Transport in 1D quantum sytems

T. Giamarchi

http://dqmp.unige.ch/giamarchi/

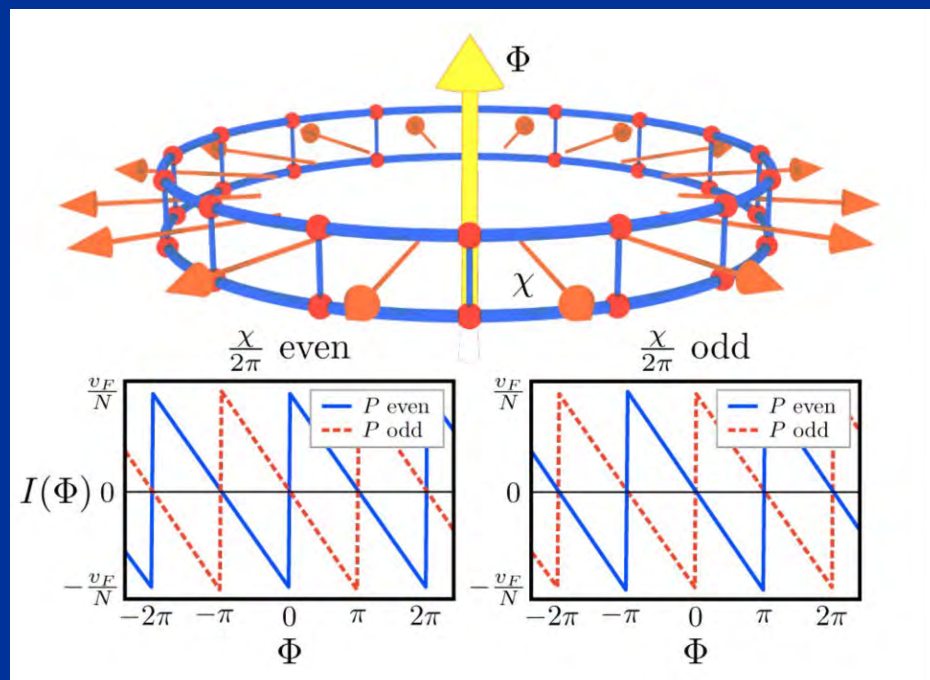
UNIVERSITÉ DE GENÈVE

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

MaNEP SWITZERLAND

A.-M. Visuri, N. Kamar, S. Greschner, F. Hartmeier, C. Bardyn, M. Filippone, C. Berthod, T. Pellegrin, N. Kestin, S. Takayoshi

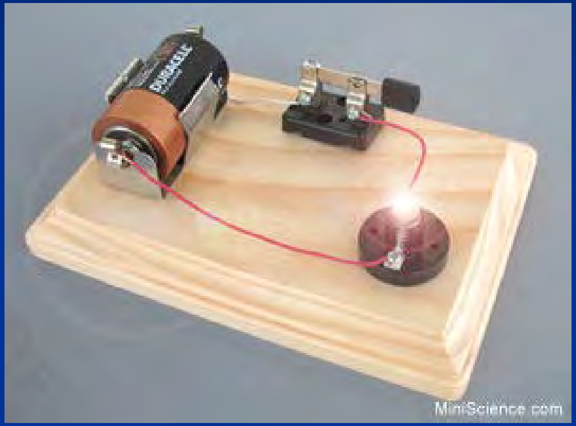
Controlled parity switch of persistent currents in quantum ladders



M. Filippone, C. Bardyn, TG, arXiv:1710.02152

Transport

- Gives informations on the excitations of the system

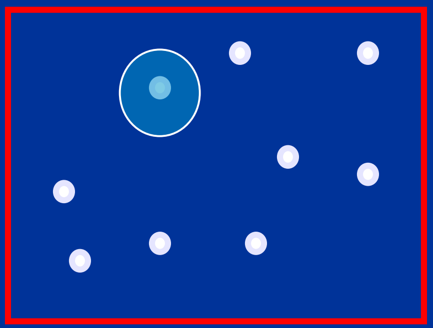


- Out of equilibrium; I = G V (linear response)

- Most cases: free excitations (e.g. Landau's quasiparticles) + lifetime

One dimension is special

- No individual excitation can exist (only collective ones)



- Strong quantum fluctuations

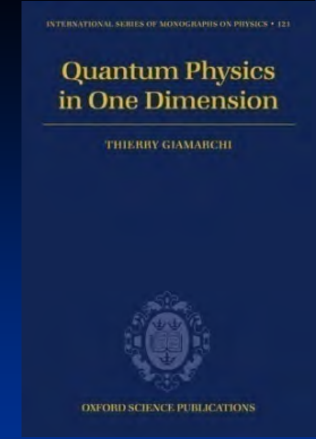
$$\psi = |\psi| e^{i\theta}$$

Difficult to order

Many beautiful things i wont talk about

- Chiral edge states (quantum hall, topological ins)
- Impurity in 1D baths
- Energy transport
- Non linear response / Quenches

References



TG, Quantum physics in one dimension, Oxford (2004)

TG in ``Understanding Q. Phase Transitions", Ed. L. Carr, CRC Press (2010)

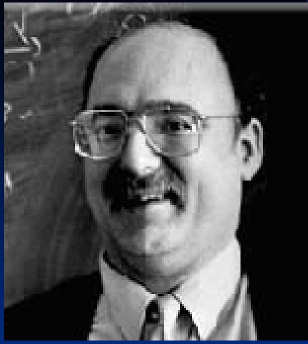
M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

TG, Int J. Mod. Phys. B 26 1244004 (2012)

TG, C. R. Acad. Sci. 17 322 (2016)

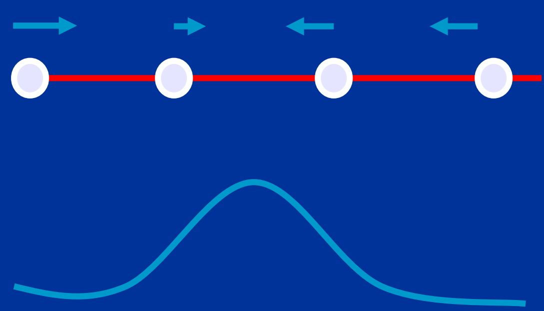
What happens in 1D ?

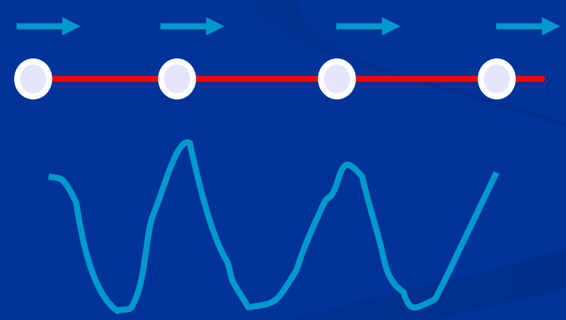
Luttinger liquid concept



- Universal low energy properties in 1D

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$





CDW

$q \sim 0$

$q \sim 2\pi\rho_0$

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

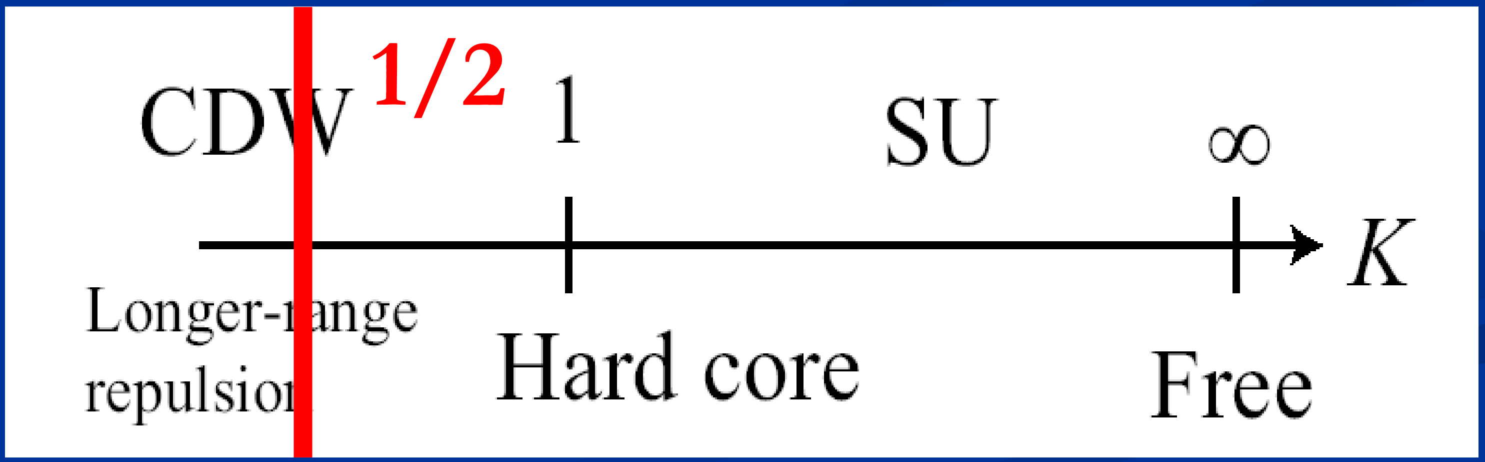
Quantum fluctuations

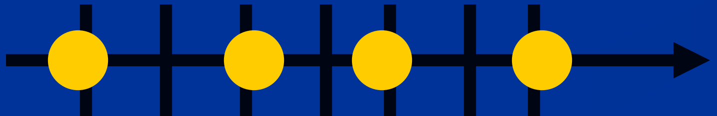
$$H = \frac{\hbar}{2\pi}\int dx [\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

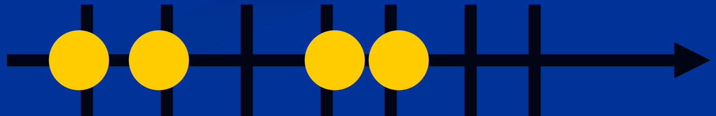
Correlations

$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$







Spins

Use boson or fermions mapping

$$S^{+} = (-1)^{i} e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^{z} = \frac{-1}{\pi} \nabla\phi + (\text{-}1)^{i} \cos(2\phi)$$

Powerlaw correlation functions

$$\langle S^z(x,0) S^z(0,0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x} \right)^{2K}$$

$$\langle S^+(x,0) S^-(0,0) \rangle = C_3 \left(\frac{1}{x} \right)^{2K + \frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x} \right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

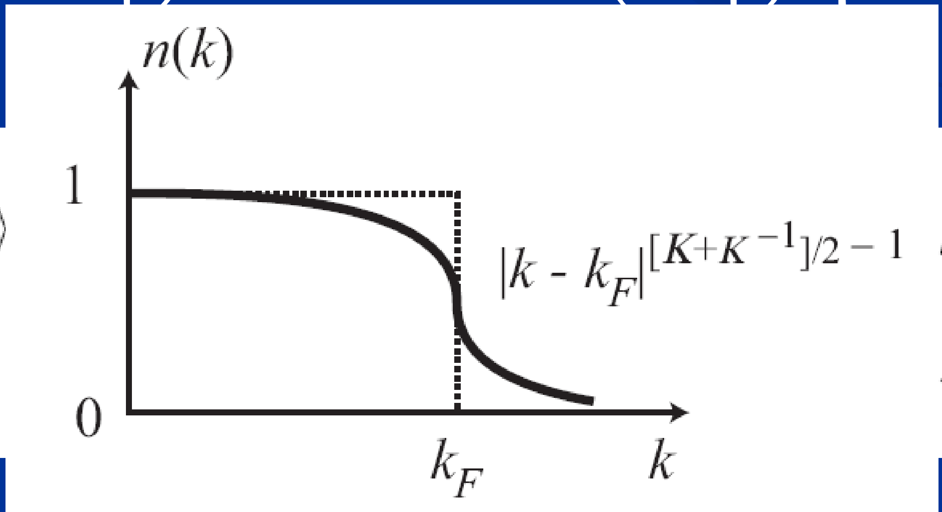
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

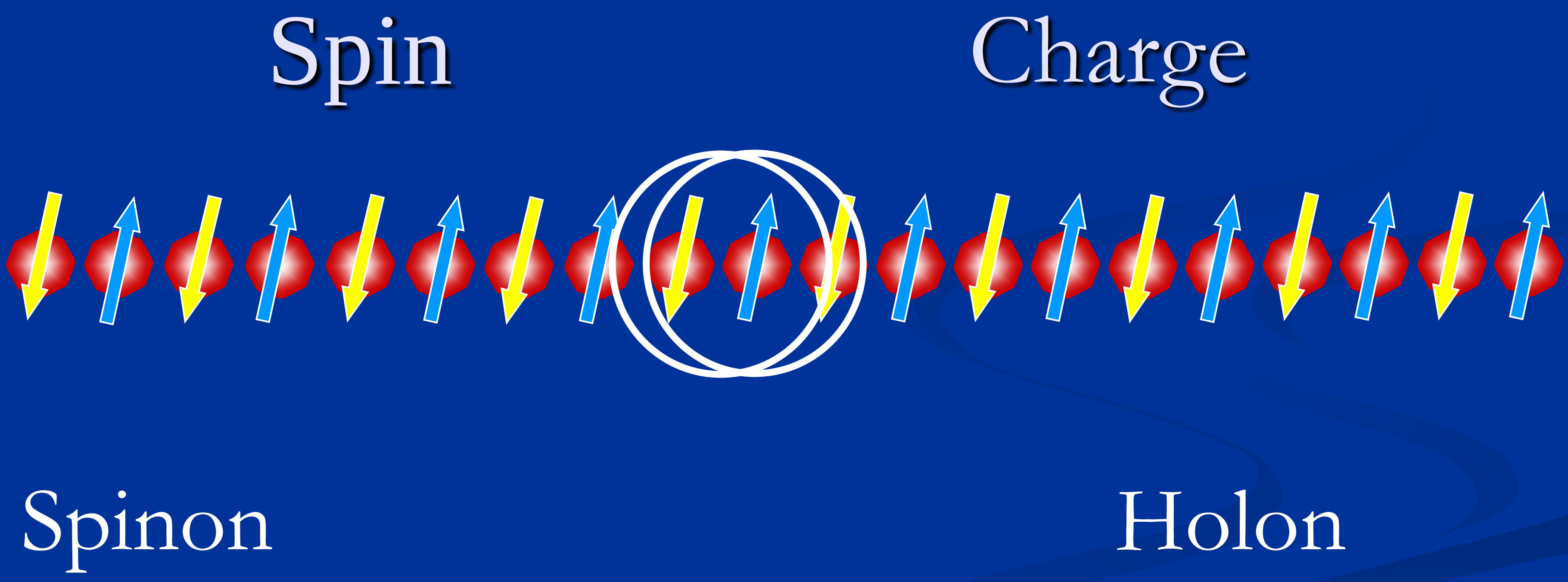
$\langle\rho(x,\tau)\rho(0)\rangle$



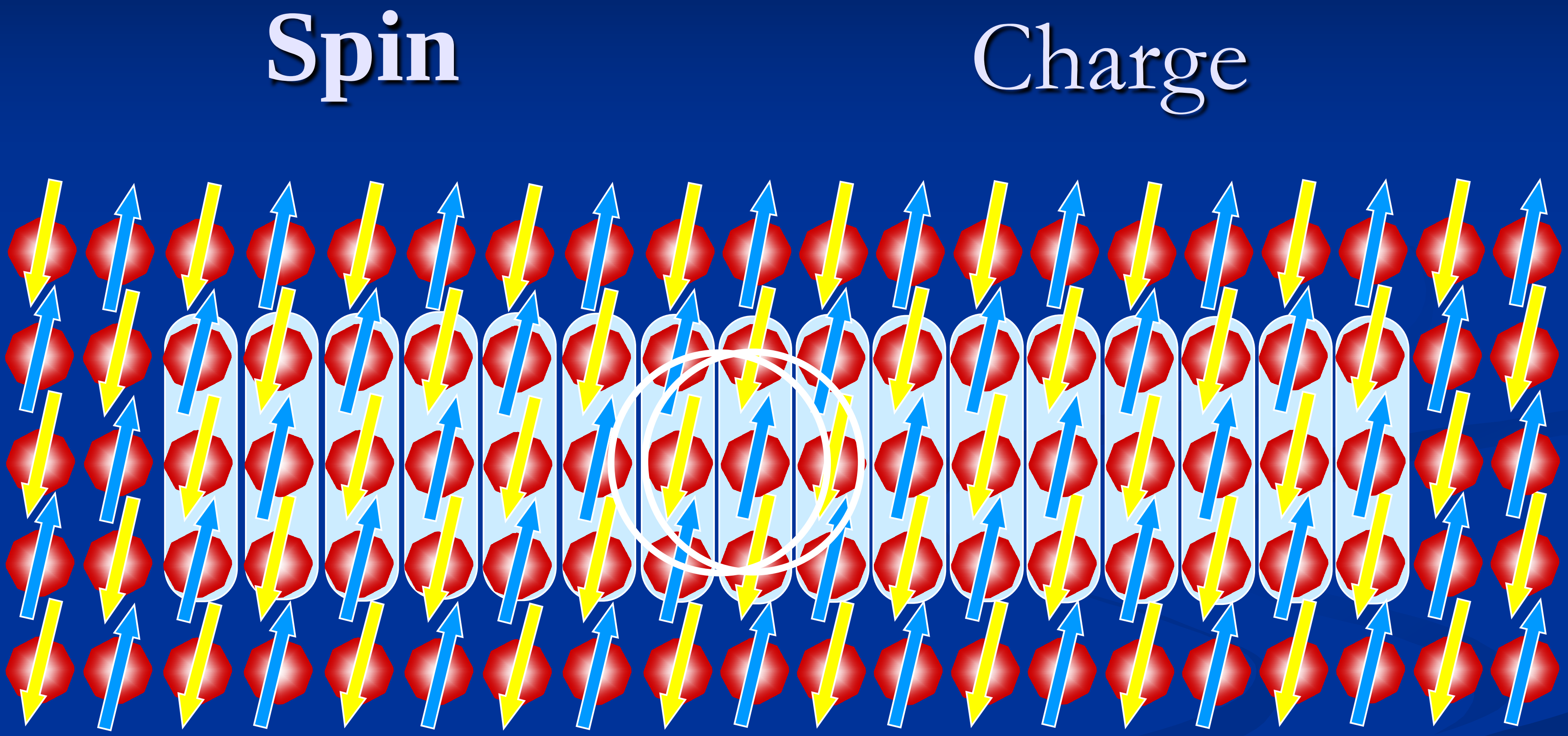
$\rho_0 x) \left(\frac{\alpha}{r}\right)^{2K}$

$\pi\rho_0 x) \left(\frac{\alpha}{r}\right)^{8K} + \cdots$

Deconstruction of the electron: spin-charge separation



Spin-Charge Separation higher D ?



Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

Transport

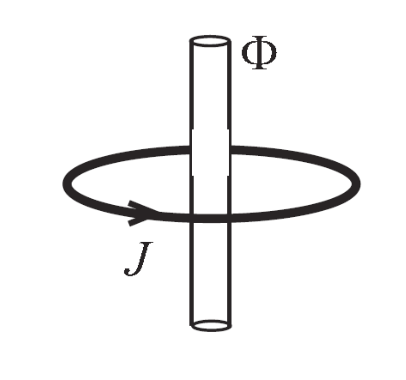
Transport

$$J = v(n_R - n_L) \propto \Pi \sim \partial_t \phi$$

$$\sigma(\omega) = -\frac{e^2}{\pi^2 \hbar} i(\omega + i\delta) \langle \phi(q=0,\omega_n)^* \phi(q=0,\omega_n) \rangle_{i\omega_n \to \omega + i\delta}$$

$$\mathrm{Re}\,\sigma(\omega) = \mathcal{D}\delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

$$\mathcal{D} = uK$$



- Memory function (TG PRB 44 2905 (1991))

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{2u_\rho K_\rho}{\pi} + \chi(\omega)\right]$$

$$\sigma(\omega) = \frac{i2u_\rho K_\rho}{\pi} \frac{1}{\omega + M(\omega)}$$

$$M(\omega) = \frac{\omega \chi(\omega)}{\chi(0) - \chi(\omega)}$$

$$F = [J, H]$$

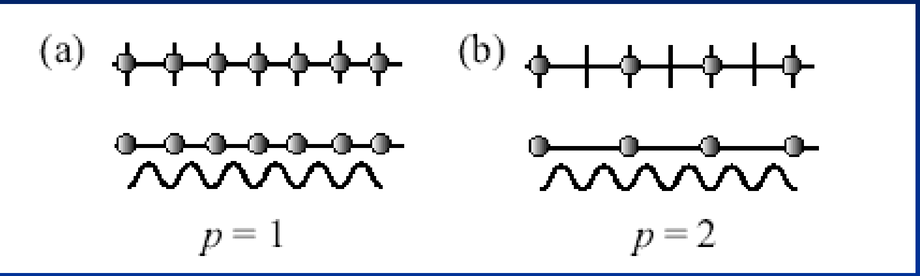
- Approximate ``hydrodynamic" solution:

$$M(\omega) \simeq \frac{[\langle F; F \rangle_\omega^0 - \langle F; F \rangle_{\omega=0}^0]/\omega}{-\chi(0)}$$

- Takes into account all vertex corrections (mandatory in 1d)
- Assumes: i) perturbation; ii) independent scattering event; Not necessarily correct

Periodic lattice

Sine-Gordon hamiltonian



$$H = \int dx V_0 \cos(Qx) \rho(x)$$

$$H = \int dx V_0 \cos(Qx) \rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x) + \delta x)$$

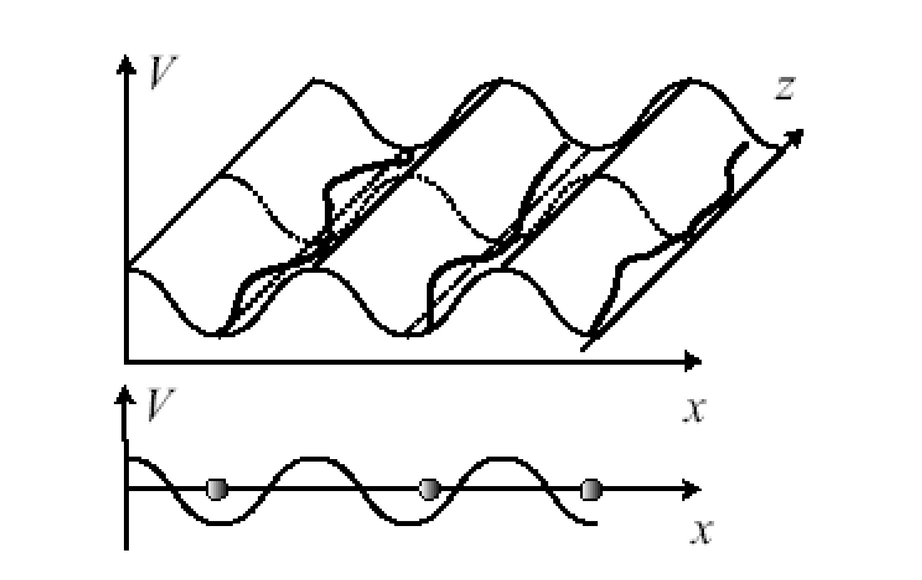
- Commensutate: $Q = 2\pi\rho_0$

$$H = \int dx \cos(2\phi(x))$$

Competition

$$S_0 = \int \frac{dx d\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dx d\tau \cos(2\phi(x))$$



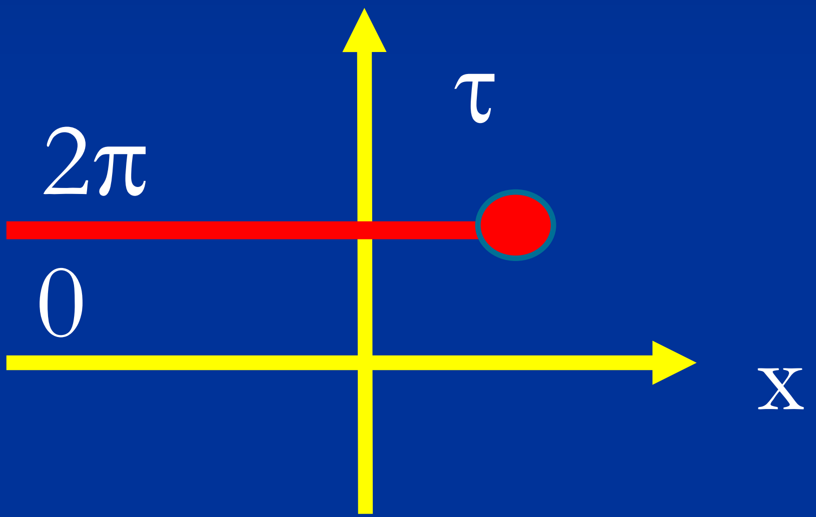
Beresinskii-Kosterlitz-Thouless transition at K=2

String order parameter

Vortex operator

$$e^{iaP}\,|\,x\rangle = |x+a\rangle$$

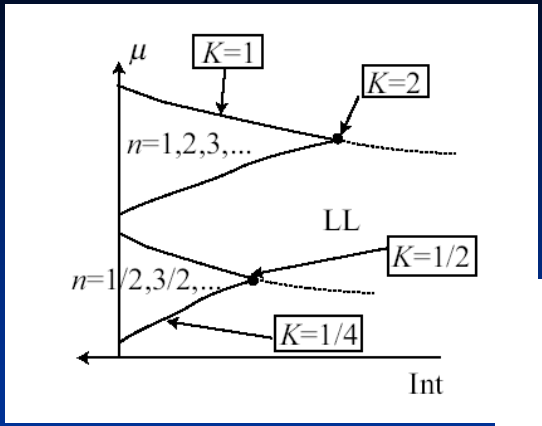
$$\phi(x,\tau) = \pi\int_{-\infty}^{x} dx'\,\Pi_\theta(x',\tau)$$

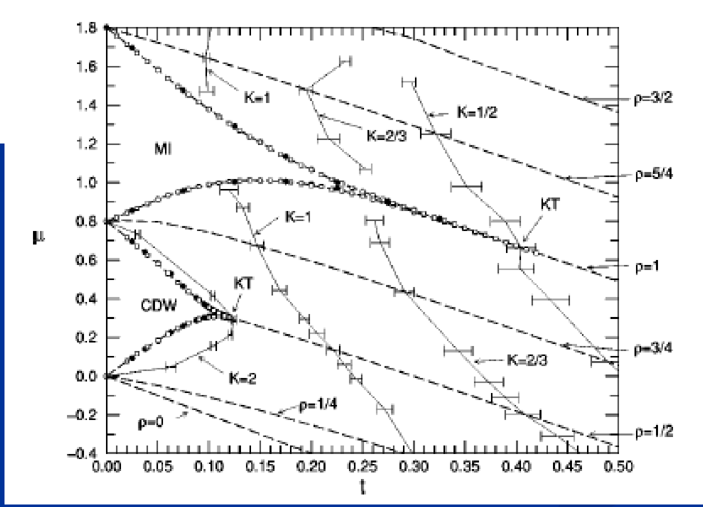


$$\cos(2\phi(x_1,\tau_1))$$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S = \frac{K}{2\pi}\int dx\,d\tau\left[\frac{1}{u}(\partial_\tau\theta) + u(\partial_x\theta)^2\right] - g\int dx\cos(2\phi)$$





Mott insulator:
ϕ is locked

Density is fixed

TG, Physica B 230 975(97): arXiv/0605472 (Salerno lectures);

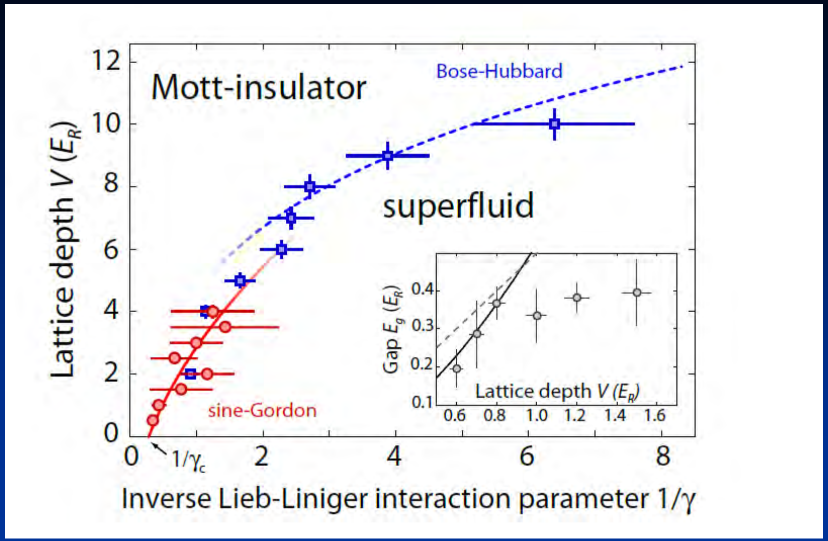
Oxford (2004);

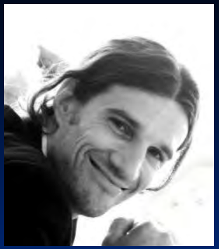
M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)

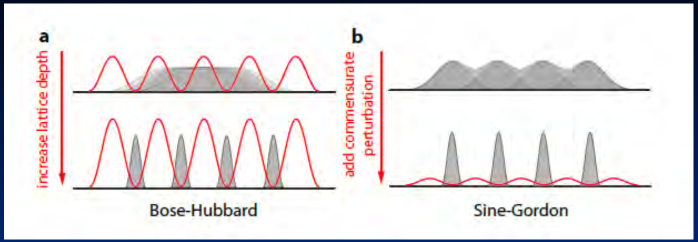
T. Kuhner et al. PRB 61 12474 (2000)

Gap in the excitation spectrum

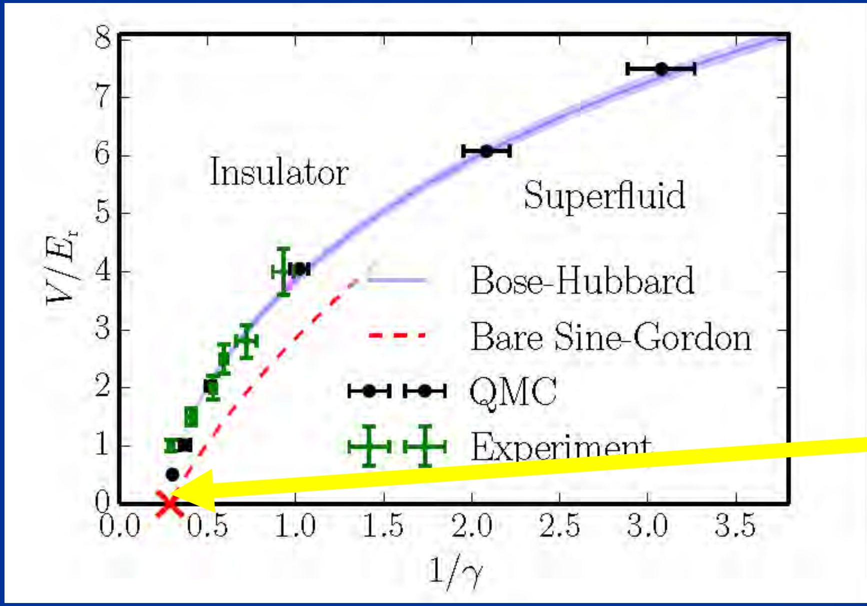
$$G(r) \propto e^{-r/\xi}$$







E. Haller et al. Nature 466 597 (2010)



Renormalized Sine-Gordon

Shows:

K* = 2

G. Boeris et al. PRA 93 93, 011601(R) (2016)

Non local (topological) order

$$\rho(x) \sim \nabla \phi(x)$$

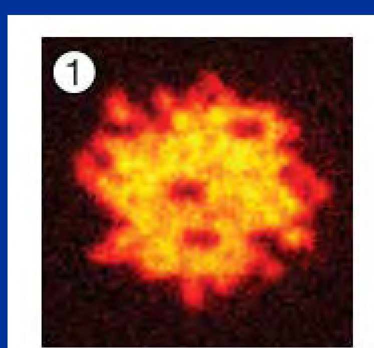
$$\mathcal{O}_{\mathrm{P}}^2 = \lim_{l\to\infty} \mathcal{O}_{\mathrm{P}}^2(l) = \lim_{l\to\infty} \left\langle \prod_{k\leq j\leq k+l} e^{i\pi\delta\hat{n}_j} \right\rangle$$

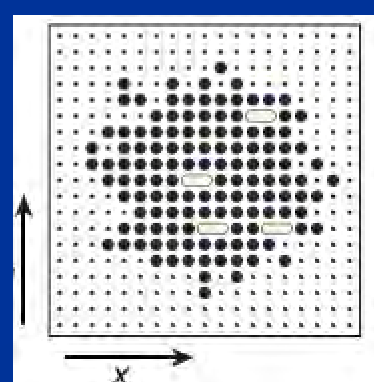
E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, *Phys. Rev. B* **77**, 245119 (2008).

