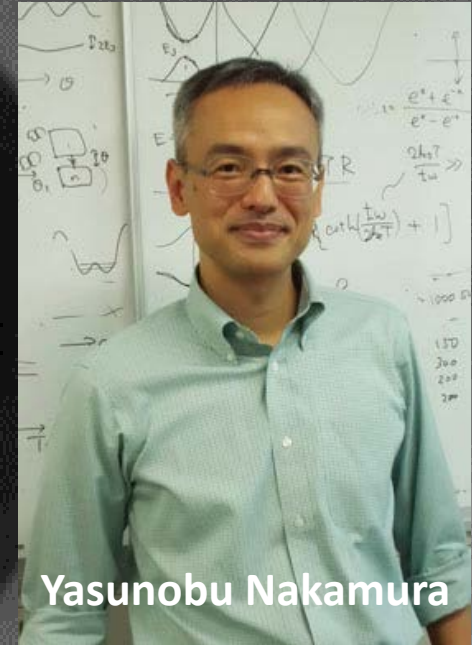
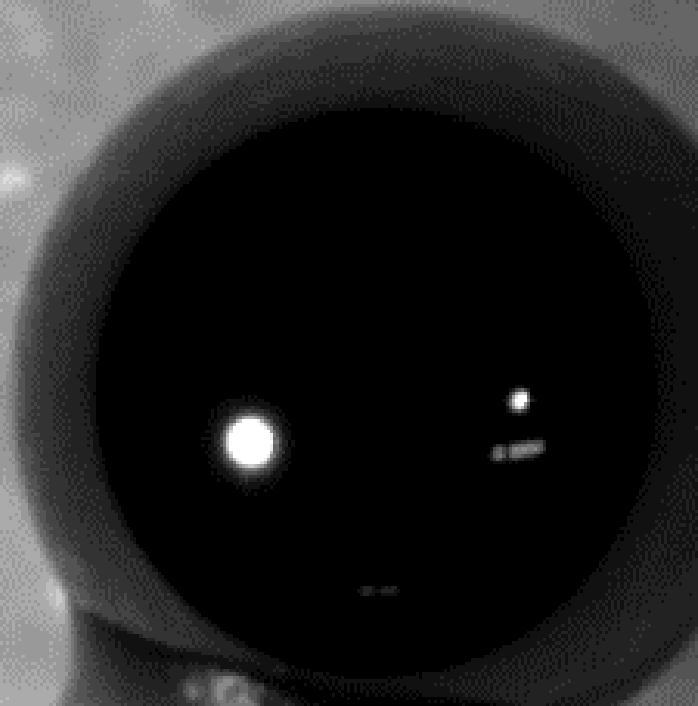


# Magnonics in the quantum regime



Yasunobu Nakamura

**Koji Usami**

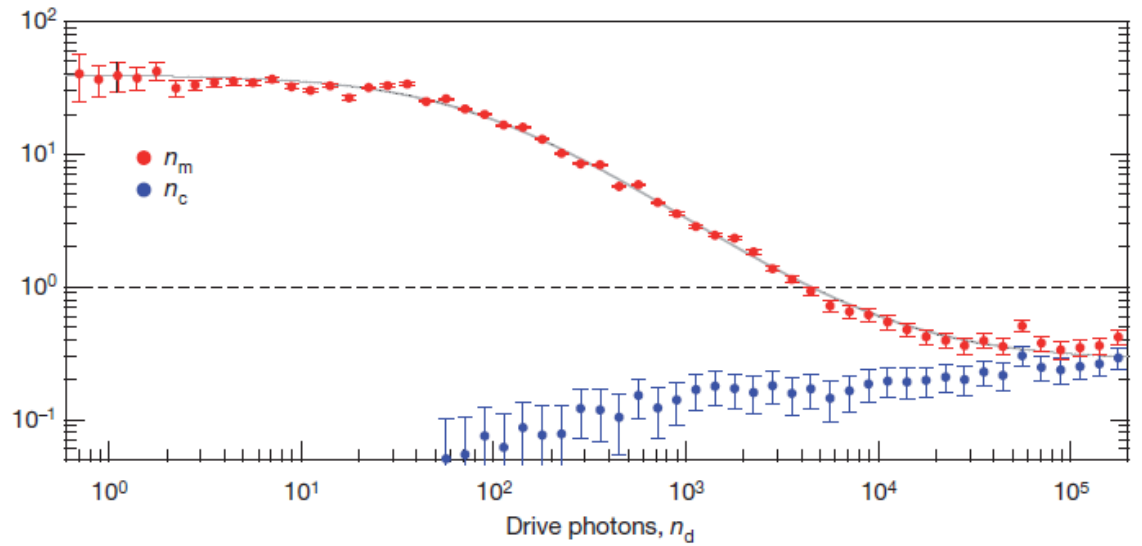
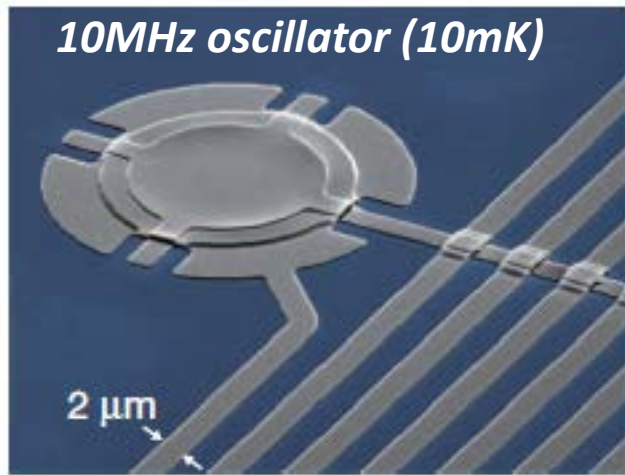
Research Center for Advanced Science and Technology (RCAST)

University of Tokyo

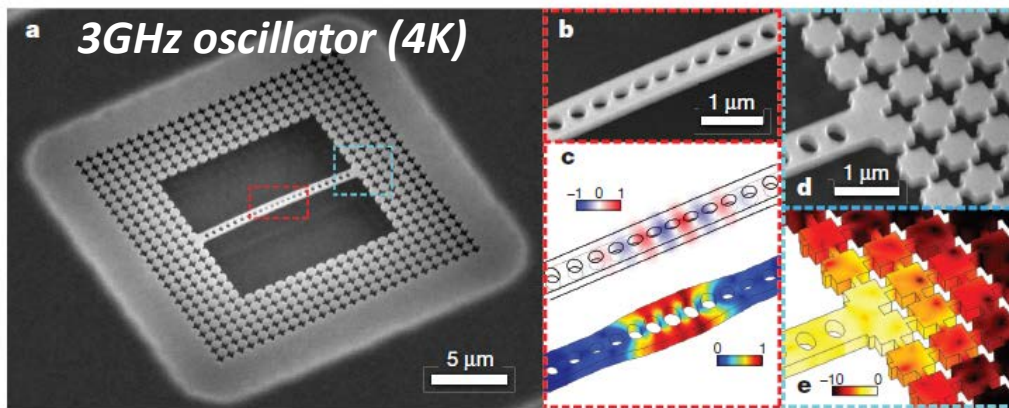
# Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

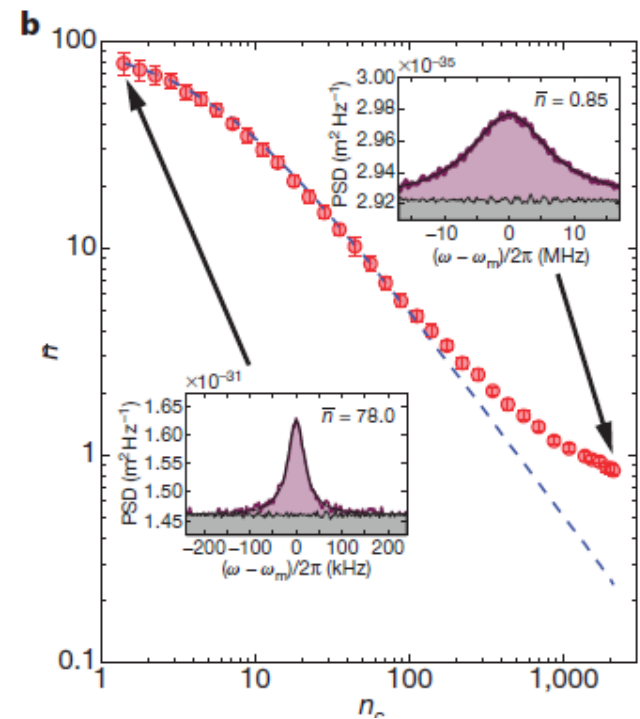
Keith C. Schwab and Michael L. Roukes



J.D.Teufel et al., Nature **475**, 359 (2011)



J.Chan et al., Nature **478**, 89 (2011)



# Putting Magnonics into Quantum Magnonics

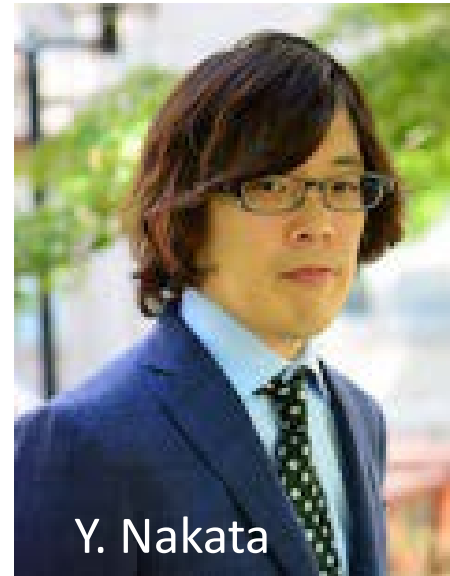
Cavity (circuit) quantum magnonics

New possibility of optomagnonics

# Posters



Magnetic quasi-vortex



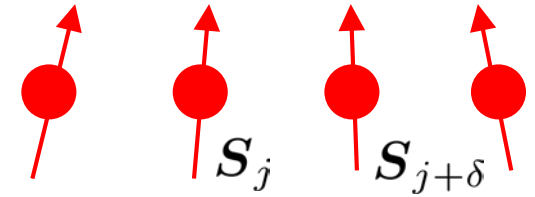
Magnonic crystal

Cavity (circuit) quantum magnonics

New possibility of optomagnonics

# Magnons

$$H = \underbrace{-J \sum_{j,\delta} \mathbf{S}_j \cdot \mathbf{S}_{j+\delta}}_{\text{Exchange}} - \underbrace{g_s \mu_B B_z \sum_j S_{jz}}_{\text{Zeeman}}$$



Exchange interaction

Holstein-Primakoff transformation:

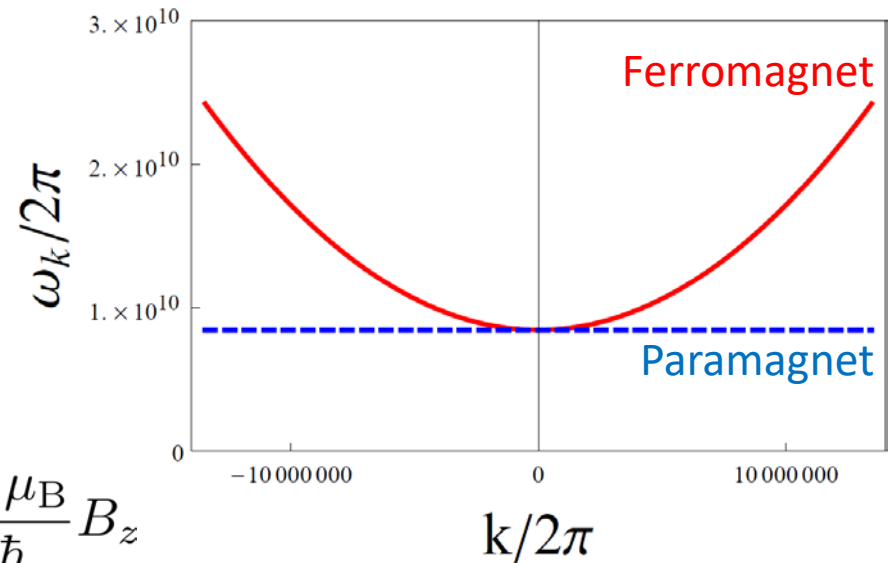
$$S_j^+ = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{-i\mathbf{k} \cdot \mathbf{x}_j} \hat{c}_{\mathbf{k}} + \dots$$

$$S_j^- = \frac{1}{\sqrt{N}} \sum_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{x}_j} \hat{c}_{\mathbf{k}}^\dagger + \dots$$

$$S_{jz} = \frac{1}{2} - \sum_{\mathbf{k}} \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}} + \dots$$

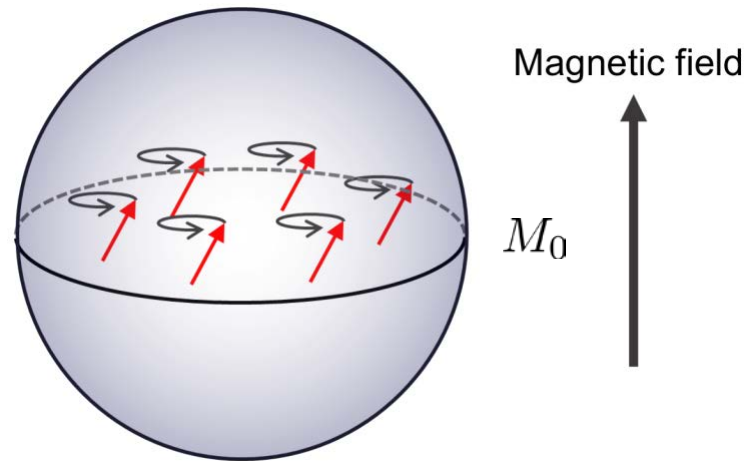
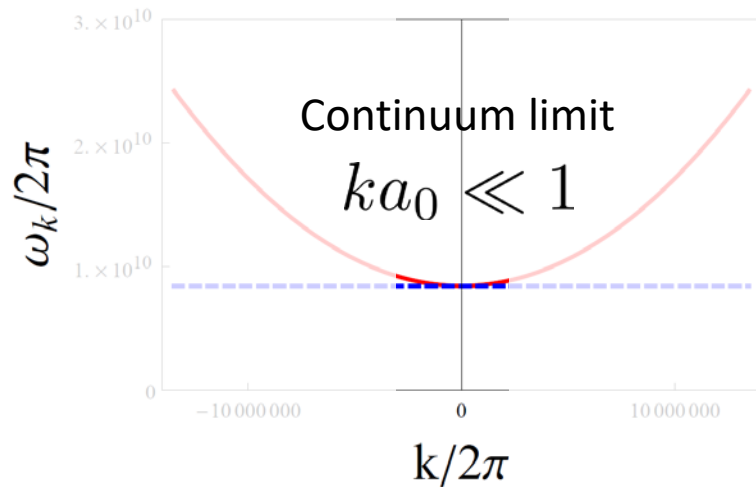
$$H_1 = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \hat{c}_{\mathbf{k}}^\dagger \hat{c}_{\mathbf{k}}$$

$$\omega_{\mathbf{k}} = \underbrace{\frac{2J}{\hbar} \sin^2 \left( \frac{\mathbf{k} a_0}{2} \right)}_{\text{Exchange}} + \underbrace{\frac{g_s \mu_B}{\hbar} B_z}_{\text{Zeeman}}$$





# Magnetostatic (Walker) modes



Walker equations  $H = \nabla\psi$

Maxwell equations

$$\nabla \times \mathbf{H} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

Landau-Lifshitz equation

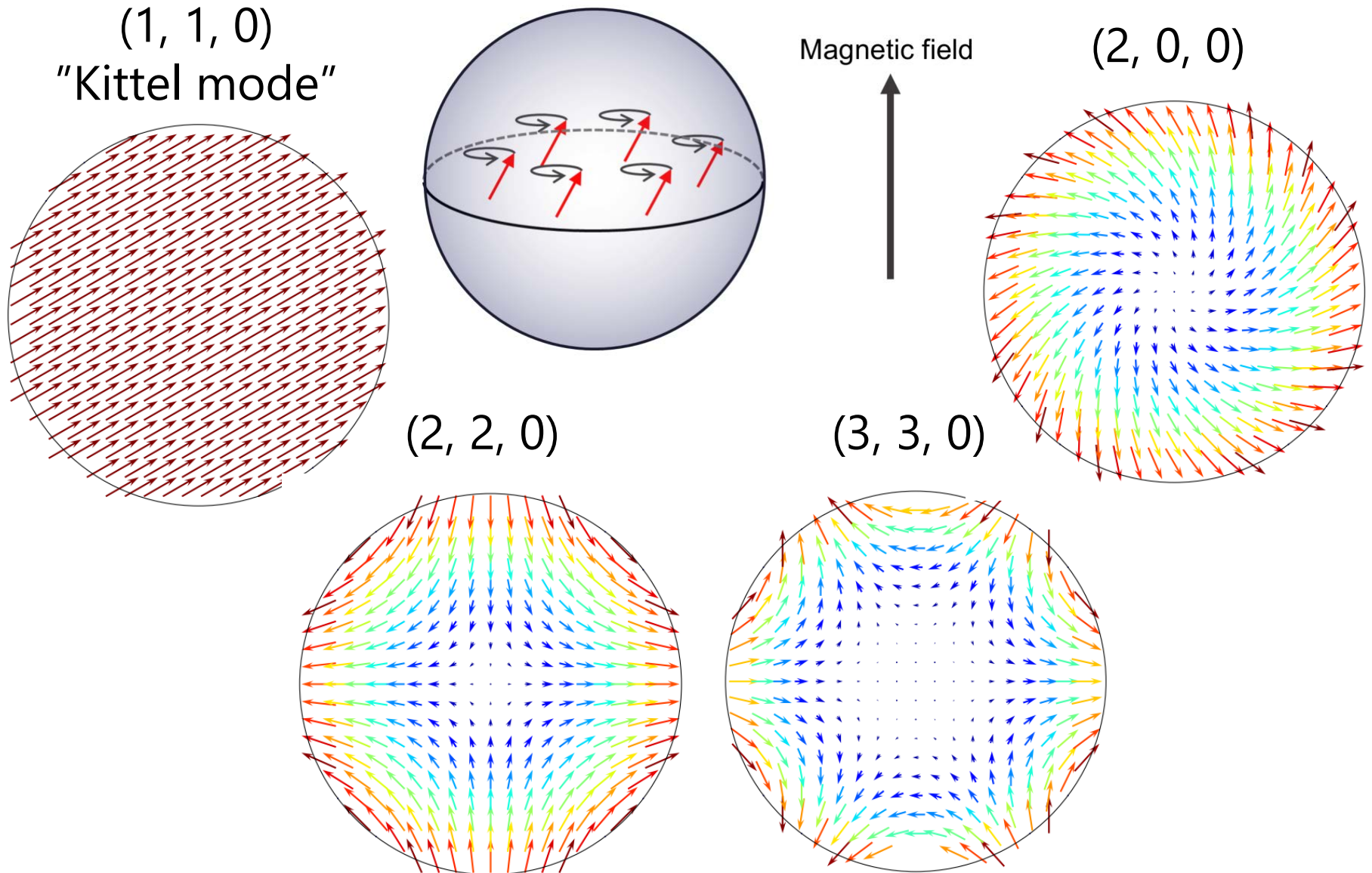
$$\frac{\partial \mathbf{M}}{\partial t} = \gamma (\mathbf{M} \times \mathbf{B})$$

$$\text{Inside: } (1 + \kappa) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi + \frac{\partial^2}{\partial z^2} \psi = 0$$

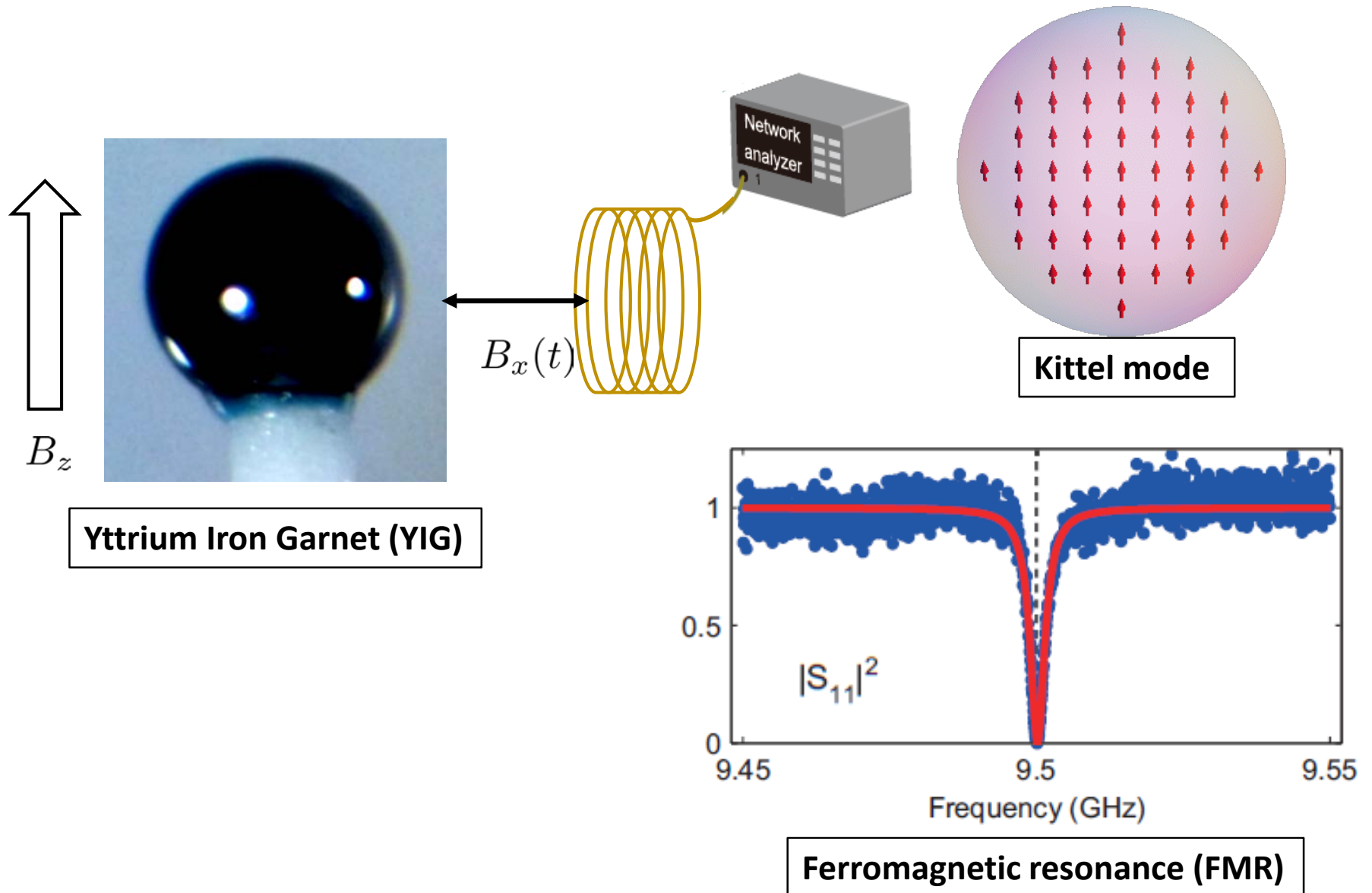
$$\text{Outside: } \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi = 0$$

$$\kappa = \frac{\Omega_H \Omega}{\Omega_H^2 - \omega^2}; \quad \Omega_H = \gamma \mu_0 H, \quad \Omega = \gamma \mu_0 M_0$$

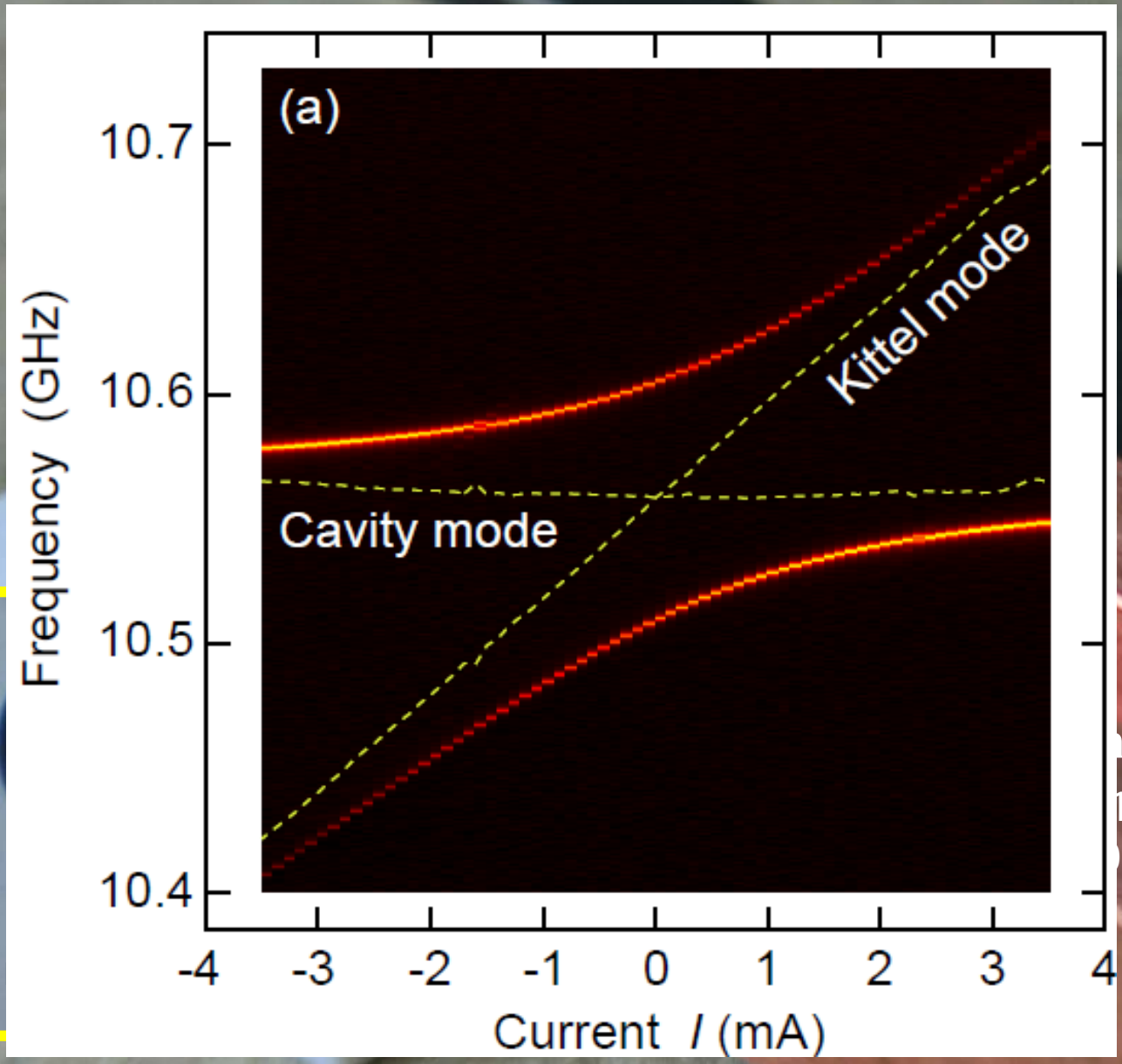
# Magnetostatic (Walker) modes



# Ferromagnetic resonance

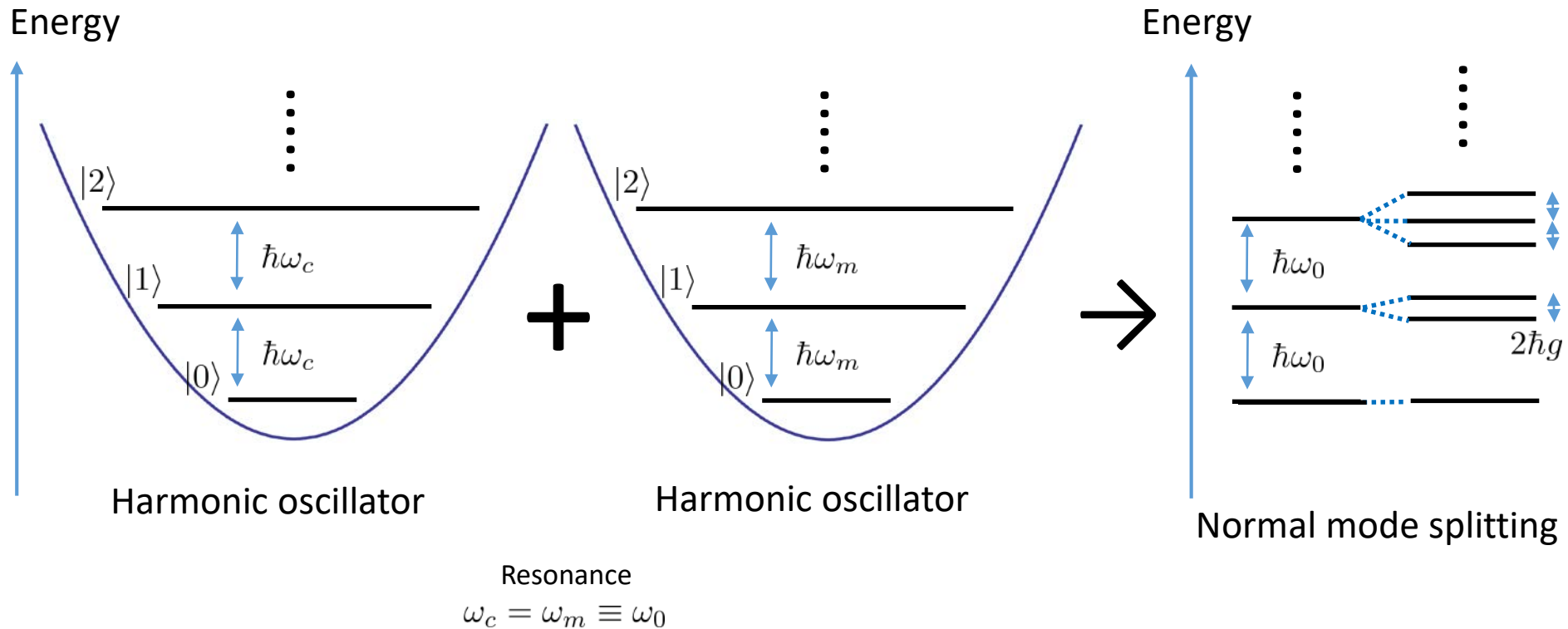


# Hybridizing magnons and microwave photons



Cavity  
101 mode  
0.5 GHz

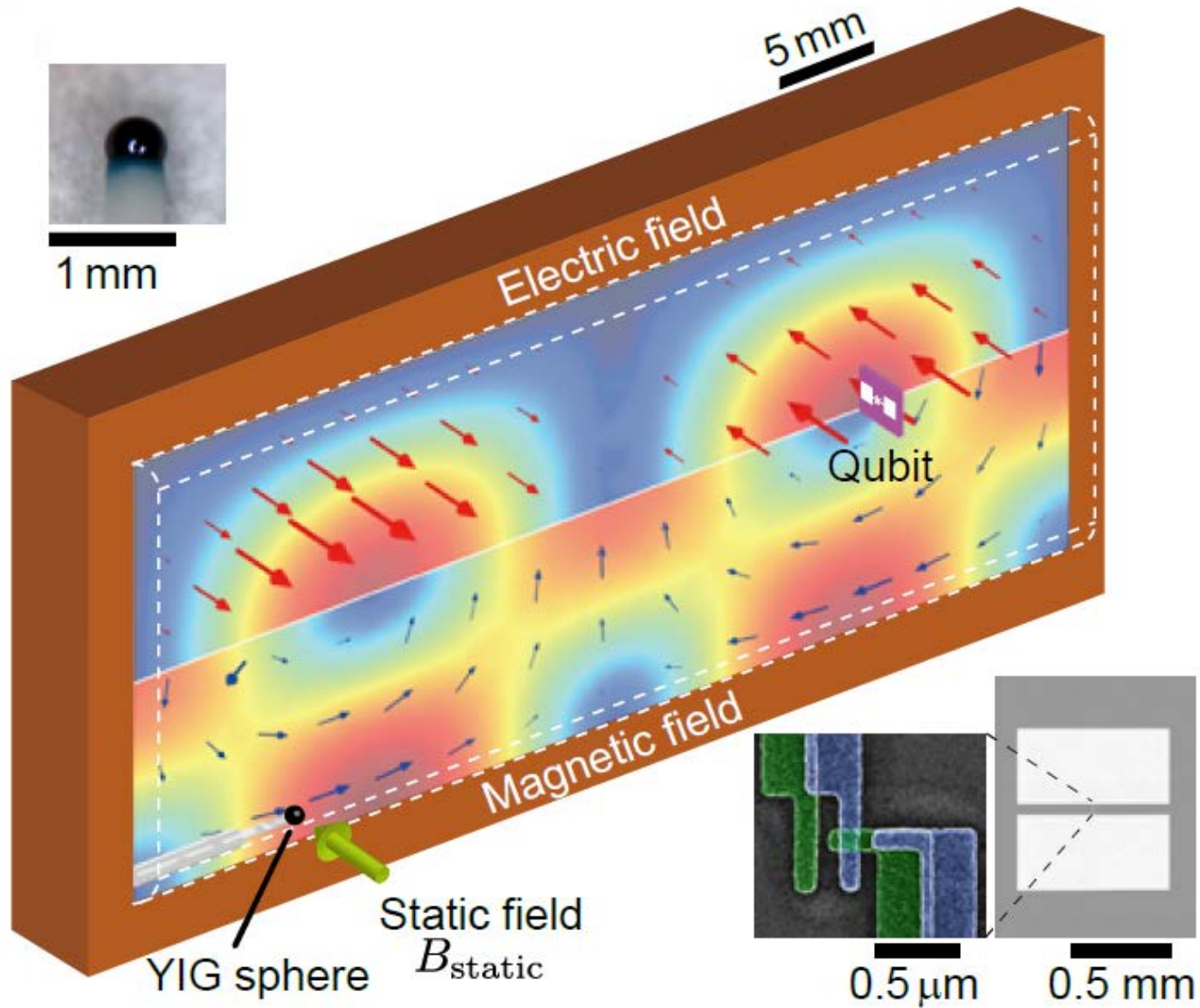
# Harmonic oscillator + harmonic oscillator



**Linear energy level structure**



# Meta cavity QED

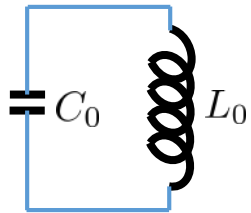


Experiment: Y. Tabuchi *et al.*, Science **349**, 405 (2015)

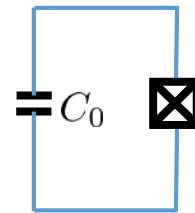
Proposal: A. Imamoglu, Phys. Rev. Lett. **102**, 083602 (2009).

