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Non-Equilibrium Dynamics of Quantum Many-Body Systems:

Irreversibility and the Quantum Butterfly Effect

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Boltzmann vs. Loschmidt: Irreversible vs. reversible dynamics

How to reconcile the second law of thermodynamics/the arrow of time with microscopic time-reversal invariance?

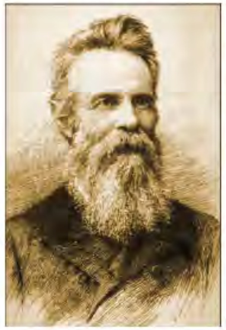
H-Theorem

$$\frac{dS}{dt} \geq 0$$

This provides an analytical proof of the Second Law [...]

[Boltzmann, 1872]

Loschmidt's paradox



Obviously, in every arbitrary system the course of events must become retrograde when the velocities of all its elements are reversed.

[Loschmidt, 1876]

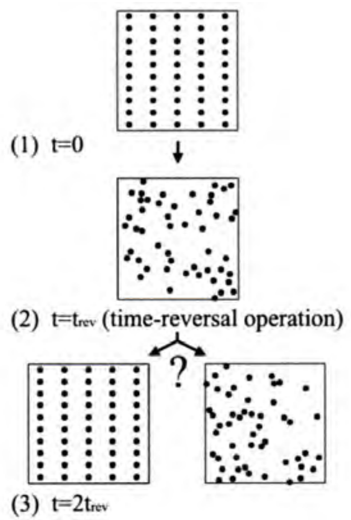
[Image source: Wikipedia]

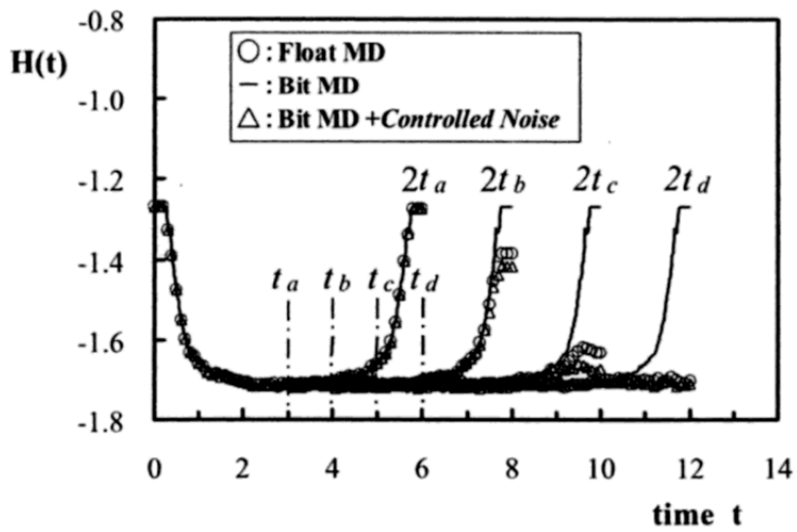
Boltzmann's reply: Then try to do it!

Thomson (1874):

If we allowed this equalization to proceed for a certain time, and then reversed the motions of all the molecules, we would observe a disequalization. However, if the number of molecules is very large, as it is in a gas, any slight deviation from absolute precision in the reversal will greatly shorten the time during which disequalization occurs.

N. Komatsu, T. Abe; Comp. Phys. Comm. 171 (2005)





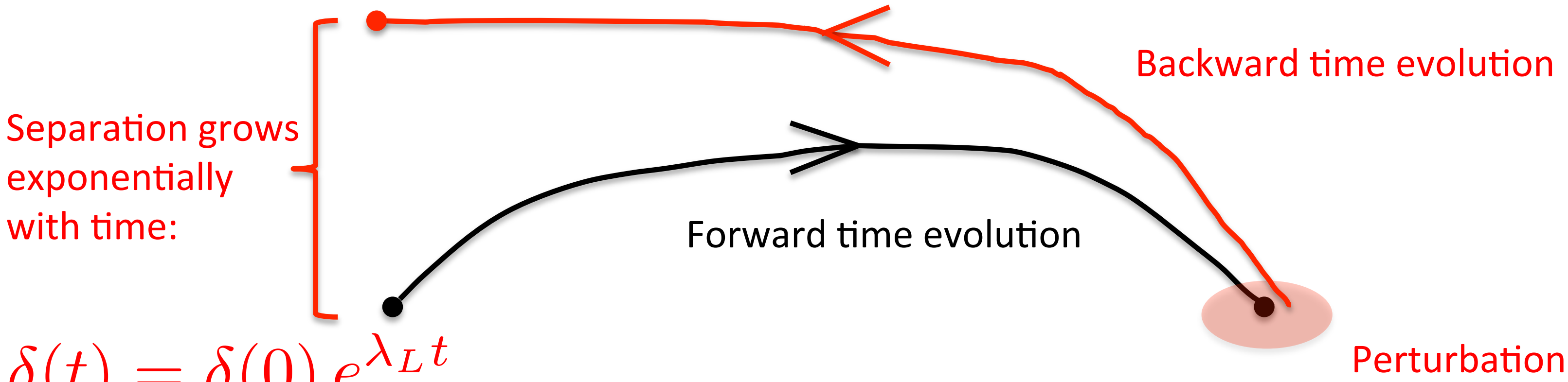
Thomson's insight in modern language:

Classical chaotic system

→ Positive Lyapunov exponent λ_L

→ Time-reversal operation requires exponentially increasing accuracy with waiting time

→ **Irreversible dynamics**



$$\delta(t) = \delta(0)\, e^{\lambda_L t}$$

Note: Lyapunov exponent λ_L determined by unperturbed system (intrinsic property)

Goal: Understand irreversibility/butterfly effect/quantum chaos in closed quantum many-body systems

Definition(s) of irreversibility for quantum systems

Loschmidt echo for characterizing quantum chaos & irreversibility (A. Peres, 1984)

Loschmidt echo $L(t) = |\langle\psi_i|\, e^{i(H+\Sigma)t}\, e^{-iHt}\, |\psi_i\rangle|^2$

↑ backward ↑ forward

time evolution

Eigenstate thermalization hypothesis (ETH):

J. M. Deutsch (1991), M. Srednicki (1994), M. Rigol et al. (since 2008)

In a non-integrable quantum many-body system few-body observables A cannot distinguish nearby many-body eigenstates (away from the edges of the spectrum)

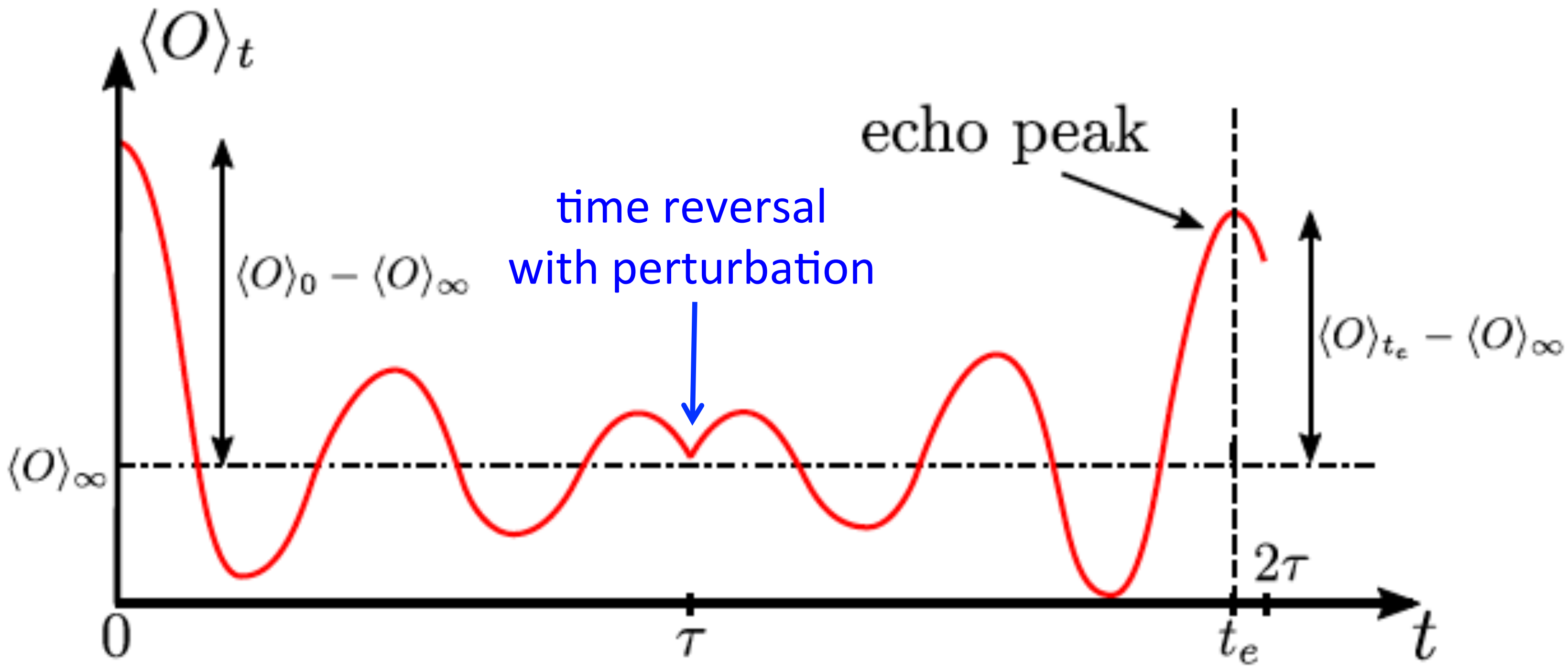
→ single eigenstates are "typical" in a given small energy window

