

# Tutorial: Magnonics

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Technische Universität Kaisers

Happy Birthday, Jairo



## Post CMOS?

### CMOS is coming to the end of Moore's law

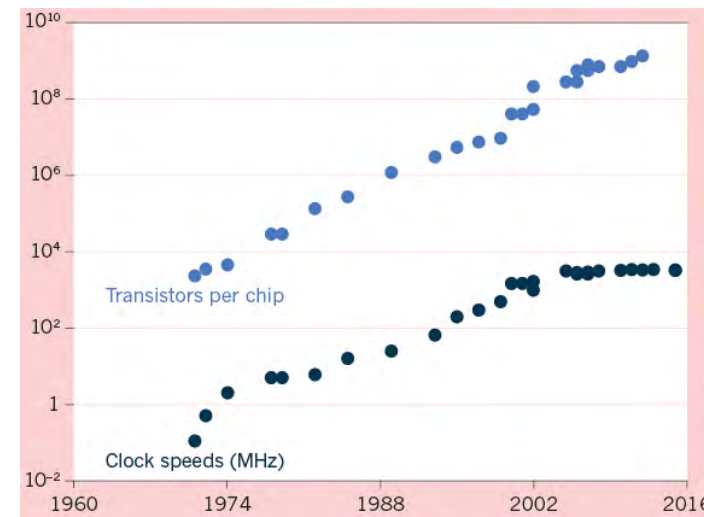
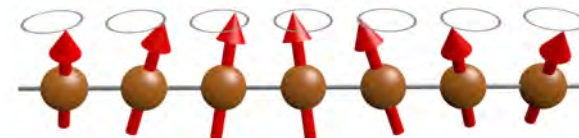
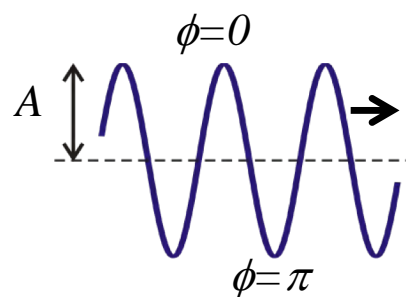
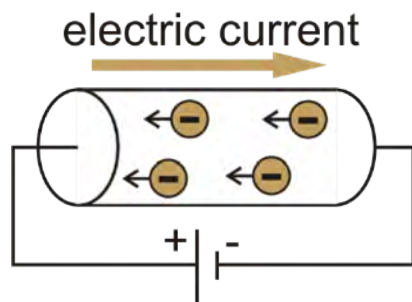
- Waste energy production
- End of scaling

### Beyond current CMOS:

- Faster computing, less energy consumption
- Same technology for logic and data
- Logic circuits with reduced footprint and/or 3D

### Novel paradigm: wave computing

Proposal: use **waves /wave packets** instead of particles (electrons) for bit representation



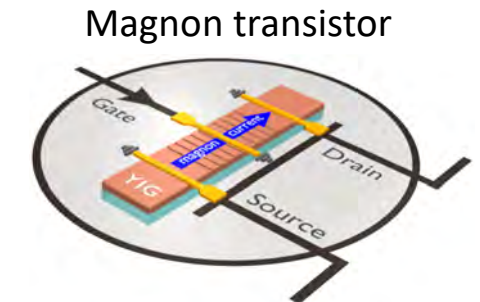
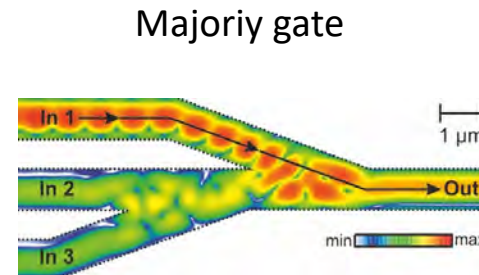
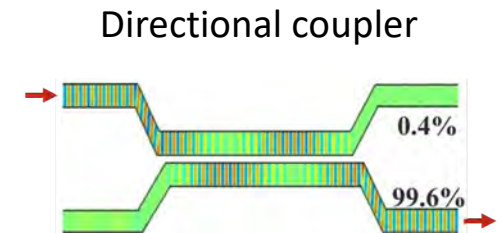
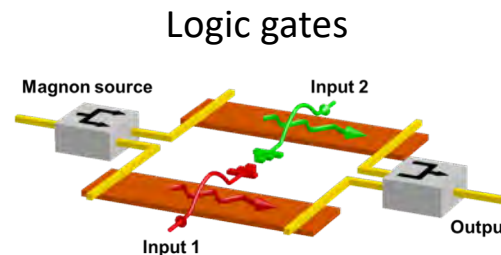
M. Mitchell, Nature **530**, 144 (2016)

# Magnon computing

## Why spin waves?

- wavelength down to nanometer, frequency up to several THz
- interference effects easily accessible
- efficient nonlinear effects
- room temperature
- no Joule heat, “insulatronics”
- wave-based computing: smaller footprint, all-wave logic
- good converters to CMOS  
→ “magnon spintronics”

## Achievements

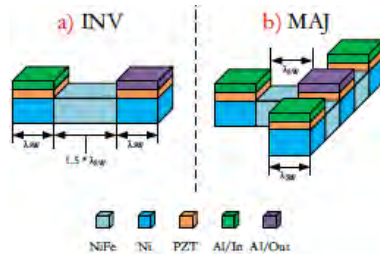


Andrii Chumak, Kaiserslautern

# Spin-wave device architectures

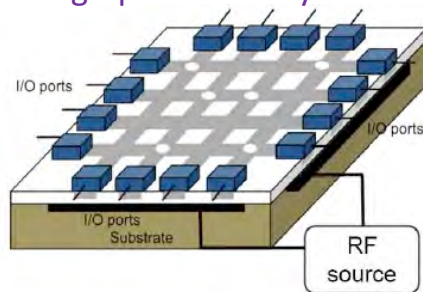
L. Amarú, P.-E. Gaillardon,  
G. De Micheli  
(EPFL, Switzerland)

Majority based synthesis for  
nanotechnologies



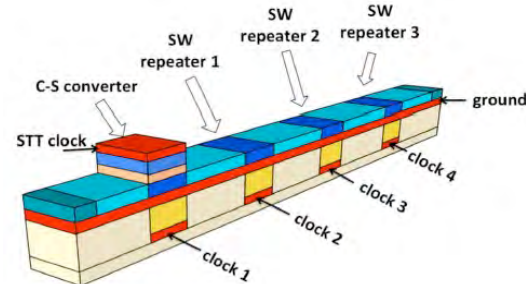
A. Khitun  
(Univ. of California Riverside, USA)

Majority gate,  
holographic memory



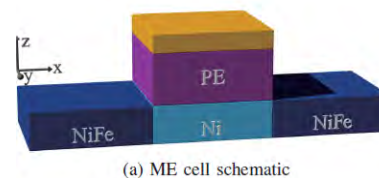
S. Dutta, A. Naeemi  
(Georgia Institute of Technology, USA)

Non-volatile clocked spin wave  
nanomagnet pipelines



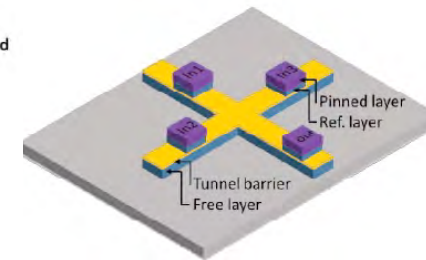
F. Ciubotaru, C. Adelmann  
(Imec, Leuven, Belgium)

Magnetoacoustic nanoresonators



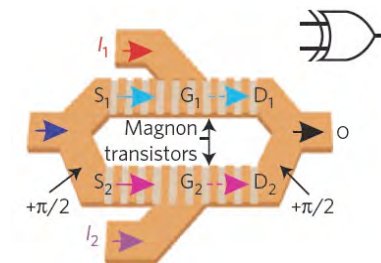
D. E. Nikonov, I. A. Young  
(Intel Corp. Hillsboro, Oregon,  
USA)

Benchmarking,  
clocked spin-wave circuits



A. Chumak  
(TU Kaiserslautern, Germany)

Magnon transistor,  
integrated magnonic circuits



# Benchmarking

## Hybrid CMOS-Spin Wave Device Circuits Compared to 10nm CMOS

TABLE IV. SUMMARY OF BENCHMARKING RESULTS

Name	Area ( $\mu m^2$ )				Energy (fJ)	Delay (ns)		Power ( $\mu W$ )		ADPP*		Impr. (x)
	SWD core	CMOS SA	SWD Total	10nm Ref.	SWD Total	SWD	10nm Ref.	SWD	10nm Ref.	SWD	10nm Ref.	
BKA264	36.48	3.12	39.60	118.55	175.50	5.07	0.21	34.62	133.92	$6.95 \cdot 10^3$	$3.33 \cdot 10^3$	0.48
HCA464	82.71	3.17	85.88	262.63	178.20	8.01	0.29	22.25	594.28	$1.53 \cdot 10^4$	$4.53 \cdot 10^4$	2.96
CSA464	78.42	3.17	81.59	240.26	178.20	7.59	1.78	23.48	663.17	$1.45 \cdot 10^4$	$2.84 \cdot 10^5$	19.51
DTM32	326.31	3.07	329.38	1183.64	172.80	14.73	0.52	11.73	3667.50	$5.69 \cdot 10^4$	$2.26 \cdot 10^6$	39.66
WTM32	264.96	3.07	268.04	1163.37	172.80	20.61	0.58	8.38	3571.90	$4.63 \cdot 10^4$	$2.41 \cdot 10^6$	52.04
DTM64	1192.69	6.14	1198.83	3459.32	345.60	18.09	0.63	19.10	12793.10	$4.14 \cdot 10^5$	$2.79 \cdot 10^7$	67.29
GFMUL	44.09	0.82	44.91	162.98	45.90	7.17	0.16	6.40	433.92	$2.06 \cdot 10^3$	$1.13 \cdot 10^4$	5.49
MAC32	295.25	3.12	298.37	1372.83	175.50	24.39	0.66	7.20	3872.10	$5.24 \cdot 10^4$	$3.51 \cdot 10^6$	67.00
DIV32	899.04	6.14	905.18	3347.73	345.60	117.21	14.00	2.95	5346.10	$3.13 \cdot 10^5$	$2.51 \cdot 10^8$	800.94
CRC32	27.61	1.54	29.14	95.88	86.40	5.07	0.22	17.04	304.30	$2.52 \cdot 10^3$	$6.42 \cdot 10^3$	2.55
Averages	324.76	3.34	328.09	1140.72	187.65	22.79	1.91	15.31	3138.03	$9.24 \cdot 10^4$	$2.87 \cdot 10^7$	105.79

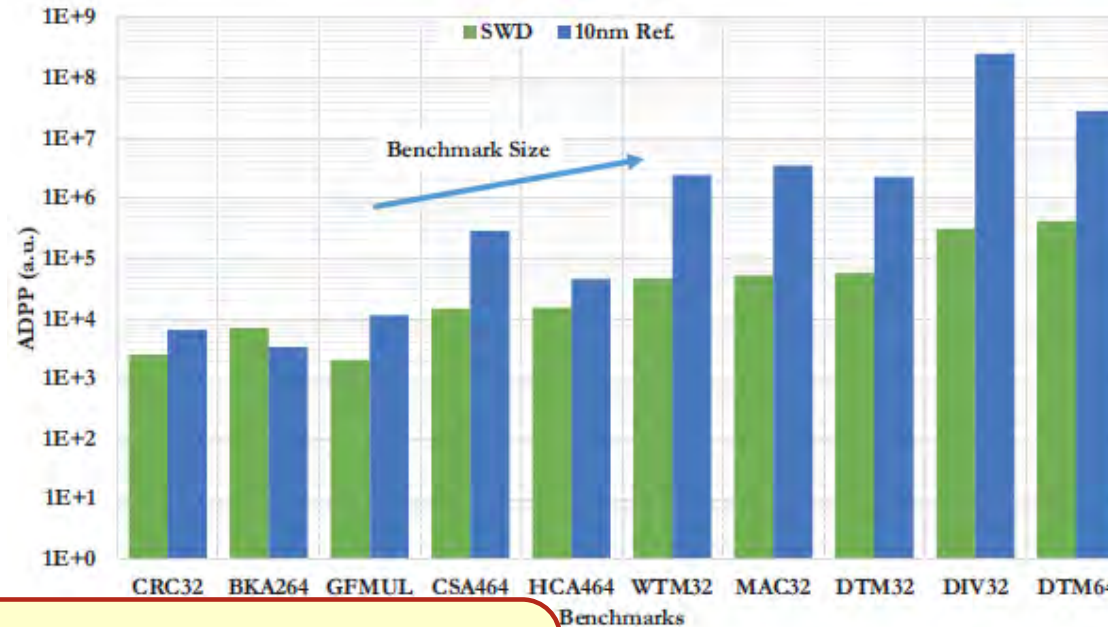
The list includes adders, multipliers, a divider, and a cyclic redundancy check module

\* Area-Delay-Power-Product (ADPP)

Zografos, et al., Proceedings of the 15<sup>th</sup> IEEE International Conference on Nanotechnology July 27-30, 2015, Rome, Italy

# Benchmarking

Hybrid CMOS-Spin Wave Device Circuits Compared to 10nm CMOS



Area: 3.5x smaller  
 Delay: 10-20x slower  
 Power consumption: 100x lower  
 ↓  
 ADPP: 50-100x better!

\* Area-Delay-Power-Product (ADPP)

Zografos, et al., Proceedings of the 15<sup>th</sup> IEEE International Conference on Nanotechnology July 27-30, 2015, Rome, Italy

# Computing principles



- Classical Computing

- Scalar variable
- Boolean logic

- Wave Packet Computing

- Vector variable
- Special task data processing

- Macroscopic Quantum State Computing

- Vector state variable

- Quantum Computing

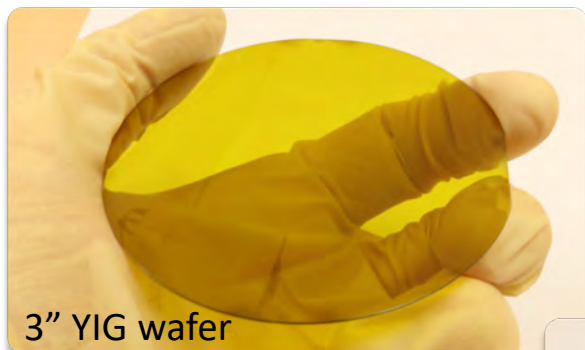
- Vector state variable
- Entanglement

Quantum-Magnonic  
Analogies



# Yttrium Iron Garnet (YIG, $Y_3Fe_5O_{12}$ )

- Room temperature ferrimagnet ( $T_C = 560$  K)
- Low phonon damping
- Magnon lifetime up to 700 ns !

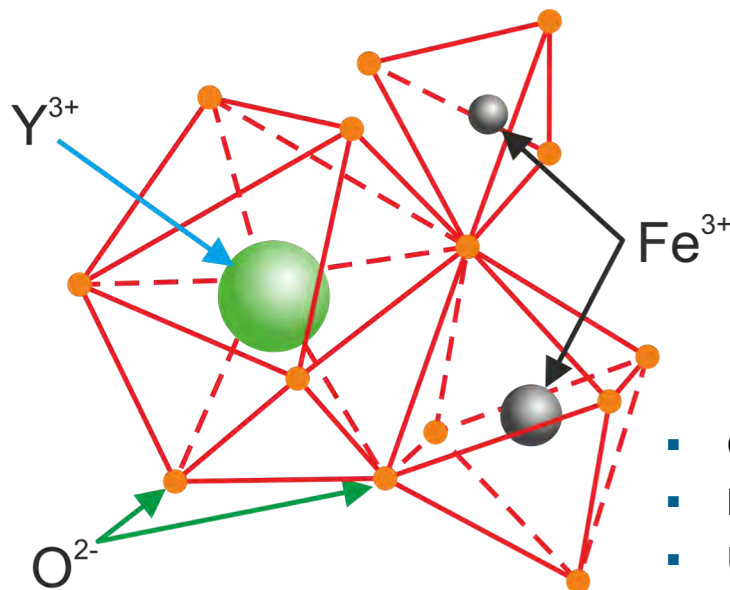


3" YIG wafer

Scientific Research Company  
"Carat", Lviv, Ukraine



YIG monocrystal



- Cubic crystal
- Lattice constant 12.376 Å
- Unit cell – 80 atoms

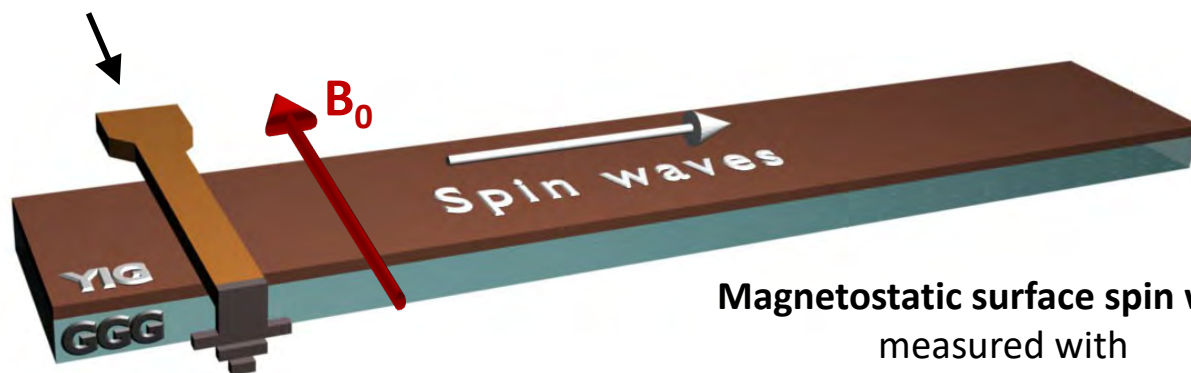
8 octahedral iron atoms (spin 5/2 up)  
12 tetrahedral iron atoms (spin 5/2 down)



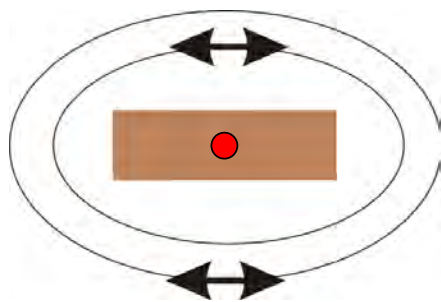
Magnetic moment of a unit cell is 20  
Bohr magnetons  $\mu_B$  at zero temperature

## Excitation of dipolar spin waves

Input microwave signal



Magnetostatic surface spin waves  
measured with  
Brillouin light scattering spectroscopy

