MEASURING HEAT CURRENT AND NOISE IN QUANTUM CIRCUITS

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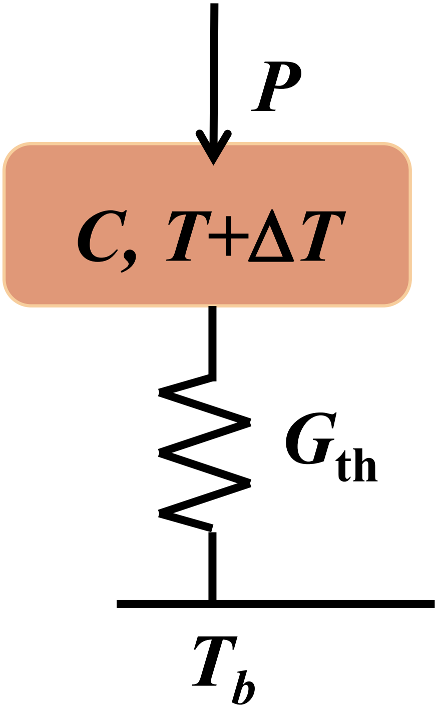
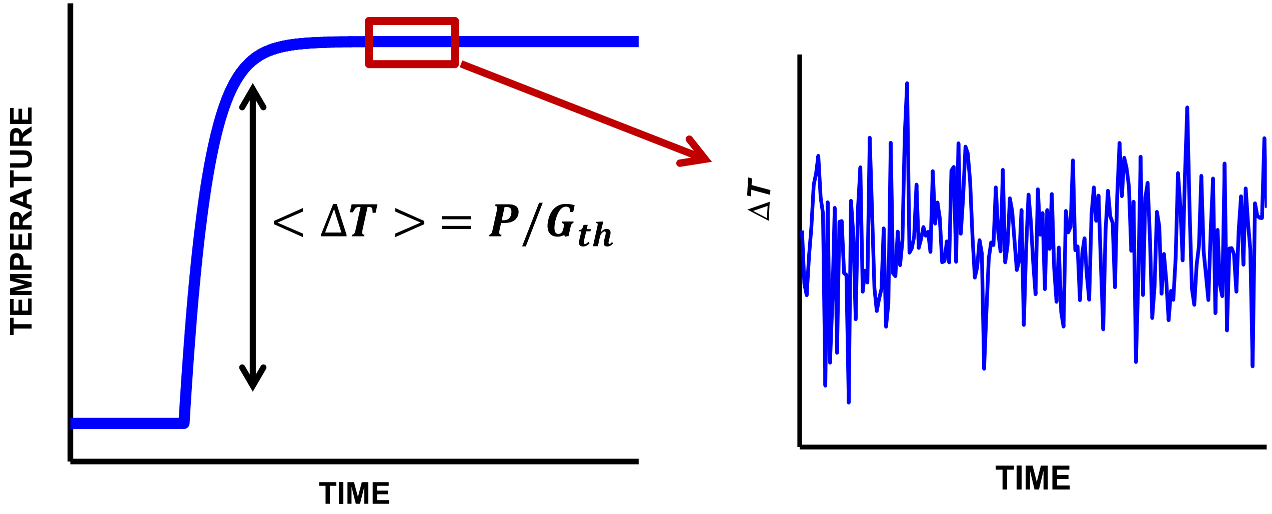
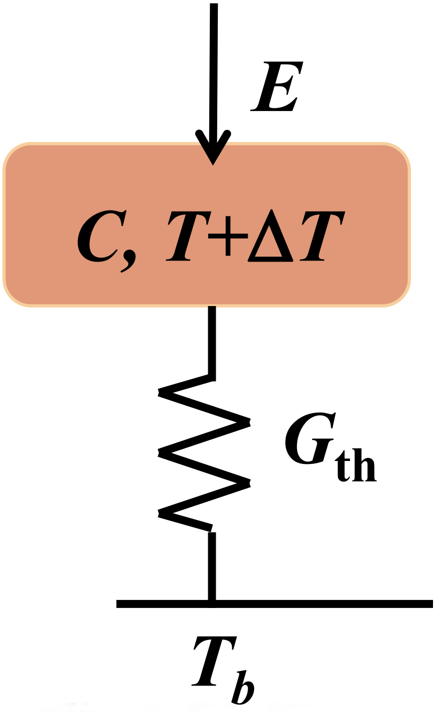
How to measure heat current?

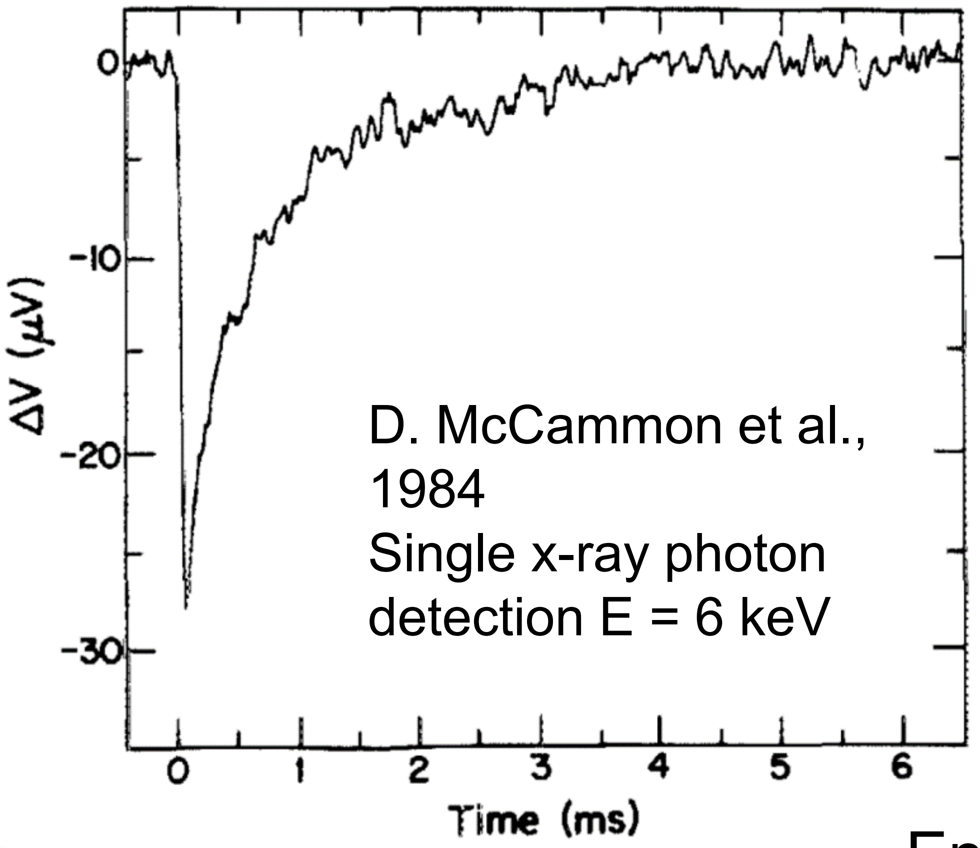


Measurement of temperature by a fast thermometer

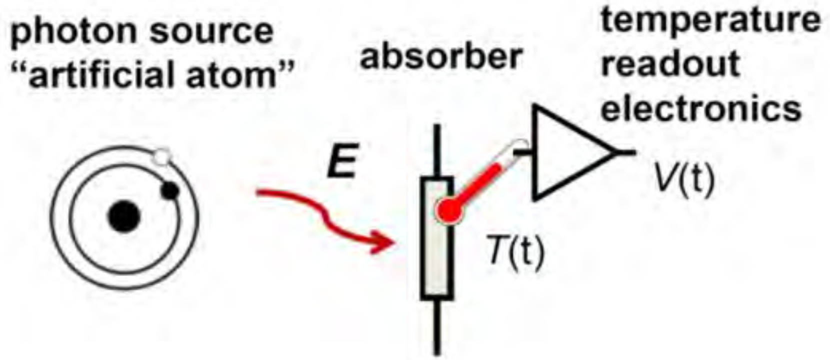


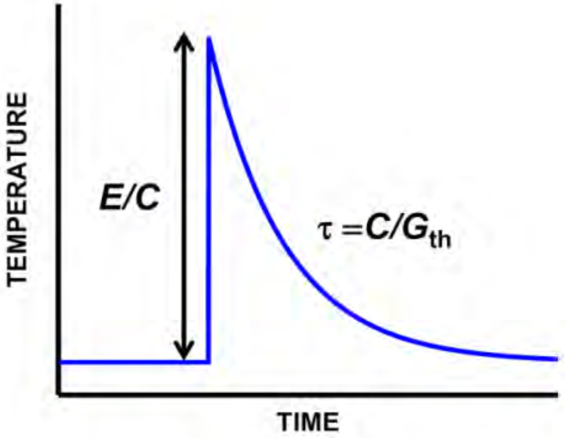
How to measure heat current?





Our goal:
Single microwave photon detection
E = 100 μeV
(10^8 times smaller energy!)



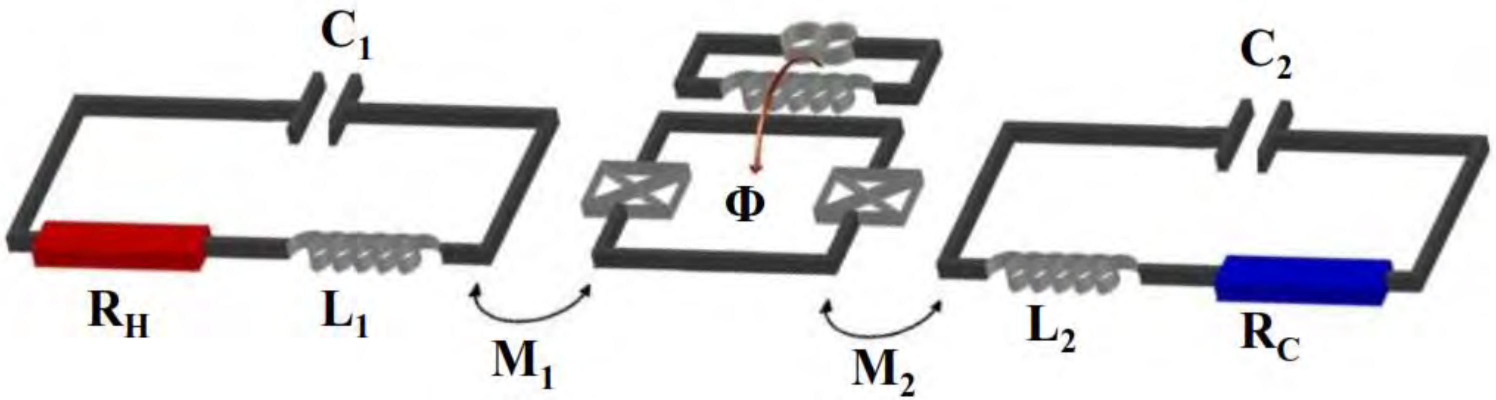


Energy resolution:

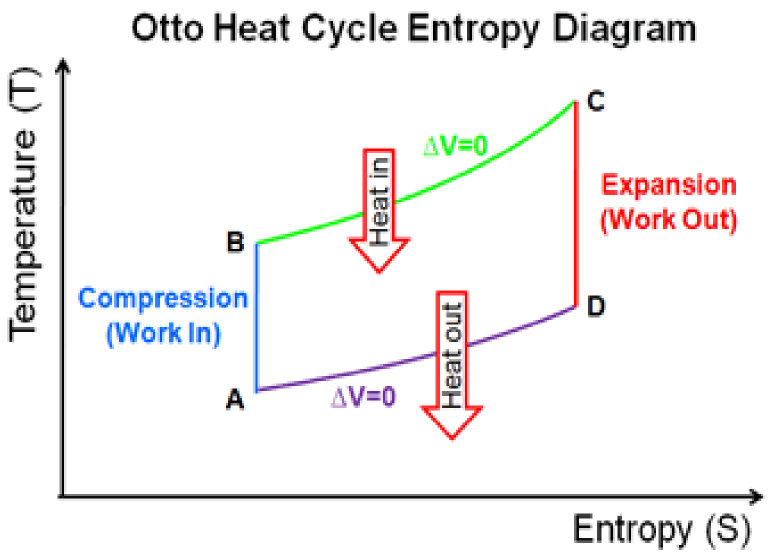
Thermometry!

