QUANTUM CHARGE AND ENERGY TRANSPORT IN MESOSCOPIC SYSTEMS AND TOPOLOGICAL NANOSTRUCTURES

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Mainz 2018

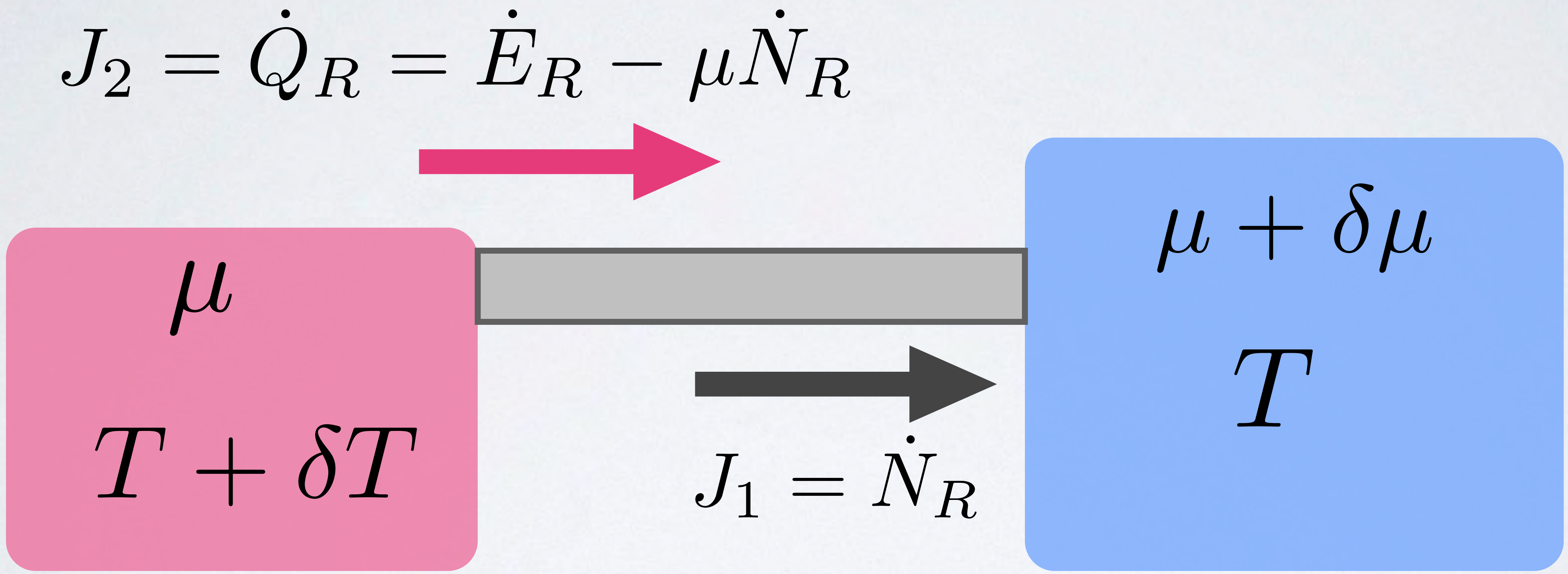
PLAN

- Thermoelectric effects in edge states of 2D topological systems: quantum Hall and quantum spin Hall.
- Dissipation of energy in driven quantum dots.
- (Role of many-body interactions)

DC HEAT-WORK CONVERSION:

THERMOELECTRICITY

TWO-TERMINAL DEVICE



Conservation laws: $\dot{N}_L = -\dot{N}_R, \qquad \dot{E}_L = -\dot{E}_R.$

Linear response:

$$J_j = \sum_{i=1}^{2} L_{ji} X_i. \qquad X_1 = \delta\mu/T \text{ and } X_2 = \delta T/T^2$$

Fluxes

Affinities

Onsager relations: linear response=>micro reversibility (encoded in transport coefficients L)

$$L_{11}(B) = L_{11}(-B), \qquad L_{12}(B) = L_{21}(-B)$$

Rate of entropy production:

$$\dot{S} = \frac{\dot{Q}_L}{T_L} + \frac{\dot{Q}_R}{T_R},$$

$$\dot{Q}_\alpha = \dot{E}_\alpha - \mu_\alpha \dot{N}_\alpha$$

2nd principle:

$$\dot{\mathcal{S}} = \mathbf{X}^{\mathrm{t}} \cdot \mathbf{L} \cdot \mathbf{X}.$$

$$L_{11}, \quad L_{22} > 0$$

$$L_{11}L_{22} - L_{12}L_{21} > 0$$

EFFICIENCY

dc- Heat engine: electrical power/heat flux

$$\eta = \frac{eTJ_1X_1}{J_2} \leq \eta_C, \qquad \eta_C = \frac{\delta T}{T}$$

dc- Heat pump: heat flux/electrical power

$$\eta = \frac{-J_2}{eTJ_1X_1} \leq \eta_C, \qquad \eta_C = \frac{T}{\delta T}$$

Maximum efficiency for a given diff of temperature :

$$\eta = \eta_C \, \frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+1} \qquad ZT = \frac{L_{12}L_{21}}{det\mathbf{L}}$$

Figure of merit

STATUS OF APPLICATIONS

Fundamental aspects of steady-state conversion of heat to work at the nanoscale

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[d] *Department of Physics, Keio University 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan*
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Desired efficiency	Necessary ZT
Carnot efficiency	∞
$9/10\times$ Carnot efficiency	360
$3/4\times$ Carnot efficiency	48
$1/2\times$ Carnot efficiency	8
$1/3\times$ Carnot efficiency	3
$1/6\times$ Carnot efficiency	$24/25 \sim 1$
$1/10\times$ Carnot efficiency	$40/81 \sim 0.5$
$1/100\times$ Carnot efficiency	$400/9801 \sim 0.04$

Table 1: Examples of the dimensionless figure of merit ZT necessary for a desired heat-engine efficiency, see Eq. (27). This connection between the maximum efficiency and ZT is convenient, as it is easier to calculate ZT from basic transport measurements than to measure the maximum efficiency directly. Current bulk semiconductor thermoelectric have $ZT \sim 1$, while a $ZT \sim 3$ would be necessary for most industrial or household applications. However the connection between maximum efficiency and ZT only exists in the linear-response regime, as ZT has no meaning outside the linear-response regime.

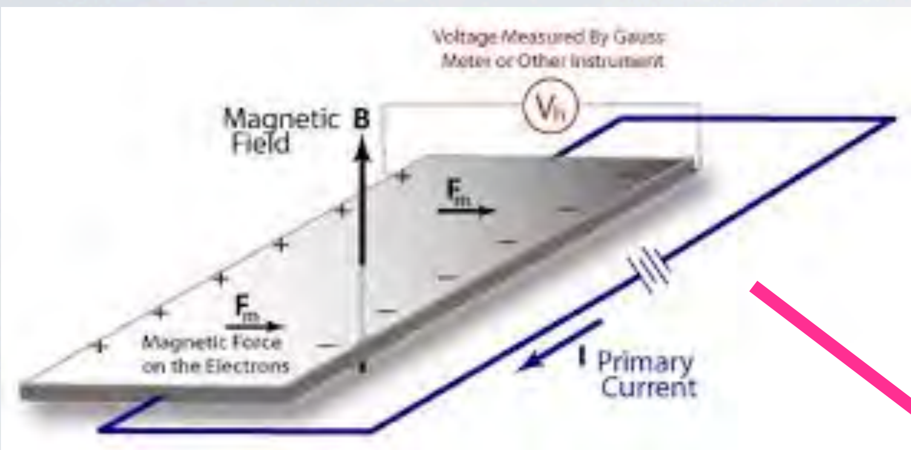
GO QUANTUM

PARADIGM:

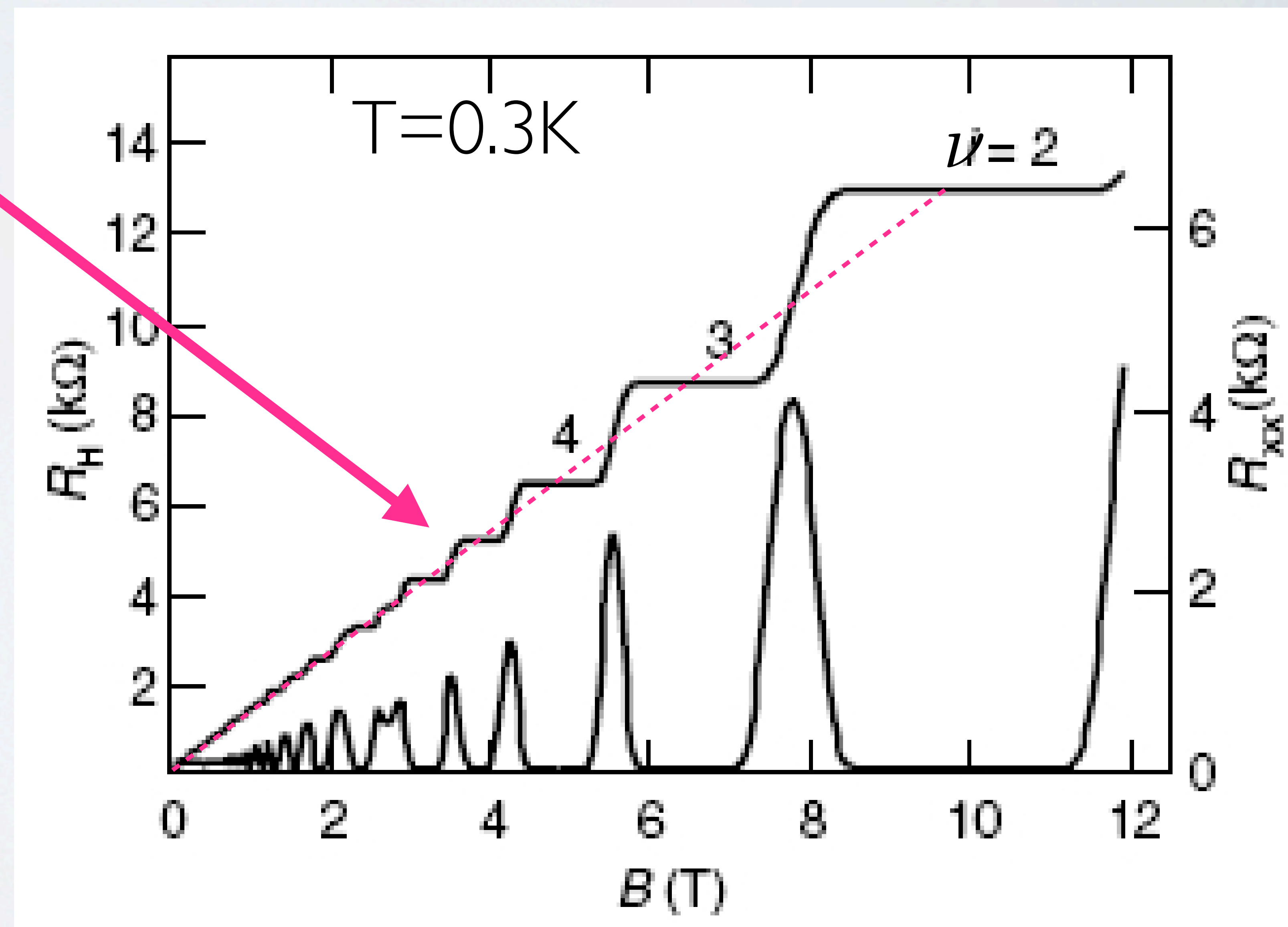
QUANTUM HALL EFFECT

QUANTUM HALL EFFECT

von Klitzing et al PRL '80



2DEG in GaAs-GaAlAs structures

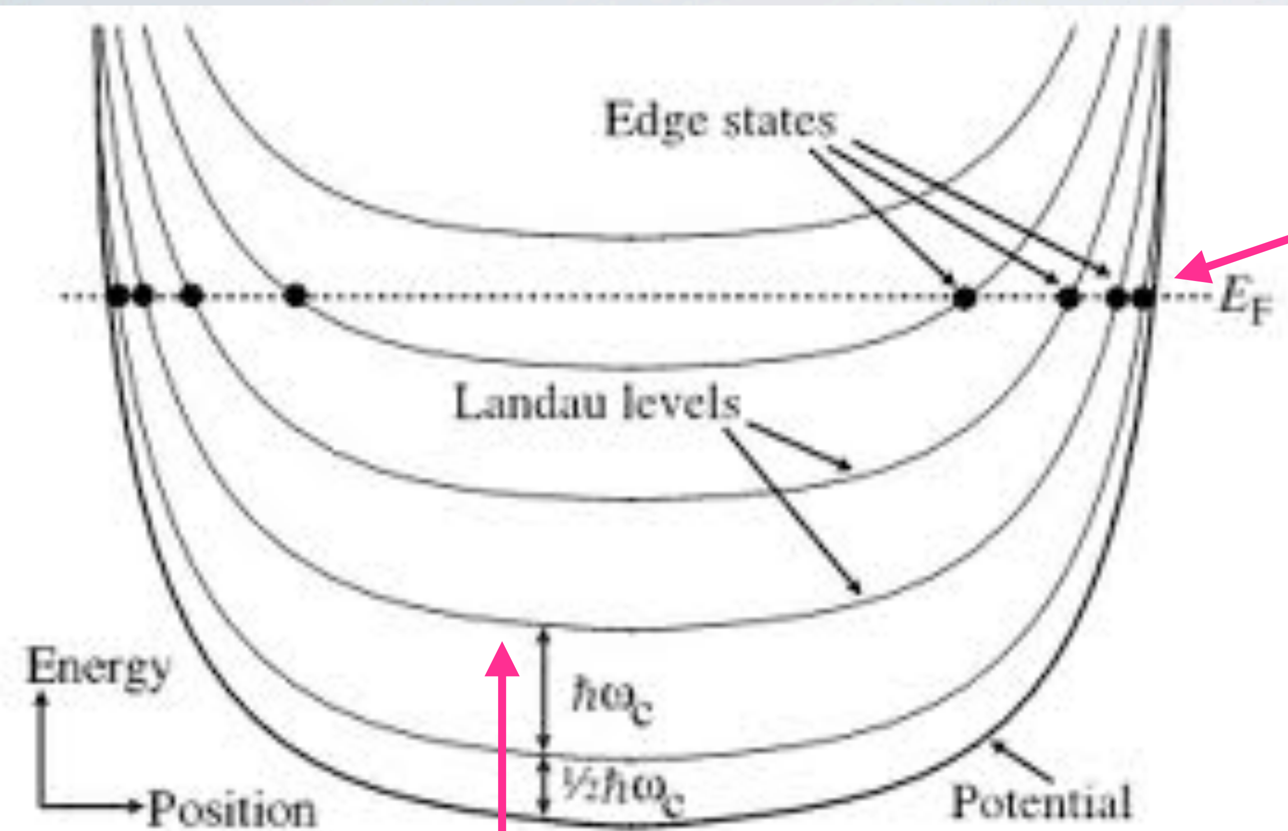


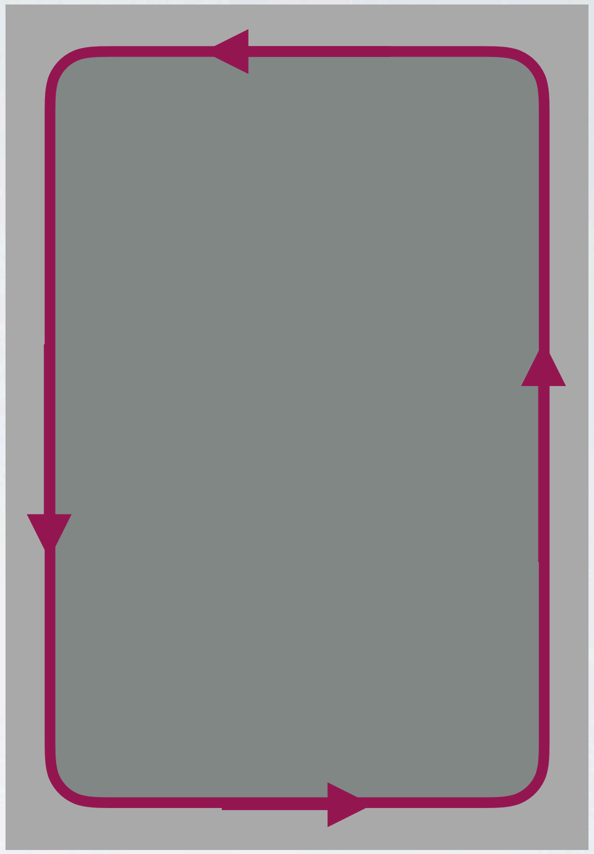
$$\nu = 1, 2, 3....$$

$$\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$$

EDGE STATES IN QUANTUM HALL EFFECT

Laughlin, PRB 23, 5632 (1981); Halperin, PRB 25, 2185 (1982)





$\nu = 1$

Edge states:

Topologically protected chiral propagation of charge

$$H = -iv \int dx \psi^\dagger(x)\, \partial_x\, \psi^\dagger(x)$$

Bulk: Landau levels

$$E_n = \hbar\omega_c(n + \frac{1}{2}), \;\; \omega_c = \frac{eB}{mc}, \;\; l_B = \sqrt{\frac{\hbar c}{eB}}$$

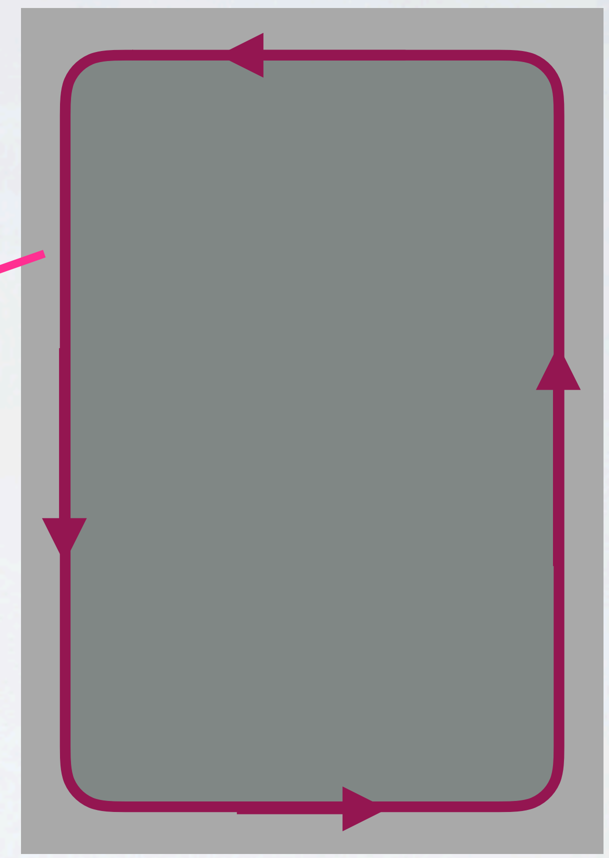
FRACTIONAL QUANTUM HALL EFFECT

Wen, PRB 41, 12838 (1990)