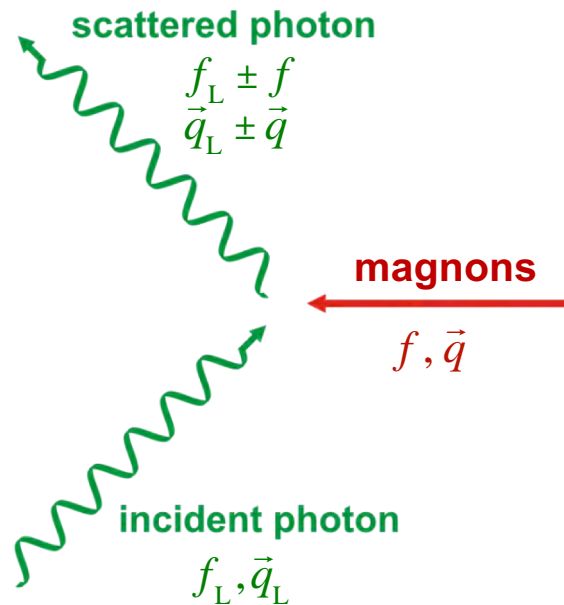


Alternating magnetic field

# Brillouin light scattering spectroscopy

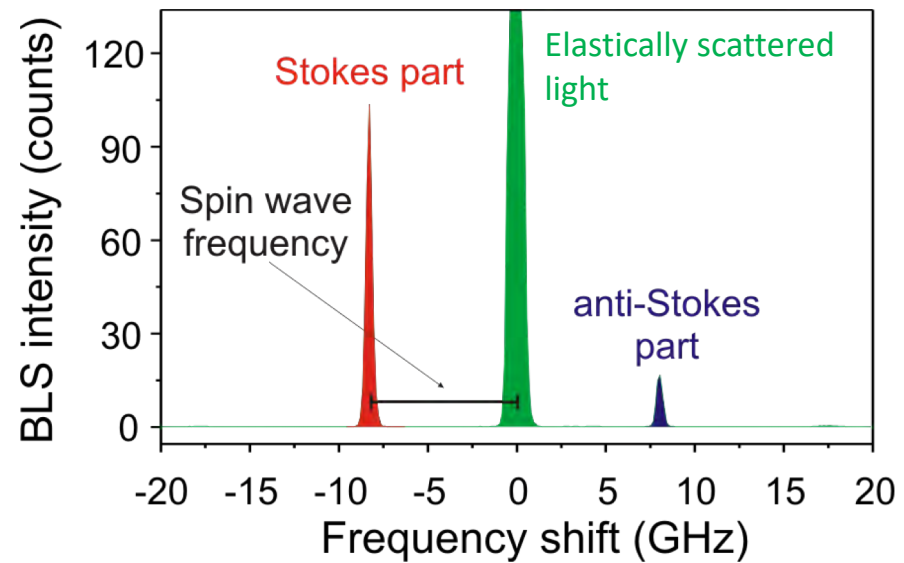
## Brillouin light scattering process

= inelastic scattering of photons from spin waves

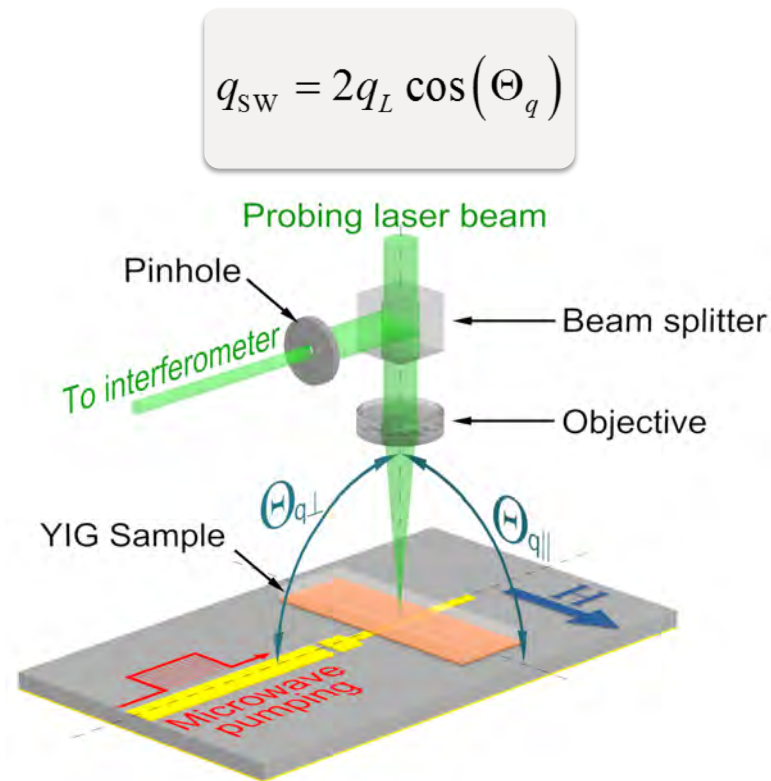
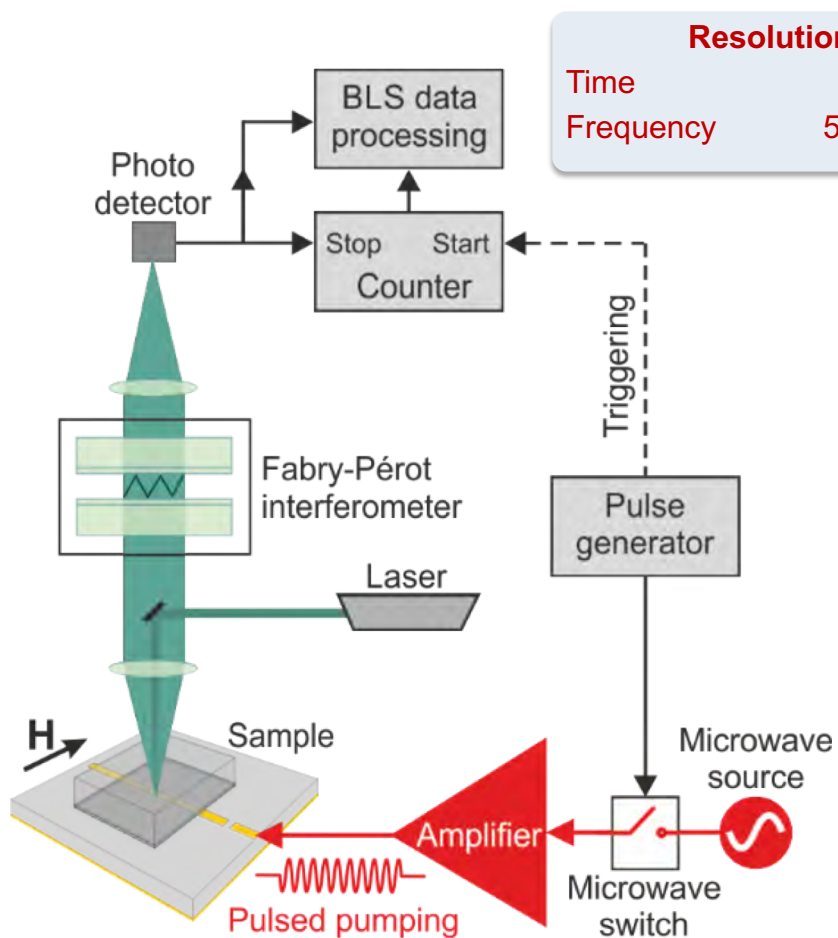


$$f_{\text{scattered L}} = f_L \pm f$$

$$\vec{q}_{\text{scattered L}} = \vec{q}_L \pm \vec{q}$$



# Time-, space- and wavevector-resolved Brillouin light scattering spectroscopy



Max wavenumber	$2.36 \times 10^5$ rad/cm
Wavenumber resolution	$2 \times 10^3$ rad/cm

## “Magnonics” team

### Kaiserslautern PI Team



**A. Chumak**



**P. Pirro**



**T. Brächer**



**V. Vasyuchka**



**A. Serga**

### Main External Collaborators

**V.S. L’vov** (Weizmann Institute of Science, Rehovot, Israel)

**G.A. Melkov** (National Taras Shevchenko University of Kyiv, Ukraine)

**E. Saitoh** (Tohoku University, Sendai, Japan)

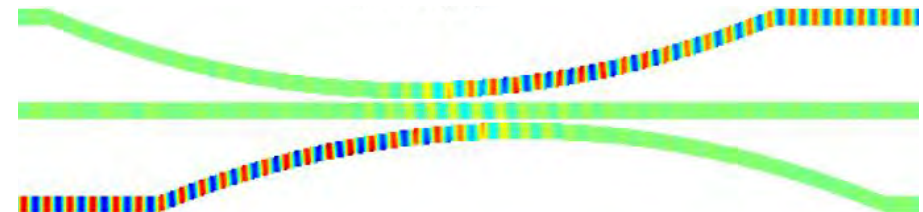
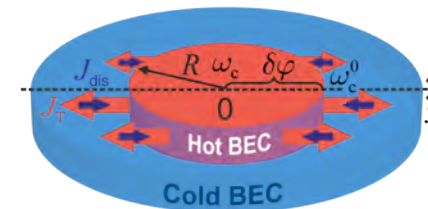
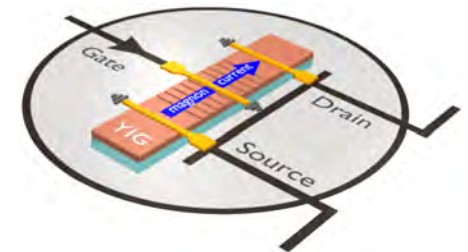
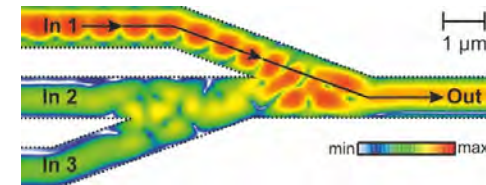
**A.N. Slavin** (Oakland University, Rochester, USA)



Prof. B. Hillebrands, Jun. Prof. A. V. Chumak, V. Lauer, Q. Wang, P. Frey, B. Heinz, L. Mihalceanu, M. Kewenig, Dr. D. A. Bozhko, M. Schneider, Dr. P. Pirro, M. Schweizer, Dr. habil. A. A. Serga, Dr. T. Langner, E. Wiedemann, A. Kreil, Dr. A. Conca Parra, S. Steinert, M. Geilen, S. Keller, H. Schäfer, T. Noack, T. Fischer, Dr. T. Meyer, Jun. Prof. E. Th. Papaioannou, F. Heussner, J. Greser, K. Fukuda (guest), Dr. V. I. Vasyuchka

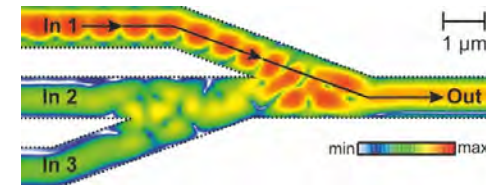
## Advanced magnonics

- I. Magnon interference logic
- II. Non-linear magnonics: Magnon transistor
- III. Magnonic macroscopic quantum state
- IV. Quantum-classical analogies in magnonics

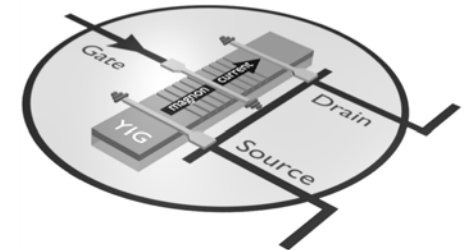


# Advanced magnonics

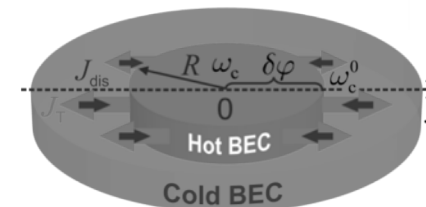
## I. Magnon interference logic



## II. Non-linear magnonics: Magnon transistor



## III. Magnonic macroscopic quantum state

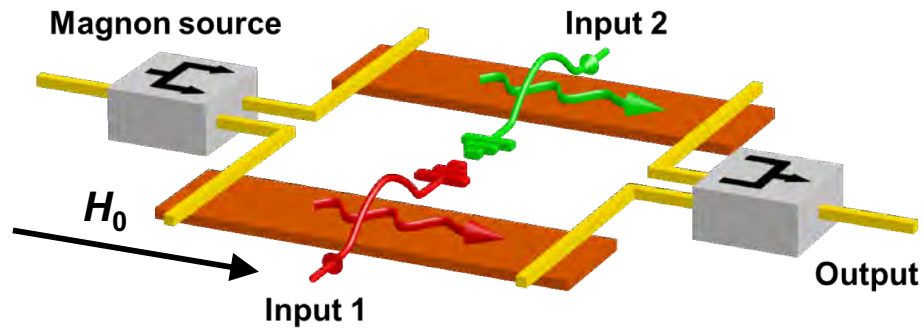


## IV. Quantum-classical analogies in magnonics

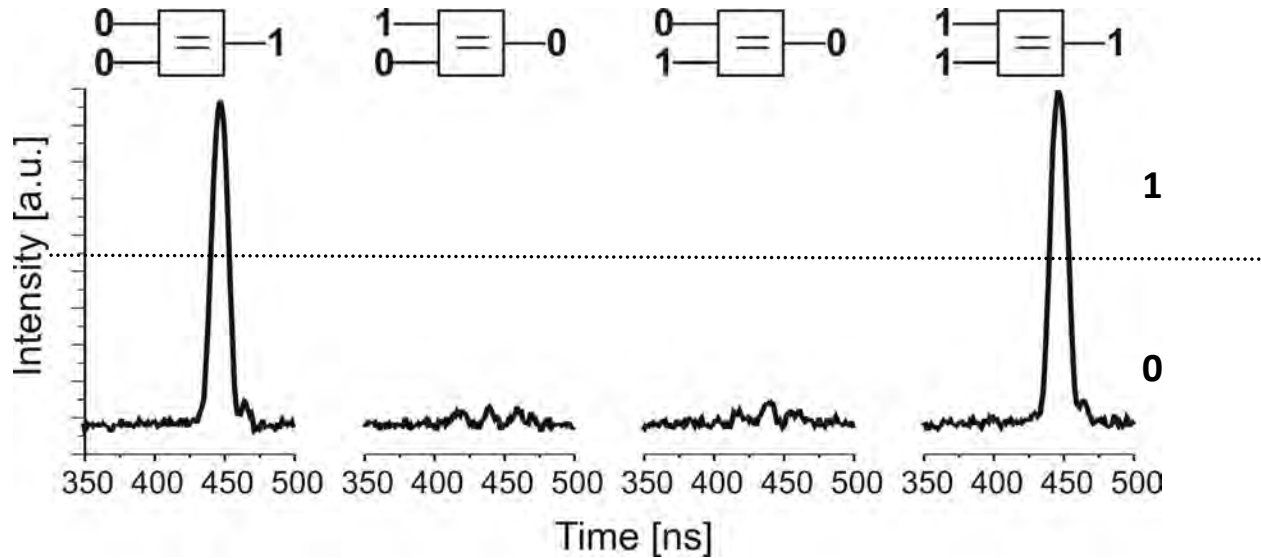




# First prototype Mach-Zehnder interferometer based spin-wave logic gate



Realization of XNOR gate



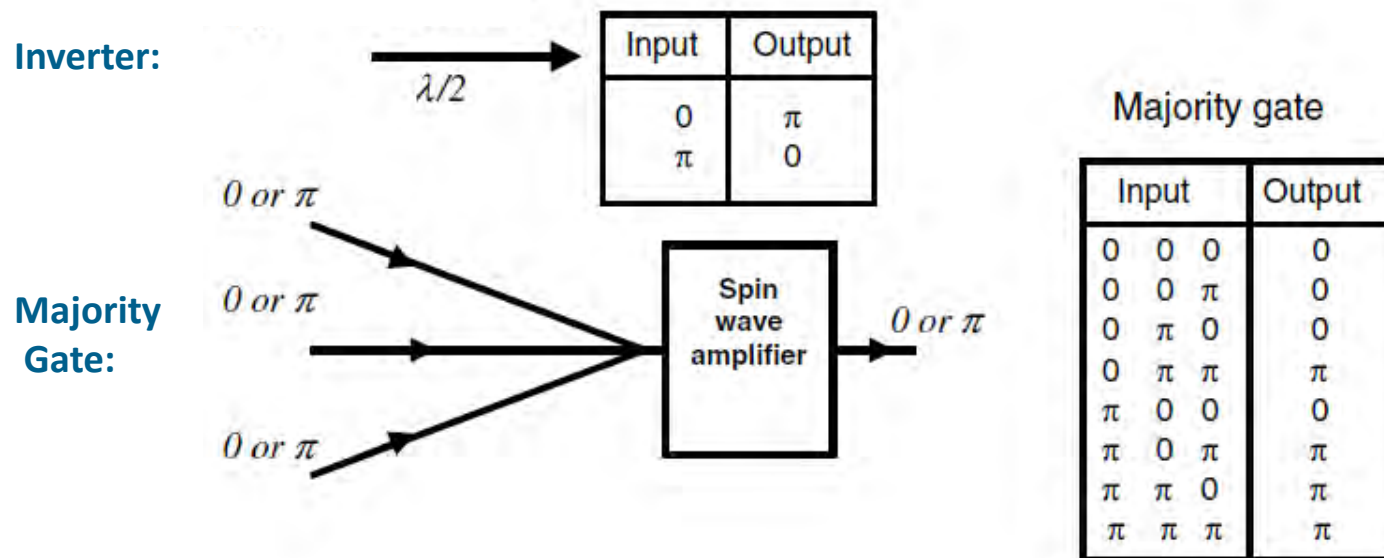
Inputs		Output
A ( $I_1$ )	B ( $I_2$ )	
0 (0)	0 (0)	1
0 (0)	1 ( $I_\pi$ )	0
1 ( $I_\pi$ )	0 (0)	0
1 ( $I_\pi$ )	1 ( $I_\pi$ )	1

Kostylev et al., *APL* **87**, 153501 (2005)

Schneider et al., *APL* **92**, 022505 (2008)

## Magnon majority gates: General idea

Data is coded into spin-wave phase

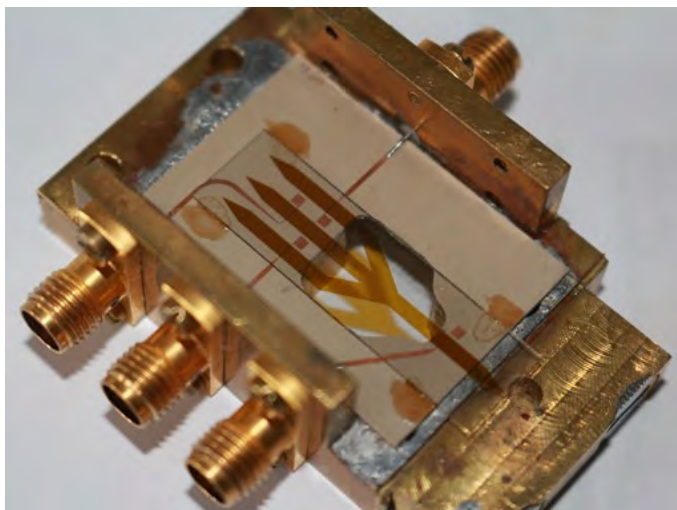


A. Khitun, et al., J. Phys. D. **43**, 264005 (2010)

- simple realization of majority gate (spin-wave combiner)
- trivial realization of NOT operation (= phase shift during  $\Delta x = \lambda/2$  propagation)
- is all-magnonic
- majority gate + inverter are building blocks for full logic functionality

## Experimental realization

### Macroscopic majority gate



YIG sample:

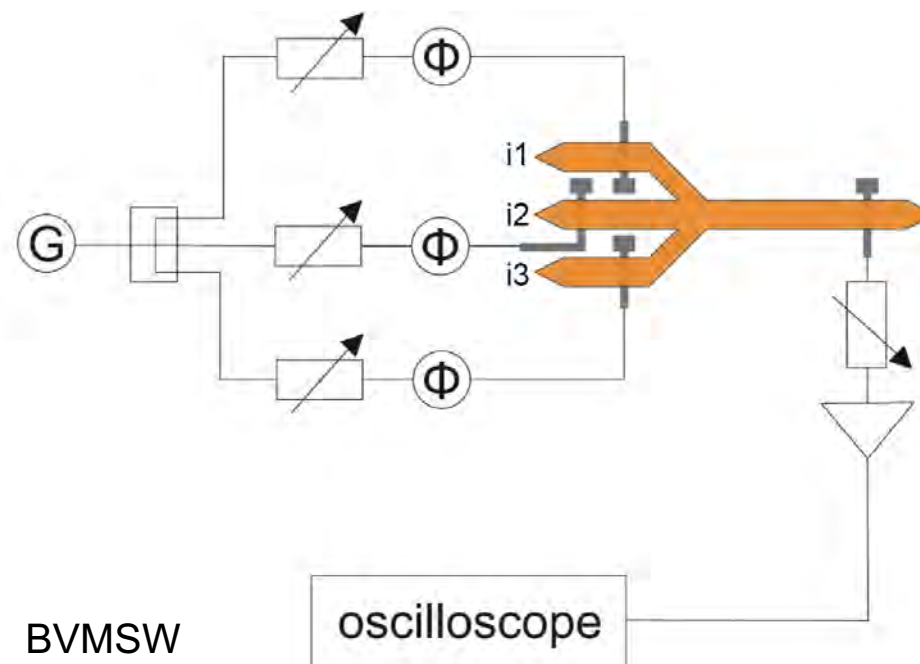
- thickness 5.4  $\mu\text{m}$
- waveguide width 1.5 mm

Geometry  
Frequency  
Magnetic field

BVMSW  
6.035 GHz  
1429 Oe

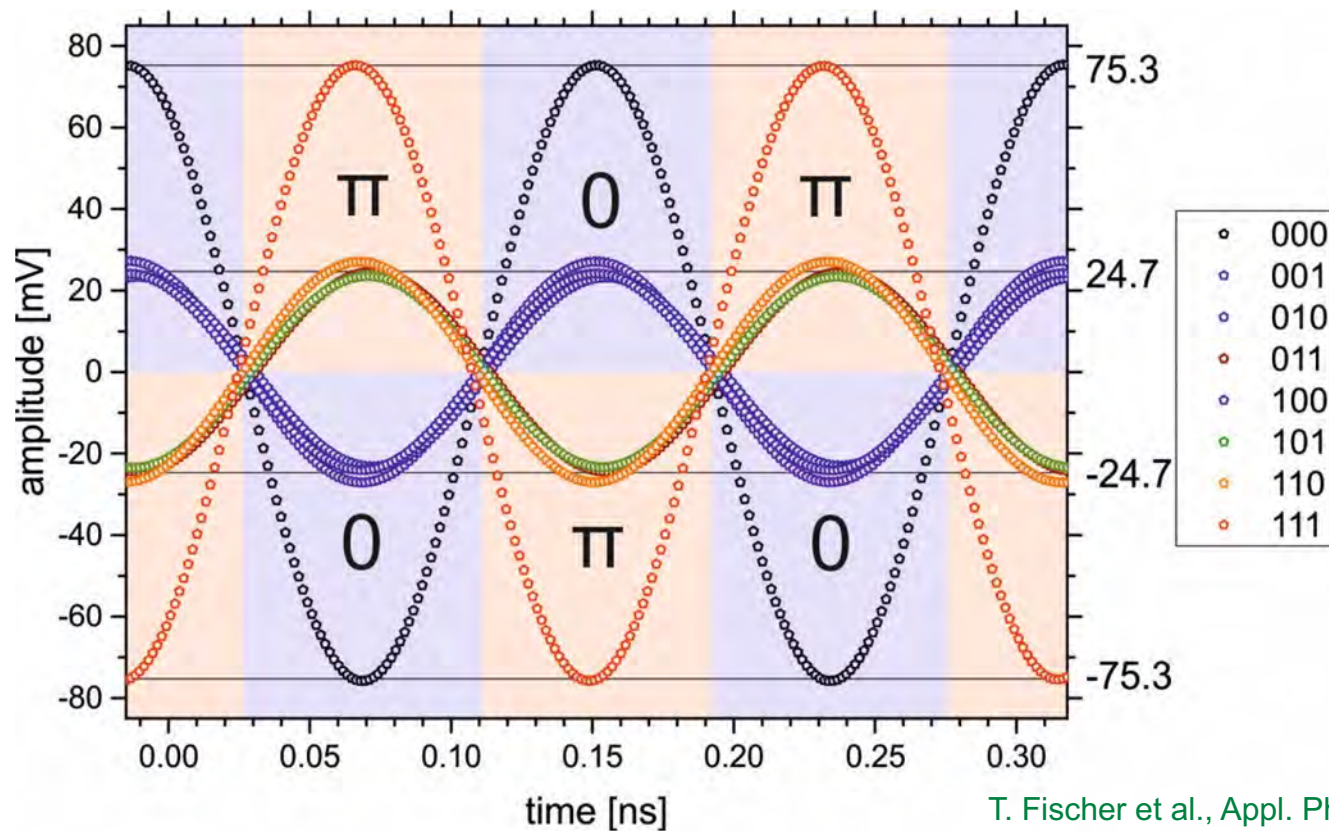
Produced by Scientific Research  
Company Carat, Lviv, Ukraine

### Experimental setup



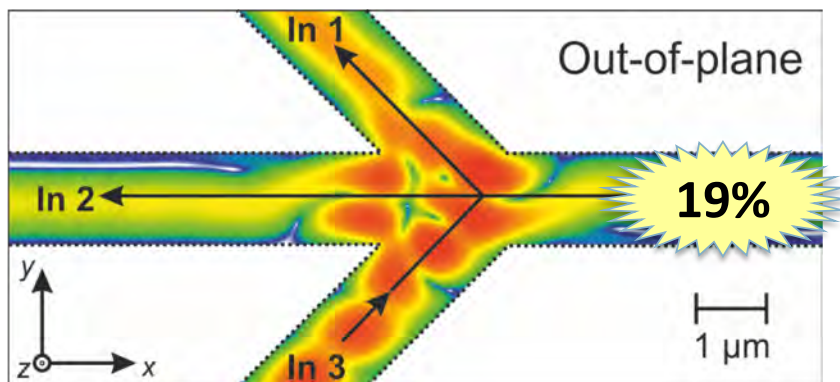
T. Fischer et al., Appl. Phys. Lett. **110**, 152401 (2017)

## Superposition of all spin-wave channels

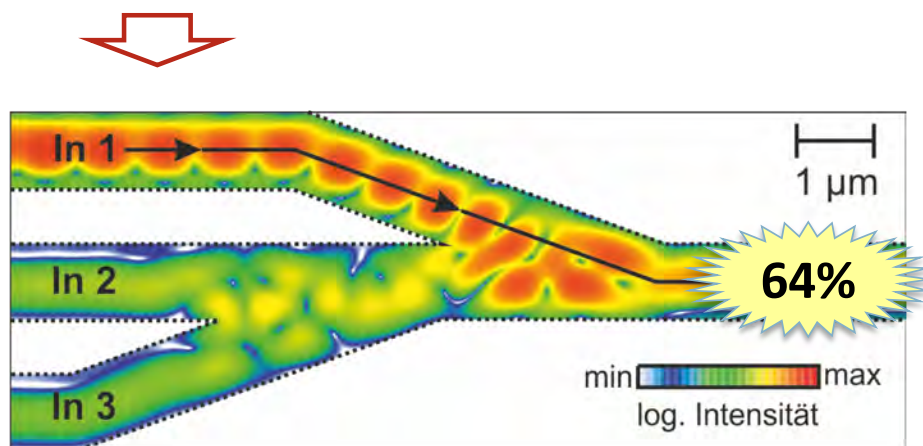


The output phase of the signal is defined by the majority of the input phases

## Majority gates: Out-of-plane magnetization



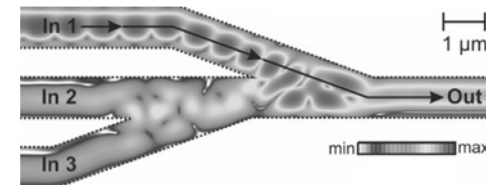
How to increase  
this value?