$$\langle E_m|A|E_m\rangle = \langle E_n|A|E_n\rangle + o(\dim \mathcal{H}^{-1})$$

→ Orthogonality of states no useful criterion for "physically different"

→ Loschmidt echo not useful for characterizing irreversibility in quantum **many-body** systems

Definition of quantum irreversibility via echo dynamics of observables



Normalized echo peak height: $$E^*_\tau[O] = \max_{t>\tau} \left| \frac{O_t - O_\infty}{O_0 - O_\infty} \right|$$

Irreversible dynamics means $E_\tau^*[O]$ decays exponentially or faster as a function of the waiting time τ, otherwise the dynamics is reversible. Perturbation strength only enters as prefactor.

Goal for this talk:

Collection of results for integrable & non-integrable quantum many-body systems

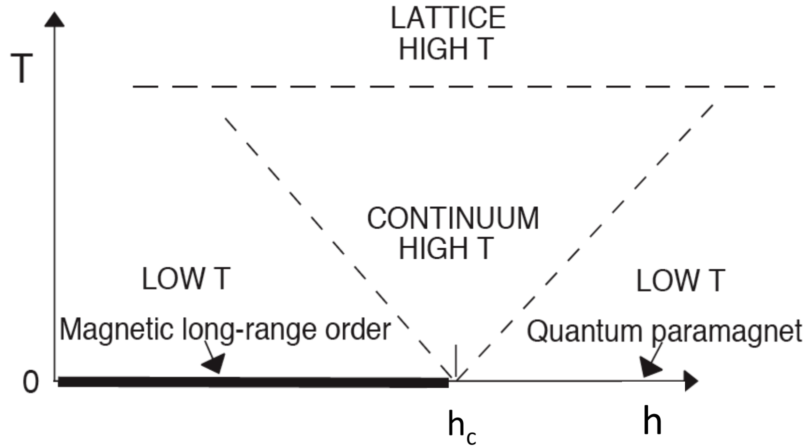
- M. Schmitt, S. Kehrein, Europhys. Lett. 115 (2016)
- M. Schmitt, S. Kehrein; arXiv:1711.00015
- M. Schmitt, D. Sels, S. Kehrein, A. Polkovnikov; arXiv:1802.06796

Transverse field Ising model

M. Schmitt, S. Kehrein, Europhys. Lett. 115 (2016)

$$H(h) = -\sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z + h \sum_{i=1}^{N} \sigma_i^x$$

Ising interaction — transverse magnetic field



S. Sachdev, Quantum Phase Transitions (Cambridge Univ. Press, 2011)

- Quantum phase transition at $h_c = 1$
- Integrable model:
  Quadratic in fermions after Jordan-Wigner transformation

Thermalization to "generalized Gibbs ensemble" (GGE) for all quenches $h_0 \rightarrow h$

Fagotti and Essler, Phys. Rev. B 87 (2013)

Reduced density matrix for n spin subsystem from time evolved initial state

$$\lim_{t\to\infty} |\psi_i(t)\rangle\langle\psi_i(t)|_n = \rho_{\mathrm{GGE},n}$$

Echo protocols:

a) Initial state: Ground state of $H(h_0)$

b) Forward time evolution (quench dynamics $h \neq h_0$): $U(\tau) = e^{-iH(h)\tau}$

c) Backward time evolution:

1) Sign change with perturbation $V(s) = e^{iH(h+\delta h)s}$

2) Loschmidt pulse $V(s) = U_P^\dagger \, e^{-iH(h)s} \, U_P$ ← approx. particle-hole trf. (equiv. to velocity reversal)

3) Generalised Hahn echo $V(s) = e^{-iH(-h)s}$

d) Observables:

transverse magnetization σ_i^x

longitudinal spin-spin correlation function (distance d) $\sigma_i^z \, \sigma_{i+d}^z$