Laughlin series: $\nu = 1/m \quad m$ odd → Charge fractionalization

$$e^* = \frac{e}{m}$$

Edge states: Chiral Luttinger liquid



$$H = \frac{v\pi}{\nu} \int dx \rho^2(x)$$

$e \rightarrow m_\alpha e^*$

$$\rho(x) = \frac{\partial_x \phi(x)}{2\pi},$$

Chiral bosons

$$[\phi(x), \phi(x')] = -i\pi\nu sg(x - x')$$

Fermions

$$\psi(x) = \frac{F}{\sqrt{2\pi a}} e^{\frac{i}{\nu}\phi(x,t)}$$

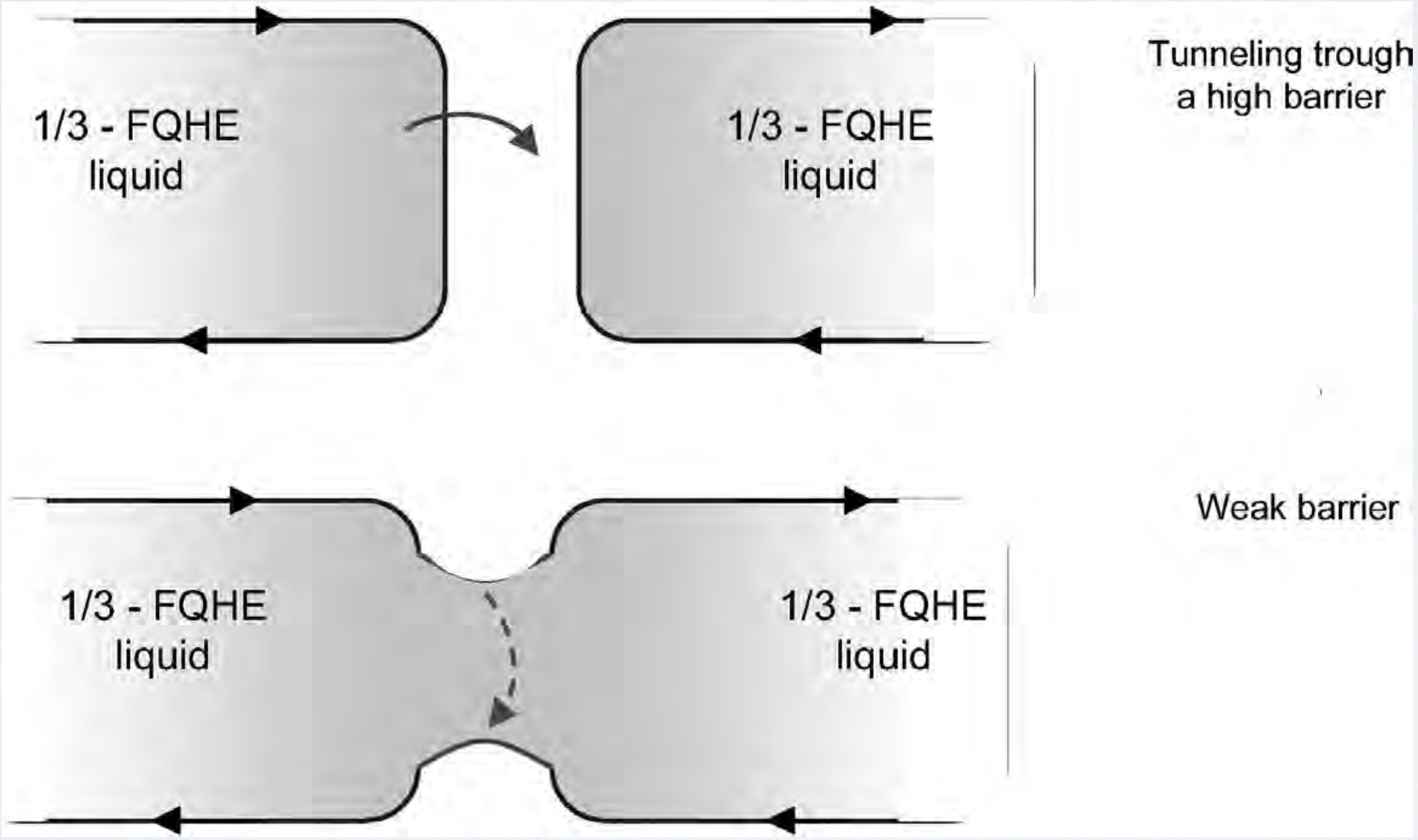
Books by E. Fradkin and by T. Giamarchi

CHARGE TRANSPORT IN TUNNELING CONTACTS OF THE QUANTUM HALL EFFECT

TUNNELING CONTACTS IN THE FRACTIONAL QUANTUM HALL EFFECT

C. Glattli, Séminaire Poincaré 2, 75 (2004)

Milliken, Umbach, Webb (1994), Chang, Pfeiffer, West (1996)



Electron tunneling e

$$G(T) \propto T^{2m-2}$$

Quasiparticle tunneling e*= e/m

$$G(T) \propto T^{2/(m-2)}$$

Figure 8: Schematic view of charge transfer in the case of a strong barrier (upper figure) and a weak barrier. In the later case the FQHE fluid is weakly perturbed and charge transfer occurs via the FQHE fluid.

THERMAL TRANSPORT IN THE QUANTUM HALL EFFECT

OBSERVATION OF CHIRAL HEAT TRANSPORT THROUGH THE EDGE STATES

G. Granger, J. P. Eisenstein and J. L. Reno, Phys. Rev.Lett. 102, 086803 (2009)

$\nu = 1$

"heater"

voltage probe $V_0(t) = \mu_0 + \sum_{k\neq 0} e^{-i(k\Omega_0 t+\varphi_k)} V_0^{(k)}/2$

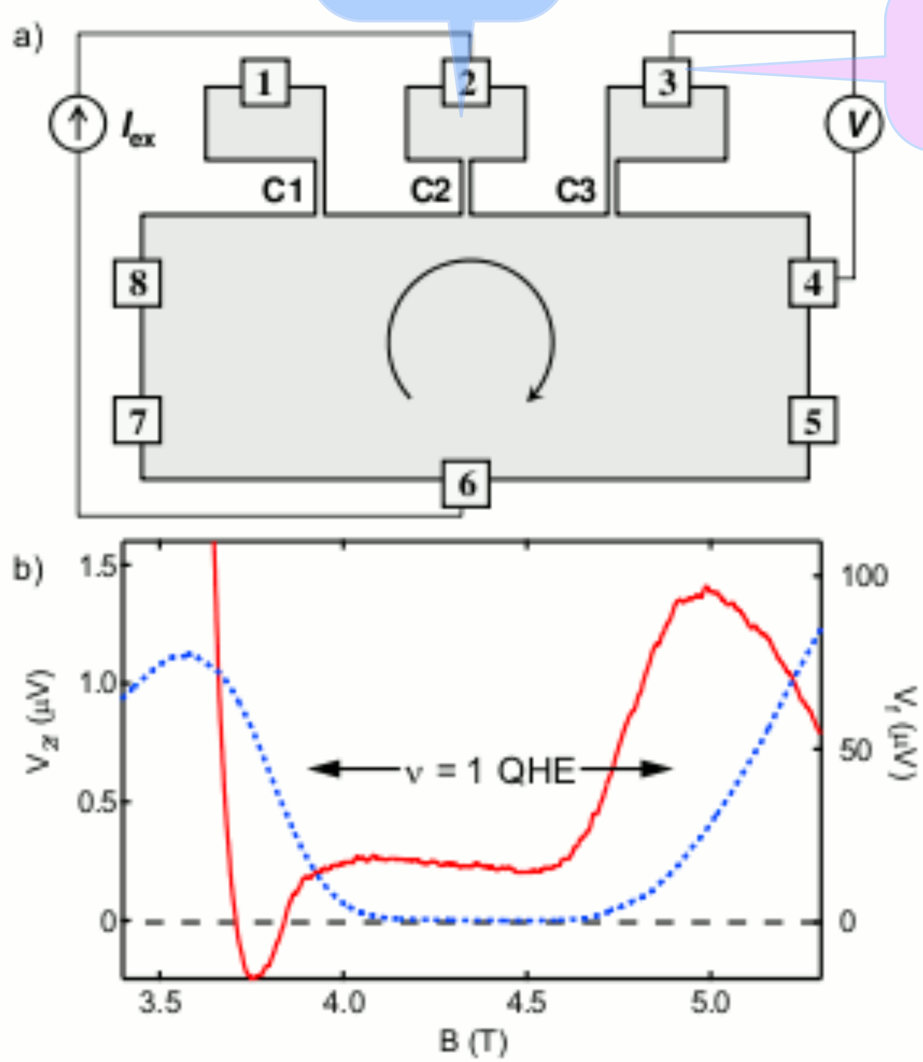


FIG. 1: (color online) (a) Schematic diagram, not to scale, of device layout. Numbered squares represent ohmic contacts; C1, C2, and C3 represent constrictions in the 2DES. (b) V_f (dotted) and V_{2f} (solid) vs. magnetic field B at $T = 0.1$ K in the $\nu = 1$ QHE for the measurement circuit indicated. Edge state chirality is clockwise as shown.

Different signals up and downstream for k=1,2
k=2 assumed to be a good sensor of heat

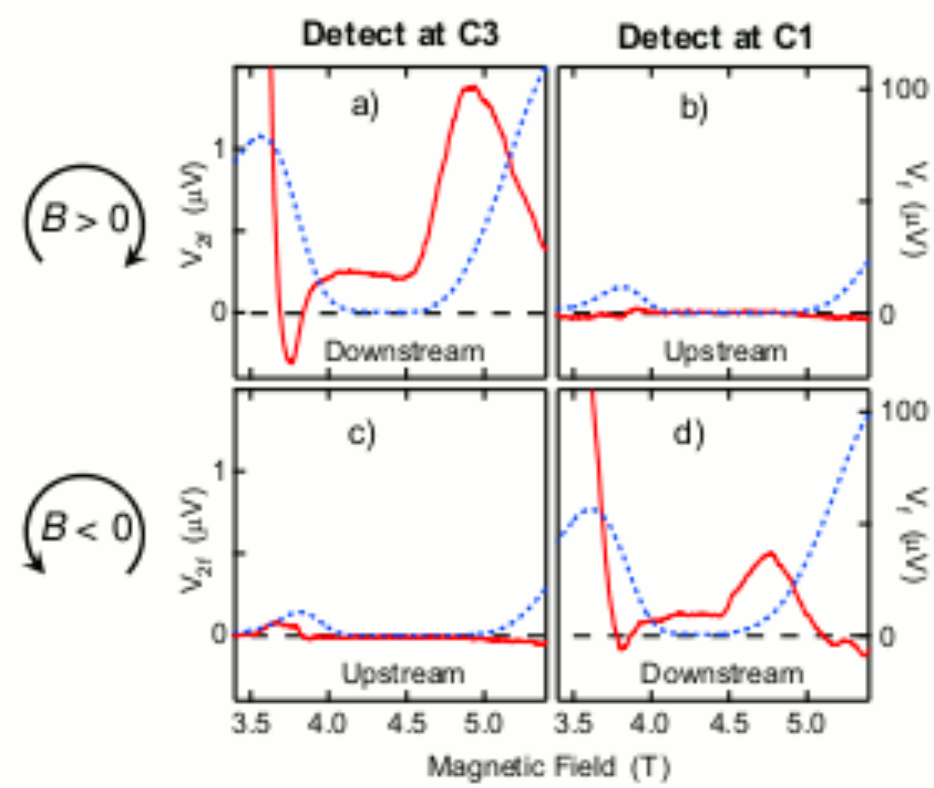


FIG. 3: (color online) Chirality of thermal transport at $\nu = 1$ at $T = 0.1$ K. a) and d): V_{2f} (solid) and V_f (dotted) observed downstream from heater constriction, C2. b) and c) V_{2f} and V_f observed upstream from heater. Edge state chirality and magnetic field directions are indicated.

Chargeless heat transport in the fractional quantum Hall regime

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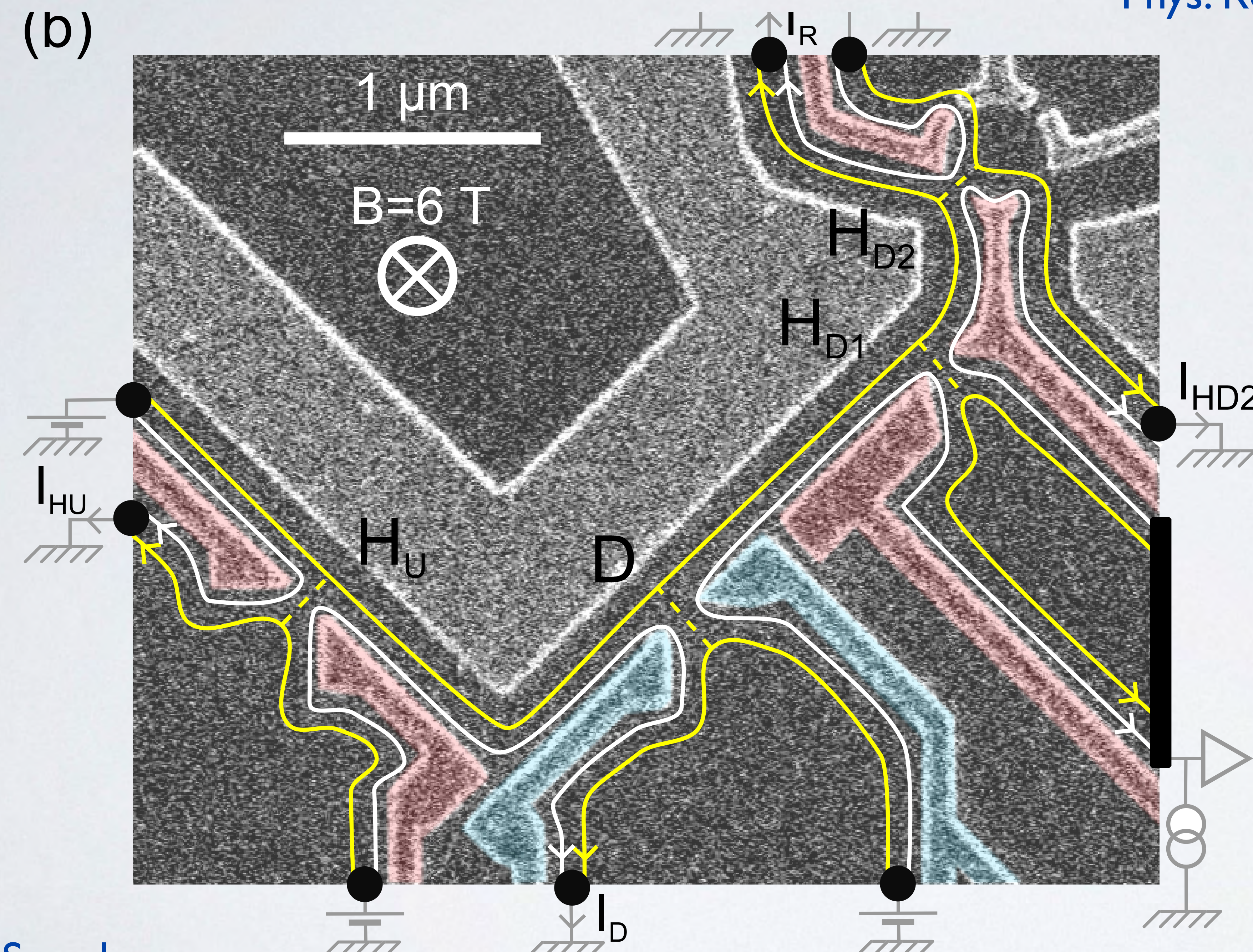
[1] *CNRS / Univ Paris Diderot (Sorbonne Paris Cité), Laboratoire de Photonique et de Nanostructures (LPN), route de Nozay, 91460 Marcoussis, France*

(Dated: February 29, 2012)

Phys. Rev.Lett. 109, 026803 (2012)



$\nu = 2/3$

Charged and neutral edge modes

See also:
V.Venkatachalam, Nat.Phys. 8, 676 (2012)
H. Inoue, et al, Nat. Comm. (2014)

Quantum Limit of Heat Flow Across a Single Electronic Channel

$$\nu = 1, 2$$

S. Jezouin,[1*] F. D. Parmentier,[1*] A. Anthore,[1,2†] U. Gennser,[1] A. Cavanna,[1] Y. Jin,[1] F. Pierre[1†]

Quantum physics predicts that there is a fundamental maximum heat conductance across a single transport channel and that this thermal conductance quantum, G_Q, is universal, independent of the type of particles carrying the heat. Such universality, combined with the relationship between heat and information, signals a general limit on information transfer. We report on the quantitative measurement of the quantum-limited heat flow for Fermi particles across a single electronic channel, using noise thermometry. The demonstrated agreement with the predicted G_Q establishes experimentally this basic building block of quantum thermal transport. The achieved accuracy of below 10% opens access to many experiments involving the quantum manipulation of heat.

$$\kappa = \frac{\pi^2 k_B}{6h} T$$

Science 342, 601 (2013)
See also M. Banerjee, et al
Nature 545 (2017)