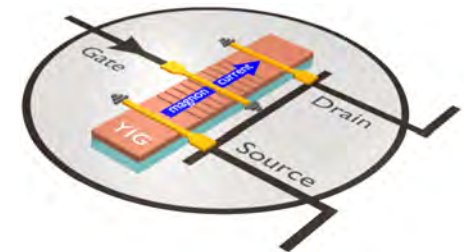
S. Klingler et al., Appl. Phys. Lett. **106**, 212406 (2015)

# Advanced magnonics

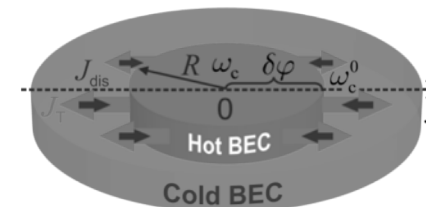
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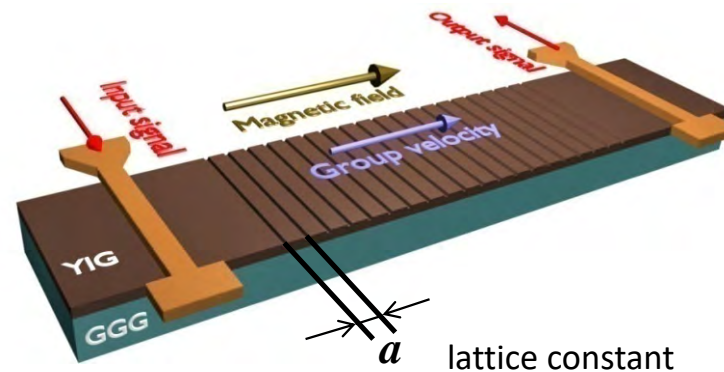


# Magnonic crystal

Magnonic crystal – magnetic meta-material:

- artificial medium with periodic lateral **variation in magnetic properties**
- Acts like magnonic Fabry-Pérot cavity characterized by quality factor

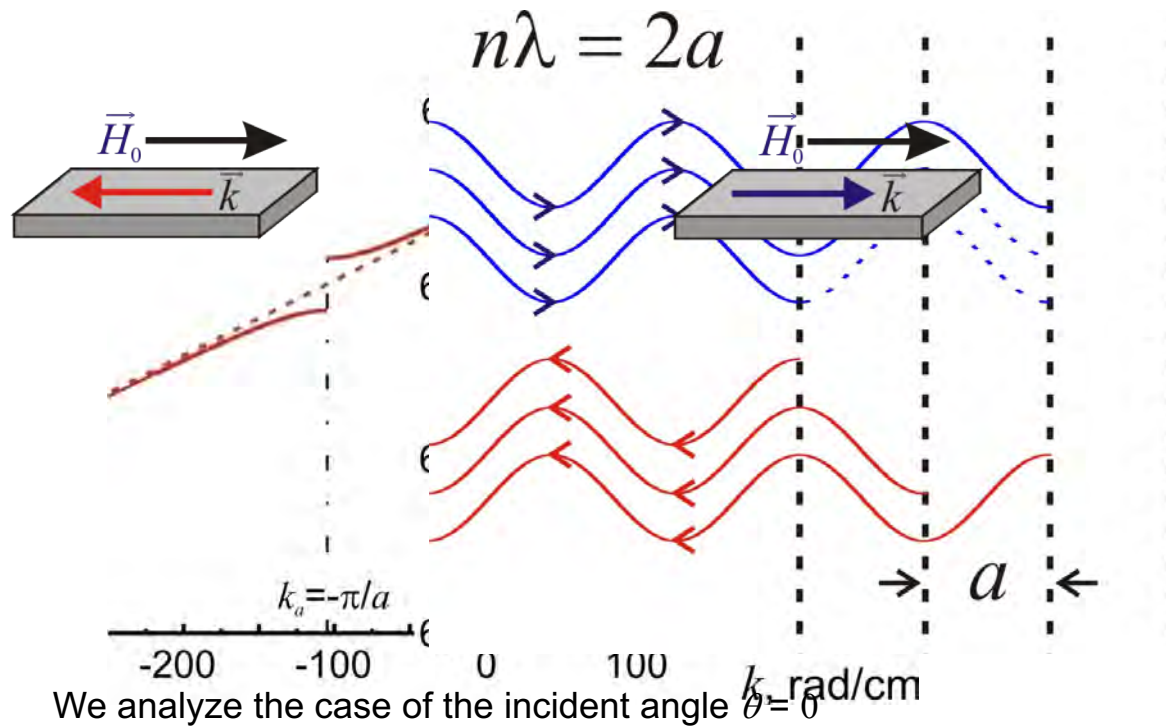
One-dimensional magnonic crystal:



- analogous to **photonic and sonic** crystals but operates with spin waves in the GHz frequency range

## Band gap

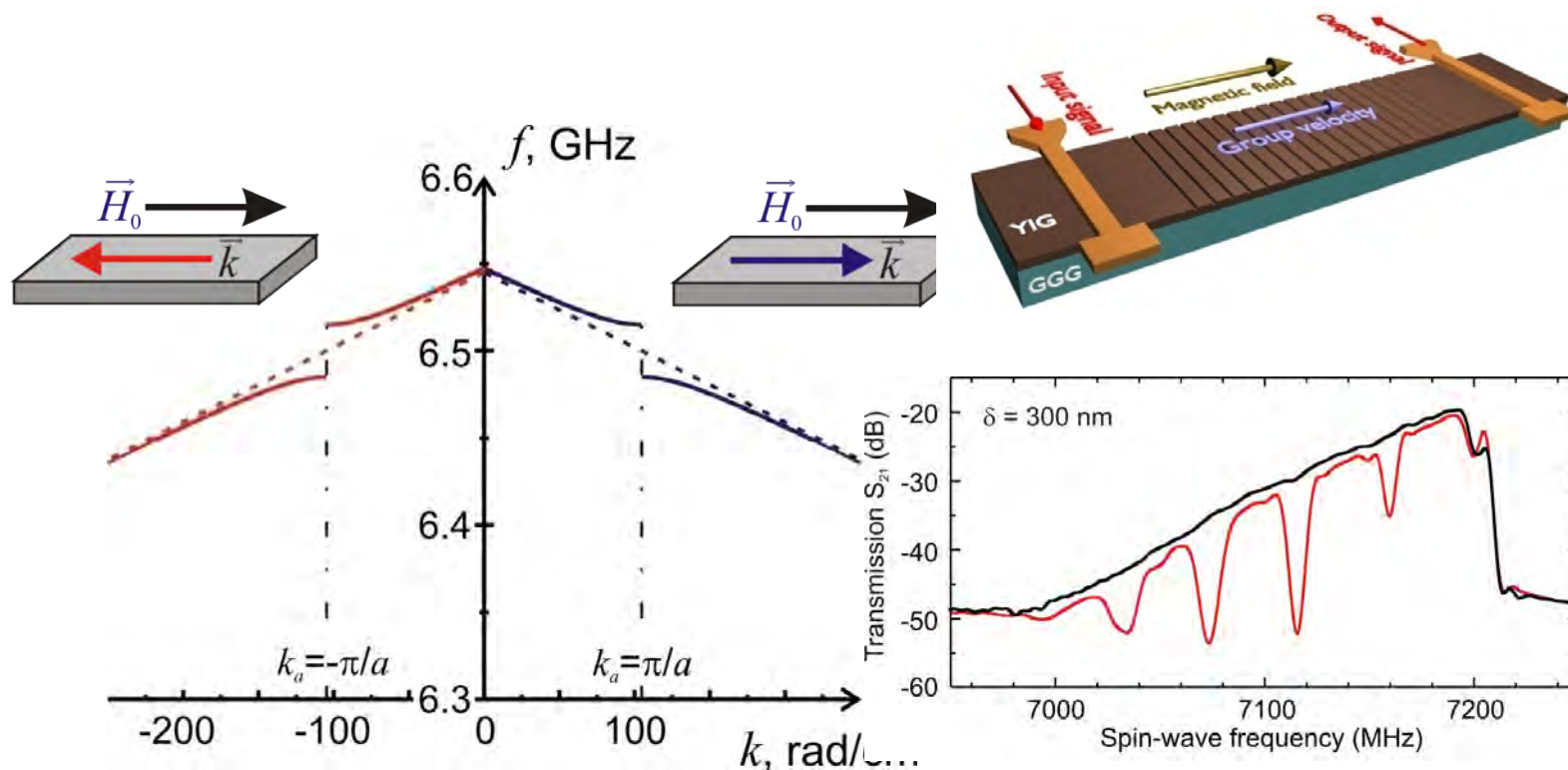
**Band gaps** – regions of the spectrum over which waves are **not allowed** to propagate





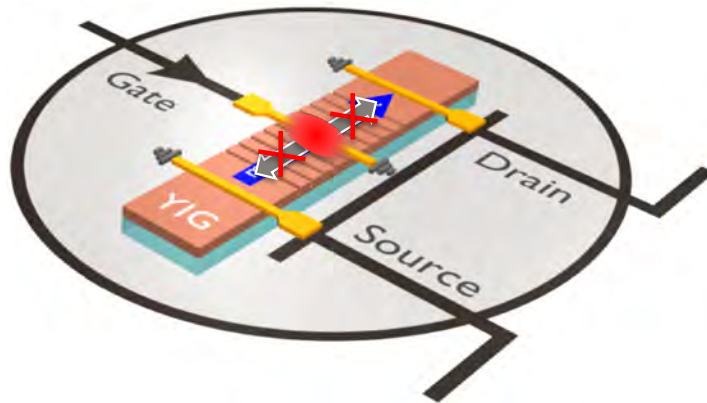
# Band gap

Band gaps – regions of the spectrum over which waves are **not allowed** to propagate

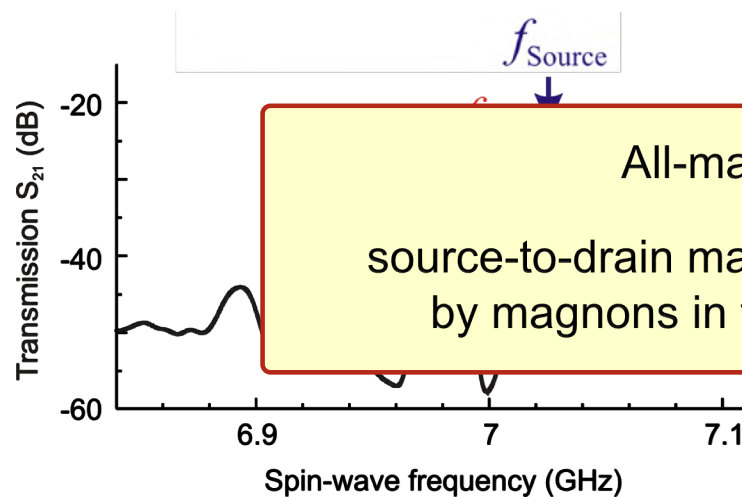
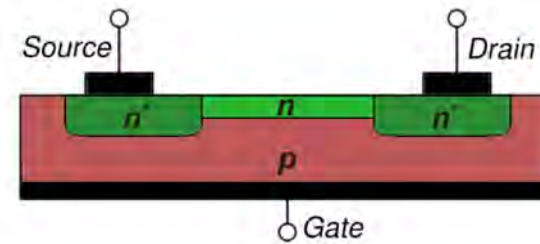


A.V. Chumak et al., Appl. Phys. Lett. **93**, 022508 (2008)

# Magnon transistor



Semiconductor field-effect transistor:



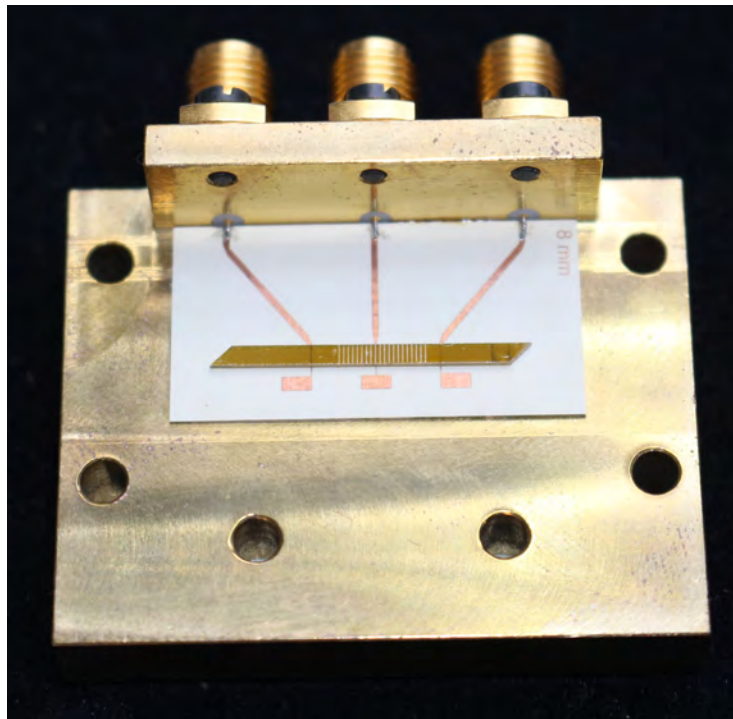
All-magnon device  
source-to-drain magnon flow  
by magnons in the transistor

Magnonic crystal acts  
as an enhancer of  
non-linear effects

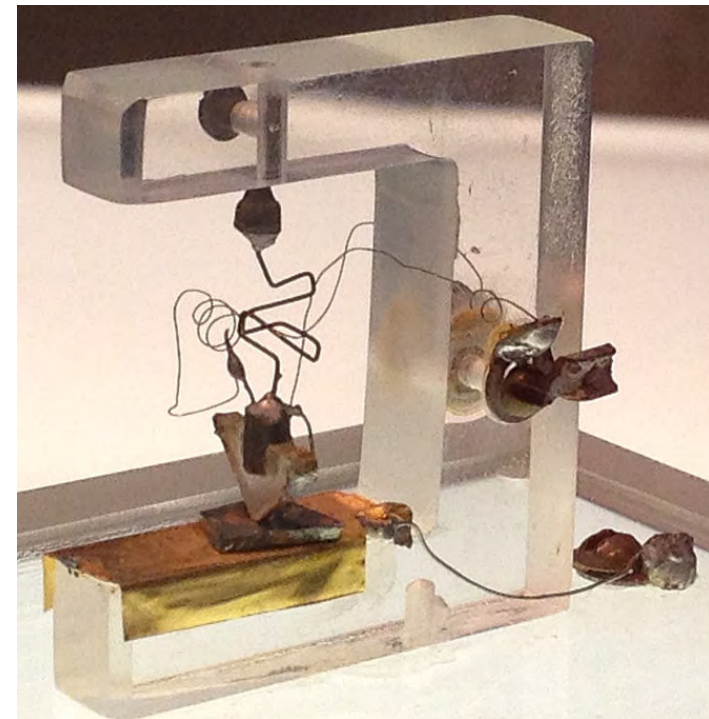
A.V. Chumak et al., Nat. Commun. 5:4700 (2014)

## First transistors

Magnon transistor prototype, 2014



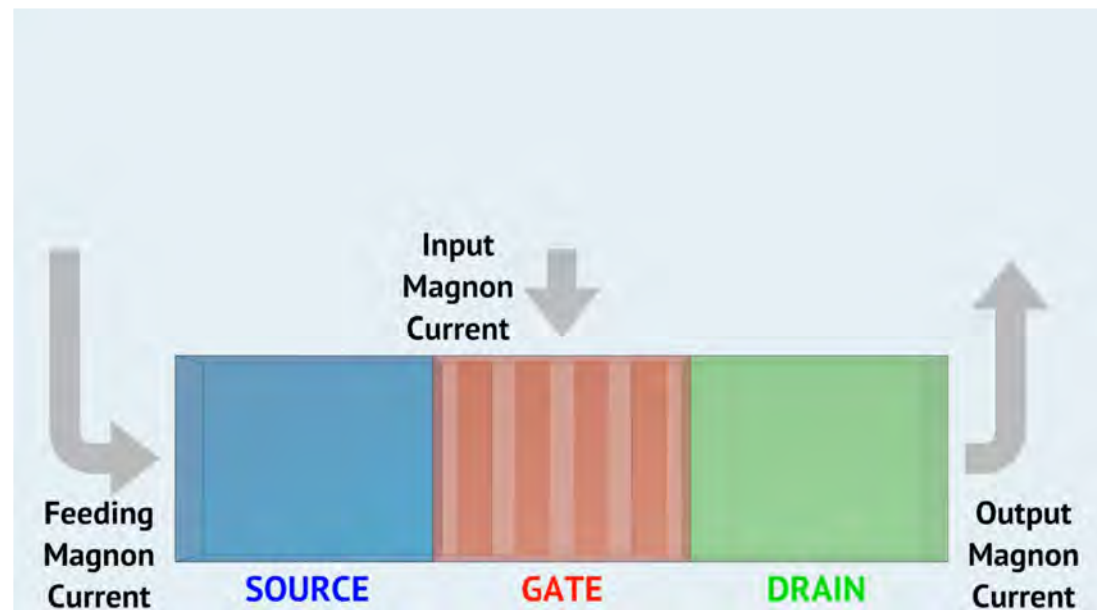
First transistor, 1947



<https://de.wikipedia.org>

# Magnon transistor

Magnon transistor allows for the control of one magnon current by another

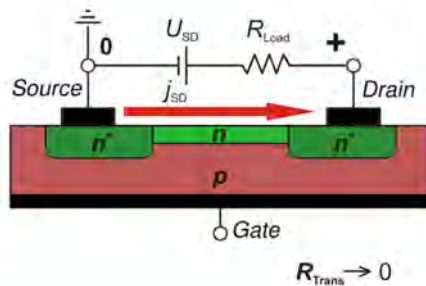
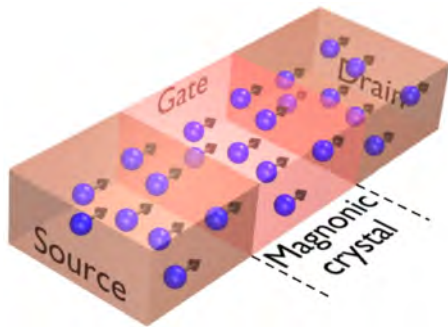


# Magnon transistor

**Opened:  $R \rightarrow 0$**

Gate magnon density

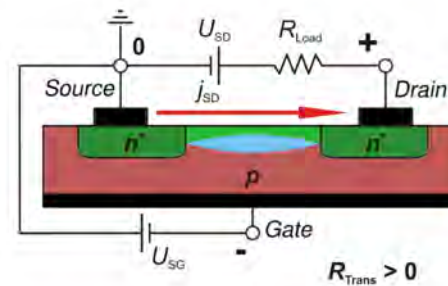
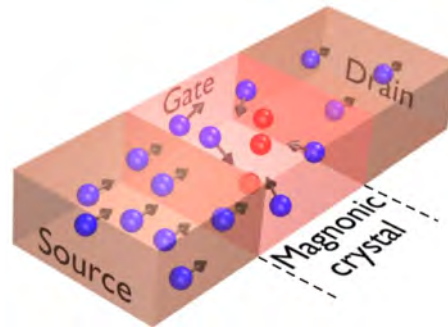
$$n_G = 0$$



**Semi-closed:  $R > 0$**

Gate magnon density

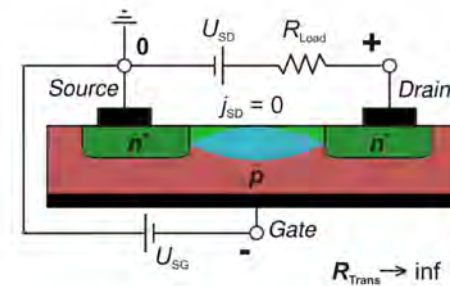
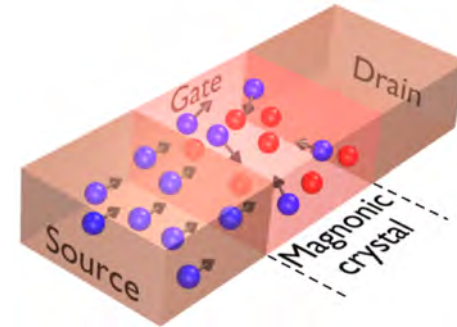
$$n_G > 0$$