# Superconducting qubit

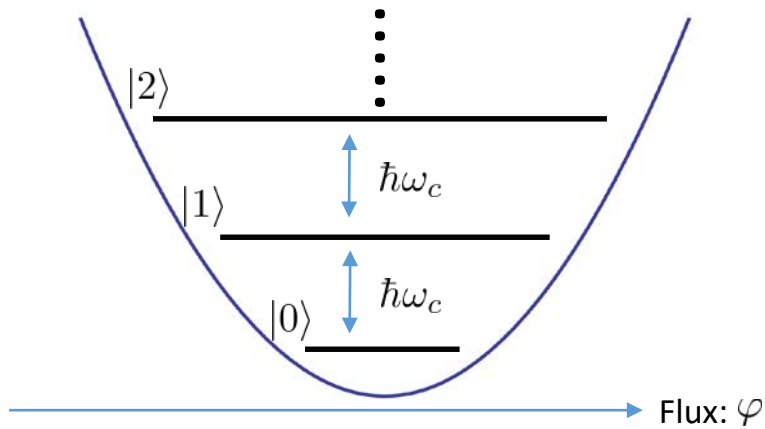
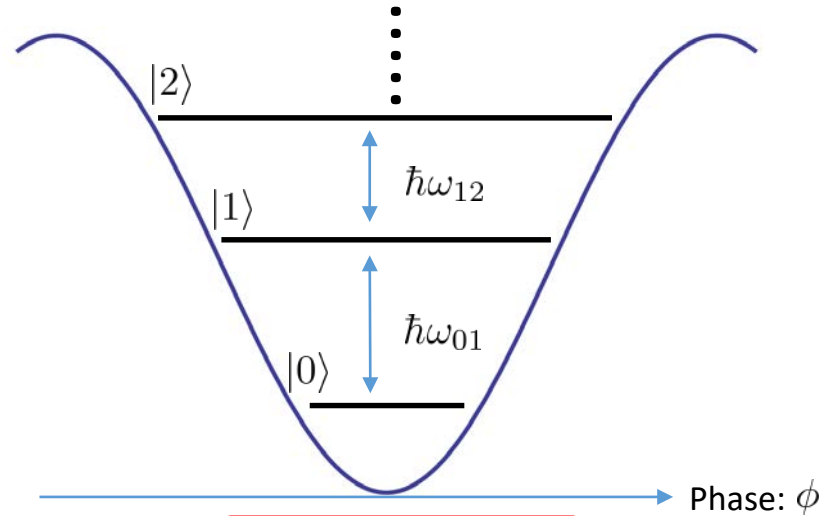
LC circuit



Capacitor with Josephson junction



$$I = I_c \sin \phi$$
$$V = \frac{\hbar}{2e} \dot{\phi}$$

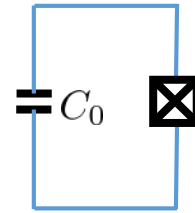


$$\ddot{\varphi} + \frac{1}{L_0 C_0} \varphi = 0$$

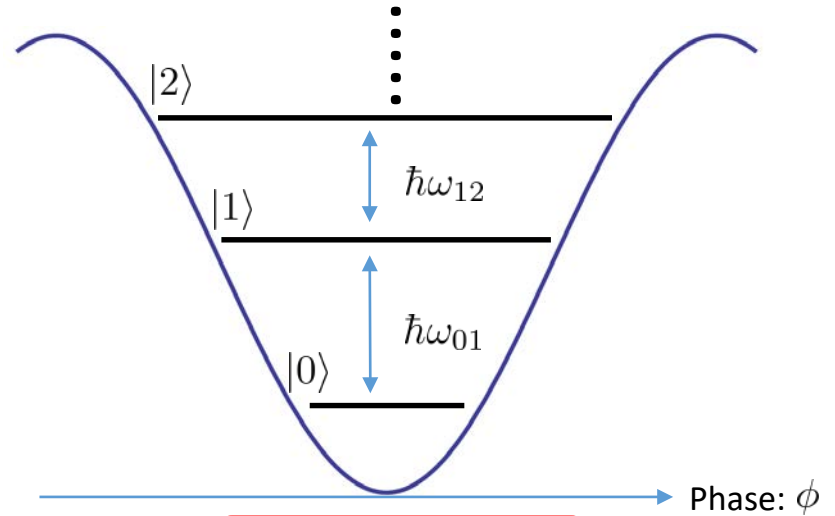
$$\ddot{\phi} + \frac{2eI_c}{\hbar C_0} \sin \phi = 0$$

# Superconducting qubit

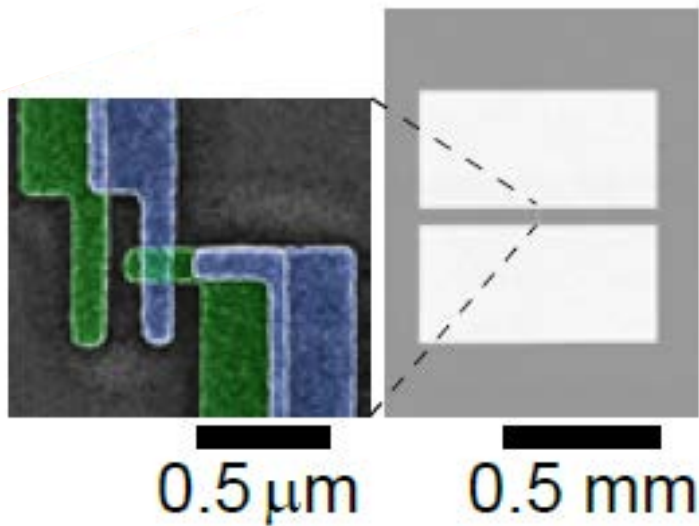
Capacitor with Josephson junction



$$I = I_c \sin \phi$$
$$V = \frac{\hbar}{2e} \dot{\phi}$$

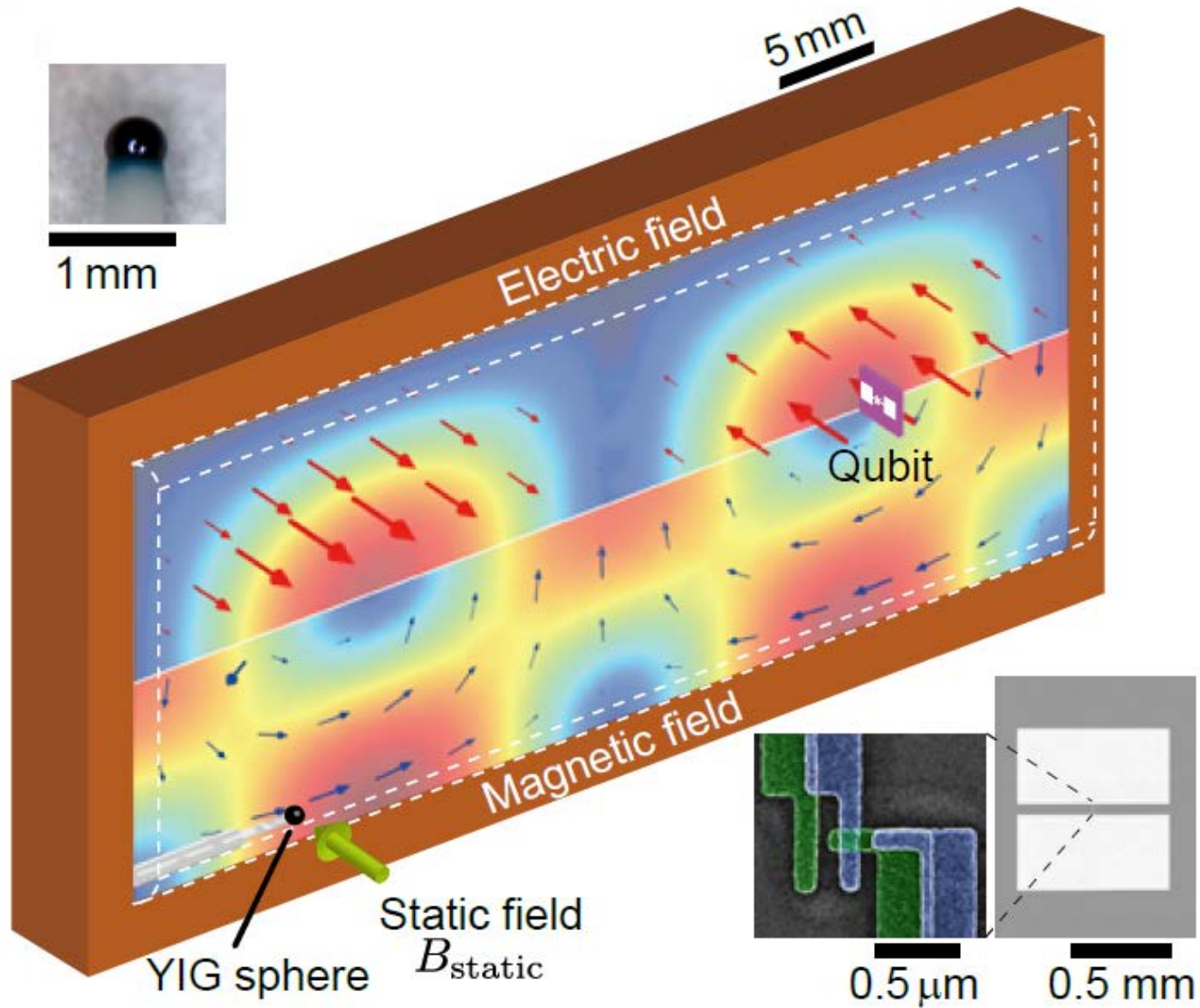


$$\ddot{\phi} + \frac{2eI_c}{\hbar C_0} \sin \phi = 0$$





# Meta cavity QED

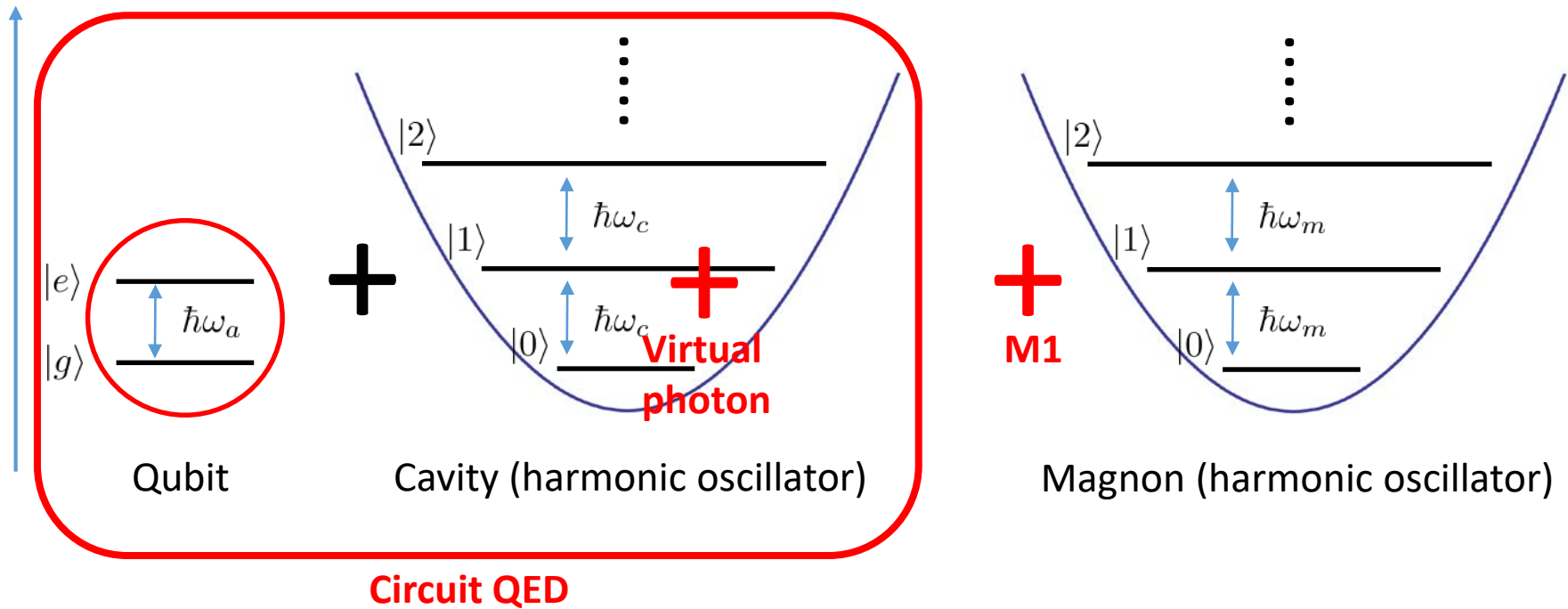


Experiment: Y. Tabuchi *et al.*, Science **349**, 405 (2015)

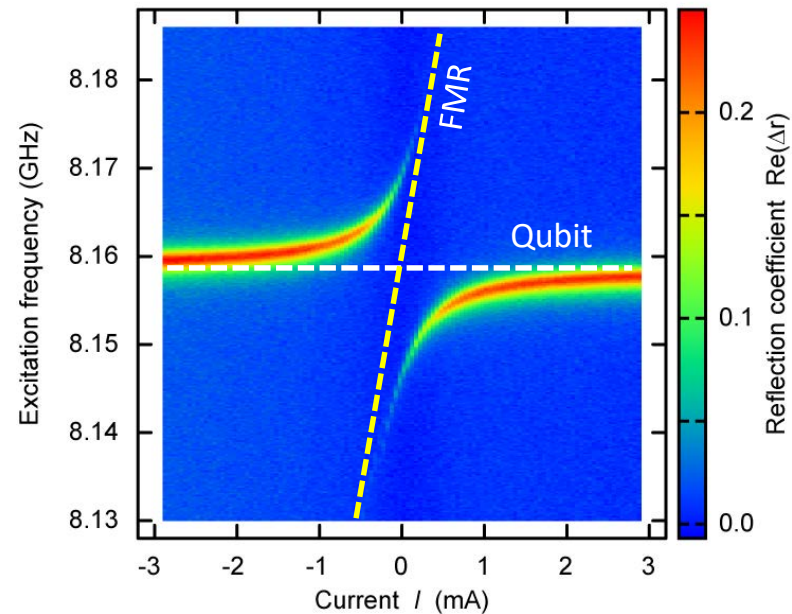
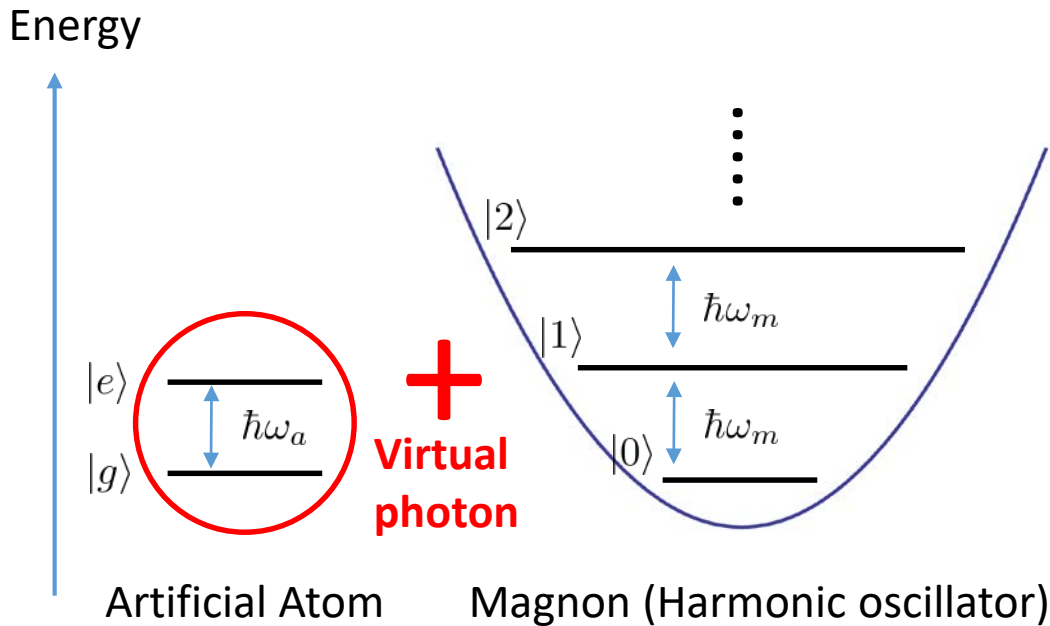
Proposal: A. Imamoglu, Phys. Rev. Lett. **102**, 083602 (2009).

# Meta cavity QED

Energy

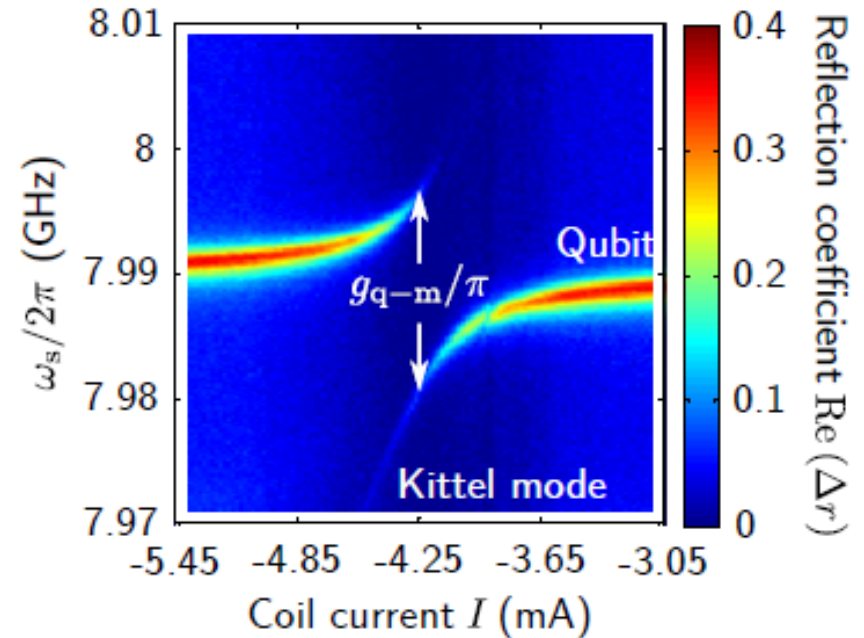
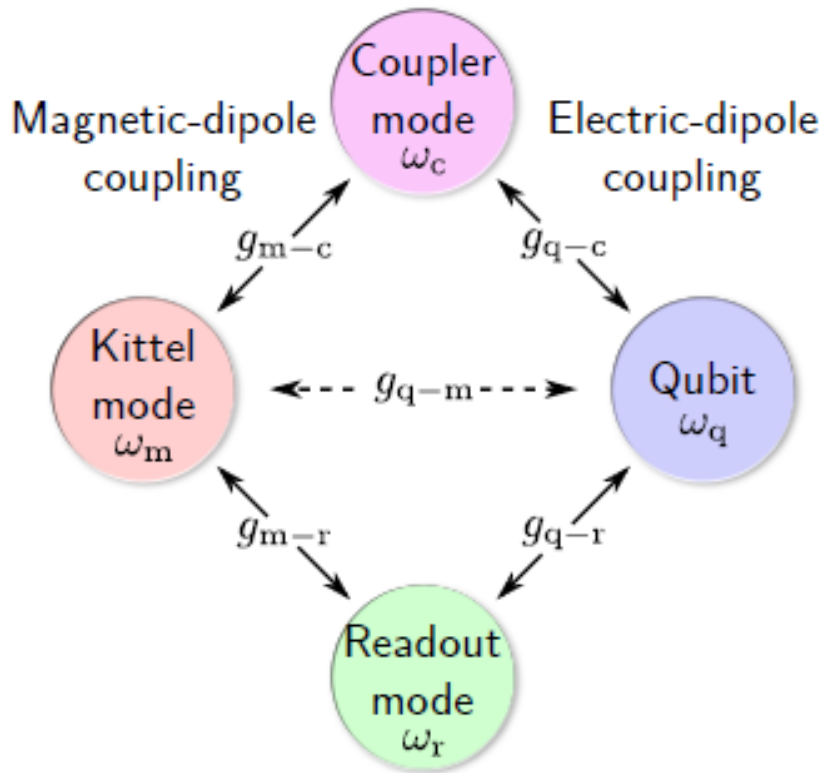


# Magnon-vacuum Rabi splitting



**Magnon-vacuum Rabi splitting**

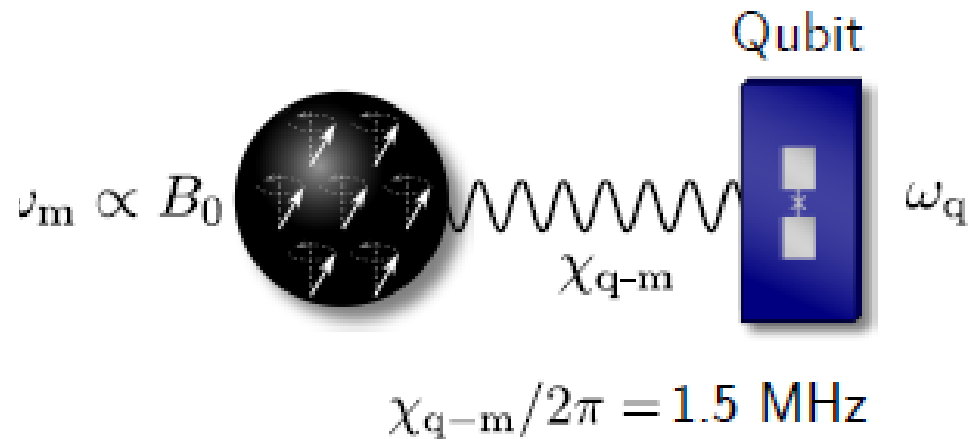
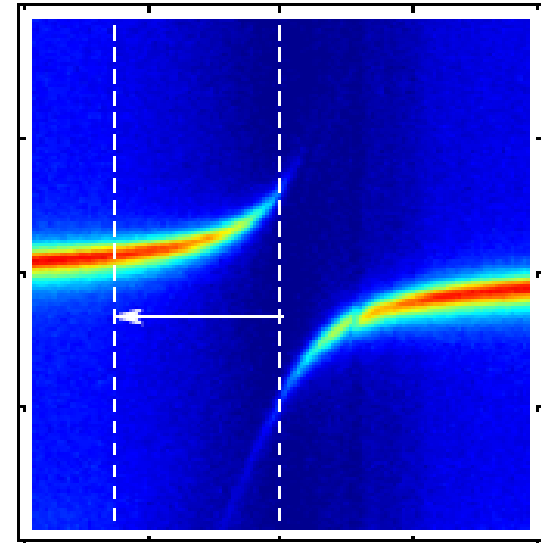
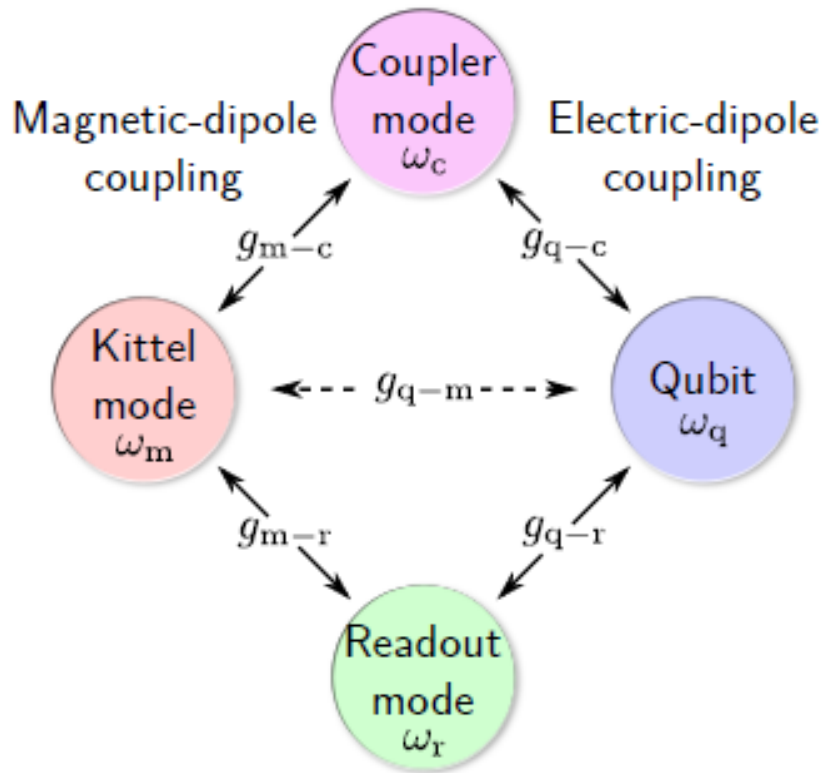
# Resonant coupling



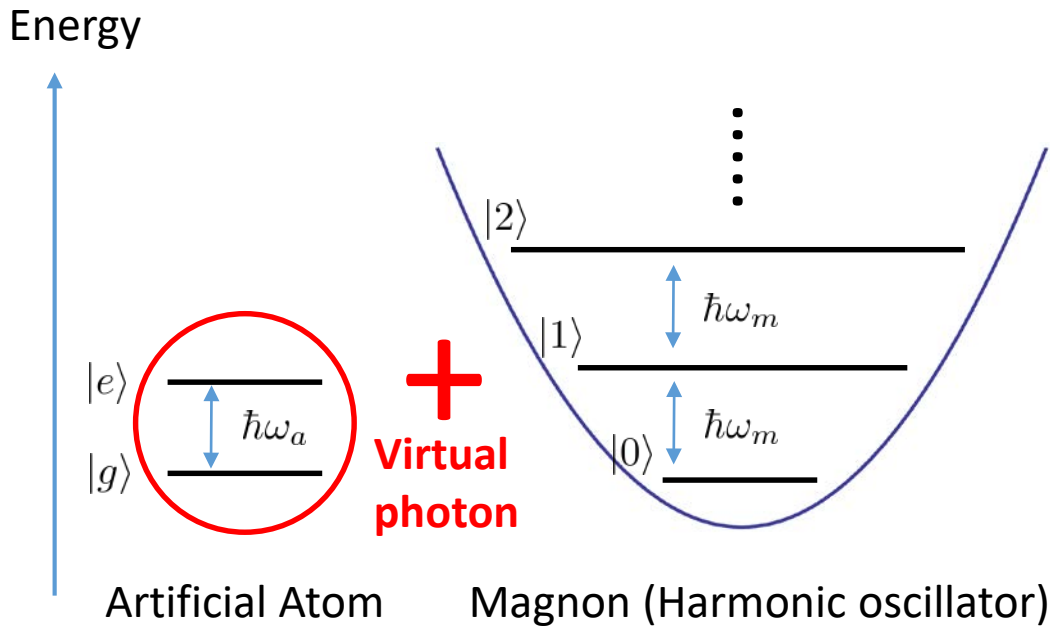
Experiment:  $g_{q-m}/2\pi = 7.79$  MHz

$$g_{q-m} = \sum_{i=c,r,\dots} \frac{g_{q-i}g_{m-i}}{\omega_i^{\text{bare}} - \omega_{q,m}^{\text{bare}}}$$

