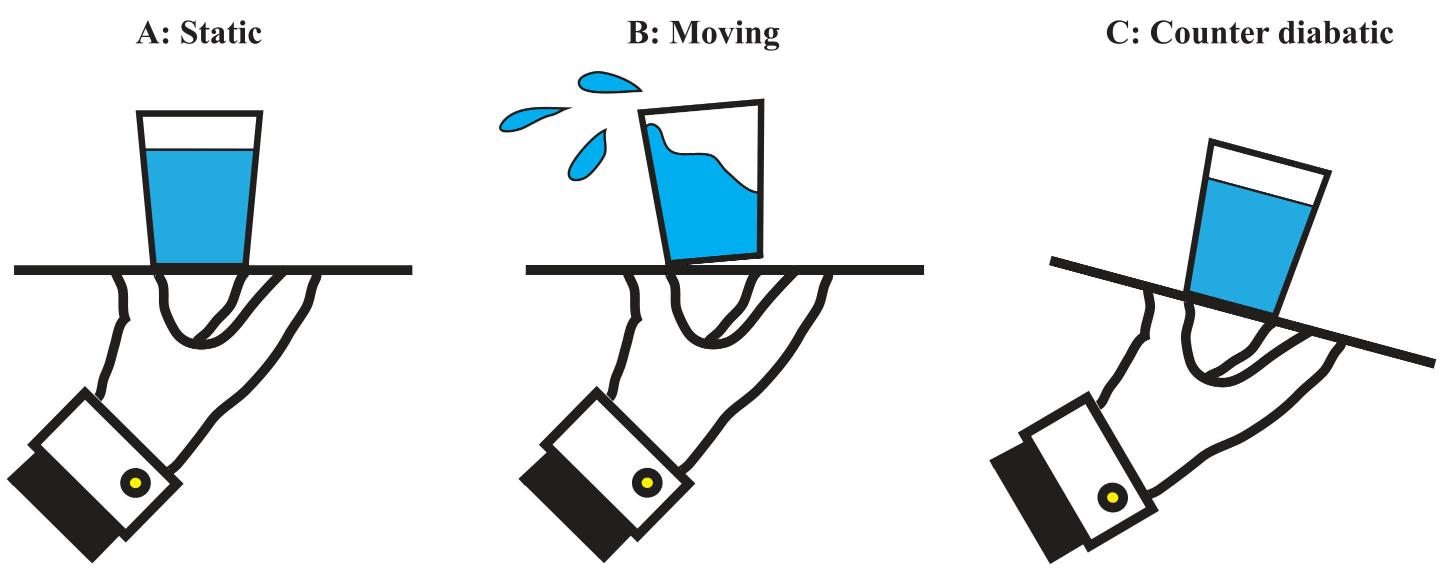
Counter-diabatic driving in quantum many body systems

Dries Sels & Anatoli Polkovnikov
Harvard-BU



fwo Opening new horizons

VE RI TAS
BOSTON UNIVERSITY

Outline

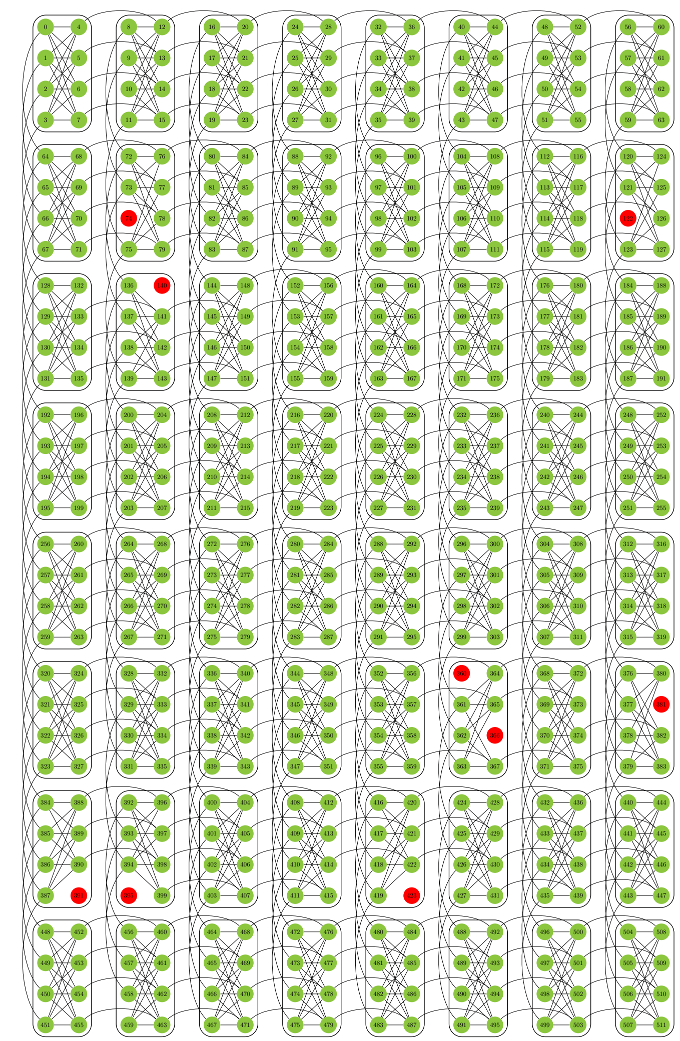
- Non-adiabatic response and quantum geometry
- Counter-diabatic driving and quantum speed limit
- Variational adiabatic gauge fields for many body systems

Motivation

- How to drive your system without exciting it?

