engineering muscle into the experimental field of quantum computing as it works to build a machine that could tackle problems beyond the reach of today's digital computers.

There is a growing optimism in the tech world that quantum computers, superpowerful devices that were once the stuff of science fiction, are possible — and may even be practical. If these machines work, they will have an impact on work in areas such as drug design and artificial intelligence, as well as offer a better understanding of the foundations of modern physics.

Microsoft's decision to move from pure research to an expensive effort to build a working prototype underscores a global competition among technology companies, including Google and IBM, which are also making significant investments in search of breakthroughs.

In the exotic world of quantum physics, Microsoft has set itself apart from its competitors by choosing a different path. The company's approach is based on "braiding" particles known as anyons — which physicists describe as existing in just two dimensions — to form the building blocks of a supercomputer …

TECHNOLOGY Microsoft Spends Big to Build a Computer Out of Science Fiction

Snapchat Plays Hard to Get With Celebrities and Influencers | BITS A World of Surveillance Doesn't Always Help to Catch a Thief | How a Putin Fan Overseas Pushed Pro-Trump Propaganda to Americans | ITINERARIES Communal TV Screens in a Binge-Watching Age? Won't Do, Airlines

TECHNOLOGY

[…]

Mr. Holmdahl's project will also include the physicists Leo Kouwenhoven of Delft University, Charles M. Marcus of the University of Copenhagen, David Reilly of the University of Sydney and Matthias Troyer of E.T.H. Zurich.

They will all become Microsoft employees as part of the Artificial Intelligence and Research Group that Microsoft recently created under the leadership of one of its top technical employees, Harry Shum.

Microsoft's newly hired physicists say the decision to try to build a topological quantum computer comes after scientific advances made in the last two years that give the scientists growing confidence that the company will be able to create more stable qubits.

"The magic recipe involves a combination of semiconductors and superconductors," Dr. Marcus said. The researchers recently made a "remarkable breakthrough" in their ability to control the materials used to form qubits, he said.

Most of the competing approaches involve…

1d topological SC by proximity

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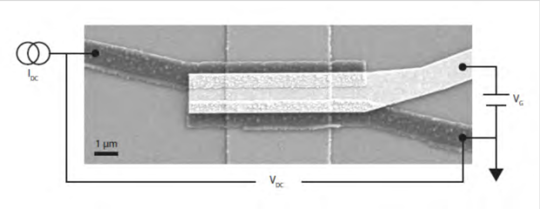
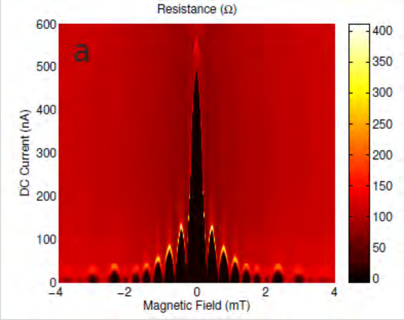
2D Topological Insulators

Fu & Kane 2008



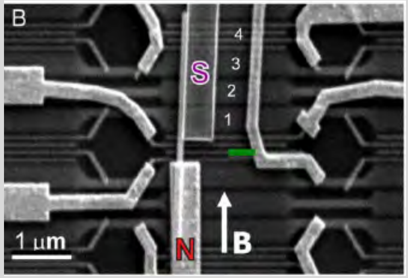
- signatures of proximity SC



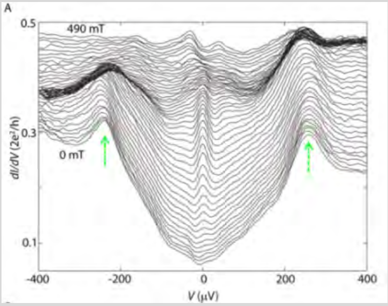
Yacoby & Molenkamp groups 2013
Kouwenhoven & Wegscheider 2014

Semiconductor Quantum Wires

Lutchyn et al. 2010
Oreg, Refael, FvO 2010



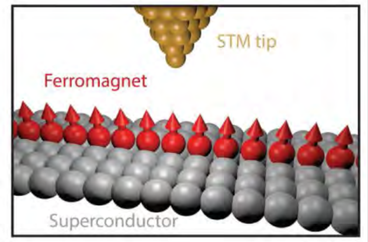
- possible signatures of Majoranas



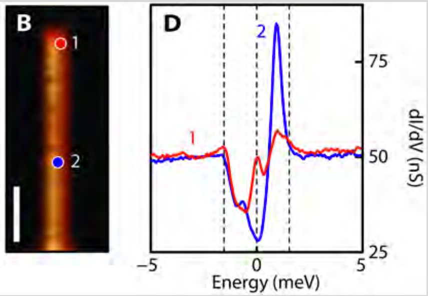
Kouwenhoven group 2012 & Xu, Heiblum, Marcus, deFrancesci etc.

Magnetic Adatom Chains

Nadj-Perge et al. 2013



- possible signatures of Majoranas



Yazdani group 2014
Franke group, Meyer group 2015

Magnetic adatoms on SC

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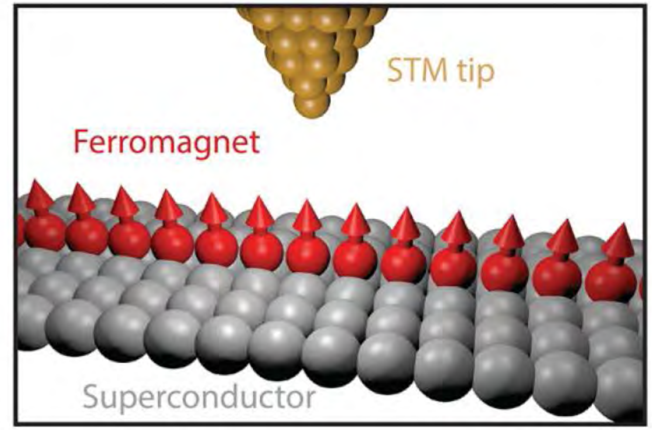
Sciencexpress Rese

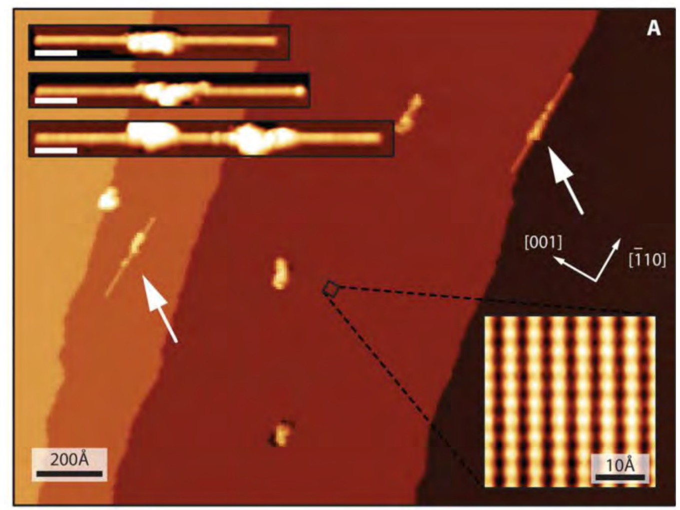
2014

Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor

Stevan Nadj-Perge,[1]* Ilya K. Drozdov,[1]* Jian Li,[1]* Hua Chen,[2]* Sangjun Jeon,[1] Jungpil Seo,[1] Allan H. MacDonald,[2] B. Andrei Bernevig,[1] Ali Yazdani[1]†

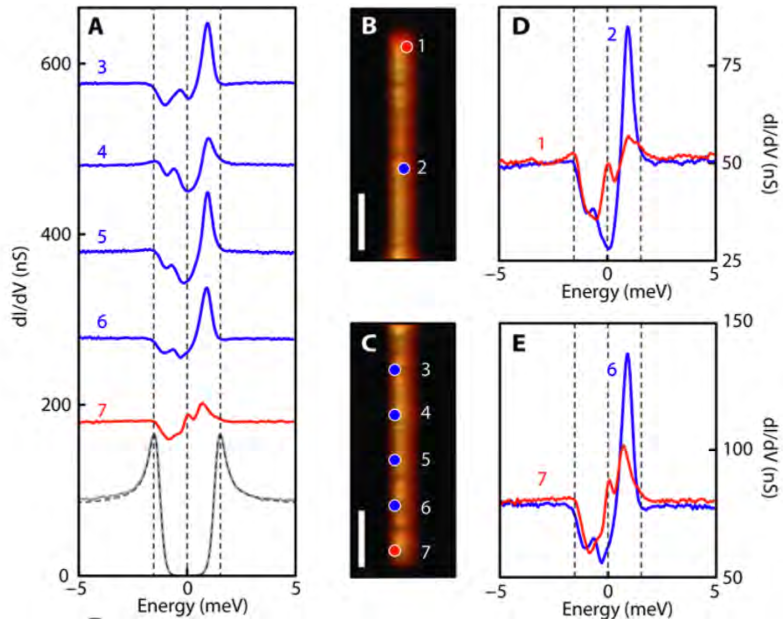


- ✓ conventional super-conducting host
- ✓ spin polarization (Fe adatoms)
- ✓ strong SO coupling (Pb substrate)

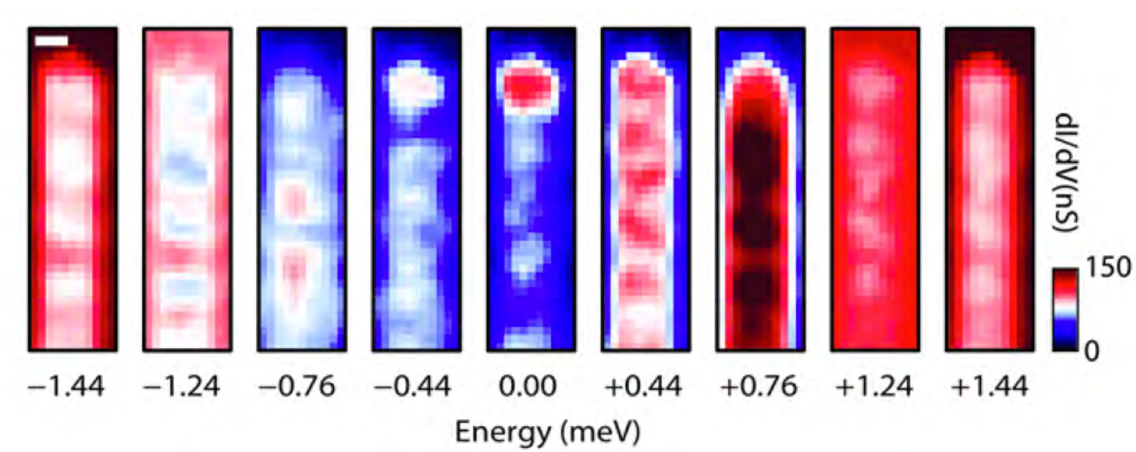


Fe on Pb 110

Observations



scale bar 1nm



- zero bias peak at chain end
- soft gap along chain
- localization of end states over a few Fe sites
- additional subgap peaks at nonzero energy
- adatoms order ferromagnetically

see also:

Ruby et al.,
PRL **115**, 197204 (2015)

