Emission noise in an interacting quantum dot

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OUTLINE

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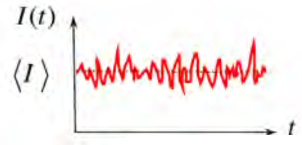
NEXT SECTION

FLUCTUATIONS

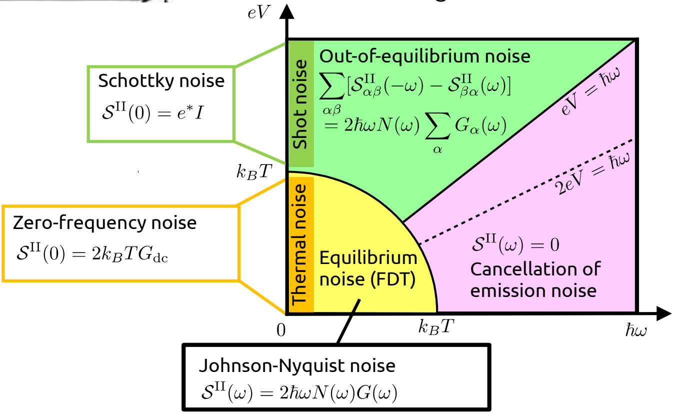
Current under DC voltage



Origin

- Thermal agitation
- Impurities/Defects
- Probabilistic nature of transfer
- Electromagnetic environment

Information contains in noise

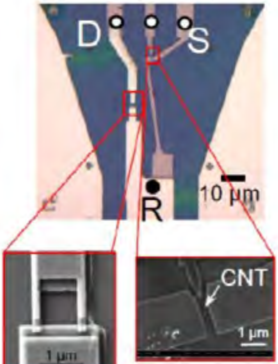


Crépieux, Eyméoud, Michelini, IEEE (2017)

FINITE-FREQUENCY NOISE MEASUREMENT

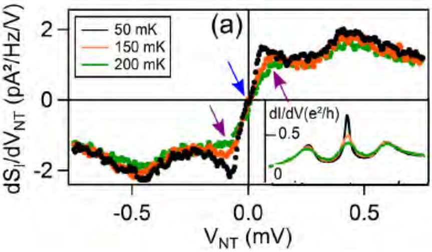
Delagrange, Basset, Bouchiat, Deblock, PRB 97, 041412R (2018)

Nanotube quantum dot in the Kondo regime



Josephson junction noise detector

CNT source of noise



$\omega = 12$ GHz
$T = 80$ mK
$T_K \approx 350$ mK
$a = \Gamma_L/\Gamma_R = 11$

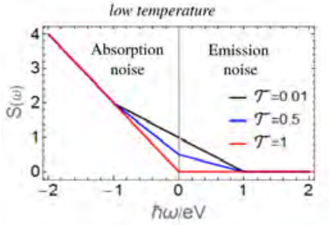
Strong asymmetry of the barriers

NON-SYMMETRIZED NOISE

Definition: $\mathcal{S}_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \Delta \hat{\mathrm{I}}_\alpha(t) \Delta \hat{\mathrm{I}}_\beta(0) \rangle$

with $\Delta \hat{\mathrm{I}}_\alpha(t) = \hat{\mathrm{I}}_\alpha(t) - \langle \hat{\mathrm{I}}_\alpha \rangle$

Simple example of noise spectrum



$\mathcal{S}(\omega > 0) \Rightarrow$ Emission noise

$\mathcal{S}(\omega < 0) \Rightarrow$ Absorption noise

Lesovik, Loosen, JETP Lett. 65, 295 (1997)

Aguado, Kouwenhoven, PRL 84, 1986 (2000)

Emission noise cancels at $\hbar\omega \gg eV$

At equilibrium

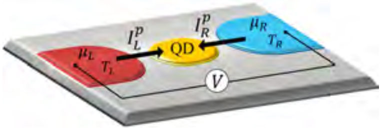
KMS: $\mathcal{S}(-\omega) = e^{\hbar\omega/k_B T} \mathcal{S}(\omega)$

$\mathcal{S}_{\mathrm{sym}}(\omega) = 2\hbar\omega \left[\frac{1}{2} + N(\omega)\right] G(\omega)$ Zero-point noise fluctuations

