Many-body quantum chaos at $\hbar \sim 1$: Analytic connection to random matrix theory

Tomaž Prosen

Faculty of mathematics and physics, University of Ljubljana, Slovenia

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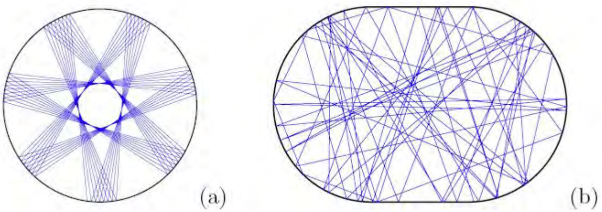
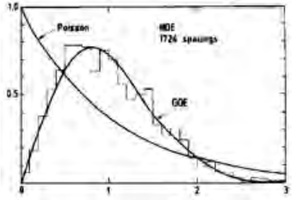
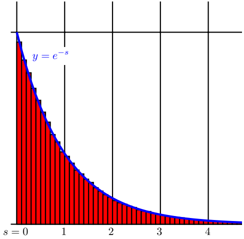
Collaborators: *Bruno Bertini, Pavel Kos, Marko Ljubotina*

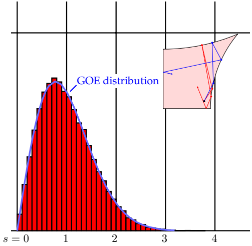
Chaos and random matrix theory

[Casati, Guarneri and Val-Gris 1980, Berry 1981, Bohigas, Giannoni and Schmit 1984, ...]









[Berry 1985, Sieber and Richter 2001, Müller, ...and Haake 2004,2005]

A simple key object: Spectral pair correlation function

$$R(\epsilon) = \frac{1}{\bar{\rho}^2}\left\langle \rho(E+\frac{\epsilon}{2\pi\bar{\rho}})\rho(E-\frac{\epsilon}{2\pi\bar{\rho}})\right\rangle - 1$$

or the spectral form factor

$$K(\tau) = \frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{d}\epsilon R(\epsilon) e^{2\mathrm{i}\epsilon\tau} \sim \left\langle \left|\sum_n e^{\mathrm{i}\epsilon_n\tau}\right|^2\right\rangle \sim \langle|\mathrm{tr}\, e^{-\mathrm{i}H\tau}|^2\rangle$$

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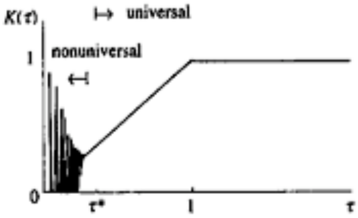
- universal described by RMT,
- non-universal described by short classical periodic orbits.



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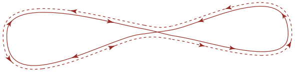

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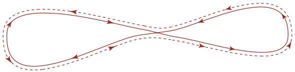
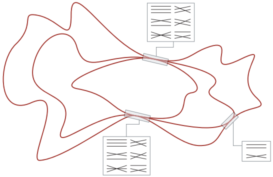
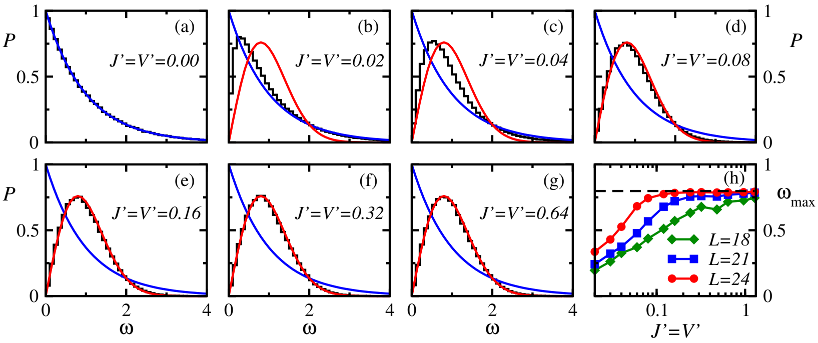


To all orders, RMT terms is reproduced by considering full combinatorics of self-encountering orbits (Müller et al, 2004)



A lot of numerics accumulated to day, showing that integrable many body systems, such as an interacting chain of spinless fermions:

$$H = \sum_{j=0}^{L-1} (-J c_j^\dagger c_{j+1} - J' c_j^\dagger c_{j+2} + \mathrm{h.c.} + V n_j n_{j+1} + V' n_j n_{j+2}), \quad n_j = c_j^\dagger c_j.$$



[from: Rigol and Santos, 2010]
More data: Montamboux et al.1993, Prosen 1999, 2002, 2005, 2007, 2014, Kollath et al. 2010, ...

