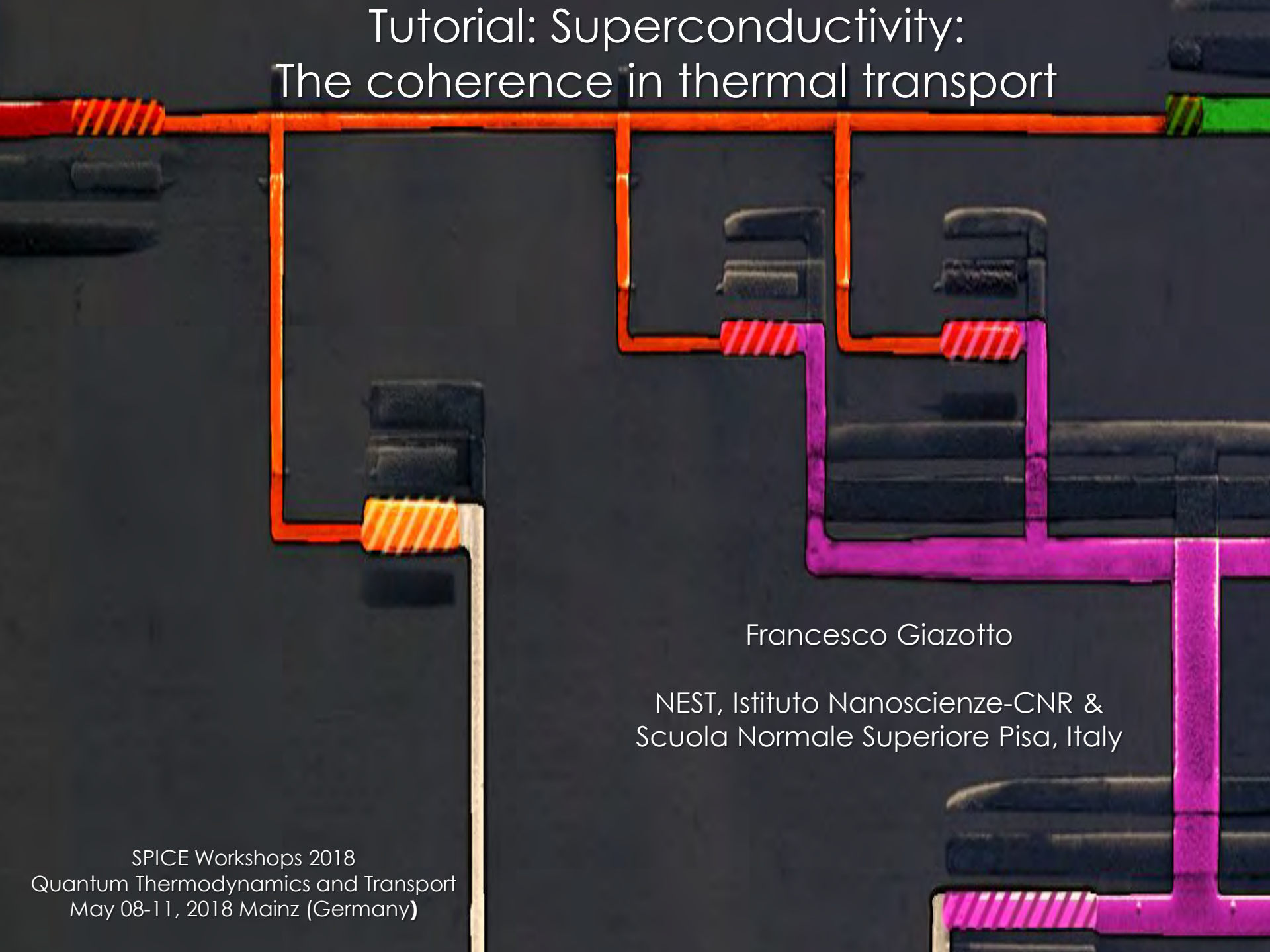


# Tutorial: Superconductivity: The coherence in thermal transport

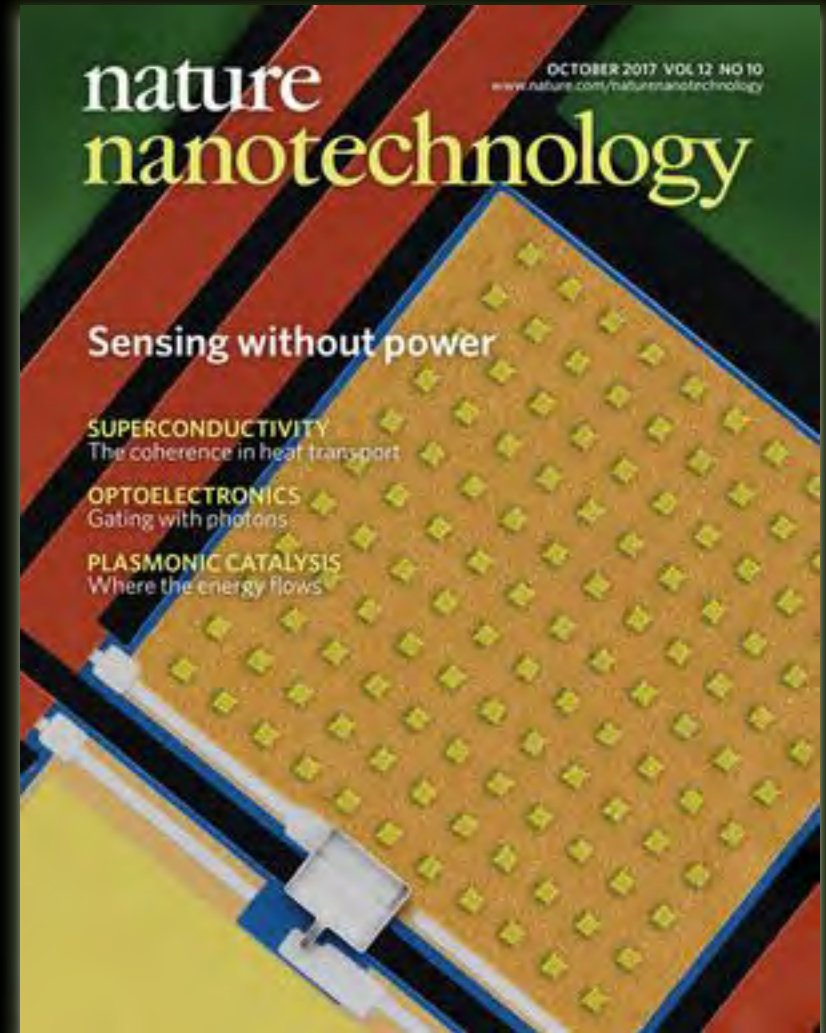


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Scuola Normale Superiore Pisa, Italy

# Outline

1. Motivation & mission
2. Overview
3. Basic concept of thermal transport in Josephson-based quantum circuits
4. Double & single-slit heat interferometers
5. Balanced thermal modulators
6. Josephson thermal  $\pi$ -junctions
7. Phase-tunable Josephson thermal routers



# Motivations & mission

- Set the experimental ground for a challenging young branch of science: the *coherent caloritronics*, i.e., the complementary of coherent electronics
- *Phase*-manipulate & master heat transfer in a solid-state environment
- Provide original & novel approaches to realize *thermal devices* (heat transistors, splitters, diodes, refrigerators, exotic quantum circuits)
- Address & understand fundamental *energy-* and *heat-related* phenomena at nanoscale (coherent dynamics, heat interference, time-dependent effects, quantum thermodynamics, decoherence)

## NEWS & VIEWS

THEME: PHYSICS

### Quantum interference heats up

A thermal effect predicted more than 40 years ago was nearly forgotten, while a related phenomenon stole the limelight. Now experimentally verified, the effect could spur the development of heat-controlling devices. [SEE LETTER p.401](#)

RAYMOND W. SIMMONDS

Wouldn't it be strange to have a material whose thermal conductivity could be changed by a magnetic field? Imagine holding the end of a rod made of this material with the other end placed in a hot fire. As long as a friend keeps a bar magnet away from the rod, you wouldn't burn your hand, but as soon as they apply a magnetic field — ouch! As odd as this seems, the rules of quantum mechanics predict this type of situation for heat transported across a pair of Josephson junctions (devices that consist of two superconductors separated by a thin insulating gap). Writing on page 401, Giazotto and Martínez-Pérez<sup>1</sup> report experiments confirming that this strange phenomenon can actually occur.

In 1962, Brian Josephson made a remarkable discovery<sup>2</sup> as a graduate student, while investigating what would happen if two superconducting metals were placed very close together without touching. He found that the 'Cooper pairs' of electrons that make up the supercurrent (a current that flows without resistance) in superconductors could miraculously jump or 'tunnel' across the gap without needing an applied electric voltage.

The size of the supercurrent flowing through this 'tunnel barrier' depends on whether the superconductors at either edge of the gap have the same or a different phase — a property of the quantum-mechanical wavefunction that describes the behaviour of Cooper pairs. In a bulk superconductor, any phase changes in the wavefunction between local regions give rise to supercurrent flow. Alternatively, forcing a supercurrent to flow produces phase differences, even across a thin non-conducting or insulating barrier.

Consider also what happens when superconductors form closed circuits, such as loops. Now the total phase that accumulates around the loop when supercurrent flows must be an integer multiple of  $2\pi$ , to maintain the continuity of the wavefunction. This causes magnetic flux in the system to be quantized. The Josephson effect can be combined with this flux quantization to produce a superconducting direct-current quantum interference device<sup>3</sup> (d.c.-SQUID). In these devices, a split

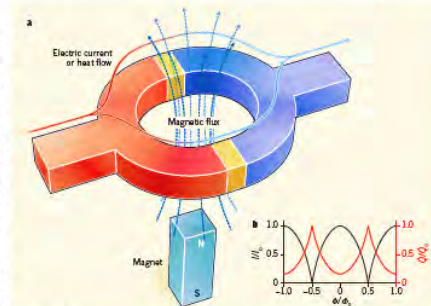


Figure 1 | A direct-current superconducting quantum interference device (d.c.-SQUID). a, In d.c.-SQUIDs, a superconducting loop contains two Josephson junctions — thin insulating barriers (yellow) sandwiched between the two superconductors (red and blue). b, The maximum electrical current ( $I_c$ , black, left axis) flowing through the device from left to right can be fully modulated by the amount of magnetic flux ( $\phi$ ) passing through the loop.  $I_c$  is the maximum current that can flow through the d.c.-SQUID.  $\phi_0$  is the magnetic flux quantum,  $2.07 \times 10^{-15}$  webers. Giazotto and Martínez-Pérez<sup>1</sup> have observed an interference effect for heat flow ( $Q$ , red, right axis) in a d.c.-SQUID. The total amount of heat passing through the device can also be modulated by an applied magnetic flux.

superconducting path with two Josephson junctions can sustain a maximum supercurrent, the amplitude of which can be modulated by the amount of magnetic flux piercing the loop (Fig. 1). Such d.c.-SQUIDs are among the most sensitive detectors of magnetic flux ever created and have found many practical applications<sup>4</sup>.

In addition to the phase-dependent supercurrent, Josephson discovered<sup>2</sup> two other currents that are present when a finite voltage difference exists across a junction. These currents were caused by the tunnelling of quasiparticles ( lone electrons from broken Cooper pairs) or of quasiparticles with Cooper pairs. The first type was similar to the flow of electrons through normal metal-metal junctions, but the second type of current was rather odd: it involved a dynamic

process in which the tunnelling occurred in conjunction with processes for breaking and recombining Cooper pairs. Because Cooper pairs are involved, this current should exhibit interference effects analogous to those seen in d.c.-SQUIDs (in which differences in the wavefunction's accumulated phase along the two paths of a loop create constructive or destructive interference). But electrical experiments that clearly quantify the behaviour of this 'interference current' have remained elusive<sup>5</sup>.

What does all this talk of electrical currents have to do with thermal properties? Well, according to the Wiedemann-Franz law, a metal's thermal conductivity is proportional to its electrical conductivity (and to temperature). This is because electrons can transport some of the heat in a metal. Only three years after

358 | NATURE | VOL 492 | 20/27 DECEMBER 2012

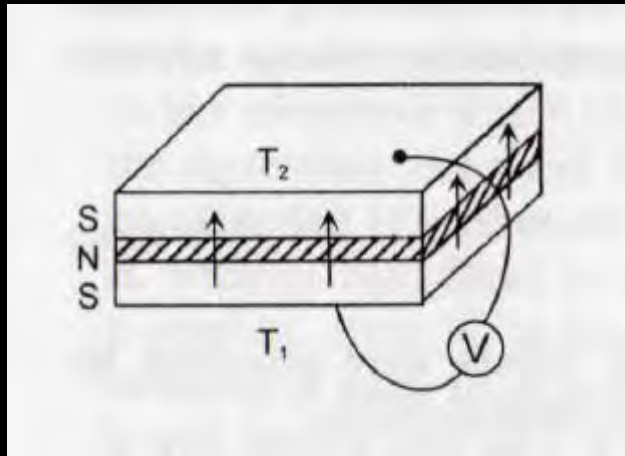
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Main goal: develop **quantum technology** for managing **heat** in nanoscale circuits

