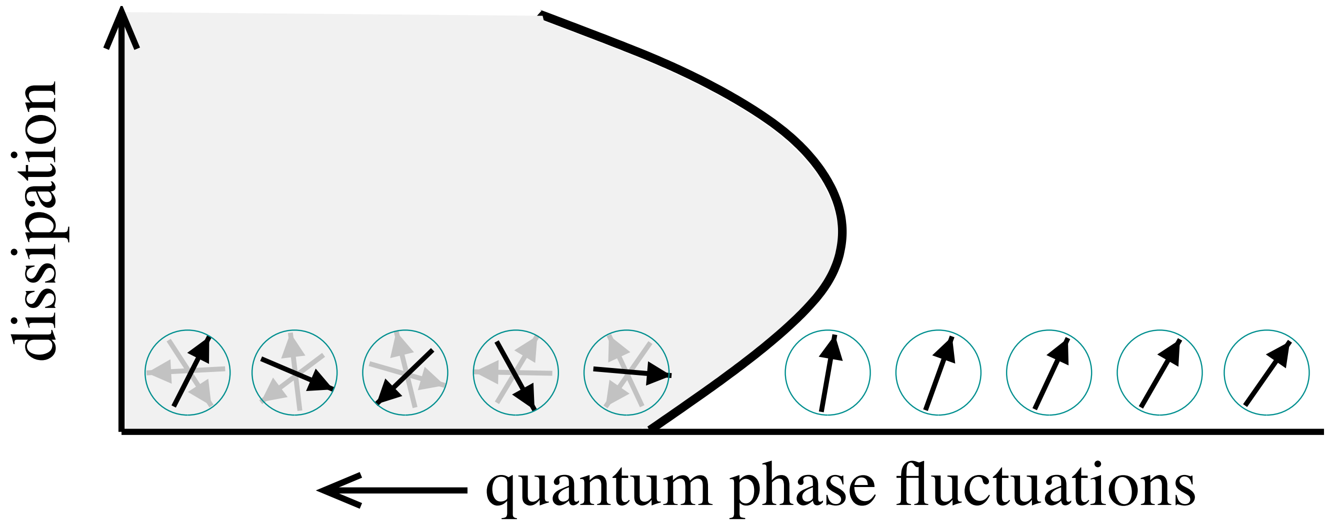
Quantum phase transition with dissipative frustration

Gianluca Rastelli

https://www.rastelli.uni-konstanz.de



Quantum Thermodynamics and Transport, May 8th, Mainz, Germany

collaborators

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Dominik Maile

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EBERHARD KARLS UNIVERSITÄT TÜBINGEN

acknowledgments

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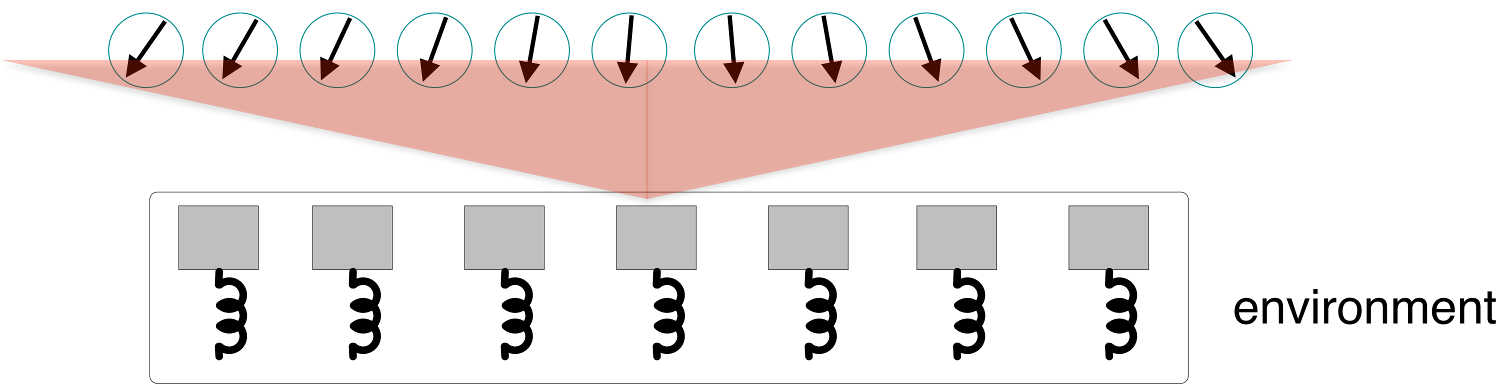
Motivation: mesoscopic many-body systems

- **Examples:** Rydberg atoms in optical lattices, trapped ions, cold atoms in cavities, coupled QED cavities, lattices of qubits, etc.

- **Artificial, synthetic quantum matter**
  - engineered interactions and tunable parameters
  - driving, state preparation and nonequilibrium dynamics
  - “macroscopic” size ⟶ open (dissipative) systems

(individual, coherent quantum units with interaction)



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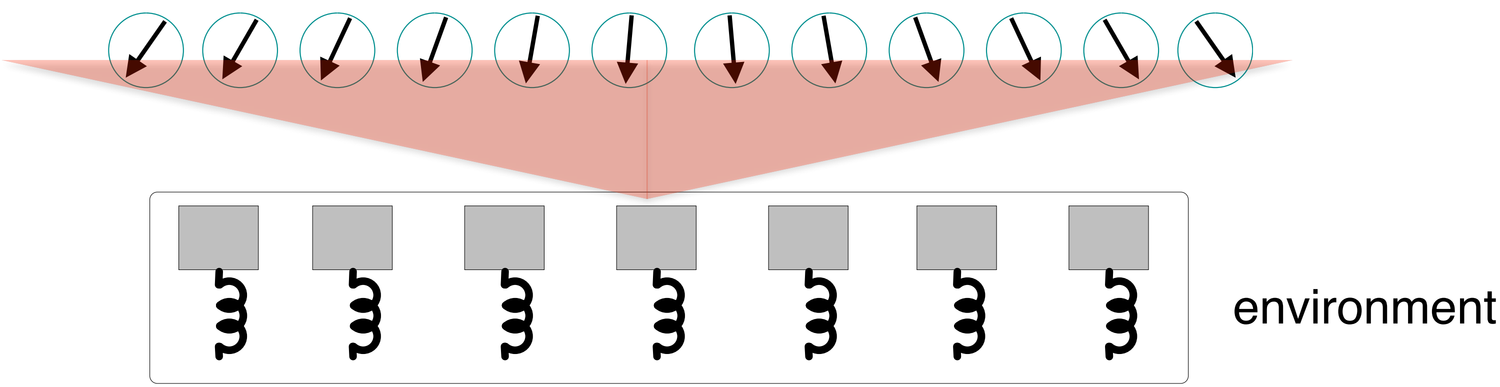
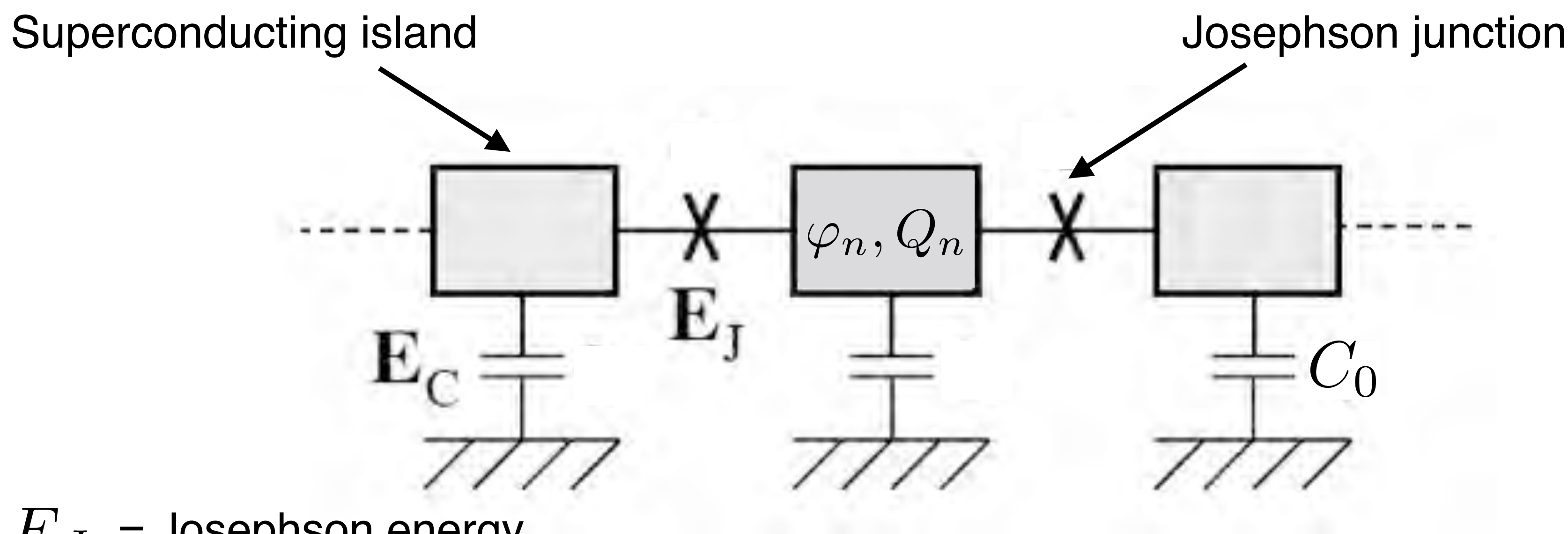


Quantum systems: dissipation affects the thermodynamical equilibrium

Chain of Josephson junctions



E_J = Josephson energy

$E_C = 4e^2/C_0$ = charging energy

$\Delta \longrightarrow \infty$ no quasi-particle excitations

Quantum phase model

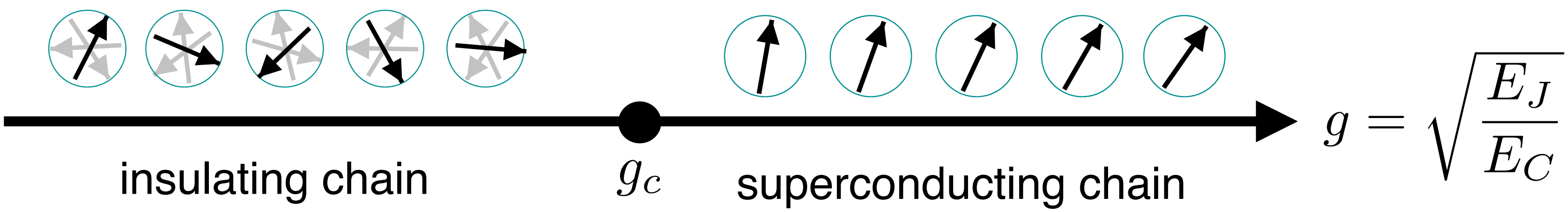
$$\hat{H} = \sum_n \left[\frac{\hat{Q}_n^2}{2C_0} - E_J \cos\left(\Delta\hat{\varphi}_n\right) \right]$$

$$\left[\hat{\varphi}_n, \hat{Q}_m\right] = 2e\, i\, \delta_{nm}$$

$$\Delta\hat{\varphi}_n = \hat{\varphi}_n - \hat{\varphi}_{n-1}$$

Quantum Phase Transition (QPT)

$T = 0K$



Bradley-Doniach Phys. Rev. B **30**, 1138 (1984)

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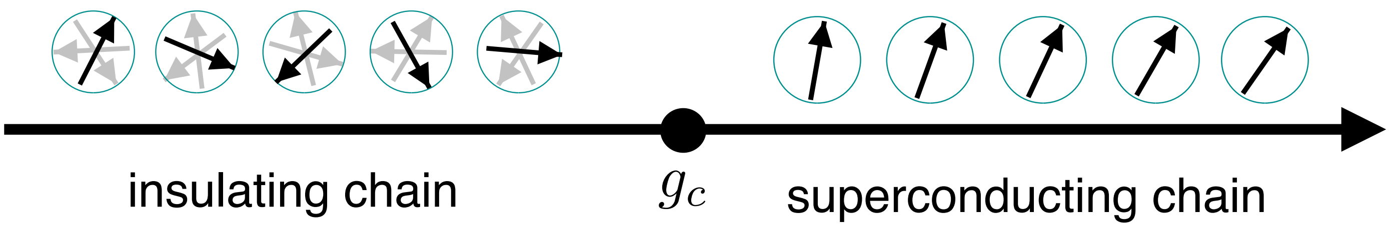
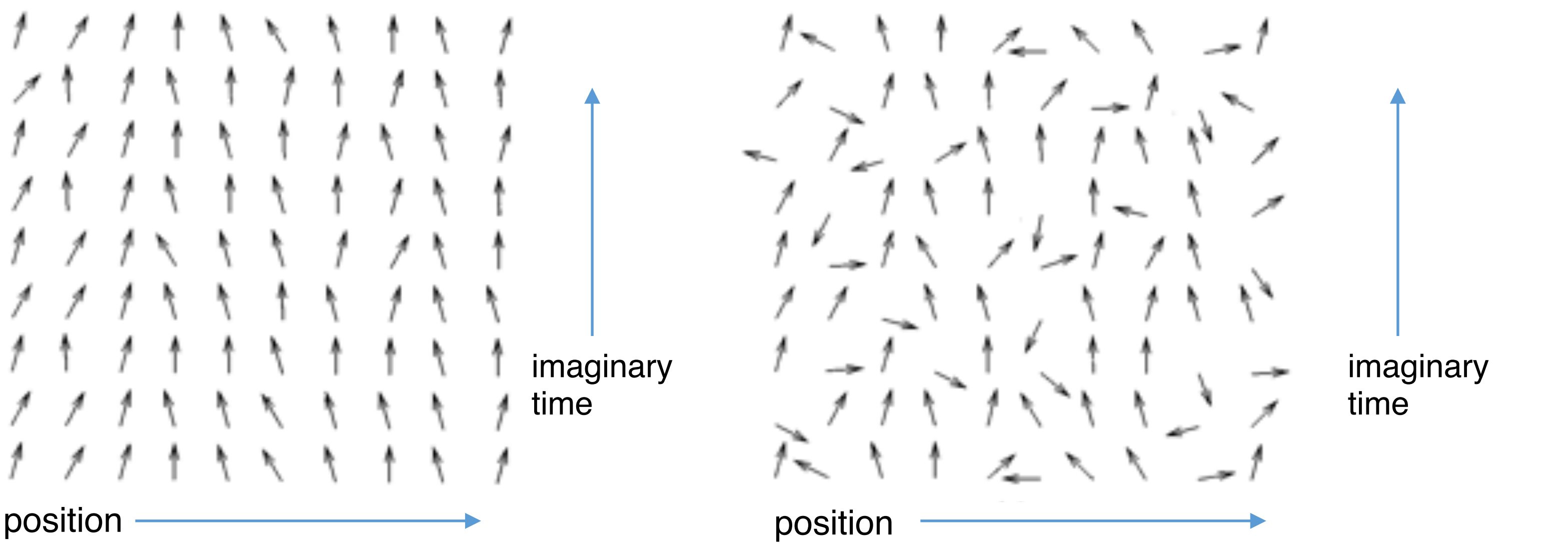


$$g = \sqrt{\frac{E_J}{E_C}}$$

Bradley-Doniach Phys. Rev. B **30**, 1138 (1984)

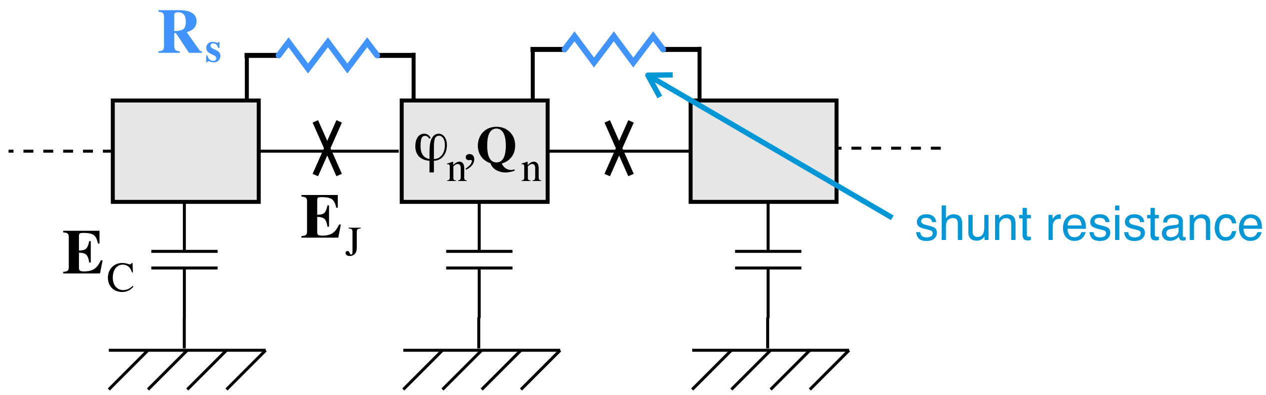
Universal class: BKT transition (1D+1 mapping, quantum ⟶ classical)



Sondhi et al, RMP **69**, 315 (1997)