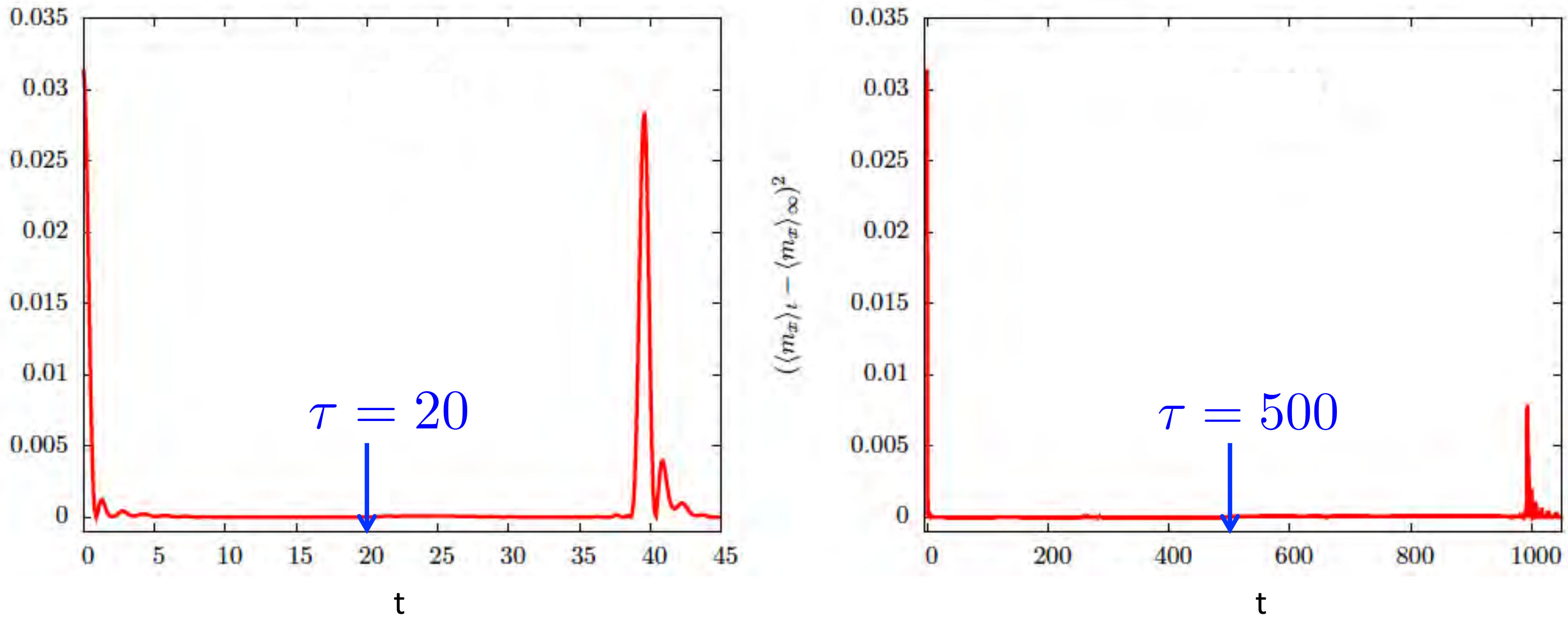
Methods:

- Numerical evaluation of Toeplitz determinants in the thermodynamic limit
- Stationary phase approximation for large waiting times (analytical result)

Echo protocol: Sign change with perturbation

Observable: Transverse magnetization

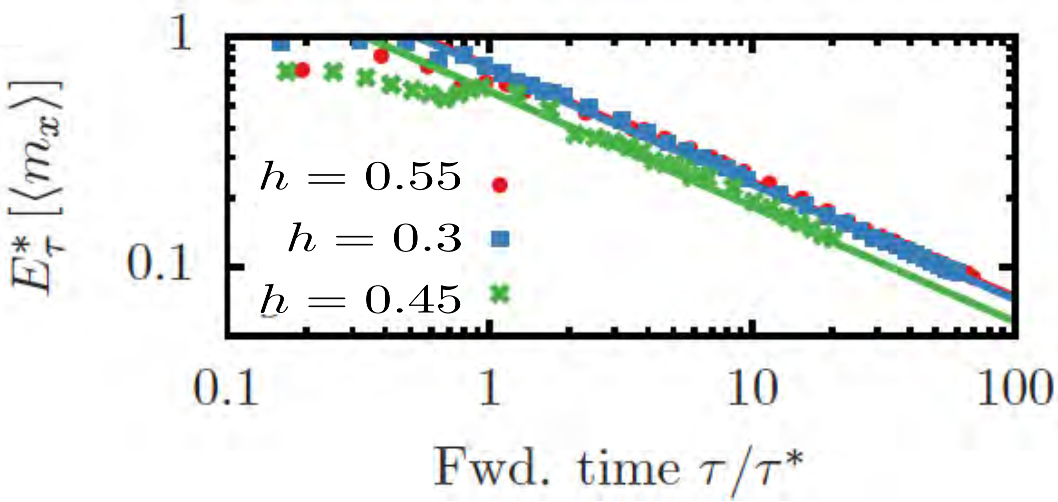
$$h_0 = 5.0,\ h = 1.1,\ \delta h = 0.04$$