**Closed:  $R \rightarrow \infty$**

Gate magnon density

$$n_G \gg 0$$



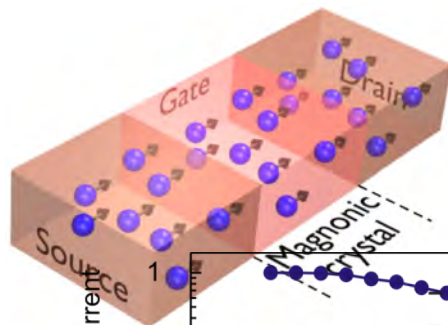
A.V. Chumak et al., Nat. Commun. 5:4700 (2014)

# Magnon transistor

**Opened:  $R \rightarrow 0$**

Gate magnon density

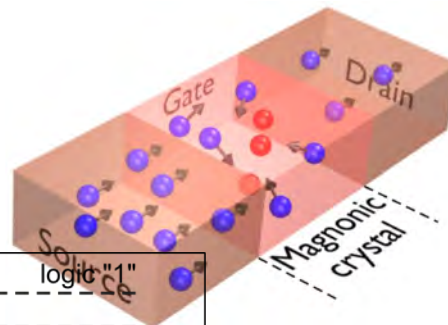
$$n_G = 0$$



**Semi-closed:  $R > 0$**

Gate magnon density

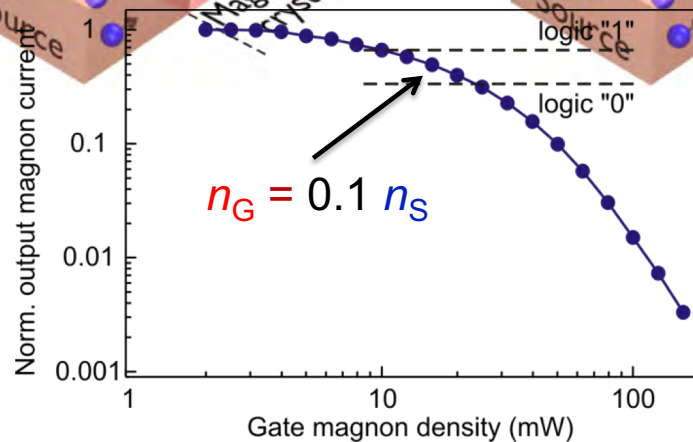
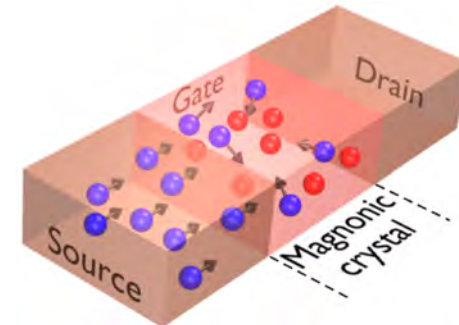
$$n_G > 0$$



**Closed:  $R \rightarrow \infty$**

Gate magnon density

$$n_G \gg 0$$

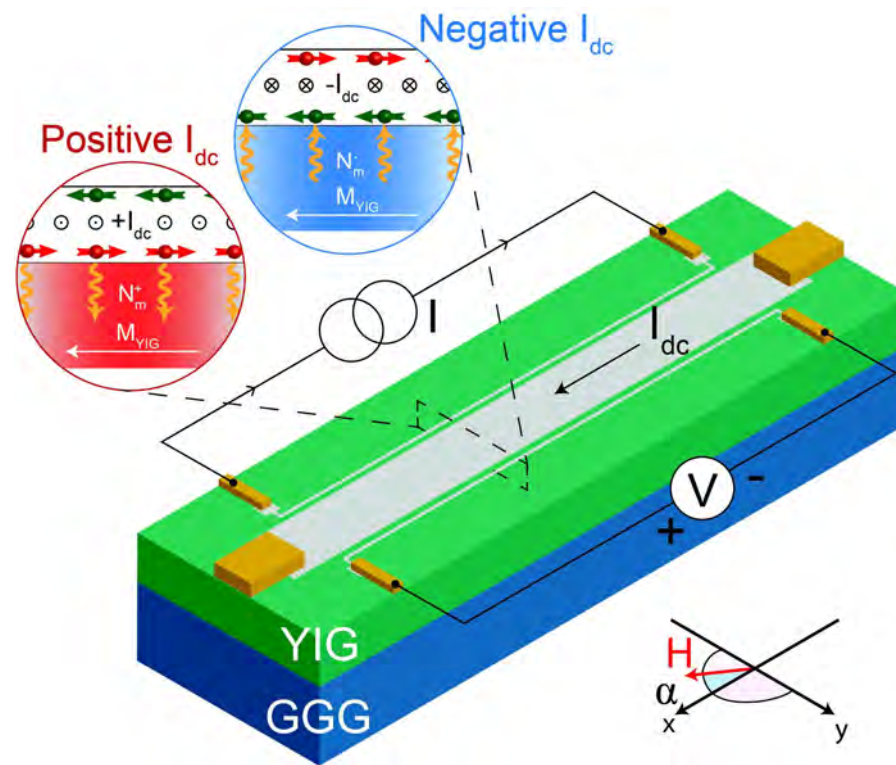


“magnon control by magnon“  
principle was realized:  
data can be processed on the  
same magnetic chip

A.V. Chumak et al., Nat. Commun. 5:4700 (2014)

# Magnon transistor based on the diffusive transport of thermal magnons

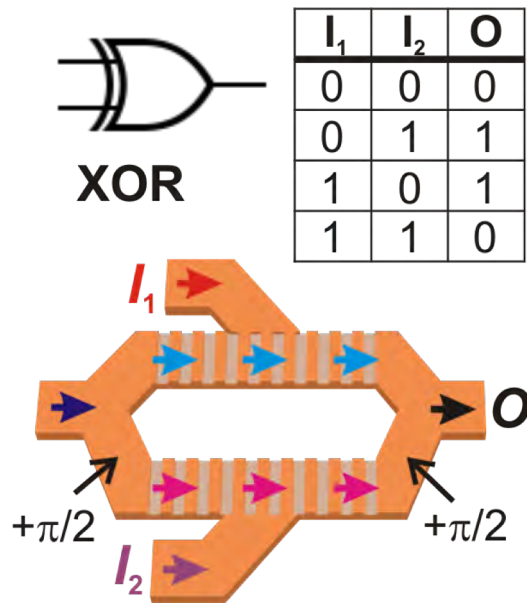
Proof of principle of a method for modulating the diffusive transport of thermal magnons



L. J. Cornelissen et al., Phys. Rev. Lett. **120**, 097702 (2018)

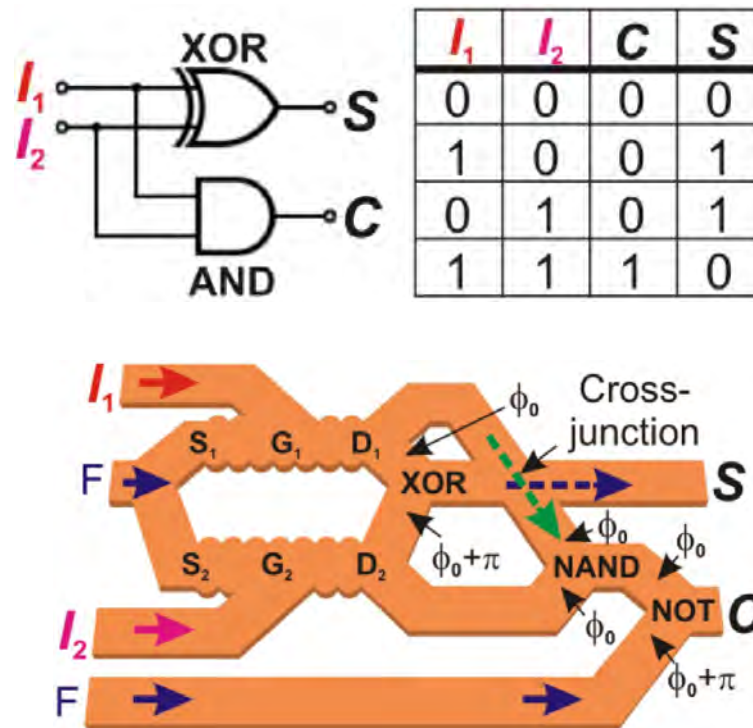
# Logic operations

XOR logic gate



XOR gate requires 2 magnon transistors instead of 8 FET in CMOS

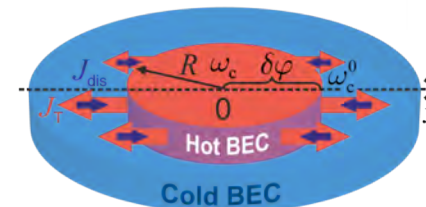
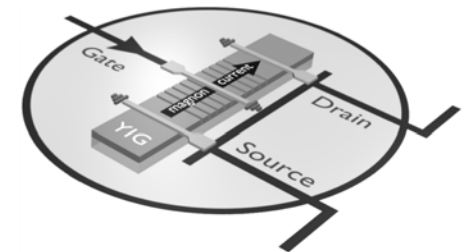
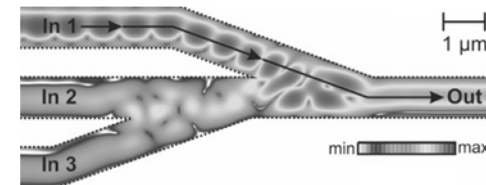
Half adder





## Advanced magnonics

- I. Magnon interference logic
- II. Non-linear magnonics: Magnon transistor
- III. Magnonic macroscopic quantum state
- IV. Quantum-classical analogies in magnonics

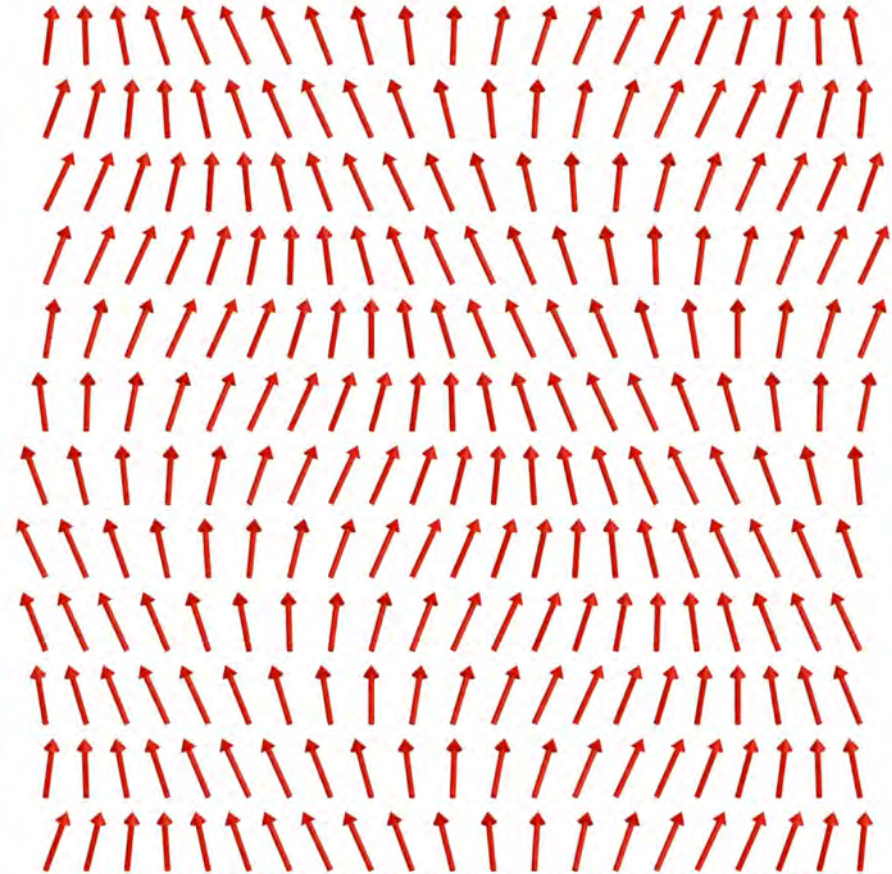


### Magnon as a quanta of spin-wave

- Energy  $\varepsilon = \hbar \omega = \frac{\eta}{\hbar} p^2$
- Momentum  $\vec{p} = \hbar \vec{q}$
- Mass  $m = \hbar / (2\eta)$
- Spin  $s = 1$
- Four- and three-magnon scattering



## Magnon gas



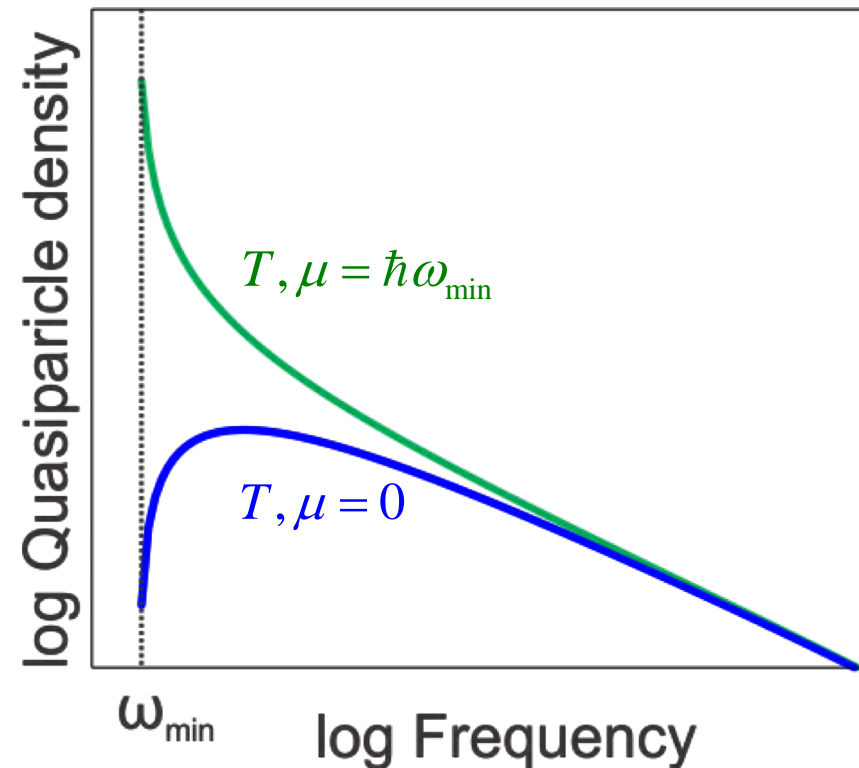
## Magnon distribution

### Bose-Einstein distribution

$$\rho(\omega) = \frac{D(\omega)}{\exp\left(\frac{\hbar\omega - \mu}{k_B T}\right) - 1}$$

$\mu$ : chemical potential

Magnons are **bosons** ( $s=1$ ) and similar to other quasi-particles are described in **thermal equilibrium** by Bose-Einstein distribution with **zero chemical potential**



# Magnon spectrum of in-plane magnetized YIG film

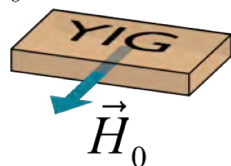
## Landau-Lifshitz equation

$$\frac{\partial \vec{M}}{\partial t} = -|\gamma| \vec{M} \times \vec{H}_{\text{eff}}$$

$$\vec{H}_{\text{eff}}(\vec{r}) = \vec{H}_0 + \int_V \tilde{G}(\vec{r}, \vec{r}') \cdot \vec{M}(\vec{r}') d\vec{r}' + \frac{\eta}{\gamma M_S} \nabla^2 \vec{M} + \dots$$

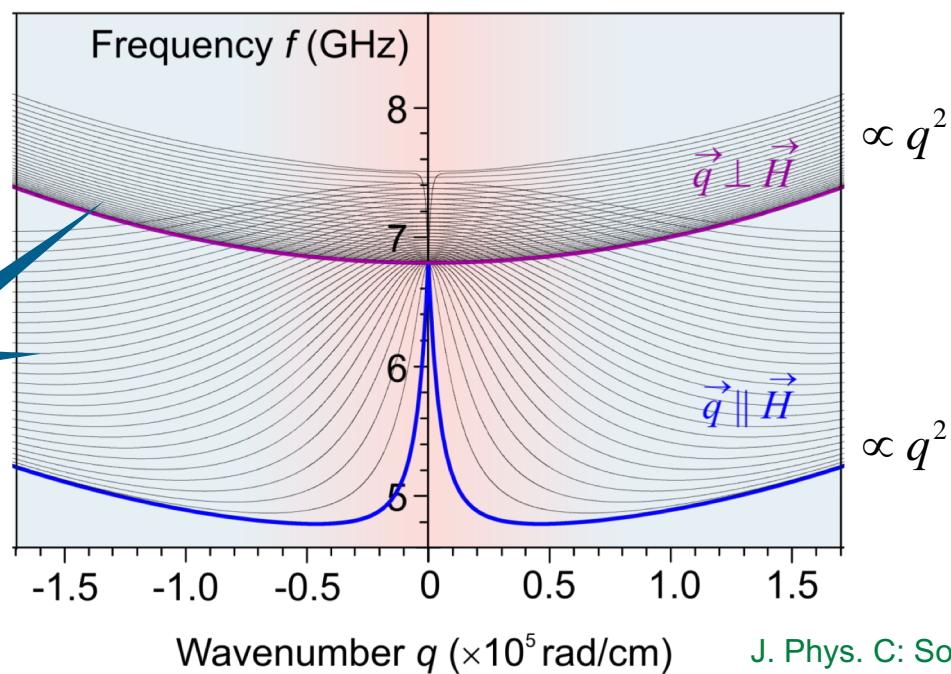
dipolar interaction      exchange interaction

$$H_0 = 1710 \text{ Oe}$$



Thickness modes having a non-uniform harmonic distribution of dynamic magnetization along the film thickness

6  $\mu\text{m}$  thick YIG film



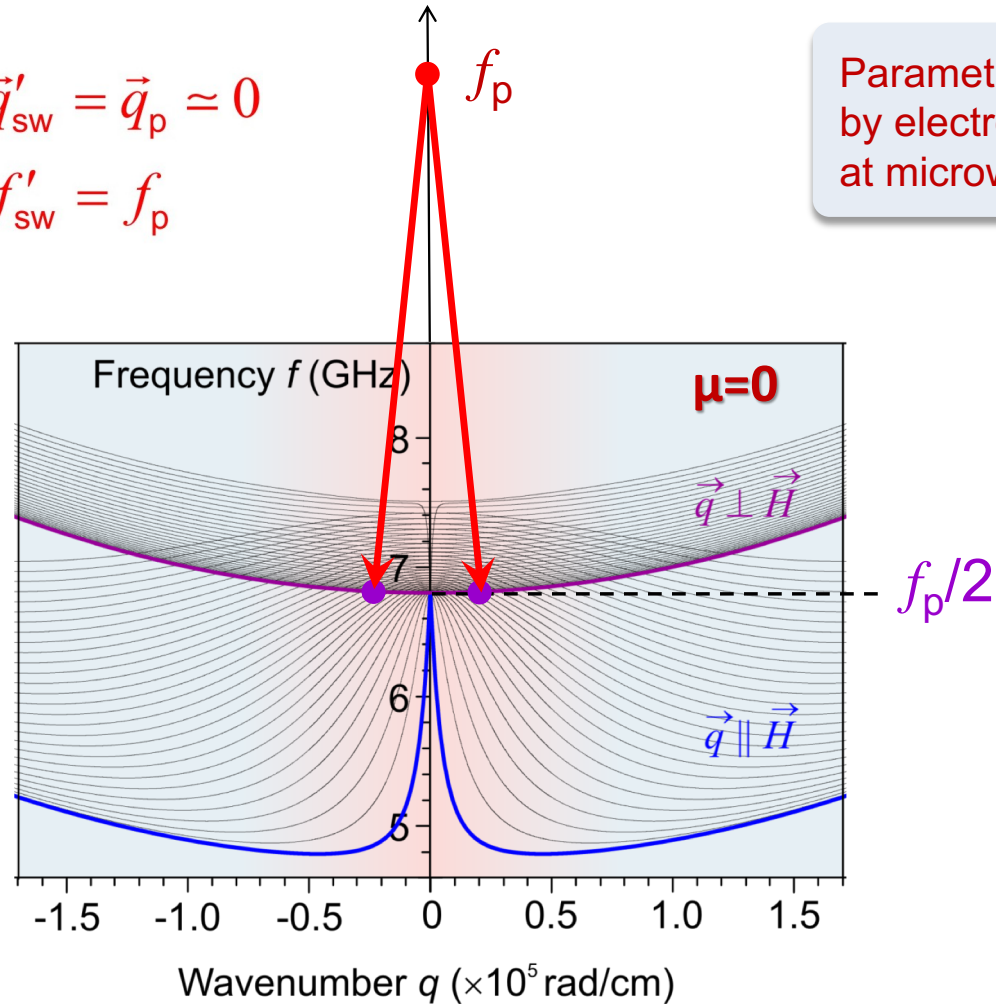
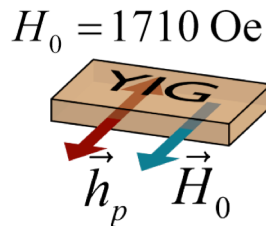
Calculations based on:  
Kalinikos and Slavin,  
J. Phys. C: Solid State Phys **19**, 7013 (1986)

# Control of magnon gas density by parametric pumping

Energy and momentum conservation laws for parametric pumping

$$\begin{cases} \vec{q}_{sw} + \vec{q}'_{sw} = \vec{q}_p \approx 0 \\ f_{sw} + f'_{sw} = f_p \end{cases}$$