Dissipative phase model

Open system $\hat{H} = \hat{H}_S + \hat{H}_{env} + \hat{H}_{int}$



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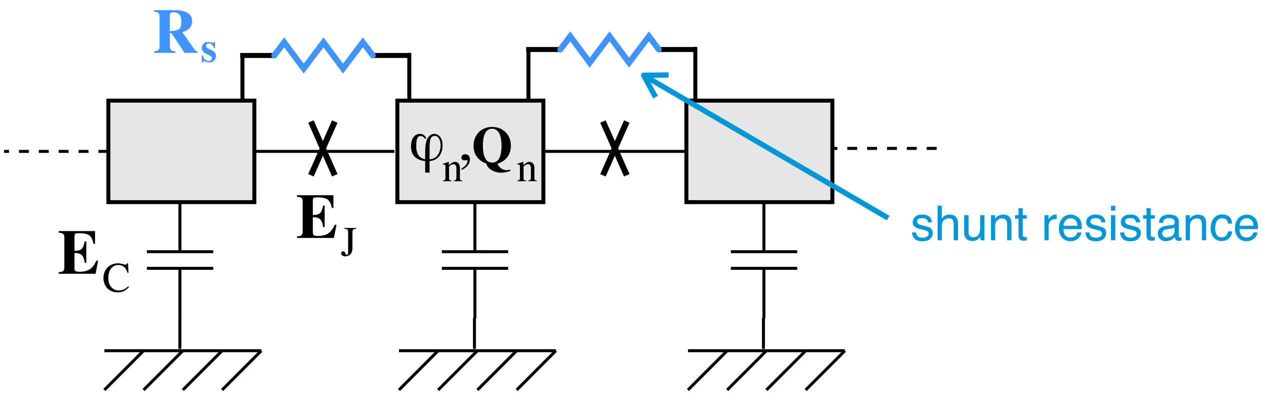
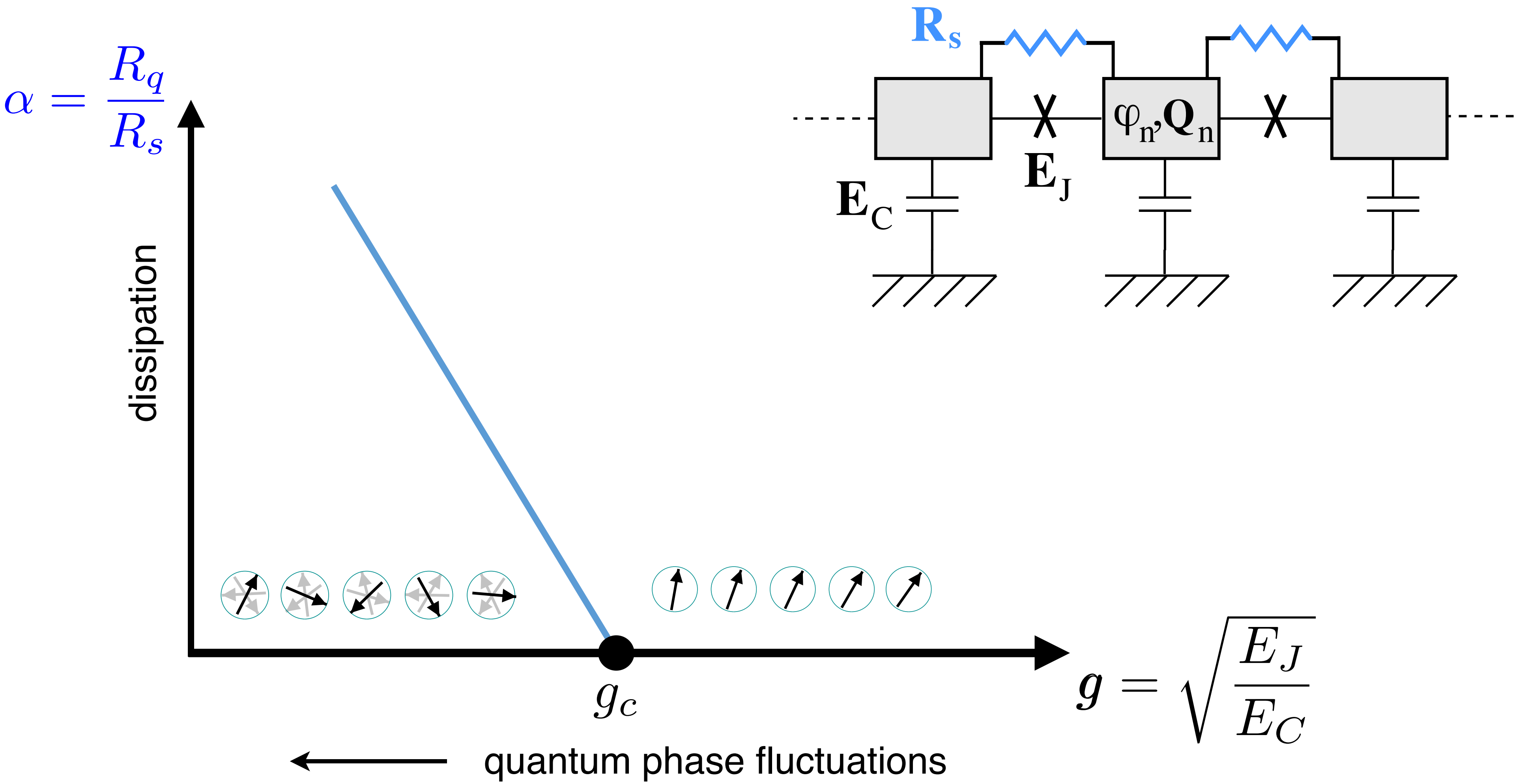


$$I = \frac{V}{R} = \frac{1}{R}\left(\frac{\hbar}{2e}\right)\frac{d\left(\varphi_i - \varphi_{i-1}\right)}{dt} \longrightarrow \alpha = \frac{R_q}{R_s}$$ coupling strength

$$R_q = \frac{h}{4e^2}$$ resistance quantum

$\alpha = h\gamma/E_C$ $\qquad \gamma = 1/(R_s C_0)$ decay rate

Dissipative QPT



Chakravarty et al., PRL **56**, 2303 (1986)

Panyukov,Zaikin, Phys.Lett.A **124**, 325 (1987)

Korshunov, EPL **9**, 107 (1989)

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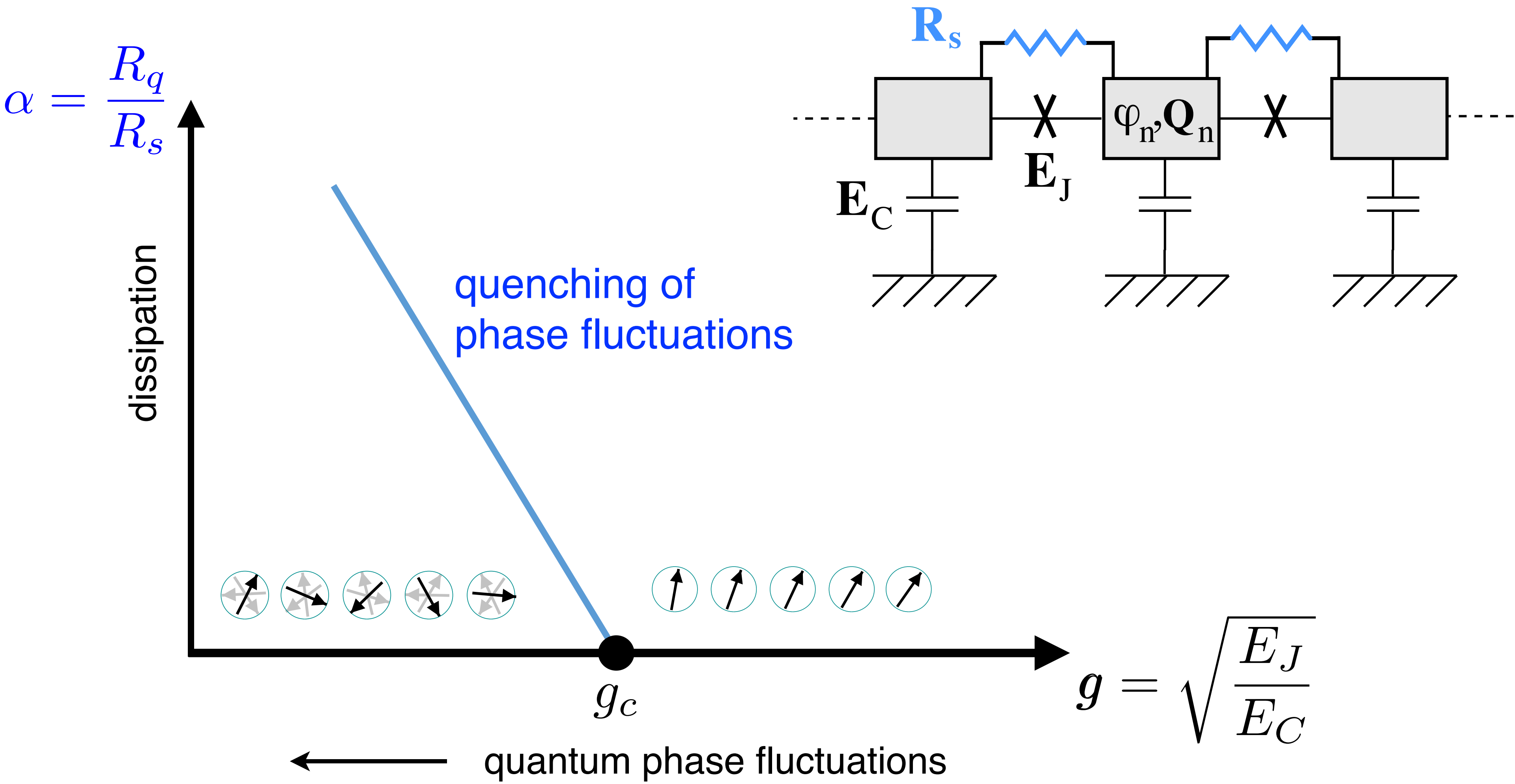
Wagenblast, PRL **79**, 2730 (1997)

Refael et al., PRB **75**, 014522 (2007)

Dissipative QPT



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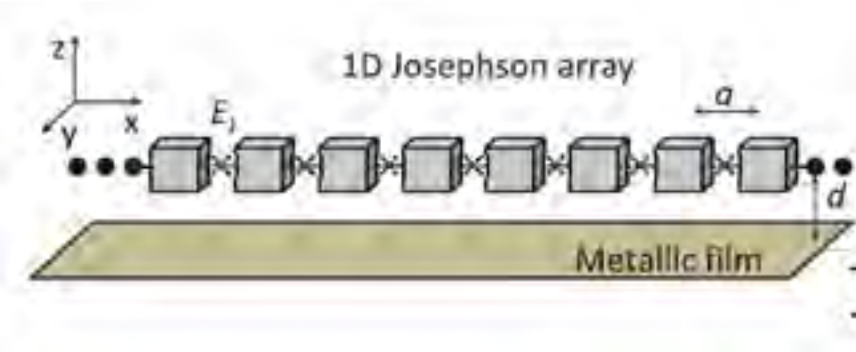
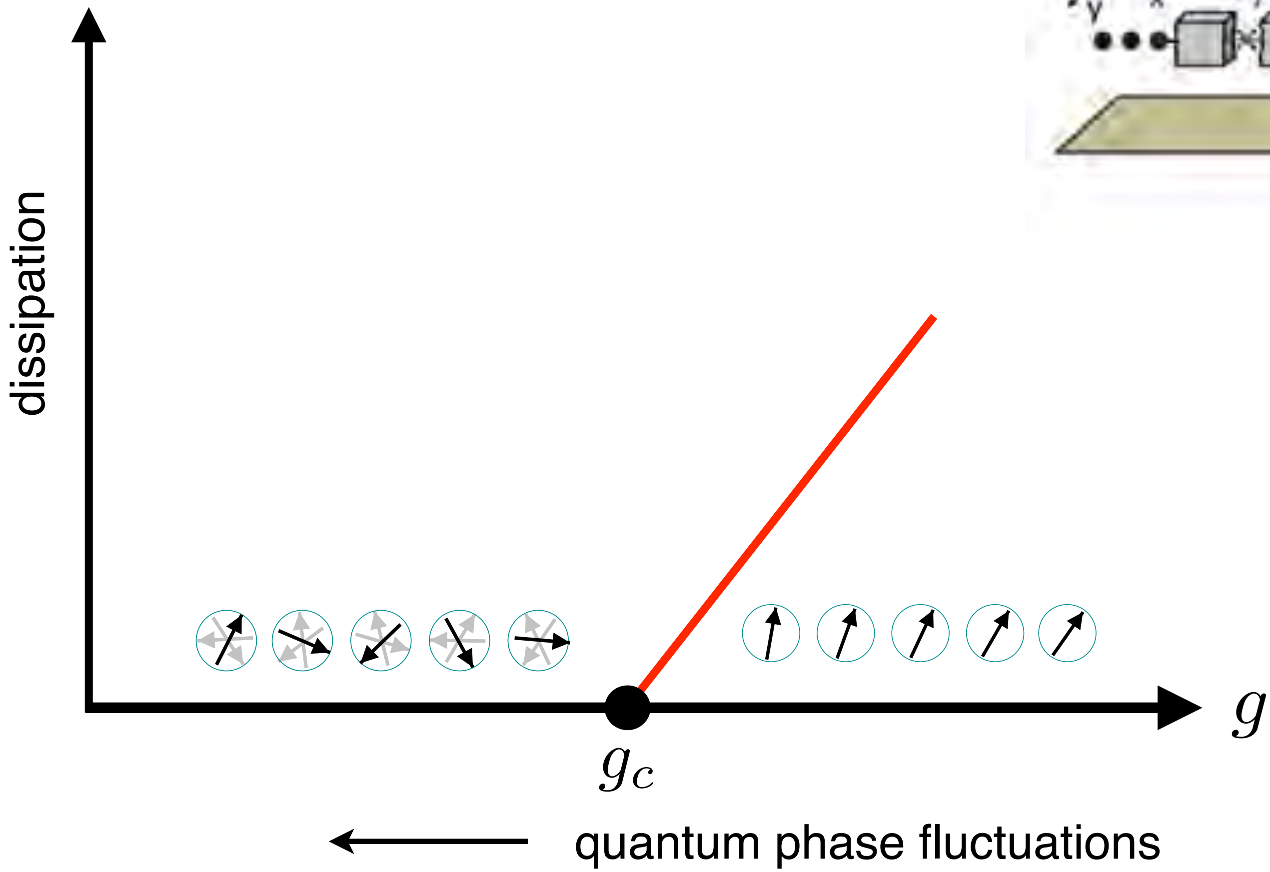
JJ chain with “charge” dissipation





Lobos, Giamarchi, Phys. Rev. B **84**, 024523 (2011)

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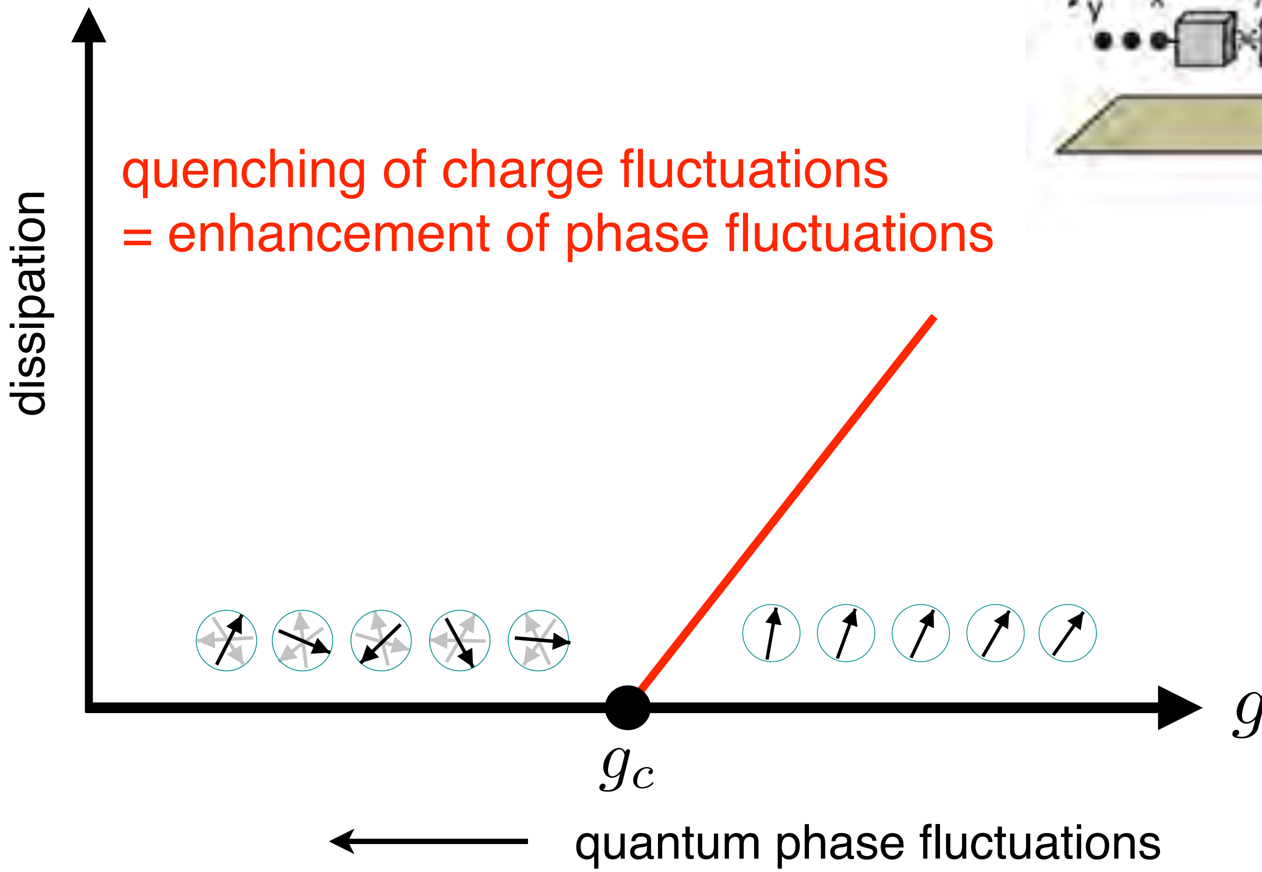

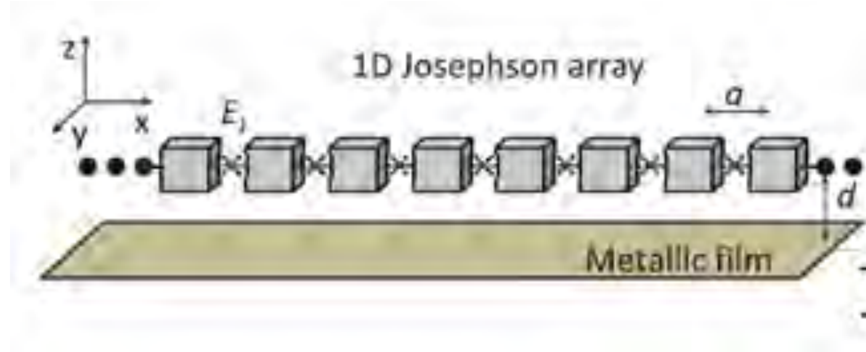

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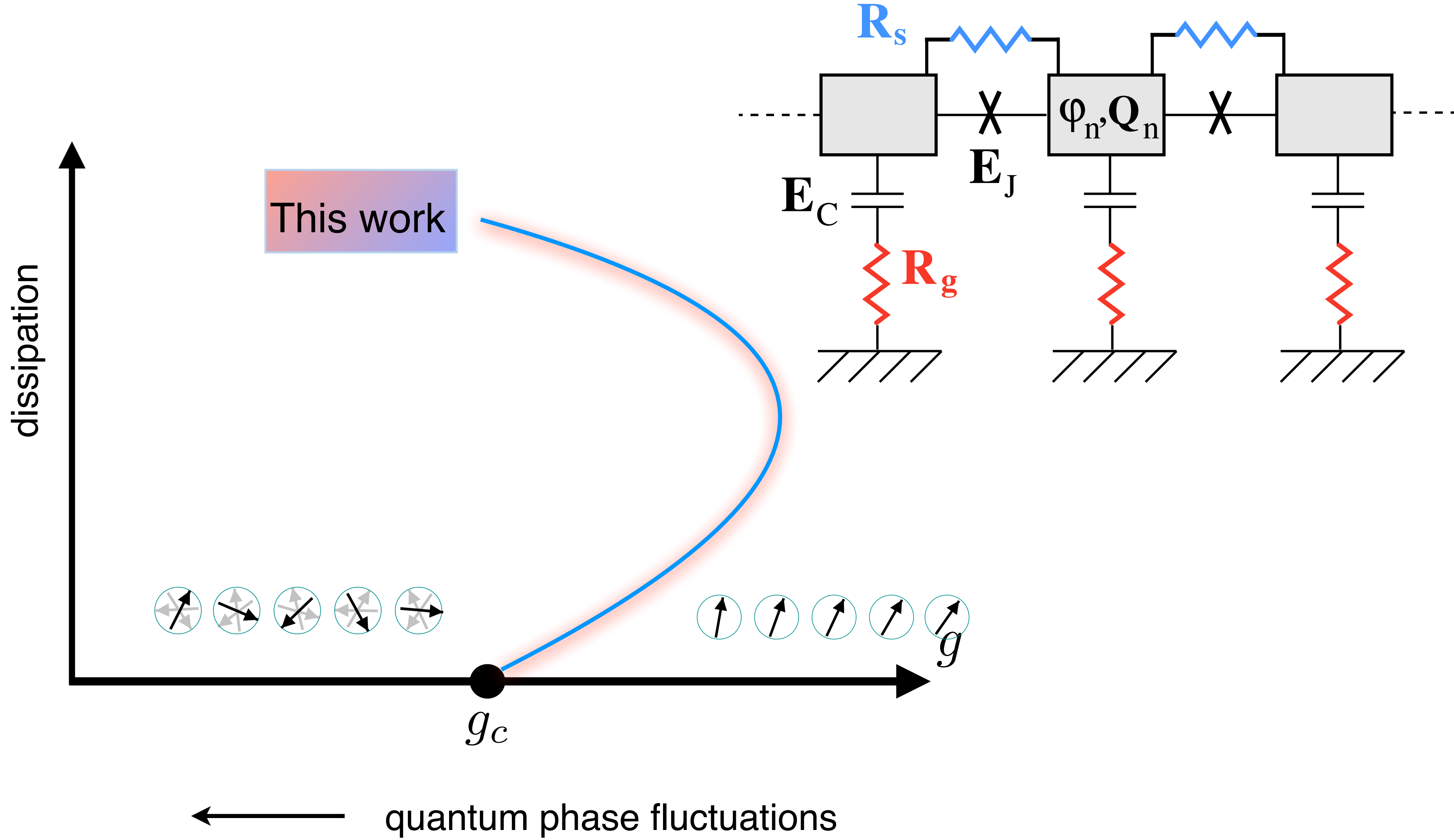
QPT with dissipative frustration