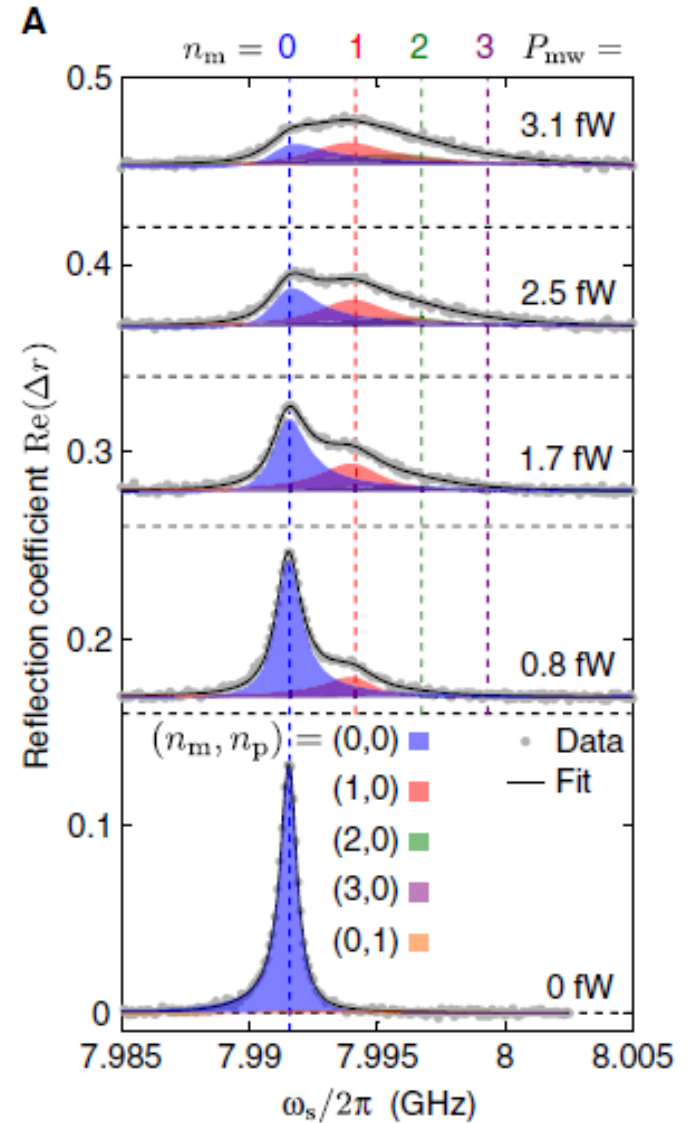
# Dispersive coupling



# “Magnon-polariton” shift



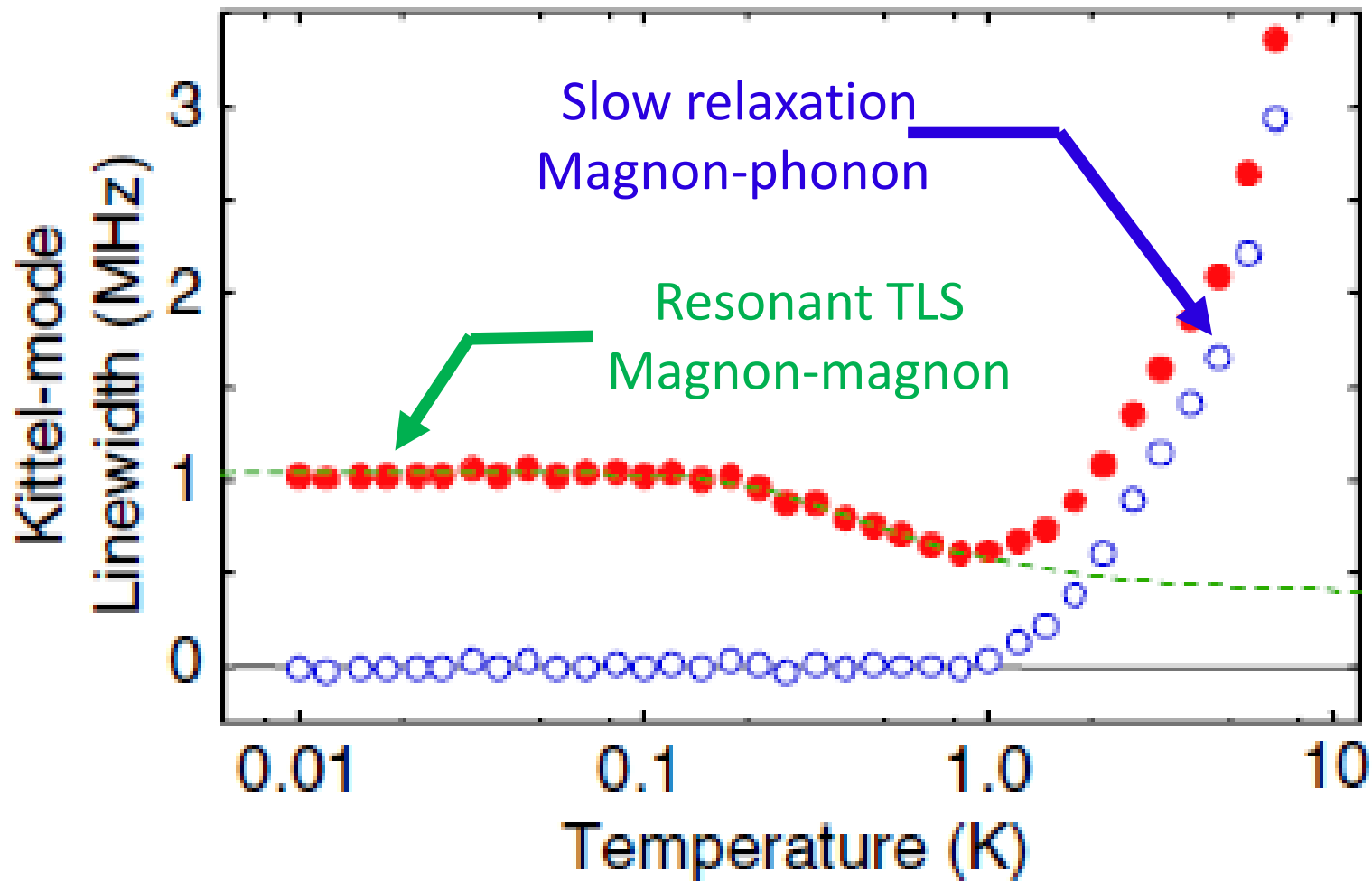
$$\chi_{q-m}/2\pi = 1.5 \text{ MHz}$$



**“Magnon-polariton” shift**

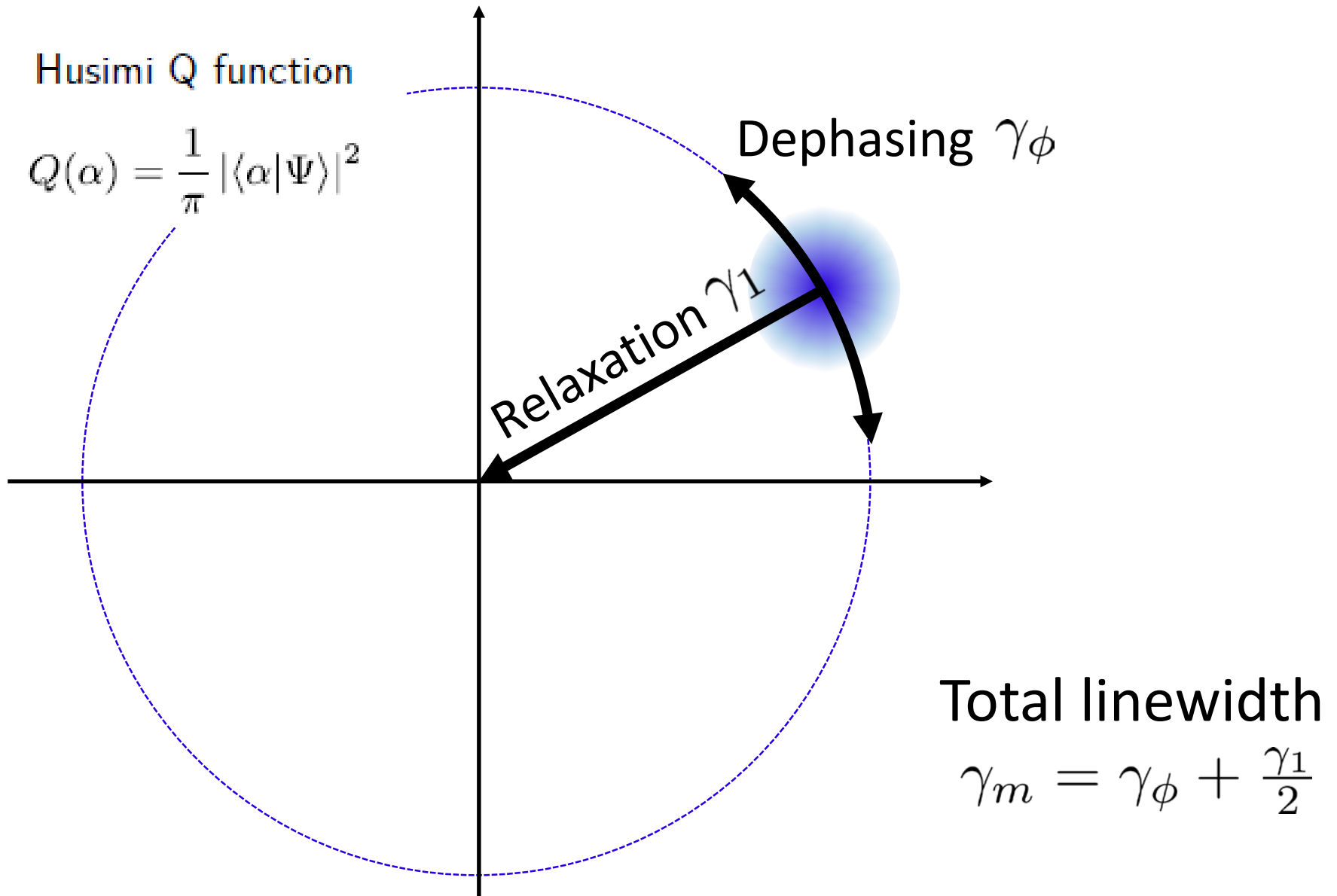
# New developments of quantum magnonics

# Magnon linewidth vs. temperature



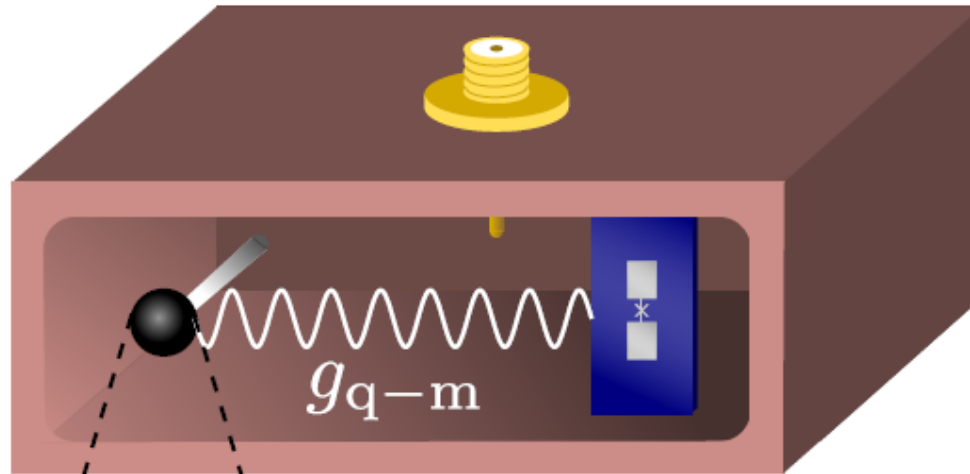


# Relaxation and dephasing



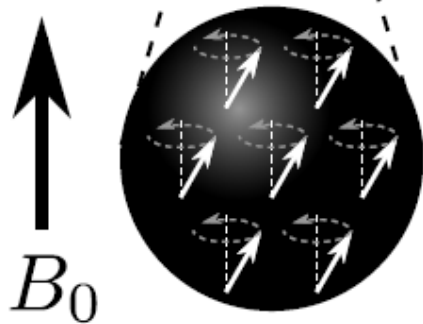
# Tomography of magnon states

Microwave cavity



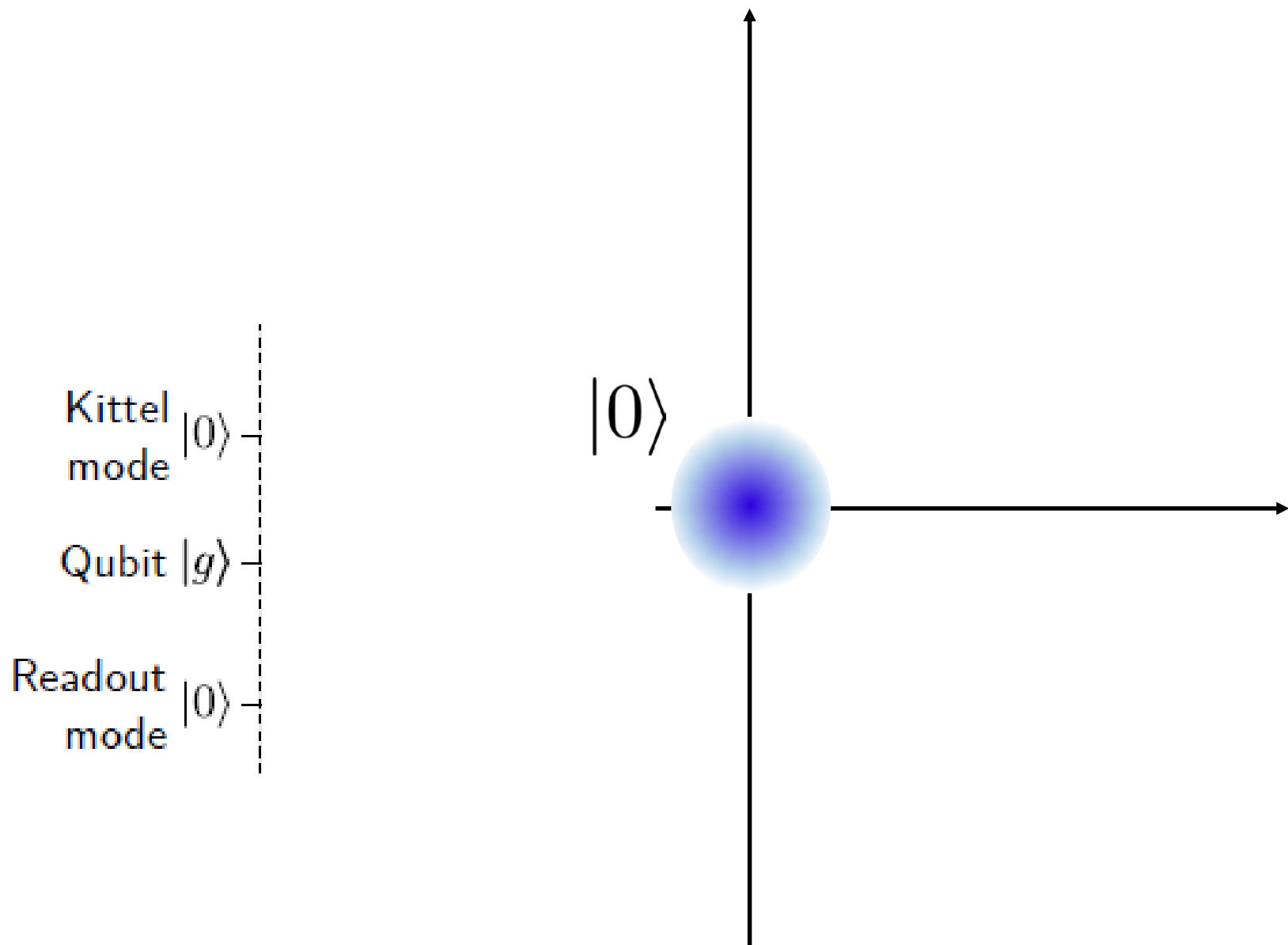
Ferromagnet

Qubit

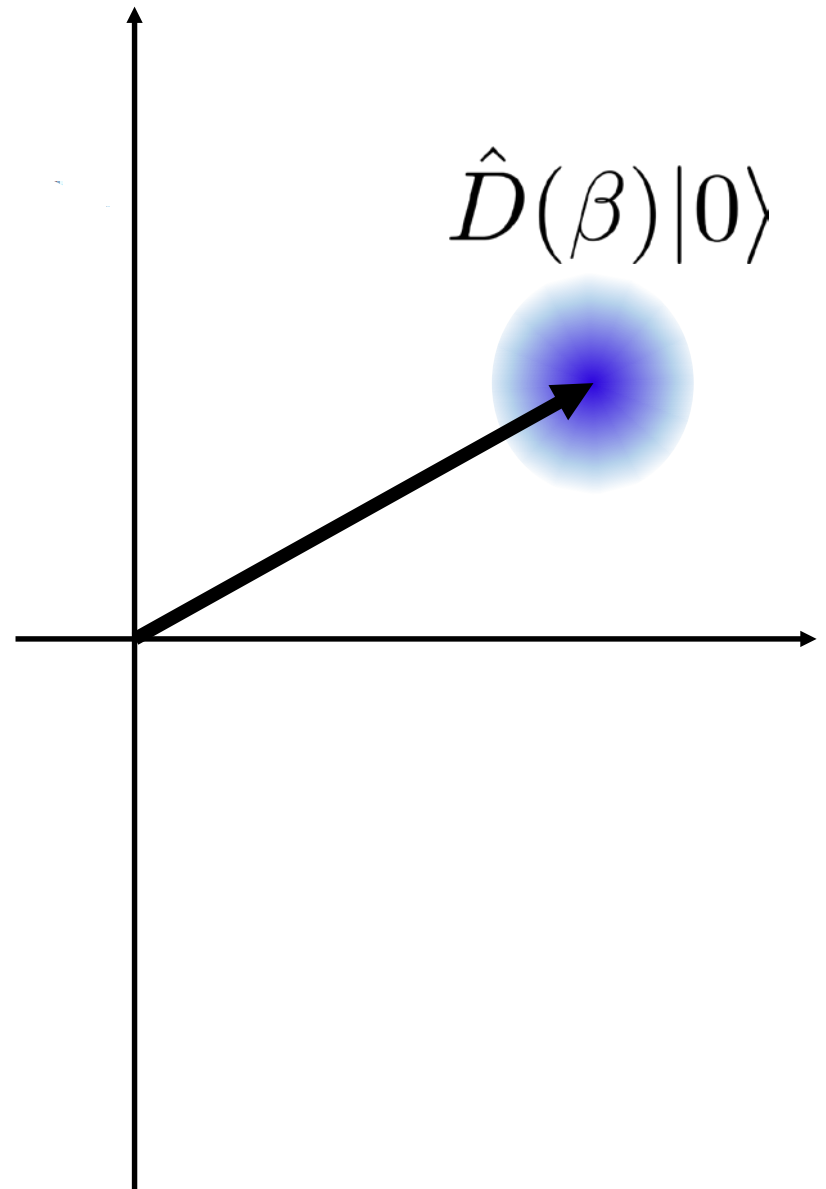
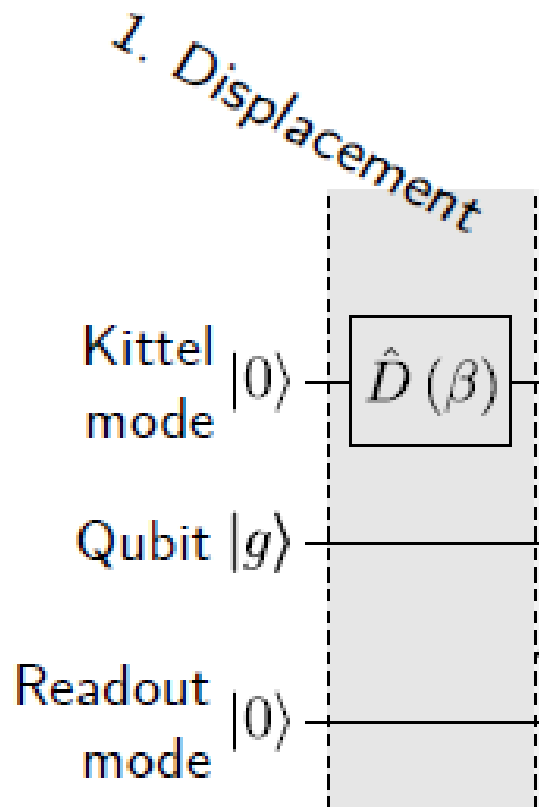


Kittel mode

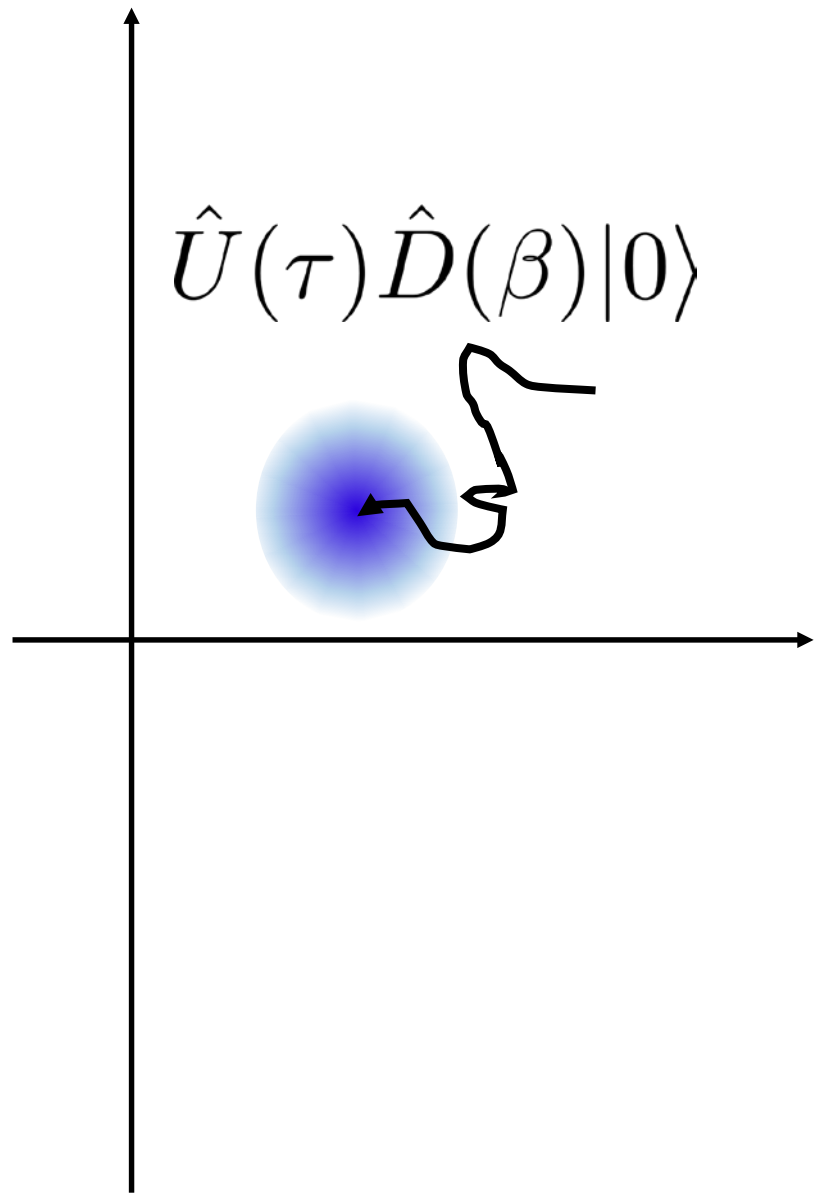
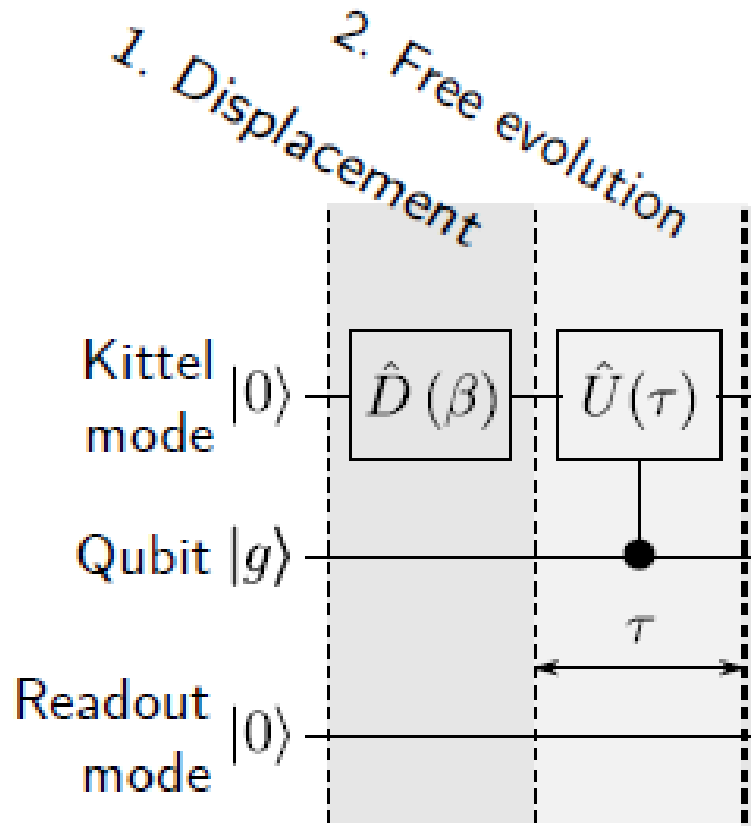
# Tomography of magnon states



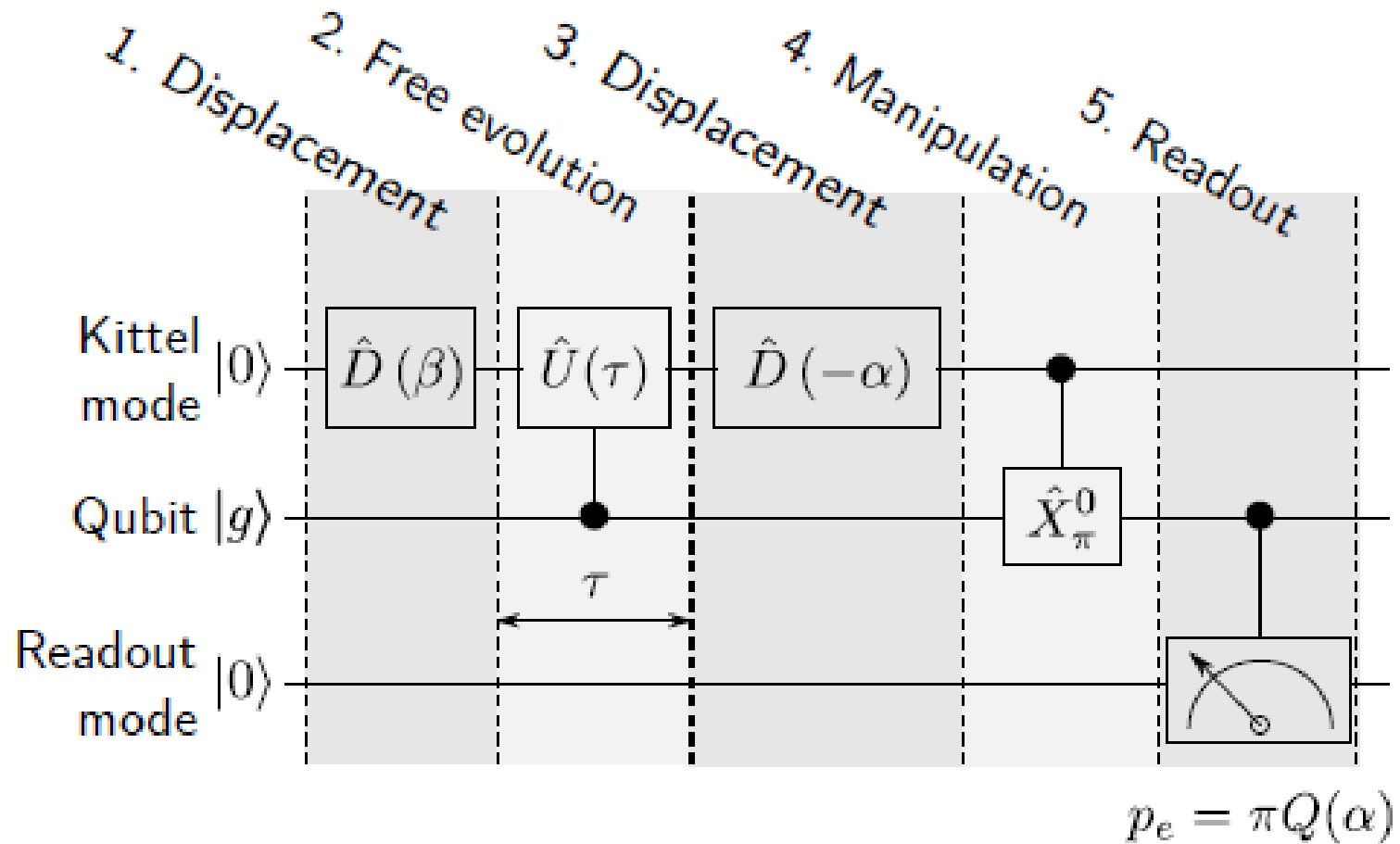
# Tomography of magnon states



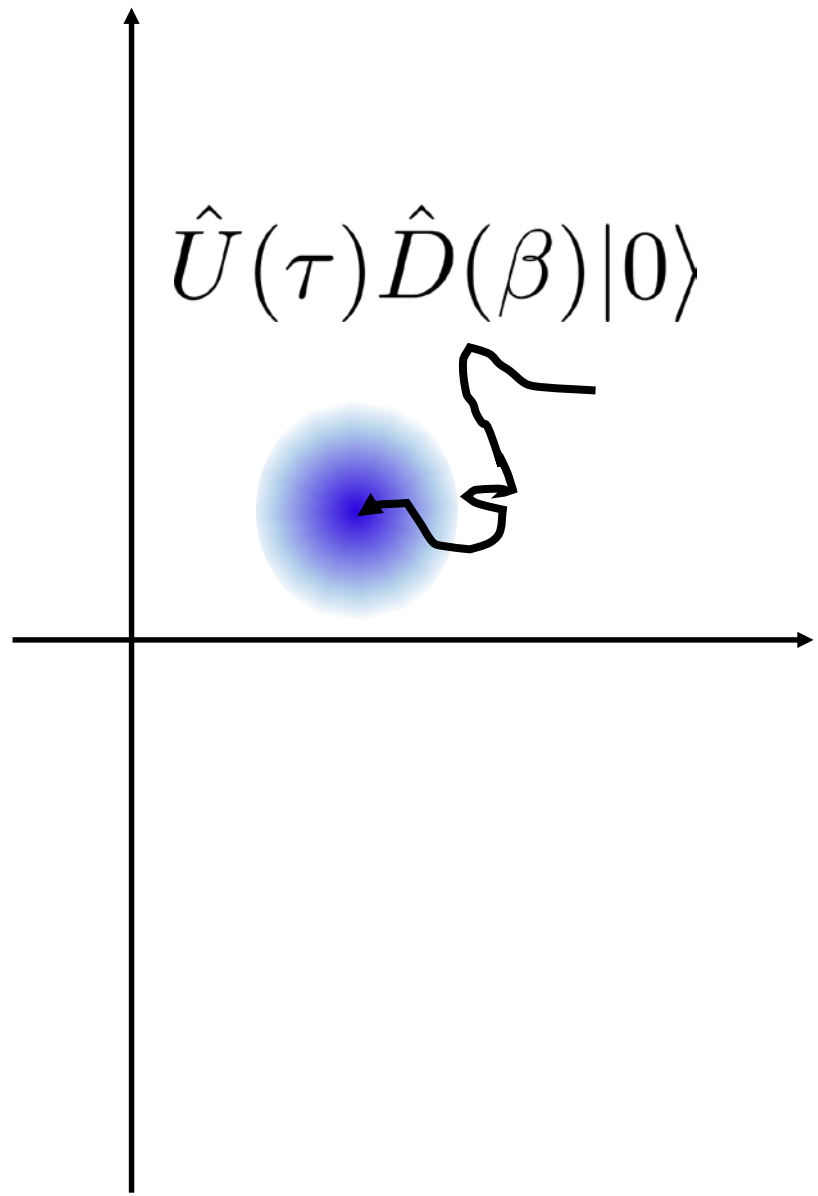
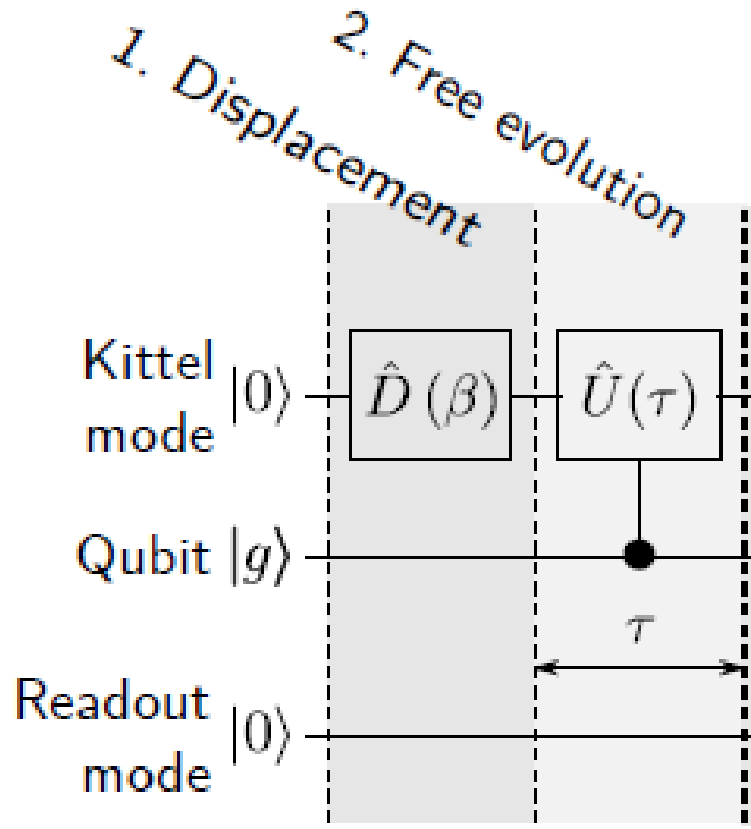
# Tomography of magnon states