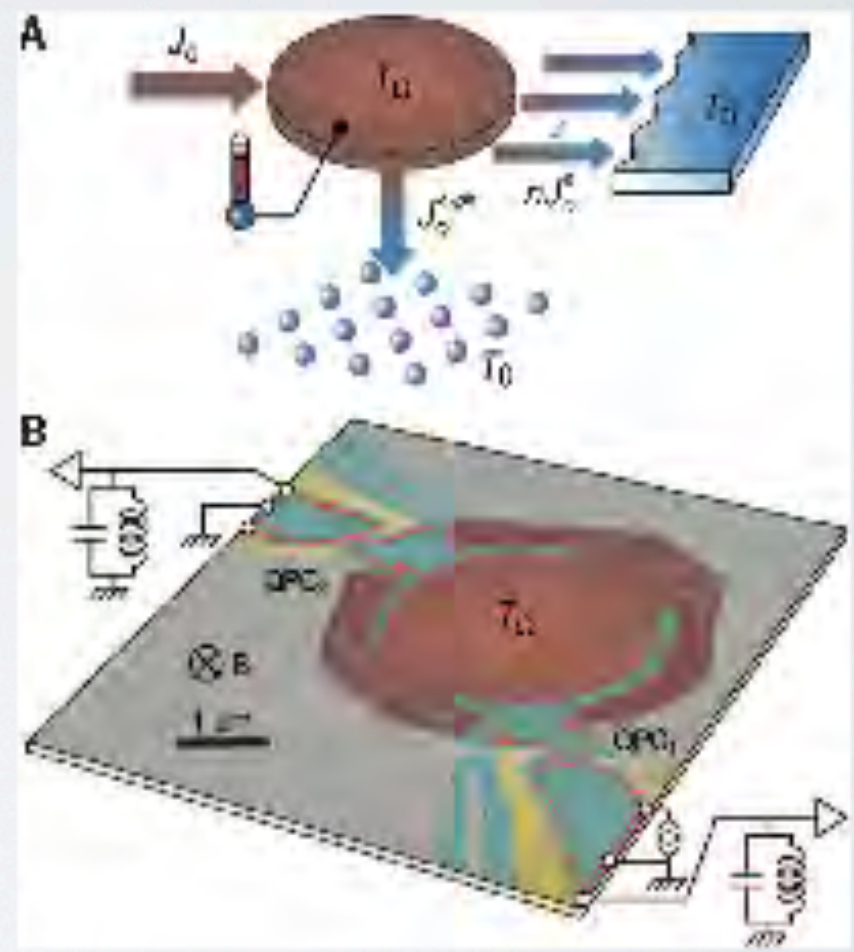


Fig. 1. Experimental principle and practical implementation. (**A**) Principle of the experiment: Electrons in a small metal plate (brown disk) are heated up to T_Ω by the injected Joule power J_Q. The large arrows symbolize injected power (J_Q) and outgoing heat flows (nJ_Q^e, $J_Q^{e\text{-}ph}$). (**B**) False-colors scanning electron micrograph of the measured sample. The Ga(Al)As 2D electron gas is highlighted in light blue, the QPC metal gates in yellow and the micrometer-sized metallic ohmic contact in brown. The light gray metal gates are polarized with a strong negative gate voltage and are not used in the experiment. The propagation direction of two copropagating edge channels (shown out of $\nu = 3$ or $\nu = 4$) is indicated by red arrows. QPC_1 is here set to fully transmit a single channel ($n_1 = 1$) and QPC_2 two channels ($n_2 = 2$), corresponding to a total number of open electronic channels $n = n_1 + n_2 = 3$. The experimental apparatus is shown as a simplified diagram. It includes two $L-C$ tanks used to perform the noise thermometry measurements around 700 kHz. The Joule power J_Q is injected on the micrometer-sized metallic electrode from the DC polarization current partly transmitted through QPC_1.

Energy Partitioning of Tunneling Currents into Luttinger Liquids

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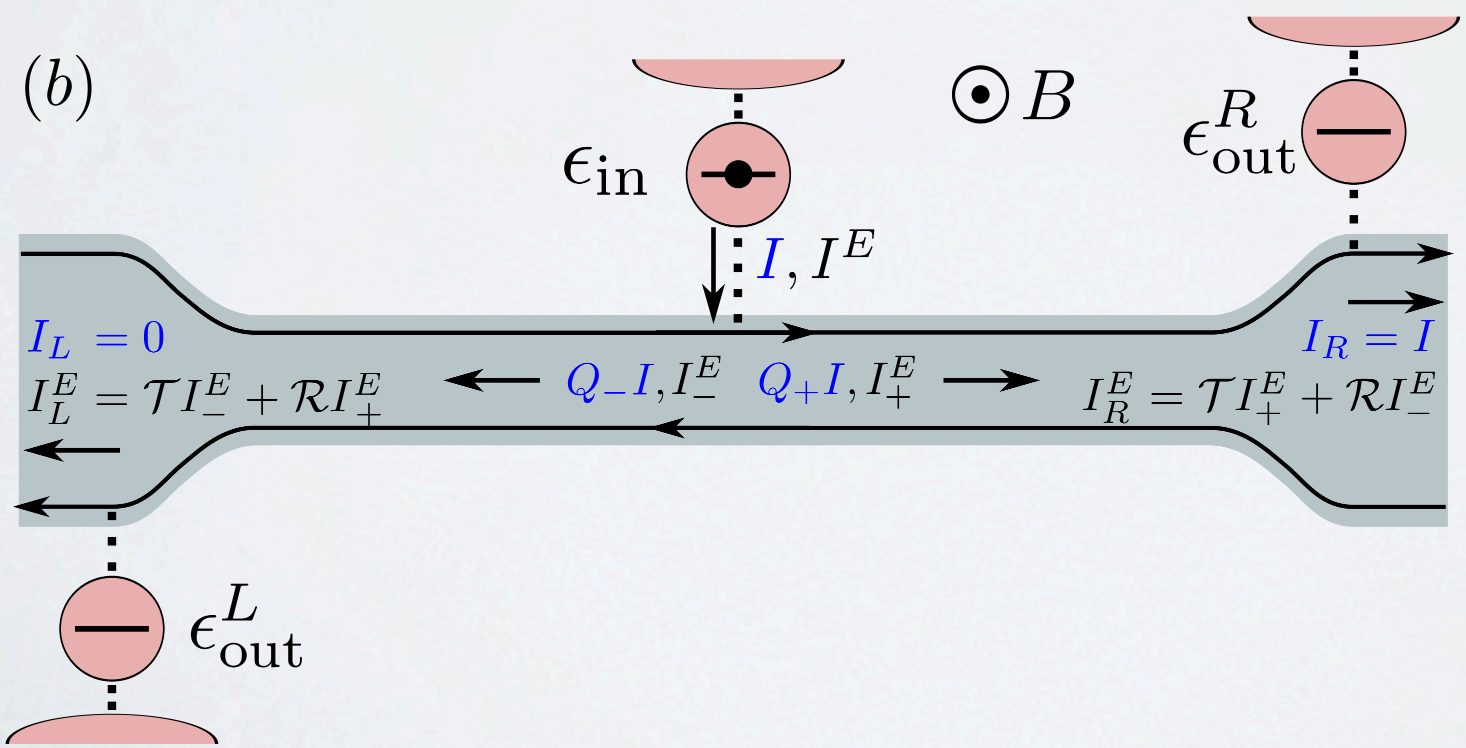
[3]*Department of Physics, Yale University, 217 Prospect Street, New Haven, Connecticut 06520, USA*

(Dated: May 3, 2018)

Tunneling of electrons of definite chirality into a quantum wire creates counterpropagating excitations, carrying both charge and energy. We find that the partitioning of energy is qualitatively different from that of charge. The partition ratio of energy depends on the excess energy of the tunneling electrons (controlled by the applied bias) and on the interaction strength within the wire (characterized by the Luttinger liquid parameter κ), while the partitioning of charge is fully determined by κ. Moreover, unlike for charge currents, the partitioning of energy current should manifest itself in *dc* experiments on wires contacted by conventional (Fermi-liquid) leads.

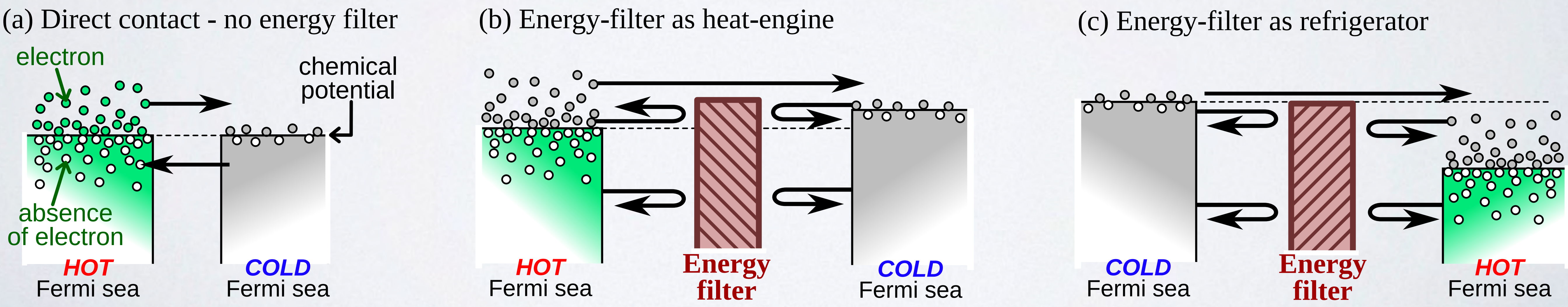
PRL 107, 176403 (2011)



THERMOELECTRIC PERFORMANCE OF QHE STRUCTURES

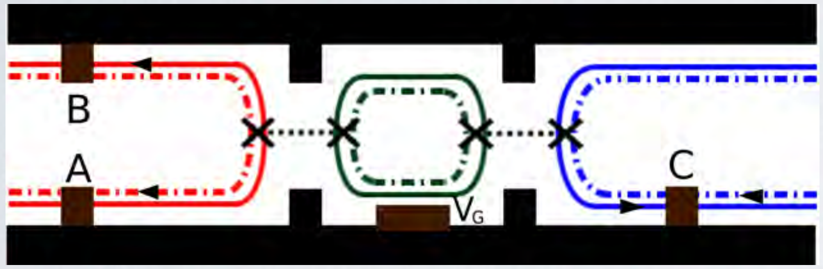
NECESSARY MICROSCOPIC INGREDIENT FOR DC THERMOELECTRICITY

Particle-hole symmetry breaking



Benenti, Casati, Saito, Whitney, Physics Reports 694, 1 (2017)

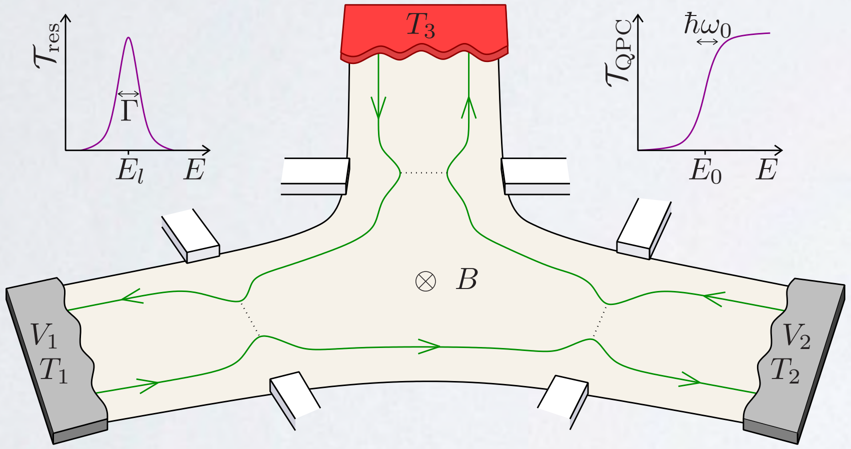
THERMOELECTRICITY IN QUANTUM HALL (THEORETICAL WORKS)



$\nu = 2/3, 5/2$

G.Viola, S.Das, E.Grosfeld, A.Stern
PRL 109, 146801 (2012)

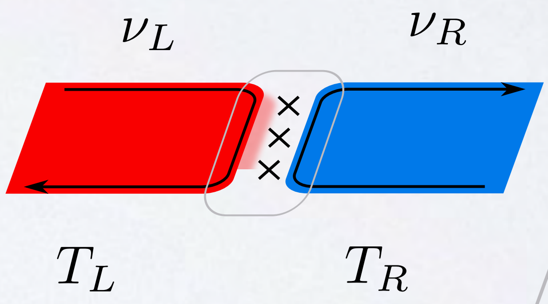
$\nu = 1/m \quad m$ odd

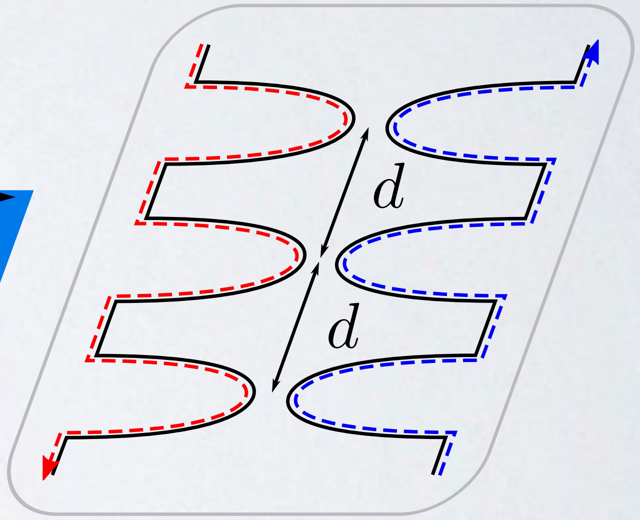


$\nu = 1$

FIG. 1: Three-terminal quantum Hall bar. A finite current is generated along the edge state between cold terminals 1 and 2 by conversion of heat injected from the hot probe terminal 3, originating from a temperature bias ΔT_3. Details of the energy-dependent scattering at the constrictions influence the thermoelectric response dramatically, revealing the chiral nature of electronic propagation in the sample.

R. Sanchez, B. Sothmann, A. Jordan
PRL 114, 146801 (2015)





L.Vannucci, F. Ronetti, G. Dolcetto, M. Carrega
M. Sassetti,
PRB 92, 075446 (2015)

Enhanced thermoelectric response in the fractional quantum Hall effect

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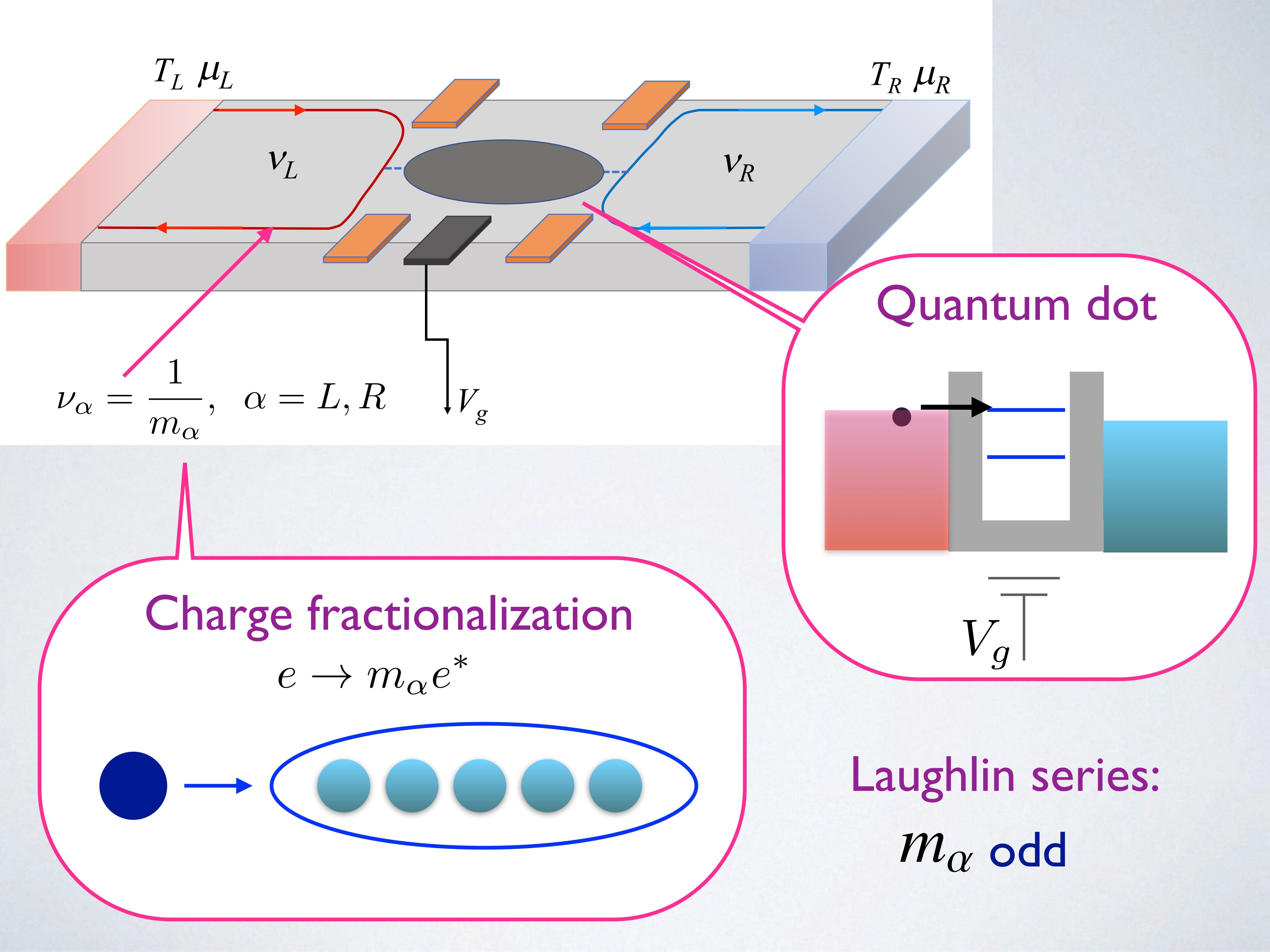
[1]*Dpto de Física, Centro Atómico Constituyentes, Comisión Nacional de Energía Atómica, CONICET, Buenos Aires, Argentina*
[2]*International Center for Advanced Studies, ECyT Universidad Nacional de San Martín, Campus Miguelete, 25 de Mayo y Francia, 1650 Buenos Aires, Argentina*
[3]*Dahlem Center for Complex Quantum Systems and Fachbereich Physik, Freie Universität Berlin, 14195 Berlin, Germany*
[4]*Department of Physics and Institute for Condensed Matter Theory, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801-3080, USA*