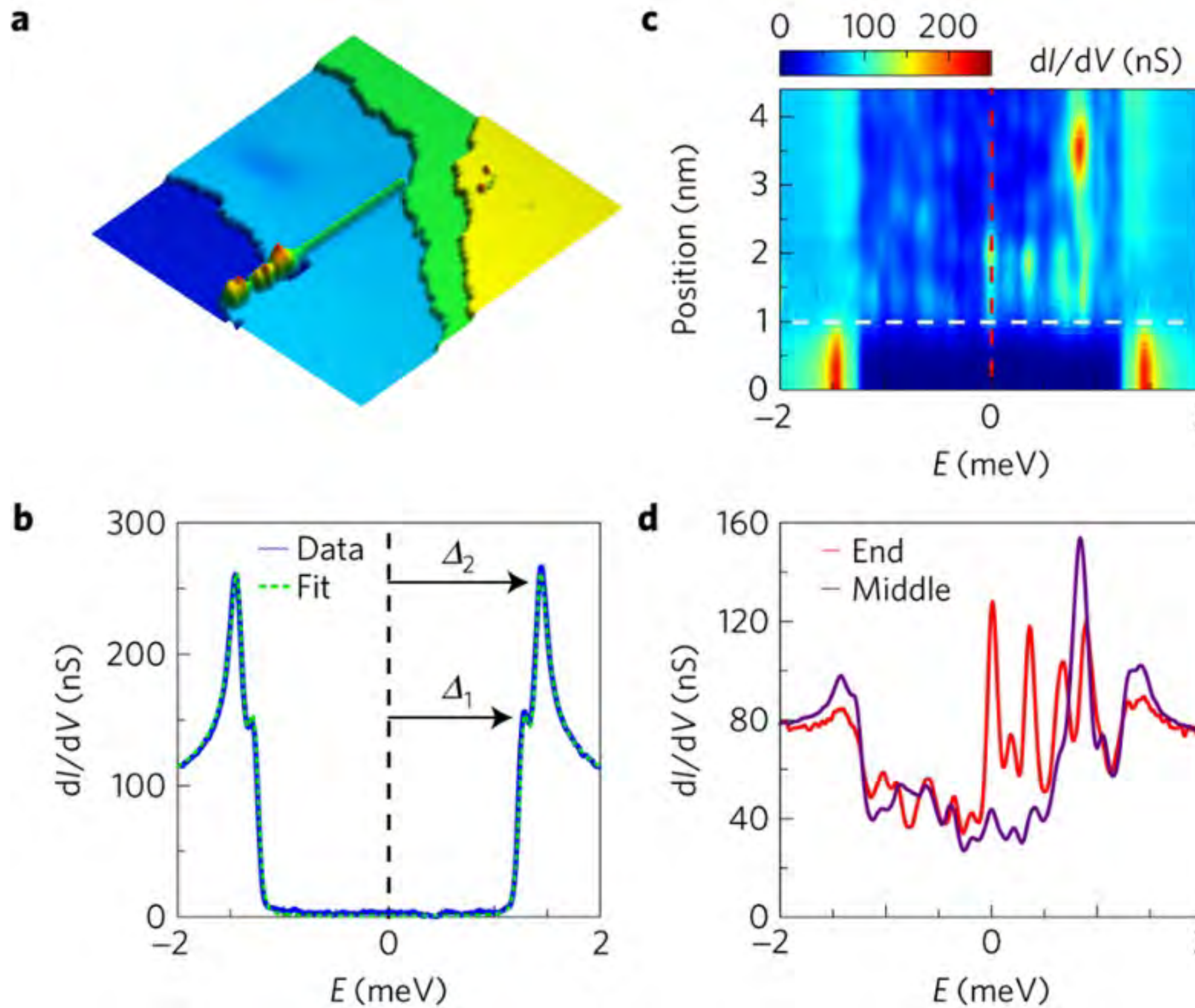
Pawlak et al.,
arXiv:1505.06078 (2015)

High resolution measurements

nature physics
ARTICLES
PUBLISHED ONLINE: 7 NOVEMBER 2016 | DOI: 10.1038/NPHYS3947

High-resolution studies of the Majorana atomic chain platform

Benjamin E. Feldman†, Mallika T. Randeria†, Jian Li†, Sangjun Jeon, Yonglong Xie, Zhijun Wang, Ilya K. Drozdov†‡, B. Andrei Bernevig and Ali Yazdani*



Majorana ?

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Majorana Fermions in Chiral Topological Ferromagnetic Nanowires arXiv:1410.5412v1

Eugene Dumitrescu[1], Brenden Roberts[1], Sumanta Tewari[1], Jay D. Sau[2], and S. Das Sarma[2]

and the prediction of a multiple Majorana based fractional Josephson effect. A critical comparison of our results with the experimental data shows a basic inconsistency in the interpretation of the Fe nanowire STM experiment in terms of Majorana zero modes– in particular, the observation of the precise localization of the Majorana zero modes at the wire ends cannot be reconciled with the extremely small topological superconducting gap (and the associated extremely weak Majorana tunneling peak) observed simultaneously. Other than this rather disturbing

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Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor.

Authors: Stevan Nadj-Perge, Ilya K. Drozdov, Jian Li, Hua Chen, Sangjun Je
Seo, Allan H. MacDonald, B. Andrei Bernevig, and Ali Yazdani , *Science E*
Published online 2 October 2014 [DOI:10.1126/science.1259327]

$$\xi_M = \frac{\hbar v_F}{\Delta_{top}} \sim \xi_{Pb}$$

Recommendation and commentary by Patrick Lee.

hybridize to form convention fermions with a finite energy splitting E_0. This expectation seems at variance with the experimental observation of a rapid decay of the end state wavefunction. While the authors offer numerical calculations to explain this rapid decay, the energy gap they used is far too large to be realistic. We believe this short distance physics is not yet fully understood, but the long distance behavior must still be controlled by the coherence length. To make further progress, it is important to go to the lower temperature to

$\xi_M \sim 1nm$

$\xi_{Pb} \sim 100nm$?

Cobalt chains

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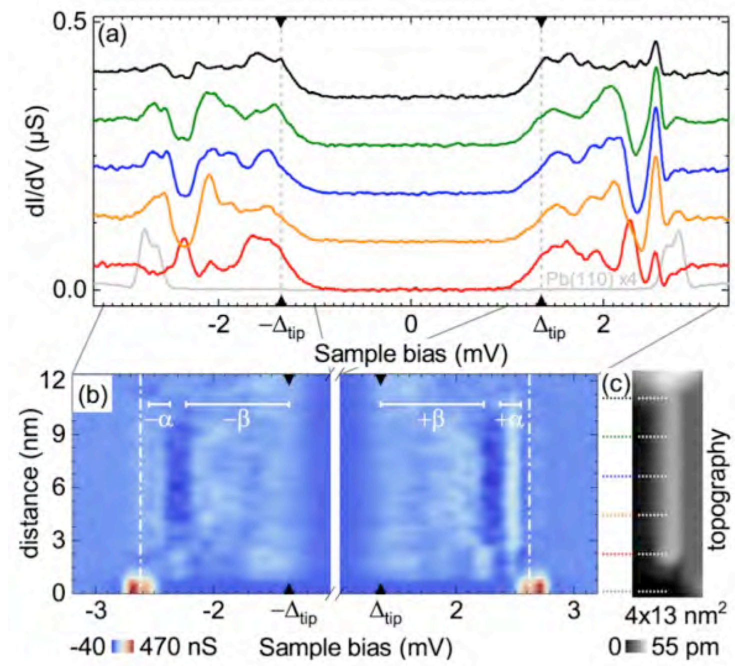
NANO LETTERS
Letter
pubs.acs.org/NanoLett

Exploring a Proximity-Coupled Co Chain on Pb(110) as a Possible Majorana Platform

Michael Ruby,[†] Benjamin W. Heinrich,[*,†] Yang Peng,[†,‡] Felix von Oppen,[†,‡] and Katharina J. Franke[†]

- ✓ conventional super-conducting host
- ✓ spin polarization (Fe adatoms)
- ✓ strong SO coupling (Pb substrate)



Why do Majoranas seem missing in Co chains?

Kitaev chain

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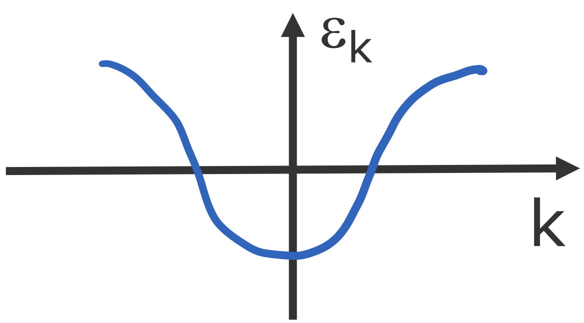
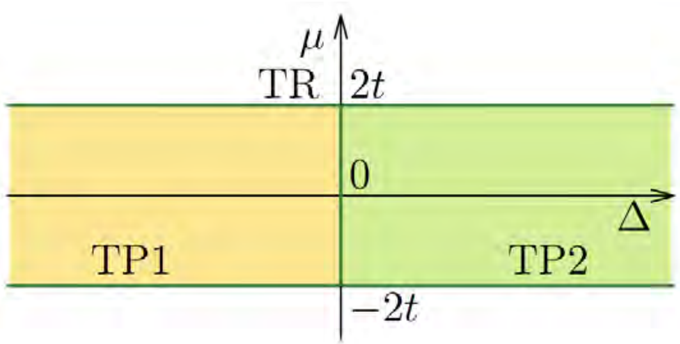
spinless p-wave superconductor in 1d

$$H = -t\sum_i [c_i{}^+ c_{i+1} + c_{i+1}{}^+ c_i] + \Delta \sum_i [c_i{}^+ c_{i+1}{}^+ + c_{i+1} c_i] - \mu \sum_i c_i{}^+ c_i$$

A. Kitaev, Phys. Usp. 44, 131 (2001)





- zero-energy Majorana bound states
- localized at domain walls between trivial and topological phase
- Majoranas can be moved by moving domain wall Alicea et al. 2011

μ <-2t $\mu = 0$

Finite Kitaev chain

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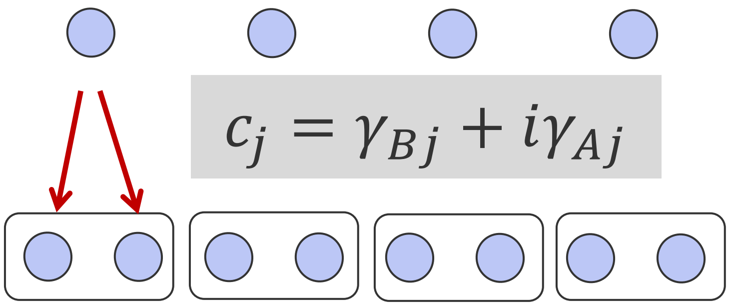
$$H = -t \sum_i [c_i{}^+ c_{i+1} + c_{i+1}{}^+ c_i] + \Delta \sum_i [c_i{}^+ c_{i+1}{}^+ + c_{i+1} c_i] - \mu \sum_i c_i{}^+ c_i$$

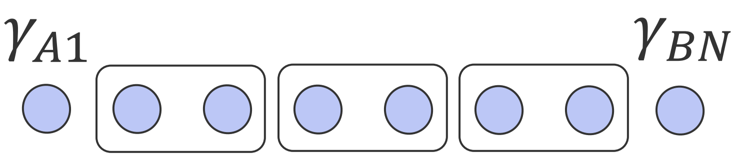
μ→−∞



$$c_j = \gamma_{Bj} + i\gamma_{Aj}$$

$$H = -\mu \sum_{j=1}^{N} i\, \gamma_{Aj} \gamma_{Bj}$$

t=Δ, μ=0



$$d_j = \gamma_{Aj+1} + i\gamma_{Bj}$$

$$H = -t \sum_{j=1}^{N} i\, \gamma_{Bj} \gamma_{Aj+1}$$

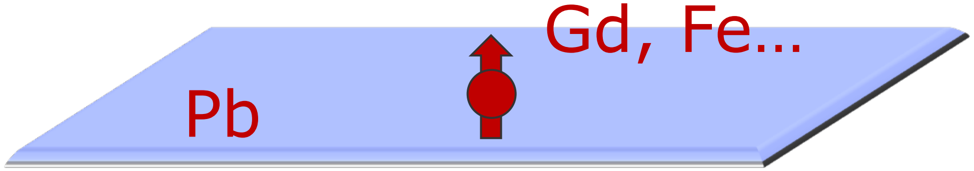
$$= t \sum_{j=1}^{N} \left(2 d_j^+ d_j - 1\right)$$

