Irreversible entropy production in non-equilibrium quantum processes

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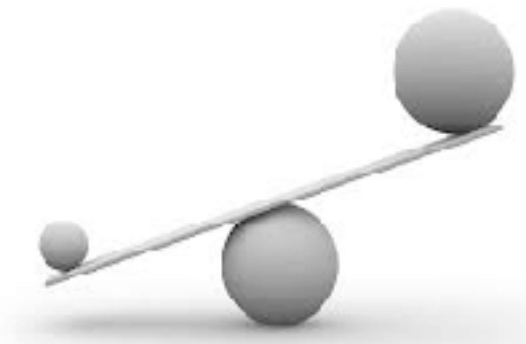
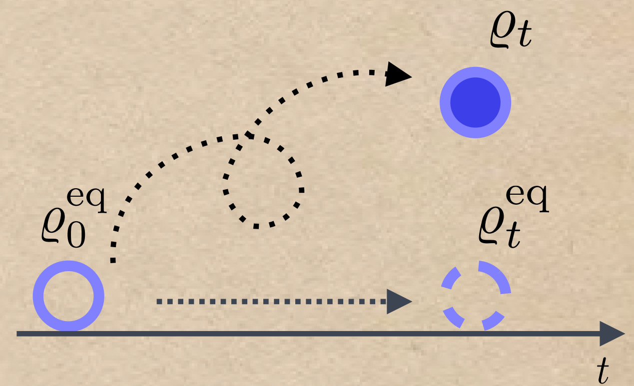
Content & structure



Irreversibility & entropy production

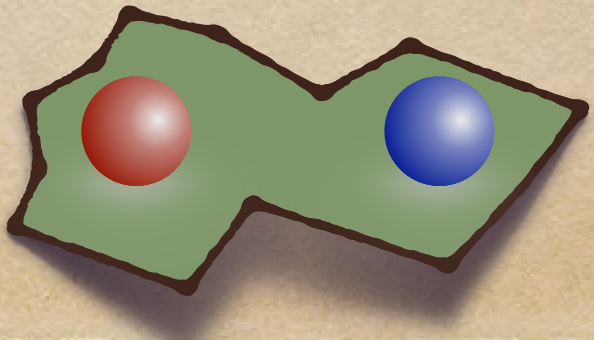
Issues with current formulations of entropy production/flux rate



$$S_2(\varrho) = -\ln \mathrm{Tr}\,\varrho^2$$

Proposal for Renyi-2 based formulation of entropy production

Observability & link to non-classical features



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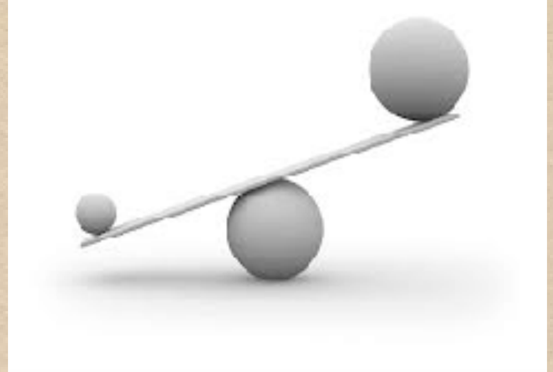
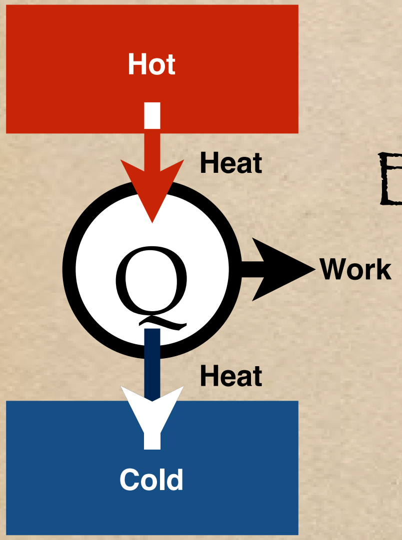
Why entropy production?

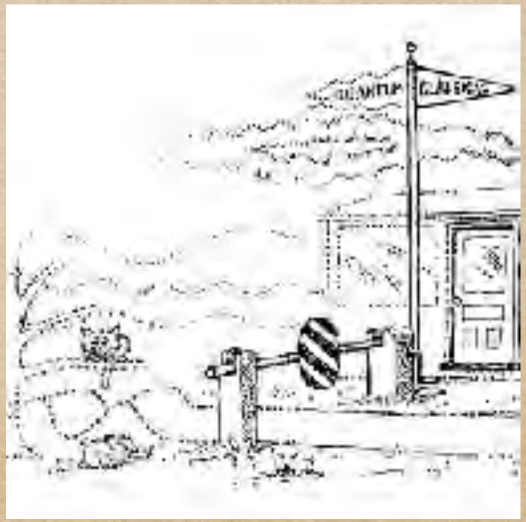
Non-equilibrium processes dissipate energy. This produces irreversible increase of entropy





Entropy production for estimating the performance of devices (exergy is reduced by irreversibility)