Parametric pumping by electromagnetic wave at microwave frequency



Bose-Einstein distribution

$$\rho(f) = \frac{D(f)}{\exp\left(\frac{hf - \mu}{k_B T}\right) - 1}$$

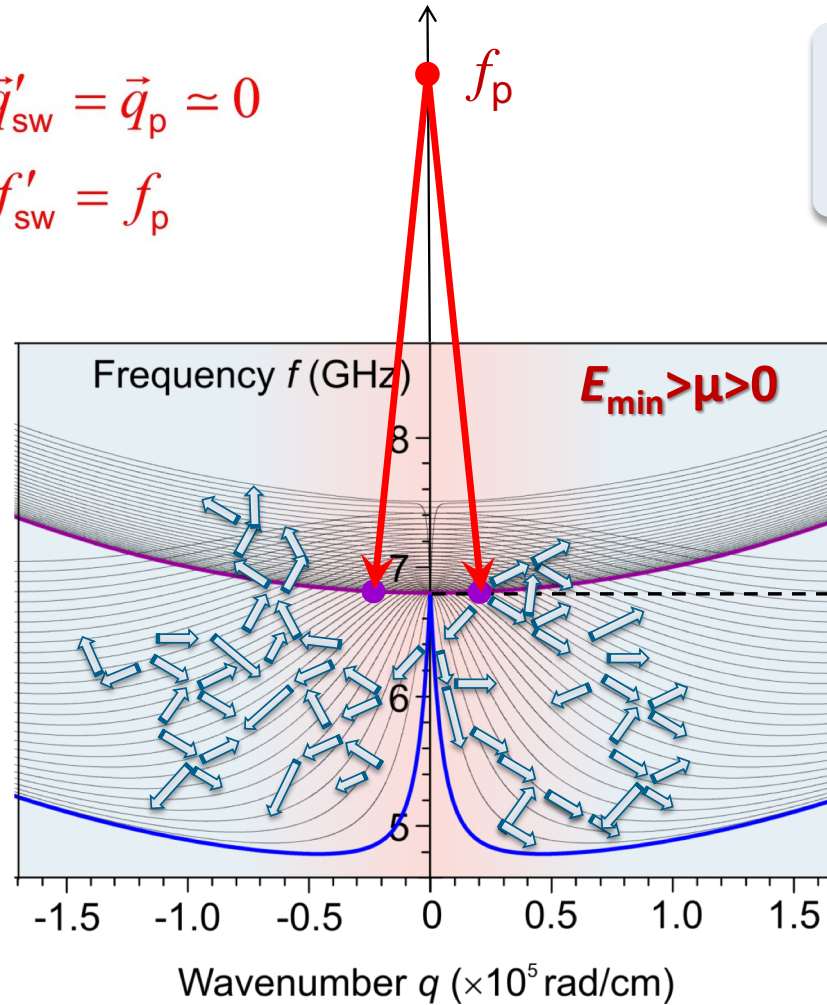
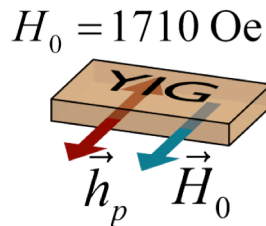
$\mu$ : chemical potential

# Control of magnon gas density by parametric pumping

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$$\begin{cases} \vec{q}_{sw} + \vec{q}'_{sw} = \vec{q}_p \approx 0 \\ f_{sw} + f'_{sw} = f_p \end{cases}$$

Parametric pumping by electromagnetic wave at microwave frequency



$f_p/2$

Magnon thermalization due to 4-particle scattering: incoherent magnon gas

Bose-Einstein distribution

$$\rho(f) = \frac{D(f)}{\exp\left(\frac{hf - \mu}{k_B T}\right) - 1}$$

$\mu$ : chemical potential

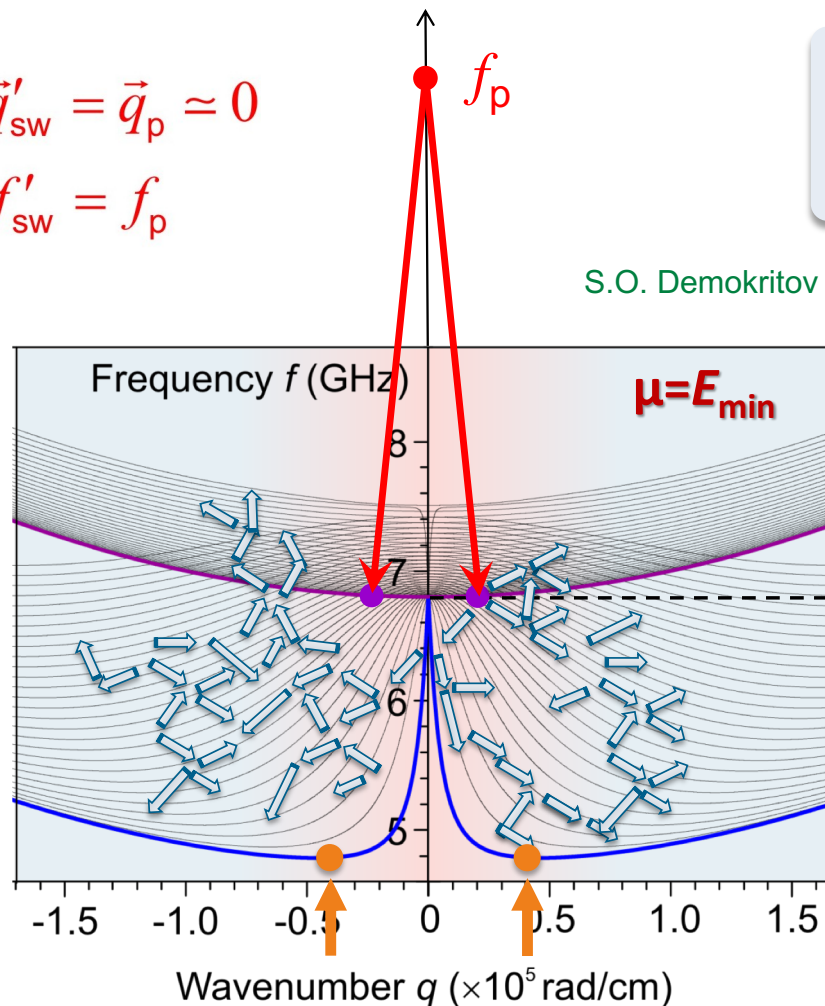
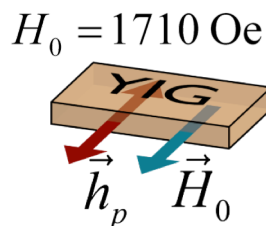
# Bose-Einstein condensation of magnons

Energy and momentum conservation laws for parametric pumping

$$\begin{cases} \vec{q}_{sw} + \vec{q}'_{sw} = \vec{q}_p \approx 0 \\ f_{sw} + f'_{sw} = f_p \end{cases}$$

Parametric pumping by electromagnetic wave at microwave frequency

S.O. Demokritov *et al.*, Nature **443**, 430 (2006)



$f_p/2$

Magnon thermalization due to 4-particle scattering: incoherent magnon gas

Bose-Einstein magnon condensate

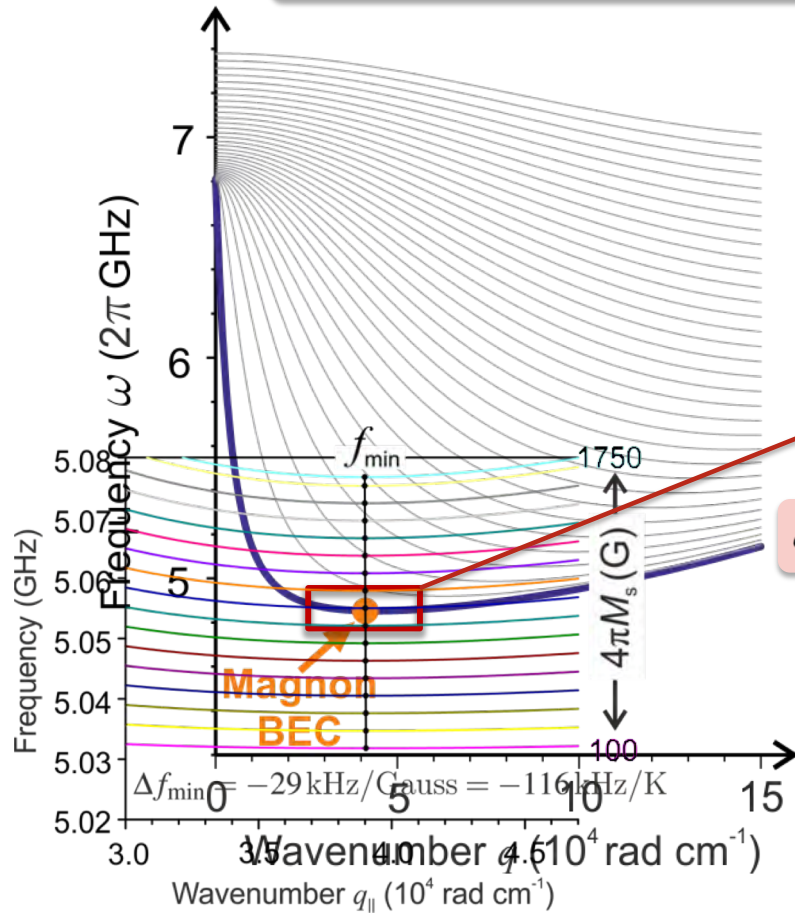
**Bose-Einstein distribution**

$$\rho(f) = \frac{D(f)}{\exp\left(\frac{hf - \mu}{k_B T}\right) - 1}$$

$\mu$ : chemical potential

# Supercurrent in magnon BEC

**Supercurrent:** Flow of particles due to **phase gradient** of the condensate's wavefunction



Complex BEC wave function:  $\psi(\vec{r}, t)$

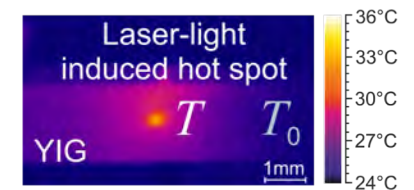
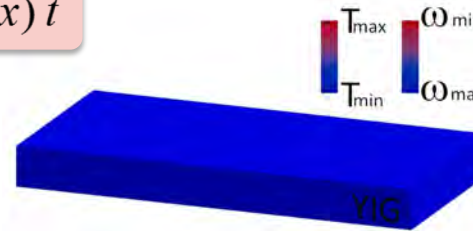
BEC density:  $N_c = |\psi|^2$

BEC phase:  $\varphi = \arg(\psi)$

**Supercurrent**

$$\vec{J}(\vec{r}, t) = \frac{\hbar}{m} N_c \nabla \varphi$$

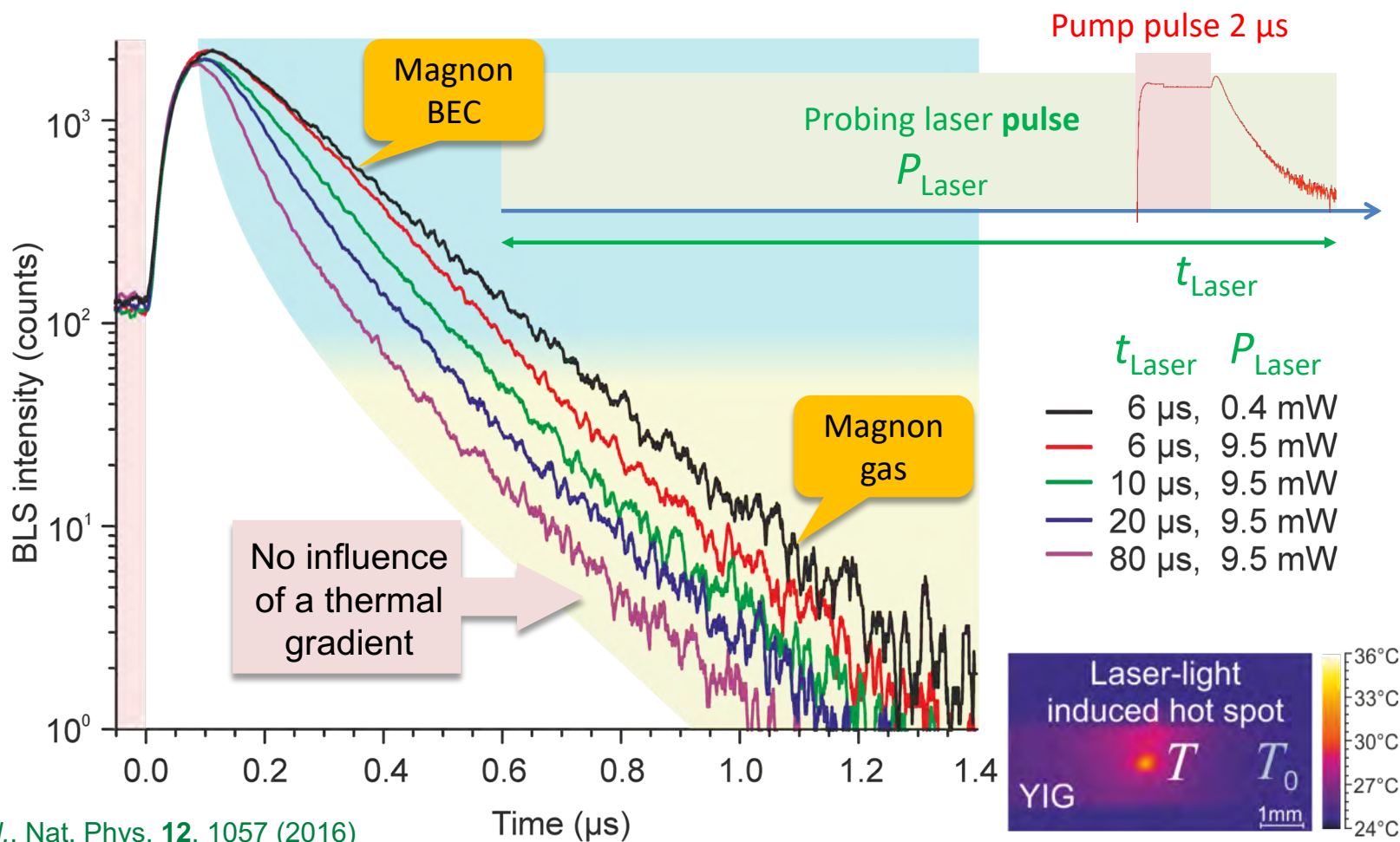
$$\delta\varphi = \delta\omega_c(x) t$$



By changing probing laser **power** or laser pulse **duration** it is possible to control the **phase** of the magnon BEC



# Dynamics of condensed magnons in thermal gradient



D. A. Bozhko *et al.*, Nat. Phys. **12**, 1057 (2016)

## Dynamics of condensed magnons in thermal gradient - theory

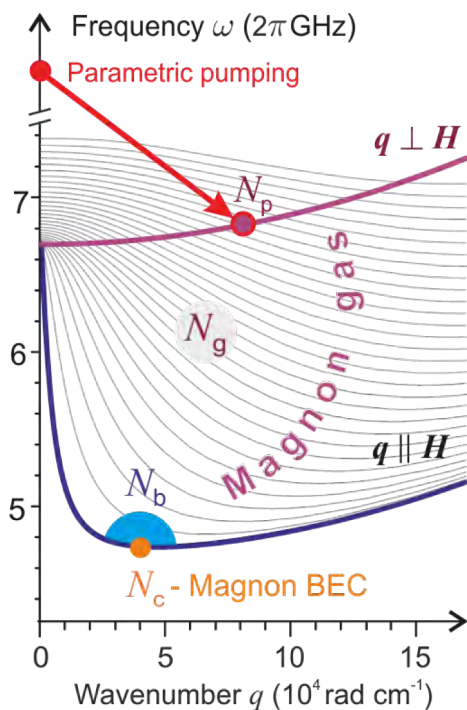
Dynamics of condensed magnons  $N_c(t)$ , magnons in gaseous states  $N_g(t)$  and gaseous magnons at the bottom of SW spectrum  $N_b(t)$  described using rate equations

Without thermal gradient →

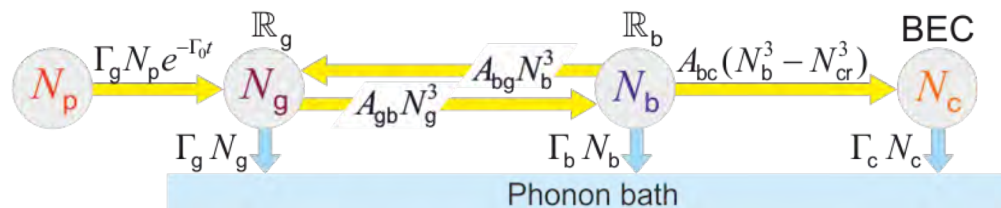
$$\frac{\partial N_g}{\partial t} = -\Gamma_g N_g + \Gamma_g N_p e^{-\Gamma_0 t} - A_{gb} N_g^3 + A_{bg} N_b^3$$

$$\frac{\partial N_b}{\partial t} = -\Gamma_b N_b + A_{gb} N_g^3 - A_{bg} N_b^3 - A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr})$$

$$\frac{\partial N_c}{\partial t} = -\Gamma_c N_c + A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr})$$



$N_{cr}$  – a critical number of magnons at which the chemical potential  $\mu$  of the magnon gas reaches  $\omega_{\min}$



D. A. Bozhko *et al.*, Nat. Phys. **12**, 1057 (2016)

## Dynamics of condensed magnons in thermal gradient - theory

Dynamics of condensed magnons  $N_c(t)$ , magnons in gaseous states  $N_g(t)$  and gaseous magnons at the bottom of SW spectrum  $N_b(t)$  described using rate equations

With thermal gradient  $\rightarrow$

### Supercurrent

$$\vec{J}(\vec{r}, t) = \frac{\hbar}{m} N_c \nabla \varphi$$

Phase  
gradient

BEC density:  $N_c = |\psi|^2$

BEC phase:  $\varphi = \arg(\psi)$

Magnon mass:  $m = \hbar \frac{\partial^2 \omega(q)}{\partial q^2}$

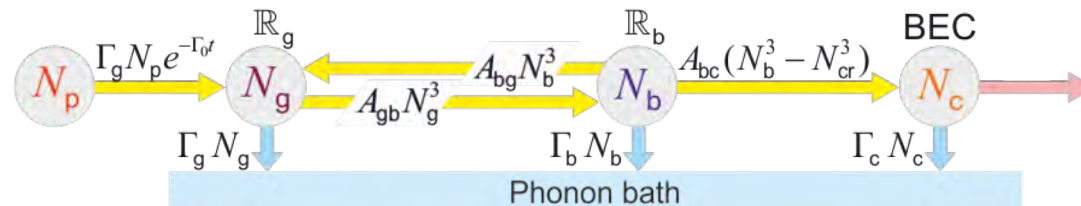
Complex BEC wave function:  $\psi(\vec{r}, t)$

$$\frac{\partial N_g}{\partial t} = -\Gamma_g N_g + \Gamma_g N_p e^{-\Gamma_0 t} - A_{gb} N_g^3 + A_{bg} N_b^3$$

$$\frac{\partial N_b}{\partial t} = -\Gamma_b N_b + A_{gb} N_g^3 - A_{bg} N_b^3 - A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr})$$

$$\frac{\partial N_c}{\partial t} = -\Gamma_c N_c + A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr}) - \frac{\partial J(\vec{r}, t)}{\partial \vec{r}}$$

Additional decrease of population of condensed magnons  $N_c(t)$  due to magnon **supercurrent**  $\vec{J}(\vec{r}, t)$



D. A. Bozhko *et al.*, Nat. Phys. **12**, 1057 (2016)

## Dynamics of condensed magnons in thermal gradient - theory

Dynamics of condensed magnons  $N_c(t)$ , magnons in gaseous states  $N_g(t)$  and gaseous magnons at the bottom of SW spectrum  $N_b(t)$  described using rate equations

With thermal gradient  $\rightarrow$

### 2D supercurrent

$$J_x = N_c D_x \frac{\partial \phi}{\partial x}$$

$$J_y = N_c D_y \frac{\partial \phi}{\partial y}$$

$$\frac{\partial N_g}{\partial t} = -\Gamma_g N_g + \Gamma_g N_p e^{-\Gamma_0 t} - A_{gb} N_g^3 + A_{bg} N_b^3$$

$$\frac{\partial N_b}{\partial t} = -\Gamma_b N_b + A_{gb} N_g^3 - A_{bg} N_b^3 - A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr})$$

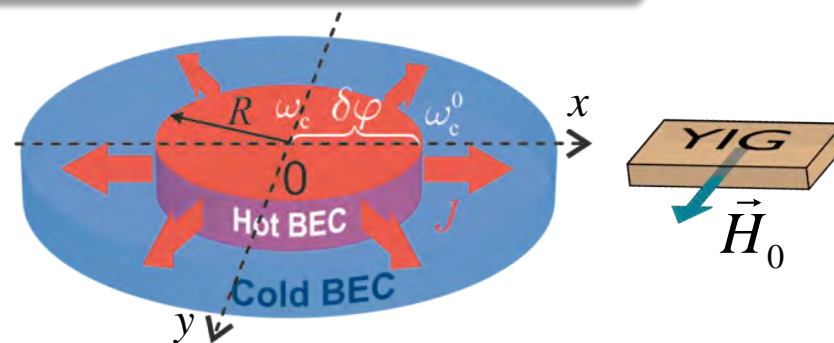
$$\frac{\partial N_c}{\partial t} = -\Gamma_c N_c + A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr}) - \frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y}$$

Additional decrease of population of condensed magnons  $N_c(t)$  due to magnon **supercurrent**  $\vec{J}(x, y, t)$

Anisotropic  
dispersion  
coefficients

$$D_x = \frac{1}{2} \frac{\partial^2 \omega(\vec{q})}{\partial q_x^2}$$

$$D_y = \frac{1}{2} \frac{\partial^2 \omega(\vec{q})}{\partial q_y^2}$$



# Dynamics of condensed magnons in thermal gradient - theory

Dynamics of condensed magnons  $N_c(t)$ , magnons in gaseous states  $N_g(t)$  and gaseous magnons at the bottom of SW spectrum  $N_b(t)$  described using rate equations

With thermal gradient  $\rightarrow$

## 2D supercurrent

$$J_x = N_c D_x \frac{\partial \phi}{\partial x}$$

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$$\frac{\partial N_c}{\partial t} = -\Gamma_c N_c + A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr}) - \frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y}$$

Additional decrease of population of condensed magnons  $N_c(t)$  due to magnon **supercurrent**  $\vec{J}(x, y, t)$

