How can one measure the entropy of a mesoscopic system ?

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The Epic Mission of Maxwell's Demon to Save Earth from Overproduction of Entropy and Environmental Disaster: Science-Based Fiction.

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26
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Wikipedia:

In statistical mechanics, **entropy** (usual symbol *S*) is related to the number of microscopic configurations Ω that a thermodynamic system can have when in a state as specified by some macroscopic variables.

$S = - k_B \Sigma p_i \log(p_i)$

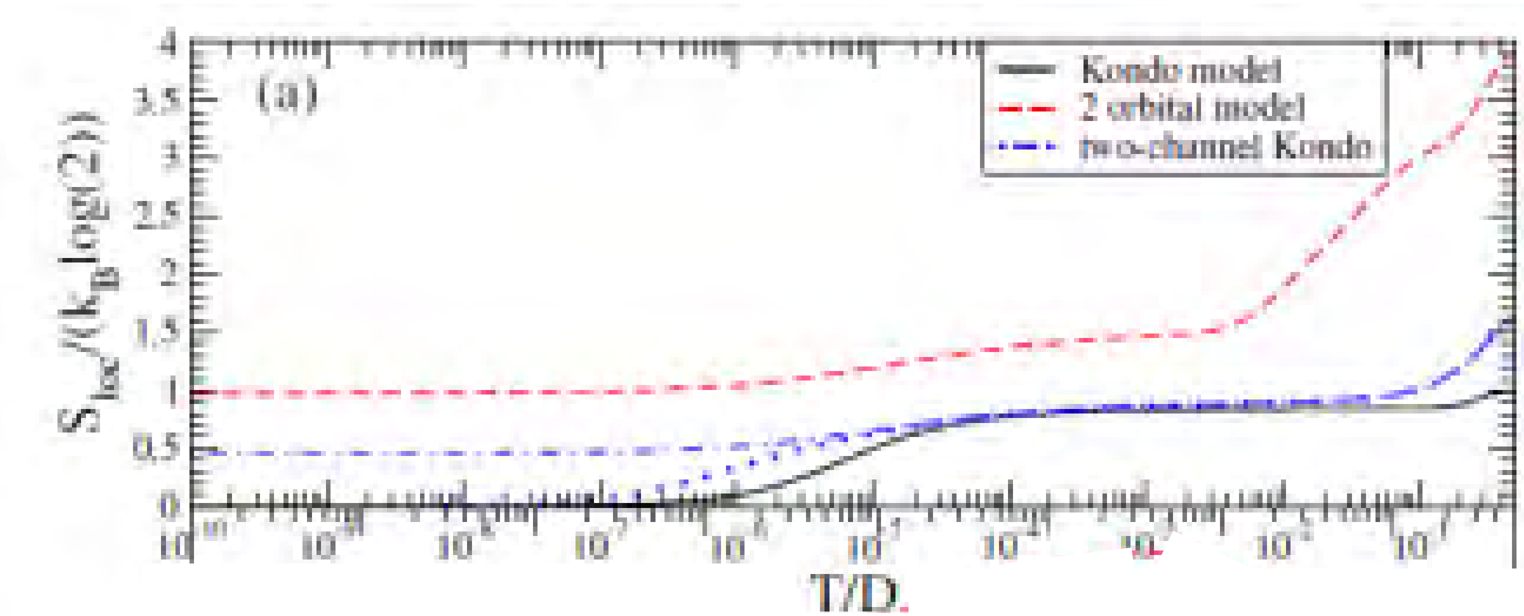
If $p_i=1/N$, then $S = \log(N)$

In particular, at T=0, S measures the degeneracy of the ground state.

Motivation – the additional entropy of a mesoscopic system coupled to thermodynamic reservoirs yields information about the nature of the ground states, and at finite temperatures about the low lying excitations.