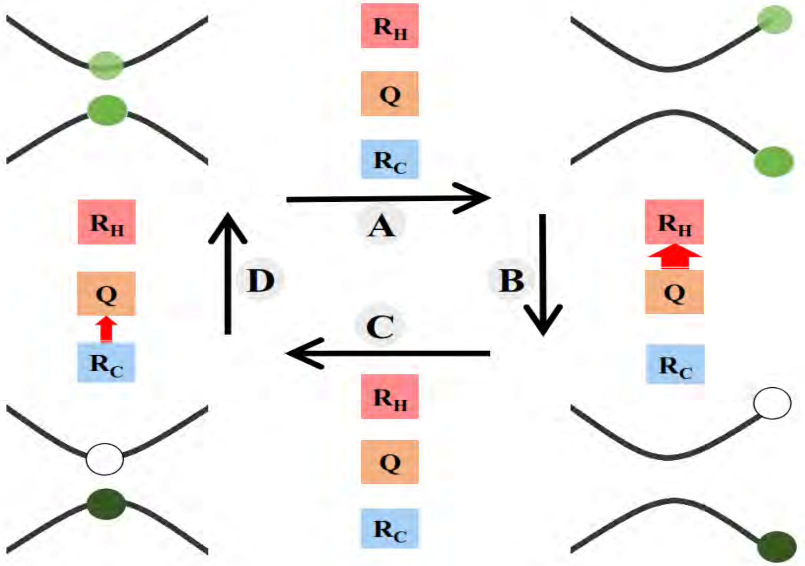
$$\delta E = \sqrt{C G_{\rm th} S_T}$$

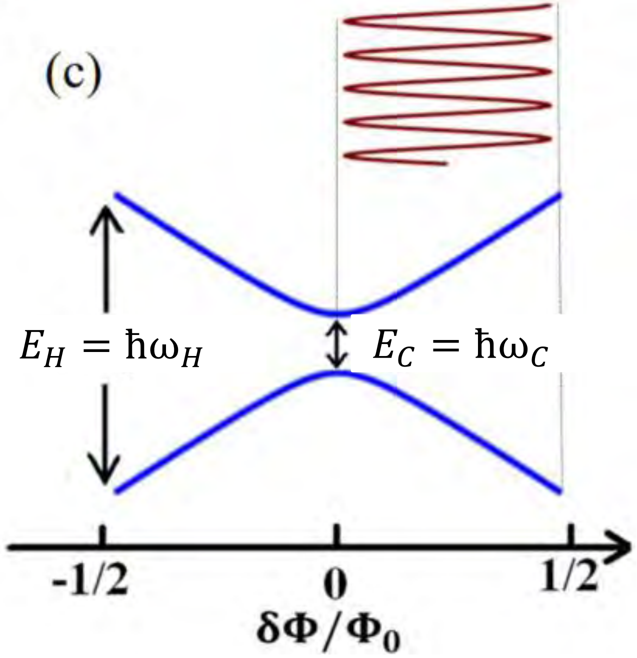
Quantum Otto refrigerator[1]



$$q \equiv \delta\Phi/\Phi_0$$
$$\delta\Phi \equiv \Phi - \Phi_0/2$$





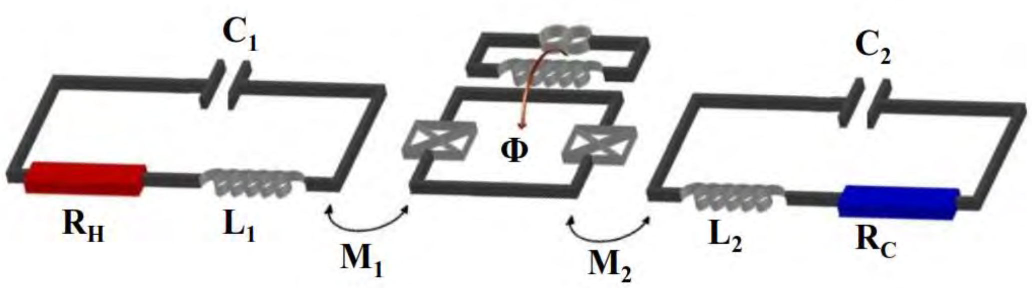


[1]**B. Karimi and J. P. Pekola, Otto refrigerator based on a superconducting qubit: classical and quantum performance, Phys. Rev. B 94, 184503 (2016).** ***Editor's suggestion***

System and Hamiltonian

The Hamiltonian of the whole set-up

$$H = H_{R_H}+H_{R_C}+H_{c_H}+H_{c_C}+H_Q$$



The Hamiltonian of the qubit

$$H_Q = -E_0(\Delta\sigma_x + q\sigma_z)$$

The transition rates between the two levels

$$\Gamma_{\downarrow\uparrow,j} = \frac{E_0^2 M_j^2}{\hbar^2\Phi_0^2}\frac{\Delta^2}{q^2+\Delta^2}S_{I,j}\left(\pm {E}/{\hbar}\right)$$

Master equation for the Qubit density matrix[1]

$$\dot{\rho}_{gg} = -\frac{\Delta}{q^2+\Delta^2}\dot{q}\,\Re e\left[\rho_{ge}\,e^{i\int_0^t E(t')dt'/\hbar}\right] - \Gamma_\Sigma\rho_{gg} + \Gamma_\downarrow$$

$$\dot{\rho}_{ge} = \frac{\Delta}{q^2+\Delta^2}\dot{q}\,(\rho_{gg}-\tfrac{1}{2})\,e^{-i\int_0^t E(t')dt'/\hbar} - \frac{1}{2}\Gamma_\Sigma\rho_{ge}$$

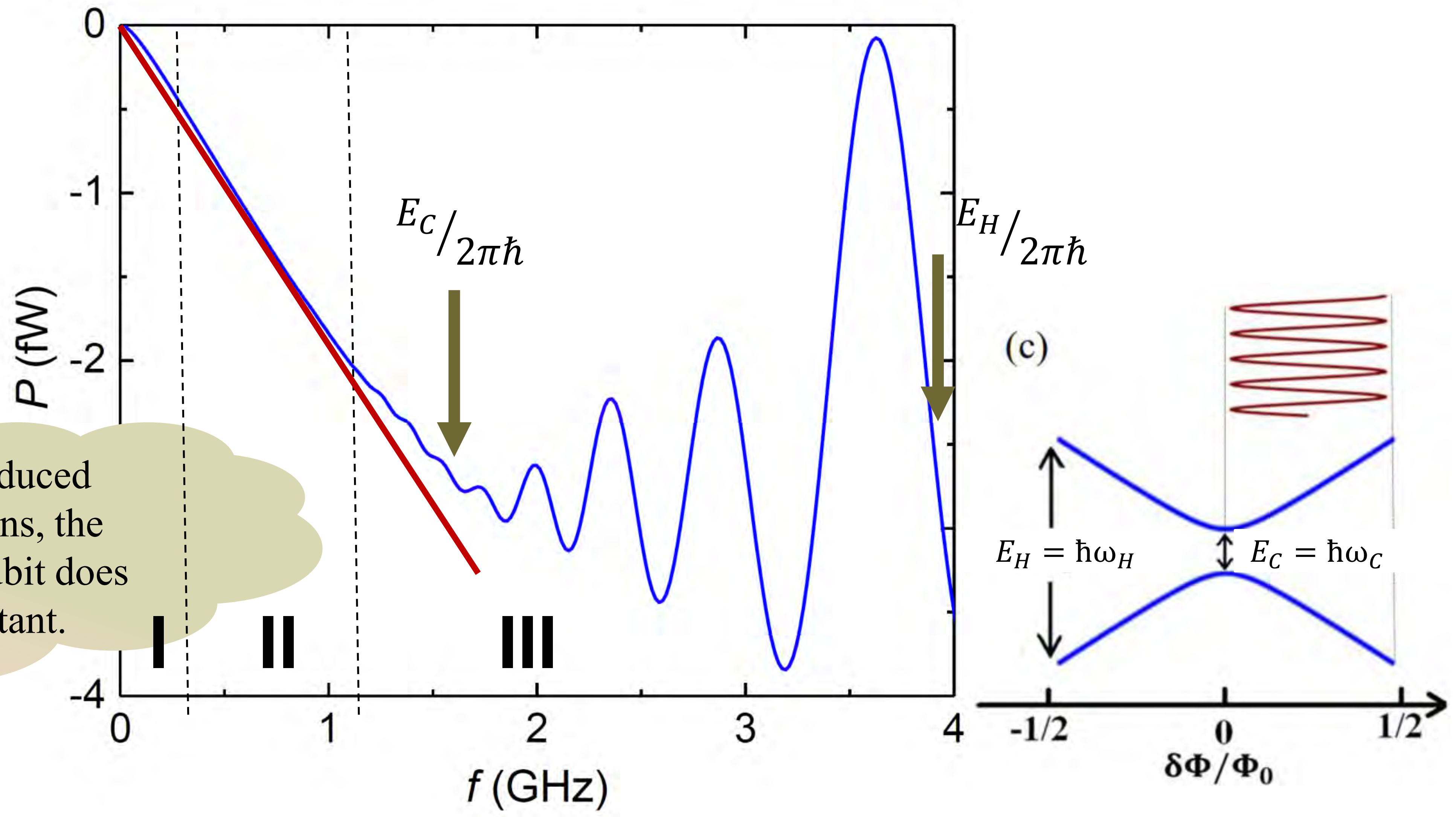
The power to the resistor j from the qubit

$$P_j = E(t)(\rho_{ee}\Gamma_{\downarrow,j} - \rho_{gg}\Gamma_{\uparrow,j})$$

[1]J.P.Pekola, D.S. Golubev, and D.A. Averin, Maxwell's demon based on a single qubit. PRB. 93,024501 (2016)

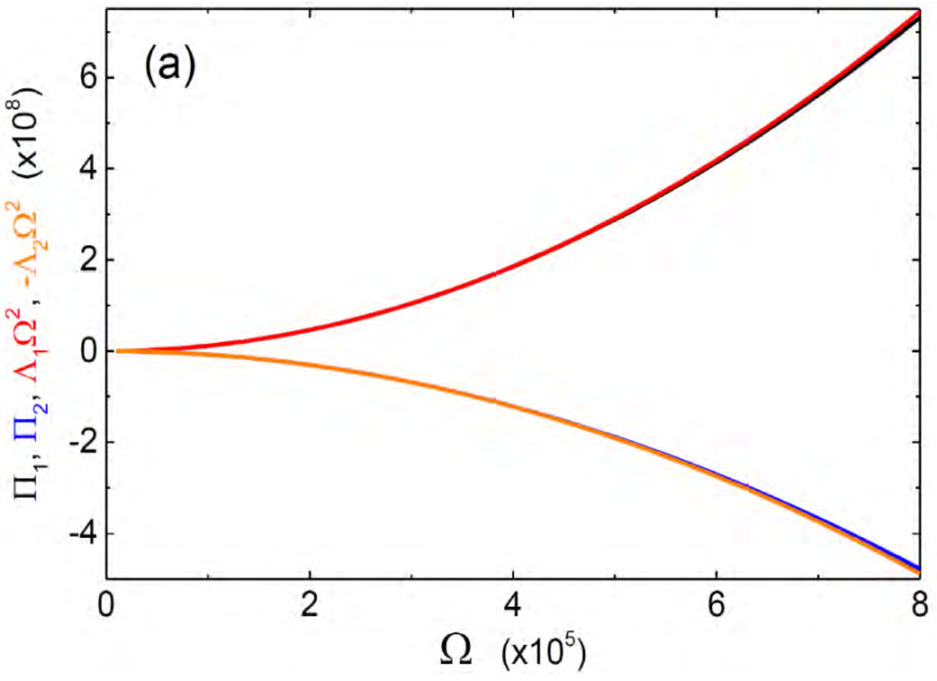
Different operation regimes

Coherent oscillations of heat current at high frequencies



Due to driving-induced coherent oscillations, the population of the qubit does not remain constant.

Nearly adiabatic regime (at very low frequencies)



$$\Pi_j^{(2)} = \Lambda_j \Omega^2$$

1. Classical rate equation: $\dot{\rho}_{gg} = -\Gamma_\Sigma \rho_{gg} + \Gamma_\downarrow$

$$\Lambda_{j,\mathrm{CL}} = -\frac{1}{\pi}\int_0^{2\pi} du \sqrt{q^2+\Delta^2}\left(\frac{\frac{d^2\rho_{\mathrm{eq},gg}}{du^2}}{\xi_\Sigma^2} - \frac{\left(\frac{d\rho_{\mathrm{eq},gg}}{du}\right)\left(\frac{d\xi_\Sigma}{du}\right)}{\xi_\Sigma^3}\right)\xi_{\Sigma,j}$$

2. Full (quantum) master equation:

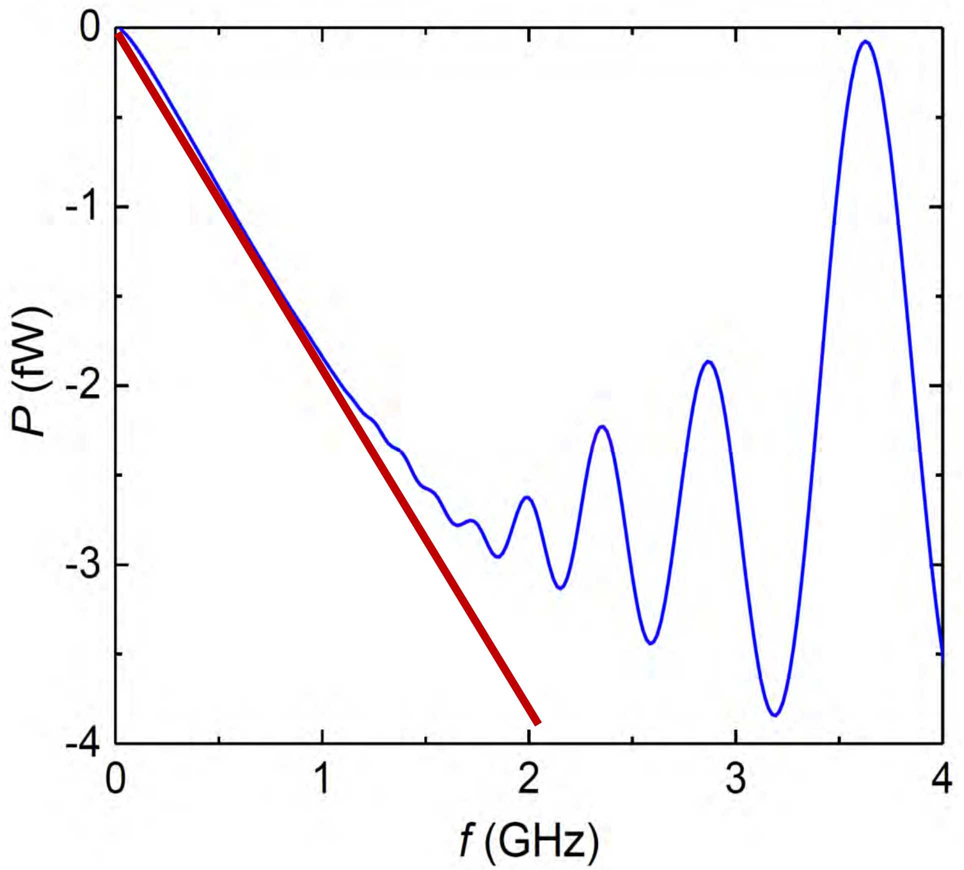
$$\Lambda_j = \Lambda_{j,\mathrm{CL}} + \delta\Lambda_{j,\mathrm{Q}}$$