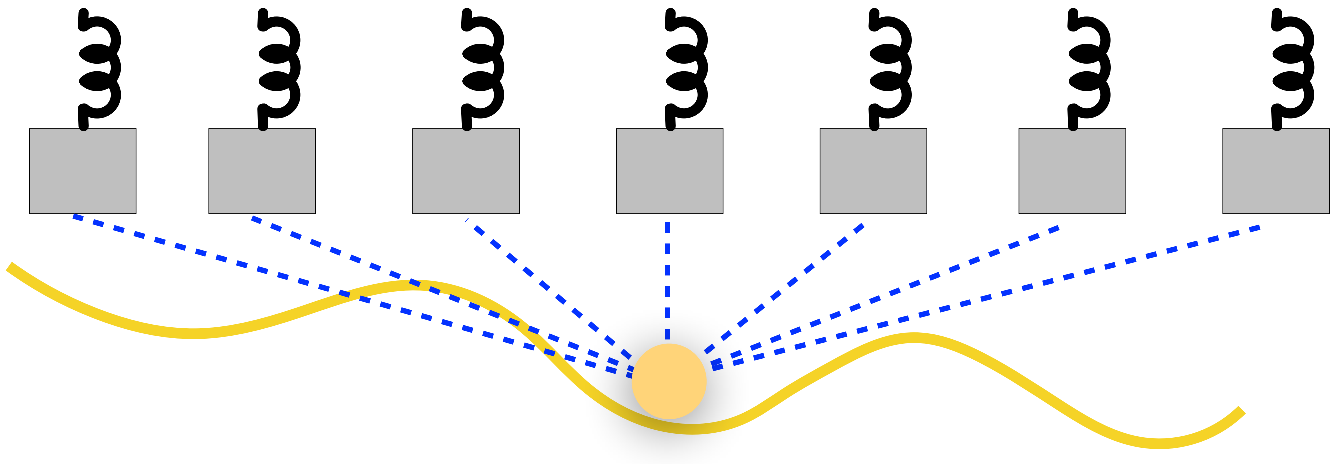


D. Maile, S. Andergassen, W. Belzig, G. Rastelli, Phys. Rev. B **97**, 155427 (2018).

Quantum Brownian motion

Caldeira-Leggett model

$$\hat{H} = \hat{H}_S + \hat{H}_{int} + \hat{H}_{bath}$$



$$\hat{H}_S = \frac{\hat{p}^2}{2m} + V(\hat{q})$$ particle of mass *m*

$\hat{H}_{bath} =$ set of independent harmonic oscillators

$$\hat{H}_{int} = \alpha\, \hat{q}\, \hat{B}$$

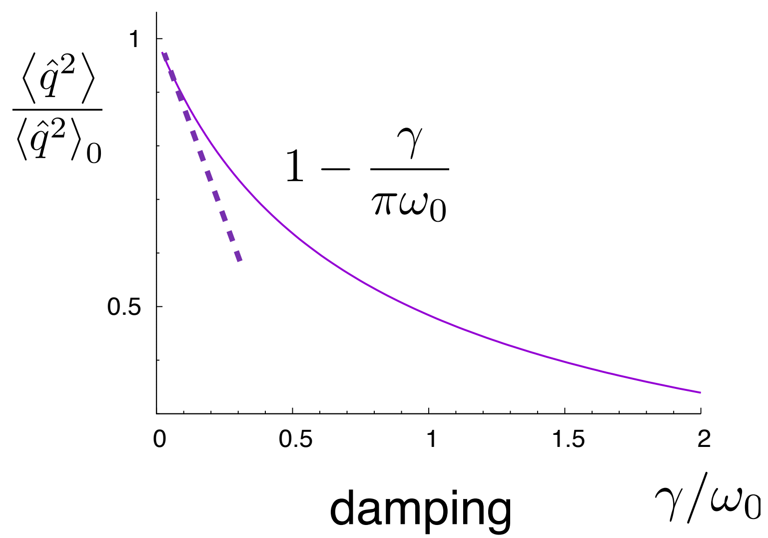
$\hat{B} =$ Bath's operator

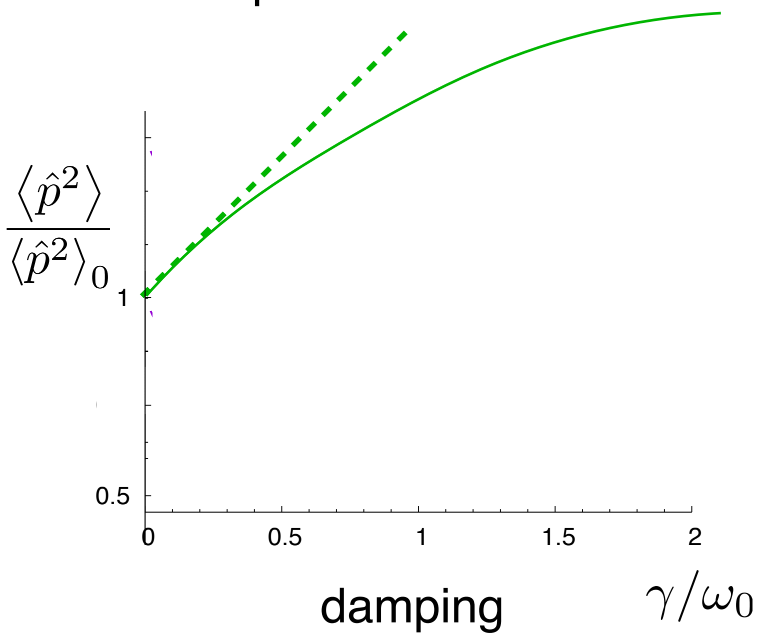
Non-commuting dissipative interactions

Example: harmonic oscillator ω_0

➡ thermodynamics and dynamics are inseparable

➡ linear damping: squeezing of the quantum fluctuations of q



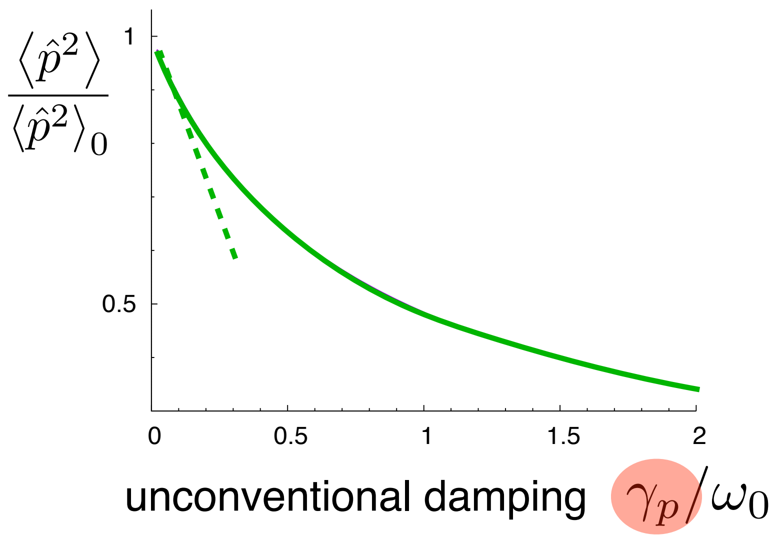


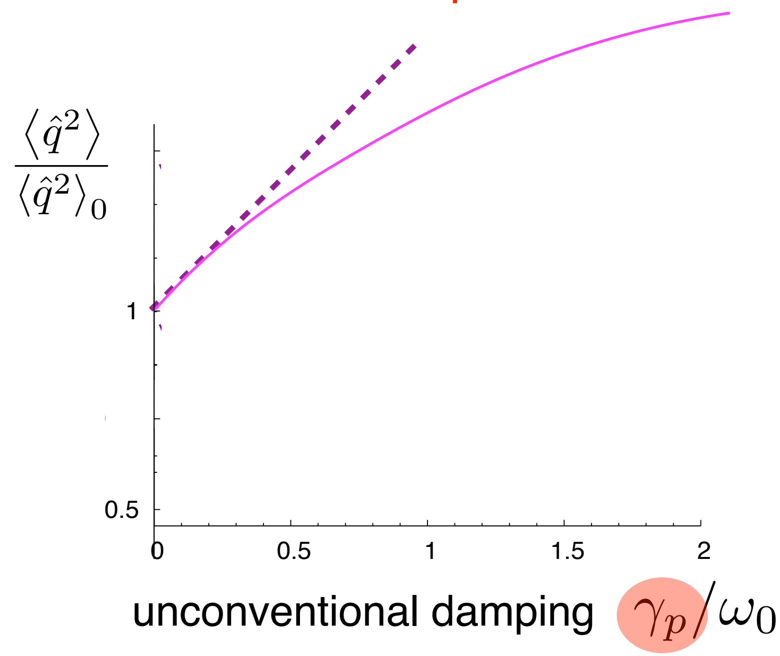
Non-commuting dissipative interactions

Example: harmonic oscillator ω_0

➡ thermodynamics and dynamics are inseparable

➡ unconventional damping: squeezing of the quantum fluctuations of p





Non-commuting dissipative interactions

Example: harmonic oscillator ω_0

➡ thermodynamics and dynamics are inseparable

➡ 1 single dissipation: squeezing of the quantum fluctuations of one quadrature

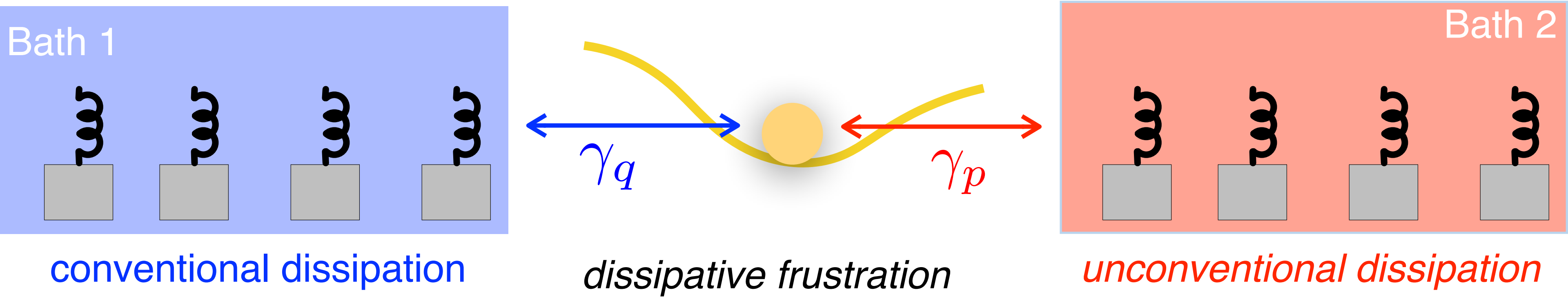
Non-commuting dissipative interactions

Example: harmonic oscillator ω_0

➡ thermodynamics and dynamics are inseparable

➡ 1 single dissipation: squeezing of the quantum fluctuations of one quadrature

➡ Coupling to two baths via two non-commuting observables? $\delta q\, \delta p \geq \hbar/2$



- harmonic oscillator $[\hat{q}, \hat{p}] = i\hbar$

H.Kohler, F.Sols, PRB **72**, 180404 (2005)

A.Cuccoli, N.Del Sette, R.Vaia, PRE **81**, 041110 (2010)

- single spin $[\hat{\sigma}_z, \hat{\sigma}_x] = i\hat{\sigma}_y$

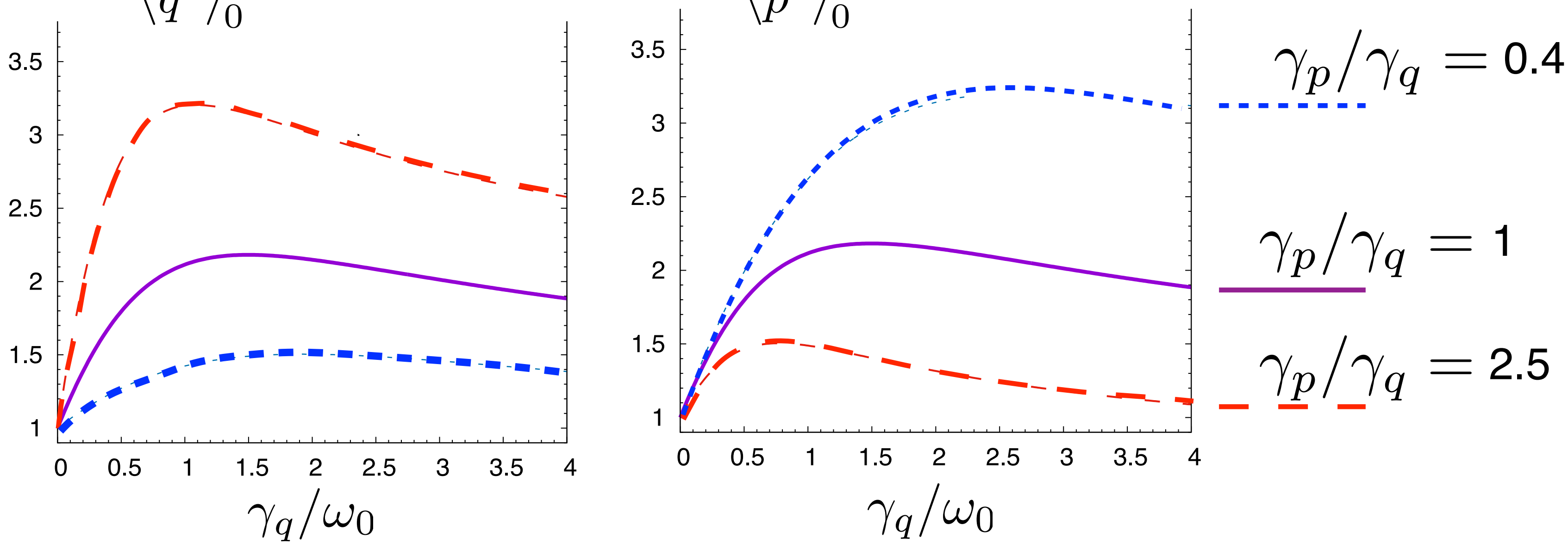
E. Novais et al., PRB **72**, 014417 (2005)

H. Castro et al., PRL **91**, 096401 (2003)

Dissipative frustration: harmonic oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{q}^2 + \hat{H}_{Bath,q} + \hat{H}_{Bath,p}$$

$$\frac{\langle \hat{q}^2 \rangle}{\langle \hat{q}^2 \rangle_0} = \sigma\left(\gamma_q, \gamma_p\right) \qquad \frac{\langle \hat{p}^2 \rangle}{\langle \hat{p}^2 \rangle_0} = \sigma\left(\gamma_p, \gamma_q\right)$$



$\gamma_q > \gamma_p$ position dissipation dominates *but* $\langle \hat{q}^2 \rangle$ are enhanced

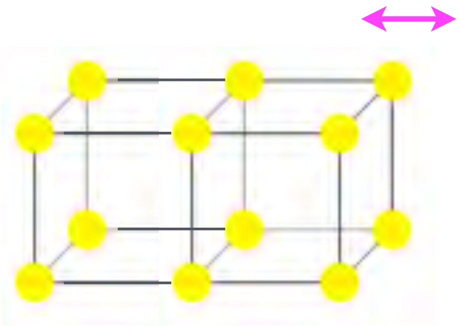
$\gamma_p > \gamma_q$ momentum dissipation dominates *but* $\langle \hat{p}^2 \rangle$ are enhanced

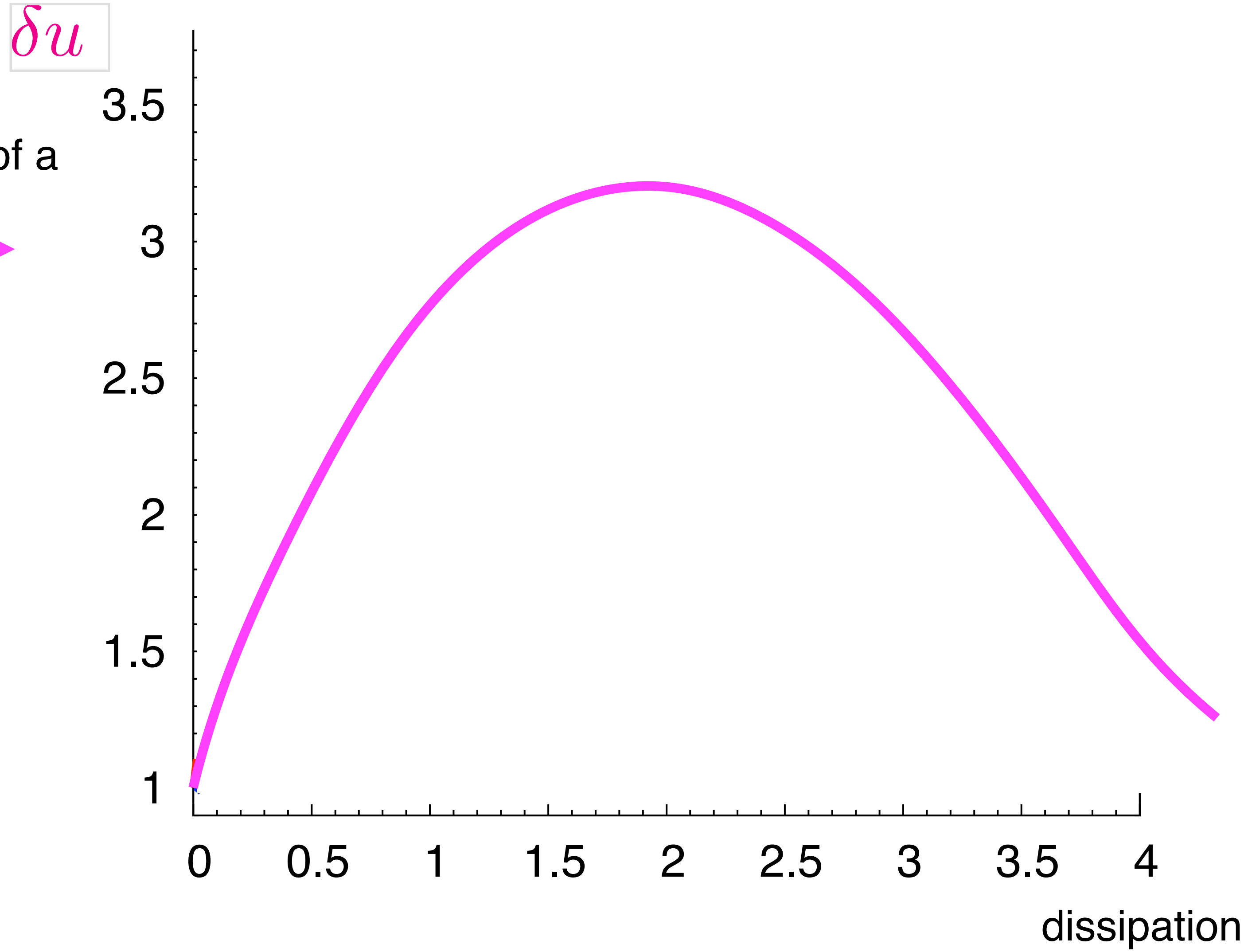
➡ **no squeezing *in both fluctuations***