Decay of normalized echo peak height $E^*_\tau[O] = \max_{t>\tau} \left| \frac{O_t - O_\infty}{O_0 - O_\infty} \right|$

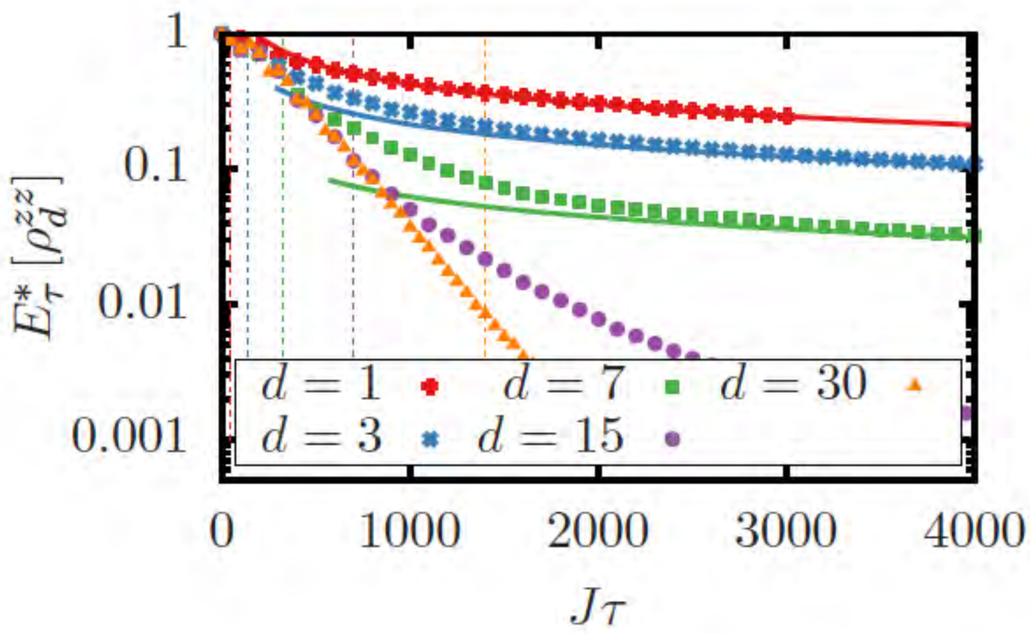
Stationary phase approximation predicts algebraic decay (for all protocols)

$$E^*_\tau[O] \propto \tau^{-1/2}$$



- with known prefactor (depending on perturbation)

$$E^*_\tau[\sigma^x] \sim c(h_0, h)\ \delta h^{-1/2} \tau^{-1/2}$$
for $\tau \gtrsim \delta h^{-1}$

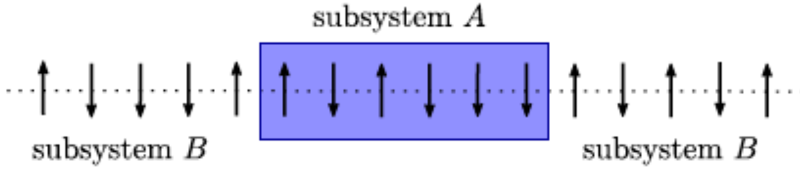


- full lines show analytical predictions
- essentially dephasing dynamics in small effective Hilbert spaces

Entanglement entropy

Bipartite pure state: $|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

Reduced density matrix: $\rho_A \overset{\text{def}}{=} \text{Tr}_{\mathcal{H}_B} |\psi\rangle\langle\psi|$