Observation of Correlated Particle-Hole Pairs and String Order in Low-Dimensional Mott Insulators

M. Endres,[1*] M. Cheneau,[1] T. Fukuhara,[1] C. Weitenberg,[1] P. Schauß,[1] C. Gross,[1] L. Mazza,[1] M. C. Bañuls,[1] L. Pollet,[2] I. Bloch,[1,3] S. Kuhr[1,4]

Science (2011)



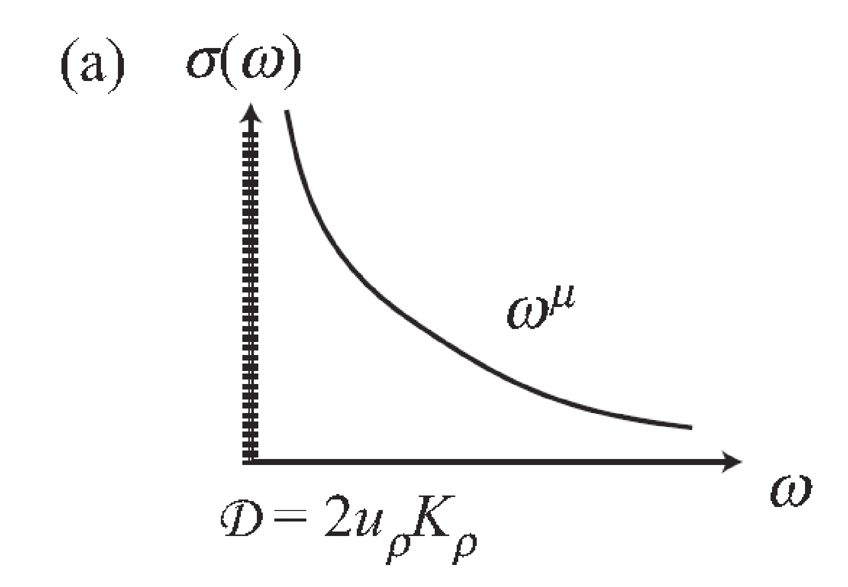


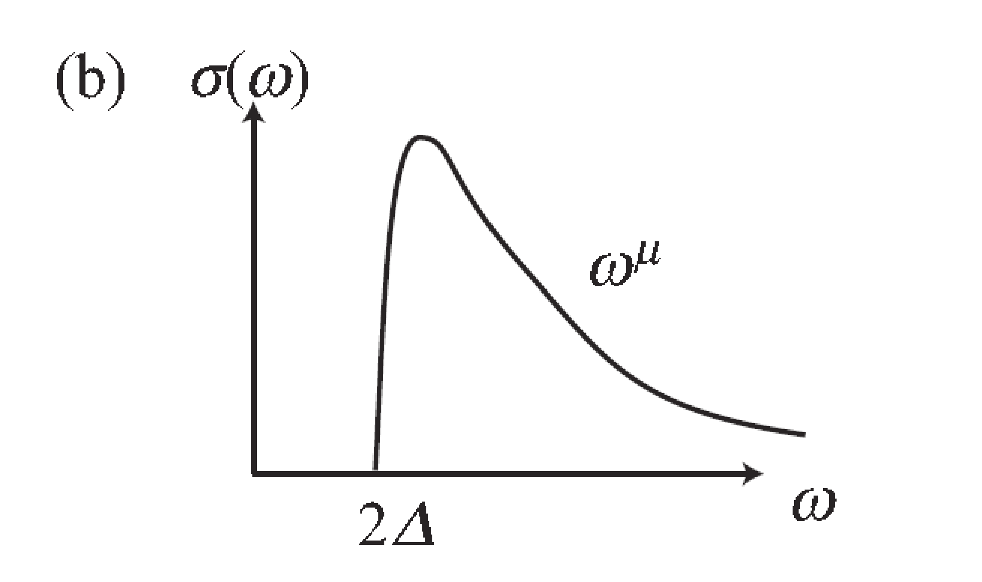
Conductivity (T=0)

$$F = [j, H] = \frac{8 g_3}{(2\pi\alpha)^2}(u_\rho K_\rho)\, i\ \sin(\sqrt{8}\phi_\rho(x,\tau) - \delta x)$$

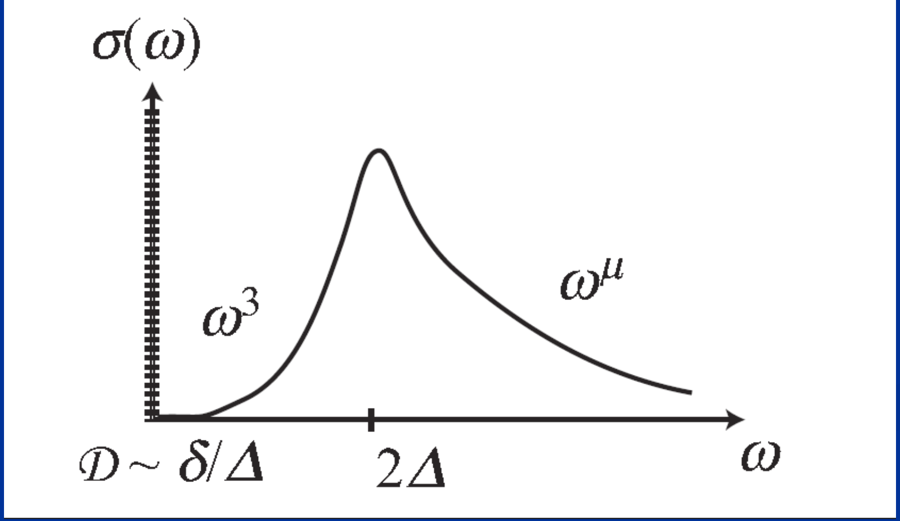
$$M(\omega) = \frac{g_3^2 K_\rho}{\pi^3 \alpha^2}\left(\frac{2\pi\alpha T}{u_\rho}\right)^{4K_\rho - 2} \frac{1}{\omega}\left[B(K_\rho - iS_+, 1 - 2K_\rho)B(K_\rho - iS_-, 1 - 2K_\rho) - B(K_\rho - iS_+^0, 1 - 2K_\rho)B(K_\rho - iS_-^0, 1 - 2K_\rho)\right] \quad (7.97)$$

$$S_\pm = (\omega \pm u_\rho \delta)/(4\pi T)$$





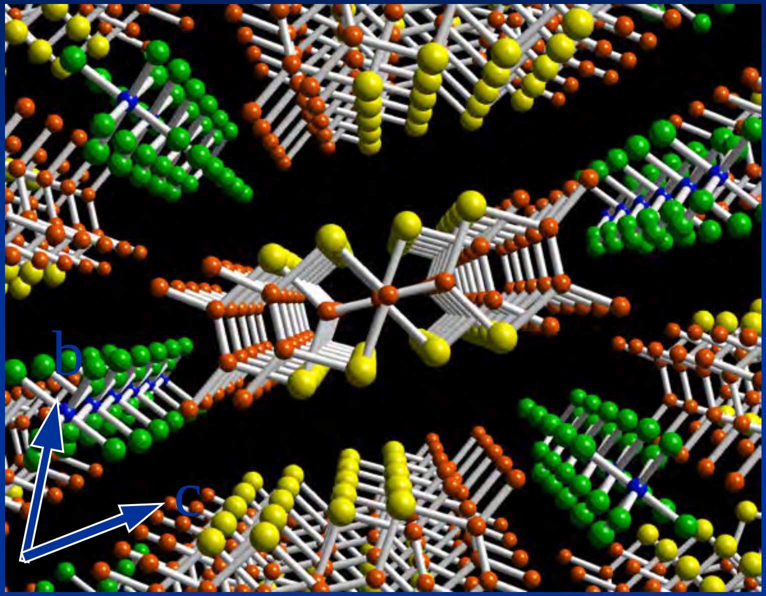
Combine memory function and RG (TG (1991))



Incommensurate:

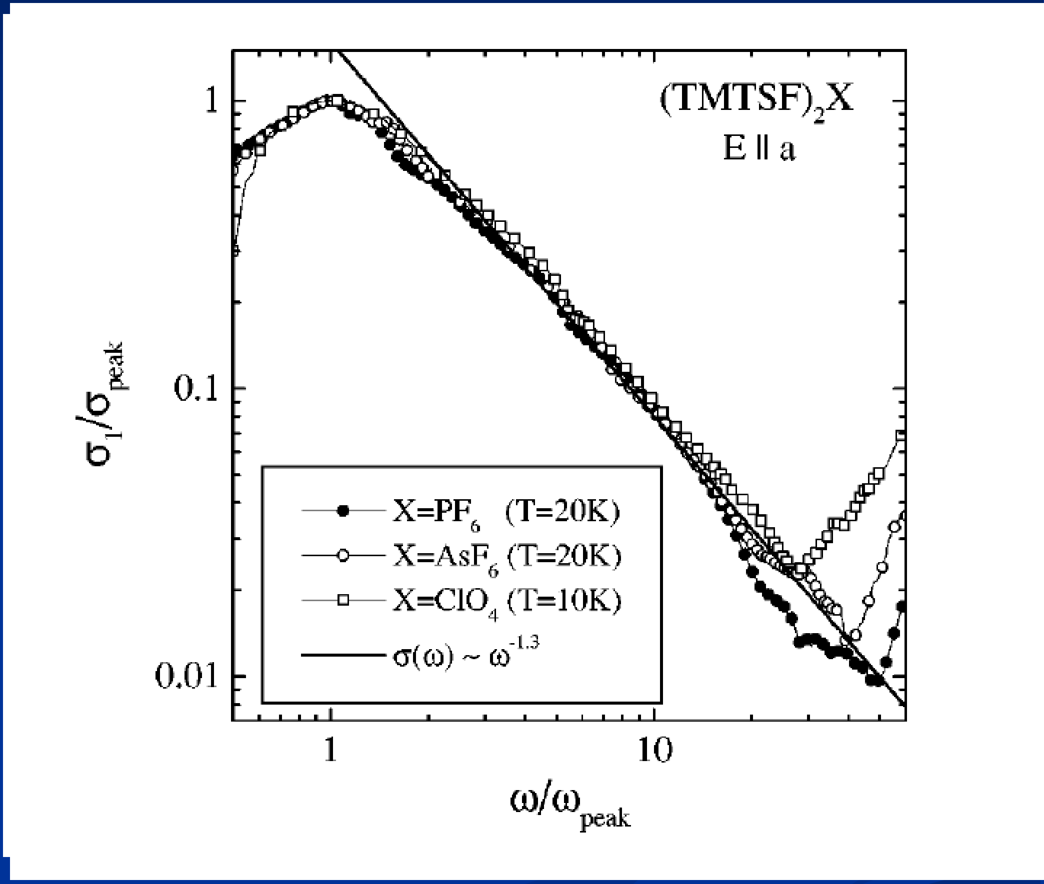
Doping δ

Organic conductors



$$\sigma(\omega) \sim \omega^{-\nu}$$

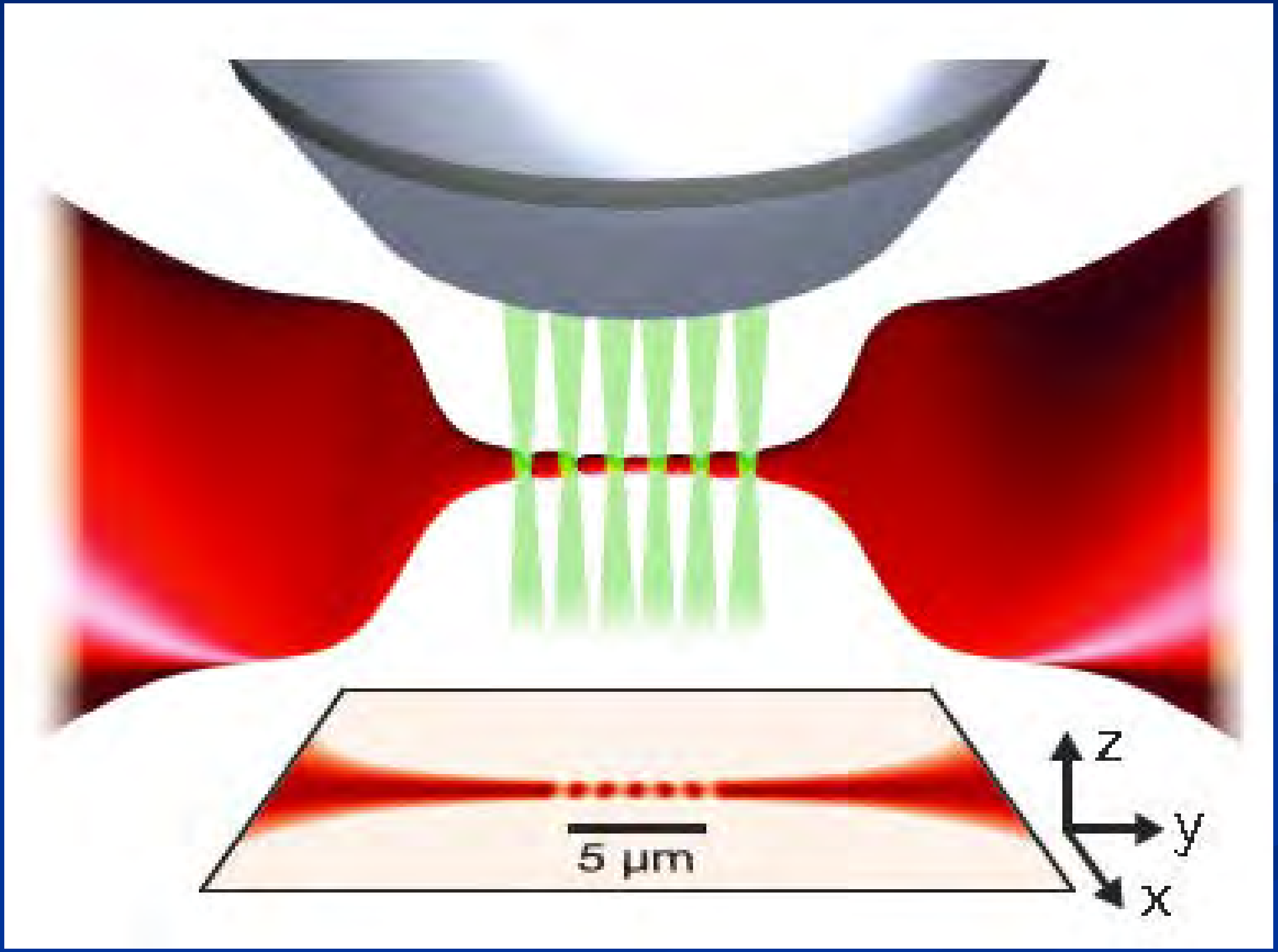
TG PRB (91) :
Physica B 230 (1996)



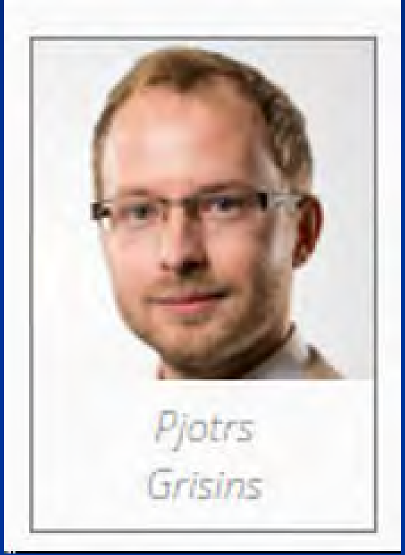
A. Schwartz et al. PRB 58 1261 (1998)

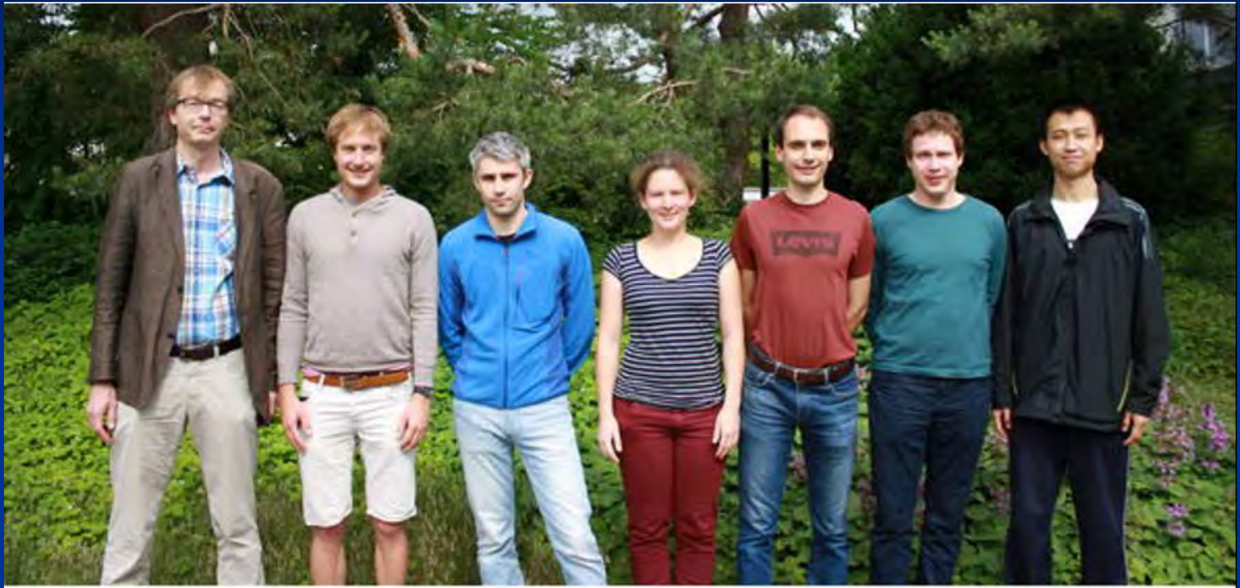
Quarter filling commensurability

Cold atoms



Cold atoms





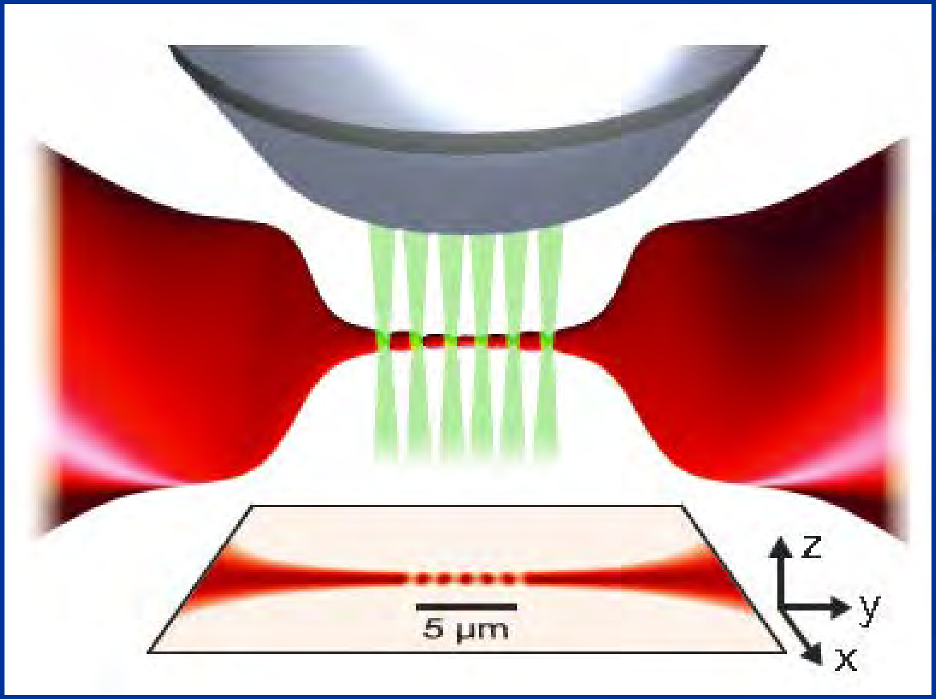
From left to right: Tilman Esslinger, Dominik Husmann, Jean-Philippe Brantut, Laura Corman, Martin Lebrat, Samuel Häusler, Muqing Xu