(Dated: November 25, 2017)

arXiv: 17.2082680
Physical Review B (RC) 081104 (2018)



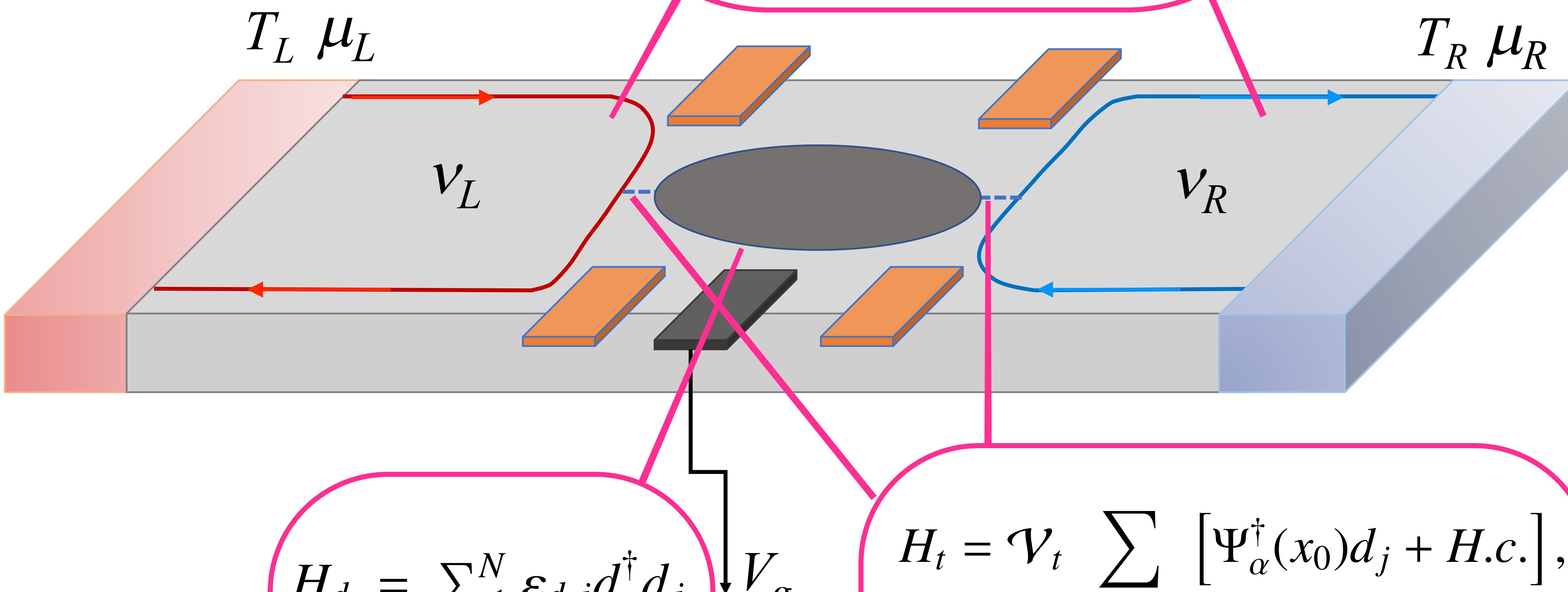
$T_L\ \mu_L$
$T_R\ \mu_R$
ν_L
ν_R
$\nu_\alpha = \frac{1}{m_\alpha}, \quad \alpha = L, R$
V_g
Quantum dot
V_g
Charge fractionalization
$e \to m_\alpha e^*$
Laughlin series:
m_α odd

$$H = \sum_{\alpha=L,R} H_\alpha + H_d + H_t.$$

$$H_\alpha = \frac{\pi v}{\nu_\alpha} \int dx\, \rho_\alpha^2(x), \quad \alpha = L, R$$

$$\rho_\alpha(x) = \partial_x \phi_\alpha(x)/(2\pi)$$

$$\left[\phi_\alpha(x), \phi_\beta(x')\right] = -i\pi\nu_\alpha \delta_{\alpha,\beta} \mathrm{sg}(x - x').$$



$$H_d = \sum_{j1}^{N} \varepsilon_{d,j} d_j^\dagger d_j,$$

$$\varepsilon_{d_j} = \Delta(j-1) - eV_g.$$

$$H_t = \mathcal{V}_t \sum_{j,\alpha=L,R} \left[\Psi_\alpha^\dagger(x_0) d_j + H.c.\right],$$

$$\Psi_\alpha(x) \equiv \frac{F_\alpha}{\sqrt{2\pi a}} e^{\pm i m_\alpha \phi_\alpha(x,t)}$$

CHARGE AND HEAT CURRENTS

$$J_C = eV_t\Big(G^<_{d,L}(t-t') - G^<_{L,d}(t-t')\Big)|_{t=t'} = -e\langle \dot{N}_L\rangle$$

$$J_E = iV_t\partial_t\Big(G^<_{d,L}(t-t') - G^<_{L,d}(t-t')\Big)|_{t=t'} = -\langle \dot{H}_L\rangle$$

$$G^<_{\alpha,d}(t,t') \;=\; i\langle d^\dagger_\sigma(t')\Psi_\alpha(xt)\rangle$$

Formalism:
Schwinger-Keldysh non-equilibrium Green's functions

T. Martin, Les Houches LXXXI, arXiv:0501208

PERTURBATION THEORY IN TUNNELING V_t

$$J_C = eV_t\Big(G^<_{d,L}(t-t')|_{t'=t} - G^<_{L,d}(t-t')|_{t'=t}\Big)$$

$$= eV_t^2 \int_{-\infty}^{+\infty} dt e^{it\mu_L}\Big(g^<_L(t)g^>_d(-t) - g^>_L(t)g^<_d(-t)\Big),$$

$$g_d^{<,>}(\varepsilon) = V_t^2|g_d^r(\varepsilon)|^2\left[\sum_{\alpha=L,R} g_\alpha^{<,>}(\varepsilon)\right] = \frac{V_t^2}{\gamma}D_d(\varepsilon)\left[\sum_{\alpha=L,R} g_\alpha^{<,>}(\varepsilon)\right],$$

$$g^{\lessgtr}_\alpha(t) = \pm\frac{i}{2\pi a}e^{\nu^{-2}D^{\lessgtr}_\alpha(t)},$$

$$D_d(\varepsilon) = |g_d^r(\varepsilon)|^2\gamma = \sum_j \frac{\gamma/N\pi}{\left(\varepsilon-\varepsilon_{d,j}\right)^2+\gamma^2}$$

$$D_\alpha^{\eta\eta'}(t,t') = \langle\hat{T}\left(\phi_\alpha(t^\eta)\phi_\alpha(t'^{\eta'})\right)\rangle - \frac{\langle\phi_\alpha(t^\eta)\rangle^2}{2} - \frac{\langle\phi_\alpha(t'^{\eta'})\rangle^2}{2}$$

$$D_\alpha^{\eta,-\eta}(t) = -\nu\ln\Big(\sinh\left(\pi T_\alpha(\eta t+ia)\right)/\sinh\left(i\pi T_\alpha a\right)\Big)$$

$$g^<_\alpha(t) = \frac{i}{2\pi a}\ \frac{\sinh^{m_\alpha}(ia\pi T_\alpha)}{\sinh^{m_\alpha}[\pi T_\alpha(t+ia)]}$$

$$g^<_\alpha(\varepsilon) = \frac{i}{2\pi a}\ a^{m_\alpha}\frac{(2\pi T_\alpha)^{m_\alpha-1}}{\Gamma(m_\alpha)}e^{-\varepsilon/2T_\alpha}\Big|\Gamma[m_\alpha/2+i\varepsilon/(2\pi T_\alpha)]\Big|^2.$$

PERTURBATION THEORY IN TUNNELING V_t

$$J_C = \frac{e}{h}\int d\varepsilon\, \tau(\varepsilon)\ \left[f_L(\varepsilon+\mu_L) - f_R(\varepsilon+\mu_R)\right],$$

$$J_E = \frac{1}{h}\int d\varepsilon\, \varepsilon\, \tau(\varepsilon)\ \left[f_L(\varepsilon+\mu_L) - f_R(\varepsilon+\mu_R)\right],$$

$$\tau(\varepsilon) = \frac{V_t^4}{\gamma}\ D_R(\varepsilon+\mu_R)\ D_d(\varepsilon)\ D_L(\varepsilon+\mu_L).$$

Transmission function

$$D_\alpha(\varepsilon) = \frac{a^{m_\alpha-1}}{2\pi}\frac{(2\pi T_\alpha)^{m_\alpha-1}}{\Gamma(m_\alpha)}\left|\frac{\Gamma\left(m_\alpha/2 + i\varepsilon/(2\pi T_\alpha)\right)}{\Gamma\left(1/2 + i\varepsilon/(2\pi T_\alpha)\right)}\right|^2$$

$$D_d(\varepsilon) = \sum_j \frac{\gamma/N\pi}{\left(\varepsilon-\varepsilon_{d,j}\right)^2+\gamma^2}$$

LINEAR RESPONSE

Currents

$$\boldsymbol{J} \equiv (J_C,\ J_Q)$$

Affinities

$$\boldsymbol{X} = \left(eV/k_BT, \Delta T/k_BT^2\right)$$

$$\boldsymbol{J} = \hat{L}\ \boldsymbol{X}$$

Onsager matrix

$$\hat{L} = -\frac{k_BT}{2h}\int d\varepsilon \begin{pmatrix} e & e\varepsilon \\ \varepsilon & \varepsilon^2 \end{pmatrix} \tau(\varepsilon)\,\frac{\partial f(\varepsilon)}{\partial \varepsilon}.$$

Transport coefficients

$$G = \frac{L_{11}}{T}, \qquad \kappa = \frac{1}{T^2}\frac{\det\hat{L}}{L_{11}}, \qquad TS = \Pi = \frac{L_{12}}{L_{11}}.$$

TRANSPORT COEFFICIENTS

Depend on $\tilde{m} = m_L + m_R$

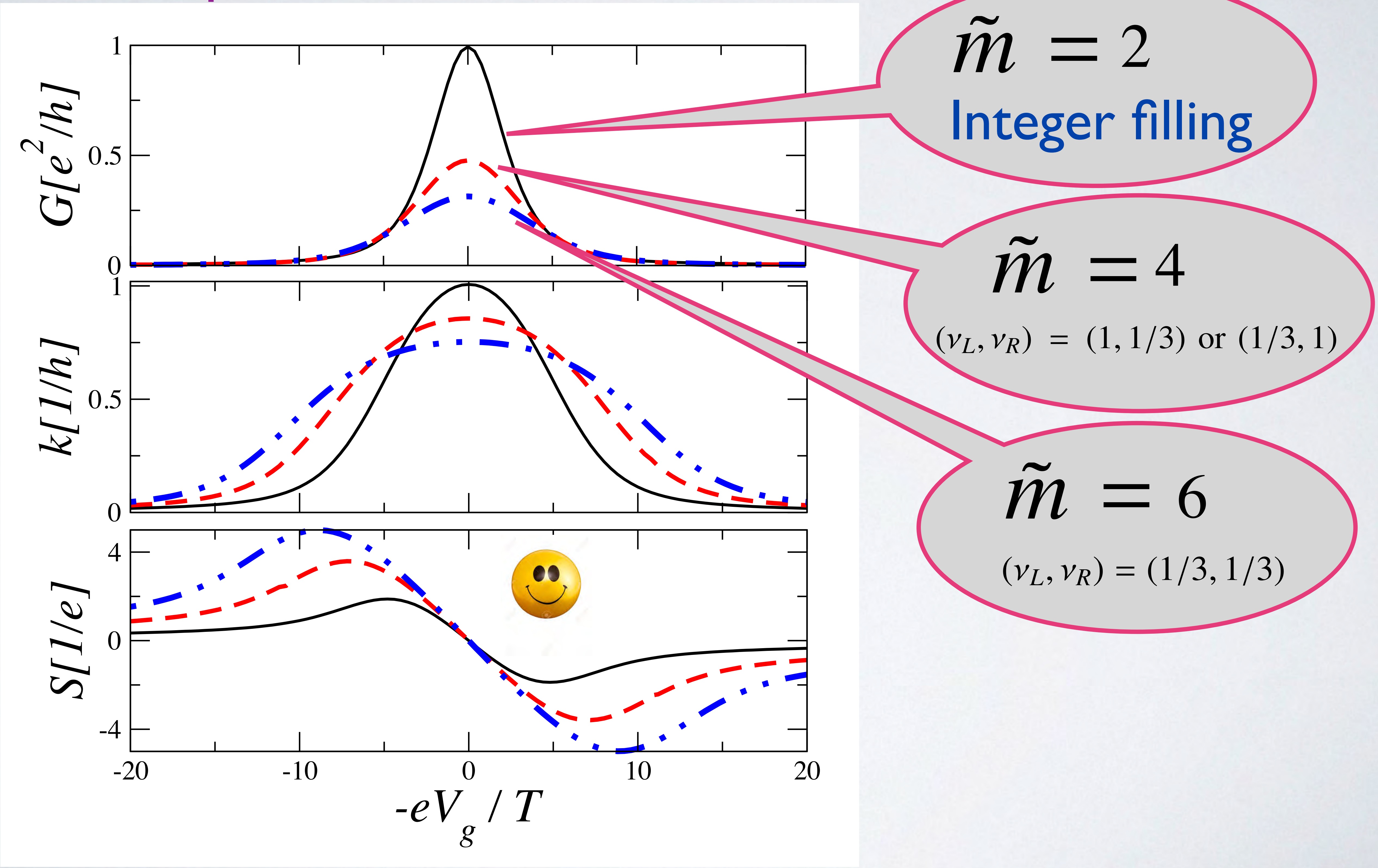
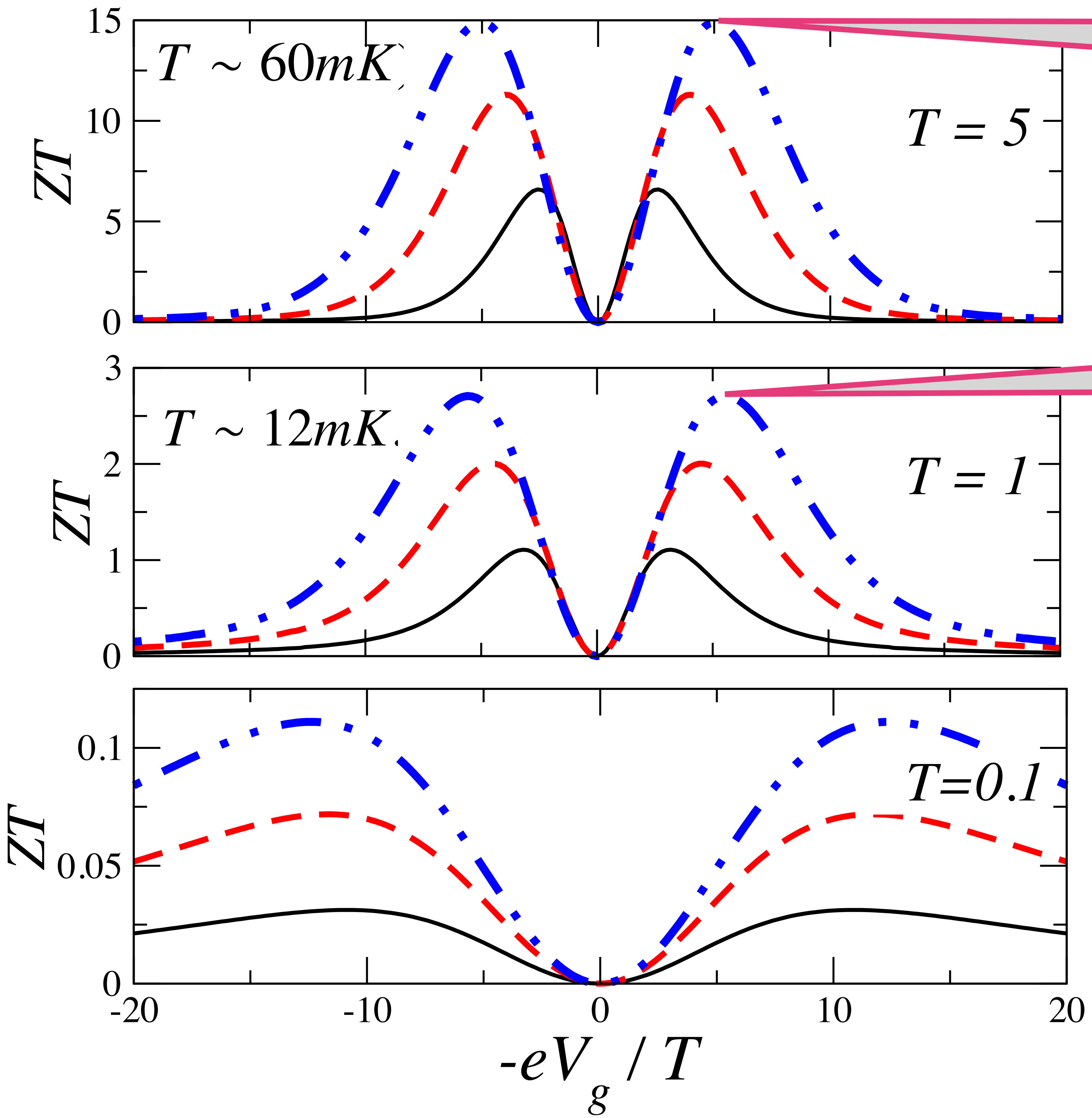
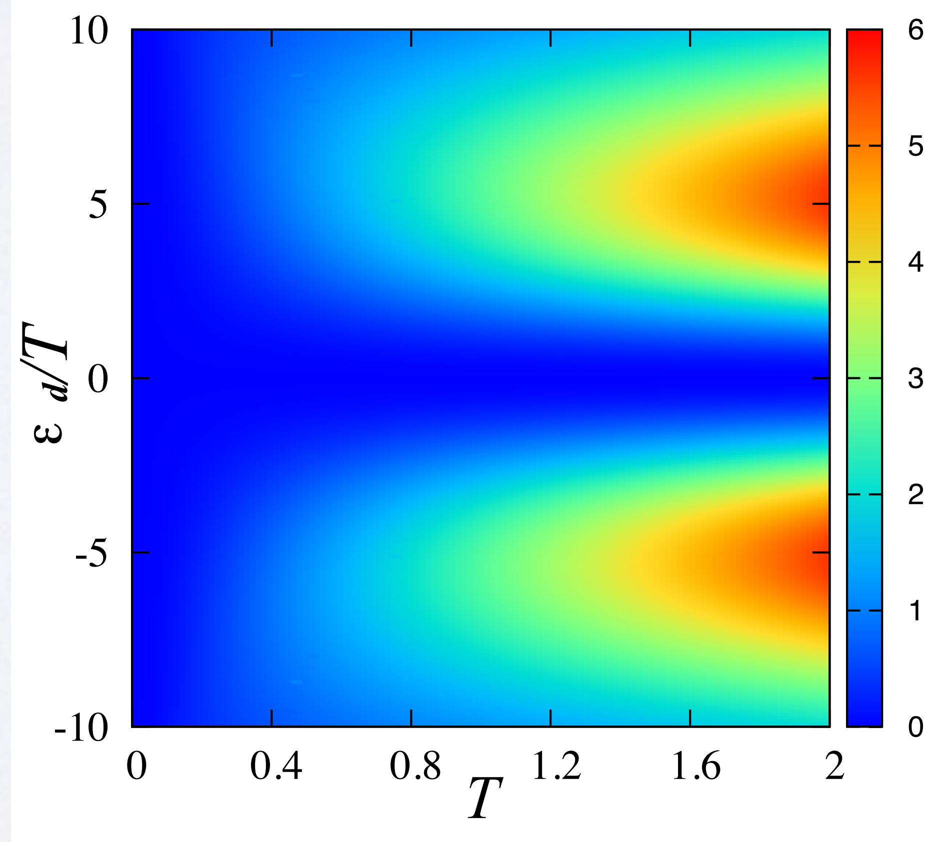