Anisotropic  
dispersion  
coefficients

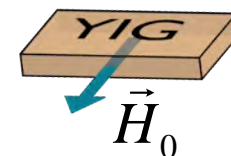
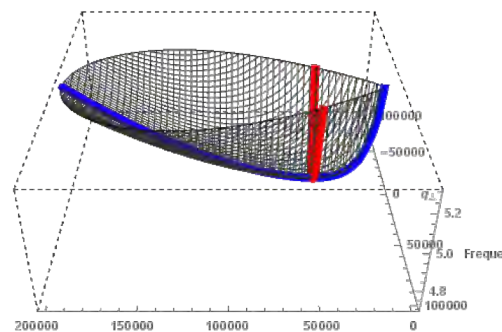
$$D_x = \frac{1}{2} \frac{\partial^2 \omega(\vec{q})}{\partial q_x^2}$$

$$D_y = \frac{1}{2} \frac{\partial^2 \omega(\vec{q})}{\partial q_y^2}$$

In experiment:

$$D_x \approx 21 D_y$$

$$J_T = J_x \gg J_y$$



## Dynamics of condensed magnons in thermal gradient - theory

Dynamics of condensed magnons  $N_c(t)$ , magnons in gaseous states  $N_g(t)$  and gaseous magnons at the bottom of SW spectrum  $N_b(t)$  described using rate equations

With thermal gradient  $\rightarrow$

**1D thermally driven  
supercurrent**

$$J_T = N_c D_x \frac{\partial \varphi}{\partial x}$$

$$\delta \varphi = \delta \omega_c(x) t$$

a weak frequency shift  
of the BEC wave function  
due to temperature change

$$\frac{\partial N_g}{\partial t} = -\Gamma_g N_g + \Gamma_g N_p e^{-\Gamma_0 t} - A_{gb} N_g^3 + A_{bg} N_b^3$$

$$\frac{\partial N_b}{\partial t} = -\Gamma_b N_b + A_{gb} N_g^3 - A_{bg} N_b^3 - A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr})$$

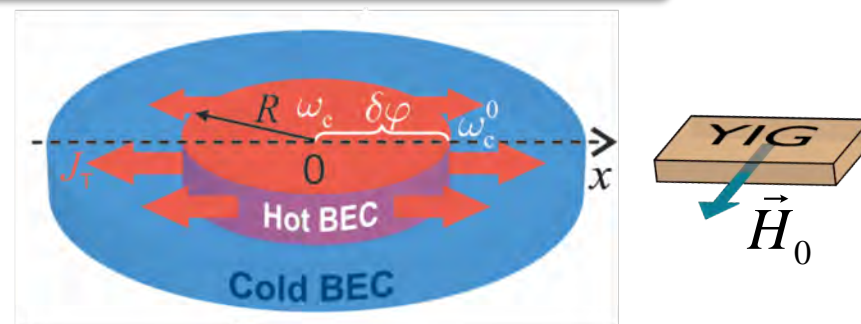
$$\frac{\partial N_c}{\partial t} = -\Gamma_c N_c + A_{bc} (N_b^3 - N_{cr}^3) \Theta(N_b - N_{cr}) - \frac{\partial J_T}{\partial x}$$

Additional decrease of population of condensed magnons  $N_c(t)$  due to magnon **supercurrent**  $J_T(x,t)$

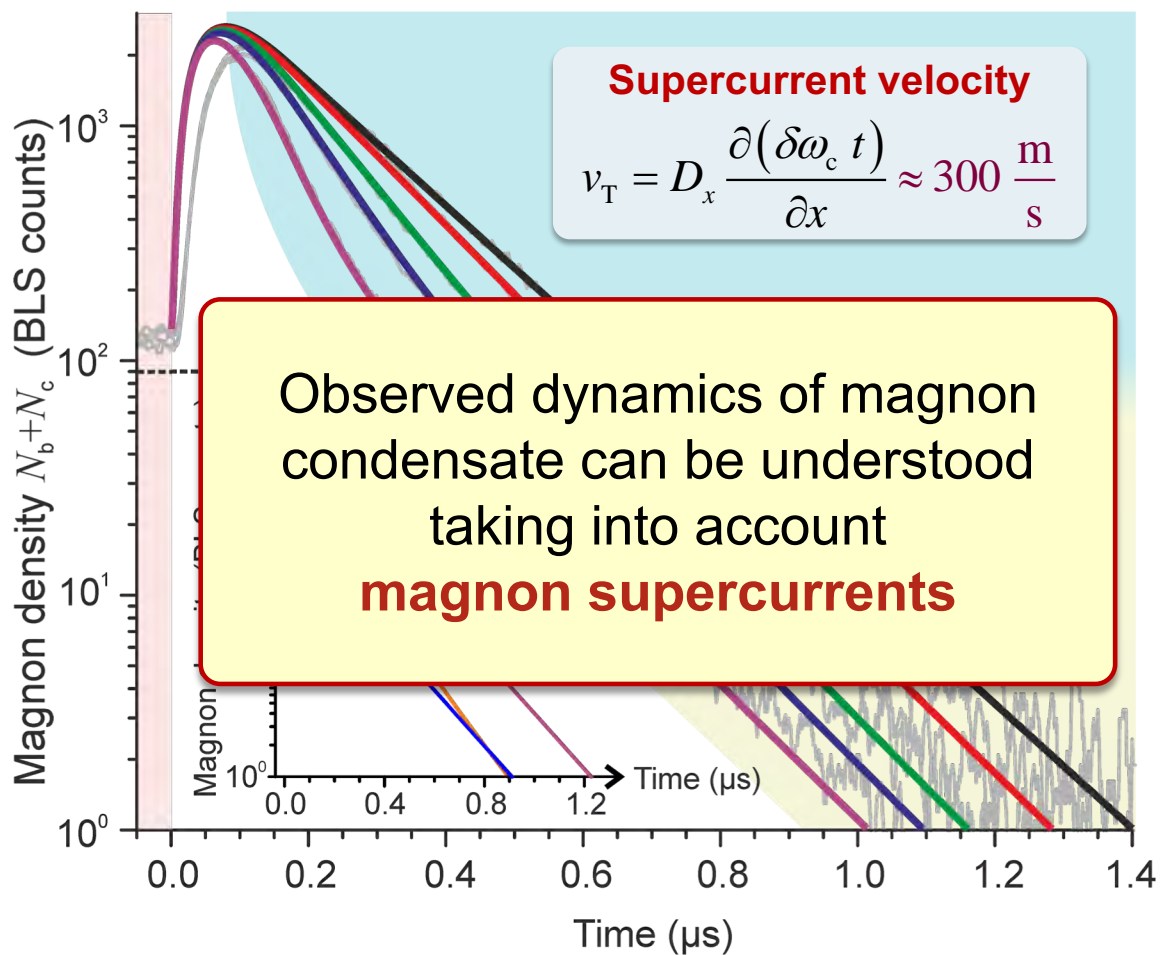
In experiment:

$$D_x \approx 21 D_y$$

$$J_T = J_x \gg J_y$$



# Dynamics of condensed magnons in thermal gradient - comparison with theory



$$\delta\varphi = \delta\omega_c(T)t$$

Thermally induced frequency shift of the magnon BEC

$$\delta\omega_c(T) / 2\pi$$

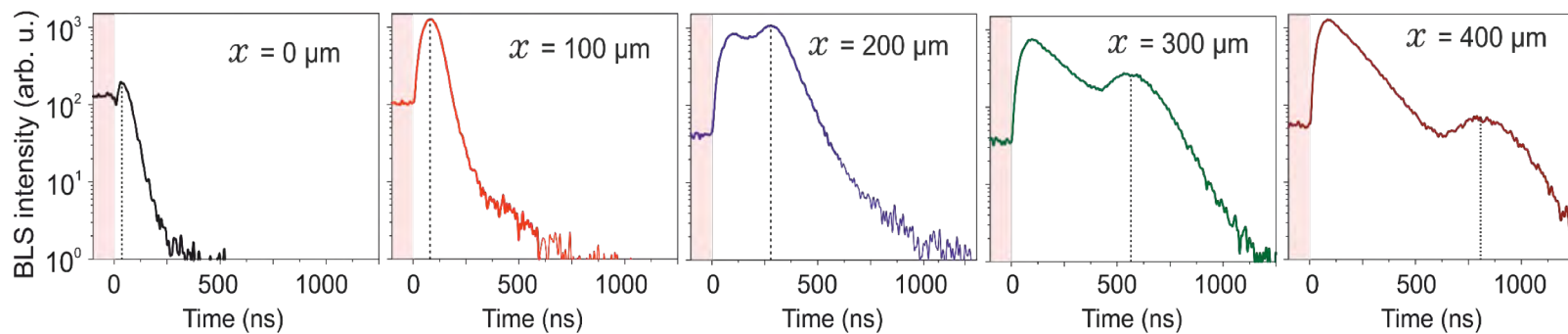
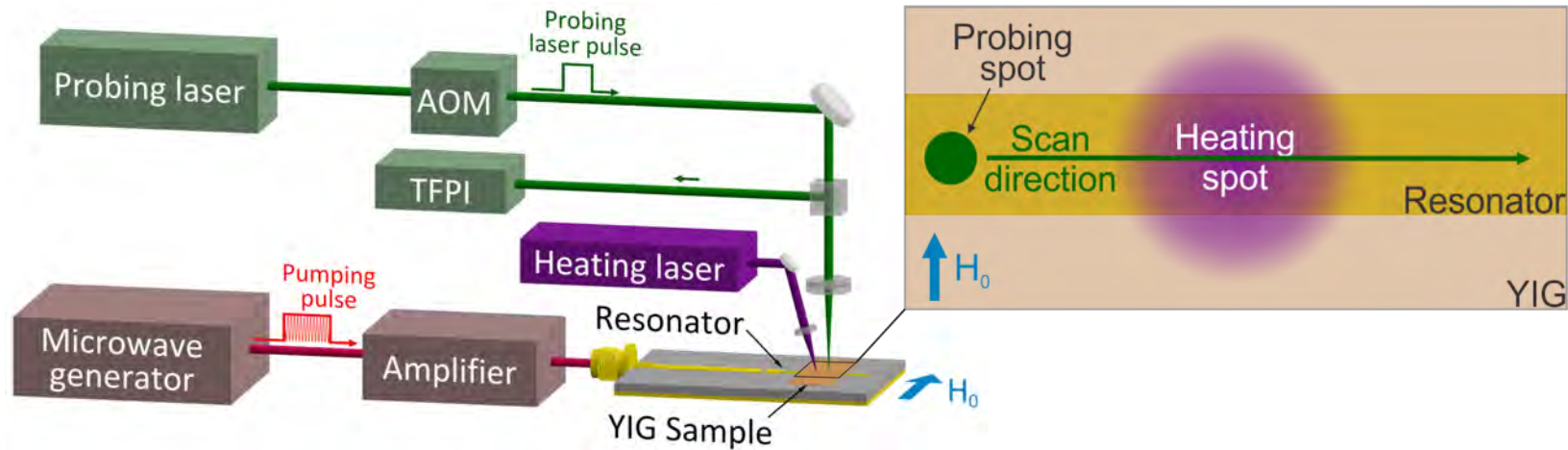
- 0 kHz
- 25 kHz
- 100 kHz
- 198 kHz
- 550 kHz



Corresponding maximal temperature change  
**4.7 K**

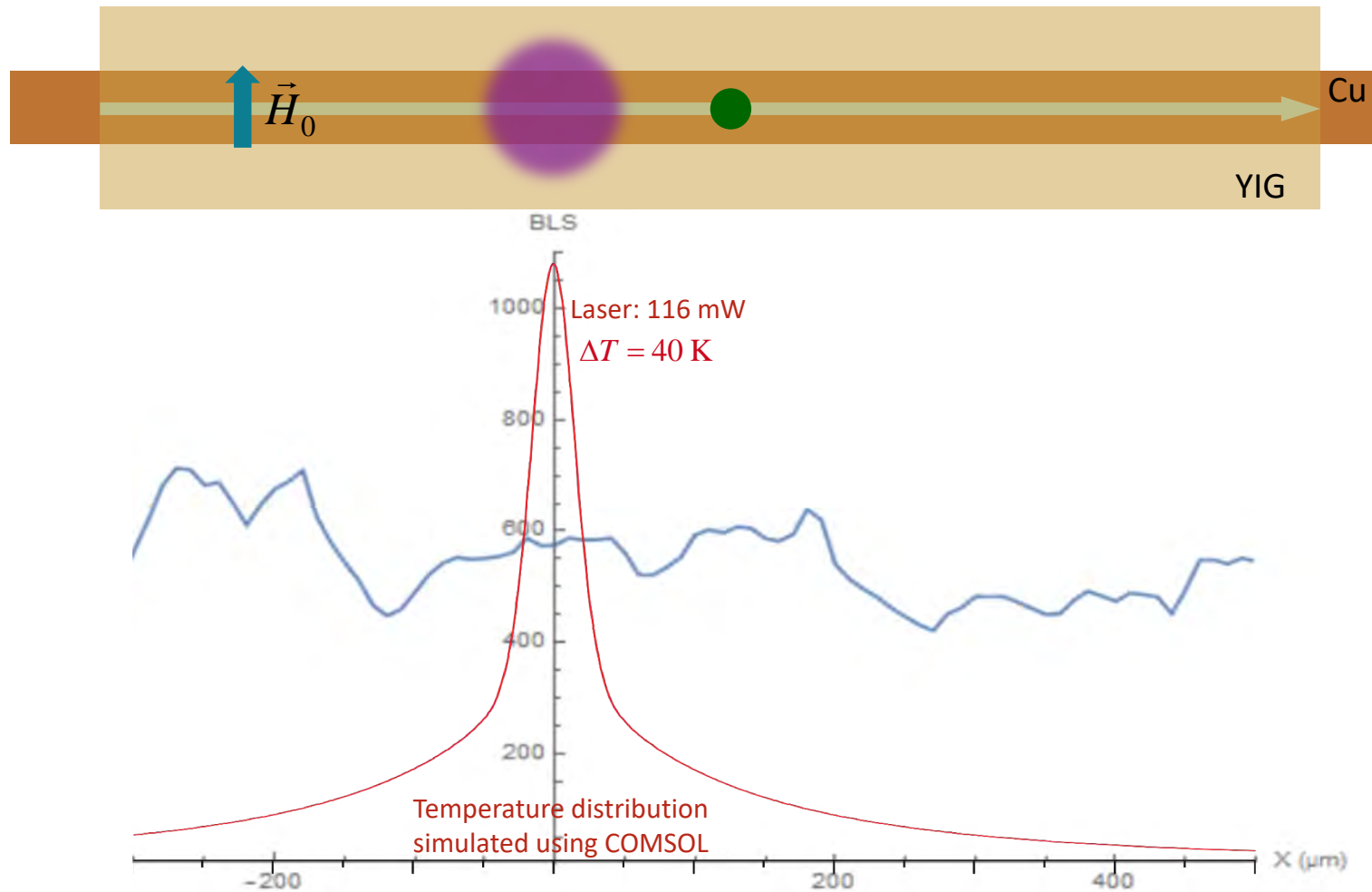
D. A. Bozhko *et al.*, Nat. Phys. **12**, 1057 (2016)

# Non-local measurement: Supercurrent magnon transport

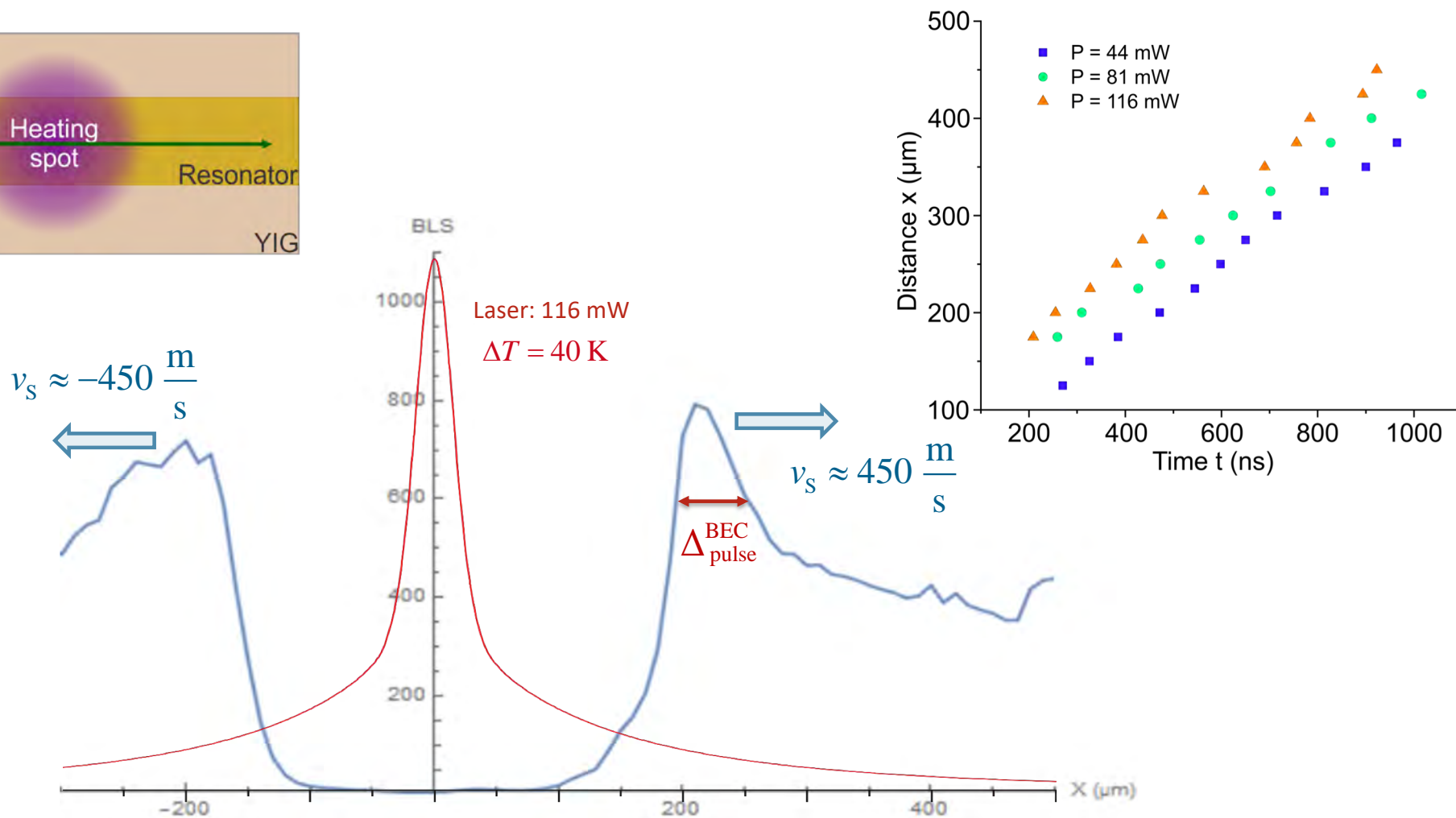
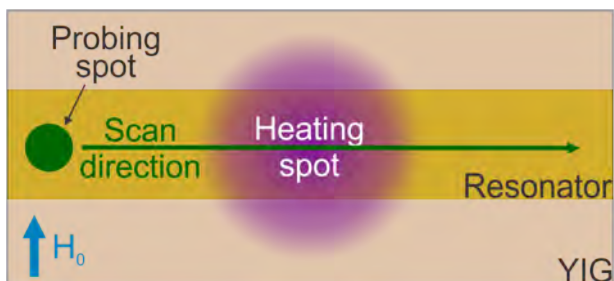




# Non-local measurement: Supercurrent magnon transport



# Non-local measurement: Supercurrent magnon transport



# Non-local measurement: Second magnonic sound scenario

Gross-Pitaevskii equation for dynamic behavior of the magnon BEC

$$\left[ i \frac{\partial}{\partial t} + D_x \frac{\partial^2}{\partial x^2} - W |\psi|^2 \right] \psi = 0$$

Dispersion coefficient  $D_x = \frac{1}{2} \frac{\partial^2 \omega(\vec{q})}{\partial q_x^2}$

Amplitude of four-wave **repulsive** interaction  $W > 0$

O. Dzyapko *et al.*, Phys. Rev. B **96**, 064438 (2017)

Stationary solution:  $\psi(x, t) = \sqrt{N_c} \exp(-iWN_c t)$

Magnon BEC density  $N_c$

Bogoliubov dispersion law for small perturbations on the background of the stationary solution  
**(second sound)**

$$\omega(q) = c_s q \sqrt{1 + D_x q^2 / 2WN_c}$$

$$D_x q^2 \ll 2WN_c$$

$$\omega(q) = c_s q$$

**second sound velocity**

$$c_s = \sqrt{2D_x WN_c}$$

$$D_x q^2 \gg 2WN_c$$

$$\omega(q) = D_x q^2$$

**Estimations** for  $c_s = v_s \approx 450$  m/s and  $D_x \approx 7.45$  cm<sup>2</sup>/s

Nonlinear frequency shift:  $2WN_c \approx 2\pi \cdot 44$  MHz

For 116 mW laser power the BEC pulse width  $\Delta_{\text{pulse}}^{\text{BEC}} \approx 44$   $\mu$ m

$q = \pi / \Delta_{\text{pulse}}^{\text{BEC}} \approx 720$  rad/cm  $D_x q^2 \approx 2\pi \cdot 0.52$  MHz