Single magnetic impurity on SC

Progress of Theoretical Physics, Vol. 40, No. 3, September 1968

Classical Spins in Superconductors

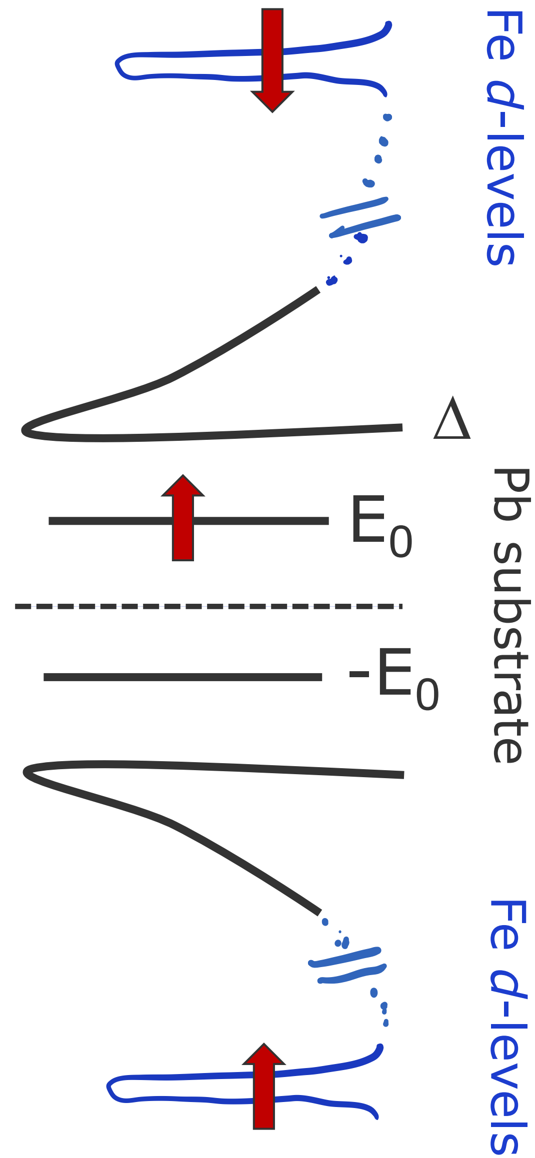
Hiroyuki SHIBA*)



$$\mathcal{H} = \xi_{\mathbf{p}}\tau_z + V\delta(\mathbf{r})\tau_z - J\mathbf{S}\cdot\boldsymbol{\sigma}\delta(\mathbf{r}) + \Delta\tau_x$$

Shiba states

- $E = \pm\Delta \dfrac{1-\alpha^2+\beta^2}{\sqrt{(1-\alpha^2+\beta^2)^2+4\alpha^2}}$
- $\psi \sim \dfrac{\cos(k_F r - \delta)}{k_F r} e^{-r/\xi_E}$
- spin polarized

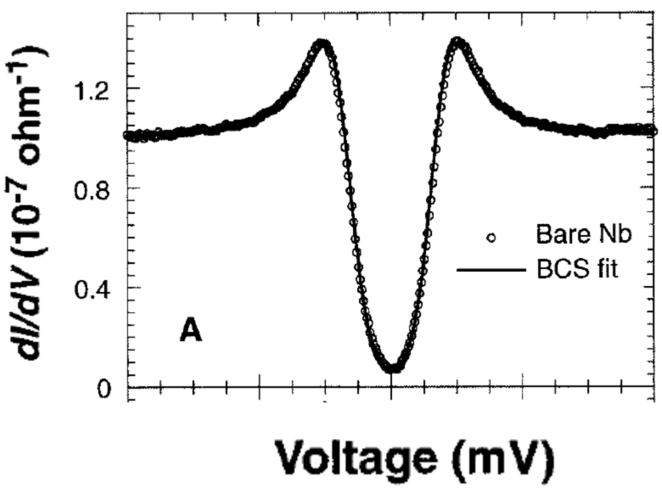


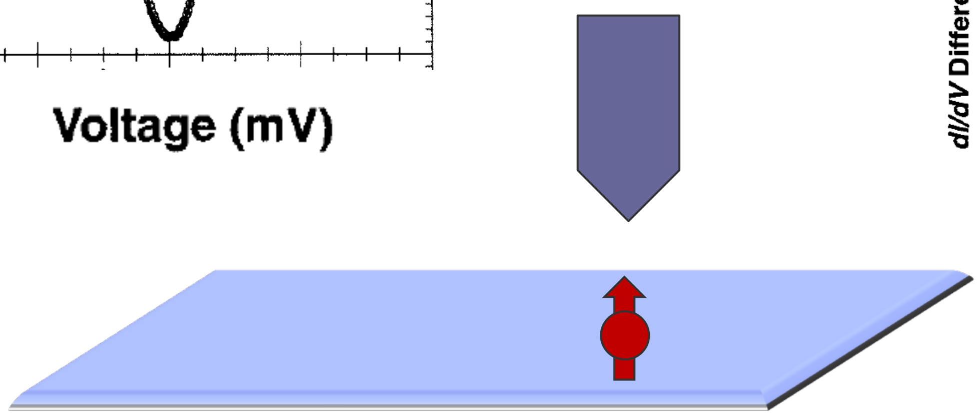
Probing Shiba bound states

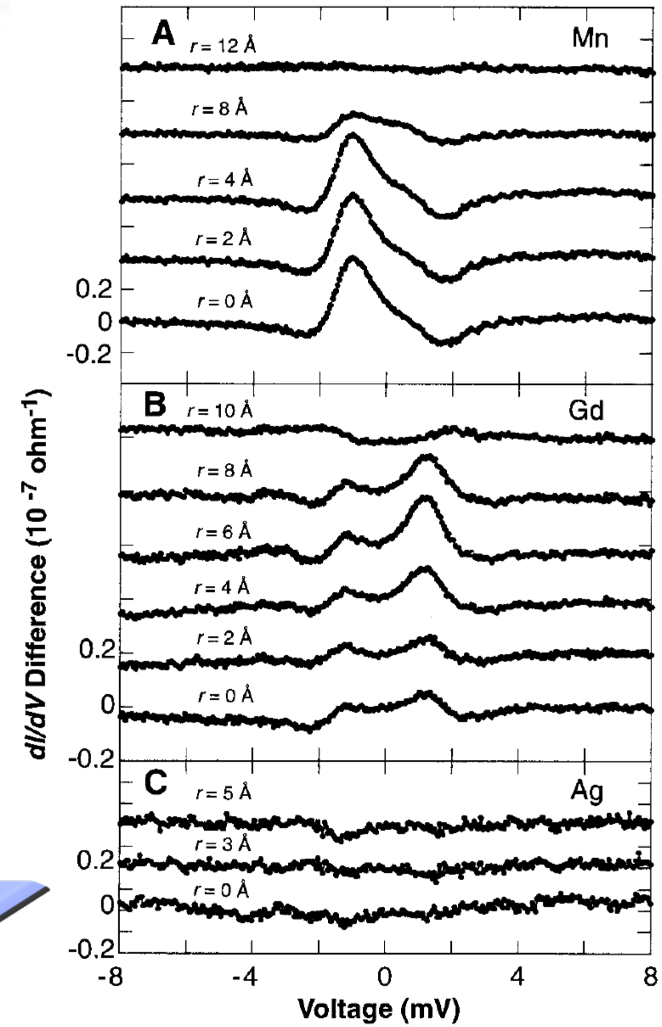
Science 1997

Probing the Local Effects of Magnetic Impurities on Superconductivity

Ali Yazdani,* B. A. Jones, C. P. Lutz, M. F. Crommie,† D. M. Eigler







Superconducting tip

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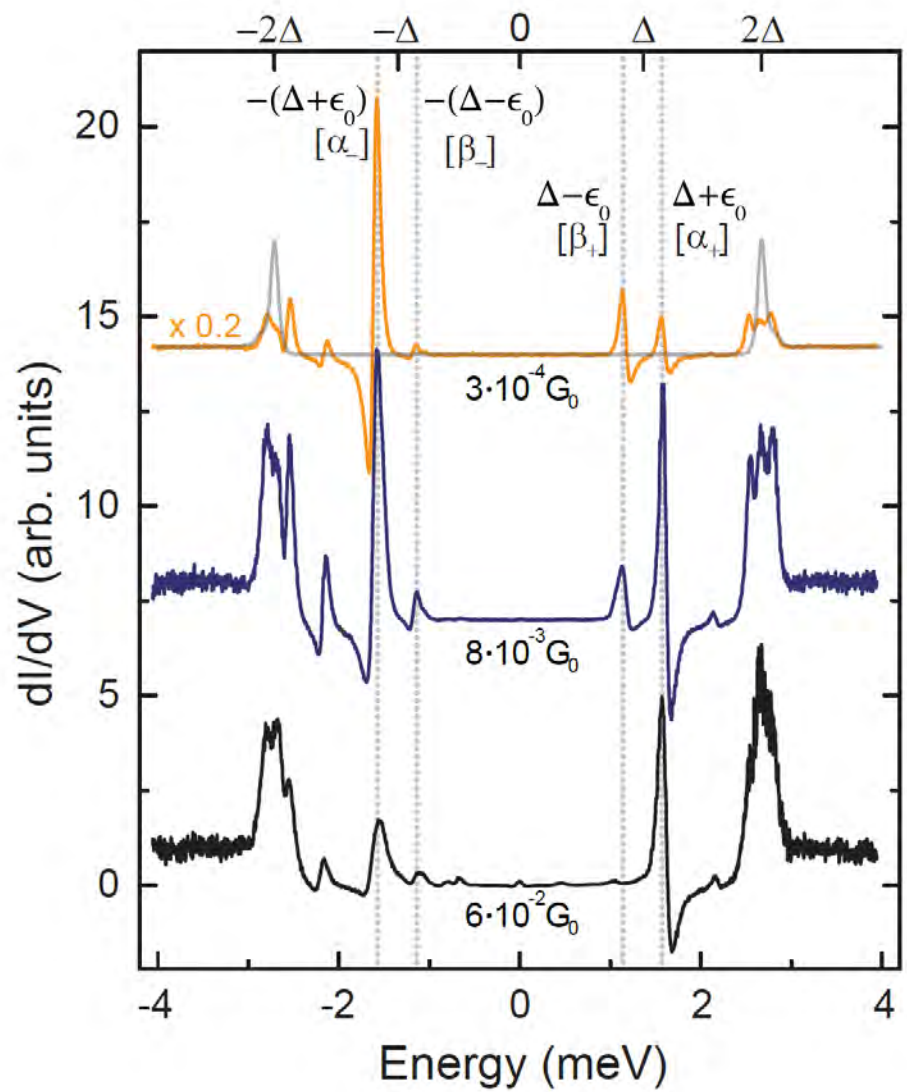
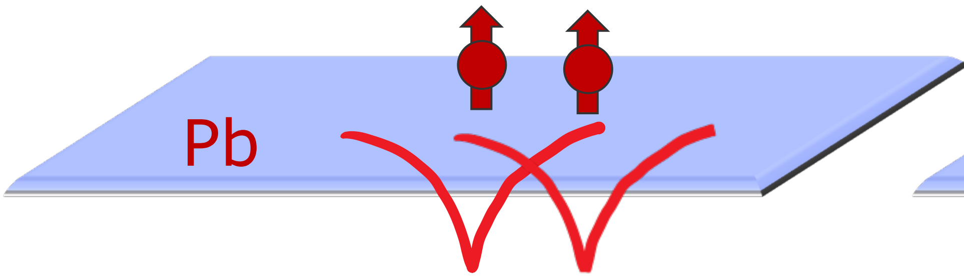


Mn on Pb(111)

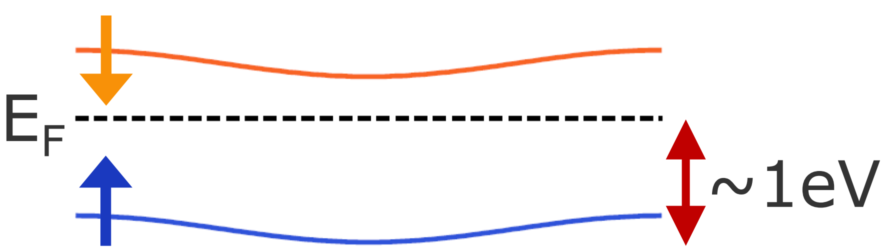
M Ruby, F Pientka, Y Peng, FvO, B Heinrich, K Franke, PRL **115**, 087001 (2015)

