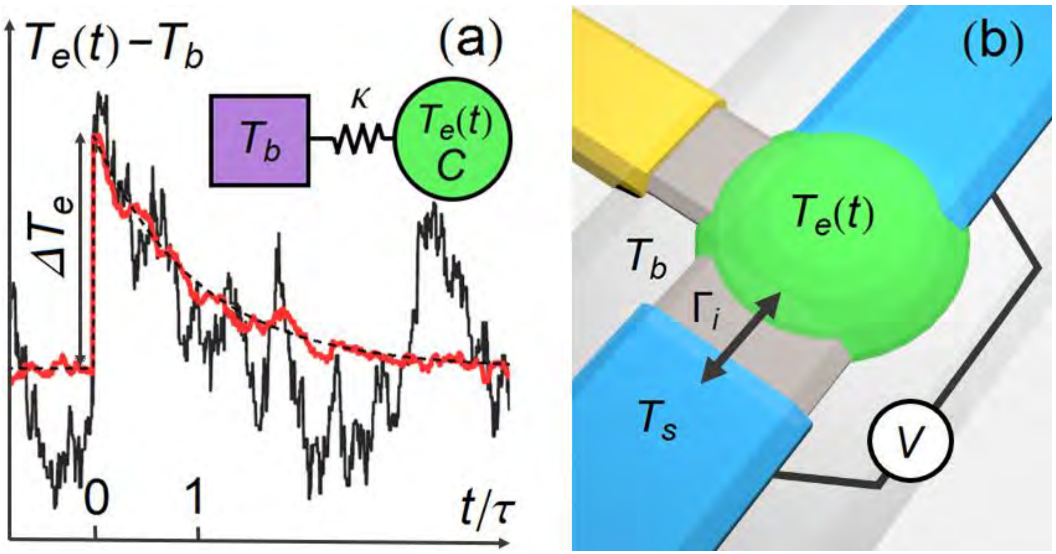
Nanoscale Quantum Calorimetry with Electronic Temperature Fluctuations

F. Brange[1], P. Samuelsson[1], B. Karimi[2], J.P. Pekola[2]

[1]Lund University, [2]Aalto University



arXiv:1805.02728

Outline

Calorimetry of heat pulses

- Calorimetry and bolometry
- Fast and sensitive temperature measurements
- Estimates for sub-meV detection

Proposed nanoscale calorimeter

- Hybrid normal-superconductor calorimeter
- Energy transfer statistics

Temperature fluctuations

- Temperature fluctuations, full statistics
- Temperature noise
- Third cumulant and back-action

Outline

Calorimetry of heat pulses

- Calorimetry and bolometry
- Fast and sensitive temperature measurements
- Estimates for sub-meV detection

Proposed nanoscale calorimeter

- Hybrid normal-superconductor calorimeter
- Energy transfer statistics

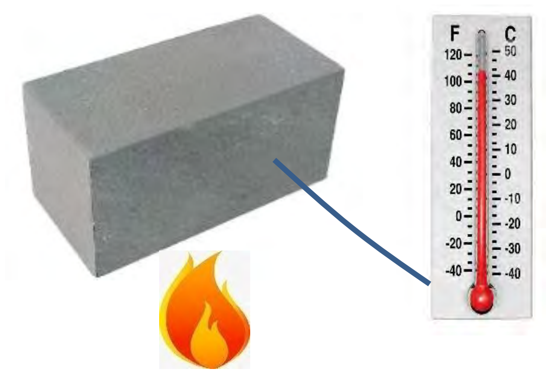
Temperature fluctuations

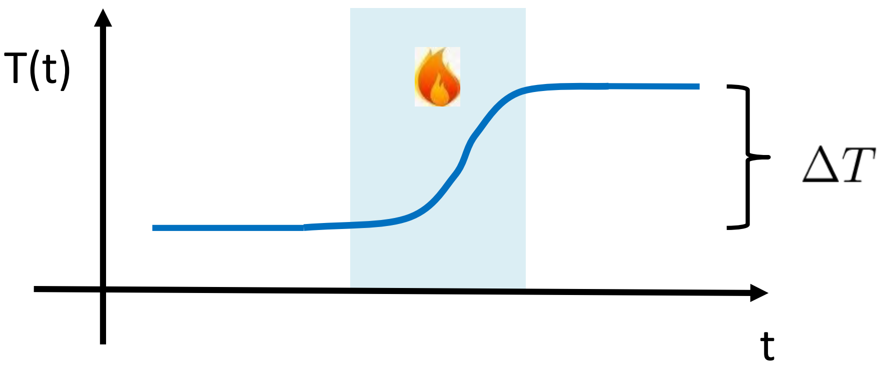
- Temperature fluctuations, full statistics
- Temperature noise
- Third cumulant and back-action

Calorimetry

"Calorimetry is the science or act of measuring changes in state variables of a body for the purpose of deriving the heat transfer associated with changes of its state due, for example, to chemical reactions, physical changes, or phase transitions under specified constraints" - Wikipedia

- Here: measurement of energy/heat transfer via temperature change





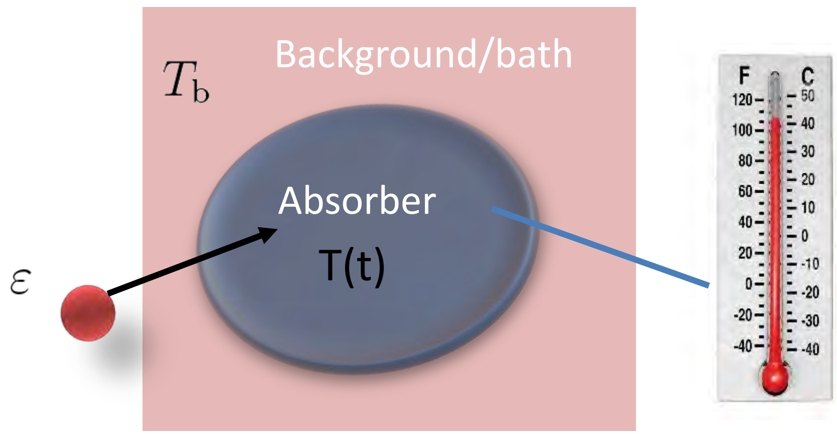
- Absorbed energy

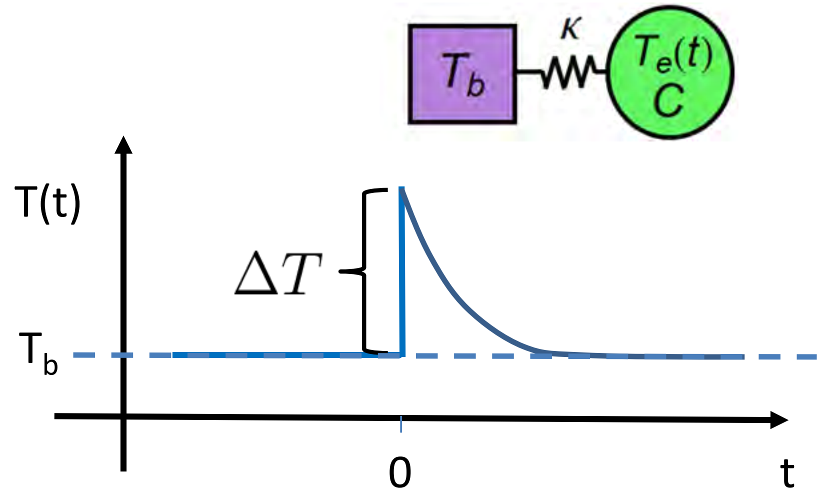
$$E = C\Delta T$$

C - heat capacity

Quantum Calorimetry

Single particle energy detection (particle physics Kilbourne et al, Phys. Today 99)