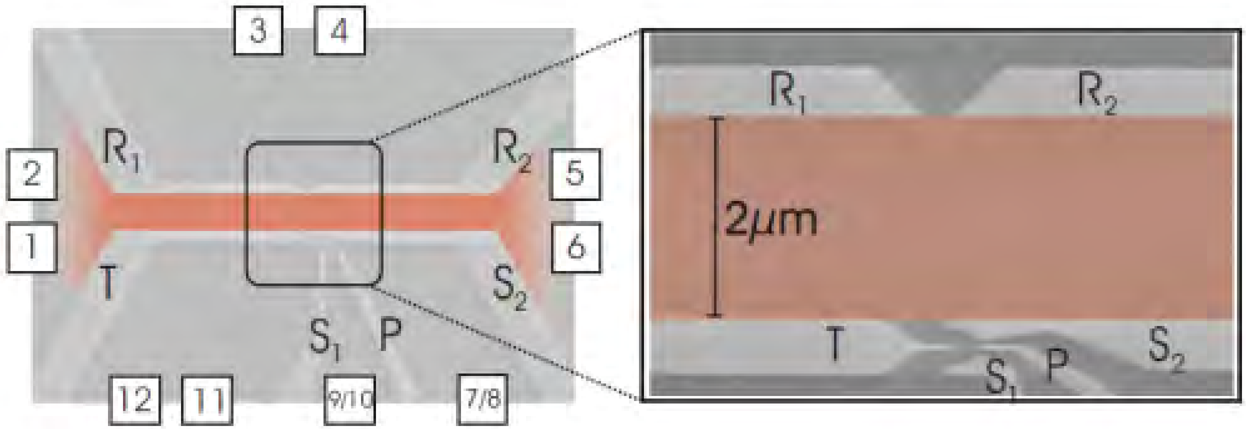
Example: Kondo/Anderson models

Problem: entropy easy to calculate, very hard to measure

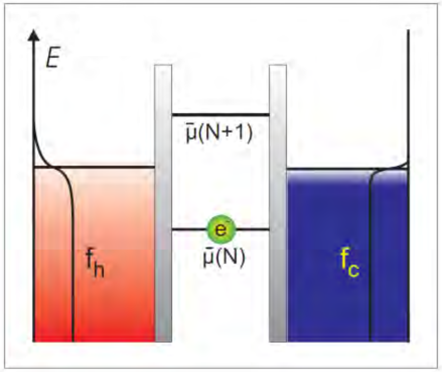


Anders 2012

Thermopower measurements (Sheibner et al., PRL 2005)



$$TP \equiv \frac{\partial I}{\partial \Delta T} / \frac{\partial I}{\partial \Delta \mu}$$



Mott Formula: $$TP = -\frac{\pi^2}{3} k_B{}^2 T \frac{\partial \ln[G(\mu)]}{\partial \mu}$$

correct at low temperatures for noninteracting systems

Transmission coefficient

Fermi-Dirac distribution

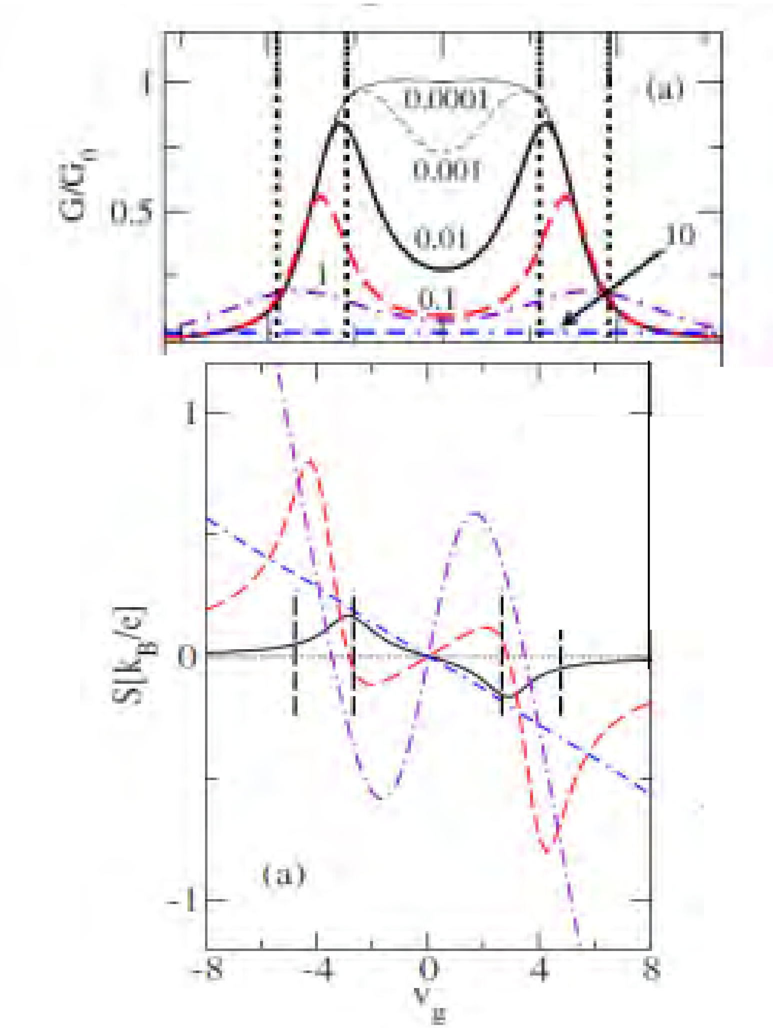
$$G = -\int t(\epsilon) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial \epsilon} d\epsilon \cong t(\mu)$$