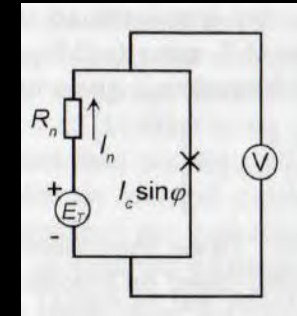


# Thermoelectric effects in Josephson junctions

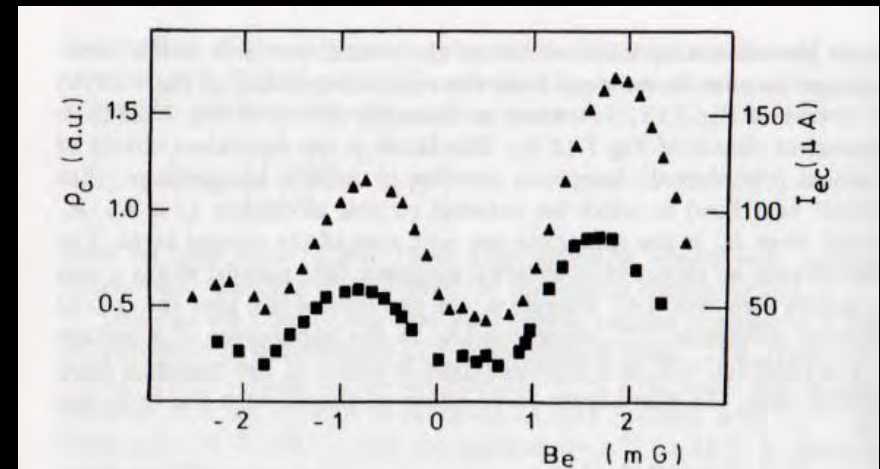
dc & ac **thermoelectric** response



SNS-like Josephson junction



Presence of a magnetic field



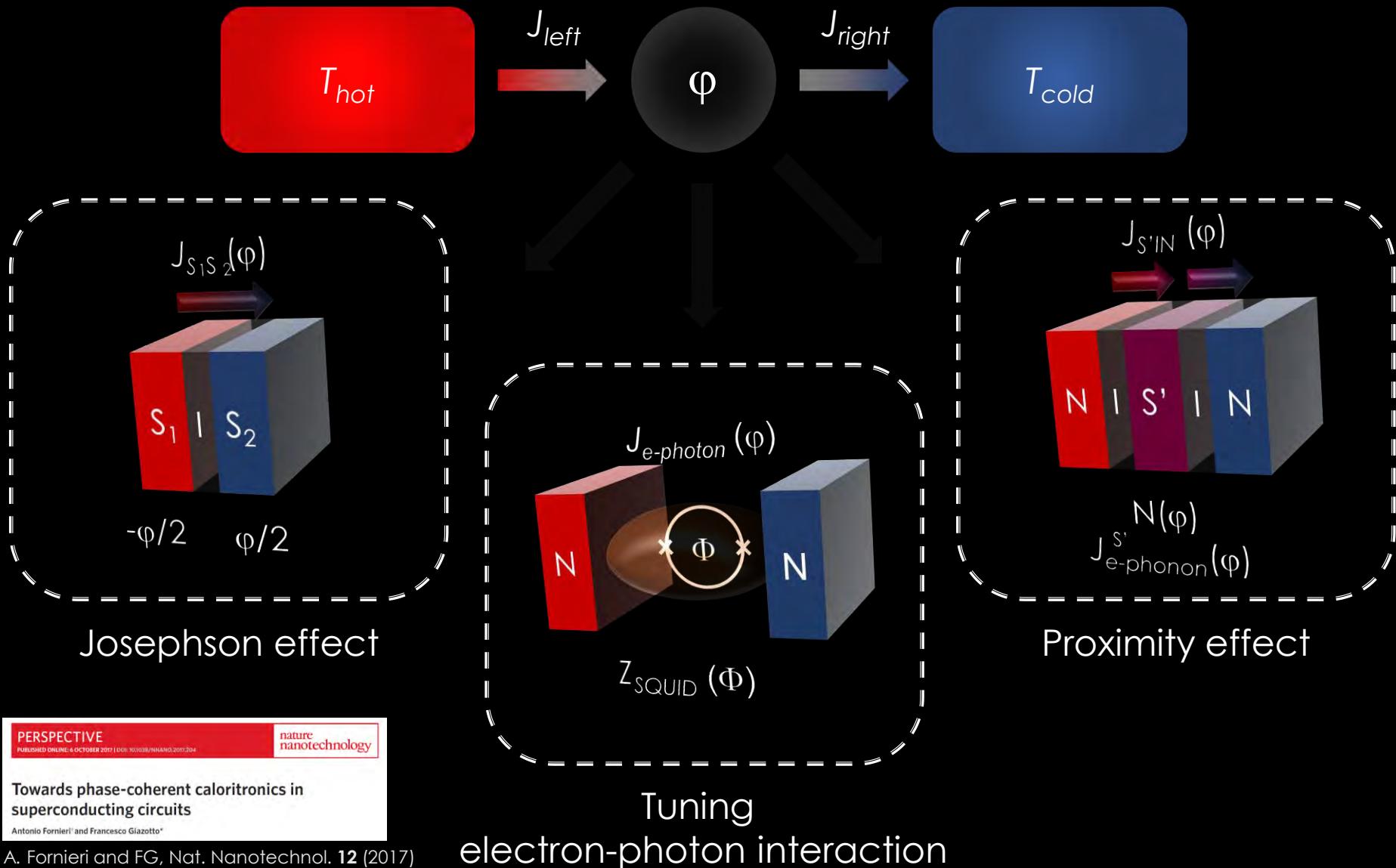
$$\omega = \frac{2e}{\hbar} R_n \left[ \left( \frac{\alpha \Delta T}{R_n} \right)^2 - I_c^2 \right]^{1/2}$$

$\alpha \sim 10^{-8}$  V/K thermopower

Aronov and Galperin, JETP Lett. **19**, 165 (1974);  
Kartsovnik, Ryazanov, and Schmidt, JETP Lett. **33**, 356 (1981);  
Ryazanov and Schmidt, Solid State Commun. **40**, 1055, (1981);  
Clarke and Freake, Phys. Rev. Lett. **29**, 588 (1982).

Panaitov, Ryazanov, Ustinov, and Schmidt, Phys. Lett. **100A**, 301 (1984);  
Schmidt, JETP Lett. **33**, 98 (1981);  
Ryazanov and Schmidt, Solid State Commun. **42**, 733 (1982);  
Huebener, Supercond. Sci. Technol. **8**, 189 (1995).

# Physical basis of coherent caloritronics



PERSPECTIVE  
PUBLISHED ONLINE: 4 OCTOBER 2017 | DOI: 10.1038/NANANO.2017.204

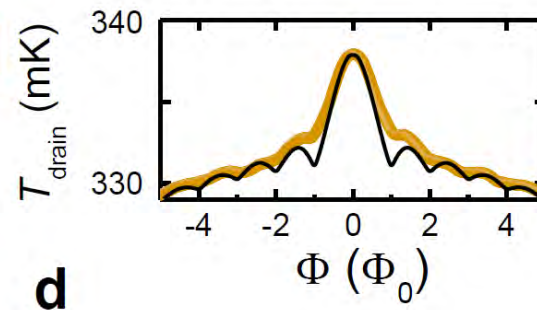
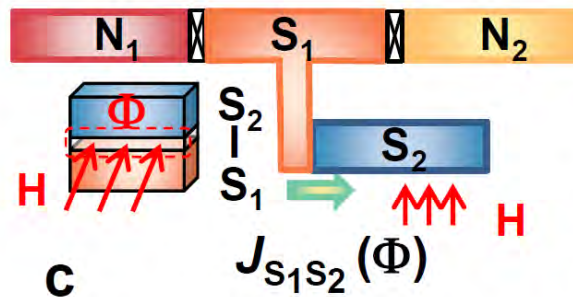
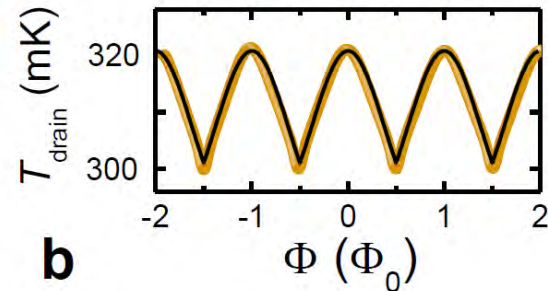
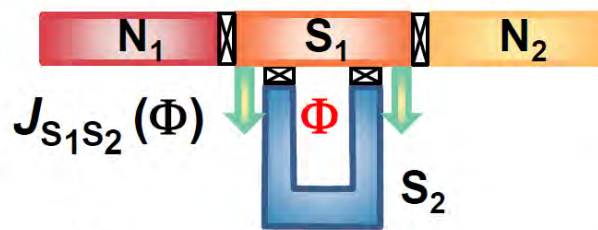
nature  
nanotechnology

Towards phase-coherent caloritronics in  
superconducting circuits

Antonio Fornieri\* and Francesco Giazotto\*

A. Fornieri and FG, Nat. Nanotechnol. **12** (2017)

# Josephson tunnel circuits



## Josephson heat interferometers

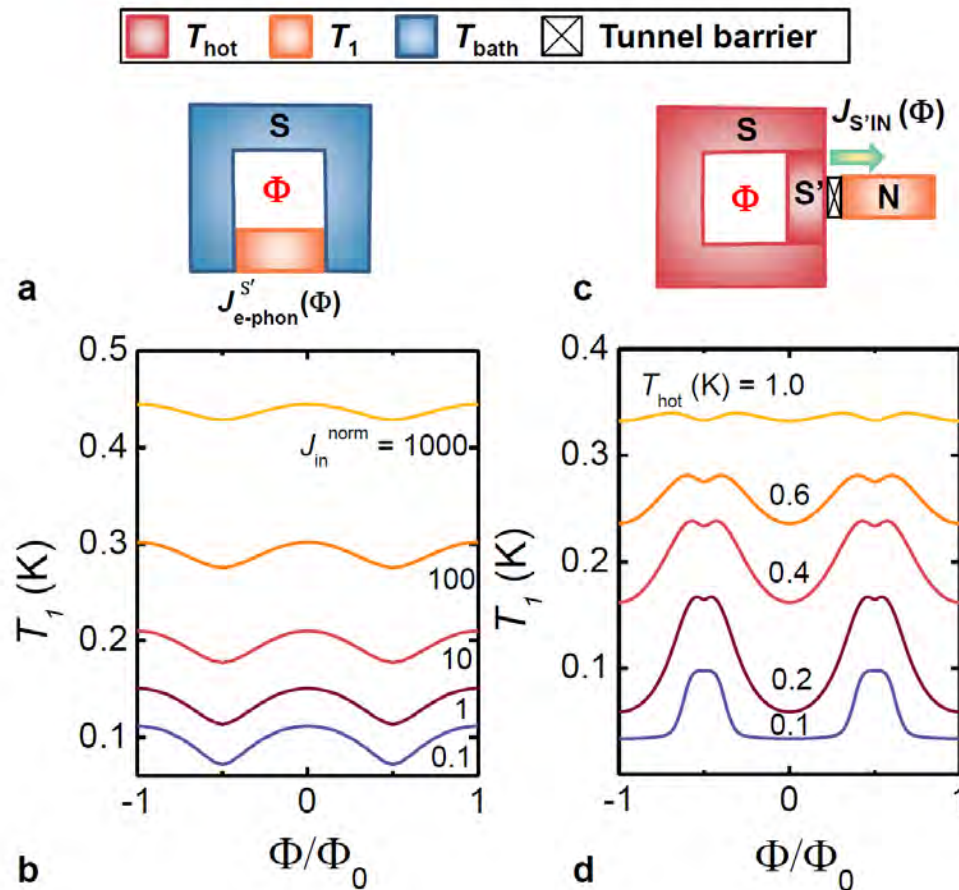
**b** – Double-slit Josephson interferometer

$$J_{\text{SQUID}}(T_1, T_{\text{bath}}, \Phi) = 2J_{\text{qp}}(T_1, T_{\text{bath}}) - 2J_{\text{int}}(T_1, T_{\text{bath}}) \left| \cos \left( \frac{\pi \Phi}{\Phi_0} \right) \right|$$

**c** – Single-slit Josephson diffractor

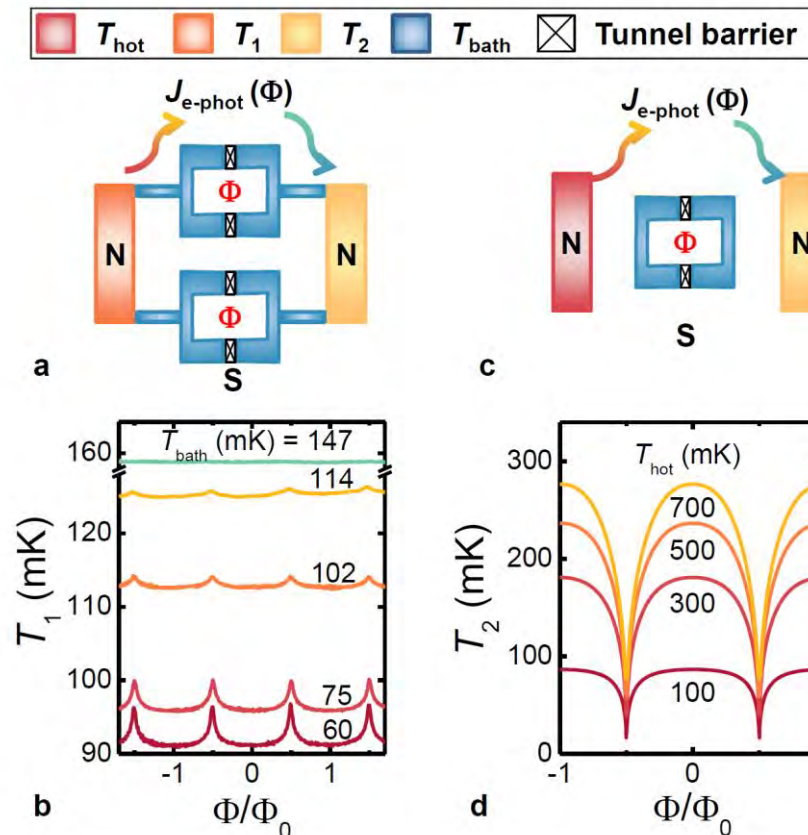
$$J_{\text{S1S2}}(T_1, T_{\text{bath}}, \Phi) = J_{\text{qp}}(T_1, T_{\text{bath}}) - J_{\text{int}}(T_1, T_{\text{bath}}) \left| \frac{\sin(\pi \Phi / \Phi_0)}{(\pi \Phi / \Phi_0)} \right|$$

# Superconducting proximity structures



- a**– Phase-dependent electron-phonon coupling, entropy, specific heat
- c** – Phase-tunable proximity thermal valve

# Photonic heat transistors



**a** – First demonstration of phase-dependent photonic heat conduction

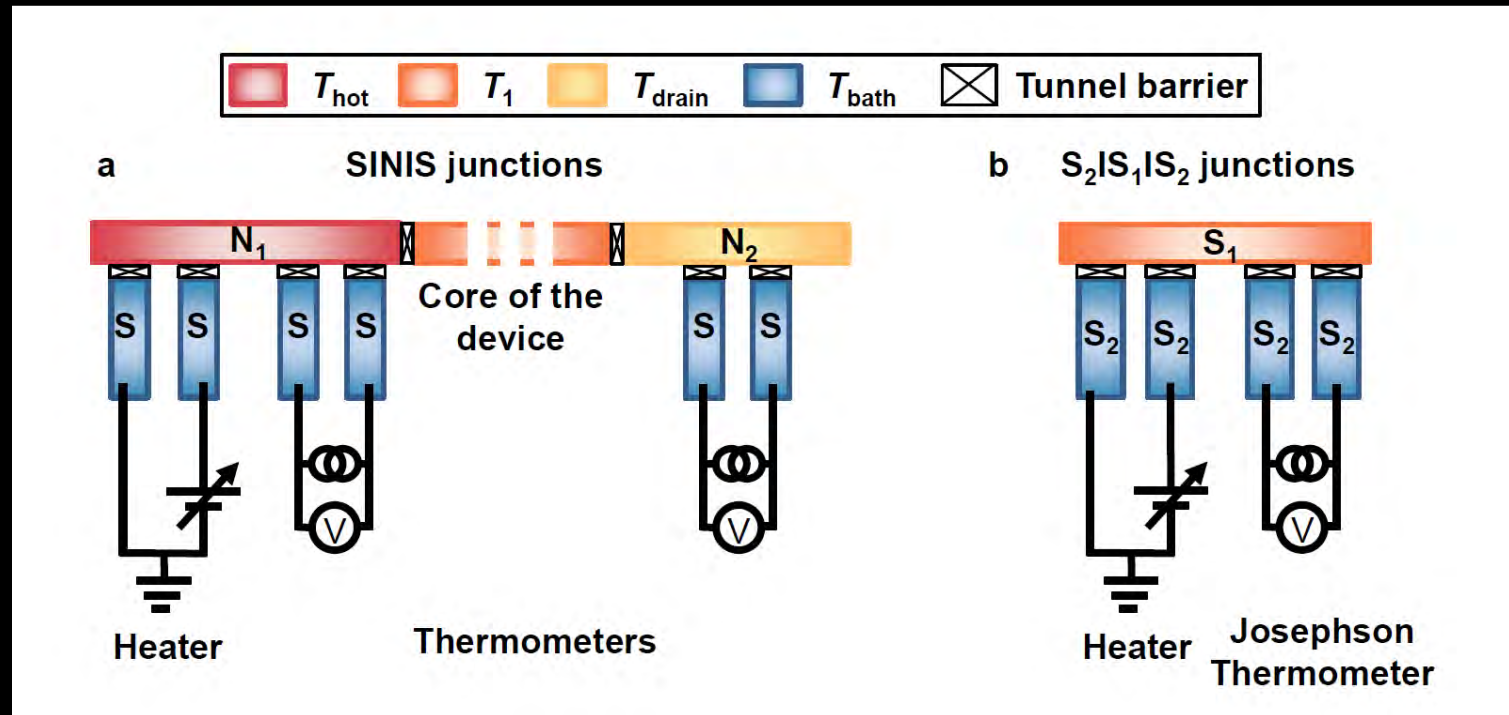
**c** – Design for a *non-galvanic* photonic thermal transistor

$$\mathcal{T}(\omega) = \frac{4\Re[Z_1(\omega)]\Re[Z_2(\omega)]}{|Z_{\text{tot}}(\omega)|^2}$$

M. Meschke, *et al.*, Nature **444**, 187 (2006);  
 A. Fornieri and FG, Nat. Nanotechnol. **12** (2017);  
 A. Ronzani, *et al.*, arXiv:1801.09312.