G.Rastelli, New Journal of Physics. **18**, 053033 (2016)

Idea: Lindemann criterion

average fluctuation of a particle in a solid





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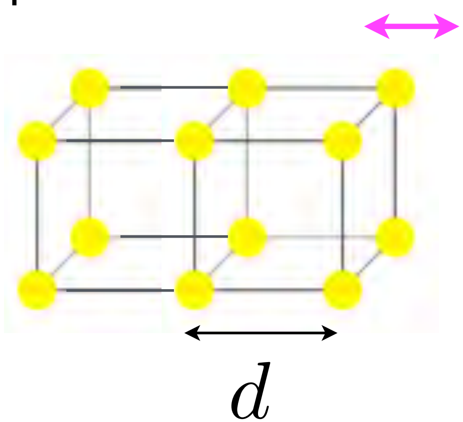
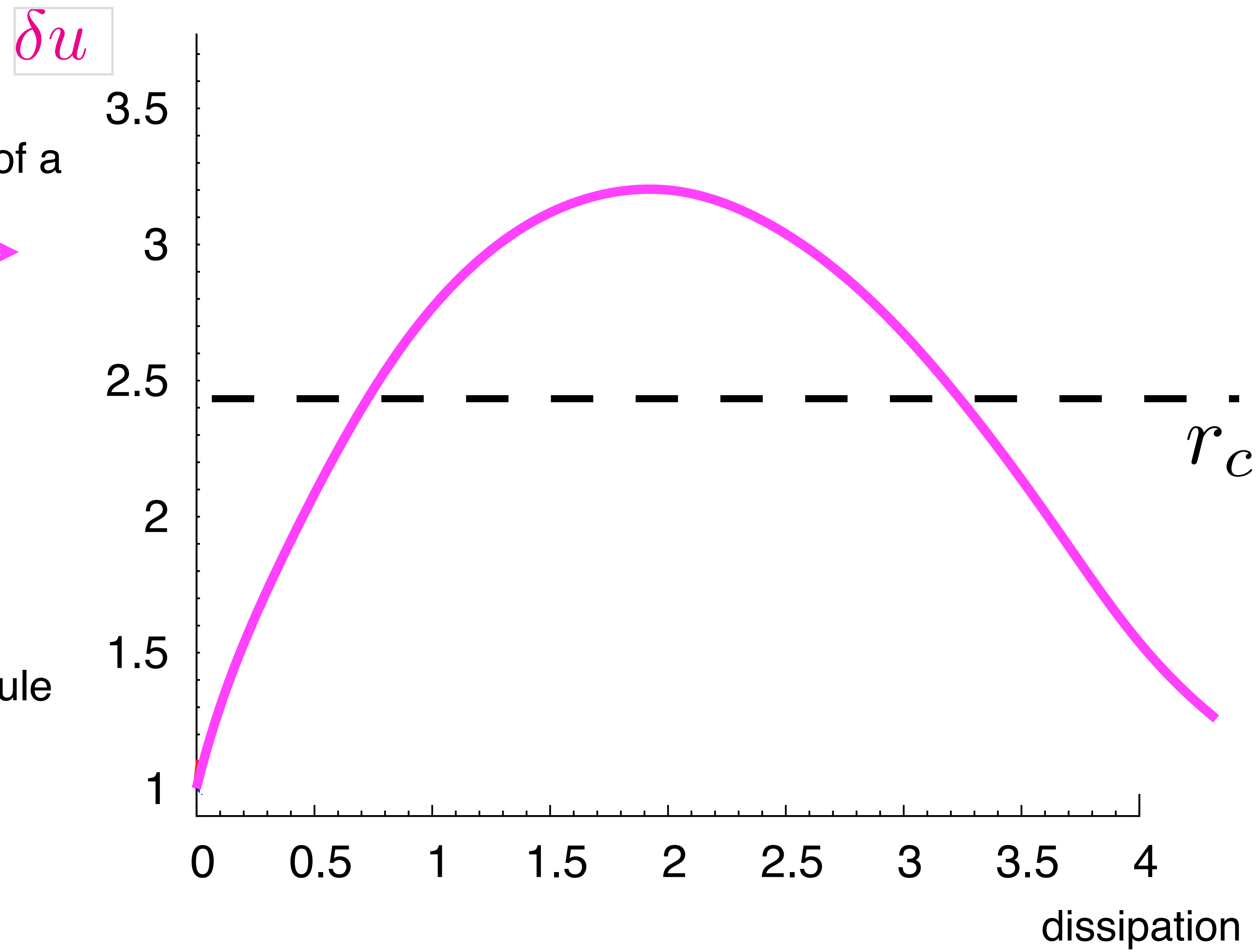


Phenomenological rule for melting

$$\frac{\delta u}{d} \leq r_c$$



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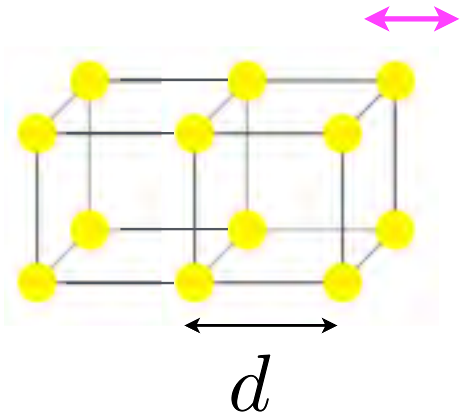

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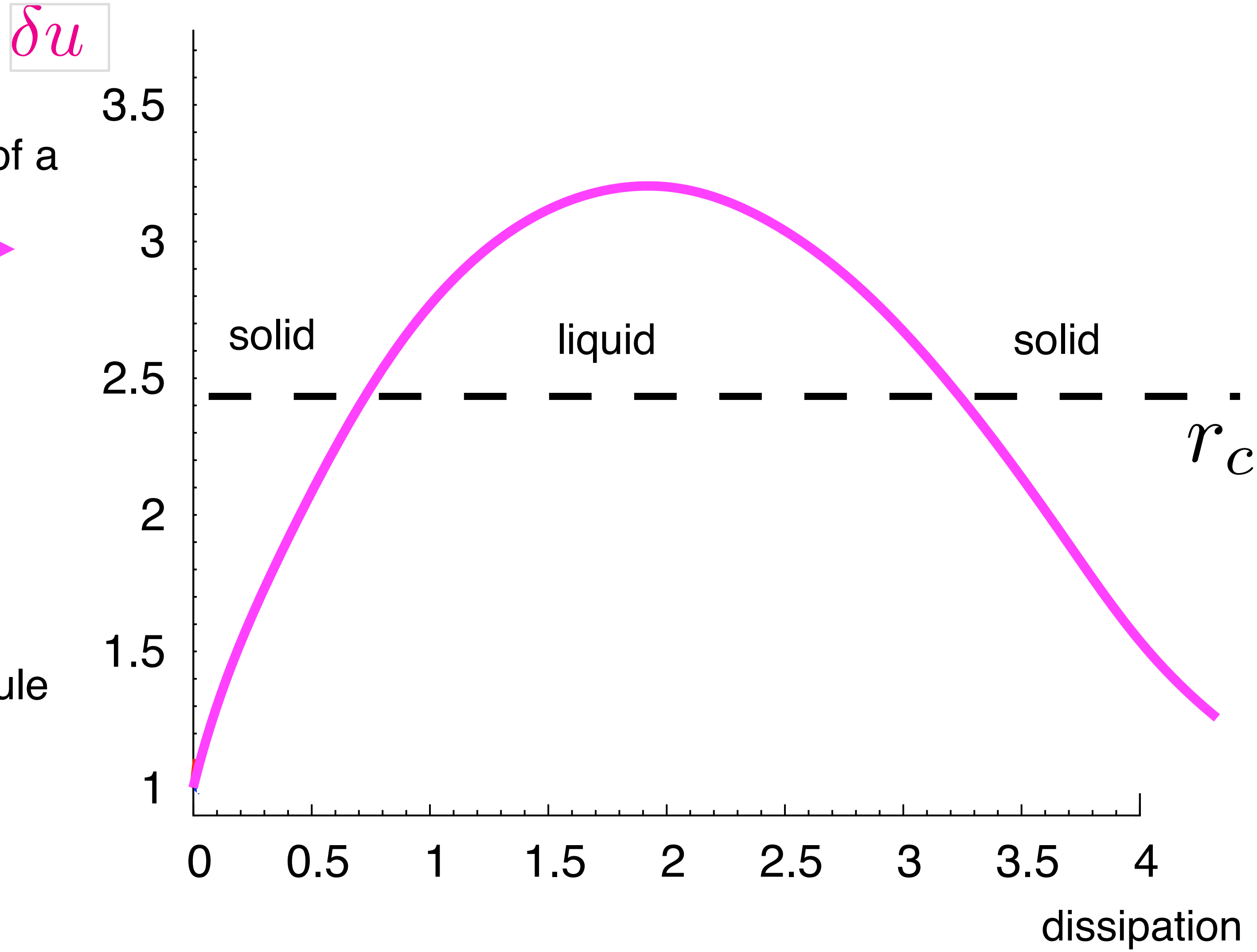
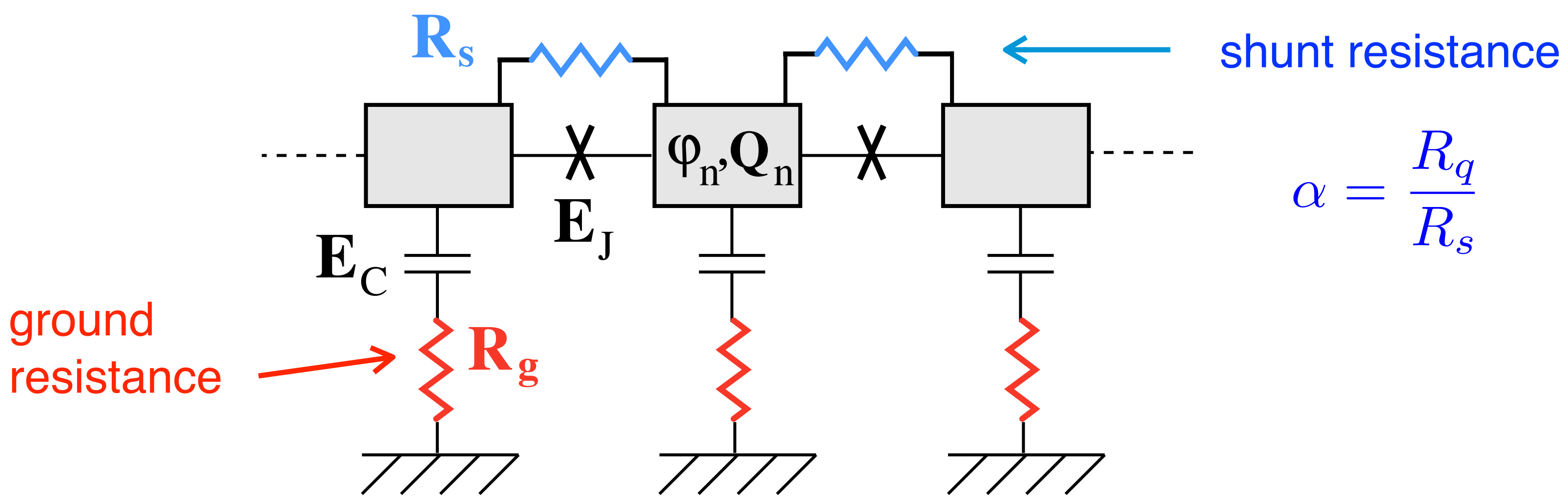


dissipative frustration → reentrant behaviour

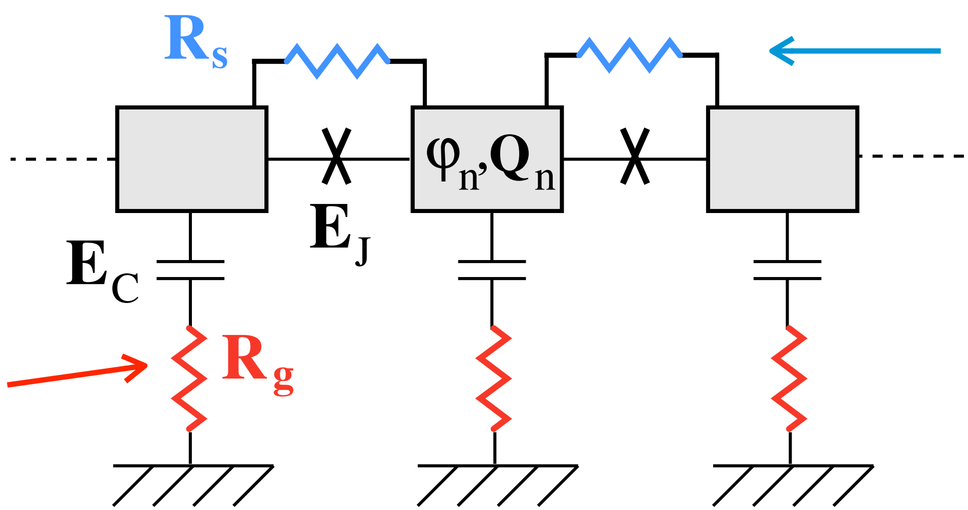
Charge dissipation in JJ chain



$$\alpha = \frac{R_q}{R_s}$$

$$R_q = \frac{h}{4e^2}$$

Charge dissipation in JJ chain



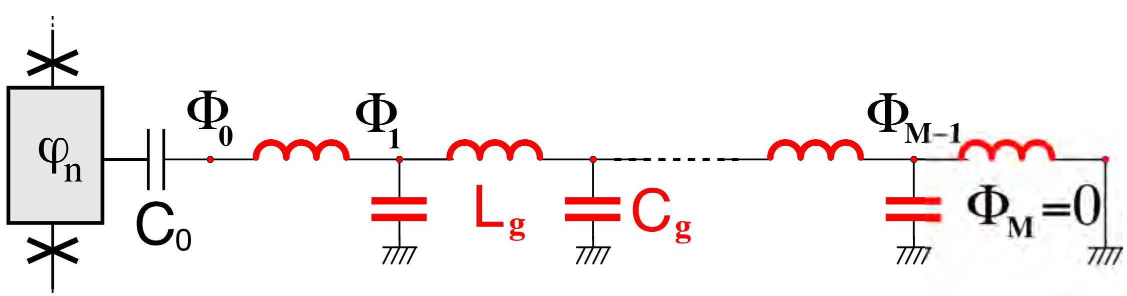
shunt resistance

ground resistance

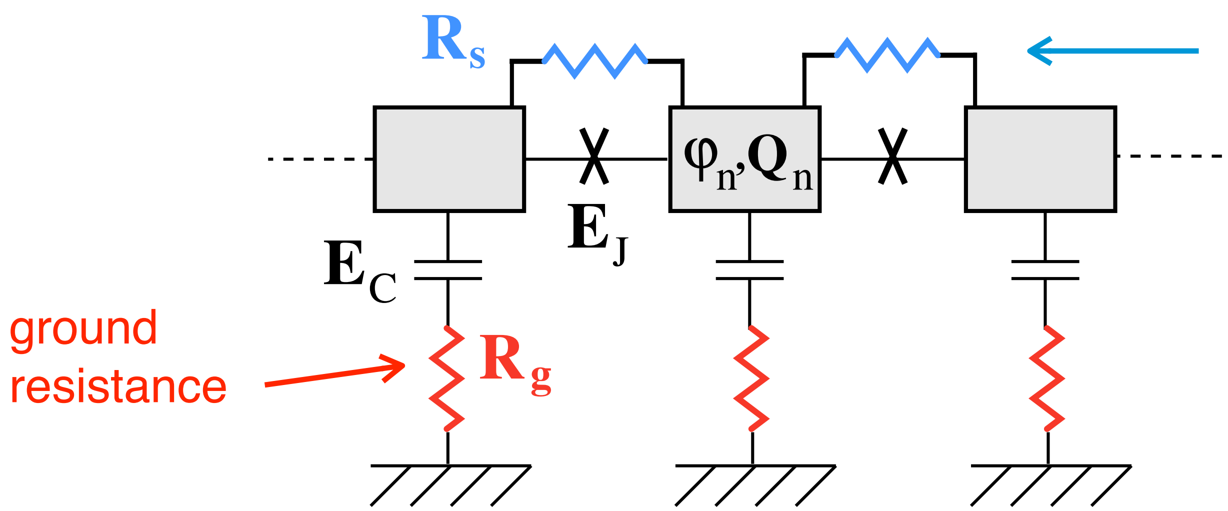
$$\alpha = \frac{R_q}{R_s} \qquad R_q = \frac{h}{4e^2}$$

$$\tilde{\alpha} = \frac{R_g}{R_q}$$ coupling strength for the charge dissipation

$$\tilde{\alpha} = \tau_g E_C / h \qquad \tau_g = R_g C_0$$ decay time



Charge dissipation in JJ chain



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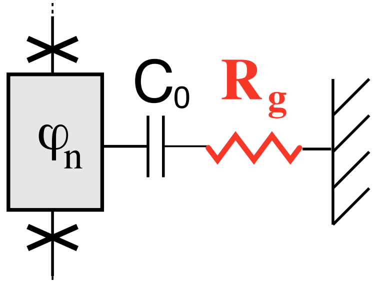