Example : D-Wave machine

$$\sum_{ij} J_{ij} s_i^z s_j^z + \sum_i h_i(\tau) s_i^x$$



Non-adiabatic forces

- Consider a particle in an optical tweezer:

$$H = \frac{p^2}{2m} + V(x - \lambda(t))$$

- Let's move with the atom: $|\psi\rangle = U(\lambda)\,|\phi\rangle$

  - Dynamics: $i\partial_t\,|\psi\rangle = H\,|\psi\rangle$

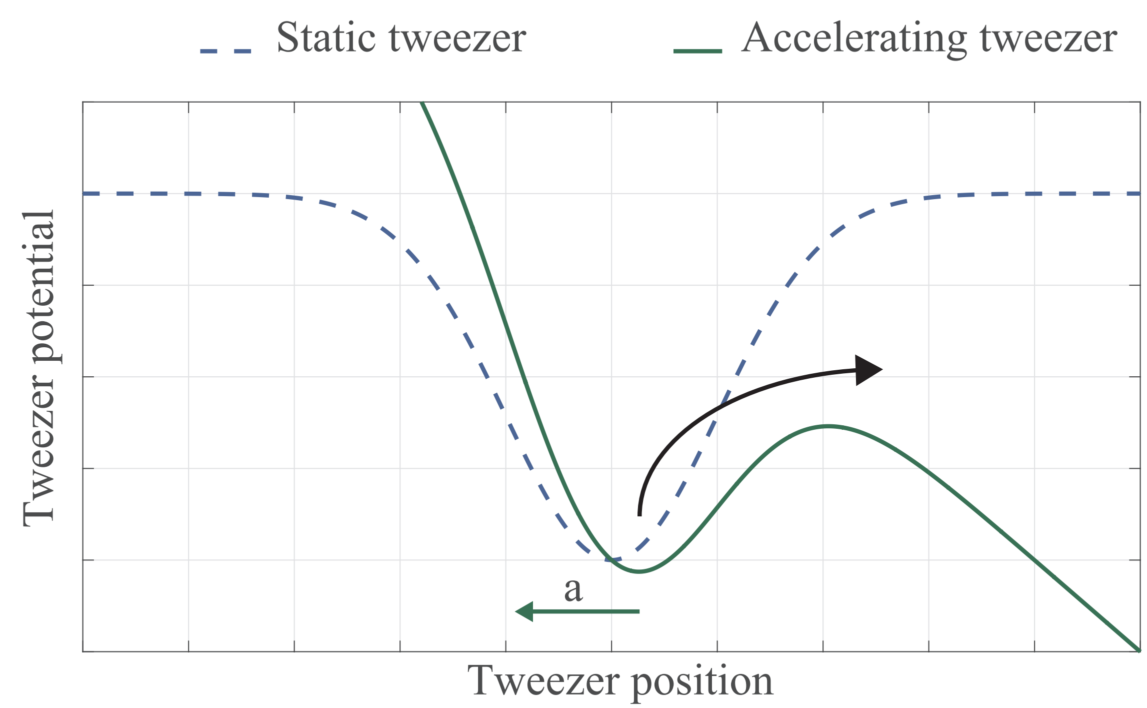
$$\Rightarrow \quad \tilde{H} = U_\lambda^\dagger H U_\lambda - iU_\lambda^\dagger \partial_t U_\lambda$$

  - Hence:

$$\tilde{H} = \frac{p^2}{2m} + V(x) - \dot{\lambda}p$$

  - Shift the momentum:

$$\tilde{H}' = \frac{p^2}{2m} + V(x) + m\ddot{\lambda}x$$



General

Time-dependent Hamiltonian:

$$H(\lambda_\mu(t))$$

Adiabatic gauge potential:

$$A_\mu = iU^\dagger \partial_{\lambda_\mu} U$$

Moving frame Hamiltonian:

$$\tilde{H} = U^\dagger H U - \dot{\lambda}_\mu A_\mu$$

Adiabatic gauge potential

- Berry connection is the expectation value of the gauge potential
$$\langle 0| A_\mu |0\rangle$$
- Berry curvature:
$$F_{\mu\nu} = -i \langle 0| [A_\mu, A_\nu] |0\rangle$$
- Metric tensor
$$g_{\mu\nu} = \frac{1}{2} \langle 0| A_\mu A_\nu + A_\nu A_\mu |0\rangle$$
    - Defines a metric on the ground state manifold. Can be used to detect and classify phase transitions.
        - Universal near second order QPT: $g \sim |\lambda - \lambda_c|^{d\nu - 2}$

Non-adiabatic response

- Let's look at the corrections to the ground state energy

General

$$\tilde{H} = U^\dagger H U - \dot{\lambda}_\mu A_\mu$$

First order:

$$\Delta E_1 = \dot{\lambda}_\mu \langle 0 | A_\mu | 0 \rangle$$

Second order:

$$\Delta E_2 = \sum_{n=0} \frac{|\langle 0 | \dot{\lambda}_\mu A_\mu | n \rangle|^2}{E_0 - E_n}$$

Translation

$$\tilde{H} = U^\dagger H U - \dot{\lambda} p$$

First order:

$$\Delta E_1 = \dot{\lambda} \langle 0 | p | 0 \rangle = 0$$

Second order:

$$\Delta E_2 = \dot{\lambda}^2 \sum_{n \neq 0} \frac{|\langle 0 | p | n \rangle|^2}{E_0 - E_n}$$