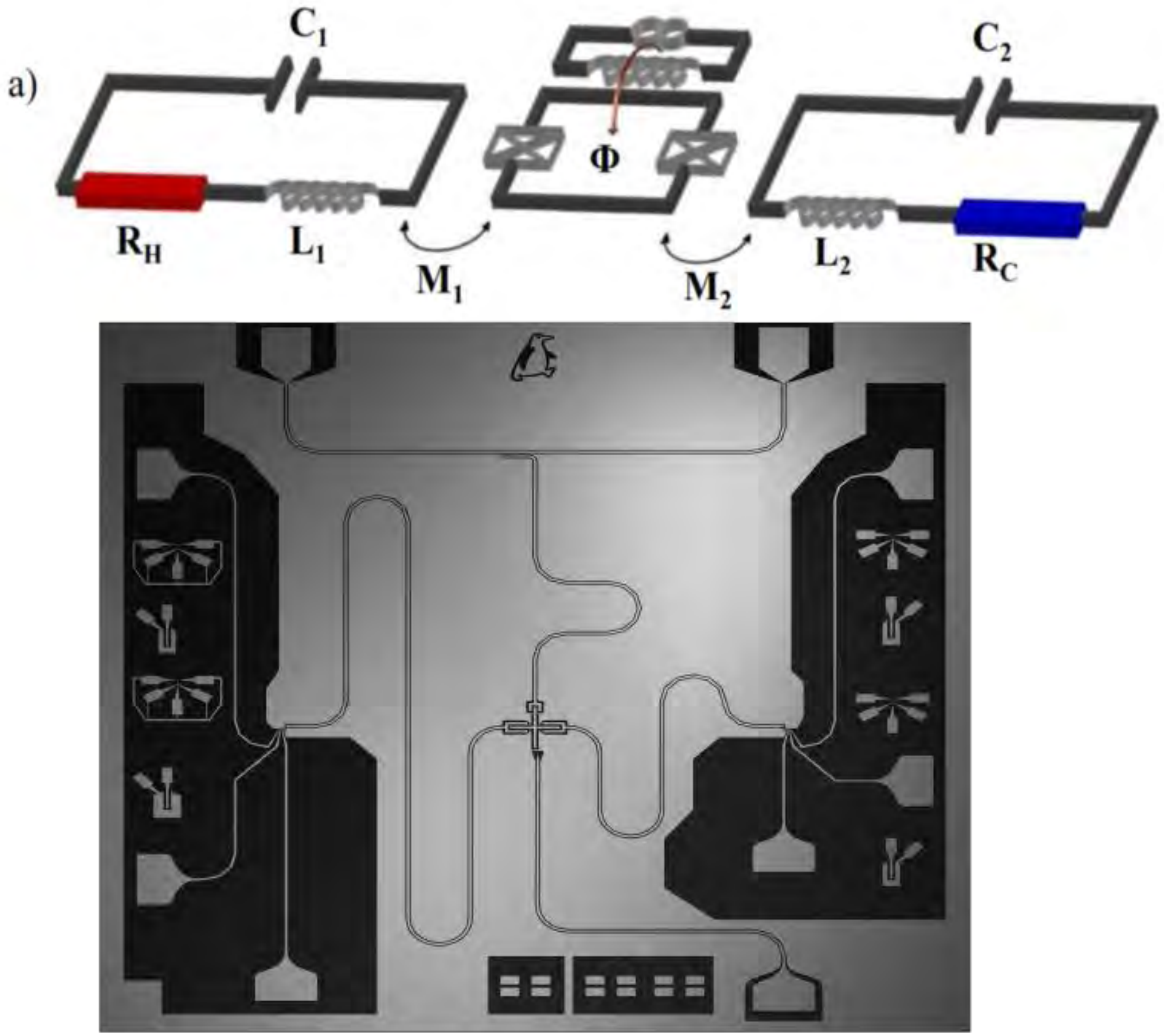
$$\delta\Lambda_{j,\mathrm{Q}} = \frac{1}{\pi}\int_0^{2\pi} du \frac{\Delta^2}{(q^2+\Delta^2)^{3/2}}\left(\frac{dq}{du}\right)^2 \frac{(\xi_\downarrow - \xi_\uparrow)\xi_{\Sigma,j}}{\xi_\Sigma[\xi_\Sigma^2 + 16(q^2+\Delta^2)]} > 0$$

Quantum coherence degrades the performance of the refrigerator

Quantum Otto refrigerator

$$P = -\frac{\hbar\omega_2}{2}\left[\tanh\left(\frac{\beta_1\hbar\omega_1}{2}\right) - \tanh\left(\frac{\beta_2\hbar\omega_2}{2}\right)\right] f$$



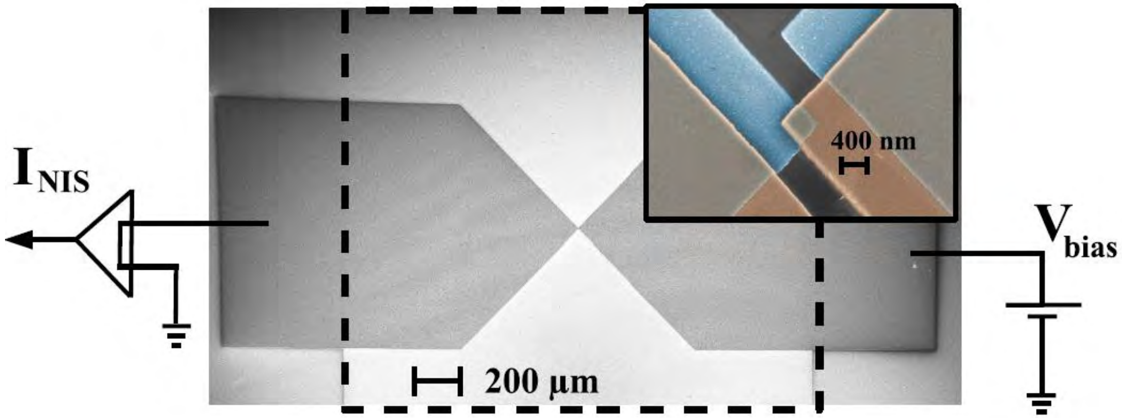


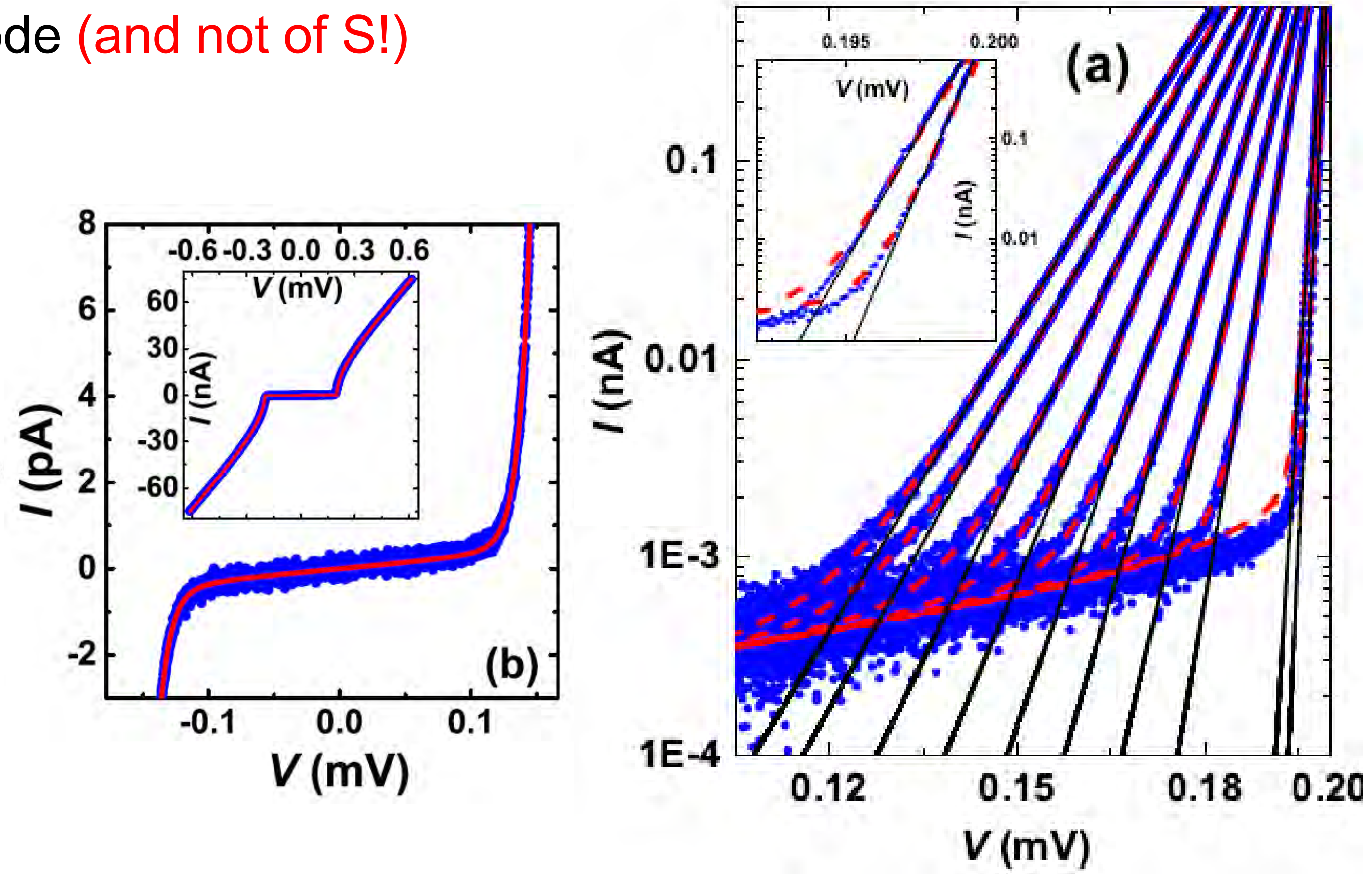
Expect about 1 fW cooling power at 1 GHz driving frequency

NIS-thermometry

$$I = \frac{1}{2eR_T} \int n_S(E)[f_N(E - eV) - f_N(E + eV)]dE$$

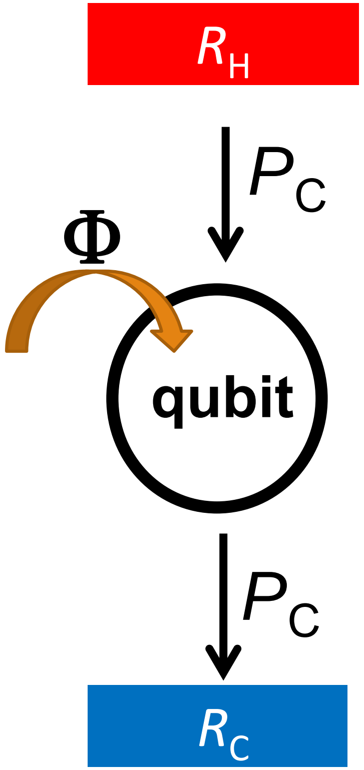
Probes electron temperature of N electrode (and not of S!)

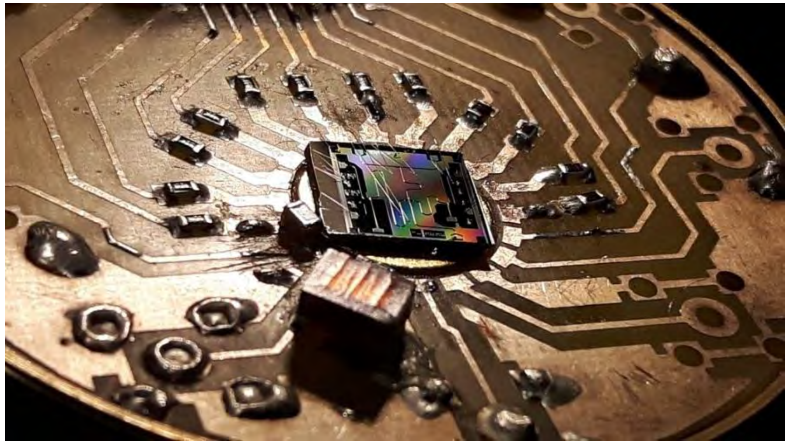


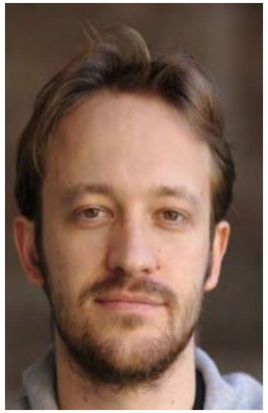


Phys. Rev. Appl. 4, 034001 (2015).

