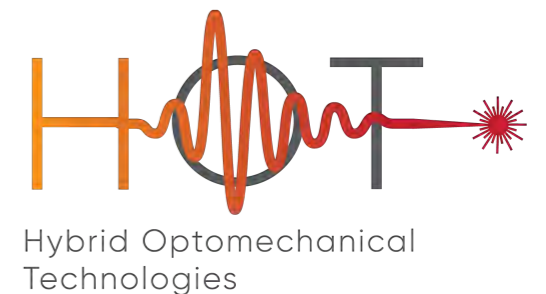
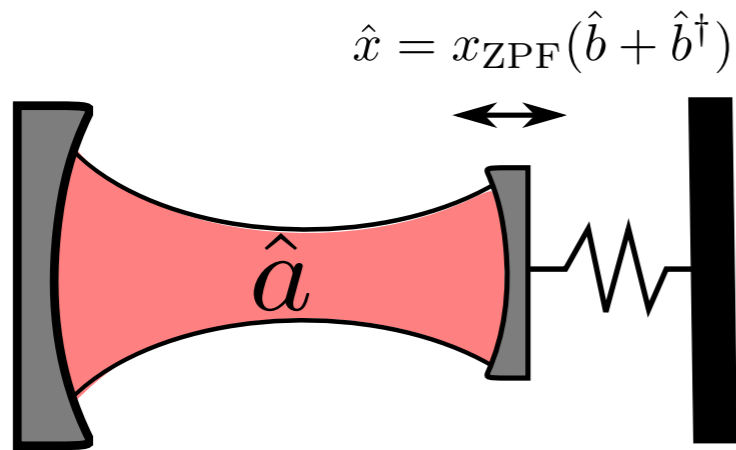


Cavity optomechanics: platform for nonreciprocity and synchronization

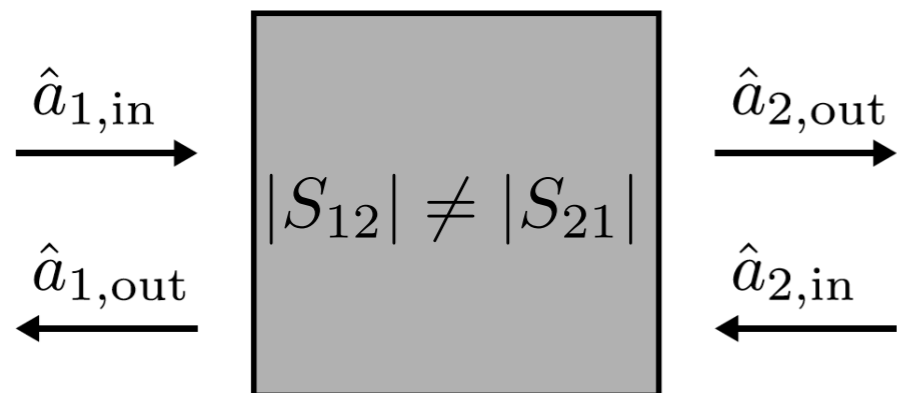
Andreas Nunnenkamp
Cavendish Laboratory



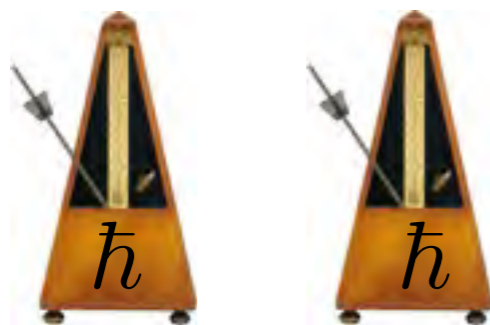
Outline



Cavity optomechanics



Nonreciprocal devices



Quantum synchronization

Cavity optomechanics

$$\hat{H} = \hbar\omega_C(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b}$$

photons

phonons

For small displacements

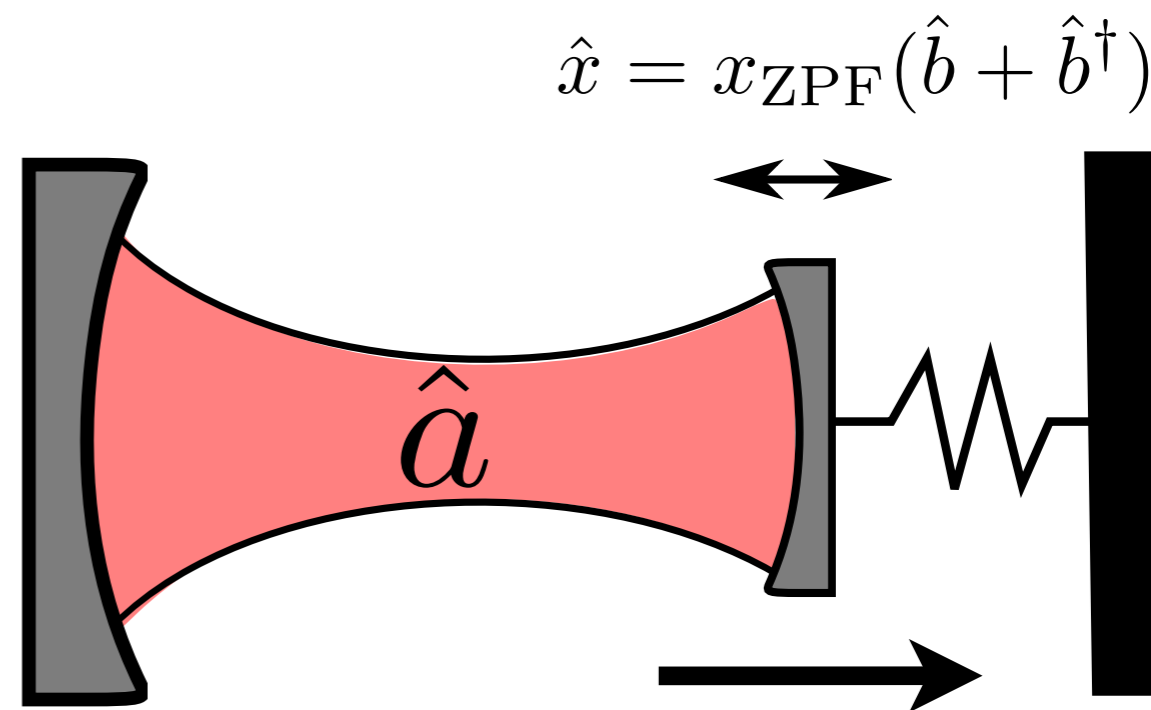
$$\omega_C(\hat{x}) = \omega_R + \frac{\partial\omega_C}{\partial x}\hat{x} + \dots$$

Two parametrically coupled oscillators

$$\hat{H}_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)$$

AN, Børkje, and Girvin, PRL 2011

First analysis of single-photon coupling



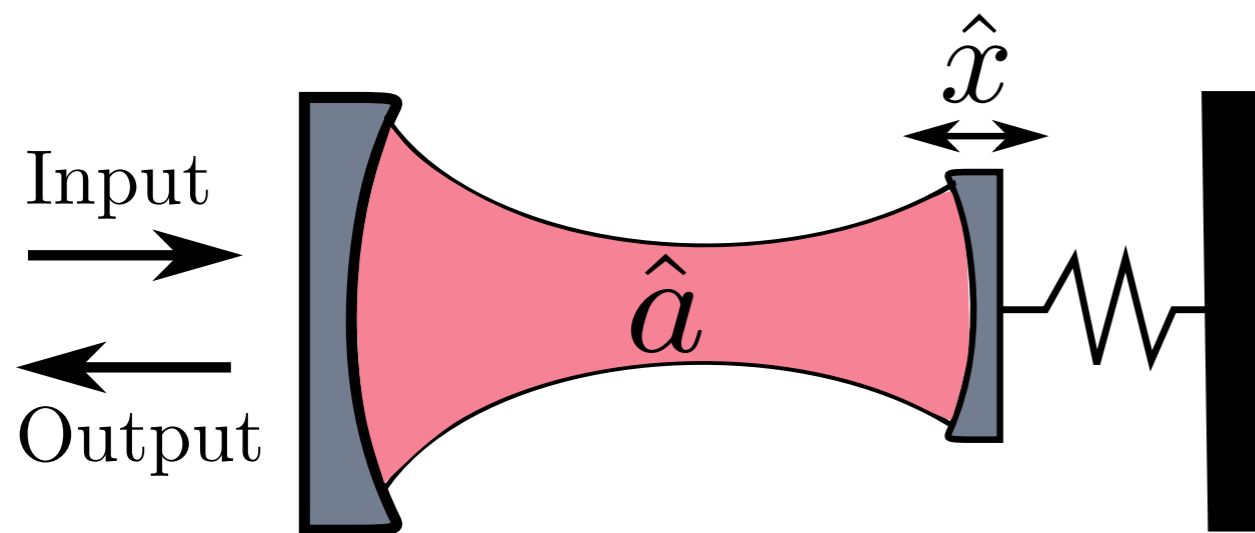
radiation-pressure force

$$\hat{H}_{\text{int}} = -\hat{F}\hat{x} \quad \hat{F} = \frac{\hbar\omega_R}{L}\hat{a}^\dagger\hat{a}$$

Børkje, AN, and Girvin, PRA 2010

Signature of radiation pressure shot noise

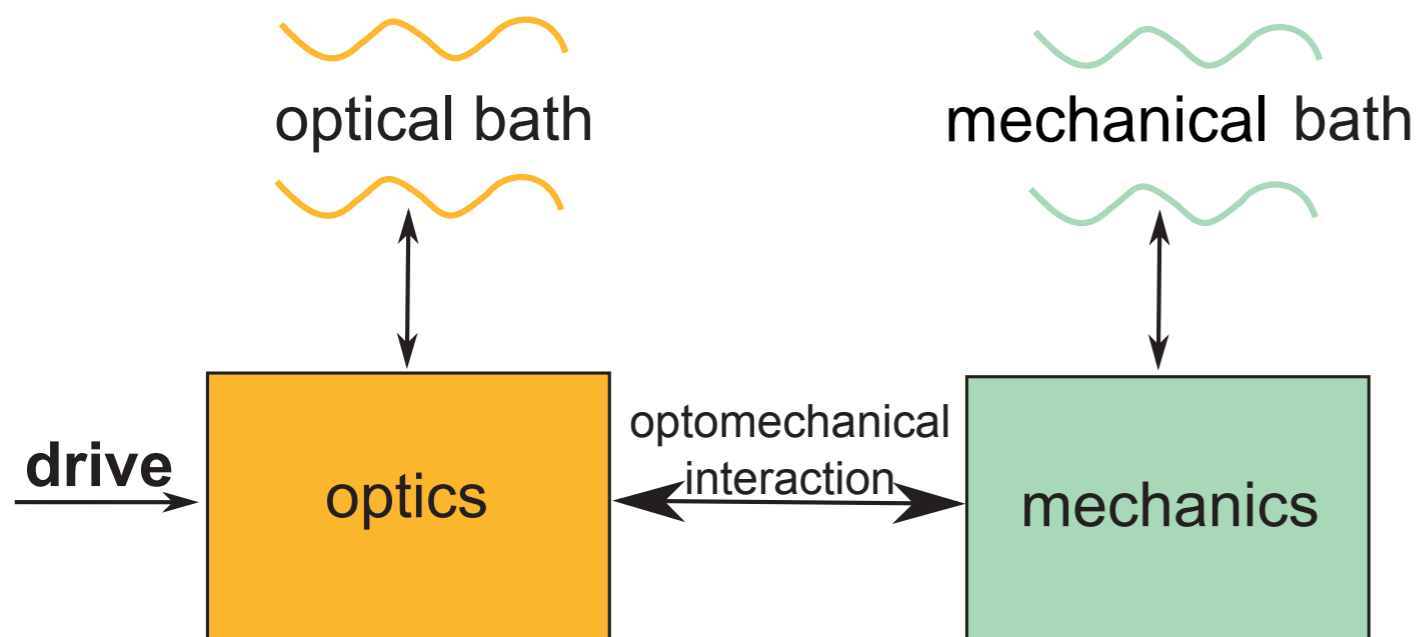
Cavity optomechanics



- open quantum system
- coherence and dissipation
- far from equilibrium

+

$$\hat{H}_{\text{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)$$



rich physics: steady states,
limit cycles, classical chaos

Cavity optomechanics

$$\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{a} = \alpha + \delta \hat{a}$$

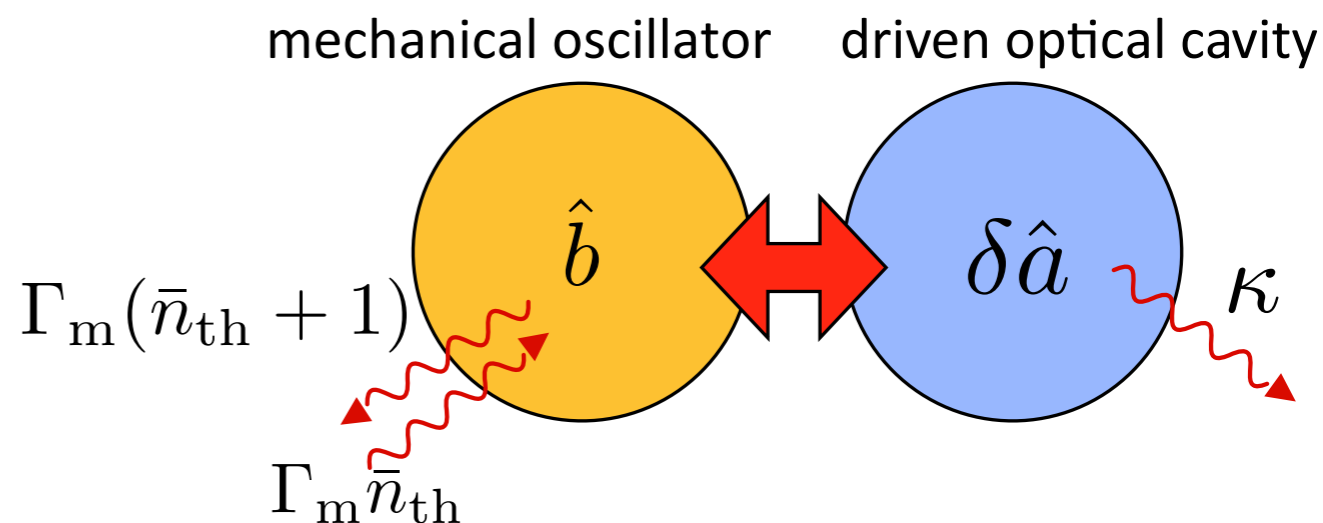
$$\hbar g_0 (\alpha \delta \hat{a}^\dagger + \alpha^* \delta \hat{a}) (\hat{b} + \hat{b}^\dagger)$$