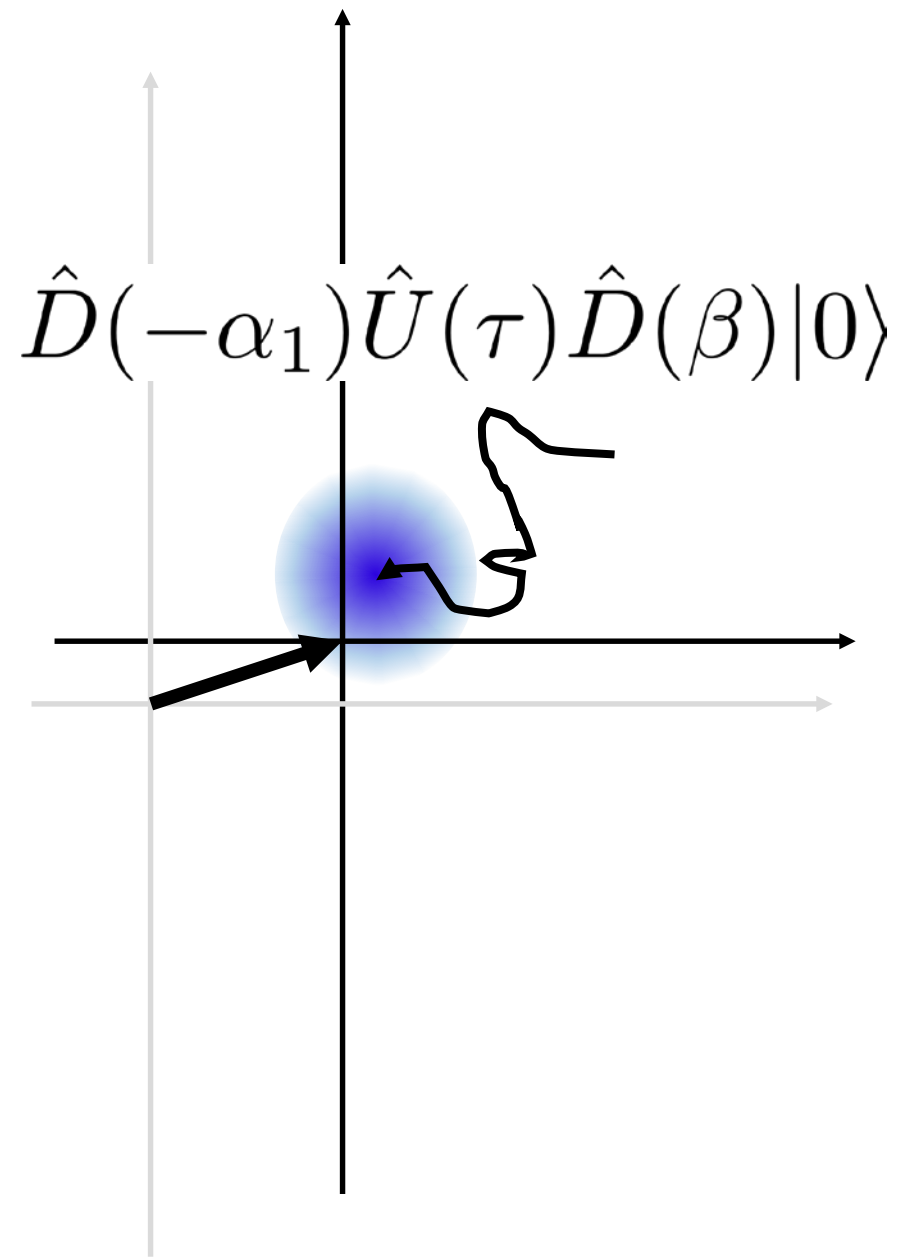
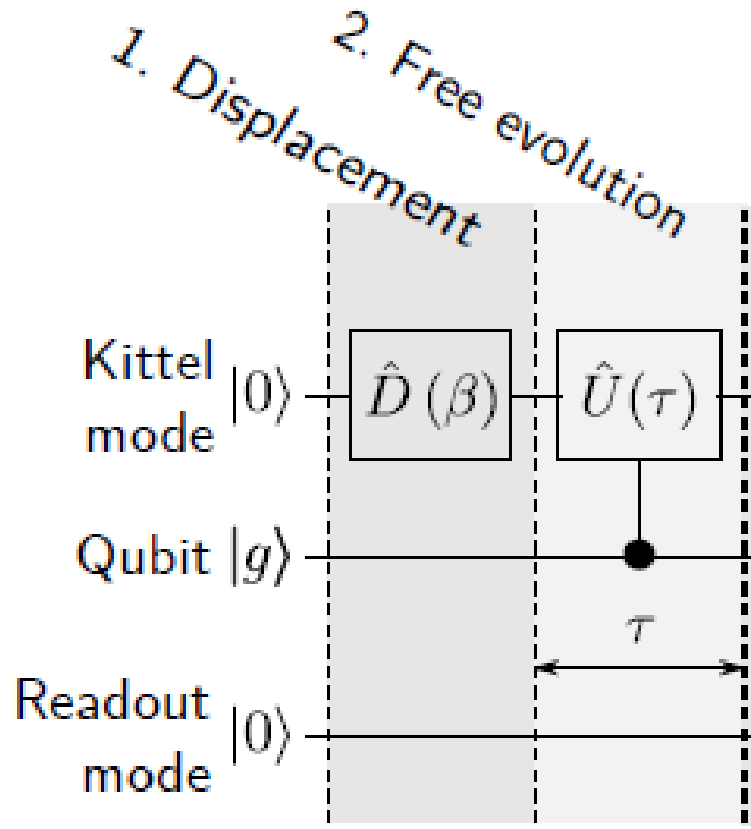
# Tomography of magnon states



# Tomography of magnon states

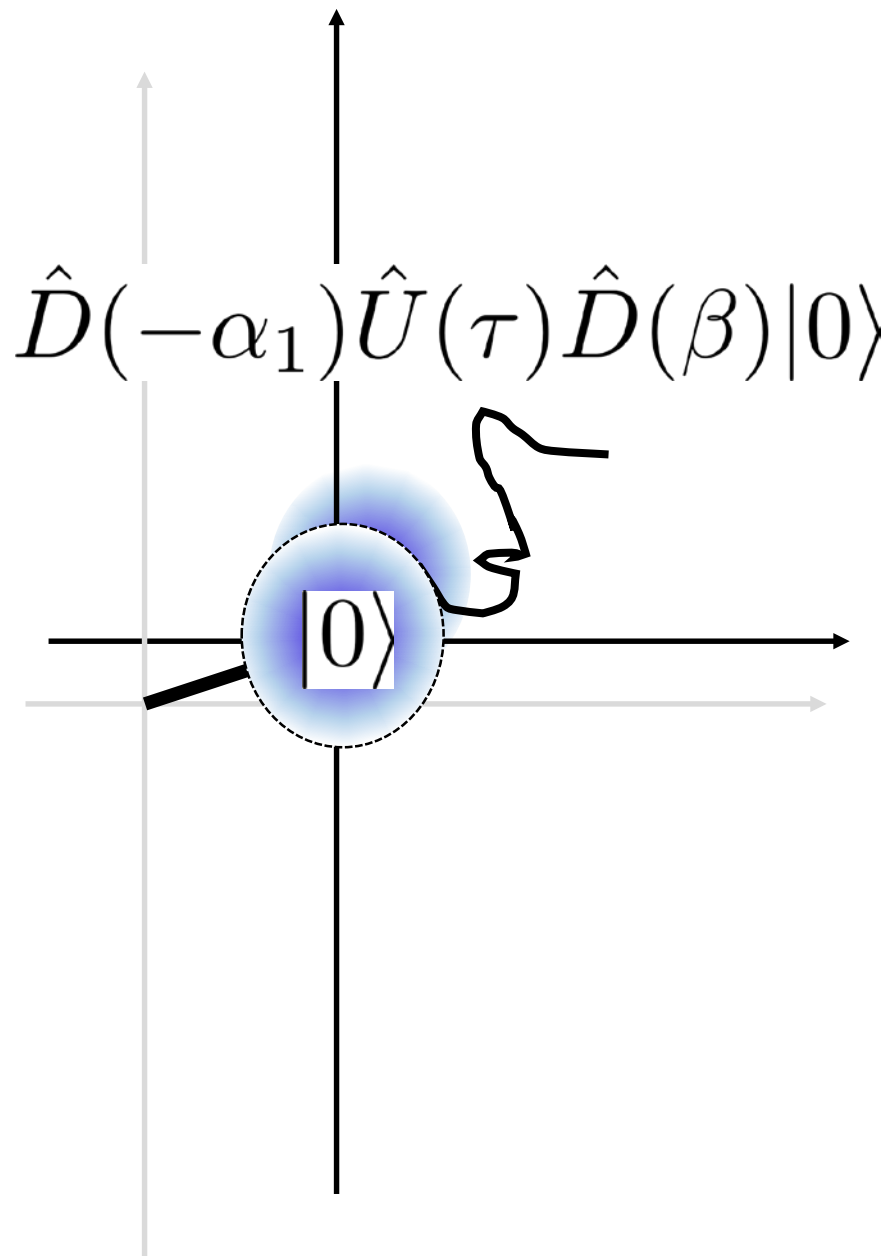
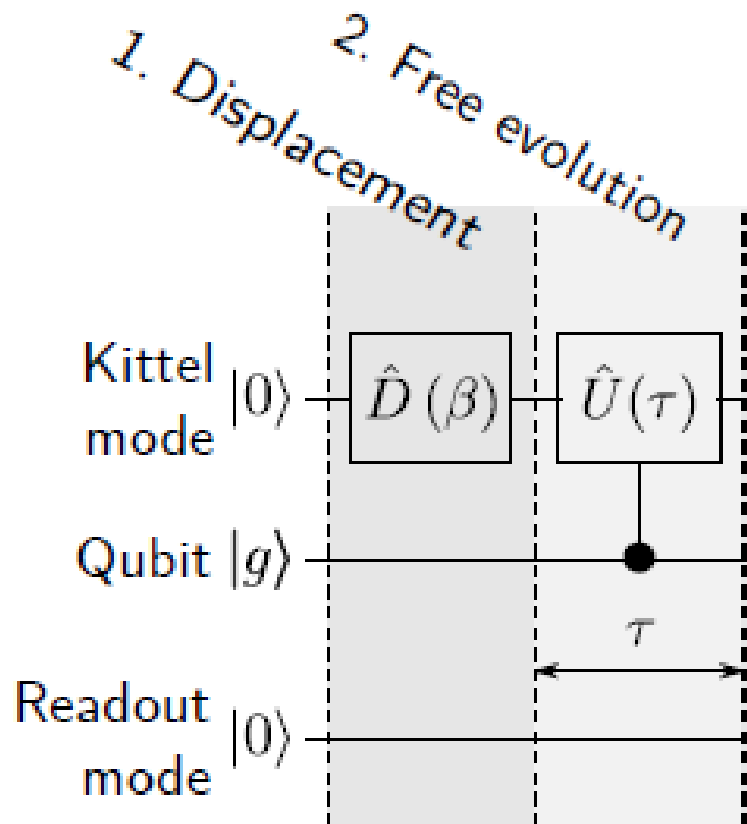


# Tomography of magnon states

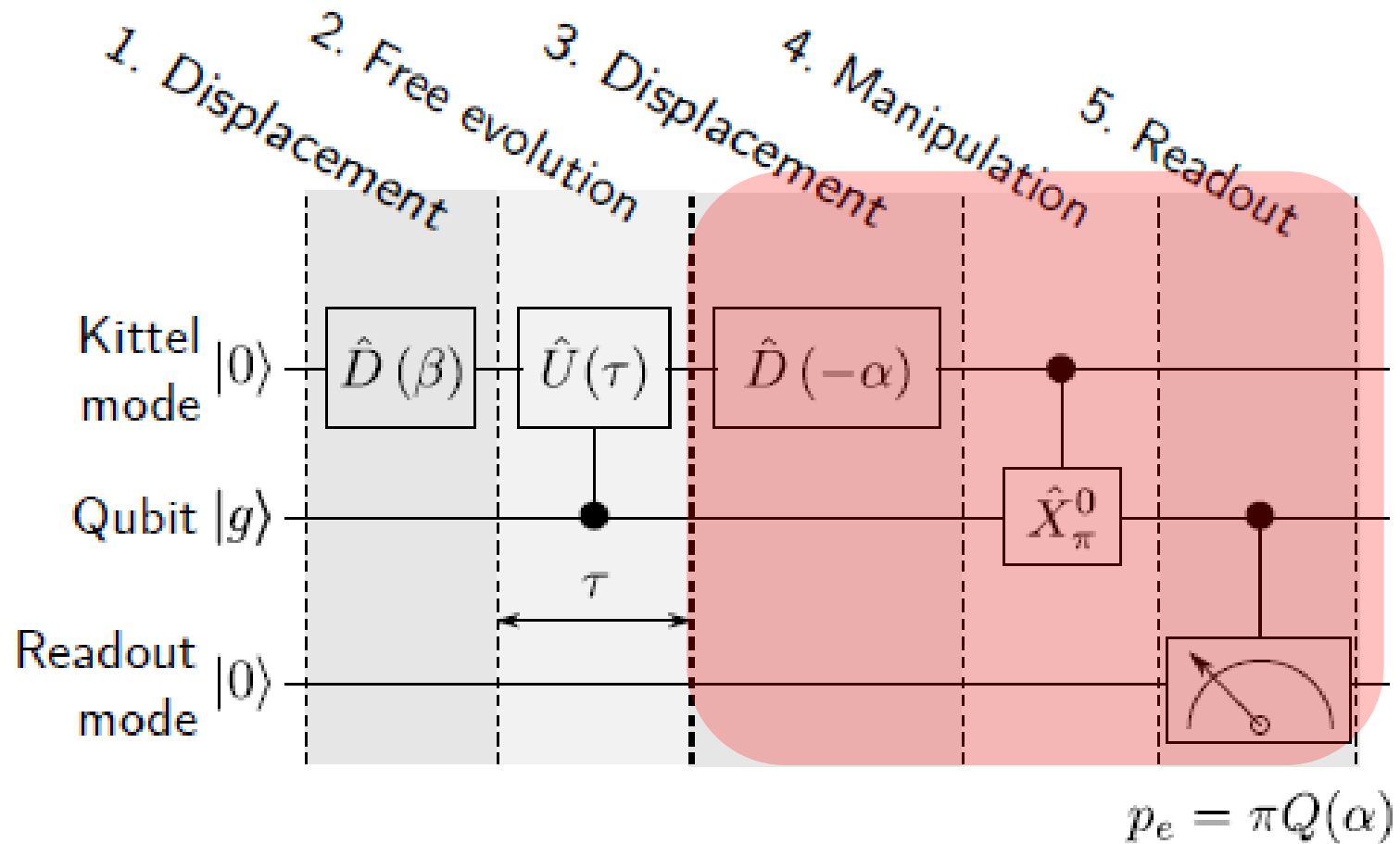




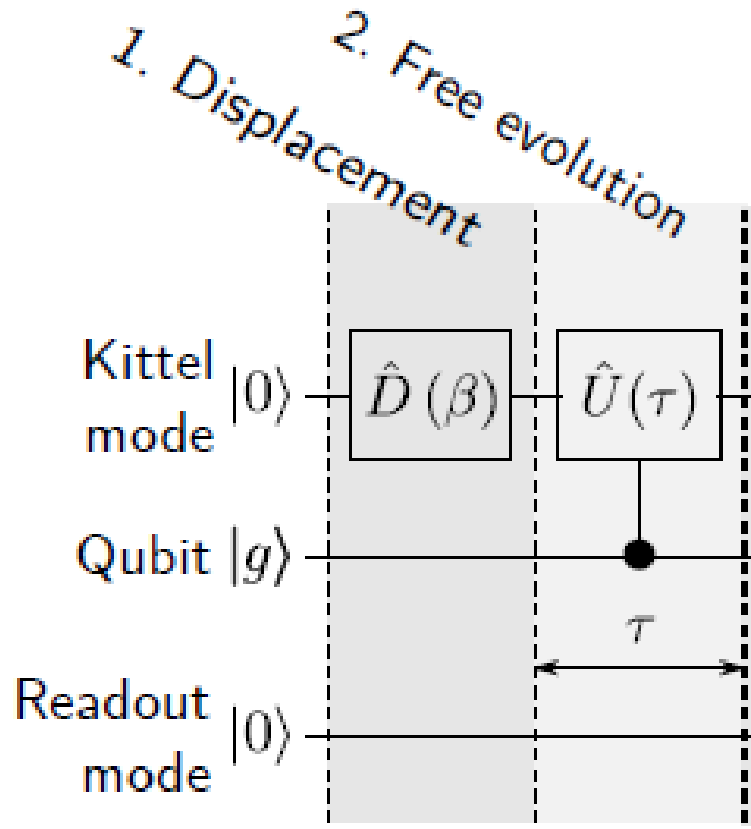
# Tomography of magnon states



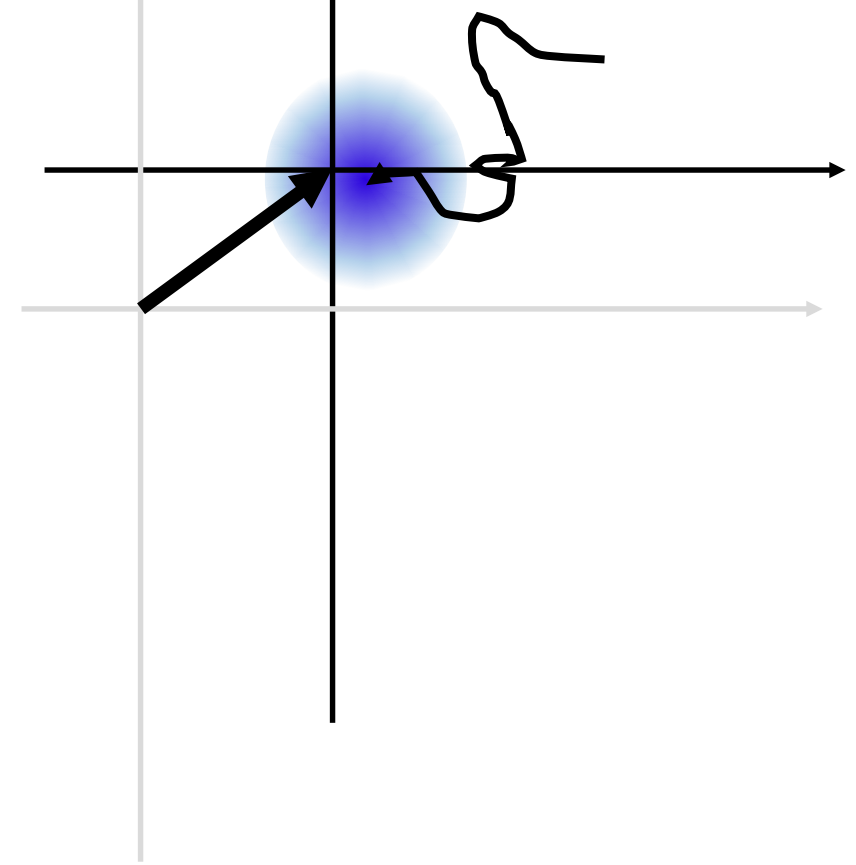
# Tomography of magnon states



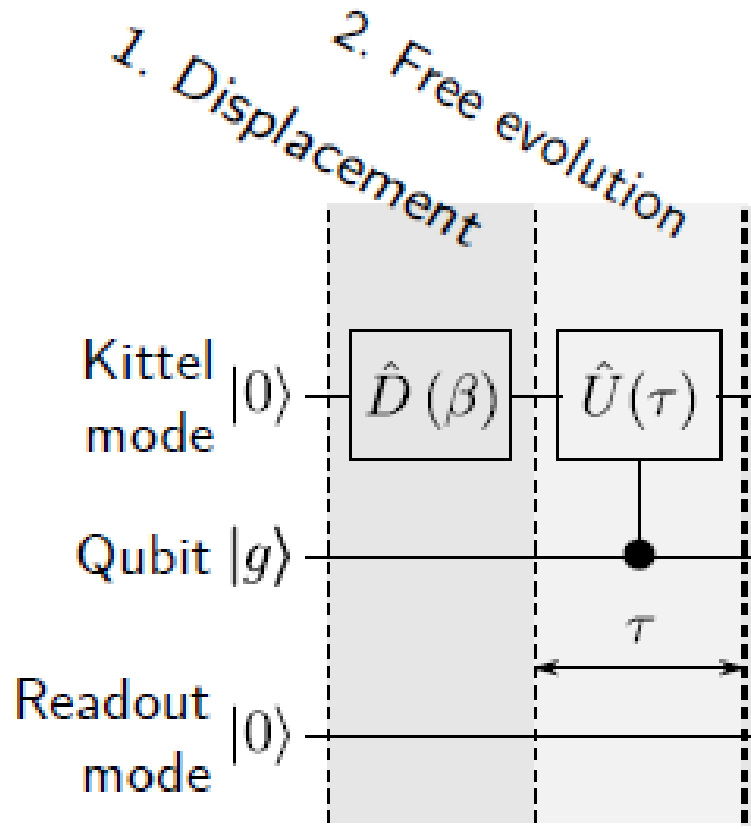
# Tomography of magnon states



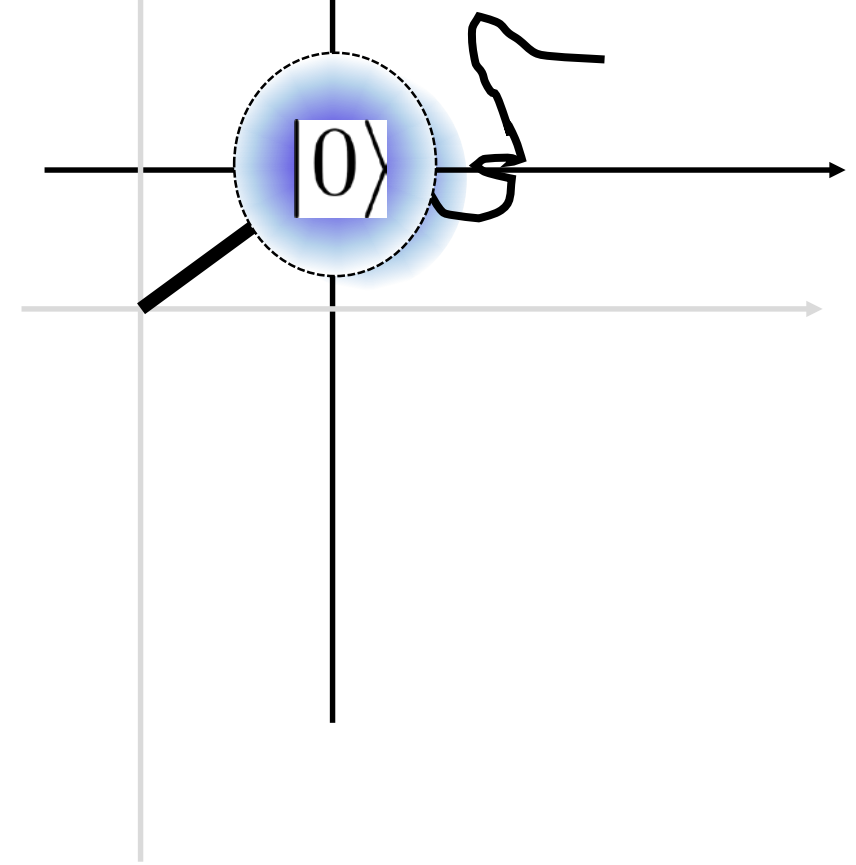
$$\hat{D}(-\alpha_2)\hat{U}(\tau)\hat{D}(\beta)|0\rangle$$



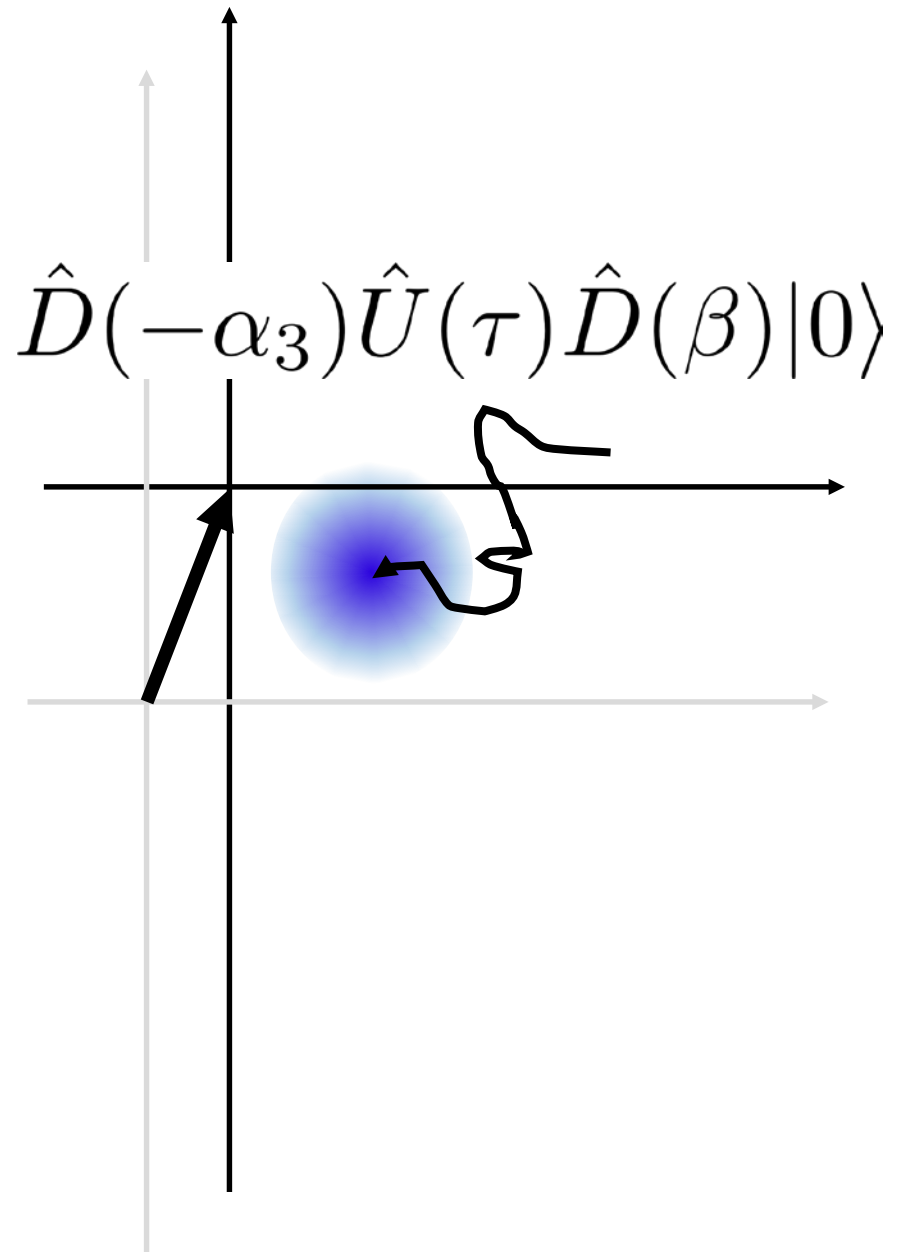
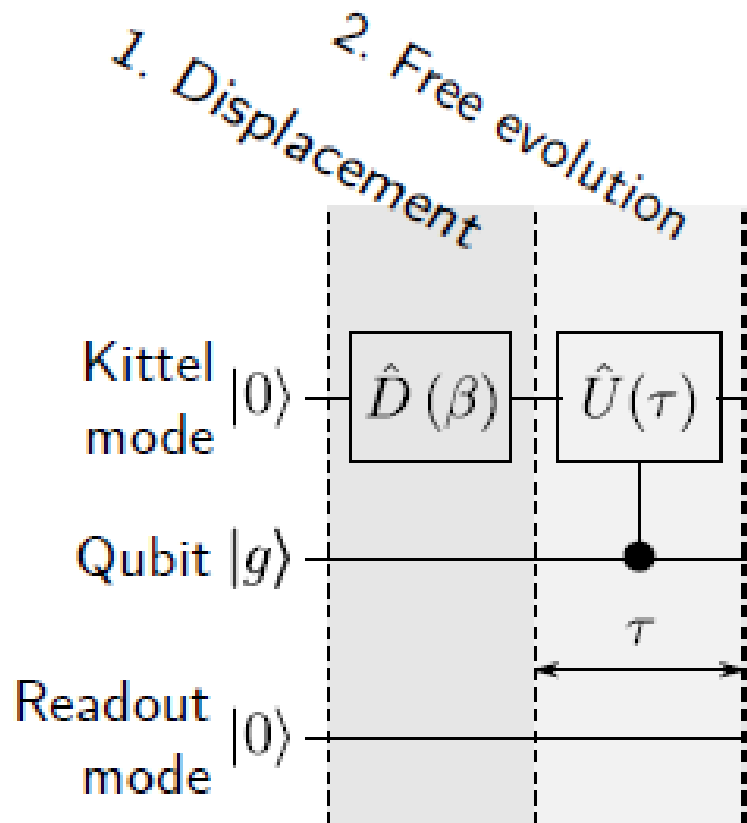
# Tomography of magnon states