... or:

Derivation of random matrix spectral form factors for non-integrable spin chains [P. Kos, M. Ljubotina and TP, arXiv:1712.02665].

Setup: Periodically kicked Ising models

$$H(t) = H_0 + H_1 \sum_{m \in \mathbb{Z}} \delta(t - m)$$

where

$$H_0 = \sum_x J_x^1 \sigma_x^{(3)} + \sum_{x<x'} J_{x,x'}^2 \sigma_x^{(3)} \sigma_{x'}^{(3)} + \cdots, \quad H_1 = h \sum_x \sigma_x^{(1)}.$$

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Floquet propagator:

$$\begin{aligned} U &= \mathcal{T}\text{-exp}\left(-\mathrm{i}\int_0^1 \mathrm{d}t\, H(t)\right) = VW, \\ W &= e^{-\mathrm{i}H_0}, \quad V = e^{-\mathrm{i}H_1} = v^{\otimes \ell}, \quad v = \begin{pmatrix} \cos h & \mathrm{i}\sin h \\ \mathrm{i}\sin h & \cos h \end{pmatrix}. \end{aligned}$$

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N.N. variants of KI model introduced and discussed in:
TP, JPA **31**, L397 (1998); Prog.Theor.Phys.Suppl. **139**, 191 (2000); PRE **65**, 036208, (2002); JPA **40**, 7881 (2007)...

- Consider computational basis of *classical* spin configurations $\underline{s} = (s_1, \ldots, s_\ell)$, $s_x \in \{0, 1\}$, where interacting propagator acts as a pure multiplicative phase

$$W|\underline{s}\rangle = e^{\mathrm{i}\theta_{\underline{s}}}|\underline{s}\rangle, \quad \theta_{\underline{s}} = -\sum_x J^1_x (-1)^{s_x} - \sum_{x<x'} J^2_{x,x'}(-1)^{s_x + s_{x'}} + \cdots$$

while the matrix elements of V factorize

$$\langle \underline{s}|V|\underline{s}'\rangle = \prod_{x=1}^{\ell} v_{s_x, s'_x}.$$

- The spectral form factor then reads ($\mathcal{N} = 2^\ell$):

$$K(t) = \langle (\mathrm{tr}\, U^t)(\mathrm{tr}\, U^t)^* \rangle - \mathcal{N}^2 \delta_{t,0}$$
$$= \sum_{\underline{s}_1, \ldots, \underline{s}_t} \sum_{\underline{s}'_1, \ldots, \underline{s}'_t} \langle e^{\mathrm{i}\sum_{\tau=1}^{t}(\theta_{\underline{s}_\tau} - \theta_{\underline{s}'_\tau})} \rangle$$
$$\times \prod_{x=1}^{\ell}\prod_{\tau=1}^{t} v_{s_{x,\tau}, s_{x,\tau+1}} v^*_{s'_{x,\tau}, s'_{x,\tau+1}}.$$

- Assume pseudo-randomness of phases:

$$\langle e^{\mathrm{i}\sum_{\tau=1}^{t}(\theta_{\underline{s}_\tau} - \theta_{\underline{s}'_\tau})} \rangle = \delta_{\langle \underline{s}_1, \ldots, \underline{s}_t\rangle, \langle \underline{s}'_1, \ldots, \underline{s}'_t\rangle} + \text{fluctuations}.$$

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- Assume further that (i) fluctuations can be neglected (at least in the limit $\ell \to \infty$), and (ii) the terms ('orbits') where some configuration $\underline{s}_\tau$ is repeated can be neglected as they are exponentially (in ℓ) rare for fixed t

$$\exists \pi \in S_t : \tau \to \pi(\tau), \quad \text{such that} \quad \underline{s}'_\tau = \underline{s}_{\pi(\tau)}.$$

- This implies the following twisted 1D Ising model representation:

$$K(t) = \sum_{\pi \in S_t} Z_\pi^\ell, \qquad \text{where} \qquad Z_\pi = \sum_{s_1,\ldots,s_t} \prod_{\tau=1}^{t} V_{s_\tau, s_{\tau+1}} V^*_{s_{\pi(\tau)}, s_{\pi(\tau+1)}}.$$

- The leading order (in the limit $\ell \to \infty$) contributions come from t cyclic permutations and t anti-cyclic permutations, i.e. all $2t$ permutations π which do not change any neigbour the sequence $\underline{s}$:

$$K(t) \simeq 2t(\mathrm{tr}\, T^t)^\ell = 2t(1 + (\cos 2h)^t)^\ell$$

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where $T = \begin{pmatrix} \cos^2 h & \sin^2 h \\ \sin^2 h & \cos^2 h \end{pmatrix}$ is 1D Ising model transfer matrix.