- Ideal operation (linear $\Delta T \ll T_\mathrm{b}$, noise free)

$$T(t) = T_\mathrm{b} + \Delta T e^{-t/\tau}, \qquad t \geq 0 \qquad\qquad \tau = C/\kappa$$

κ - thermal conductivity, absorber-bath

Particle energy

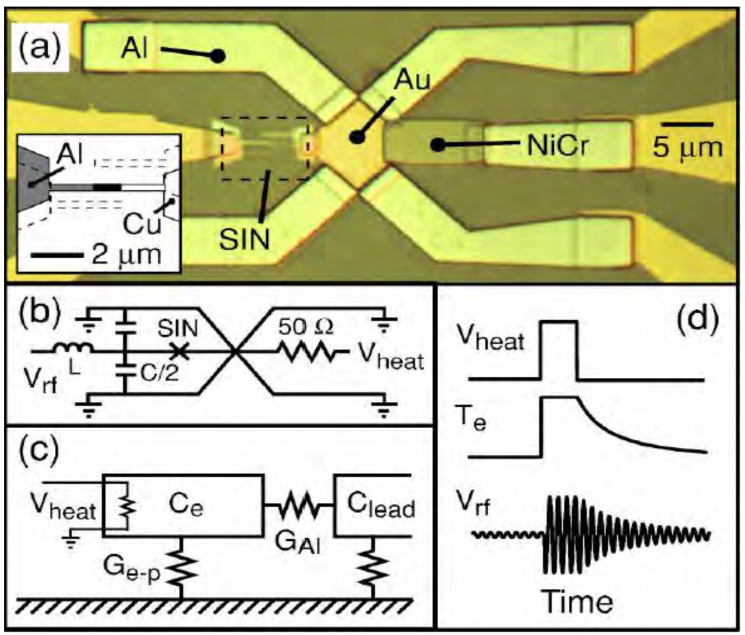
$$\varepsilon = C\Delta T$$

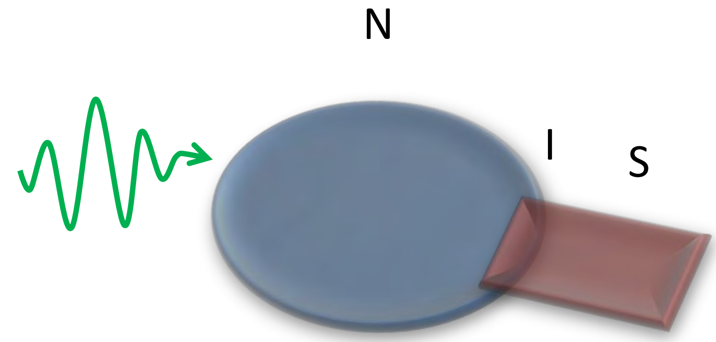
Nanoscale calorimeters and bolometers

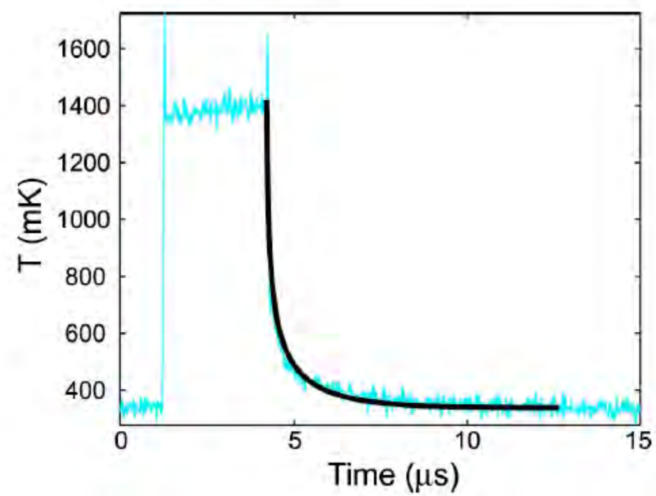
Early experiments

Electron temperature in metal bolometers, $\tau_{\text{e-e}} \ll \tau$

- X-ray detection Nahum, Martinis, APL 95
- RF readout Schmidt et al, PRB 04







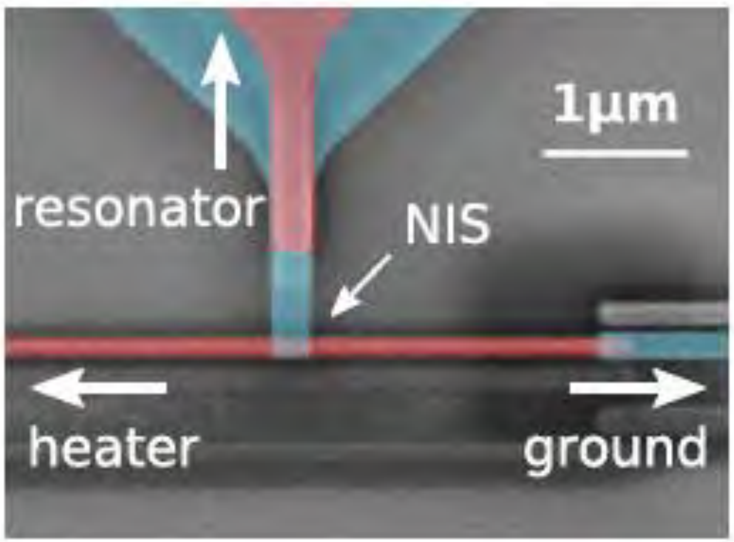
- Large sensitivity Schmidt et al, APL 05