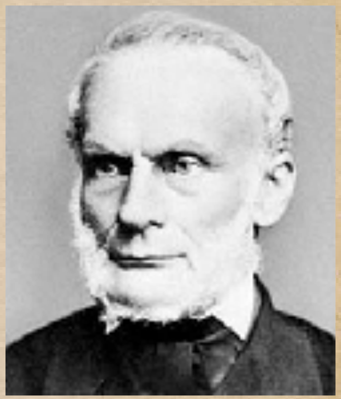
Fantastic framework for pinpointing the quantum-to-classical transition



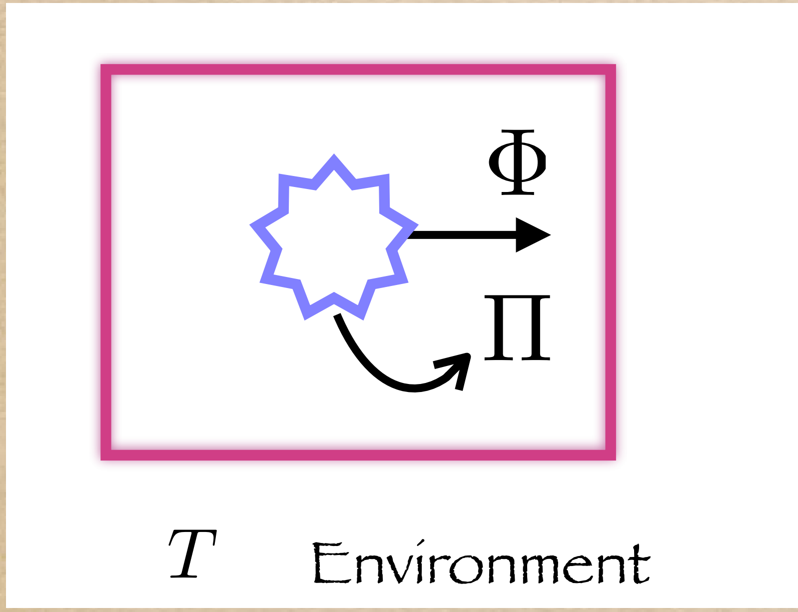
Entropy production

Second Law: $\Delta S \geq \int \frac{\delta Q}{T}$ $\qquad$ $\Delta S = \Sigma + \int \frac{\delta Q}{T}$

Clausius: "Uncompensated transformation" $\qquad$ Entropy production



Rudolf Clausius



$$\frac{dS}{dt} = \Phi(t) + \Pi(t)$$

$\Pi(t)$ Entropy production rate

$\Phi(t)$ Entropy flux rate

Stationary state $\Pi_s = -\Phi_s$

Important figure of merit for optimisation of thermal machines

Role of quantum fluctuations in entropy production

Entropy production

Π, Φ are not observable. No continuity equation for entropy

No unifying theory of entropy production, to date

$$\Pi(t)$$

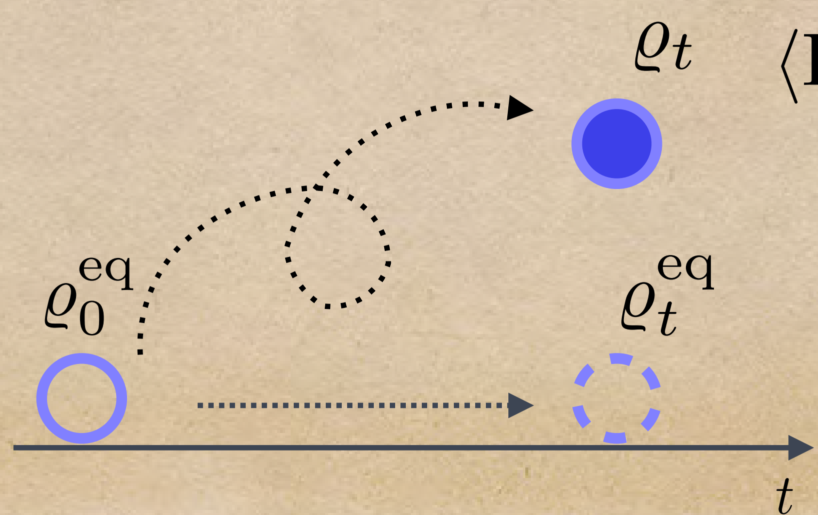
L Onsager
(theory of chemical kinetics)
related to energy currents

J Schnakenberger
transition rates of a system
governed by a master equation

G Gallavotti & E Cohen
integral fluctuation theorem
$\langle e^{-\Sigma} \rangle = 1$

Ch. Jarzynski
& Tasaki-Crooks

$$\frac{P_{\mathrm{f}}(+\Sigma)}{P_{\mathrm{b}}(-\Sigma)} = e^{\Sigma}$$



$$\langle \Pi(t) \rangle = -\partial_t S(\varrho_t || \varrho_t^{\mathrm{eq}}) - \mathrm{Tr}[\varrho_t \partial_t \ln \varrho_t^{\mathrm{eq}}]$$

H Spohn, J Lebowitz

S Deffner, E Lutz

H-P Breuer

Entropy production which entropy to use?