$$g = g_0 \alpha$$

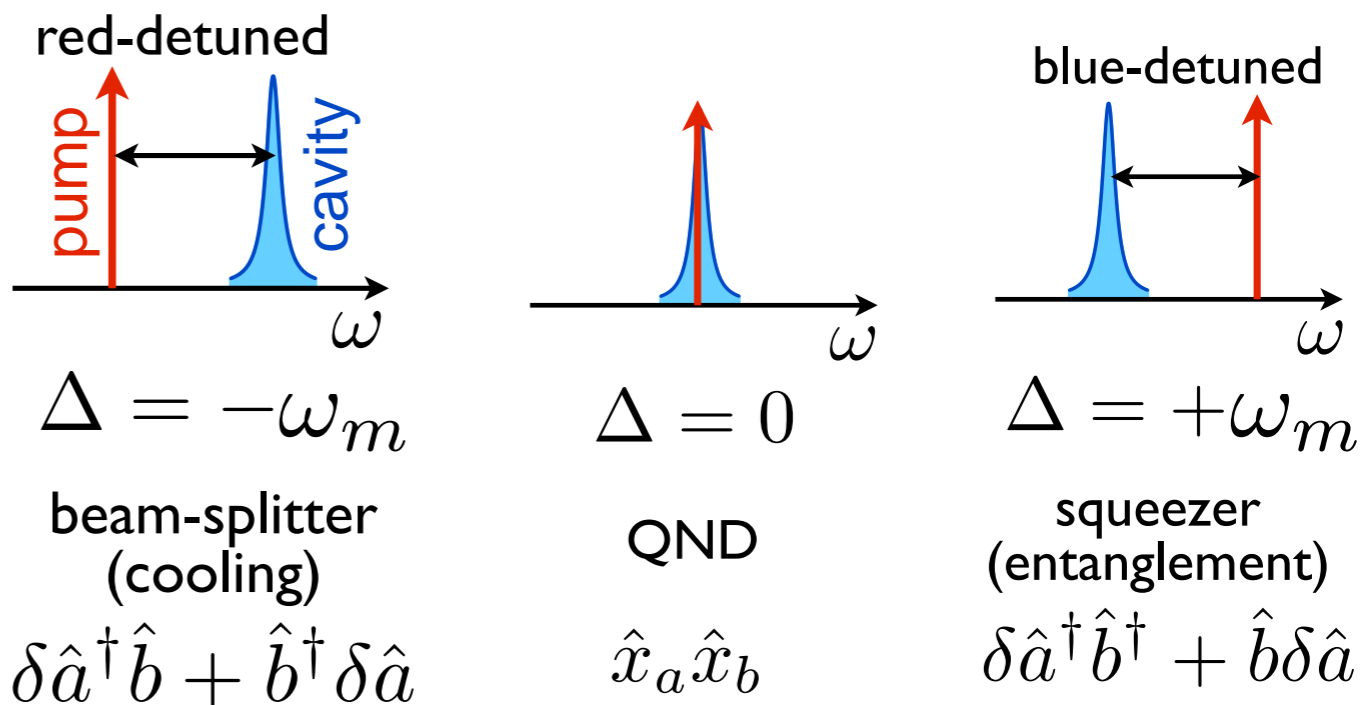
enhanced, reconfigurable
coupling with tunable phase

Input-output scattering matrix

$$\hat{a}_{\text{out}} = S(\omega) \hat{a}_{\text{in}}$$

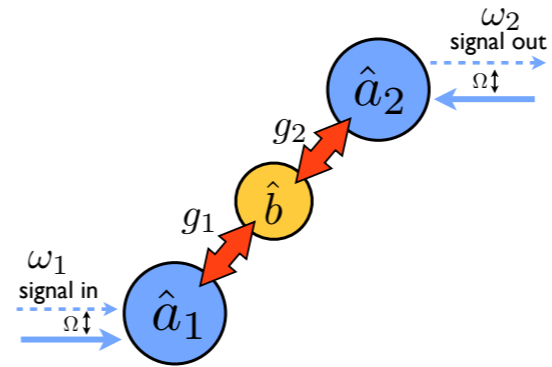


Quantum optomechanics toolbox

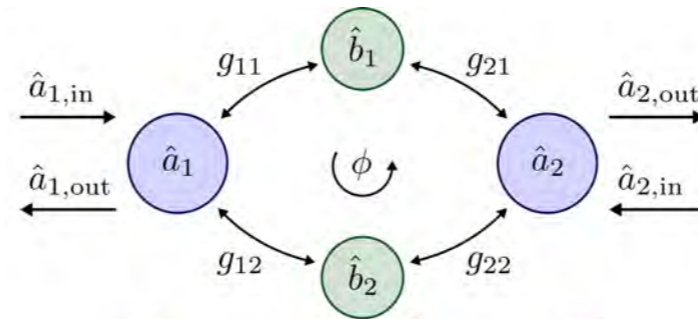


Hybrid optomechanical technologies 'HOT'

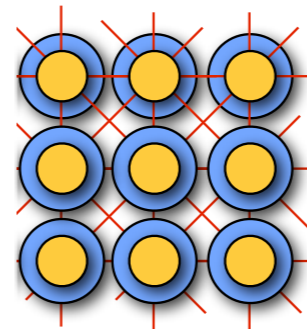
Quantum-limited transducer
(microwave to optical link)



Nonreciprocal devices
(isolator, circulator, amplifier)



Mechanical "metamaterials"
Topological light and sound



Hybrid Optomechanical
Technologies

€10Mio, 4 years,
17 partners

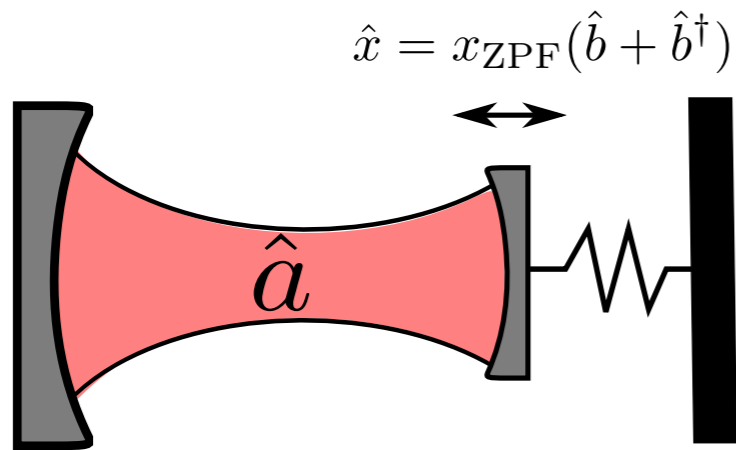


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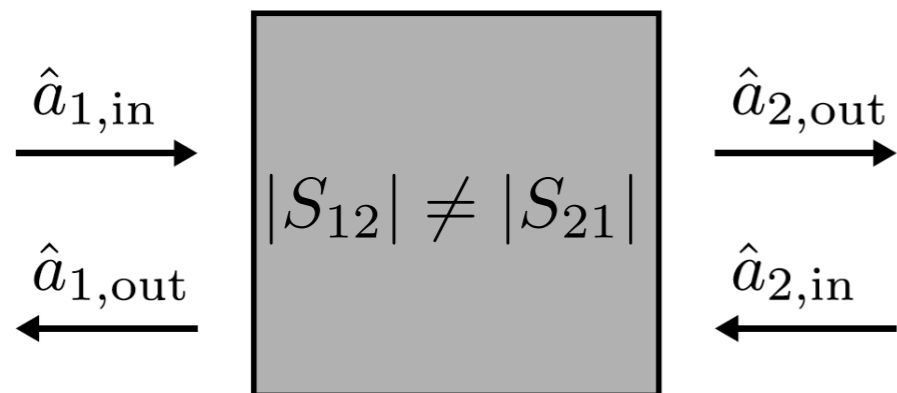


HITACHI
Inspire the Next

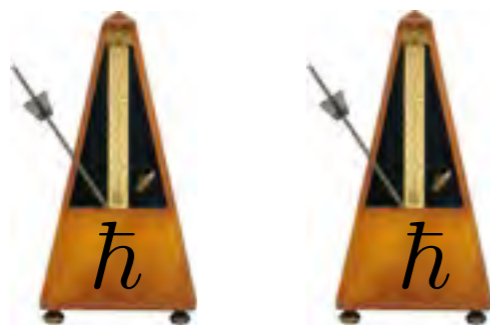
Outline



Cavity optomechanics

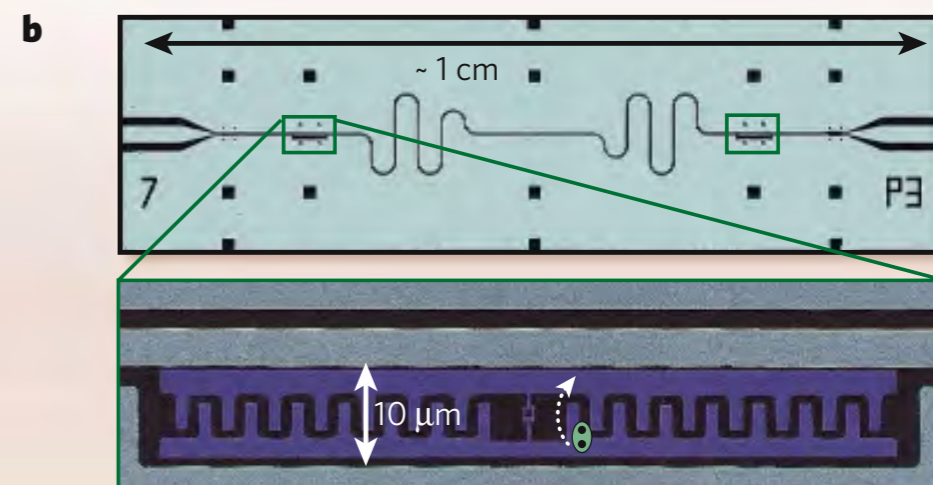
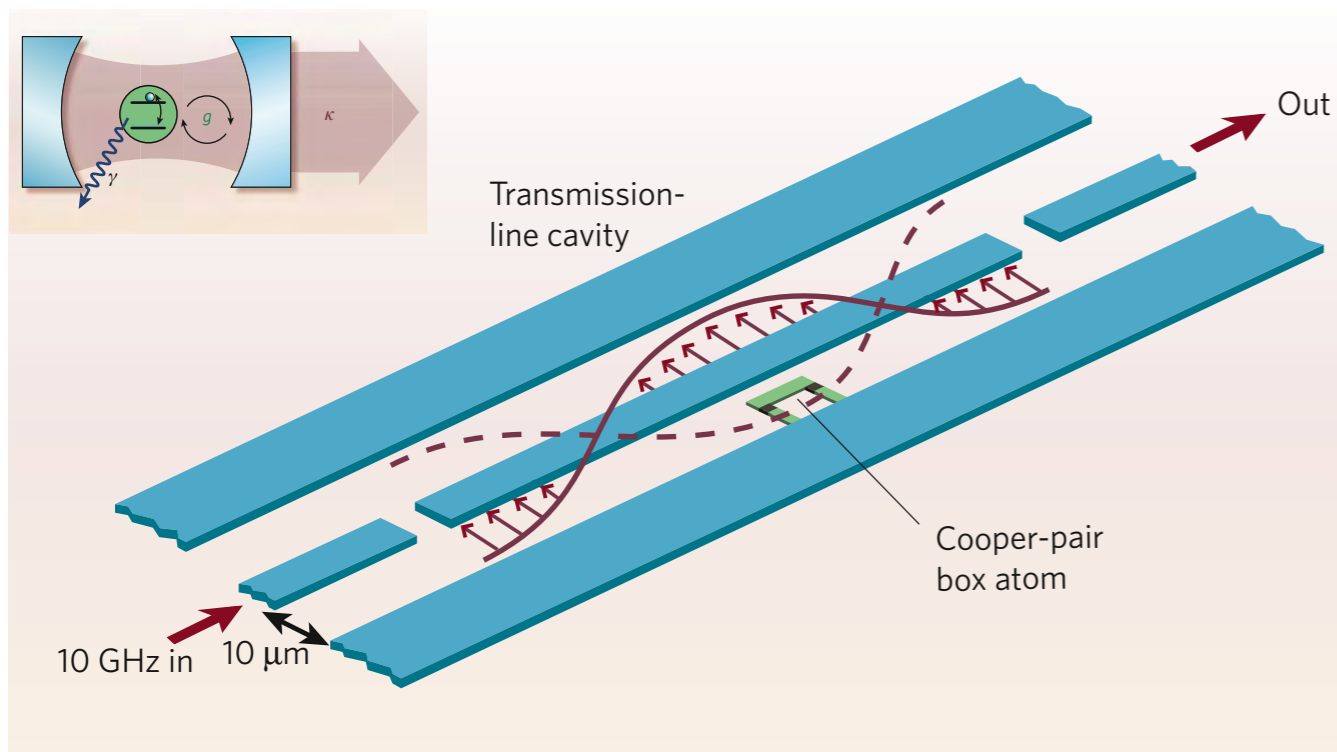


Nonreciprocal devices



Quantum synchronization

Nonreciprocal devices by reservoir engineering



Schoelkopf and Girvin, Nature **451**, 664 (2008)

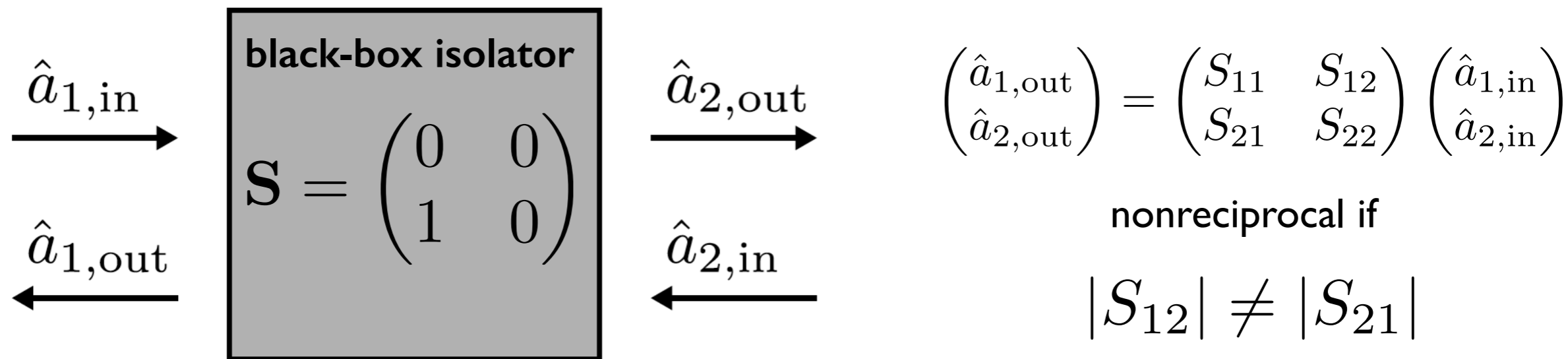


Magneto-optical effect (YIG) breaks Lorentz reciprocity.