- This is exactly the leading term of the Random-Matrix-Theory result!

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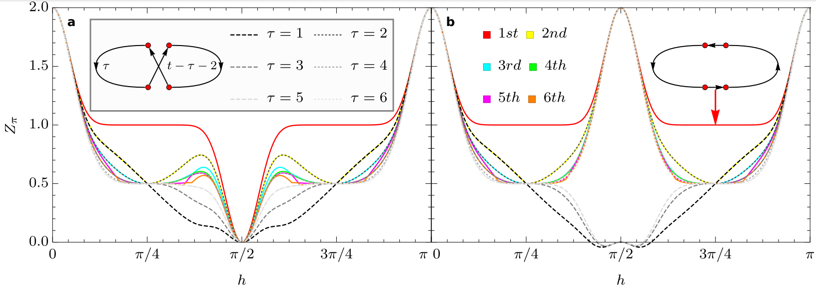
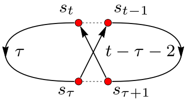
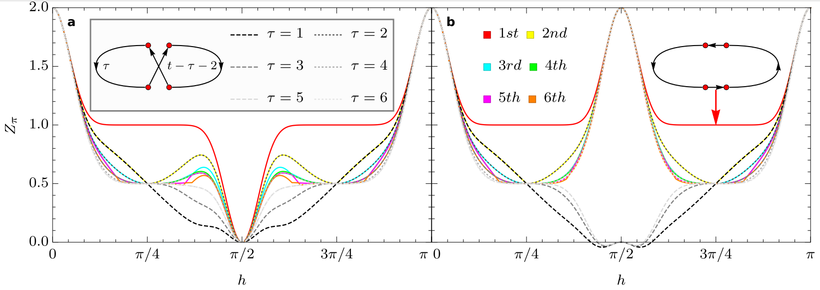


- Most important subleading contributions yield exponentially small (in ℓ) corrections and can be computed using a diagrammatic technique.
- The leading corrections are given by single-cross diagrams ($\lambda = \cos 2h$):

$$Z_X(\tau) = \sum_{s_\tau, s_{\tau+1}, s_{t-1}, s_t} T^{\tau}_{s_t, s_\tau} T^{t-\tau-2}_{s_{\tau+1}, s_{t-1}} v_{s_\tau, s_{\tau+1}} v^*_{s_\tau, s_{t-1}} v_{s_{t-1}, s_t} v^*_{s_{\tau+1}, s_t} = \frac{1}{2}\left(1+\lambda^\tau+\lambda^{t-\tau-2}-\lambda^{t-2}+\lambda^t\right)$$



- Similarly one can derive 2nd order correction in time $-2t^2/\mathcal{N}$ by considering sequences – 'orbits' – $\underline{s}_1, \ldots \underline{s}_t$ with exactly one repetition (analogues of self-encountering Sieber-Richter periodic orbit doublets).



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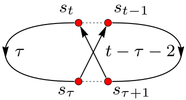

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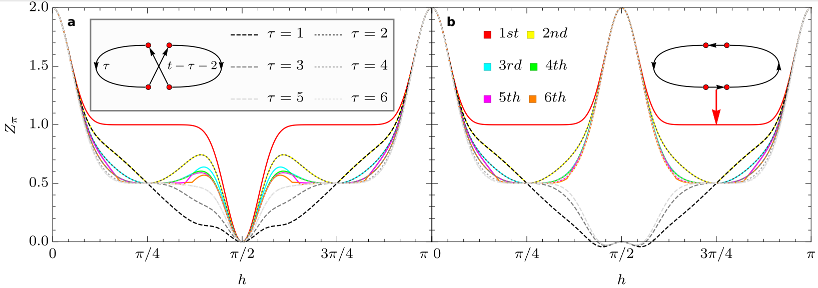

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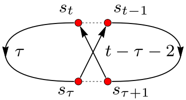

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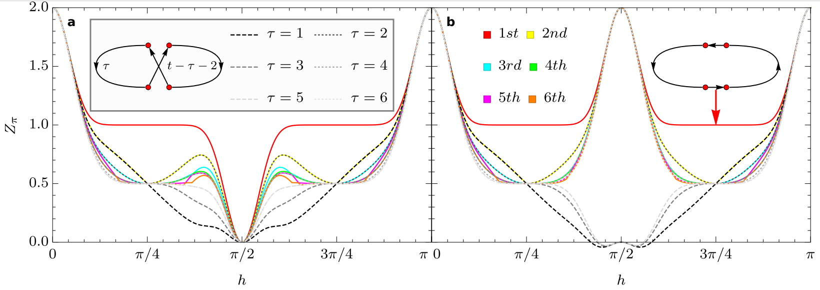