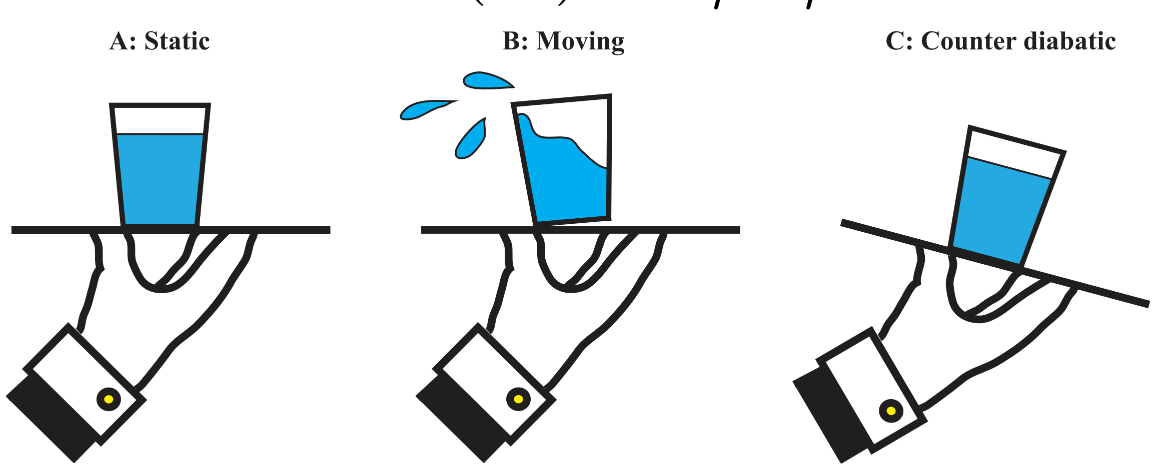
Effective mass of the classical parameter: $\Delta E_2 = -\frac{m\dot{\lambda}^2}{2}$

Counter-diabatic drive

- Can we keep the system adiabatic even if we change our parameters fast?

  - Just cancel out the gauge potential!

$$H_{CD} = H(\lambda_t) + \dot{\lambda}_\mu A_\mu$$



  - Rotate back into H if possible

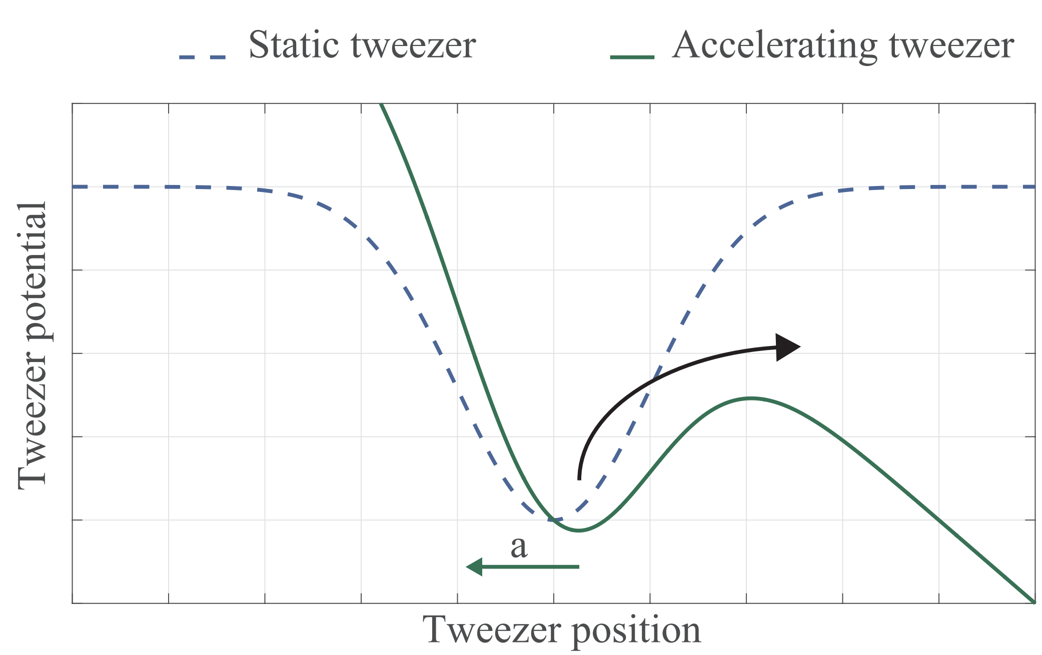
$$H_{FF} = R^\dagger H_{CD}(\lambda_t) R - i R^\dagger \partial_t R$$