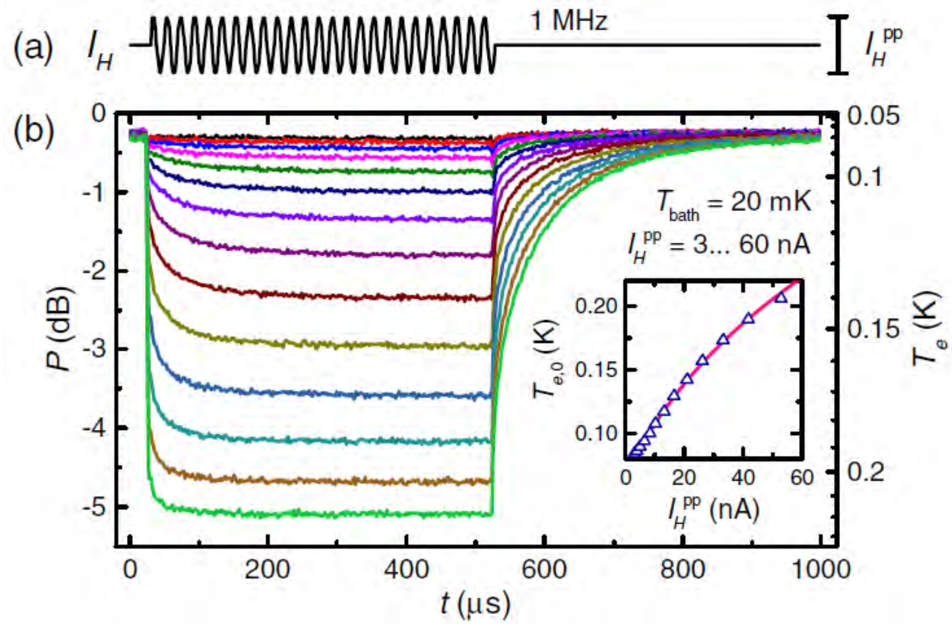
Largely space application driven

Recent Aalto results

(also B. Karimi presentation, Friday)

Gasparinetti et al, PR App 14, Govenius et al, PRL 16, Viisanen and Pekola, PRB 2018,....





- Fast, sensitive thermometry, effectively non-invasive.
- Small absorber volume ⇨ small heat capacity.
- Small background noise.

Typical parameters

$C \sim 10^3 - 10^5 k_{\rm B}$ $\quad\quad$ $\tau \sim 1 - 10 \mu s$

$T_{\rm b} \sim 30 - 100 \text{ m}K$ $\quad$ (effective bath)

Can we extend to quantum calorimetry for sub meV-energies?

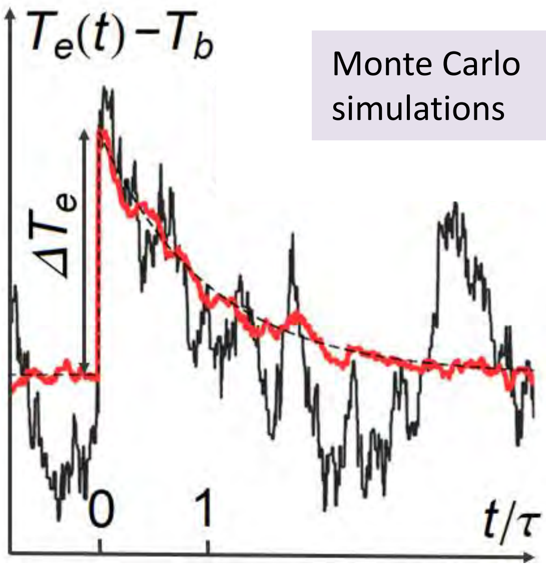
Energy quanta detection and fluctuations

Fluctuation-dissipation like relation

$$\langle \delta T_{\mathrm{e}}(t)\delta T_{\mathrm{e}}(t')\rangle = \frac{k_{\mathrm{B}}T_{\mathrm{b}}^2}{C}e^{-|t-t'|/\tau},$$

⇨ amplitude of fluctuations

$$\sqrt{\langle \delta T_{\mathrm{e}}^2(t)\rangle} = T_{\mathrm{b}}(k_{\mathrm{B}}/C)^{1/2}$$



Signal-to-noise ratio (SNR), $\Delta T_{\mathrm{e}} = \varepsilon/C$

$$\Delta T_{\mathrm{e}}/\sqrt{\langle \delta T_{\mathrm{e}}^2\rangle} = \varepsilon/[T_{\mathrm{b}}\sqrt{k_{\mathrm{B}}C}]$$

Typical parameters for single energy quanta detection

$$\varepsilon = 200\ \mu eV, C = 10^3 k_{\mathrm{B}} \begin{cases} T_{\mathrm{b}} = 5\ \mathrm{m}K \\ T_{\mathrm{b}} = 30\ \mathrm{m}K \end{cases}$$

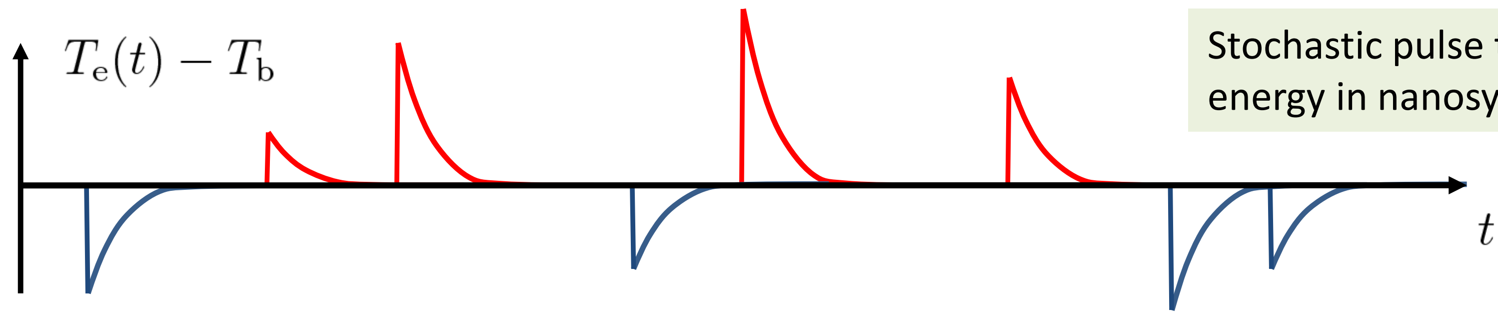
SNR 15

SNR 2.4

Ex: Al gap $\Delta \approx 200\mu eV$, 50 GHz microwave photon

Careful treatment of fluctuations needed!

Stochastic treatment of all transfer events



Stochastic pulse time and energy in nanosystems

Probability distribution of total energy transfer (during time t_0)

$$P_\sigma(E, T_\mathrm{e}) = \frac{1}{2\pi} \int d\xi_\sigma e^{-iE\xi_\sigma + t_0 F_\sigma(\xi_\sigma, T_\mathrm{e})} \qquad \sigma = \mathrm{i,b}$$

Injection of particles (i) and absorber-bath (b) transfers, T_e constant.