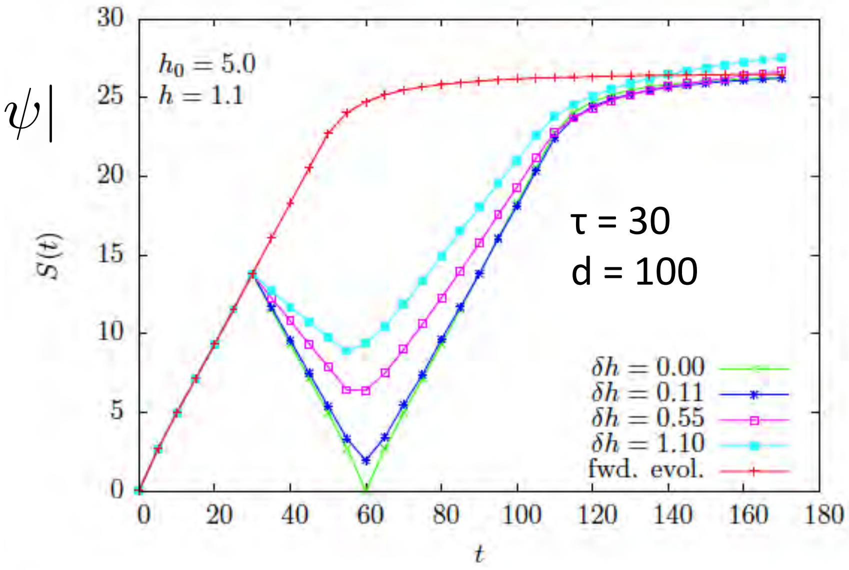
Entanglement entropy:

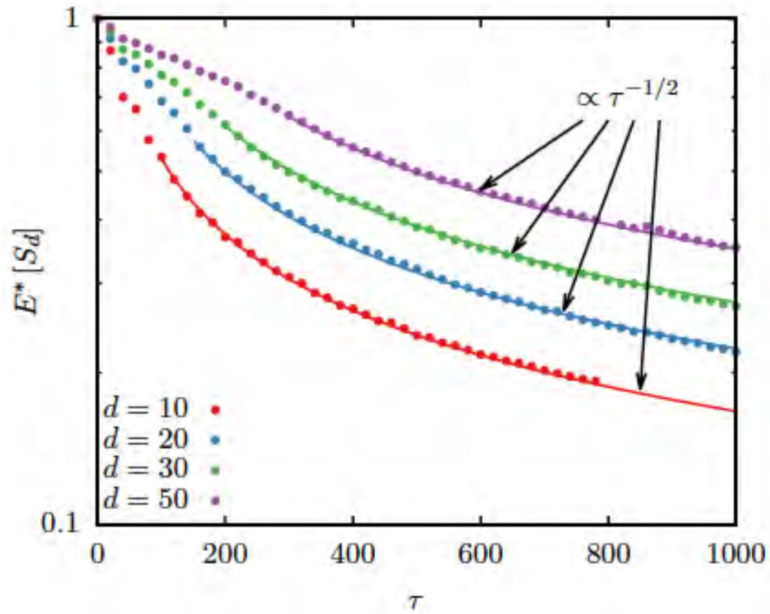
$$S^A_{\text{ent}} = -\text{Tr}_{\mathcal{H}_A} \left(\rho_A \, \ln \rho_A \right)$$

Measure entanglement entropy $S^d_{\text{ent}}(t)$ for subsystem with d spins:

Algebraic decay of normalised echo peak height of the entanglement entropy (analytical result):

$$E^*_\tau [S^d_{\text{ent}}] \propto \tau^{-1/2}$$





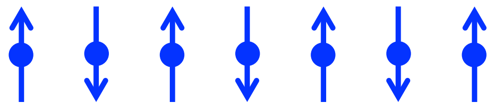
Non-integrable XY-models

M. Schmitt, S. Kehrein; arXiv:1711.00015

Method: Exact diagonalization

XY-models (local [nn and nnn] or fully connected [random]):

$$H = \sum_{i,j} J_{ij} \left(S_i^+ S_j^- + S_i^- S_j^+ \right)$$



<u>Protocol:</u> 1. Time evolution $|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$ (from Neel state)

2. Perturbation (δt small) $|\tilde{\psi}(t)\rangle = e^{-i\tilde{H}\delta t} |\psi(t)\rangle$

with $\tilde{H} = \sum_{i,j} \tilde{J}_{ij} \left(S_i^+ S_j^- + S_i^- S_j^+ \right)$ ← different random distribution

Subtract component parallel to $|\psi(0)\rangle$
(exponentially suppressed for large N)

3. Backward time evolution (echo for $t' \approx t$) $|\tilde{\psi}(t+t')\rangle = e^{iHt'} |\tilde{\psi}_\perp(t)\rangle$

