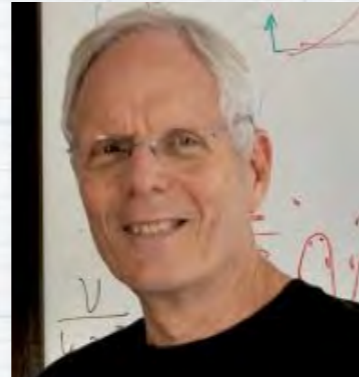
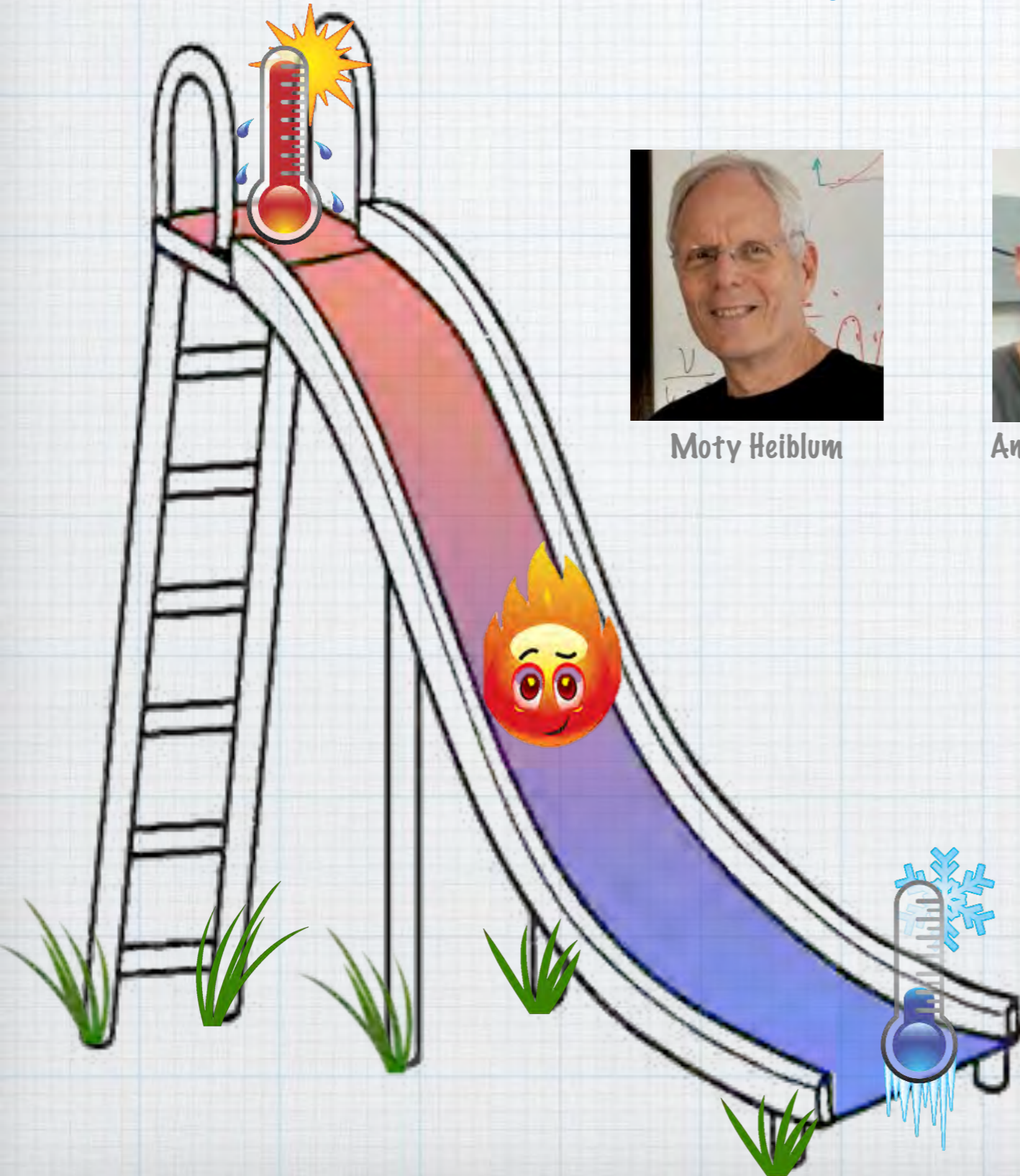


Quantization of Heat Flow in Fractional Quantum Hall States



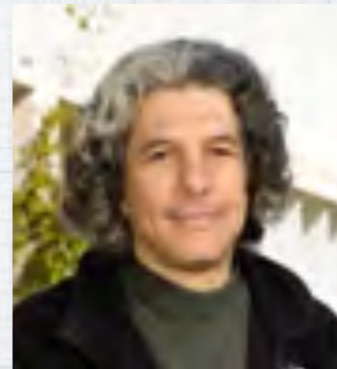
Moty Heiblum



Amir Rosenblatt



Vladimir Umansky



Yuval Oreg



Ady Stern



Dima Feldman

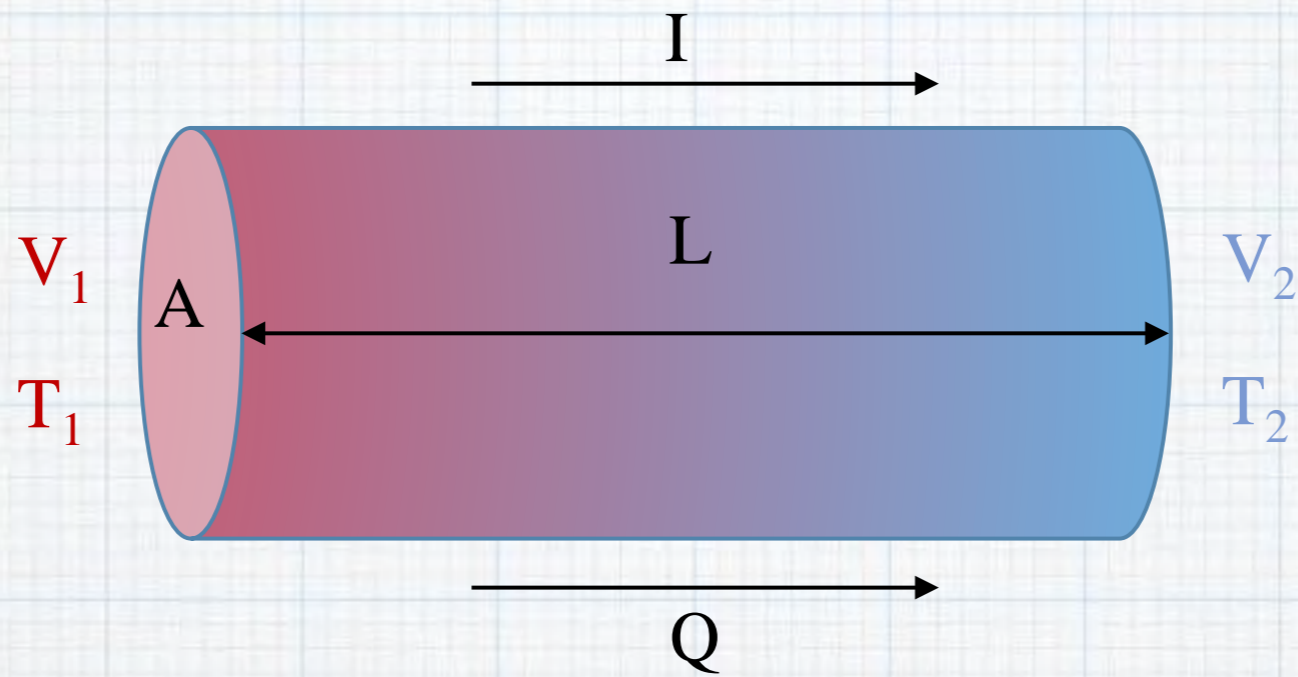
Mitali Banerjee

Braun Center for Submicron Research



מכון ויצמן למדע

WEIZMANN INSTITUTE OF SCIENCE



$$\sigma = \frac{I}{V_1 - V_2} \cdot \frac{L}{A}$$

$$\kappa = \frac{Q}{T_1 - T_2} \cdot \frac{L}{A}$$

Wiedemann - Franz Law

$$\frac{\kappa}{\sigma} = \frac{\pi^2 k_B^2}{3e^2} T$$

Lorentz No.

heat flow in 1-D ballistic channel

J B Pendry

Quantum limits to the flow of information and entropy

J. Phys. A: Math. Gen. **16** (1983) 2161-2171

thermal energy = temperature X entropy

together with energy uncertainty

sets an universal upper limit on energy/heat transfer

**universality of quantum (upper) limit of heat flow
per channel for all non-interacting particles**

$$KT \leq \kappa_0 T$$

1D ballistic transport

$$\kappa_0 \approx 9.5 \times 10^{-13} \text{ W / K}^2$$

$$\frac{dJ_{th}}{dT} = \kappa_0 T$$

$$\kappa_0 = \frac{\pi^2 k_B^2}{3h}$$

$$J_{th} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2)$$

Wiedemann - Franz ballistic 1D channel

for non-interacting electrons

$$G_{th} = K_0 T \qquad G_e = \frac{e^2}{h}$$

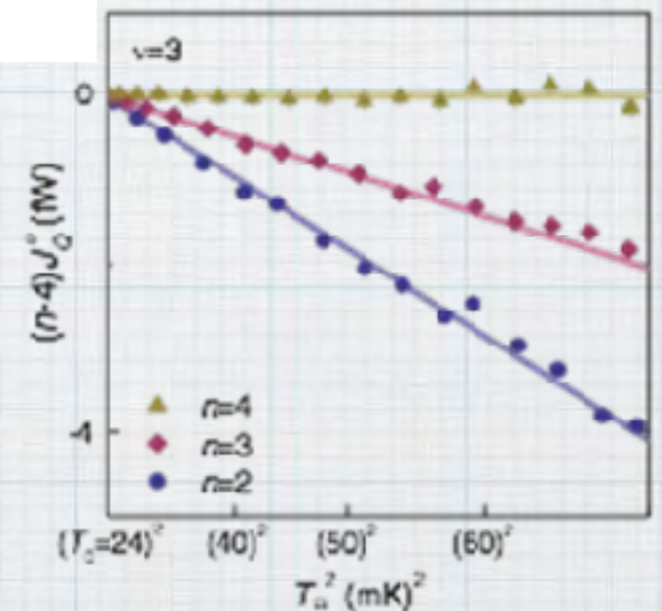
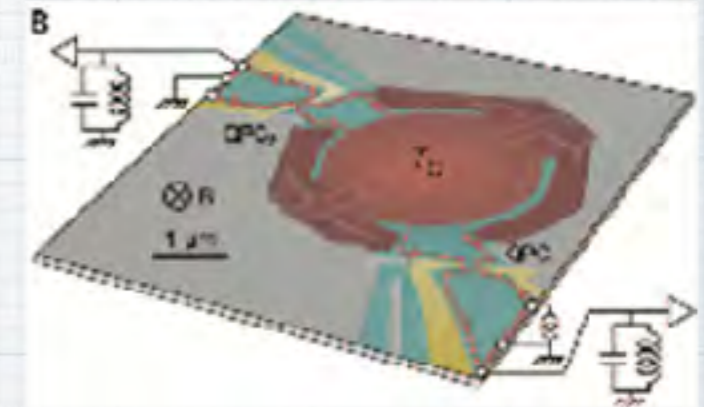
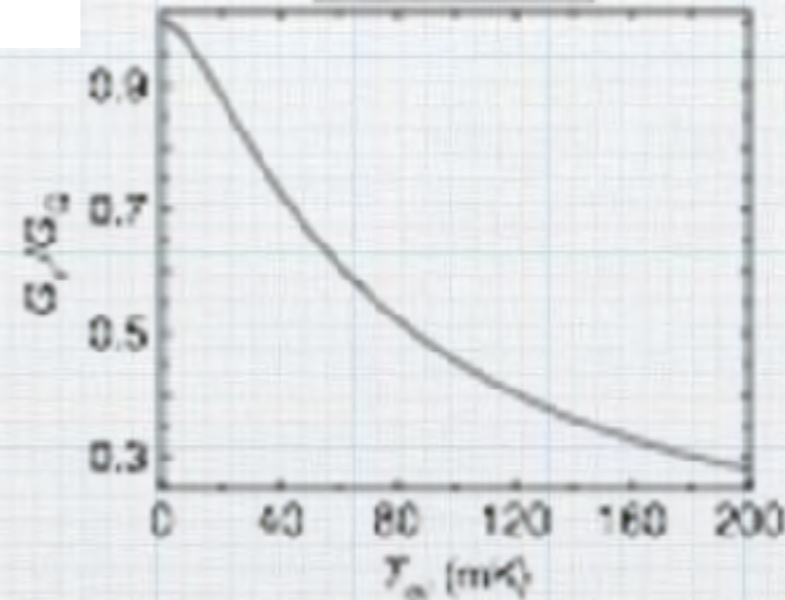
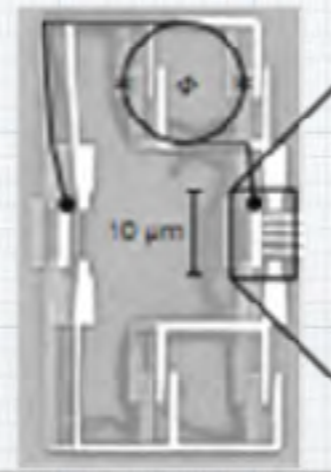
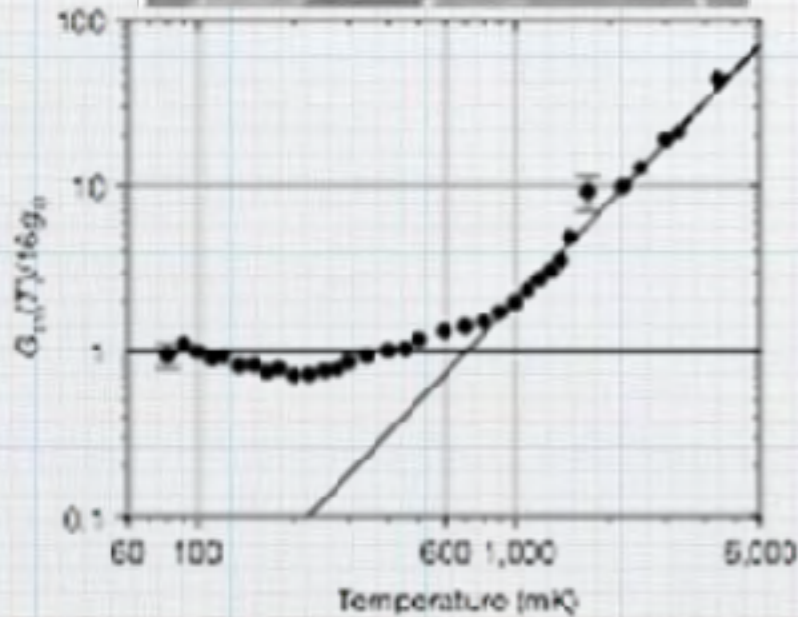
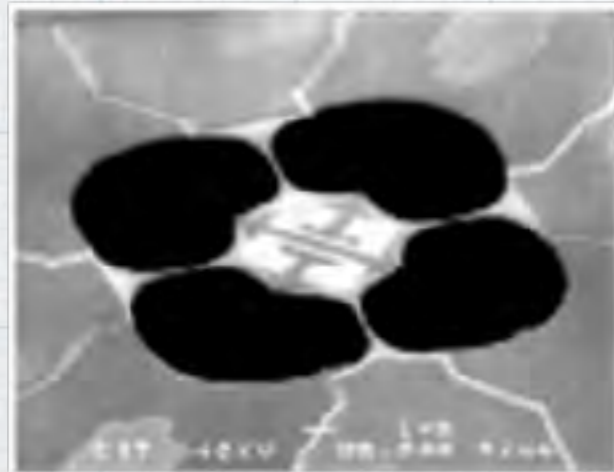
$$\frac{G_{th}}{G_e} = l_{Lorentz} T = \frac{\pi^2 k_B^2}{3e^2} T$$

past experiments.... in accord with theory

16 phonon modes
Schwab et al, 2000

single photon mode
Meschke et al, 2006

single electron mode
Jezouin et al, 2013



Non-interacting bosons and fermions both carries the same amount of heat

Interactions....

Pendry's theory extended for **interacting** particles

Kane, C. L. & Fisher, M. P. A.

Quantized thermal transport in the fractional quantum Hall effect

Phys. Rev. B **55**, 15832–15837 (1997)

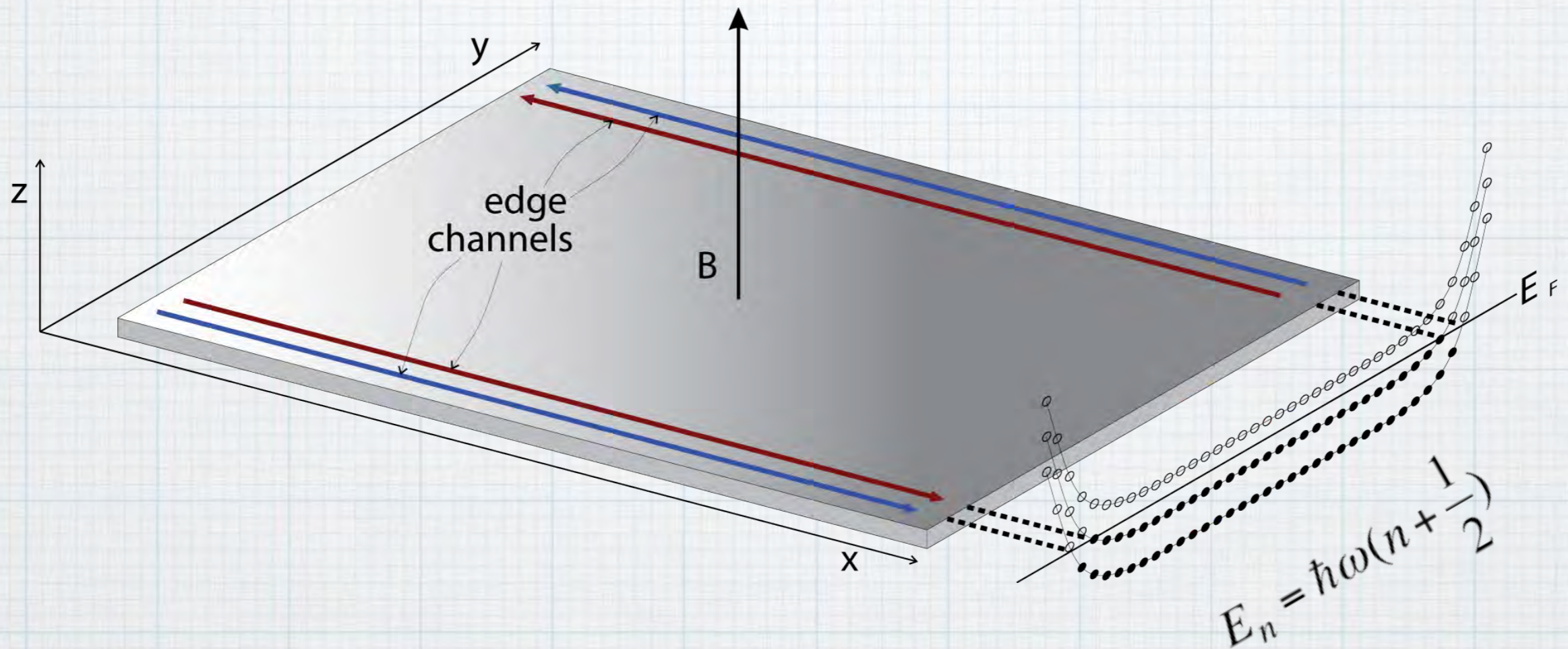
interactions should not effect quantum of thermal conductance !!!

$$K = \kappa_0$$

Wiedemann - Franz law breaks down

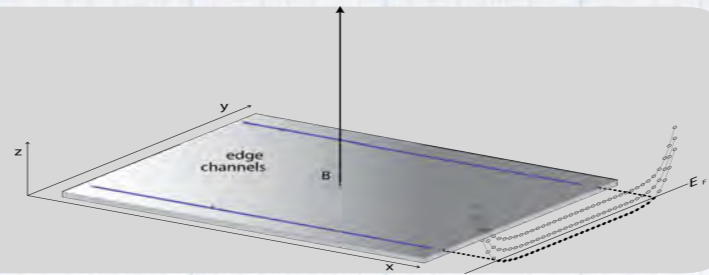
our **1D** interacting system.....**FQHE**

Quantum Hall effect : chiral edge modes



each edge mode carries $I = \frac{e^2}{h} V$

1D modes in QHE



bulk of QHEinsulating localized quasiparticles

edge of IQHEinteger 1D chiral edge modes $G_H = \nu e^2/h$ $\nu = 1, 2, 3, \dots$