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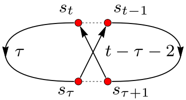

- Similarly one can derive 2nd order correction in time $-2t^2/\mathcal{N}$ by considering sequences – ‘orbits’ – $\underline{s}_1, \ldots \underline{s}_t$ with exactly one repetition (analogues of self-encountering Sieber-Richter periodic orbit doublets).

- Consider two-body clean Ising model with power-law decaying interactions

$$J^1_x = a + \frac{N_1 b}{x^\alpha}, \quad J^2_{x,x'} = \frac{N_2 J}{(x'-x)^\alpha}, \quad J^{k>2}_{x,x'\ldots} \equiv 0,$$

with normalization constants defined as

$$\frac{1}{N_1} = \sum_x \frac{1}{x^\alpha}, \quad \frac{1}{N_2} = \frac{1}{\ell - 1} \sum_{x<x'} \frac{1}{(x'-x)^\alpha}$$

and interaction effectively being *short range* $N_{1,2} = \mathcal{O}(\ell^0)$ for $\alpha > 1$.

- Compute integrated spectral form factor

$$K_{\rm int}(t) = \sum_{\tau=1}^{t} K(\tau)$$

which is expected to be self-averaging.

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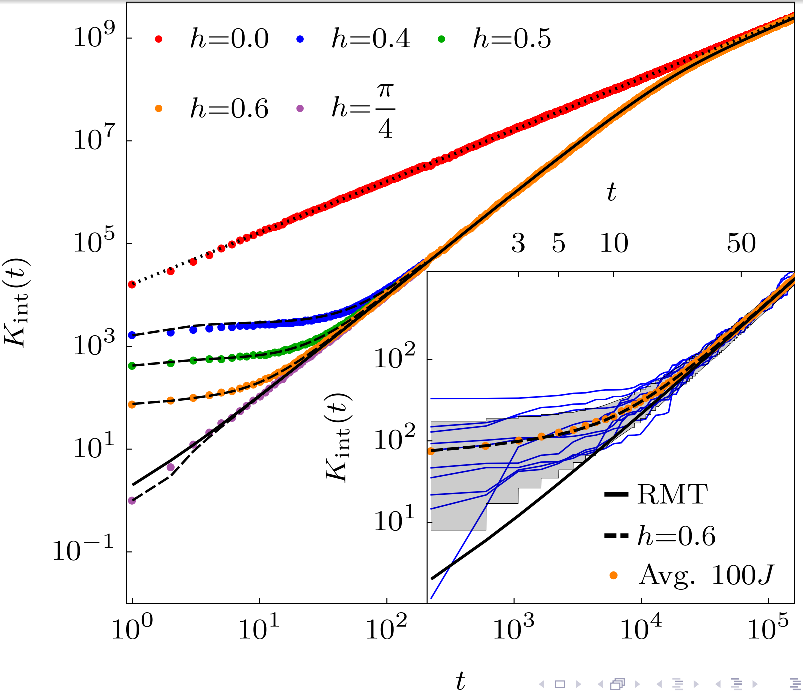
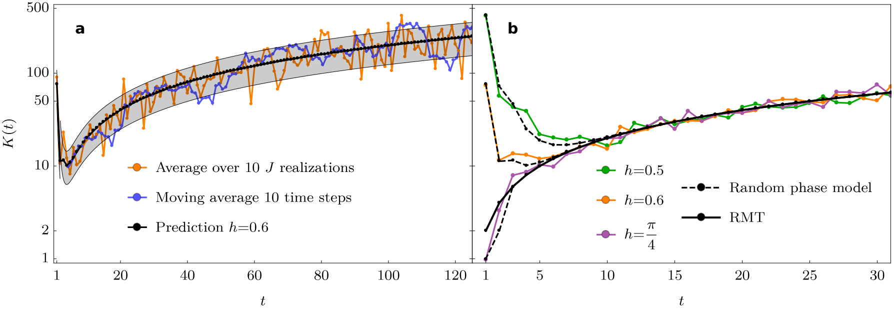
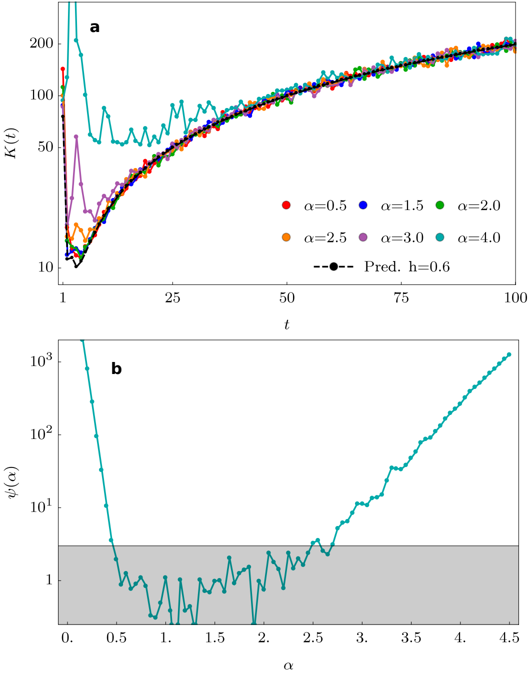