Towards impurity chains

Shiba limit:
weak hopping

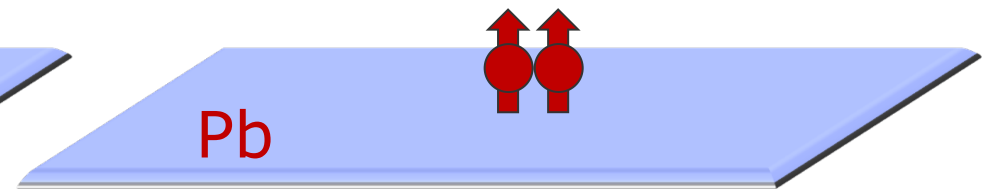


- Weakly coupled *d*-levels
- Shiba-state hybridization
- Impuritiy chain: Shiba bands

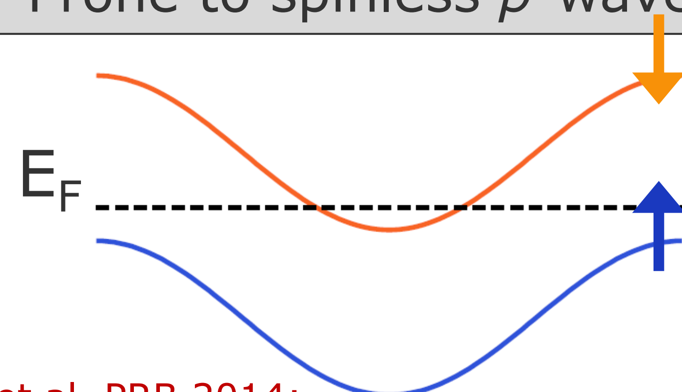


Nadj-Perge et al. PRB 2013;
Pientka, Glazman, FvO, PRB 2013 & 2014

Wire limit:
strong hopping



- Strong *d*-level hybridization
- *d*-levels no longer frozen out
- Prone to spinless *p*-wave SC



Li et al. PRB 2014;
Peng, Pientka, Glazman, FvO, PRL 2015

Chain of Anderson impurities

Hamiltonian

$$H = H_d + H_{SC} + H_{hyb}$$

$$H_d = \sum_{j\sigma} \epsilon_d d_{j\sigma}{}^{+} d_{j\sigma} + \sum_{j} U n_{j\uparrow} n_{j\downarrow} - w \sum_{j\sigma} \left[d_{j+1,\sigma}{}^{+} d_{j\sigma} + d_{j\sigma}{}^{+} d_{j+1,\sigma} \right]$$

$$H_{hyb} = -t \sum_{j\sigma} \left[\psi_{\sigma}{}^{+}(R_j) d_{j\sigma} + d_{j\sigma}{}^{+} \psi_{j\sigma}(R_j) \right]$$

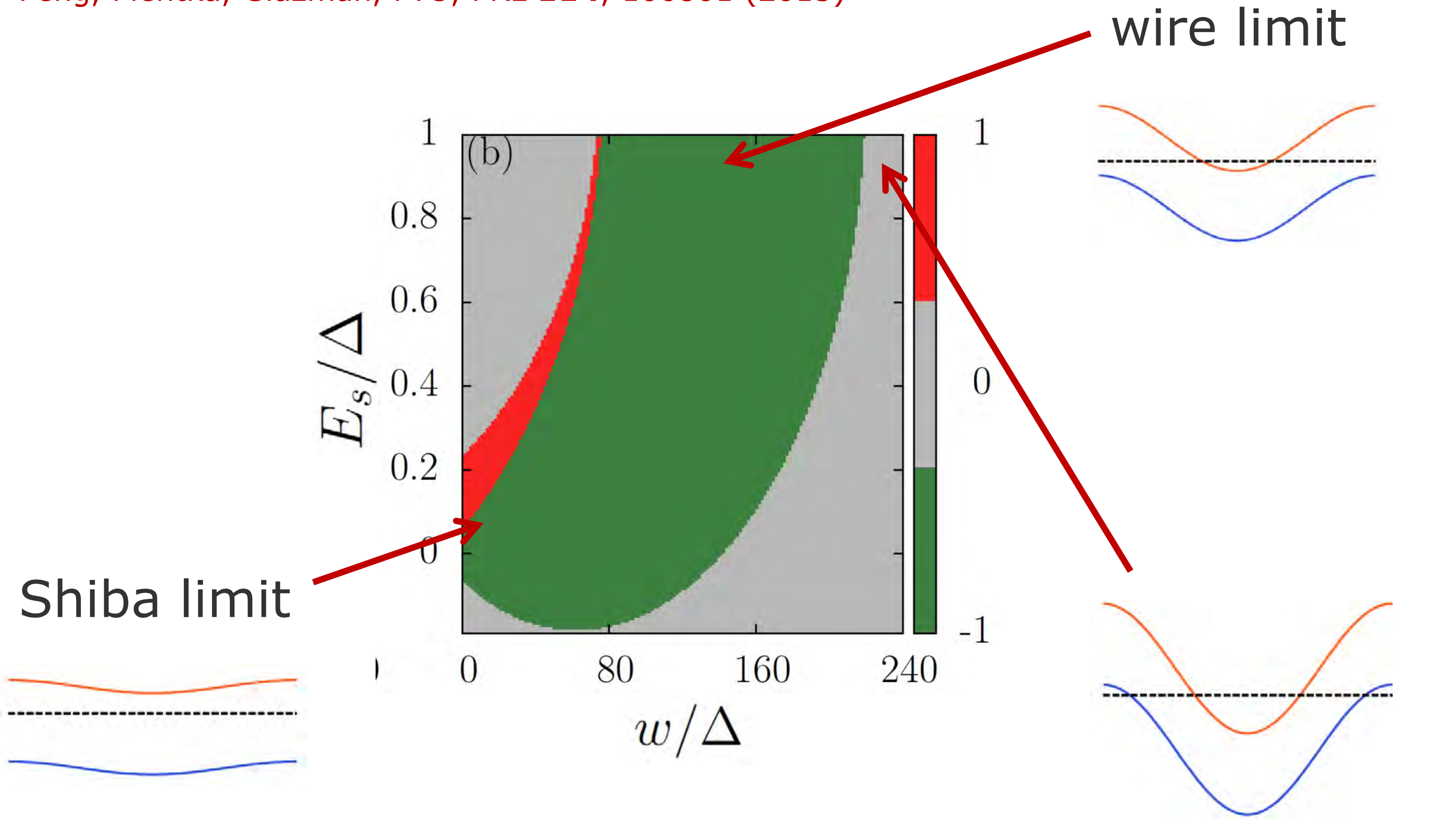
- mean-field

$$\sum_j U n_{j\uparrow} n_{j\downarrow} \rightarrow \sum_j U \langle n_{j\uparrow} \rangle n_{j\downarrow} + \sum_j U n_{j\uparrow} \langle n_{j\downarrow} \rangle$$

✓ take FM ‘order‘ as experimental fact; no self consistency

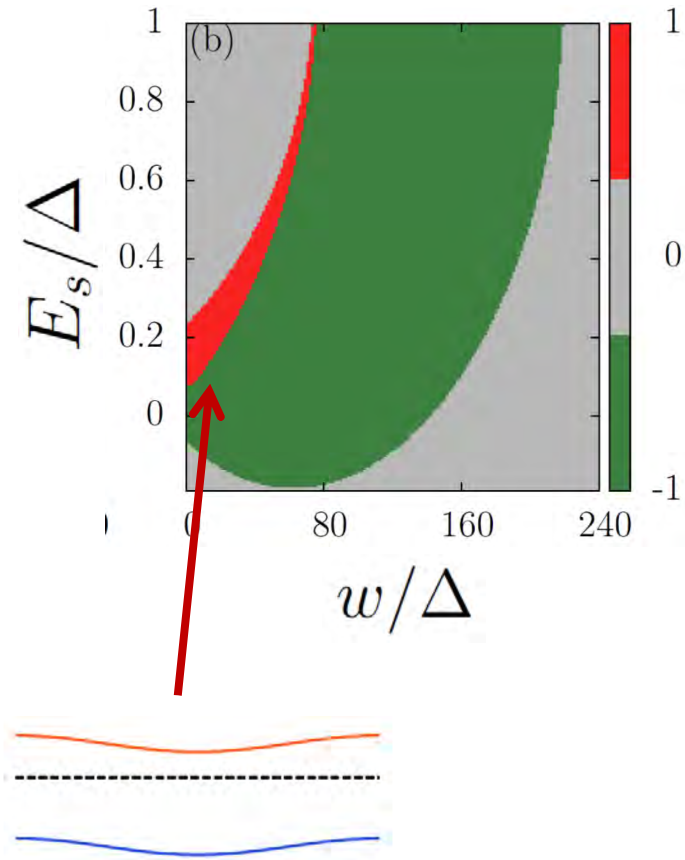
Topological phase diagram

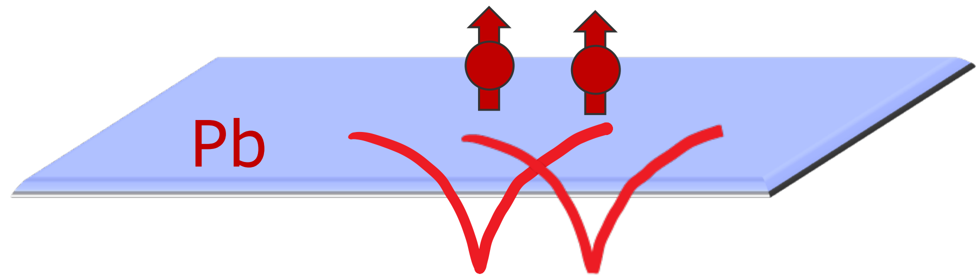
Peng, Pientka, Glazman, FvO, PRL **114**, 106801 (2015)



Shiba limit

Pientka, Glazman, FvO, PRB **88**, 155420 (2013) & PRB **89**, 180505(R) (2014)



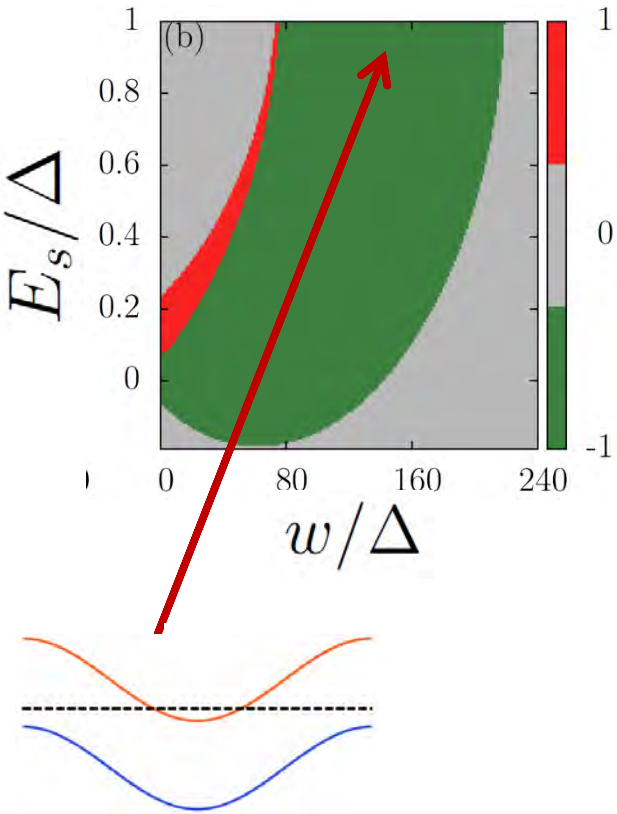


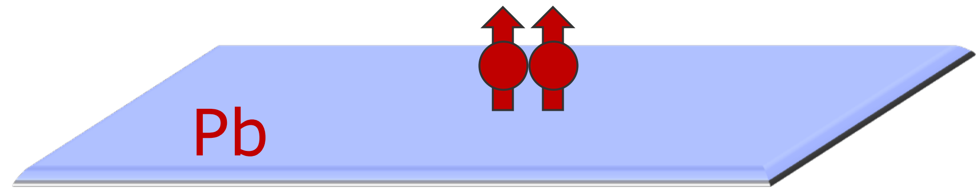
- deep Shibas: project out qp continuum
- weakly hybridized Shiba stands form Shiba bands
- proximity-induced p-wave pairing in Shiba bands when Shiba bands cross Fermi energy (Kitaev chain)
- ALSO: long range hopping & pairing

$$\mathcal{H} = E_0 \sum_j c_j^\dagger c_j - t \sum_j [c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1}] + \Delta \sum_j [c_{j+1} c_i + c_j^\dagger c_{j+1}^\dagger]$$