Poisson particle transfer statistics: cumulant generating function

van den Berg et al NJP 15

$$F_\sigma(\xi_\sigma, T_\mathrm{e}) = \Gamma_\sigma(T_\mathrm{e}) \left[\int d\varepsilon e^{i\varepsilon\xi_\sigma} P_\sigma(\varepsilon, T_\mathrm{e}) - 1 \right],$$

Particle transfer rate

Particle energy distribution

Outline

Calorimetry of heat pulses

- Calorimetry and bolometry
- Fast and sensitive temperature measurements
- Estimates for sub-meV detection

Proposed nanoscale calorimeter

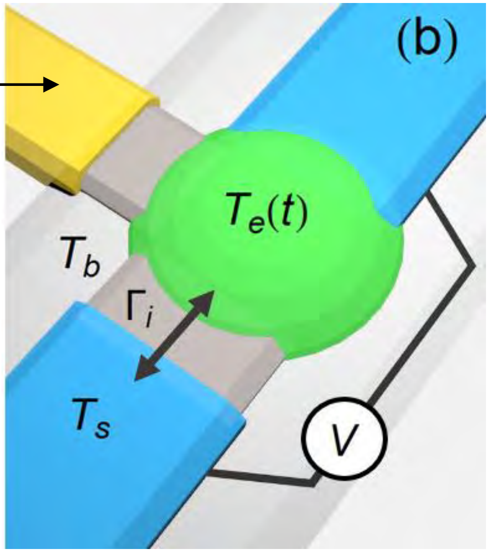
- Hybrid normal-superconductor calorimeter
- Energy transfer statistics

Temperature fluctuations

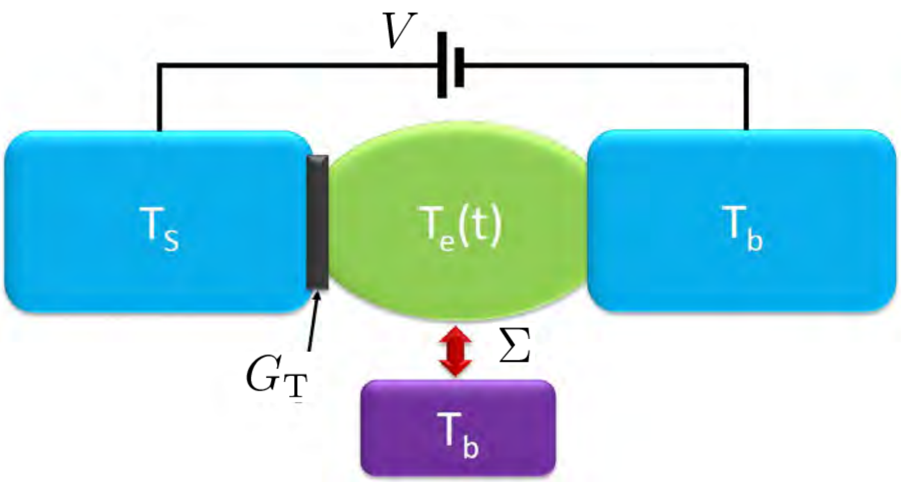
- Temperature fluctuations, full statistics
- Temperature noise
- Third cumulant and back-action

Normal-superconductor set-up

Thermometer,
"non invasive"



Circuit



Parameters

- Tunneling conductance G_{T}
- Phonon coupling constant Σ
- Temperatures $T_{\mathrm{s}}, T_{\mathrm{b}}, T_{\mathrm{e}}(t)$
- Applied bias V

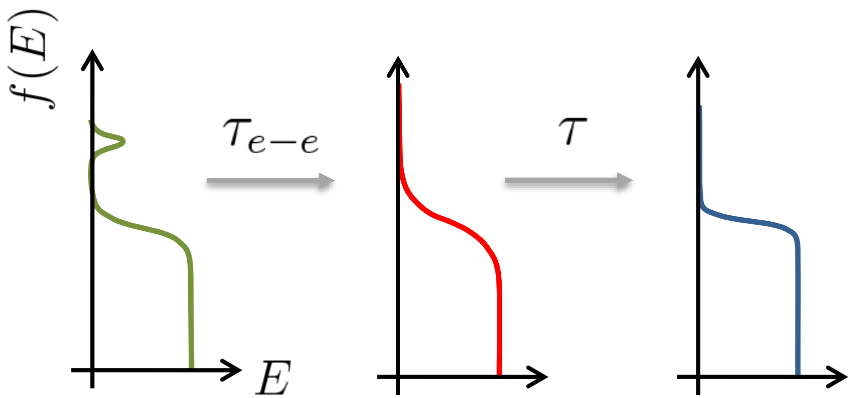
Right superconductor

- Transparent, ohmic contact
- Suppresses potential fluctuations
- Perfect heat mirror

Time scales and assumptions

Hot-electron regime

Absorber relaxation times



Electronic thermalization

Lattice thermalization

Quasi-equilibrium

$$\tau_{\text{e-e}} \ll \tau\,,\ 1/\Gamma_{\text{i}}$$

- Electronic distribution

$$f_e(E) = \left[1 + e^{E/k_B T_e(t)}\right]^{-1}$$

- Well defined $T_{\text{e}}(t)$

Well separated pulses

$$\Gamma_{\text{i}}\tau \ll 1$$

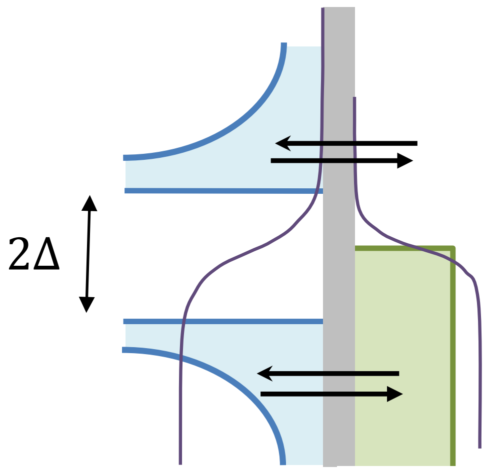
Not needed in general

Additional assumptions

- No standard and inverse proximity effect
- No unwanted heating, V does not affect T_{s}

