Non-equilibrium quasi-particles in disordered superconductors

Julia S. Meyer

with

Anton Bespalov ($\rightarrow$ Nizhni-Novgorod),

Manuel Houzet (Grenoble), and **Yuli Nazarov** (TU Delft)

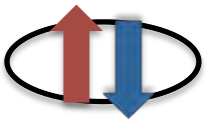
SPICE Workshop: Quantum Thermodynamics and Transport

Mainz – May 10, 2018

Quasiparticles in superconductors

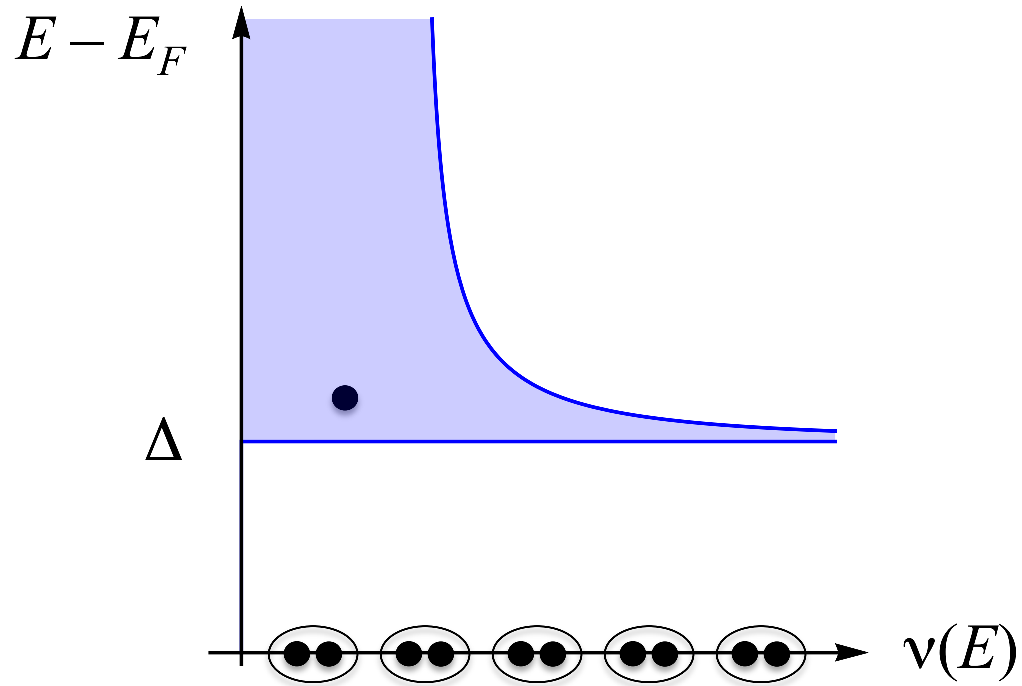
conventional superconductors (e.g. Aluminium):

- spin-singlet
- s-wave
- fully gapped density of states



at low temperatures ($T \ll \Delta$):
exponentially small
thermal concentration
of quasiparticles

$$c_{\mathrm{eq}}(T) \simeq \nu_0 \sqrt{8\pi k_B T \Delta}\, e^{-\Delta/k_B T}$$

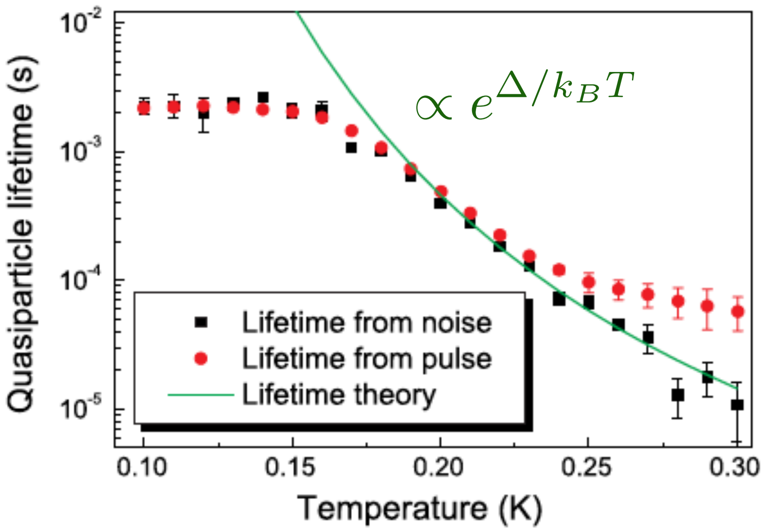


Experiment: Excess quasiparticles

residual quasiparticle concentration at low temperatures: $c \gg c_{\mathrm{eq}}$

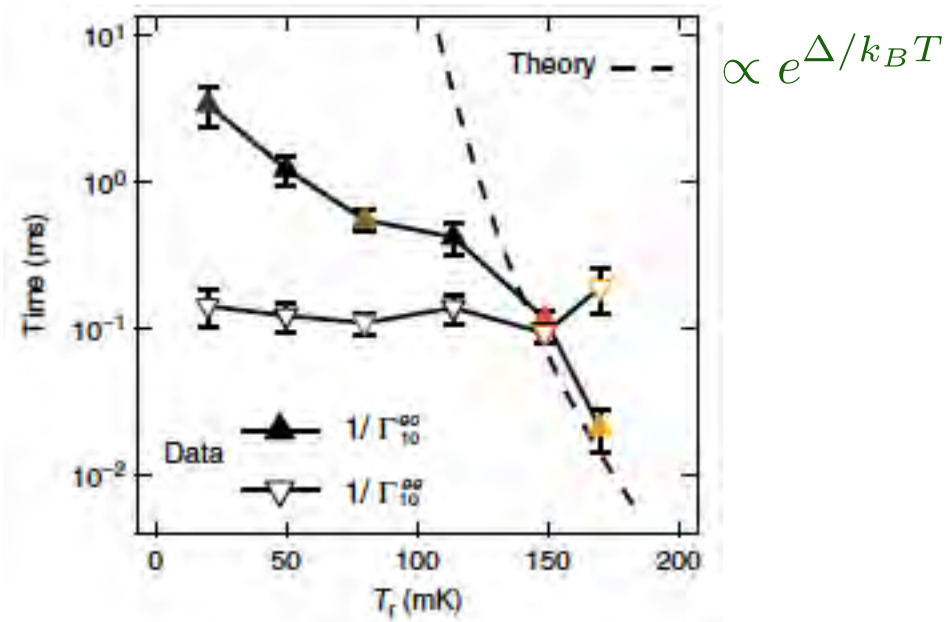
$\rightarrow$ saturation:

lifetime of superconducting resonators



de Visser *et al.*, PRL 2011

coherence time of superconducting qubits



Ristè *et al.*, Nat. Commun. 2013

UNIVERSITÉ Grenoble Alpes

Main results

observation:

excess quasiparticles in virtually all superconducting devices which limits their performances

our work: generation-recombination model

$\rightarrow$ residual quasiparticle concentration

- for delocalized quasiparticles above the superconducting gap $c \propto \sqrt{A}$
- for localized quasiparticles

  at mesoscopic fluctuations of the gap edge $c \propto \dfrac{1}{\ln^3(1/A)}$

  $\Rightarrow$ **poor efficiency of shielding**

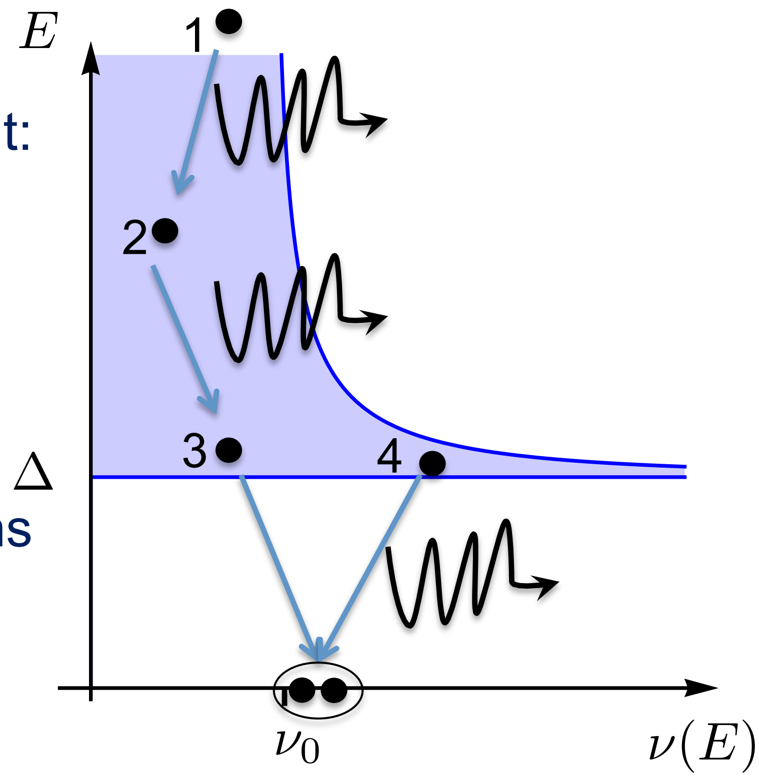
where A generation rate due to non-equilibrium agent

- spontaneous polarization of residual quasiparticles

  in mesoscopic systems

Non-equilibrium quasiparticles

- generation due to a non-equilibrium agent:
  - EM and blackbody radiation
  - cosmic rays
  - natural radioactivity
  - …



- fast energy relaxation by emitting phonons
- slow annihilation of two quasiparticles near the gap edge with rate

$$\Gamma_{34} = \bar{\Gamma} \int d\mathbf{r}\, p_3(\mathbf{r}) p_4(\mathbf{r})$$

balance between generation (rate A) and annihilation for delocalized quasiparticles near gap edge:

$$A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$$

UNIVERSITÉ Grenoble Alpes

Disordered superconductors

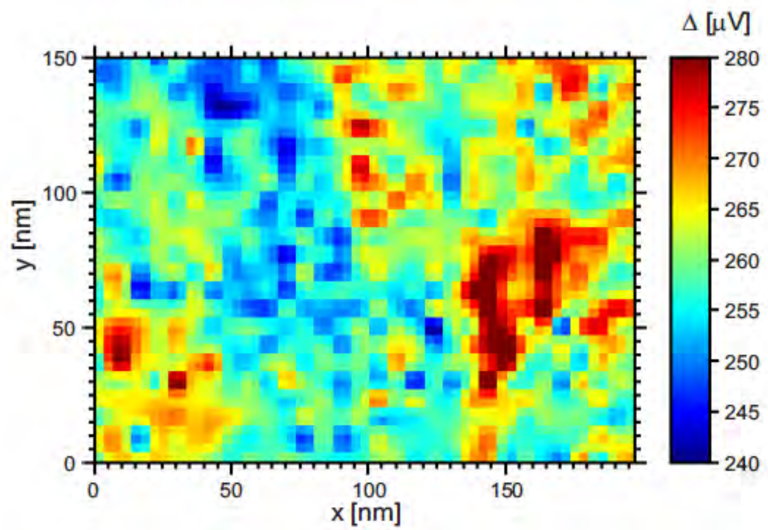
"Anderson theorem" (mean field):

Δ is unaffected by non-magnetic disorder and remains spatially uniform

Abrikosov-Gorkov 1958
Anderson 1959

at larger disorder:

Sacépé *et al.*,
PRL 2008



STM study
of TiN films

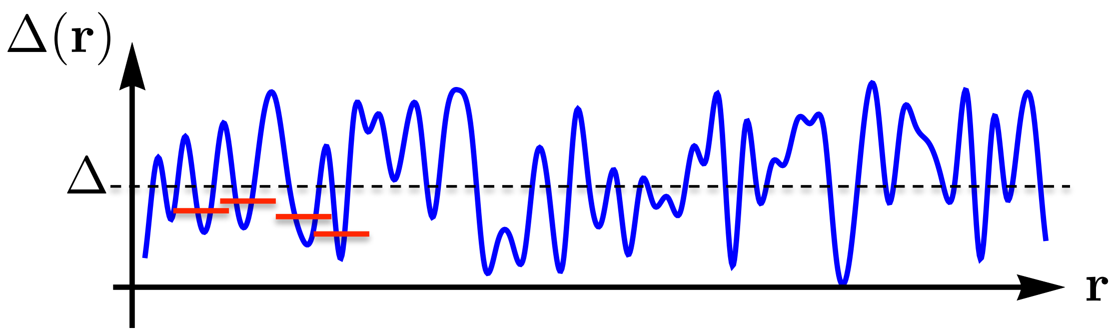
Disordered superconductors

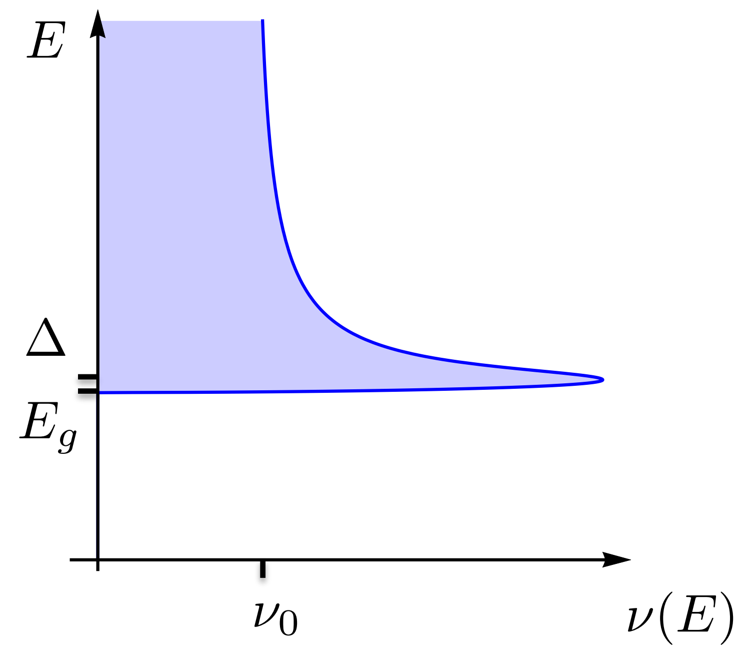
mesoscopic fluctuations of the gap:

Larkin and Ovchinnikov, JETP 1972

$$\delta\Delta(\mathbf{r}) = \Delta(\mathbf{r}) - \Delta$$

$$\langle \delta\Delta(\mathbf{r})\delta\Delta(\mathbf{r}')\rangle = (\delta\Delta)^2\delta(\mathbf{r}-\mathbf{r}')$$ with correlation radius $< \xi$ (coherence length)





- reduced gap
  due to overlapping bound states
- rounding of the BCS singularity

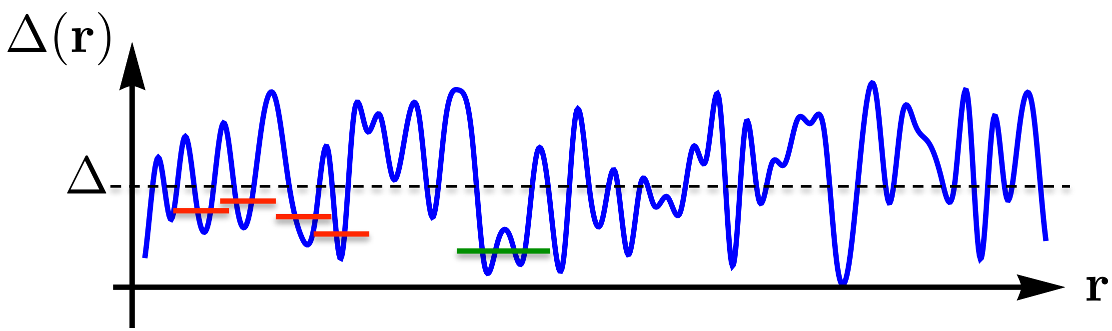
Disordered superconductors

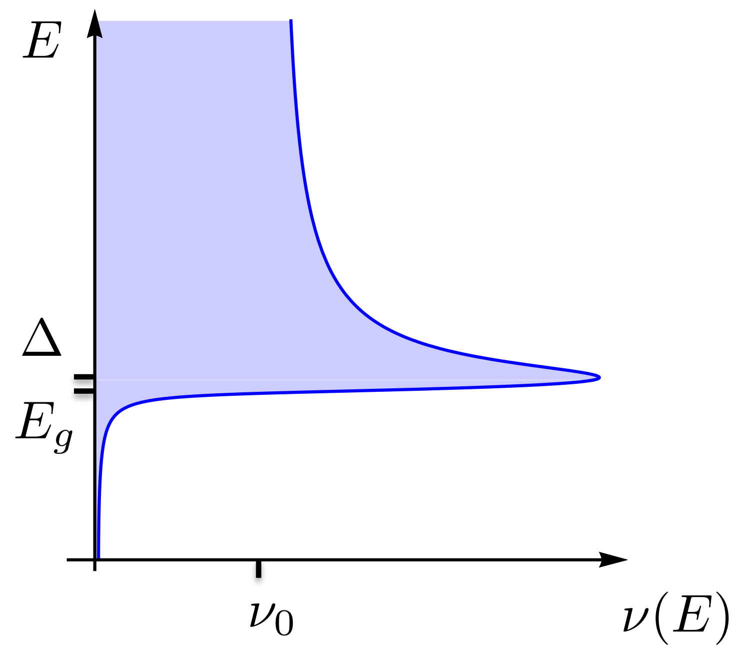
mesoscopic fluctuations of the gap:

Larkin and Ovchinnikov, JETP 1972

$$\delta\Delta(\mathbf{r}) = \Delta(\mathbf{r}) - \Delta$$

$\langle \delta\Delta(\mathbf{r})\delta\Delta(\mathbf{r}')\rangle = (\delta\Delta)^2\delta(\mathbf{r}-\mathbf{r}')$ with correlation radius $< \xi$ (coherence length)





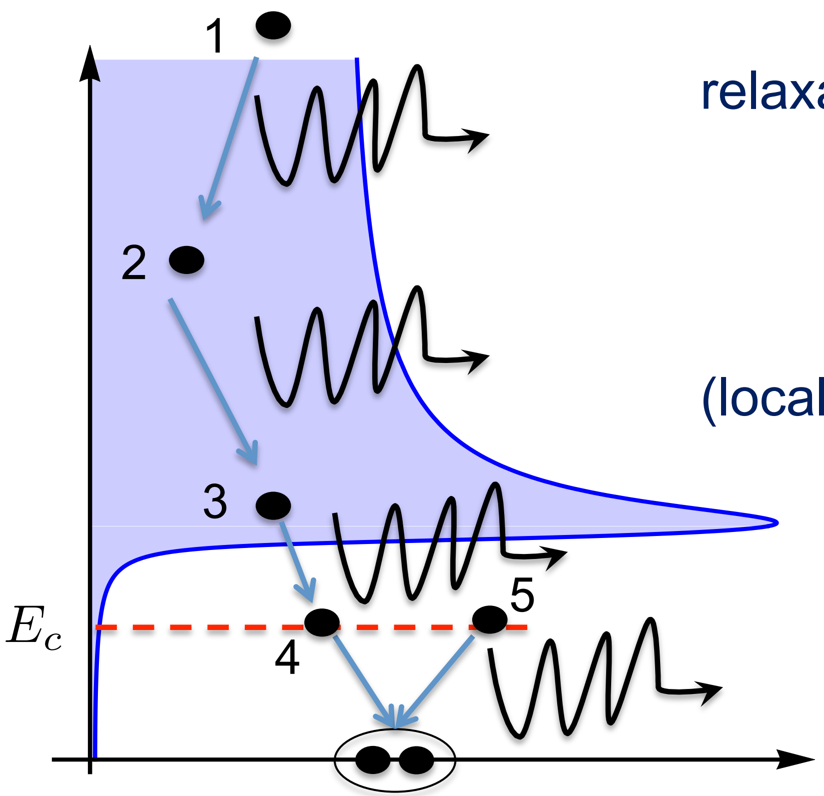
- rare optimal fluctuations

  generate localized tail states

  with energies $E < E_g$

Bottleneck for relaxation

fast energy relaxation by emitting phonons does not stop at Δ:
quasiparticles relax into the localized tail states