Experiment on quantum heat switch







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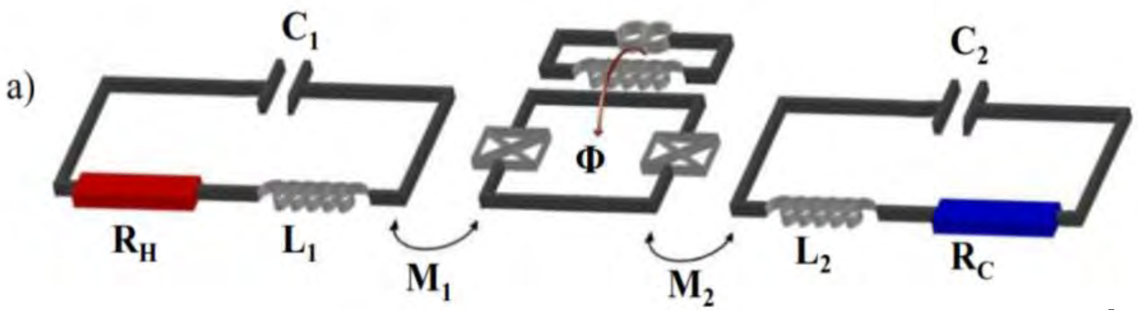


B. Karimi, J. Pekola, M. Campisi, and R. Fazio, Quantum Science and Technology **2**, 044007 (2017).

A. Ronzani, B. Karimi, J. Senior, Y. C.Chang, J. T. Peltonen, C. D. Chen, and J. P. Pekola, arxiv:1801.09312

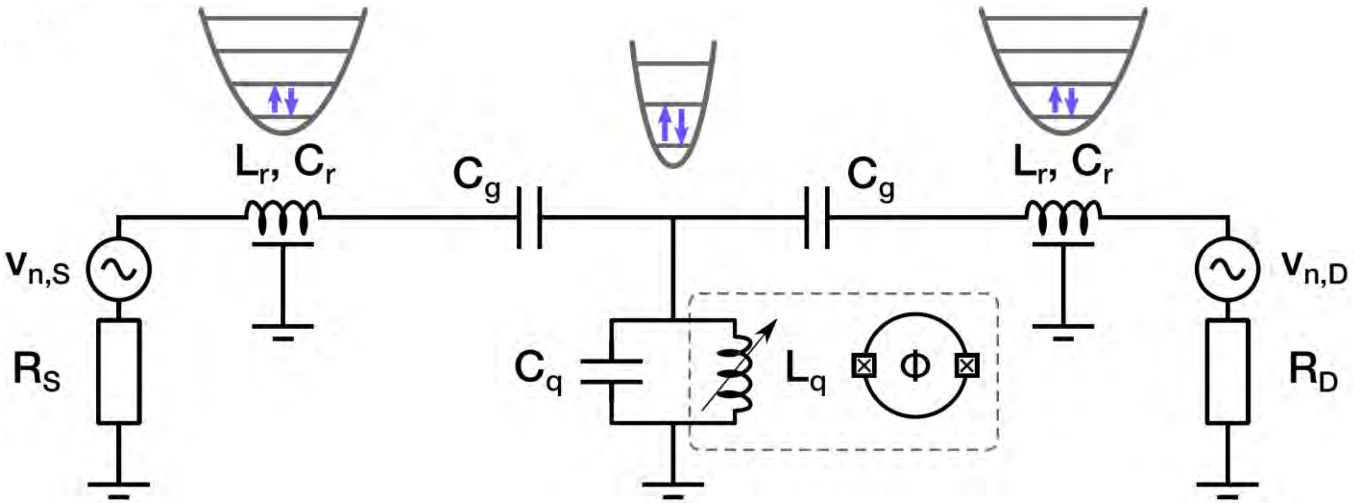
Experimental realization: Quantum heat switch[1]

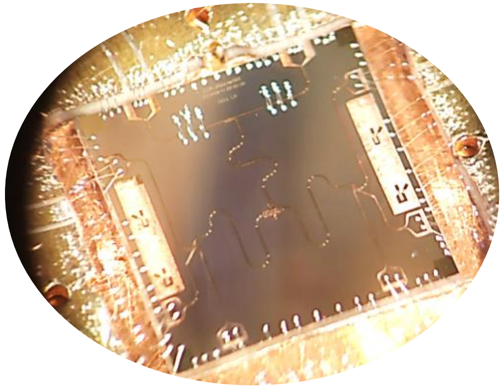
[1]A. Ronzani, B. Karimi, et al, Realisation of a quantum heat valve, arXiv: 1801.09312 (2018).

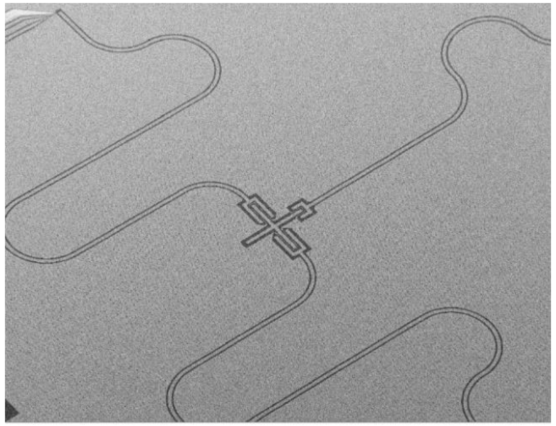
Schmidt et al., PRL 93, 045901 (2004)

Timofeev et al., PRL 102, 200801 (2009)

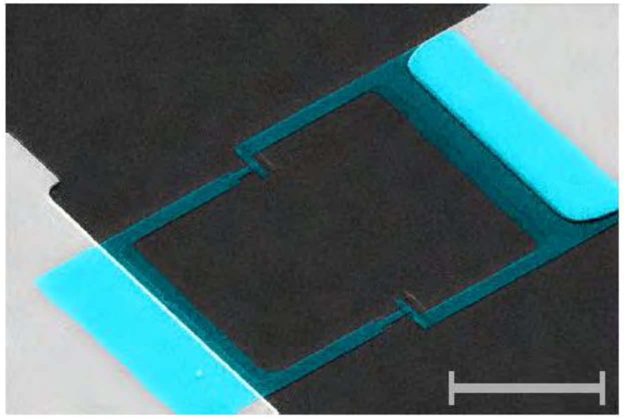
M. Partanen et al., Nature Physics 12, 460 (2016).





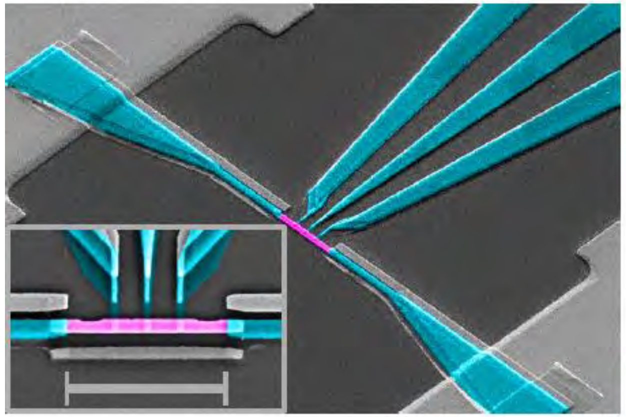


1 mm



10 μm

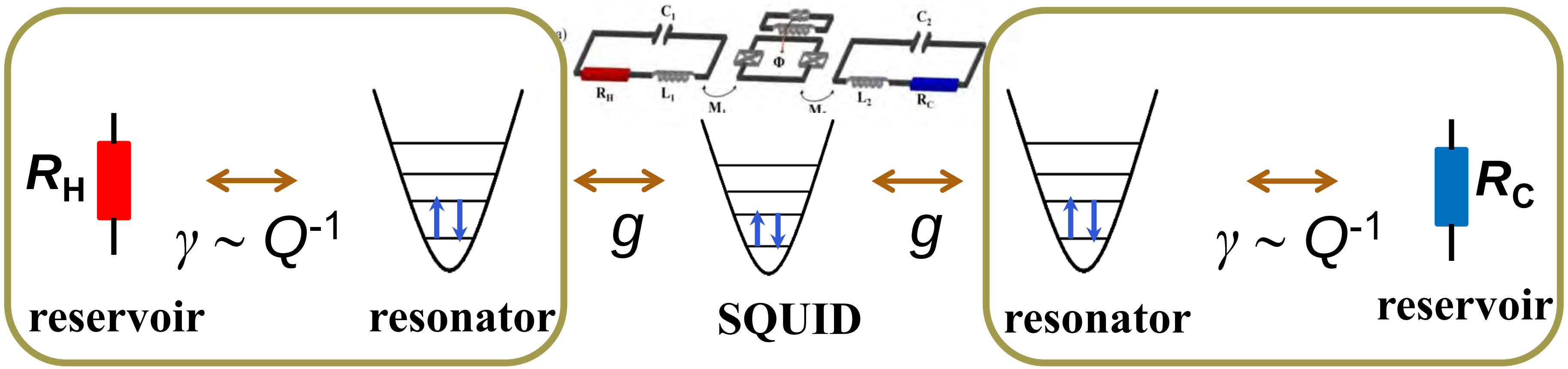
TRANSMON QUBIT



3 μm

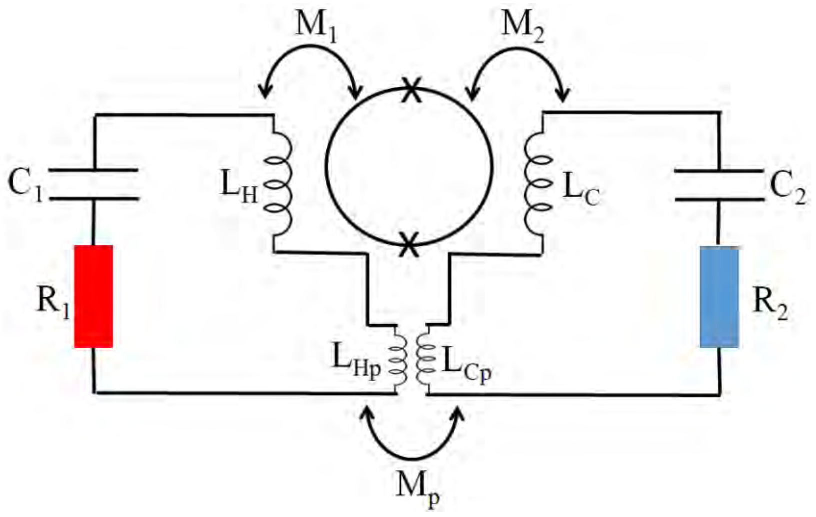
RESERVOIR AND THERMOMETERS

Theory vs. experiment: non-Hamiltonian



$gQ << 1$

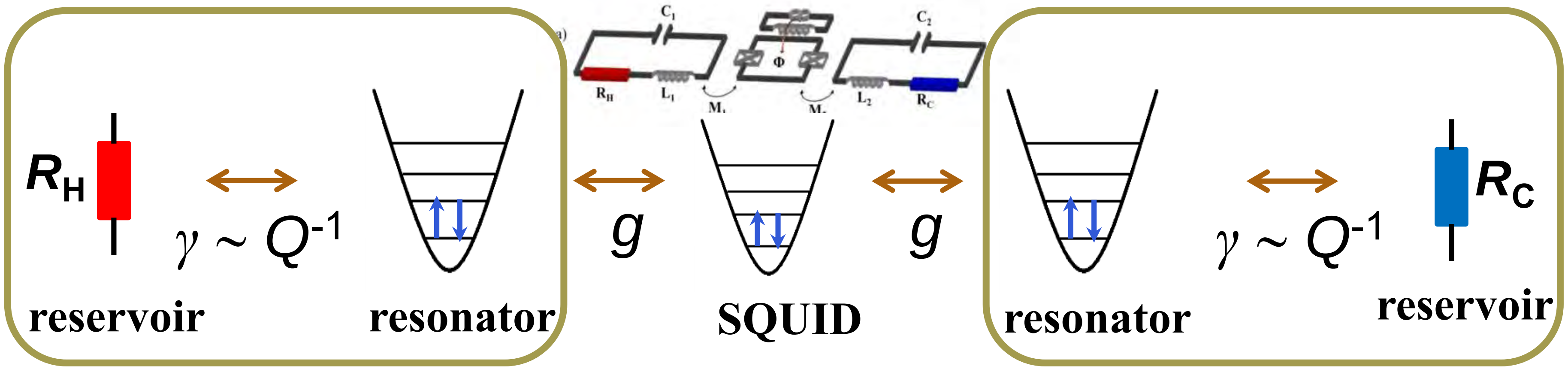
$$P_{\mathrm{D}} = \pi h g f_{\mathrm{r}}^2 \frac{n(\beta_{\mathrm{S}} h f_{\mathrm{q}}) - n(\beta_{\mathrm{D}} h f_{\mathrm{q}})}{[1+Q_{\mathrm{r}}^2(r-1/r)^2][\coth(\beta_{\mathrm{S}} h f_{\mathrm{q}}/2)+\coth(\beta_{\mathrm{D}} h f_{\mathrm{q}}/2)]} + \pi h \kappa f_{\mathrm{r}}^2 \int_0^\infty \frac{n(x\,\beta_{\mathrm{S}} h f_{\mathrm{r}}) - n(x\,\beta_{\mathrm{D}} h f_{\mathrm{r}})}{[1+Q_{\mathrm{r}}^2(x-1/x)^2]^2} x^3 \mathrm{d}x$$