Method and approximations

Path integral: Euclidean Action

$$S = \sum_n \int_0^\beta d\tau \left[\frac{\hbar^2}{2E_C} \dot{\varphi}_n^2 - E_J \cos\left(\Delta\varphi_n(\tau)\right) \right]$$

$$+ \sum_n \iint_0^\beta d\tau d\tau' \left[F(\tau - \tau') \left|\Delta\varphi_n(\tau) - \Delta\varphi_n(\tau')\right|^2 + \tilde{F}(\tau - \tau') \dot{\varphi}_n(\tau) \dot{\varphi}_n(\tau') \right]$$

$$\alpha = \frac{R_q}{R_s} \qquad \widetilde{\alpha} = \frac{R_g}{R_q}$$

<u>Self consistent harmonic approximation</u>

Bogoliubov inequality for free energy: $F \leq F_v$

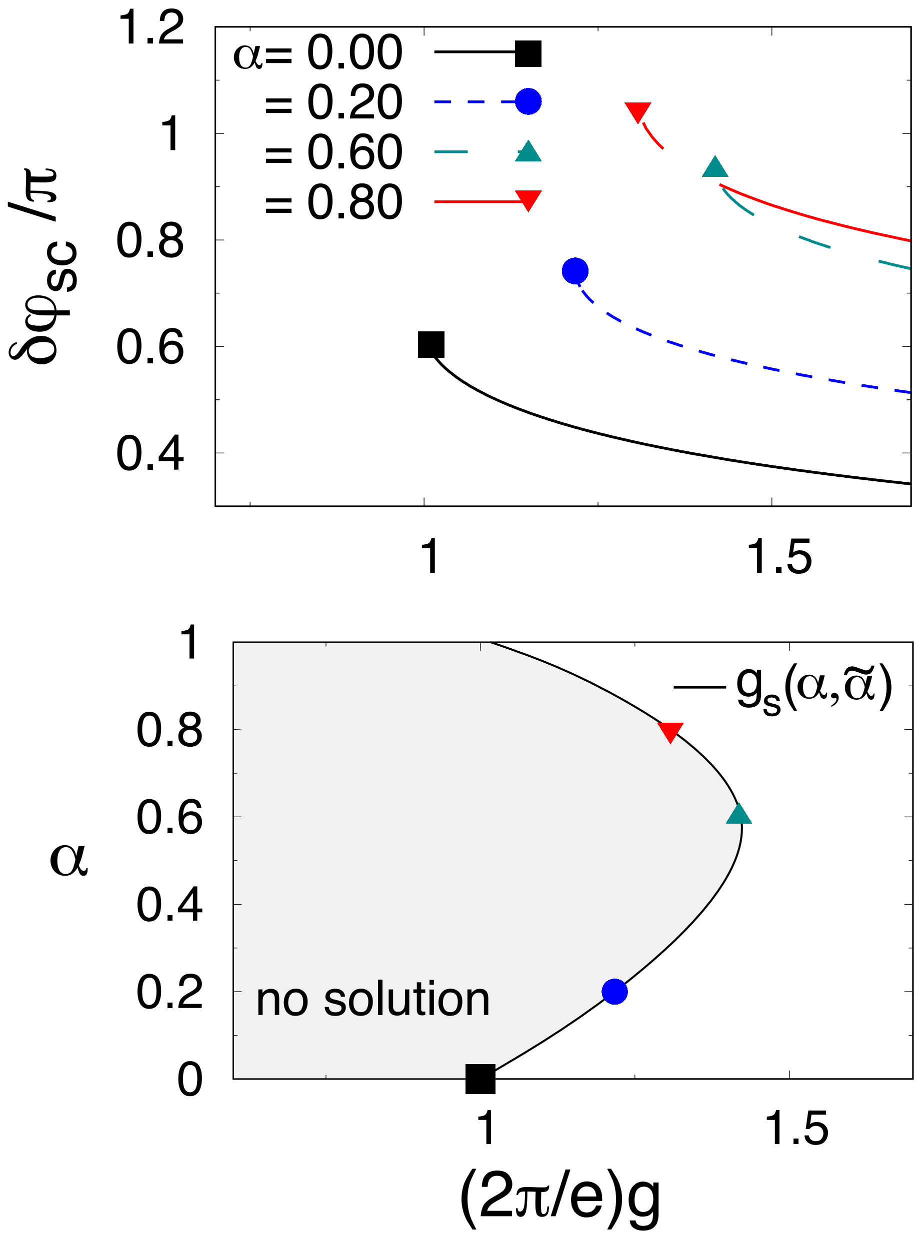
$$F_v = F_{V_T} - \frac{1}{\beta} \langle \Delta S \rangle_{V_T} \qquad \Delta S = \sum_n \int_0^\beta d\tau \left[-E_J \cos\left(\Delta\varphi_n(\tau)\right) - \frac{V_T}{2} \Delta\varphi_n^2(\tau) \right]$$

$$\frac{dF_v}{dV_T} \stackrel{!}{=} 0 \longrightarrow V_{sc} = E_J \; e^{-\frac{1}{2}\delta\varphi_{sc}}$$

$$\delta\varphi_{sc} = \sqrt{\langle \Delta\varphi^2 \rangle_{V_{sc}}}$$

zero temperature fluctuations of the phase difference

Example: solution of self-consistent equation



$\tilde{\alpha}/\alpha = 0.3$

$g = \sqrt{\frac{E_J}{E_C}}$

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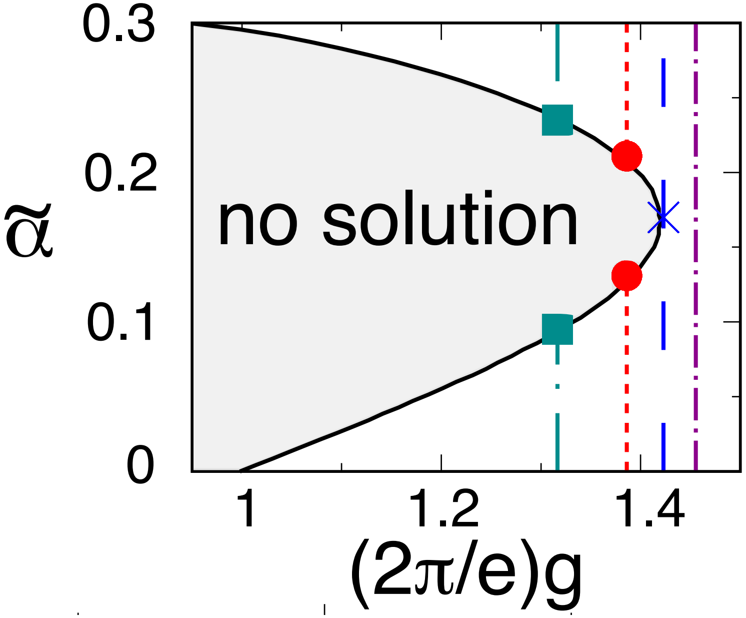

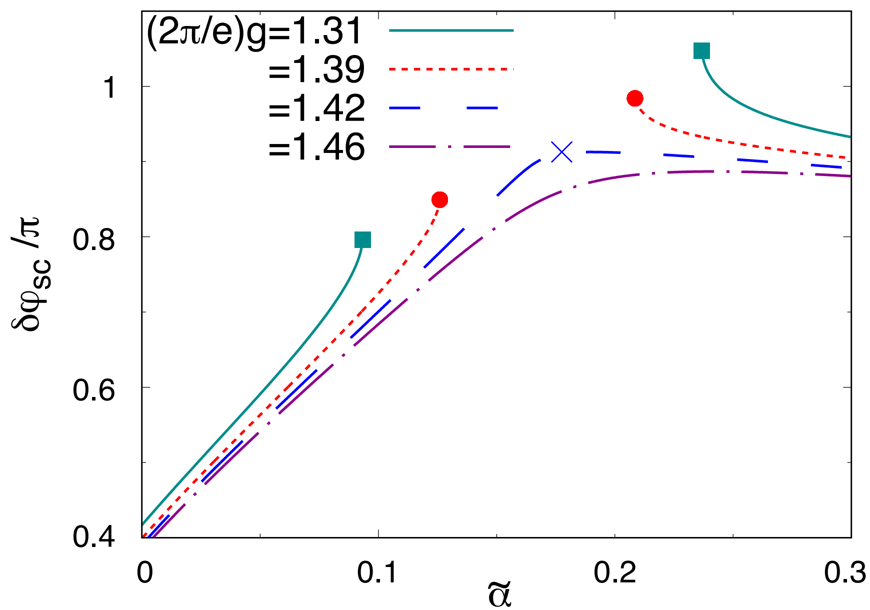

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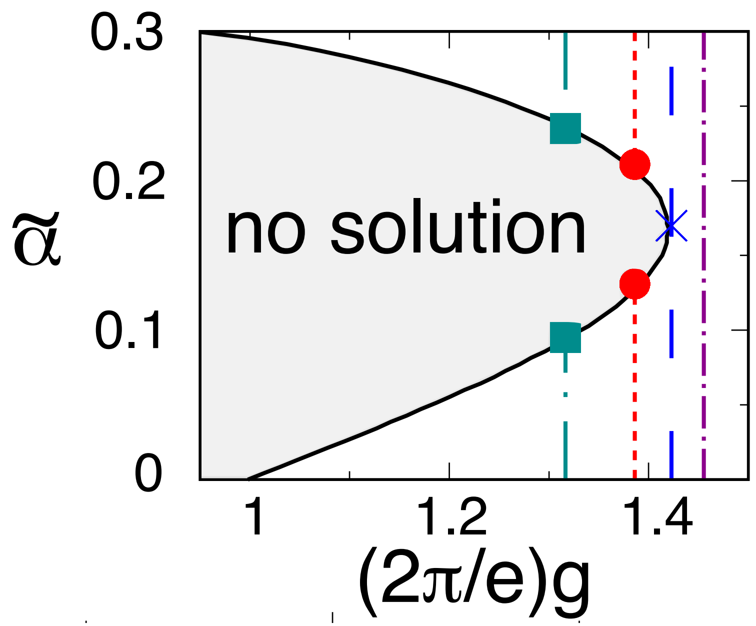

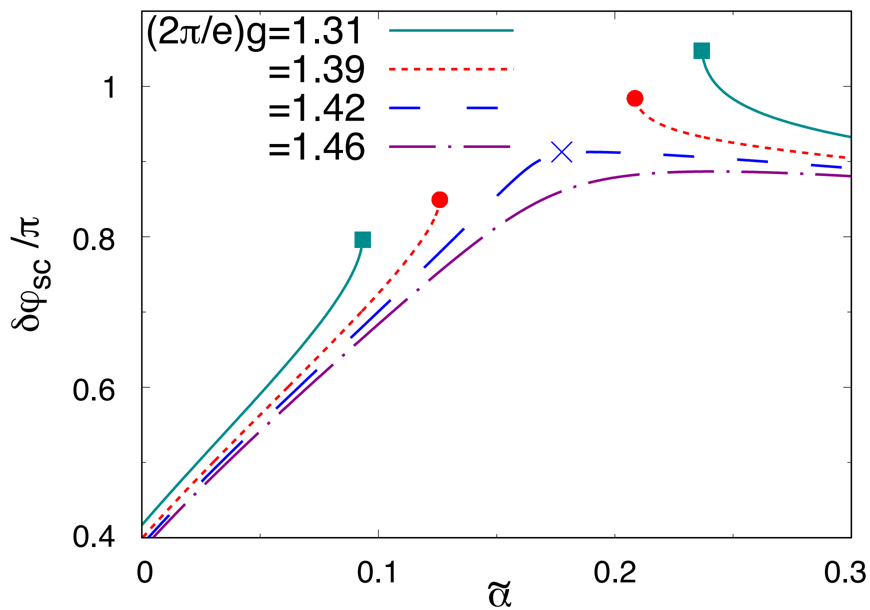

➡ non-monotonic critical line

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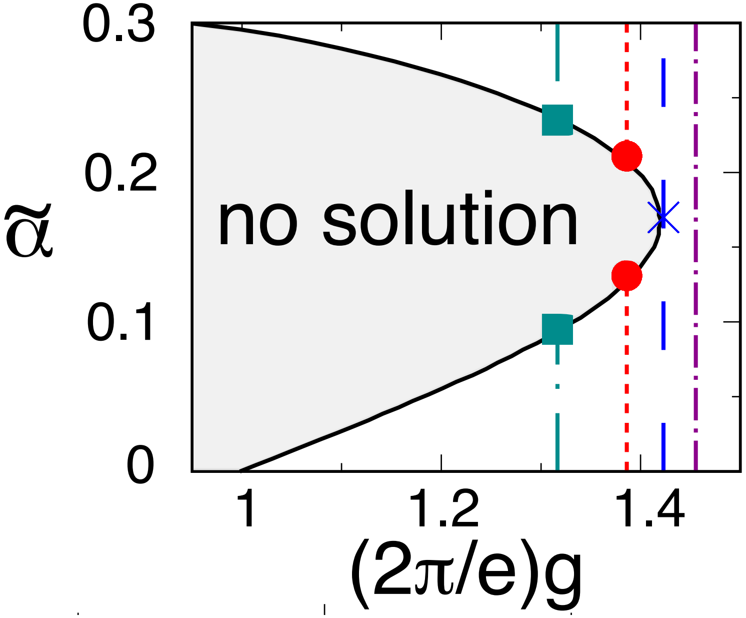

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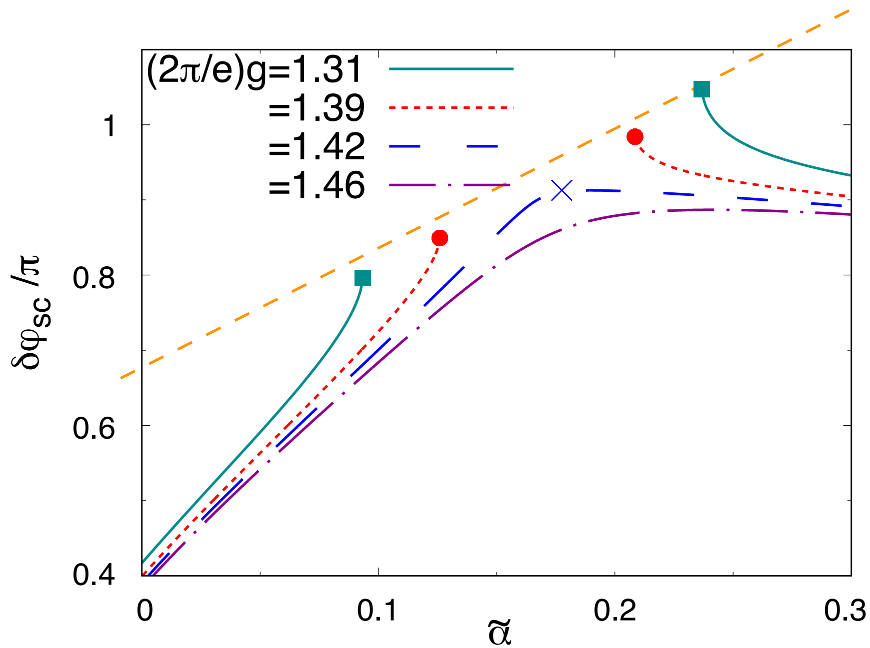
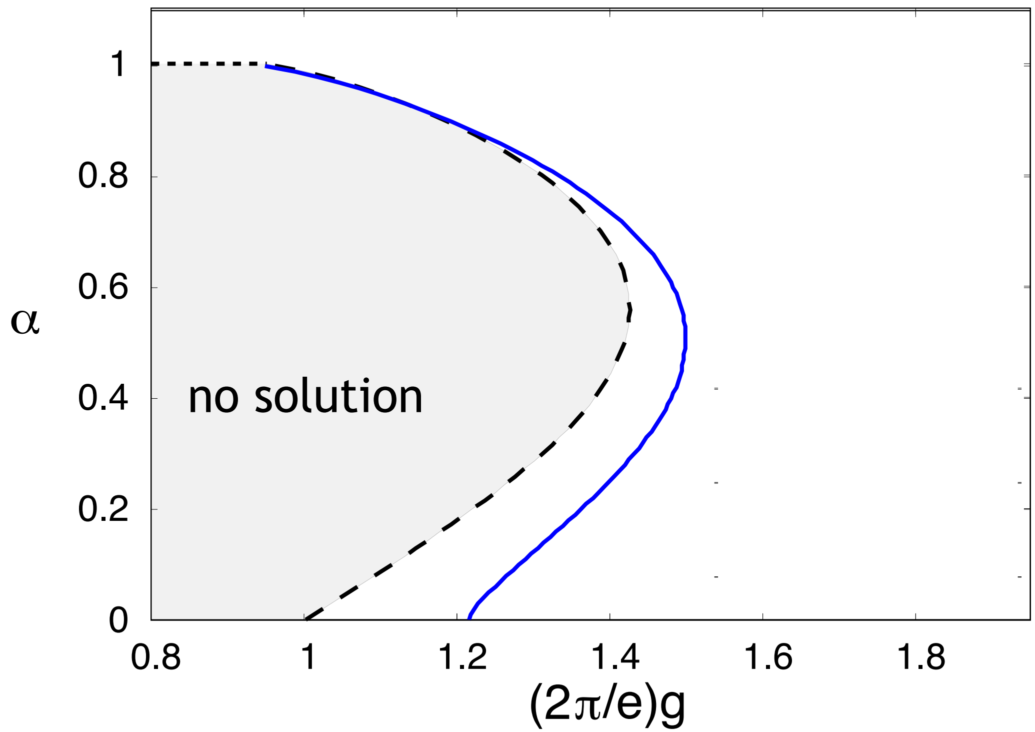


➡ non-monotonic critical line

➡ ~ comparison with the Lindemann's rule

Phase diagram

Variational approach: transition order/disorder when $$F_v(V_{sc}) = F_v(0)$$



$$\widetilde{\alpha}/\alpha = 0.3$$

Phase diagram

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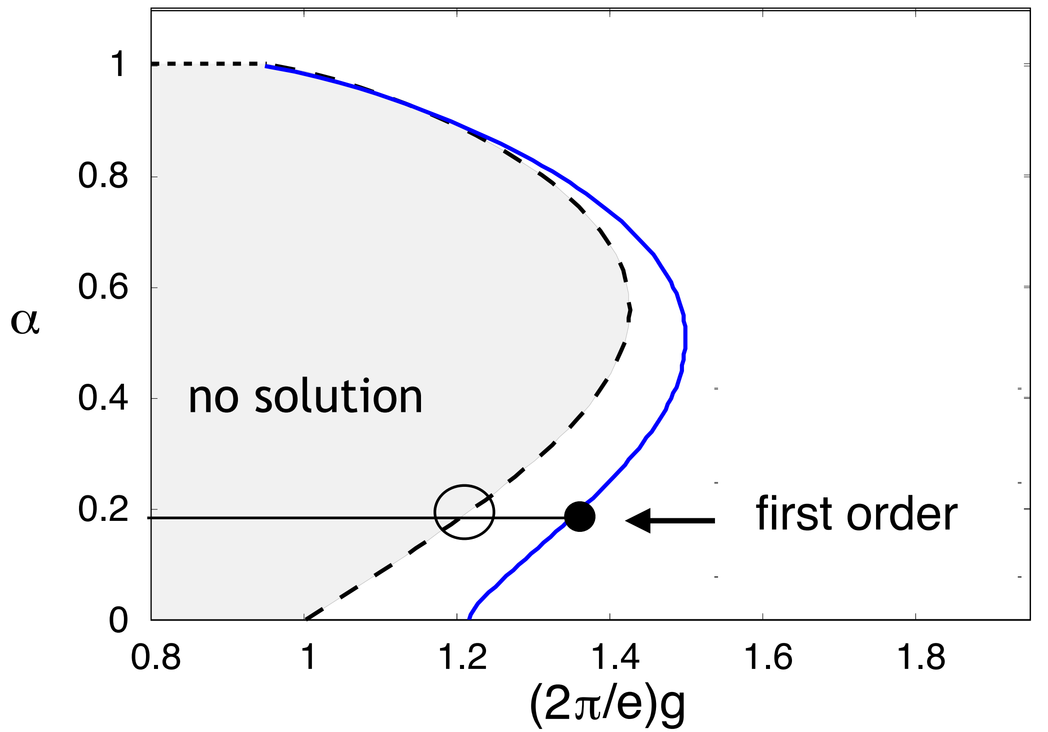
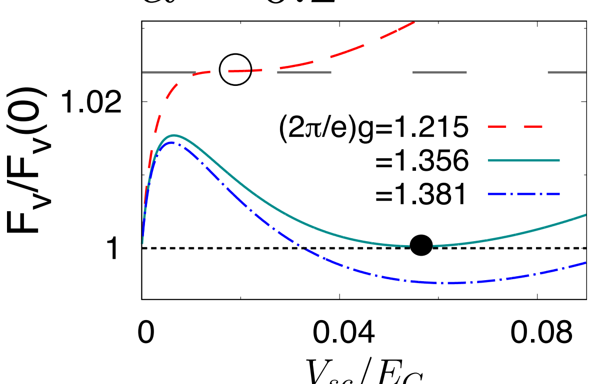