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}(\rho)$$

$$\Pi_{vN}(t) = -\partial_t S_{vN}(\rho|\rho_t^*)$$

Spohn, Lebowitz
Deffner & Lutz
Donald, Breuer

For thermal bath: $\Pi_{vN}(t) = \dfrac{dS_{vN}}{dt} + \Phi_{vN}(t)$

$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$

Rudolf Clausius

Energy flux from system to environment

Entropy production which entropy to use?

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}(\rho)$$

$$\Pi_{vN}(t) = -\partial_t S_{vN}(\rho|\rho_t^*)$$

Spohn, Lebowitz

Deffner & Lutz

Donald, Breuer

For thermal bath: $\Pi_{vN}(t) = \frac{dS_{vN}}{dt} + \Phi_{vN}(t)$

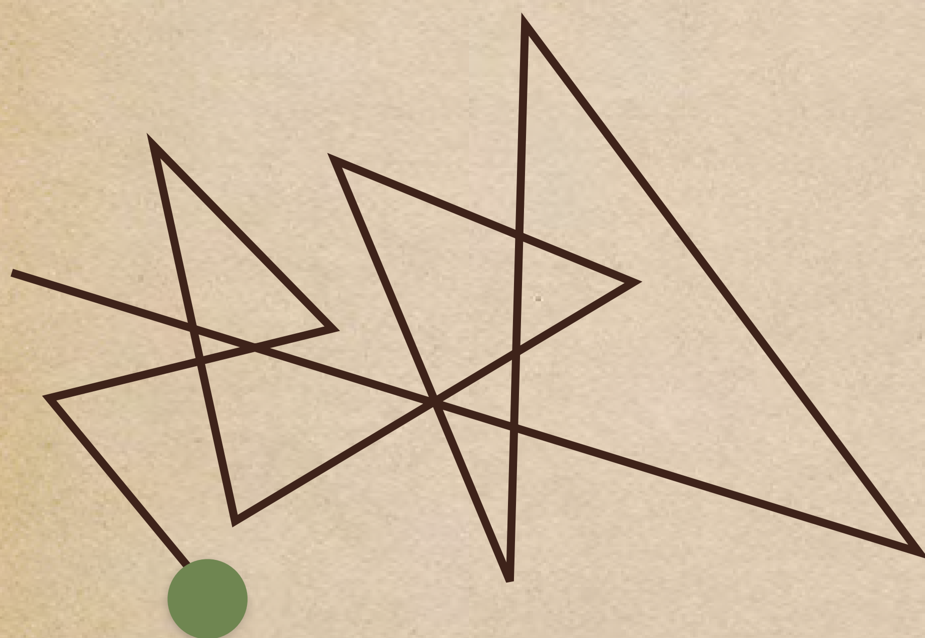
$$= \frac{dS_{vN}}{dt} + \frac{\Phi_E(t)}{T}$$

$\Pi(t), \Phi(t)$ diverge as $T \to 0$

Idealised large heat reservoirs

Several attempts at fixing it....

Entropy production
which entropy to use?



$$\Sigma(t) \equiv D[\rho(t)||\rho_s(t)\prod_r \rho_r^{\mathrm{eq}}] \geq 0$$

Nice physical interpretation:
how far is the state of the compound
a factorised system-environment state?

However: it does not increase monotonically in time (signature of recurrence?). Monotonicity only for large environments

Esposito et al. NJP 12, 013013 ('10); Pucci et al. J Stat P04005 ('13)

Our proposal for q-harmonic systems

$$S = -\int d^2\alpha \, W(\alpha) \ln W(\alpha)$$

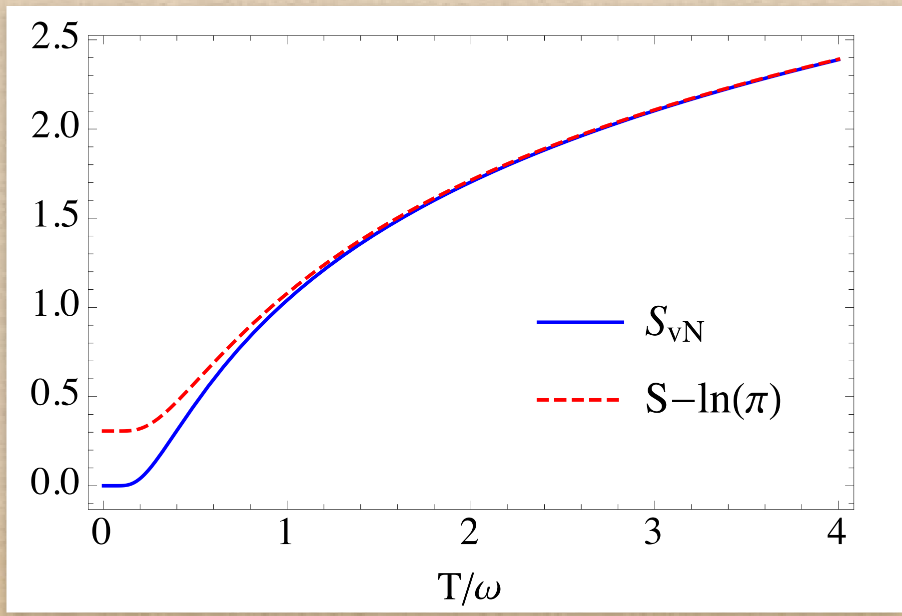
Entropy of the Wigner function

For Gaussian states:

- coincides with Rènyi-2 entropy $S_2(\varrho) = -\ln \mathrm{Tr}\varrho^2$ → can be directly related to free energy difference
  J C Baez, arXiv 1182.2098 (2011)

- satisfies the strong sub-additivity inequality → can be used to construct correlation measures $\mathcal{I}_2(\varrho_{a:b})$
  Adesso, Girolami, Serafini (2014)

- for thermal states:



$$\Pi(t) = -\partial_t S(W(t)|W_{eq}) \geq 0 \quad \text{(Gaussian states)}$$

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

Why it makes sense

$$\Pi(t) = -\int \mathrm{d}^2\alpha\, \mathcal{D}(W) \ln(W/W_{\mathrm{eq}})$$

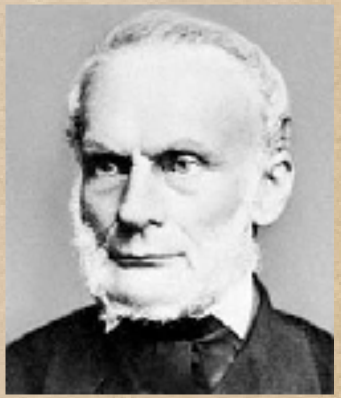
For a single harmonic oscillator in a thermal bath:

$$\Phi(t) = \frac{\gamma}{\bar{n}+1/2}(\langle a^\dagger a\rangle - \bar{n})$$

Observable!!

$$= \frac{\Phi_E}{\omega(\bar{n}+1/2)} \simeq \frac{\Phi_E}{T}$$

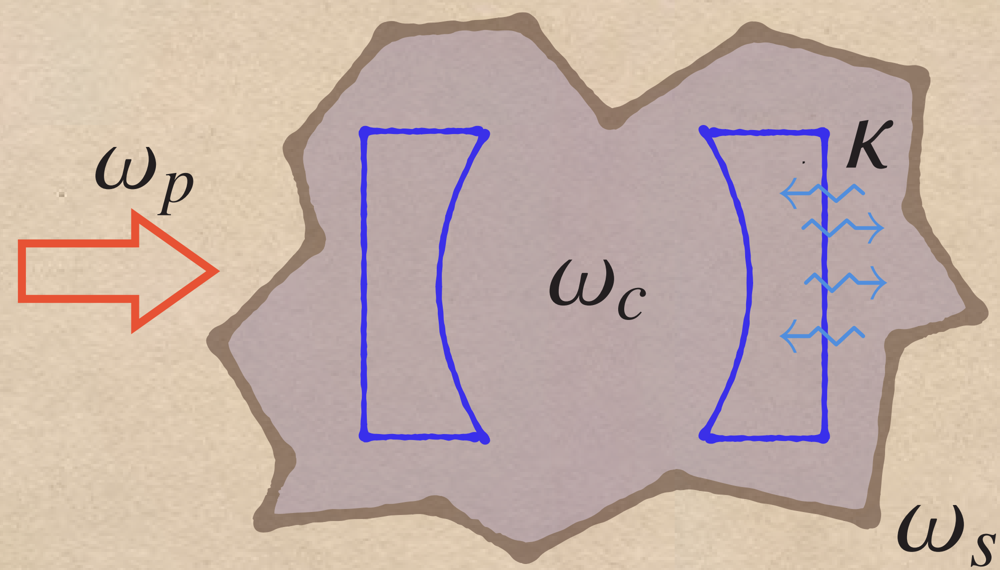
but no divergence at zero-temperature



Rudolf Clausius

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

For a squeezed bath



$$\Phi(t) = \frac{\gamma}{\bar{n} + 1/2}\left(\langle a^\dagger a \rangle - \bar{n}\right)$$

$$\Pi = \frac{2\kappa\Delta_{sc}^2}{\kappa^2 + \Delta_{sc}^2}\sinh^2(2r) + \frac{4\kappa|\mathcal{E}|^2}{\kappa^2 + \Delta_{cp}^2}\cosh(2r) + 4\kappa\mathrm{Re}\left[\frac{\mathcal{E}^2 e^{-i(2\Delta_{ps}t+\theta)}}{(\kappa + i\Delta_{cp})^2}\right]\sinh(2r).$$