Injector – absorber, electron tunnelling

Quasiparticle picture



superconductor   Normal absorber

Spectral tunneling rates

Standard expressions

$$\Gamma^{\mathrm{i}}_{\pm}(\varepsilon) = (G_{\mathrm{T}}/e^2)\nu_{\mathrm{S}}(\varepsilon - eV) f_{\pm}(\varepsilon - eV, T_{\mathrm{s}}) f_{\mp}(\varepsilon, T_{\mathrm{e}})$$

with

$$\nu_{\mathrm{S}}(\varepsilon) = |\varepsilon|/\sqrt{\varepsilon^2 - \Delta^2}\theta(|\varepsilon| - \Delta)$$

$$f_{+}(\varepsilon, T) = (e^{\varepsilon/[k_{\mathrm{B}}T]} + 1)^{-1}, f_{-}(\varepsilon, T) = 1 - f_{+}(\varepsilon, T)$$

Cumulant generating function

Energy counting factors

$$F_{\mathrm{i}}(\xi_{\mathrm{i}}, T_{\mathrm{e}}) = \int d\varepsilon \left[\Gamma^{\mathrm{i}}_{+}\left(e^{i\xi_{\mathrm{i}}\varepsilon} - 1\right) + \Gamma^{\mathrm{i}}_{-}\left(e^{-i\xi_{\mathrm{i}}\varepsilon} - 1\right)\right]$$

Rate into absorber

Rate out of absorber

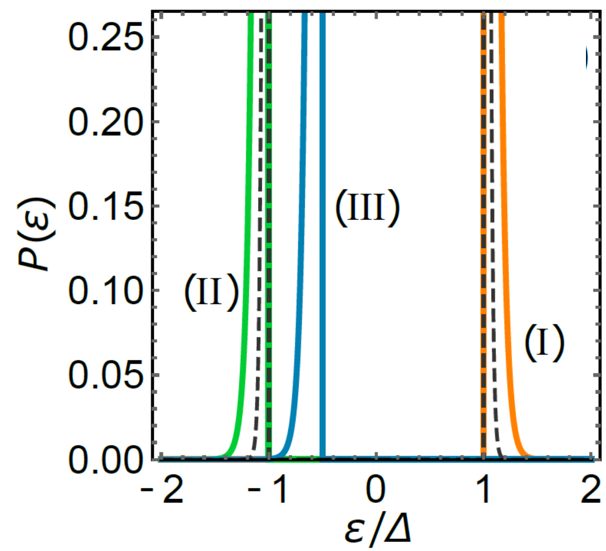
Cumulants from $(-i)^n \partial^n_{\xi_{\mathrm{i}}} F(\xi_{\mathrm{i}}, T_{\mathrm{e}})|_{\xi_i=0}$, giving current and noise

$$I^E_{\mathrm{i}} = \int d\varepsilon\varepsilon \left[\Gamma^{\mathrm{i}}_{+} - \Gamma^{\mathrm{i}}_{-}\right] \qquad S^E_{\mathrm{i}} = \int d\varepsilon\varepsilon^2 \left[\Gamma^{\mathrm{i}}_{+} + \Gamma^{\mathrm{i}}_{-}\right]$$

Tunneling rate and energy distribution

$$\Gamma_{\rm i}(T_{\rm e}) = \int d\varepsilon\, \left[\Gamma^{\rm i}_{+}(\varepsilon) + \Gamma^{\rm i}_{-}(\varepsilon)\right]$$

$$P_{\rm i}(\varepsilon, T_{\rm e}) = \left[\Gamma^{\rm i}_{+}(\varepsilon) + \Gamma^{\rm i}_{-}(-\varepsilon)\right]/\Gamma_{\rm i}$$



Single energy injection

Relevant regime $k_{\rm B}T_{\rm s}, k_{\rm B}T_{\rm e} \ll \Delta$, three cases with well defined energies

$$\varepsilon_{\rm I} = \Delta,\ \varepsilon_{\rm II} = -\Delta,\ \varepsilon_{\rm III} = eV - \Delta$$

for

$$V = 0, T_{\rm s} \gg T_{\rm e} \quad \text{(I)} \qquad V = 0, T_{\rm s} \ll T_{\rm e} \quad \text{(II)}$$

$$T_{\rm s}(1 - e|V|/\Delta) \ll T_{\rm e} \ll e|V|/k_{\rm B} \quad \text{(III)}$$

$\Rightarrow$

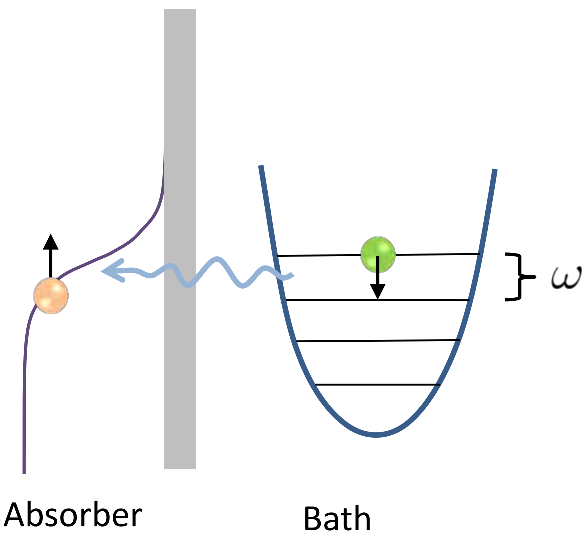
generating function $g = \sqrt{2\pi} G_{\rm T}\Delta/e^2$, c_α constant

$$F^{(\alpha)}_{\rm i}(\xi_{\rm i}, T_{\rm e}) = g c_\alpha \left(e^{i\varepsilon_\alpha \xi_{\rm i}} - 1\right), \quad \alpha = \text{I,II,III}$$

Uncorrelated/Poisson injection of particles with energy ε_α

Bath – absorber, phonons

Bath phonon picture



Spectral rates

Standard expressions, 3D-phonons in metals

$$\Gamma^{\mathrm{b}}_{\pm}(\varepsilon) = -\Sigma\mathcal{V}/[24k_{\mathrm{B}}^5\zeta(5)]\varepsilon^3 n(\pm\varepsilon, T_{\mathrm{b}})n(\mp\varepsilon, T_{\mathrm{e}})$$

with Riemann $\zeta(x)$,

$$n(\varepsilon, T) = (e^{\varepsilon/[k_{\mathrm{B}}T]} - 1)^{-1}$$

and the absorber volume $\mathcal{V}$.