M. Demirplak, S. A. Rice (2003), M. Berry (2009), S. Deffner, A. Del Campo, C. Jarzynski (2014+), …

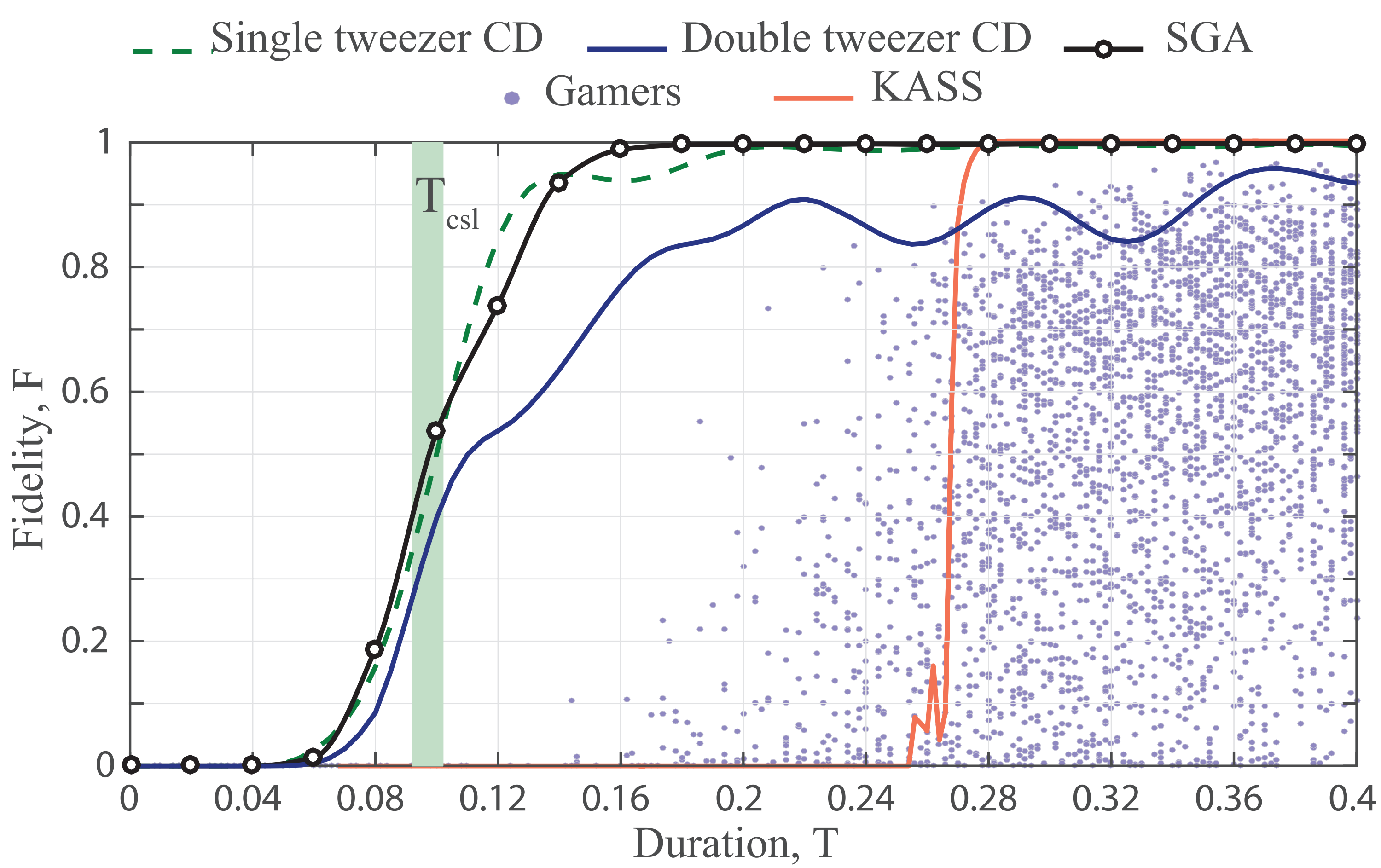
Let’s play a game

J. J. W. H. Sørensen et al., Nature 532, 210–213; 2016

Effective translation

- System is described by: $H = \frac{p^2}{2m} + V_0(x) + V_1(x-\lambda)$
- Let's assume: $A_\lambda = v(\lambda)(xp + px)$
- Transform to: $H_{FF} = H - \partial_t(v(\lambda)\dot{\lambda})x \approx \frac{p^2}{2m} + V_0(x) + V_1(x - \lambda_{FF}(t))$
- 

Effective translation

- System is described by: $H = \dfrac{p^2}{2m} + V_0(x) + V_1(x-\lambda)$
- Let's assume: $A_\lambda = v(\lambda)(xp+px)$
- Transform to: $H_{FF} = H - \partial_t(v(\lambda)\dot{\lambda})x \approx \dfrac{p^2}{2m} + V_0(x) + V_1(x-\lambda_{FF}(t))$
- 

Many body systems

- Whole other ballgame

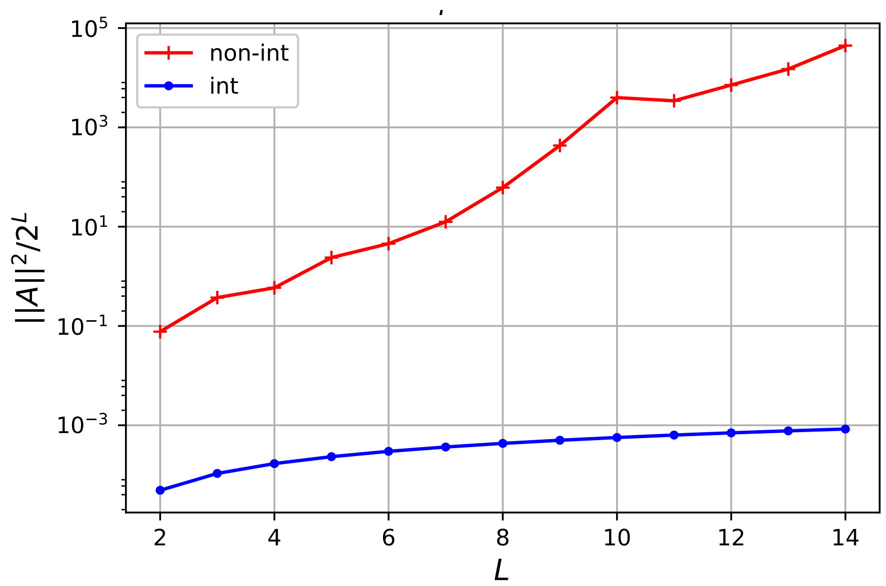
$$\langle n| A_\lambda |m\rangle = \langle n| U^\dagger i\partial_\lambda U |m\rangle = \langle n(\lambda)| i\partial_\lambda |m(\lambda)\rangle$$

- Hence

$$\langle n| A_\lambda |m\rangle = -i\frac{\langle n| \partial_\lambda H |m\rangle}{E_n - E_m}$$

- If the system is ergodic we are in trouble:

$$|\langle n|A_\lambda|m\rangle = \frac{|\langle n|\partial_\lambda H|m\rangle}{|E_n - E_m|} \sim \frac{e^{-S/2}}{e^{-S}} = e^{S/2}$$



What now?

- We should be less stringent. Eigenstates of the same energy density are locally indistinguishable anyway.

- Here is our trick:

  - Observe: $\langle n| A_\lambda |m\rangle = -i\frac{\langle n| \partial_\lambda H |m\rangle}{E_n - E_m}$

$$\Rightarrow \quad \partial_\lambda H + i[A, H] = F_{BO}$$

$$\Rightarrow \quad [\partial_\lambda H + i[A, H], H] = 0$$

$$S(O) = \mathrm{Tr}(\partial_\lambda H + i[O, H])^2 \to A = \mathrm{argmin} S(O)$$

Variational method

$$S(O) = \mathrm{Tr}(\partial_\lambda H + i[O,H])^2 \rightarrow A = \mathrm{argmin} S(O)$$

- Now we can just restrict to sensible, local, experimentally relevant operators.