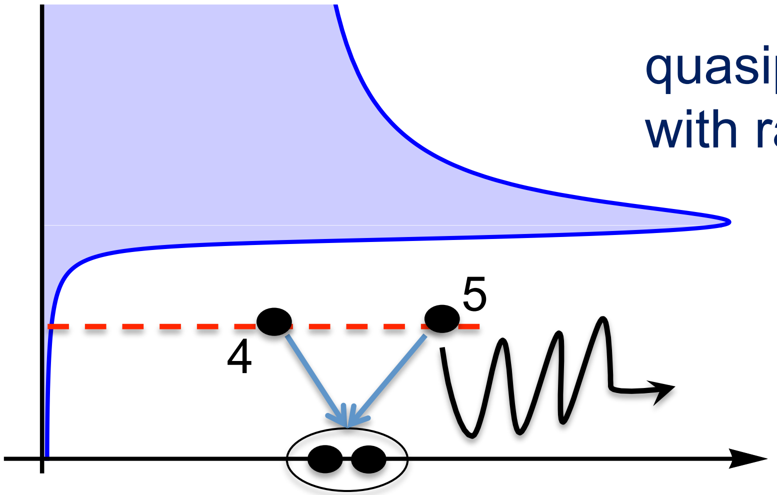


relaxation stops when
no more overlapping states
with lower energy are available

(localization radius at E_c: $r_c \sim$ few ξ)

Université Grenoble Alpes

Generation/recombination model for localized states



quasiparticles are generated at random points with rate A per unit volume

- they keep their positions
- they annihilate pairwise with the rate

$$\Gamma(\mathbf{R}) = \bar{\Gamma} \int d\mathbf{r}\, p_c(\mathbf{r}) p_c(\mathbf{r}+\mathbf{R})$$

most probable shape of the localized state at energy E_c

balance between generation and annihilation

$\rightarrow$ typical distance r between quasiparticles:

- dense limit $r \ll r_c$: $A = \bar{\Gamma} c^2 \implies c = \sqrt{A/\bar{\Gamma}}$
- dilute limit $r \gg r_c$: $Ar^3 \sim \dfrac{\bar{\Gamma}}{r_c^3} e^{-r/r_c} \implies c \propto \dfrac{1}{r_c^3 \ln^3\left(\frac{\bar{\Gamma}}{A r_c^6}\right)}$

Simplified model: Bursting bubbles

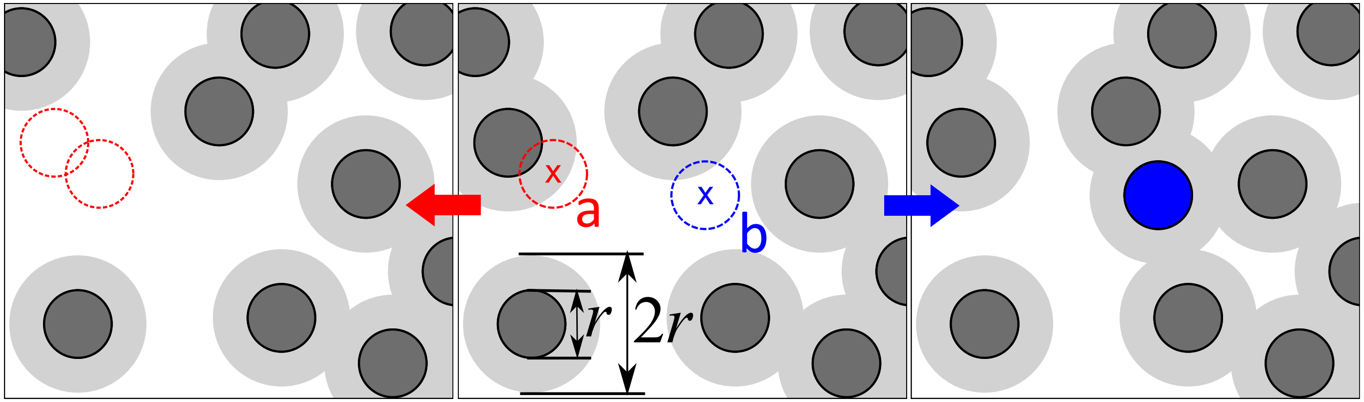
characteristic length scale $\frac{r}{r_c} \approx \ln\left(\frac{\bar{\Gamma}}{A r_c^6}\right)$

annihilation rate varies very quickly

with distance d_{ij} between two quasiparticles:

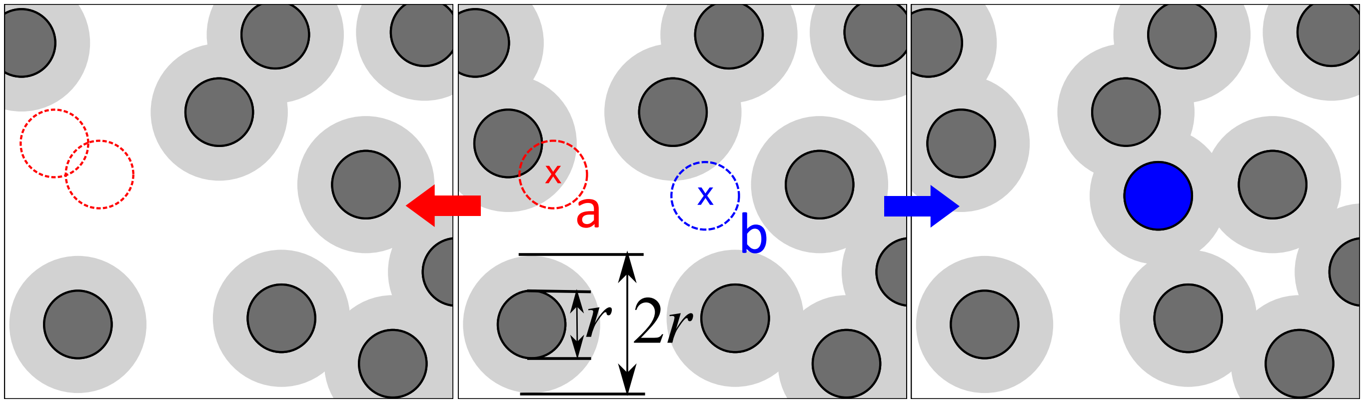
- rapid annihilation, if $d_{ij} < r$
- slow annihilation, if $d_{ij} > r$