FIGURE OF MERIT



$\eta \sim 3/5\eta_C$

$\eta \sim \eta_C/3$



LOW-TEMPERATURE BEHAVIOR

$$L_{11} \sim c_{m_L,m_R}(T)\frac{D_d(0)}{(\tilde{m}-1)}(k_BT)^{\tilde{m}-1},$$
$$L_{12} \sim c_{m_L,m_R}(T)\frac{D'_d(0)}{(\tilde{m}+1)}(k_BT)^{\tilde{m}+1},$$
$$L_{22} \sim c_{m_L,m_R}(T)\frac{D_d(0)}{(\tilde{m}+1)}(k_BT)^{\tilde{m}+1},$$

$$\tilde{m} = m_L + m_R$$

$$S \simeq -2\frac{(\tilde{m}-1)}{(\tilde{m}+1)}\frac{\varepsilon_d\, k_BT}{\varepsilon_d^2+\gamma^2}, \qquad T < \gamma.$$

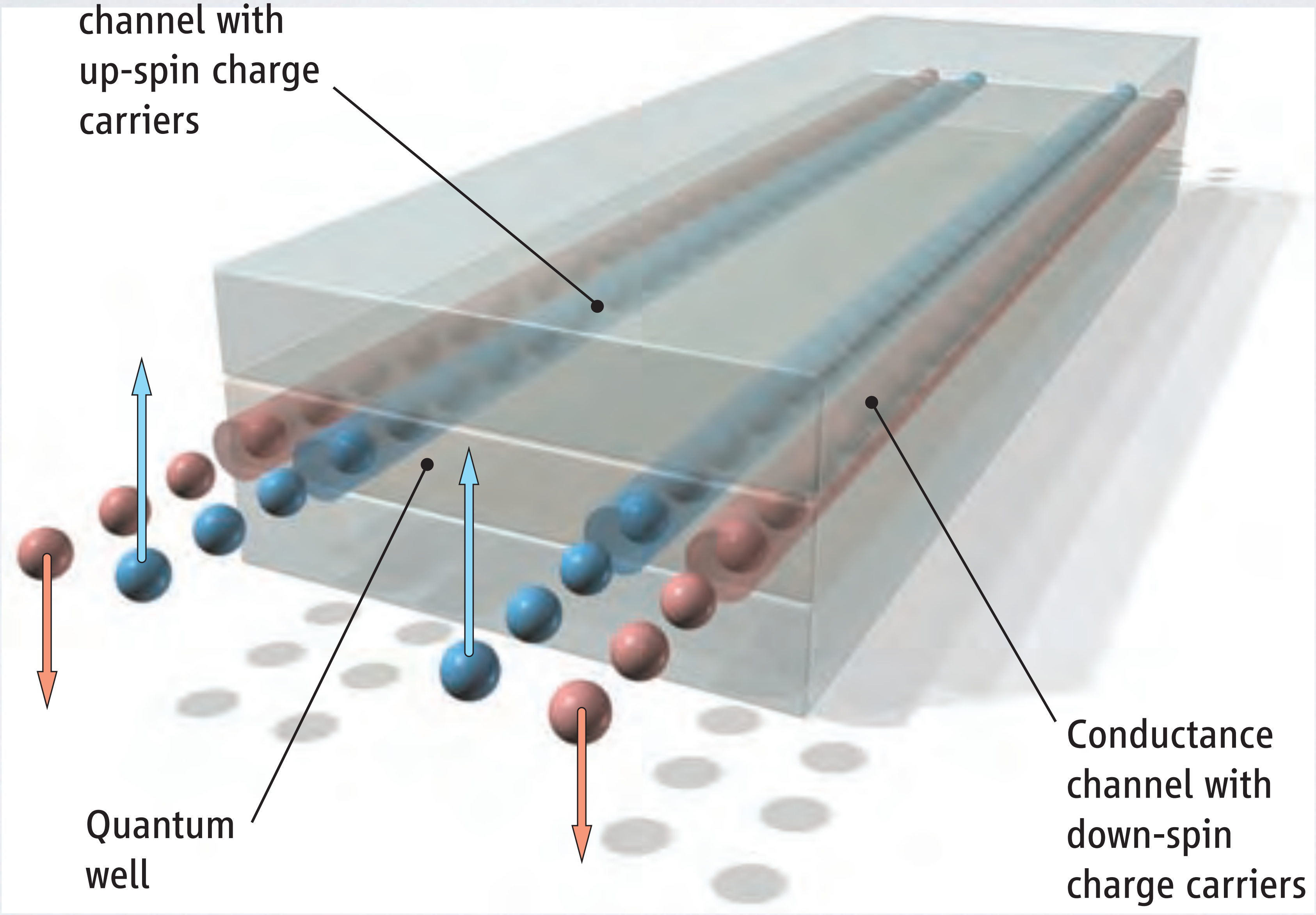
$$ZT = \frac{1}{\alpha-1}, \qquad \alpha = \frac{\left(\varepsilon_d^2+\gamma^2\right)^2}{(2k_BT\varepsilon_d)^2}\frac{(\tilde{m}+1)}{(\tilde{m}-1)},$$

HELICAL THERMOELECTRICITY IN QUANTUM SPIN HALL NANOSTRUCTURES

Quantum Spin Hall Insulator State in HgTe Quantum Wells

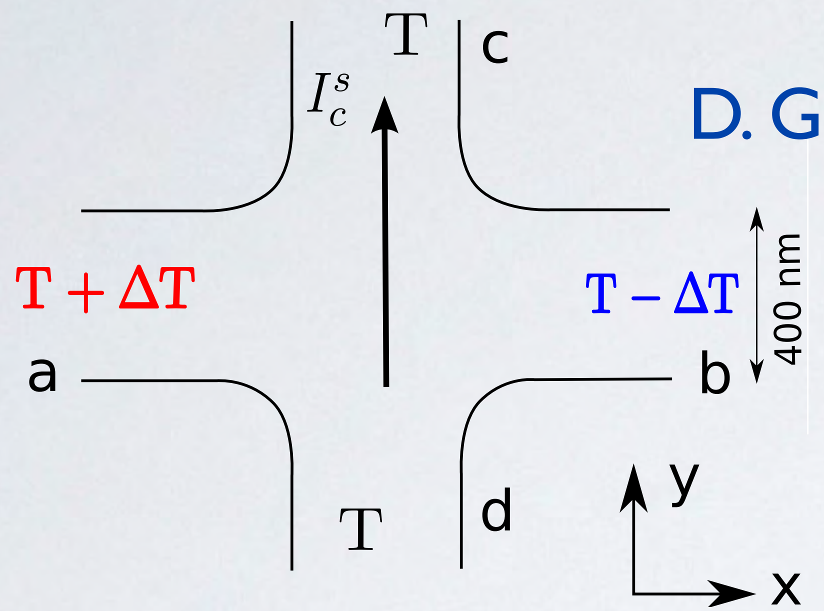
Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1]* Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]

Science 318, 766 (2007)



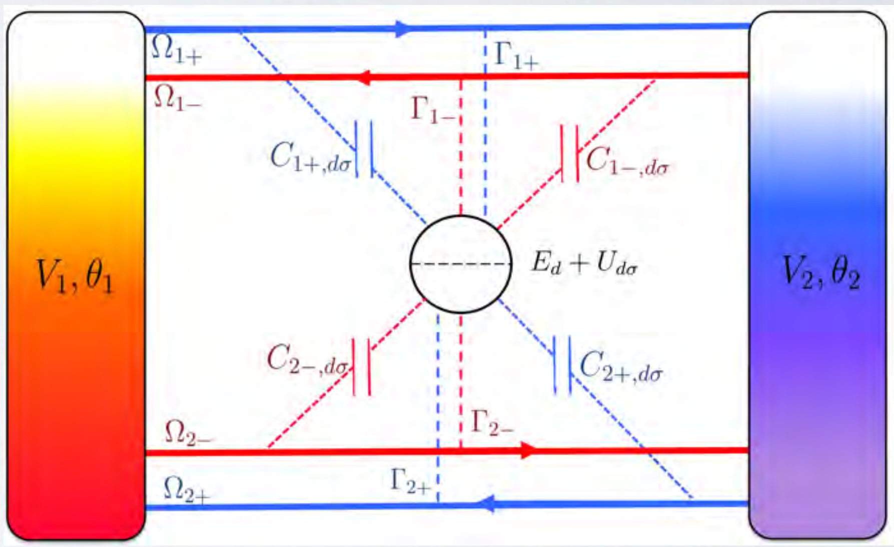
Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

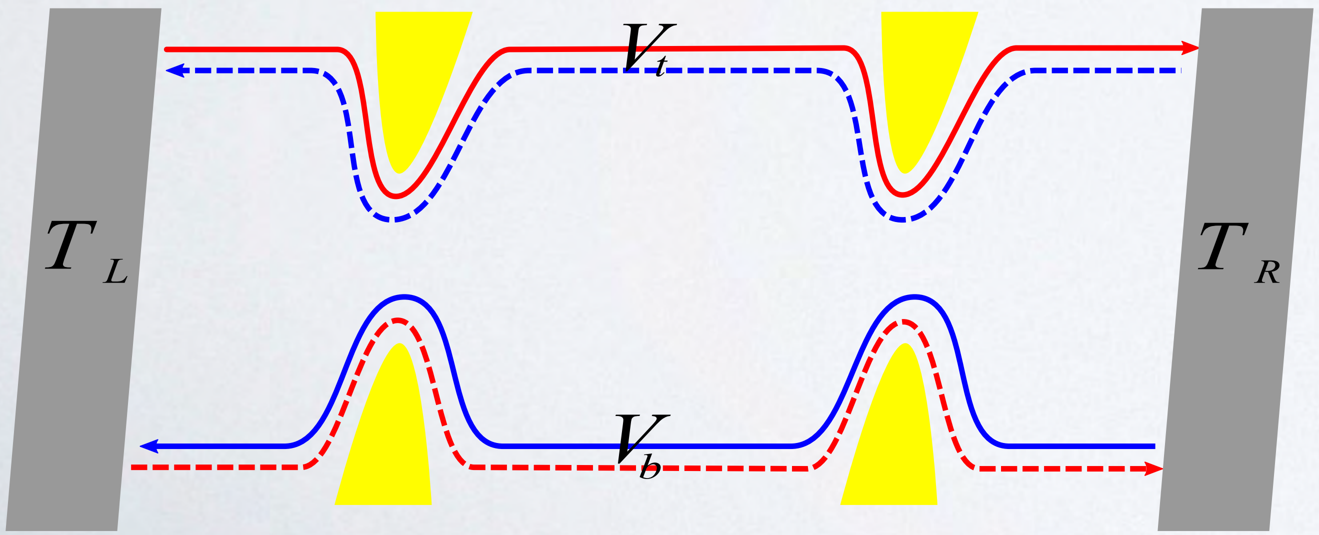
THERMOELECTRICITY IN QUANTUM SPIN HALL (THEORETICAL WORKS)



D. G.Rothe, E. M. Mankiewicz, B. Trauzettel, M. Guigou
PRB 86, 165434 (2012)

S-Y Hwang, R. Lopez, M. Lee, D. Sanchez
PRB 90, 115301 (2014)

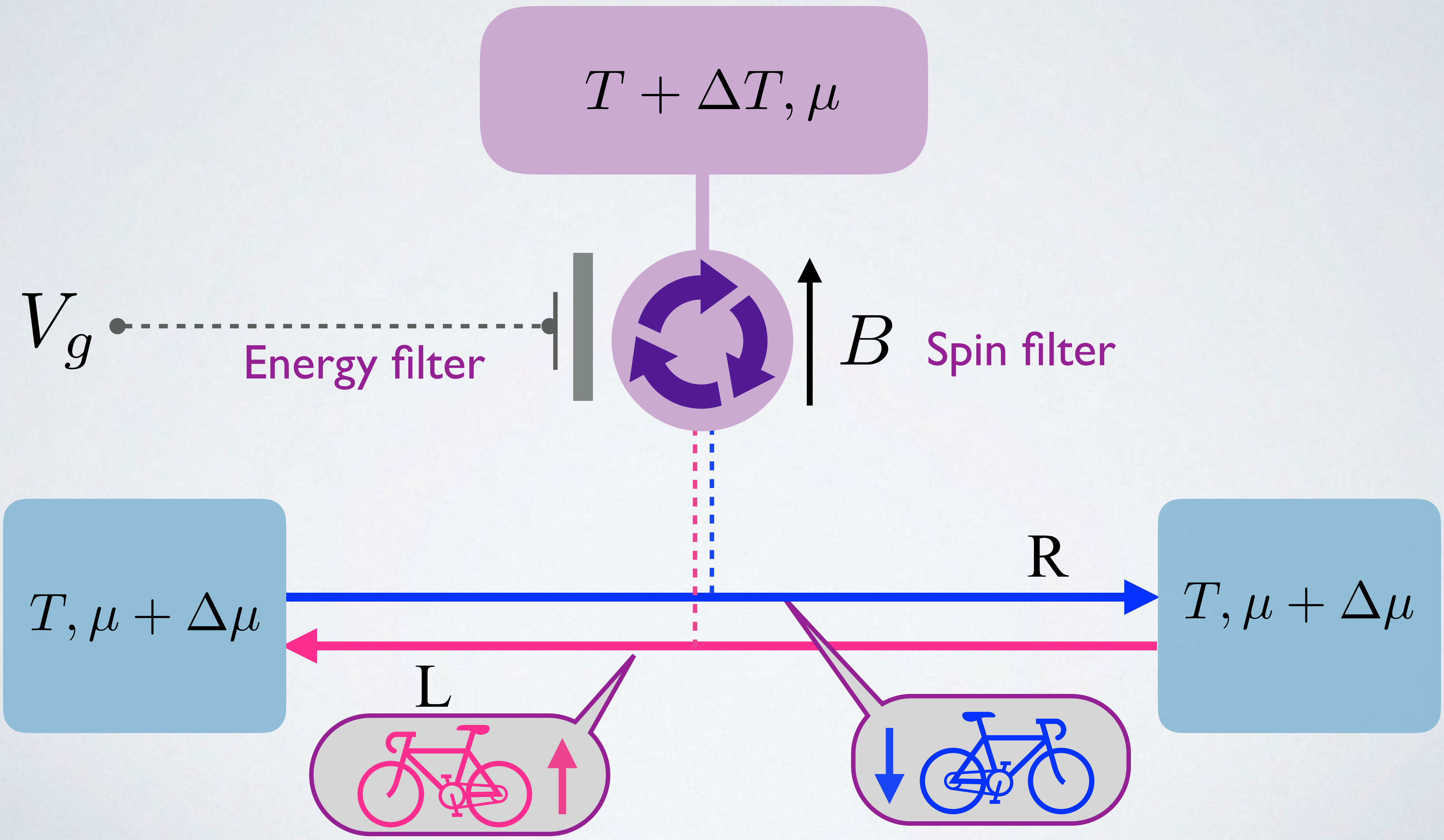




L. Vannucci, F. Ronetti, G. Dolcetto,
M. Carrega, M. Sassetti,
PRB 93, 165414 (2016)

HELICAL THERMOELECTRICITY

To be submitted



MODEL

$$H_d = \left(eV_g + \frac{B}{2}\right) n_{d\uparrow} + \left(eV_g - \frac{B}{2}\right) n_{d\downarrow}$$

Quantum dot

$$H_{LL} = \frac{v}{4\pi K} \int dx \left[\left(\partial_x \phi_L(x)\right)^2 + \left(\partial_x \phi_R(x)\right)^2 \right]$$

Helical edge states
Luttiger liquid

Interactions: $K \neq 1$

$L \equiv \uparrow$

$R \equiv \downarrow$

$$H_{tun} = w_L d_\uparrow \psi_L + w_R d_\downarrow \psi_R + H.c.$$

Tunneling

$$\psi_{R,L}(x) = \frac{F_{R,L}}{\sqrt{2\pi a}} e^{i[K_\pm \phi_R(x) + K_\mp \phi_L(x)]} \qquad K_\pm = (K^{-1} \pm 1)/2$$

$$\{\psi_\alpha(x), \psi^\dagger_{\alpha'}(x')\} = \delta_{\alpha\alpha'}\delta(x - x')$$

$$\begin{aligned}
[\phi_R(x), \phi_R(x')] &= -[\phi_L(x), \phi_L(x')] = i\pi K \mathrm{sgn}(x - x') \\
[\phi_R(x), \partial_{x'}\phi_R(x')] &= -[\phi_L(x), \partial_{x'}\phi_L(x')] = -2\pi K i \delta(x - x') \\
\rho_{L,R}(x) &= \pm \partial_x \phi_{L,R}(x)/2\pi
\end{aligned}$$