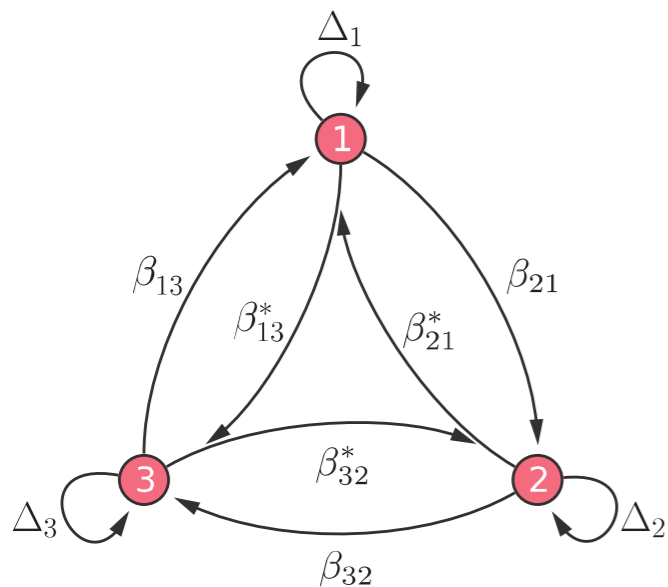
But: bulky, lossy, large magnetic fields
 → searching for on-chip, reconfigurable, magnetic-field-free, nonreciprocal device

Jalas *et al.*, Nature Photonics **7**, 579 (2013)
 Verhagen and Alu, Nature Physics **13**, 922 (2017)

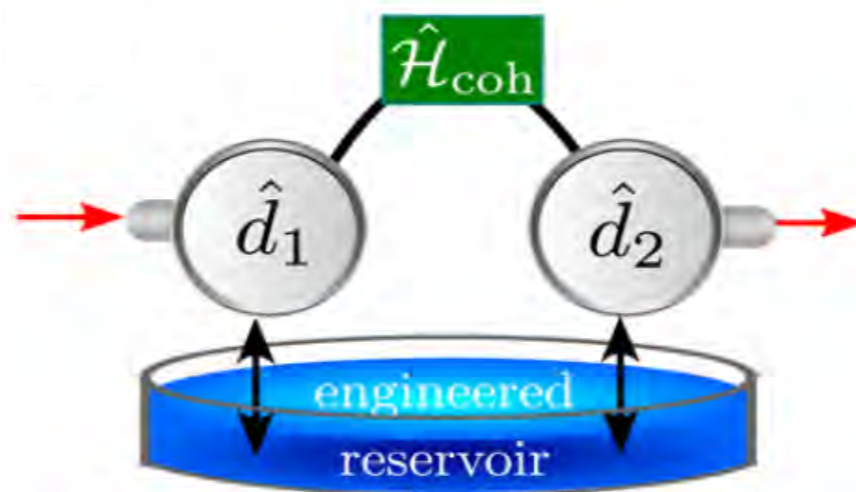
Nonreciprocal devices by reservoir engineering



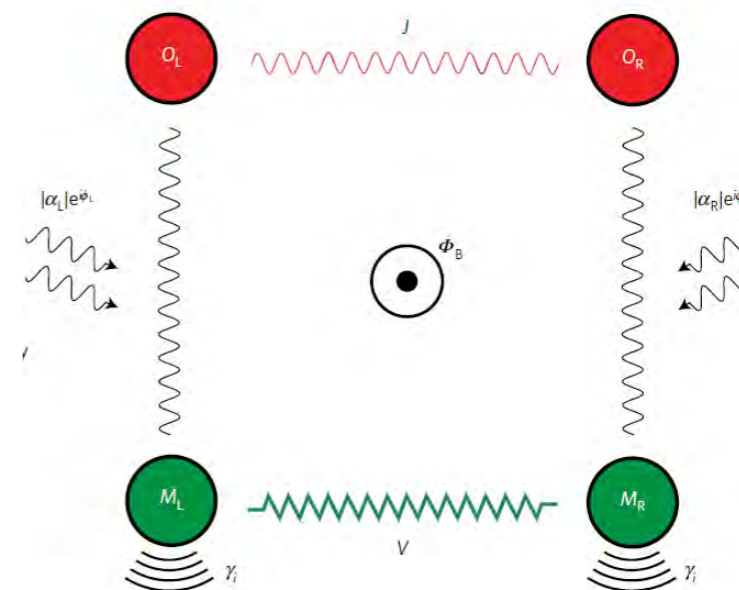
Some of the recent work on non-reciprocity using reservoir engineering



Ranzani and Aumentado, NJP 17, 023024 (2015)

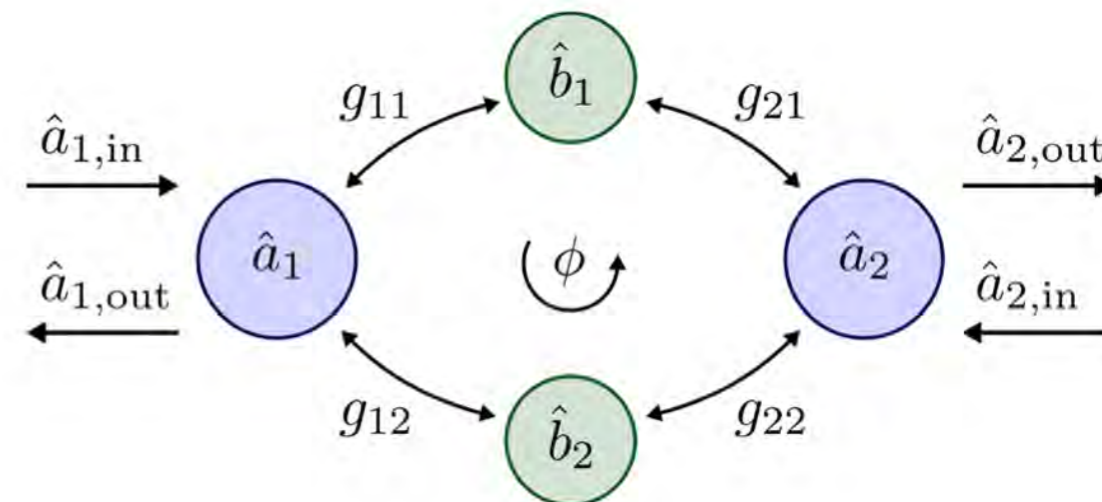
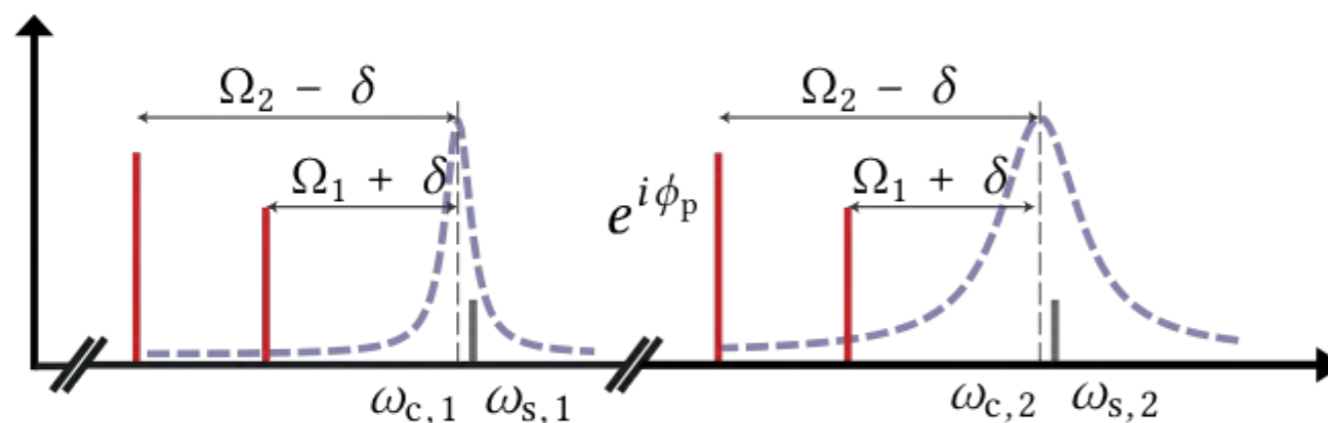


Metelmann and Clerk, PRX 5, 021025 (2015)



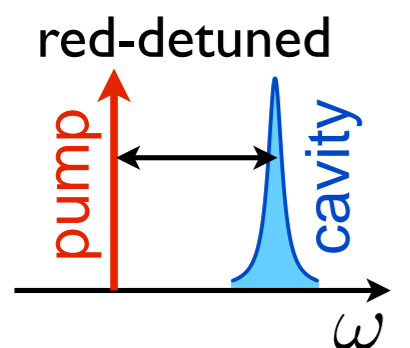
Fang, ..., Painter Nat. Phys. 13, 465 (2017)

Nonreciprocal devices by reservoir engineering



two cavities, **two** mechanics, **four** coherent drives

optomechanical plaquette
with gauge-invariant phase



$$\Delta = -\omega_m$$

beam-splitter
(cooling)

$$\hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}$$

$$H = -\delta \hat{b}_1^\dagger \hat{b}_1 + \delta \hat{b}_2^\dagger \hat{b}_2 + g_{11} (\hat{a}_1 \hat{b}_1^\dagger + \hat{a}_1^\dagger \hat{b}_1) + g_{21} (\hat{a}_2 \hat{b}_1^\dagger + \hat{a}_2^\dagger \hat{b}_1) + g_{12} (\hat{a}_1 \hat{b}_2^\dagger + \hat{a}_1^\dagger \hat{b}_2) + g_{22} (e^{i\phi} \hat{a}_2 \hat{b}_2^\dagger + e^{-i\phi} \hat{a}_2^\dagger \hat{b}_2)$$

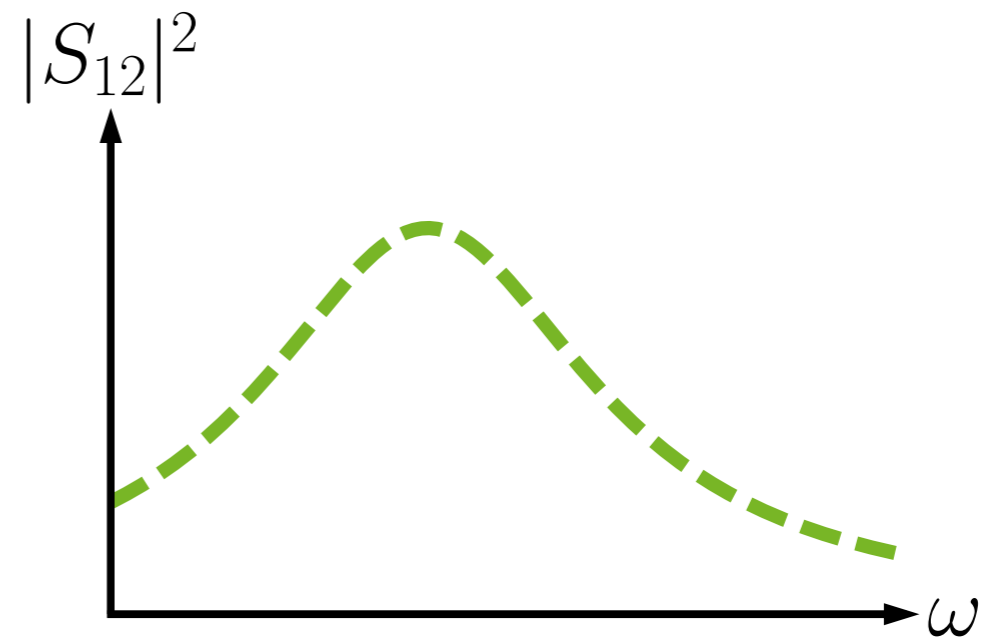
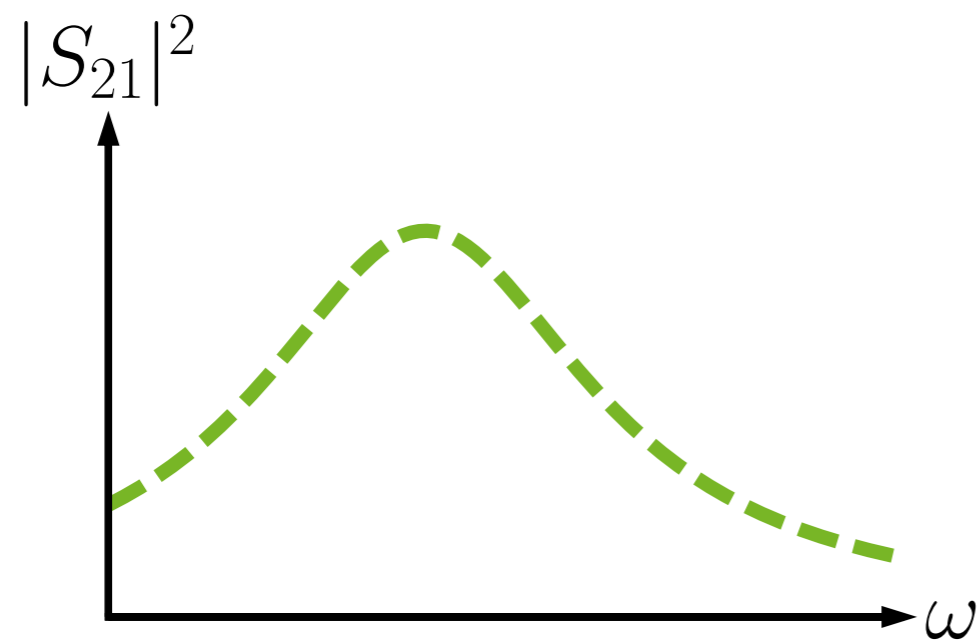
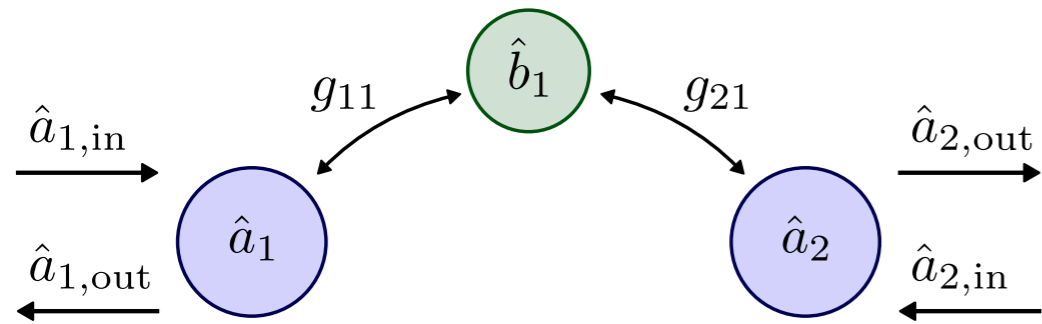
Artificial magnetic field for photons and phonons in a
“synthetic dimension” by drives (broken time reversal)

Bernier et al., Nat. Commun. 8, 604 (2017)

also: Peterson et al., Phys. Rev. X 7, 031001 (2017)