Sommerfeld

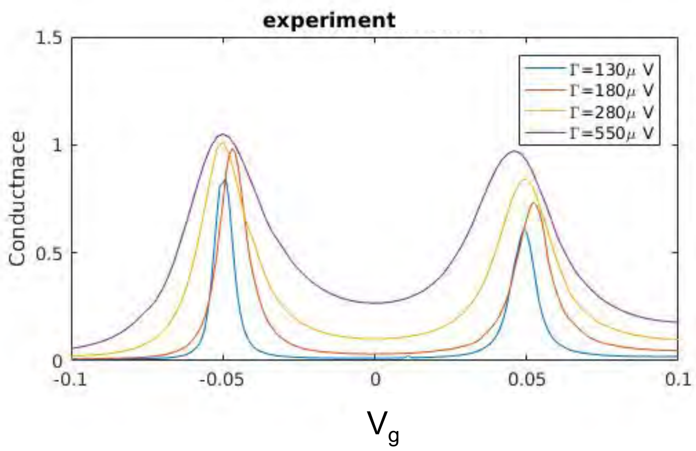
$$TP = -\frac{1}{G} \int t(\epsilon) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial T} d\epsilon \cong -\frac{1}{G} \frac{\pi^2}{3} k_B{}^2 T \left. \frac{\partial \mathrm{t}(\epsilon)}{\partial \epsilon} \right|_{\epsilon = \mu}$$

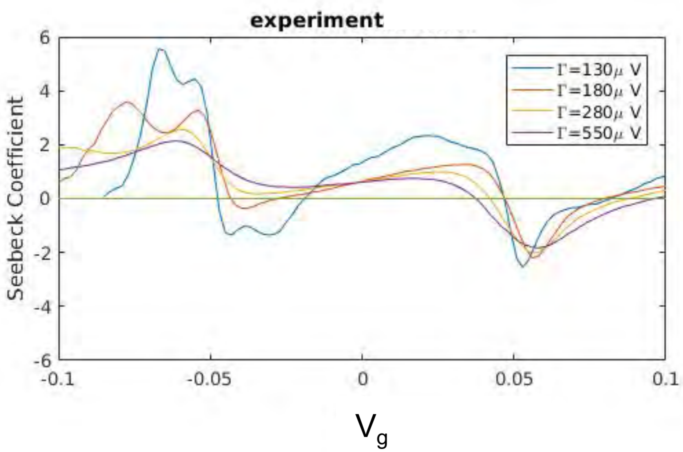
Thermopower of the single-impurity Anderson model: Numerical Renormalization Group

Costi & Zlatic, PRB 2010



Thermopower measurements (Sheibner et al., PRL 2005)
(Thierschmann, Buhmann, Molenkamp, unpublished)





Revisiting Mott Law:

$$TP = -\frac{\pi^2}{3} {k_B}^2 T \frac{\partial \ln[G(\mu)]}{\partial \mu}$$

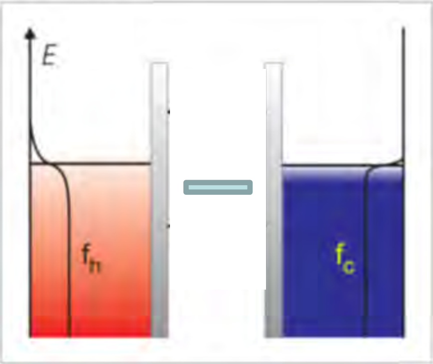
$$G = -\int t(\epsilon; \mu, T) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial \epsilon} d\epsilon \;\cong t(\mu; \mu, T)$$

$$TP = -\frac{1}{G} \int t(\epsilon; \mu, T) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial T} d\epsilon \cong -\frac{\pi^2}{3} {k_B}^2 T \left. \frac{\partial \mathrm{t}(\epsilon; \mu, T)}{\partial \epsilon} \right|_{\epsilon = \mu}$$

$$\frac{\partial G(\mu)}{\partial \mu} = \left. \frac{\partial \mathrm{t}(\epsilon; \mu, T)}{\partial \epsilon} \right|_{\epsilon = \mu} + \left. \frac{\partial \mathrm{t}(\epsilon; \mu, T)}{\partial \mu} \right|_{\epsilon = \mu}$$

an additional term

Explicit calculation of the extra term:



Non-interacting model:

$$t_0(\varepsilon) = \frac{\Gamma^2}{(\varepsilon - \varepsilon_0)^2 + \Gamma^2}$$

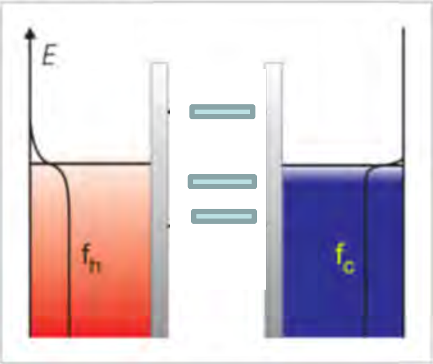
$$G = -\int t_0(\epsilon) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial \epsilon} d\epsilon$$

$$TP = -\frac{1}{G} \int t_0(\epsilon) \frac{\partial \mathrm{f}(\epsilon - \mu, T)}{\partial T} d\epsilon$$