J. Santos, G. Landi, and M Paternostro, Phys Rev Lett 118, 220601 (2017)

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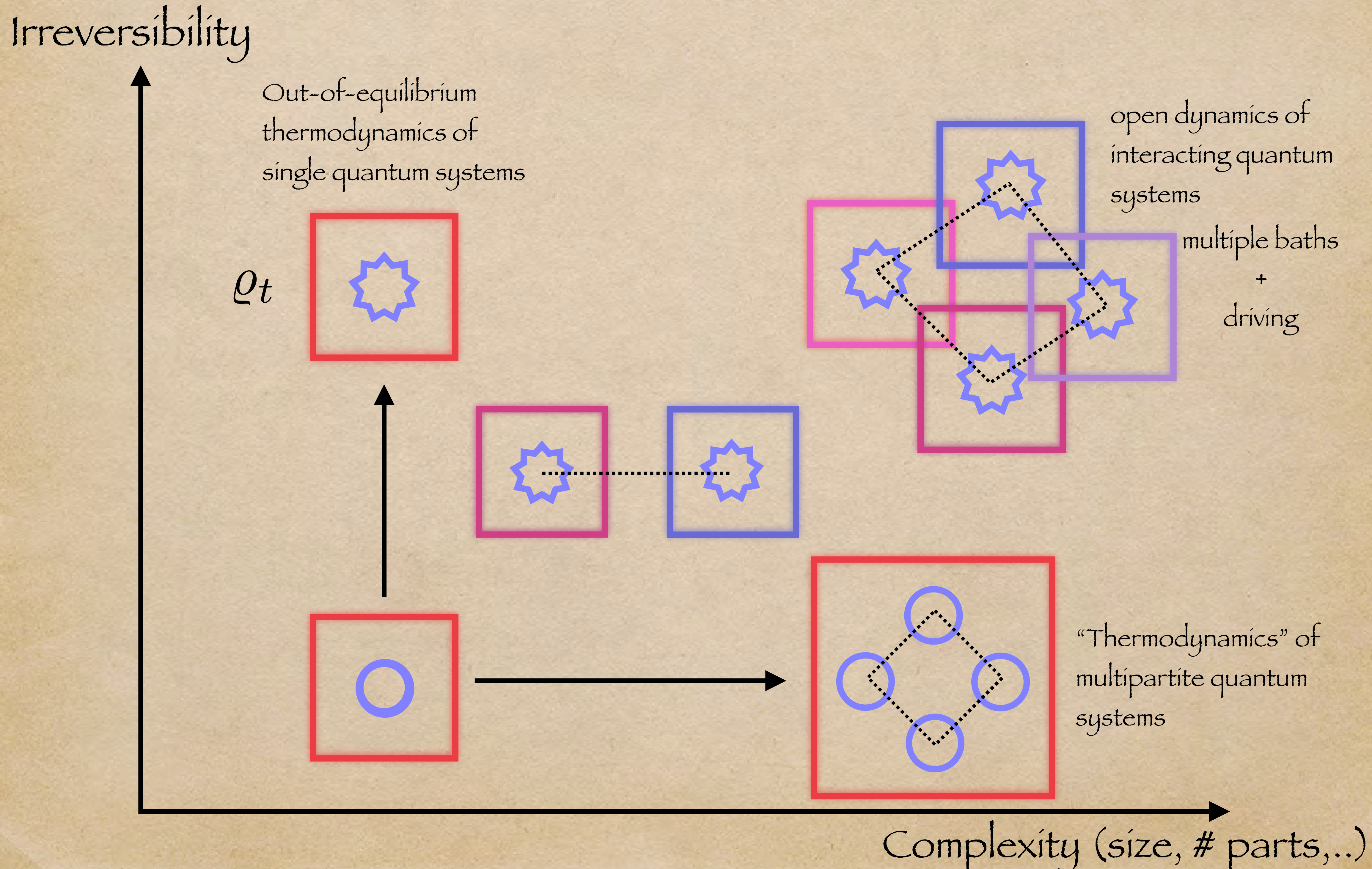
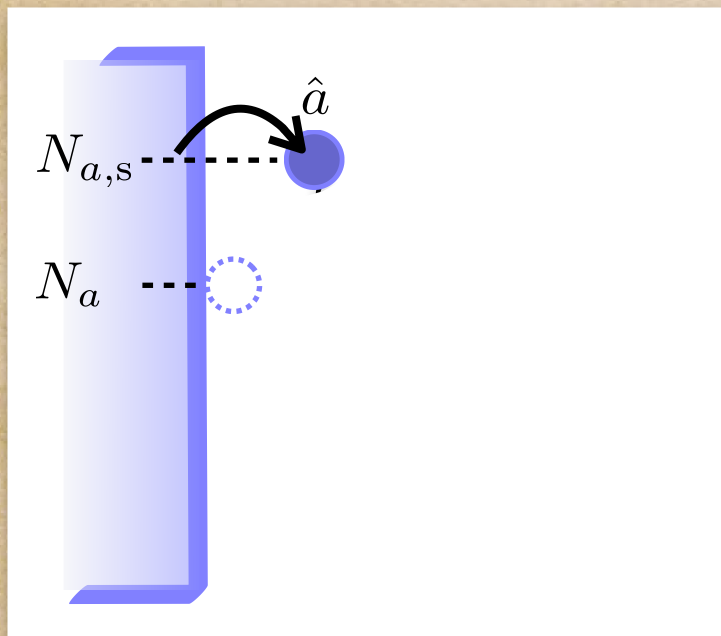
Entropy production: open questions