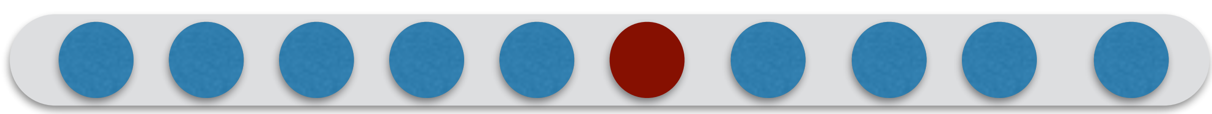
- No local approximation can ever be close to the exact result, so is minimizing S sensible?

  - Yes! S(O) is a measure for the rate of increase of the energy fluctuations:

$$\frac{\partial \delta E_n^2}{\partial t} \propto \langle n| \left(\partial_\lambda H + i[A,H]\right)^2 |n\rangle$$

Free fermions

- Let's see what happens for free fermions



$$H = -\sum_i (c_{i+1}^\dagger c_i + h.c.) + \sum_i V(i,\lambda) c_i^\dagger c_i$$

- Let's take the most local Ansatz

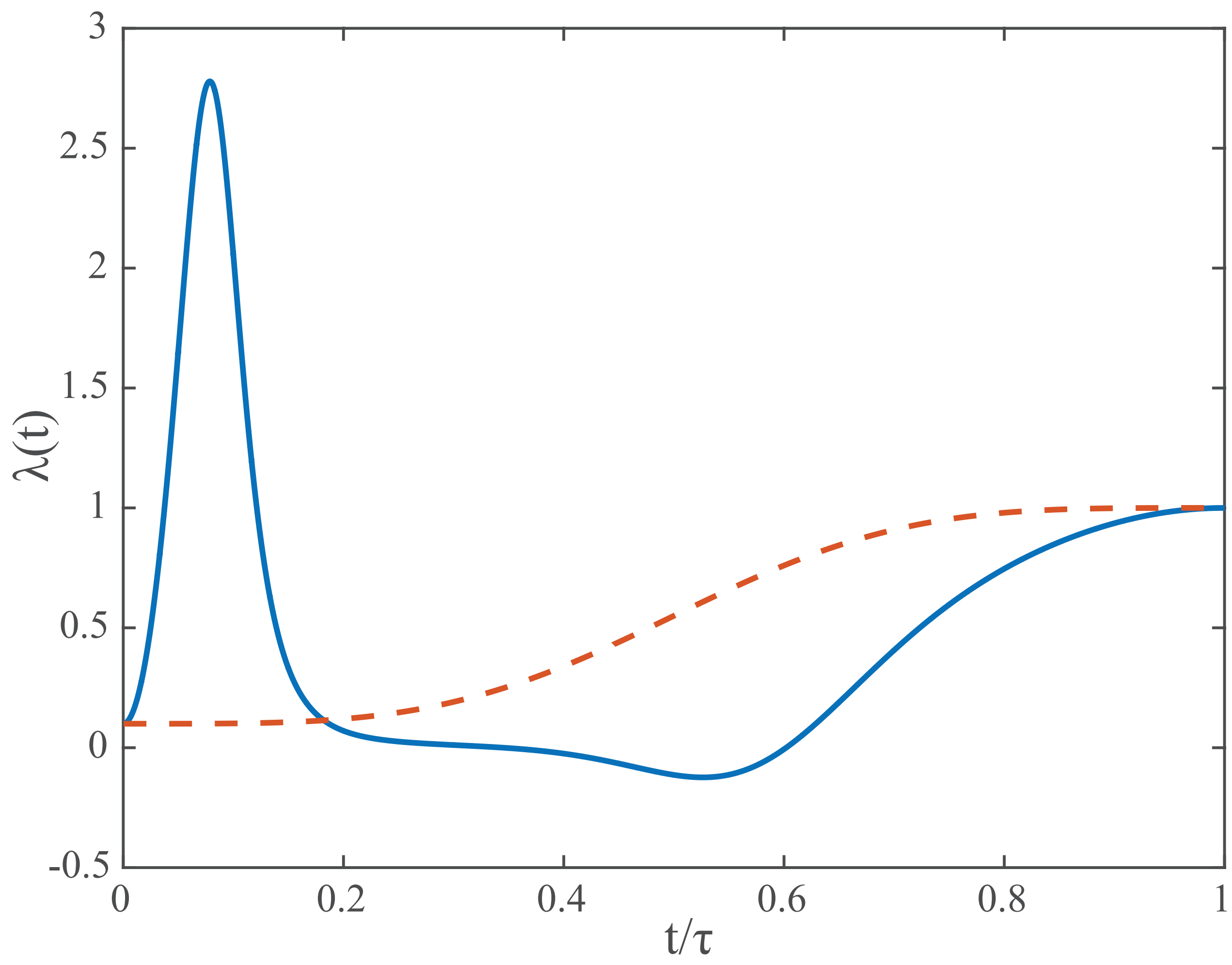
$$A = i \sum_i \alpha_i (c_{i+1}^\dagger c_i - h.c.)$$

- Compute some traces

$$-3\partial_i^2 \alpha + (\partial_i V)^2 \alpha = \partial_i \partial_\lambda V$$