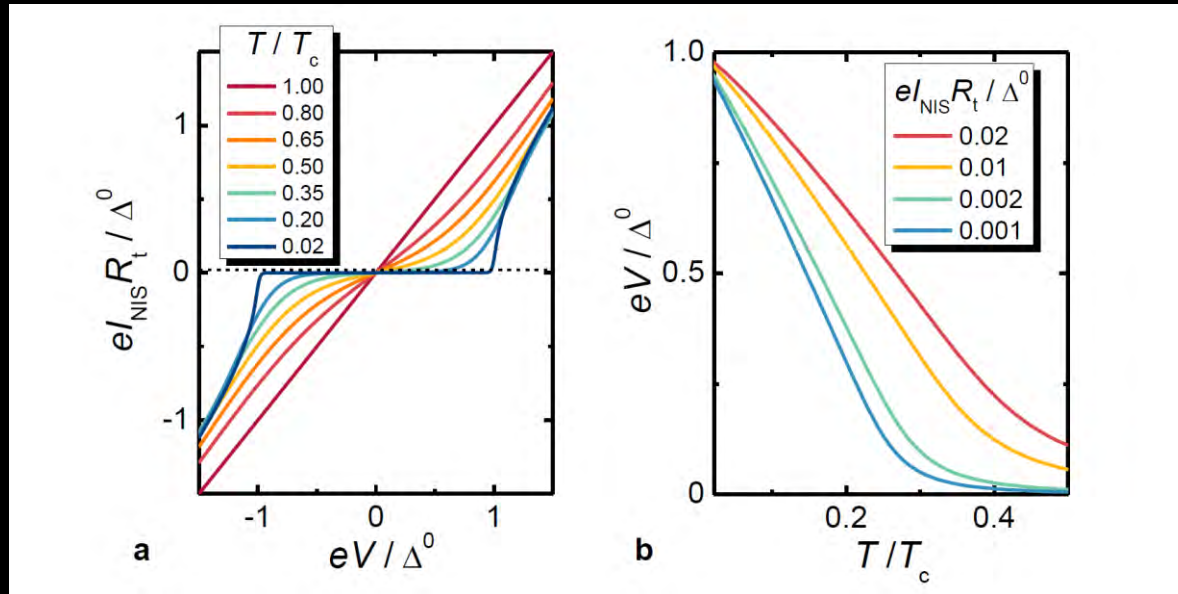
# Experimental setups



**b** – DC & RF electron thermometry through SINIS tunnel junctions

**c** – Electron thermometry through temperature dependence of the critical current, or through quasiparticle current

# Electric transport in superconducting tunnel junctions (NIS)



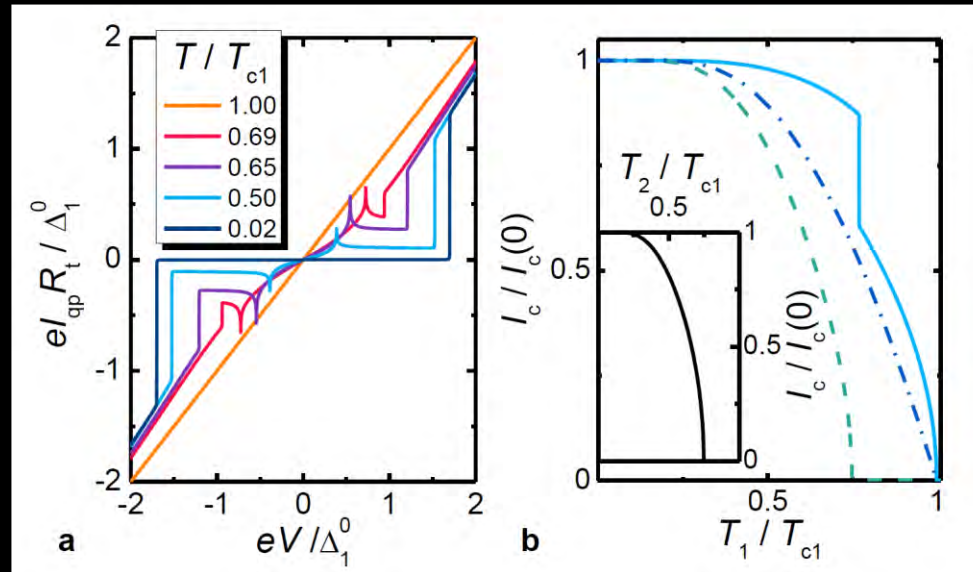
$$I_{\text{NIS}}(V, T_1, T_2) = \frac{1}{eR_t} \int_{-\infty}^{\infty} dE \mathcal{N}(E, T_2) [f_1(E - eV, T_1) - f_2(E, T_2)],$$

$$= \frac{1}{2eR_t} \int_{-\infty}^{\infty} dE \mathcal{N}(E, T_2) [f_1(E - eV, T_1) - f_1(E + eV, T_1)]$$

**a** –  $I/V$  characteristics of a NIS junction

**b** – Voltage response of the junction vs  $T$  at given  $I_{\text{bias}}$ : sensitive electron thermometry

# Electric transport in superconducting tunnel junctions (SIS)



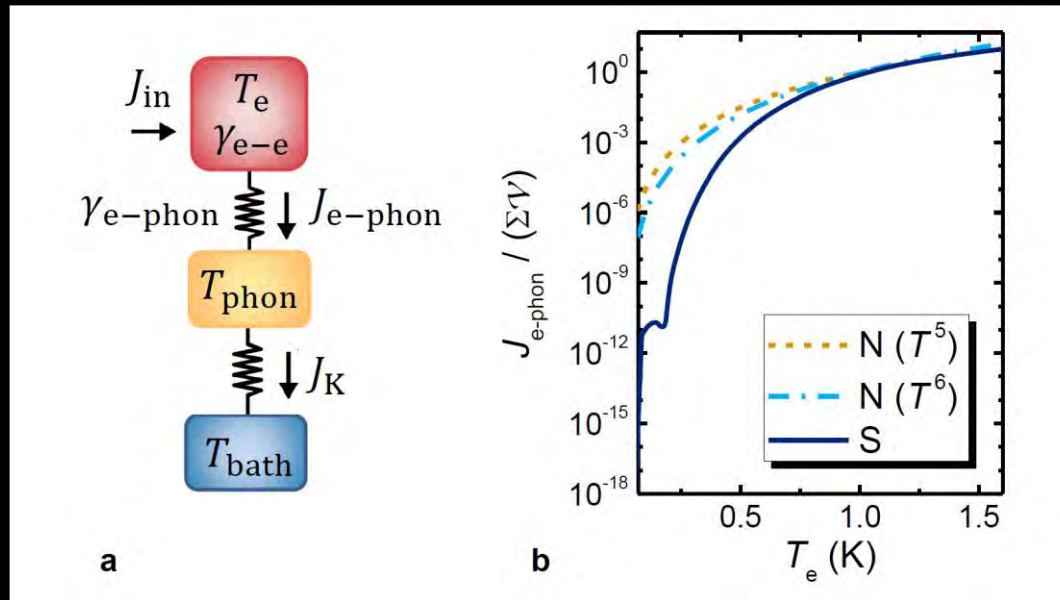
$$I_j(V, T_1, T_2, \varphi) = I_c(T_1, T_2) \sin \varphi + I_{\text{int}}(V, T_1, T_2) \cos \varphi$$

$$I_c(T_1, T_2) = \frac{1}{2eR_t} \left| \int_{-\infty}^{\infty} dE \{ \mathbf{f}(E, T_1) \Re[\mathcal{F}_1(E, T_1)] \Im[\mathcal{F}_2(E, T_2)] + \mathbf{f}(E, T_2) \Re[\mathcal{F}_2(E, T_2)] \Im[\mathcal{F}_1(E, T_1)] \} \right|,$$

**a** –  $I/V$  quasiparticle characteristics of a SIS junction: more complicated thermometry

**b** – Temperature dependence of the Josephson current: non-dissipative thermometry

# Quasiequilibrium regime in mesoscopic circuits



**a** – Scheme of N or S film on a substrate

**b** – Electron-phonon coupling in N and S

$$J_{e-\text{phon}}^N = \Sigma \mathcal{V} (T_e^5 - T_{\text{bath}}^5)$$

clean metal

$$J_{e-\text{phon}}^{\text{AlMn}} = \Sigma_{\text{AlMn}} \mathcal{V} (T_e^6 - T_{\text{bath}}^6)$$

disordered metal

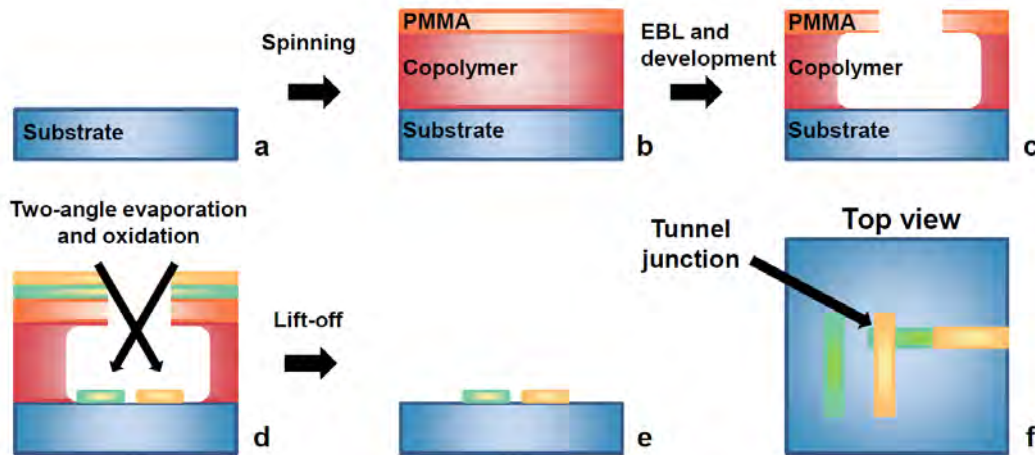
$$J_{e-\text{phon}}^S(T_e, T_{\text{bath}}) = -\frac{\Sigma \mathcal{V}}{96\zeta(5)k_B^5} \int_{-\infty}^{\infty} dE E \int_{-\infty}^{\infty} d\epsilon \epsilon^2 \text{sgn}(\epsilon) L(E, E + \epsilon, T_e) \left\{ \coth\left(\frac{\epsilon}{2k_B T_{\text{bath}}}\right) [f^{(1)}(E, T_e) - f^{(1)}(E + \epsilon, T_e)] - f^{(1)}(E, T_e) f^{(1)}(E + \epsilon, T_e) + 1 \right\}.$$

superconductor

FG, T. T. Heikkilä, A. Luukanen, A. M. Savin,  
and J. P. Pekola, Rev. Mod. Phys. **78**, 217 (2006);  
A. Fornieri and FG, Nat. Nanotechnol. **12** (2017);  
A. V. Timofeev, *et al.*, Phys. Rev. Lett. **102**, 017003 (2009)

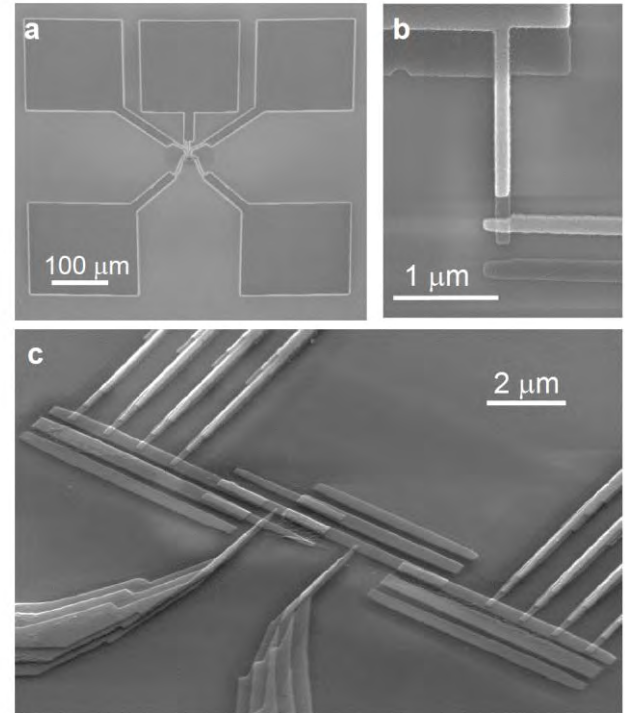


# Nanofabrication techniques

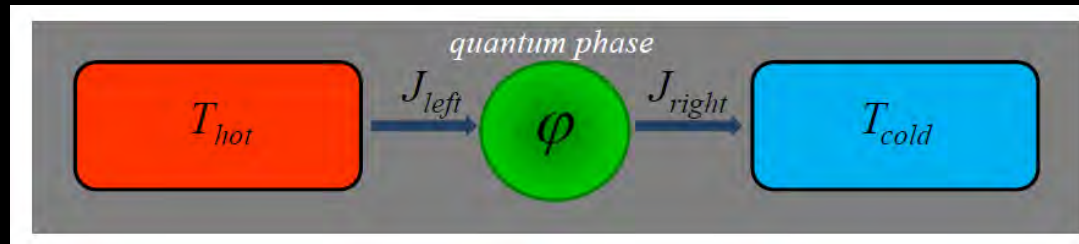


Angle evaporation and *in-situ* oxidation

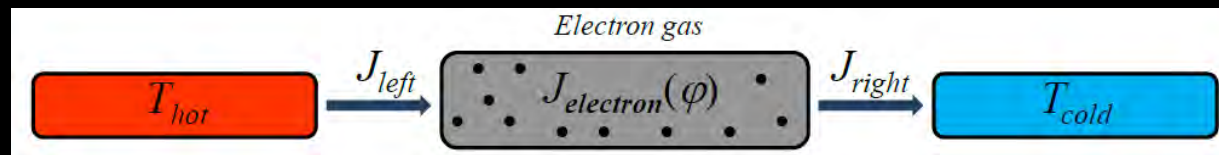
Typical shadow-mask evaporated structures