Wire limit

Nadj-Perge et al. Science 2014; Li et al. PRB 2014





- focus on spin-polarized (!) bands near E_F
- proximity-induced p-wave pairing
- description by continuum version of Kitaev chain (spinless p-wave SC)
- ALSO: strong coupling to SC

$$\mathcal{H} = \int dx \left\{ \psi^\dagger(x) \left(\frac{p^2}{2m} - \mu \right) \psi(x) + \Delta' [\psi^\dagger(x) \partial_x \psi^\dagger(x) + \text{h.c.}] \right\}$$

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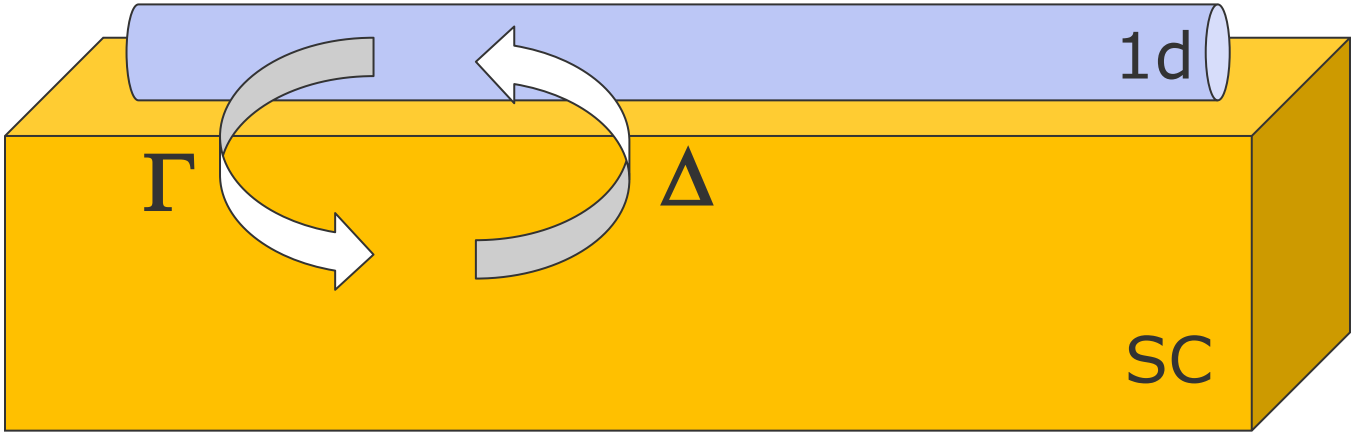
Majorana localization:

$$\xi_M = \frac{\hbar v_F}{\Delta_{top}}$$

Physical idea:

- SC acts as delay line for subgap excitations of ‘wire‘
- renormalization of Fermi velocity by $\Delta/\Gamma \sim 10^{-3}$



Peng, Pientka, Glazman, FvO, PRL **114**, 106801 (2015)

& more formally

Green function of wire electrons

$$G(k,E) = [E - v_F k\tau_z - \Sigma(k,E)]^{-1}$$

coupling to the bulk SC:

$$\Sigma(k,E) = -\Gamma \frac{E + \Delta\tau_x}{\sqrt{\Delta^2 - E^2}}$$

$$\Sigma(k,\omega) = \sum_{\mathbf{k}_\perp} t^2 G_{SC}(\mathbf{k},\omega)$$

$E \ll \Delta$

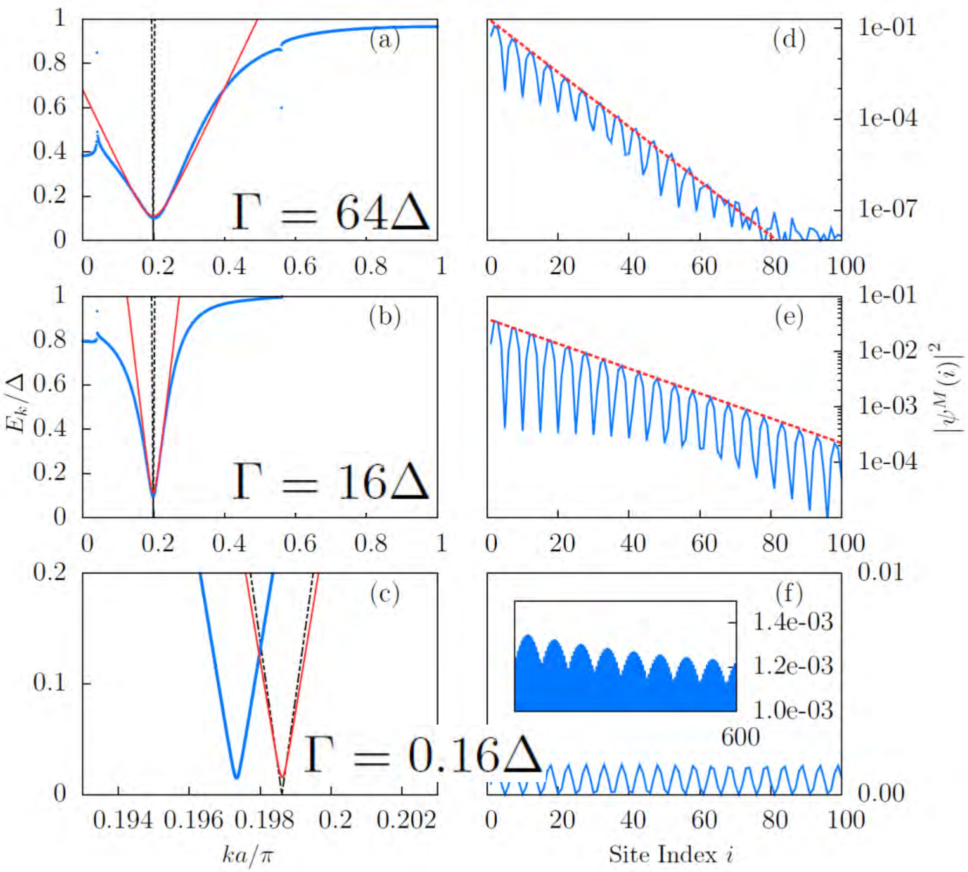
$$G(k,E) = \frac{Z}{E - Z v_F k\tau_z - Z\Gamma\tau_x}$$

$$Z = \frac{1}{1 + \Gamma/\Delta}$$

$$v_F \to \tilde{v}_F = Z v_F$$

$$\Delta_{ind} = Z\Gamma$$

Explicit model calculations

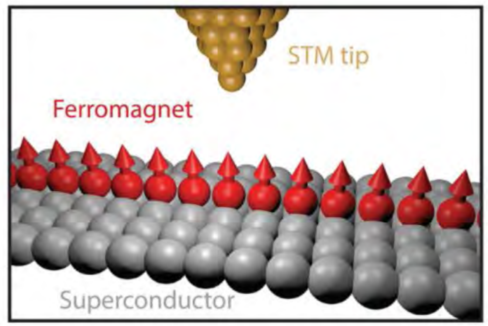


- $\xi_M \sim \xi_0(\Delta/\Gamma)(\Delta/\Delta_{\text{ind}})$
- small compared to ξ_0
- of order adatom spacing for Γ~1eV, Δ~10K, and Δ_{ind}~0.1Δ
- additional power-law tail which grows w/ Γ

consistent with experiment …

Probing Majorana bound states

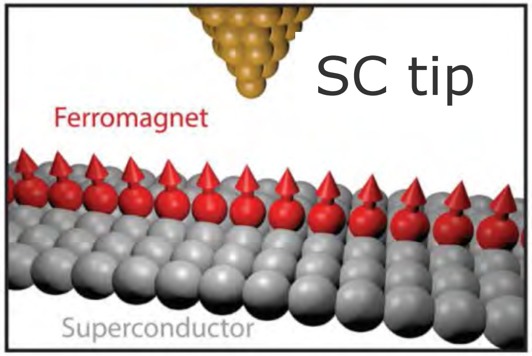
... but is it really Majorana bound state in experiment ?

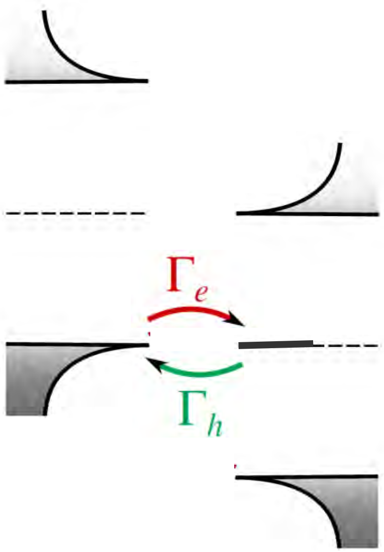


STM probes of Majorana:

- spectral & spatial resolution ✓
- quantized conductance: $2e^2/h$ ✗
  - thermal broadening
  - quasiparticle poisoning

Superconducting tip/lead





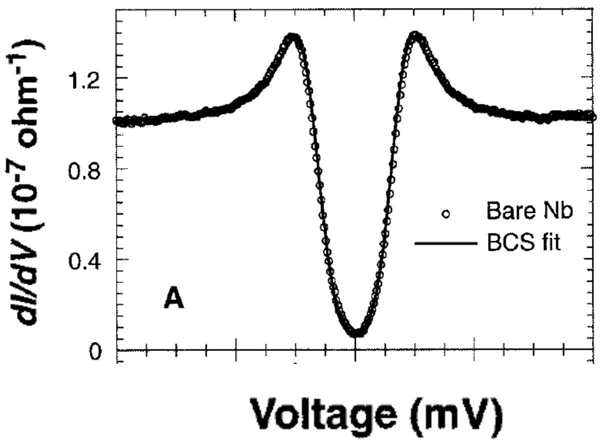
- thermal quasiparticles suppressed exponentially by SC gap
- broadening by quasiparticle poisoning less malignent due to coupling to BCS singularity
- double resonance of Andreev processes for Majorana state
- two Majorana peaks: $eV = \pm\Delta_t$

> ➢ peak quantization ?
> ➢ symmetry ?