Books by T. Giamarchi and E. Fradkin

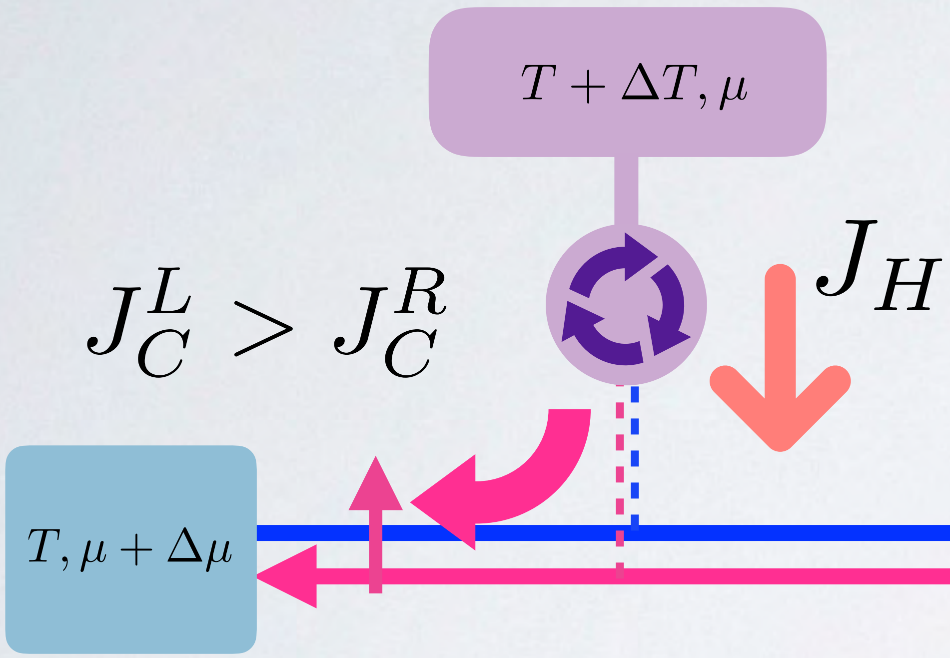
REGIMES

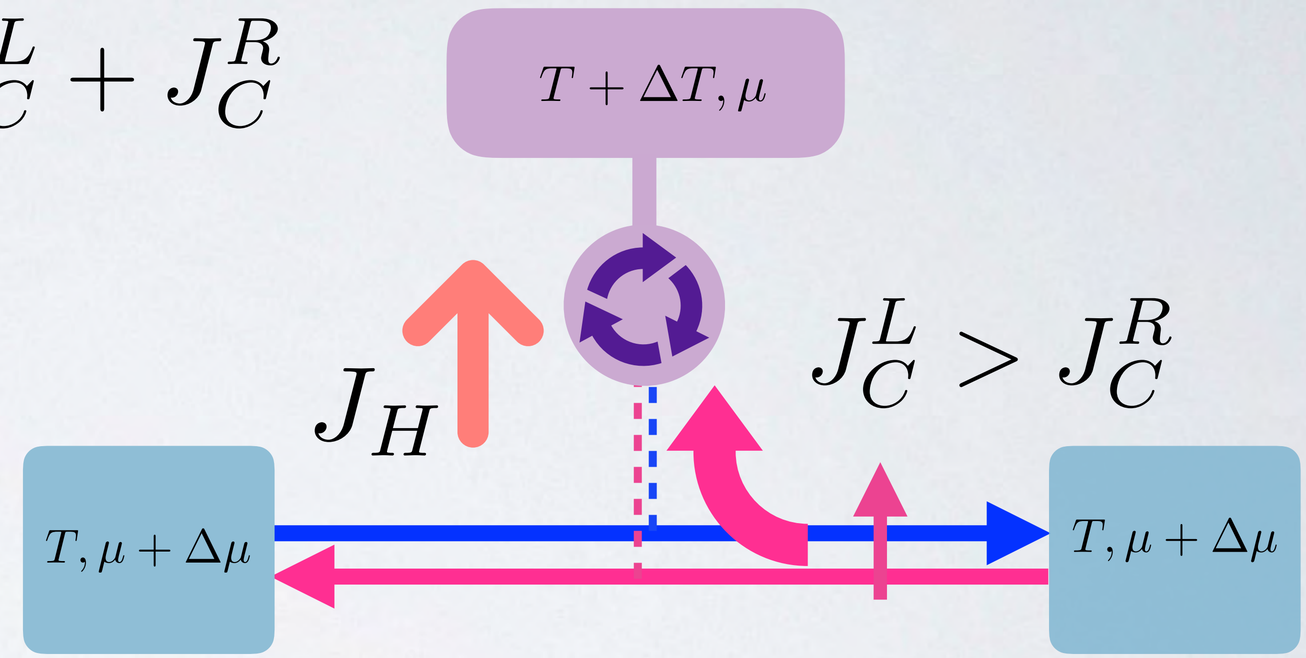
Spin heat engine

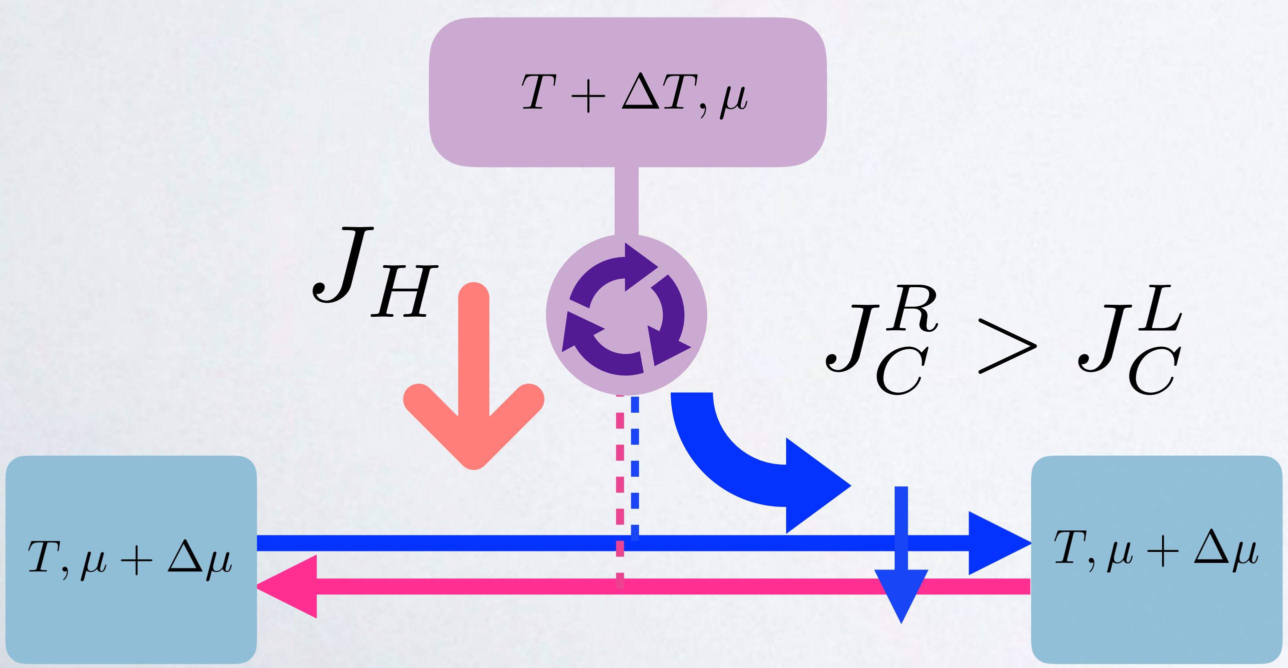
Spin refrigerator

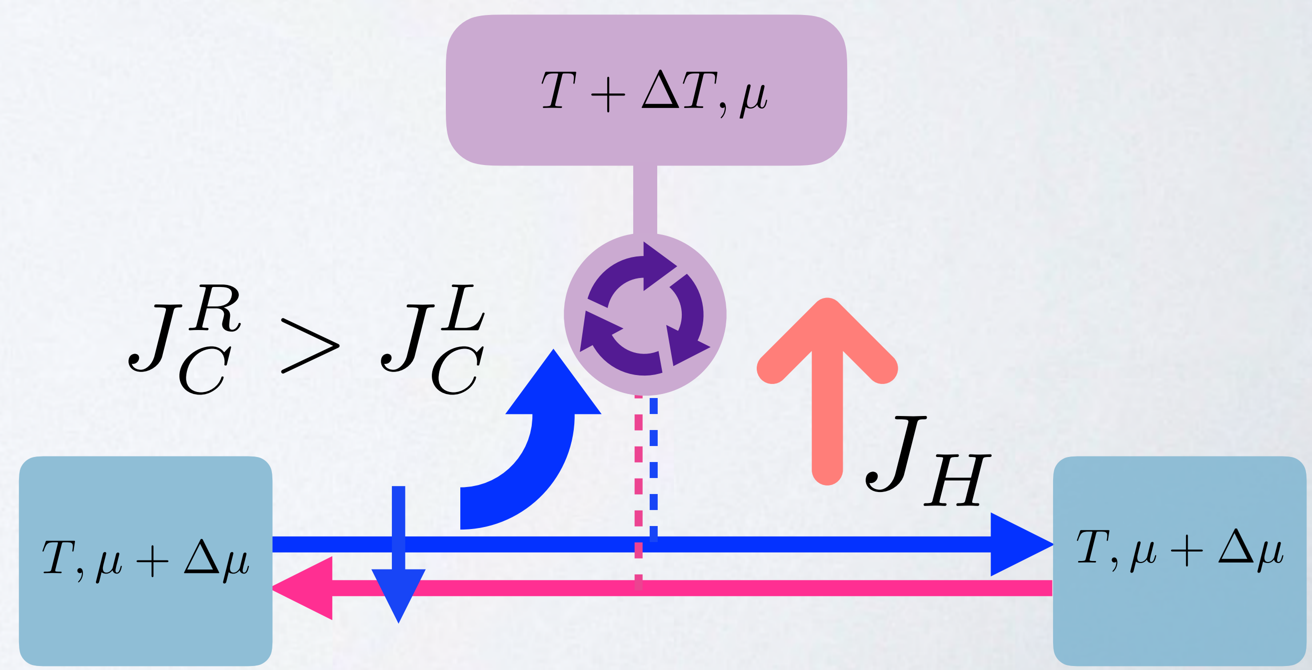
$$J_C^- = J_C^L - J_C^R$$

$$J_H = J_C^L + J_C^R$$









LINEAR RESPONSE + PERTURBATION THEORY

Currents

$$\mathbf{J}_\alpha \equiv (J_C^\alpha, J_H^\alpha)$$

Affinities

$$\boldsymbol{X} = \left(eV/k_B T, \Delta T/k_B T^2\right)$$

$$\mathbf{J}_\alpha = \hat{L}_\alpha \mathbf{X}$$

Onsager matrix

$$\hat{L}_\alpha = -\frac{k_B T}{2h} \int d\varepsilon \begin{pmatrix} e & e\varepsilon \\ \varepsilon & \varepsilon^2 \end{pmatrix} \tau_\alpha(\varepsilon) \frac{\partial f(\varepsilon)}{\partial \varepsilon}.$$

$$\tau_\alpha(\varepsilon) = 4\pi w_\alpha^2 \rho_{d\alpha}(\varepsilon) \rho_\alpha(\varepsilon)$$

$$\bar{K} \equiv (K_+^2 + K_-^2)K = \frac{1}{2}(\frac{1}{K} + K)$$

$$\rho_{d,\uparrow,\downarrow}(\omega) = \frac{\gamma}{(\omega - eV_g \pm B/2)^2 + \gamma^2}$$

$$\rho_\alpha(\omega) = a^{\bar{K}-1} \frac{(2\pi T_\alpha)^{\bar{K}-1}}{\Gamma(\bar{K})} \left| \frac{\Gamma(\bar{K}/2 + i\omega/2\pi T_\alpha)}{\Gamma(1/2 + i\omega/2\pi T_\alpha)} \right|^2$$

PERFORMANCE

$$J_C^- = \mathcal{L}_{11} X_1 + \mathcal{L}_{12} X_2$$
$$J_H = \mathcal{L}_{21} X_1 + \mathcal{L}_{22} X_2$$

$$\mathcal{L}_{1j} = \xi \Lambda_{ij}^-, \qquad \xi = sgn\left(\Lambda_{11}^-\right),$$
$$\mathcal{L}_{2j} = \Lambda_{ij}^+ \qquad \Lambda_{ij}^\pm = L_{ij}^{\mathrm{L}} \pm L_{ij}^{\mathrm{R}}$$

$L_{ij}^\sigma(B) = L_{ij}^{\overline{\sigma}}(-B)$ $\qquad$ $\mathcal{L}_{12} \neq \mathcal{L}_{21}$ Nontrivialy related

Figure of merit $\qquad ZT = \dfrac{\mathcal{L}_{12}\mathcal{L}_{21}}{det[\hat{\mathcal{L}}]}$

COEFFICIENTS

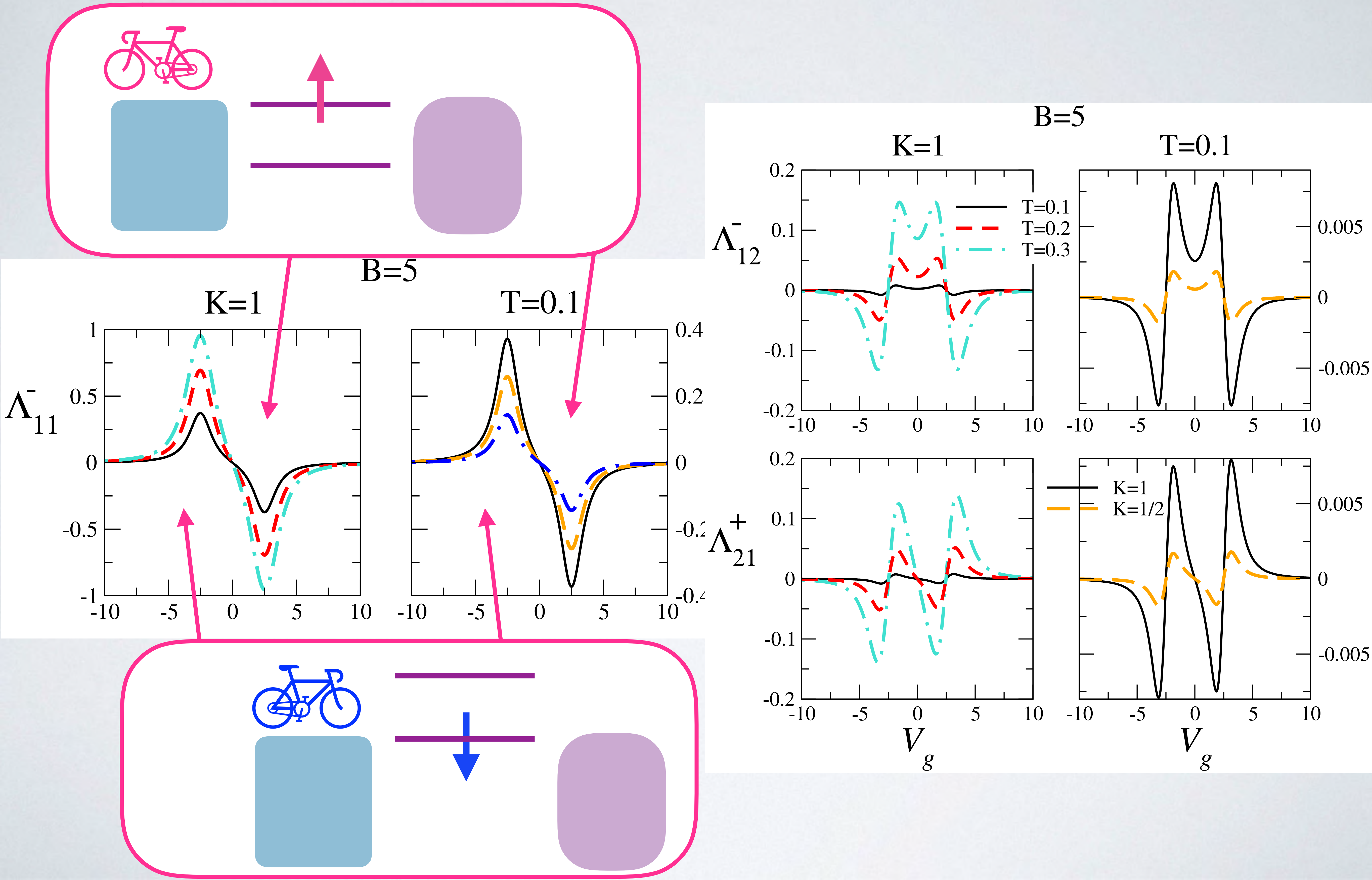
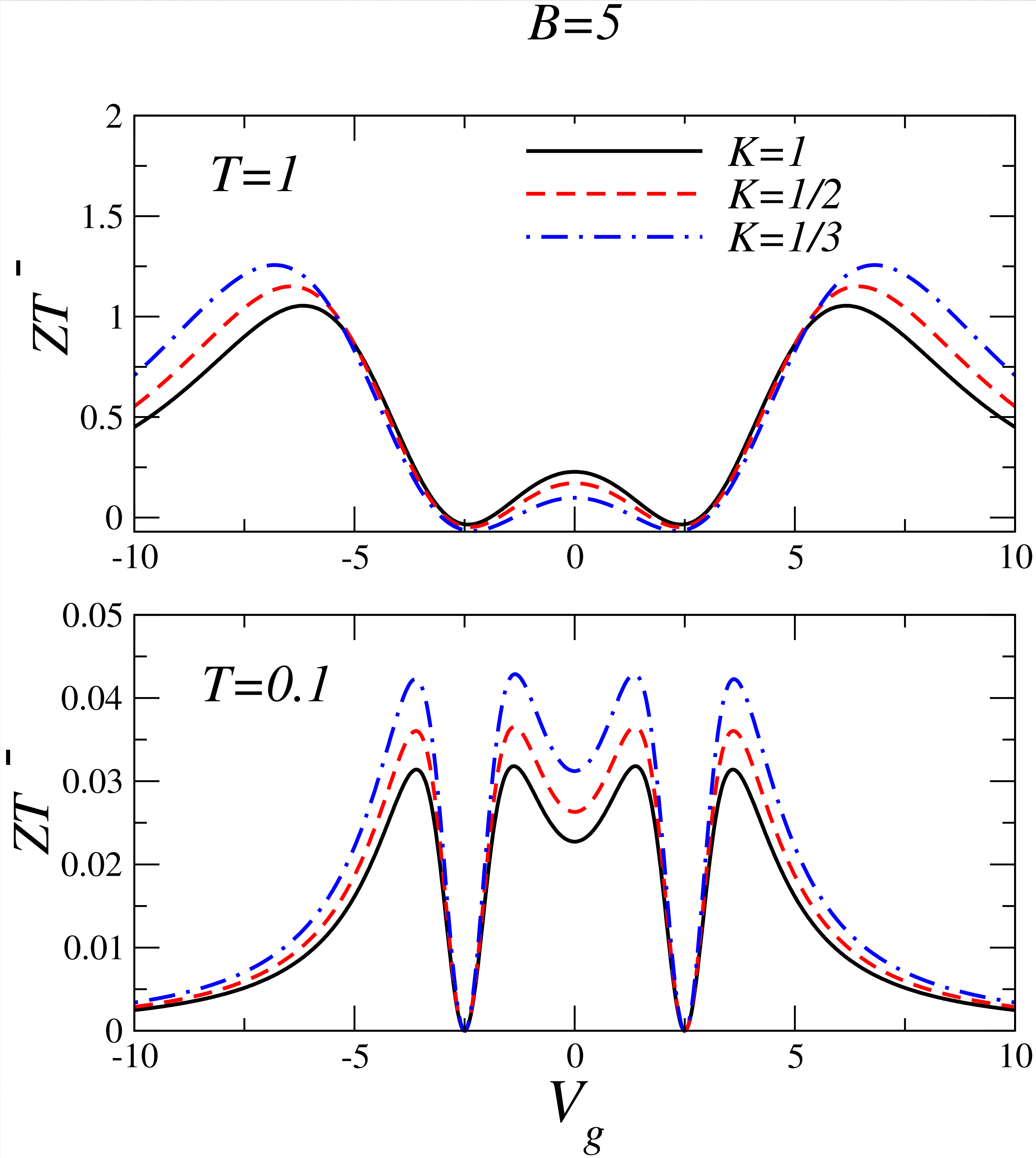