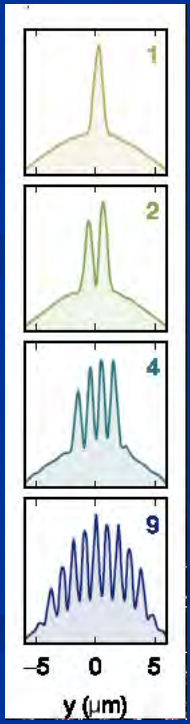
Th: Unige

Expt: J.P Brantut and T. Esslinger's group (ETHZ-EPFL)

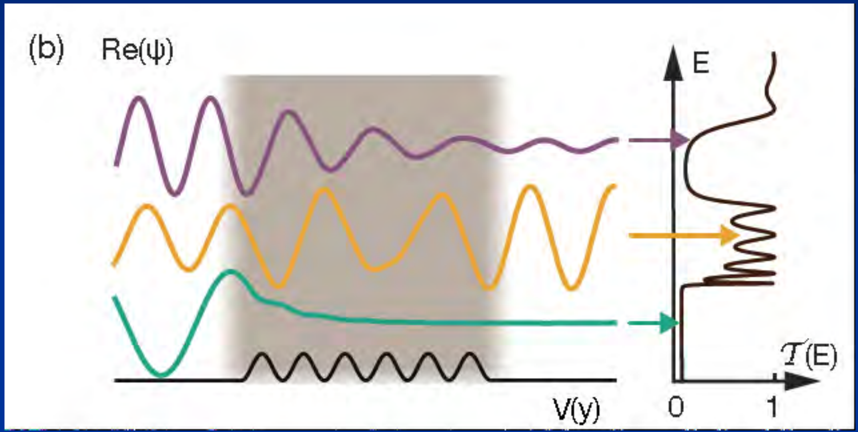
Periodic one dimensional structure

M. Lebrat, P. Grisins et al., Phys. Rev. X 8, 011053 (2018)

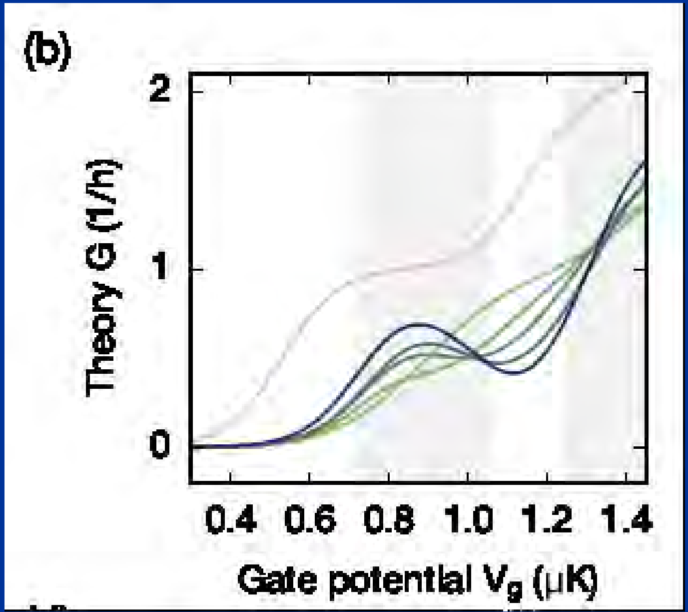




No interactions: band insulator

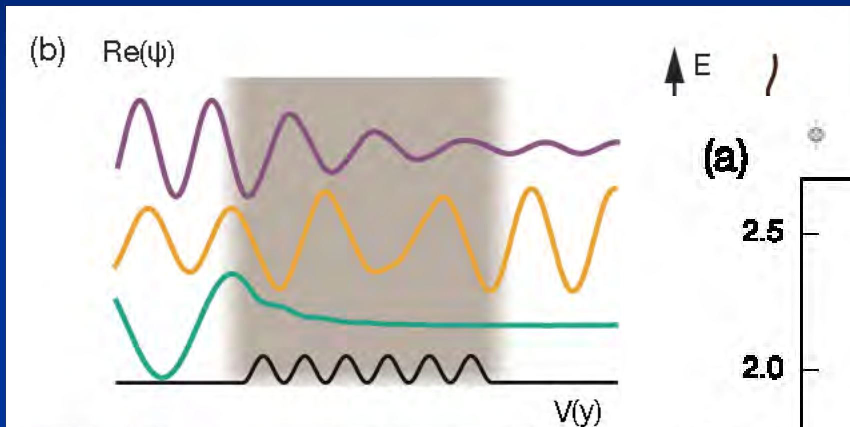


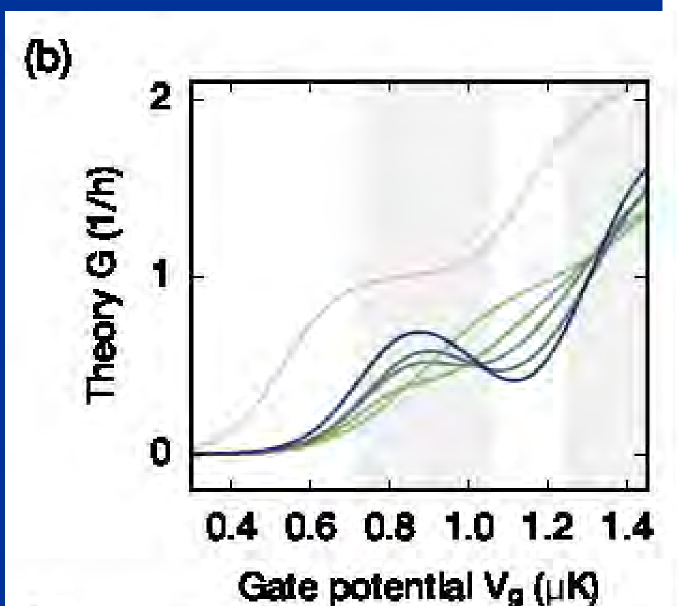
$$G = \frac{1}{h \cdot \Delta\mu} \sum_{n,m} \int_{-\infty}^{\infty} dE\, \mathcal{T}_{n,m}(E) [f_L(E) - f_R(E)].$$

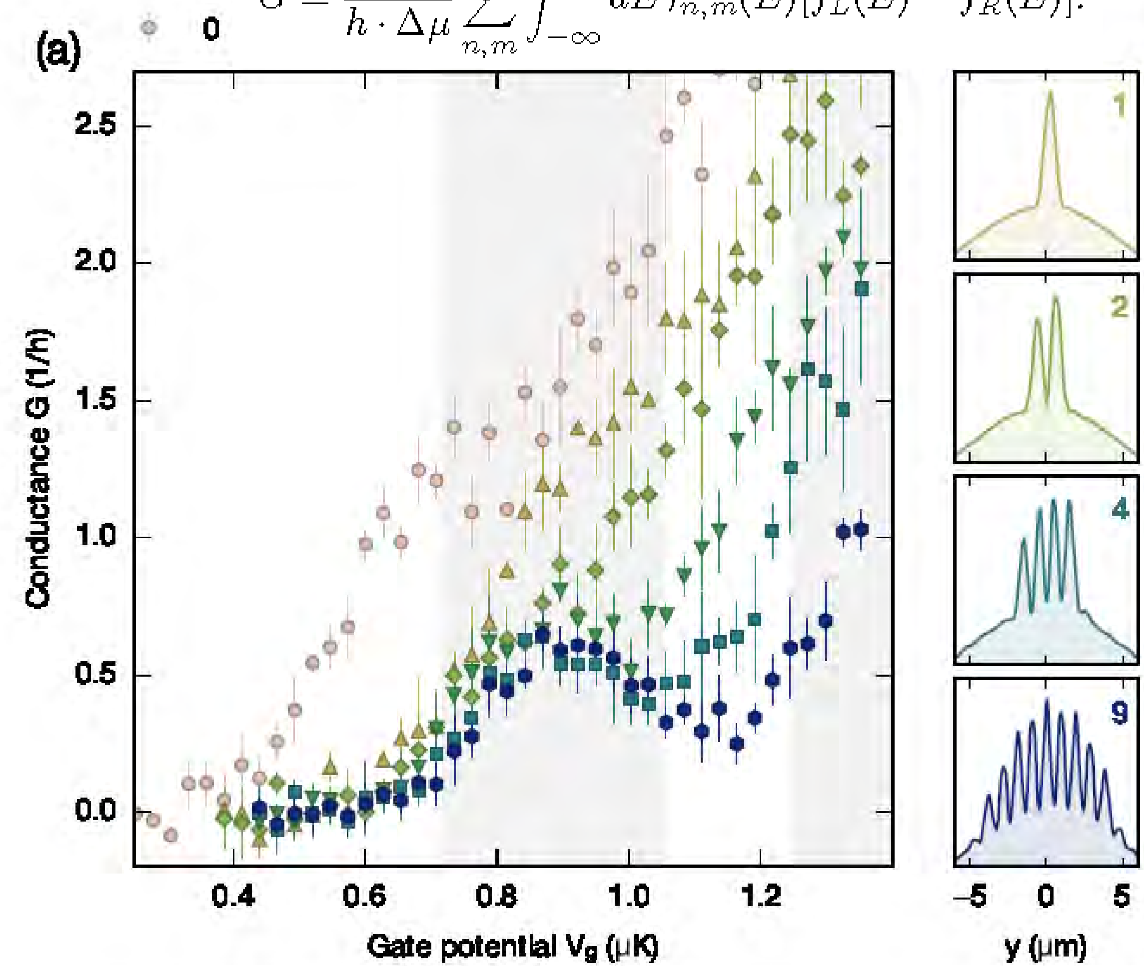


No interactions: band insulator

$$G = \frac{1}{h \cdot \Delta\mu} \sum_{n,m} \int_{-\infty}^{\infty} dE\, \mathcal{T}_{n,m}(E) [f_L(E) - f_R(E)].$$







Attractive interactions and (weak) periodic lattice

- Attraction: singlet superconductor

- Superconductor resists “scattering” (disorder, potentials, etc.)

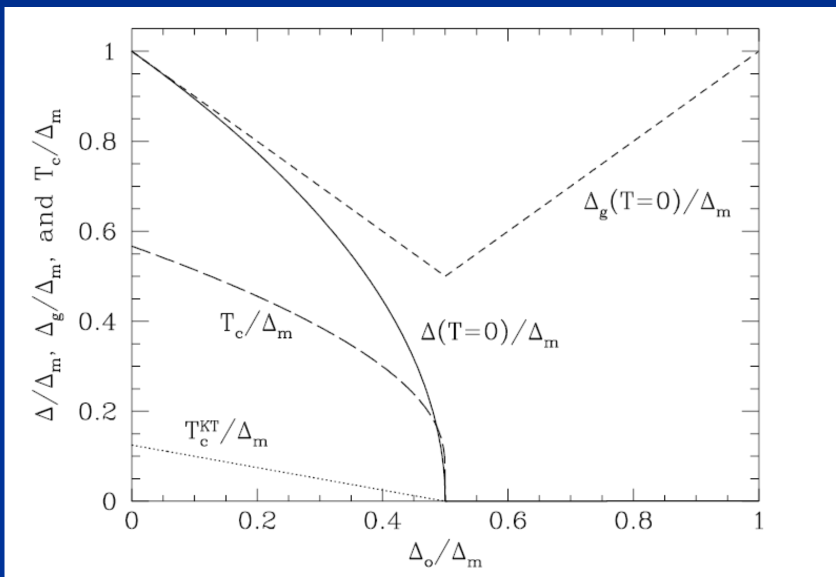
- Expect a competition band insulator-superconductivity

Eur. Phys. J. B **10**, 649–662 (1999)

THE EUROPEAN PHYSICAL JOURNAL B
EDP Sciences

From semiconductors to superconductors: a simple model for pseudogaps

P. Nozières and F. Pistolesi[a]



- Mean-Field
- Superconductor-Band insulator transition

- Always true ?
- How to test experimentally ?

What happens in 1D ?

$$H = H_{\rm GY} + H_{\rm lattice},$$

$$H_{\rm GY} = -\frac{\hbar^2}{2m}\sum_i \frac{\partial^2}{\partial y_i^2} + g_1 \sum_{i<j} \delta(y_i - y_j),$$

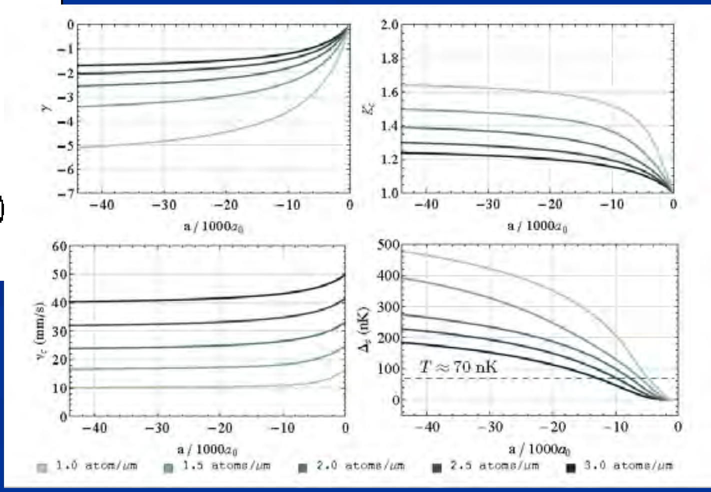
$$H_{\rm lattice} = \int dy\, V(y)\rho(y),$$

Bosonization

$$\mathcal{H}_c = \frac{1}{2\pi}\left[v_c K_c (\nabla\theta_c)^2 + \frac{v_c}{K_c}(\nabla\phi_c)^2\right] - V(y)\rho(y),$$

$$\mathcal{H}_s = \frac{1}{2\pi}\left[v_s K_s (\nabla\theta_s)^2 + \frac{v_s}{K_s}(\nabla\phi_s)^2\right] + \frac{2g_1}{(2\pi\alpha)^2}\cos(\sqrt{8}\phi_s)$$

$$\rho(y) = \rho_0 - \frac{\sqrt{2}}{\pi}\nabla\phi_c(y) + \rho_0\left[e^{i\left(2k_F y - \sqrt{2}\phi_c(y)\right)}\cos(\sqrt{2}\phi_s(y)) + c.c.\right] + C\rho_0\left[e^{i\left(4k_F y - 2\sqrt{2}\phi_c(y)\right)} + c.c.\right]. \quad \text{(D}$$



Luther-Emery liquid

- Gap in the spin sector (singlet pairing)

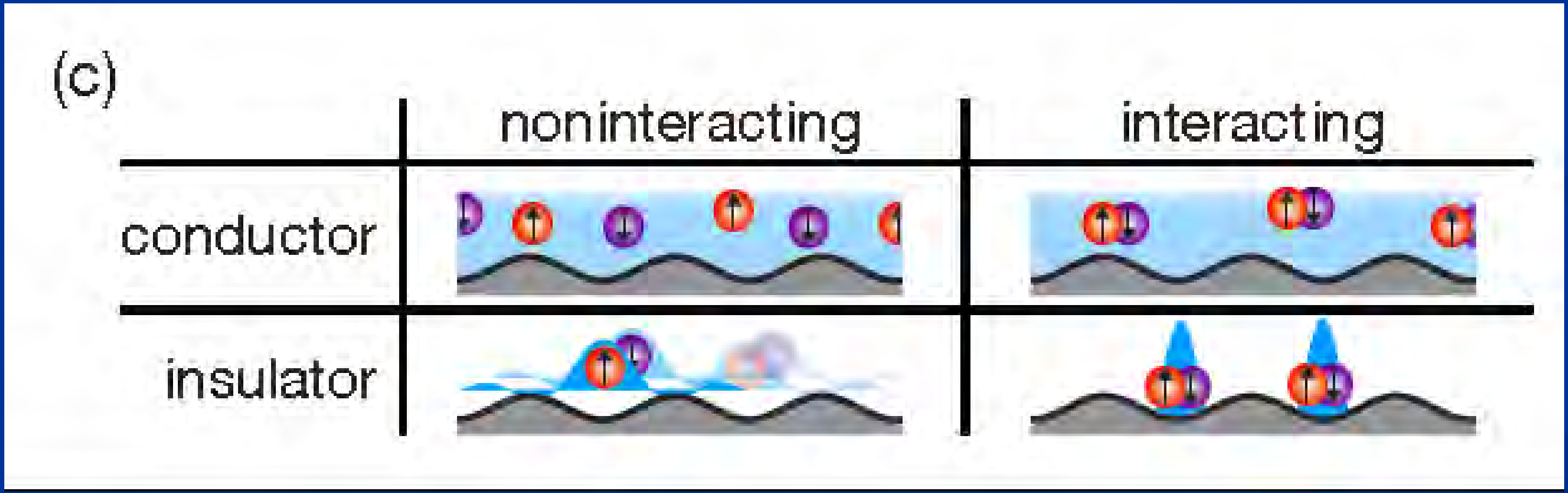
$$\rho(y) = \rho_0 - \frac{\sqrt{2}}{\pi}\nabla\phi_c(y) + 2\rho_0 f_s \cos\left(2k_F y - \sqrt{2}\phi_c(y)\right) + 2C\rho_0 \cos\left(4k_F y - 2\sqrt{2}\phi_c(y)\right),$$