Entropy production in open systems



For a single harmonic oscillator in a thermal bath:

$$\Pi_s = 2\kappa_a\left(\frac{\langle\hat{q}_a^2\rangle_s + \langle\hat{p}_a^2\rangle_s}{2N_a+1} - 1\right)$$

$$\Pi_\mathrm{s} = 2\kappa_a\left(\frac{\langle\hat{q}_a^2\rangle_\mathrm{s} + \langle\hat{p}_a^2\rangle_\mathrm{s}}{2N_a+1} - 1\right) + 2\kappa_b\left(\frac{\langle\hat{q}_b^2\rangle_\mathrm{s} + \langle\hat{p}_b^2\rangle_\mathrm{s}}{2N_b+1} - 1\right)$$

Experimentally testable (and indeed tested!)

M Brunelli and MP, arXiv:1610.01172 (2016)

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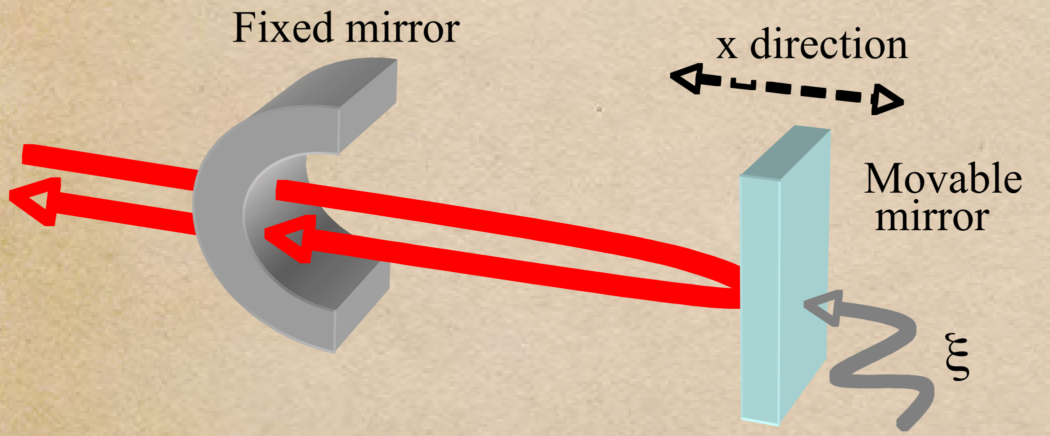
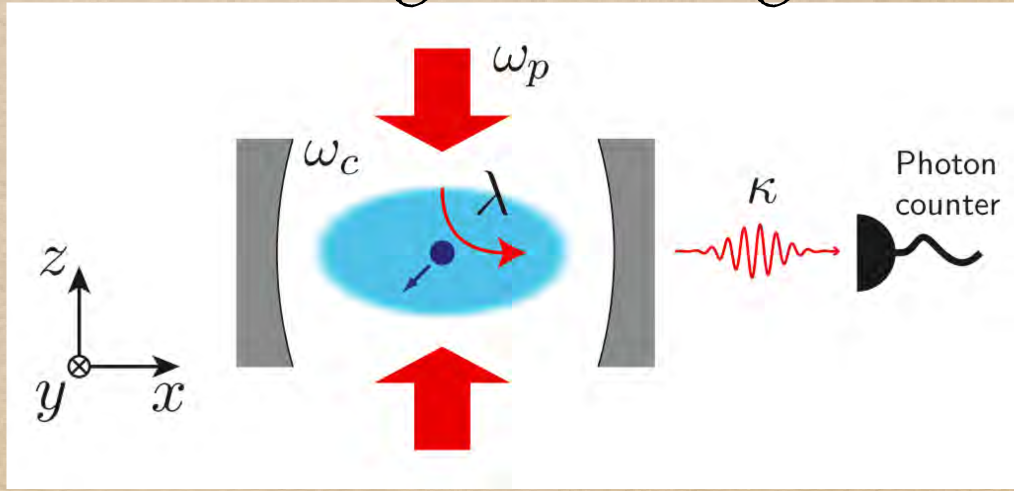
Entropy production & mesoscopics

Optomechanics



$$H = \frac{\hbar\omega}{2}(p^2 + q^2) + \hbar(\omega_c - gq)a^\dagger a$$
$$+ i\hbar\mathcal{E}(a^\dagger e^{-i\omega_0 t} - a e^{i\omega_0 t})$$