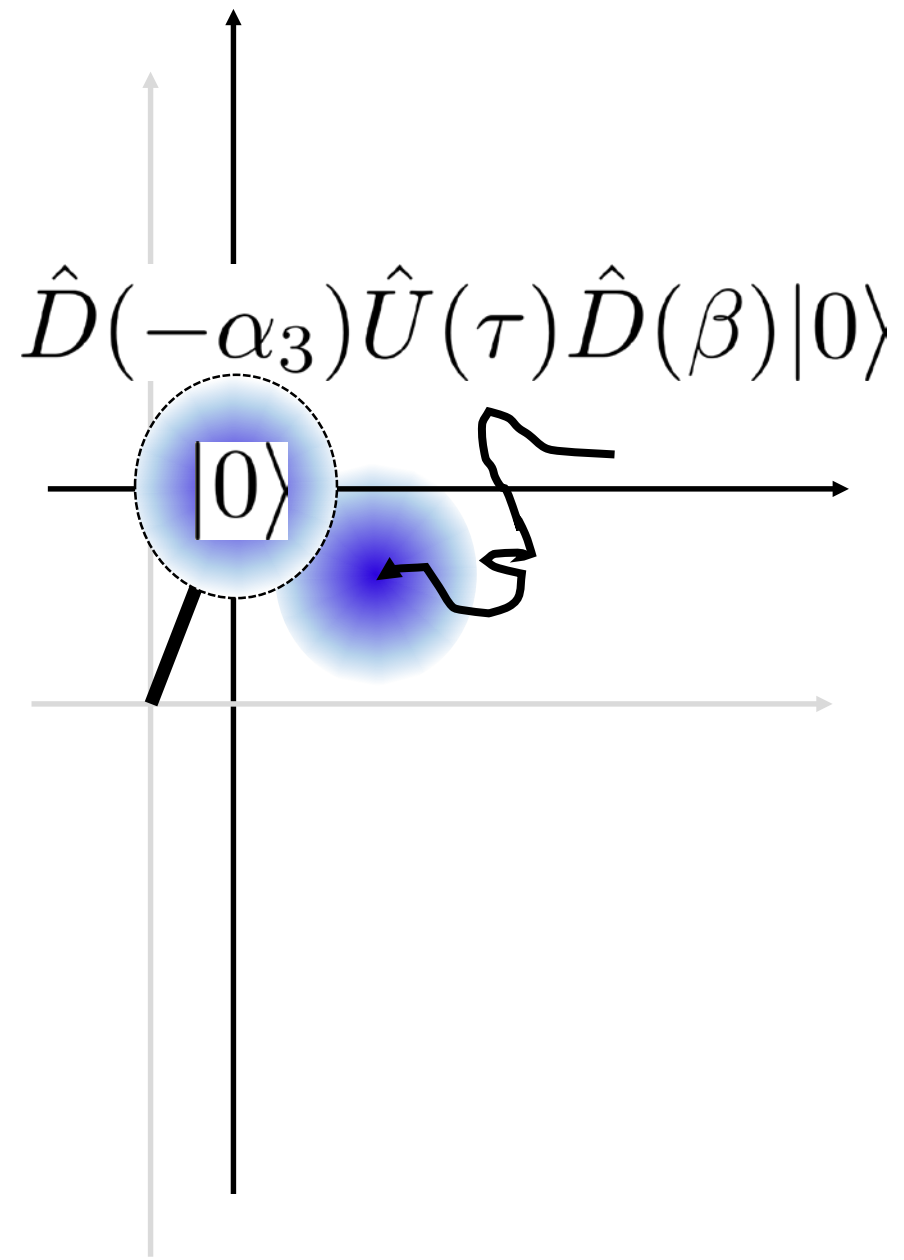
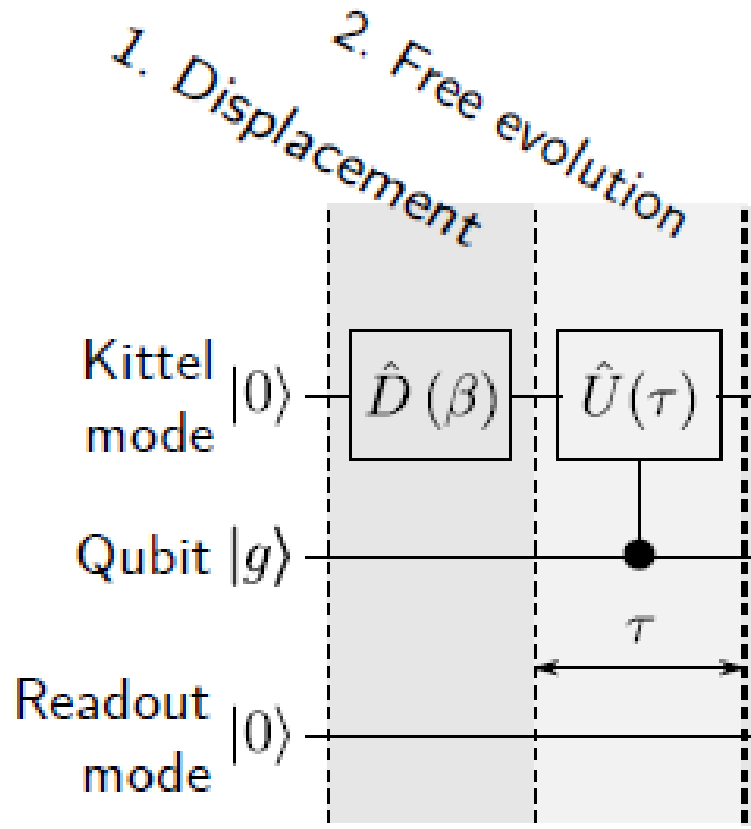
$$\hat{D}(-\alpha_2)\hat{U}(\tau)\hat{D}(\beta)|0\rangle$$



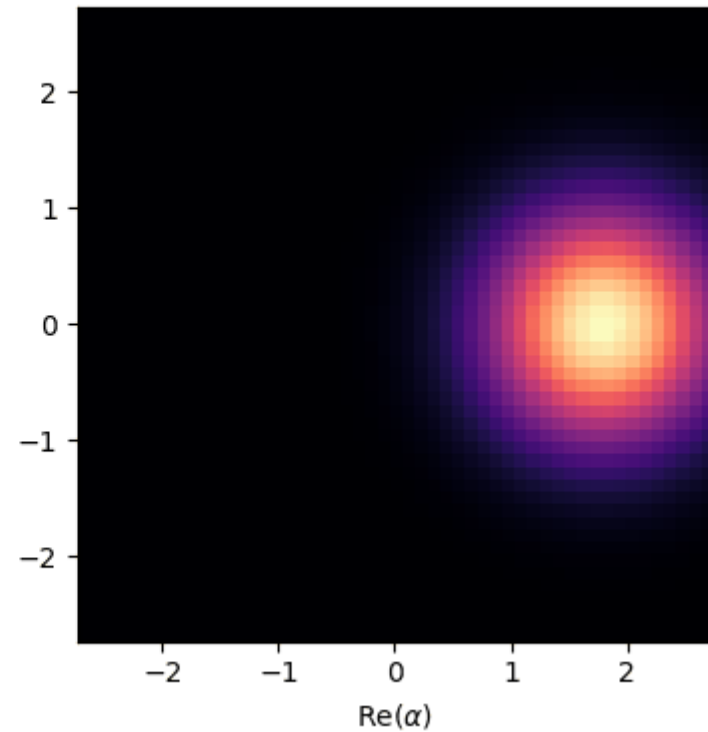
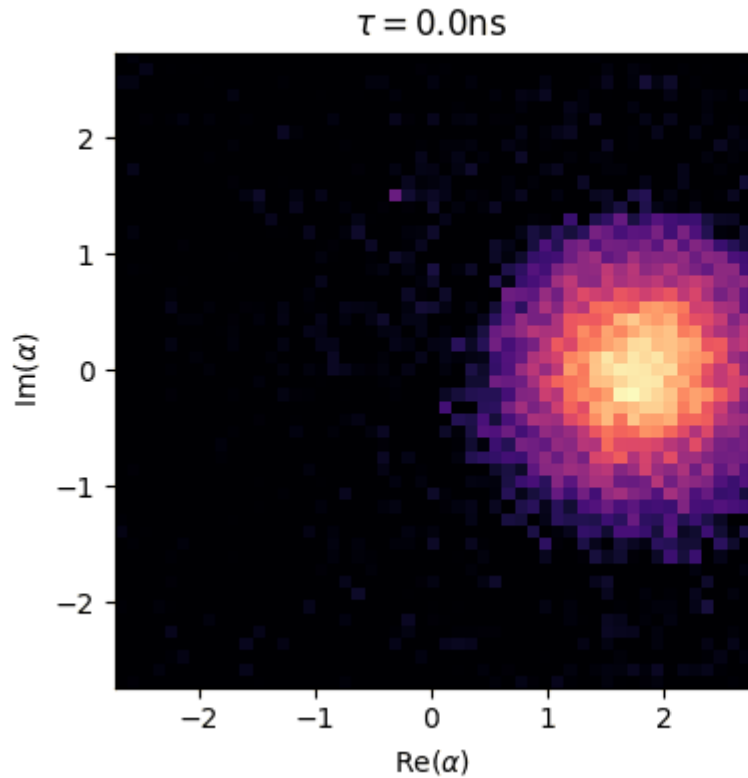
# Tomography of magnon states



# Tomography of magnon states

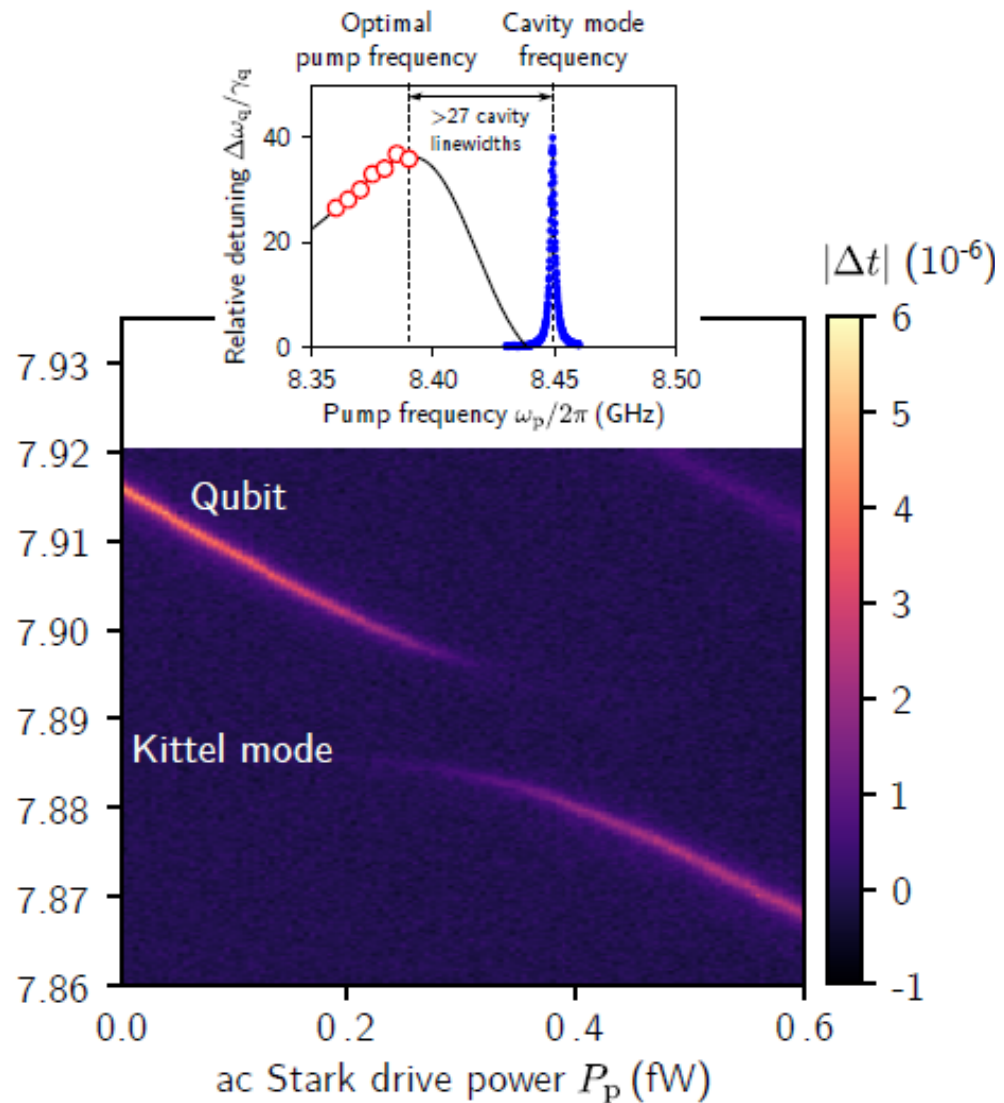
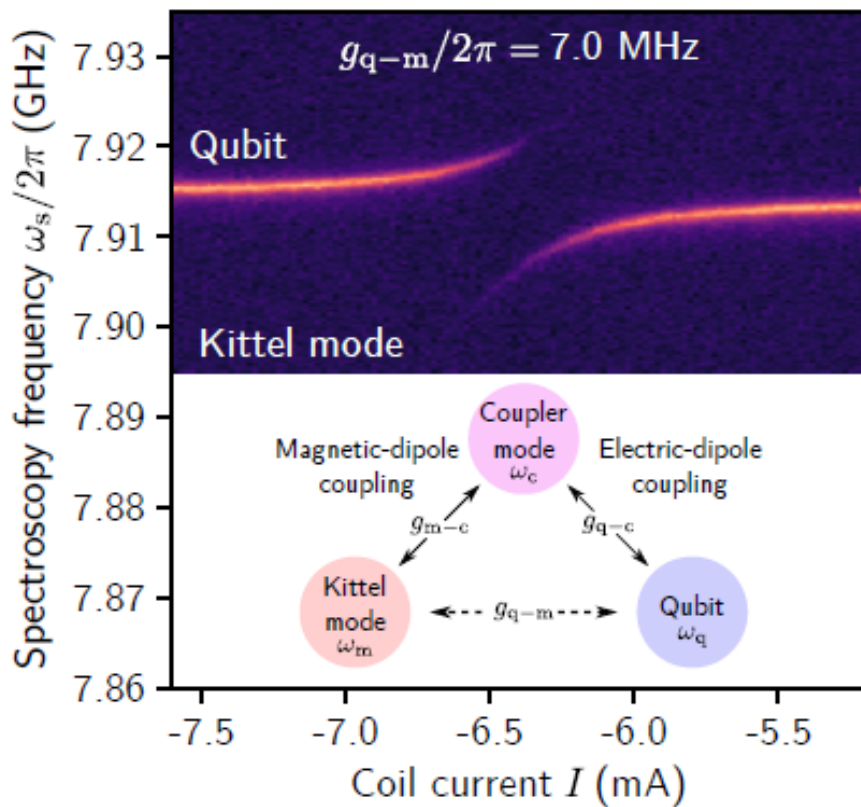


# Tomography of magnon states



# From coil current to ac-Stark drive

Statically tunable detuning



Idea from Patrice Bertet, thanks!



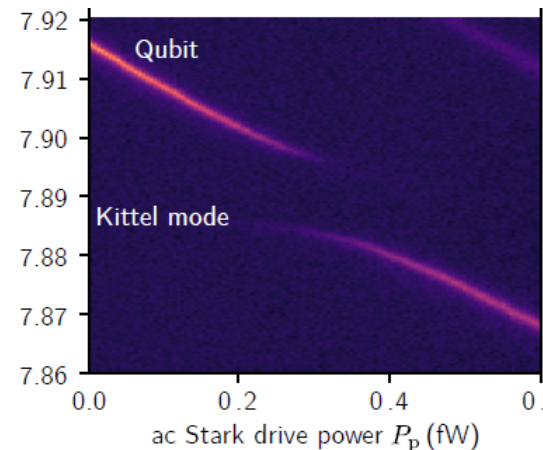
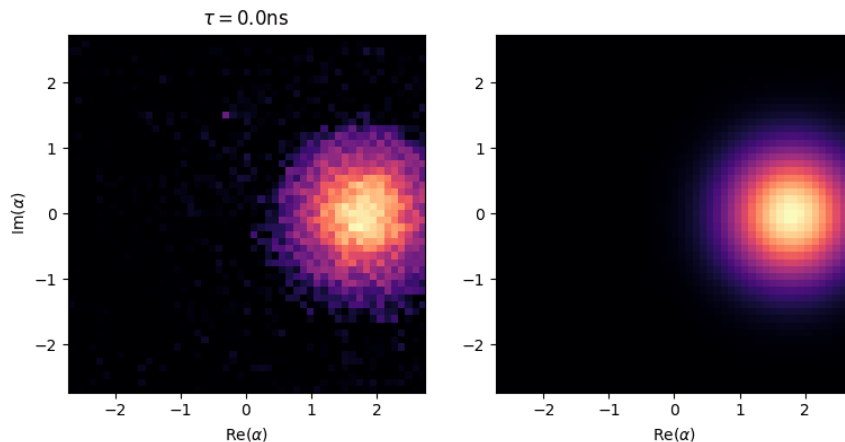
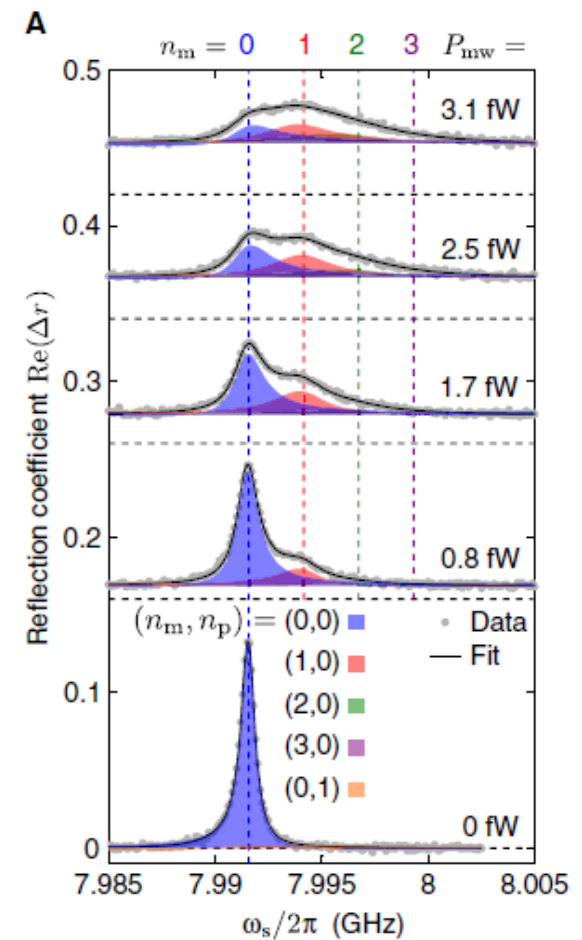
# Summary

## Quantum magnonics with ferromagnet

- Strong coupling with microwave cavity
- Vacuum Rabi splitting
- Magnon-number-resolving spectroscopy

## In progress

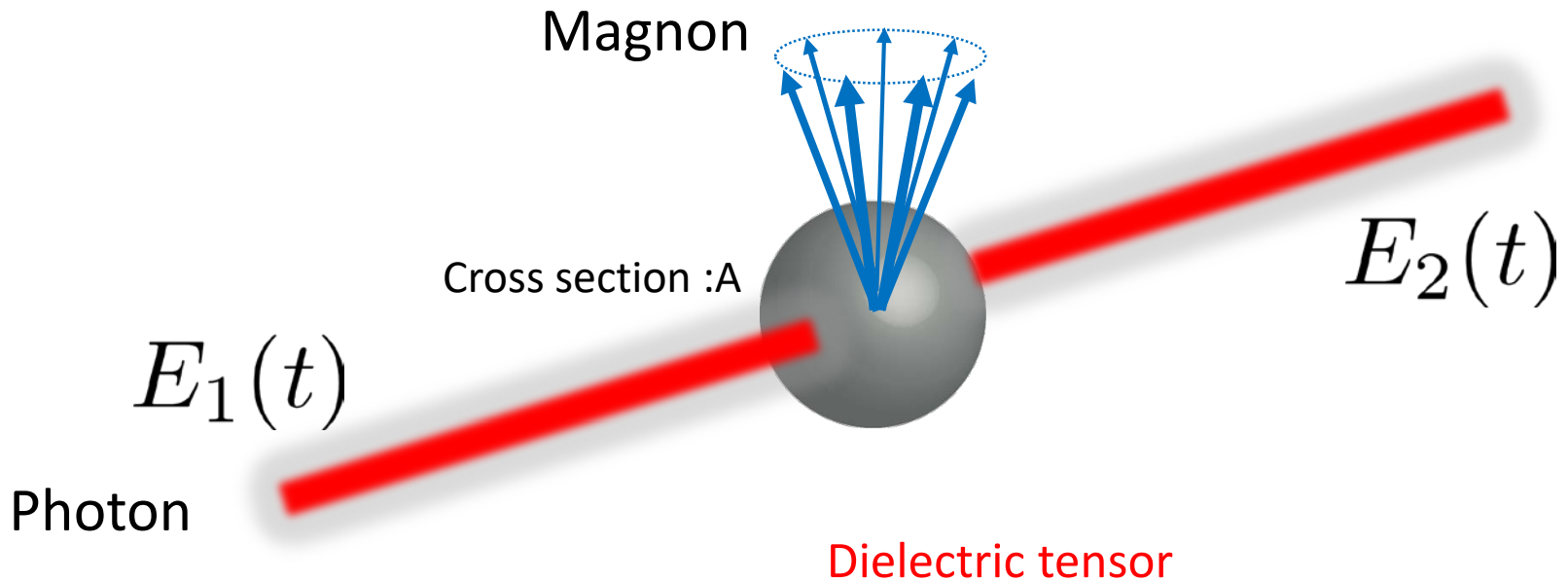
- Magnon-state tomography
- Quantum magnon state generation



Cavity (circuit) quantum magnonics

New possibility of optomagnonics

# Effective Hamiltonian



$$H_I = \frac{1}{2} \int_0^T E_2^*(t) \epsilon(\tilde{t}) E_1(t) A c dt$$

W. Happer and B.S. Mathur, Phys. Rev. **163**, 12 (1967);

J.M. Geremia, J.K. Stockton, and H. Mabuchi, PRA **73**, 042112 (2006).

K. Hammerer, A.S.Sorensen, and E.S.Polzik, Rev. Mod. Phys. **82**, 1041 (2010).

# Symmetry requirements of $\tilde{\epsilon}$

Hermiticity (losslessness):

$$\begin{aligned}\tilde{\epsilon} &= \tilde{\epsilon}_R + i\tilde{\epsilon}_I \\ \tilde{\epsilon}^\dagger &= \tilde{\epsilon}_R^\dagger - i\tilde{\epsilon}_I^\dagger\end{aligned}$$

$$\tilde{\epsilon}_R = \tilde{\epsilon}_R^\dagger \quad (\text{symmetric})$$

$$\tilde{\epsilon}_I = -\tilde{\epsilon}_I^\dagger \quad (\text{anti-symmetric})$$

# Symmetry requirements of $\tilde{\epsilon}$

Reciprocity:

$$\tilde{\epsilon}_{ij}(\mathbf{M}) = \tilde{\epsilon}_{ji}(-\mathbf{M})$$

Symmetric part:  $\tilde{\epsilon}_R \Rightarrow$  even powers of  $\mathbf{M}$

Anti-symmetric part:  $\tilde{\epsilon}_I \Rightarrow$  odd powers of  $\mathbf{M}$

# Dielectric tensor

$$\tilde{\epsilon} = \tilde{\epsilon}_0 + \tilde{\epsilon}_1 + \tilde{\epsilon}_2,$$

## Scalar light shift (symmetric)

$$\tilde{\epsilon}_0 = \epsilon_0 \epsilon_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Sigma_{xx} = M_x M_x$$

$$\Sigma_{yy} = M_y M_y$$

$$\Sigma_{zz} = M_z M_z$$

$$\Sigma_{xy} = \frac{1}{2} (M_x M_y + M_y M_x)$$

$$\Sigma_{yz} = \frac{1}{2} (M_y M_z + M_z M_y)$$

$$\Sigma_{zx} = \frac{1}{2} (M_z M_x + M_x M_z)$$

## Vector light shift (anti-symmetric)

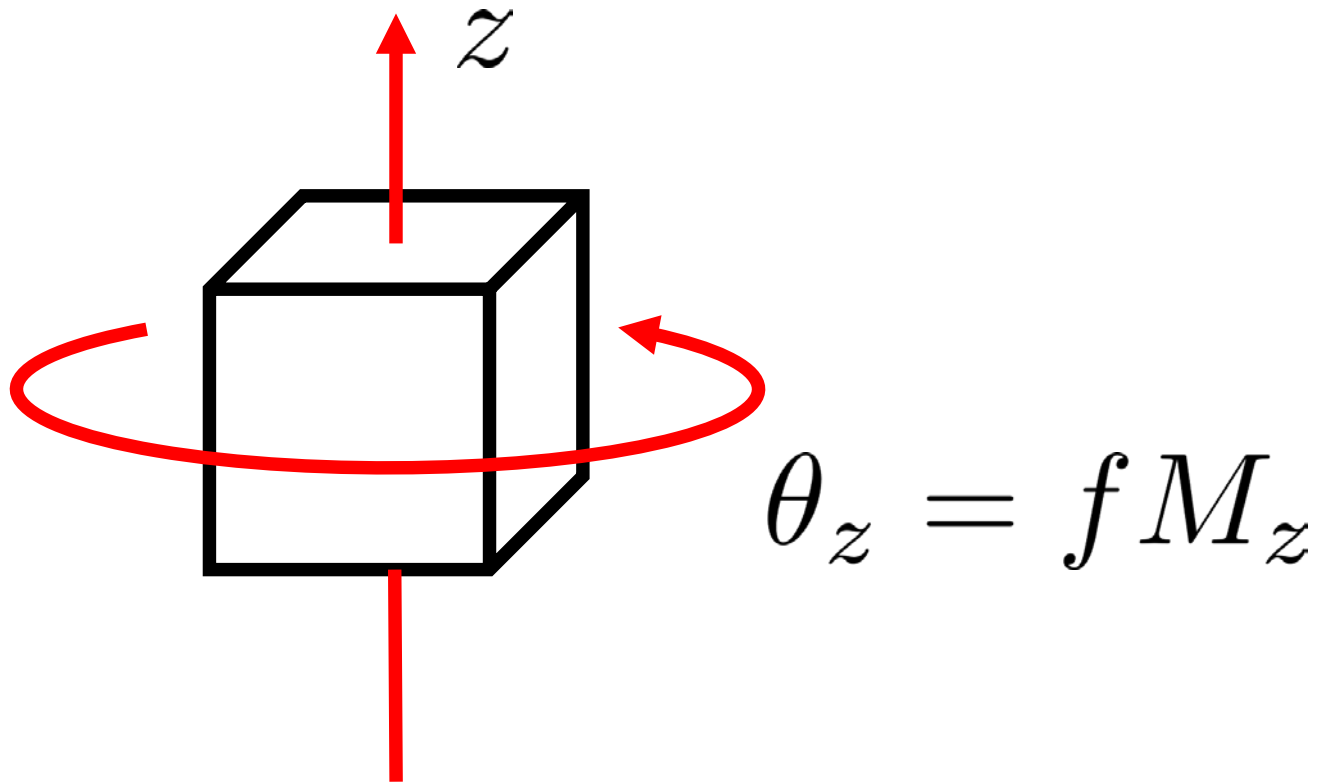
$$\tilde{\epsilon}_1 = \begin{bmatrix} 0 & -ifM_z & ifM_y \\ ifM_z & 0 & -ifM_x \\ -ifM_y & ifM_x & 0 \end{bmatrix}$$

## Tensor light shift (symmetric)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

# Vector light shift (Faraday effect)

$$\tilde{\epsilon}_1 = \begin{bmatrix} 0 & -ifM_z & ifM_y \\ ifM_z & 0 & -ifM_x \\ -ifM_y & ifM_x & 0 \end{bmatrix} \quad \textit{Rotation matrix}$$



# Tensor light shift (Cotton-Mouton effect)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

*Stress tensor*

*Stiffness matrix*

*Strain tensor*

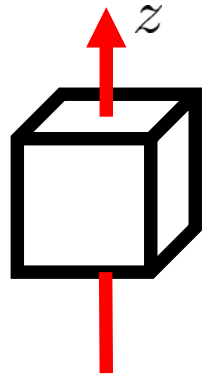
$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{12} & 0 & 0 & 0 \\ G_{12} & G_{11} & G_{12} & 0 & 0 & 0 \\ G_{12} & G_{12} & G_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{44} \end{bmatrix} \begin{bmatrix} \Sigma_{xx} \\ \Sigma_{yy} \\ \Sigma_{zz} \\ 2\Sigma_{yz} \\ 2\Sigma_{zx} \\ 2\Sigma_{xy} \end{bmatrix}$$

For **cubic** crystal like YIG (H // 100)



DC response

# DC response



## Scalar light shift

$$\tilde{\epsilon}_0 = \epsilon_0 \epsilon_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Vector light shift (anti-symmetric tensor)

$$\tilde{\epsilon}_1 = \begin{bmatrix} 0 & -ifM_z & ifM_y \\ ifM_z & 0 & -ifM_x \\ -ifM_y & ifM_x & 0 \end{bmatrix}$$

## Tensor light shift (symmetric tensor)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

$$\Sigma_{xx} = M_x M_x$$

$$\Sigma_{yy} = M_y M_y$$

$$\Sigma_{zz} = M_z M_z$$

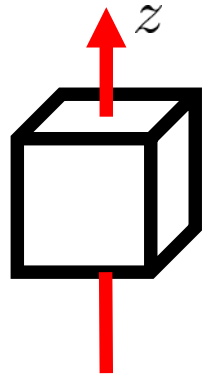
$$\Sigma_{xy} = \frac{1}{2} (M_x M_y + M_y M_x)$$

$$\Sigma_{yz} = \frac{1}{2} (M_y M_z + M_z M_y)$$

$$\Sigma_{zx} = \frac{1}{2} (M_z M_x + M_x M_z)$$

One-magnon processes

# One-magnon process



## Scalar light shift

$$\tilde{\epsilon}_0 = \epsilon_0 \epsilon_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Vector light shift (anti-symmetric tensor)

$$\tilde{\epsilon}_1 = \begin{bmatrix} 0 & -ifM_z & ifM_y \\ ifM_z & 0 & -ifM_x \\ -ifM_y & ifM_x & 0 \end{bmatrix}$$