- Conductance determined by the charge sector

$$\mathcal{H}_\mu(y) = -\mu(y)\rho(y) = \mu(y)\frac{\sqrt{2}}{\pi}\nabla\phi_c,$$

$$I_{\uparrow\downarrow}(y) = \frac{\sqrt{2}}{\pi}\partial_t\phi_c(y,t)$$

Many-body insulator ``pinned" L.E. liquid



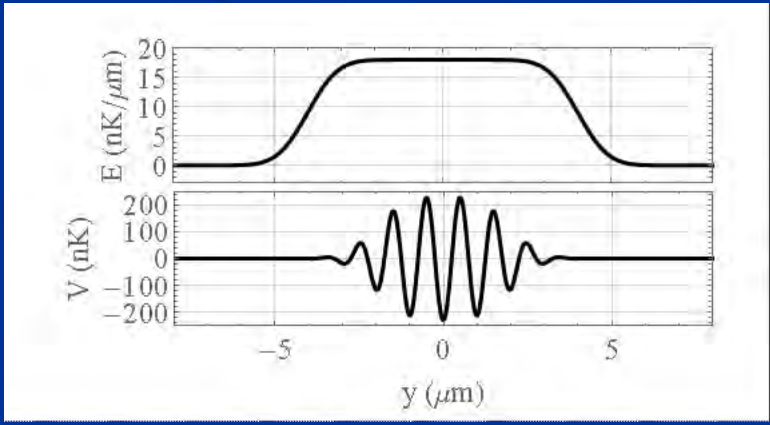
Calculation of transport

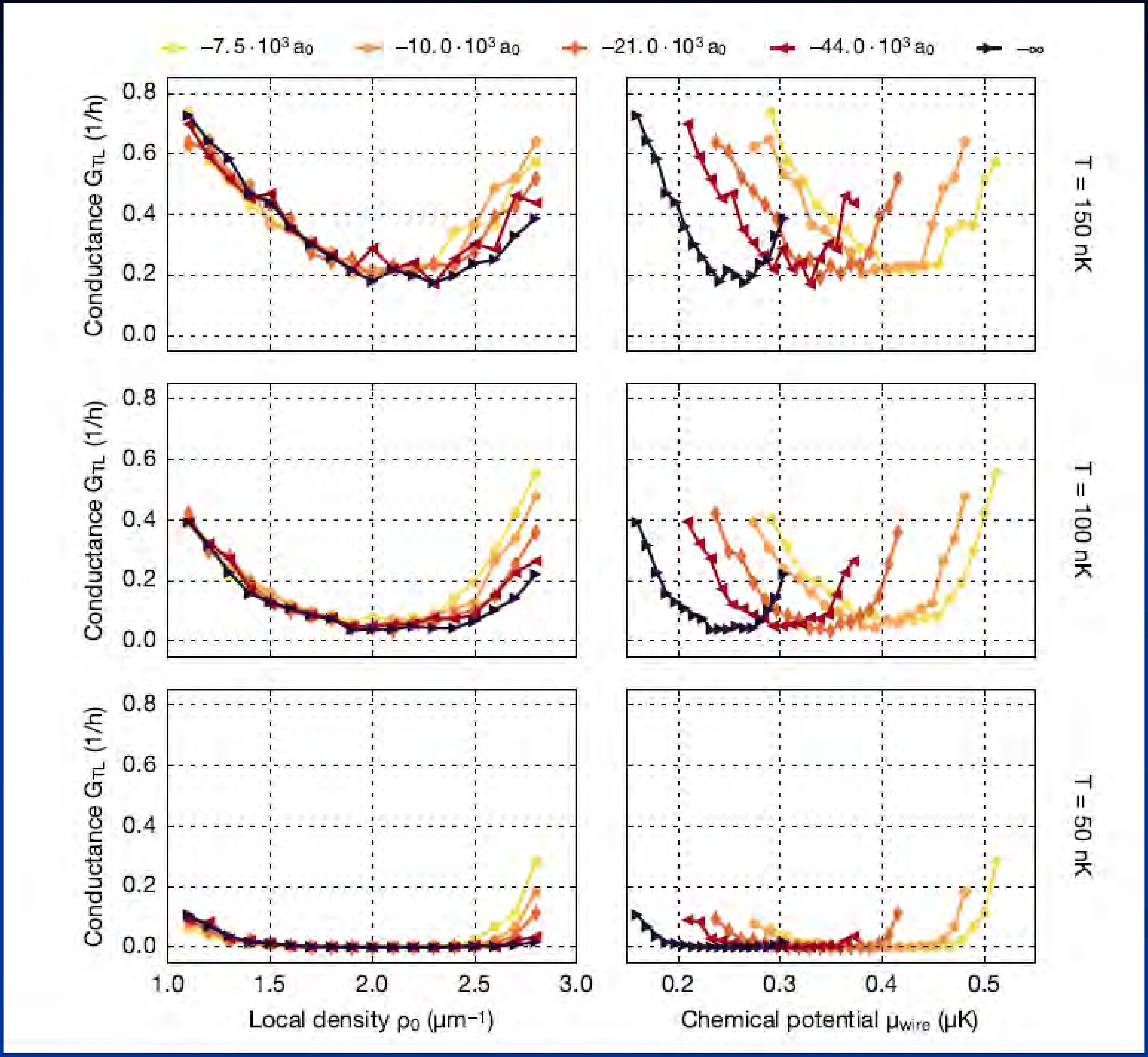
- Solve classical equation of motion

$$-2\sqrt{2}V(y)\rho_0\left[2\sin\left(2\sqrt{2}\phi_c(y,t)-4k_Fy\right)+f_s\sin\left(\sqrt{2}\phi_c(y,t)-2k_Fy\right)\right]=$$
$$=\frac{v_c}{\pi K_c}\partial_{yy}\phi_c(y,t)-\frac{1}{\pi v_c K_c}\partial_{tt}\phi_c(y,t)-\frac{\sqrt{2}V'(y)}{\pi}+\frac{\sqrt{2}}{\pi}E(y).$$

- Finite region of size L
- Boundary condition and thermal noise

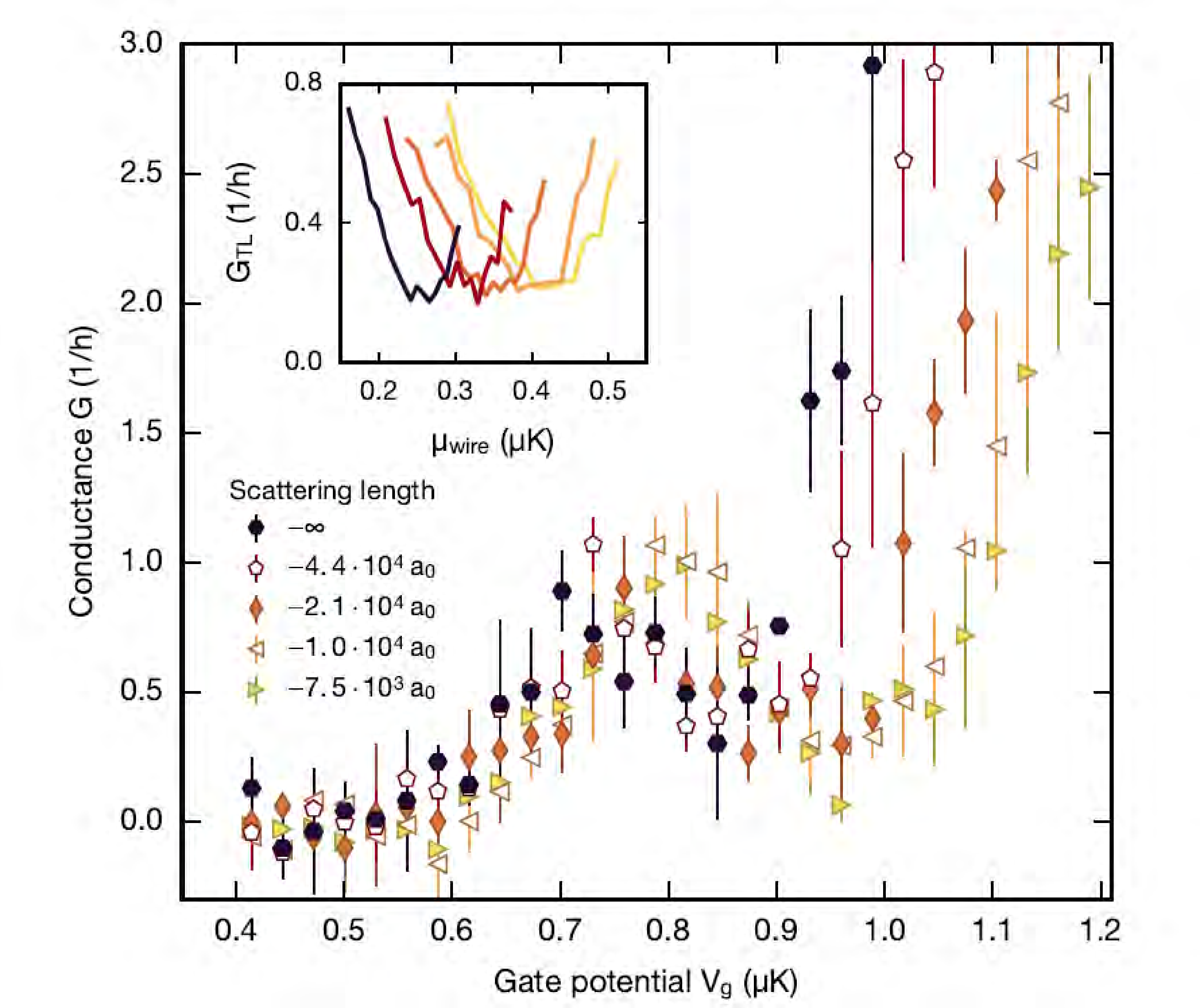
$$d\phi_c(L,t)+v_c\nabla\phi_c(L,t)\,dt=\sigma_T\,dW_L(t),$$
$$d\phi_c(0,t)-v_c\nabla\phi_c(0,t)\,dt=\sigma_T\,dW_0(t),$$





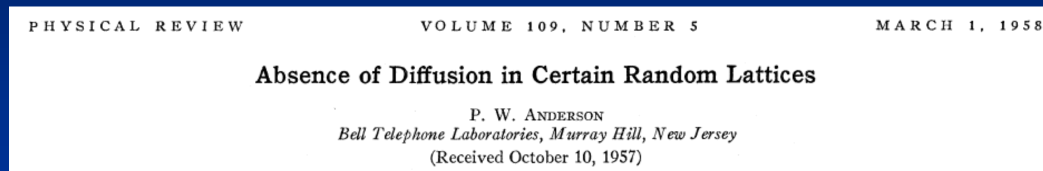
−7.5 · 10³ a₀
−10.0 · 10³ a₀
−21.0 · 10³ a₀
−44.0 · 10³ a₀
−∞
Conductance G_TL (1/h)
T = 150 nK
T = 100 nK
T = 50 nK
Local density ρ₀ (μm⁻¹)
Chemical potential μ_wire (μK)

Experimental evidence for L.E. liquid



Disorder

Anderson Localization



PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

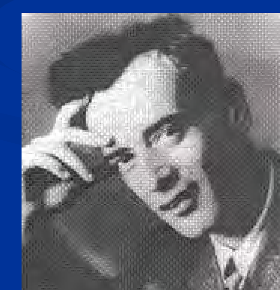
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Light, sound, electrons, etc……. waves

www.andersonlocalization.com

Interactions *and* disorder ?

- U>0 Landau Fermi liquid $m \rightarrow m^*$



- U < 0 Supraconductivity Insensitive to disorder ?

Disorder and Interactions

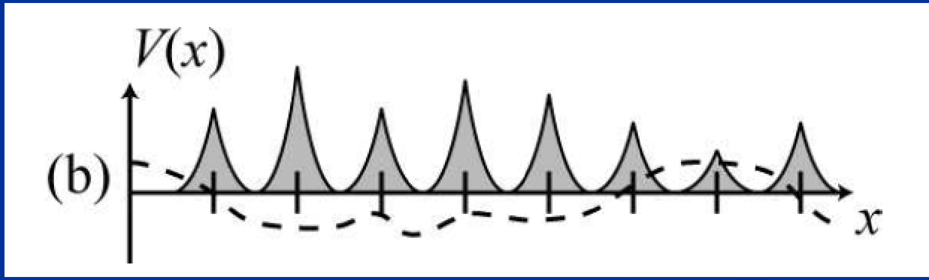
- Fermions: reinforcement of interactions by disorder
  perturbative: Altshuler-Aronov-Lee (80)
  RG: Finkelstein (84); TG+Schulz (88)

Localization ? Phases (electron glass) ? Transport ? …..

- Bosons: competition between superfluidity/localization

Free Bosons: pathological

$$H = \frac{1}{2m}\left(\frac{1}{L}\right)^2 - V_0$$



Interactions needed from the start

Example: localization of interacting bosons

Dirty bosons

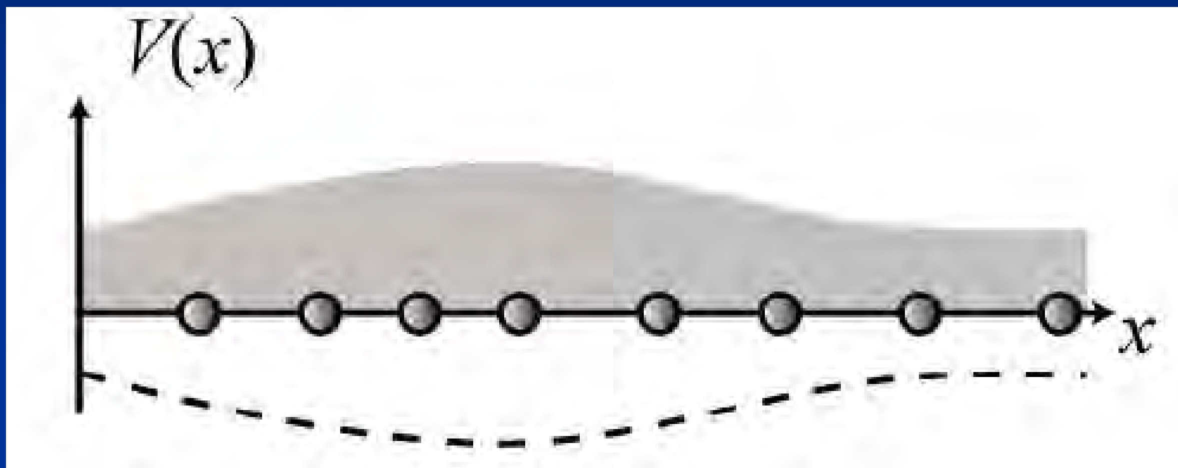
TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

$$H_{\text{dis}} = \int dx\, V(x) \rho(x)$$

$$H_{\text{dis}} = \int dx\, V(x) [-\frac{1}{\pi} \nabla \phi(x) + \rho_0 (e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$

``Two” fourier components of disorder

Forward scattering (q » 0)

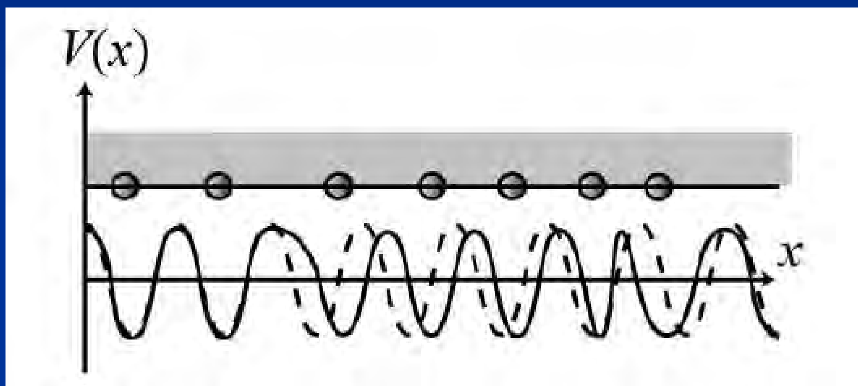


Random (smooth) chemical potential

No localization

Can break commensurate phases

Backward scattering (q » 2 π ρ_0)



$$\frac{dK}{dl} = -\frac{K^2}{2}\tilde{D}_\mathrm{b}$$
$$\frac{dD}{dl} = (3-2K)\tilde{D}_\mathrm{b}$$

Responsible for localization (Bose Glass)