FIGURE OF MERIT



LOW-TEMPERATURE BEHAVIOR ROLE OF INTERACTIONS

$$L_{11}^{\alpha} \sim \frac{D_{d\alpha}(0)}{\tilde{K}} (k_B T)^{\tilde{K}}$$

$$L_{12}^{\alpha} \sim \frac{D'_{d\alpha}(0)}{\tilde{K}+2} (k_B T)^{\tilde{K}+2}$$

$$L_{22}^{\alpha} \sim \frac{D_{d\alpha}(0)}{\tilde{K}+2} (k_B T)^{\tilde{K}+2}$$

Factor of enhancement

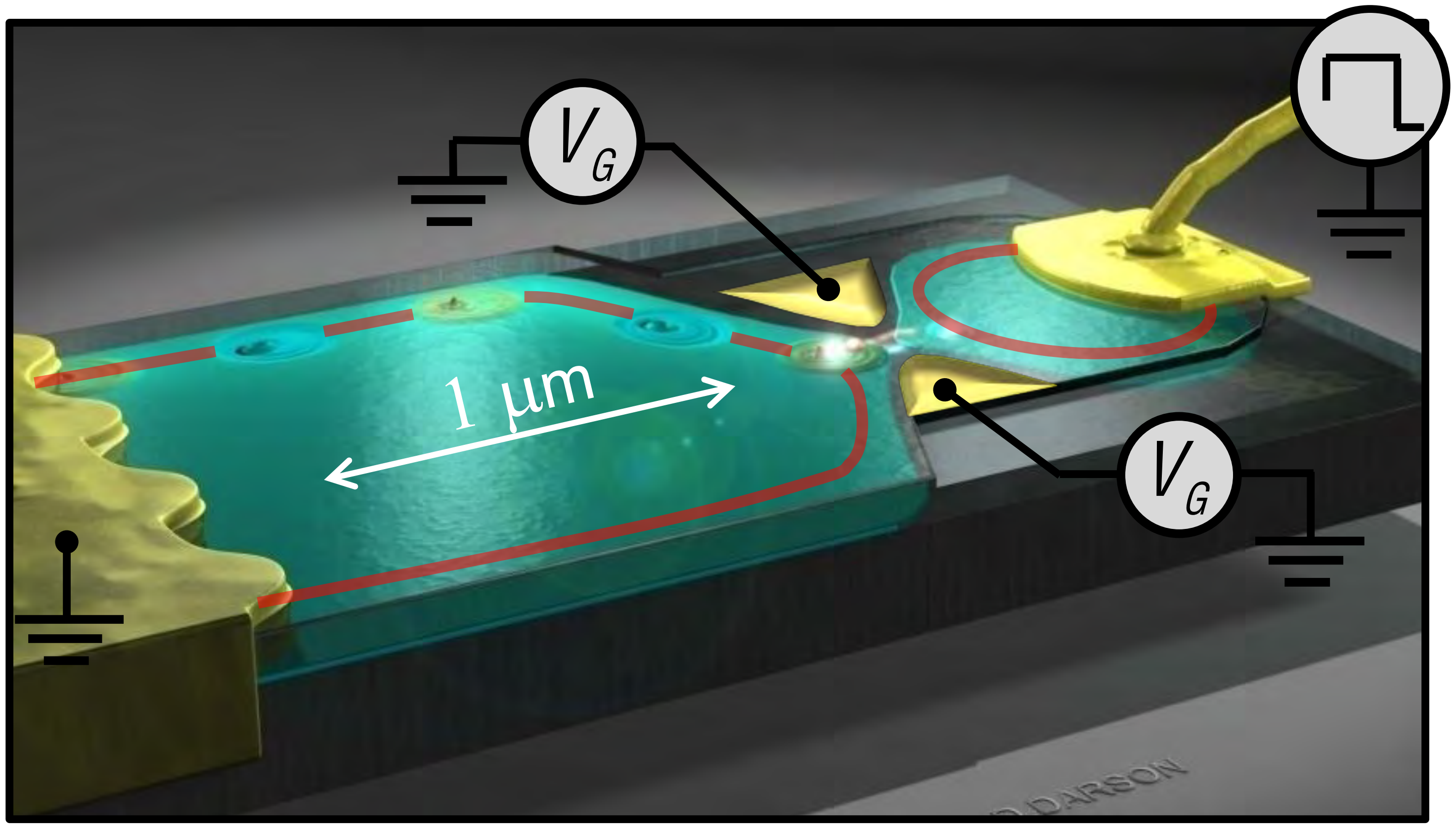
$$F = \frac{\mathcal{S}(K)}{\mathcal{S}(K=1)} \sim \frac{\tilde{K}+2}{2\tilde{K}}, \qquad \mathcal{S} = \frac{\mathcal{L}_{12}}{\mathcal{L}_{11}}$$

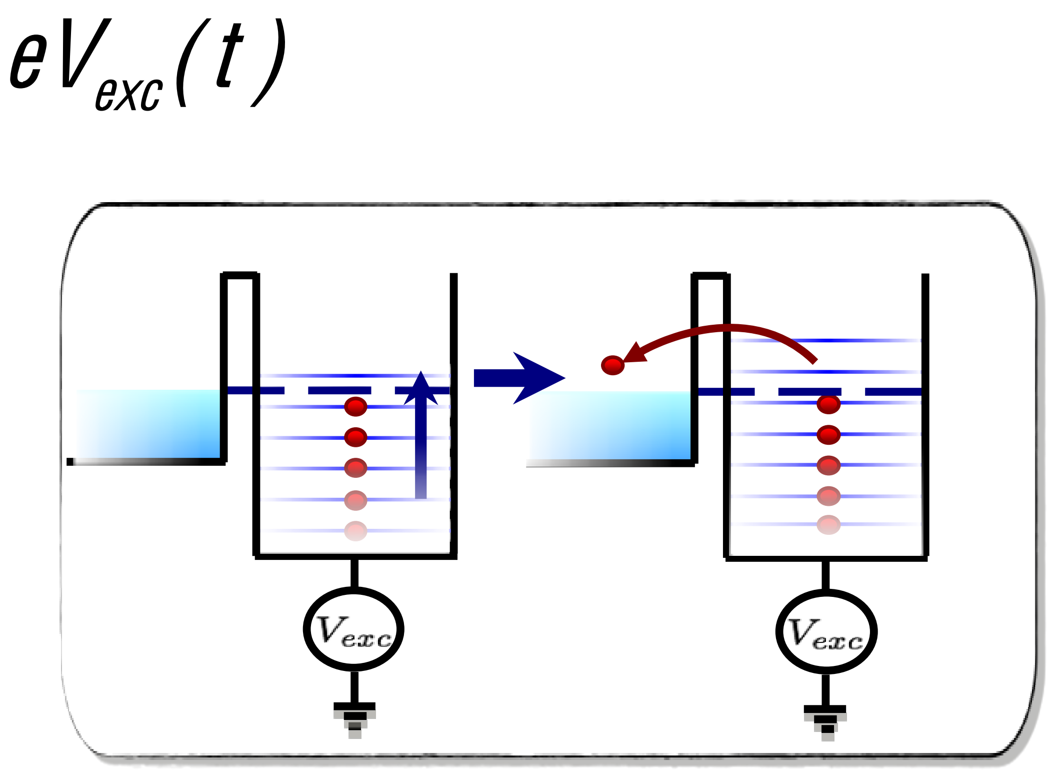
OUTLOOK

- Edge states of the fractional quantum Hall effect: chiral Luttinger liquids.
- Suppression of transport coefficients in tunnel contacts: power laws in T. However different behavior of heat and charge channels. Enhancement of the thermoelectric performance in the fractional quantum Hall effect.
- Edge states of quantum spin Hall structures: Helical Luttinger liquids.
- Thermoelectric heat to work conversion manipulating spin. Enhancement of the thermoelectric performance with interactions.

NON-LINEAR CHARGE AND ENERGY DYNAMICS OF ADIABATICALLY DRIVEN QUANTUM DOTS

Quantum capacitors and single-particle emiters





G. Fève et al., Science **316**, 1169 (2007)

Non-linear charge and energy dynamics of an adiabatically driven interacting quantum dot

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(Dated: May 15, 2017)

We formulate a general theory to study the time-dependent charge and energy transport of an adiabatically driven interacting quantum dot in contact to a reservoir for arbitrary amplitudes of the driving potential. We study within this framework the Anderson impurity model with a local ac gate voltage. We show that the exact adiabatic quantum dynamics of this system is fully determined by the behavior of the charge susceptibility of the frozen problem. At $T = 0$, we evaluate the dynamic response functions with the numerical renormalization group (NRG). The time-resolved heat production exhibits a pronounced feature described by an instantaneous Joule law characterized by an universal resistance quantum $R_0 = h/(2e^2)$ for each spin channel. We show that this law holds in non-interacting as well as in the interacting system and also when the system is spin-polarized. In addition, in the presence of a static magnetic field, the interplay between many-body interactions and spin polarization leads to a non-trivial energy exchange between electrons with different spin components.

PACS numbers: 73.23.-b, 73.63.Kv,72.15.Qm

PRB 95, 235117 (2017)

ADIABATIC RESPONSE

MF. Ludovico, F. Battista, F.von Oppen, LA, PRB 93, 075136 (2016)

Time-periodic Hamiltonian with $\mathcal{T} = 2\pi/\omega$

$$\mathcal{H} = \mathcal{H}(\mathbf{V}(t)) \qquad \mathbf{V}(t) = \mathbf{V}(t+\mathcal{T}) = (V_1(t), V_2(t), \ldots)$$

Evolution operator for linear response in $\dot{\mathbf{V}}(t)$

$$\hat{U}(t,t_0) \simeq \mathrm{T}\exp\{-i\hat{\mathcal{H}}_t(t-t_0) - i\int_{t_0}^{t} dt'(t-t')\hat{\mathbf{F}}\cdot\dot{\mathbf{V}}(t)\}$$

Force

$$\hat{\mathbf{F}}(t) = -\frac{\partial\hat{\mathcal{H}}(t)}{\partial\mathbf{V}(t)}$$

Generalized velocity

Mean value of an observable

$$O(t) \simeq \langle \hat{O} \rangle_t - i \int_{t_0}^{t} dt'(t-t')\langle \left[\hat{O}(t), \hat{\mathbf{F}}(t')\right] \rangle_t \dot{\mathbf{V}}(t)$$

$$= \langle \hat{O} \rangle_t + \mathbf{\Lambda}_t^{O\mathbf{F}} \cdot \dot{\mathbf{V}}(t).$$

Evaluated with frozen $\hat{\rho}_t$

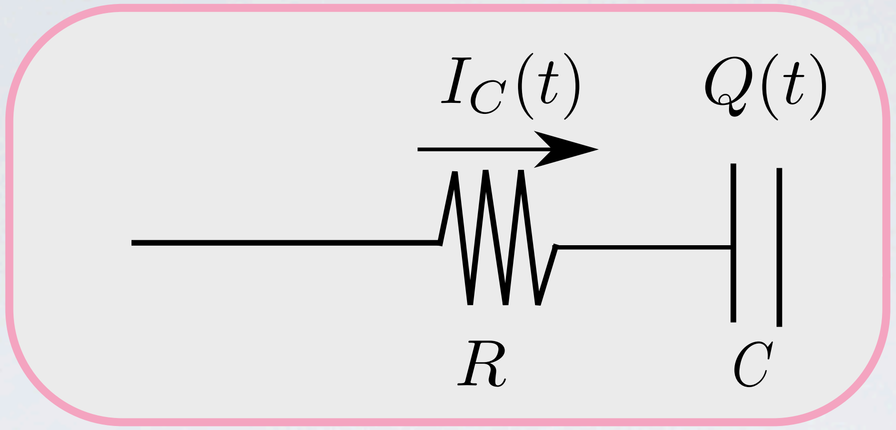
Linear response coefficient:

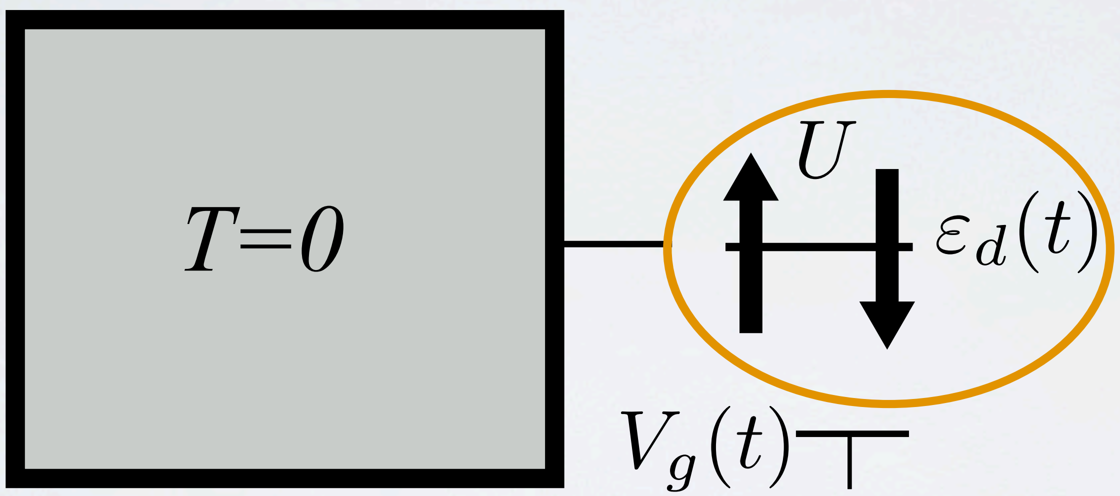
$$\Lambda^{O\mathbf{F}} = \int_{-\infty}^{+\infty} d\tau \tau \chi_t^{O\mathbf{F}}(\tau) = \lim_{\Omega \to 0} \frac{\mathrm{Im}\left[\chi_t^{O\mathbf{F}}(\Omega)\right]}{\Omega}$$

Equilibrium (Kubo-like) susceptibility:

$$\chi_t^{O,\mathbf{F}}(t-t') = -i\theta(t-t')\langle [\hat{O}(t), \hat{\mathbf{F}}(t')] \rangle_t$$

MODEL AND CLASSICAL ANALOG





$$H(t) = H_{\text{dot}}(t) + H_{\text{res}} + H_{\text{T}}.$$

$$H_{\text{dot}}(t) = \sum_{\sigma} \varepsilon_{d,\sigma}(t) n_{d\sigma} + U\left(n_{\uparrow} - \frac{1}{2}\right)\left(n_{\downarrow} - \frac{1}{2}\right),$$

$$\varepsilon_{d,\sigma}(t) = \varepsilon_0 \pm \frac{\delta_Z}{2} + \mathcal{V}_g(t)$$

$$\mathcal{V}_g(t) = eV_g(t) = V_0 \sin(\Omega t)$$

ADIABATIC DYNAMICS

$$\Lambda_\sigma(t) = -\lim_{\omega\to 0}\frac{\mathrm{Im}[\chi_t^{\sigma\sigma}(\omega)+\chi_t^{\sigma\bar{\sigma}}(\omega)]}{\hbar\omega}, \quad \chi_t^{\sigma\sigma'}(t-t') = -i\theta(t-t')\langle[n_{d\sigma}(t), n_{d\sigma'}(t')]\rangle_t$$

$$n_{d\sigma}(t) = n_{f\sigma}(t) + e\Lambda_\sigma(t)\dot{V}_g(t),$$

Charge current

$$e\dot{n}_d(t) = e\sum_\sigma \dot{n}_{d\sigma}(t) = \sum_\sigma I_{C,\sigma}(t),$$