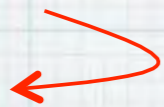
edge of FQHEfractional 1D chiral edge modes

abelian states $G_H = \nu e^2/h$ $\nu = 1/3, 2/5, \dots, 2/3, 3/5, 4/7, \dots$

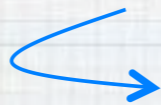
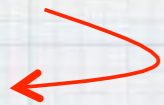
non-abelian states (?) $G_H = \nu e^2/h$ $\nu = 5/2, 12/5, \dots$

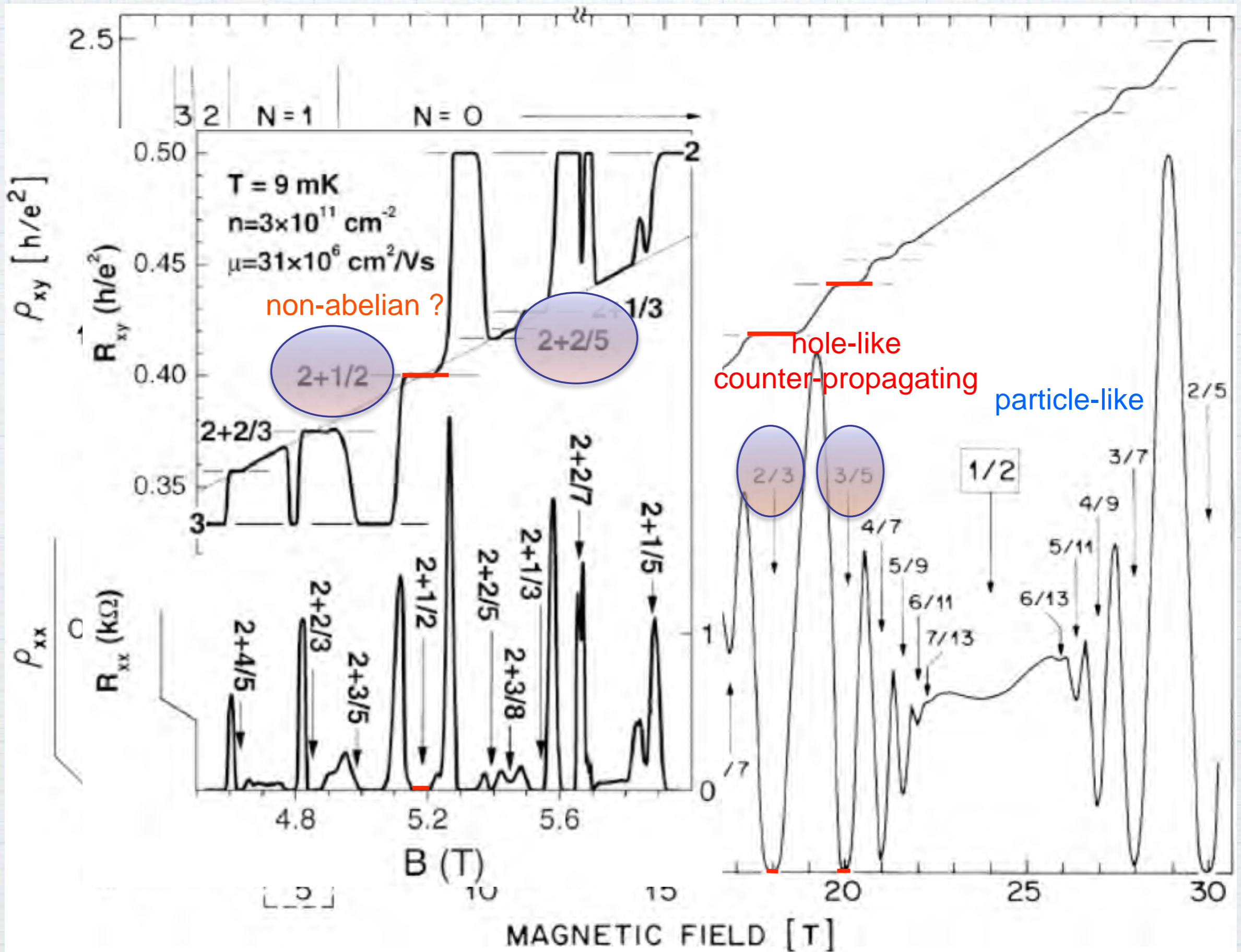
1D modes in FQHE

➤ *downstream* charge.....particle - like

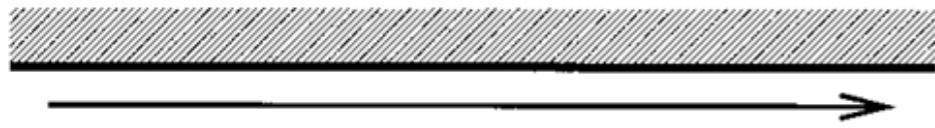


➤ *downstream* charge + *upstream* neutral ...hole – conjugate & non - abelian





K in lowest $LL\dots$ Kane & Fisher 1997



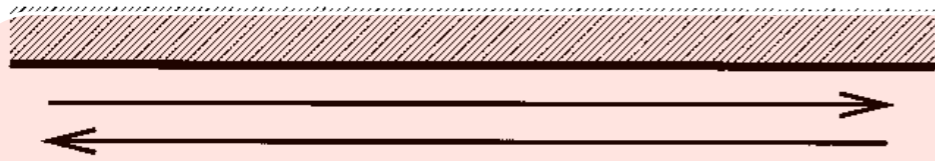
$$\nu = 1/3 \rightarrow K_0$$

1 composite fermion mode



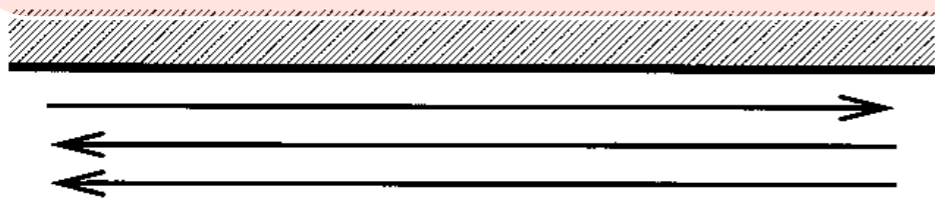
$$\nu = 2/5 \rightarrow 2K_0$$

2 composite fermion modes



$$\nu = 2/3 \rightarrow 0$$

1 charge down - 1 neutral up



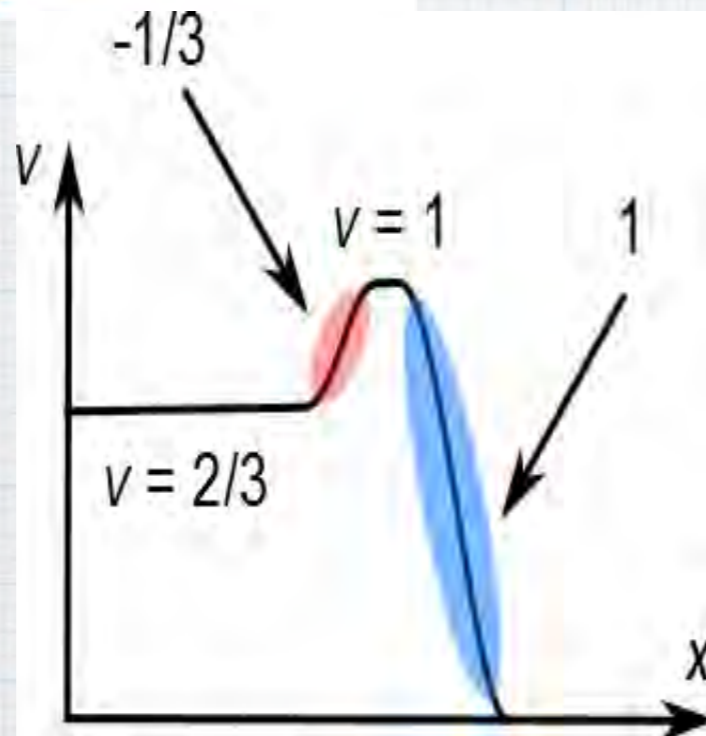
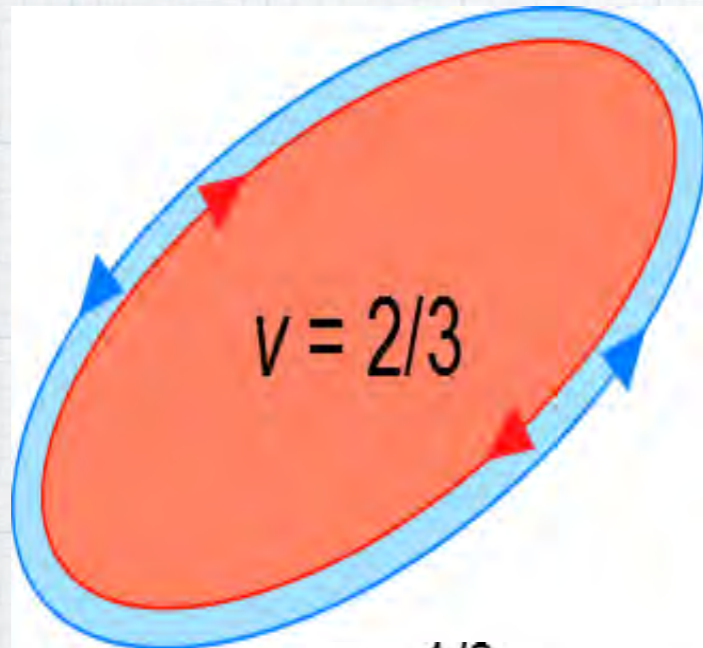
$$\nu = 3/5 \rightarrow -K_0$$

1 charge down - 2 neutral up

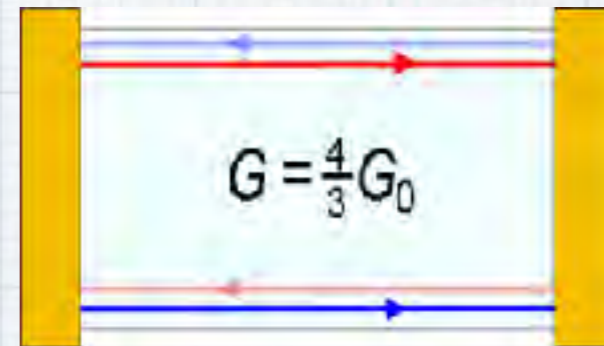
hole - like $\nu = 2/3$ non-equilibrated

non-equilibrated

$$\nu = 2/3 = 1 - 1/3_{\text{upstream}}$$



K=0



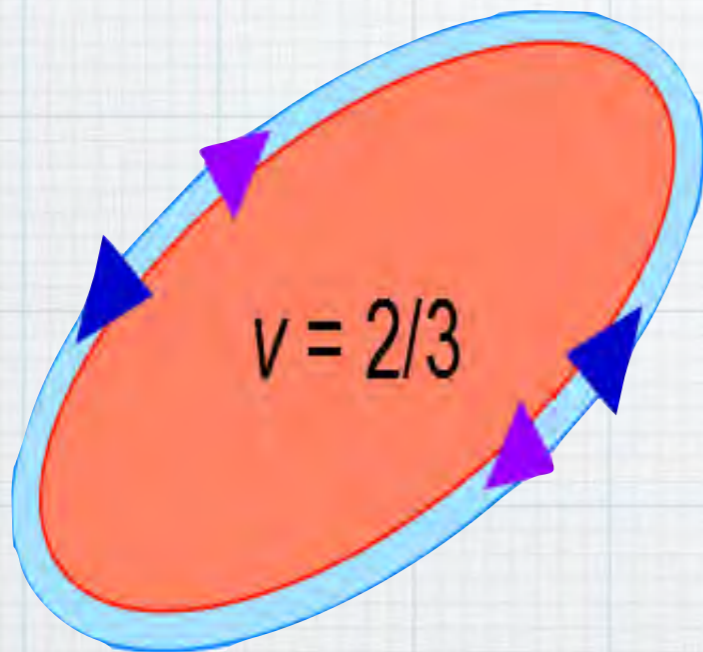
MacDonald, A. H.
Edge states in the fractional quantum Hall effect regime
Phys. Rev. Lett. **64**, 220–223 (1990)

hole - like $\nu = 2/3$ equilibrated

equilibrated

$\nu = 2/3 = 2/3 - \text{neutral}_{\text{upstream}}$

K=0



Kane, C. L., Fisher, M. P. A. & Polchinski, J.
Randomness at the edge:
theory of quantum Hall transport at filling 2/3
Phys. Rev. Lett. **72**, 4129–4132 (1994)

neutral modes....carrying energy w/o net charge

equilibration of **counter-propagating** charge modes



topological **neutral modes**

- * **invisible** in conductance measurements
- * bosonic thermal conductance κ_0
- * associated only with particular **FQHE** states

Why thermal conductance in FQHE?

- * **topological constant** : determined by bulk wave-function
 - * reveals **NET** chirality of modes (down-up)
- * insensitive to edge reconstruction

these are true for abelian particles

however

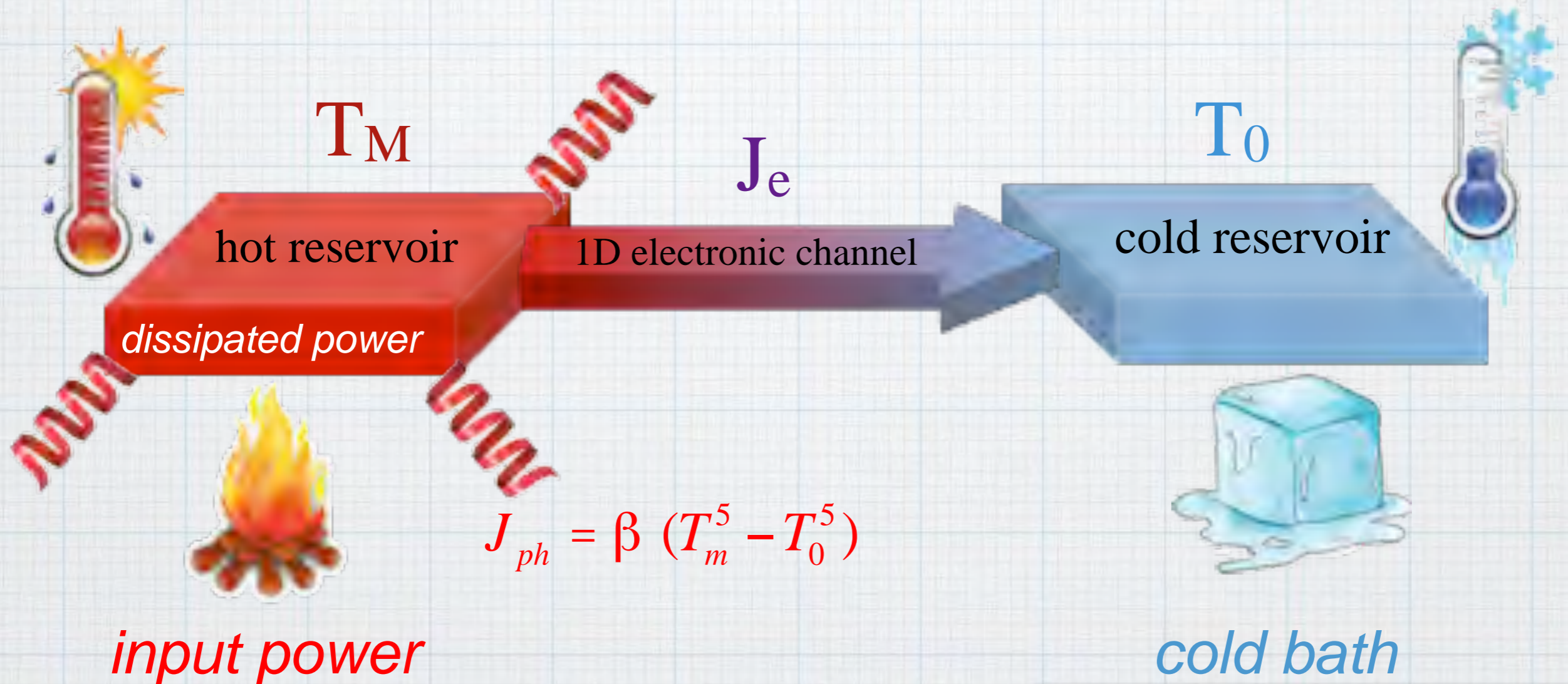
$$K_{non-abelian} = \left(n + \frac{1}{2}\right) \kappa_0 \quad \text{(Majorana)}$$

The experiment

Working principle :

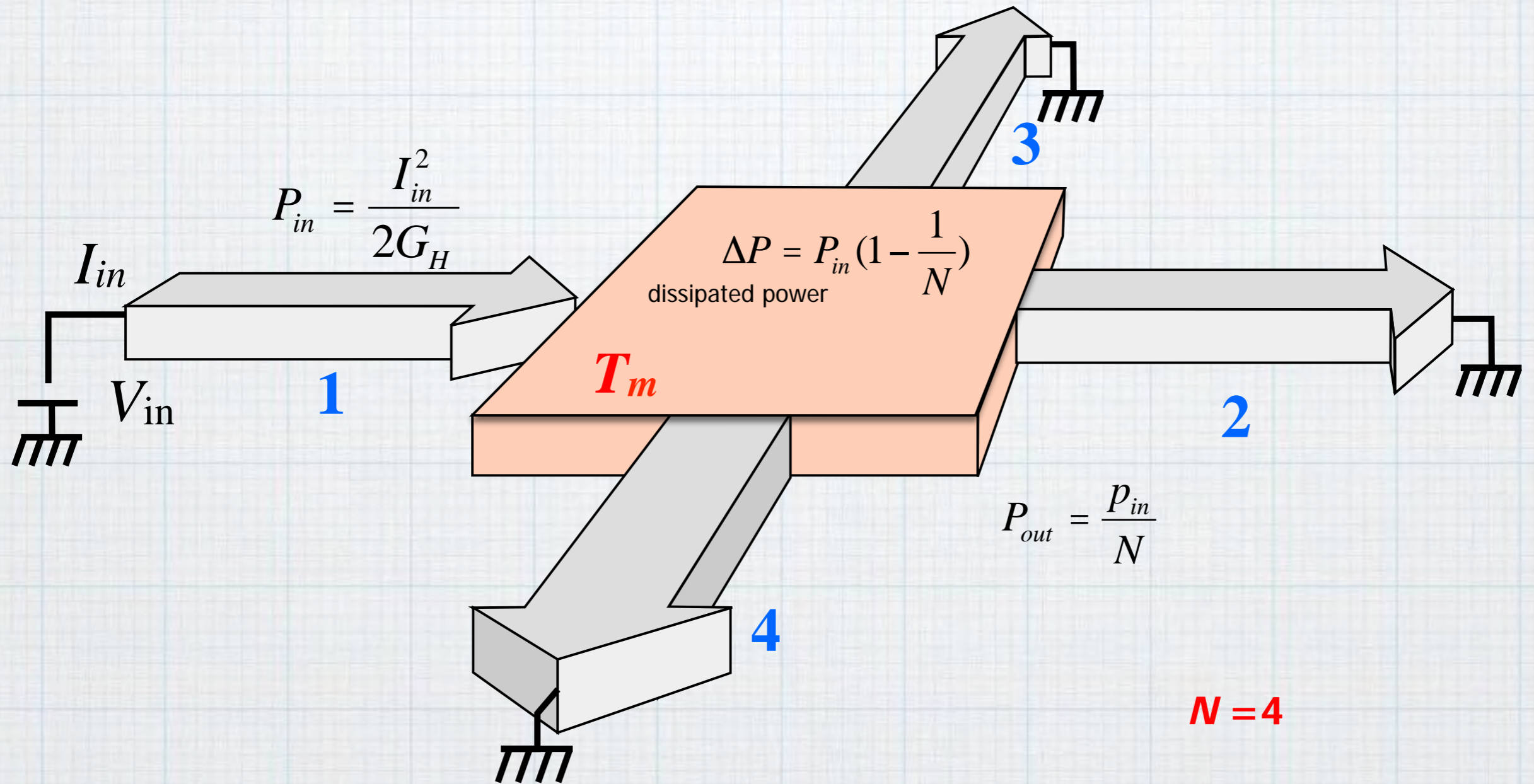
flow of dissipated power..... $J_{tot} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2) + J_{ph}$

electrons *phonons*



Wellstood, F. C., Urbina, C. & Clarke, J.
Hot-electron effects in metals.
Phys. Rev. B **49**, 5942–5955 (1994)

N - arm device



we measure only temperature...