NEXT SECTION

INTERACTING QUANTUM DOT

System



Single-level interacting QD
connected to 2 reservoirs

Bias voltage: $eV = \mu_L - \mu_R$

Coupling asymmetry: $a = \Gamma_L / \Gamma_R$

Anderson Hamiltonian

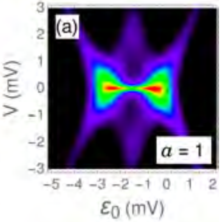
$$H = \sum_{k\in\{L,R\},\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_\sigma \varepsilon_0 d^\dagger_\sigma d_\sigma + U n_\uparrow n_\downarrow + \sum_{k\in\{L,R\},\sigma} V_k c^\dagger_{k\sigma} d_\sigma + h.c.$$

Green function

$$G^r_\sigma(\varepsilon) = \frac{1 - \langle n_{\bar\sigma}\rangle}{\varepsilon - \varepsilon_0 - \Sigma^0_\sigma(\varepsilon) - \Pi^{(1)}_\sigma(\varepsilon)} + \frac{\langle n_{\bar\sigma}\rangle}{\varepsilon - \varepsilon_0 - U - \Sigma^0_\sigma(\varepsilon) - \Pi^{(2)}_\sigma(\varepsilon)}$$

Roermund, Shiau, Lavagna, PRB 81, 165115 (2010)

Differential conductance



NOISE CALCULATION

$$\boxed{\mathcal{S}_{\alpha\beta}(\omega) = \int_{-\infty}^{\infty} dt e^{-i\omega t}\langle \Delta\hat{\mathrm{I}}_\alpha(t)\Delta\hat{\mathrm{I}}_\beta(0)\rangle} \text{ with } \Delta\hat{\mathrm{I}}_\alpha(t) = \hat{\mathrm{I}}_\alpha(t) - \langle\hat{\mathrm{I}}_\alpha\rangle$$

Current operator

$$\hat{\mathrm{I}}_\alpha = \frac{ie}{\hbar}\sum_{k\in\alpha,\sigma}\left(V_k c_{k\sigma}^\dagger d_\sigma - V_k^* d_\sigma^\dagger c_{k\sigma}\right)$$

Two-particle Keldysh Green functions

$$\begin{aligned}
G_1^{cd,>}(t,t') &= i^2\langle c_{k\sigma}^\dagger(t)d_\sigma(t)c_{k'\sigma'}^\dagger(t')d_\sigma(t')\rangle\\
G_2^{cd,>}(t,t') &= i^2\langle c_{k\sigma}^\dagger(t)d_\sigma(t)d_\sigma^\dagger(t')c_{k'\sigma'}(t')\rangle\\
G_3^{cd,>}(t,t') &= i^2\langle d_\sigma^\dagger(t)c_{k\sigma}(t)c_{k'\sigma'}^\dagger(t')d_\sigma(t')\rangle\\
G_4^{cd,>}(t,t') &= i^2\langle d_\sigma^\dagger(t)c_{k\sigma}(t)d_\sigma^\dagger(t')c_{k'\sigma'}(t')\rangle
\end{aligned}$$

Steps

- Calculation of $G_{1,2,3,4}^{cd,T}(t,t')$ + Decoupling
- Many electron-hole pair processes are neglected
- Langreth rules to get Keldysh Green functions
- Flat wide-band approximation

RESULT

$$\mathcal{S}_{\alpha\beta}(\omega) = \frac{e^2}{h} \sum_{\gamma\delta} \int_{-\infty}^{\infty} d\varepsilon M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\omega) f_\gamma^e(\varepsilon) f_\delta^h(\varepsilon - \hbar\omega)$$