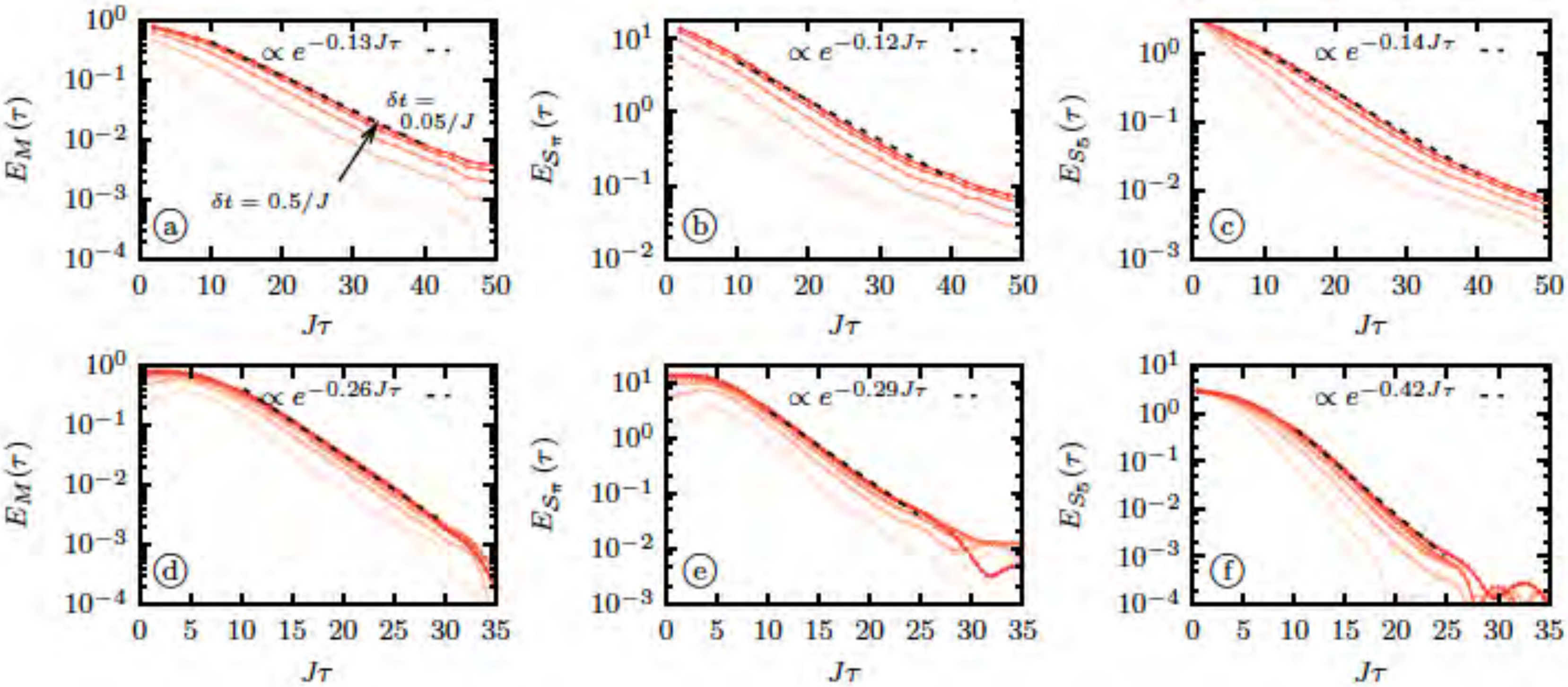


FIG. 3. Decay of the echo peak heights of staggered magnetization M, spin structure factor $\mathcal{S}_\pi$, and entanglement entropy of five consecutive spins S_5 after imperfect effective time reversal for both the local Hamiltonian $\hat{H}_{\rm loc}$ with $N = 24$ (a)-(c) and the fully connected Hamiltonian $\hat{H}_{\rm fc}$ with $N = 22$ (d)-(f). The perturbation Hamiltonian is the same realization of $\hat{H}_p$ in all cases, whereas the plotted perturbation strengths are $J\delta t = 0.5, 0.35, 0.25, 0.15, 0.05$. The dashed lines indicate exponential fits to the results for $\delta t = 0.05/J$.