# Principle of phase-dependent heat current control



Exploitation of quantum phase to control heat current flow



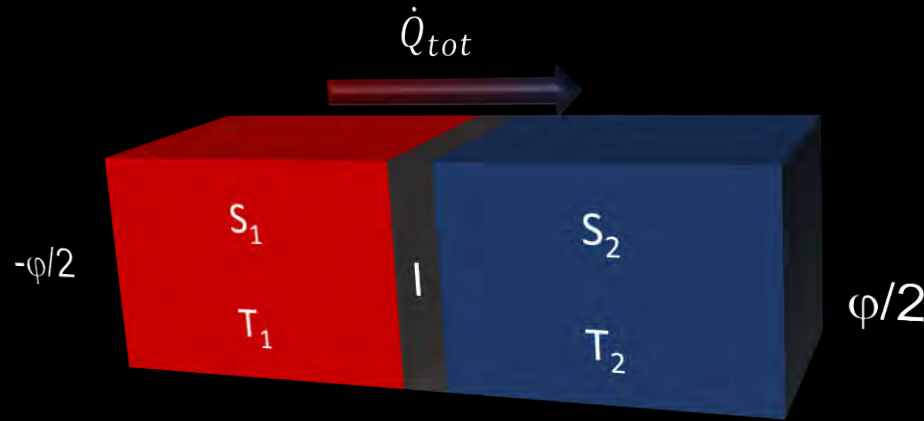
Temperature-biased Josephson tunnel junction



Heat current is predicted to be phase dependent and stationary

Maki and Griffin, PRL **15**, 921 (1965);  
Zhao et al., PRL **91**, 077003 (2003);  
Zhao et al., PRB **69**, 134503 (2004)

# Heat current in a temperature-biased JJ



Maki and Griffin, PRL **15**, 921 (1965);  
Zhao et al., PRL **91**, 077003 (2003);  
Zhao et al., PRB **69**, 134503 (2004)

$$\dot{Q}_{tot} = \dot{Q}_{qp}(T_1, T_2) - \dot{Q}_{int}(T_1, T_2) \cos \varphi$$

$$\dot{Q}_{qp}(T_1, T_2) = \frac{2}{e^2 R_T} \int_0^\infty E \aleph_1(E, T_1) \aleph_2(E, T_2) [f_1(E, T_1) - f_2(E, T_2)] dE$$

quasiparticle

$$\dot{Q}_{int}(T_1, T_2) = \frac{2}{e^2 R_T} \int_0^\infty E \mathcal{M}_1(E, T_1) \mathcal{M}_2(E, T_2) [f_1(E, T_1) - f_2(E, T_2)] dE$$

interference

$$\aleph_{1,2}(E, T_{1,2}) = |E| / \sqrt{E^2 - \Delta_{1,2}(T_{1,2})^2} \theta [E^2 - \Delta_{1,2}(T_{1,2})^2]$$

$$\mathcal{M}_{1,2}(E, T_{1,2}) = |\Delta_{1,2}(T_{1,2})| / \sqrt{E^2 - \Delta_{1,2}(T_{1,2})^2} \theta [E^2 - \Delta_{1,2}(T_{1,2})^2]$$

$$f_{1,2}(E, T_{1,2}) = [1 + e^{E/k_B T_{1,2}}]^{-1}$$

$$\begin{cases} \dot{Q}_{qp} = 0 \\ \dot{Q}_{int} = 0 \end{cases} \quad \text{if } T_1 = T_2$$

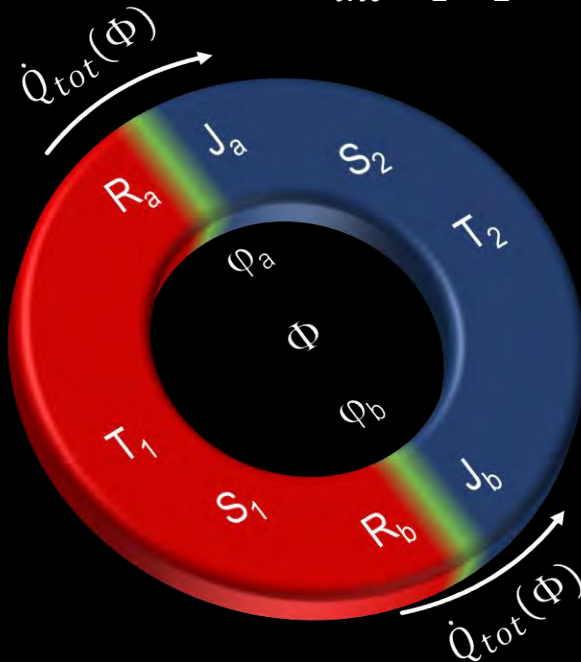
$$\dot{Q}_{int} = 0 \quad \text{if } S_1 \text{ or } S_2 \text{ in normal state}$$

# Temperature-biased DC-SQUID: theory (i)

$$\dot{Q}_{tot} = \dot{Q}_{qp}(T_1, T_2) - \dot{Q}_{int}(T_1, T_2, \varphi_a, \varphi_b)$$

$$\dot{Q}_{qp}(T_1, T_2) = \dot{Q}_{qp}^a(T_1, T_2) + \dot{Q}_{qp}^b(T_1, T_2)$$

$$\dot{Q}_{int}(T_1, T_2) = \dot{Q}_{int}^a(T_1, T_2) \cos \varphi_a + \dot{Q}_{qp}^b(T_1, T_2) \cos \varphi_b$$



$$\varphi_a + \varphi_b + 2\pi \Phi / \Phi_0 = 2k\pi$$

Flux quantization

$$I_j^a \sin \varphi_a = I_j^b \sin \varphi_b$$

Current conservation

$$\cos \varphi_a = \frac{r + \cos(2\pi x)}{\sqrt{1 + r^2 + 2r \cos(2\pi x)}}$$

$$x = \Phi / \Phi_0$$

$$\cos \varphi_b = \frac{1 + \cos(2\pi x)}{\sqrt{1 + r^2 + 2r \cos(2\pi x)}}$$

$$r = I_j^a / I_j^b$$

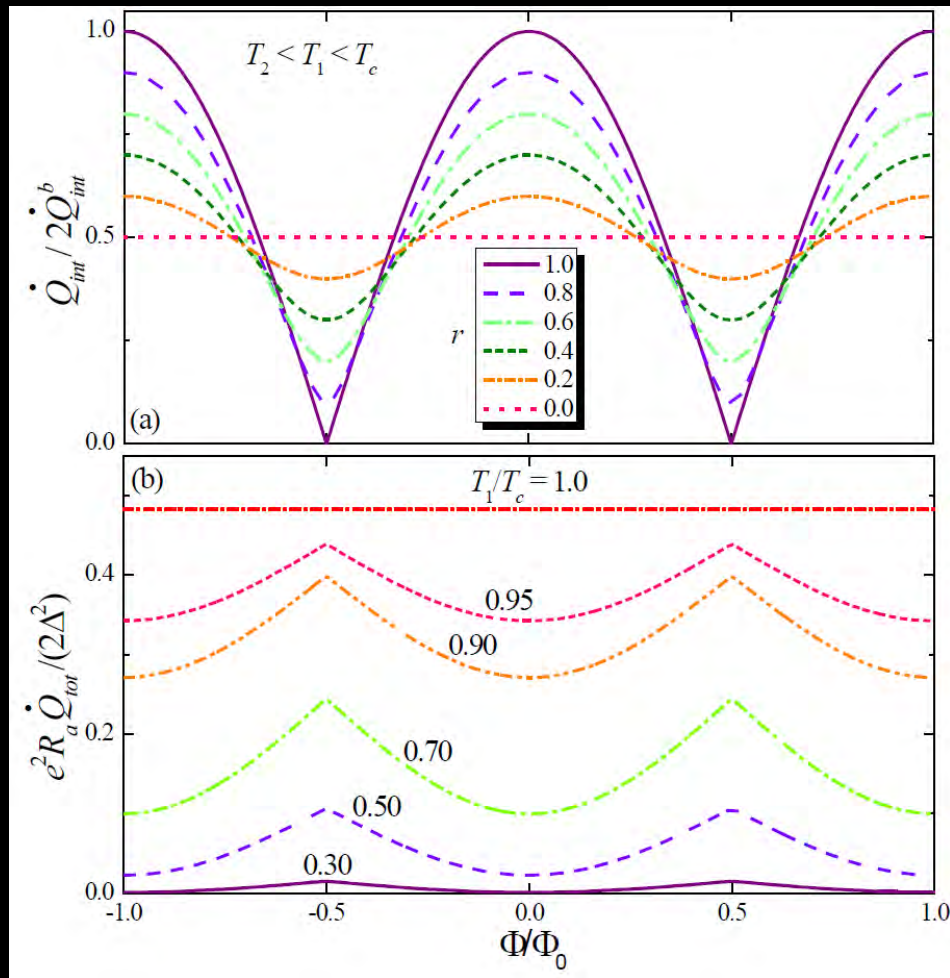
$$\dot{Q}_{int} = \dot{Q}_{int}^b(T_1, T_2) \sqrt{1 + r^2 + 2r \cos\left(\frac{2\pi\Phi}{\Phi_0}\right)}$$

Symmetric SQUID

$$\dot{Q}_{int} = 2\dot{Q}_{int}^b(T_1, T_2) \left| \cos\left(\frac{\pi\Phi}{\Phi_0}\right) \right|$$



# Temperature-biased DC-SQUID: theory (ii)



Role of critical current asymmetry

Maximum  $\dot{Q}_{int}^b (1 + r)$

Minimum  $\dot{Q}_{int}^b (1 - r)$

Total heat current behavior  
(symmetric SQUID)

$$T_2 = 0.1T_c$$

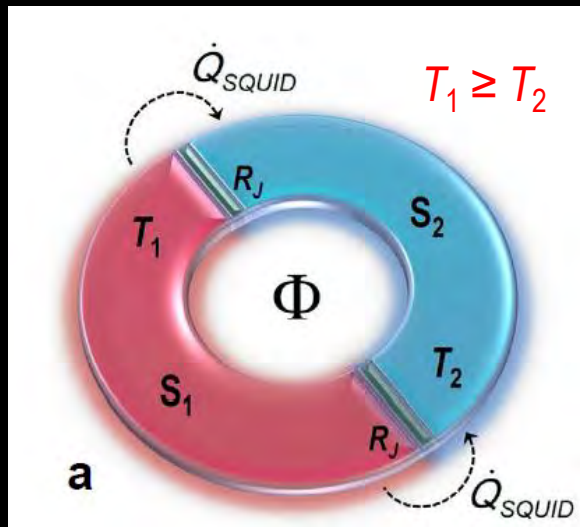
# “Josephson heat interferometer”: setup (i)

## LETTER

doi:10.1038/nature11702

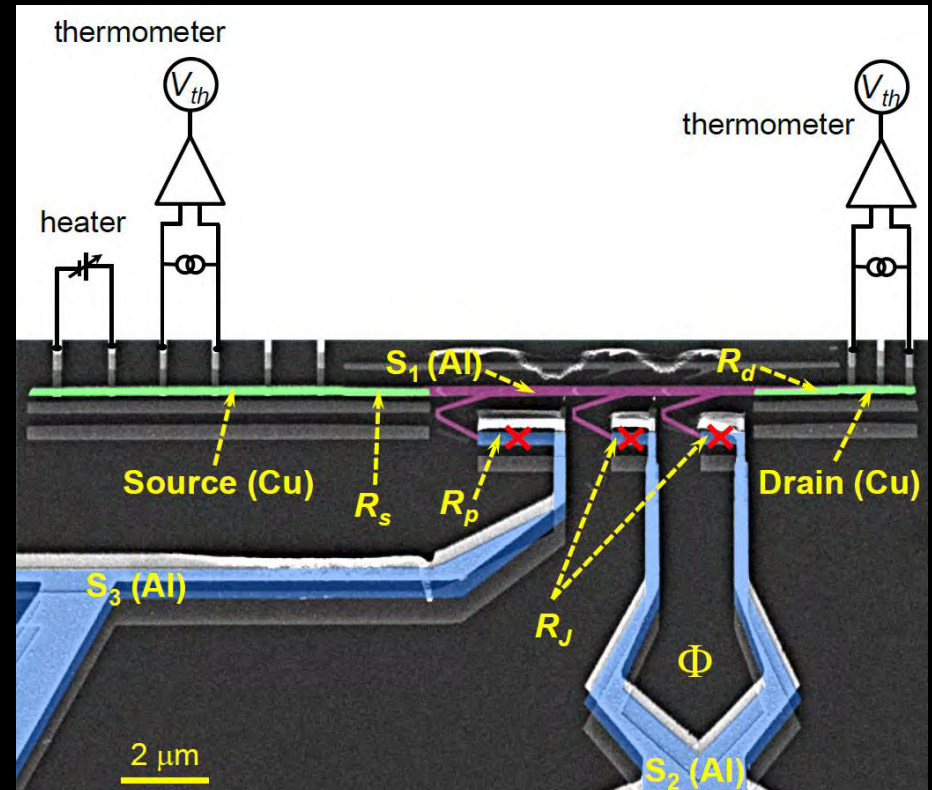
### The Josephson heat interferometer

Francesco Giazotto<sup>1</sup> & María José Martínez-Pérez<sup>1</sup>

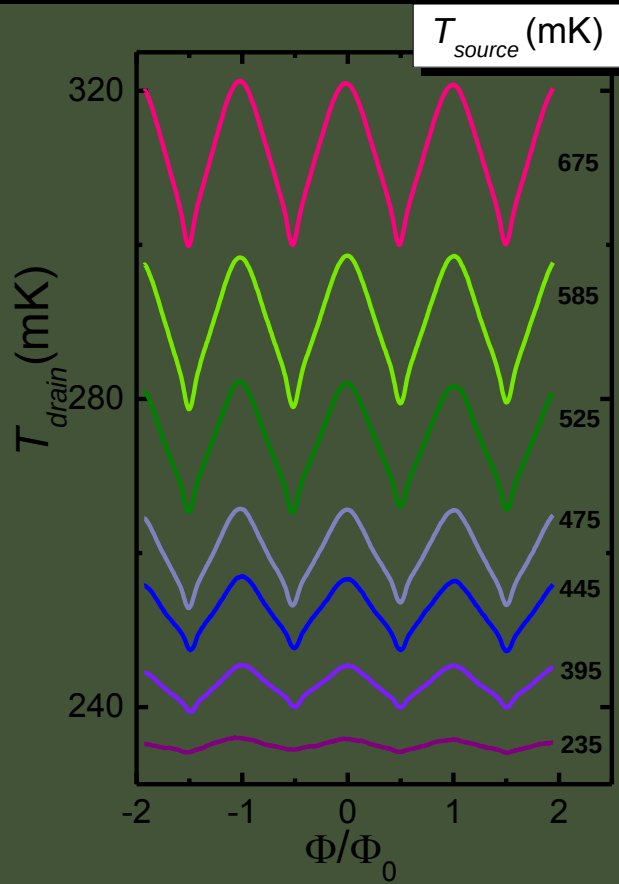


Symmetric SQUID ( $r = 1$ )

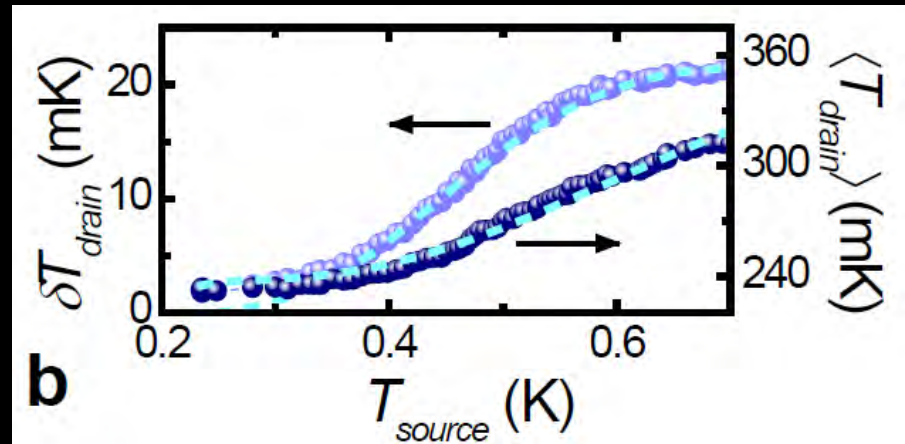
$$\dot{Q}_{SQUID}(\Phi) = 2\dot{Q}_{qp} - 2\dot{Q}_{int} \left| \cos \left( \frac{\pi \Phi}{\Phi_0} \right) \right|$$