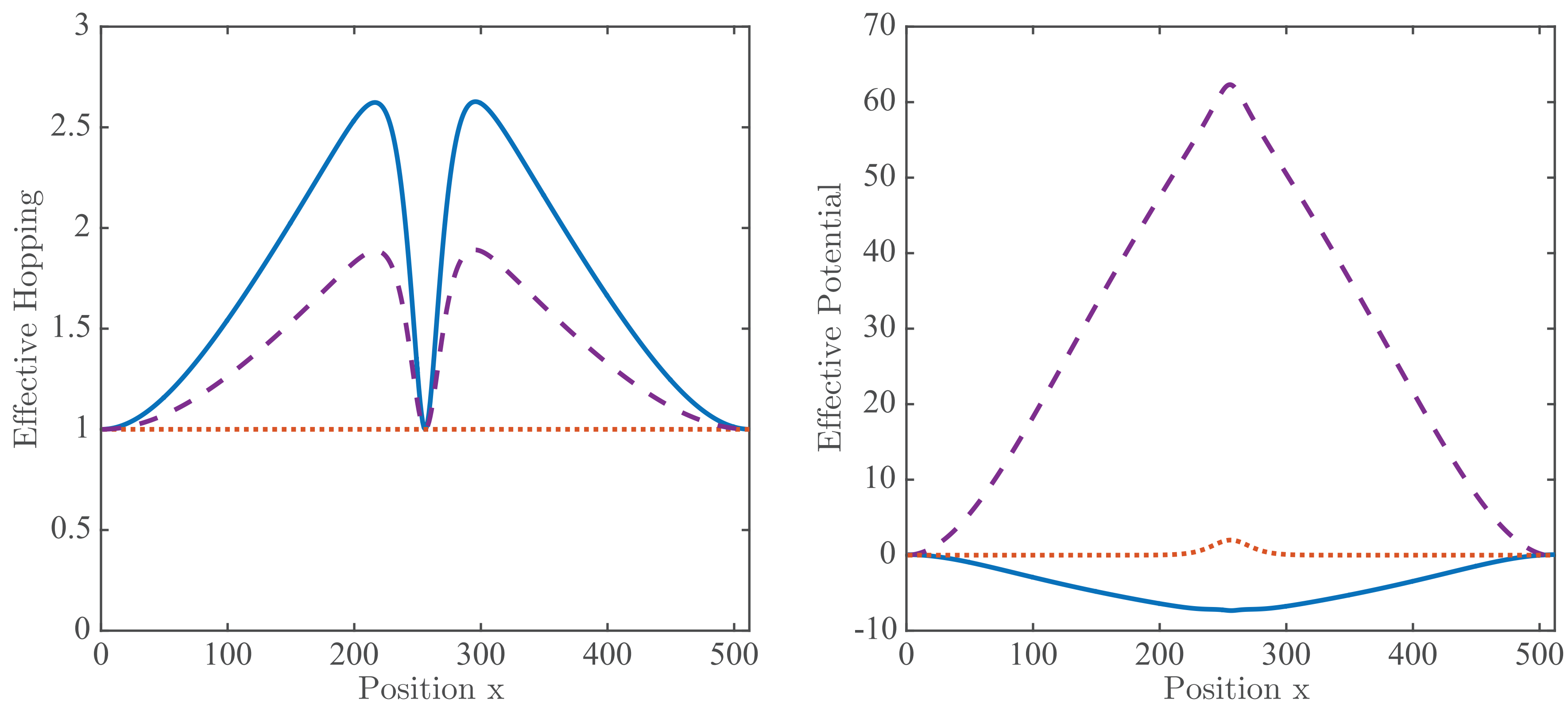
  - Exact solution for linear potential

Compensating electric field



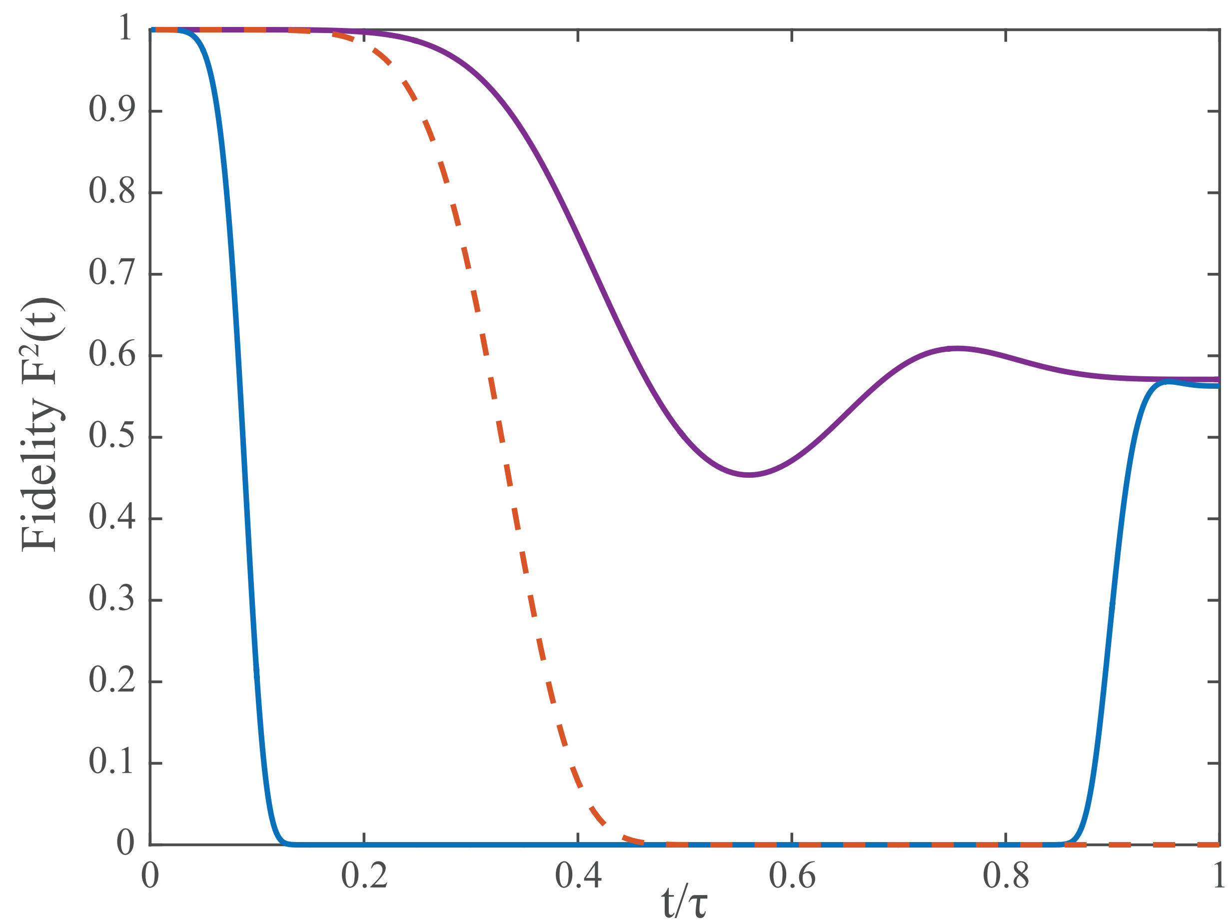
Anderson orthogonality I

- Recall:
$$H_{CD} = H + \dot{\lambda} A_\lambda = -\sum_i ((1 - i\alpha_i) c_i^\dagger c_i + h.c.) + \sum_i V(i,\lambda) c_i^\dagger c_i$$