electron temperature in grounded contacts..... T_0

electron temperature in heated reservoir..... T_m

$$\Delta P = J_{th}^{total} = 0.5 K (T_m^2 - T_0^2) + \beta (T_m^5 - T_0^5)$$

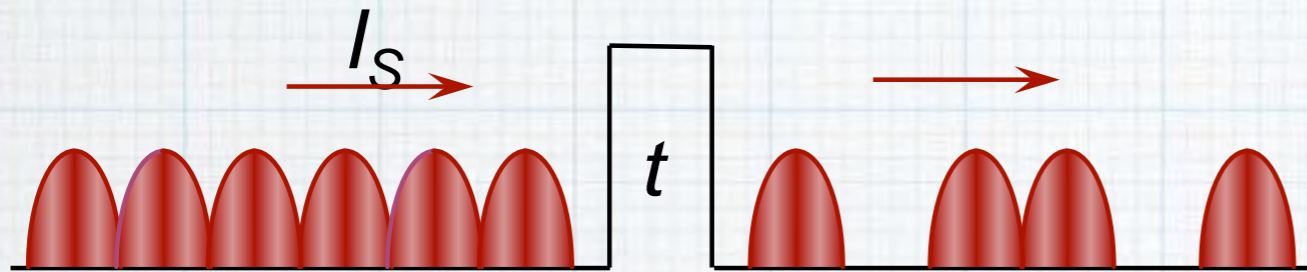
- small Tphonon term irrelevant
- high Tphonon term subtracted
- K determined

measuring temperature

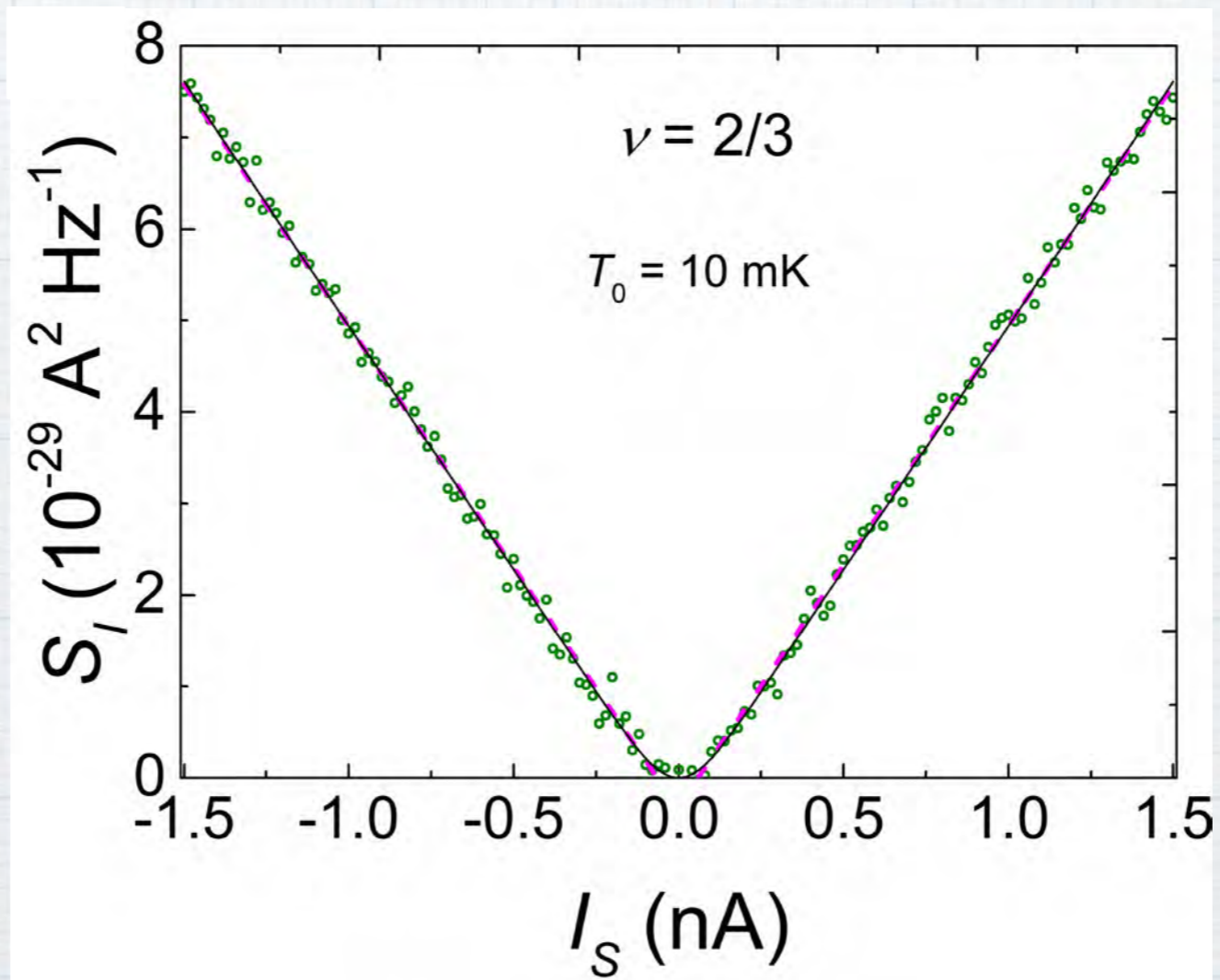
temperature in grounded contacts..... T_0
shot noise

excess temperature in heated reservoir..... $T_m - T_0$
thermal noise

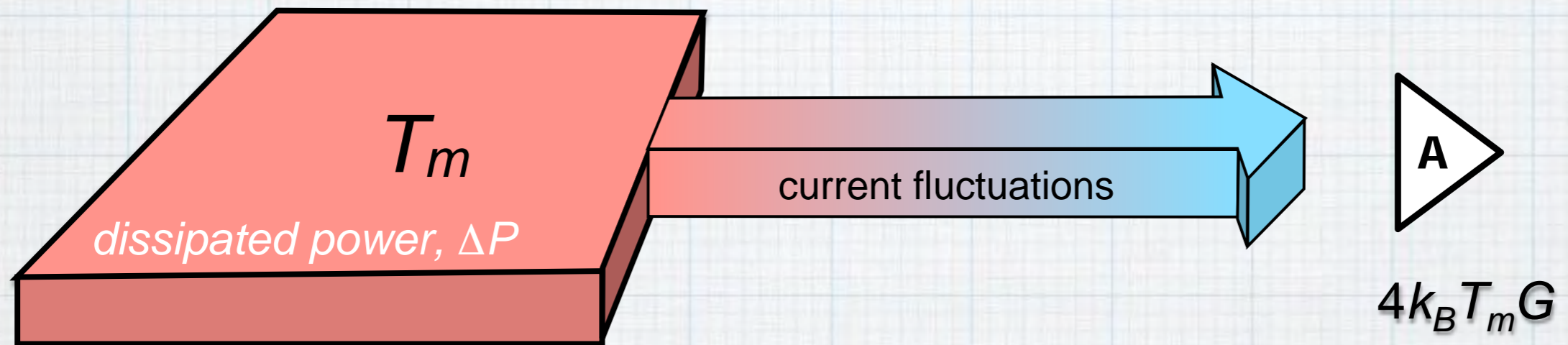
measuring T_0 shot noise



$$S_i(\omega : 0) = 2e^* I_s t(1-t) \cdot \mathfrak{S}(V_S, T) + 4k_B T G$$



measuring T_m Johnson-Nyquist noise

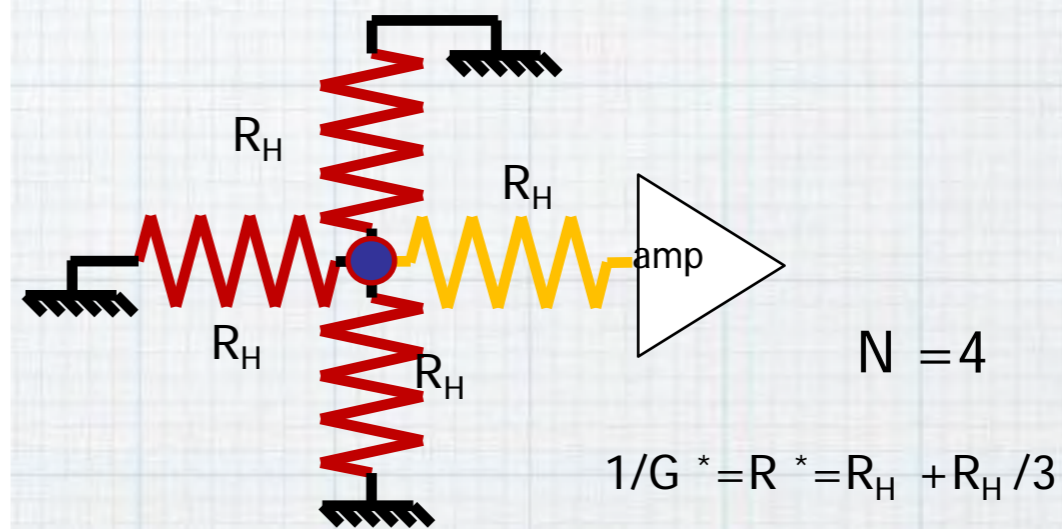
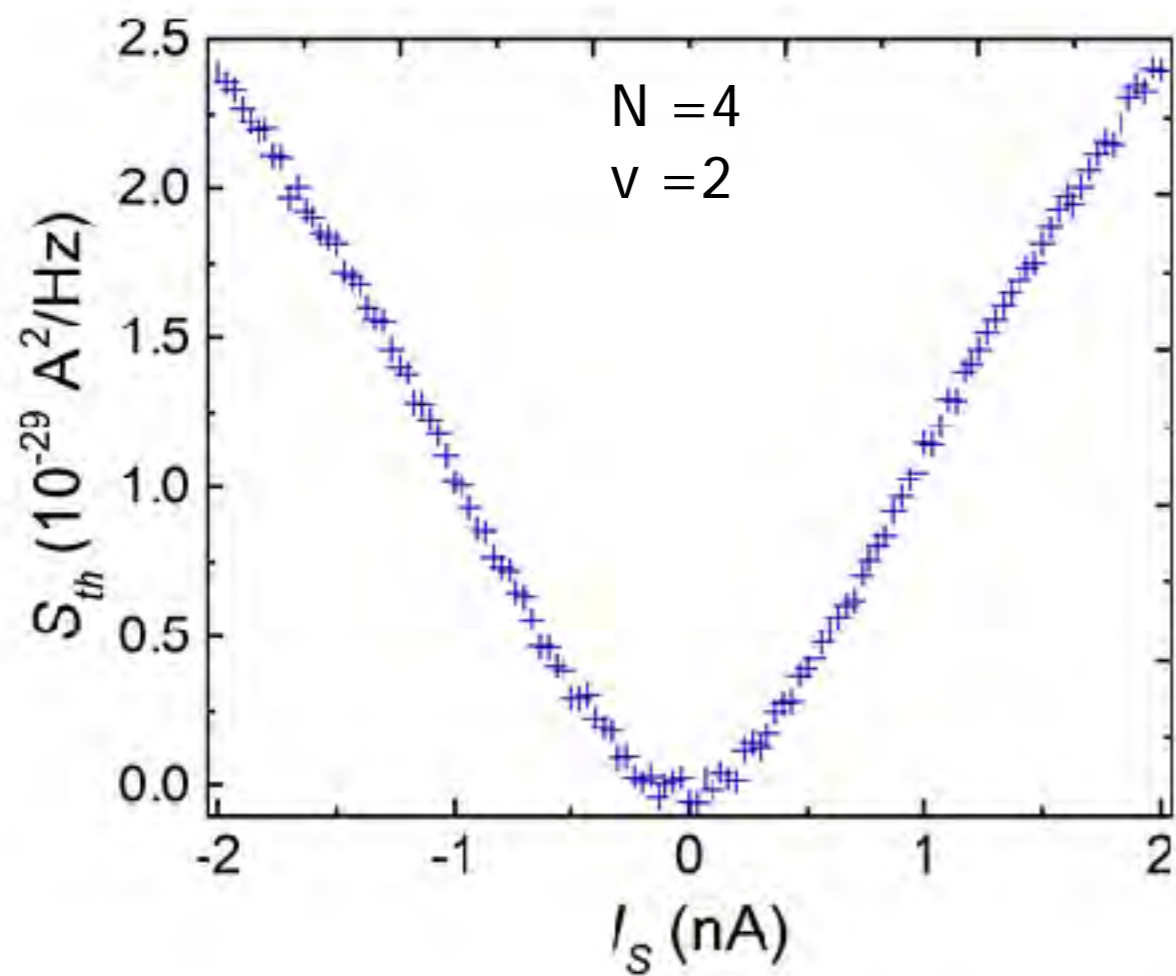


- modes leave contact with noise $4k_B T_m G$
- even if modes cool down with distance...

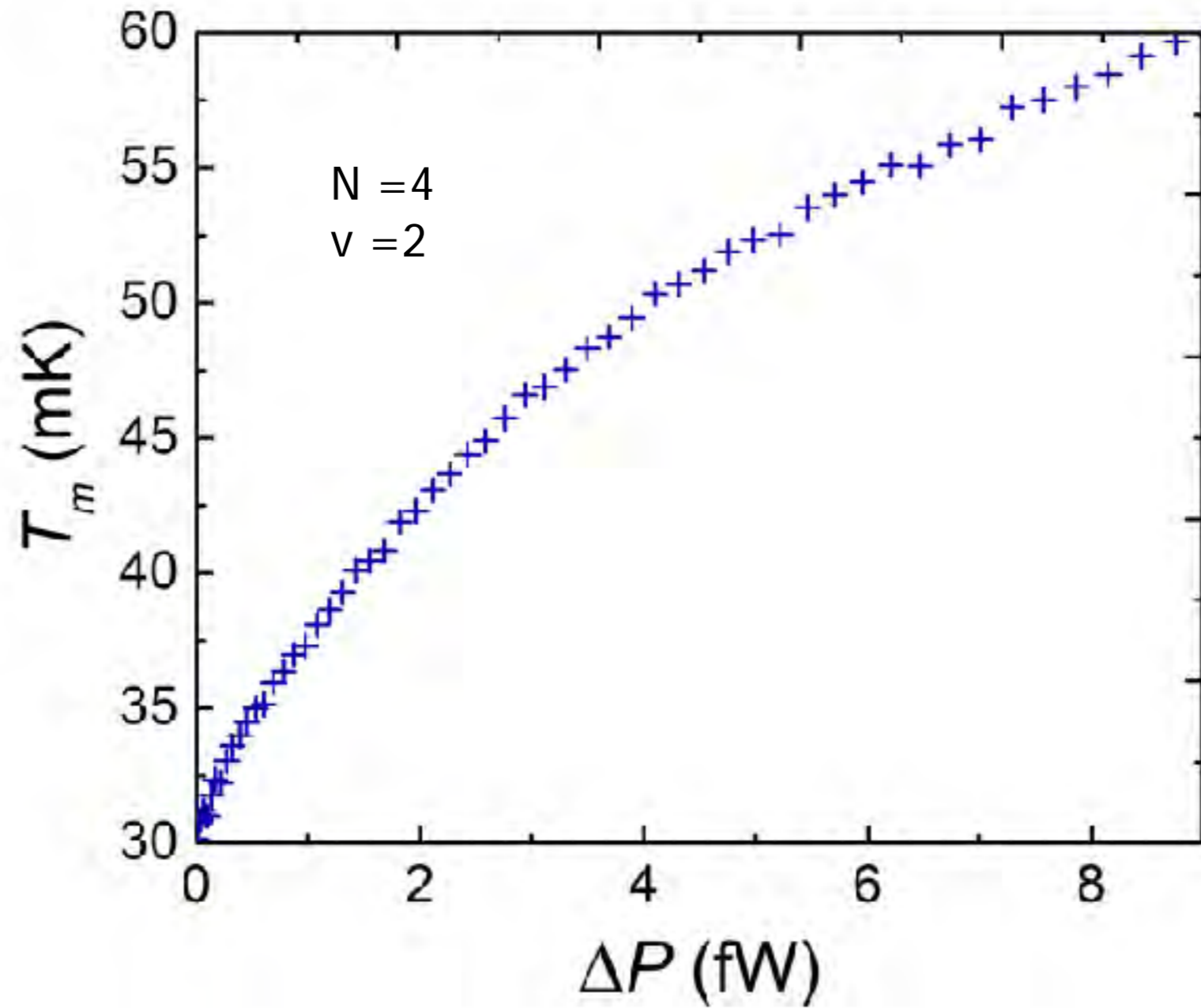
low frequency current fluctuations conserved

measuring T_m Johnson-Nyquist noise

excess Johnson - Nyquist noise ... $2k_B G^*(T_m - T_0)$

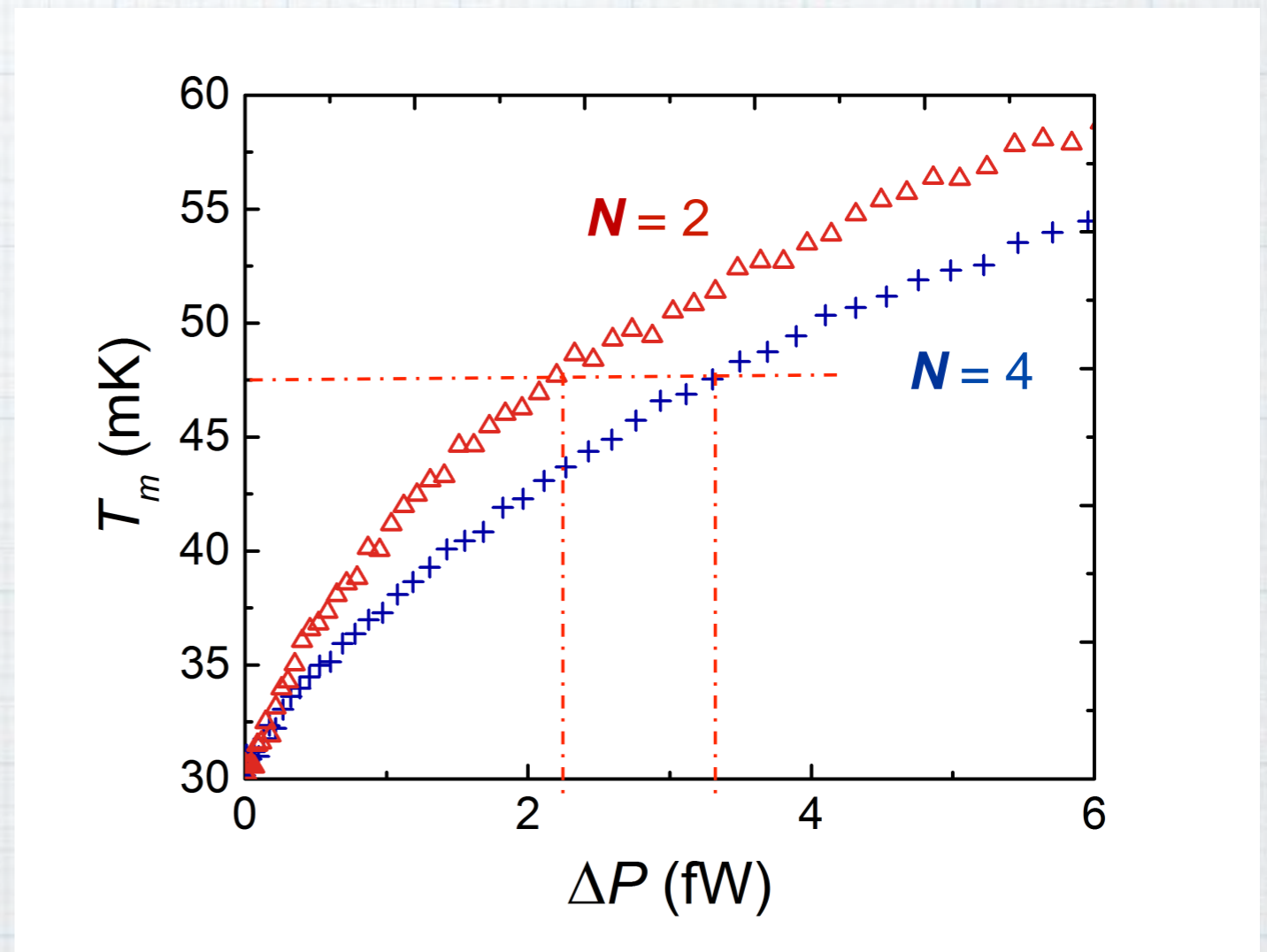
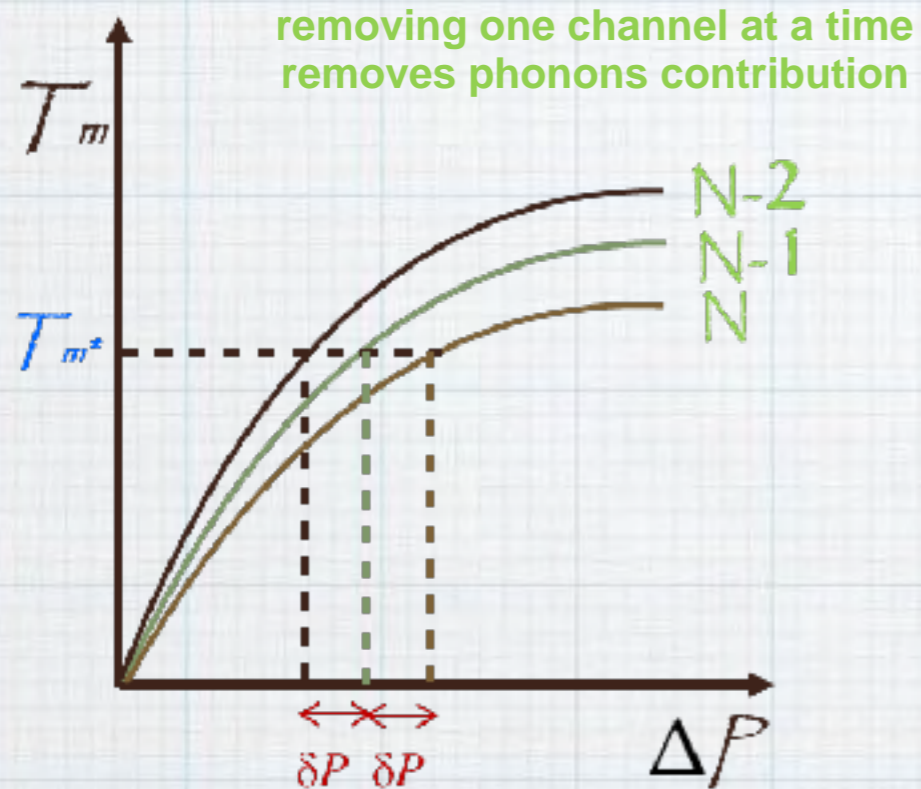