Cumulant generating function

$$F_{\mathrm{b}}(\xi_{\mathrm{b}}, T_{\mathrm{e}}) = \int d\varepsilon\, \left[\Gamma^{\mathrm{b}}_{+}\left(e^{i\xi_{\mathrm{b}}\varepsilon} - 1\right) + \Gamma^{\mathrm{b}}_{-}\left(e^{-i\xi_{\mathrm{b}}\varepsilon} - 1\right)\right]$$

Cumulants $S_{\mathrm{b}}^{(n)} = \partial^n_{\xi_{\mathrm{b}}} F_{\mathrm{b}}(\xi_{\mathrm{b}}, T_{\mathrm{e}})|_{\xi_{\mathrm{b}}=0}$ are ($n_{\pm} = n + (7 \pm 1)/2,\ n = 1, 2...$)

$$S_{\mathrm{b}}^{(n)} = \Sigma\mathcal{V}k_{\mathrm{B}}^{n-1}\frac{\zeta(n_{\pm})(n+3)!}{24\zeta(5)}\left(T_{\mathrm{e}}^{n+4} \pm T_{\mathrm{b}}^{n+4}\right),$$

- Odd n exact
- Even n within 2%

Bath – absorber, phonons

Energy current and noise

$$S_{\mathrm{b}}^{(1)} = \Sigma\mathcal{V}(T_{\mathrm{e}}^5 - T_{\mathrm{b}}^5)$$

Wellstood et al, PRB 94

$$S_{\mathrm{b}}^{(2)} \propto \Sigma\mathcal{V}(T_{\mathrm{e}}^6 + T_{\mathrm{b}}^6)$$

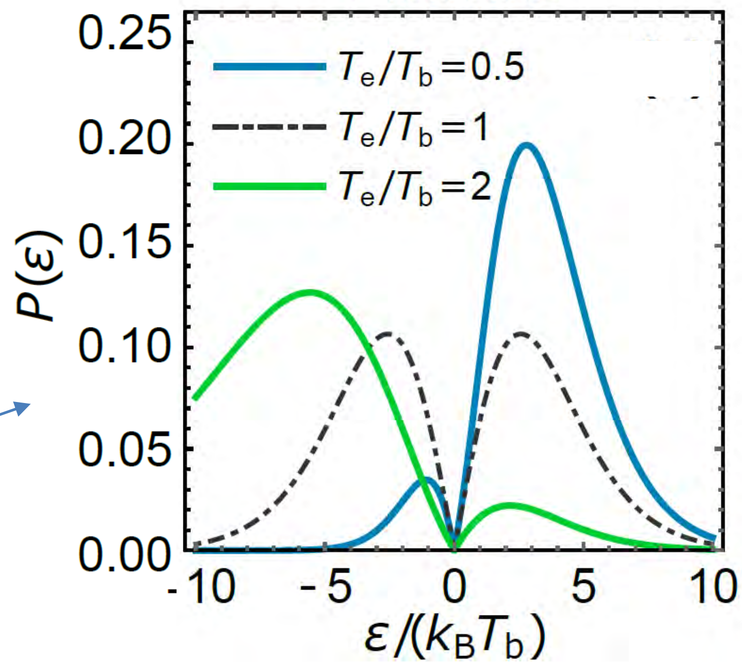
Karimi, Pekola, JLTP 18

Tunneling rate and energy distribution

$$\Gamma_{\mathrm{b}}(T_{\mathrm{e}}) = \int d\varepsilon\, [\Gamma_+^{\mathrm{b}}(\varepsilon) + \Gamma_-^{\mathrm{b}}(\varepsilon)]$$

$$P_{\mathrm{b}}(\varepsilon, T_{\mathrm{e}}) = [\Gamma_+^{\mathrm{b}}(\varepsilon) + \Gamma_-^{\mathrm{b}}(-\varepsilon)]/\Gamma_{\mathrm{b}}$$

Emission and absorption of phonons



Outline

Calorimetry of heat pulses

- Calorimetry and bolometry
- Fast and sensitive temperature measurements
- Estimates for sub-meV detection

Proposed nanoscale calorimeter

- Hybrid normal-superconductor calorimeter
- Energy transfer statistics

Temperature fluctuations

- Temperature fluctuations, full statistics
- Temperature noise
- Third cumulant and back-action

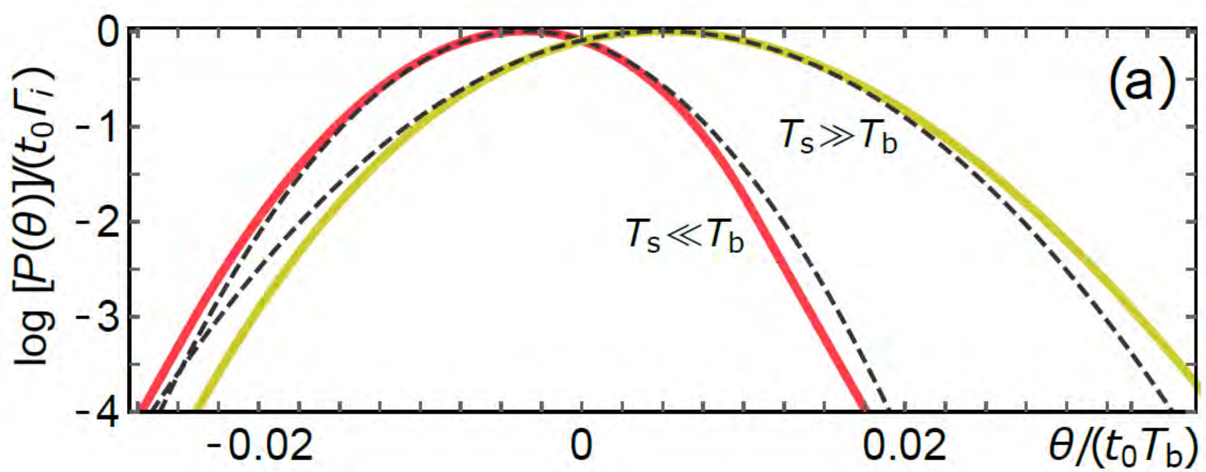
Full temperature statistics

Rates $\Gamma_\mathrm{i}(T_\mathrm{e})$ and $\Gamma_\mathrm{b}(T_\mathrm{e})$ depend on electron temperature ⇨
Back-action of heat transfer induced T_e-fluctuations on rates.

Resulting, full, temperature fluctuations investigated via

$$P(\theta)\,, \qquad \theta = \int [T_\mathrm{e}(t) - \overline{T}_\mathrm{e}]dt$$

Stochastic path integral approach Jordan et al, J. Mat. Phys 04, Battista et al, PRL 13.



---- best Gaussian fit

- Shifted away from 0.
- Non-Gaussian fluctuations.