Nonreciprocal devices by reservoir engineering

single path: reciprocal frequency conversion

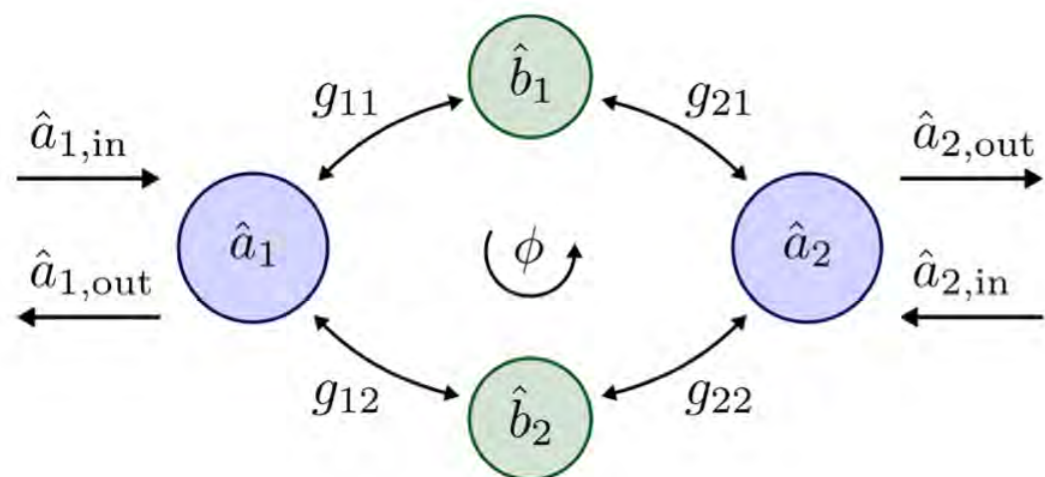


Andrews *et al.*, Nature Physics **10**, 321 (2014)

Lecocq *et al.*, PRL **116**, 043601 (2016)

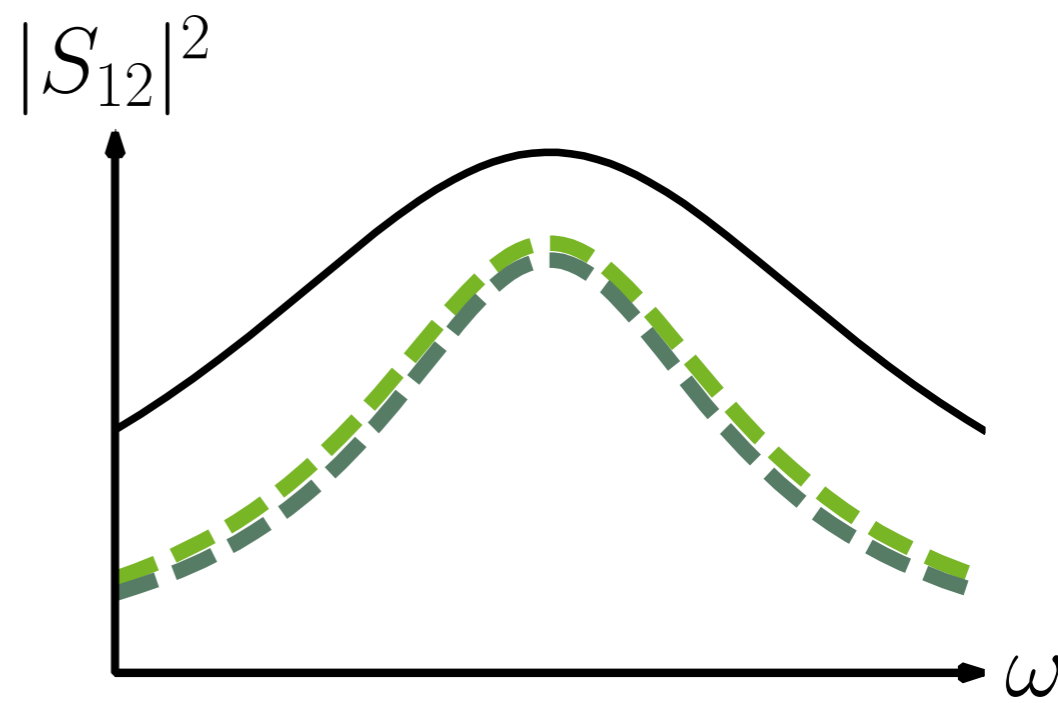
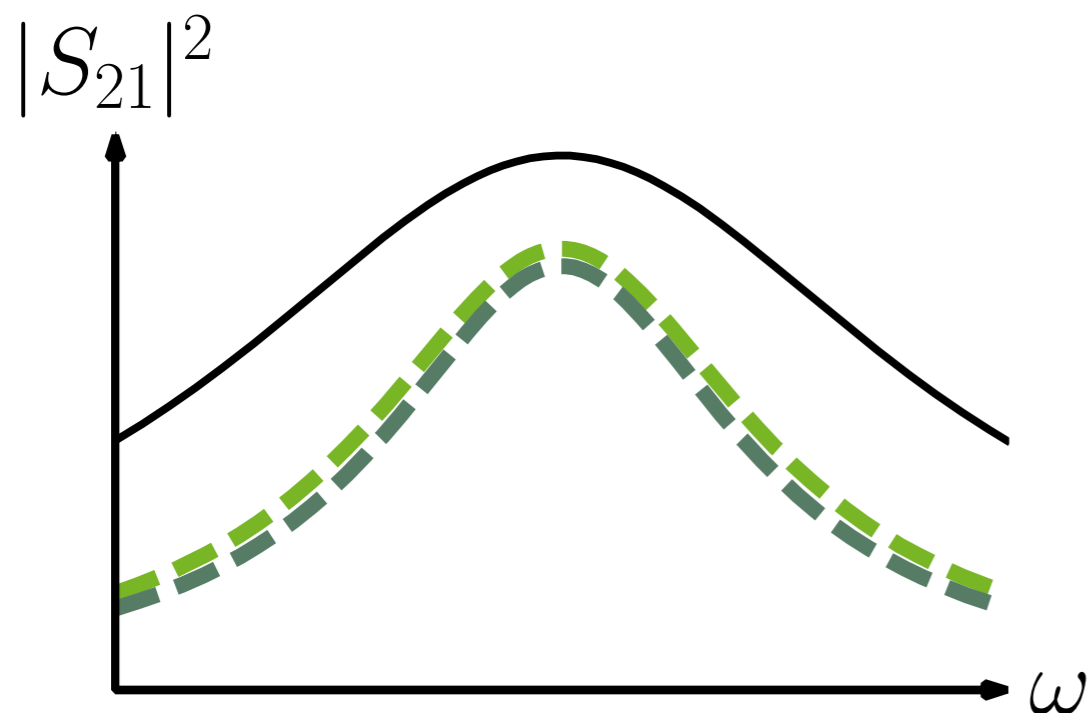
Nonreciprocal devices by reservoir engineering

two symmetric paths : still reciprocal frequency conversion



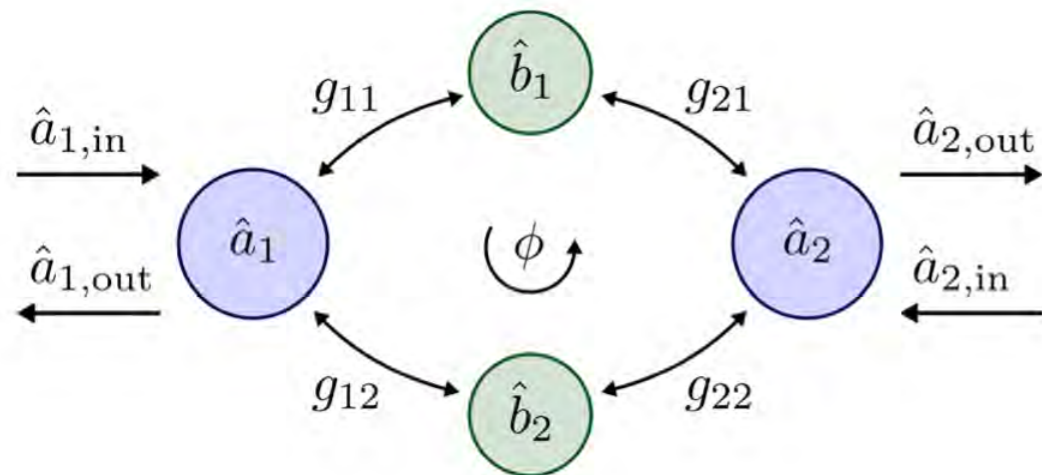
Interference of two coupling amplitudes

$$\frac{S_{12}(\omega)}{S_{21}(\omega)} = \frac{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{+i\phi}}{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{-i\phi}}$$



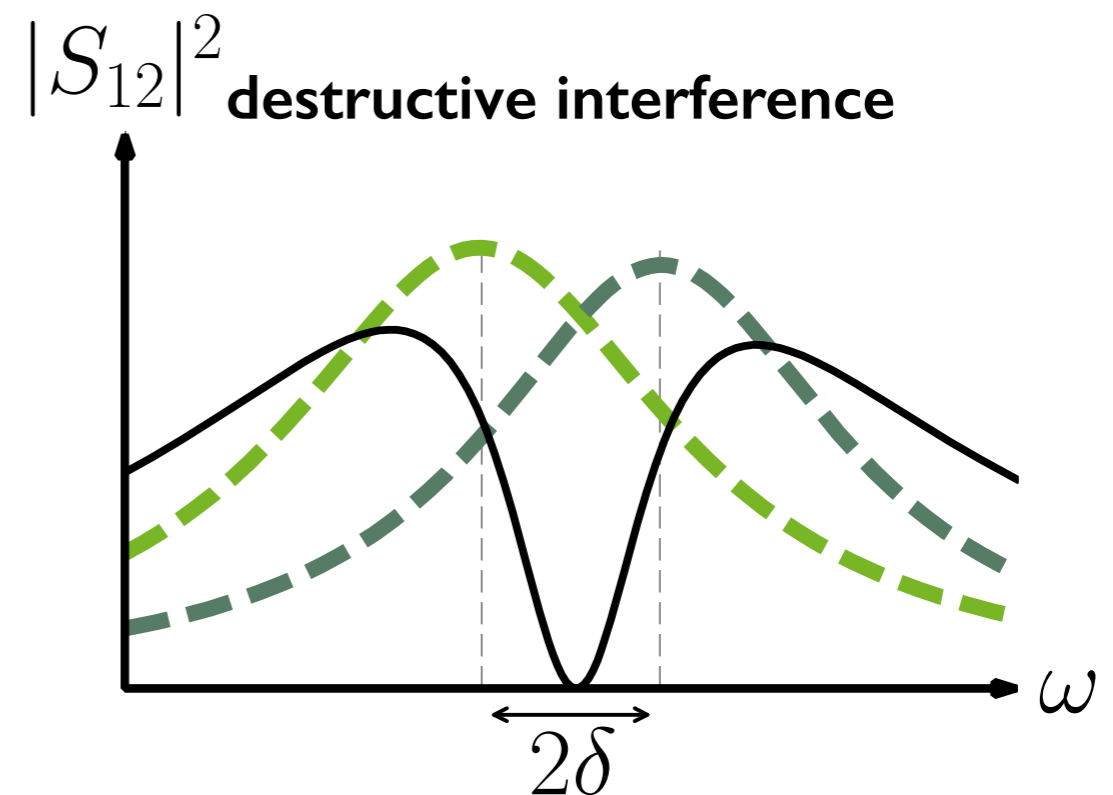
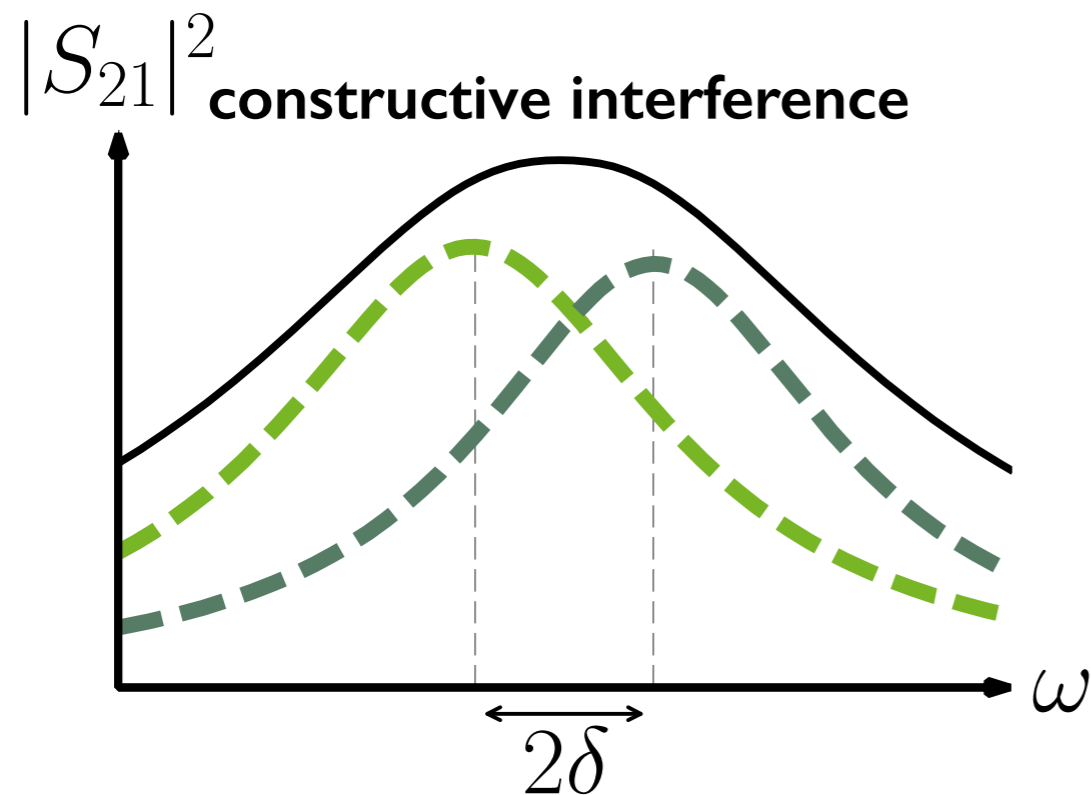
Nonreciprocal devices by reservoir engineering

two **a**symmetric paths: **non**-reciprocal frequency conversion



Interference of two coupling amplitudes
(broken time reversal, dissipation, asymmetry)

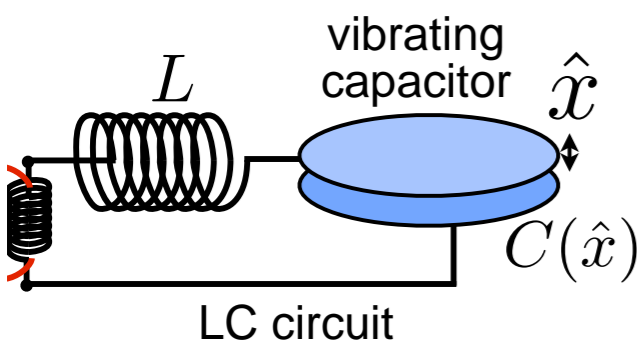
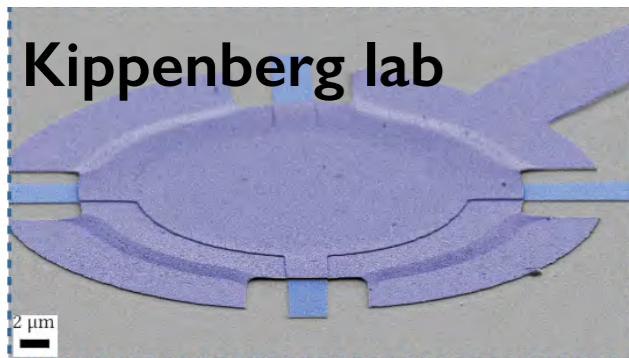
$$\frac{S_{12}(\omega)}{S_{21}(\omega)} = \frac{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{+i\phi}}{g_{11}\chi_1(\omega)g_{21} + g_{12}\chi_2(\omega)g_{22}e^{-i\phi}}$$