T_m vs dissipated power

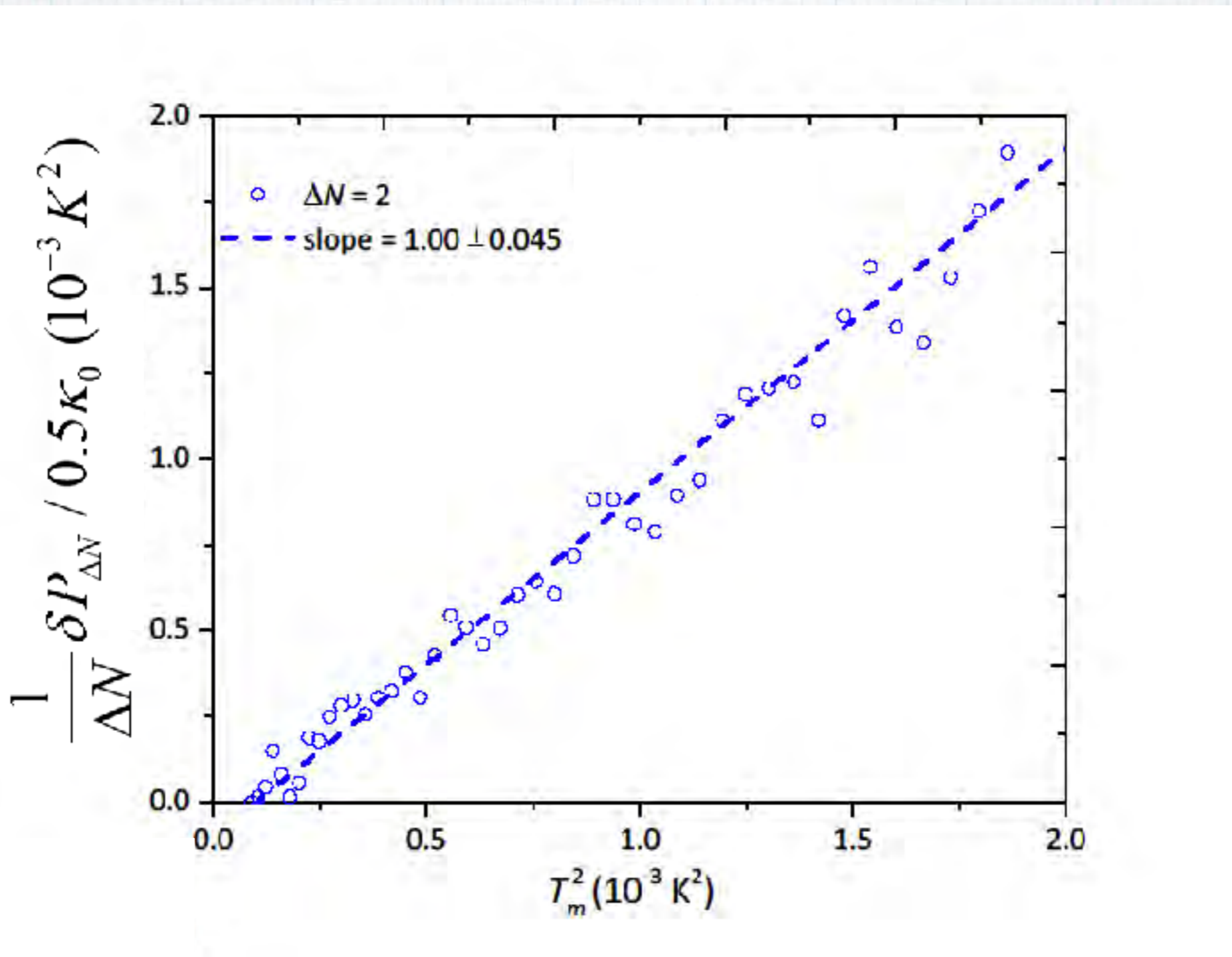


actual analysis

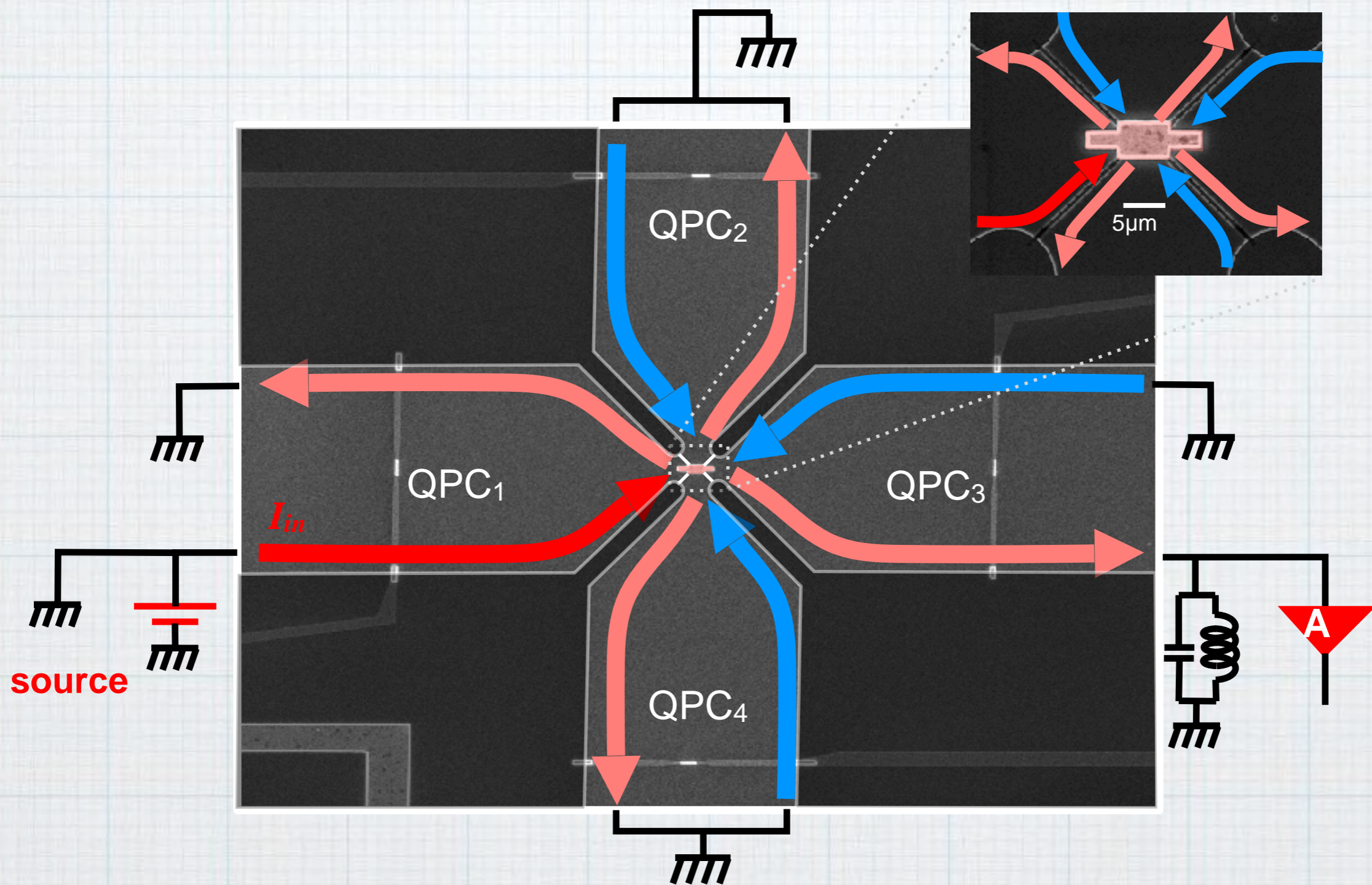
$$J_{tot} = \frac{1}{2} \kappa_0 (T_m^2 - T_0^2) + J_{ph}$$



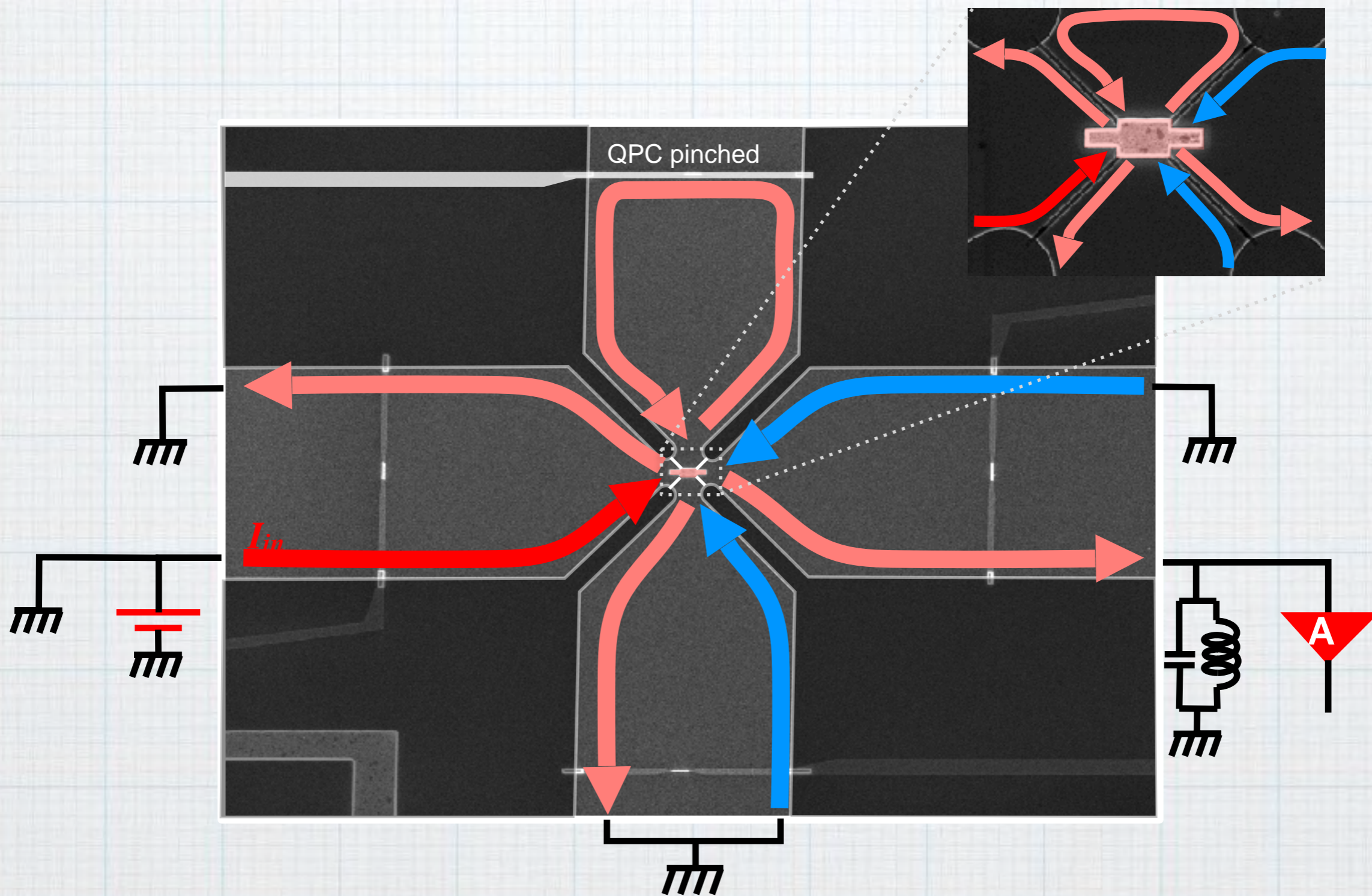
getting κ



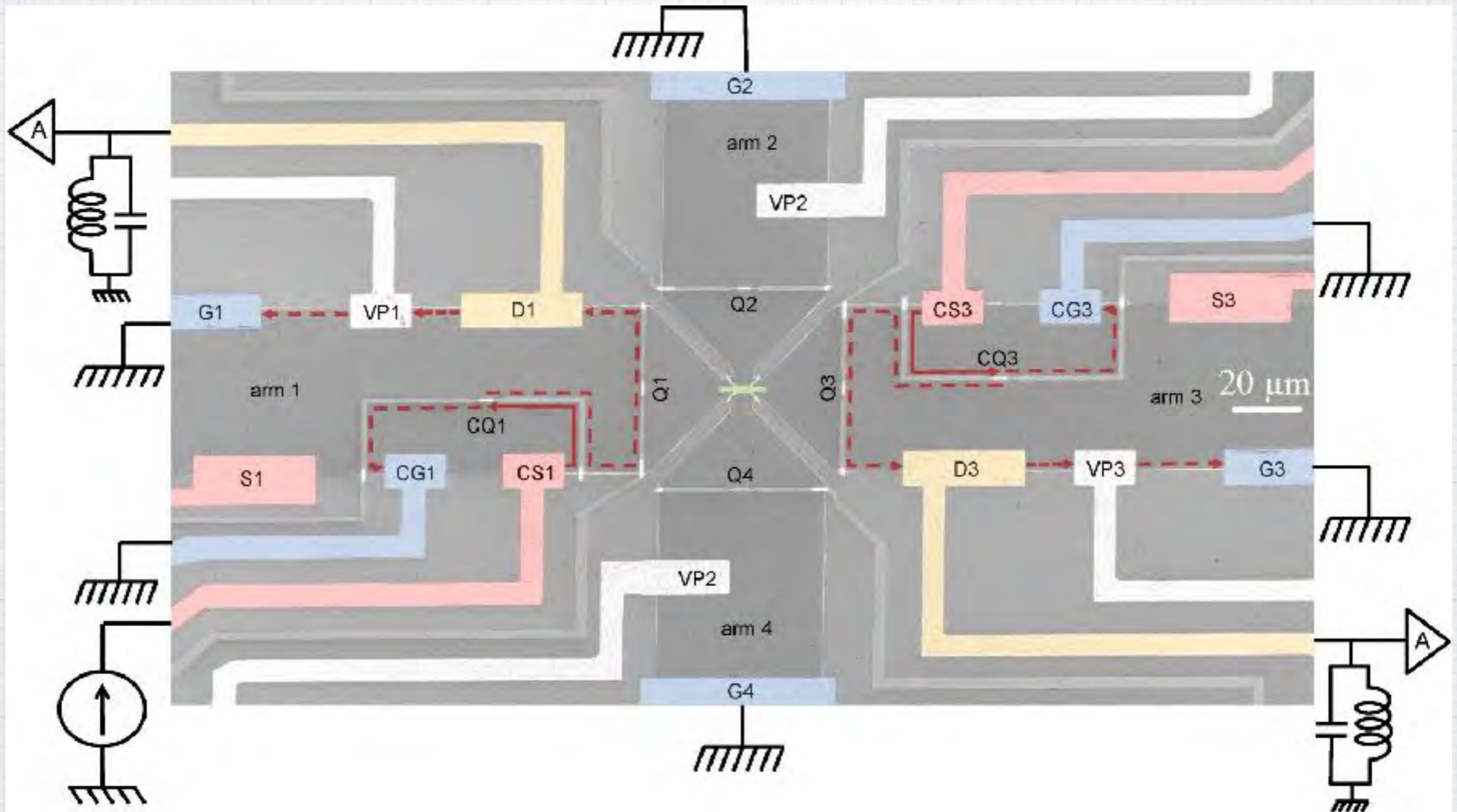
realization..... $N = 4$



$N = 3$



realized structure



points of consideration *not an easy experiment*

- electrons fully equilibrate in the small floating reservoir T_m
- outgoing charge channels carry **only** Johnson-Nyquist noise
without shot noise
- no presence of bulk energy modes (may increase the *apparent* thermal conductance)
- length of arms is limited (~150 μm , temperature equilibration between up-down modes)
- equal splitting between arms, amplifier gain determination, contacts' resistance, ...

* weak interaction regime (IQHE) $\nu = 2, 1$

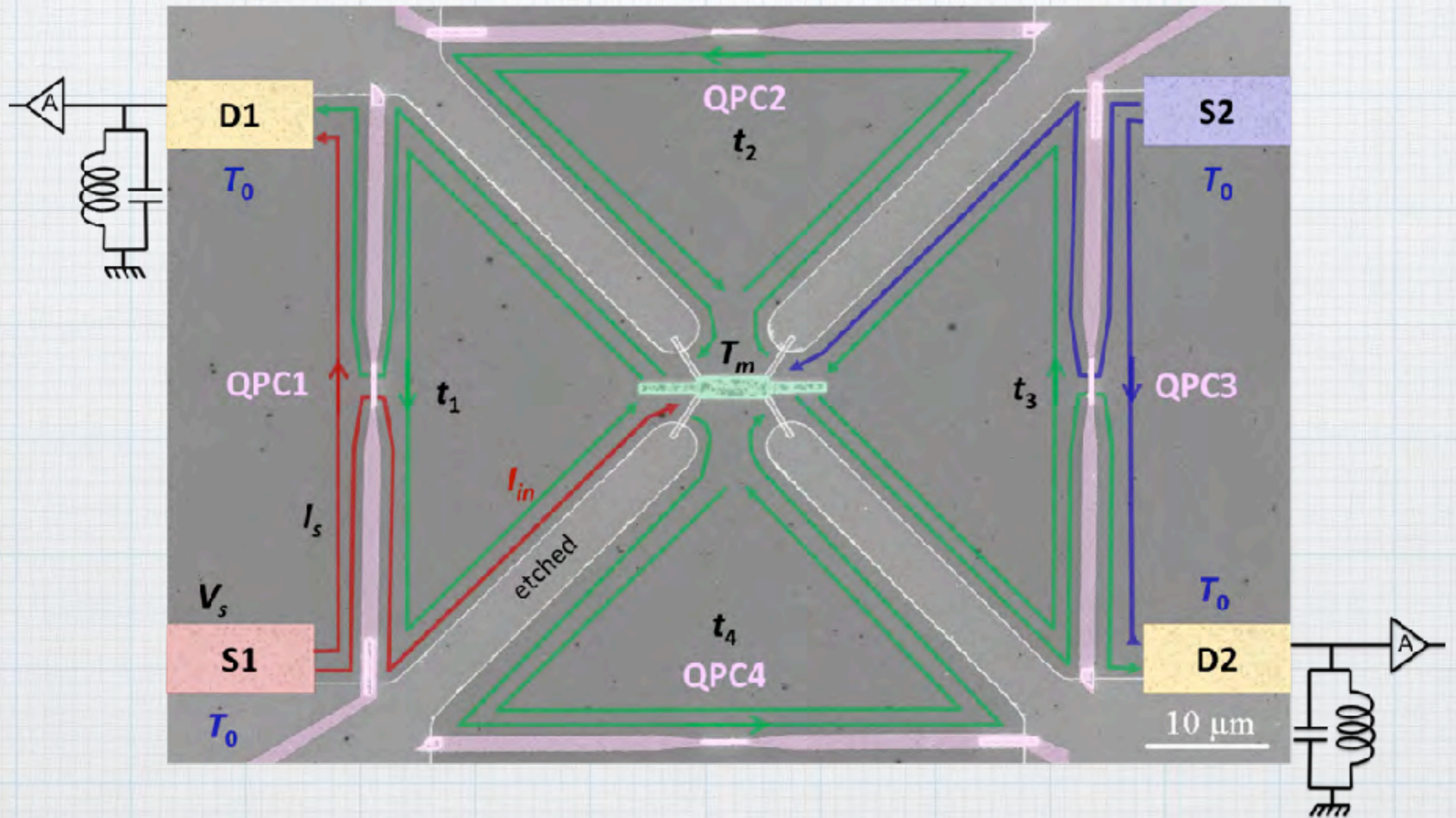
* strong interaction regime (FQHE)