$\rightarrow$ describe quasiparticles as bubbles with radius $r/2$ that cannot overlap



UNIVERSITÉ Grenoble Alpes

Simplified model: Bursting bubbles



quasiparticle concentration: $$c = \frac{C_p}{(4\pi/3)r^3}$$

with packing coefficient C_p

- naïve estimate: $C_p = 0.5$
- simulation: $C_p \approx 0.605 \pm 0.008$

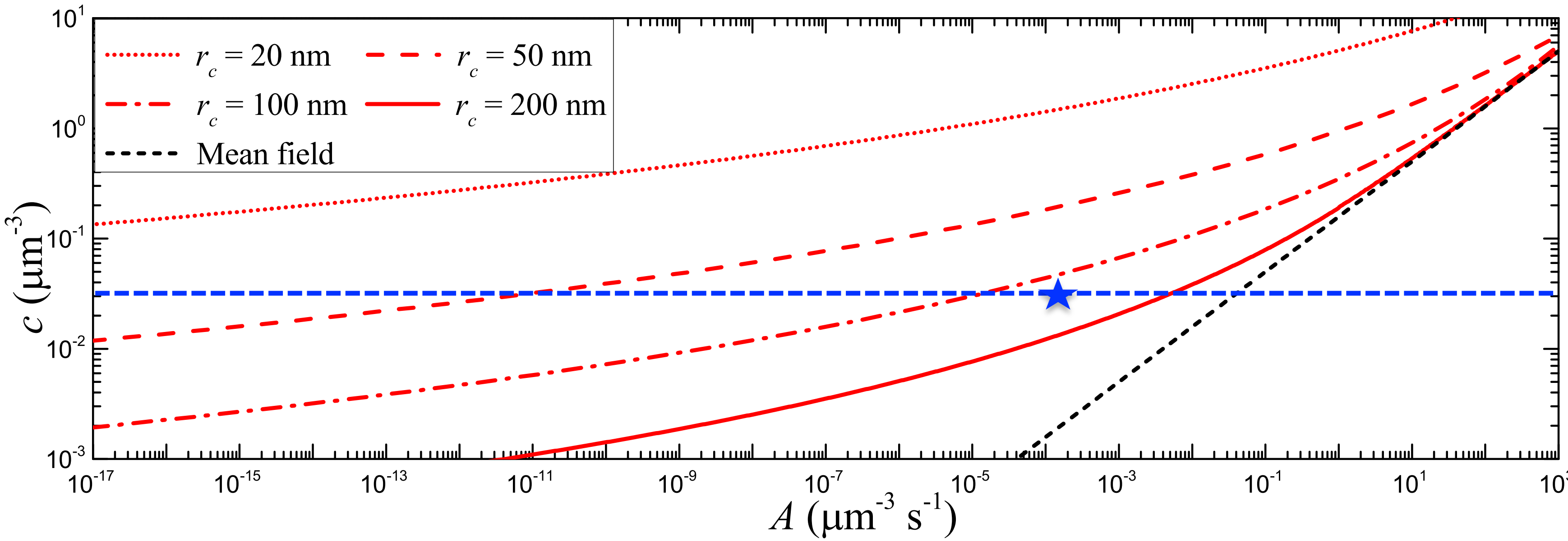
$$c = \frac{0.605}{(4\pi/3)r_c^3 \ln^3(\bar{\Gamma}/Ar_c^6)}$$

UNIVERSITÉ Grenoble Alpes

Full dynamical simulation: Results for Aluminium

$\bar{\Gamma} = 40\,\mathrm{s}^{-1}\mu\mathrm{m}^3$

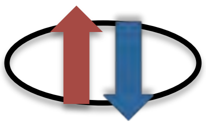
different values for r_c correspond to different disorder strengths:



cosmic radiation at sea level dominated by muons with:

- mean energy in the GeV range
- flux of 1 muon/cm²/min $\rightarrow A \sim 10^{-4}\mathrm{s}^{-1}\mu\mathrm{m}^{-3}$
- stopping power in Al of $\sim$ 1 $\mathrm{MeV/cm}$

Spin?

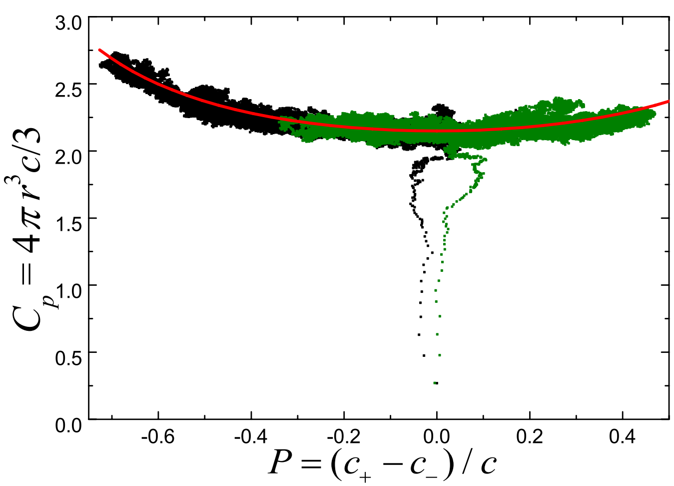
Cooper pair: 