Nonreciprocal devices by reservoir engineering



ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE



$$\hat{H} = \hbar\omega_C(\hat{x})\hat{a}^\dagger\hat{a}$$

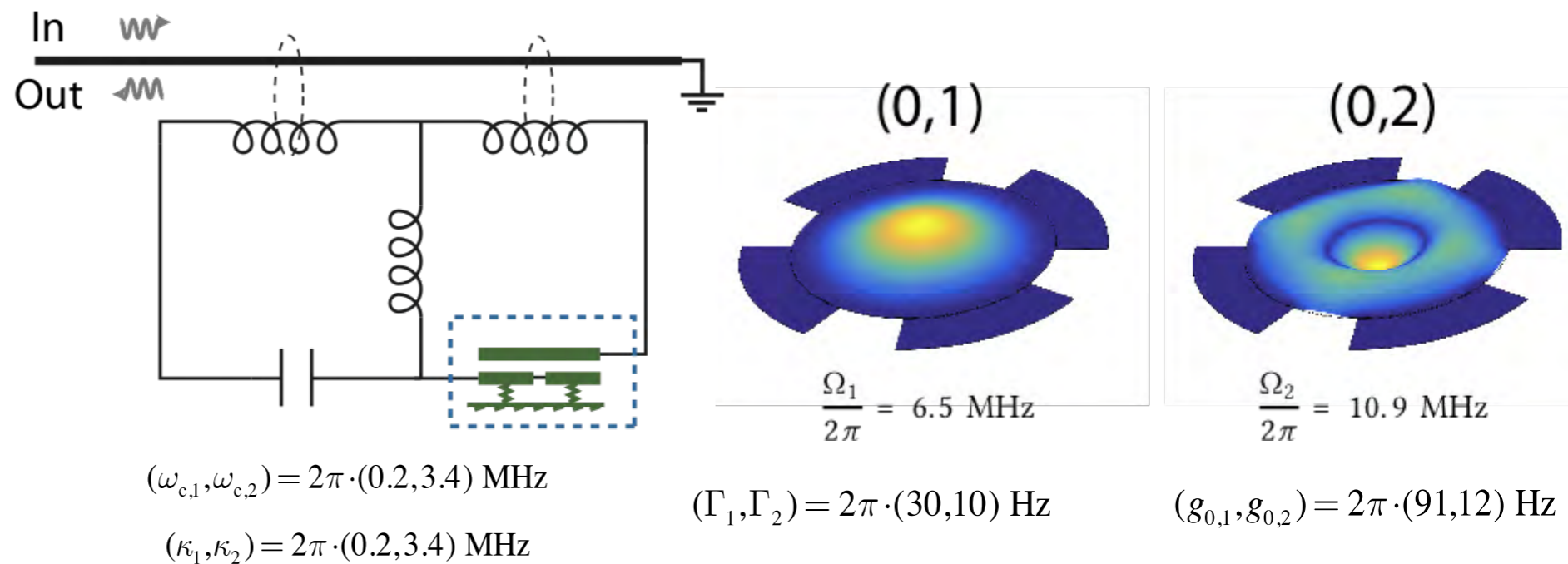
nature physics

ARTICLES

PUBLISHED ONLINE: 15 MAY 2017 | DOI: 10.1038/NPHYS4121

A dissipative quantum reservoir for microwave light using a mechanical oscillator

L. D. Tóth^{1†}, N. R. Bernier^{1†}, A. Nunnenkamp², A. K. Feofanov^{1*} and T. J. Kippenberg^{1*}

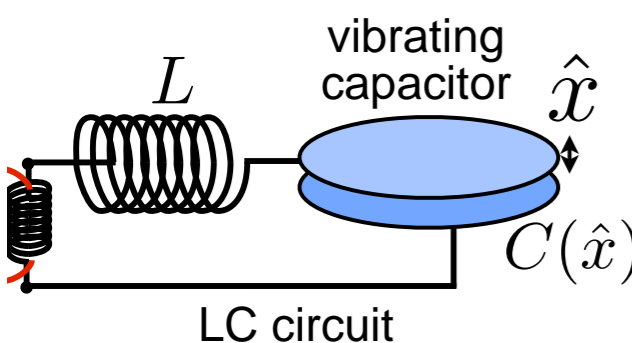
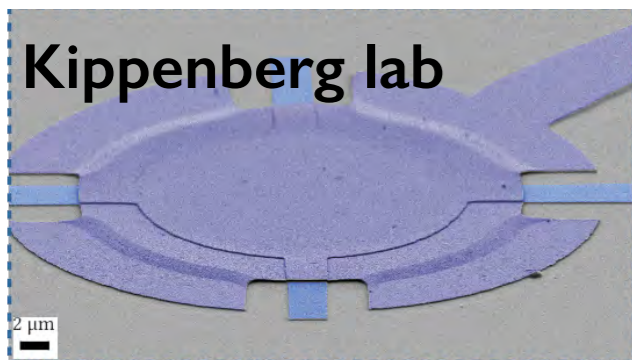


Bernier... Malz, AN..., Nat. Commun. 8, 604 (2017)

Nonreciprocal devices by reservoir engineering

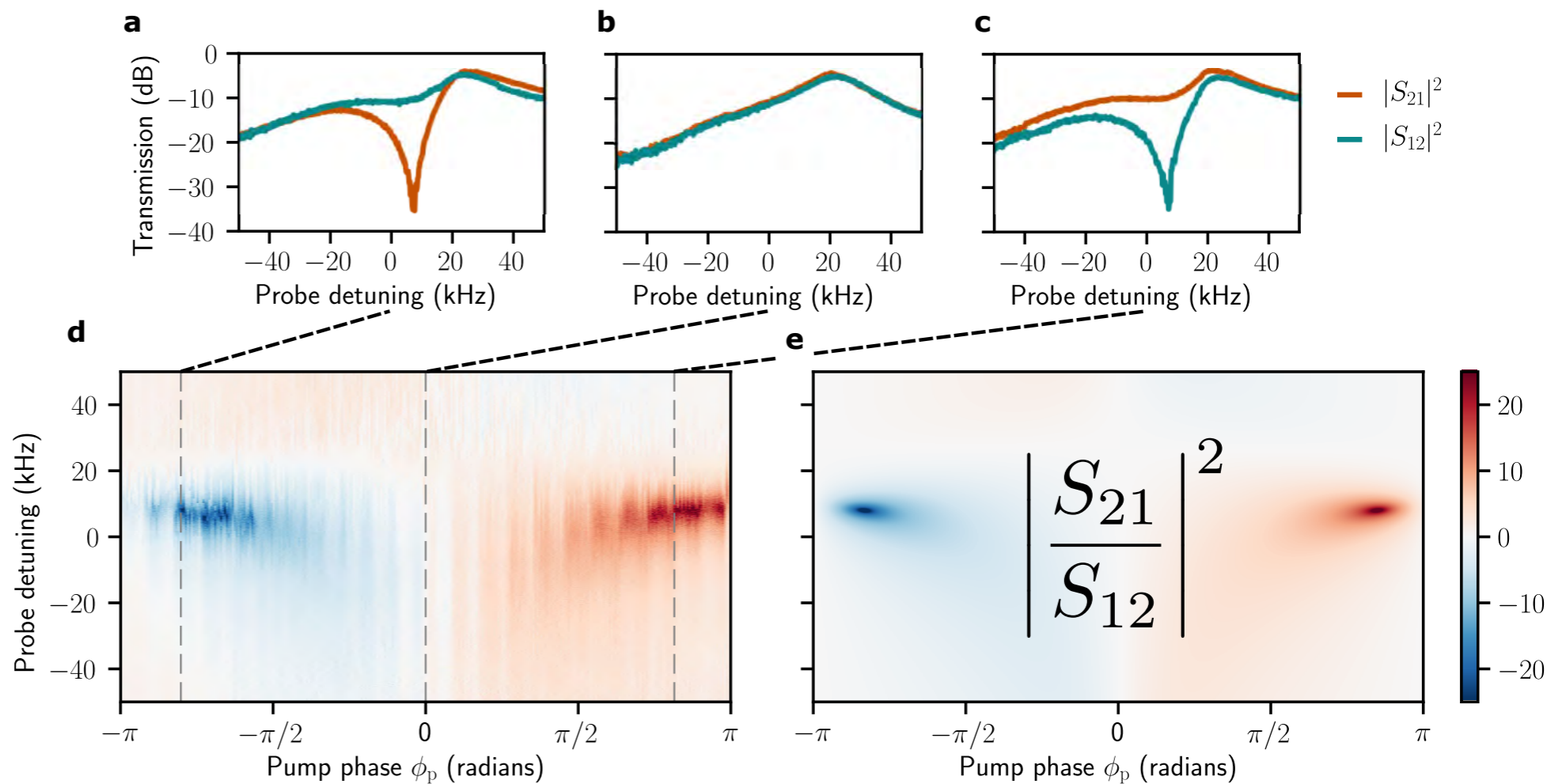


ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE



$$\hat{H} = \hbar\omega_C(\hat{x})\hat{a}^\dagger\hat{a}$$

low insertion loss,
low added noise



**Nonreciprocal (20dB), reconfigurable,
on-chip electro-mechanical circuit!**

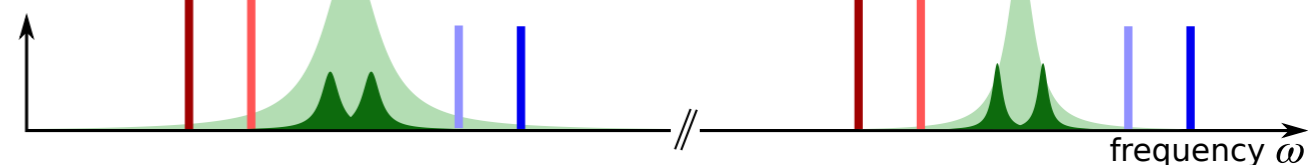
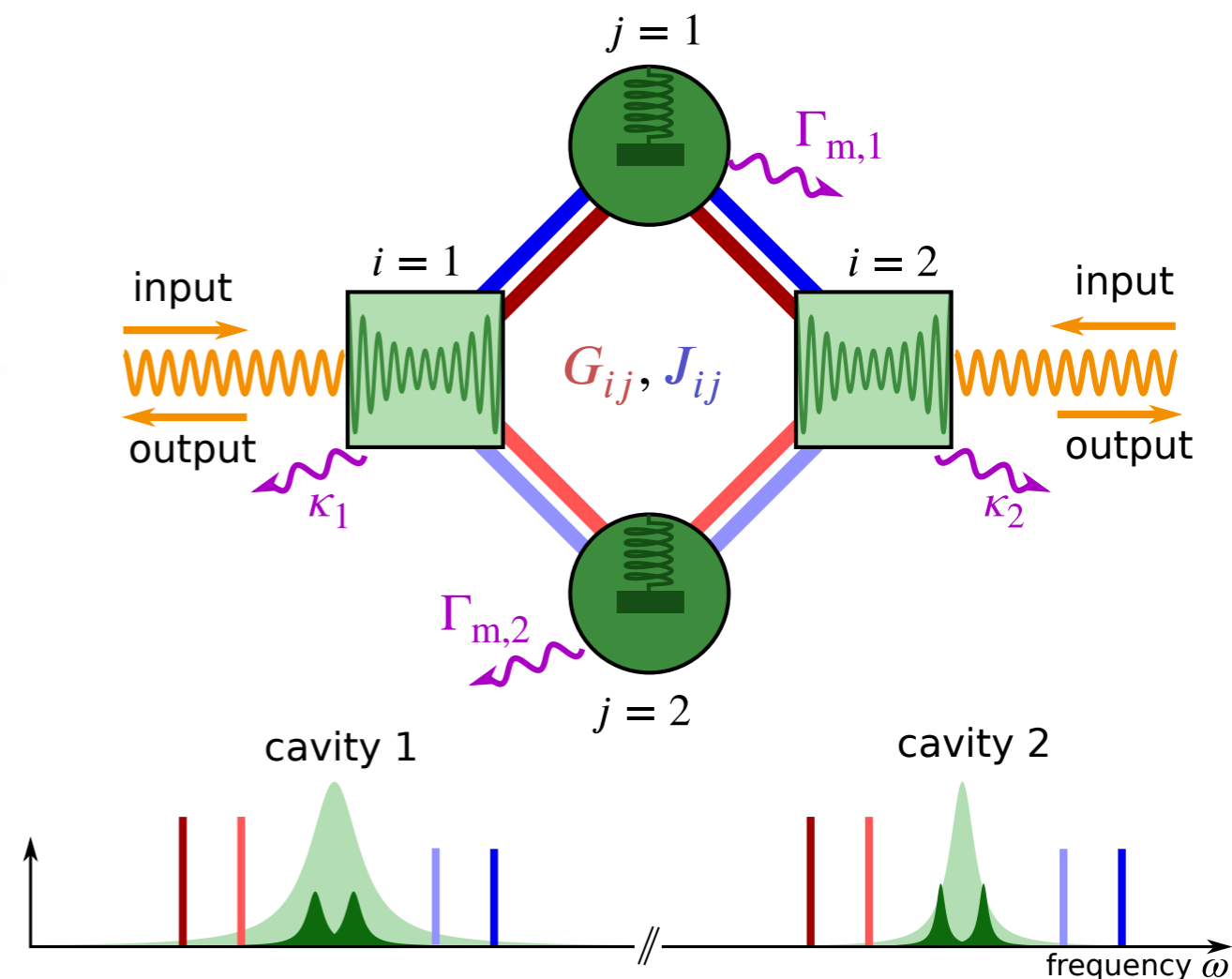
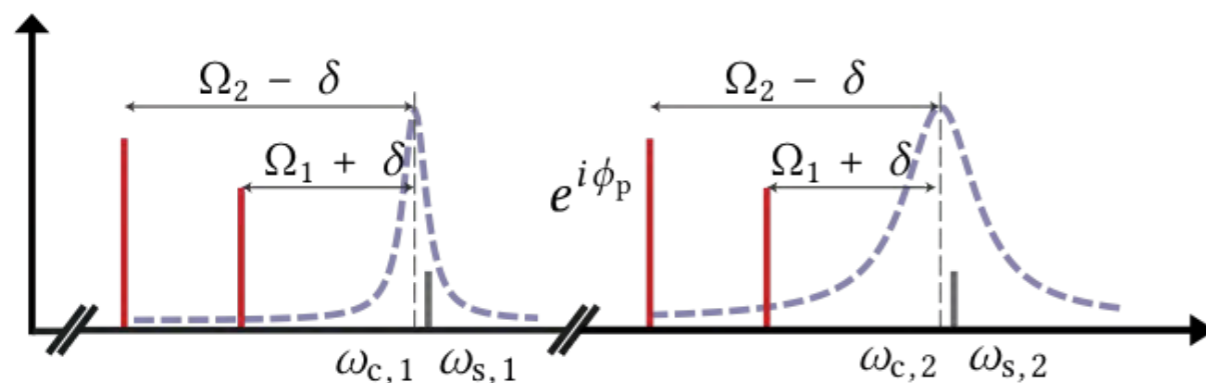
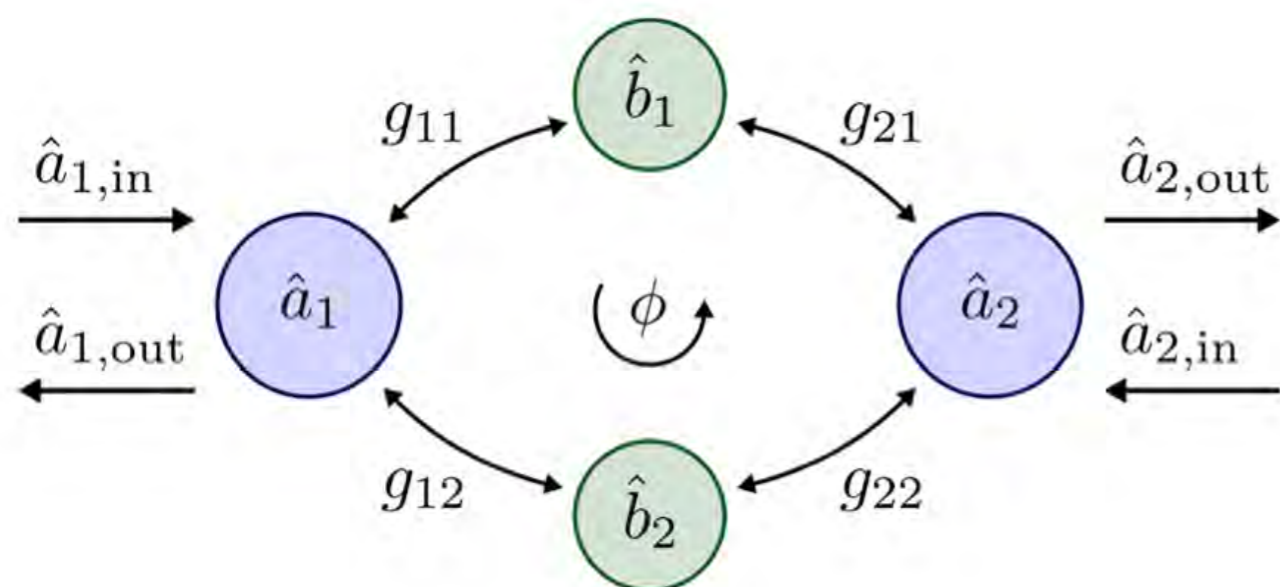
Quantum thermodynamics