h=0.0
h=0.4
h=0.5
h=0.6
h=π/4
$K_{\rm int}(t)$
10^9
10^7
10^5
10^3
10^1
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
t
3
5
10
50
RMT
h=0.6
Avg. 100J


a
b
500
100
50
10
2
1
K(t)
1
20
40
60
80
100
120
t
Average over 10 J realizations
Moving average 10 time steps
Prediction h=0.6
1
5
10
15
20
25
30
h=0.5
h=0.6
h=π/4
Random phase model
RMT


a
$K(t)$
200
100
50
10
1
25
50
75
100
t
α=0.5
α=1.5
α=2.0
α=2.5
α=3.0
α=4.0
Pred. h=0.6
b
$\psi(\alpha)$
10^3
10^2
10^1
1
0.
0.5
1.
1.5
2.
2.5
3.
3.5
4.
4.5
α

What next?

1. Build rigorous control over the (pseudo)random-phase argument.
2. Consider the case of local interactions $\alpha = \infty$ which needs to be studied separately as the phases $\theta_{\underline{s}}$ are clearly insufficiently pseudo-random then.
3. Introduce quenched disorder and study the ergodicty – MBL transition from the ergodic side.
4. Study spectral correlations beyond 2-point.
5. Adapt the method to study other RMT-like objects in many-body physics, e.g. the entanglement spectrum, dynamical correlation functions, ETH...

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B. Bertini, P. Kos, TP, arXiv:1805.00931

$$U_{\mathrm{KI}} = e^{-iH_{\mathrm{I}}} e^{-iH_{\mathrm{K}}}, \quad H_{\mathrm{I}}[\boldsymbol{h}] \equiv \sum_{x=1}^{L} \left\{ J\sigma_x^{(3)}\sigma_{x+1}^{(3)} + h_x \sigma_x^{(3)} \right\}, \quad H_{\mathrm{K}} \equiv b \sum_{x=1}^{L} \sigma_j^{(1)},$$

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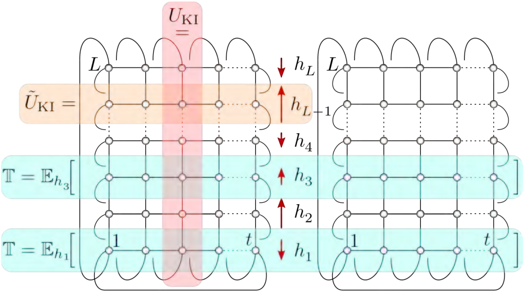
Dual transfer matrix formulation of spectral form factor

$$\bar{K}(t) = \mathbb{E}_{\{h_x\}} \left[({\rm tr}\, U_{\rm KI}^t)({\rm tr}\, U_{\rm KI}^t)^* \right]$$



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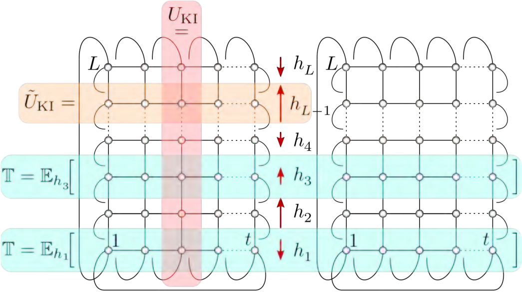

We proved (irrespective of distribution of i.i.d. $\{h_x\}$):

$$\lim_{L\to\infty} \bar{K}(t) = \begin{cases} 2t-1, & t \le 5 \\ 2t, & t \ge 7 \end{cases}, \quad t \text{ odd}.$$