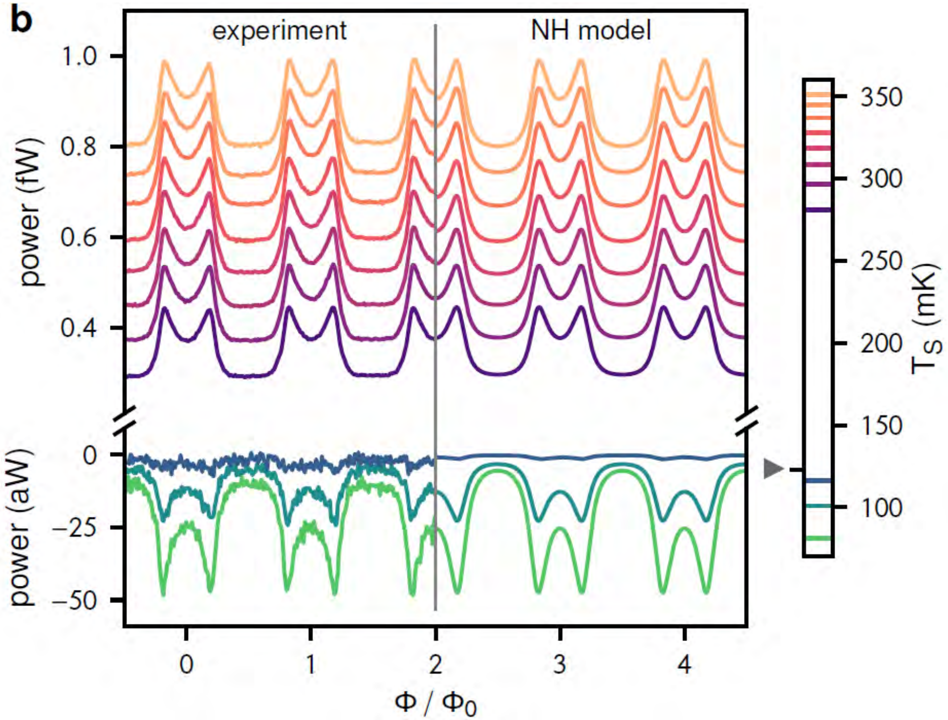
$$n(\beta_{\mathrm{S/D}} h f) = 1/(\exp(\beta_{\mathrm{S/D}} h f) - 1)$$

$$f_{\mathrm{q}} \equiv r f_{\mathrm{r}}$$

Theory vs. experiment: non-Hamiltonian

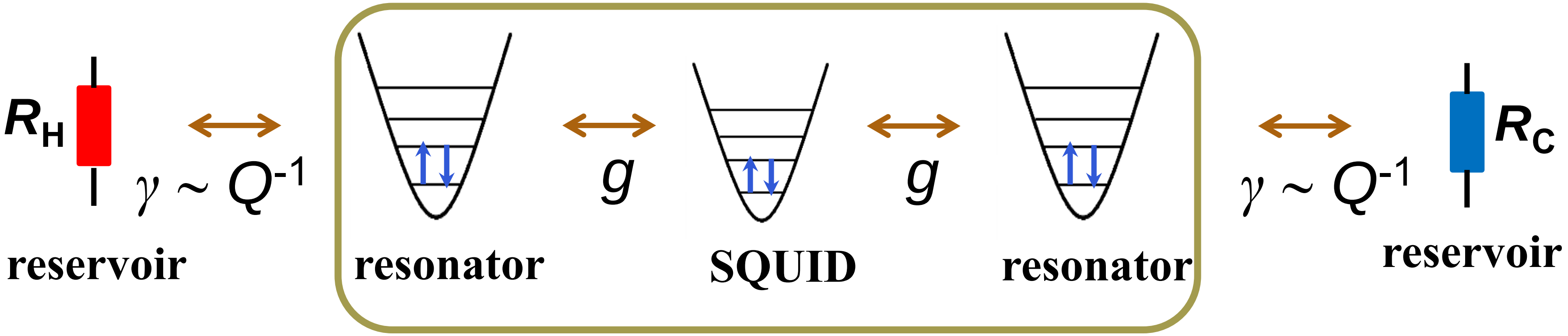


$gQ << 1$



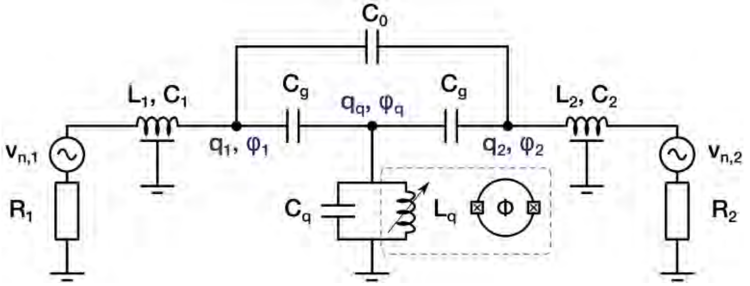
Cooling at distance of 4 mm by mw photons

Theory vs. experiment: quasi-Hamiltonian



$gQ \sim 1$

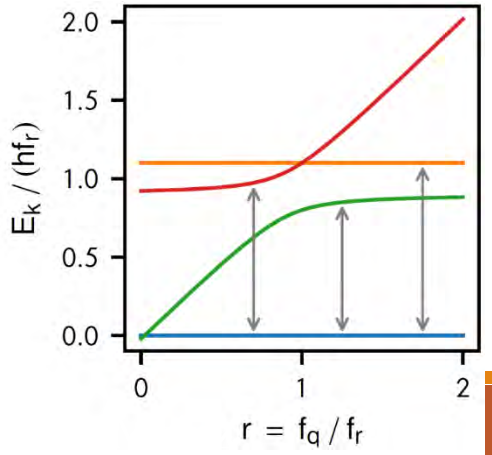
$$\mathfrak{L}(\varphi_1,\dot\varphi_1,\varphi_\mathrm{q},\dot\varphi_\mathrm{q},\varphi_2,\dot\varphi_2)=\frac{1}{2}\left(C_1\dot\varphi_1^2+C_\mathrm{g}(\dot\varphi_\mathrm{q}-\dot\varphi_1)^2+C_\mathrm{q}\dot\varphi_\mathrm{q}^2+C_\mathrm{g}(\dot\varphi_\mathrm{q}-\dot\varphi_2)^2+C_0(\dot\varphi_1-\dot\varphi_2)^2+C_2\dot\varphi_2^2\right)-\frac{1}{2}\left(\frac{\varphi_1^2}{L_1}+\frac{\varphi_\mathrm{q}^2}{L_\mathrm{q}}+\frac{\varphi_2^2}{L_2}\right)$$

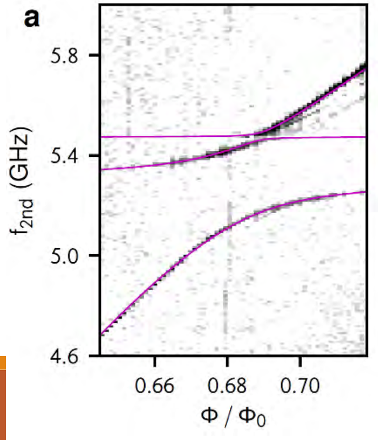


$$\hat q_\mathrm{i}=-i\sqrt{\frac{\hbar}{2Z_0}}(\hat a_\mathrm{i}-\hat a_\mathrm{i}^\dagger)\ \text{and}\ \hat q_\mathrm{q}=-i\sqrt{\frac{\hbar}{2Z_0}}(\hat b-\hat b^\dagger)$$

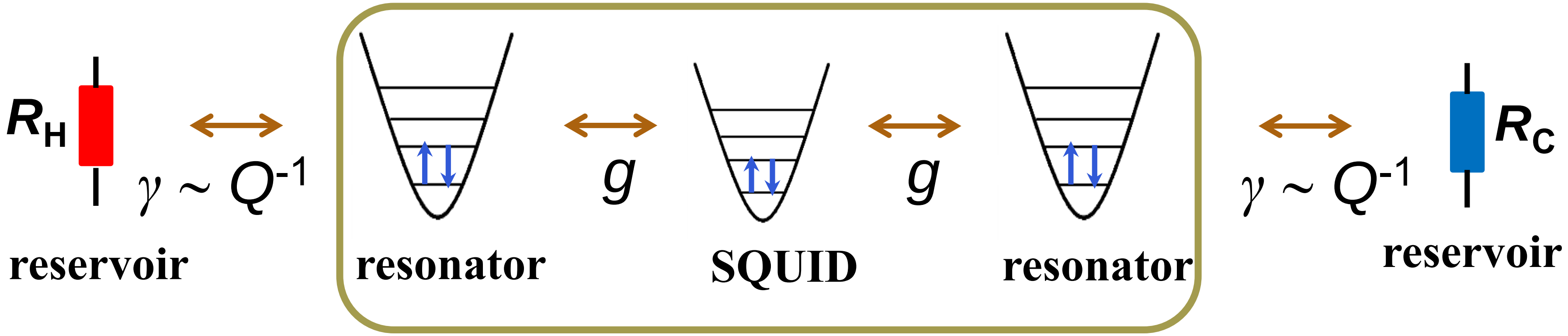
$$(hf_\mathrm{r})^{-1}\hat H=\hat a_1^\dagger\hat a_1+\hat a_2^\dagger\hat a_2+r\hat b^\dagger\hat b+g(\hat b\hat a_1^\dagger+\hat b^\dagger\hat a_1+\hat b\hat a_2^\dagger+\hat b^\dagger\hat a_2)+\tilde g(\hat a_1\hat a_2^\dagger+\hat a_1^\dagger\hat a_2)$$

$$H=hf_r\begin{pmatrix}0&0&0&0\\0&1+a/2&g&\tilde g\\0&g&r&g\\0&\tilde g&g&1-a/2\end{pmatrix}$$





Theory vs. experiment: quasi-Hamiltonian

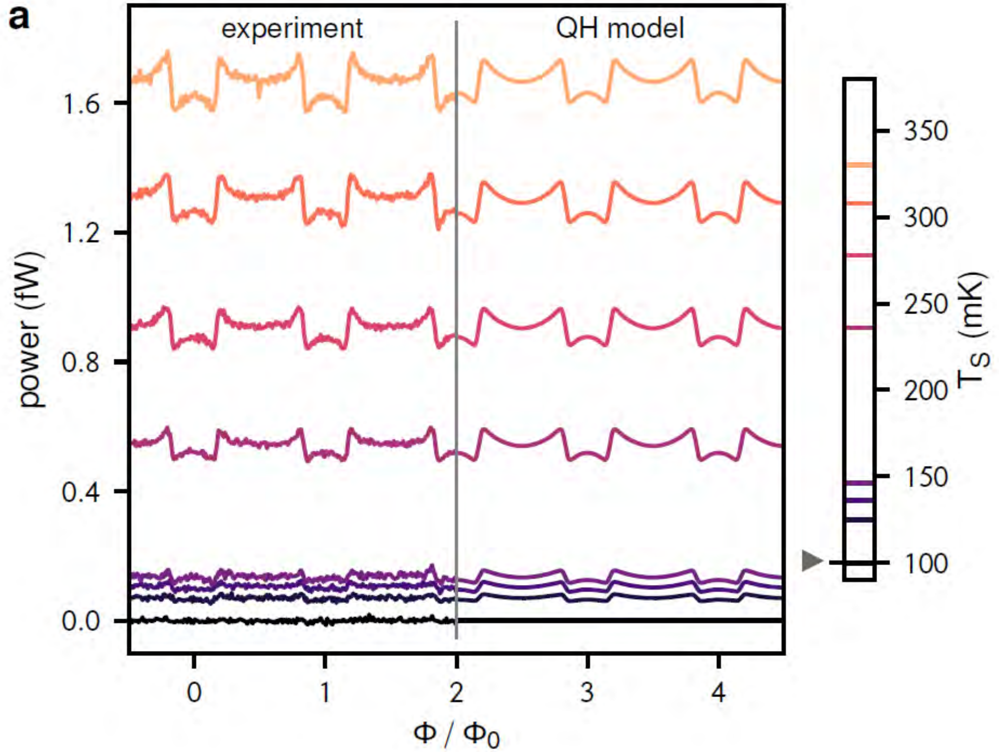


$gQ \sim 1$

$$S_v(f) = \frac{1}{1+Q^2(f/f_\mathrm{r} - f_\mathrm{r}/f)^2}\,\frac{2Rhf}{1-e^{-\beta hf}}$$

$$\Gamma_{k\to l,\mathrm{D}} = \frac{2\pi}{Q_\mathrm{D}}\,\frac{|\langle k|\hat{a}_\mathrm{D} - \hat{a}_\mathrm{D}^\dagger|l\rangle|^2}{1+Q_\mathrm{D}^2(\frac{f_{kl}}{f_\mathrm{r}} - \frac{f_\mathrm{r}}{f_{kl}})^2}\,\frac{f_{kl}}{1-e^{-\beta_\mathrm{D} h f_{kl}}};$$

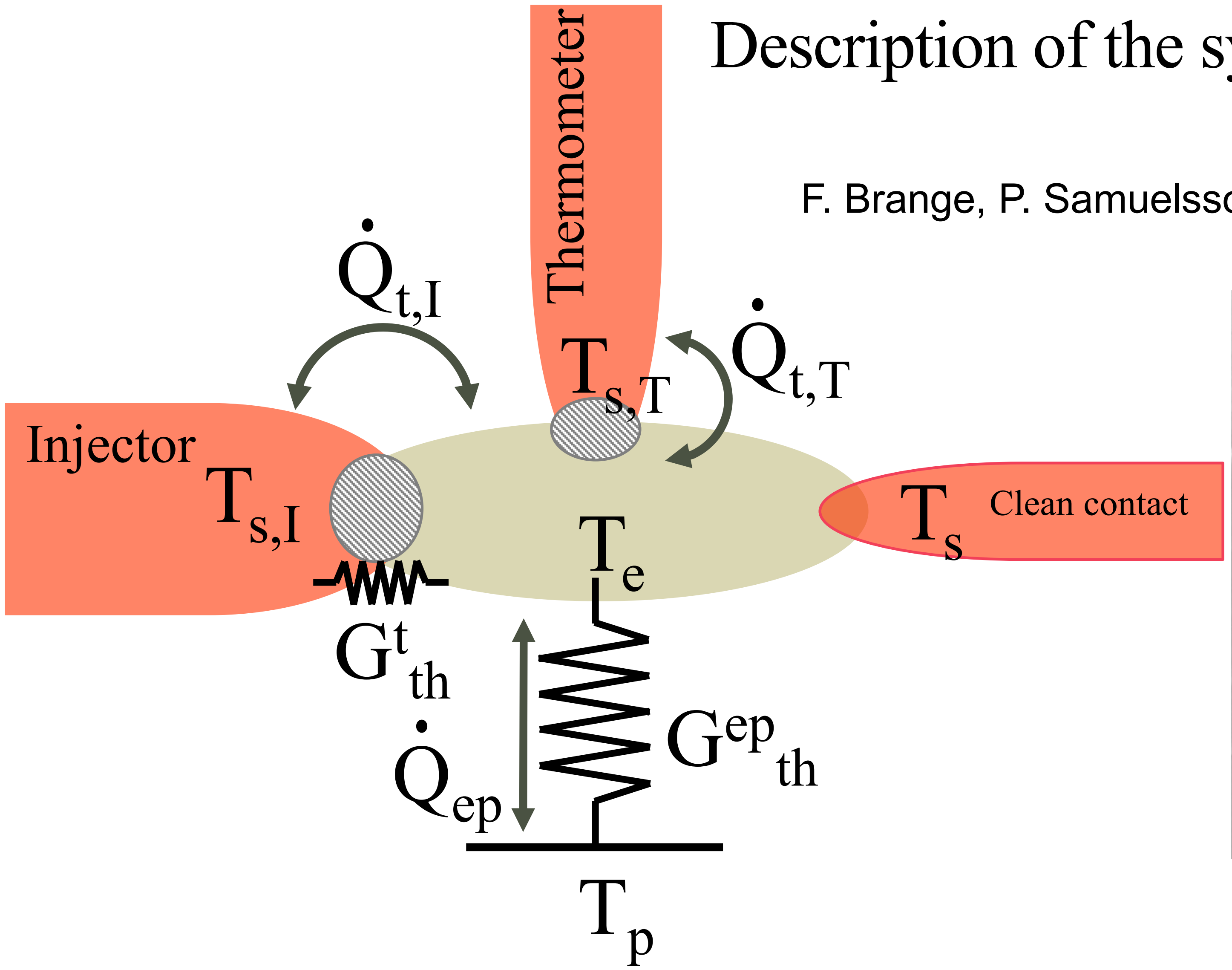
$$P_\mathrm{D} = \frac{2\pi h f_\mathrm{r}^2}{Q_\mathrm{r}} \sum_{k,l} \frac{|\langle k|\hat{a}_\mathrm{D} - \hat{a}_\mathrm{D}^\dagger|l\rangle|^2}{1+Q_\mathrm{r}^2(\frac{f_{kl}}{f_\mathrm{r}} - \frac{f_\mathrm{r}}{f_{kl}})^2}\,\frac{(E_{kl}/hf_r)^2}{1-e^{-\beta_\mathrm{D} E_{kl}}}\rho_{kk}$$