Cumulant expansion

Cumulant expansion, in terms of energy current cumulants

$$\langle\langle \mathcal{E}^n(T_\mathrm{e})\rangle\rangle = (-i)^n \partial_\xi^n F(\xi, T_\mathrm{e})|_{\xi=0} \qquad F(\xi, T_\mathrm{e}) = F_\mathrm{i}(\xi, T_\mathrm{e}) + F_\mathrm{b}(\xi, T_\mathrm{e})$$

gives average temperature $\overline{T}_\mathrm{e}$ from

$$\langle \mathcal{E}(\overline{T}_\mathrm{e})\rangle = 0.$$

The temperature noise is, with $\kappa(T_\mathrm{e}) = i\partial_{T_\mathrm{e}}\partial_\xi F(\xi, T_\mathrm{e})|_{\xi=0}$,

$$S_{\mathrm{T_e}}^{(2)} = \frac{1}{\kappa^2}\langle\langle \mathcal{E}^2(T_\mathrm{e})\rangle\rangle$$

never measured!

and the third cumulant

$$S_{\mathrm{T_e}}^{(3)} = \frac{1}{\kappa^3}\left[\langle\langle \mathcal{E}^3(T_\mathrm{e})\rangle\rangle + 3\langle\langle \mathcal{E}^2(T_\mathrm{e})\rangle\rangle \frac{d}{dT_\mathrm{e}} \frac{\langle\langle \mathcal{E}^2(T_\mathrm{e})\rangle\rangle}{\kappa(T_\mathrm{e})}\right]$$

all evaluated at $\overline{T}_\mathrm{e}$.

back action term

Average temperature

The average temperature equation is

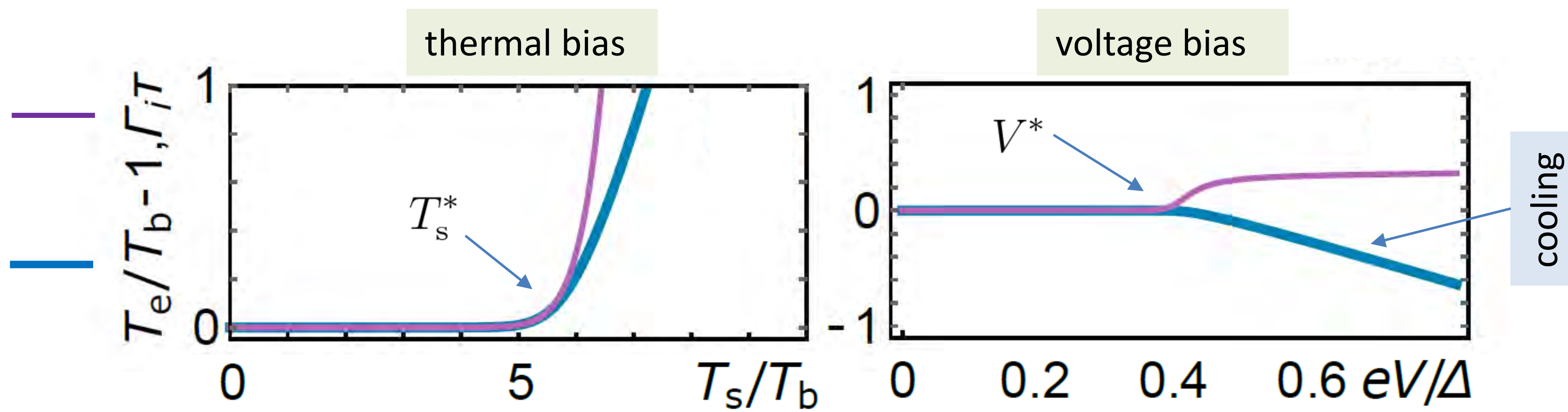
$$h(T_\mathrm{s}) + h(\overline{T}_\mathrm{e})\left[-\cosh\left(\frac{eV}{k_\mathrm{B}\overline{T}_\mathrm{e}}\right) + \frac{eV}{\Delta}\sinh\left(\frac{eV}{k_\mathrm{B}\overline{T}_\mathrm{e}}\right)\right] = \frac{1}{5r}\left(\frac{\overline{T}_\mathrm{e}^5}{T_\mathrm{b}^5} - 1\right)$$

where

$$h(T) = \sqrt{\frac{k_\mathrm{B}T}{\Delta}}e^{-\frac{\Delta}{k_\mathrm{B}T}} \qquad r = \frac{\sqrt{2\pi}G_\mathrm{T}\Delta^2}{T_\mathrm{b}e^2\kappa}$$

Injection tunnel rates

$$\Gamma_\mathrm{i} = g\left[h(T_\mathrm{s}) + h(T_\mathrm{b})\cosh\left(eV/k_\mathrm{B}T_\mathrm{b}\right)\right]$$



Well separated injection events $\Gamma_\mathrm{i}\tau \ll 1$ gives bias limits T_s^*, V^*.

Temperature noise

Thermal bias $V = 0$

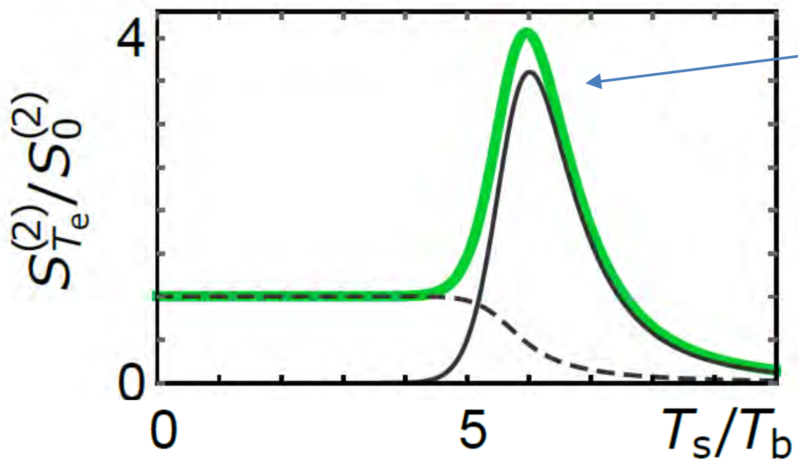
The second cumulant, for $\beta \gg \ln(r) \gg 1$, $\beta = \Delta/(k_\mathrm{B} T_\mathrm{b})$, $q = \overline{T}_\mathrm{e}/T_\mathrm{b}$

$$S^{(2)}_{T_\mathrm{e}}/S^{(2)}_0 = \frac{1+q^6}{2q^8} + \frac{\beta(q^5-1)}{10q^8}$$