Interacting system, Γ<<T

$$t(\varepsilon,\mu,T) \cong \sum_{i,j} (P_i + P_j) \underbrace{\sum_{m,n} \Gamma_{n,m} \langle \psi_j | \mathbf{d}_n^\dagger | \psi_i \rangle \langle \psi_i | \mathbf{d}_m | \psi_j \rangle \delta(\varepsilon - E_i + E_j)}_{t_0(\epsilon)}$$

Meir, Wingreen (1992)



Very simple example: N degenerate states, infinite U:

$$t(\varepsilon,\mu,T) \cong \frac{1+e^{-\beta(\varepsilon_0-\mu)}}{1+Ne^{-\beta(\varepsilon_0-\mu)}} \frac{\Gamma^2}{(\varepsilon-\varepsilon_0)^2+\Gamma^2} = \frac{1}{1+(N-1)\, f(\varepsilon-\mu,T)} t_0(\varepsilon)$$

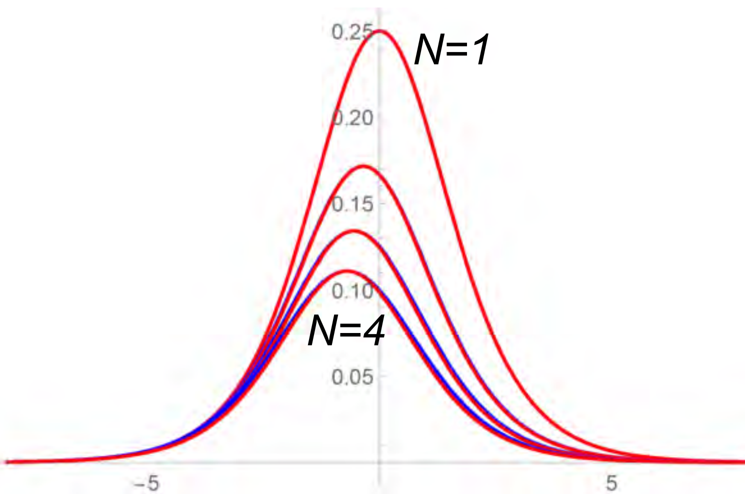
$$G = -\int t(\epsilon;\mu,T) \frac{\partial \mathrm{f}(\epsilon-\mu,T)}{\partial \epsilon} d\epsilon \qquad TP = -\frac{1}{G}\int t(\epsilon;\mu,T) \frac{\partial \mathrm{f}(\epsilon-\mu,T)}{\partial T} d\epsilon$$

Can I write: $G = -a \int t_0(\epsilon) \frac{\partial \tilde{f}(\epsilon-\mu,T)}{\partial \epsilon} d\epsilon$?

Yes !

$$\tilde{f}(\epsilon-\mu,T) = \frac{1}{\log N} \log[1 + (N-1) f(\epsilon-\mu,T)]$$

$$\approx f\left(\epsilon - \mu - \frac{1}{2} T \log N \,, T\right)$$



Viola et al., PRL (2012)

Result:

$$TP - TP_{Mott} = G\,\Delta\mu / \mathrm{T} = \tfrac{1}{2}\, G\, \mathrm{Log}\, N$$

More generally, when tunneling between a g_{n-1} and a g_n -degenerate states

$$TP - TP_{Mott} = G\,\Delta\mu / T = \tfrac{1}{2}\, G\, \mathrm{Log}(g_n/g_{n-1})$$

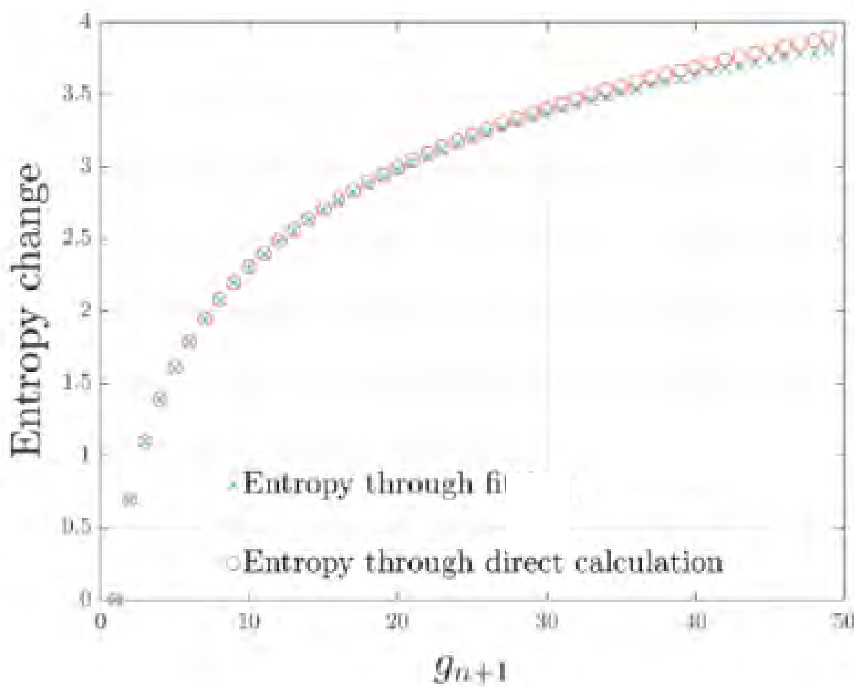
Result:

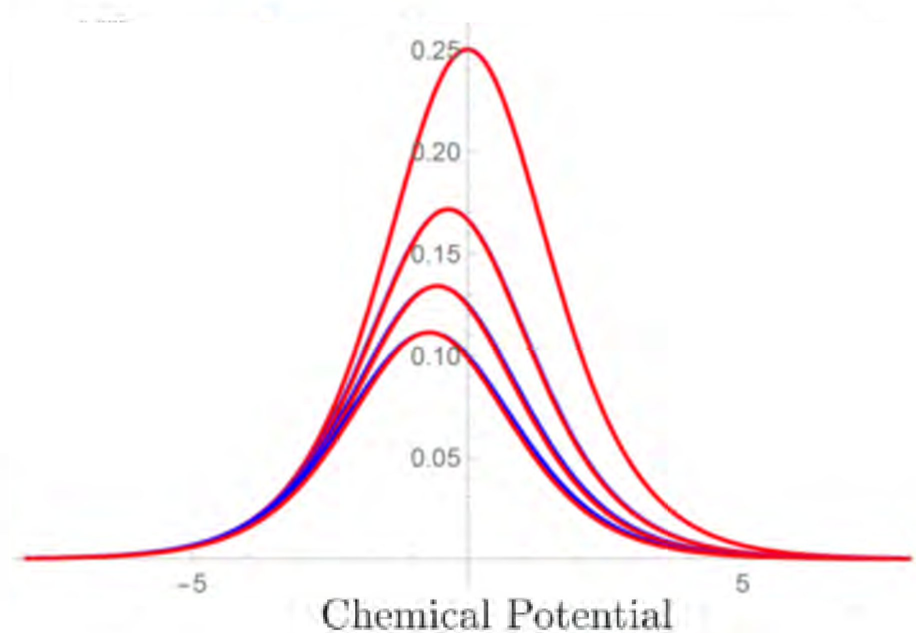
$TP - TP_{Mott}$ = $G\ \Delta\mu\ /\ T$ = ½ G Log N

More generally, when tunneling between a g_{n-1} and a g_n -degenerate states

$TP - TP_{Mott}$ = $G\ \Delta\mu\ /\ T$ = ½ G Log(g_n/g_{n-1})

Example: SU(ń) quantum dot

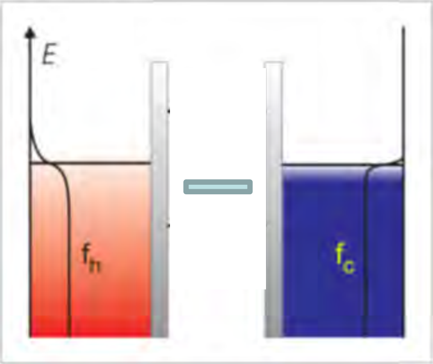




Interacting system, Γ<<T

$$t(\varepsilon,\mu,T) \cong \sum_{i,j}(P_i+P_j)\underbrace{\sum_{m,n}\Gamma_{n,m}\langle\psi_j|\mathbf{d}_n^\dagger|\psi_i\rangle\langle\psi_i|\mathbf{d}_m|\psi_j\rangle\delta(\varepsilon-E_i+E_j)}_{t_0(\epsilon)}$$

Meir, Wingreen (1992)



Arbitrary number of levels:

$$G_{ij} = -a\int t_0(\epsilon)\frac{\partial f(\epsilon-\mu-\Delta\mu_{ij},T)}{\partial\epsilon}d\epsilon$$