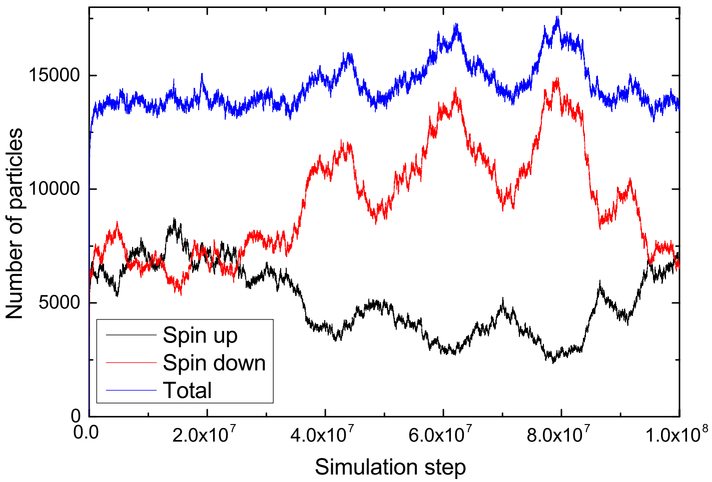
→ 2 quasiparticle can only annihilate, if they are in a singlet state

first attempt: bursting bubbles model with classical spin

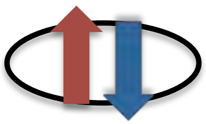
- without spin-flip: $C_p \approx 2.19 \pm 0.05$

but large fluctuations …





Classical Spin

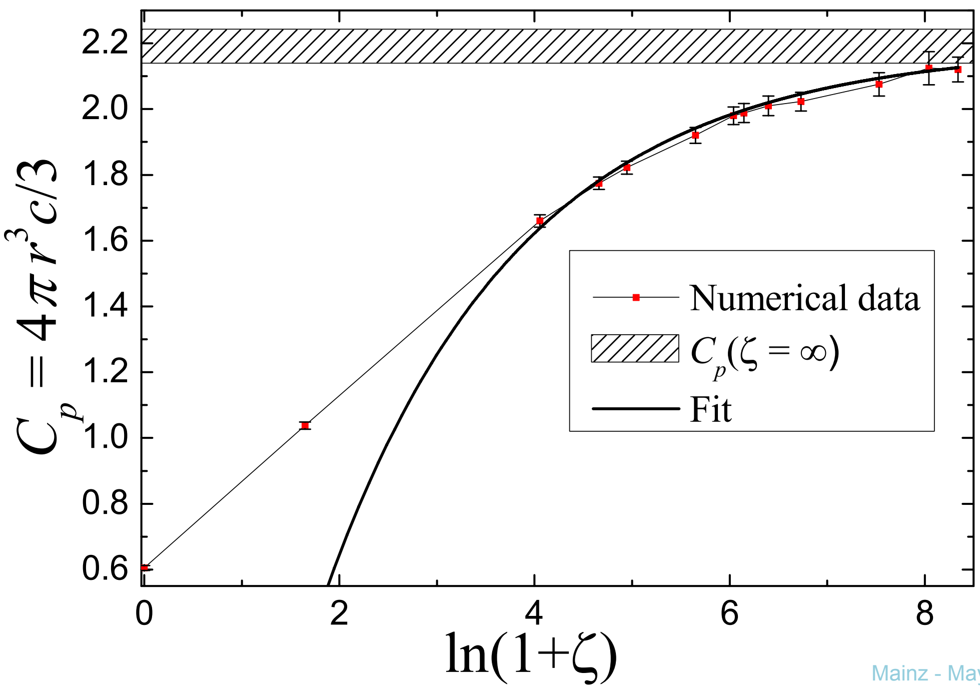
Cooper pair: 

$\rightarrow$ 2 quasiparticles can only annihilate, if they are in a singlet state

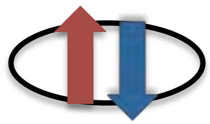
first attempt: bursting bubbles model with classical spin

- with spin-flip rate τ_{sf}:
  $\zeta = 4\pi A r^3 \tau_{\mathrm{sf}}/3$

spinless result recovered in the limit $\zeta \rightarrow 0$



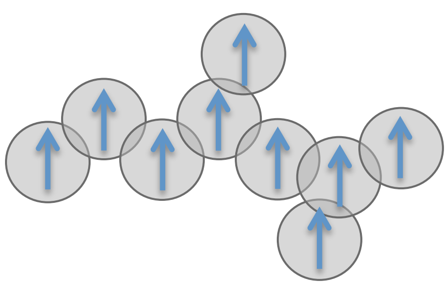
Spin?

Cooper pair: 

$\rightarrow$ 2 quasiparticles can only annihilate, if they are in a singlet state

second attempt: bursting bubbles model with quantum spin

- cluster = bubbles connected through a chain of overlapping bubbles

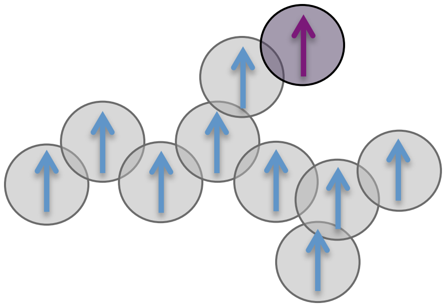


- no decay, if each pair of bubbles is in a spin-triplet state

  $\rightarrow$ cluster of N particles in maximal-spin state with $S = N/2$

Quantum Spin (in progress)

- add a particle to an existing cluster:

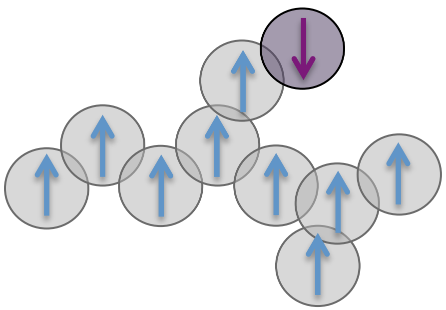


if parallel spin

$\rightarrow$ cluster with $N+1$ particles

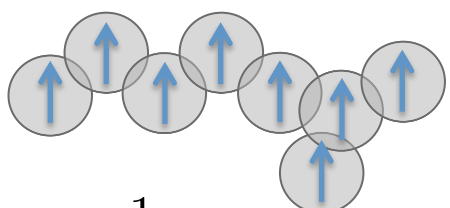
Quantum Spin (in progress)

- add a particle to an existing cluster:



if parallel spin

$\rightarrow$ cluster with $N+1$ particles



if antiparallel spin:

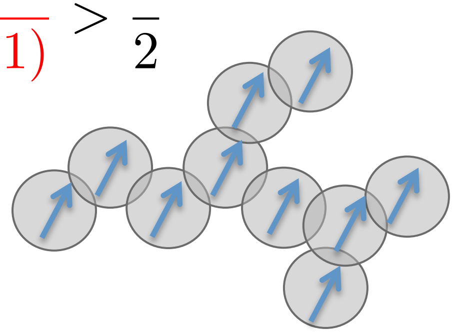
either

annihilation with a partner

$\rightarrow$ cluster with $N-1$ particles

$$p_{N+1} = \frac{1}{2} + \frac{1}{2(N+1)} > \frac{1}{2}$$

or



new cluster with $N+1$ particles

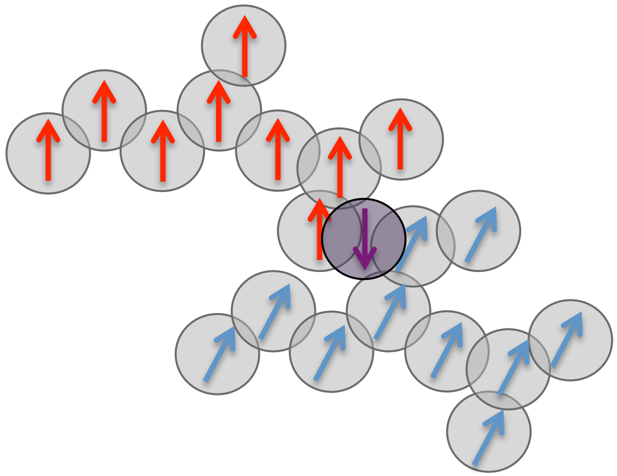
polarized along a tilted axis

UNIVERSITÉ Grenoble Alpes

Quantum Spin (in progress)

$p_{N+1} > 1/2 \;\rightarrow$ growth of cluster !

limited by interaction with other clusters …



BUT :

stabilization of a single spin-polarized cluster possible in small systems

→ spontaneous polarization of excess quasiparticles !

(confirmed by simulations)

Quantum Spin: General toy model

more general toy model:

- use fully polarized state as a starting point

  N number of particles

  m number of flipped spins: $S = N/2 - m$

possible processes:

incoming quasiparticles

$N \rightarrow N + 1,\ m \rightarrow m$ with rate $AV(1 + 1/N)/2$

$N \rightarrow N + 1,\ m \rightarrow m + 1$ with rate $AV(1 - 1/N)/2$

singlet annihilation

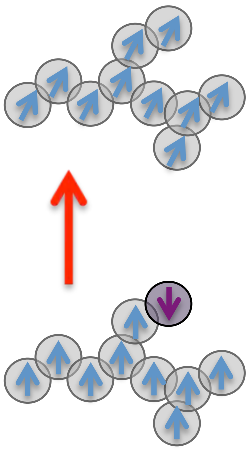
$N \rightarrow N - 2,\ m \rightarrow m - 1$ with rate $\bar{\Gamma} mN/V$

spin flip

$N \rightarrow N,\ m \rightarrow m + 1$ with rate N/τ_{sf}

triplet annihilation

$N \rightarrow N - 2,\ m \rightarrow m$ with rate $\bar{\Gamma}_T N^2/(2V)$



Quantum Spin: General toy model

more general toy model:

- use fully polarized state as a starting point

  N number of particles

  m number of flipped spins

$\rightarrow$ balance equation:

$$\frac{dN}{dt} = AV - 2\bar{\Gamma} m \frac{N}{V} - \bar{\Gamma}_T \frac{N^2}{V}$$

$$\frac{dm}{dt} = \frac{AV}{2}\left(1 - \frac{1}{N}\right) - \bar{\Gamma} m \frac{N}{V} + N\tau_{sf}^{-1}$$

stationary solution:

if spin relaxation dominates:

$$N = \sqrt{\frac{AV\tau_{sf}}{2}} \gg 1, \quad m = \frac{AV^2}{2\bar{\Gamma}N} \ll N \qquad \text{for} \quad \frac{1}{A\tau_{sf}} \ll V \ll \bar{\Gamma}\tau_{sf}$$

$\rightarrow$ polarized state with a large number of particles

UNIVERSITÉ Grenoble Alpes

Quantum Spin: General toy model

more general toy model:

- use fully polarized state as a starting point

  N number of particles

  m number of flipped spins

$\rightarrow$ balance equation:

$$\frac{dN}{dt} = AV - 2\bar{\Gamma} m \frac{N}{V} - \bar{\Gamma}_T \frac{N^2}{V}$$

$$\frac{dm}{dt} = \frac{AV}{2}\left(1 - \frac{1}{N}\right) - \bar{\Gamma} m \frac{N}{V} + N\tau_{sf}^{-1}$$

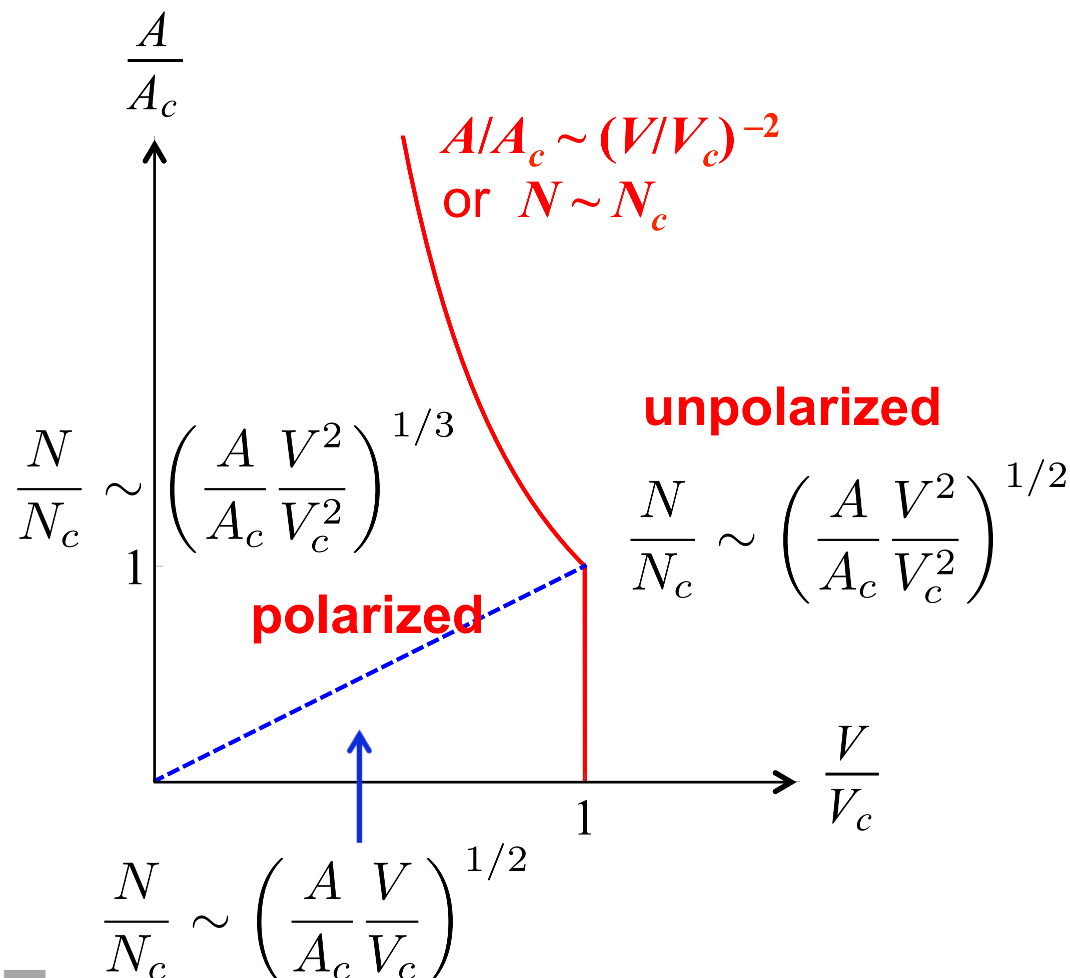
stationary solution:

if triplet annihilation dominates:

$$N = \left(\frac{AV^2}{\bar{\Gamma}_T}\right)^{1/3} \gg 1, \quad m = \frac{AV^2}{2\bar{\Gamma} N} \ll N \quad \text{for} \quad \sqrt{\frac{\bar{\Gamma}_T}{A}} \ll V \ll \frac{\bar{\Gamma}^{3/2}}{\bar{\Gamma}_T \sqrt{A}}$$

$\rightarrow$ polarized state with a large number of particles

Quantum spin: Phase diagram



maximal number of polarized particles:

$$N_c \sim \frac{\bar{\Gamma}}{\bar{\Gamma}_T} \sim \frac{1}{\alpha_{so}^2} \sim 10^4$$

volume limited by spin flips:

$$V_c \sim \bar{\Gamma}\tau_{sf} \sim \frac{\xi^3}{g}\frac{1}{\alpha_{so}^2\left(\frac{\delta E}{\Delta}\right)^2}$$

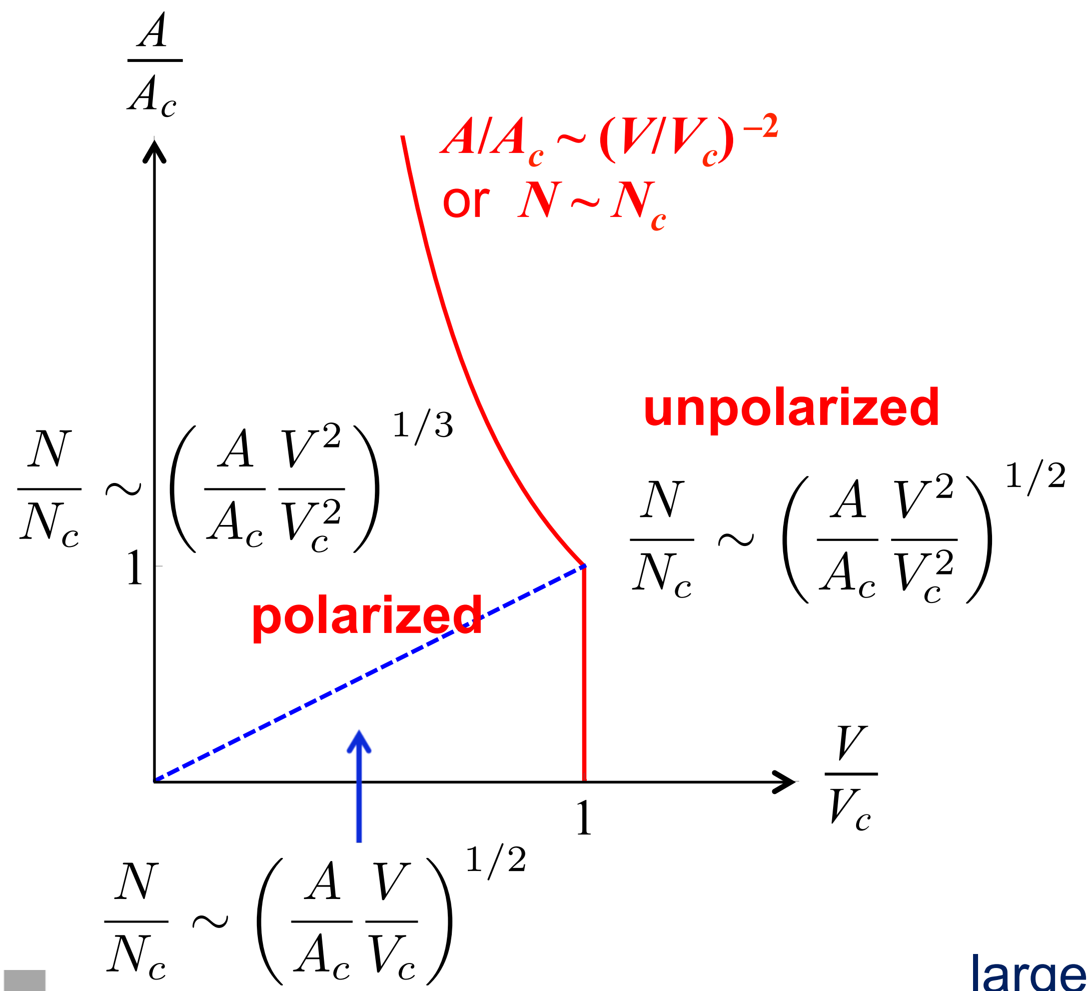
$$\sim (10^2 - 10^4)\xi^3$$

injection limited by triplet annihilation:

$$A_c \sim \frac{\bar{\Gamma}}{\bar{\Gamma}_T^2\tau_{sf}^2} \sim \bar{\Gamma}\frac{N_c^2}{V_c^2}$$

$$\sim (10^7 - 10^{11})\mathrm{s}^{-1}\mu\mathrm{m}^{-3}$$

Quantum spin: Phase diagram



large fluctuations: $\langle\langle N^2 \rangle\rangle \sim \langle N \rangle$

Conclusion

extremely slow annihilation of quasiparticles trapped in localized states

→ large concentration of excess quasiparticles
in moderately disordered superconductors

- strategies to reduce their concentration
  - shielding of the relevant non-equilibrium source
  - cleaner superconductors
  - excite quasiparticles to delocalized states ?
- physical observables ?
  - EM absorption …
  - role of large space and time fluctuations ?
- role of spin: polarized state in small islands (in progress)
  - non-equilibrium / mesoscopic scenario
    for spontaneous spin polarization
  - more excess quasiparticles in small islands than in the bulk
  - low-frequency flux noise ?

THANK YOU !

Ref: A. Bespalov *et al.*, PRL **117**, 117002 (2016)
+ *in progress*