Barzanjeh, ..., Xuereb,

PRL 120, 060601 (2018)

Bernier... Malz, AN..., Nat. Commun. 8, 604 (2017)

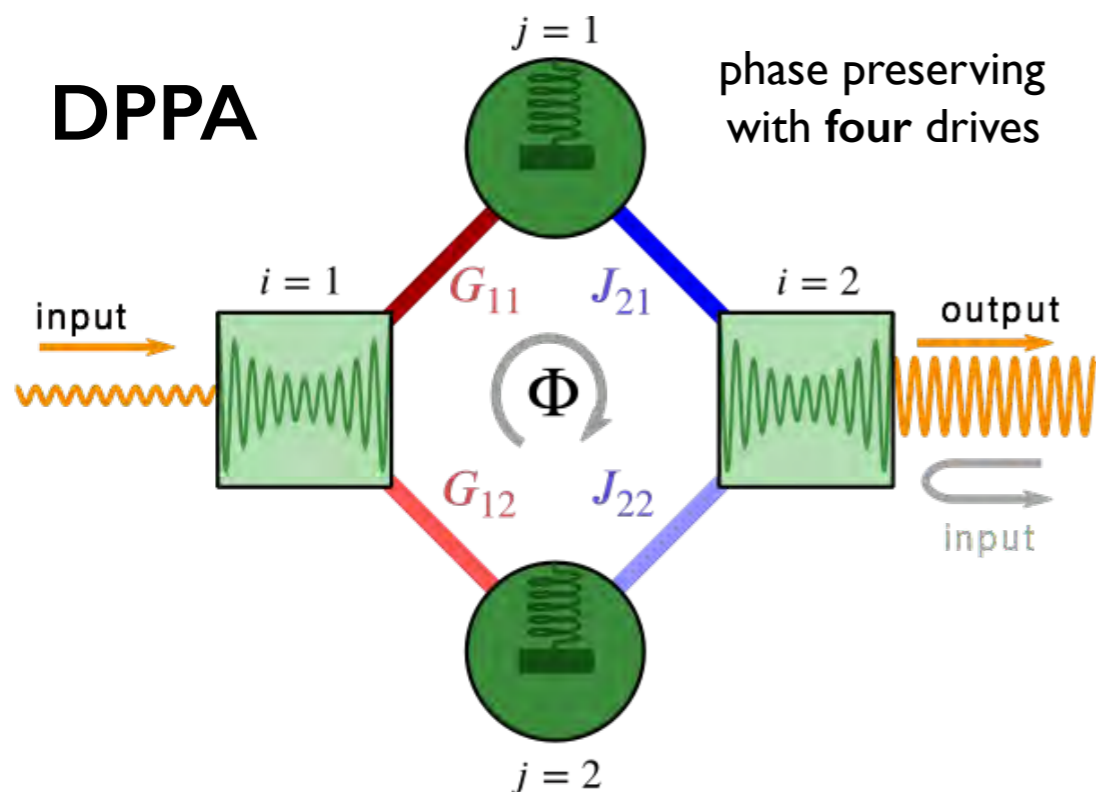
Nonreciprocal devices by reservoir engineering



Bernier et al., Nat. Commun. **8**, 604 (2017)
 Peterson et al., Phys. Rev. X **7**, 031001 (2017)
 Barzanjeh et al., Nat. Commun. **8**, 953 (2017)

Malz et al., PRL **120**, 023601 (2018)

Nonreciprocal devices by reservoir engineering

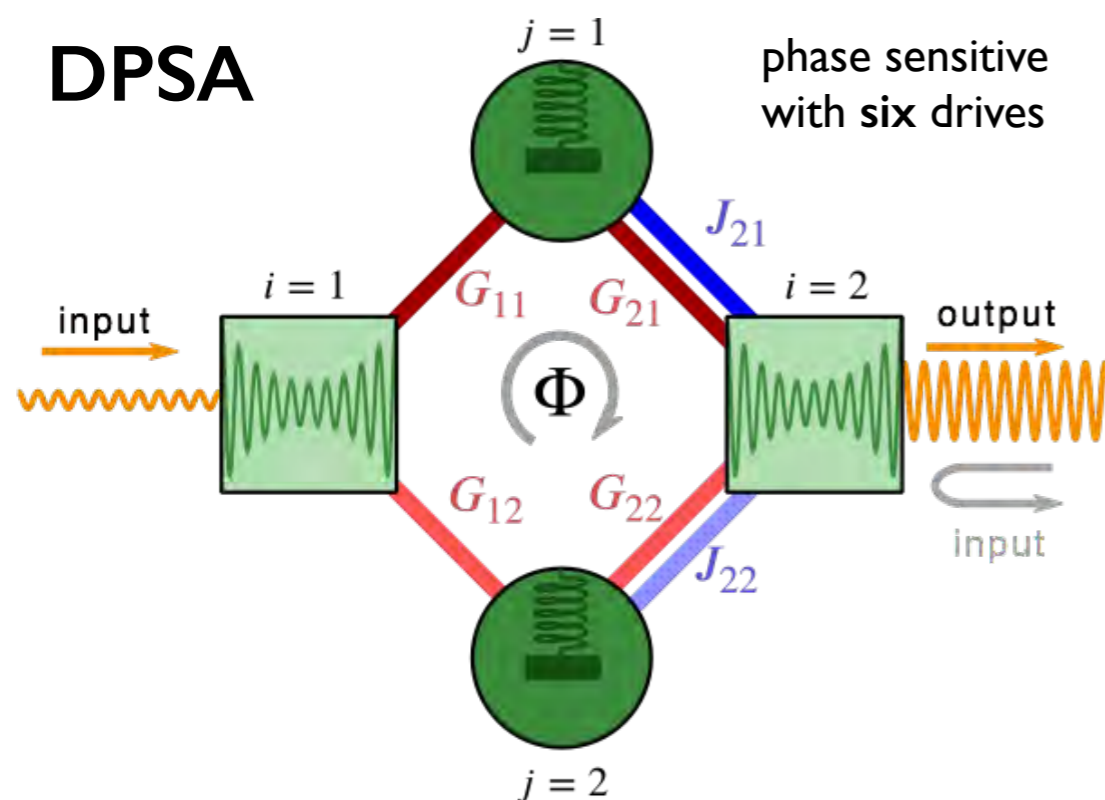


- isolation and “impedance matching”
- unlimited gain $\mathcal{G} = \frac{4C_1 C_2}{(C_1 - C_2)^2}$
- quantum limited $\mathcal{N}_{\text{DPPA}} \rightarrow \frac{1}{2}$

**Scattering matrix
(rich behavior
off-resonance)**

$$\begin{pmatrix} a_{1,\text{out}}(0) \\ a_{2,\text{out}}^\dagger(0) \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i\sqrt{\mathcal{G}}}{\sqrt{4C_1}} & \frac{i\sqrt{\mathcal{G}}}{\sqrt{4C_1}} & -\sqrt{\mathcal{G}} & \frac{C_1+C_2}{C_2-C_1} \end{pmatrix} \begin{pmatrix} b_{1,\text{in}}(0) \\ b_{2,\text{in}}(0) \\ a_{1,\text{in}}(0) \\ a_{2,\text{in}}^\dagger(0) \end{pmatrix}$$

Nonreciprocal devices by reservoir engineering



- QND measurement couples one optical quadrature to the mechanical oscillators

- unlimited gain $\mathcal{G} = \frac{8\mathcal{C}_2(2\mathcal{C}_1 - 1)}{\mathcal{C}_1^2}$

- unlimited gain-bandwidth product

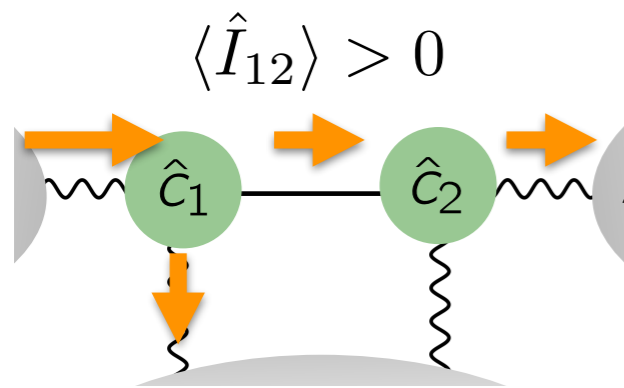
- quantum limited $\mathcal{N}_{\text{DPSA}} \rightarrow 0$

$$\begin{pmatrix} U_{1,\text{out}} \\ V_{1,\text{out}} \\ U_{2,\text{out}} \\ V_{2,\text{out}} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & \sqrt{\mathcal{G}} & 0 & -1 \end{pmatrix} \begin{pmatrix} U_{1,\text{in}} \\ V_{1,\text{in}} \\ U_{2,\text{in}} \\ V_{2,\text{in}} \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ \sqrt{2\mathcal{F}} & 0 & \sqrt{2\mathcal{F}} & 0 \end{pmatrix} \begin{pmatrix} X_{1,\text{in}} \\ P_{1,\text{in}} \\ X_{2,\text{in}} \\ P_{2,\text{in}} \end{pmatrix}$$

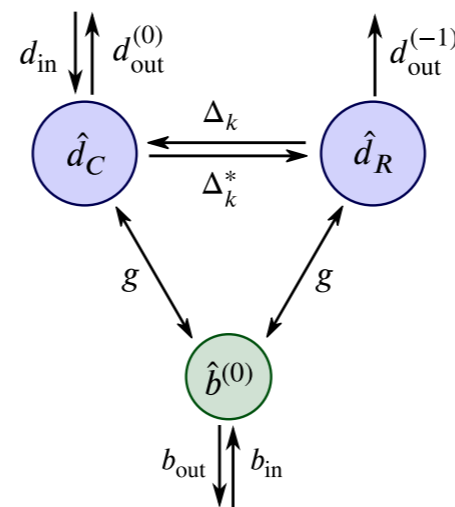
with noise scattering intensity $\mathcal{F} \equiv 4\mathcal{C}_2/\mathcal{C}_1^2$,

Malz *et al.*, PRL **120**, 023601 (2018)

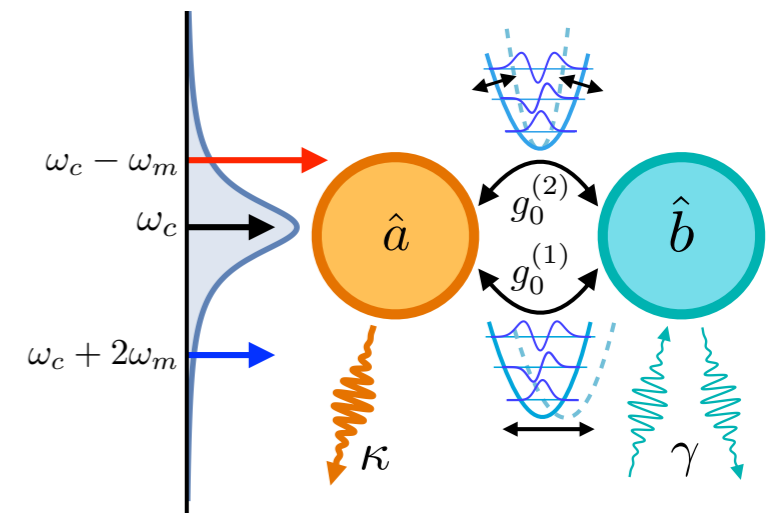
Nonreciprocal devices by reservoir engineering



Current rectification in serial spinless double quantum dot
 Malz and AN,
 PRB 97, 165308 (2018)



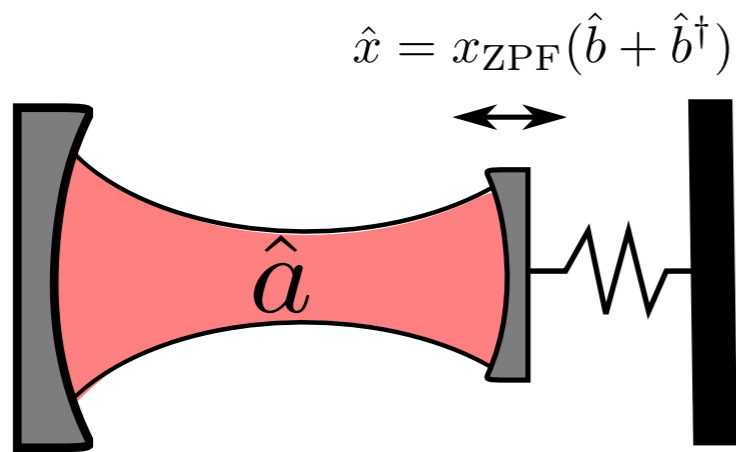
Motional Sideband Asymmetry and Kerr-type nonlinearities
 ...,Malz, AN,...,
 in preparation