Power ac

$$P(t) = n_d(t)\dot{V}_g(t),$$

Conservative

$$P_{\mathrm{cons},\sigma}(t) = en_{f\sigma}(t)\dot{V}_g(t),$$

Dissipative

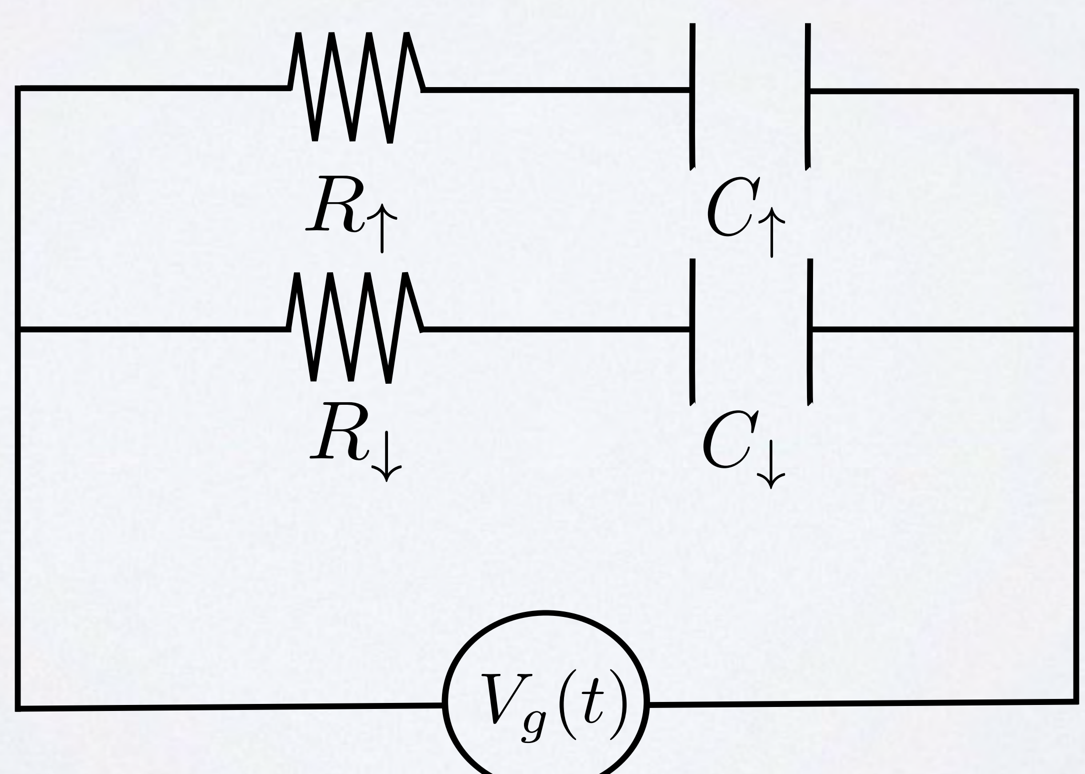
$$P_\sigma(t) = e^2\Lambda_\sigma(t)[\dot{V}_g(t)]^2.$$

ANALOGY TO CLASSICAL CIRCUIT

Similar to Moskalets, Samuelson Büttiker, PRL 100, 086601 (2008)

$$I_{C,\sigma}(t) = e\frac{dn_{f,\sigma}}{dV_g}\dot{V}_g(t) + e^2\frac{d\left[\Lambda_\sigma(t)\dot{V}_g(t)\right]}{dt},$$

$$I_{C,\sigma}(t) = -C_\sigma(t)\dot{V}_g(t) + e^2\frac{d\left[R_\sigma(t)C_\sigma(t)^2\dot{V}_g(t)\right]}{dt}.$$



NONINTERCTING LIMIT

M.F. Ludovico, J.S. Lin, M. Moskalets, LA and D. Sanchez
PRB 89, 161306 (R) 2013; PRB 94, 035436 (2016)

$$C_\sigma(t) = e\rho_{f,\sigma}(t,\mu).$$

$$\Lambda_\sigma(t) = \frac{h}{2}[\rho_{f,\sigma}(t,\mu)]^2 = \frac{h}{2}[\chi_t^{\sigma\sigma}(0)]^2,$$

$$\rho_{f,\sigma}(t,\epsilon) = (\Delta_\sigma/\pi)/\left[(\epsilon - \varepsilon_{d,\sigma}(t))^2 + \Delta_\sigma^2\right].$$

$$R_0 = h/(2e^2).$$

Büttiker resistance

Instantaneous Joule law!

$$P_\sigma(t) = R_0[I_{C,\sigma}(t)]^2$$

INTERACTING CASE IN FERMI-LIQUID REGIME

ZERO MAGNETIC FIELD

Fermi liquid → Instantaneous Korringa-Shiba law

(Linear response: Lee, López, Choi, Jonckheere, Martin, PRB (2011); Filippone, LeHur, Mora, PRL (2011))

$$\lim_{\omega\to 0}\frac{\mathrm{Im}\,[\chi_t^c(\omega)]}{\hbar\omega} = -\frac{h}{2}\sum_\sigma [\chi_t^{\sigma\sigma}(0)]^2.$$

$$P_{\mathrm{diss}}(t) = \frac{e^2 h}{2}\sum_\sigma [\chi_t^{\sigma\sigma}(0)]^2 \dot{V}_g(t)^2$$

$$I_{C,\sigma}(t) \simeq e\frac{\partial n_{f\sigma}(t)}{\partial V_g(t)}\dot{V}_g(t) = e\chi_t^{\sigma\sigma}(0)\dot{V}_g(t).$$

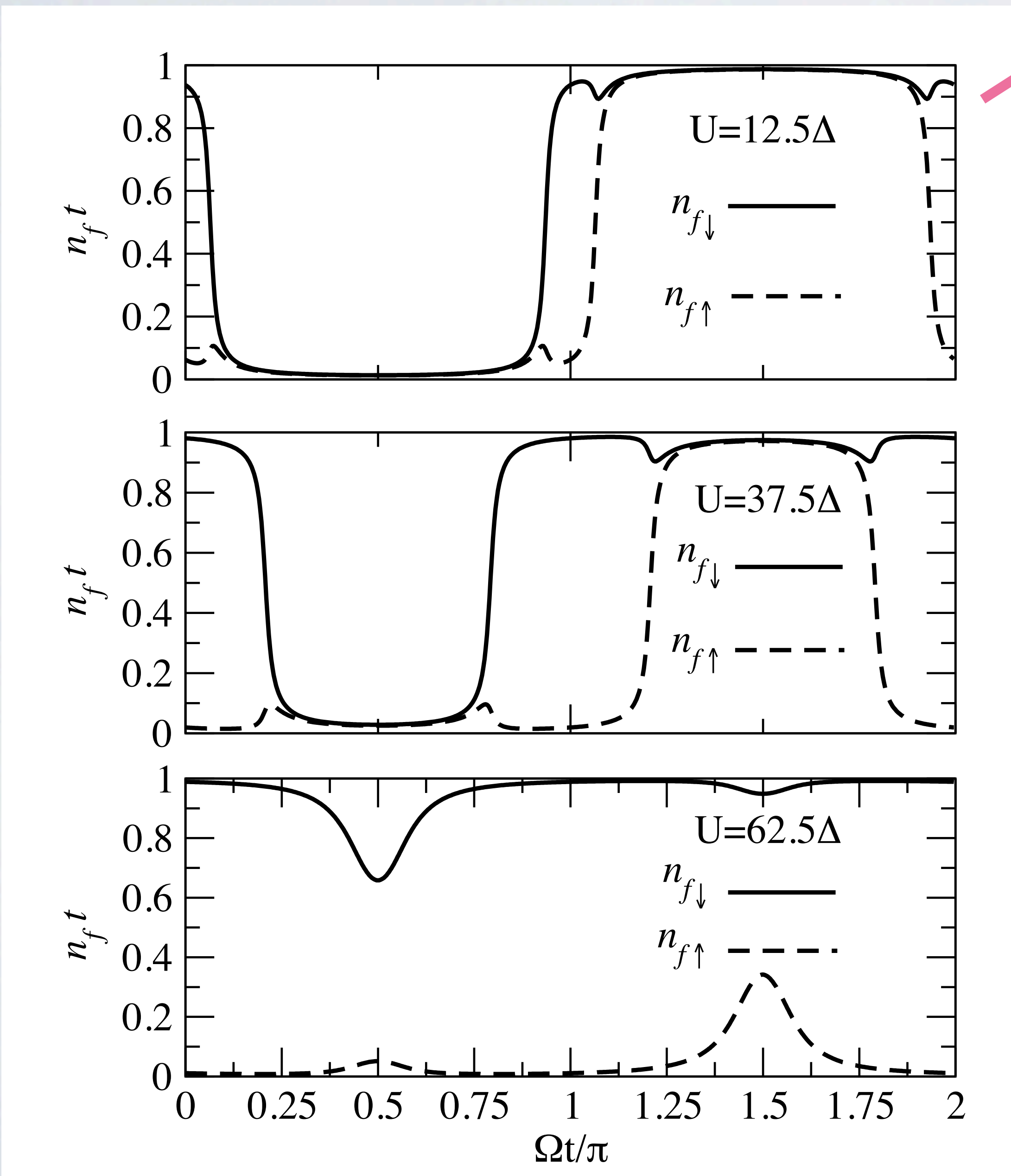
$$\boxed{P_{\mathrm{Joule}}(t) = R_q[I_C(t)]^2.}$$

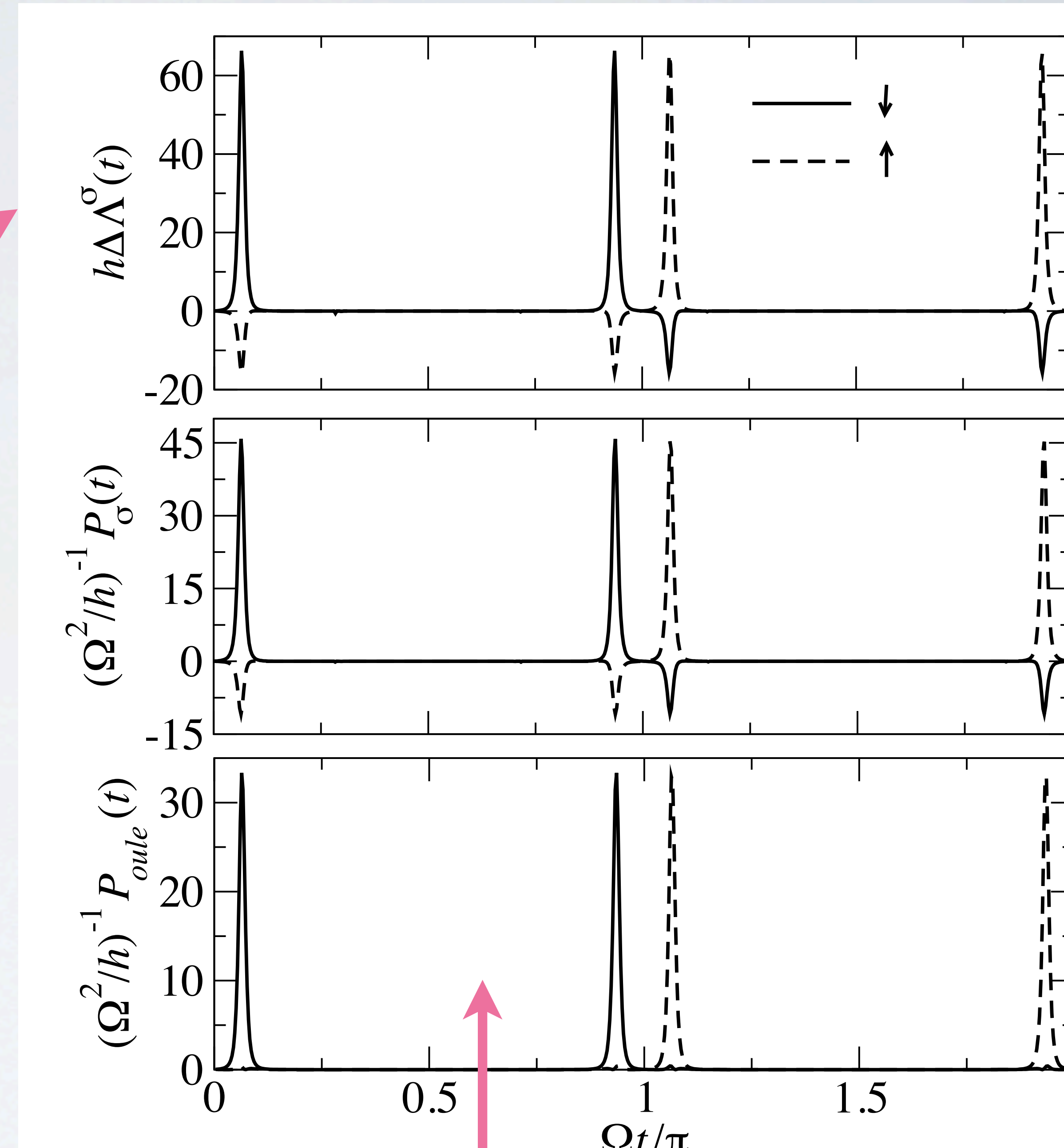
$$R_q = \frac{R_0}{2}$$

Instantaneous Joule law!

INTERACTIONS + MAGNETIC FIELD

Instantaneous NRG





Total dissipation =
Same instantaneous Joule law!

Anomalous Joule law in the adiabatic dynamics of a normal-superconductor quantum dot

Liliana Arrachea[1,2] and Rosa López[3]

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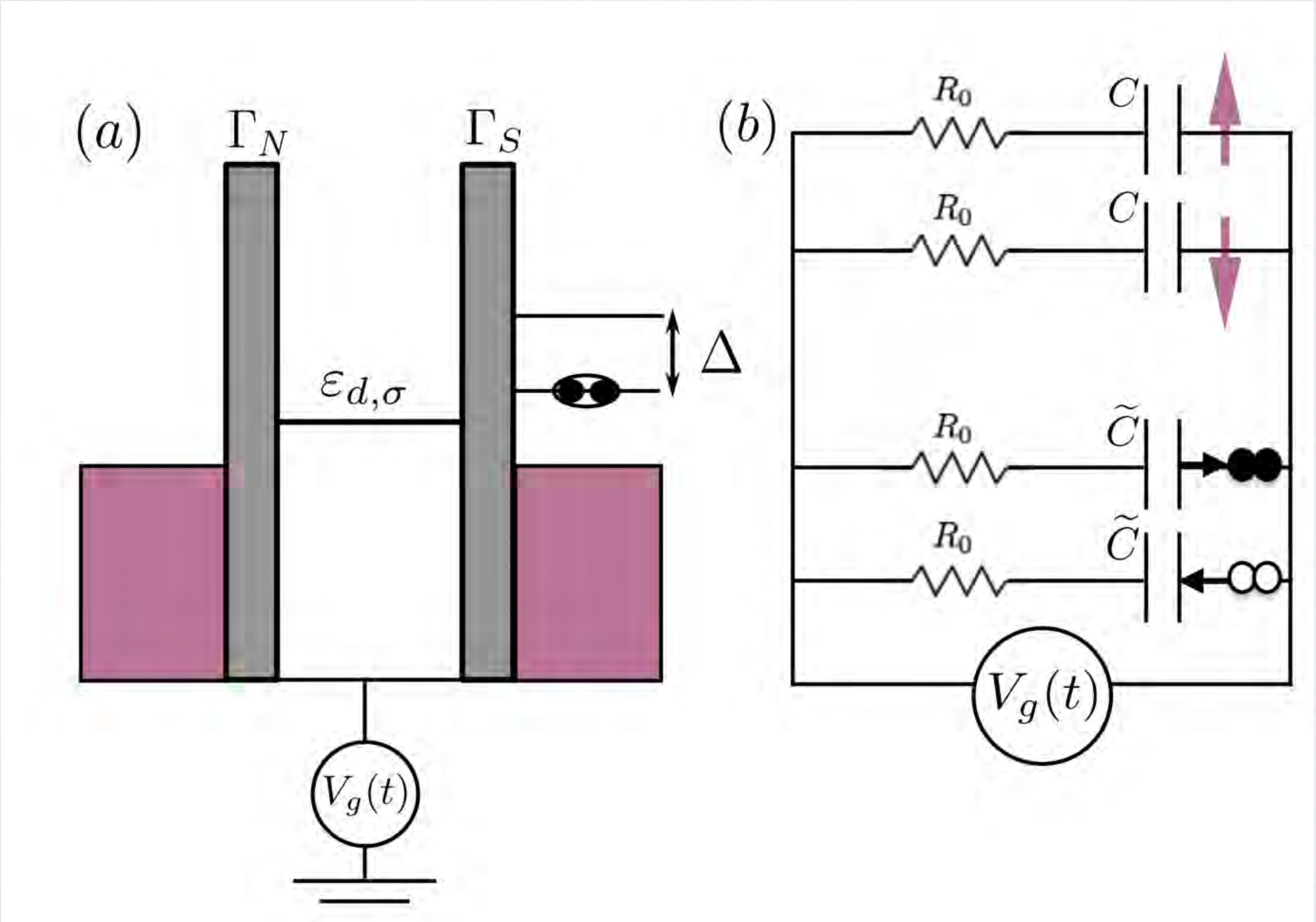
(Dated: March 21, 2018)

arXiv: 1803.10035



OUTLOOK

- Extended Kubo formalism for the adiabatic dynamics. Application: Dissipation in driven quantum dots in the adiabatic regime. Exact numerical results in combination with NRG.

- Instantaneous Joule law with Büttiker universal resistance. Satisfied by each spin channel in the non-interacting and in the interacting regime. Globally satisfied in the presence of a magnetic field, where exchange of power between electrons with up and down spins takes place. Generalizes in proximity to superconductors and has an additional anomalous contribution.

COLLABORATORS

- Pablo Roura-Bas (Bariloche)
- Eduardo Fradkin (Urbana-Illinois)
- María Florencia Ludovico (ex PhD student), Francesca Battista (ex Postdoc), Javier Romero (ex Postdoc)
- Armando Aligia (Bariloche)
- Michael Moskalets (Karkhiv)
- David Sanchez and Rosa Lopez (Illes Balears)
- Felix von Oppen (Berlin)

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THANK YOU!

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Xul Solar, Argentina, 1937-1963