Symmetry of Majorana peaks

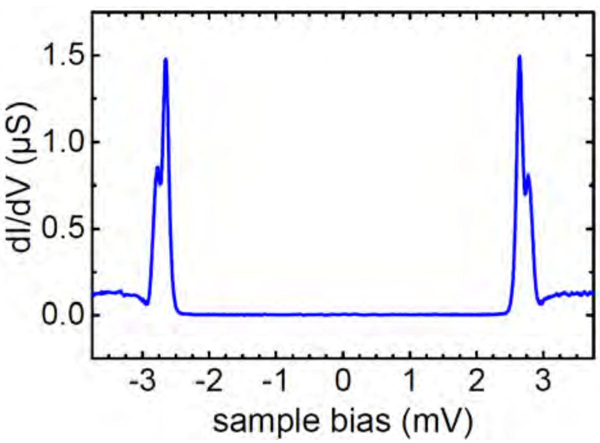
- Majorana wf is particle-hole symmetric: u=v*
- symmetry of BCS peaks in DOS

normal tip



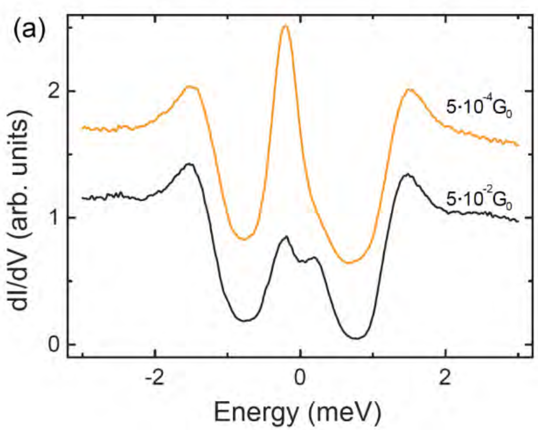
Yazdani et al. Science 1997

SC tip (Pb on Pb)



Franke group

Shiba states

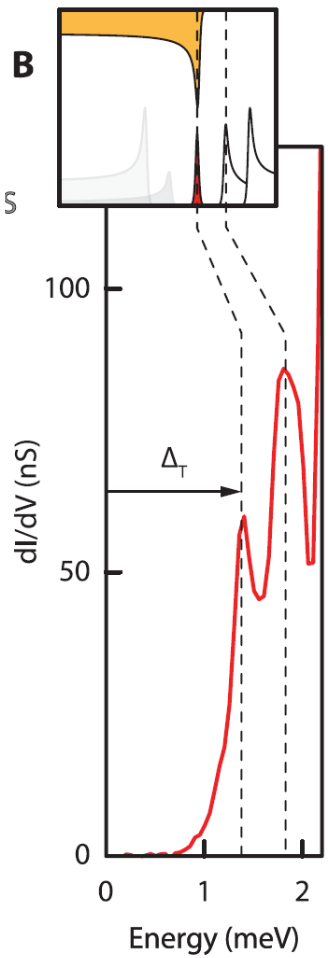


Franke group 2015

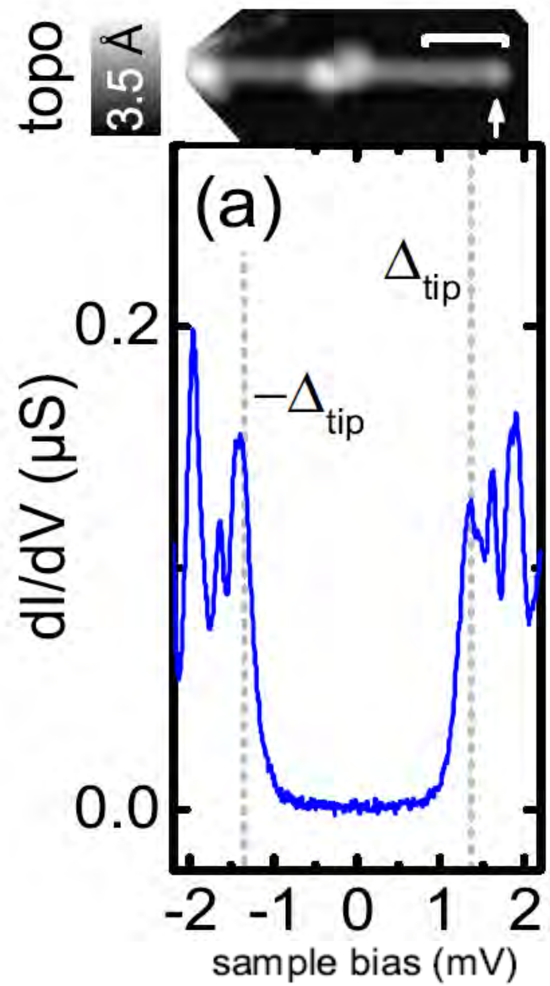
Majorana peaks should be symmetric!

Majorana symmetry

Freie Universität Berlin



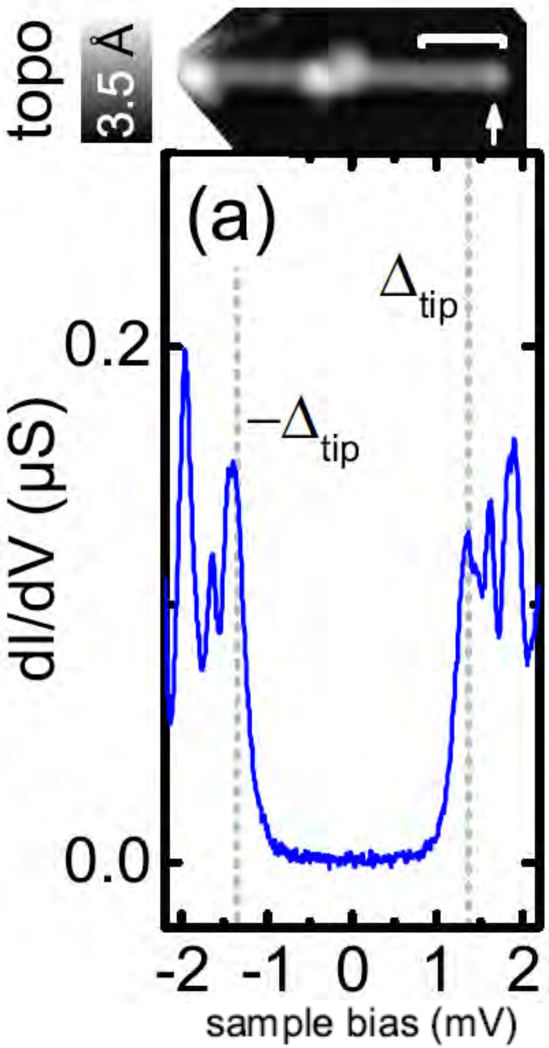
?

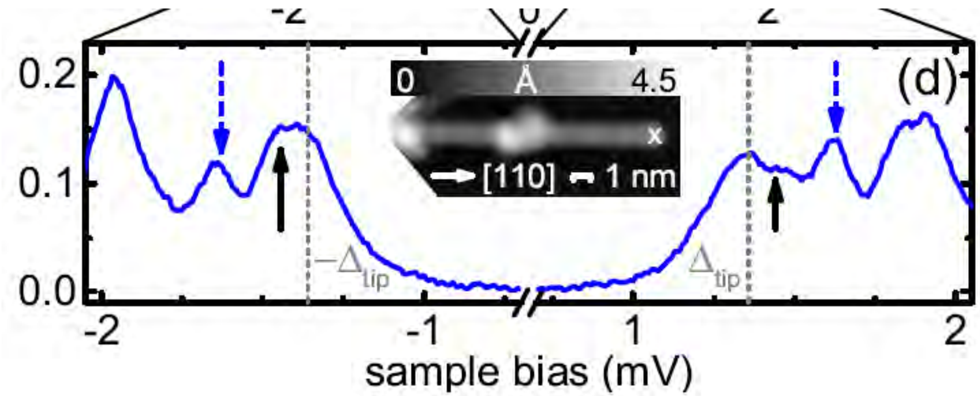


Yazdani group
Science 2014

Ruby, Pientka, Peng, FvO, Heinrich, Franke,
PRL **115**, 197204 (2015)

Experiment





- Majorana peaks overlap w/ finite-energy peaks at ~80μeV
- previously unresolved
- indication for smaller induced gap
- BUT: could explain asymmetry

Ruby, Pientka, Peng, FvO, Heinrich, Franke, PRL **115**, 197204 (2015)

Yazdani lab

Freie Universität Berlin

ARTICLES

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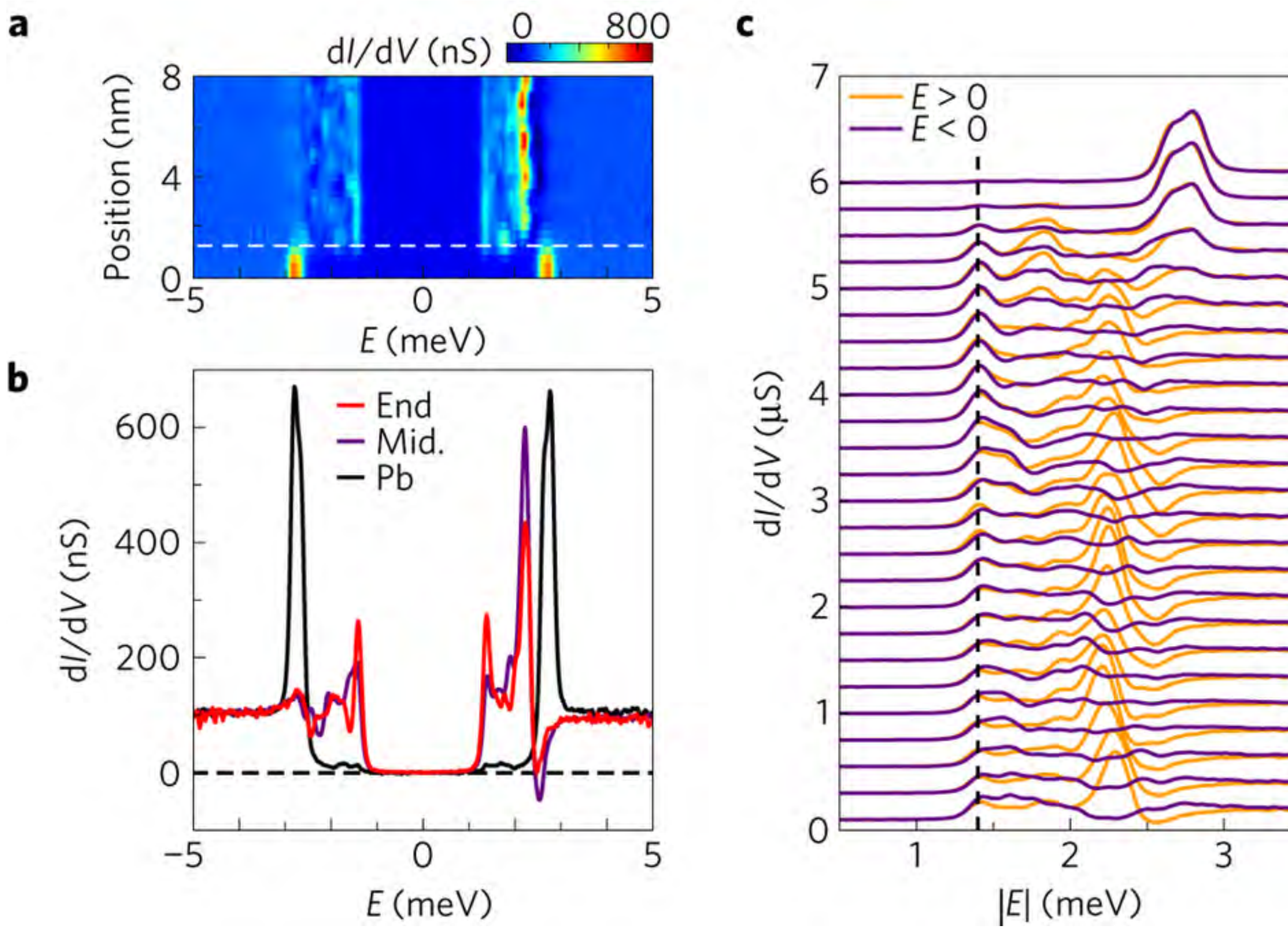
nature physics

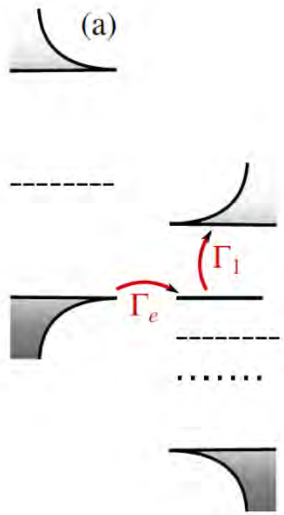
High-resolution studies of the Majorana atomic chain platform

Benjamin E. Feldman[†], Mallika T. Randeria[†], Jian Li[†], Sangjun Jeon, Yonglong Xie, Zhijun Wang, Ilya K. Drozdov[†‡], B. Andrei Bernevig and Ali Yazdani*