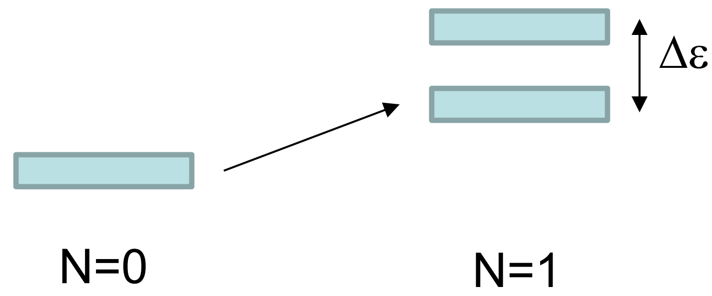
$$\Delta\mu_{ij}(T) = \frac{\epsilon_{n+1,j}-\epsilon_{n,i}}{2} + \frac{T}{2}\log\left[\frac{\sum_j g_{n+1,j}e^{-\epsilon_{n+1,j}/T}}{\sum_i g_{n,i}e^{-\epsilon_{n,i}/T}}\right]$$

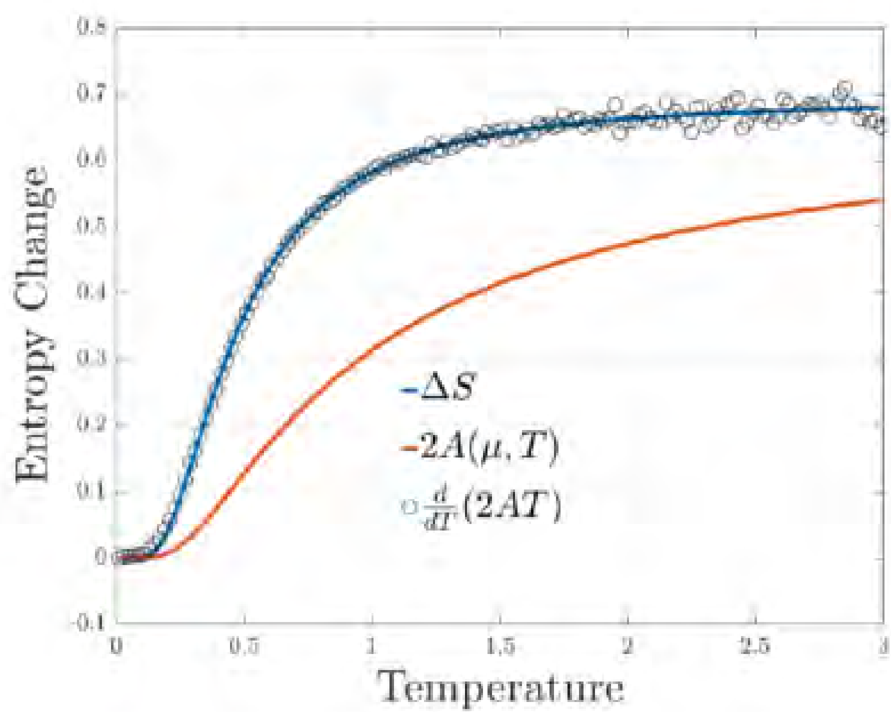
If transport is dominated by a single transition:

$$TP - TP_{Mott} = G_{ij}\,\Delta\mu_{ij}/T$$

$$\Delta S(T) = \frac{d}{dT}\left\{T\log\left[\frac{\sum_i g_{n,i}e^{-\epsilon_{n,i}/T}}{\sum_j g_{n,j}e^{-\epsilon_{n,j}/T}}\right]\right\} = \frac{d}{dT}2\Delta\mu_{ij}(\mu,T)$$

Example:

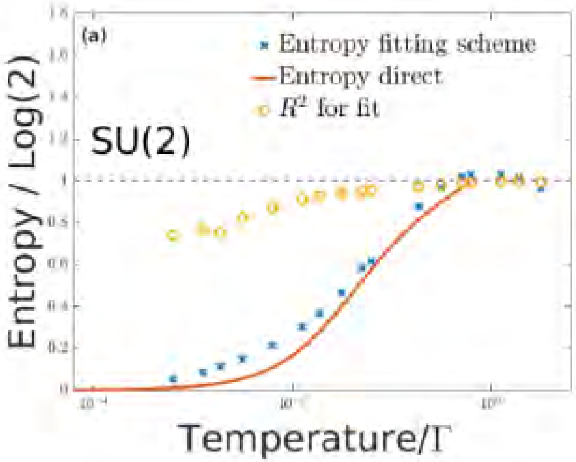




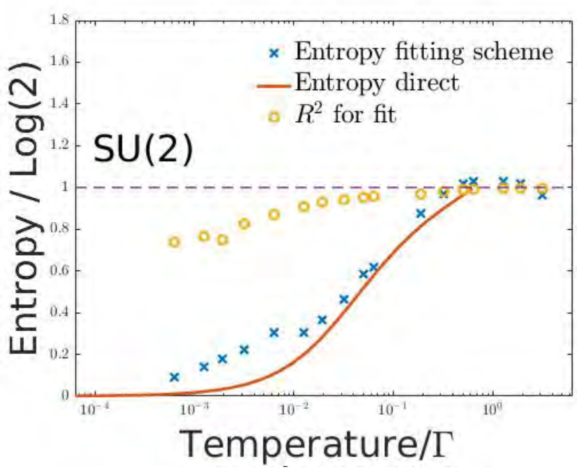
$A = \Delta\mu_{ij}/T$

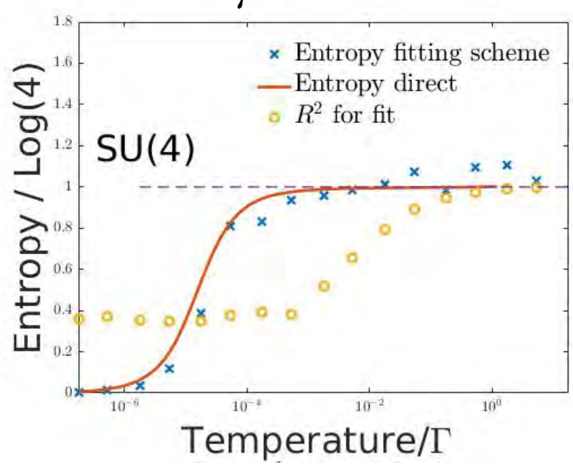
The procedure was derived for T>>Γ: $TP(\mu,T) - TP_{Mott}\,(\mu,T) = G(\mu,T)\ \Delta\mu(T)/\,T$

Empirically is works even for T~Γ/10 :

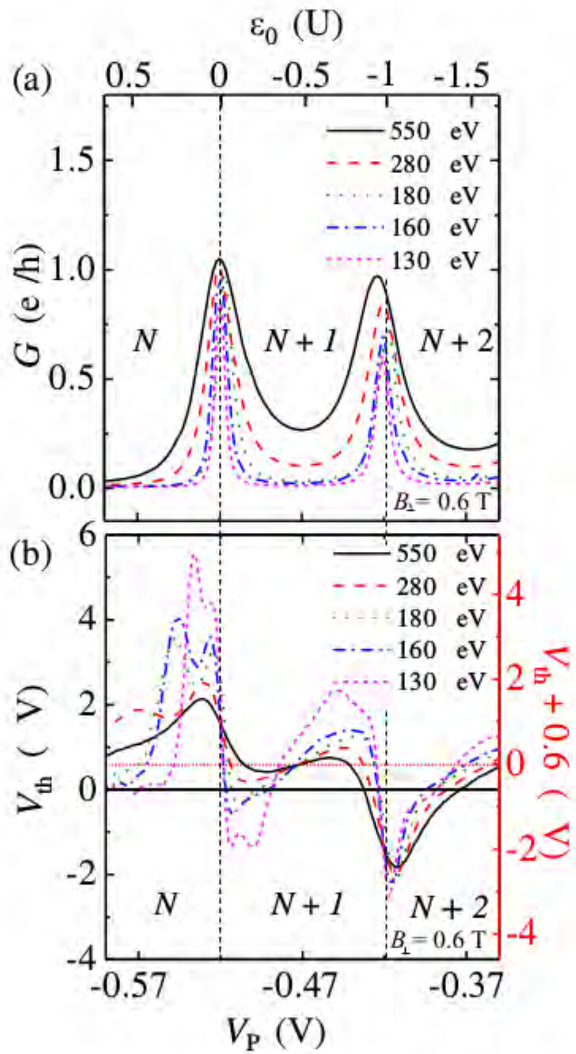


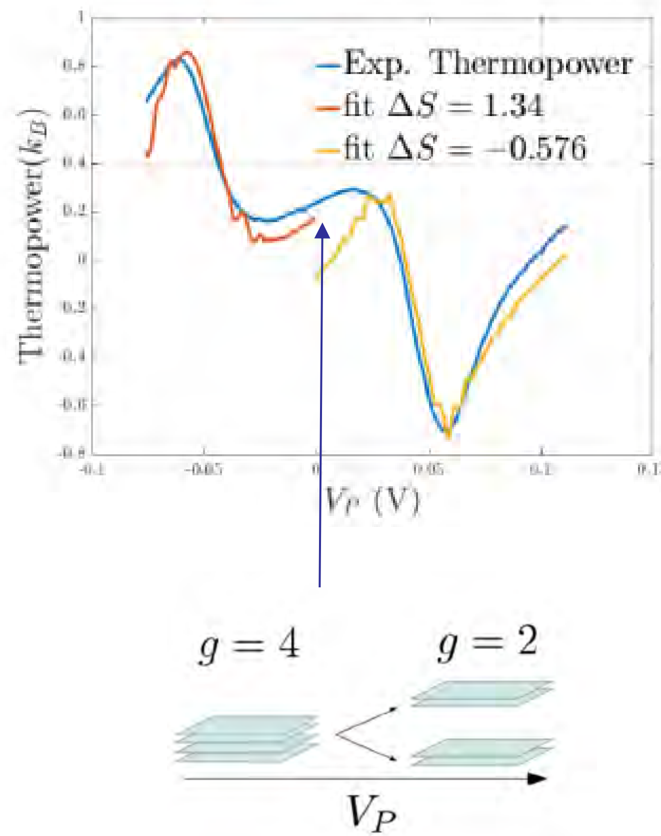
Low temperature scheme: $\Delta\mu(T) = T\ \underset{\mu}{\mathrm{Max}}[(TP(\mu,T) - TP_{Mott}\,(\mu,T))/G(\mu,T)]$