# Non-local measurement: Second magnonic sound scenario

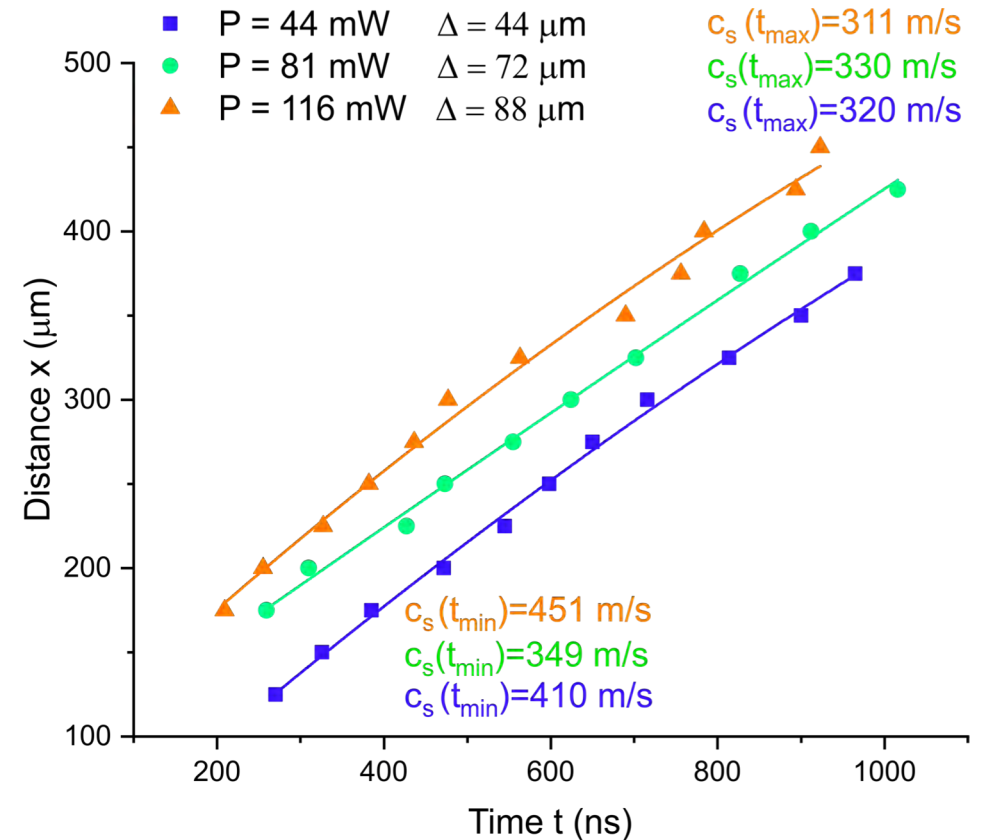
$$\omega(q) = c_s q, \quad c_s = \sqrt{2D_x W N_c}$$

- The sound velocity  $c_s$  must be **independent** on the excitation conditions i.e. the heating laser power, which determine the BEC pulse width  $\Delta_{\text{pulse}}^{\text{BEC}}$

100% change in  $\Delta_{\text{pulse}}^{\text{BEC}}$   $\longleftrightarrow$  9% change in  $c_s$

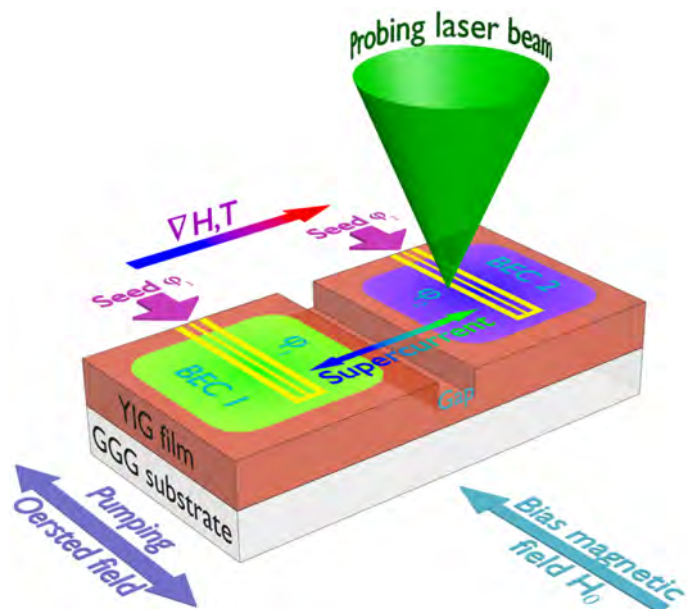
- During the pulse propagation, the amplitude of the background condensate decays and the sound velocity also has to decay

The sound velocity decrease is clear visible from parabolic fits !



Both statements are well satisfied

# Outlook 1: AC/DC magnon Josephson effect



## 1. Use of magnon supercurrent for applications

### Magnon supercurrent between two spatially separated condensates

- Phase difference is realized using initial seeding of each condensate
- Coupling strength of BECs can be varied by changing the size of the gap between the condensates
- Additional gradients of magnetic field or temperature can be applied to induce time-dependent phase shift

### Modified complex Ginzburg-Landau equation

Phase accumulation from external potential	Condensate motion	Nonlinear phase accumulation	Magnon damping	BEC seeding
---	----------------------	---------------------------------	-------------------	----------------

$$i\hbar \frac{\partial \psi}{\partial t} = \left( \hbar \omega_0(x, T, H) - \frac{\hbar^2}{2m} \nabla^2 + \hbar N |\psi|^2 \right) \psi + i\hbar(\eta - \beta |\psi|^2) \psi + if(\mathbf{r}, t)$$

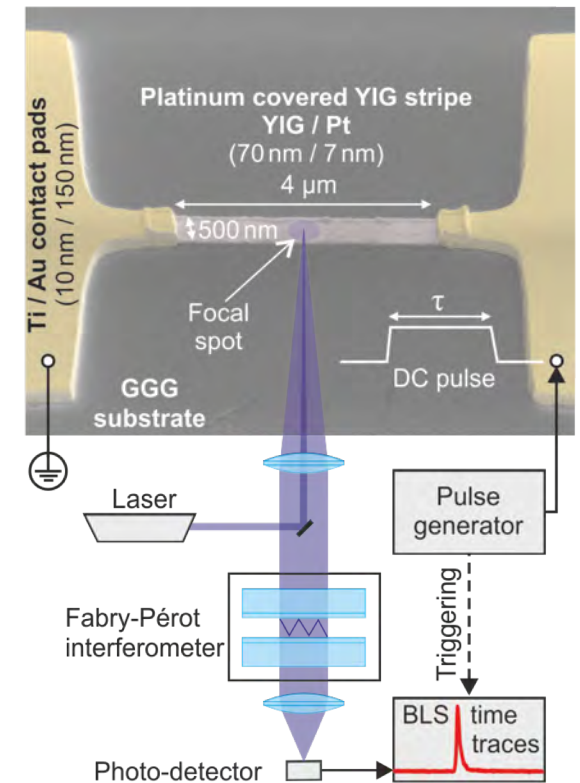
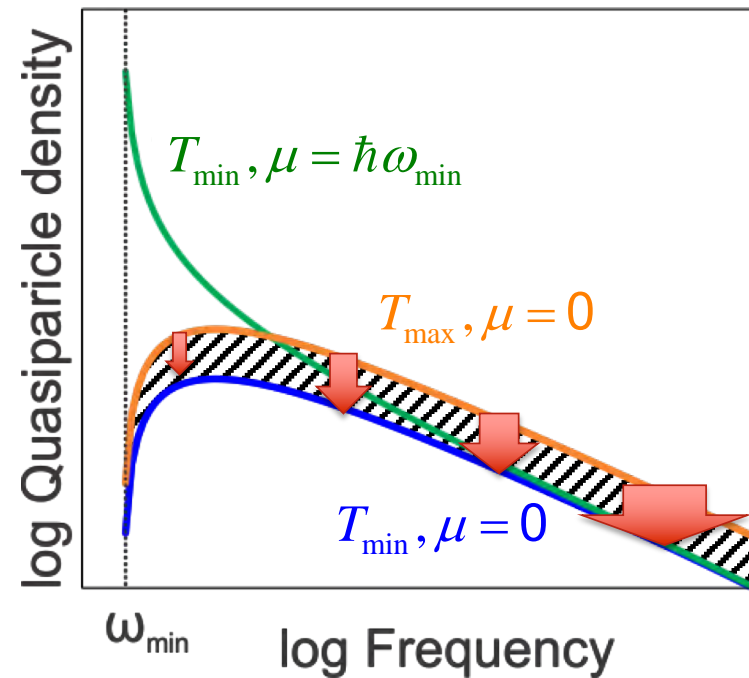
## Outlook 2: Microwave-free creation of a magnon BEC

### 2. Are there other ways how to create a magnon BEC state in YIG?

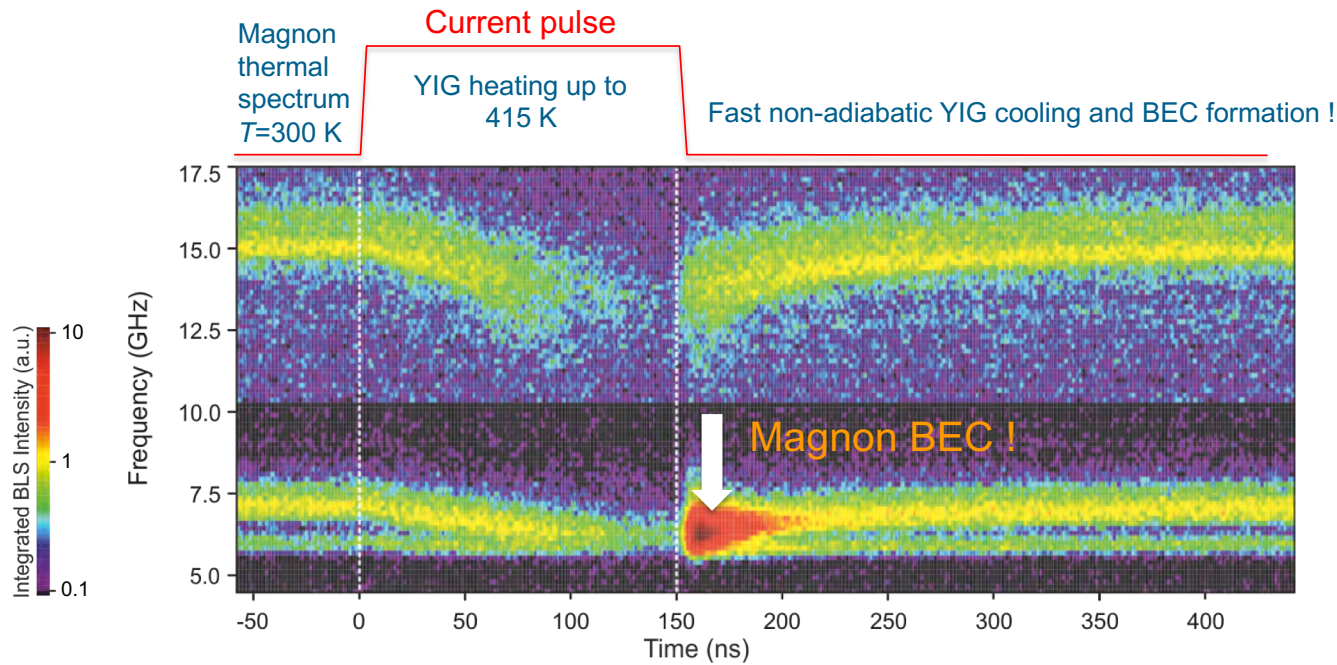
- Yes, by rapid cooling of a magnon-carrying specimen

#### Bose-Einstein distribution

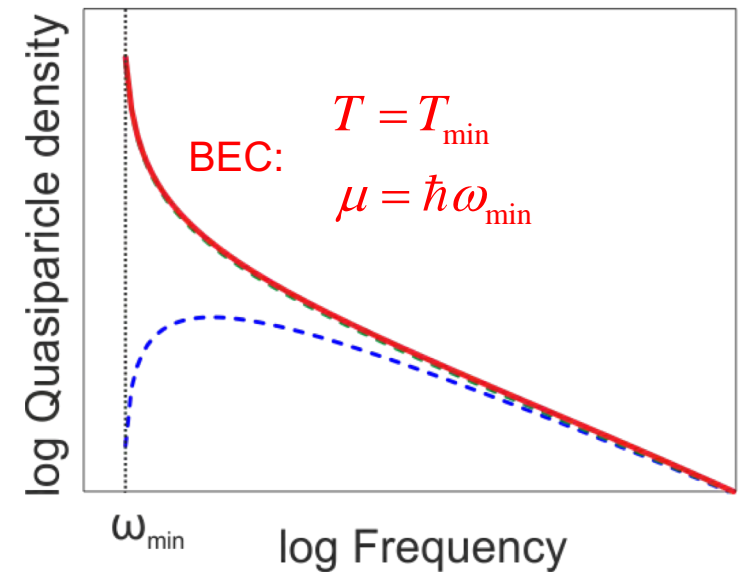
$$\rho(\omega) = \frac{D(\omega)}{\exp\left(\frac{\hbar\omega - \mu}{k_B T}\right) - 1}$$



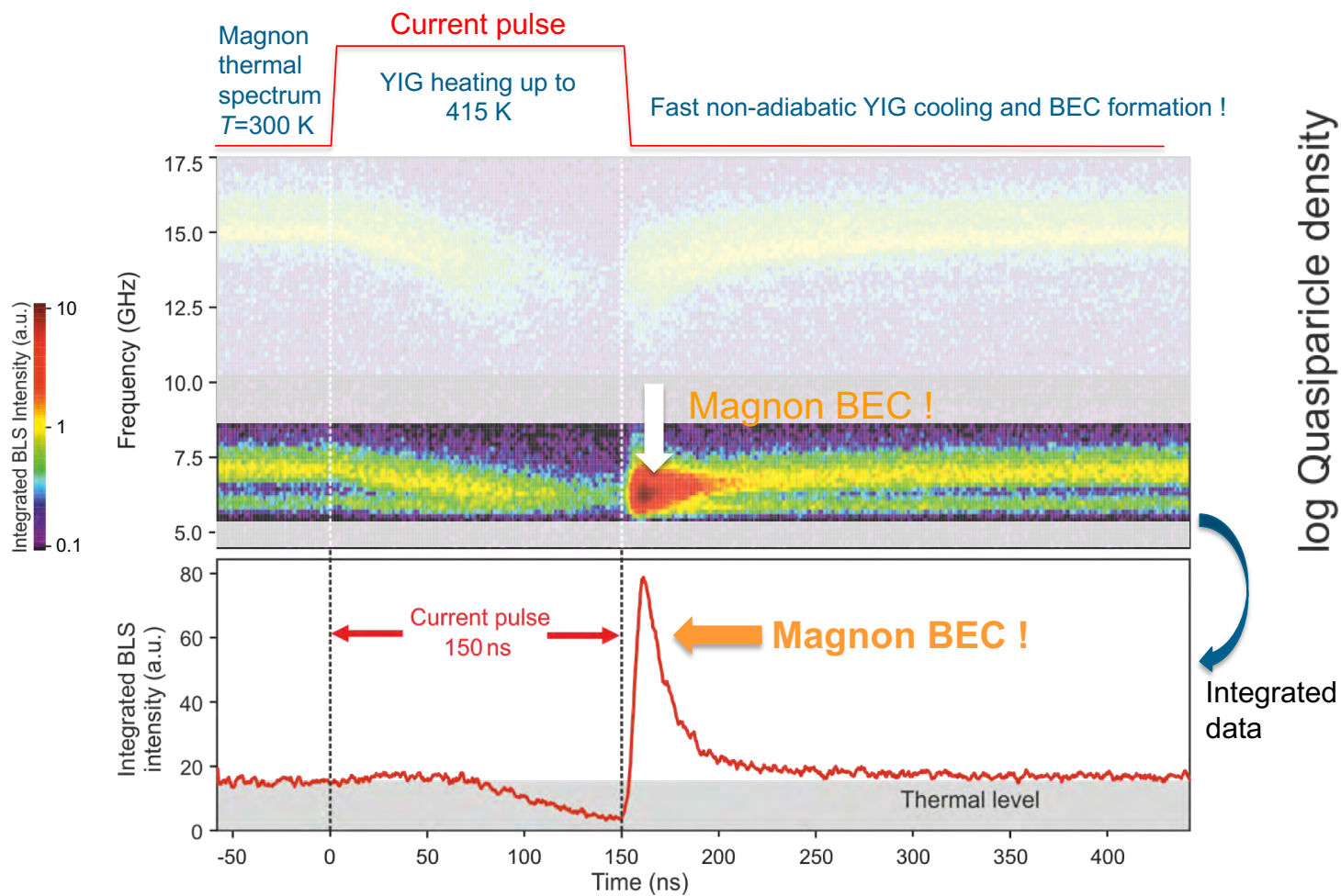
# BEC in rapidly cooled magnon gas



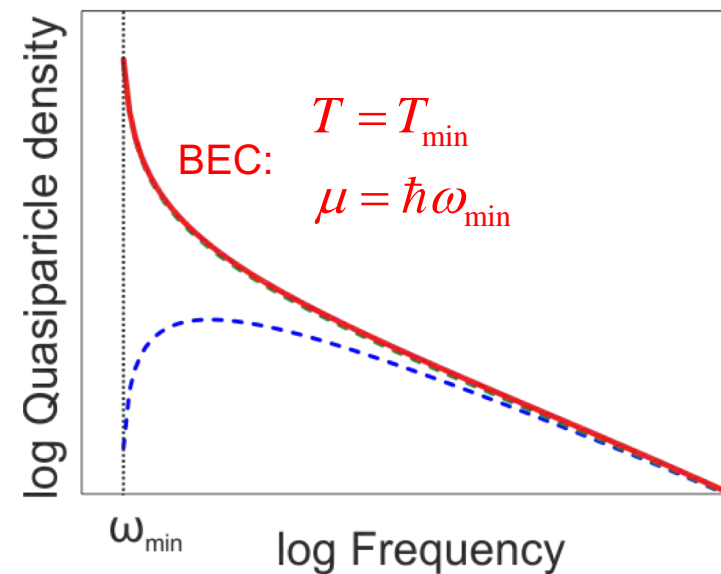
## Magnon population distribution



# BEC in rapidly cooled magnon gas



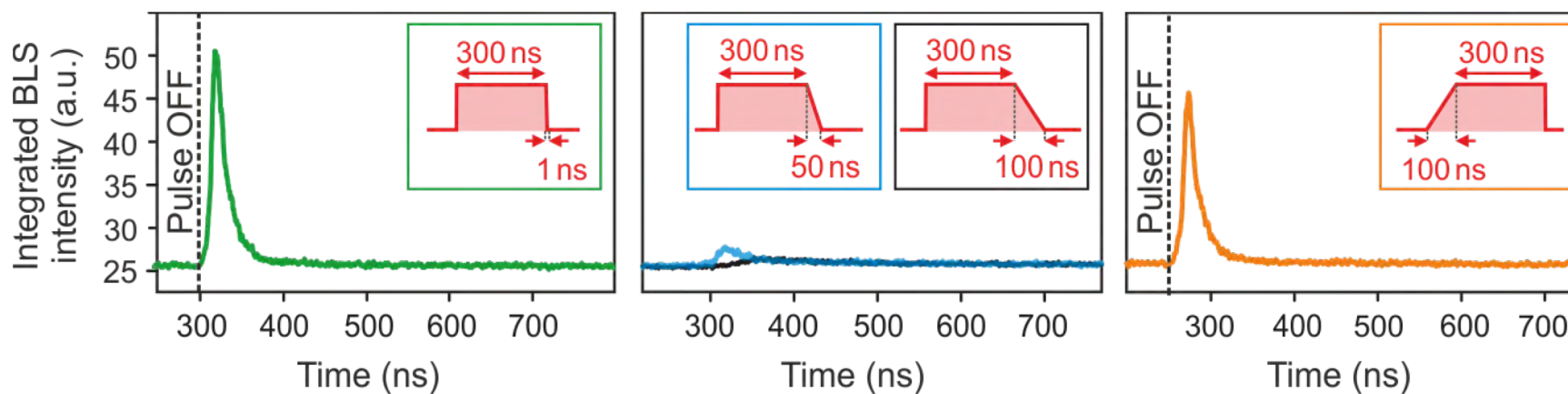
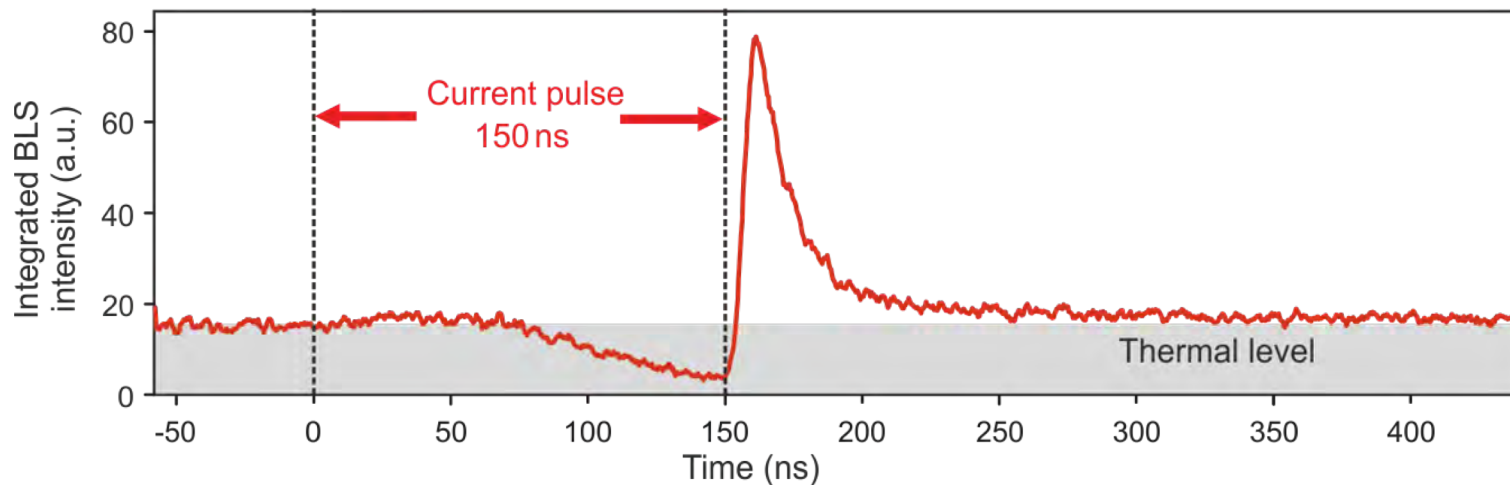
## Magnon population distribution



Integrated data

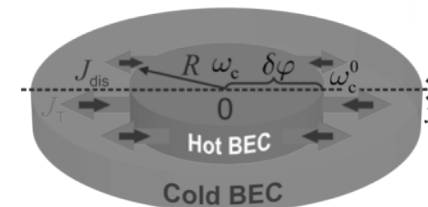
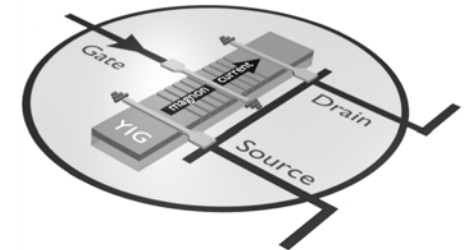
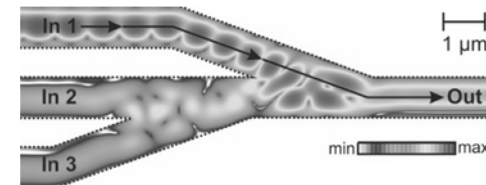