- Gauge transform
$$H_{FF} = -\sum_i (J_{eff}(i) c_i^\dagger c_i + h.c.) + \sum_i V_{eff}(i,\lambda) c_i^\dagger c_i$$

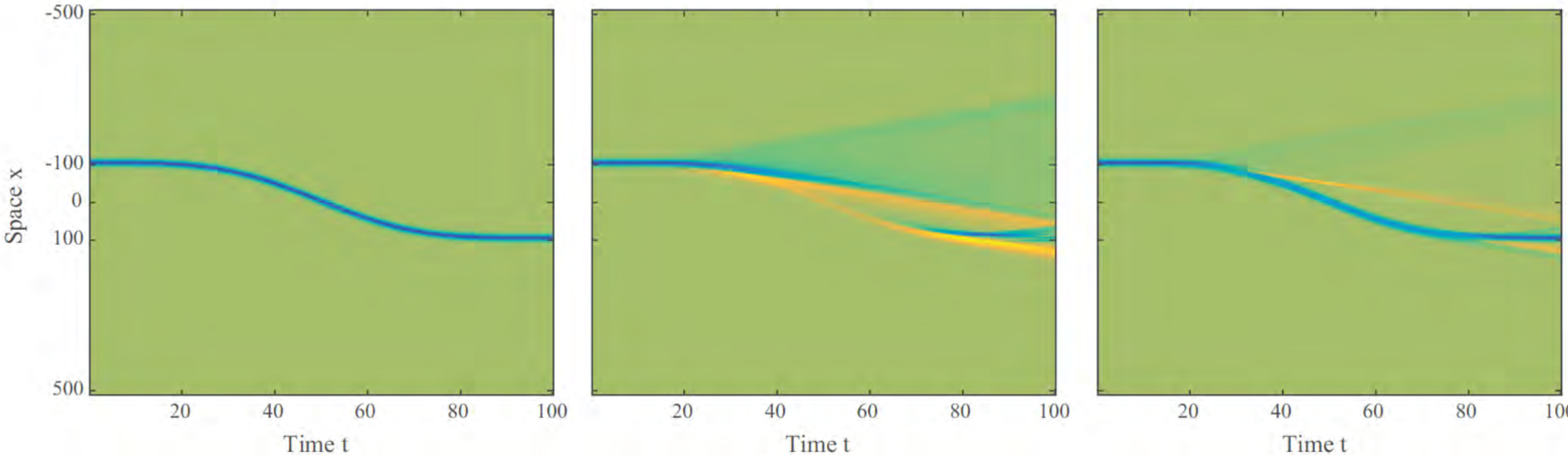


Anderson II

- Chain of 512 sites, half filling, final strength is 2J and time is 10/J.