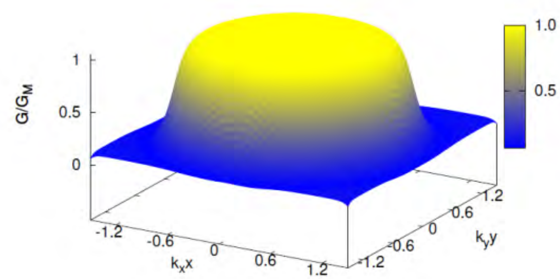


Conductance quantization

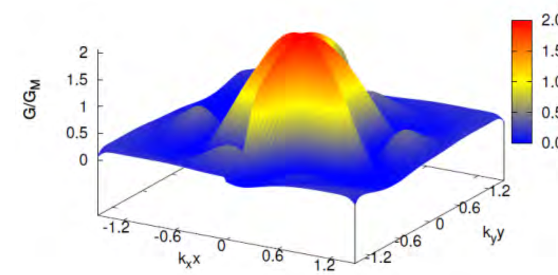
single particle



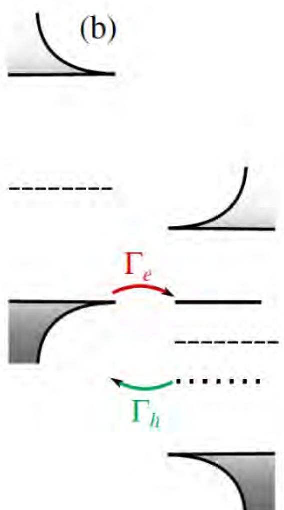
Majorana



non-Majorana

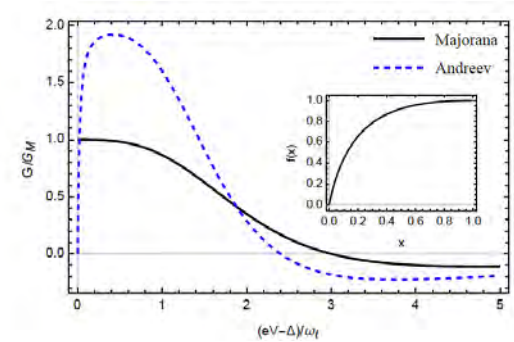


Andreev



Threshold conductance

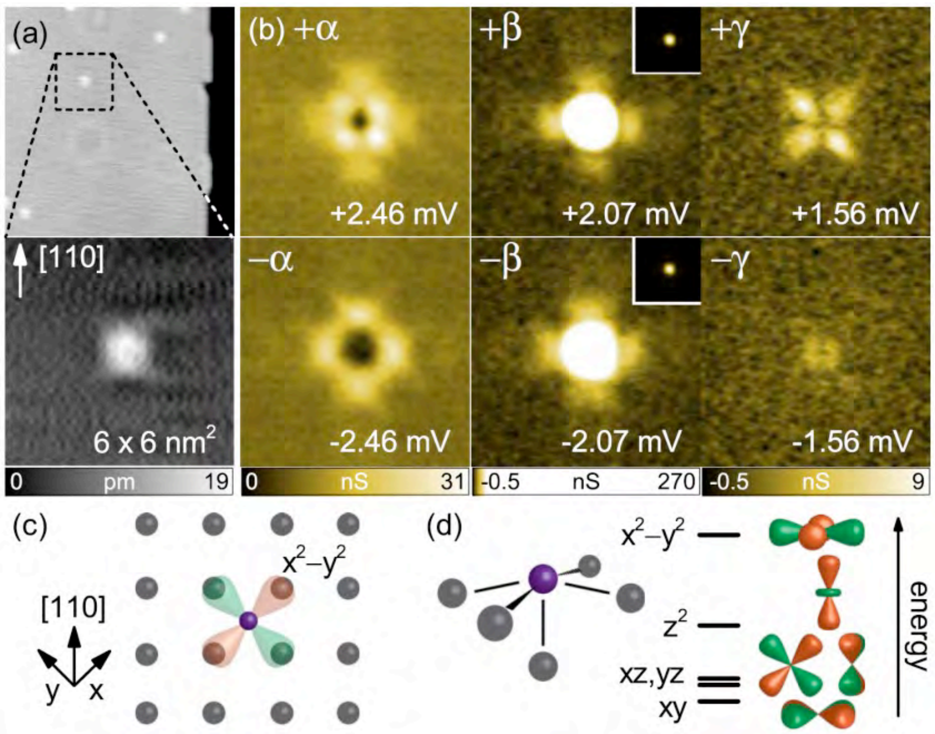
$$G_{\mathrm{M}} = (4-\pi)\frac{2e^2}{h}$$



Y Peng, F Pientka, Y Vinkler-Aviv, L Glazman, FvO, PRL **115**, 266804 (2015)

Orbital picture of Shiba multiplets

Mn on Pb (001)

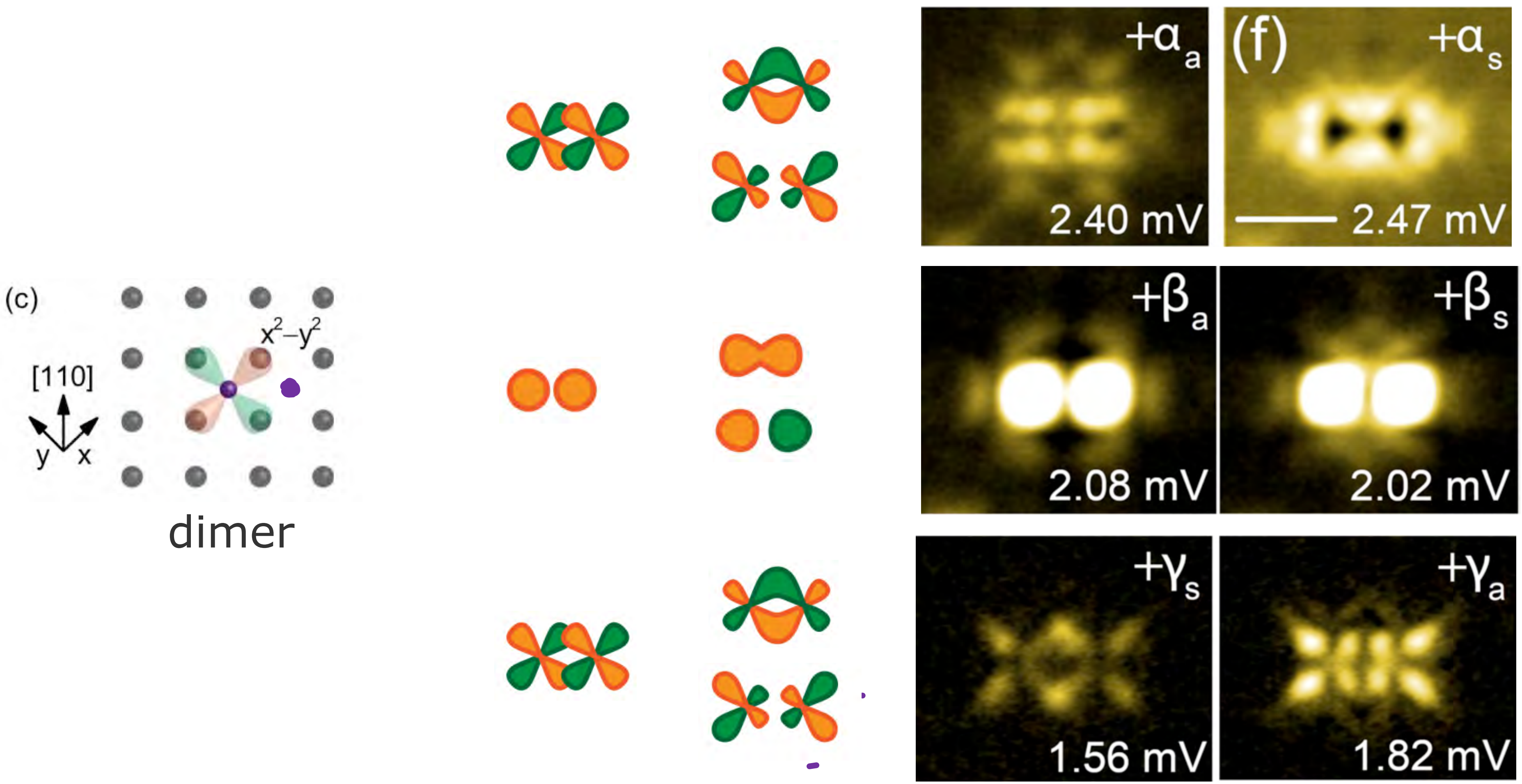


Multiplicity of Shiba states:

- Mn^{++} in $^6S_{5/2}$ configuration
- scatters in l=2 channel
- splitting according to crystal field
- explains d-orbital shape & Shiba multiplicity
- ALSO: Mn on Pb(111)

Ruby, Peng, FvO, Heinrich, Franke, PRL **117**, 186801 (2016)

Shiba molecules



Ruby, Peng, FvO, Heinrich, Franke, unpublished

Fe *d*-levels in adatom chains

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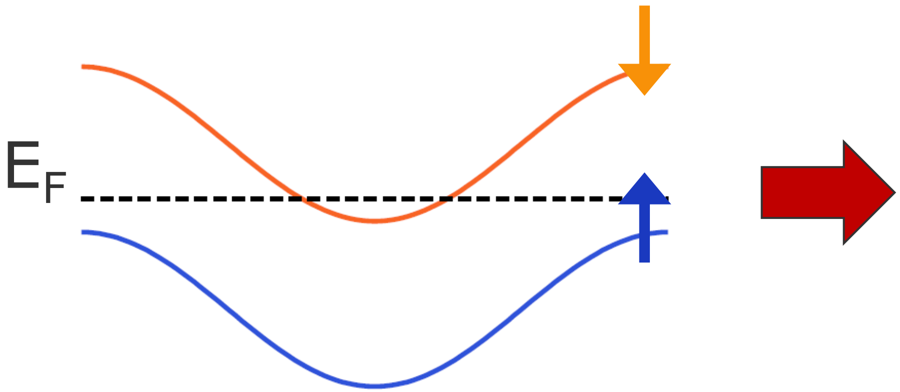
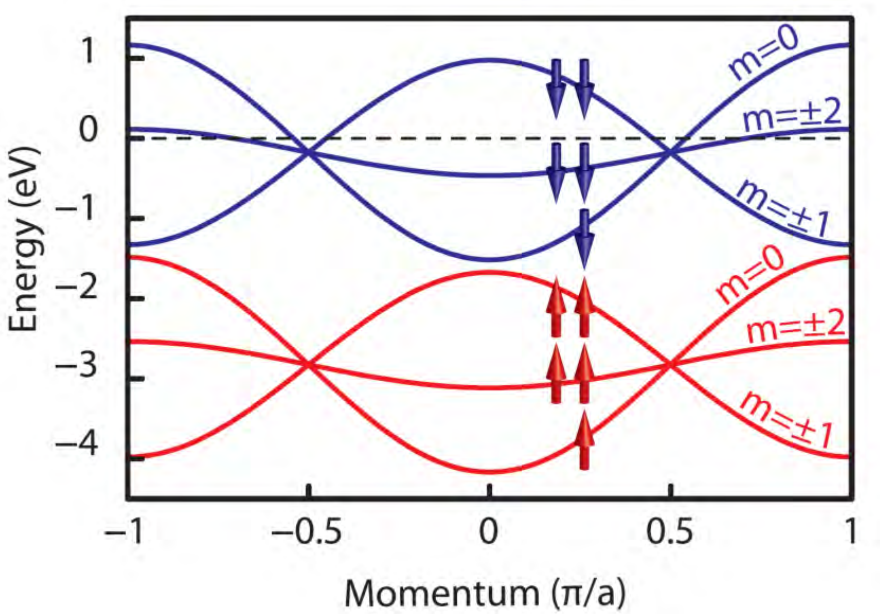


Nadj-Perge et al. Science 2014;