# Magnon BEC intensity and cooling rate



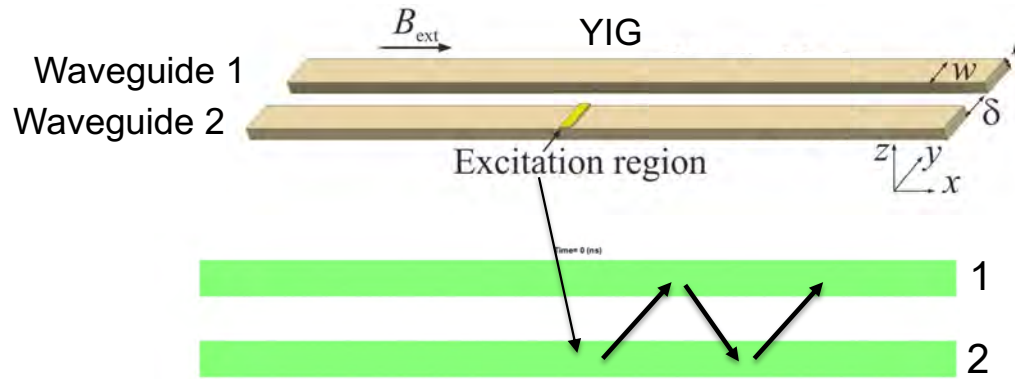
## Advanced magnonics

- I. Magnon interference logic
- II. Non-linear magnonics: Magnon transistor
- III. Magnonic macroscopic quantum state
- IV. Quantum-classical analogies in magnonics



## Coupled waveguides

Micromagnetic simulation:



Yttrium Iron Garnet (YIG):

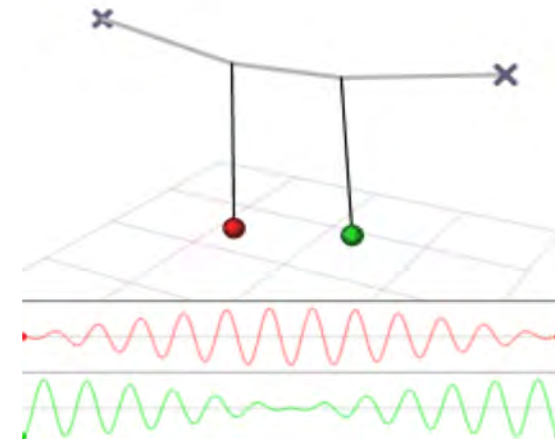
$$M_s = 1.4 \times 10^5 \text{ A/m} \quad t = 50 \text{ nm}$$

$$A = 3.5 \text{ pJ} \quad w = 100 \text{ nm}$$

$$\alpha = 2 \times 10^{-4} \quad \delta = 100 \text{ nm}$$



Coupled oscillators

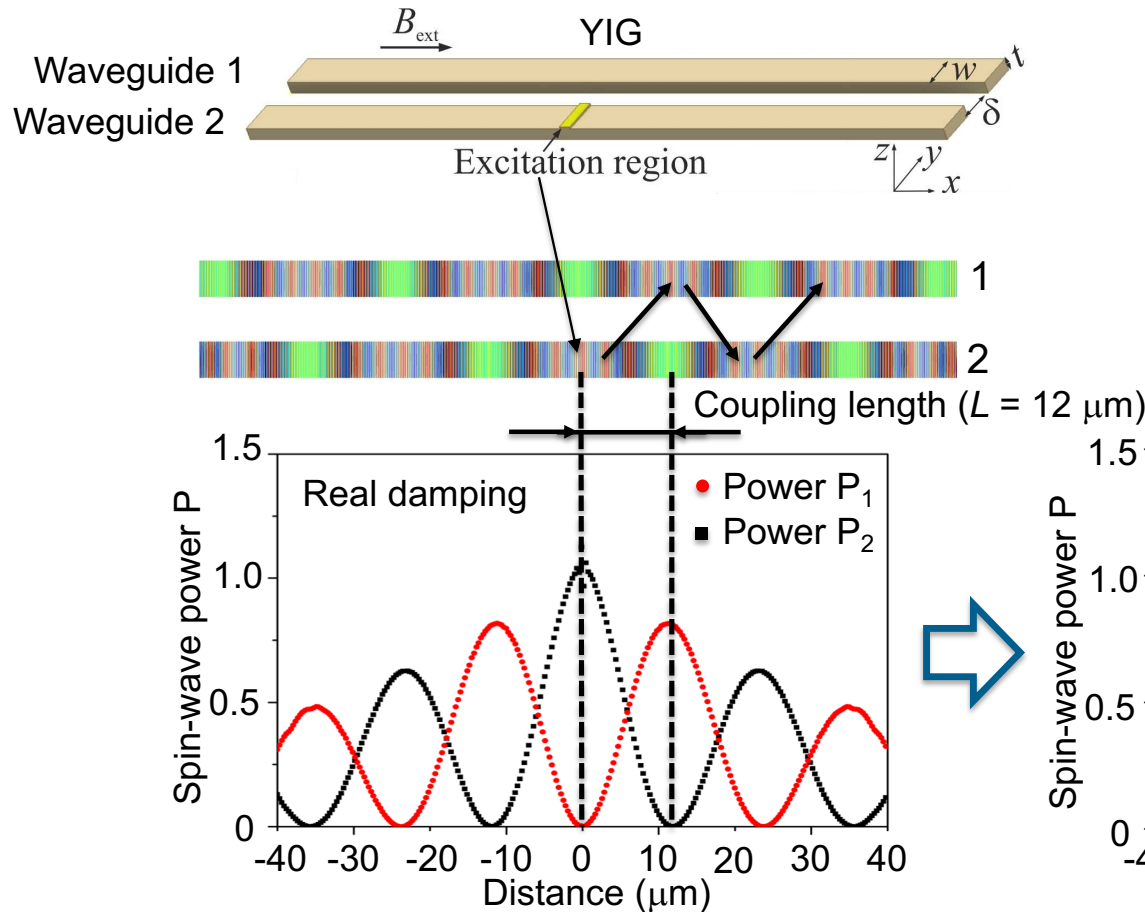


Simulation package Mumax3: A. Vansteenkiste, et al., *AIP Advance* **4**, 107113 (2014)

Source: Wikipedia

# Coupled waveguides

Micromagnetic simulation:

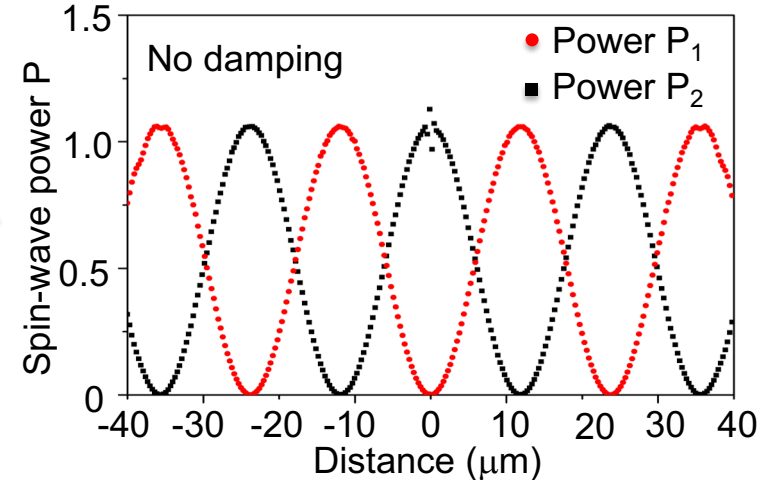


Yttrium Iron Garnet (YIG):

$$M_s = 1.4 \times 10^5 \text{ A/m} \quad t = 50 \text{ nm}$$

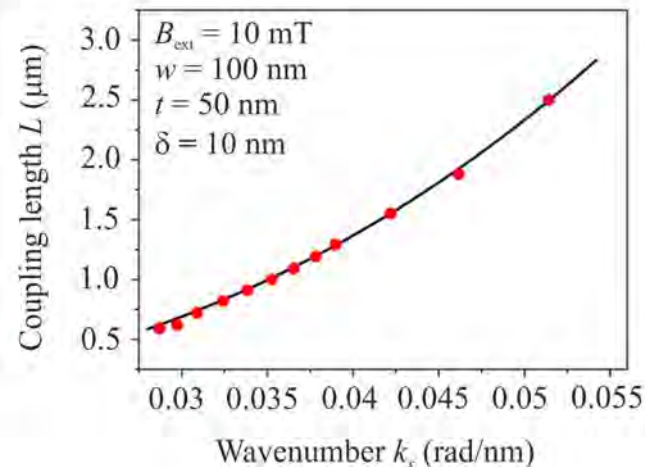
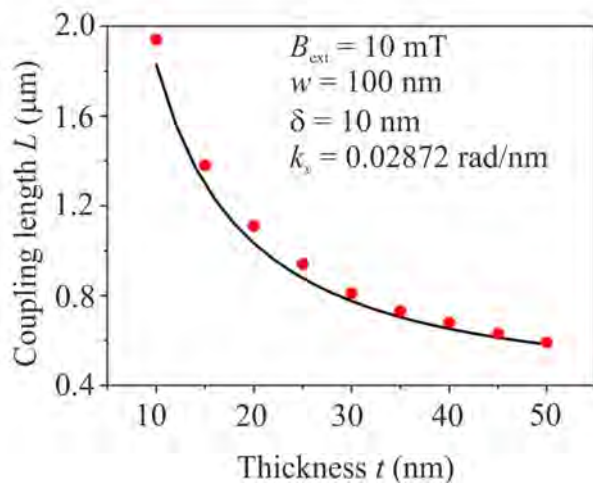
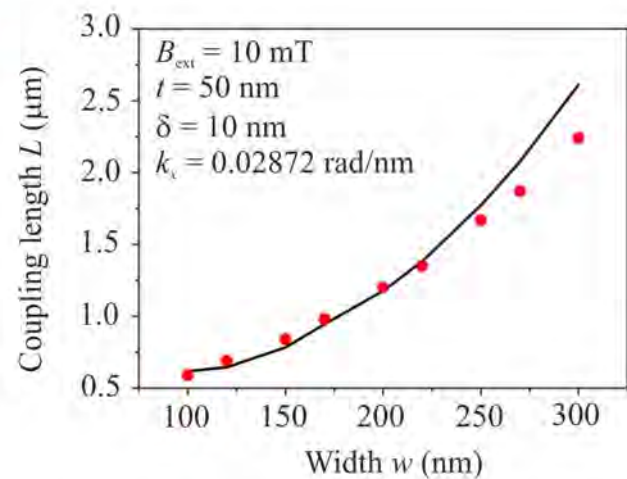
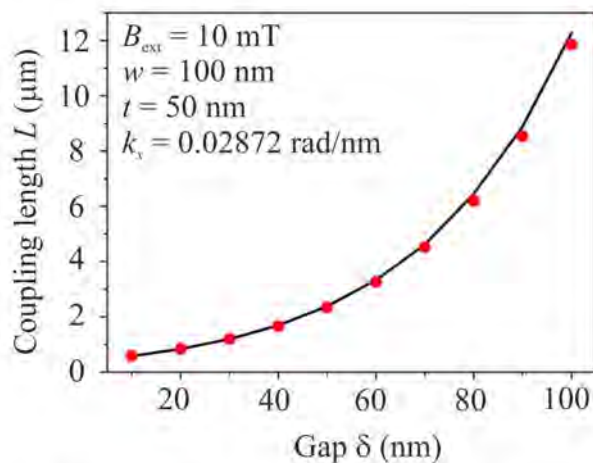
$$A = 3.5 \text{ pJ} \quad w = 100 \text{ nm}$$

$$\alpha = 2 \times 10^{-4} \quad \delta = 100 \text{ nm}$$

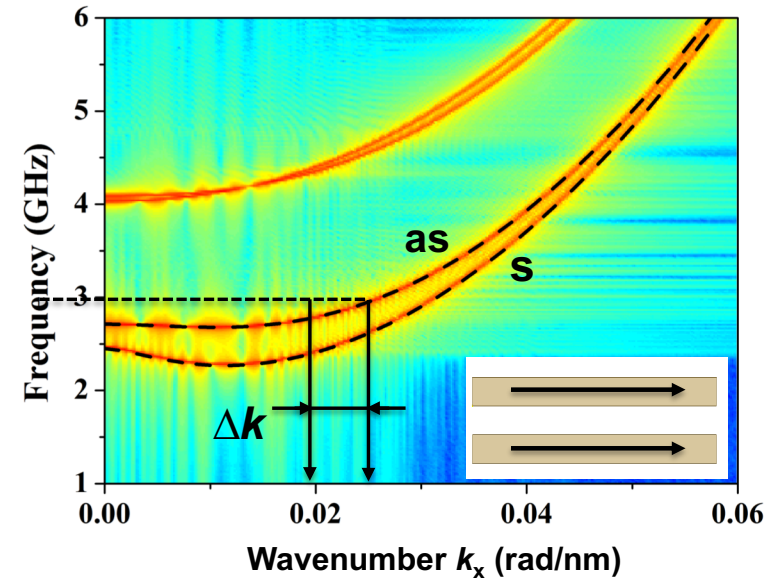
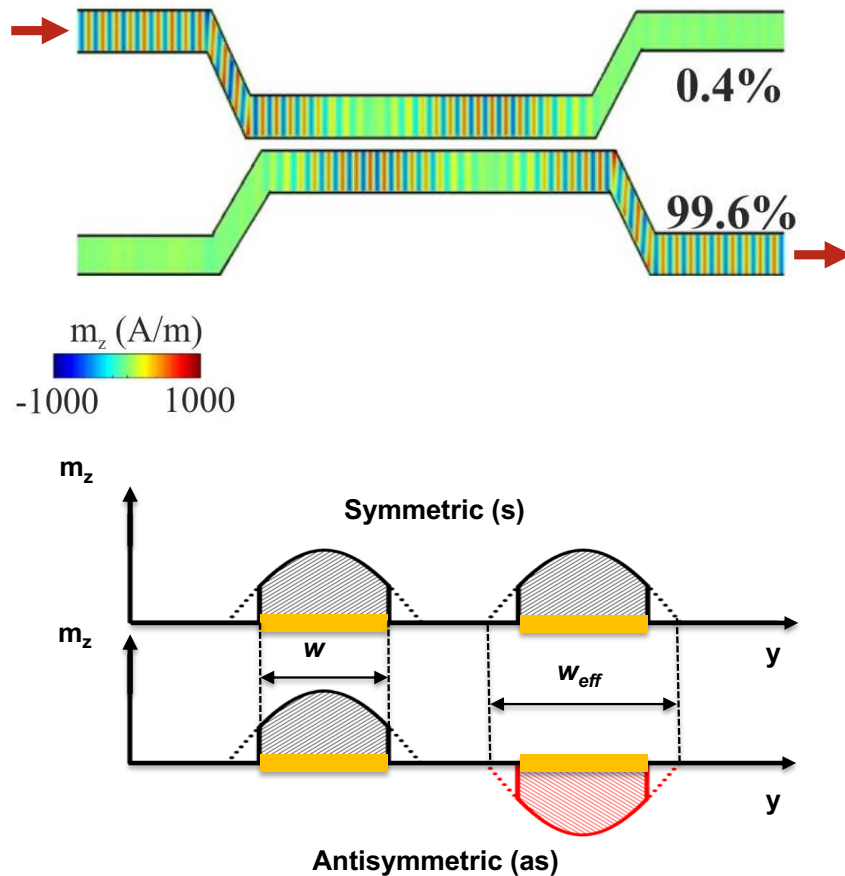


Simulation package Mumax3: A. Vansteenkiste, et al., *AIP Advance* **4**, 107113 (2014)

## Coupling length as a function of system parameters



# Magnon directional coupler

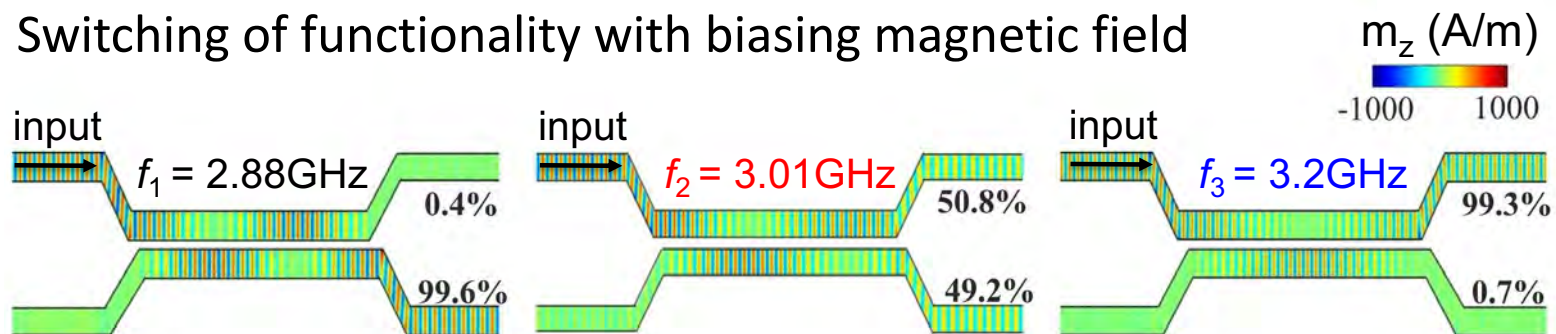


Coupling length: 
$$L = \frac{\pi}{\Delta k}$$

Coupling strength: 
$$\kappa = \frac{1}{L}$$

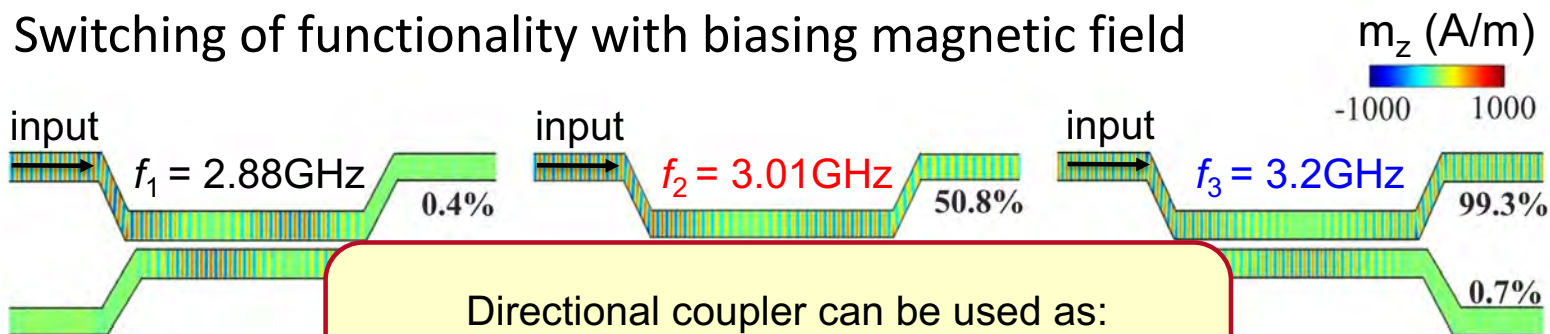
Q. Wang, et al., Sci. Adv. **4**, e1701517 (2018)

## Functionalities of directional coupler



Q. Wang, et al., Sci. Adv. **4**, e1701517 (2018)

# Functionalities of directional coupler

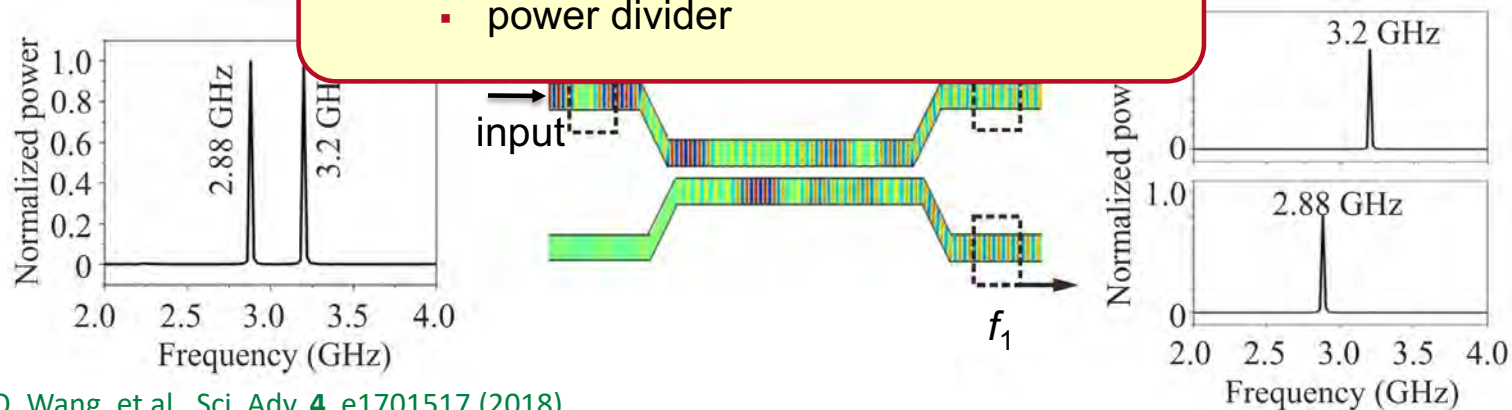


Directional coupler can be used as:

- interconnector
- multiplexer
- power divider

Utilization of t

ultiplexer



Q. Wang, et al., *Sci. Adv.* **4**, e1701517 (2018)



## Summary

- Magnon interference logic allows for data processing fully in the magnonic system
- Magnon transistor is an important nonlinear building block of magnonics for future wave-based technology
- Magnon Bose-Einstein condensate (BEC) with zero group velocity can be used for coherent spin transport
- First experimental evidence of creation of a magnon BEC by rapid cooling of a micro-scale YIG/Pt structure
- A magnonic quantum-classical analogy device has been demonstrated (magnonic STIRAP process)