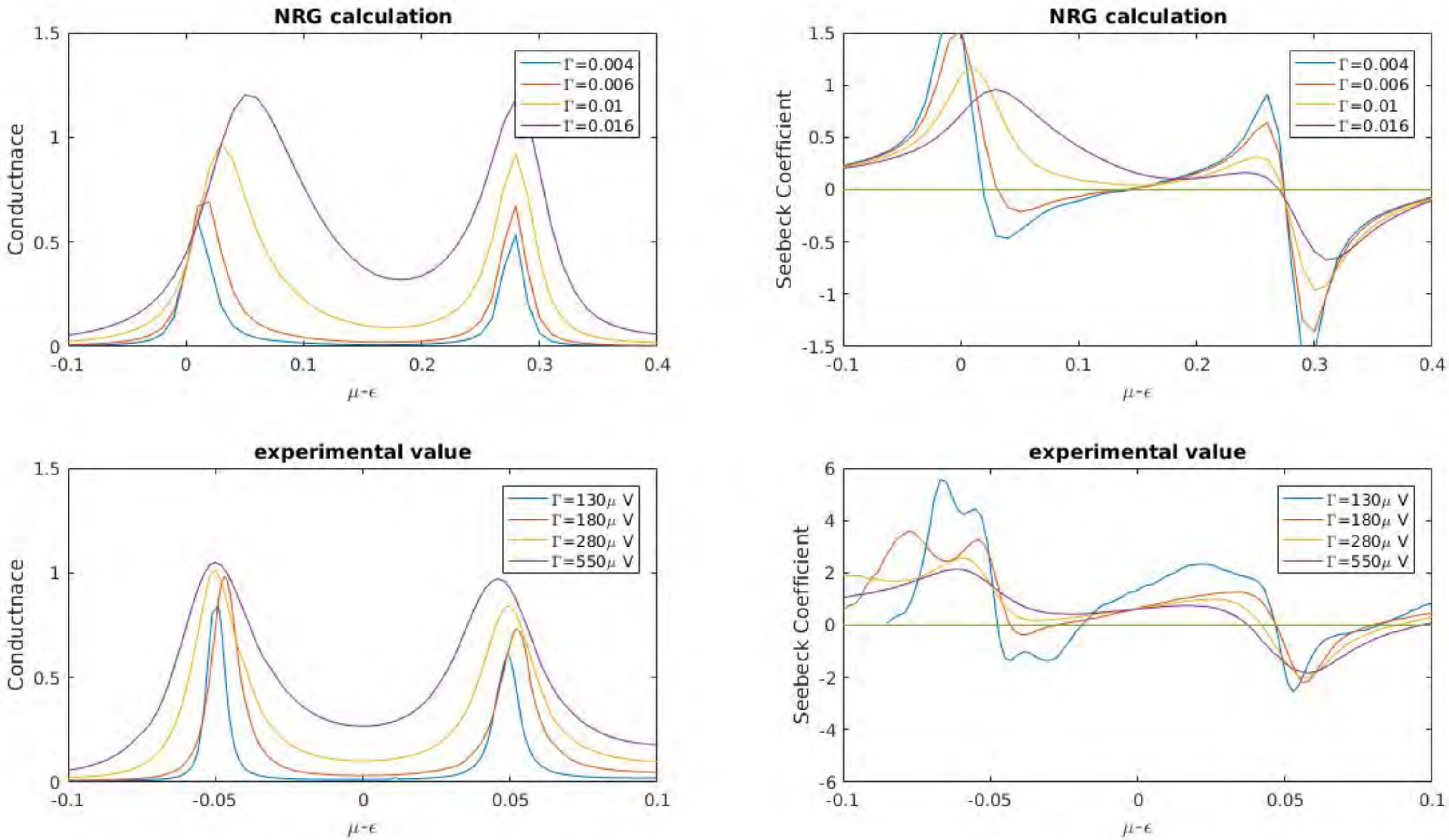


Back to experiments:
(Thierschmann, Buhmann, Molenkamp, unpublished)





Interpretation of the data:



Thank you

Yaakov Kleeorin