* particle - like : $\nu = \frac{1}{3}$

* hole - like : $\nu = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}$

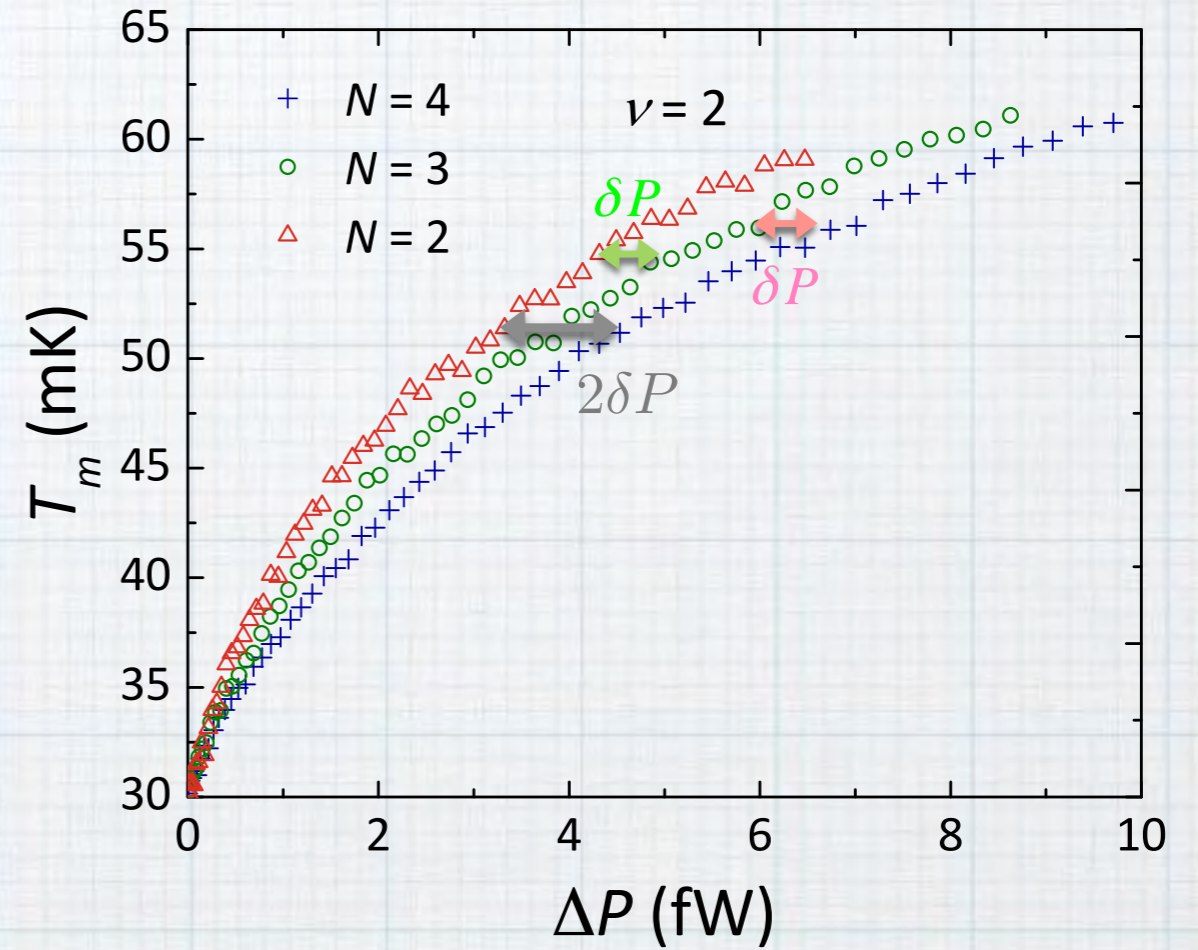
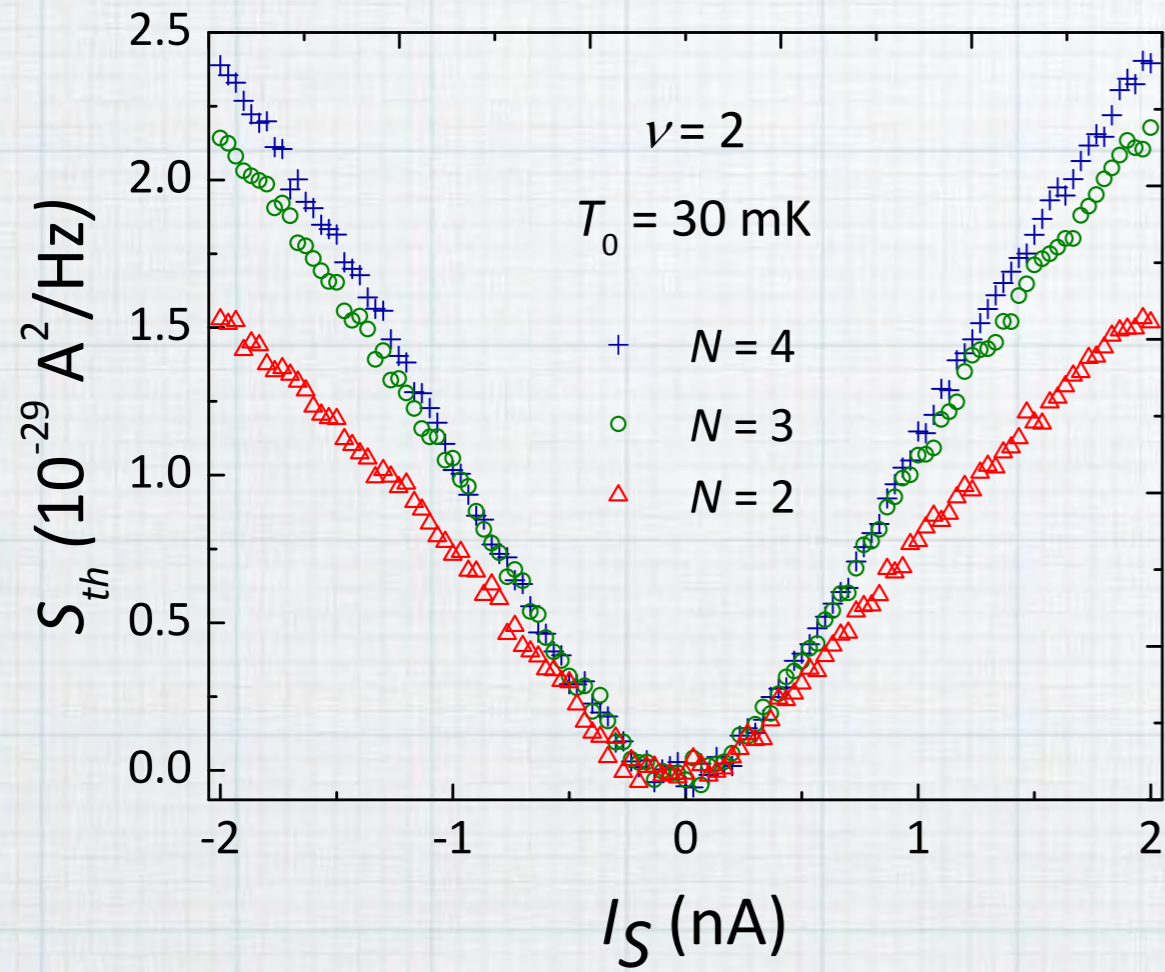
$$v = 2 \quad V_{\text{QPC}} = 1$$

$$N = 2$$



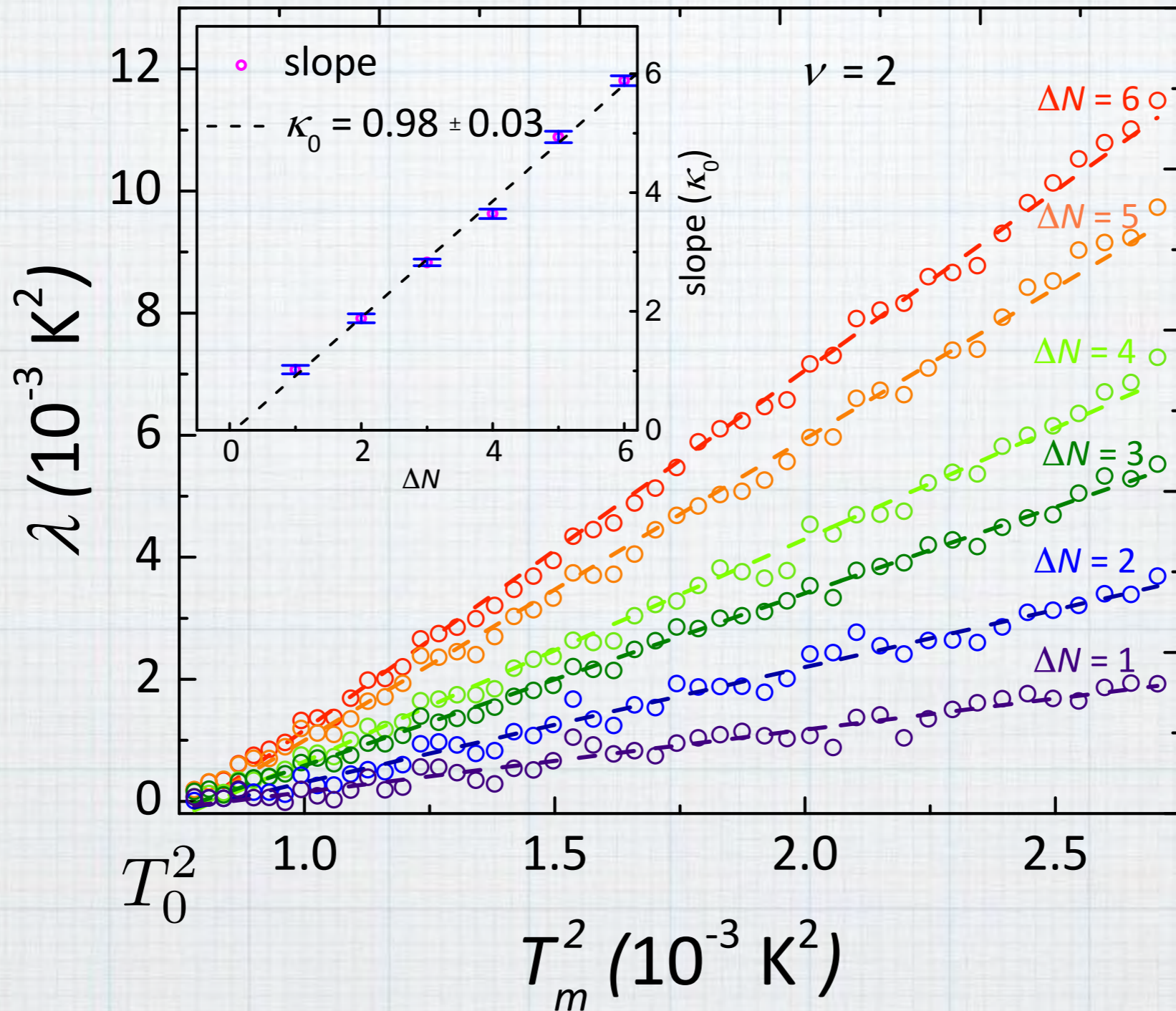
Results :

$$\nu = 2$$



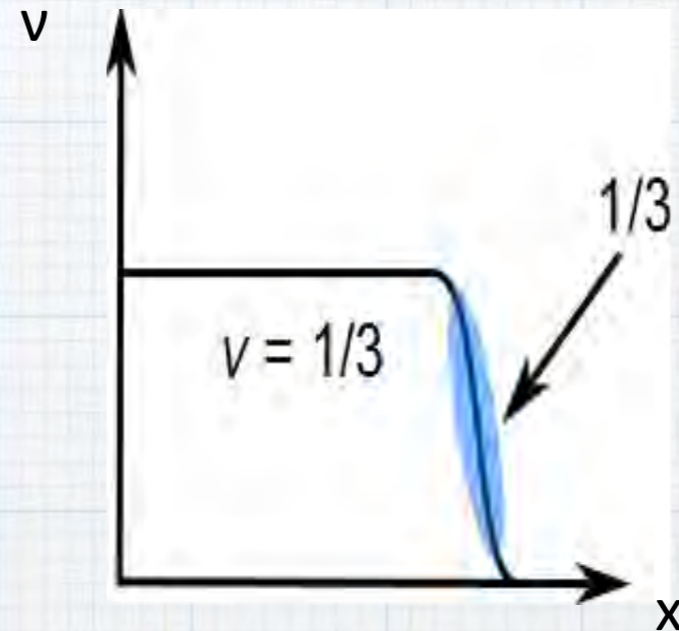
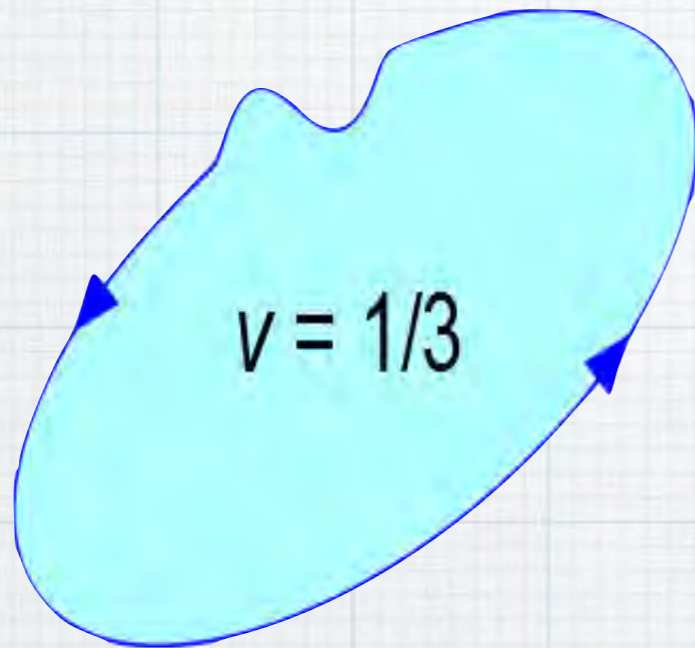
Results :

$$\nu = 2$$



$$\lambda = \Delta P / \kappa$$

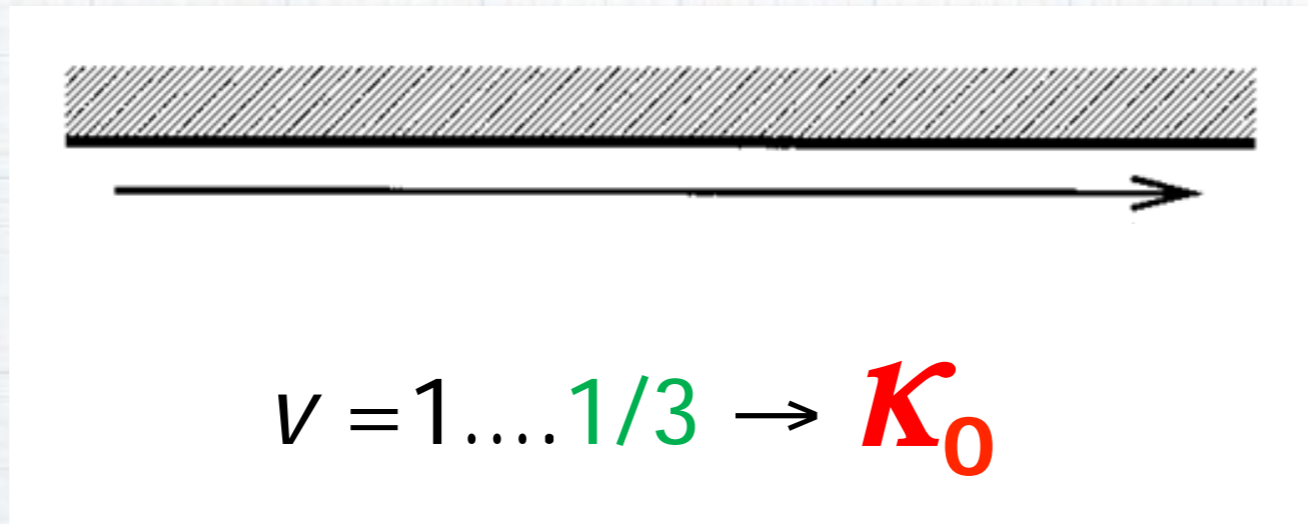
particle - like $\nu = 1/3$



bulk: gapped - incompressible liquid

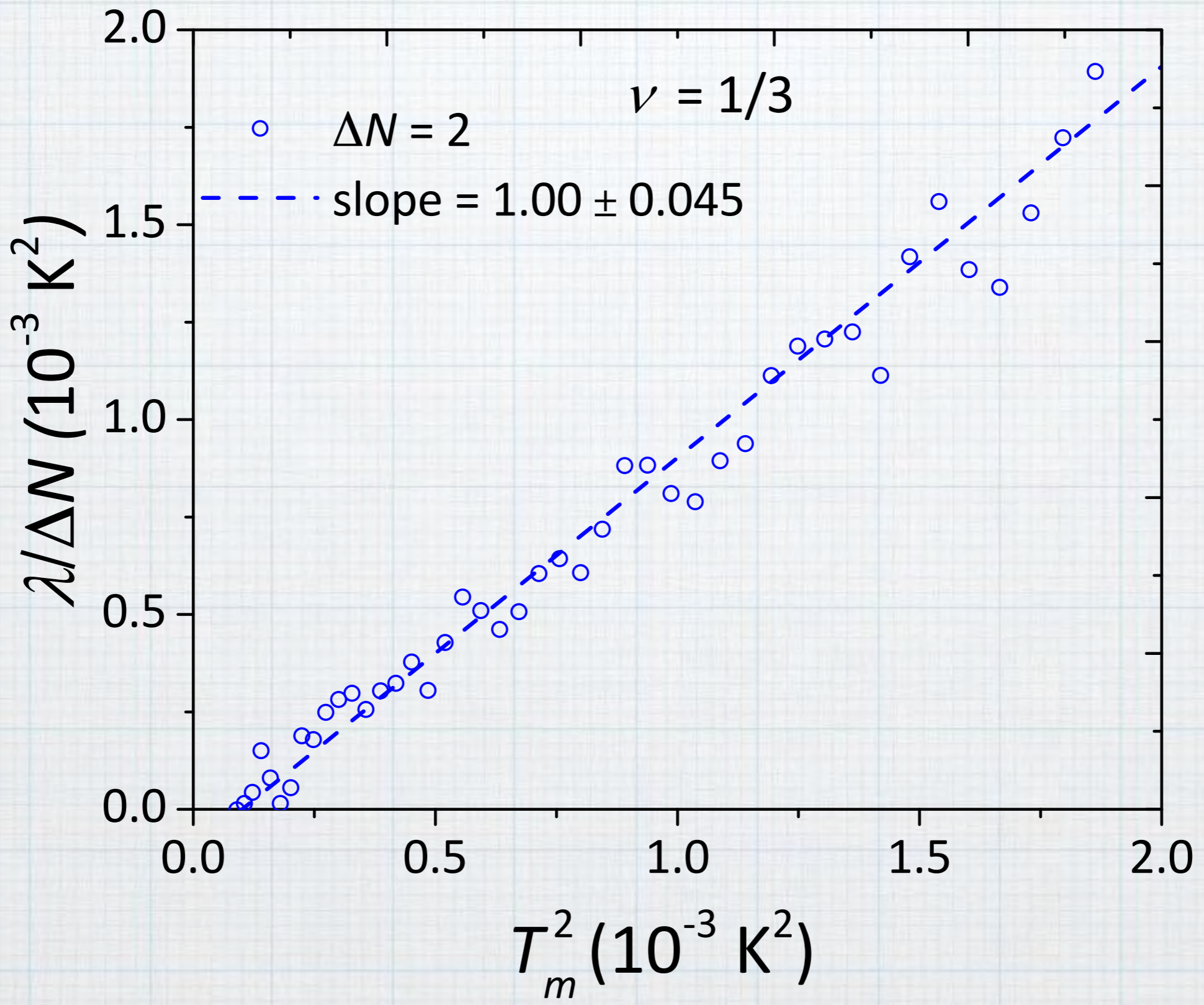
edge: single charge mode $G = G_0/3$

$$K = \kappa_0$$



$$v = 1 \dots 1/3 \rightarrow \mathbf{K}_0$$

1 electron mode....**1** composite fermion mode



interactions do not affect K

what about K of neutral modes ?

more complex fractions...