Intra-cavity atomic systems

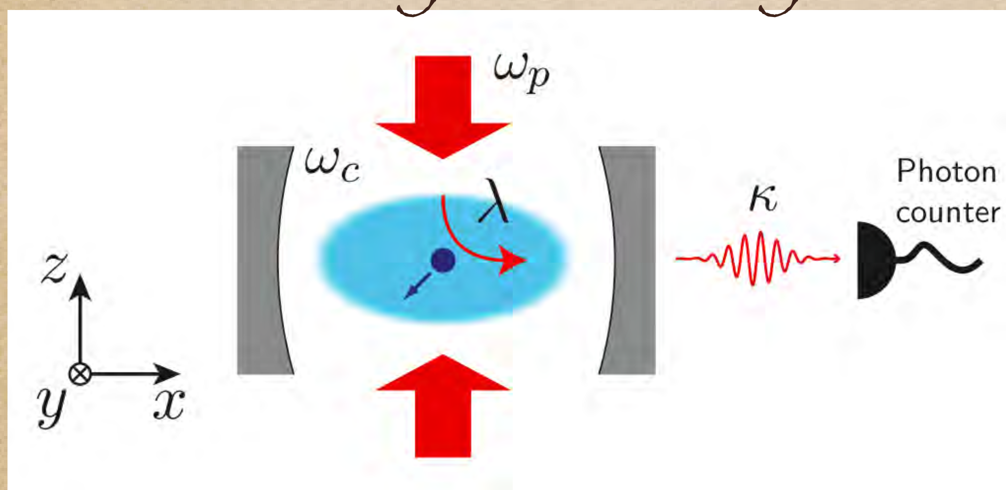


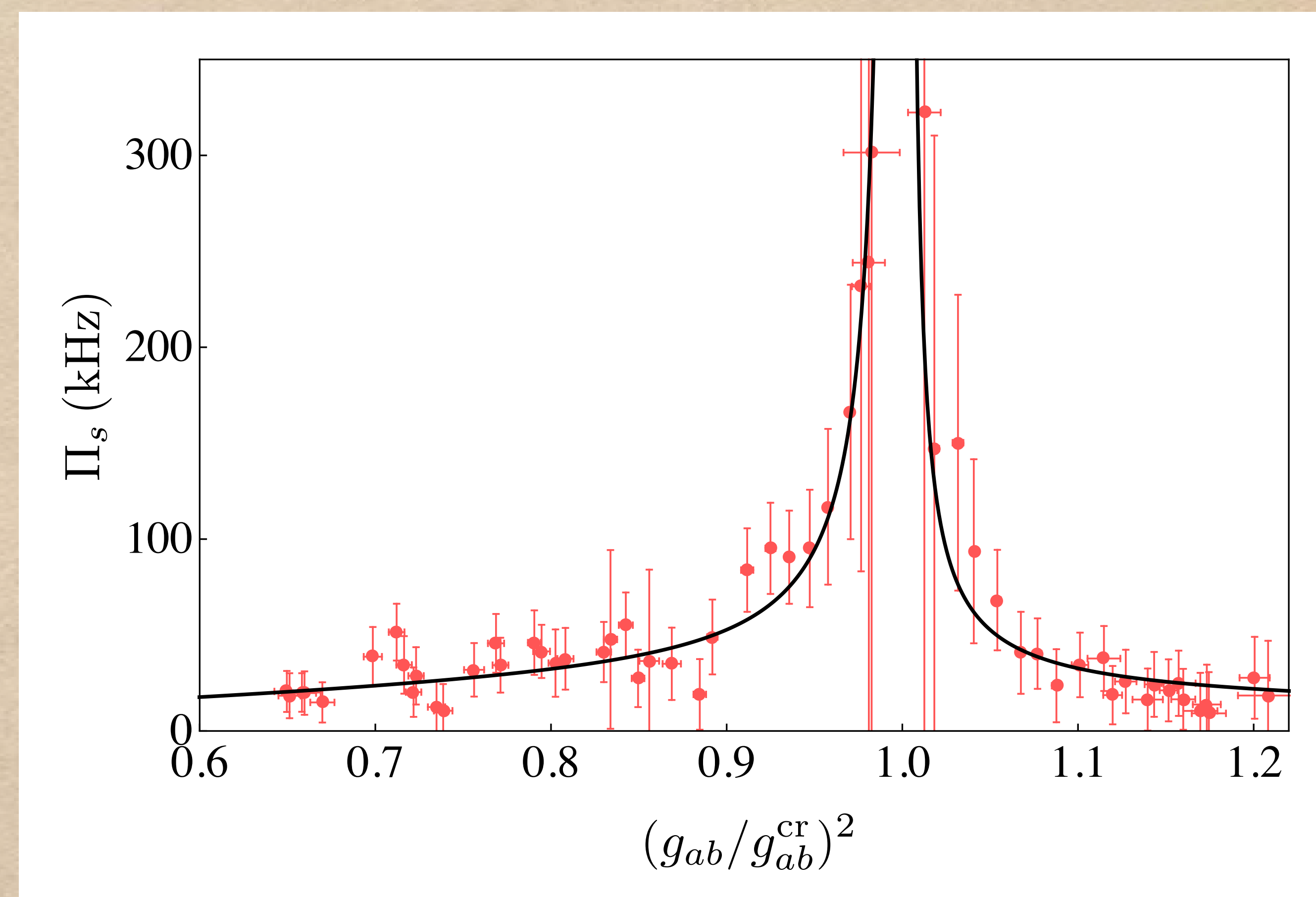
$$\hat{H} = \omega_0 \hat{J}_z + \omega \hat{a}^\dagger \hat{a} + \frac{2\lambda}{\sqrt{N}}\left(\hat{a} + \hat{a}^\dagger\right)\hat{J}_x$$