<u>Conclusion:</u>

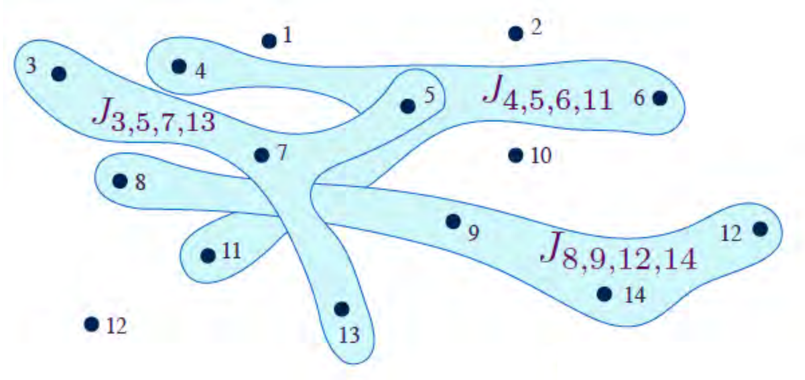
Numerical indication of exponential echo decay with rate independent of perturbation

Sachdev-Ye-Kitaev (SYK) model

M. Schmitt, D. Sels, S. Kehrein, A. Polkovnikov; arXiv:1802.06796

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

Gaussian random variable
(zero mean, variance fixed)



from: S. Sachdev, Talk at
St. John's College Oxford, 2016

- Exactly solvable in the large-N limit, but non-integrable
- Maximally chaotic in the sense of saturating the Maldacena-Shenker-Stanford bound

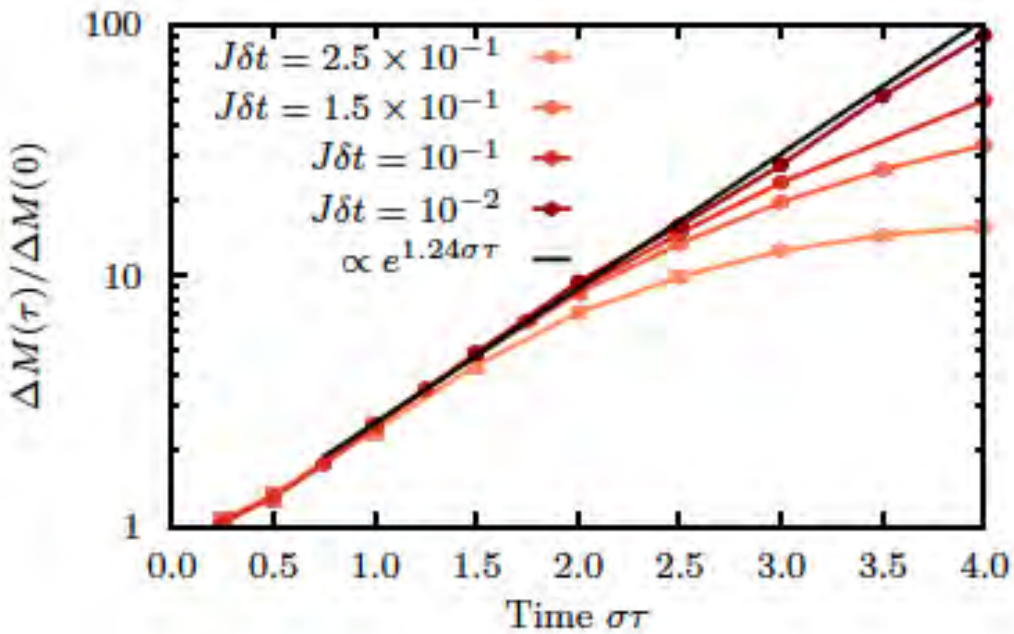
$$\lambda_L \leq 2\pi T$$

- Approximate IR-conformal symmetry indicating the existence of a holographic dual
- Model for strange metal physics

- Truncated Wigner approximation (TWA): Results in thermodynamic limit
- Initial state: A fraction m/N of the orbitals are occupied, the others empty
- Observable: Occupation imbalance $M = \frac{1}{m}\sum_{i=1}^{N}\langle c_i^\dagger c_i(t=0)\rangle \left(2c_i^\dagger c_i - 1\right)$
- Perturbation with random hopping Hamiltonian acting for time δt

$$H_{\rm pert} = \sum_i J_i \left(c_i^\dagger c_{i+1} + \text{h.c.}\right)$$

- Here: Initial decay of echo peak
- Short time exponential decay of M with rate consistent with classical Lyapunov exponent



Conclusions

- Definition of irreversibility for quantum many-body systems based on decay of echoes in observables
- Non-interacting model (transverse field Ising model):

  Algebraic decay of echoes due to dephasing
  → Reversible dynamics
- Non-integrable models:

  Exponential decay of echoes
  → Irreversible dynamics with rate of exponential decay determined by unperturbed Hamiltonian (like Lyapunov exponent)

Needed: Investigation of other integrable & non-integrable models

Outlook:

1) Connection to out-of-time-order correlators
   or: How to measure OTOCs with echoes
2) Connection to scrambling
3) Why are there echoes in the first place?
4) What determines the rate of the exponential decay?