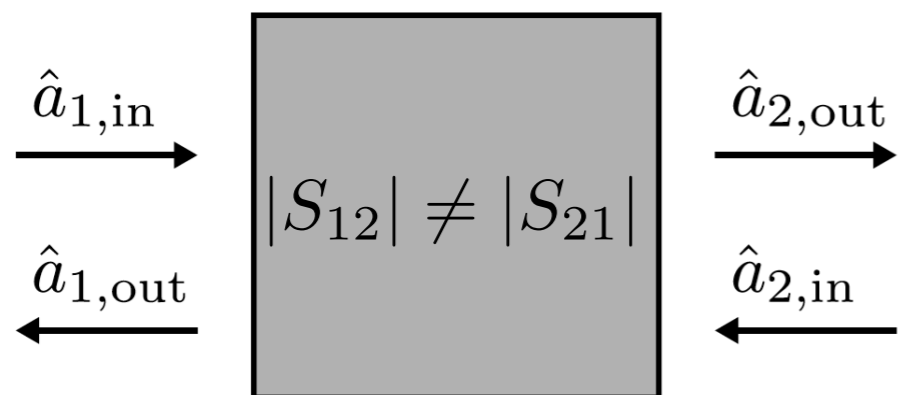
Unconditional preparation of nonclassical massive states
 Brunelli,...,AN,...,
 arXiv:1804.00014



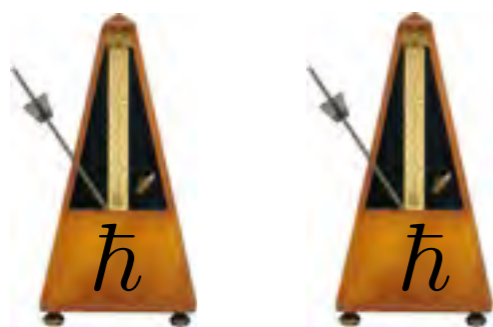
Outline



Cavity optomechanics



Nonreciprocal devices

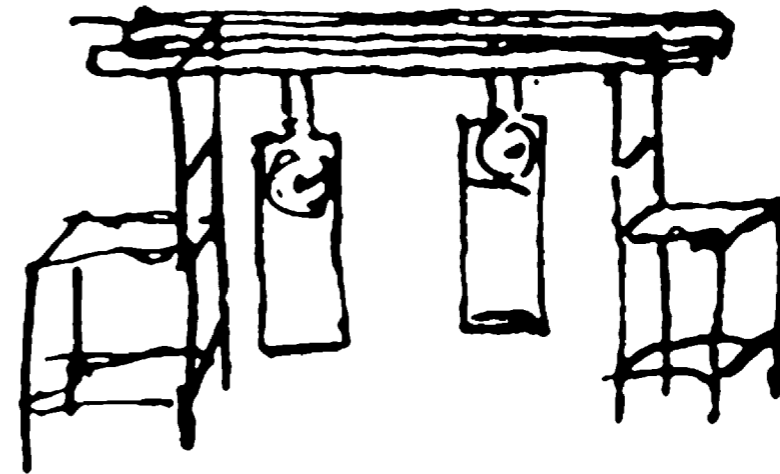


Quantum synchronization

Quantum synchronization



Christiaan Huygens
(1629 – 1695)



Letter to Royal Society of London
“adjustment of rhythms of oscillating
objects due to weak interaction”

- fireflies, heart, neurons, algae...
- Josephson junctions, lasers,
spin torque oscillators...
- power networks, GPS, ...

**Synchronization
is ubiquitous!**

Quantum synchronization

self-oscillator
(limit-cycle) oscillator



ω_0

- driven into oscillation by an energy source
- characterized by a natural frequency
- NB. It is not a harmonic degree of freedom.

synchronization



ω_1



ω_2

+ coupling



$\omega_{1'}$



$\omega_{2'}$



injection locking
(synchronization to external force)



ω_0

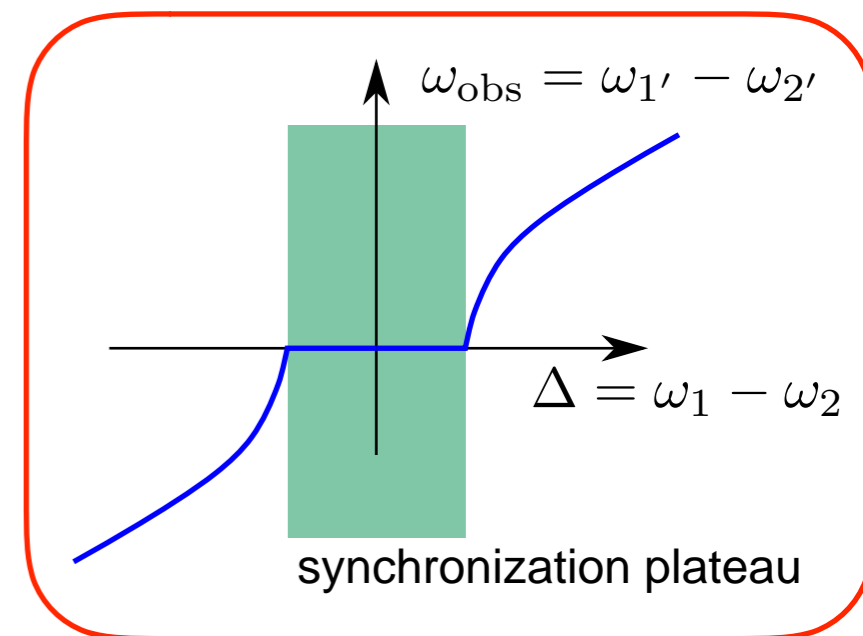
+ external force



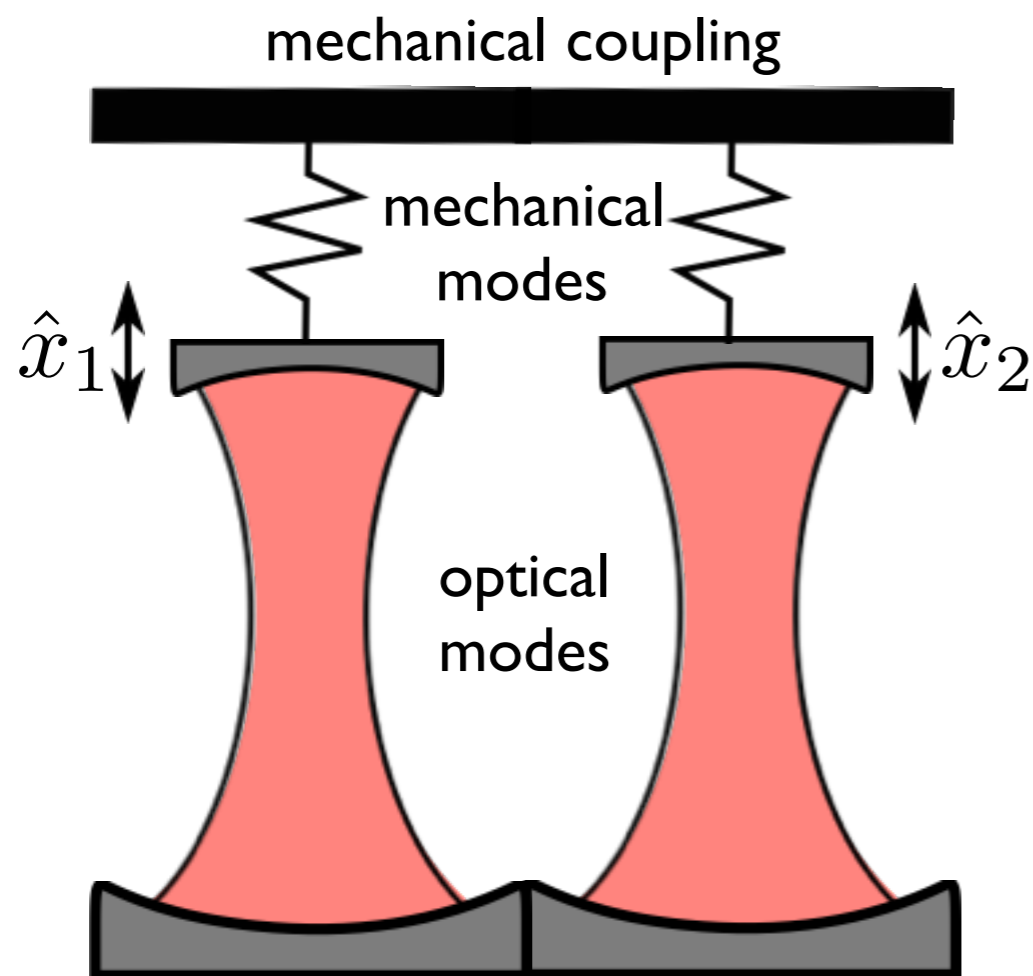
$F_0 \cos \omega_d t$



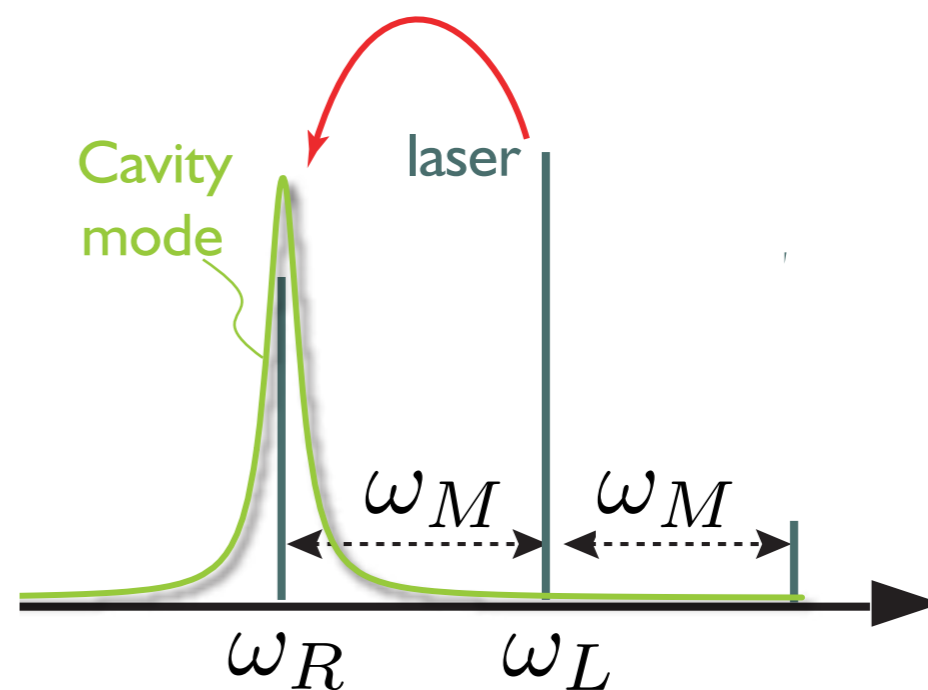
ω'_0



Quantum synchronization

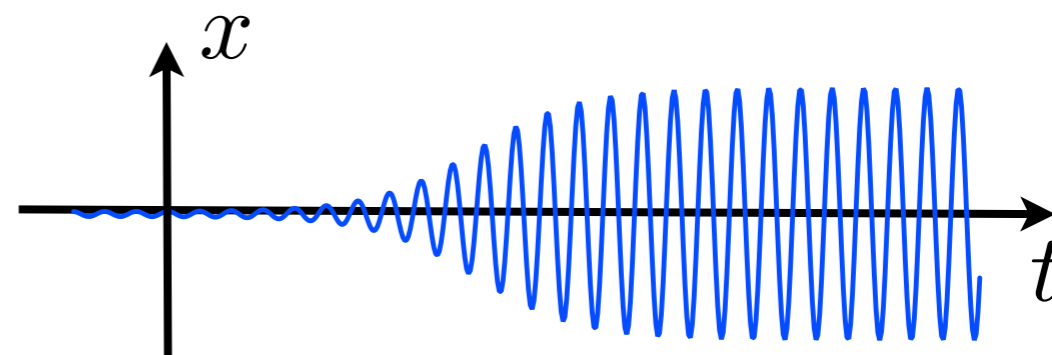


What is the fate of synchronization in the quantum regime?



Self-sustained oscillations

$$\Gamma_m + \Gamma_{\text{opt}} < 0$$



Heinrich, ..., Marquardt, PRL **107**, 043603 (2011)

Ludwig and Marquardt, PRL **111**, 073603 (2013)

Mari, ..., Fazio, PRL **111**, 103605 (2013) & more

Quantum synchronization

simplest model: *driven van der Pol oscillator*

$$\ddot{x} + (-\gamma_1 + \gamma_2 x^2)\dot{x} + \omega_0^2 x = \Omega \cos(\omega_d t)$$

γ_1 **negative damping** (“battery”)

γ_2 **nonlinear damping**

Ω **strength of the external driving field**

Δ **detuning** $\Delta = \omega_d - \omega_0$



Balthasar van der Pol



Sir Edward Appleton

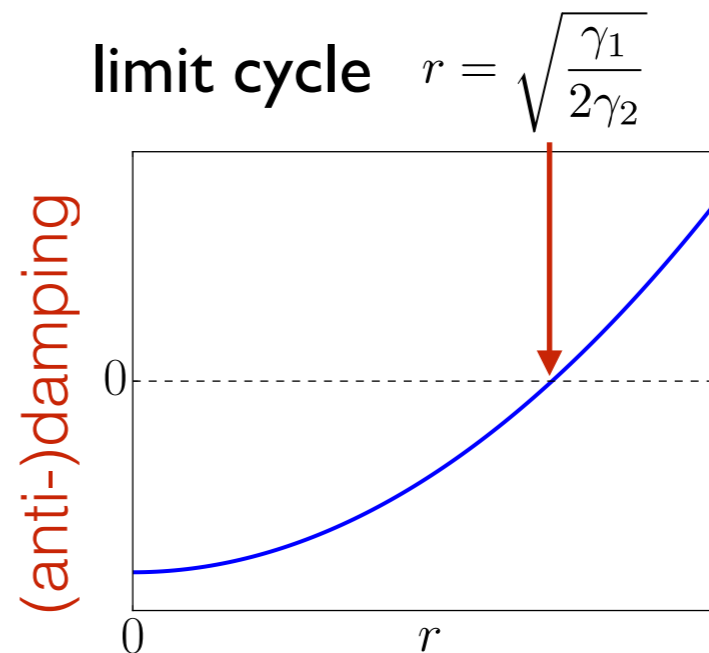
Quantum synchronization

Phase space amplitude
in the rotating frame

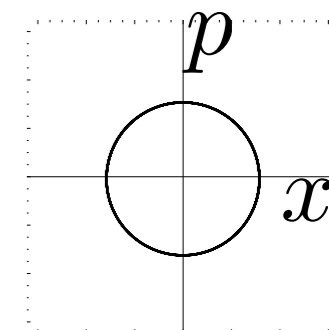
$$\begin{aligned} \text{Re}[\beta(t)] &\sim x(t) \\ \text{Im}[\beta(t)] &\sim p(t) \end{aligned}$$

$$\dot{\beta} = \underbrace{i\Delta\beta}_{\text{detuning}} + \underbrace{\frac{\gamma_1}{2}\beta}_{\text{negative damping}} - \underbrace{\gamma_2|\beta|^2\beta}_{\text{non-linear damping}} - \underbrace{\Omega}_{\text{drive}}$$

$$\Delta = \omega_d - \omega_0$$



not synchronized



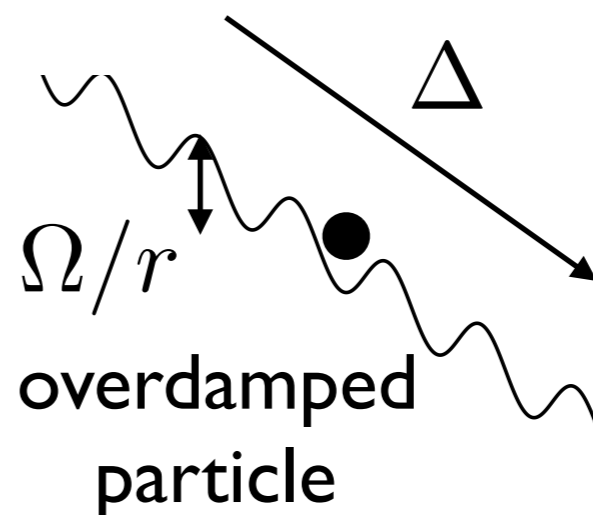
$$\Omega/r < |\Delta|$$

Using polar coordinates

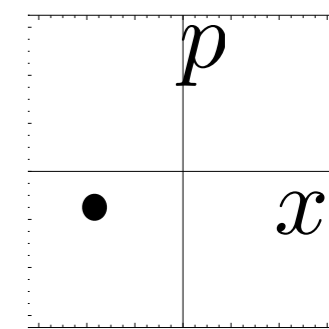
$$\beta = r e^{i\phi}$$

$$\begin{aligned} \dot{r} &= \left(\frac{\gamma_1}{2} - \gamma_2 r^2 \right) r - \Omega \cos \phi \\ \dot{\phi} &= \Delta + \frac{\Omega}{r} \sin \phi \end{aligned}$$