M Brunelli et al. arXiv:1602.06958 (2016)

Entropy production & mesoscopics

Intra-cavity atomic systems





M Brunelli et al. arXiv:1602.06958 (2016)

What makes this framework quantum?

$$F(\rho) = F_{\mathrm{eq}} + TS(\rho||\rho_{\mathrm{eq}})$$ Non-equilibrium free energy

$$F(\rho) \geq F_{\mathrm{eq}}$$ Equilibration implies decrease of free energy

$$\Pi = -\frac{1}{T}\frac{\mathrm{d}F(\rho)}{\mathrm{d}t} \geq 0 \quad \Pi = 0 \text{ iff } \rho = \rho_{\mathrm{eq}}$$

$$S(\rho||\rho_{\mathrm{eq}}) = \mathcal{S}(p||p_{\mathrm{eq}}) + \mathcal{C}(\rho)$$

diagonal entropy

relative entropy of coherence (Baumgratz, Cramer, Plenio)

G Francica, J Goold, and F Plastina, arXiv:1707.06950 (2017)

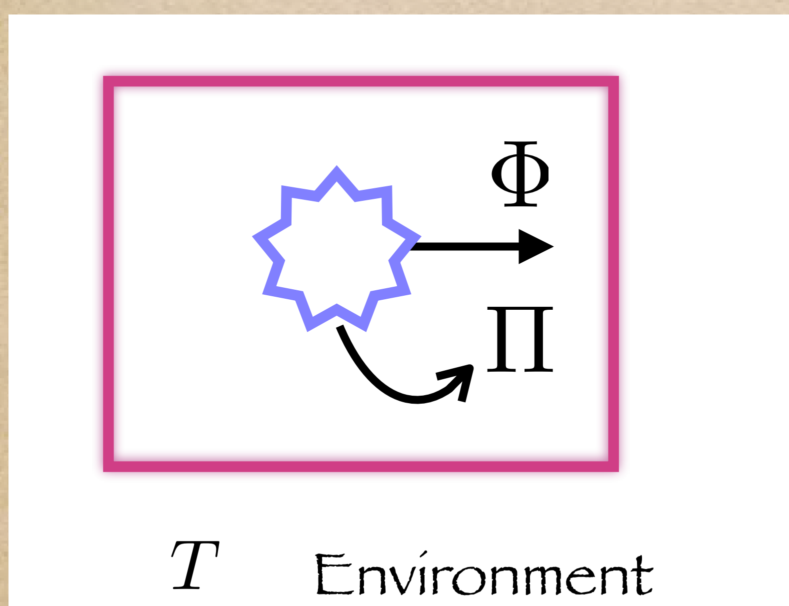
J Santos, L Celeri, G T Landi, and M Paternostro, arXiv:1707.08946 (2017)

What makes this framework quantum?

$$\Pi = \Pi_d + \Upsilon$$

$$-\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{S}(p||p_{\mathrm{eq}}) \qquad -\frac{\mathrm{d}\mathcal{C}(\rho)}{\mathrm{d}t}$$

Interpretation to the mismatch between entropy production in quantum and classical settings



$$\Phi = \Pi - \frac{\mathrm{d}S}{\mathrm{d}t} = \sum_n \frac{\mathrm{d}p_n}{\mathrm{d}t}\ln p_{\mathrm{eq}}^n$$

entropy flux has no contribution arising from quantum coherences

G Francica, J Goold, and F Plastina, arXiv:1707.06950 (2017)
J Santos, L Celeri, G T Landi, and M Paternostro, arXiv:1707.08946 (2017)

Changing perspective

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = 2\kappa\left[a\rho_S a^\dagger - \frac{1}{2}\{a^\dagger a, \rho_S\}\right]$$

$$H_T = \omega a^\dagger a + \sum_k \Omega_k b_k^\dagger b_k + \sum_k \gamma_k (a^\dagger b_k + b_k^\dagger a)$$

$$\partial_t W_E = \sum_k \partial_{\beta_k} J_k + \partial_{\beta_k^*} J_k^*$$

$$\Pi = \frac{\mathrm{d}\mathcal{I}_{SE}}{\mathrm{d}t} + \frac{\mathrm{d}S(W_E || W_E(0))}{\mathrm{d}t}$$

J Santos, A de Paula, R Drumond, G. T. Landi, and M Paternostro

arXiv:1804.02970 (2018)—To Appear in PRA as a Rapid Communication

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The Belfast crew

Go raibh maith agaibh

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