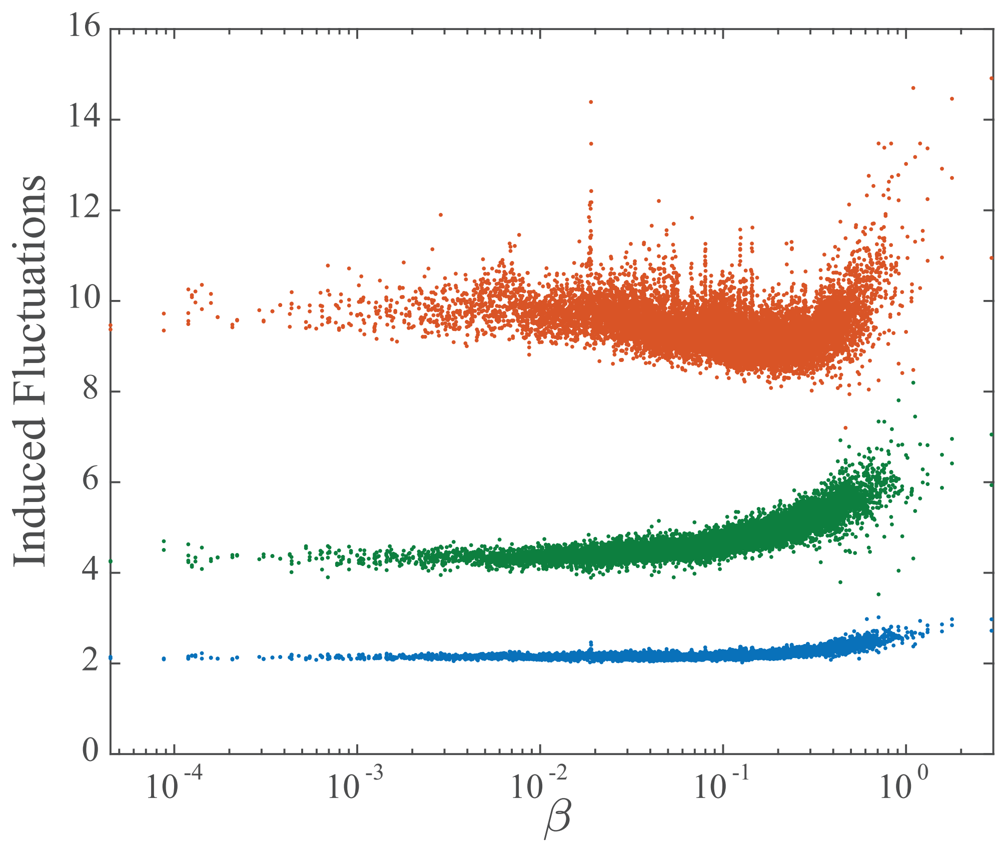


Moving impurity



Flipping spins

- Consider
$$H = \sum_i \left(\sigma^z_{i+1}\sigma^z_i + 0.8\sigma^z_j + 0.9\sigma^x_j + h_x(t)\sigma^x_0\right)$$
- Then we can expand in terms of strings of spins
$$H = \sum_i \alpha_i \sigma^y_i + \sum_{ij}\left(\beta_{i,j}\sigma^x_i\sigma^y_j + \gamma_{i,j}\sigma^z_i\sigma^y_i\right)$$



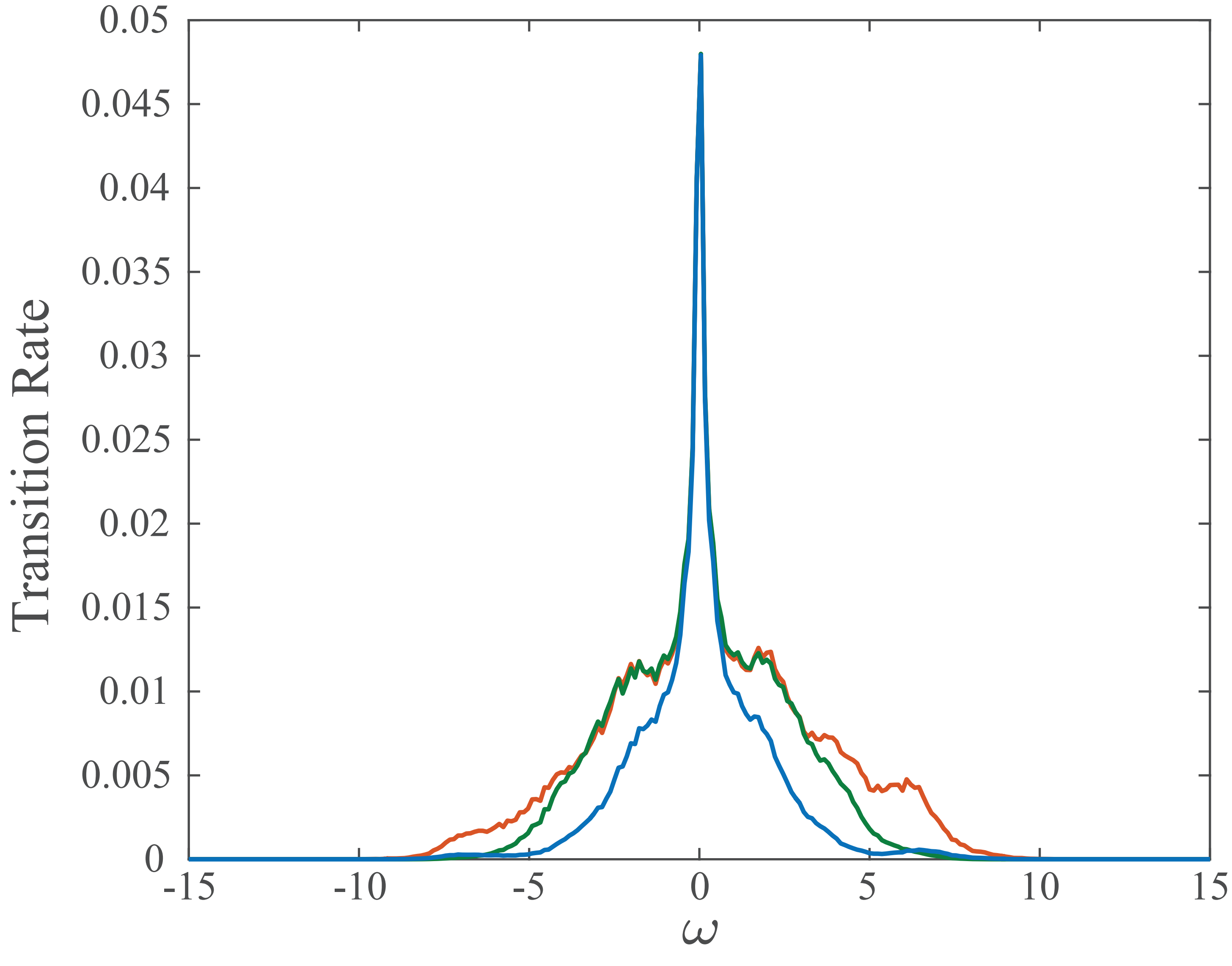
15 Site chain

No CD drive

1 Body CD

2 body CD

Transition rate



15 Site chain

No CD drive

1 Body CD

2 body CD

Conclusion

- Deep connection between non-adiabatic response and geometry
- Counter-diabatic driving can be used to suppress dissipation in many-body systems
- Simple local expansion for gapped systems
- Physical gauge potential for ergodic system seems only polynomially complex

DS and AP, PNAS, Volume 114, 20 (2017)
DS, PRA 97, 040302(R), (2018)
M. Kolodrubetz, DS, Pankaj Mehta, AP, Physics Reports 697, 1-88 (2017)