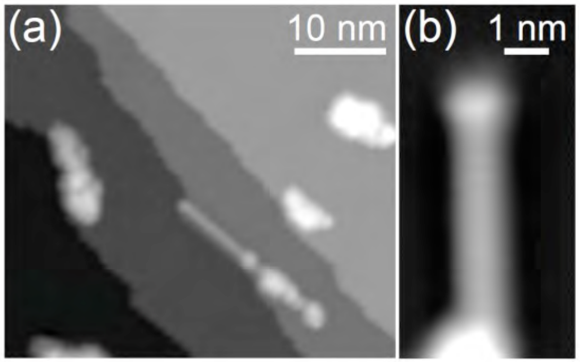
Li et al. PRB 2014



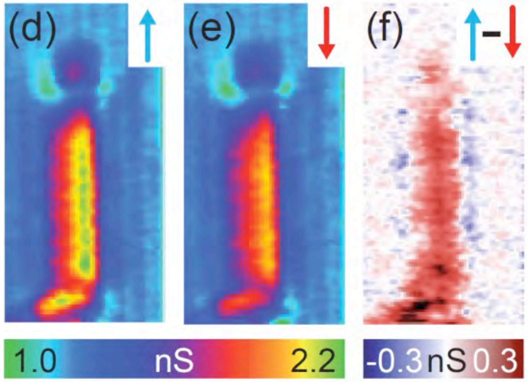
- overlap of Fe *d*-levels!
- Slater-Coster parameters
- odd number of channels

Cobalt chains – magn. properties

Co on Pb(110)

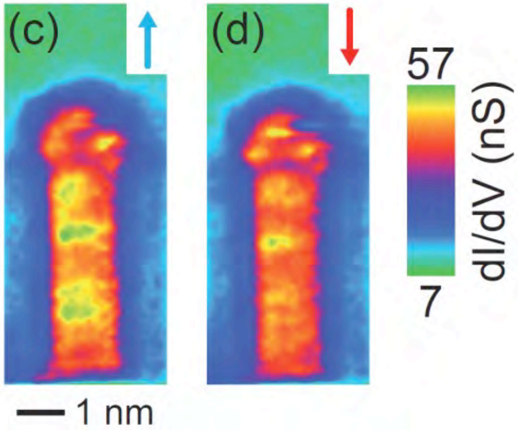


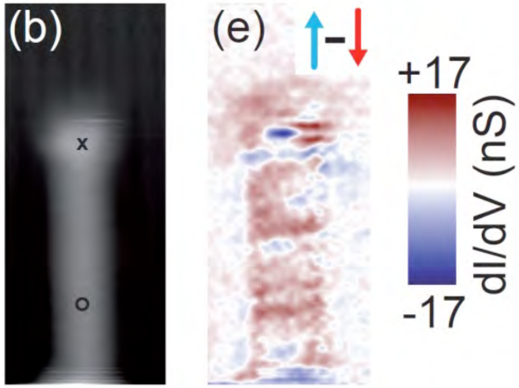
Spin polarization: eV=170meV



spin-polarized d bands

Spin polarization: eV=-850μeV





spin-polarized Shiba states

Cobalt chains

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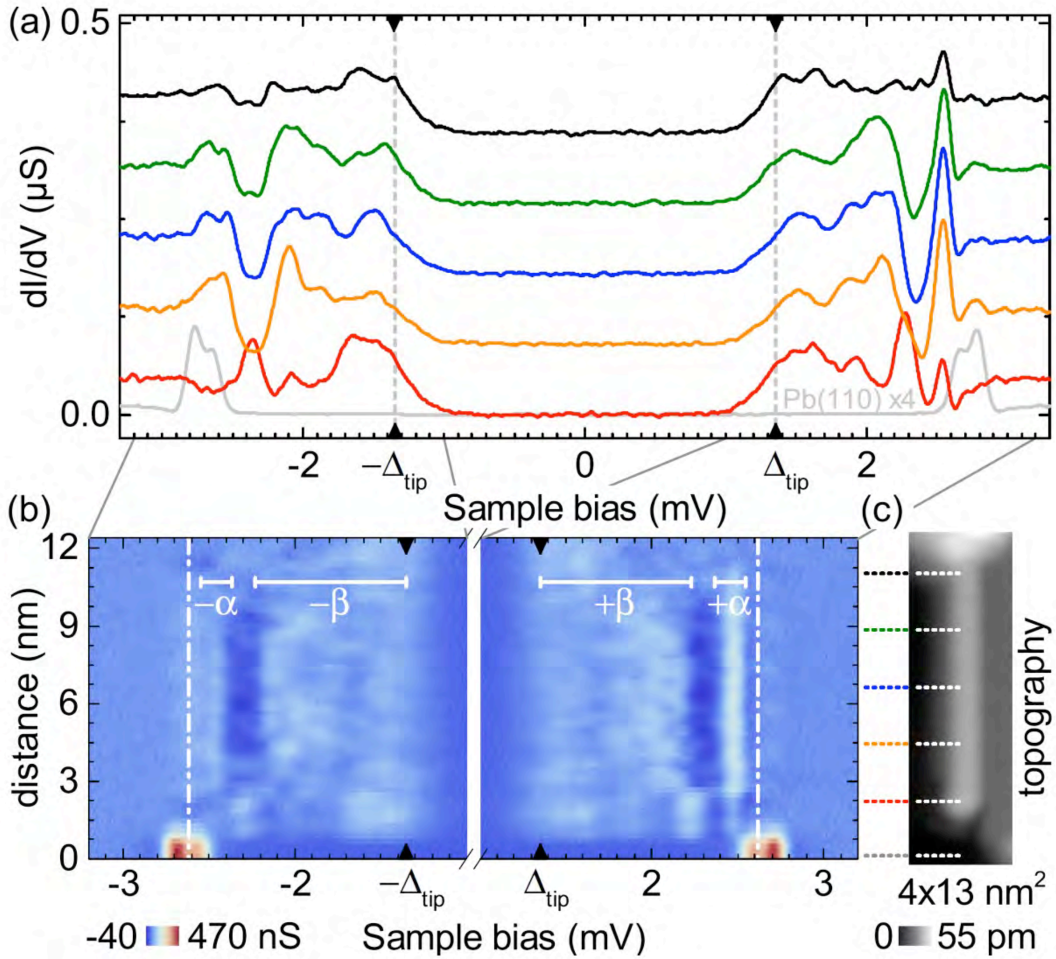
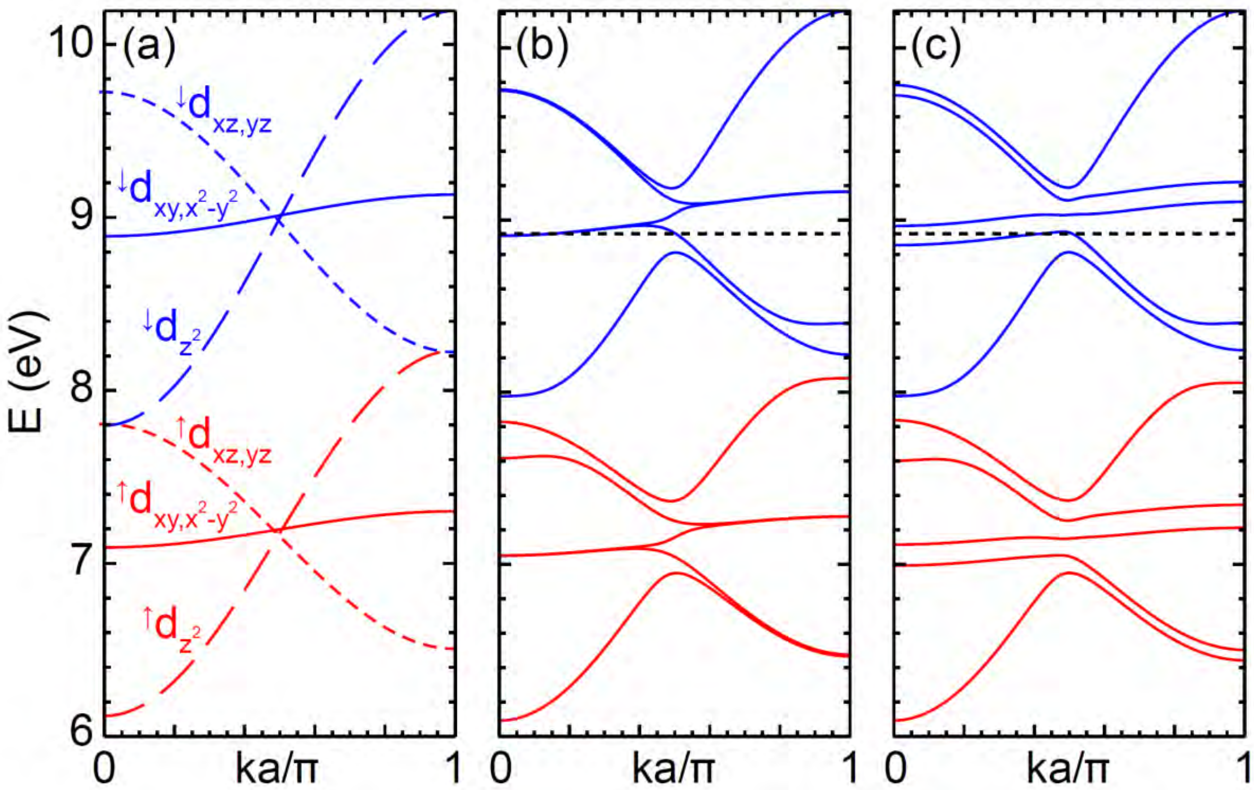


M Ruby, B Heinrich, Y Peng, FvO, K Franke, Nano Lett. **17**, 6 (2017)

Cobalt chains

Freie Universität Berlin



Cobalt:
[Ar] $3d^7\ 4s^2$

Compare: Fe
[Ar] $3d^6\ 4s^2$

1d band structure (see also Li et al. PRB 2014)

- Slater-Coster tight binding
- s-bands empty into Pb substrate
- even number of channels

M Ruby, B Heinrich, Y Peng, FvO, K Franke, Nano Lett. **17**, 6 (2017)

Towards nonabelian statistics

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topological QC

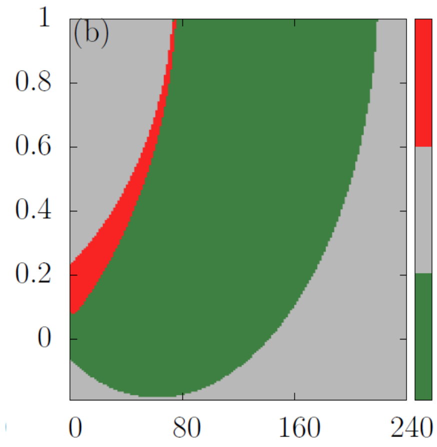
non-Abelian statistics

zero-energy Majoranas

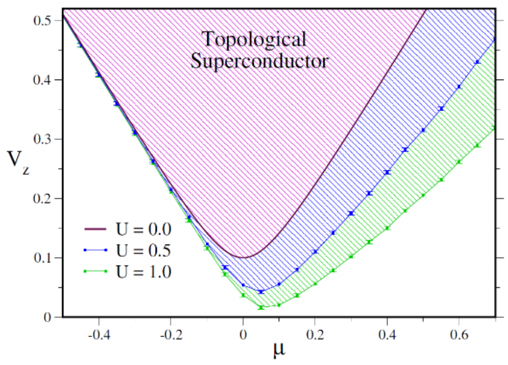
?

2012

Adatom chain



Semiconductor quantum wires



1eV typ. energies 1meV

Conclusions

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- Localization of Majorana bound states: strong coupling to superconductor strongly renormalizes Majorana localization length

  Y Peng, F Pientka, L Glazman, FvO, PRL **114**, 106801 (2015)

- Superconducting lead/tip: symmetry of conductance peaks at $eV=\pm\Delta$

  M Ruby, F Pientka, Y Peng, FvO, B Heinrich, K Franke, PRL **115**, 197204 (2015)
  M Ruby, F Pientka, Y Peng, FvO, B Heinrich, K Franke, PRL **115**, 087001 (2015)

- Superconducting lead/tip: universal conductance peaks at $eV=\pm\Delta$ and plateau behavior

  Y Peng, F Pientka, Y Vinkler-Aviv, L Glazman, FvO, PRL **115**, 266804 (2015)

- Orbital picture of Yu-Shiba-Rusinov multiplets

  M Ruby, Y Peng, FvO, B Heinrich, K Franke, PRL 186801 (2016)
  & unpublished

- Apparent absence of Majorana bound states in cobalt chains

  M Ruby, B Heinrich, Y Peng, FvO, K Franke, Nano Lett. **17**, 6 (2017)