where

$M_{\alpha\beta}^{\gamma\delta}(\varepsilon,\nu)$	$\gamma=\delta=L$	$\gamma=\delta=R$	$\gamma=L,\ \delta=R$	$\gamma=R,\ \delta=L$
$\alpha=L$ $\beta=L$	$\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon-\hbar\omega)$ $+\lvert t_{LL}(\varepsilon)-t_{LL}(\varepsilon-\hbar\omega)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-\hbar\omega)$	$[1-\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon-\hbar\omega)$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon-\hbar\omega)]$
$\alpha=R$ $\beta=R$	$\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-\hbar\omega)$	$\mathcal{T}_{LR}^{\text{eff},R}(\varepsilon)\mathcal{T}_{LR}^{\text{eff},R}(\varepsilon-\hbar\omega)$ $+\lvert t_{RR}(\varepsilon)-t_{RR}(\varepsilon-\hbar\omega)\rvert^2$	$\mathcal{T}_{LR}(\varepsilon)[1-\mathcal{T}_{LR}^{\text{eff},R}(\varepsilon-\hbar\omega)]$	$[1-\mathcal{T}_{LR}^{\text{eff},R}(\varepsilon)]\mathcal{T}_{LR}(\varepsilon-\hbar\omega)$
$\alpha=L$ $\beta=R$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-\hbar\omega)$ $\times[r_{LL}^*(\varepsilon)r_{LL}(\varepsilon-\hbar\omega)-1]$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-\hbar\omega)$ $\times[r_{RR}(\varepsilon)r_{RR}^*(\varepsilon-\hbar\omega)-1]$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-\hbar\omega)$ $\times r_{LL}^*(\varepsilon)r_{RR}^*(\varepsilon-\hbar\omega)$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-\hbar\omega)$ $\times r_{RR}(\varepsilon)r_{LL}(\varepsilon-\hbar\omega)$
$\alpha=R$ $\beta=L$	$t_{LR}^*(\varepsilon)t_{LR}(\varepsilon-\hbar\omega)$ $\times[r_{LL}(\varepsilon)r_{LL}^*(\varepsilon-\hbar\omega)-1]$	$t_{LR}(\varepsilon)t_{LR}^*(\varepsilon-\hbar\omega)$ $\times[r_{RR}^*(\varepsilon)r_{RR}(\varepsilon-\hbar\omega)-1]$	$t_{LR}^*(\varepsilon)t_{LR}^*(\varepsilon-\hbar\omega)$ $\times r_{LL}(\varepsilon)r_{RR}(\varepsilon-\hbar\omega)$	$t_{LR}(\varepsilon)t_{LR}(\varepsilon-\hbar\omega)$ $\times r_{RR}^*(\varepsilon)r_{LL}^*(\varepsilon-\hbar\omega)$

and

- $f_\gamma^e(\varepsilon)$, $f_\delta^h(\varepsilon)$ = Distribution function for electrons, holes
- $t_{\alpha\beta}(\varepsilon) = i\sqrt{\Gamma_\alpha\Gamma_\beta}\,G_{\text{dot}}^r(\varepsilon)$ and $\mathcal{T}_{\alpha\beta}(\varepsilon) = \lvert t_{\alpha\beta}(\varepsilon)\rvert^2$
- $\mathcal{T}_{LR}^{\text{eff},\alpha}(\varepsilon) = 2\text{Re}\{t_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon)$

Effective transmission coefficient includes inelastic contributions

ELASTIC CONTRIBUTIONS ONLY

Optical theorem

Unitary of the S-matrix: $\mathbf{SS}^+ = \mathbf{1}$ with $\mathbf{S} = \mathbf{1}+i\mathbf{T}$

$$\mathbf{T} = \begin{pmatrix} it_{LL}(\varepsilon) & it_{LR}(\varepsilon) \\ it_{RL}(\varepsilon) & it_{RR}(\varepsilon) \end{pmatrix} \Rightarrow \quad 2\mathrm{Re}\{t_{\alpha\alpha}(\varepsilon)\} - \mathcal{T}_{\alpha\alpha}(\varepsilon) = \mathcal{T}_{LR}(\varepsilon)$$

$$\Rightarrow \boxed{\mathcal{T}_{LR}^{\mathrm{eff},\alpha}(\varepsilon) = \mathcal{T}_{LR}(\varepsilon)}$$