$$S^{(2)}_0 = 2k_\mathrm{B} T_\mathrm{b}^2/\kappa$$

bath noise

injector noise



$T_\mathrm{b} = 0.01\Delta/k_\mathrm{B}, C = 20\Delta/T_\mathrm{b}$

peak around T_s^*

$$S^{(2)}_{T_\mathrm{e},\mathrm{max}}/S^{(2)}_0 \approx 0.035\beta$$

injector component

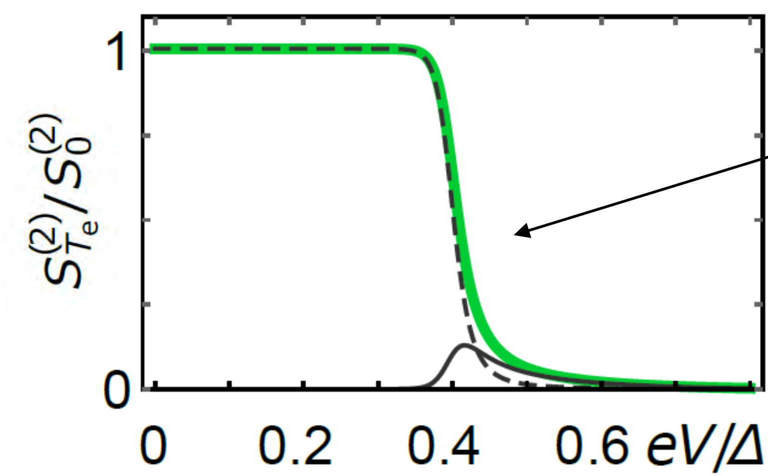
bath component

suppression from increasing $\kappa(\overline{T}_\mathrm{e})$

Voltage bias $T_\mathrm{s} = T_\mathrm{b}$

The second cumulant, with $\tilde{\beta} = \beta(1 - eV/\Delta)$

$$\frac{S^{(2)}_{\mathrm{T_e}}}{S^{(2)}_0} = \frac{q^4}{2}\frac{\overbrace{1 + q^6}^{\text{bath term}} + \overbrace{(\tilde{\beta}/5)(1-q^5)}^{\text{injector term}}}{\left(q^6 + (\tilde{\beta}/5)(1-q^5)\right)^2}.$$

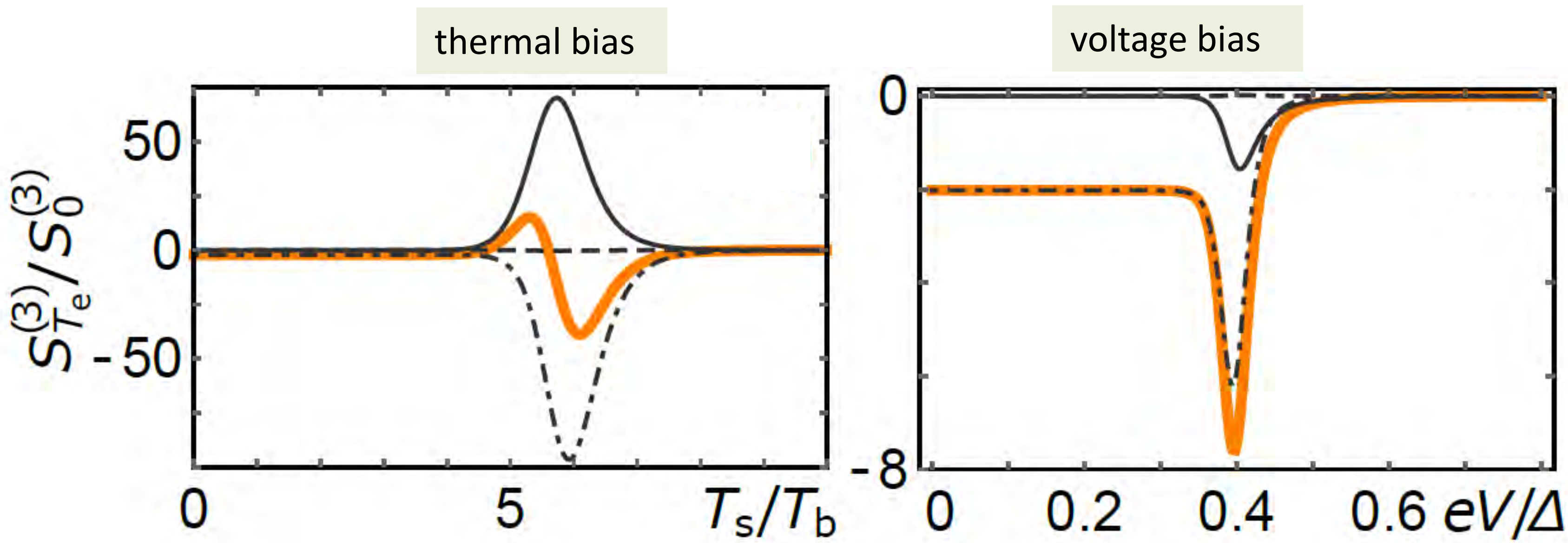


rapid decay after V^*
from increasing $\kappa(\overline{T}_\mathrm{e})$

injector component
bath component

Third cumulant

Three terms: bath, injector and back-action $S_0^{(3)} = 6k_B^2 T_b^3/\kappa^2$



injector component

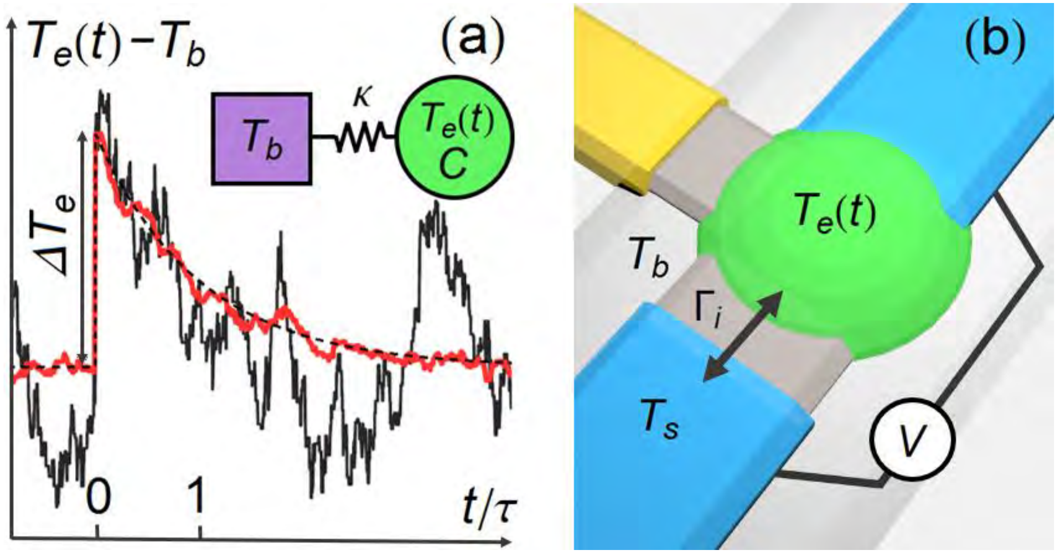
bath component

back-action component

- Features around T_s^*, V^*
- Decays due to inreasing $\kappa(\overline{T}_e)$

Conclusions

- Towards single energy quantum calorimetry
- Hybrid normal-superconductor system
- Full temperature statistics
- Parameter estimates and limits



arXiv:1805.02728