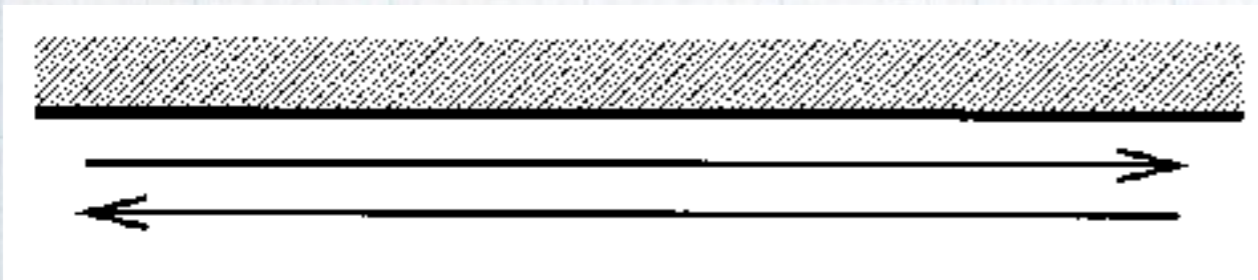
fractional hole-conjugate states..... $1/2 < \nu < 1$

full Landau level with holes
hence, counter-propagating modes

always with upstream neutral modes

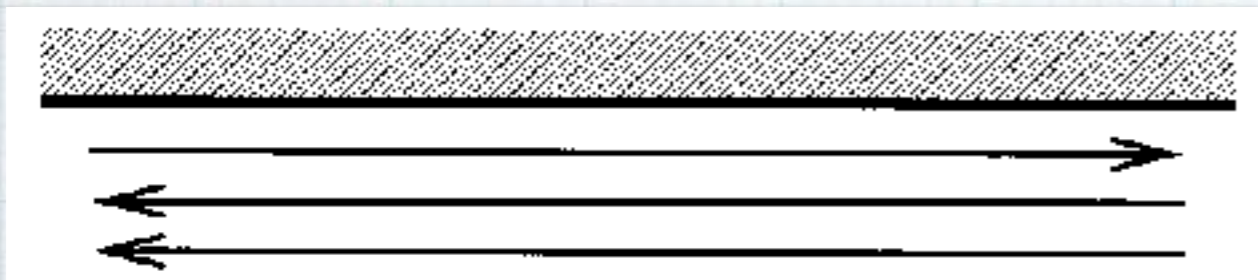
$\nu = 2/3, 3/5, 4/7$

K of hole - states... Kane & Fisher 1997



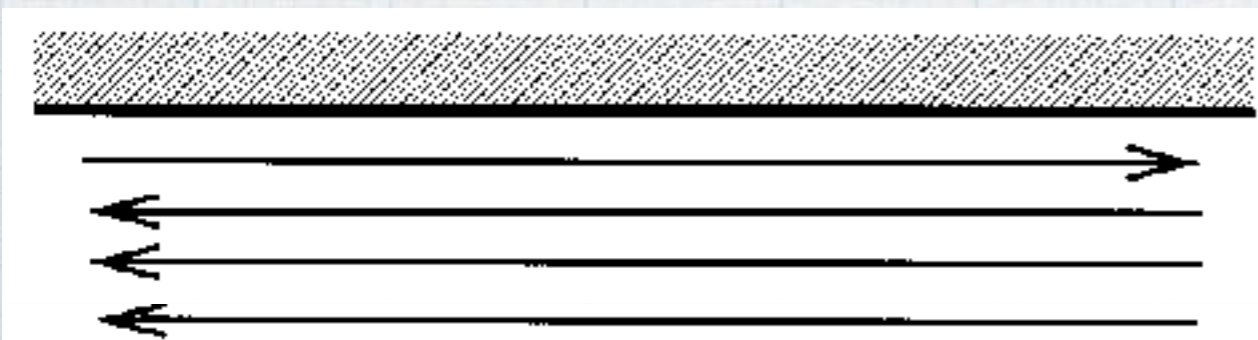
$$\nu = 2/3 \rightarrow K=0$$

1 charge down - 1 *neutral* up



$$\nu = 3/5 \rightarrow -K_0$$

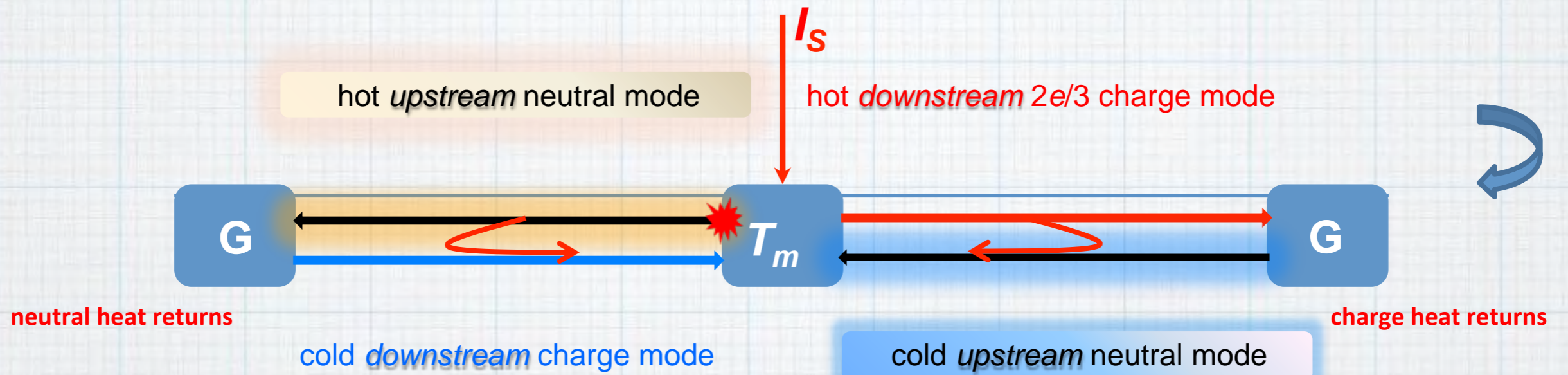
1 charge down - 2 *neutrals* up



$$\nu = 4/7 \rightarrow -2K_0$$

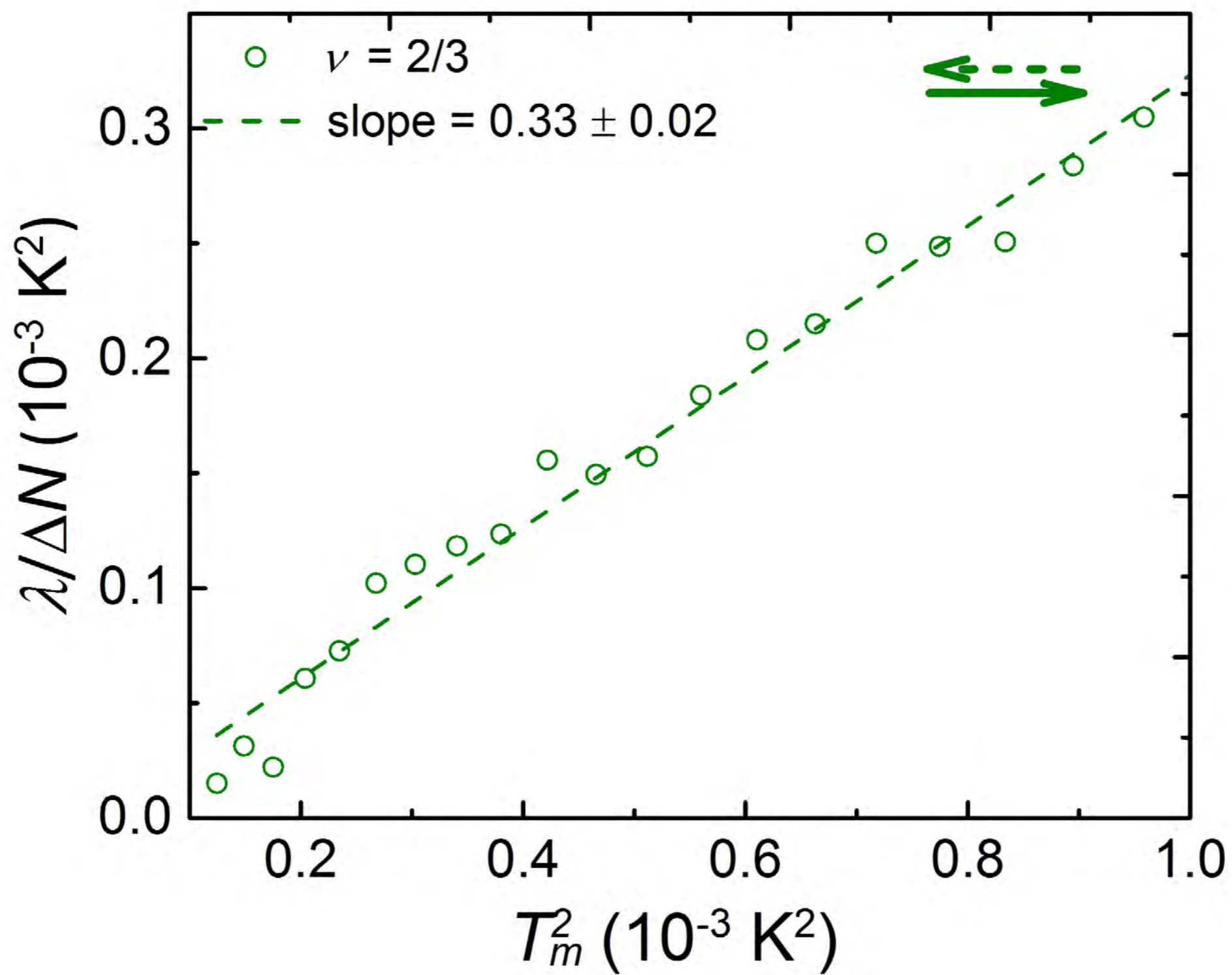
1 down charge - 3 *neutrals* up

$\nu = 2/3$ why $K = 0$?

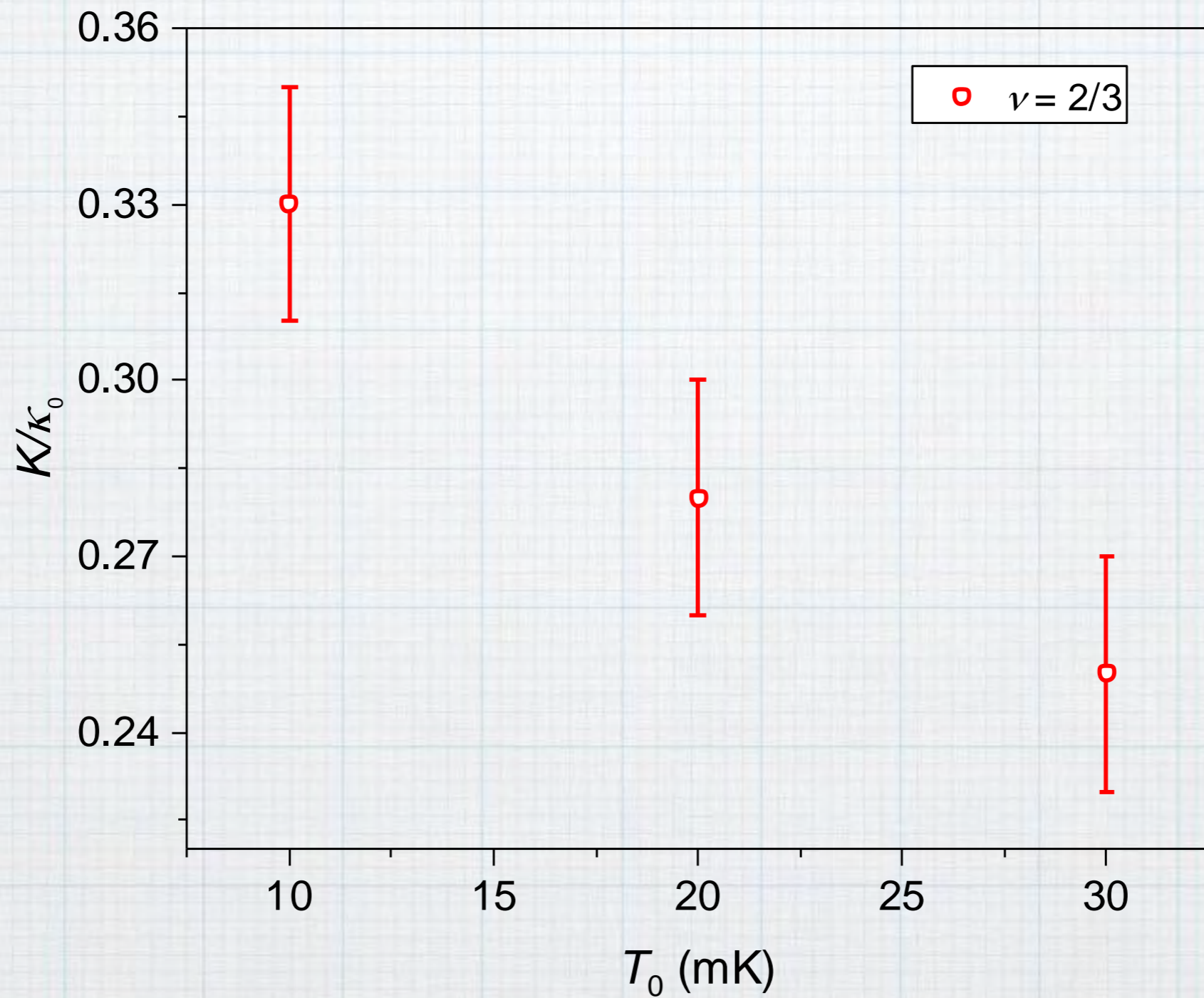


equal number of **down** and **up** modes

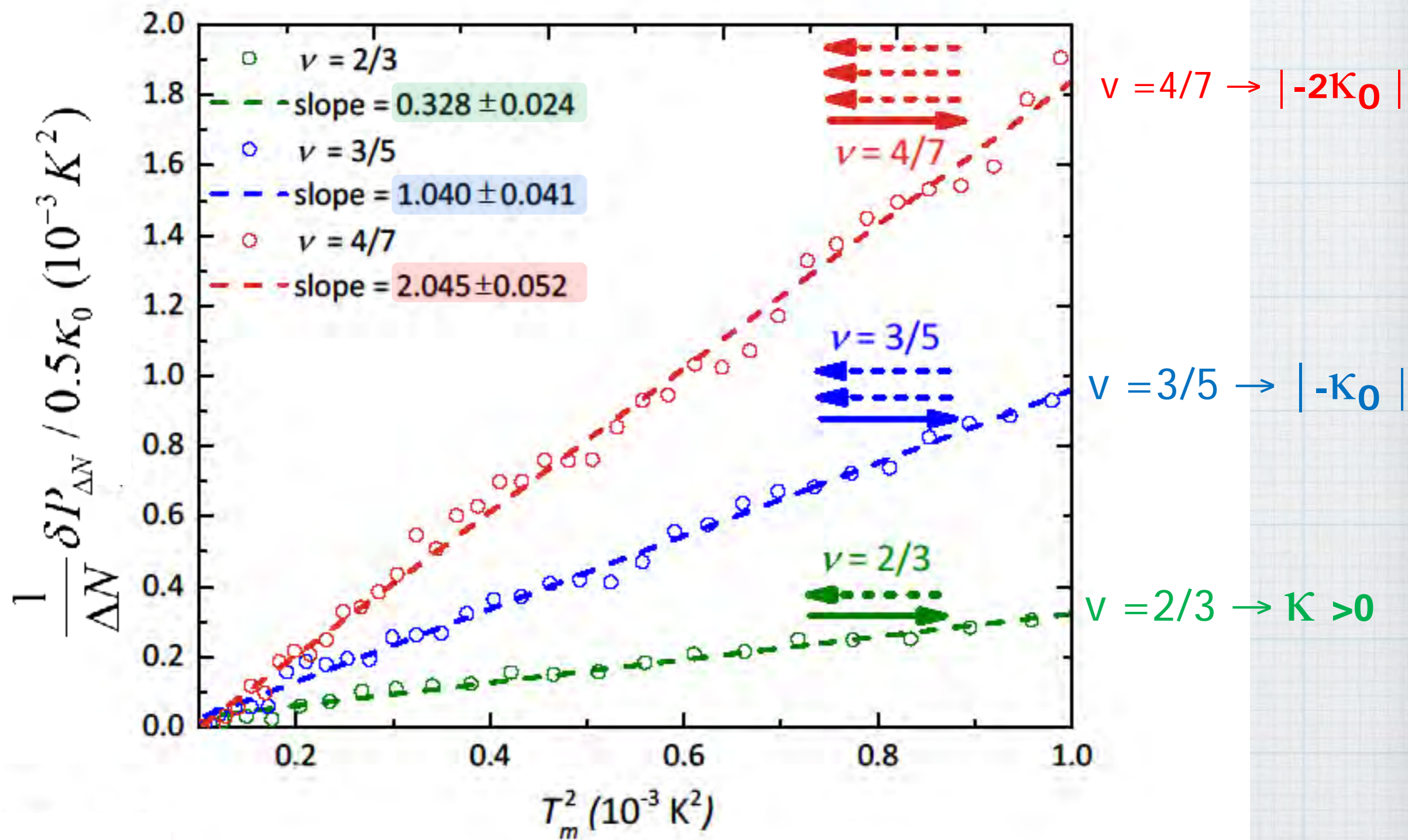
full equilibration ONLY at large length.....**all** emitted heat **returns**