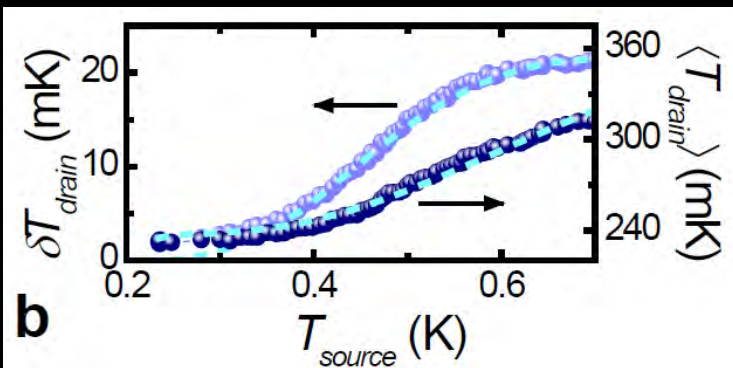
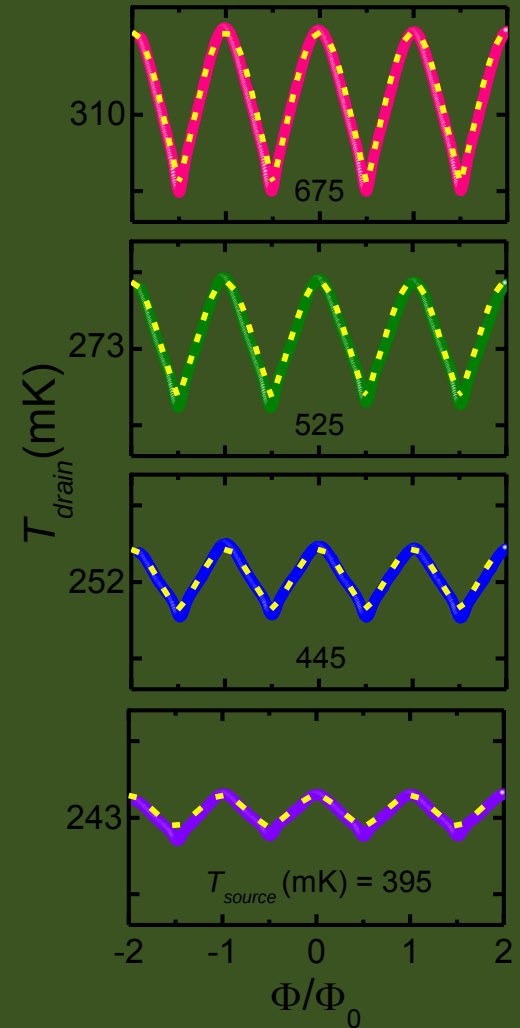
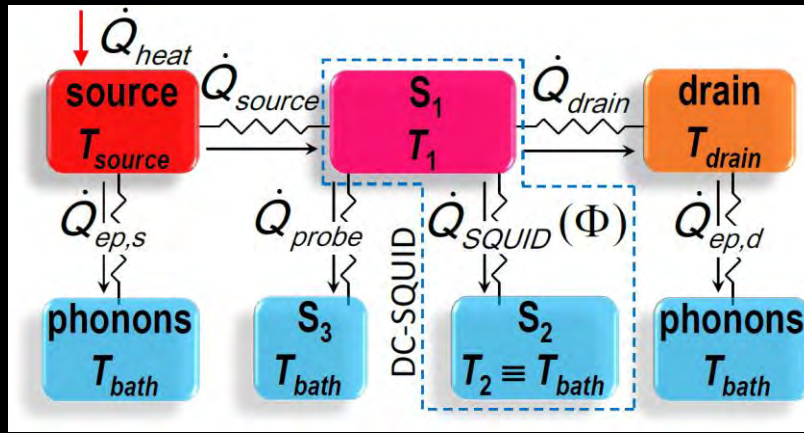
# Behavior @ 235 mK (i)



$\delta T_{\text{drain}} \sim 21$  mK  
9% relative  
modulation amplitude

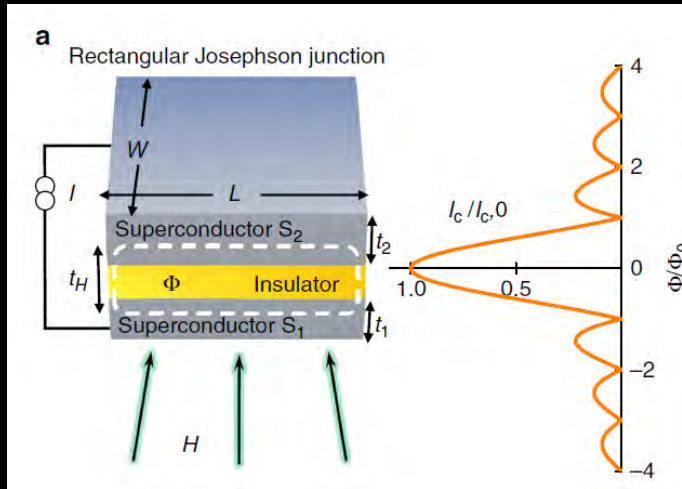


# Comparison to theory

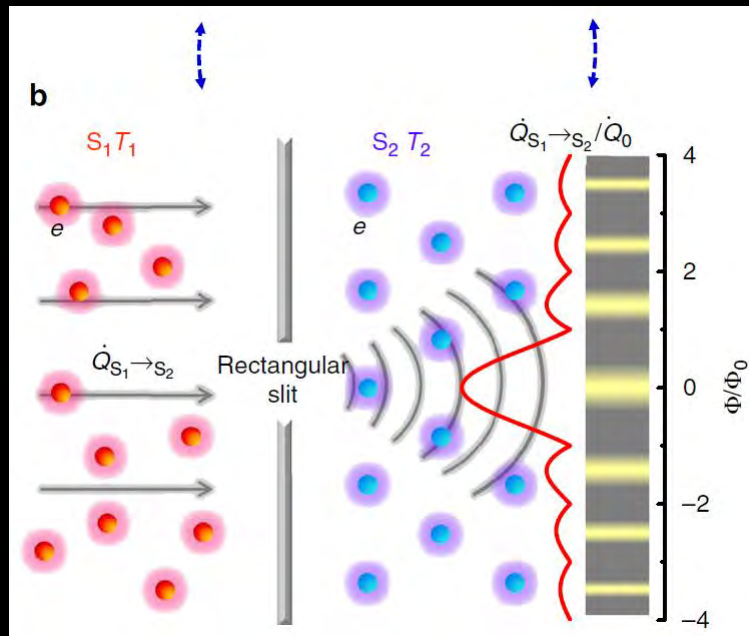


Good agreement with theoretical prediction

# Electric vs thermal quantum diffraction



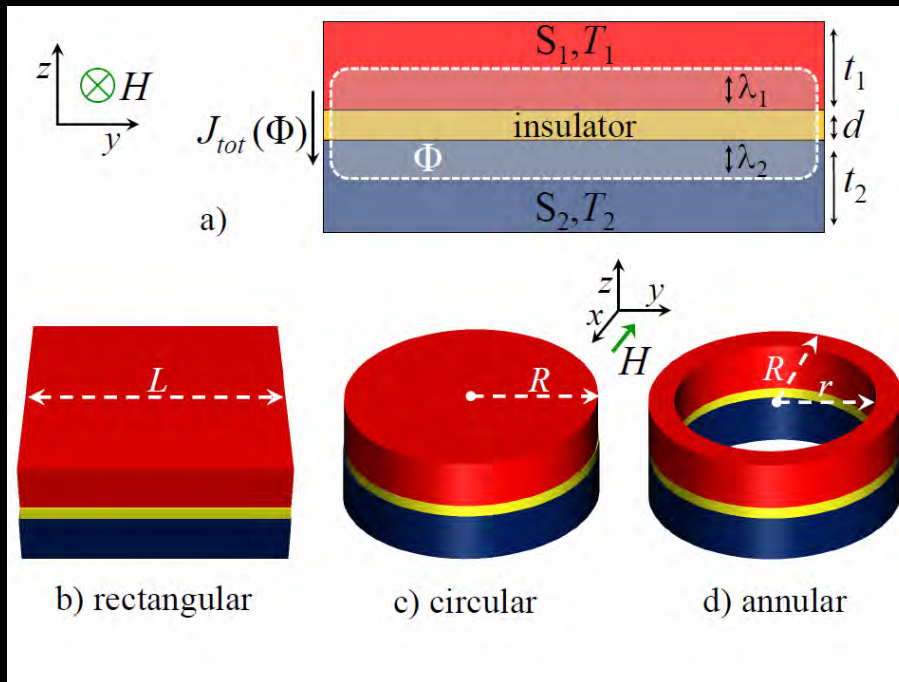
Electric diffraction through a rectangular slit



Diffraction of heat current through a rectangular slit



# Heat current quantum diffraction in extended short JJs



$$\tilde{t} = d + \lambda_1 \tanh \frac{t_1}{2\lambda_1} + \lambda_2 \tanh \frac{t_2}{2\lambda_2}$$

$$L \ll \lambda_J \equiv \sqrt{\frac{\Phi_0 W L}{2\mu_0 I_c \tilde{t}}}$$

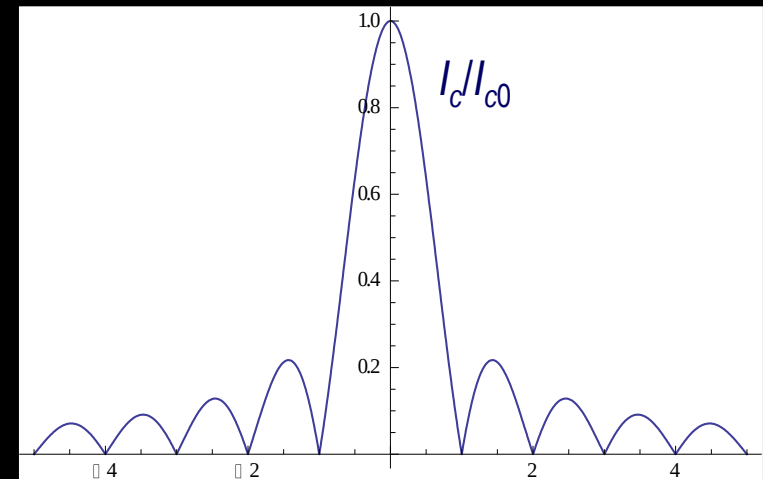
Josephson critical current

$$I_c(\Phi) = |2I_{c0} \int_{-\infty}^{\infty} f(y) \cos\left(\frac{2\pi\Phi}{\Phi_0} \frac{y}{L}\right) dy|$$

Critical current Fraunhofer pattern for a rectangular JJ

$$\frac{I_c}{I_{c0}} = |\sin(\pi\Phi/\Phi_0)/(\pi\Phi/\Phi_0)|$$

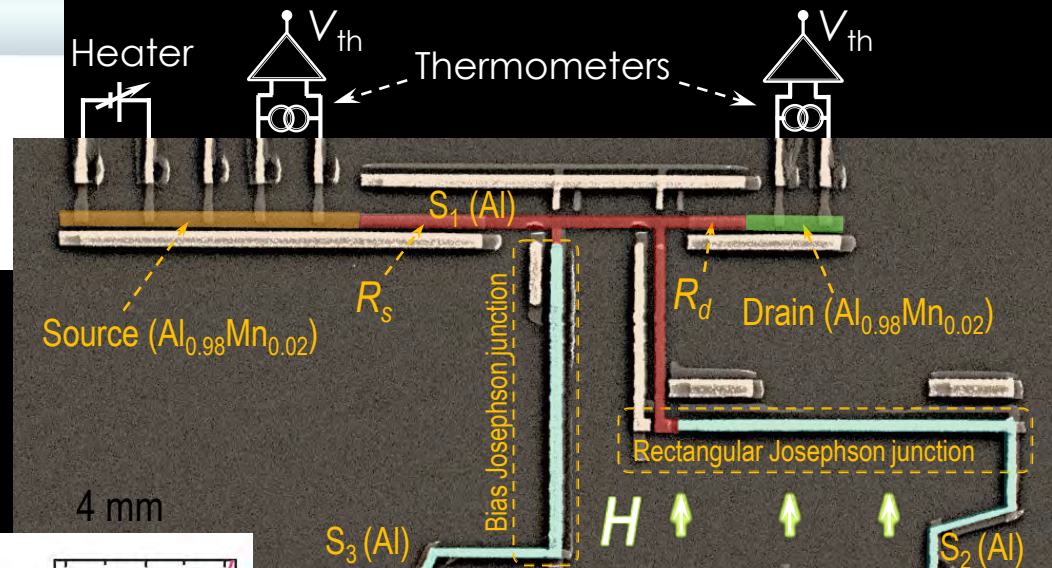
$\Phi/\Phi_0$



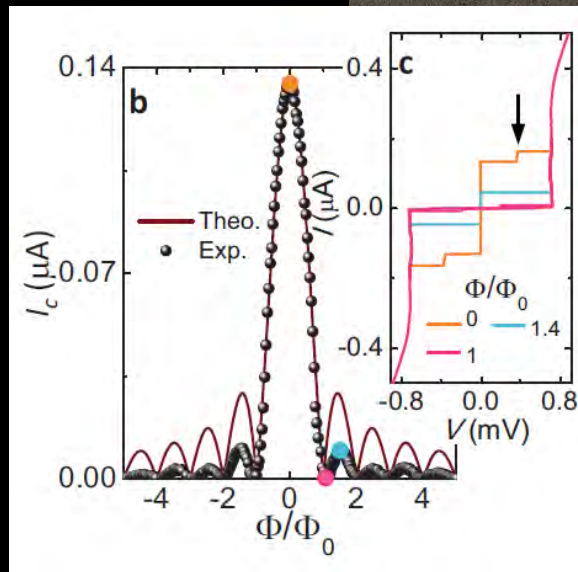
# A “quantum diffractor” for thermal flux: experimental setup