## Tensor light shift (symmetric tensor)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

$$\Sigma_{xx} = M_x M_x$$

$$\Sigma_{yy} = M_y M_y$$

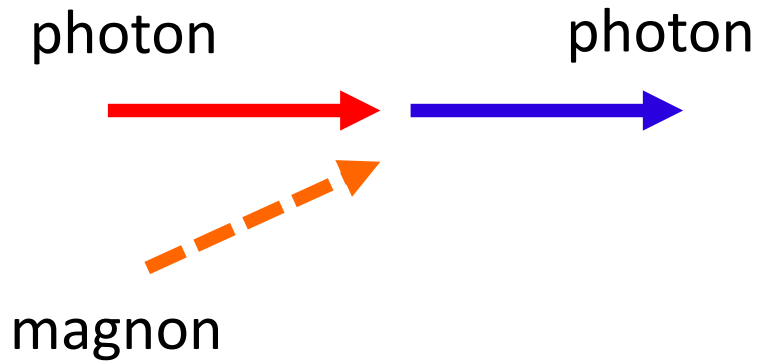
$$\Sigma_{zz} = M_z M_z$$

$$\Sigma_{xy} = \frac{1}{2} (M_x M_y + M_y M_x)$$

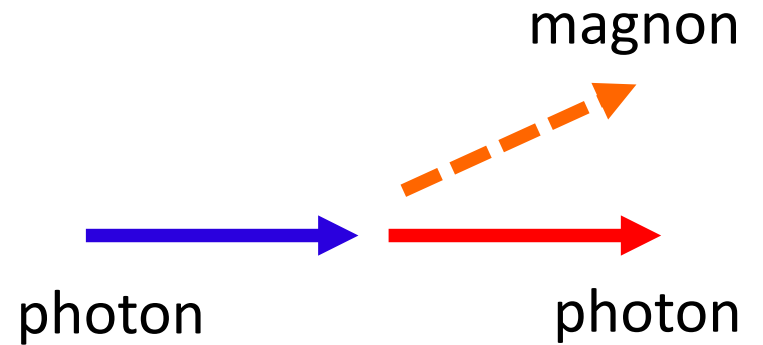
$$\Sigma_{yz} = \frac{1}{2} (M_y M_z + M_z M_y)$$

$$\Sigma_{zx} = \frac{1}{2} (M_z M_x + M_x M_z)$$

# one-magnon processes

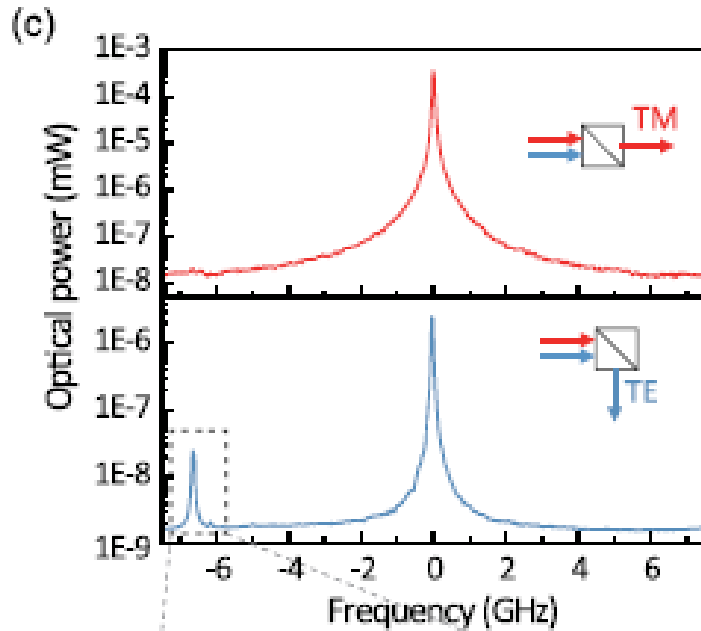


Anti-Stokes scattering

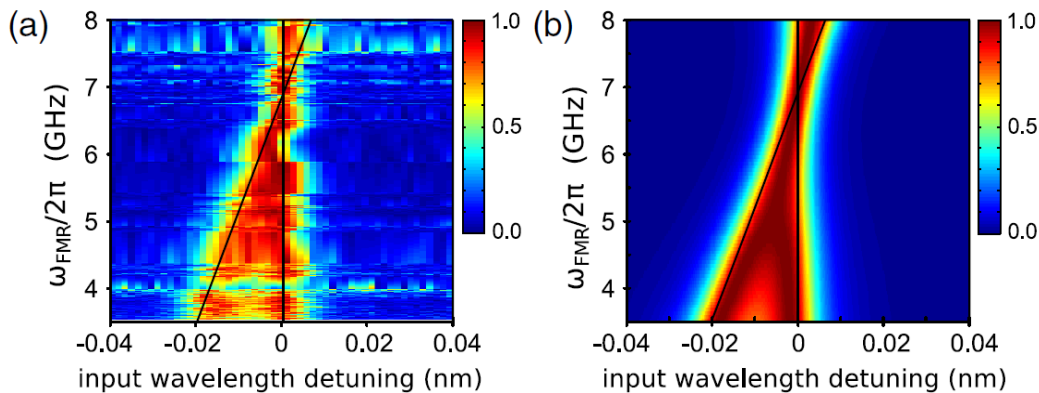


Stokes scattering

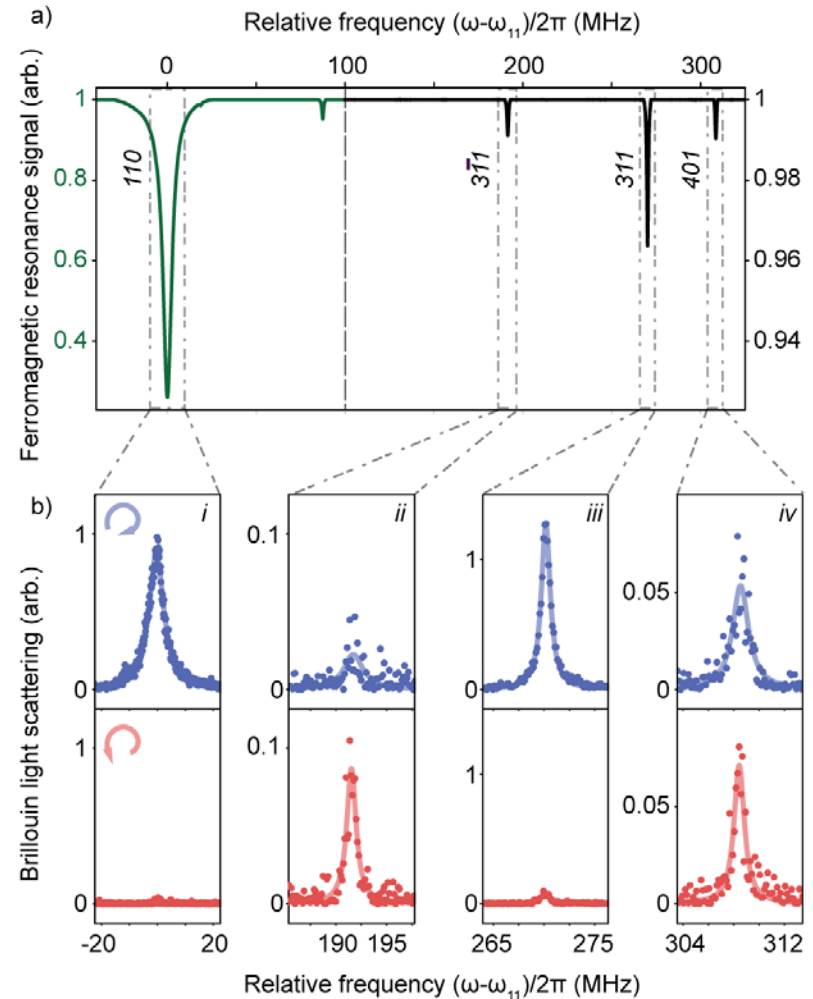
# Cavity optomagnonics



X. Zhang *et al.*, PRL **117**, 123605 (2016)



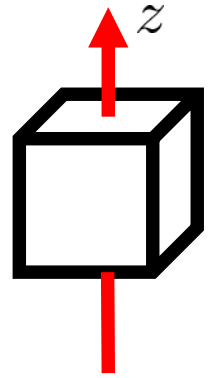
J.A. Haigh *et al.*, PRL **117**, 133602 (2016)



A. Osada *et al.*, PRL **120**, 133602 (2018)

Two-magnon processes

# Two-magnon process



## Scalar part

$$\tilde{\epsilon}_0 = \epsilon_0 \epsilon_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Vector part (anti-symmetric)

$$\tilde{\epsilon}_1 = \begin{bmatrix} 0 & -ifM_z & ifM_y \\ ifM_z & 0 & -ifM_x \\ -ifM_y & ifM_x & 0 \end{bmatrix}$$

## Tensor part (symmetric)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

$$\Sigma_{xx} = M_x M_x$$

$$\Sigma_{yy} = M_y M_y$$

$$\Sigma_{zz} = M_z M_z$$

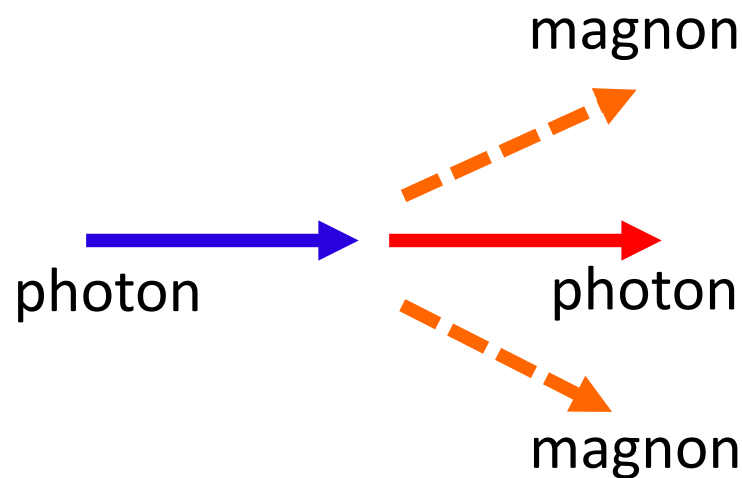
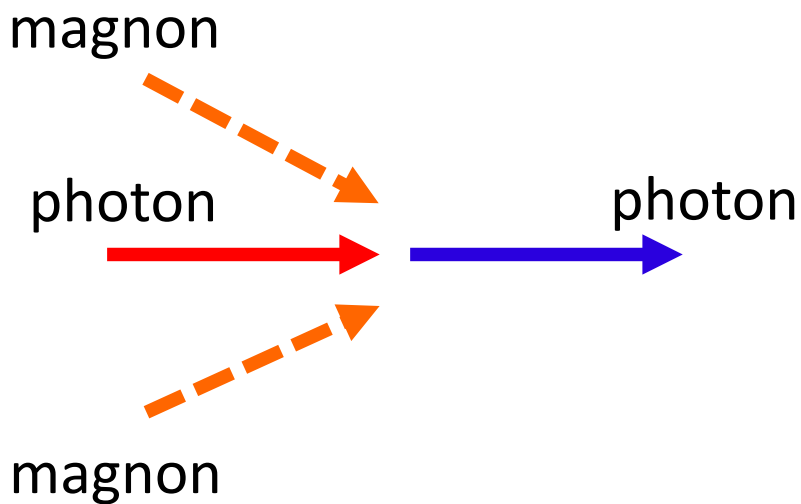
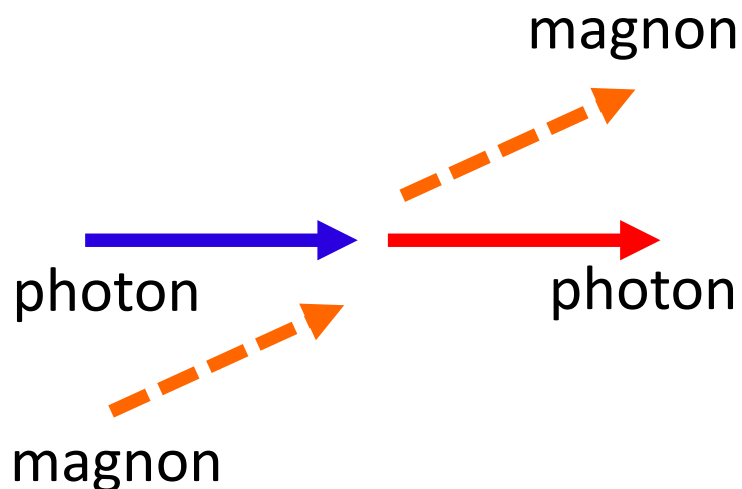
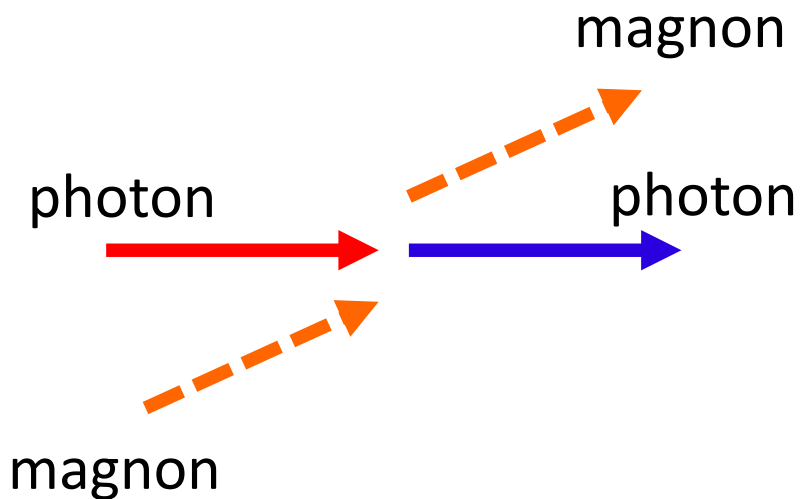
$$\Sigma_{xy} = \frac{1}{2} (M_x M_y + M_y M_x)$$

$$\Sigma_{yz} = \frac{1}{2} (M_y M_z + M_z M_y)$$

$$\Sigma_{zx} = \frac{1}{2} (M_z M_x + M_x M_z)$$

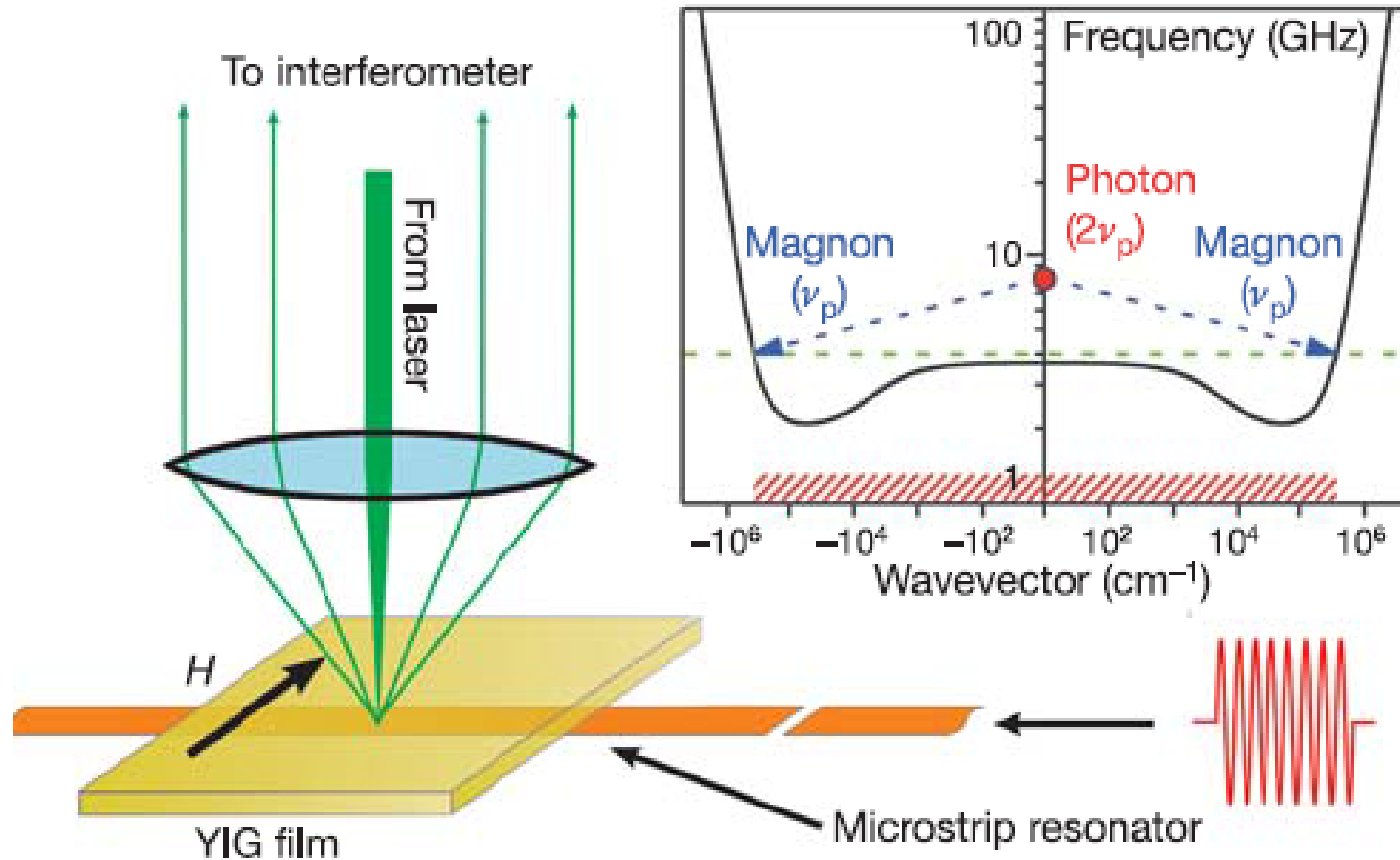


# Two-magnon processes

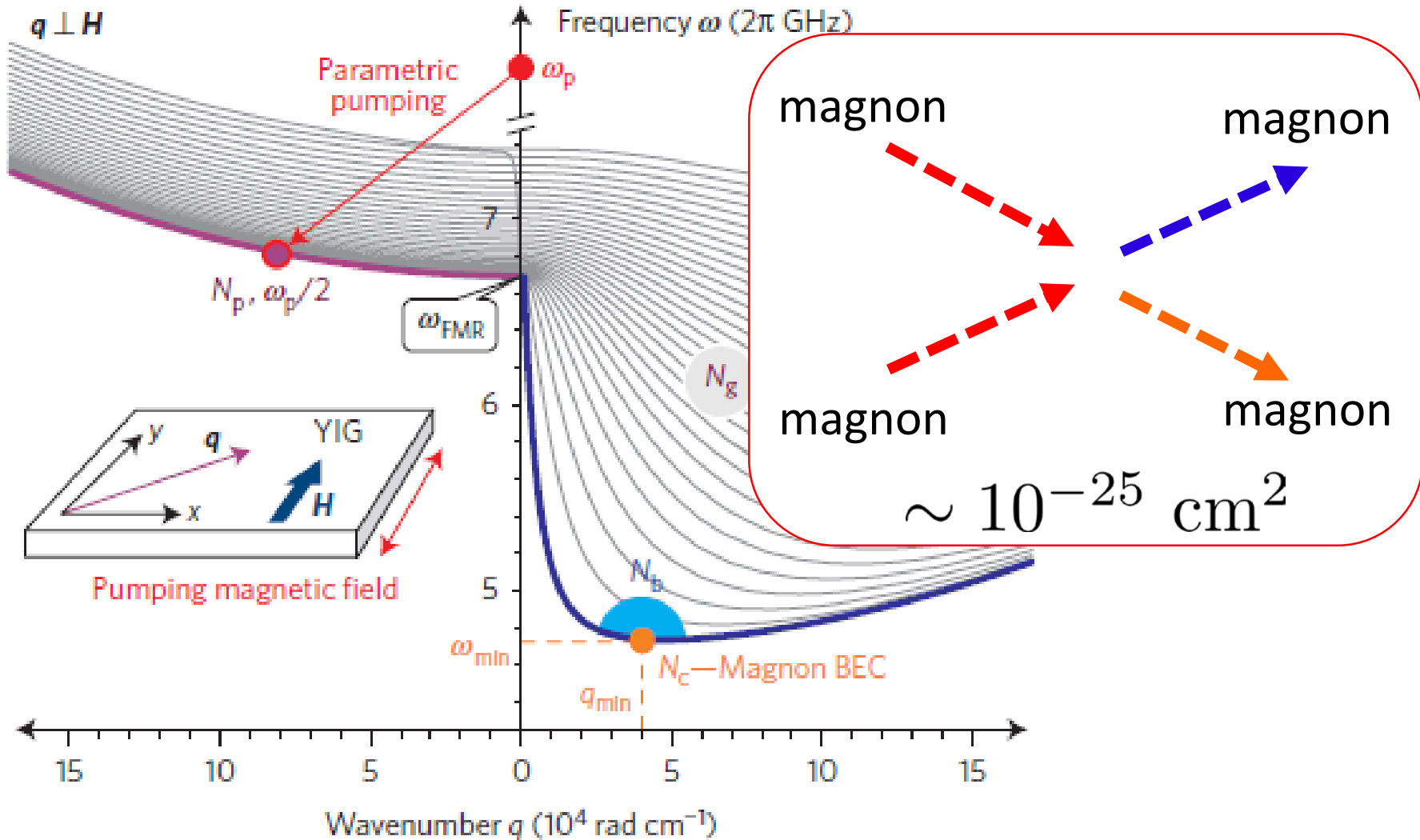


What is it good for?

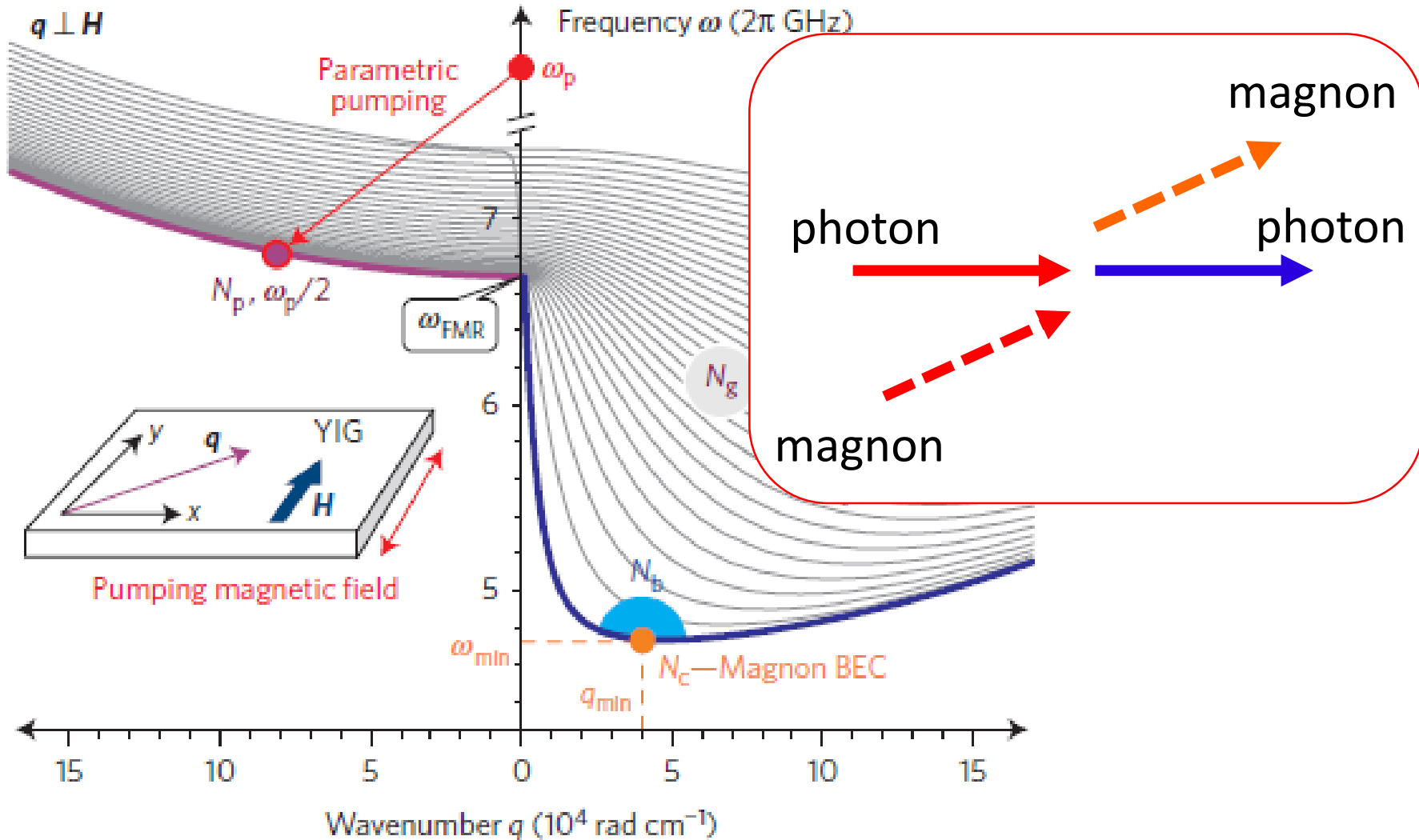
# Magnon BEC



# Magnon BEC



# Magnon BEC



Who have ever observed it?

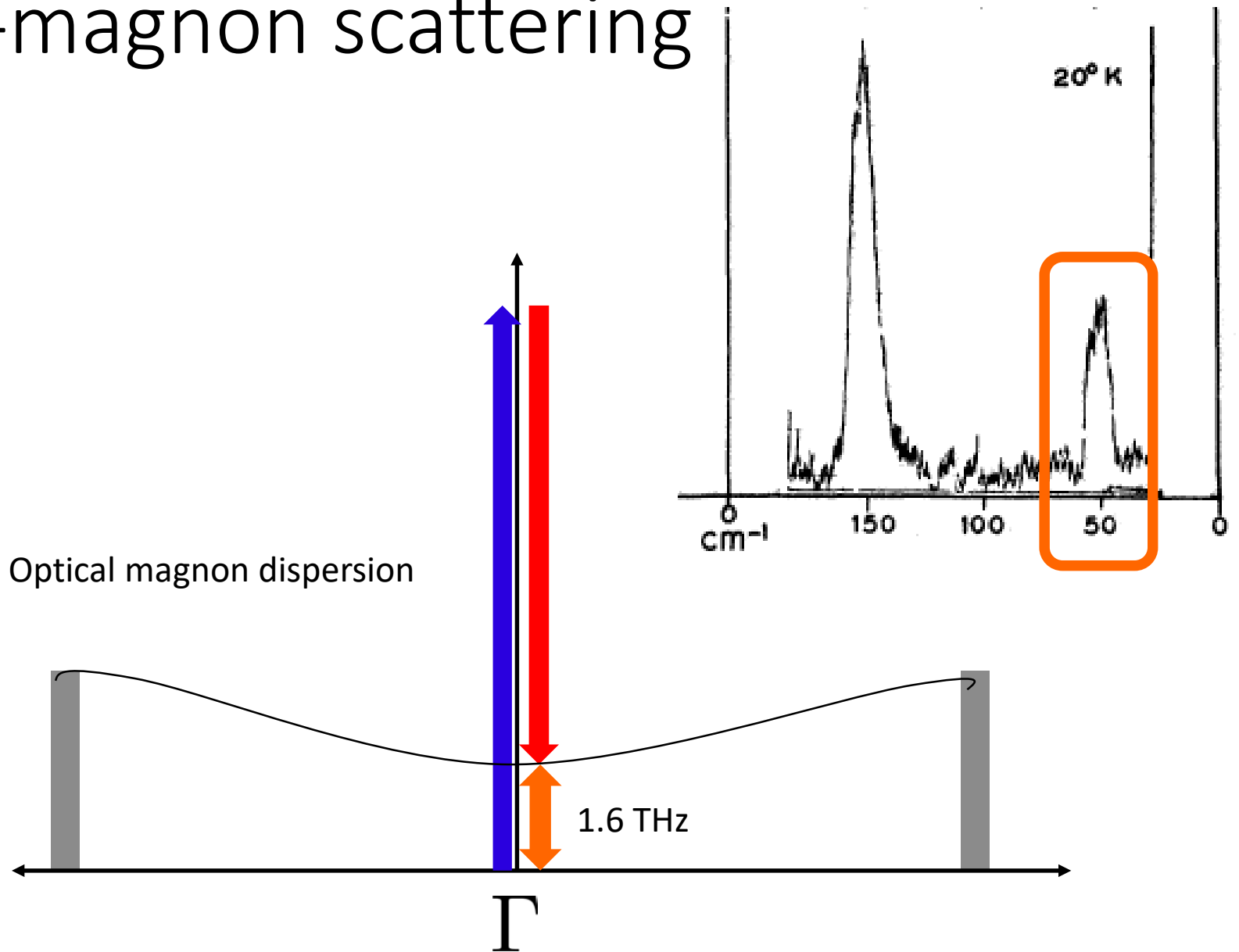
Antiferromagnet

LIGHT SCATTERING BY SPIN WAVES IN  $\text{FeF}_2$

P. A. Fleury, S. P. S. Porto, L. E. Cheesman, and H. J. Guggenheim  
Bell Telephone Laboratories, Murray Hill, New Jersey

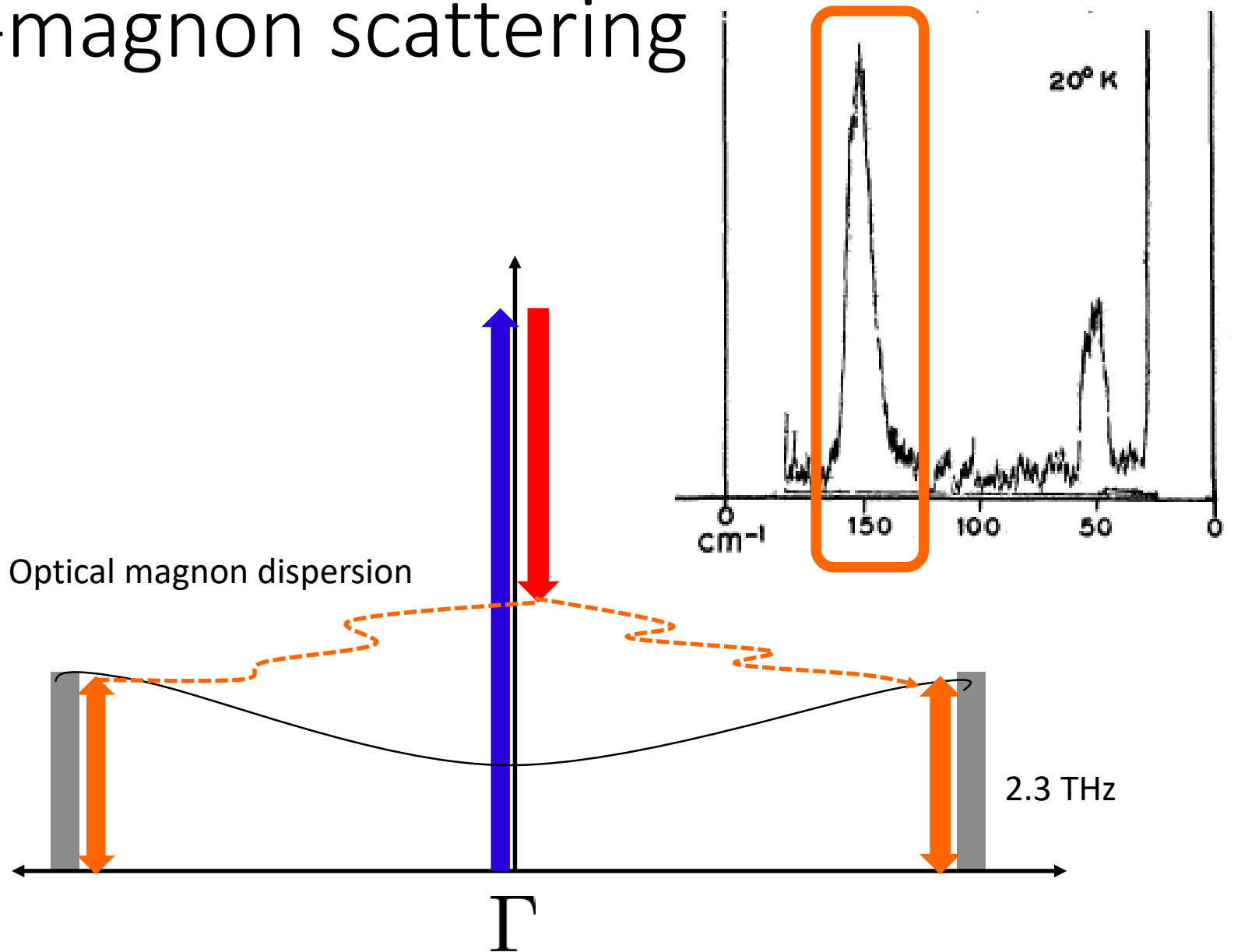
(Received 27 May 1966)