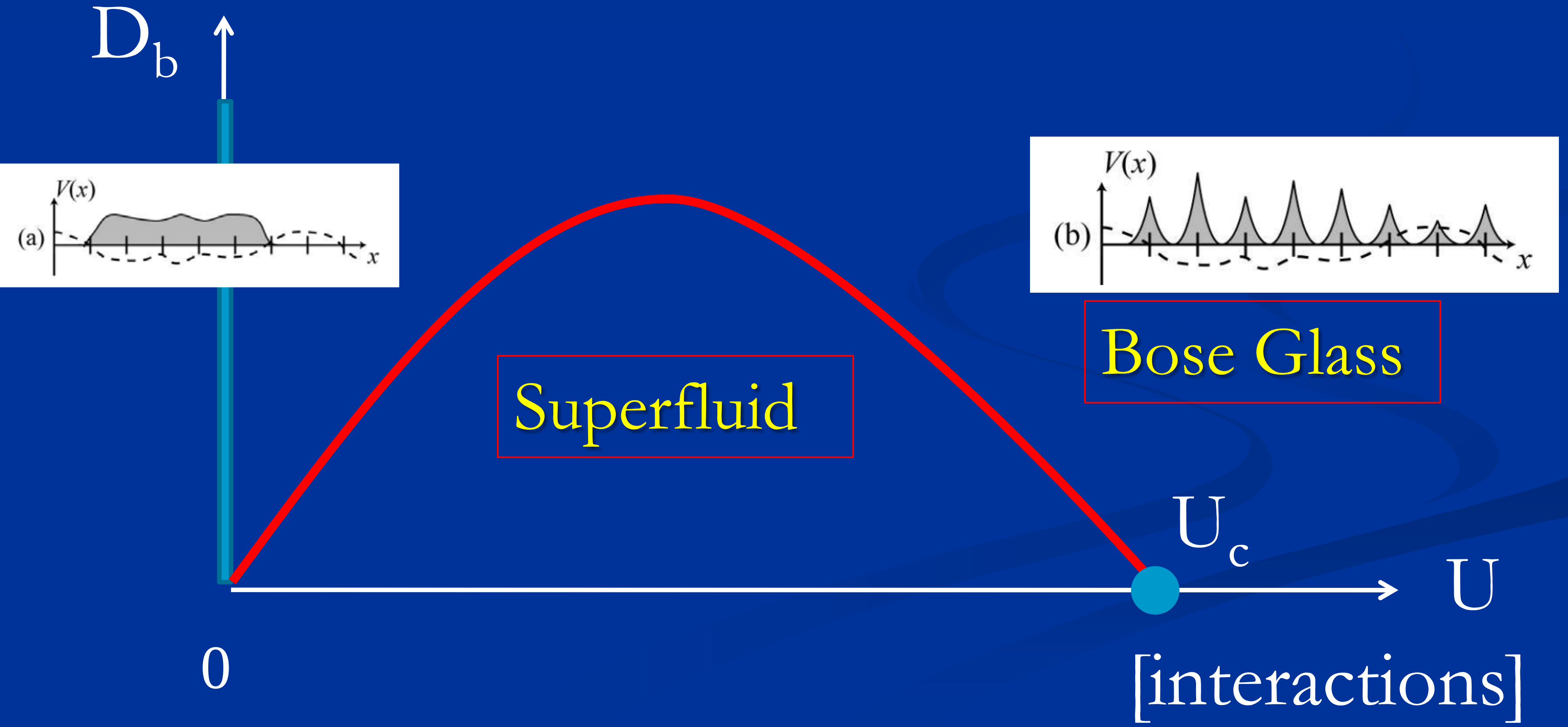
Pinning of a CDW of bosons

$$-D\sum_{ab}\int d\tau d\tau' dx \cos(2\phi_a(x\tau) - 2\phi_b(x\tau'))$$

Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);

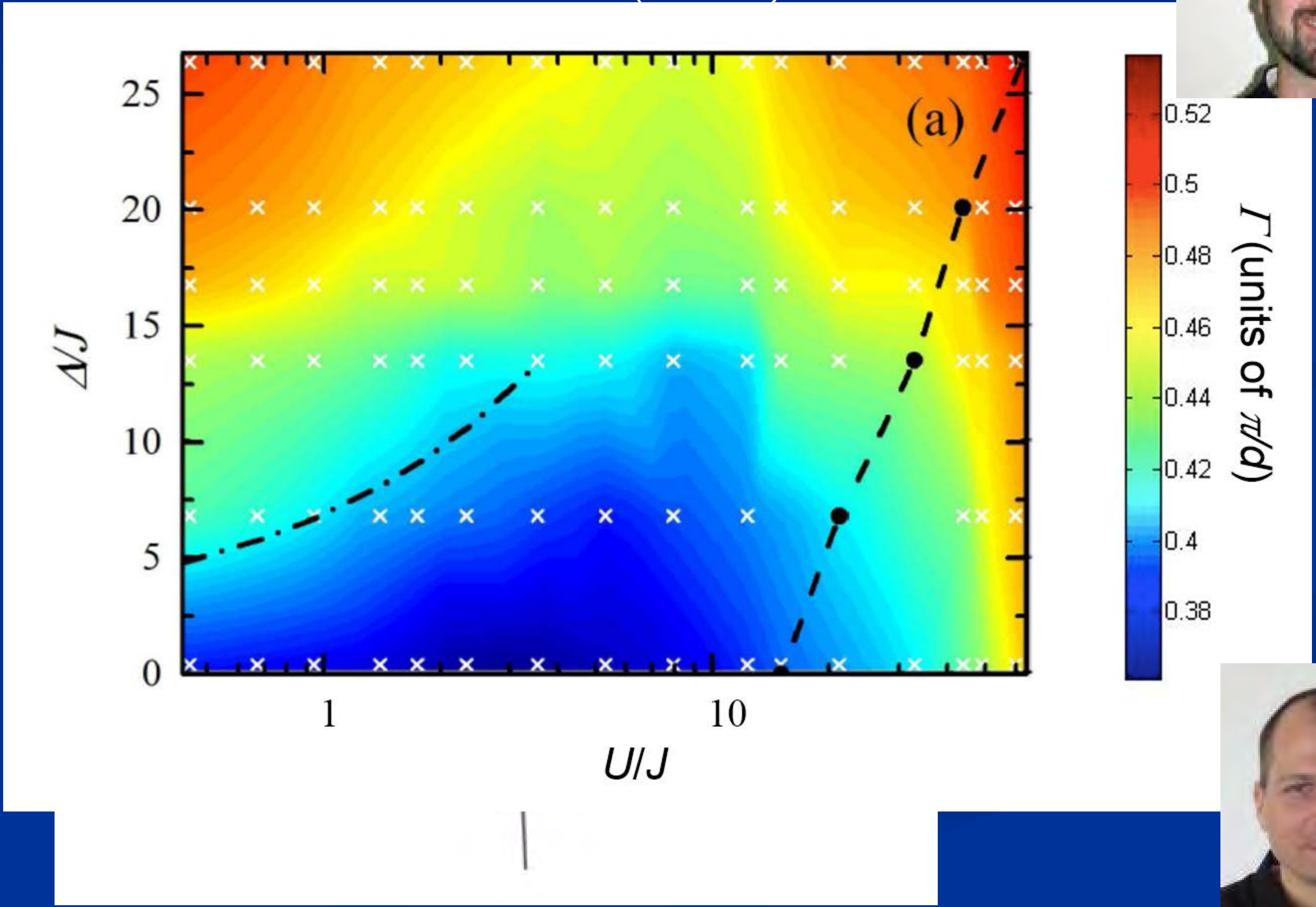
M.P.A. Fisher et al. PRB 40 546 (1989)



Quasi-periodics and interactions

C. D'Errico, E. Lucioni et al. PRL (2014);

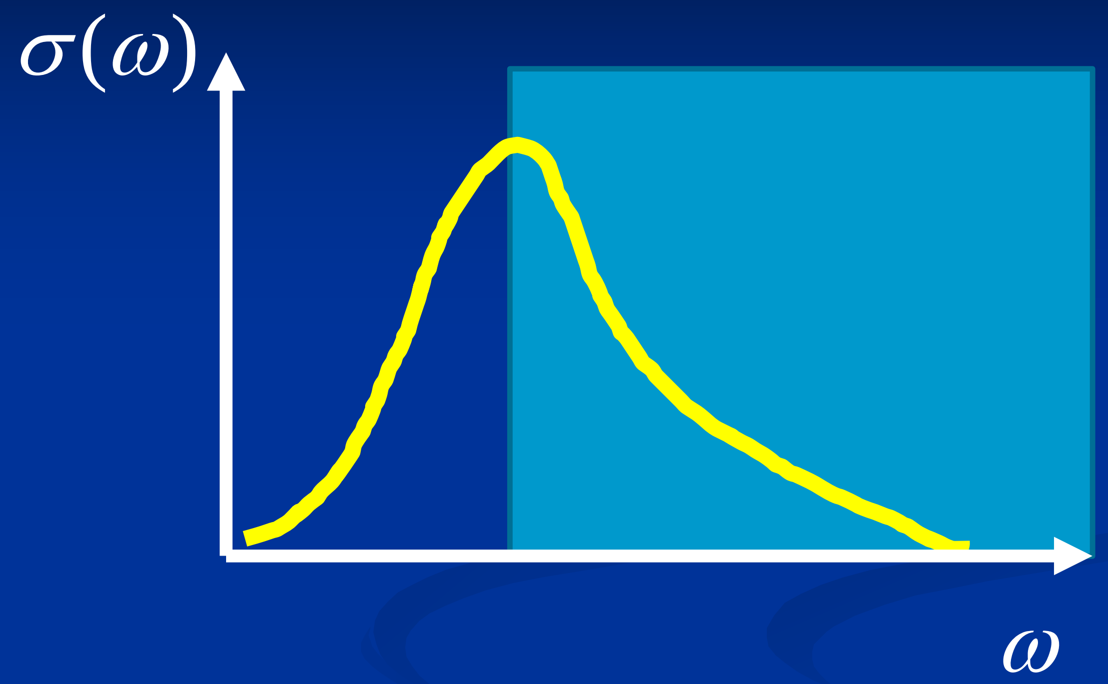
L. Gori et al PRA 93 033650 (2016)



Transport

a.c. transport

$$\xi_{\rm loc} \sim \alpha \left(\frac{1}{K^2 \tilde{D}_{\rm b}} \right)^{\frac{1}{3-2K}}$$



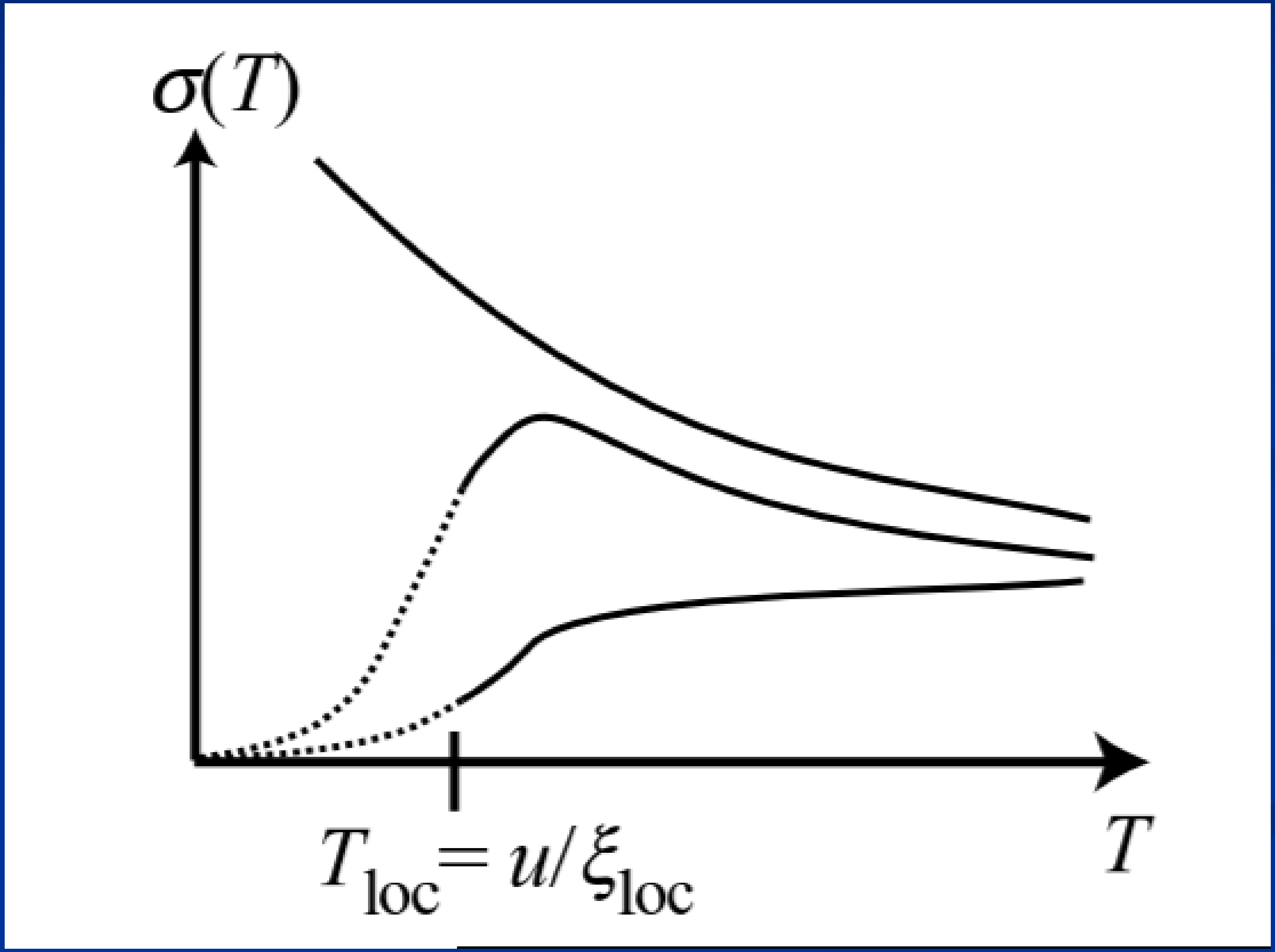
$\omega > \omega_{loc}$ $\sigma(\omega) \propto \omega^{-\nu}$

$\omega < \omega_{loc}$ $\sigma(\omega) \propto \omega^{2}$

Finite temperature

- Assumption of the contact with a thermal bath
- Equilibrium is reached ``after some time”
  (Localization of interacting particles)
- Might have to worry about aging, long relaxation times
- No thermal bath: question of ergodicity
- Many-body localization

d.c. transport (high T)



d.c transport (Low T)

T. Nattermann, TG, P. Le doussal, PRL 91, 056603 (2003)

arXiv:cond-mat/0403487

$$\frac{S}{\hbar} = \int_0^L dx \int_0^{\beta\hbar u} dy \frac{1}{2\pi K}[(\partial_y \phi)^2 + (\partial_x \phi)^2],$$

$$S_{\rm dis}/\hbar = -\frac{1}{2}\int \frac{dxdyA(x)}{2\pi K\alpha^2} e^{i[\phi(x,y)-\zeta(x)]} + \text{H.c.},$$

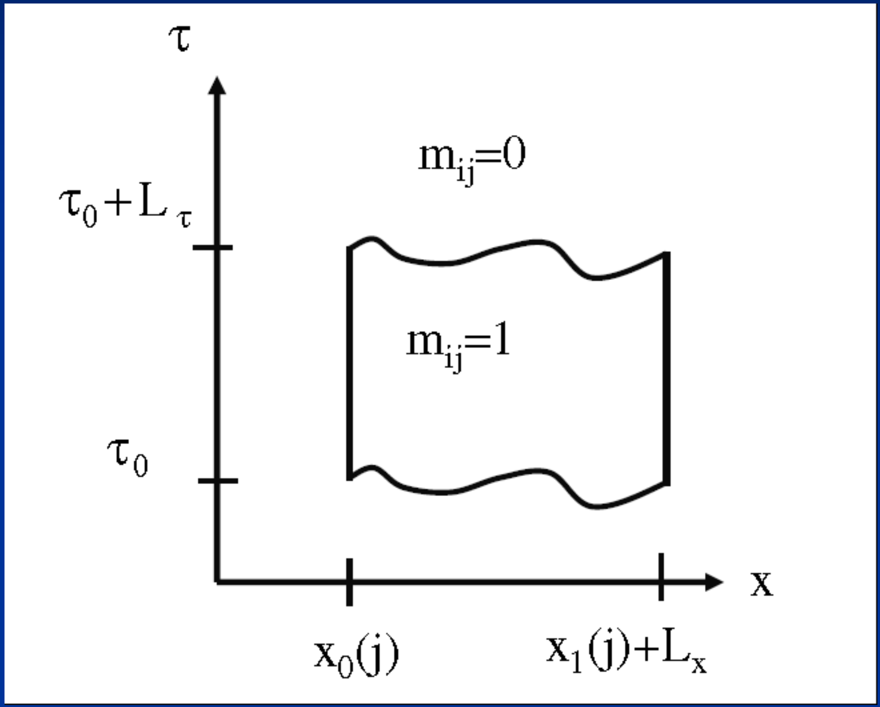
$$S_E/\hbar = \int dxdy\tilde{E}\phi(x,y),$$