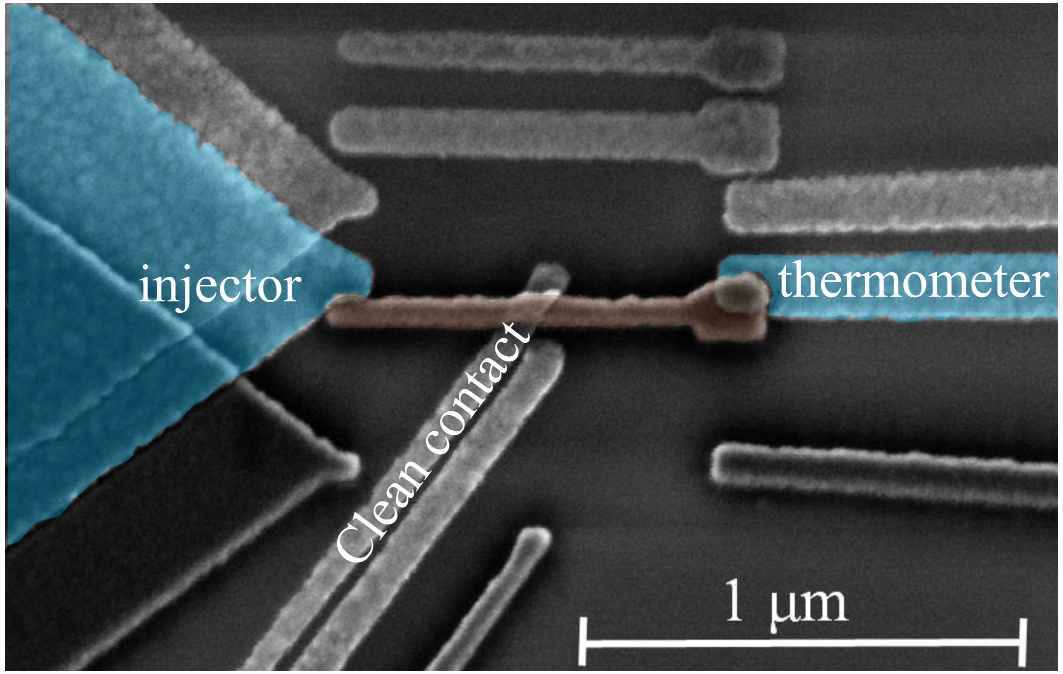


Theoretical estimation of heat current noise of a small metallic island

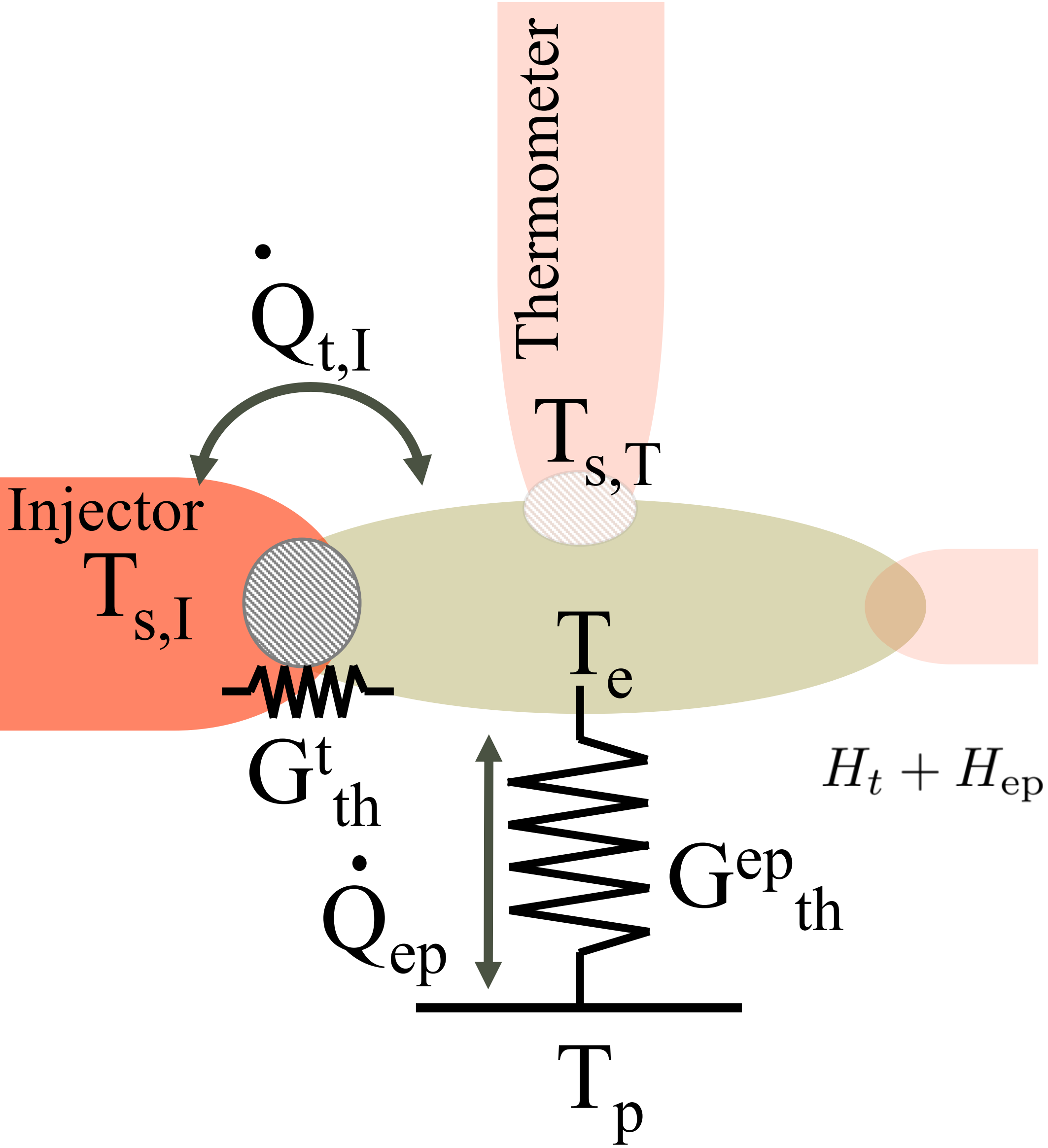
Description of the system

F. Brange, P. Samuelsson, B. Karimi, and J. P. Pekola, arXiv:1805.02728





Description of the system



- Hamiltonian of the system

$$H = H_e + H_s + H_p + H_{ep} + H_t$$

- The unperturbed Hamiltonian $H_0 = H_e + H_s + H_p$

$$H_0 = \sum_e \epsilon_e a_e^\dagger a_e + \sum_s \epsilon_s a_s^\dagger a_s + \sum_q \hbar\omega_p c_p^\dagger c_p$$

- Considering weak coupling

$$H_t + H_{\text{ep}} = \sum_{e,s}(t_{es} a_e^\dagger a_s + t_{se} a_s^\dagger a_e) + \gamma \sum_{e,p} \omega_p^{1/2}(a_e^\dagger a_{e-p} c_p + a_{e-p}^\dagger a_e c_p^\dagger)$$

Electron-phonon coupling to the bath

- The operator of heat flux from the electron system to phonons due to ep coupling

$$\dot{H}_{ep} = \frac{i}{\hbar}[H_{\mathrm{ep}}, H_p] = i\gamma \sum_{k,q} \omega_q^{3/2}(a_k^\dagger a_{k-q} c_q - a_{k-q}^\dagger a_k c_q^\dagger)$$

  - Heat current into the phonon bath and thermal conductance of the ep coupling

$$\dot{Q}_{ep} = \Sigma\mathcal{V}(T_e^5 - T_p^5)$$

$$G_{th}^{ep} = 5\Sigma\Omega T_{\mathrm{e}}^4$$

F. C. Wellstood, C. Urbina, and John Clarke, Phys. Rev. B **49**, 5942 (1994)

- Spectral density of noise due to ep coupling

$$S_{\dot{Q}_{ep}}(\omega) = \frac{\Sigma\mathcal{V}}{96\zeta(5)k_B^5} \int_0^\infty d\epsilon\, \epsilon^2 \Big[(2\epsilon - \hbar\omega)^2 \frac{1}{1 - e^{-\beta_p \epsilon}} \frac{\epsilon - \hbar\omega}{e^{\beta_e(\epsilon - \hbar\omega)} - 1} + (2\epsilon + \hbar\omega)^2 \frac{1}{e^{\beta_p \epsilon} - 1} \frac{\epsilon + \hbar\omega}{1 - e^{-\beta_e(\epsilon + \hbar\omega)}}\Big]$$

[1]**J. P. Pekola and B. Karimi, Quantum noise of electron–phonon heat current, J. Low Temp. Phys. doi.org/10.1007/s10909-018-1854-y**

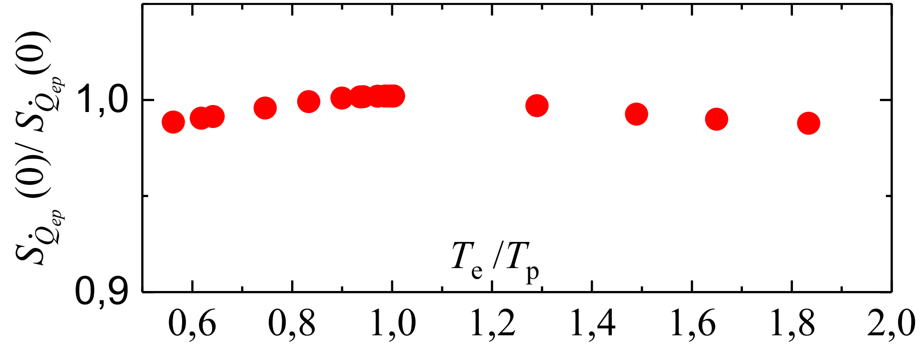
Electron-phonon coupling to the bath

- Spectral density of noise due to ep coupling

$$S_{\dot{Q}_{ep}}(\omega) = \frac{\Sigma\mathcal{V}}{96\zeta(5)k_B^5}\int_0^\infty d\epsilon\,\epsilon^2\Big[(2\epsilon-\hbar\omega)^2\frac{1}{1-e^{-\beta_p\epsilon}}\;\frac{\epsilon-\hbar\omega}{e^{\beta_e(\epsilon-\hbar\omega)}-1}$$
$$+(2\epsilon+\hbar\omega)^2\frac{1}{e^{\beta_p\epsilon}-1}\;\frac{\epsilon+\hbar\omega}{1-e^{-\beta_e(\epsilon+\hbar\omega)}}\Big]$$

$$S_{\dot{Q}_{ep}}(0) \approx 5\Sigma\mathcal{V}k_\mathrm{B}(T_\mathrm{e}^6+T_\mathrm{p}^6)$$

FDT ⟶ $S_{\dot{Q}_{ep}}(0) = 2k_\mathrm{B}T^2G_{ep}^{th}$



- Non-vanishing noise at zero temperature

$$S_{\dot{Q}_{ep}}(\omega) = \frac{\Sigma\mathcal{V}}{96\zeta(5)k_B^5}\,\frac{(\hbar\omega)^6}{60}$$

Tunneling

- The operator of heat flux from the superconductor to electrons system due to tunneling

$$\dot{H}_{et} = \frac{i}{\hbar}[H_t, H_e] = \frac{i}{\hbar}\sum_{k,l}\epsilon_k[t_{lk}b_l^\dagger a_k - t_{lk}^* b_l a_k^\dagger]$$