Rectangular JJ

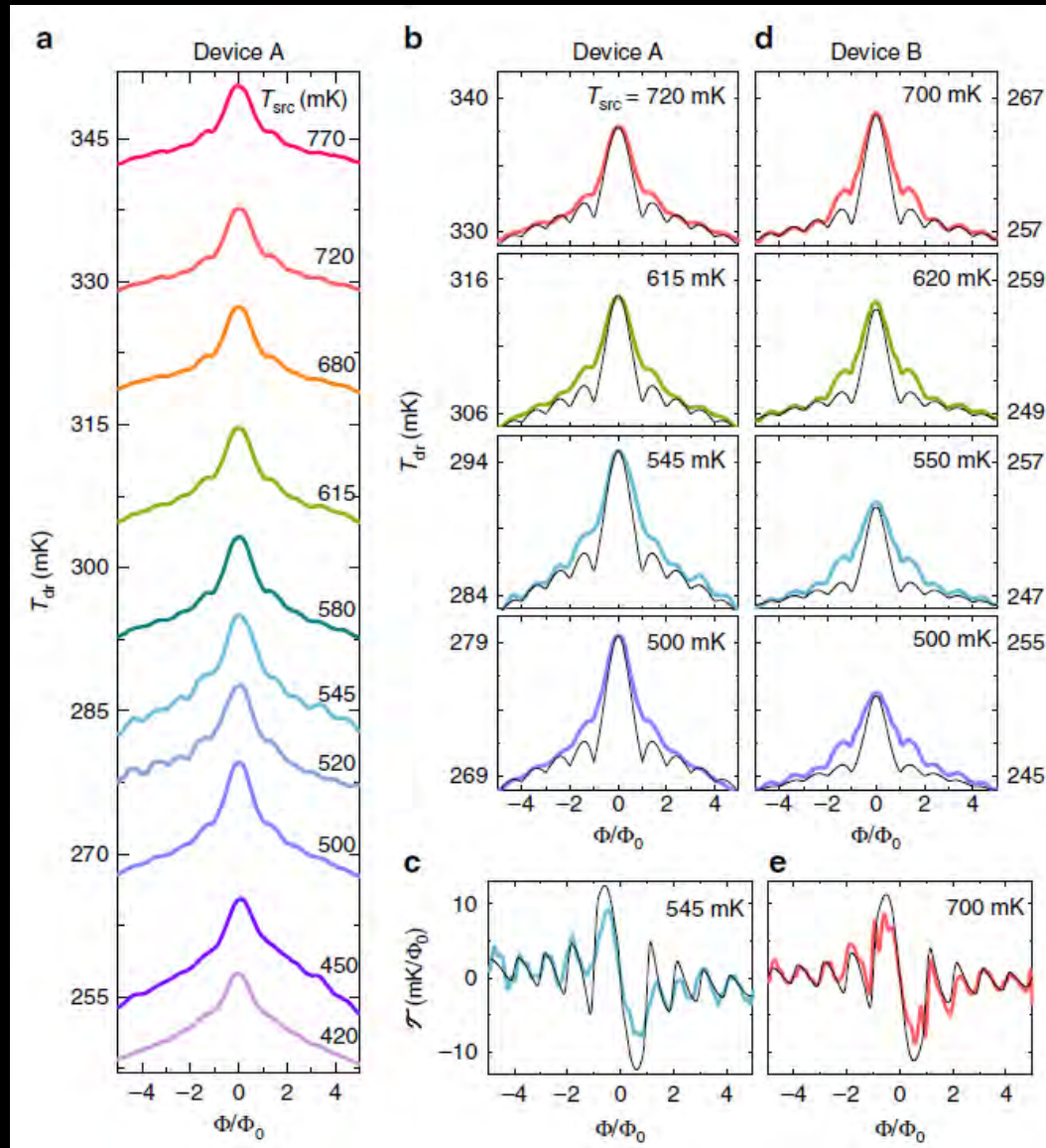


Magnetic interference pattern



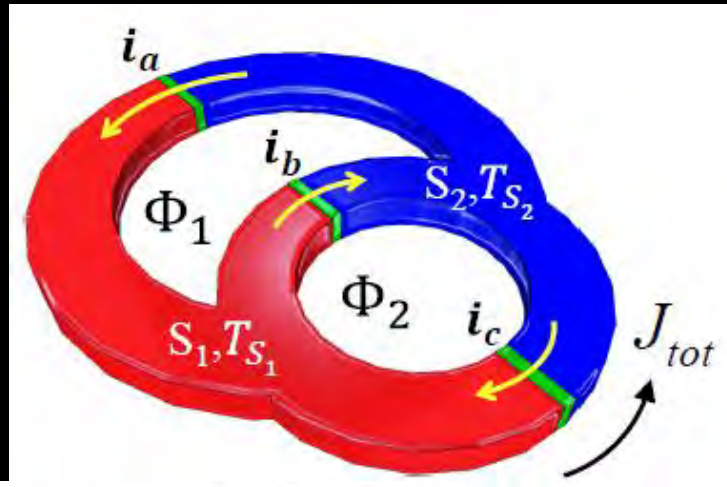
Josephson current behavior

# Temperature diffraction pattern @ 240 mK



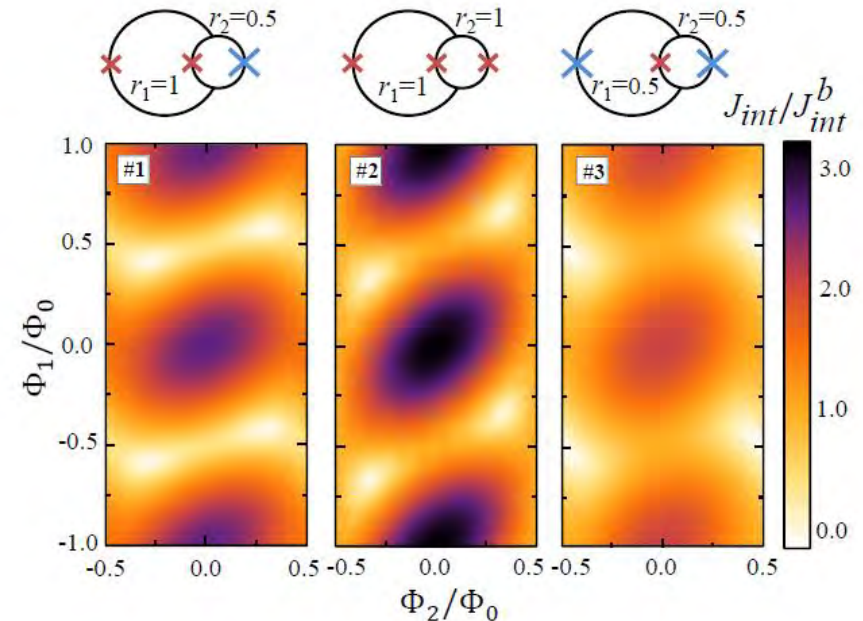
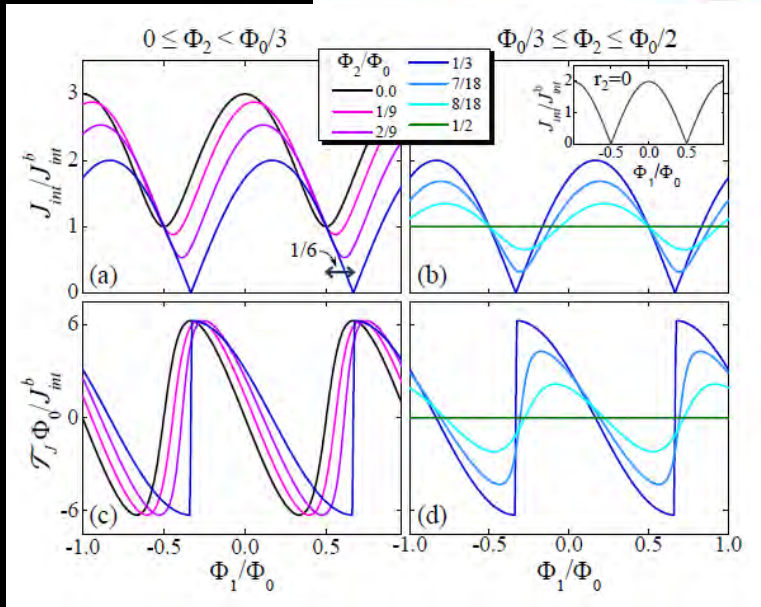


# Fully-balanced heat interferometer



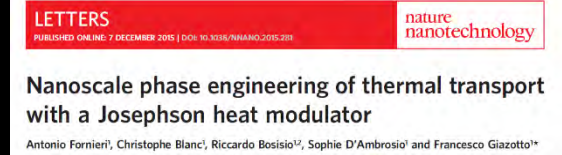
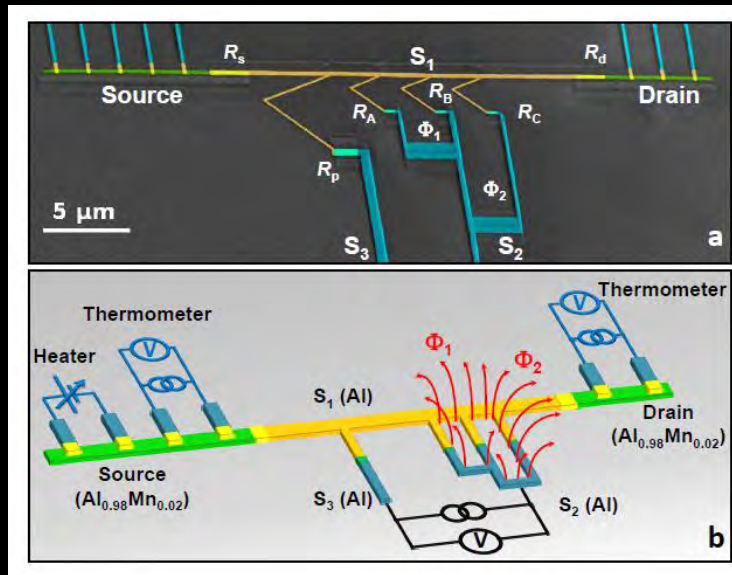
- Enhanced control over the flux-to heat current transfer function
- Complete suppression of the phase-coherent part

$$i_a - i_c \leq i_b \leq i_a + i_c$$

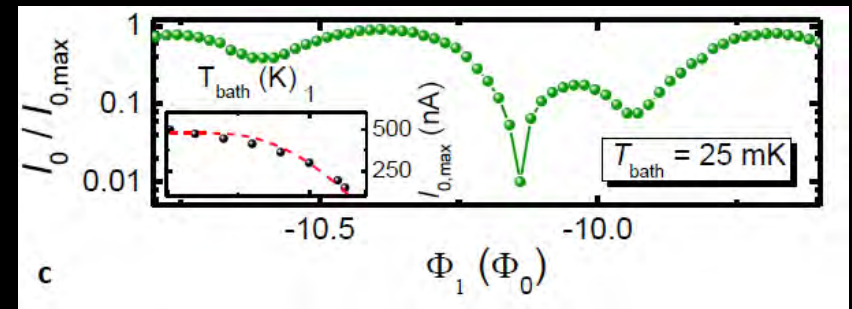
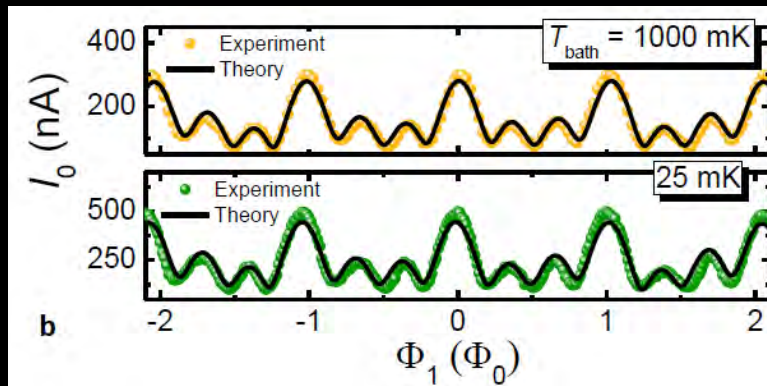
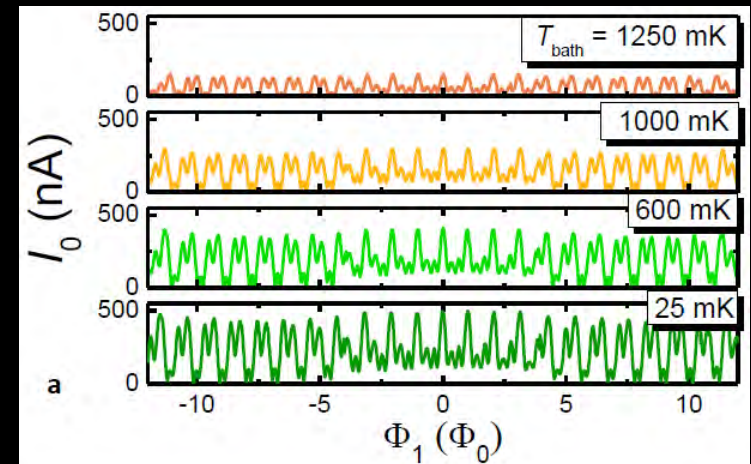


# Nanoscale phase-engineering of thermal transport

## i) Electrical response



Fully-balanced quantum thermal modulator structure:  
full phase-engineering of heat currents