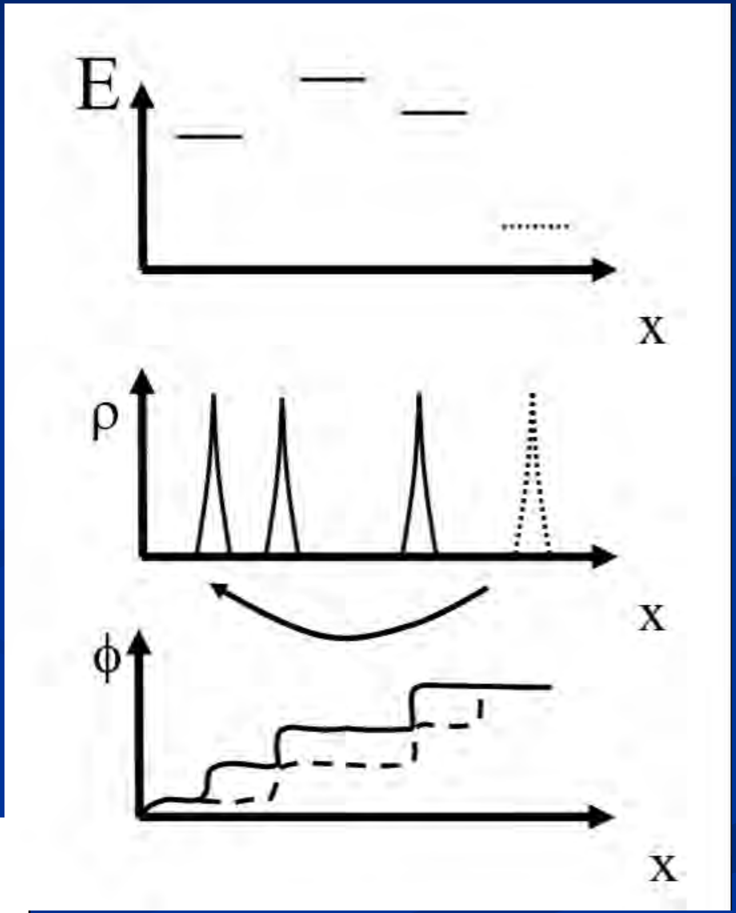
$$\frac{H}{u^*\hbar} = \frac{1}{2\pi K^*\alpha}\sum_{i=1}^{N}[(\phi_{i+1} - \phi_i)^2 - A^*\cos(\phi_i - \zeta_i)],$$

$$\frac{H}{u^*\hbar} = \frac{2\pi}{K^*\alpha}\sum_{i=1}^{N}(n_{i+1} - n_i - f_i)^2 - \frac{N}{2\pi K^*\alpha},$$

$$n_i^0 = m_0 + \sum_{j<i}[f_i],$$

VRH: solitonic excitations



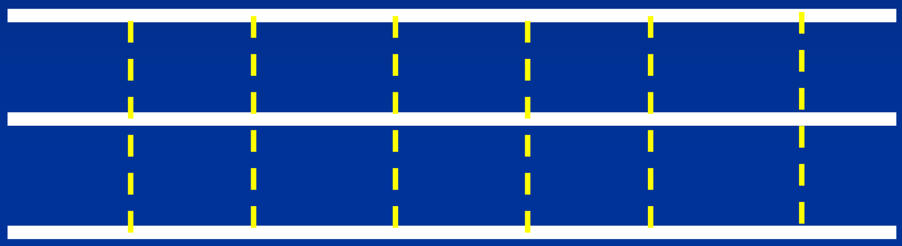


$$\sigma(T) \propto e^{-(S^*/\hbar)} = \exp\left[-\frac{4\pi}{K^*}\sqrt{2\beta\Delta}\right].$$

Beyond a single chain

- Largely open physics
- Strong difficulties to treat (analytics, numeric)
- Allows to incorporate SOC and magnetic field effects
- Many studies limited to non-interacting case
- Relevant for many experimental systems

Transverse transport

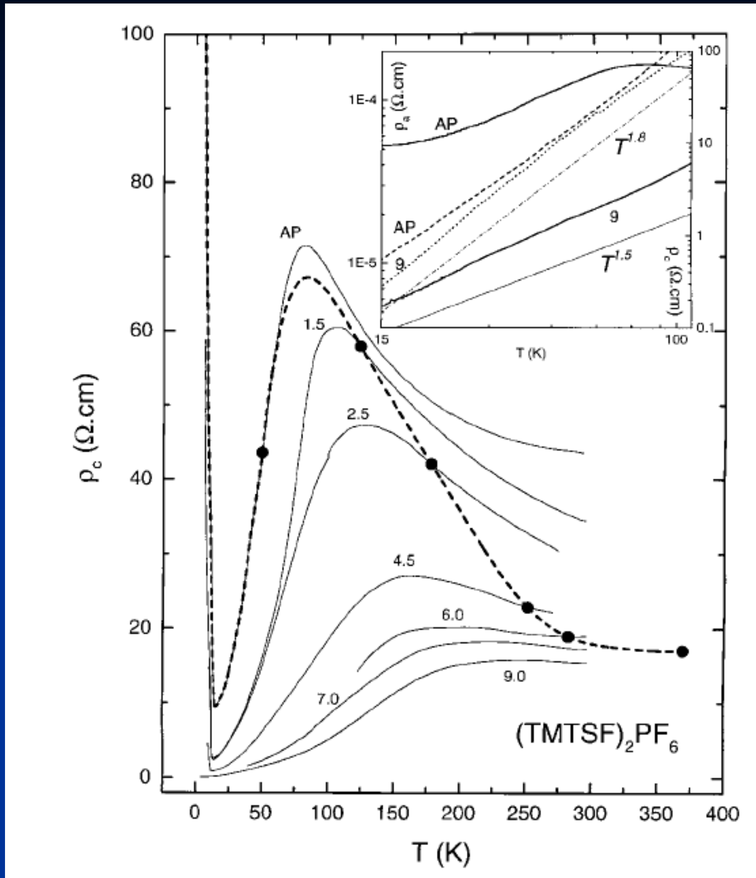


Perpendicular hopping t’

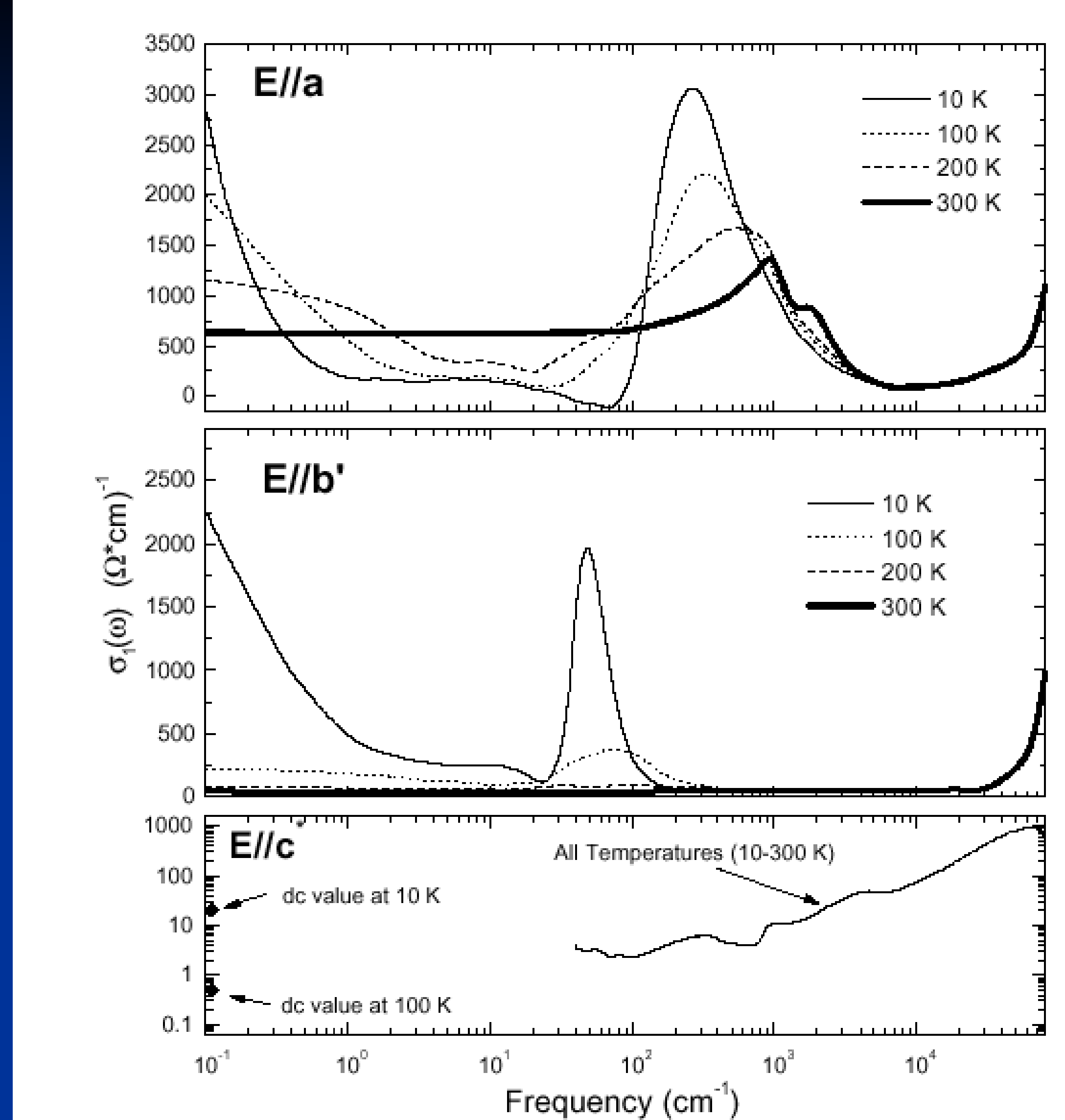
T > t’: tunneling, not usual transport

$$\sigma(\omega,T) \propto (\omega,T)^{2\alpha-1}$$

$$\alpha = \frac{1}{4}(K + K^{-1}) - \frac{1}{2}$$



J. Moser et al. Euro Phys. J. B 1 39 (1998)

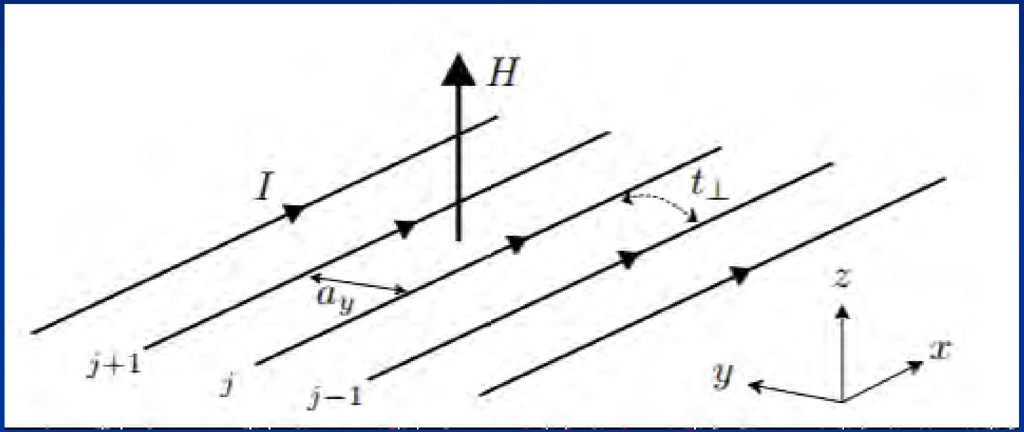


V. Vescoli et al. Euro Phys J B 11 365 (1999)

Hall effect

A. Lopatin, A. Georges, TG PRB 63 075109 (2001)
G. Leon, C. Berthod, TG PRB 75, 195123 (2007)
G. Leon, C. Berthod, TG, A.J. Millis PRB 78, 085105 (2008)

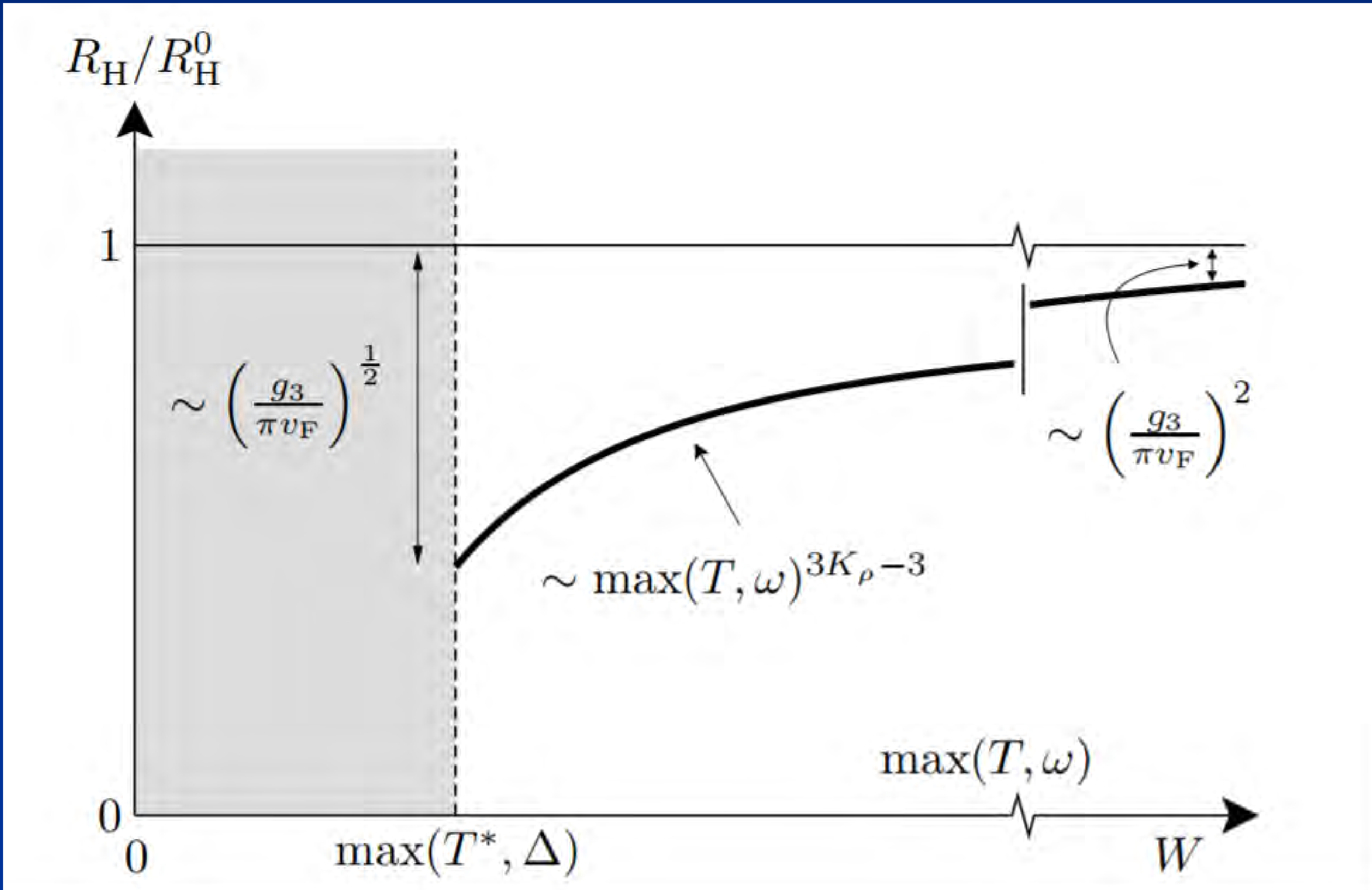
Why difficult ?

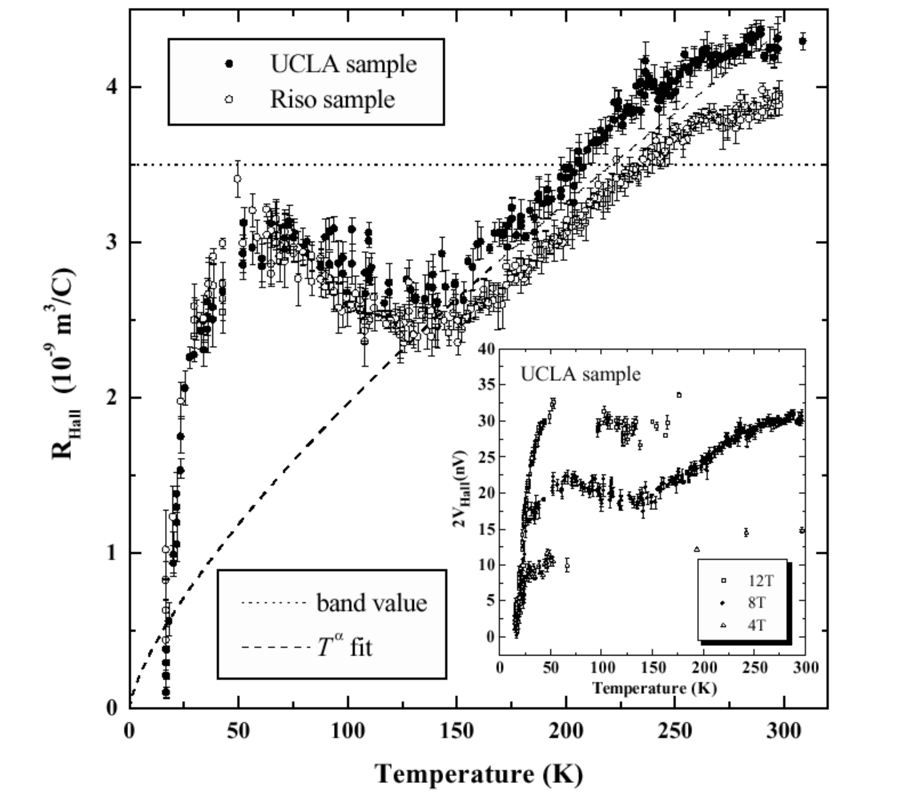


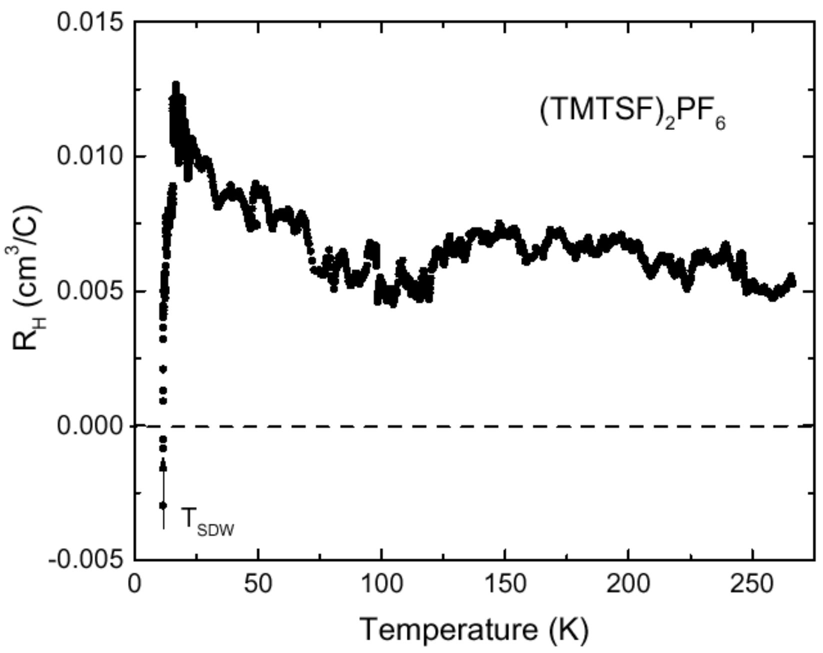
Need to go beyond TLL approximation:

- Band curvature needed (no particle-hole symmetry)
- Perturbation in : curvature, hopping, umklapp, magnetic field….