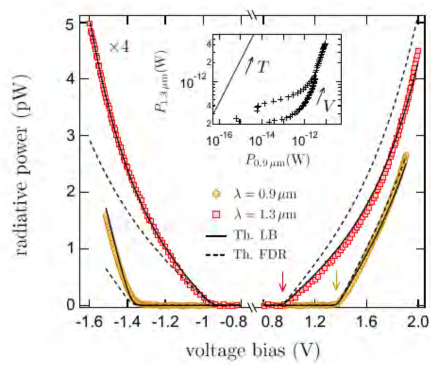
Noise in the L–reservoir

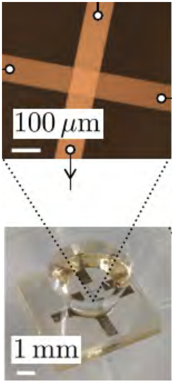
$$\begin{aligned} \mathcal{S}_{LL}(\omega) = \; & \frac{e^2}{h}\int_{-\infty}^{\infty} d\varepsilon \Big[\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-\hbar\omega) f_R^e(\varepsilon) f_R^h(\varepsilon-\hbar\omega) \\ & + [1-\mathcal{T}_{LR}(\varepsilon)]\,\mathcal{T}_{LR}(\varepsilon-\hbar\omega) f_L^e(\varepsilon) f_R^h(\varepsilon-\hbar\omega) \\ & + \mathcal{T}_{LR}(\varepsilon)\,[1-\mathcal{T}_{LR}(\varepsilon-\hbar\omega)]\, f_R^e(\varepsilon) f_L^h(\varepsilon-\hbar\omega) \\ & + \left[\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-\hbar\omega) + |t_{LL}(\varepsilon)-t_{LL}(\varepsilon-\hbar\omega)|^2\right] f_L^e(\varepsilon) f_L^h(\varepsilon-\hbar\omega) \Big] \end{aligned}$$

In agreement with *Büttiker (1992)* and *Hammer, Belzig (2011)*

NEXT SECTION

TUNNEL JUNCTION (Al/AlO$_x$/Al)





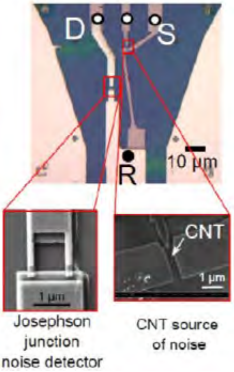
Février, Gabelli, arXiv:1707.03803

$T = 100$ K

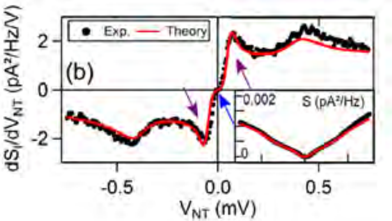
$$P = \mathcal{S}_{LL}(\omega) + \mathcal{S}_{RR}(\omega) - \Big[\mathcal{S}_{LR}(\omega) + \mathcal{S}_{RL}(\omega)\Big]$$

with $\mathcal{S}_{LL}(\omega) \neq \mathcal{S}_{RR}(\omega)$ and $\mathcal{S}_{LL}(\omega) \neq -\mathcal{S}_{LR}(\omega)$

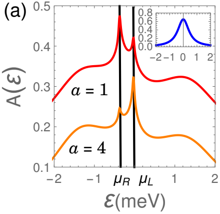
NANOTUBE QUANTUM DOT



Delagrange, Basset Bouchiat, Deblock PRB 97, 041412R (2018)



- Plateau at $eV < \hbar\omega$
- Kondo peak at $eV \approx \pm\hbar\omega$
- Coulomb blockade at $eV \approx \pm U/2$



NEXT SECTION

NOISE IN THE L-RESERVOIR

$$\mathcal{S}_{LL}(\omega) = \frac{e^2}{h}\int_{-\infty}^{\infty} d\varepsilon \Big[\mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon-\hbar\omega)f_R^e(\varepsilon)f_R^h(\varepsilon-\hbar\omega)$$
$$+\left[1-\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)\right]\mathcal{T}_{LR}(\varepsilon-\hbar\omega)f_L^e(\varepsilon)f_R^h(\varepsilon-\hbar\omega)$$
$$+\mathcal{T}_{LR}(\varepsilon)\left[1-\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon-\hbar\omega)\right]f_R^e(\varepsilon)f_L^h(\varepsilon-\hbar\omega)$$
$$+\left[\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon)\mathcal{T}_{LR}^{\text{eff},L}(\varepsilon-\hbar\omega)+|t_{LL}(\varepsilon)-t_{LL}(\varepsilon-\hbar\omega)|^2\right]f_L^e(\varepsilon)f_L^h(\varepsilon-\hbar\omega)\Big]$$

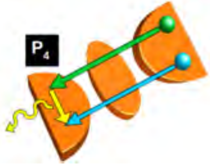
Each of the 4 contributions to the noise is associated to a **different initial configuration for the e-h pair**

Is it possible to identify the **physical processes** that lead to each of these 4 contributions?

YES!