# One-magnon scattering



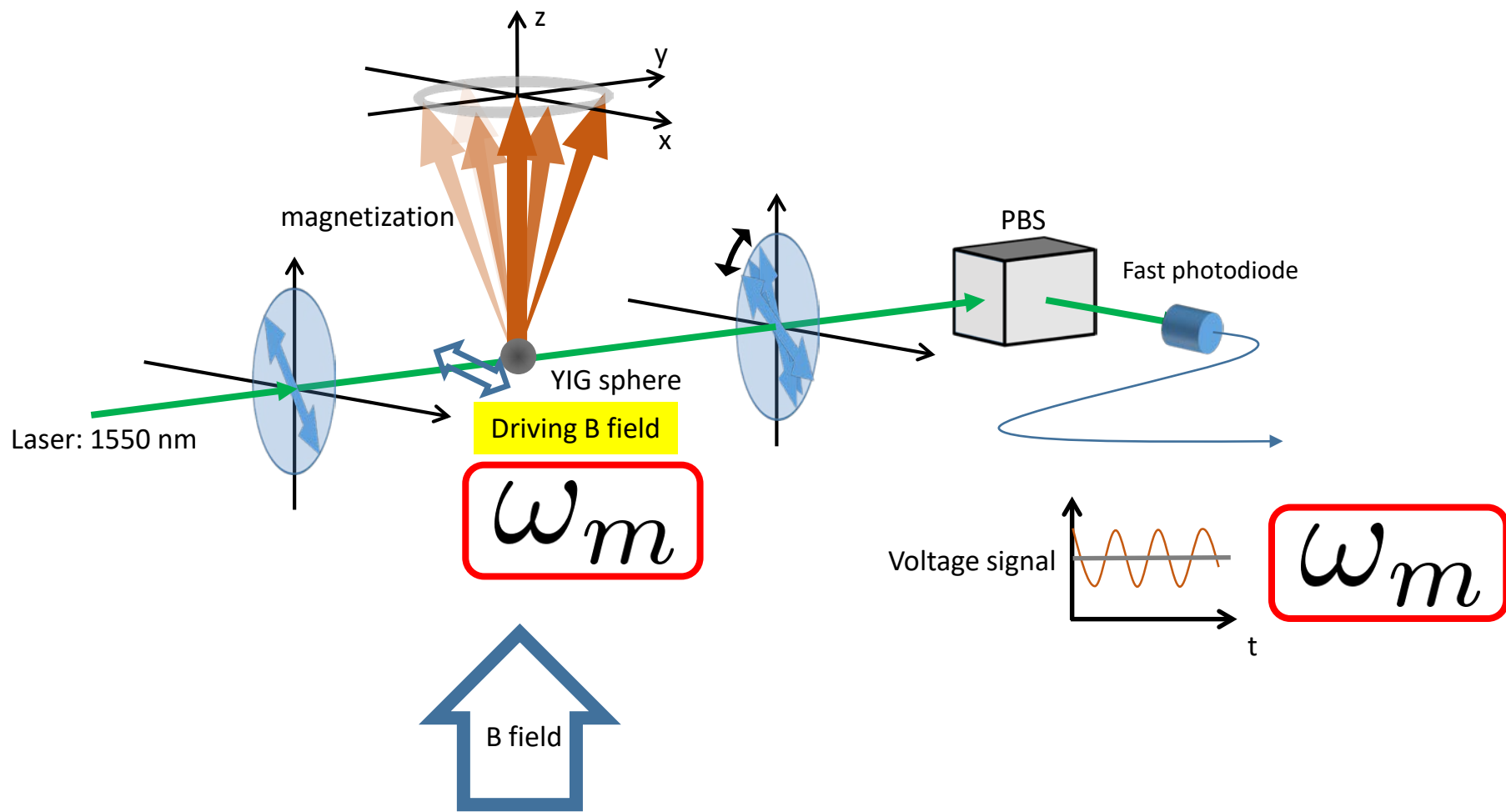


# Two-magnon scattering

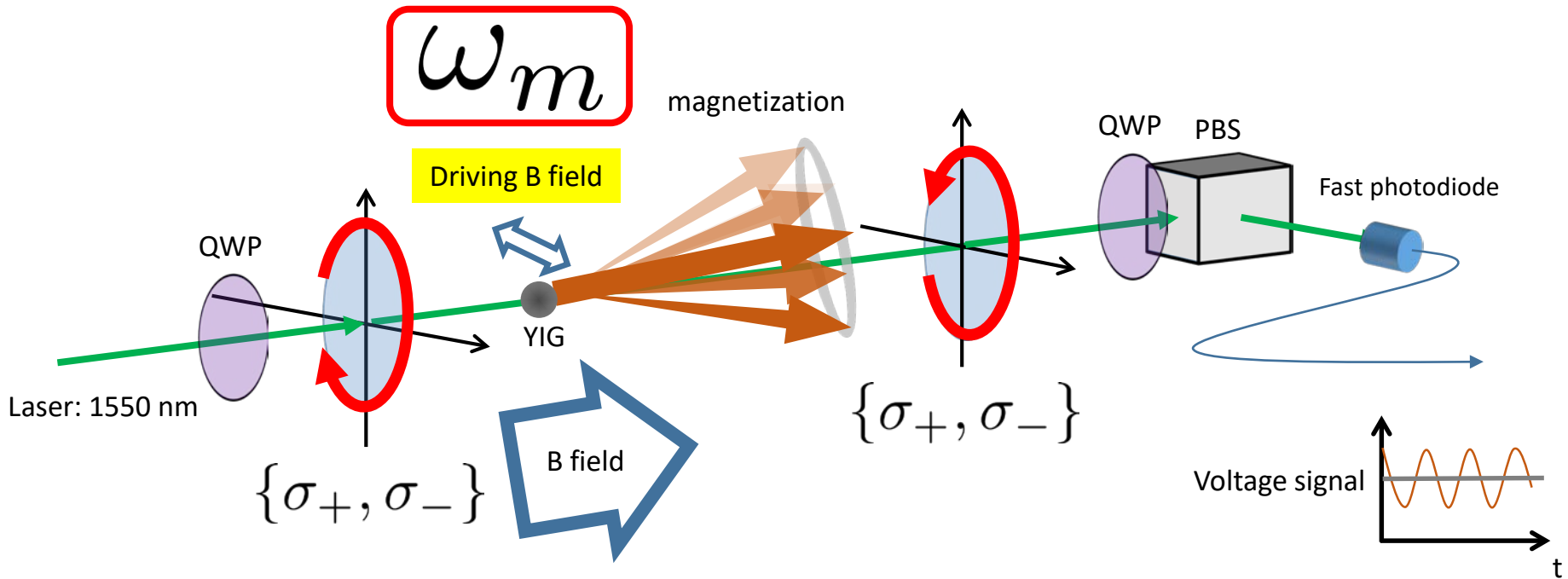


What about ferromagnets?

# One-magnon scattering

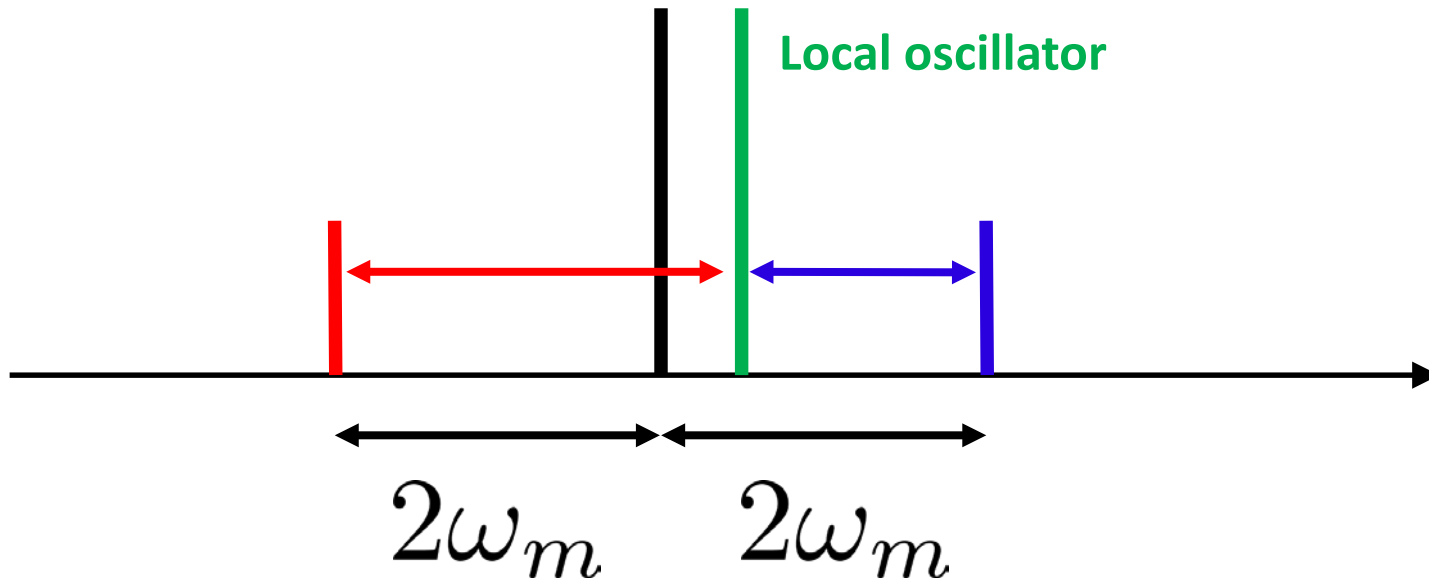
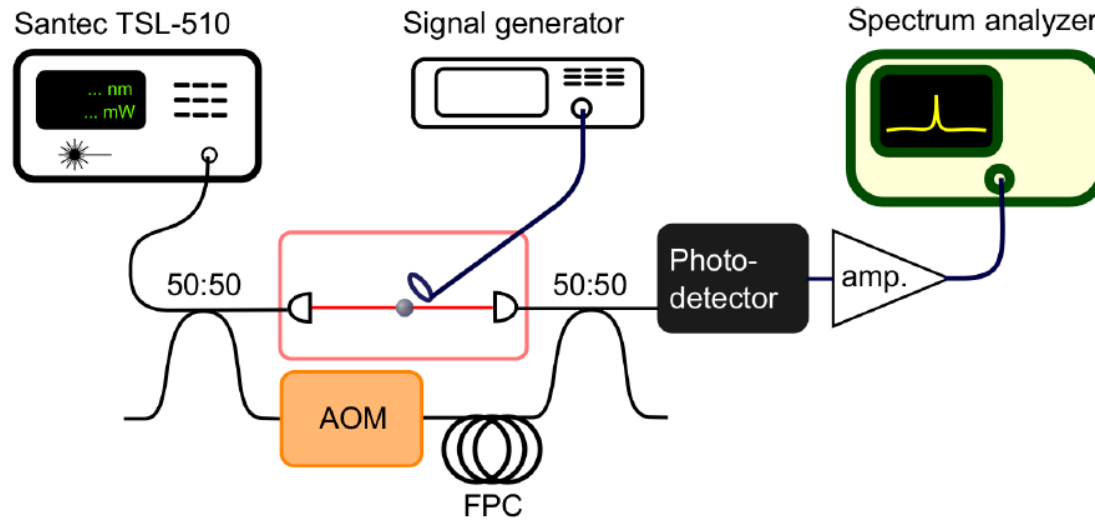


# Two-magnon scattering

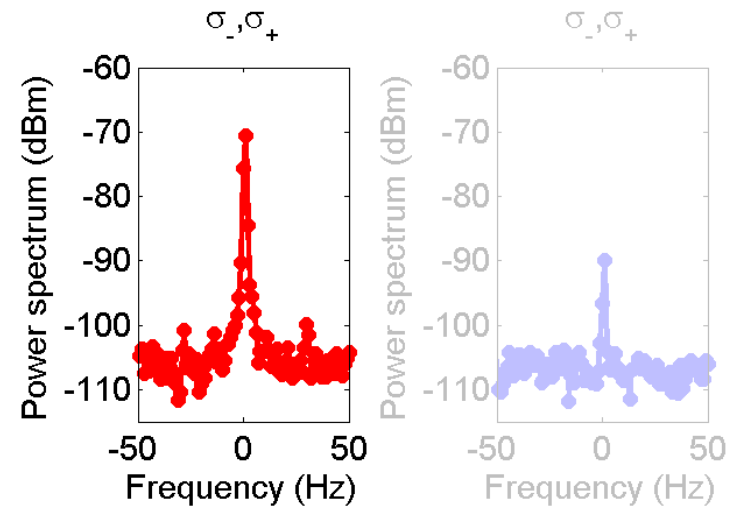
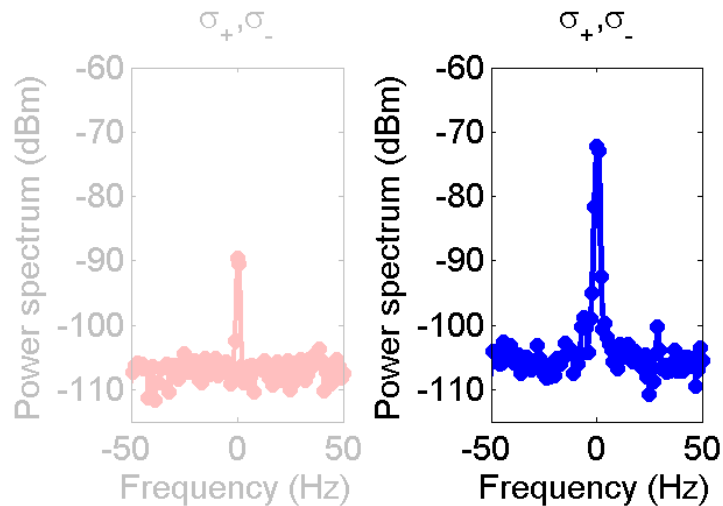
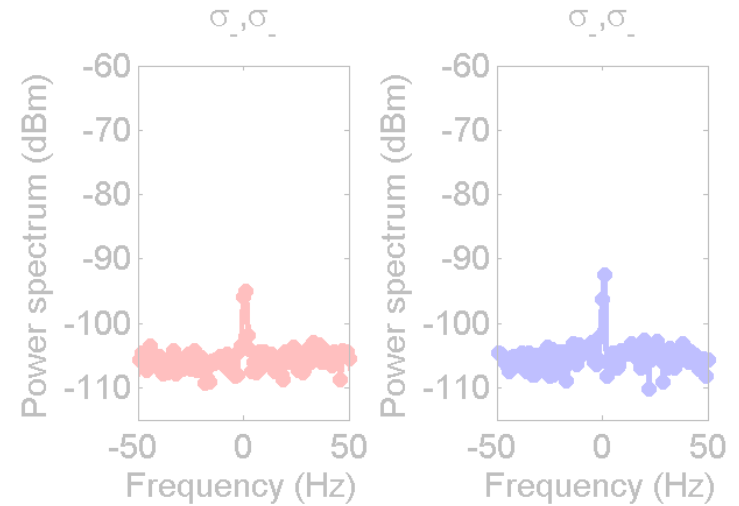
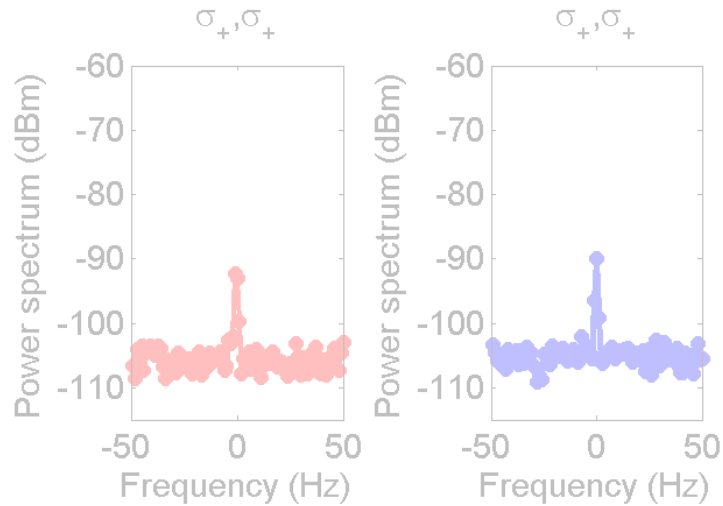


$$2\omega_m$$

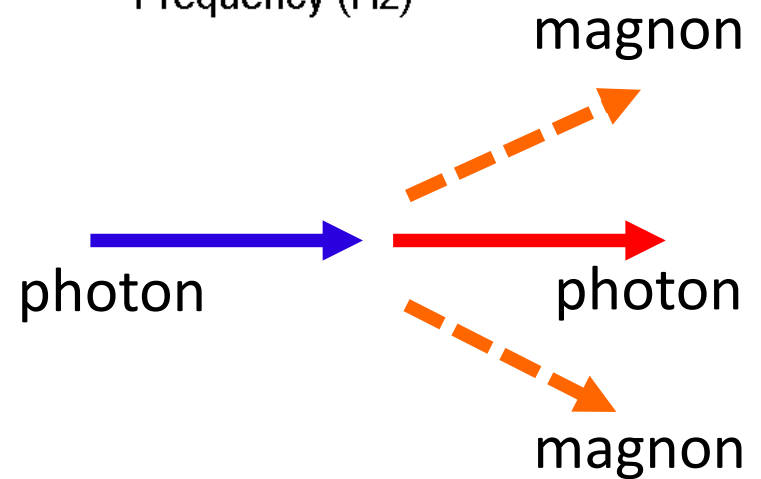
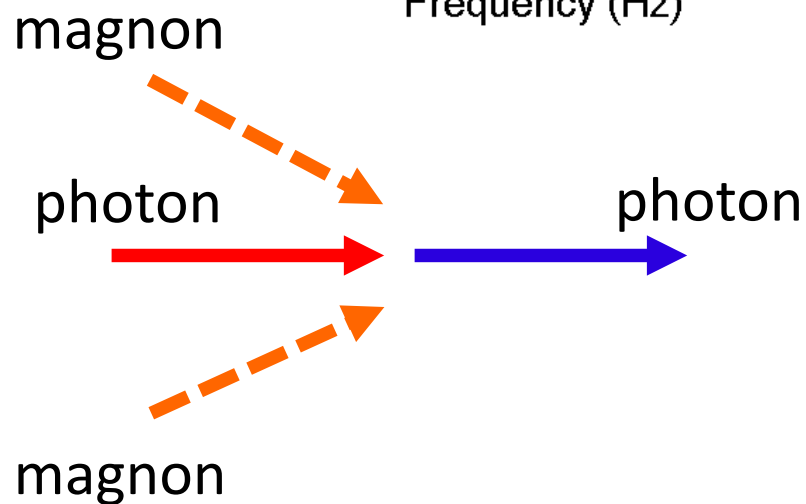
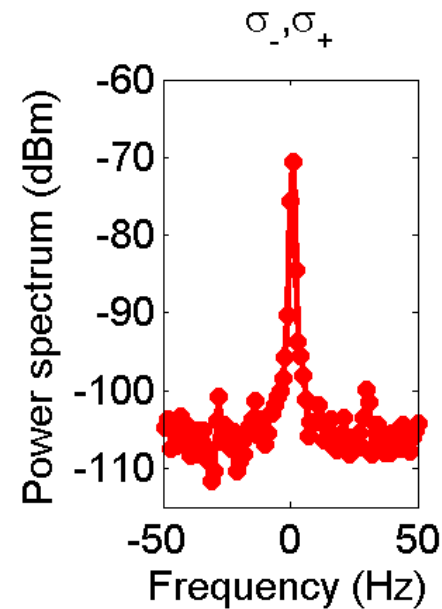
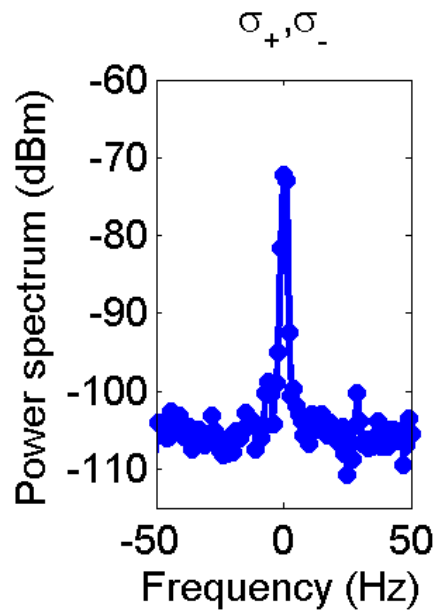
# Sideband separation



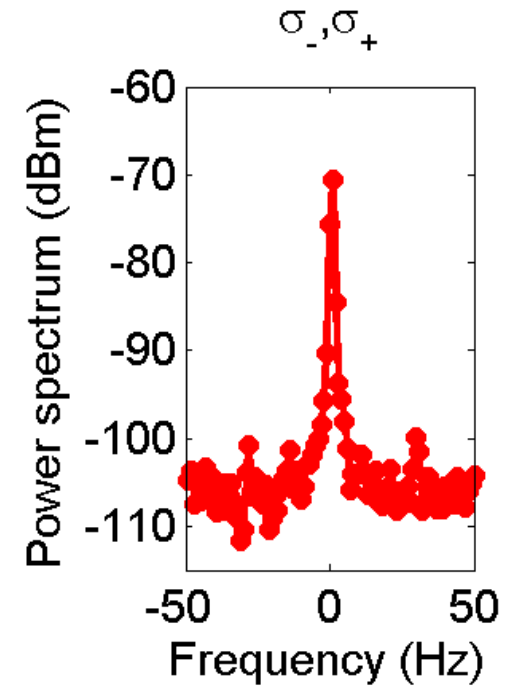
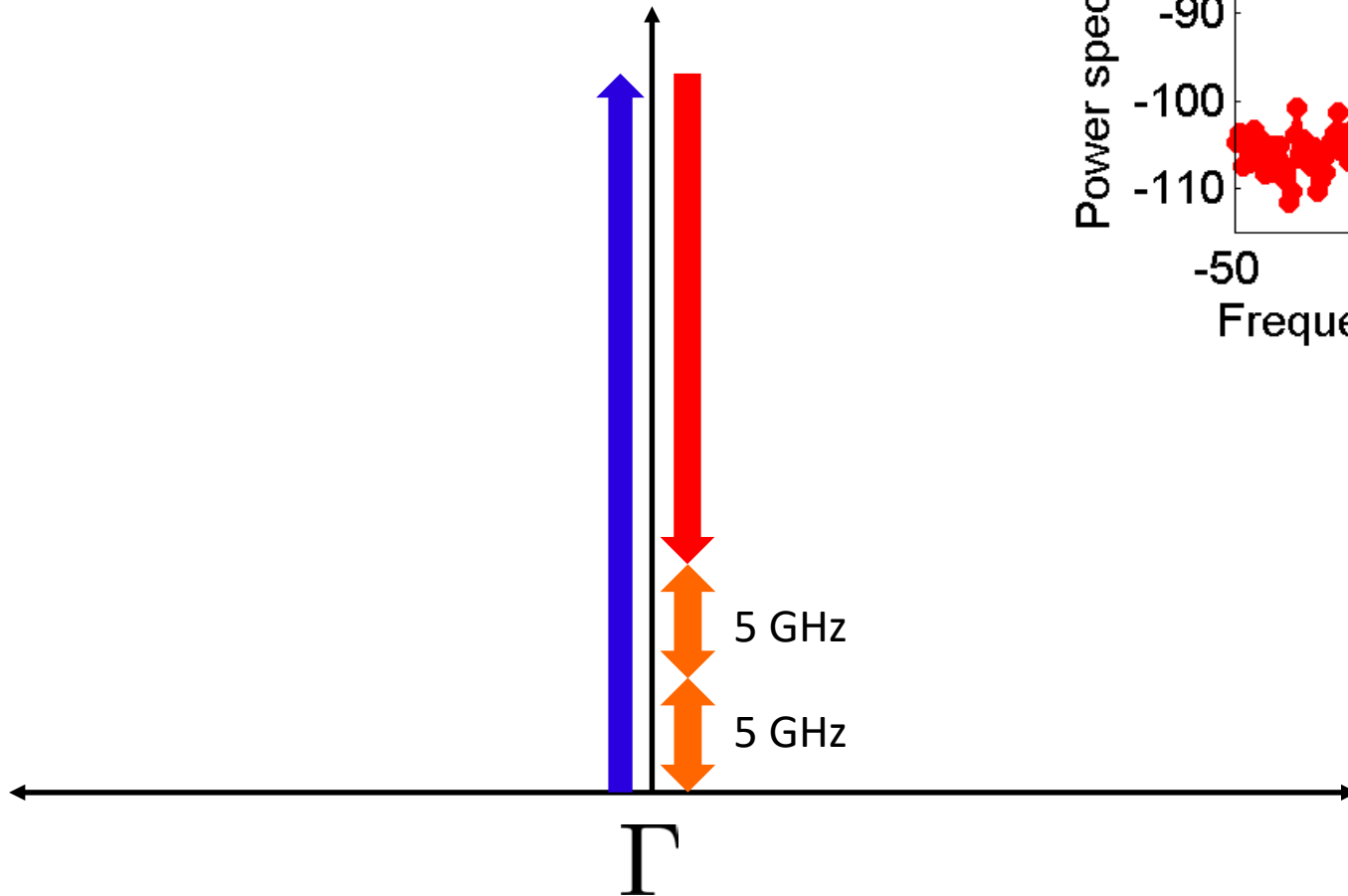
# 2 Kittel magnon scattering (100 // H)



# 2 Kittel magnon scattering (100 // H)

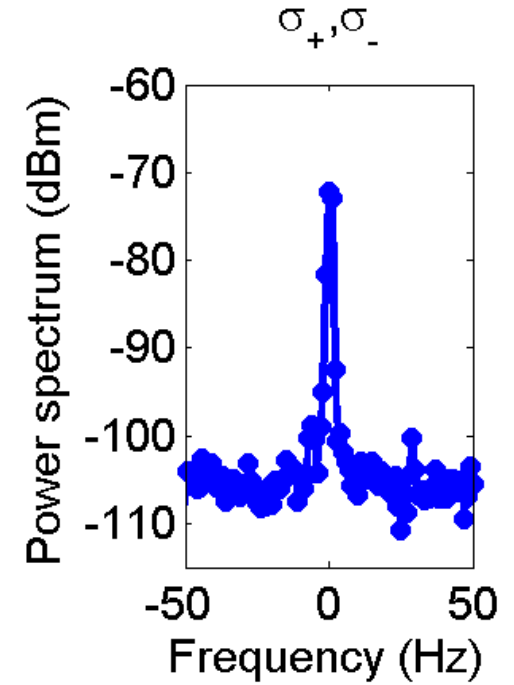
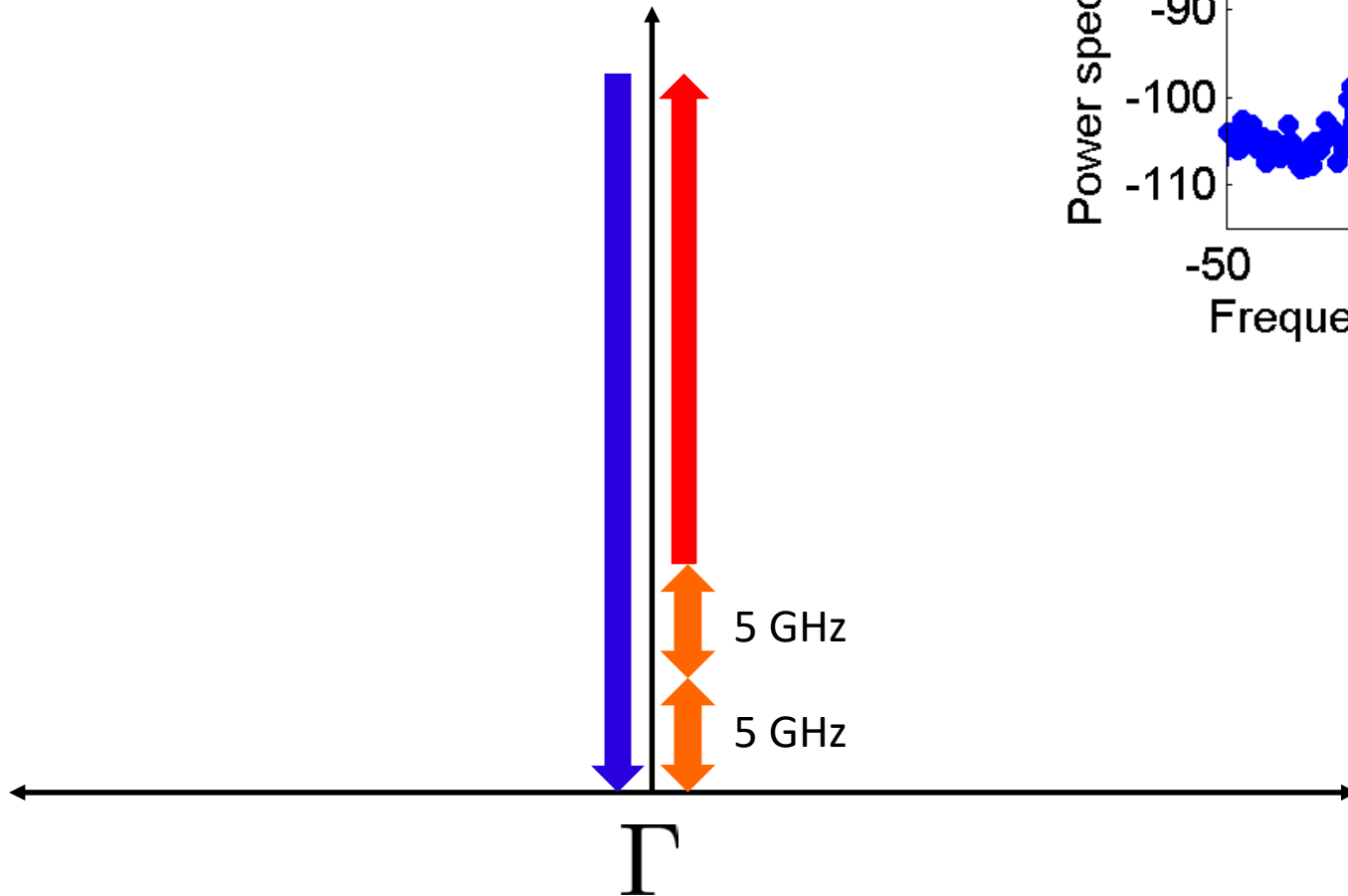