High temperature regime







G. Mihaly et al., PRL 84 2670 (00)

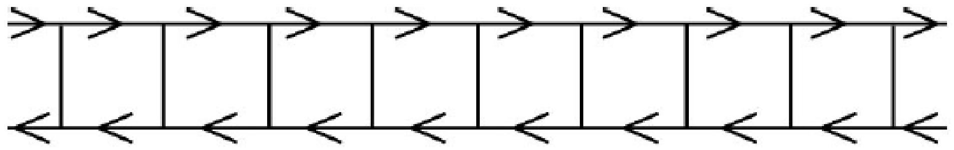
J. Moser et al., PRL 84 2674 (00)

Ladder physics

- Contains the interplay of lattice vs gauge fields
- Interactions treatable by bosonization, numerics, etc.
- Additional beautiful one-dimensional physics

Meissner effect in bosonic ladders

E. Orignac, TG, PRB 64 144515 (2001)



(a)

$$H = -t_{\parallel} \sum_{i,p=1,2} (b_{i+1,p}^{\dagger} e^{ie^* a A_{\parallel,p}(i)} b_{i,p} + b_{i,p}^{\dagger} e^{-ie^* a A_{\parallel,p}(i)} b_{i+1,p})$$

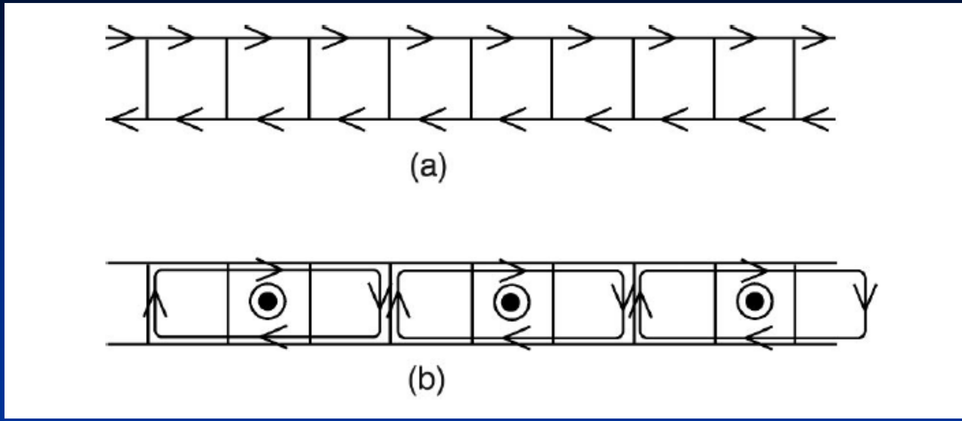
$$-t_{\perp} \sum_{i} (b_{i,2}^{\dagger} e^{ie^* A_{\perp}(i)} b_{i,1} + b_{i,1}^{\dagger} e^{-ie^* A_{\perp}(i)} b_{i,2})$$

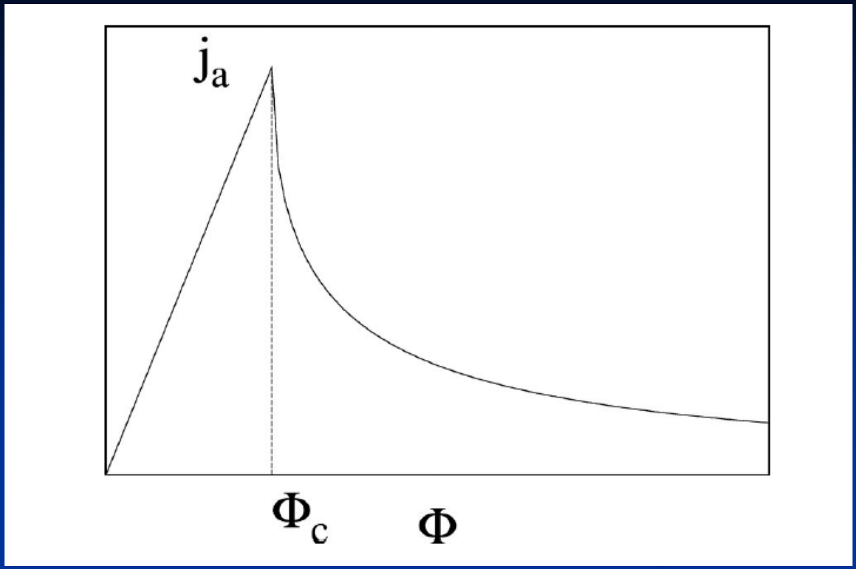
$$+U \sum_{i,p} n_{i,p}(n_{i,p} - 1) + V n_{i,1} n_{i,2}, \quad (1)$$

$$\int \vec{A} \cdot \overrightarrow{dl} = \Phi$$

$$H = H_s^0 + H_a^0 - \frac{t_{\perp}}{\pi a} \int dx \cos[\sqrt{2}\,\theta_a + e^* A_{\perp}(x)]$$

$$+ \frac{2Va}{(2\pi a)^2} \int dx \cos\sqrt{8}\,\phi_a,$$



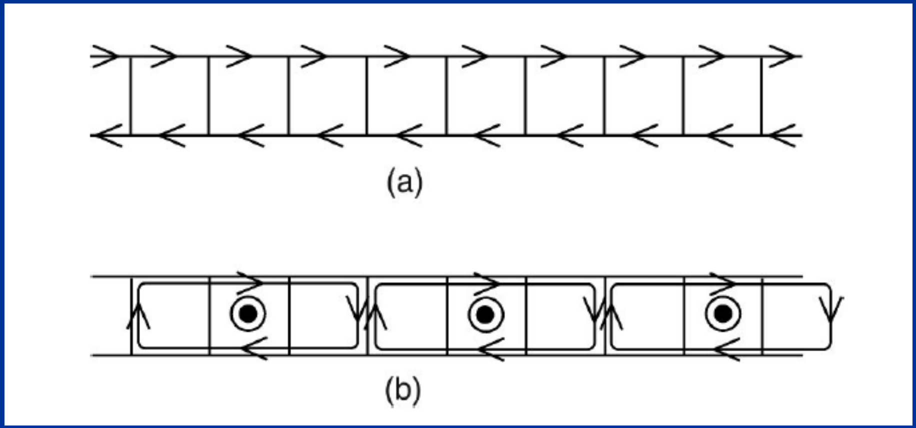


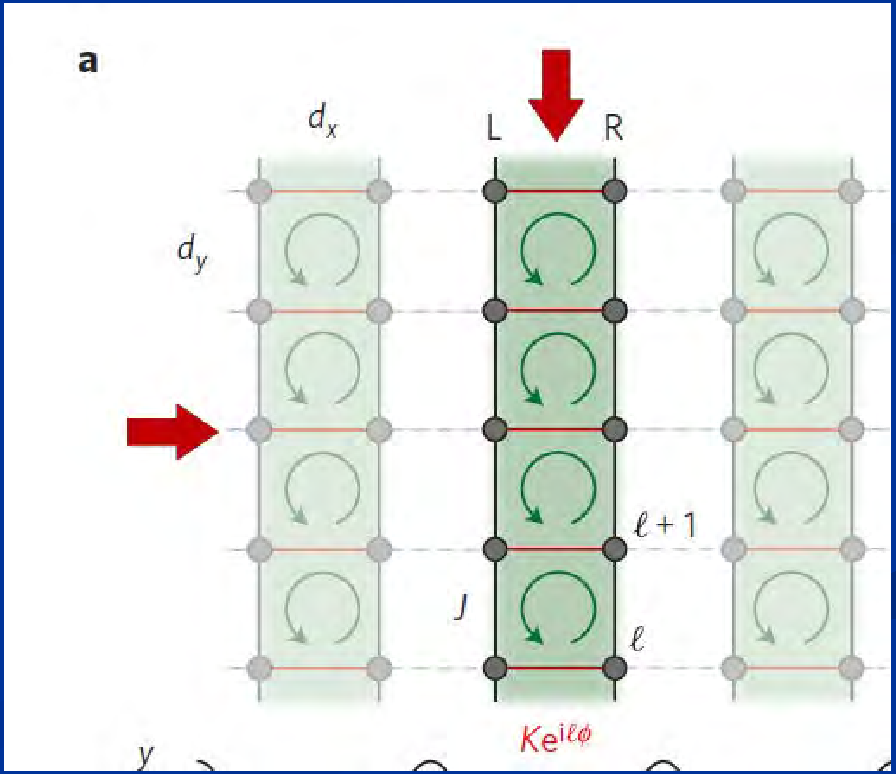
Orbital currents (``Meissner" effect)

Field ``Hc1": appearance of vortices

Artificial gauge field (cold atoms)

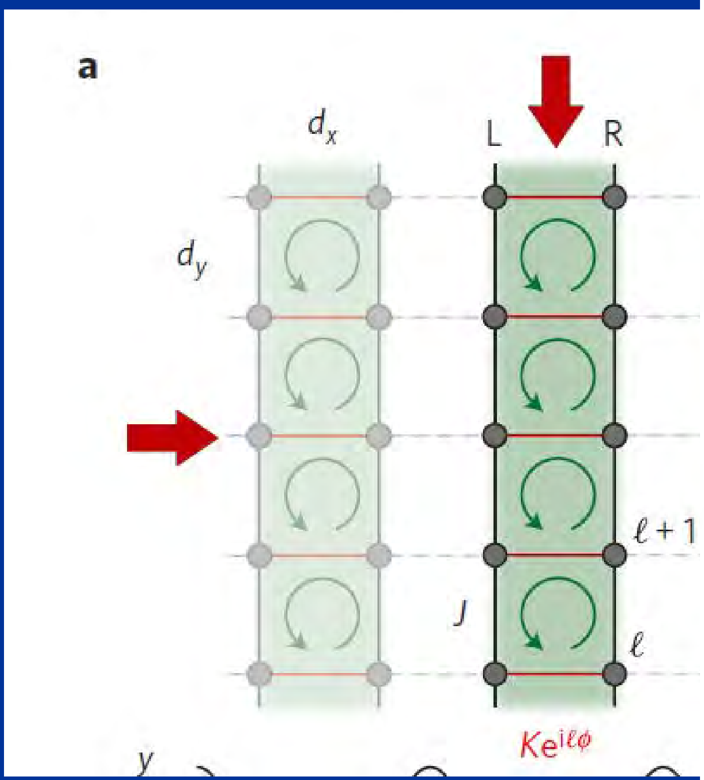
M Atala et al. Nat Phys, 10 588 (2014)

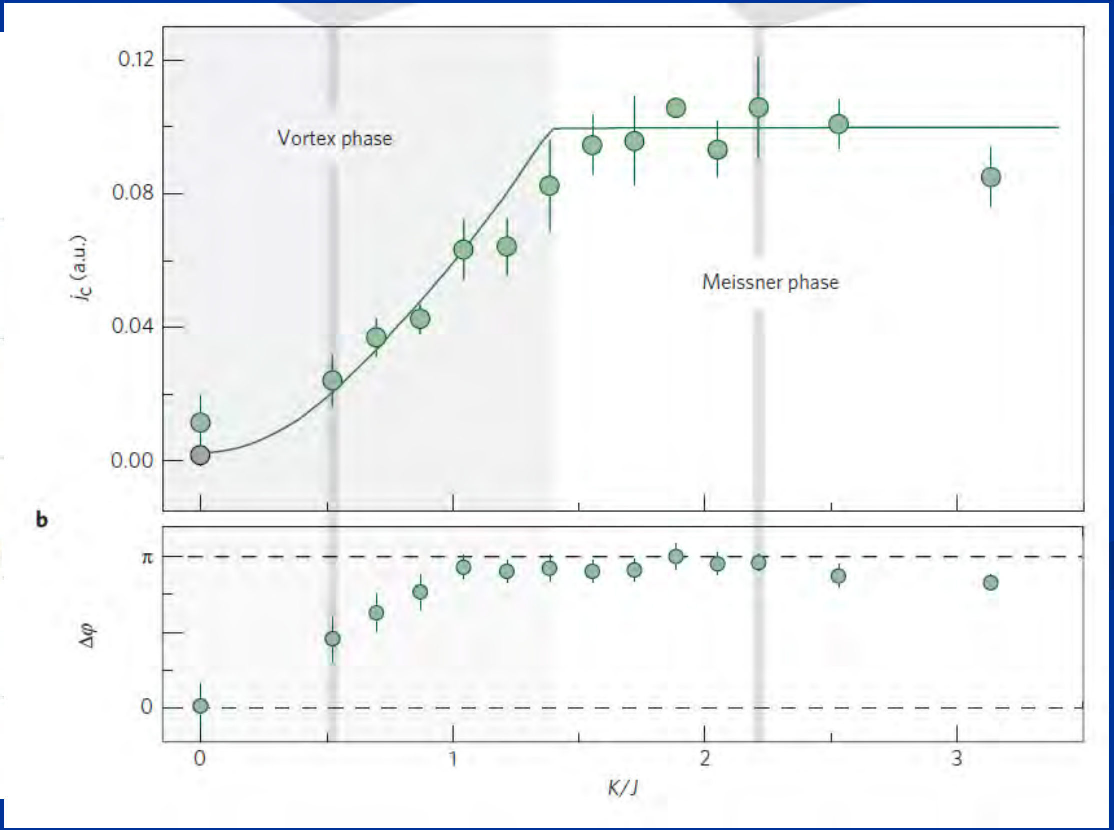




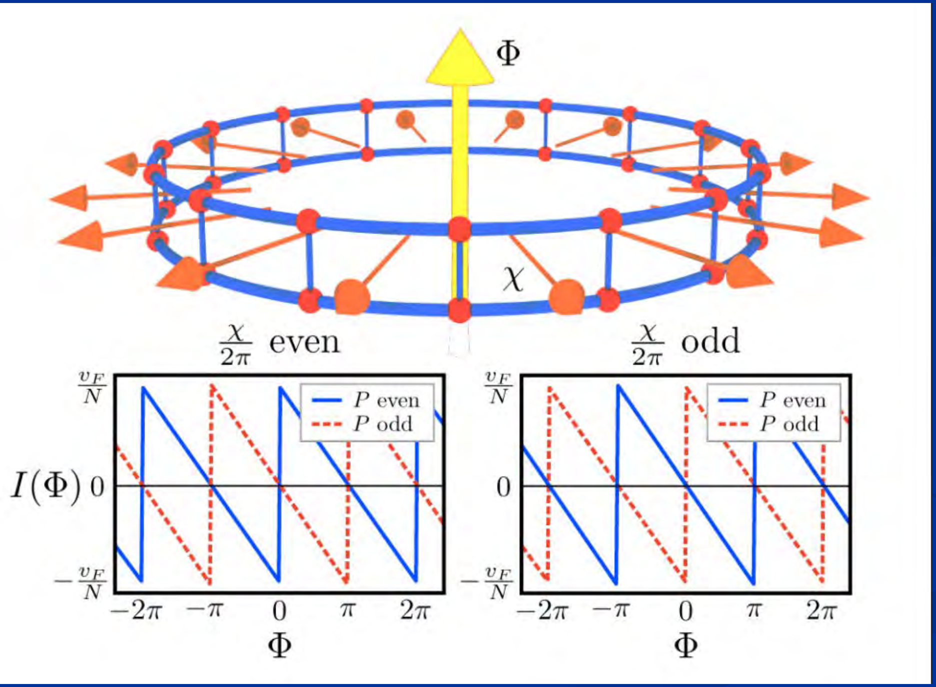
Artificial gauge field (cold atoms)

M Atala et al. Nat Phys, 10 588 (2014)





Controlled parity switch of persistent currents in quantum ladders



M. Filippone, C. Bardyn, TG, arXiv:1710.02152

Conclusions / Perspectives

- Many exotic properties of transport in 1D
- Linked to the collective nature of the excitations
- Good analytical methods to tackle this problem
- Relevant for experimental situations
- Many open questions: finite temperature, coupled chains, Hall effect, transport of energy, etc.