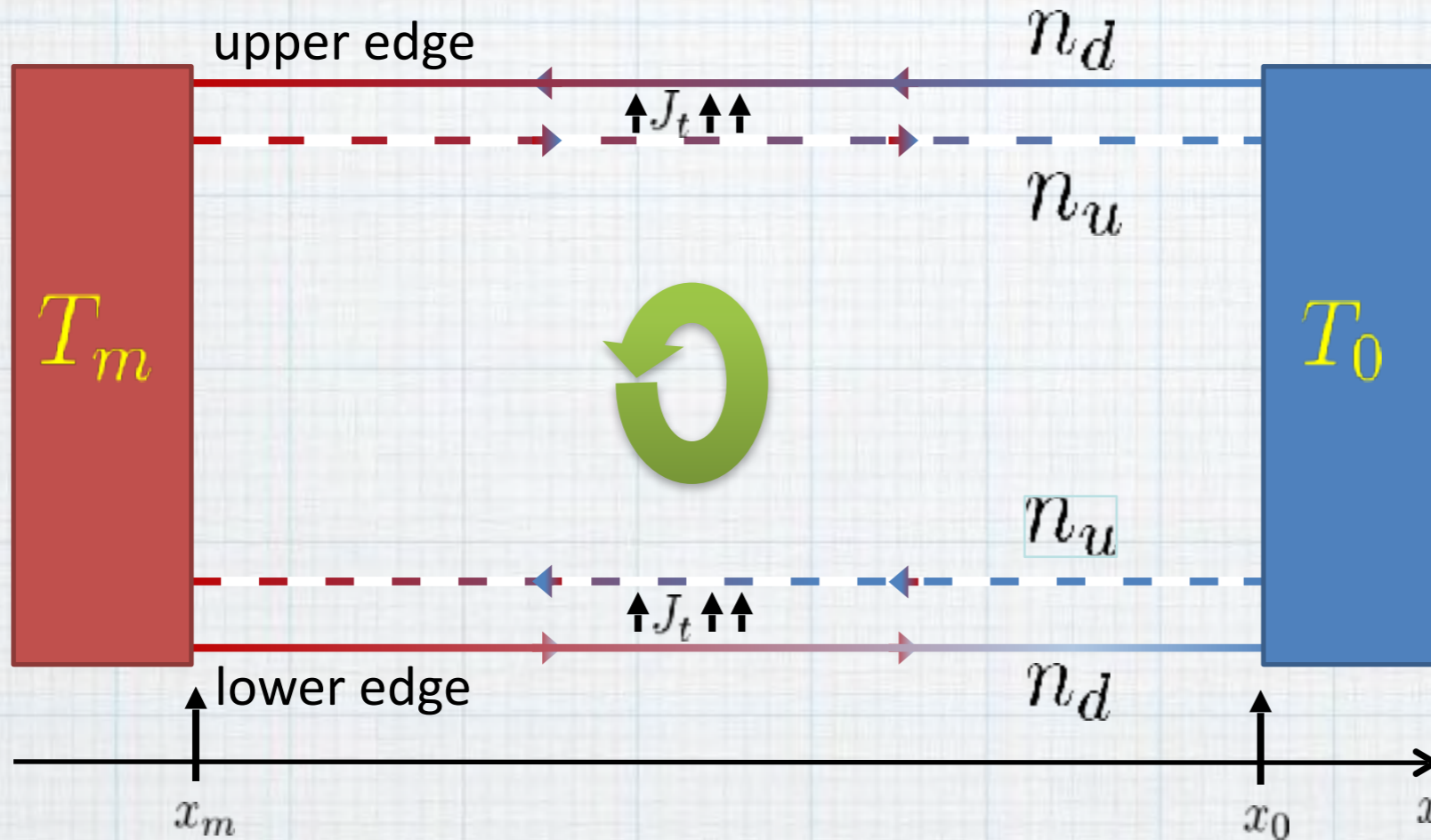
Temperature dependence



hole-states with more upstream neutral modes



calculating $T(x)$ & K $v=2/3$



$$n_d = n_u = 1$$

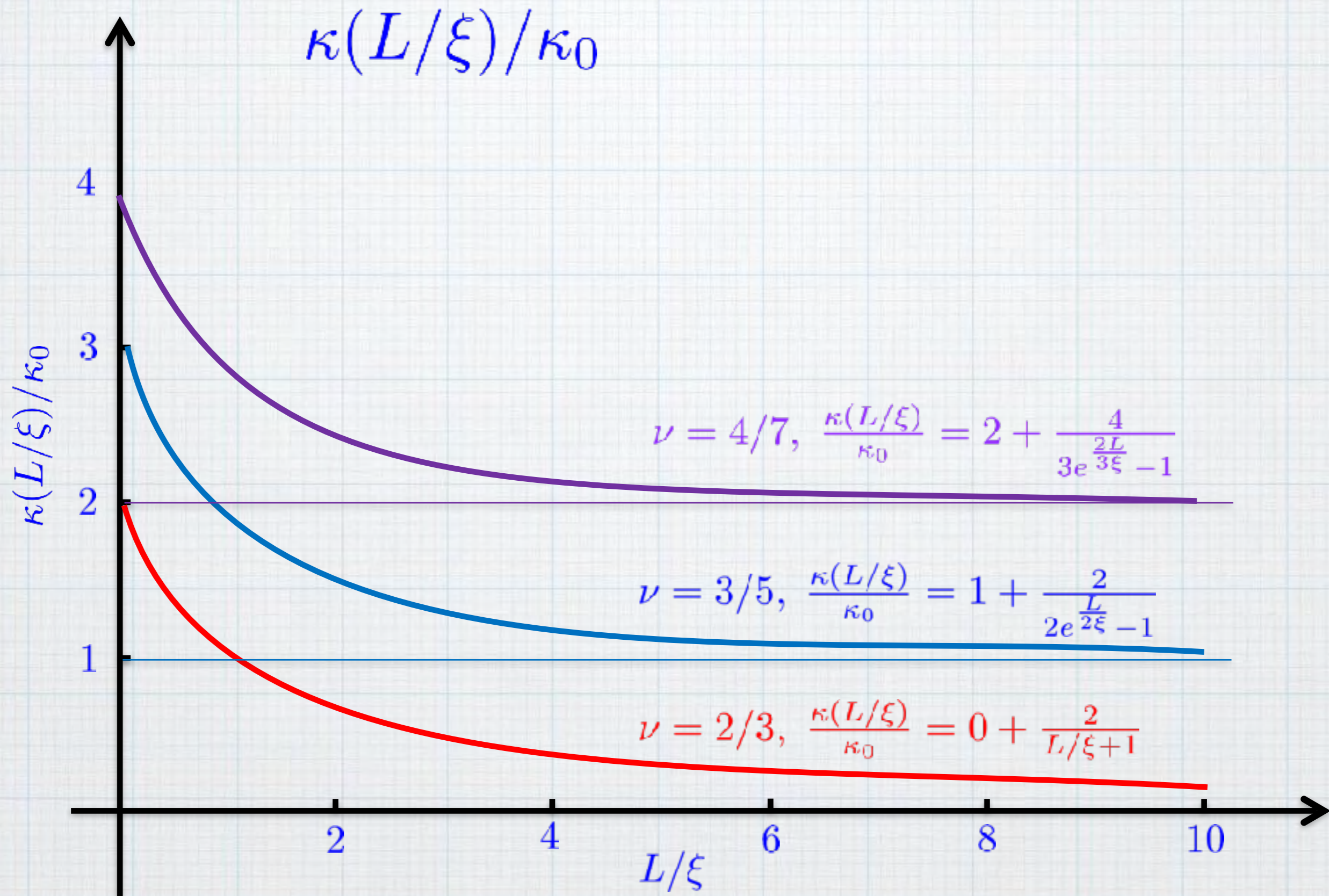
$$J = KT^2$$

$$0.5n_u\kappa_0\partial_x T_u^2(x) = -j_t(x)$$

$$0.5n_d\kappa_0\partial_x T_d^2(x) = -j_t(x)$$

Newton's law of cooling

$$j_t(x) = \frac{\kappa_0}{2\xi_T} \left(T_d^2(x) - T_u^2(x) \right)$$



Summary :

1D electron modes

1D fractional modes

1D neutral modes



quantized thermal conductance

LETTER

doi:10.1038/nature22052

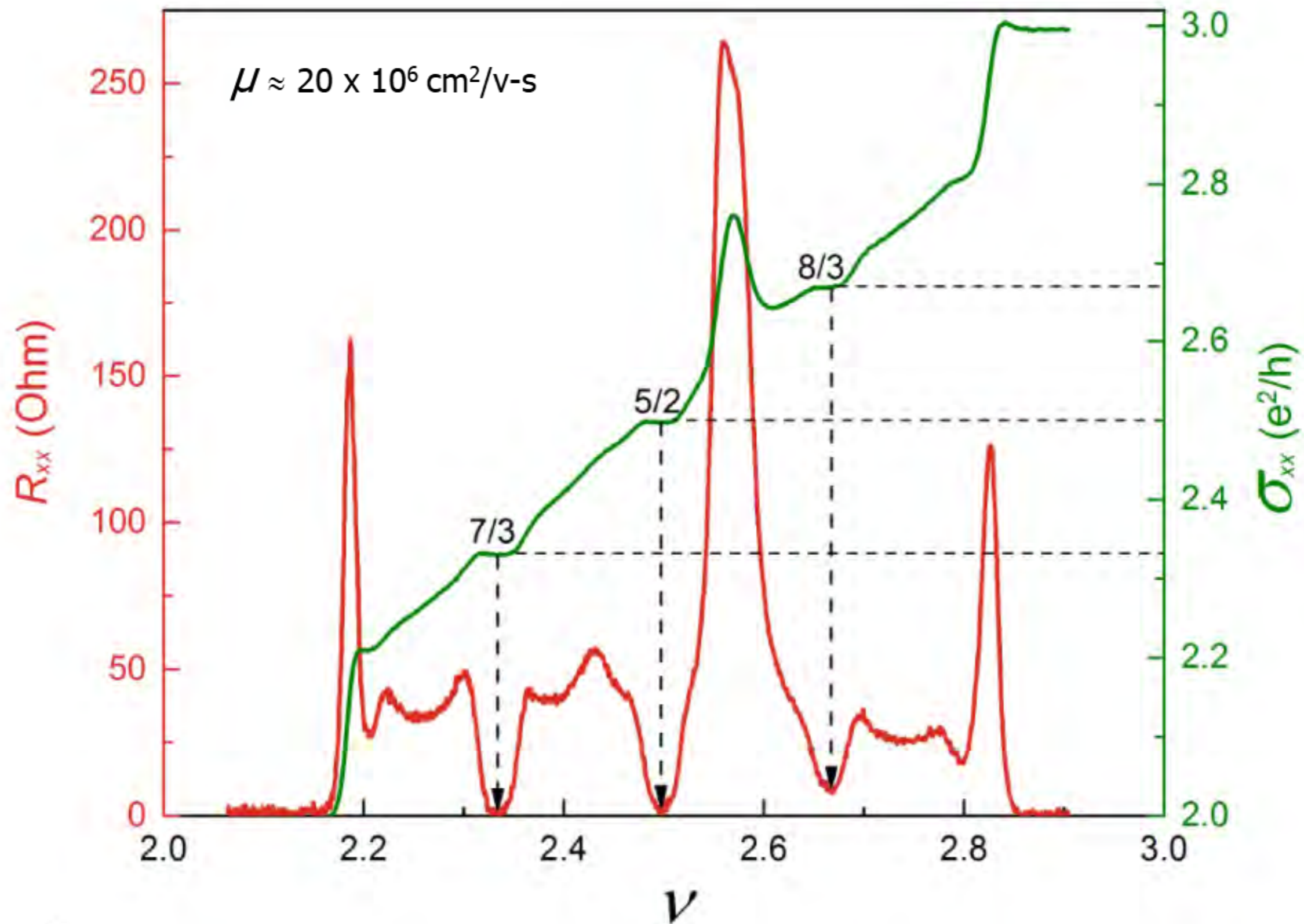
Observed quantization of anyonic heat flow

Mitali Banerjee¹, Moty Heiblum¹, Amir Rosenblatt¹, Yuval Oreg¹, Dima E. Feldman², Ady Stern¹ & Vladimir Umansky¹

fractional states in first excited Landau level

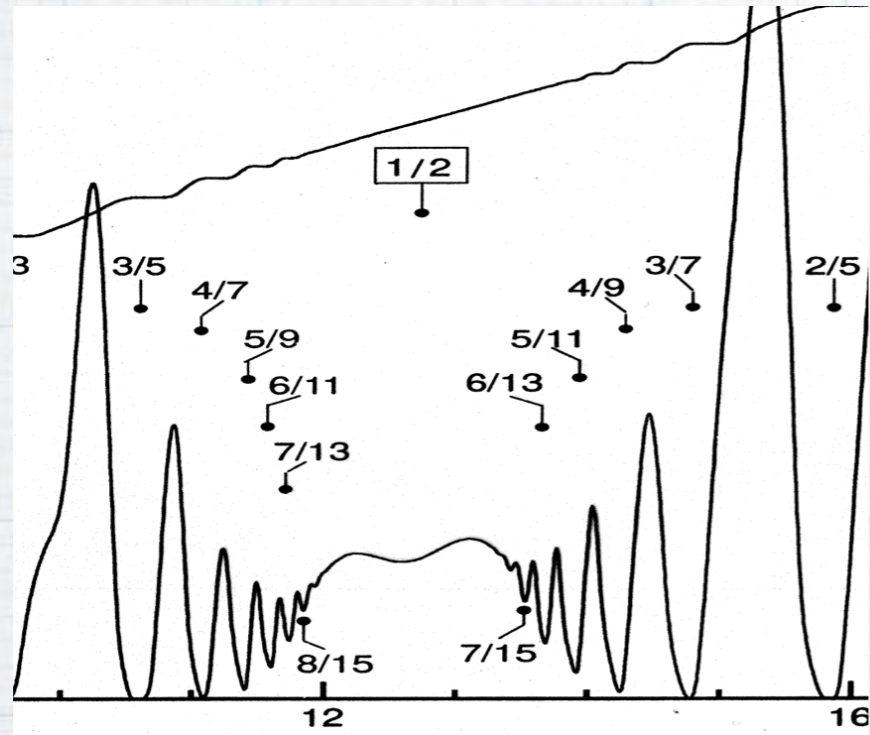
$$\nu = 2 + \eta$$

$$\nu = 7/3, 5/2, 8/3$$



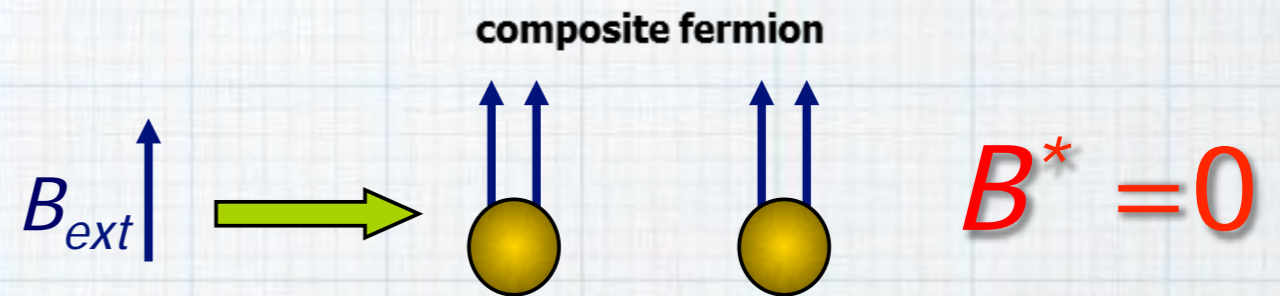
$\nu = 5/2$ state

Moore - Read 1991

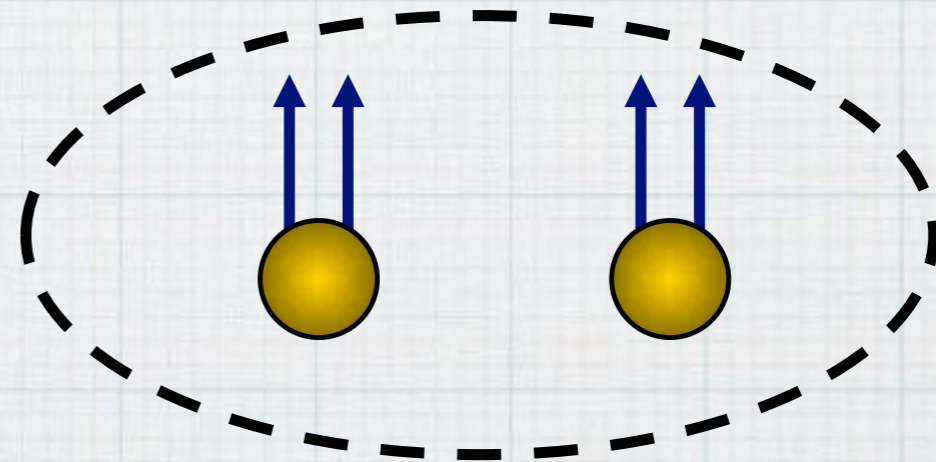


at $\nu = 5/2 \dots R_{xx} = 0$

BCS of **polarized** composite fermions
w/ **odd orbital angular momentum**

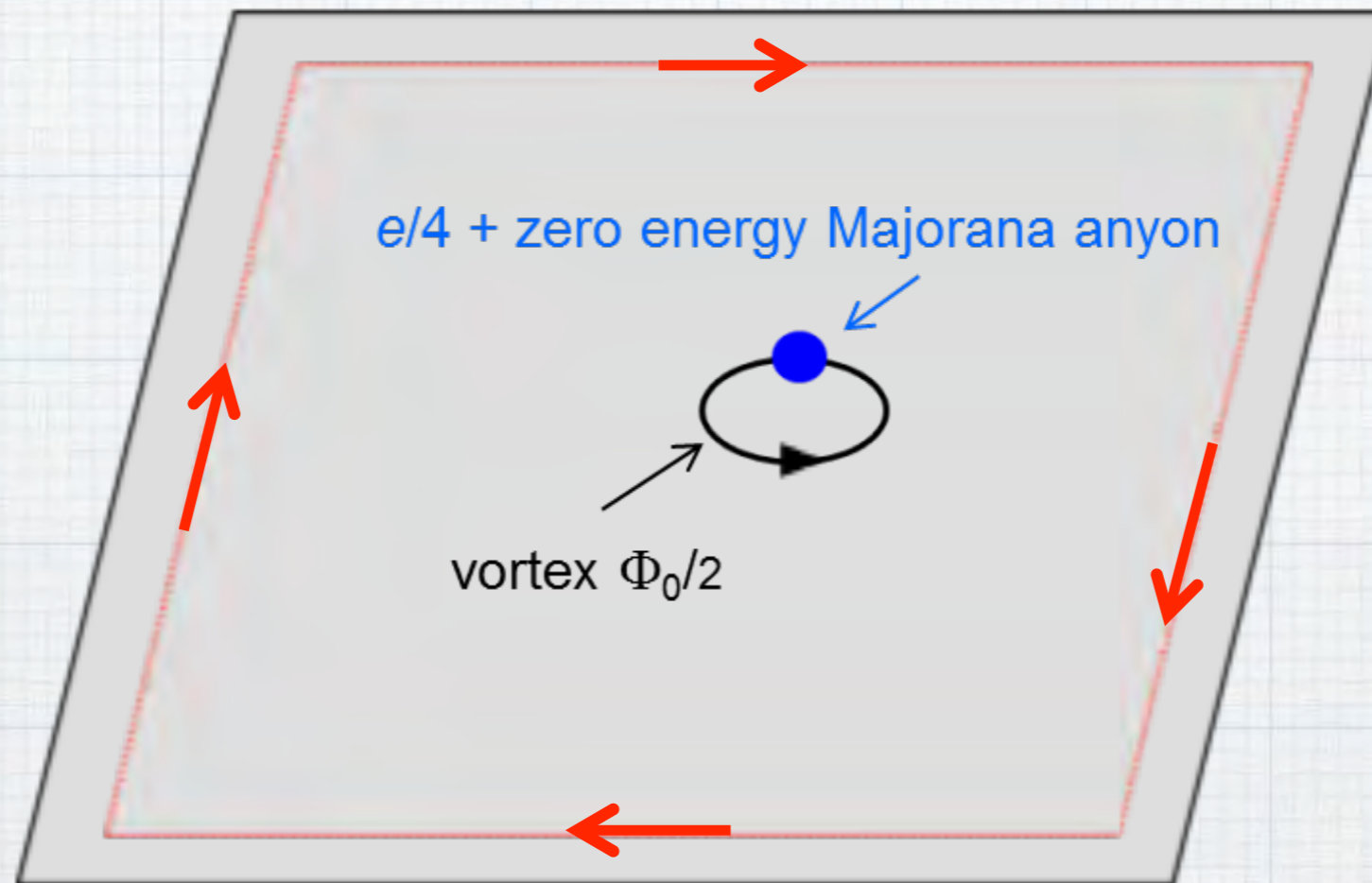


p-wave Cooper pair



5/2 state Moore – Read, Pfaffian state

bulk – edge correspondence



1D edge liquid of $e/4 +$ Majorana anyon

Majorana \rightarrow half fermion.... $K = \kappa_0 / 2$

already known for $\nu = 5/2$

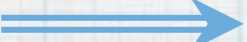



- charge $e/4$
- upstream neutral modes
- spin polarized

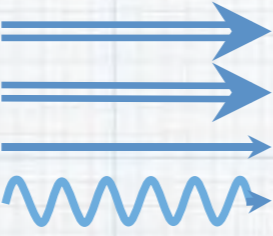


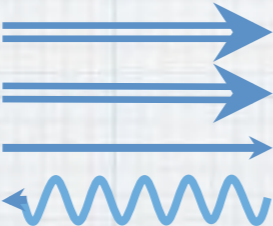

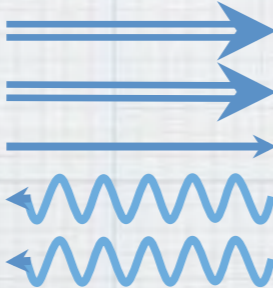
abelian or non-abelian ?

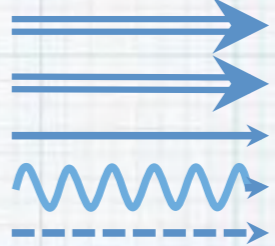
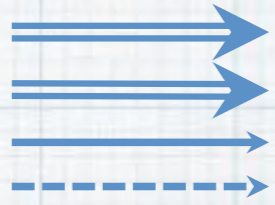



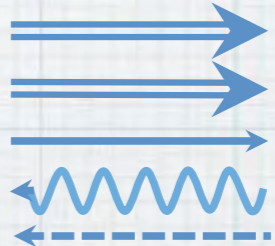


fractional state..... $\nu = 5/2$

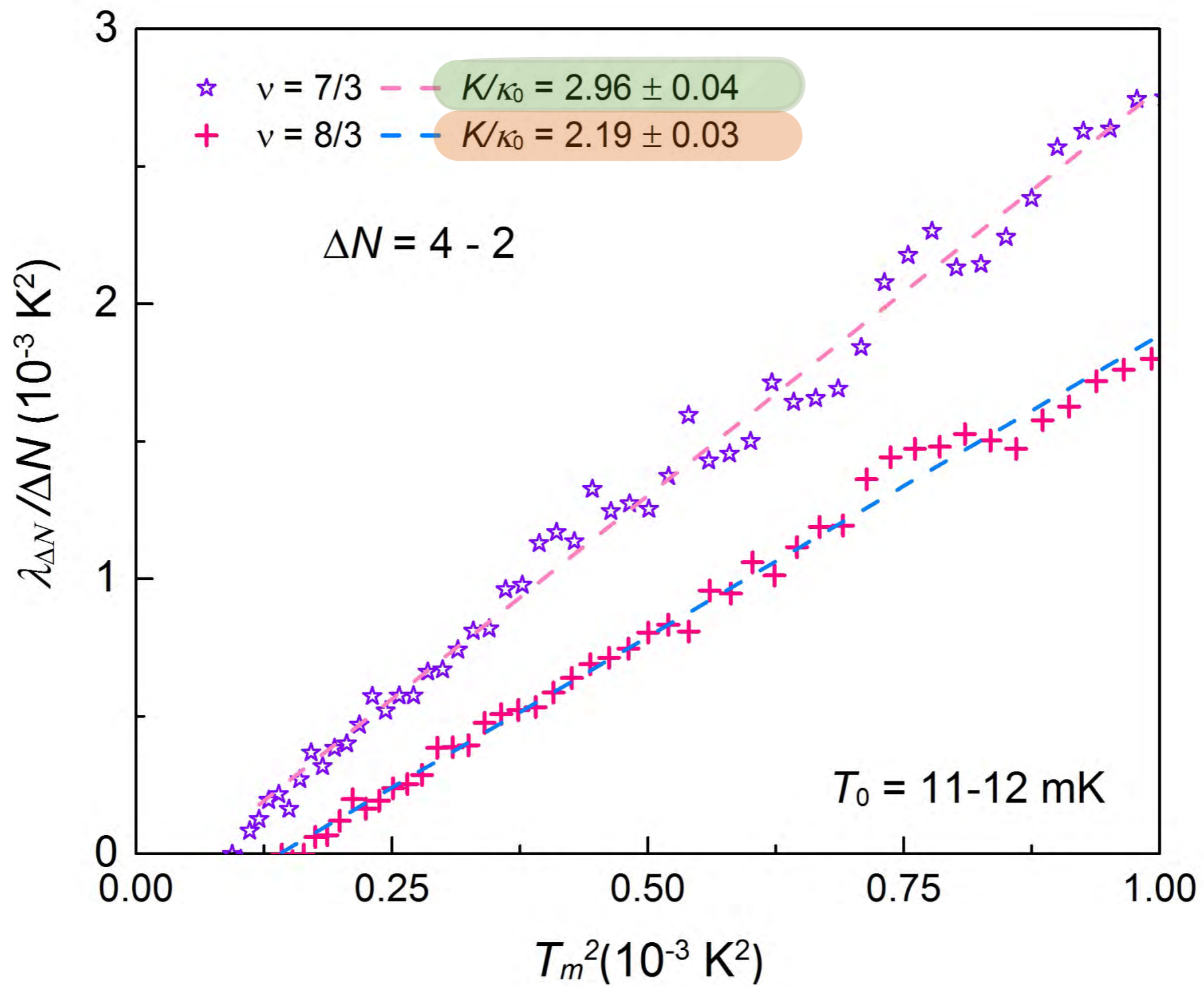
a

if non-abelian..... $K = (n \pm 0.5)\kappa_0$

integer, $e, \kappa = 1$

 fraction, $e/4, \kappa = 1$

 neutral, $0, \kappa = 1$

 Majorana, $0, \kappa = 0.5$


331		$\kappa = 4$
$K = 8$		$\kappa = 3$
113 		$\kappa = 2$
Anti-331 		$\kappa = 1$