# Two-magnon scattering

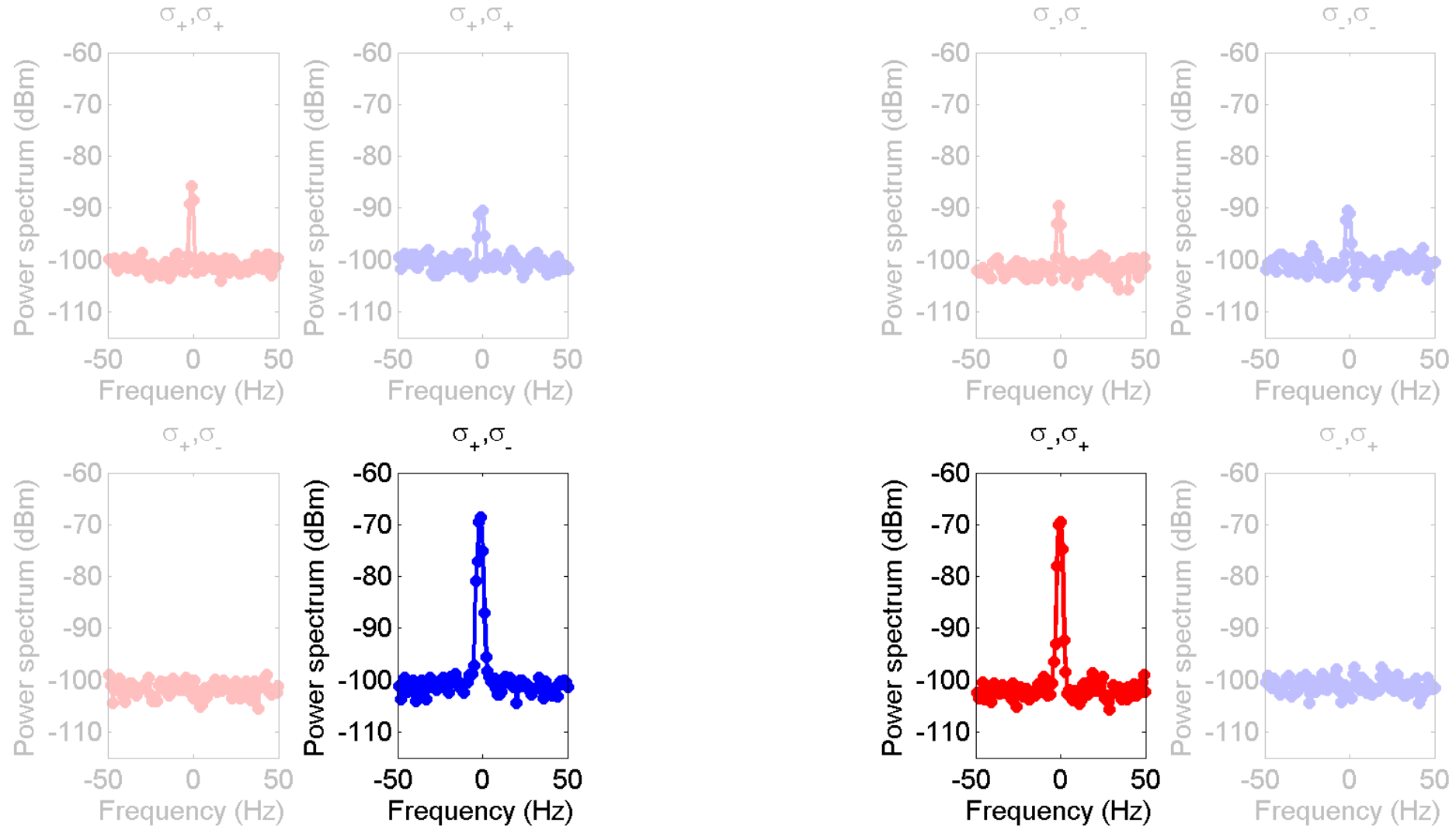




# Two-magnon scattering



# 2-Kittel magnon scattering (111 // H)



# Tensor light shift (Cotton-Mouton effect)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{12} & 0 & 0 & 0 \\ G_{12} & G_{11} & G_{12} & 0 & 0 & 0 \\ G_{12} & G_{12} & G_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{44} \end{bmatrix} \begin{bmatrix} \Sigma_{xx} \\ \Sigma_{yy} \\ \Sigma_{zz} \\ 2\Sigma_{yz} \\ 2\Sigma_{zx} \\ 2\Sigma_{xy} \end{bmatrix}$$

For **cubic** crystal like YIG (H // 100)

# Tensor light shift (Cotton-Mouton effect)

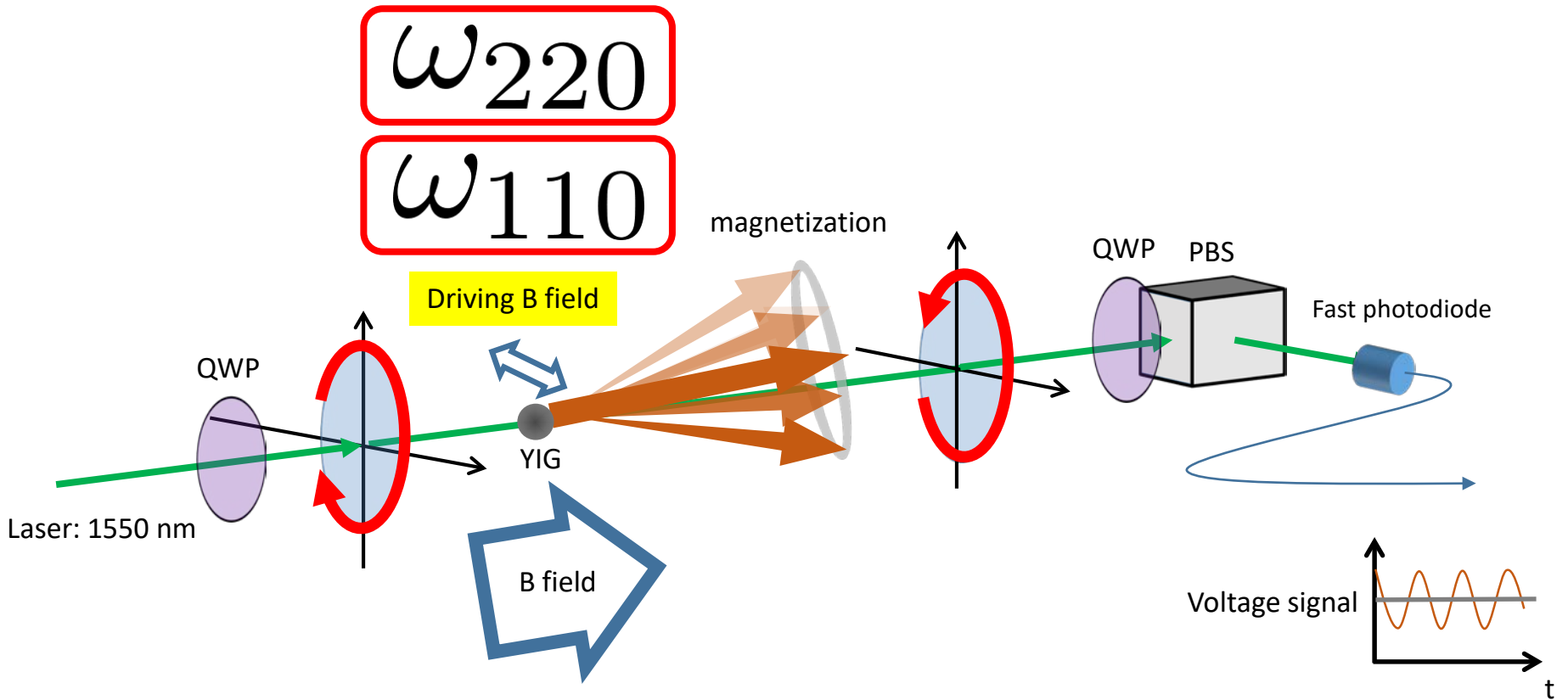
$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \epsilon_{yz} \\ \epsilon_{zx} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} 2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \Sigma_{xx} \\ \Sigma_{yy} \\ \Sigma_{zz} \\ 2\Sigma_{yz} \\ 2\Sigma_{zx} \\ 2\Sigma_{xy} \end{bmatrix}$$

**Isotropic**

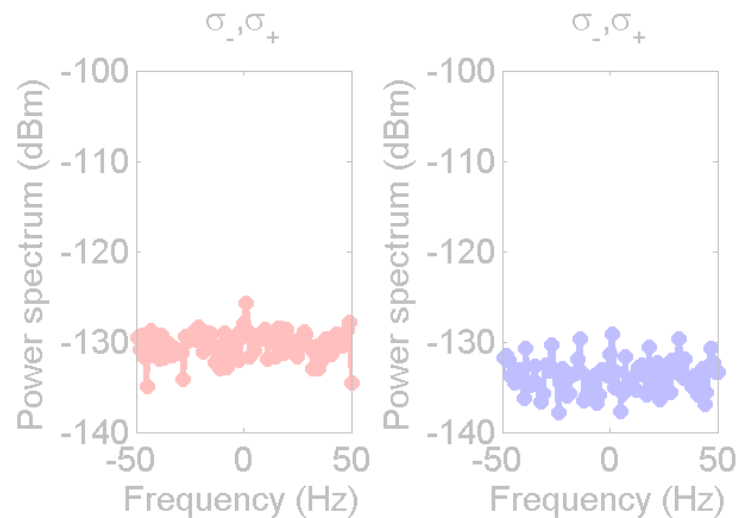
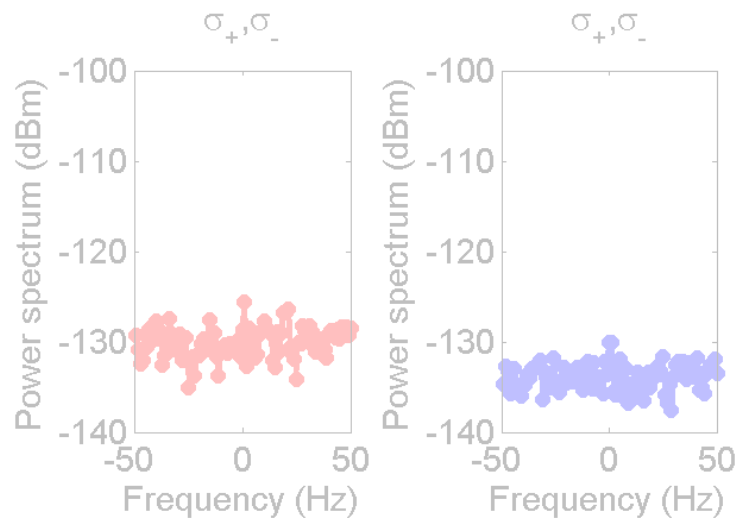
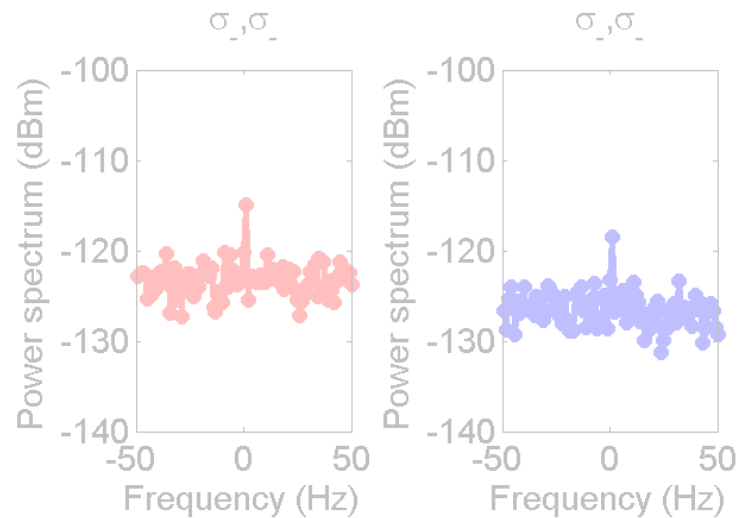
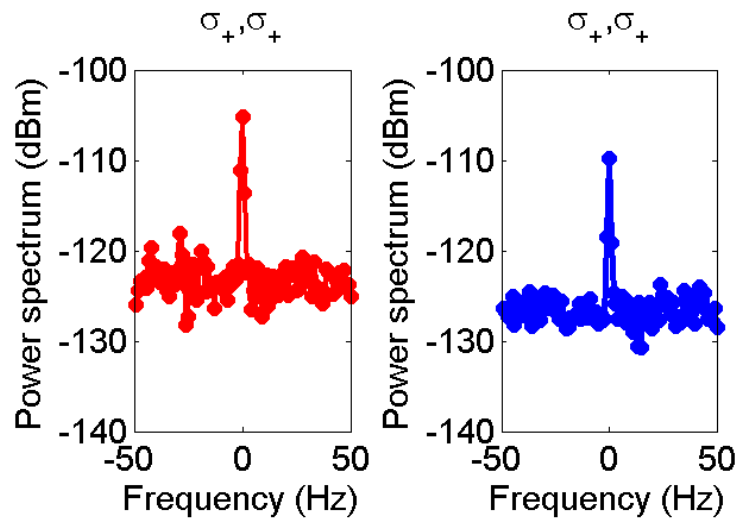
Magnon-mode transfer

# Dynamic Cotton-Mouton effect

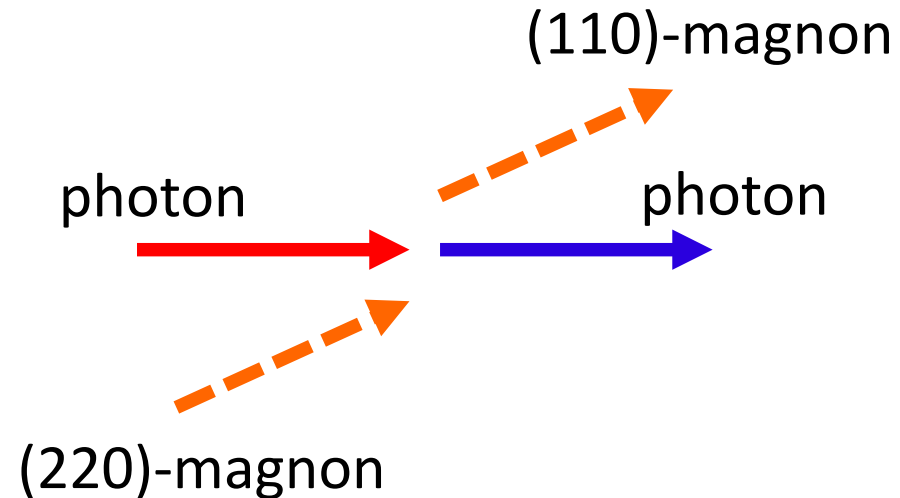
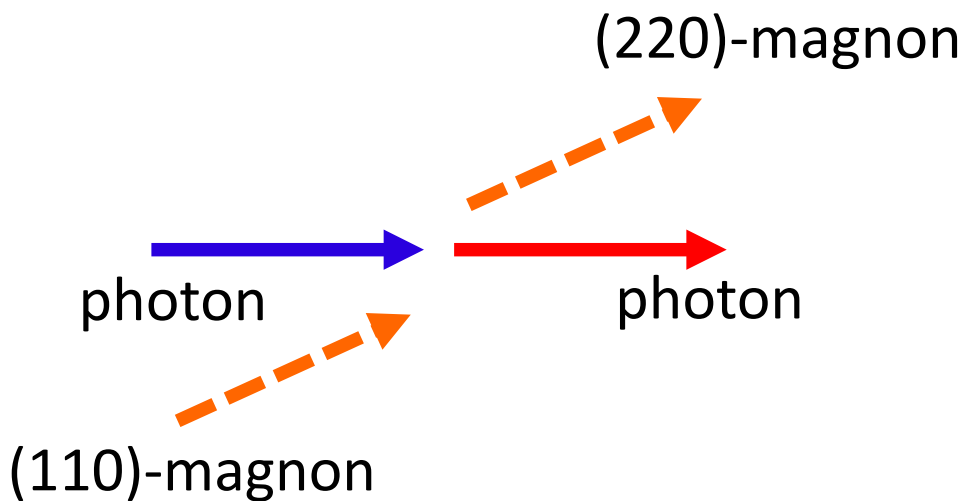
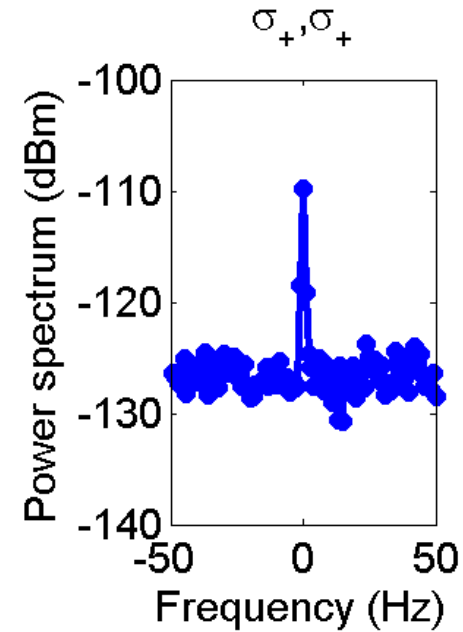
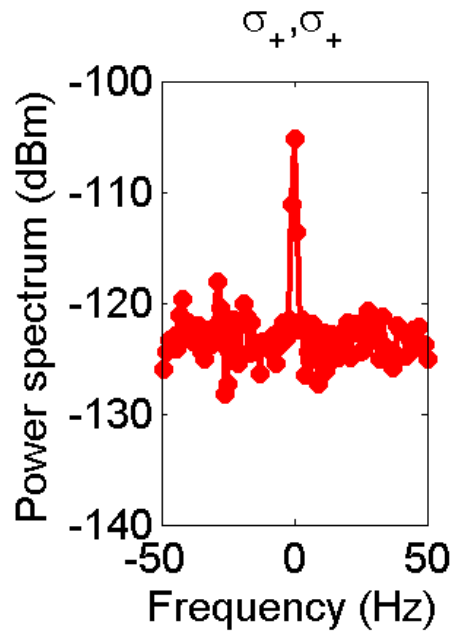


$$\pm(\omega_{110} - \omega_{220})$$

# (220)-magnons $\Leftrightarrow$ (110)-magnons

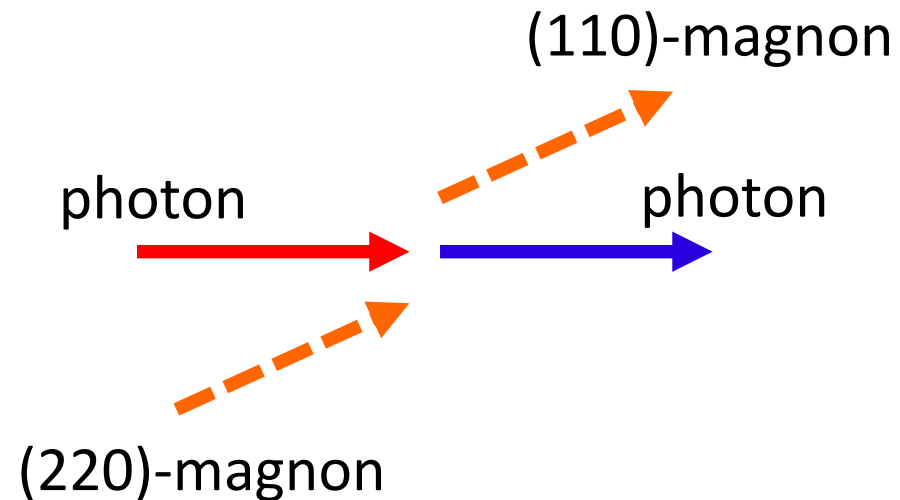
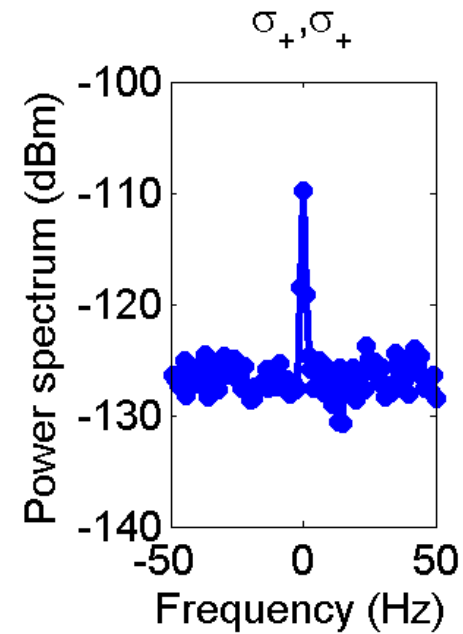
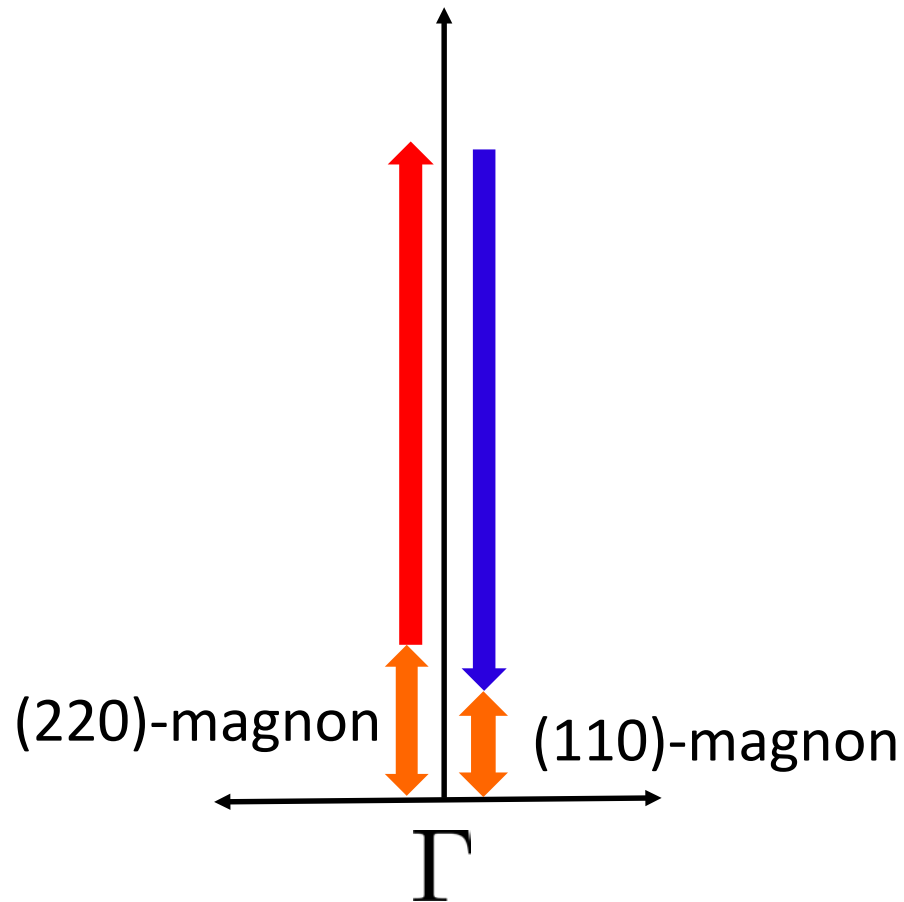


# (220)-magnons $\Leftrightarrow$ (110)-magnons

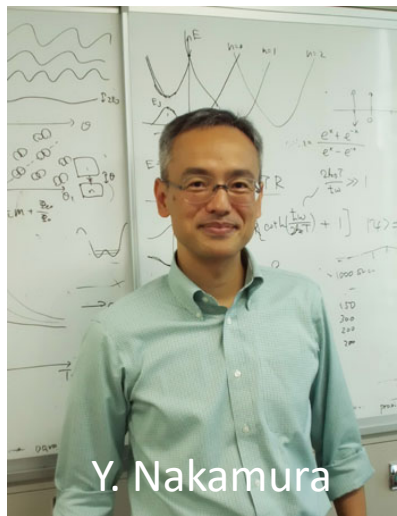




# Two-magnon scattering



# Team



Y. Nakamura



Y. Tabuchi



S. Ishino



R. Hisatomi



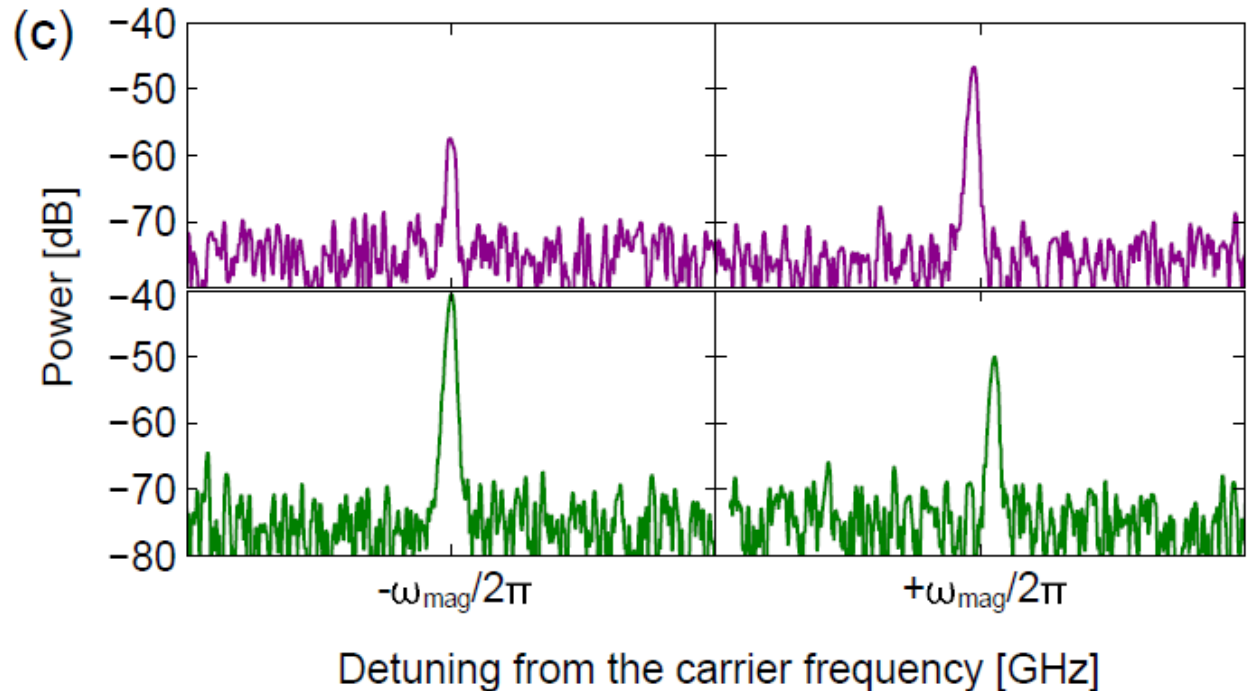
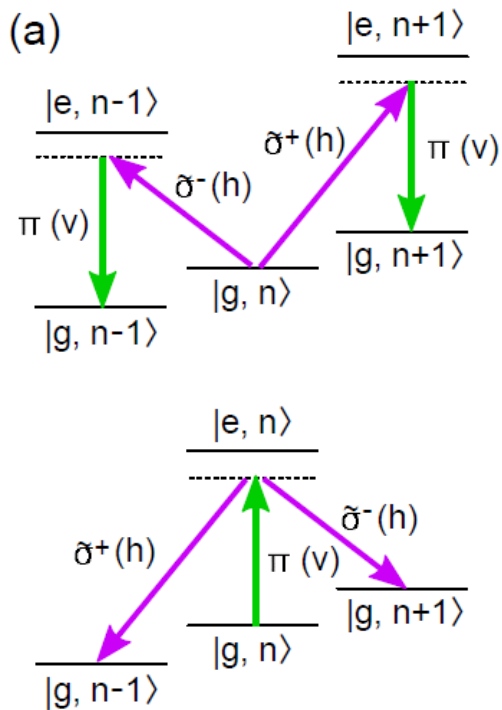
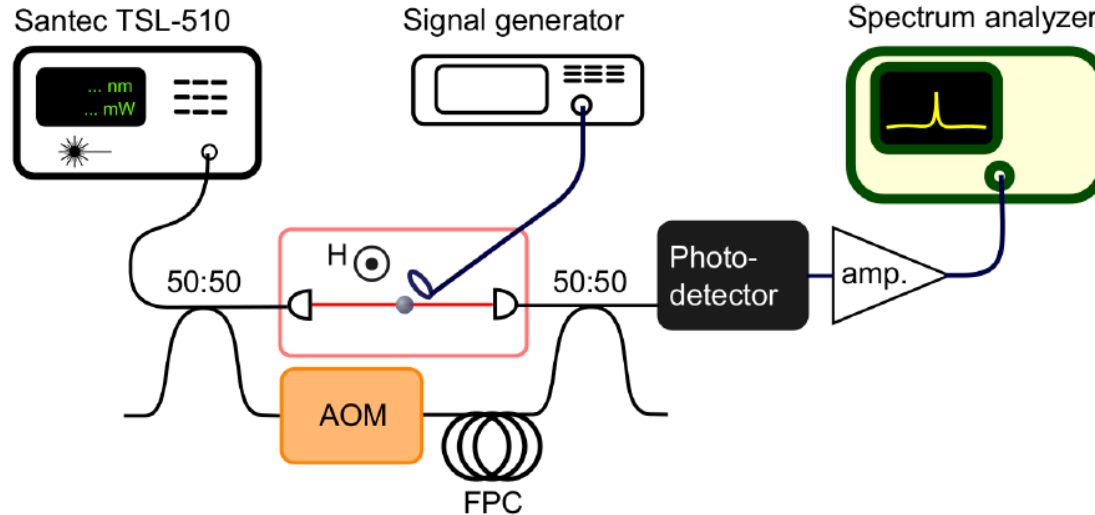
D. Lachance-Quirion



How large?

Look at one-magnon process again

# “Intrinsic” sideband asymmetry



# One-magnon process

$$\tilde{\epsilon} = \tilde{\epsilon}_0 + \tilde{\epsilon}_1 + \tilde{\epsilon}_2,$$

Scalar part

Saturation magnetization

$$\tilde{\epsilon}_0 = \epsilon_0 \epsilon_r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Sigma_{ij} = M_i M_j$$

Vector

$$\tilde{\epsilon}_1 = \begin{bmatrix} if \\ -if \\ -i \end{bmatrix}$$

$$f \sim 2G_{44} M_z$$

$$(y M_x)$$

$$(z M_y)$$

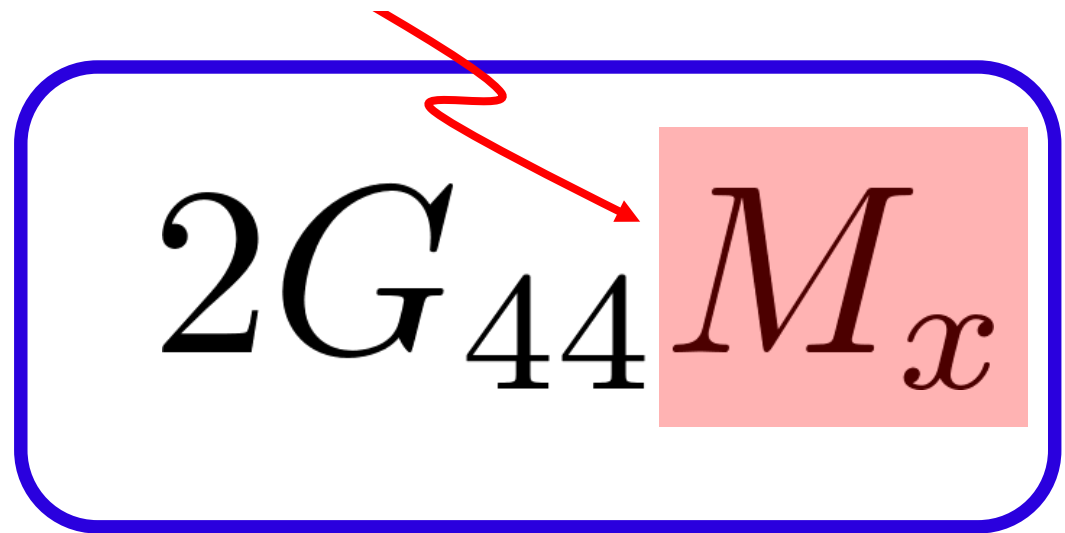
$$(x M_z)$$

Tensor part (symmetric)

$$\tilde{\epsilon}_2 = \begin{bmatrix} G_{11}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{xy} & 2G_{44}\Sigma_{zx} \\ 2G_{44}\Sigma_{xy} & G_{12}\Sigma_{xx} + G_{11}\Sigma_{yy} + G_{12}\Sigma_{zz} & 2G_{44}\Sigma_{yz} \\ 2G_{44}\Sigma_{zx} & 2G_{44}\Sigma_{yz} & G_{12}\Sigma_{xx} + G_{12}\Sigma_{yy} + G_{11}\Sigma_{zz} \end{bmatrix}$$

# Two-magnon process

magnetization due to magnon



The diagram shows the equation  $2G_{44}M_x$  enclosed in a blue rounded rectangle. A red arrow points from the text 'magnetization due to magnon' to the term  $M_x$ . The term  $M_x$  is highlighted with a light red background.

$$2G_{44}M_x$$

$$M_x \sim 10^{-3} M_z$$