- Heat current into the phonon bath and thermal conductance of the tunneling

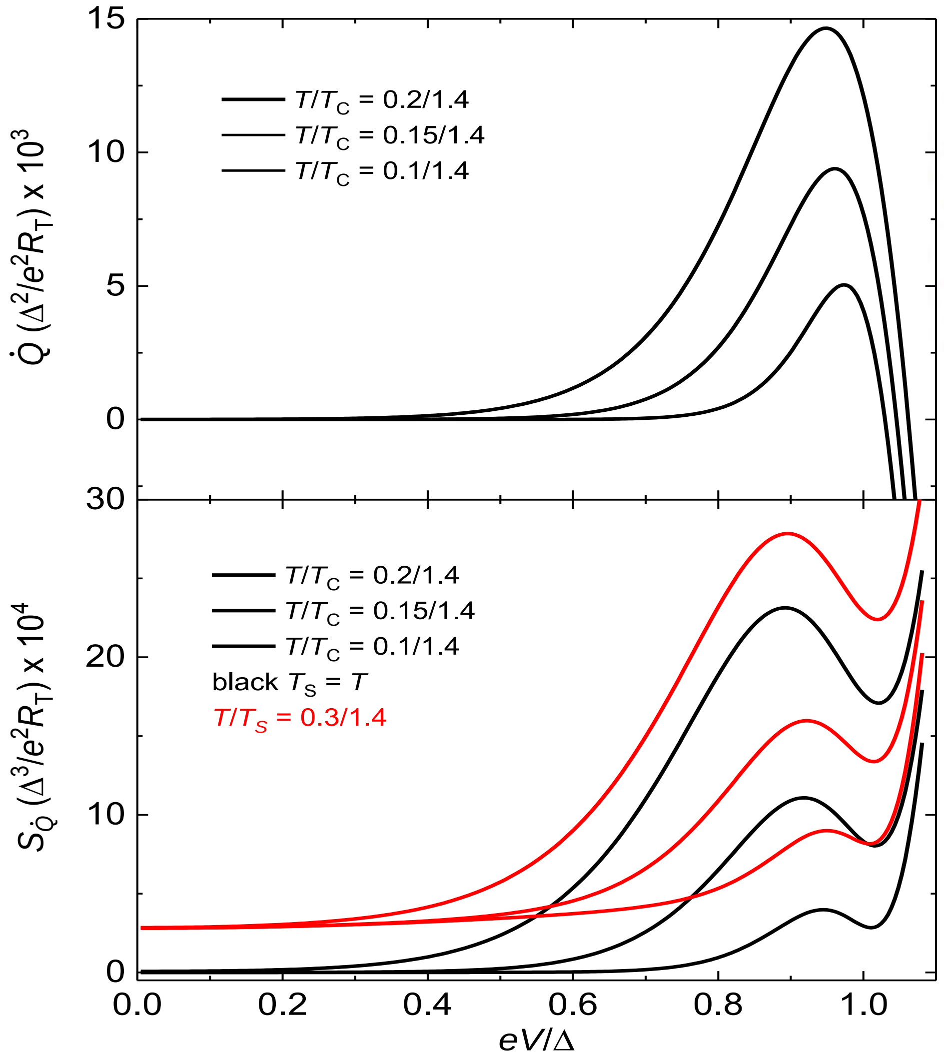
$$\dot{Q}_t = \frac{\Delta^2}{e^2 R_T}\int du\; n_S(u)\,(u-v)[f_N(u-v) - f_S(u)]$$

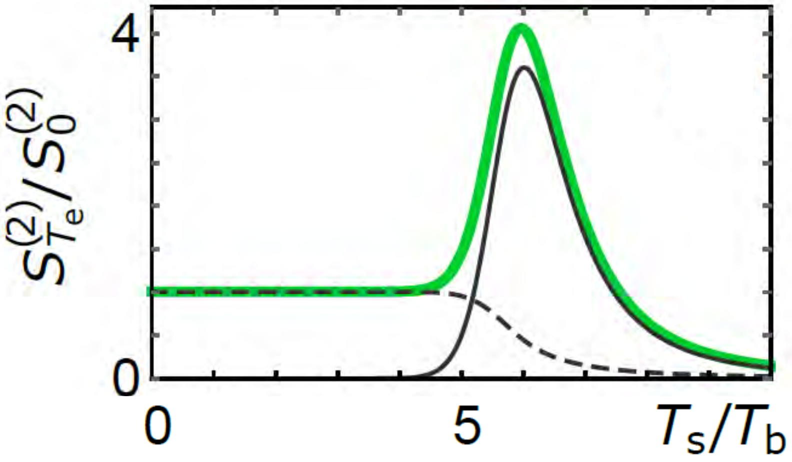
$$G_{th}^t = \frac{\Delta^3}{e^2 R_T k_\mathrm{B} T^2}\int du\; n_S(u)\; u^2 f(u)[1-f(u)]$$

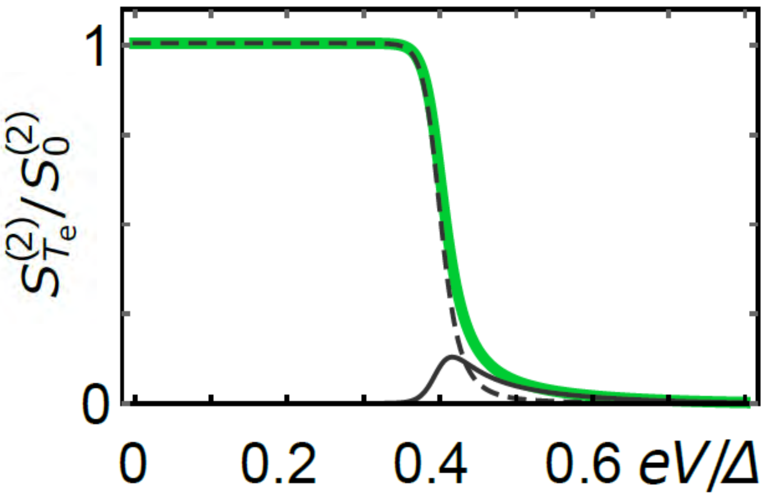
- Spectral density of noise due to tunneling $u = E/\Delta$ and $v = eV/\Delta$

$$S_{\dot{Q}_t}(0) = \frac{\Delta^3}{e^2 R_T}\int du\; n_S(u)\,(u-v)^2\{f_S(u)[1-f_N(u-v)] + f_N(u-v)[1-f_S(u)]\}$$

Cooling power and noise – numerical results



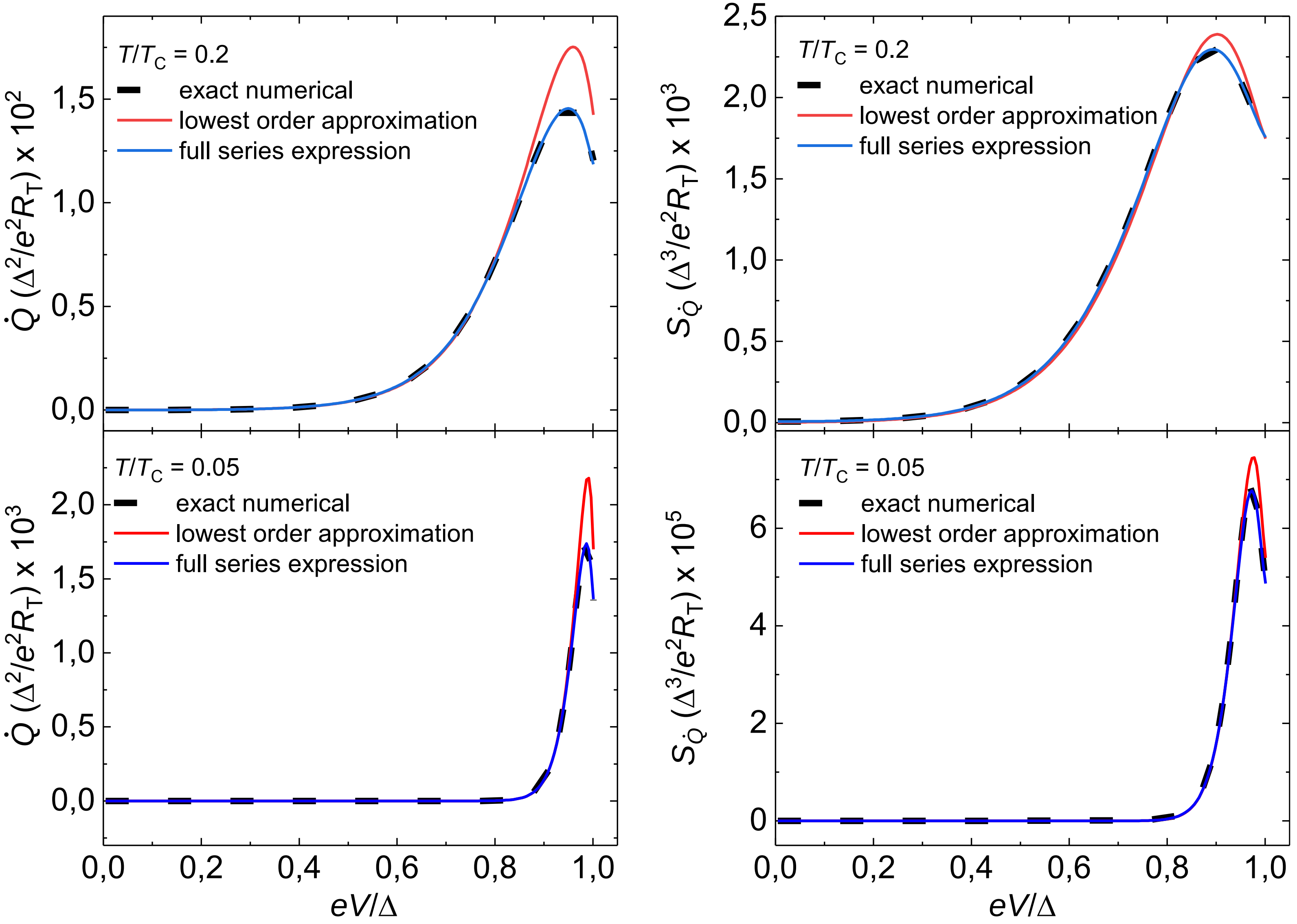




$$S_{T_e}(\omega) = \frac{S_{\dot{Q}_N}}{G_{th}^2 + \omega^2 C^2}$$

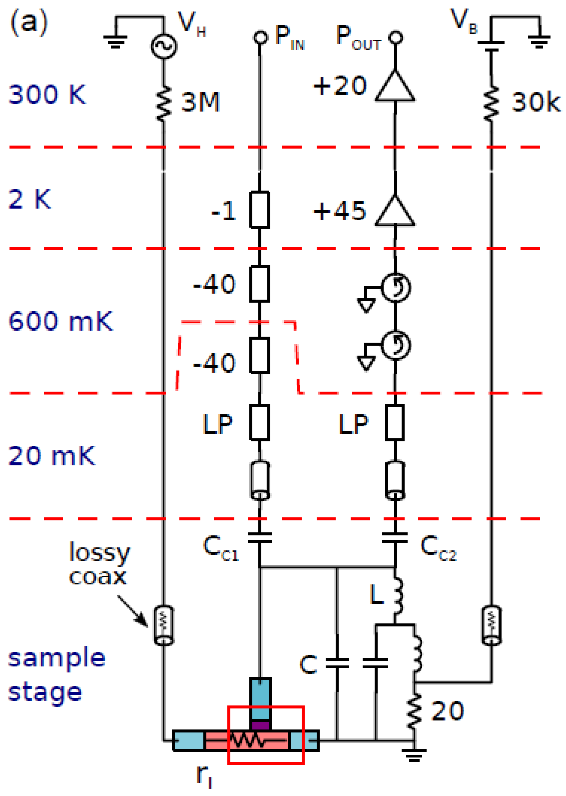
F. Brange, P. Samuelsson, B. Karimi, and J. P. Pekola, arXiv:1805.02728