Adler (1946)



synchronized



$$\Omega/r > |\Delta|$$

Quantum synchronization

classical van der Pol oscillator

$$\frac{d}{dt}\beta = \underbrace{i\Delta\beta}_{\text{detuning}} + \underbrace{\frac{\gamma_1}{2}\beta}_{\text{negative damping}} - \underbrace{\gamma_2|\beta|^2\beta}_{\text{non-linear damping}} - \underbrace{\Omega}_{\text{drive}}$$

quantum van der Pol oscillator

$$\frac{d\rho}{dt} = -i \left[\underbrace{-\Delta\hat{b}^\dagger\hat{b}}_{\text{detuning}} + \underbrace{i\Omega(\hat{b} - \hat{b}^\dagger)}_{\text{drive}}, \rho \right] + \underbrace{\gamma_1\mathcal{D}[\hat{b}^\dagger]\rho}_{\text{negative damping}} + \underbrace{\gamma_2\mathcal{D}[\hat{b}^2]\rho}_{\text{non-linear damping}}$$

$$\mathcal{D}[O]\rho = O\rho O^\dagger - \frac{1}{2}\{O^\dagger O, \rho\}$$

Quantum synchronization

quantum van der Pol oscillator

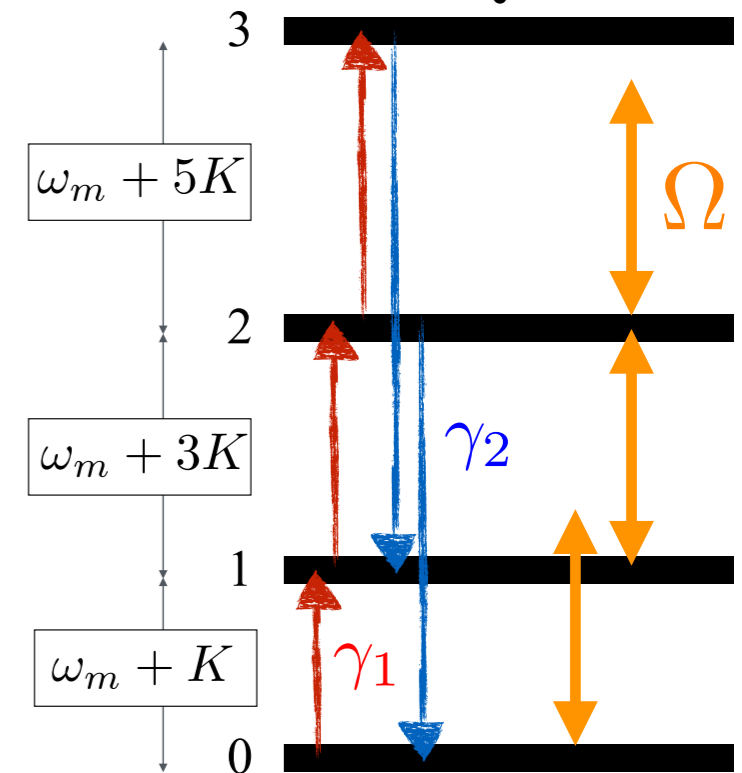
$$\frac{d\rho}{dt} = -i \left[\underbrace{-\Delta \hat{b}^\dagger \hat{b}}_{\text{detuning}} + \underbrace{i\Omega(\hat{b} - \hat{b}^\dagger)}_{\text{drive}}, \rho \right] + \underbrace{\gamma_1 \mathcal{D}[\hat{b}^\dagger]}_{\text{negative damping}} \rho + \underbrace{\gamma_2 \mathcal{D}[\hat{b}^2]}_{\text{non-linear damping}} \rho$$

$$\mathcal{D}[O]\rho = O\rho O^\dagger - \frac{1}{2} \{O^\dagger O, \rho\}$$

Add $\hat{H}_{\text{Kerr}} = K(\hat{b}^\dagger \hat{b})^2$

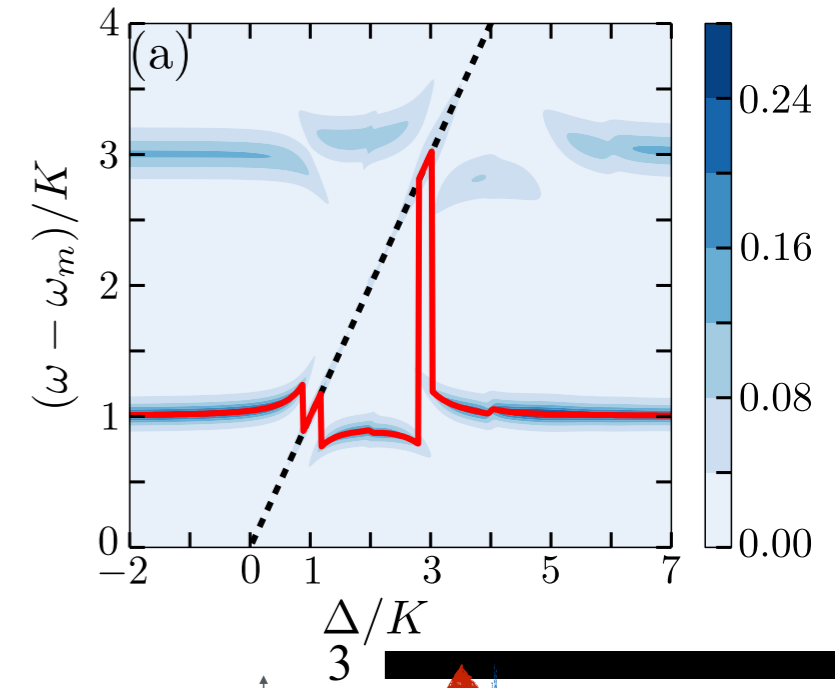
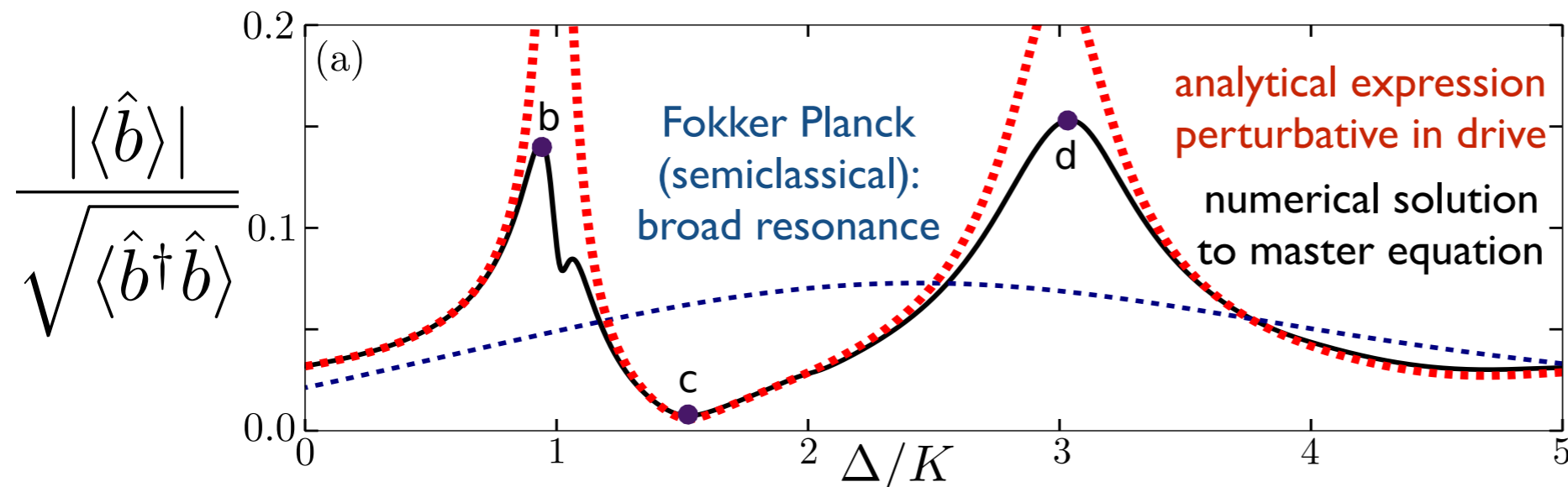
Genuine quantum features:

- footprint of quantized Fock levels
- negative steady-state Wigner density

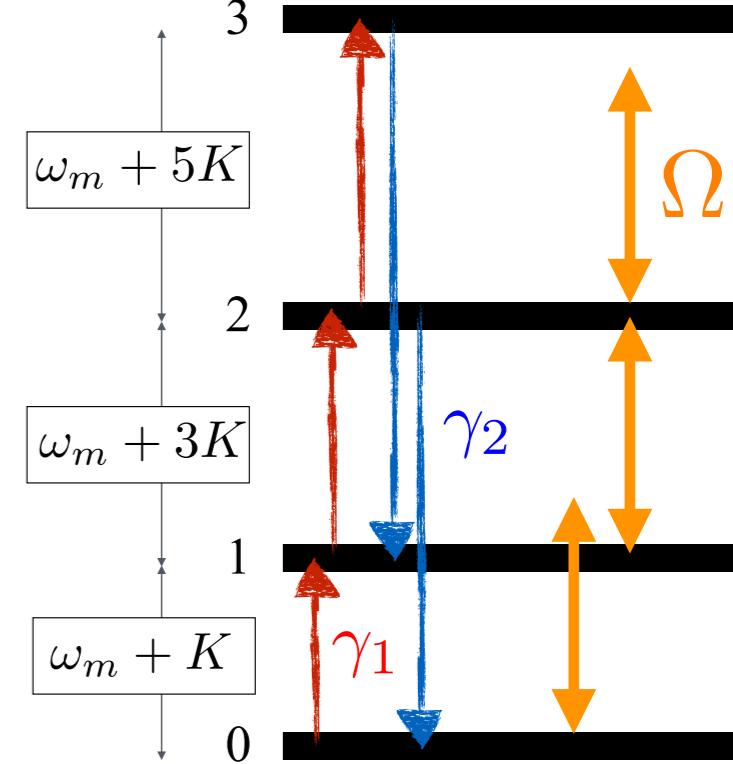


Quantum synchronization

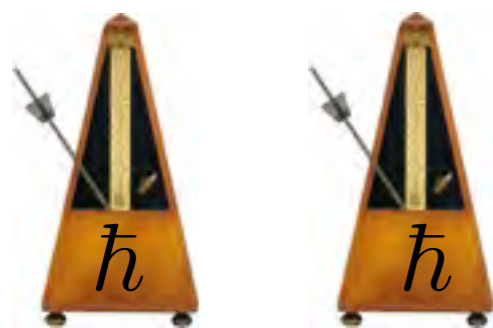
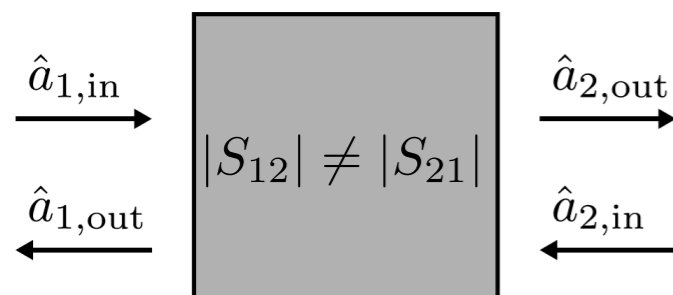
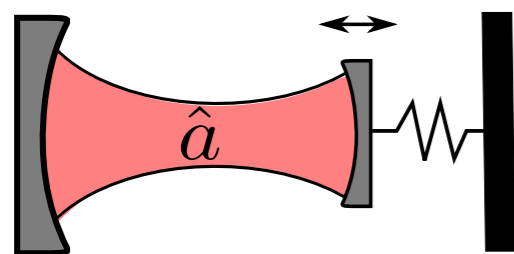
Aim: To study the tendency to phase locking



Multiple phase locking resonances are footprint of quantized energy levels.



Cavity optomechanics



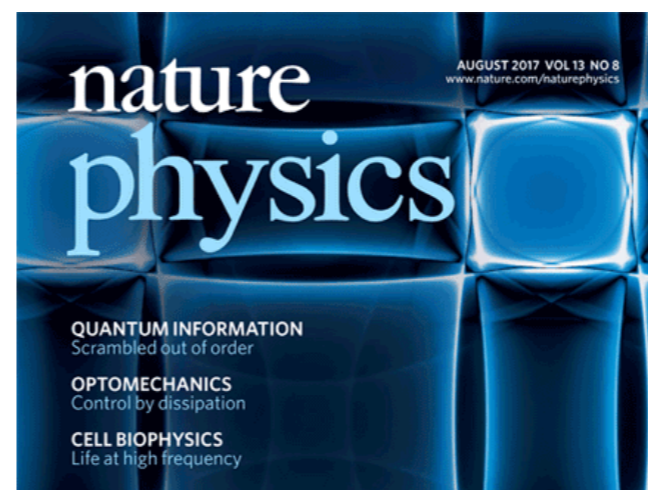
New platform for quantum science and technology

Non-reciprocal devices with reservoir engineering

- Optomechanical isolator and circulator proposal
- Proposal for optomechanical directional amplifier
- Proposal for current rectification in double QD

Platform for synchronization in quantum regime

- Quantum noise generically destroys locking
- Genuine quantum features in sync identified
- “Quantum synchronization blockade”



Nat. Phys. 13, 787 (2017)
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 PRL 117, 073601 (2016)
 PRL 118, 243602 (2017)