$\widetilde{\alpha}/\alpha = 0.3$

$\alpha = 0.2$



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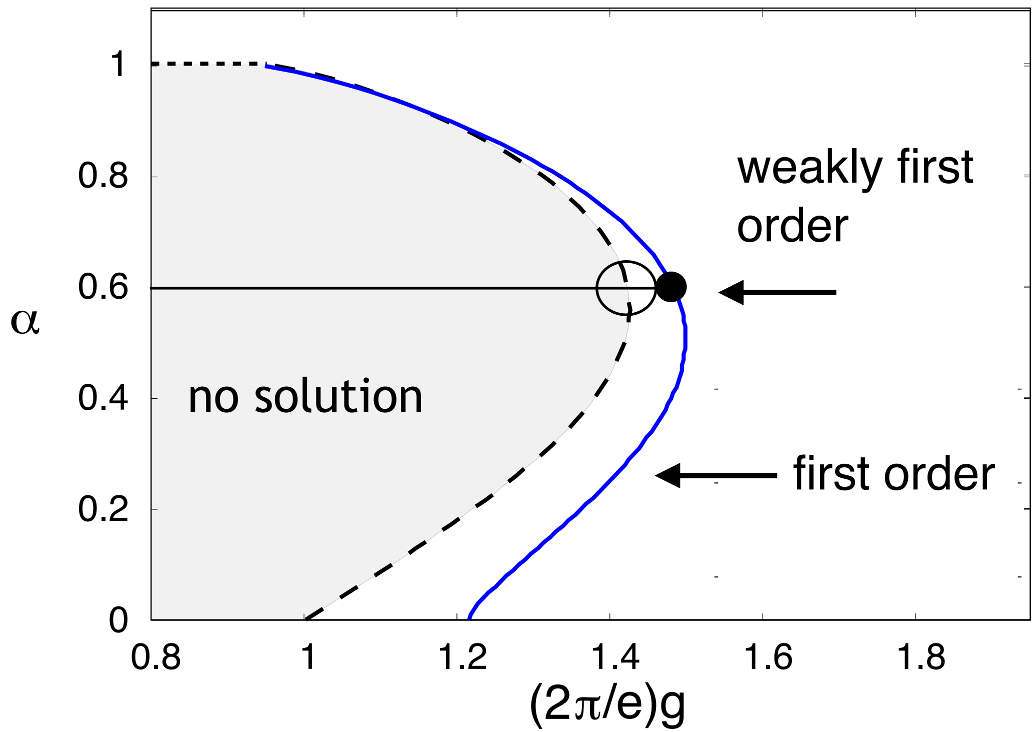

$\widetilde{\alpha}/\alpha = 0.3$

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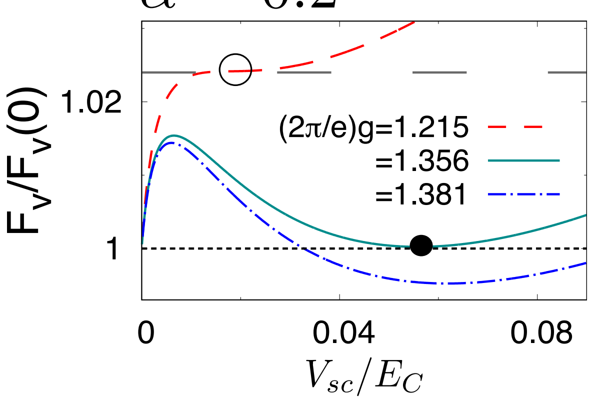
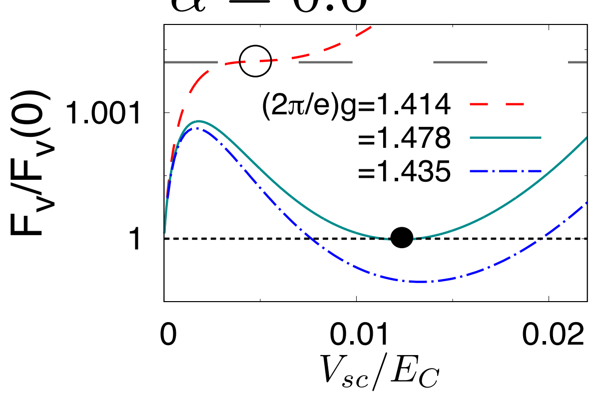
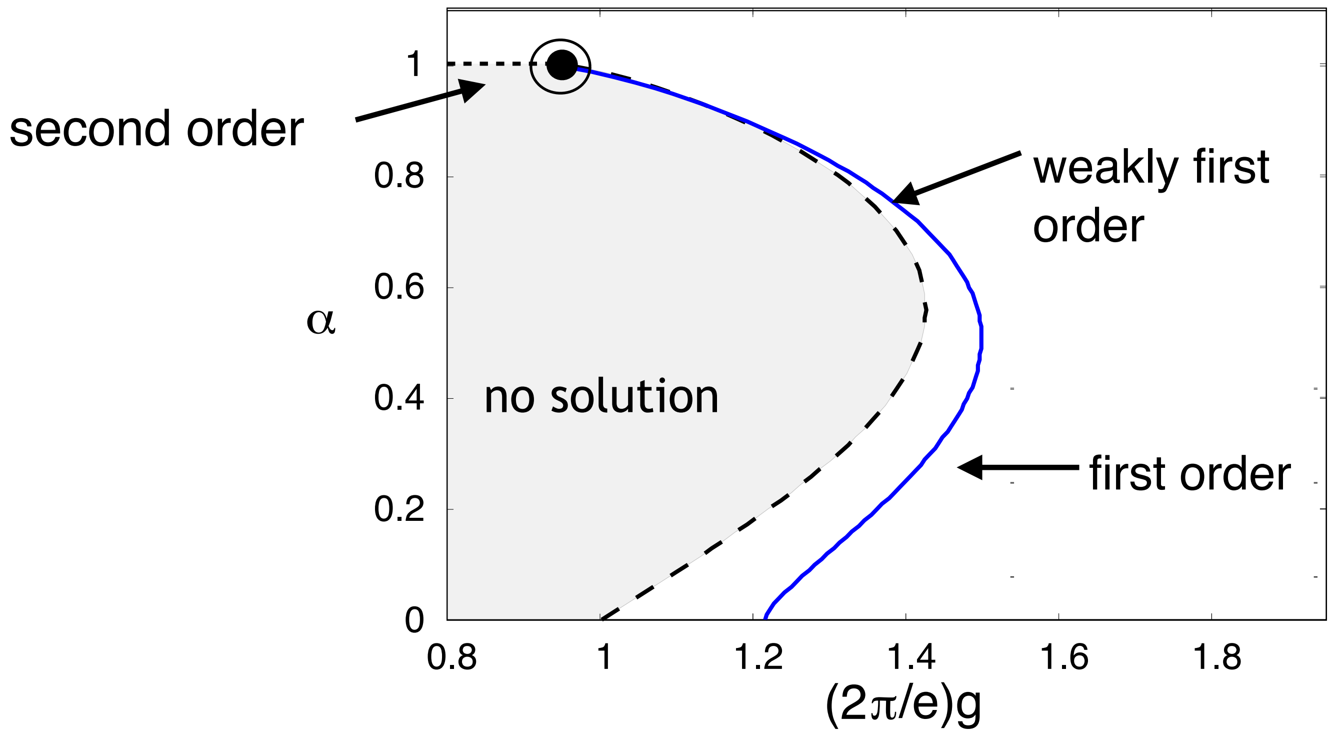


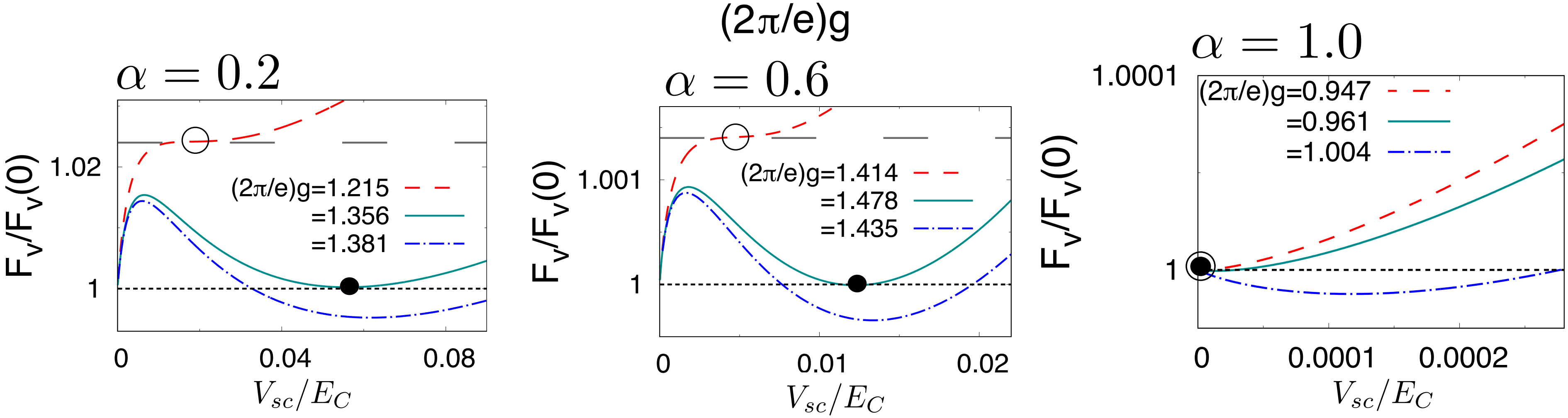
$\alpha = 0.6$



Phase diagram

Variational approach: transition order/disorder when $F_v(V_{sc}) = F_v(0)$





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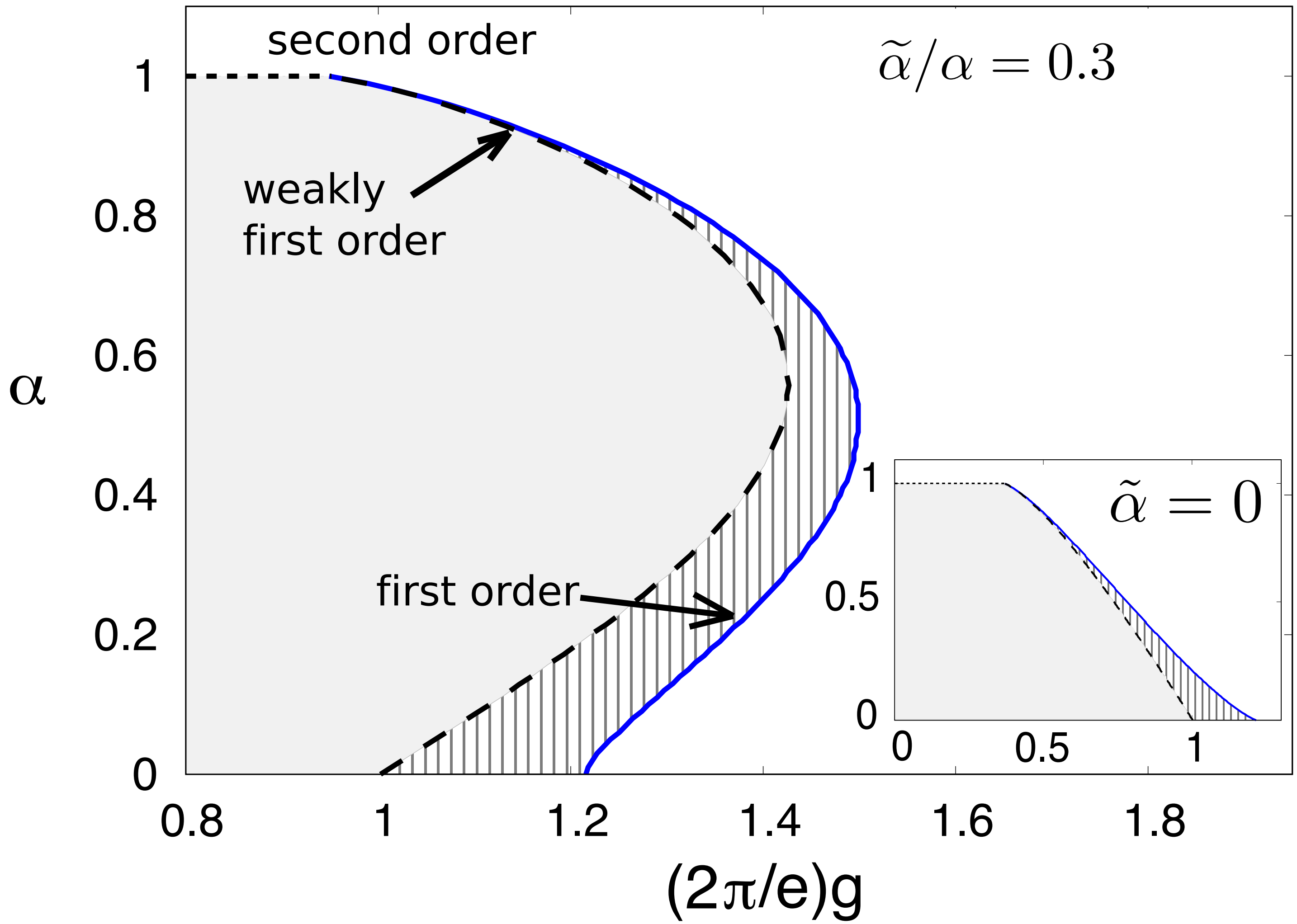

→ Dissipative frustration leads to a non-monotonic phase diagram

Thermodynamical quantities

Quantum phase transition ⟶ (mathematical) mapping to classical systems

QPT: quantum thermodynamical properties with no analog in classical systems

"Scaling of entanglement close to a quantum phase transition"

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⟶ purity

⟶ entanglement

Purity

System-environment correlation

$$P = \mathrm{tr}\left(\hat{\rho}^2\right) = \prod_k \sqrt{\frac{\langle|\varphi_k|^2\rangle_0 \langle|\dot{\varphi}_k|^2\rangle_0}{\langle|\varphi_k|^2\rangle \langle|\dot{\varphi}_k|^2\rangle}}$$

k = index for the normal modes

$\langle|\varphi_k|^2\rangle$ phase fluctuations with dissipation

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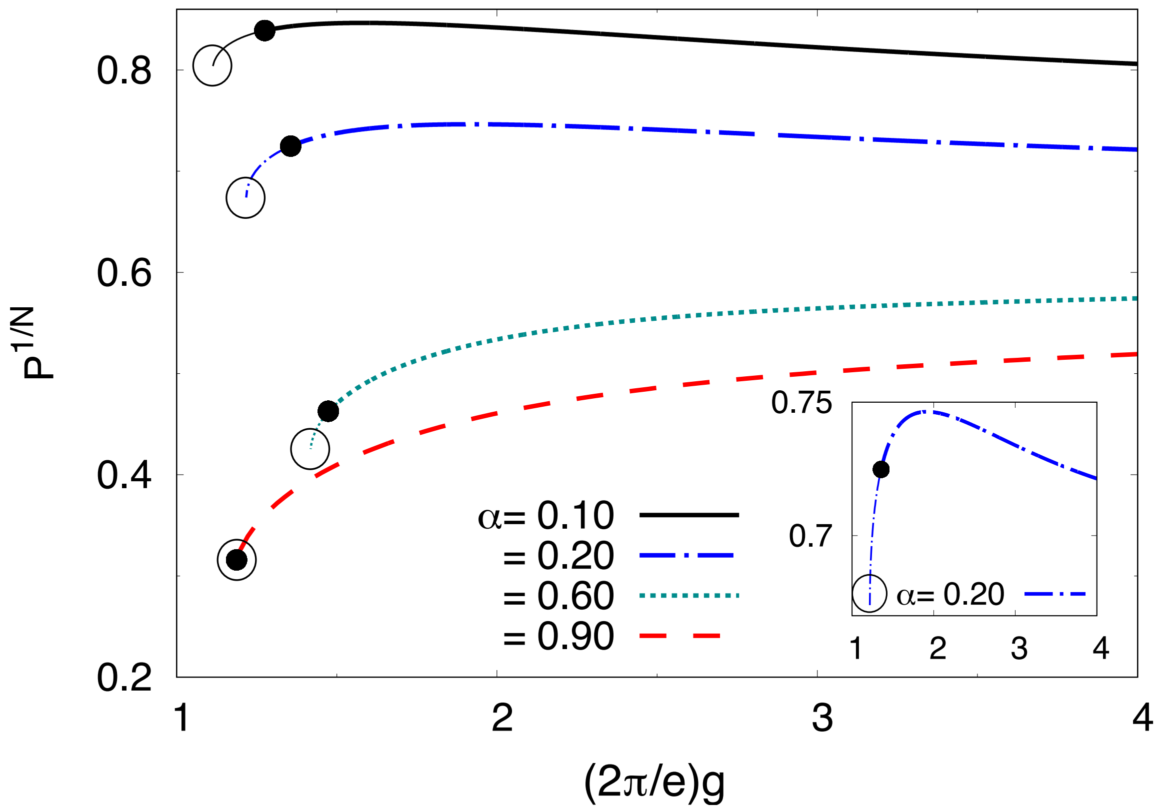

$$\widetilde{\alpha}/\alpha = 0.3$$

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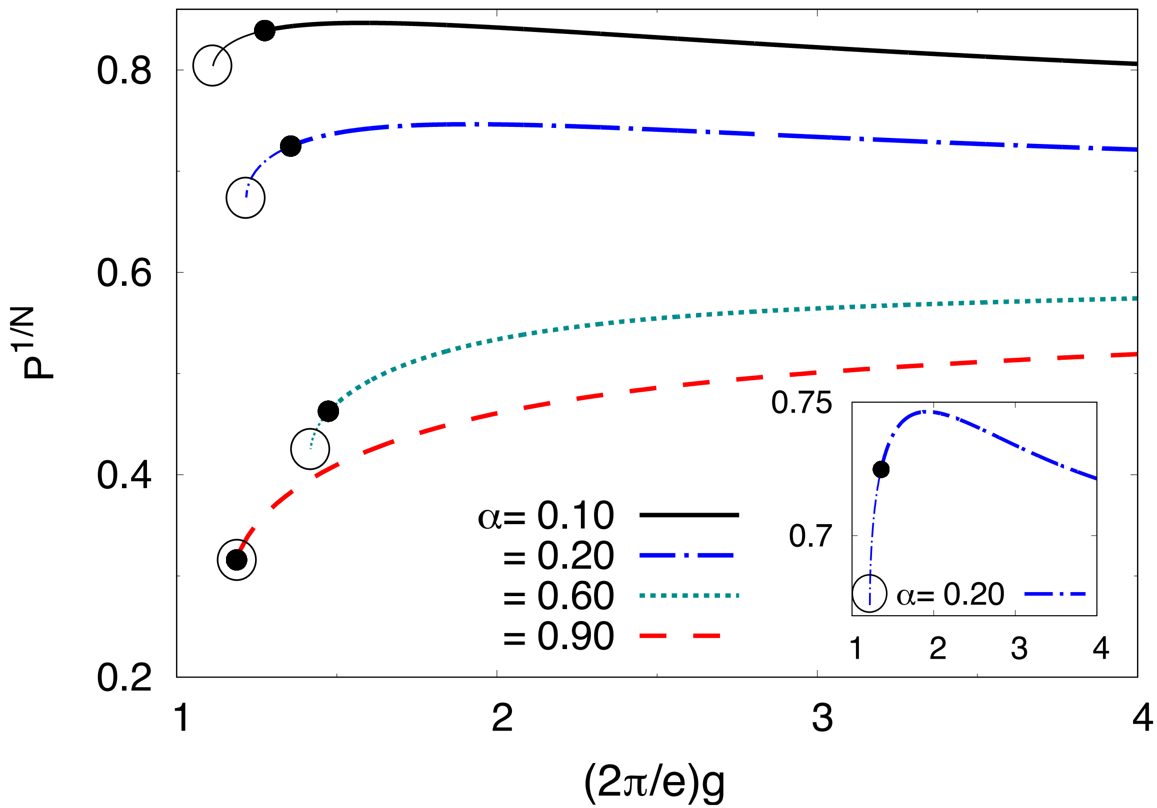