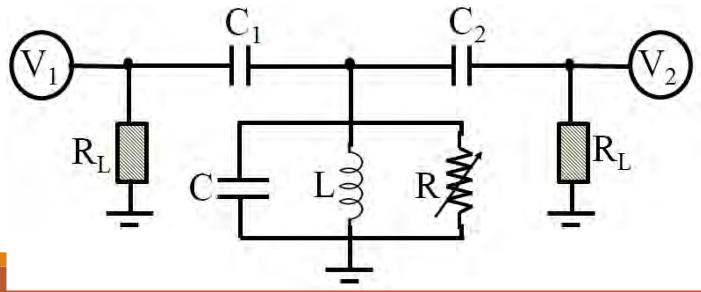
Analytical results vs. numerics

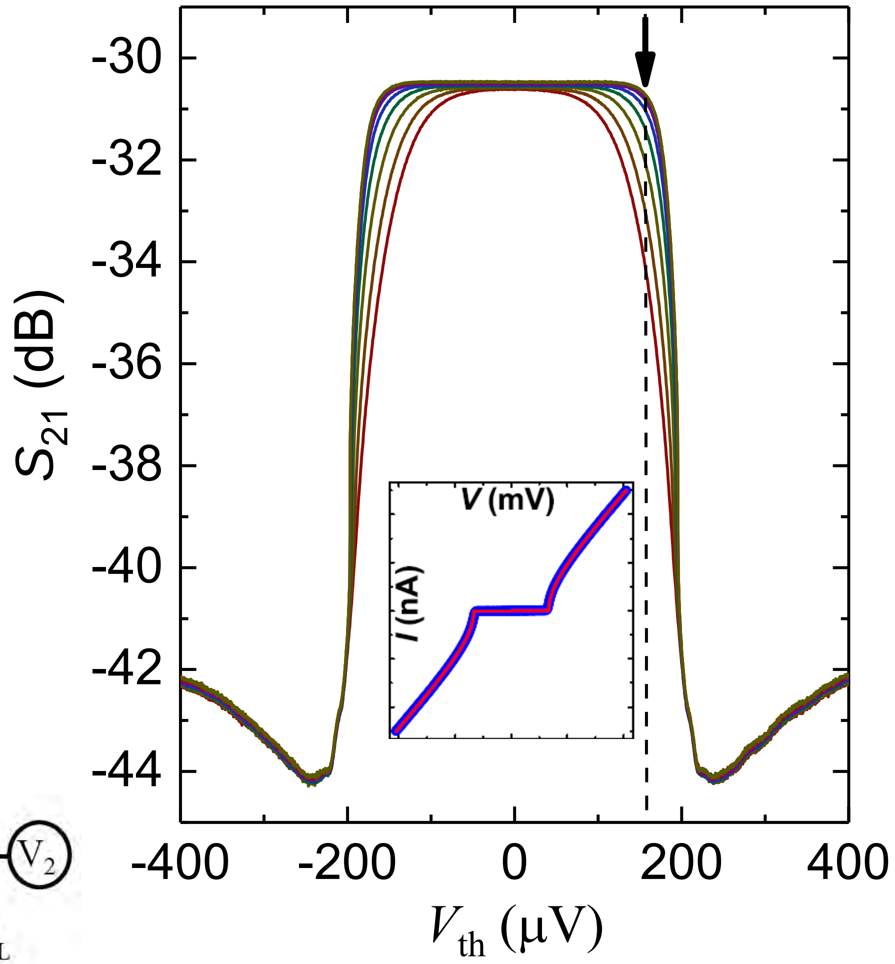


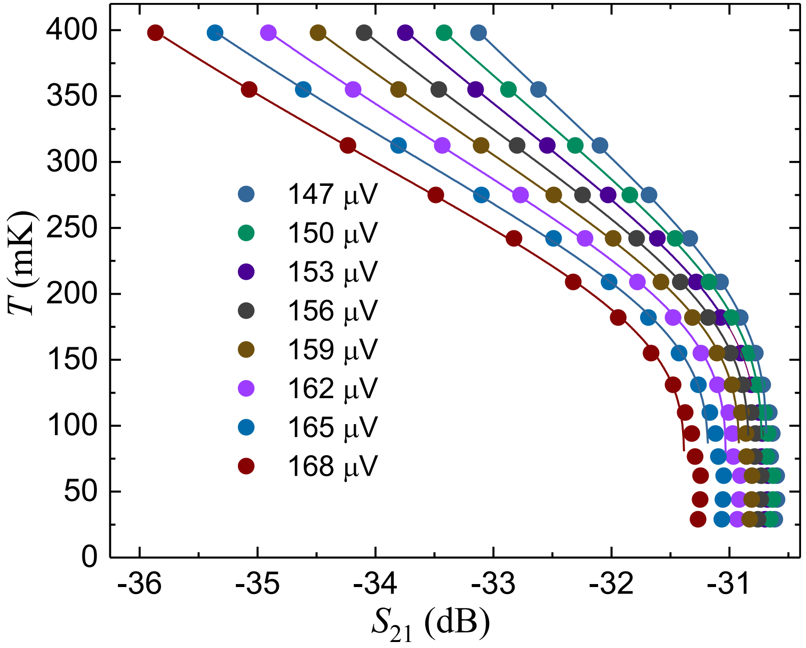
Fast NIS thermometry on electrons

Read-out at 600 MHz of a NIS junction, 10 MHz bandwidth





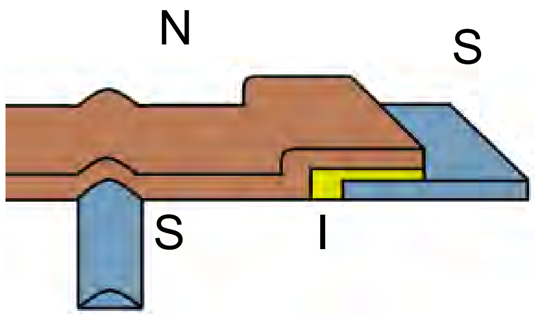




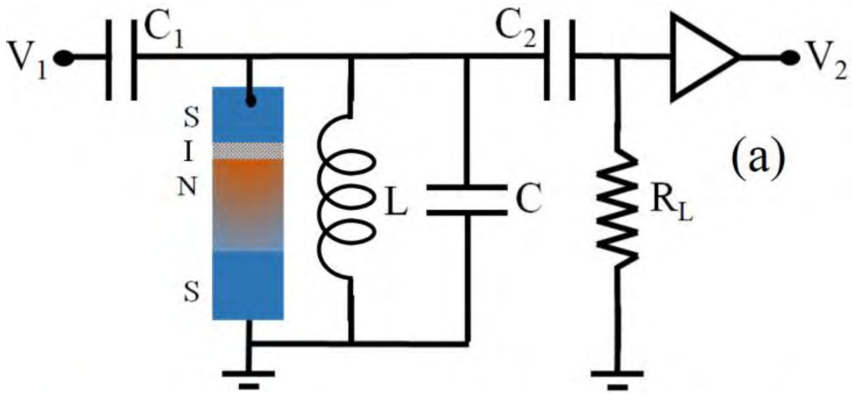
S. Gasparinetti et al., Phys. Rev. Applied 3, 014007 (2015);
B. Karimi and J. Pekola, in preparation

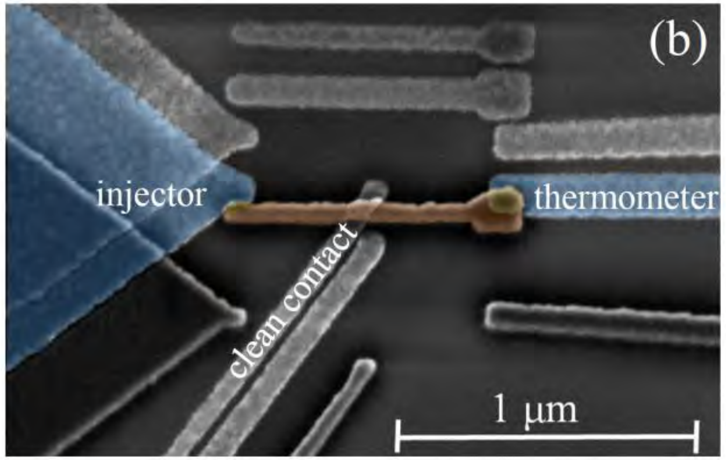
Proof of concept: D. Schmidt et al., Appl. Phys. Lett. 83, 1002 (2003).

ZBA based thermometry

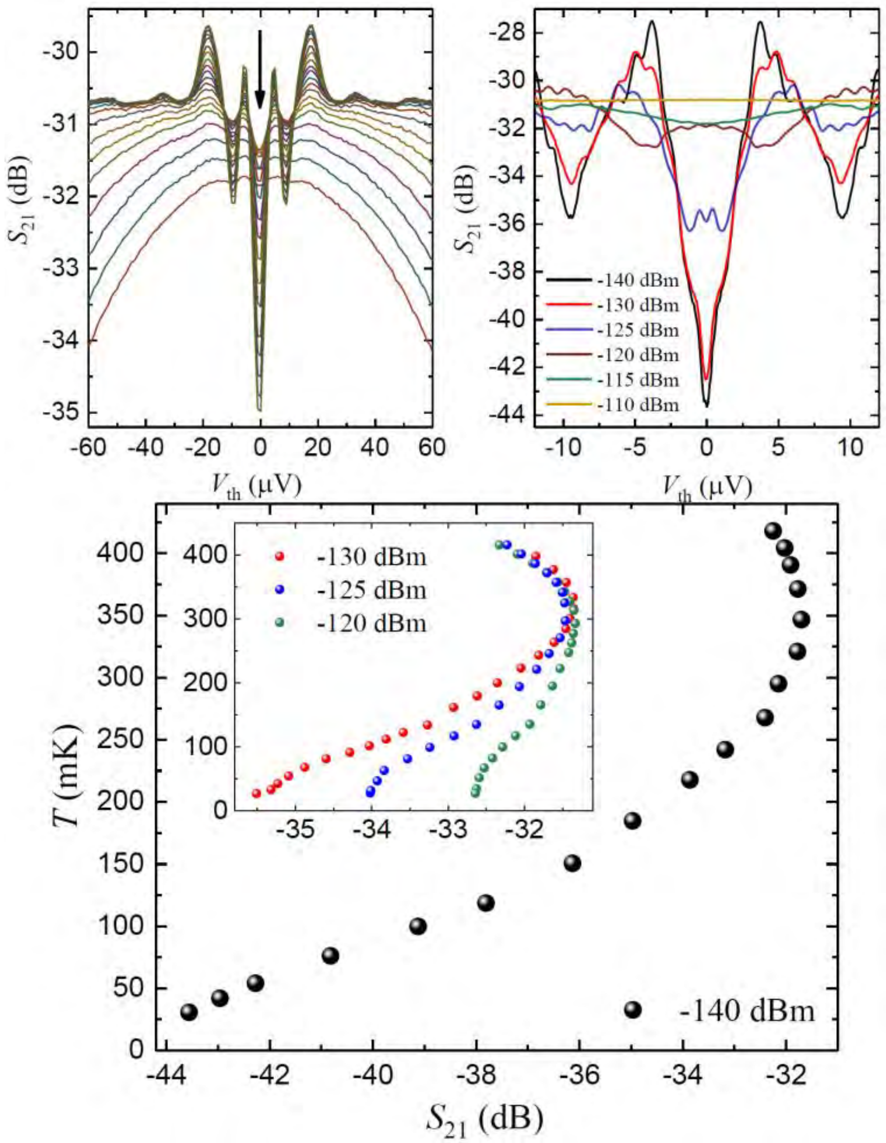


Proximity NIS junction





- non-invasive
- operates at low temperature

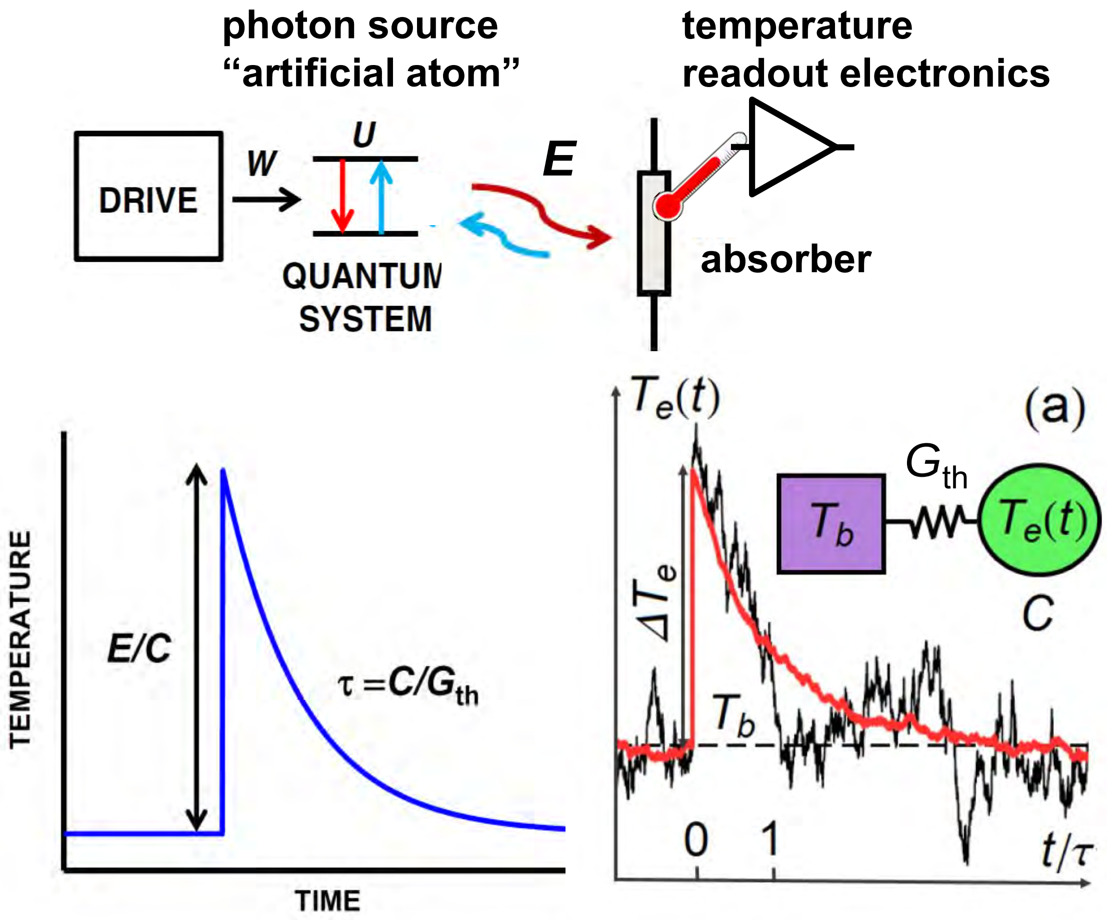


See also, O.-P. Saira et al., Phys. Rev. Appl. 6, 024005 (2016);
J. Govenius et al., PRL 117, 030802 (2016)

B. Karimi and J. Pekola, in preparation

Calorimetry for measuring mw photons

Requirements for calorimetry on single microwave quantum level:



Typical parameters

Operating temperature
$T = 0.03$ K

$E/k_B = 0.3...1$ K, $C = 300...1000k_B$

$\Delta T \sim 1...3$ mK, $\tau \sim 0.01...1$ ms

NET = 10 μK/(Hz)$^{1/2}$ is sufficient for single photon detection

$\delta E = \mathrm{NET}\,(C\,G_{th})^{1/2}$

J. Pekola, P. Solinas, A. Shnirman, and D. V. Averin., NJP **15**, 115006 (2013);
F. Brange, P. Samuelsson, B. Karimi, J. P. Pekola., arXiv:1805.2728.

Summary

- Measurement of heat and noise in circuits
- Presented quantum Otto refrigerator
- Quantum heat switch based on a superconducting qubit realized and analyzed; two regimes of operation observed and theoretically explained arxiv:1801.09312
- Non-invasive and fast thermometry down to 25mK demonstrated