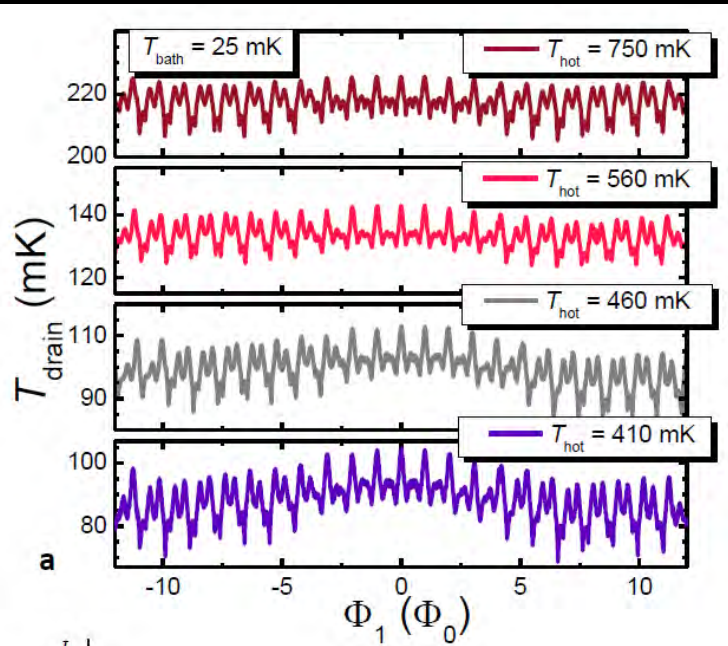


$I_c$  suppression  $\sim 99\%$



# Nanoscale phase-engineering of thermal transport

## ii) Thermal response at base $T_{\text{bath}}$

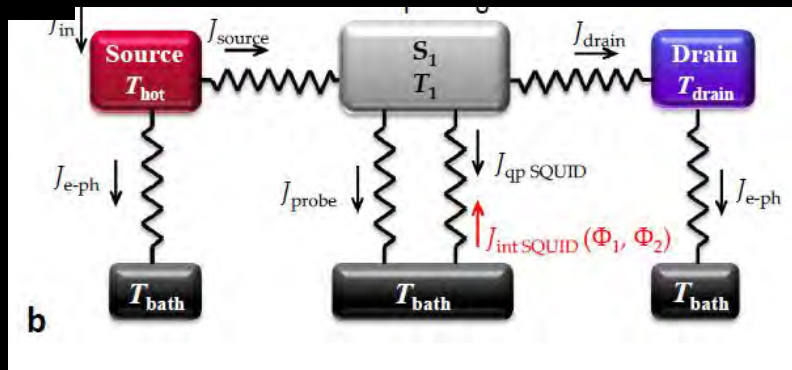


~ 40mK temperature swing

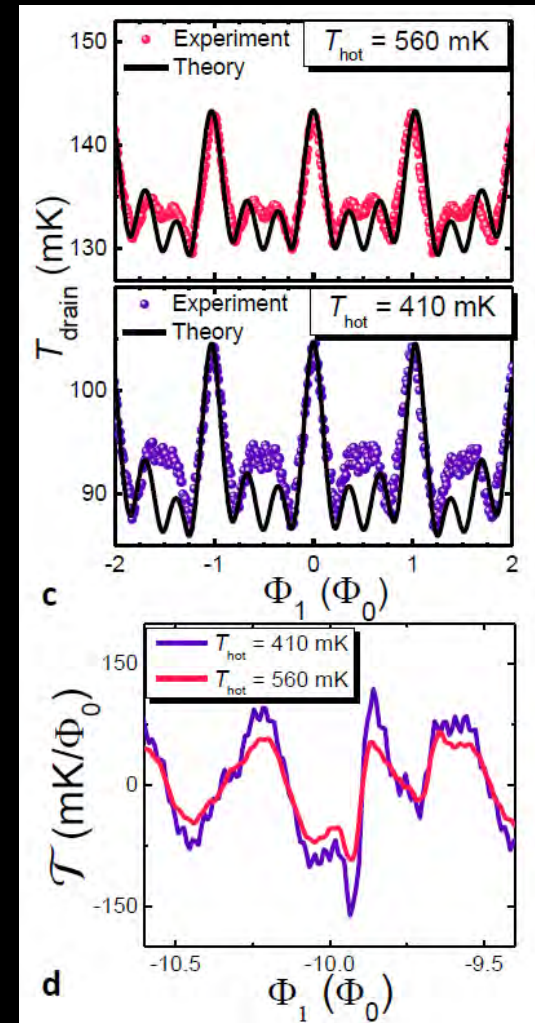
$J_{\text{int}}$  suppression ~ 99%

$\tau \sim 200\text{mK}/\Phi_0 @ 25\text{mK}$

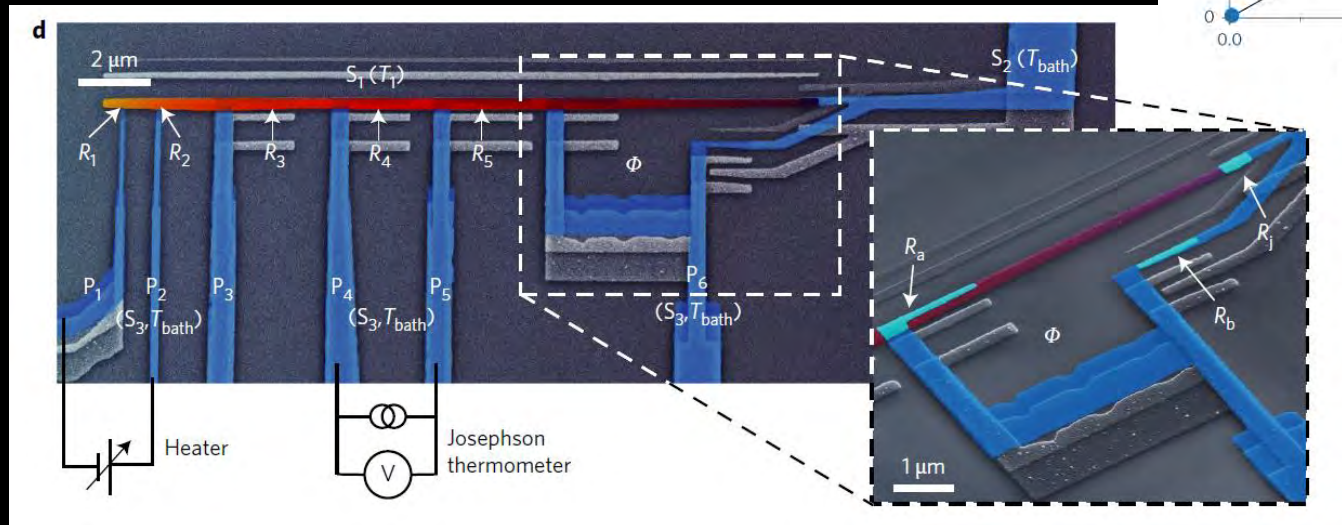
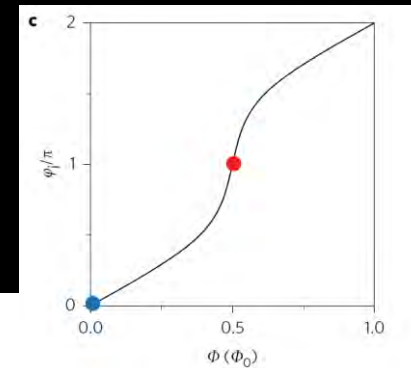
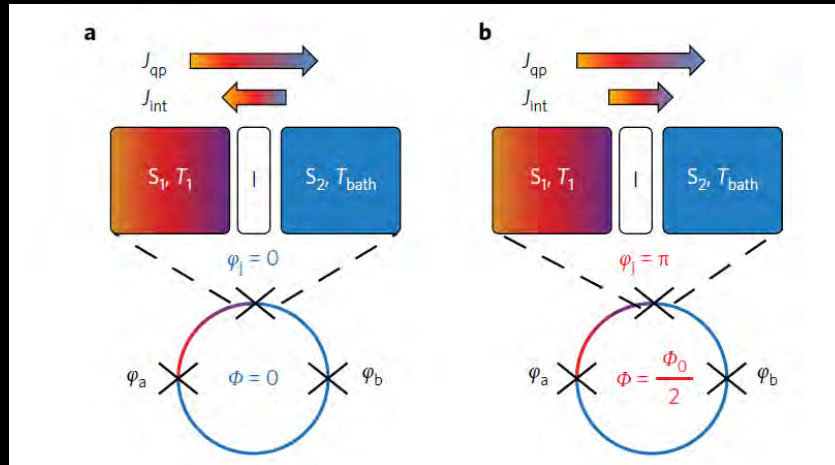
X 3 previous exps



Thermal model

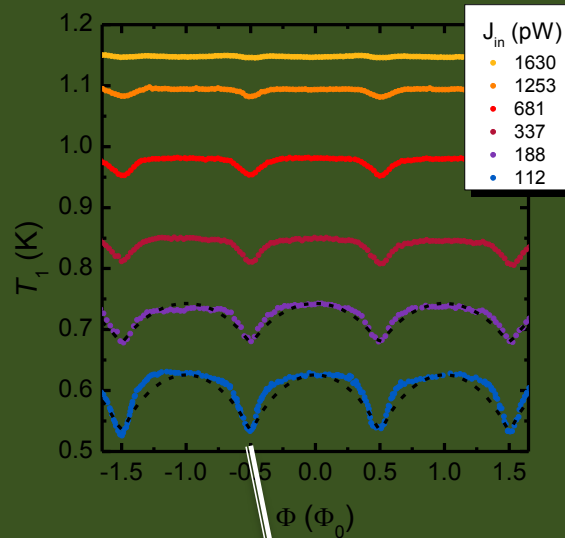


# Phase-controllable $0-\pi$ thermal Josephson junction

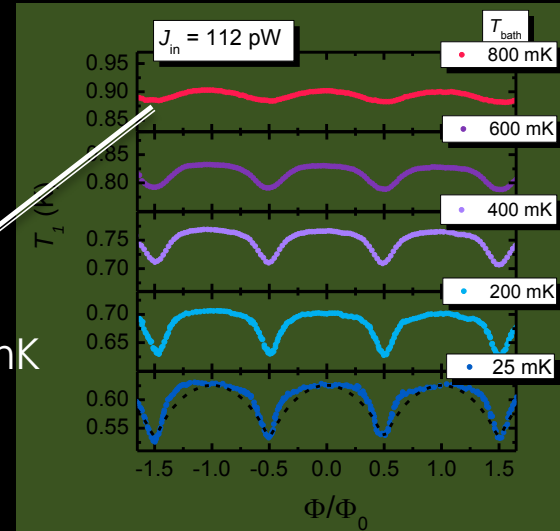


A. Fornieri, G. Timossi, P. Solinas, P. Virtanen, and FG, Nat. Nanotechnol. **12**, 425-429 (2017);  
A. Fornieri, G. Timossi, R. Bosisio, P. Solinas, and FG, Phys. Rev. B **93**, 134508 (2016)

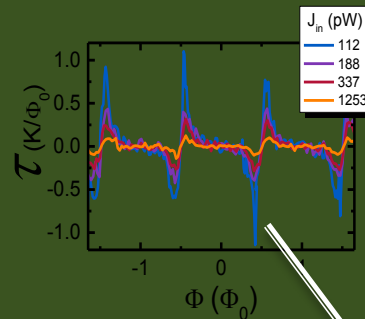
# 0- $\pi$ thermal Josephson junction: thermal behavior



$\delta T \sim 20$  mK @ 800 mK



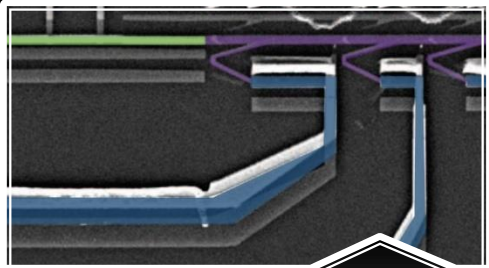
$\delta T$  up to 100 mK @ 25 mK



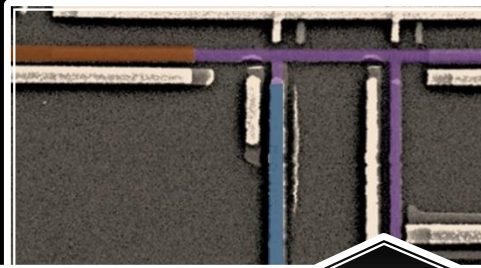
$$\mathcal{T} \equiv \partial T_{drain} / \partial \Phi$$

$\sim 1$  mK/ $\Phi_0$  @ 550 mK

# Single output caloritronic devices



The Josephson heat interferometer  
Nature **492**, 401 (2012)



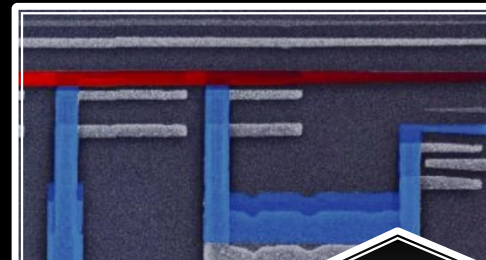
A quantum diffractor for thermal flux  
Nat. Commun. **5**, 3579 (2014)



Nanoscale phase engineering of thermal transport with a Josephson heat modulator  
Nat. Nanotech. **11**, 258 (2016)



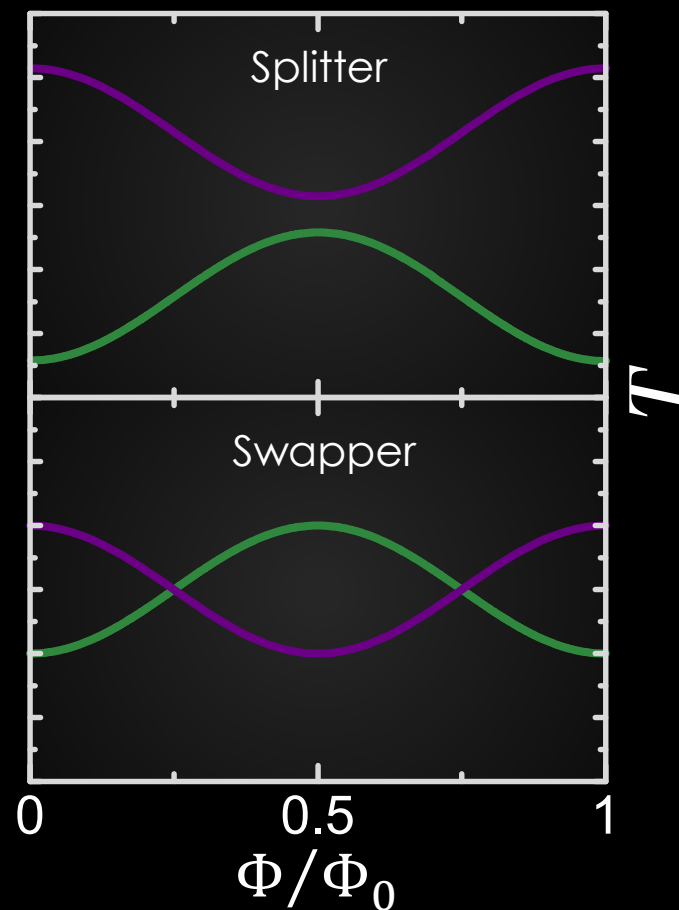
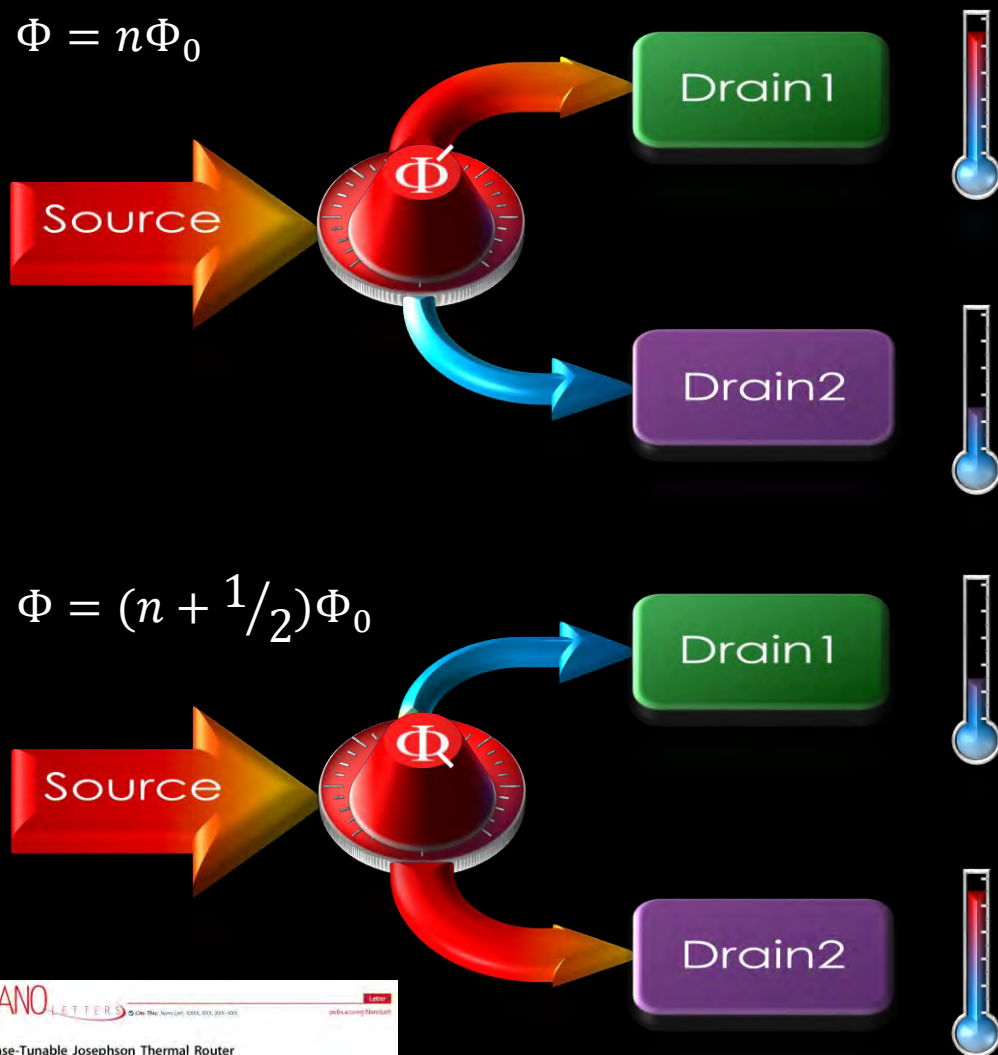
Rectification of electronic heat current by a hybrid thermal diode  
Nat. Nanotech. **10**, 303 (2015)