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→ peaks close to the critical point only for $\alpha \neq 0$ and $\tilde{\alpha} \neq 0$

Entanglement

Logarithmic negativity

$$E_N(\hat{\rho}) = \log_2 \left(1 - 2 \sum_{\lambda_k < 0} \lambda_k \left[\hat{\rho}^{T_A} \right] \right)$$

negative eigenvalues of the partially transposed density matrix

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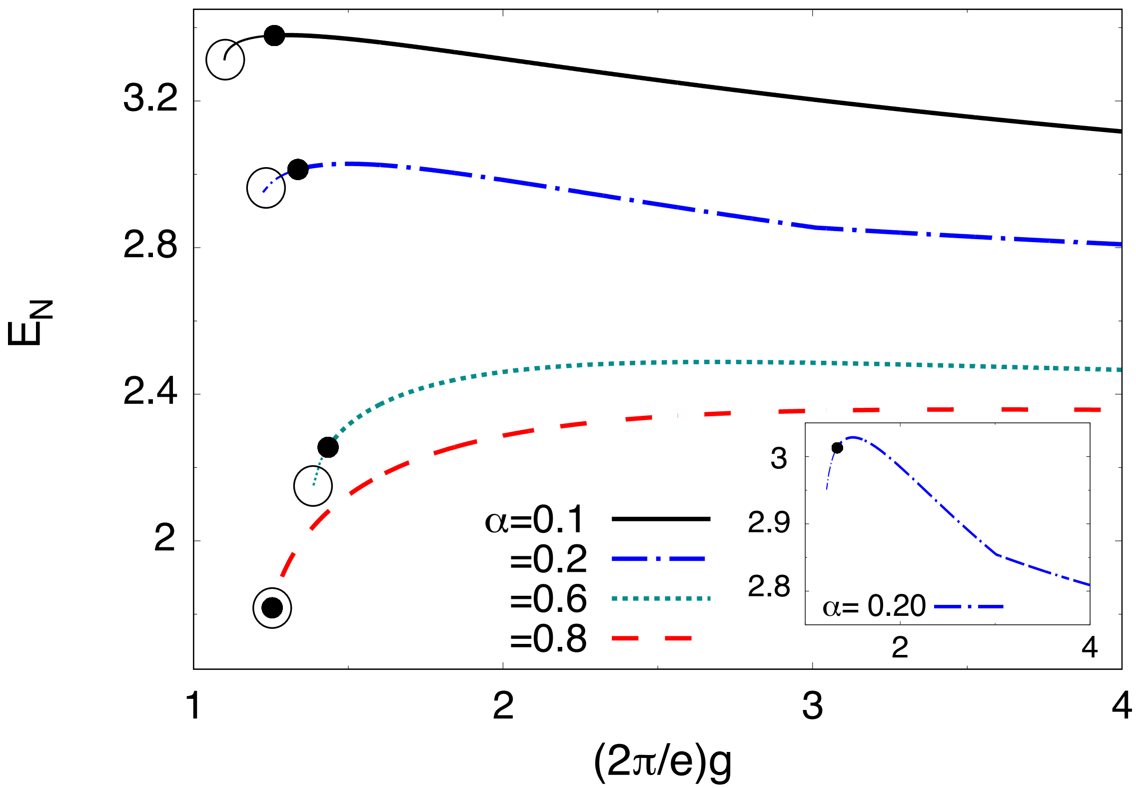

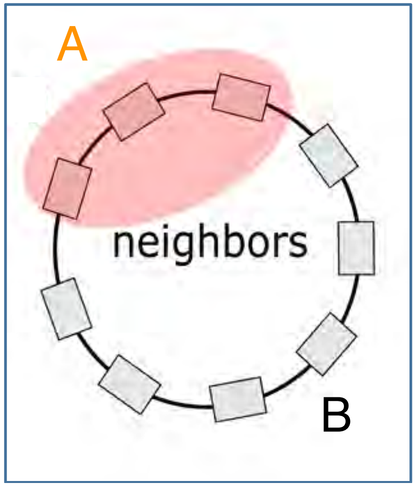

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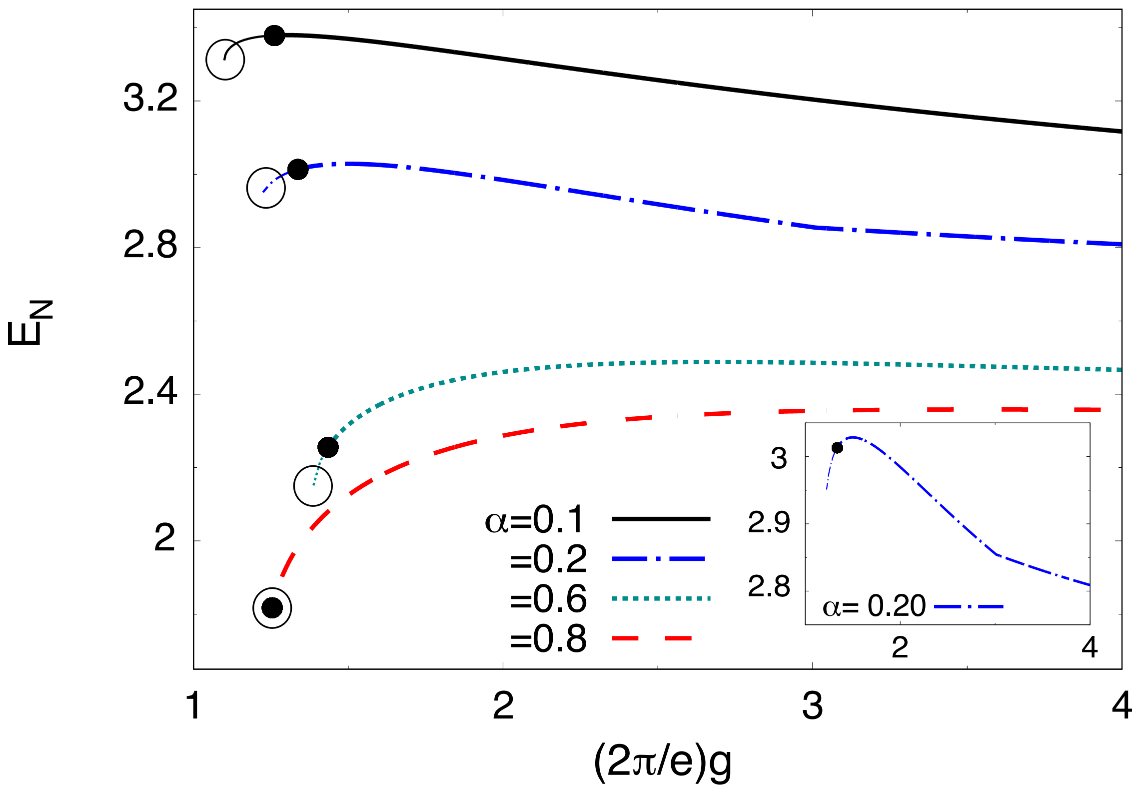

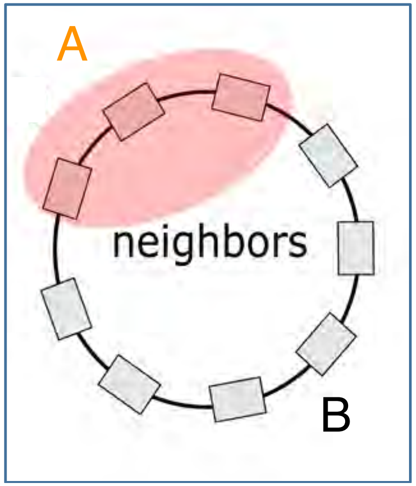

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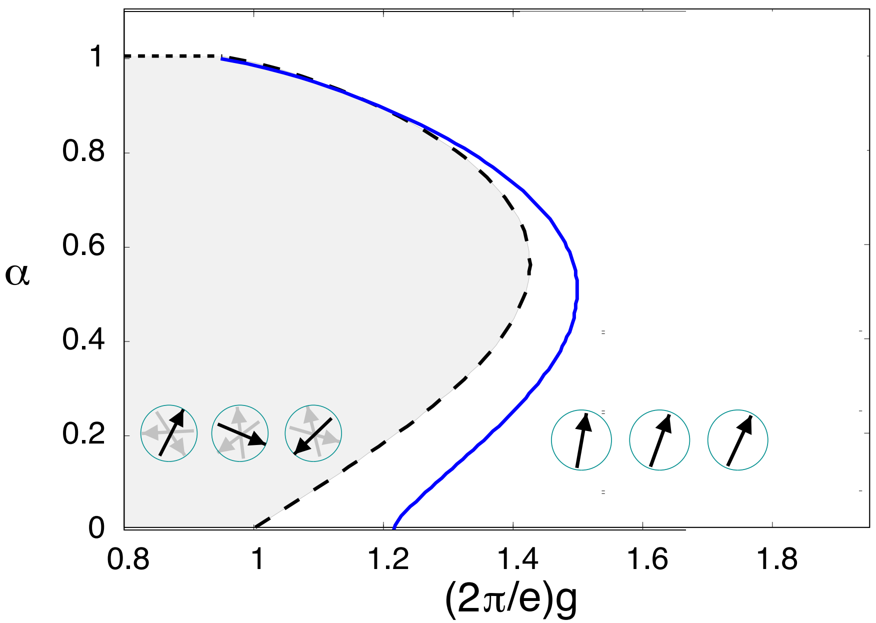
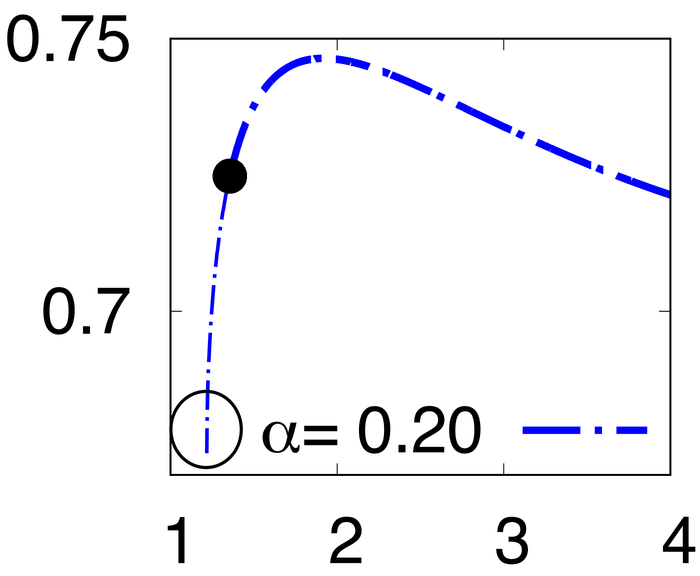
Summary

- In the quantum phase model realized by JJ chains with tailored dissipation, dissipative frustration leads to a non-monotonic phase diagram
- The purity and the logarithmic negativity show a peculiar behavior close to the critical point in presence of dissipative frustration



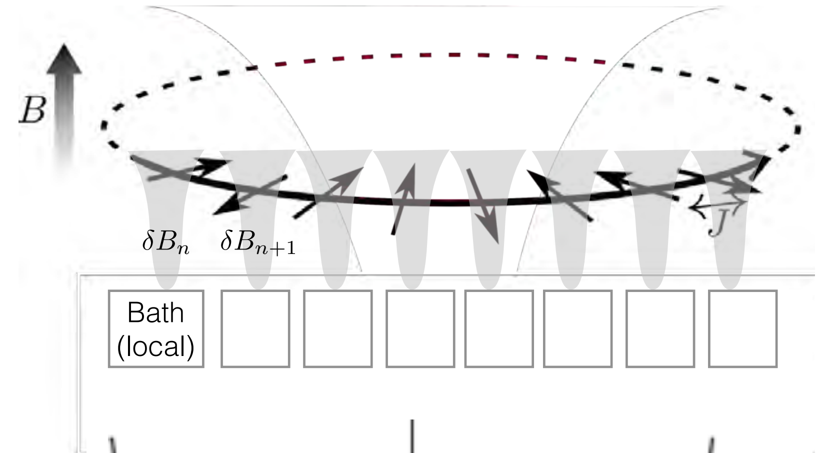


Phys. Rev. B **97**, 155427 (2018).

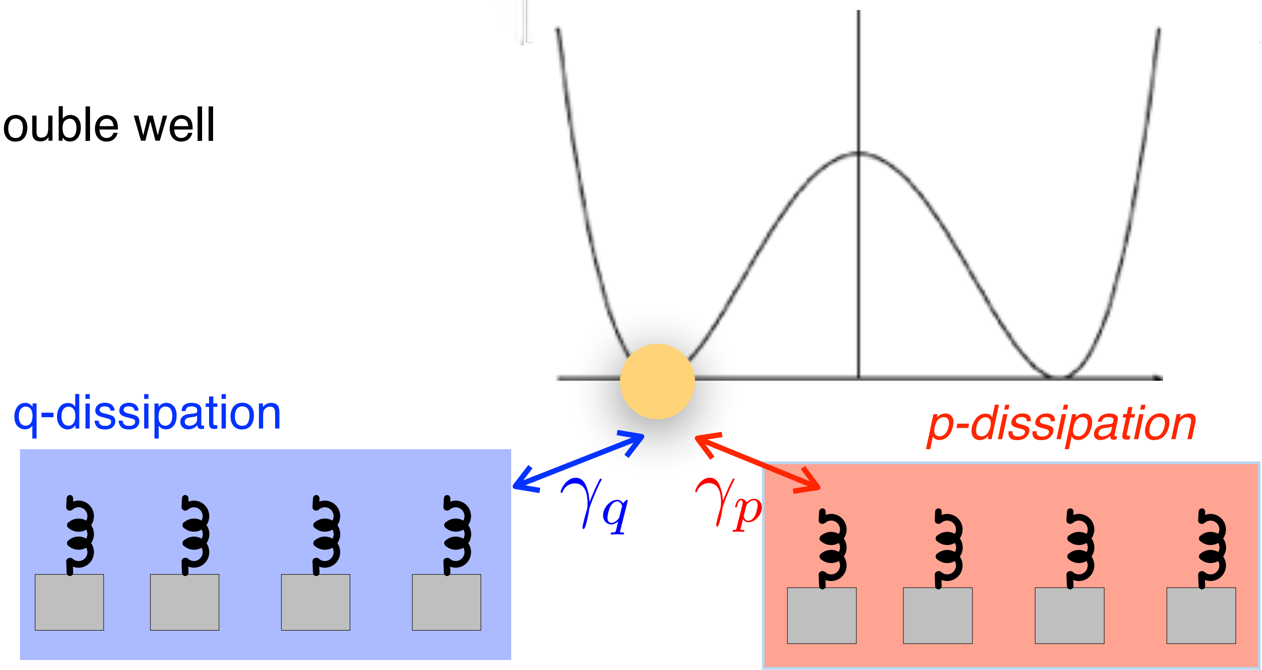
Outlook

Decoherence in the quantum Ising chain in pure dephasing regime

arXiv:1804.07559



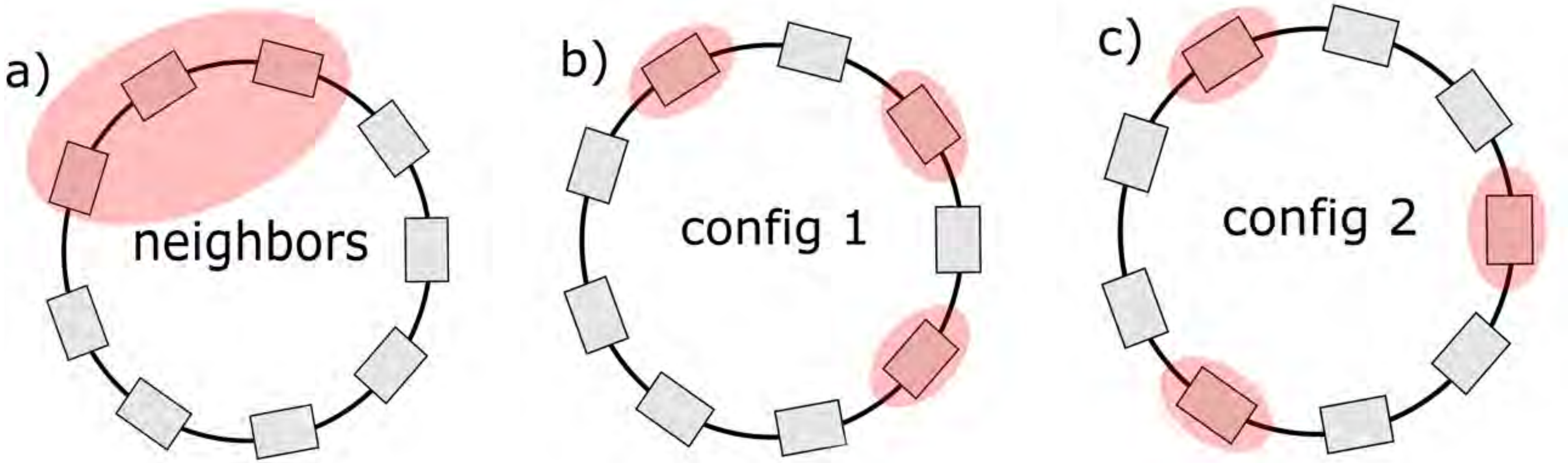
Dissipative frustration in a double well (instanton-technique)