CONTRIBUTIONS TO $\mathcal{S}_{LL}(\omega)$

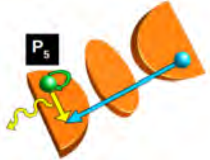
e-h pair initially in the R-reservoir



$$t_4 = t_{RL}(\varepsilon)t^*_{RL}(\varepsilon - \hbar\omega)$$

$$\Rightarrow |t_4|^2 = \mathcal{T}_{LR}(\varepsilon)\mathcal{T}_{LR}(\varepsilon - \hbar\omega)$$

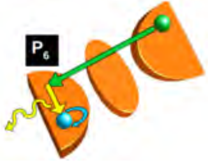
electron in L & hole in R



$$t_5 = r_{LL}(\varepsilon)t^*_{RL}(\varepsilon - \hbar\omega)$$

$$\Rightarrow |t_5|^2 = \left[1 - \mathcal{T}^{\mathrm{eff},L}_{LR}(\varepsilon)\right]\mathcal{T}_{LR}(\varepsilon - \hbar\omega)$$

electron in R & hole in L



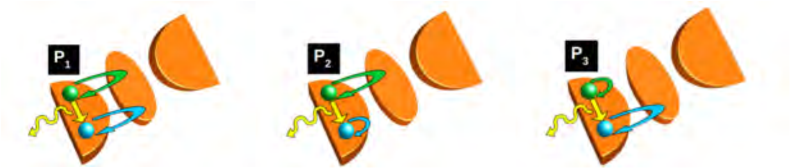
$$t_6 = t_{RL}(\varepsilon)r^*_{LL}(\varepsilon - \hbar\omega)$$

$$\Rightarrow |t_6|^2 = \mathcal{T}_{LR}(\varepsilon)\left[1 - \mathcal{T}^{\mathrm{eff},L}_{LR}(\varepsilon - \hbar\omega)\right]$$

CONTRIBUTIONS TO $\mathcal{S}_{LL}(\omega)$

e-h pair initially in L

We identify 3 processes: P_1, P_2 and P_3



$t_1 = t_{LL}(\varepsilon)t^*_{LL}(\varepsilon - \hbar\omega)$ $\quad$ $t_2 = t_{LL}(\varepsilon)r^*_{LL}(\varepsilon - \hbar\omega)$ $\quad$ $t_3 = r_{LL}(\varepsilon)t^*_{LL}(\varepsilon - \hbar\omega)$

When more than one process involved $\Rightarrow$ Take the coherent superposition

$$|t_1 + t_2 + t_3|^2 = \mathcal{T}^{\text{eff},L}_{LR}(\varepsilon)\mathcal{T}^{\text{eff},L}_{LR}(\varepsilon - \hbar\omega) + |t_{LL}(\varepsilon) - t_{LL}(\varepsilon - \hbar\omega)|^2$$

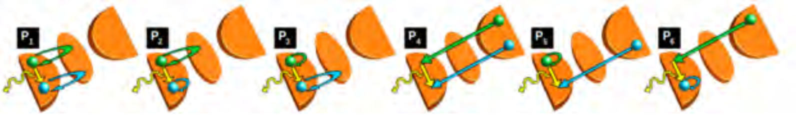
$\Rightarrow$ **Powerful method to get $\mathcal{S}_{LL}(\omega)$ without heavy calculation !**

Crépieux, Sahoo, Duong, Zamoum, Lavagna, PRL 120, 107702 (2018)

NEXT SECTION

CONCLUSION

1. Expression for the noise which includes **inelastic contributions**
2. Each contribution to the noise can be interpreted in terms of e-h pair transmission and **energy exchange with the environment**



3. More than one physical process (starting from the same initial state) $\Rightarrow$ Take the **coherent superposition of scattering paths**
4. **Good agreement with experimental results**
   - Tunnel junction
   - Nanotube Kondo quantum dot

OUTLOOKS

Find a similar interpretation for **heat noise** and **mixed noise**

CO-AUTHORS

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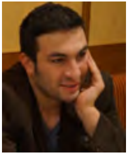

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PUBLICATIONS

A. Crépieux, S. Sahoo, T. Duong, R. Zamoum, M. Lavagna, PRL 120, 107702 (2018)

R. Zamoum, M. Lavagna, A. Crépieux, PRB 93, 235449 (2016)

R. Zamoum, M. Lavagna, A. Crépieux, JSTAT 054013 (2016)