0- $\pi$  phase-controllable thermal Josephson junction  
Nat. Nanotech. **12**, 425 (2017)



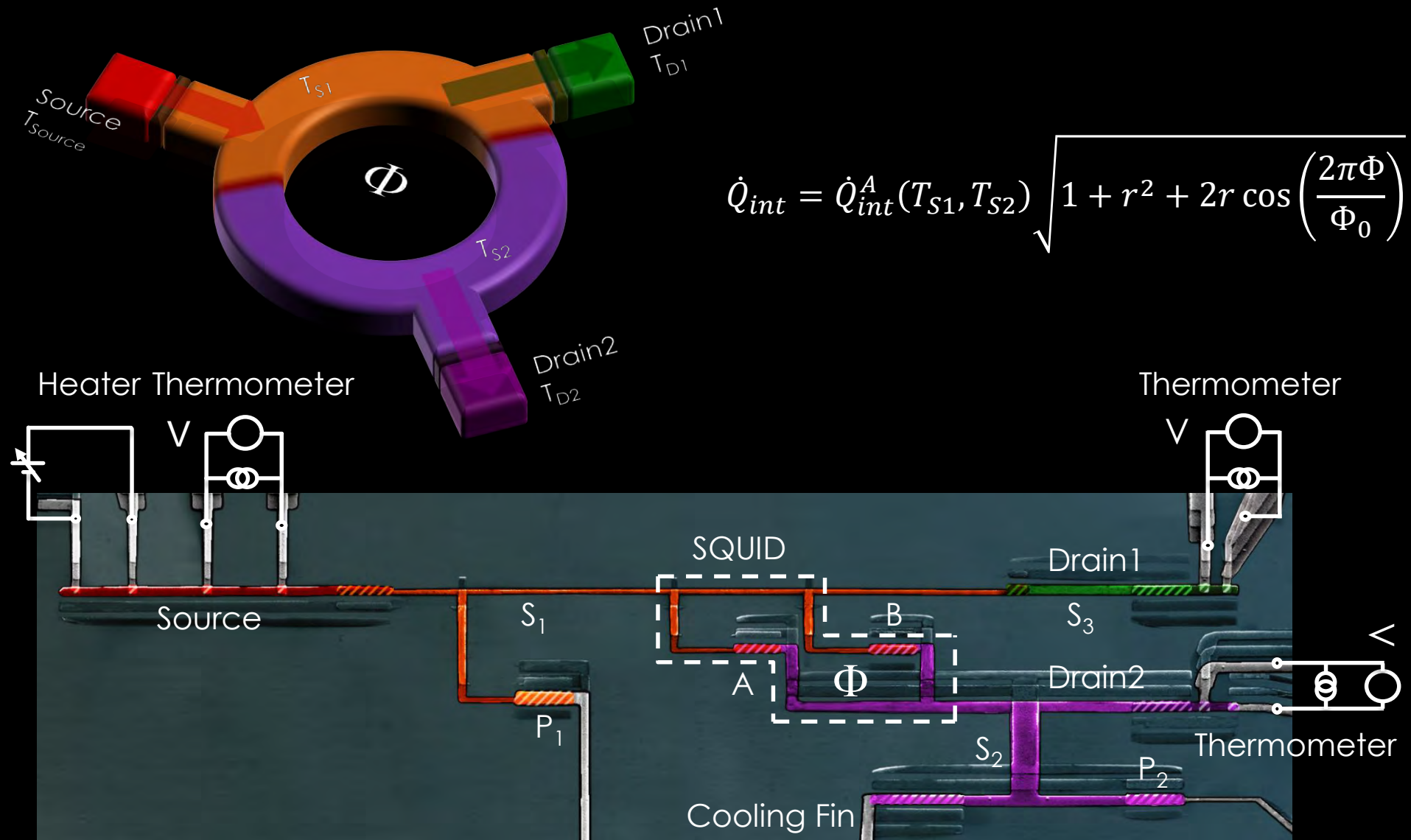
# Phase-tunable thermal router: General scheme



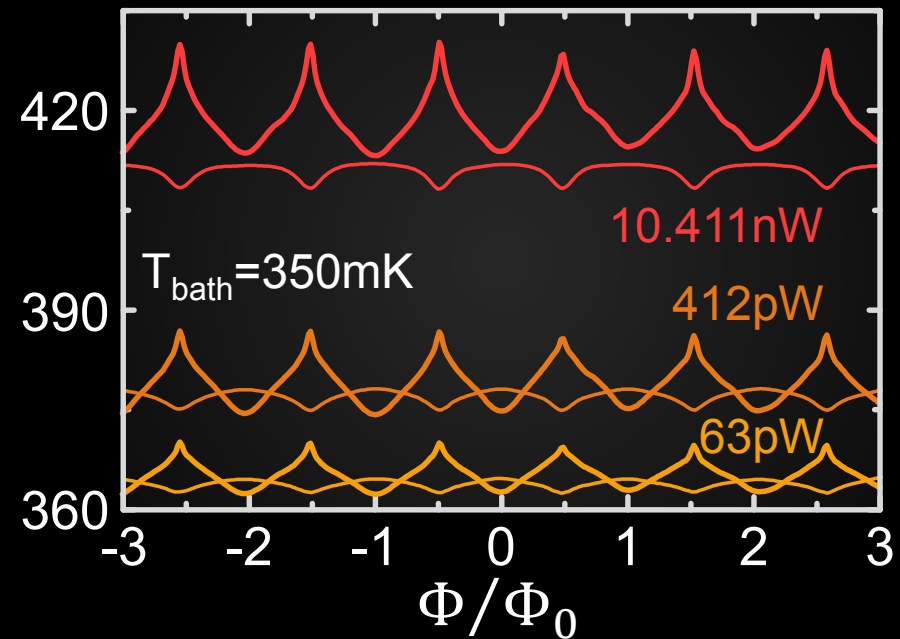
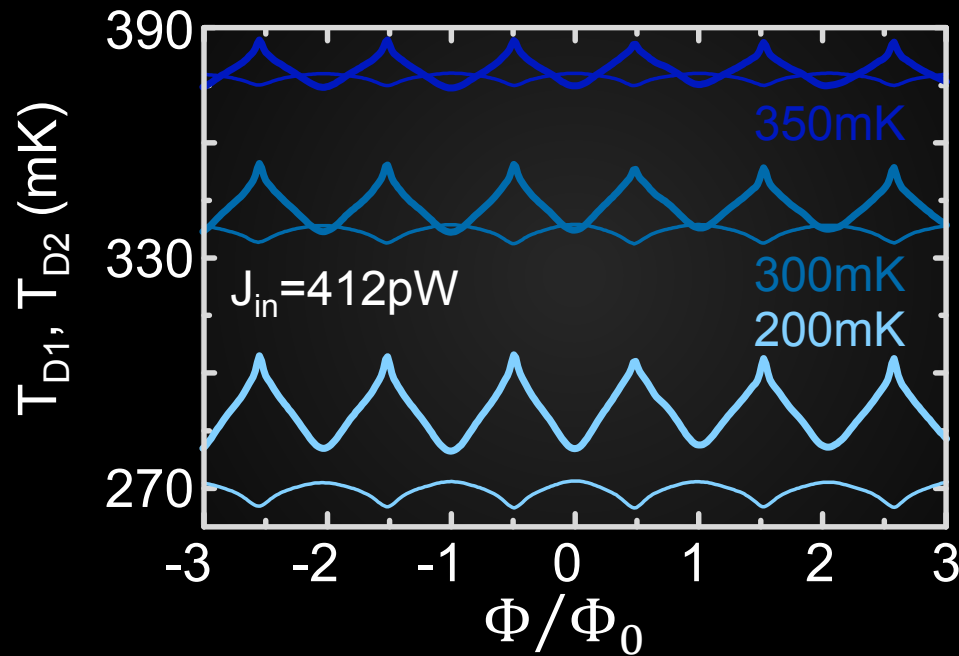
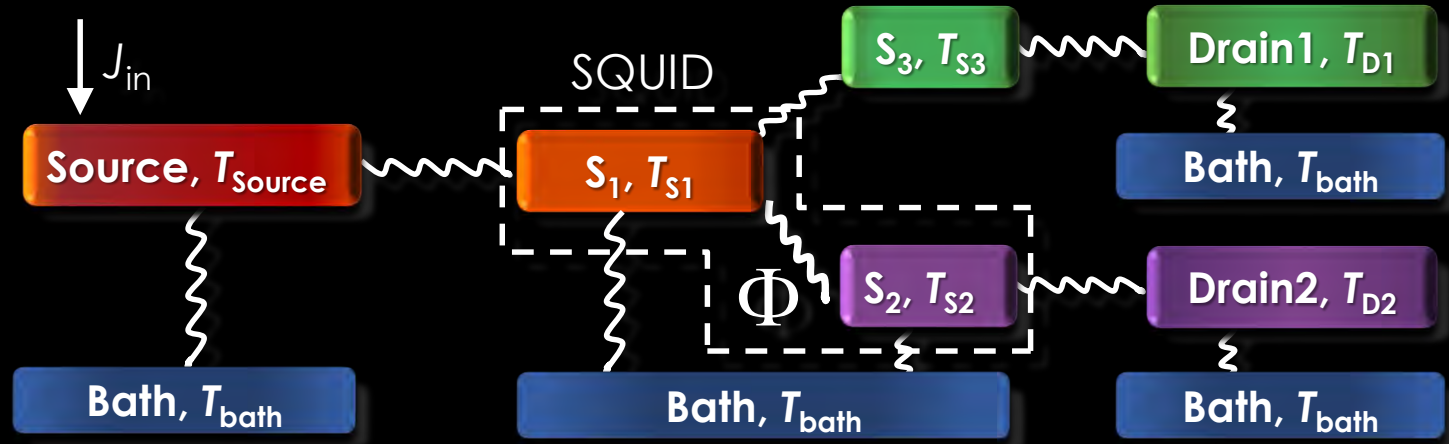
**NANO LETTERS**  
 Phase-Tunable Josephson Thermal Router  
 Giuliano Francesco Tinozzi,<sup>1,2</sup> Antonio Fornari,<sup>1,2</sup> Federico Paolucci,<sup>1</sup> Claudio Puglisi,<sup>2,3</sup>  
 and Francesco Giazetto<sup>1,†</sup>  
<sup>1</sup>NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, Piazza S. Silvestro 12, I-56127 Pisa, Italy  
<sup>2</sup>Dipartimento di Fisica dell'Università di Pisa, Largo Pontecorvo 3, I-56127 Pisa, Italy

DOI: 10.1021/acs.nanolett.7b04906

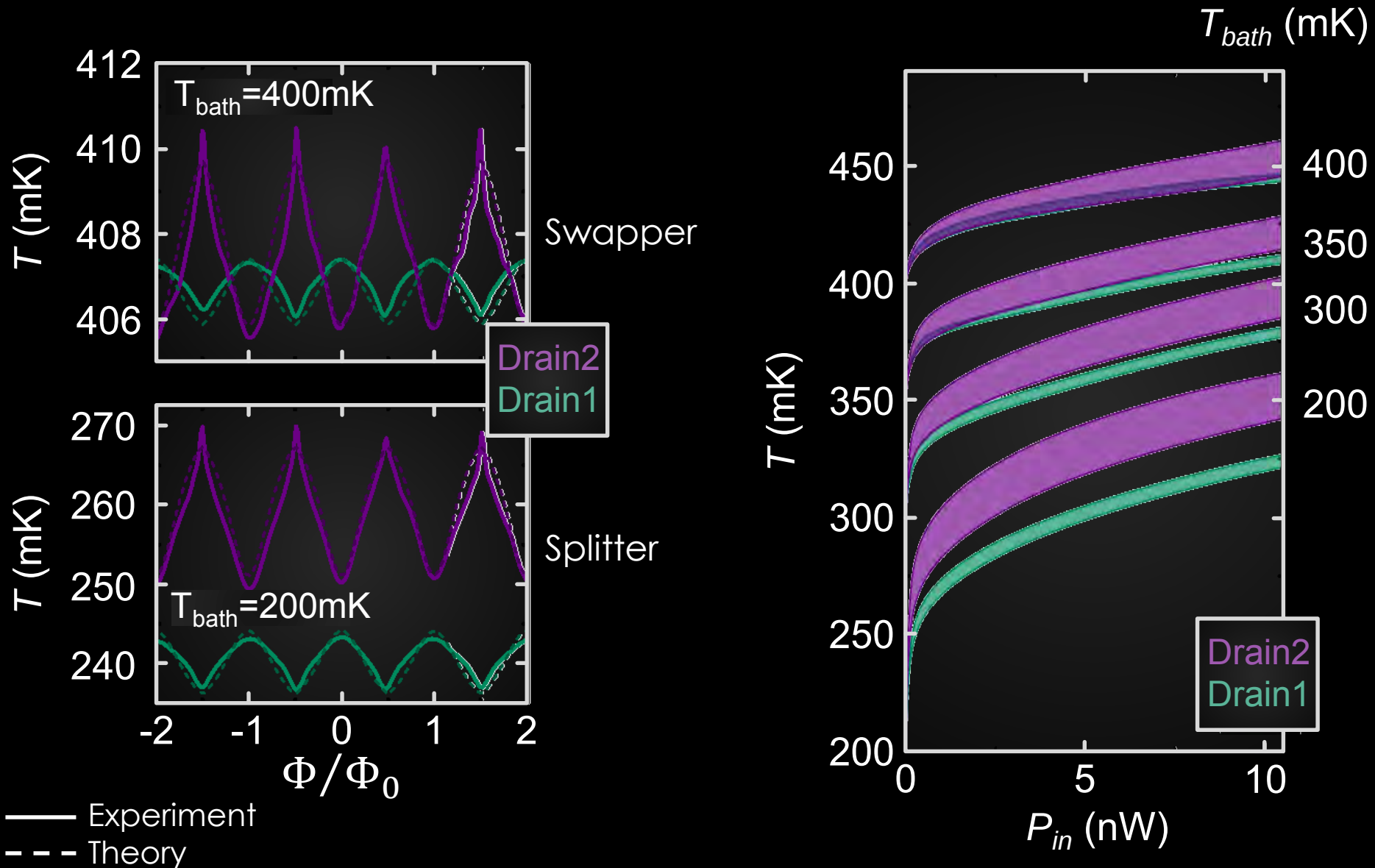
# Phase-tunable thermal router: Device structure



# Phase-tunable thermal router: Experiment



# Phase-tunable thermal router: Experiment



G. Timossi, A. Fornieri, F. Paolucci, C. Puglia, and FG, Nano Lett. **18**, 1764 (2018)



# Conclusions

1. Realization of the first heat interferometer
2. Confirmation of the existence, magnitude and sign of the phase-dependent heat current
3. Realization of the first quantum diffractor for thermal flux, complementary proof of the “thermal” Josephson effect
4. Double-loop Josephson thermal modulator: complete phase-engineering of electronic heat current at the nanoscale
5. Realization of the first controllable  $0-\pi$  thermal Josephson junction
6. Realization of the first phase-tunable Josephson thermal router with large  $T$  separation and sizeable  $T$  inversion: gateway to realize mesoscopic “thermal machines”

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FARFAS 2014-Project SCIADRO



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