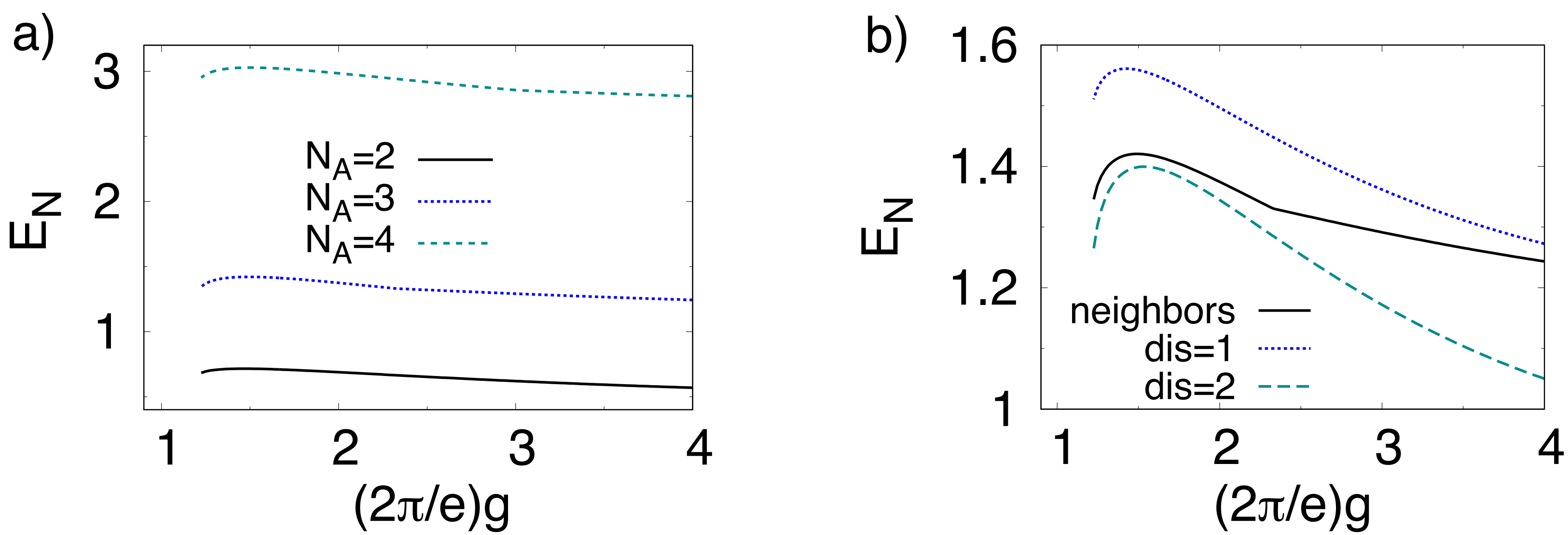


details: https://www.rastelli.uni-konstanz.de

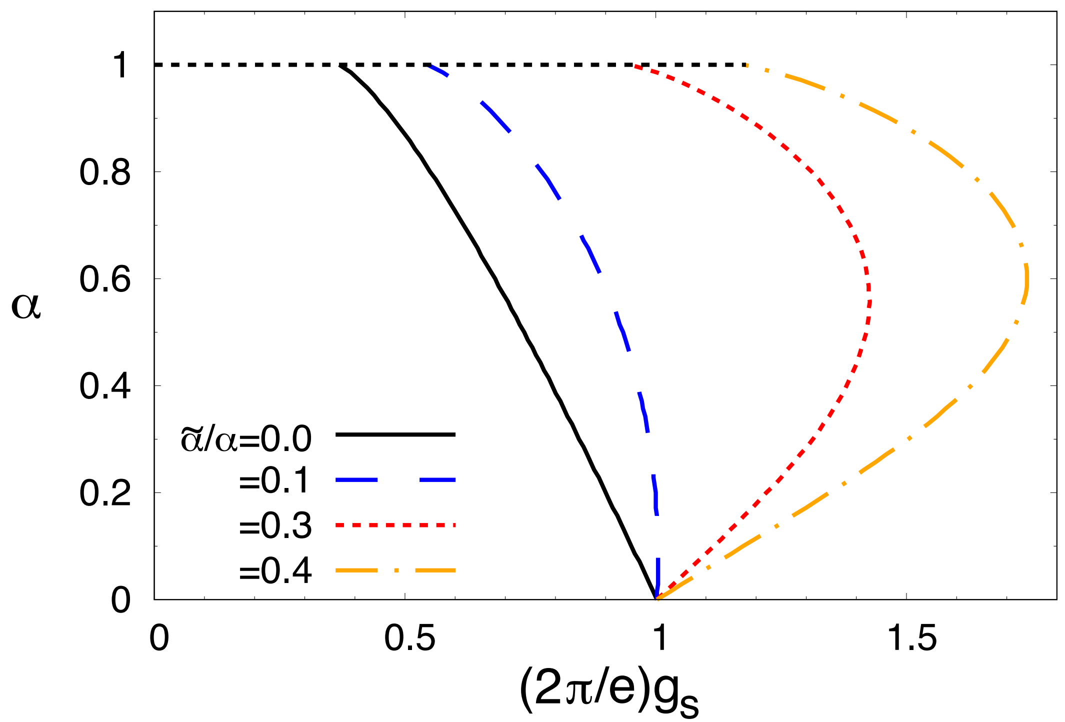
Supplemental slides

Entanglement

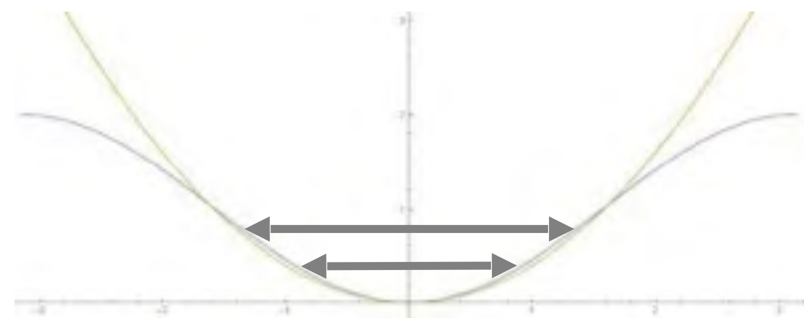




Cross-over



Self-Consistent Harmonic Approximation



$$\Delta S = S_{eff} - S_s$$
$$= -\int_0^\beta d\tau \sum_{n=0}^{N-1} V\cos\left(\varphi_{n+1}(\tau) - \varphi_n(\tau)\right) - \int_0^\beta d\tau \sum_{n=0}^{N-1} \frac{V_s}{2}\left(\varphi_{n+1}(\tau) - \varphi_n(\tau)\right)^2$$

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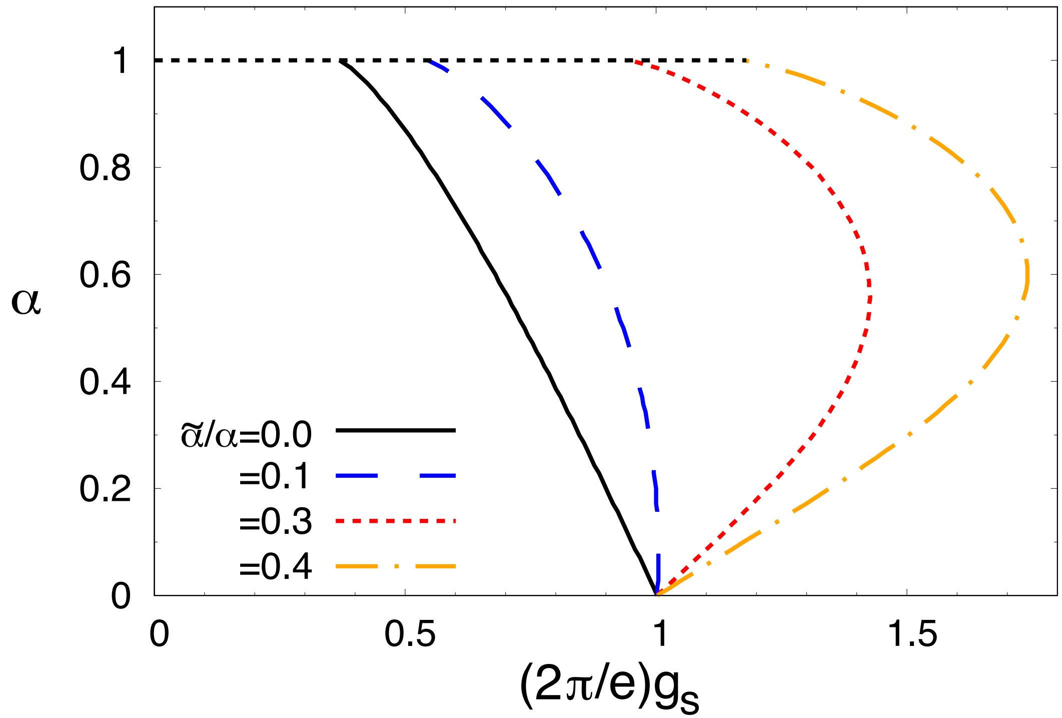

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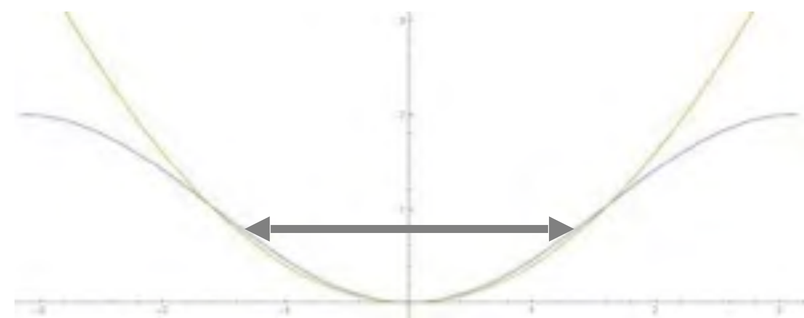

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Analytic formula

$$\left\langle \hat{Q}^2 \right\rangle = \frac{2k_B T}{\hbar\omega_0} + \frac{2\omega_0}{\pi\omega_c} \sum_{i=1}^{4} A_i \, \Psi\left(1 + \frac{\hbar\Omega_i}{2\pi k_B T}\right)$$

$$\left\langle \hat{Q}^2 \right\rangle = \frac{\langle \hat{q}^2 \rangle}{\langle \hat{q}^2 \rangle_0}$$

$-\Omega_i$ = roots of the quartic polynomial

$$\prod_{i=1}^{4}(\omega + \Omega_i) = (\omega^2 + \omega_0^2)(\omega + \omega_c)^2 + \omega_c(\gamma_q + \gamma_p)(\omega_c + \omega)\omega + \omega_c^2 \frac{\gamma_q\gamma_p}{\omega_0^2}\omega^2$$

A_i = coefficients related to the roots

$\Psi(x)$ = Digamma function

Symmetry: $\langle \hat{Q}^2 \rangle = \sigma(\gamma_q, \gamma_p)\,, \quad \langle \hat{P}^2 \rangle = \sigma(\gamma_p, \gamma_q)\,.$

G.Rastelli, New J. Phys. **18**, 053033 (2016).

Formula

$$\Gamma = \frac{\gamma_q + \gamma_p}{2\omega_0} \qquad \rho = \frac{\sqrt{\gamma_q \gamma_p}}{\omega_0}$$

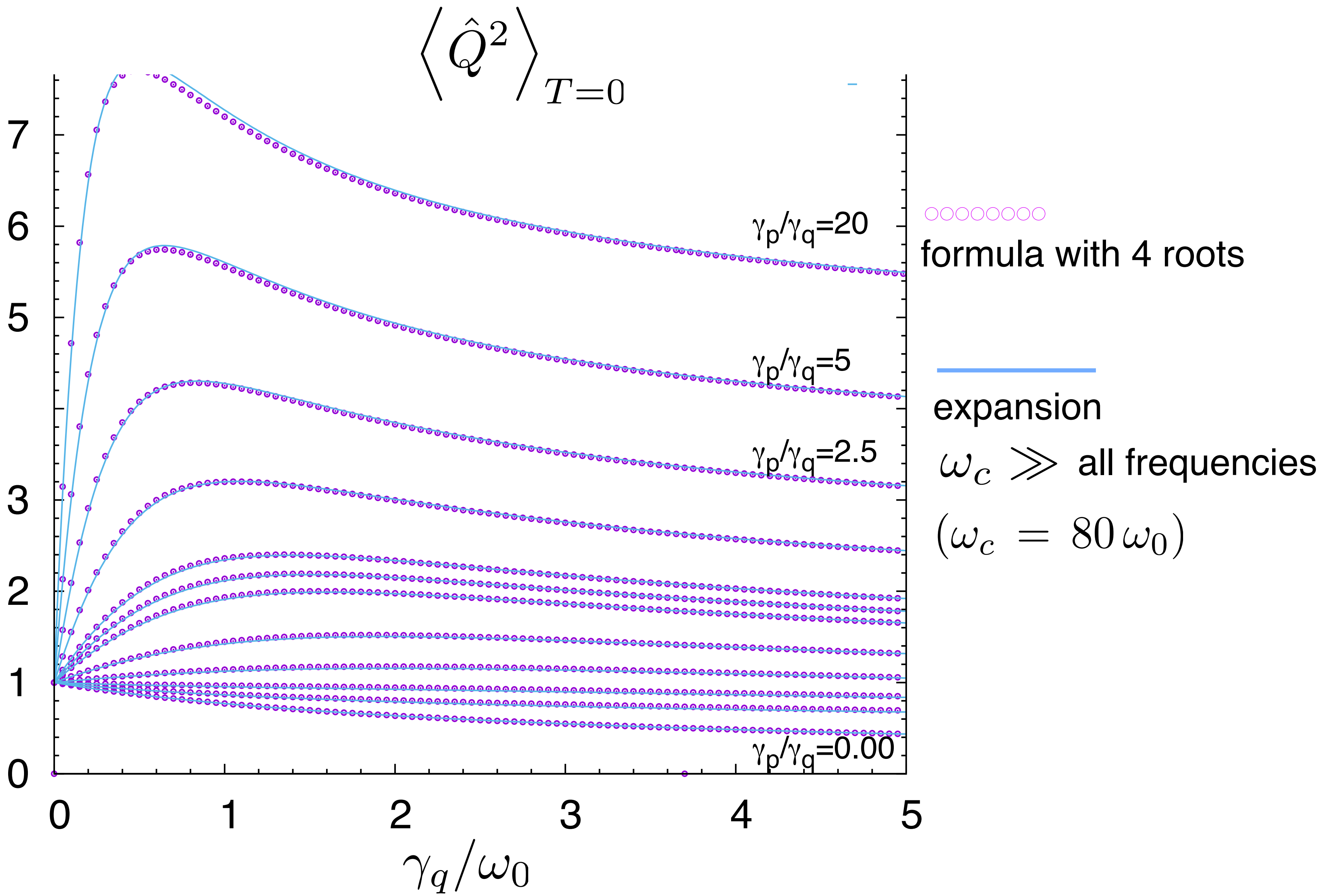
$$\Delta\Gamma = \frac{\gamma_q - \gamma_p}{2\omega_0}$$

$$\left\langle \hat{Q} \right\rangle_{T=0} = \frac{2}{\pi(1+\rho^2)} \left[\frac{\gamma_p}{\omega_0} \left(\ln\left(\frac{\omega_c}{\omega_0}\right) + \rho \arctan(\rho) + \ln(1+\rho^2) \right) + \frac{\left(1 + \frac{\gamma_p}{\omega_0}\Delta\Gamma\right)}{\sqrt{|1-\Delta\Gamma^2|}} \Theta \right]$$

$$\Theta = \begin{cases} \arctan\left(\sqrt{1-\Delta\Gamma^2}/\Gamma\right) & |\Delta\Gamma| < 1 \\ \arctan\left(\sqrt{\Delta\Gamma^2 - 1}/\Gamma\right) & |\Delta\Gamma| > 1 \end{cases}$$

G.Rastelli, New J. Phys. **18**, 053033 (2016).

Quantum fluctuations T=0



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Low temperature limit

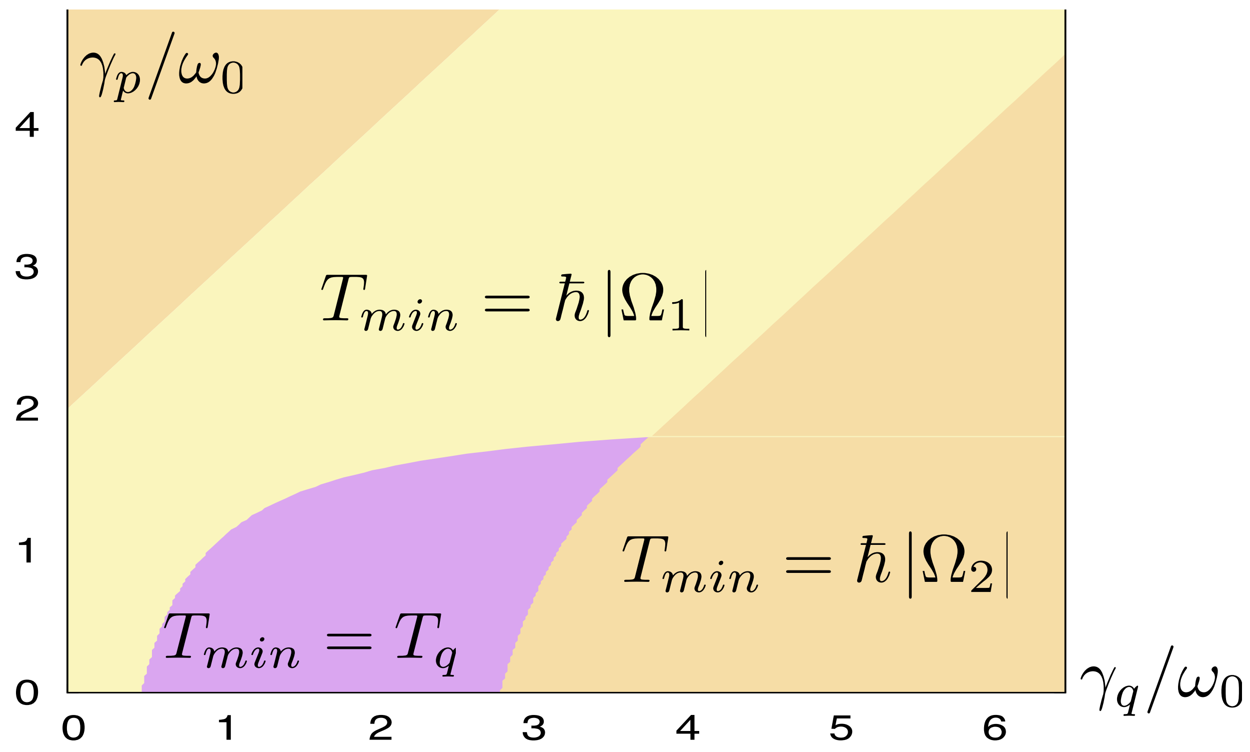
Expansion $k_B T \ll \hbar|\Omega_i|\,, \hbar\omega_0$ $\qquad k_B T_q = \hbar\omega_0 \sqrt{\dfrac{\omega_0}{2\pi\gamma_q}}$

$$\left\langle \hat{Q}^2 \right\rangle \simeq \left\langle \hat{Q}^2 \right\rangle_{T=0} + \frac{1}{3}\left(\frac{T}{T_q}\right)^2$$

finite temperature (quantum) corrections

quantum fluctuation (zero-point motion)

$$T < T_{min}$$



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Dissipative frustration: qualitative ideas

Single harmonic oscillator $$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2}{2}\hat{q}^2 + \hat{H}_{Bath,q} + \hat{H}_{Bath,p}$$

➡ fluctuations of both quadratures increase

➡ but the purity P always decreases (statistical mixture)

Example: fluctuations of q $$\rho(q) \sim \int dq_0 \, P_{\gamma_p}(q_0) \;\; \rho_{\gamma_q}(q - q_0)$$