$SU(2)_2$		$\kappa = 4.5$
Pfaffian		$\kappa = 3.5$
PH - Pfaffian 		$\kappa = 2.5$
Anti - Pfaffian 		$\kappa = 1.5$
Anti - $SU(2)_2$ 		$\kappa = 0.5$



$$\nu = 7/3$$

$$\nu = 2 + 1/3$$

particle like, downstream

measured

$$K = 3\kappa_0$$

$$\nu = 8/3$$

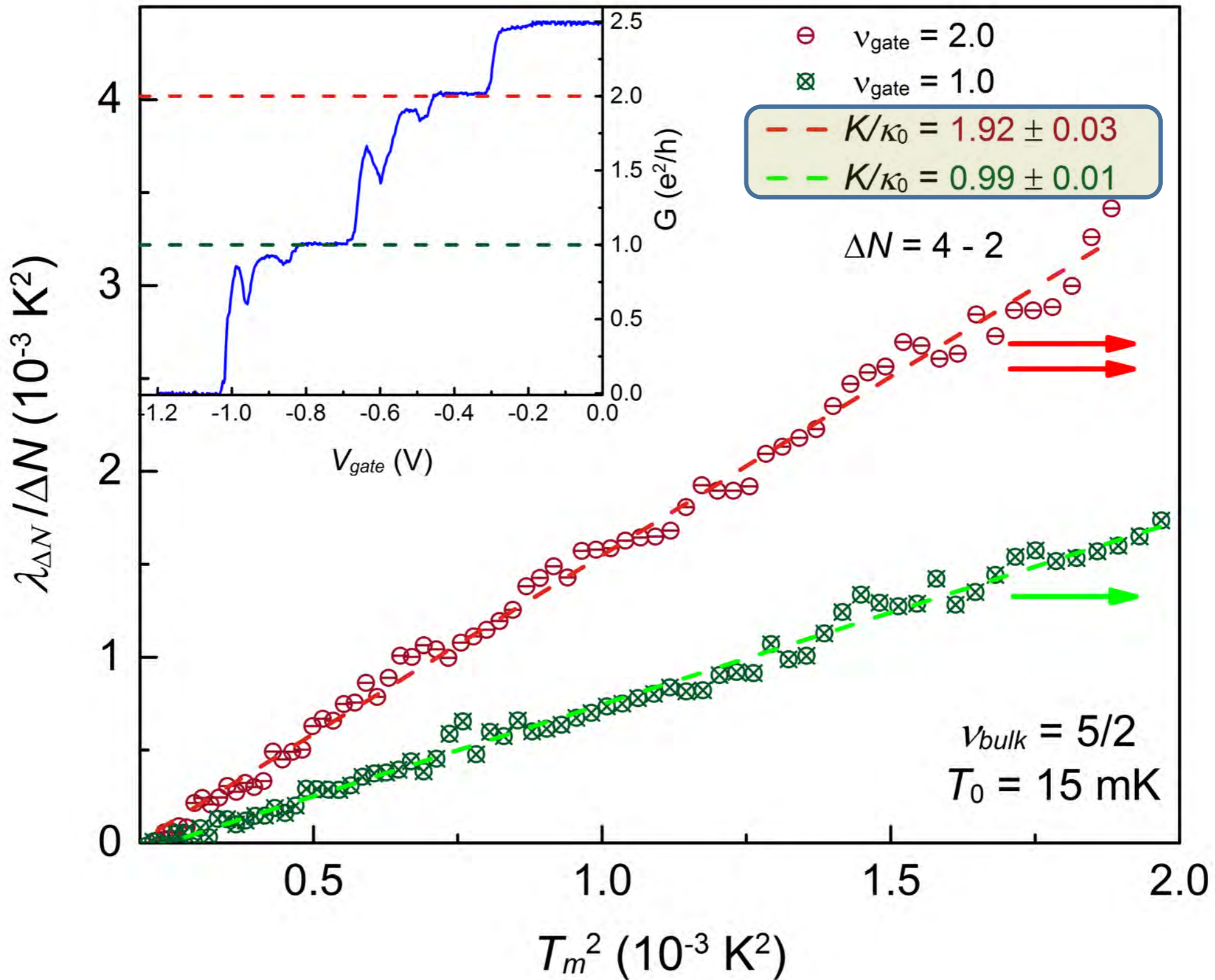
$$\nu = 2 + 2/3$$

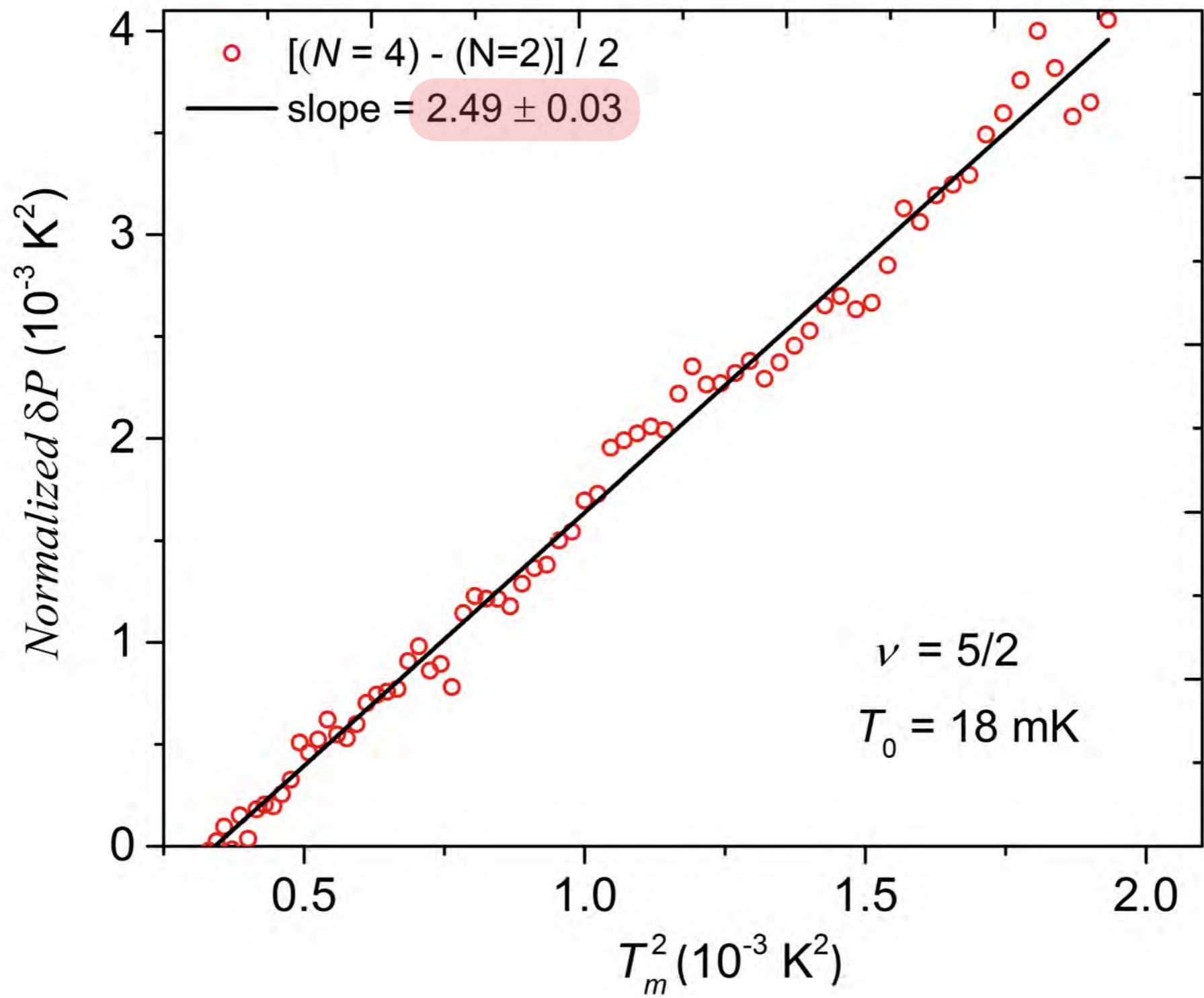
hole-like, down - up

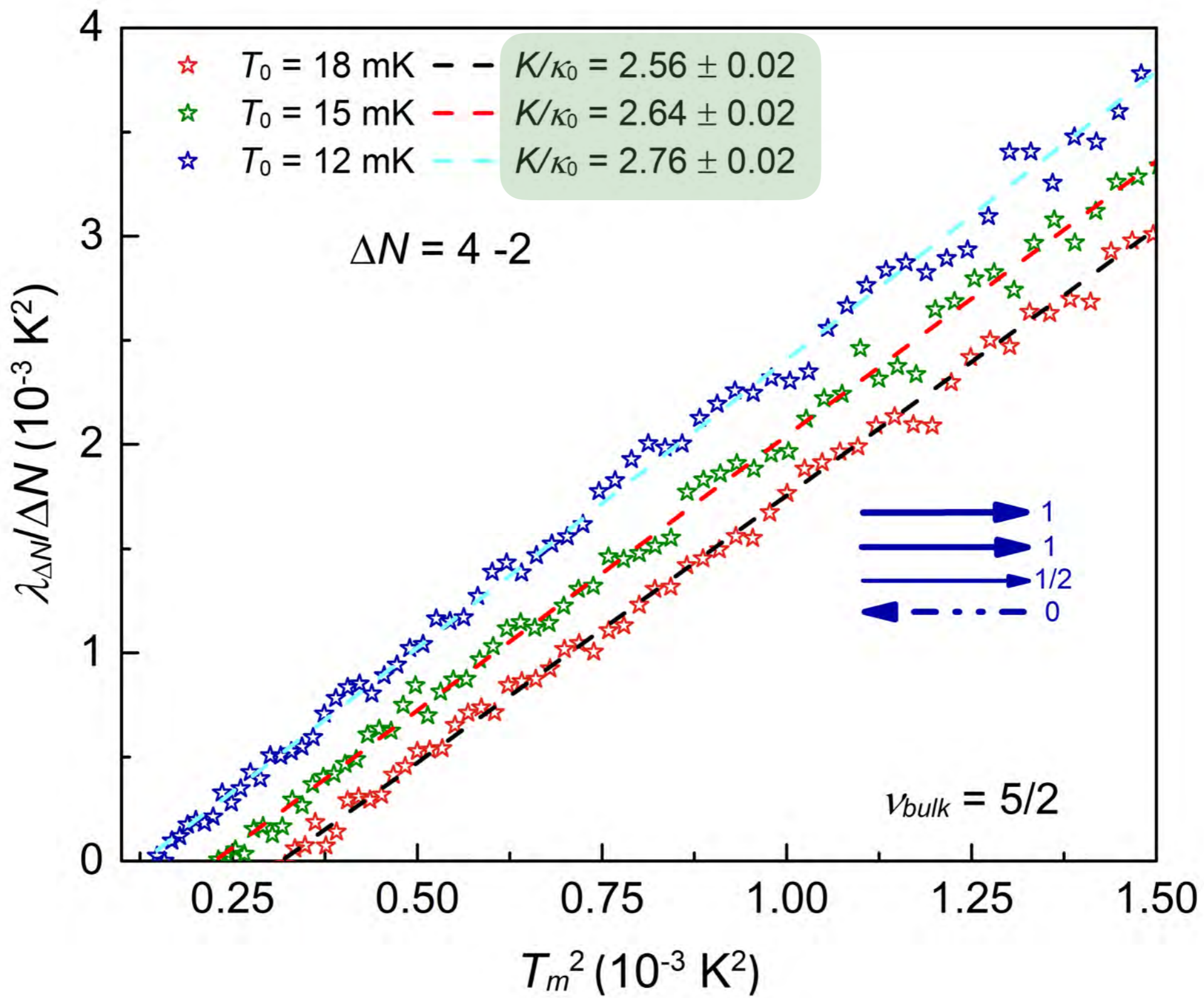
$$K = (2 + \varepsilon)\kappa_0$$

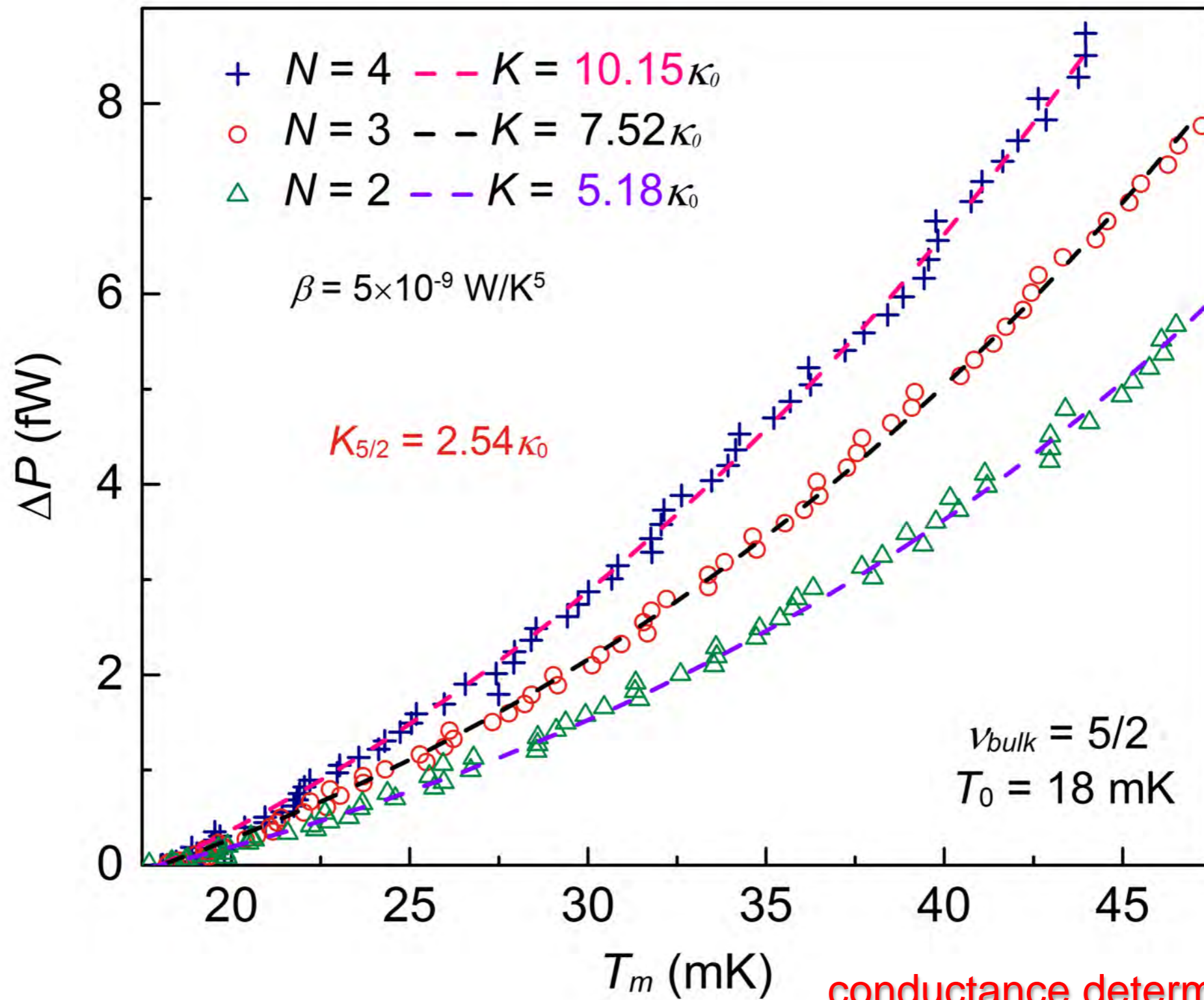
$$\nu = 5/2$$

measuring $\nu = 1, 2$ @ $\nu_B = 5/2$

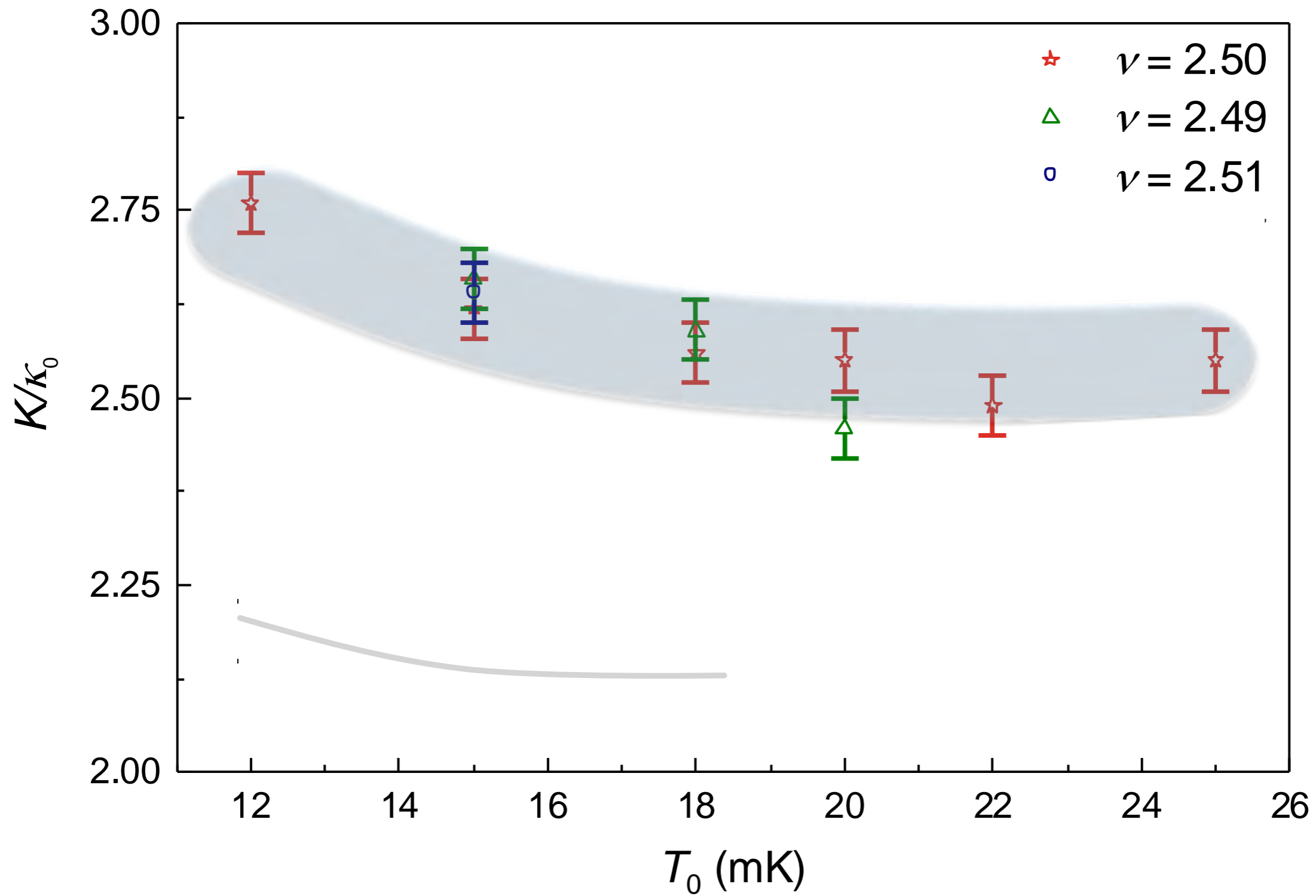


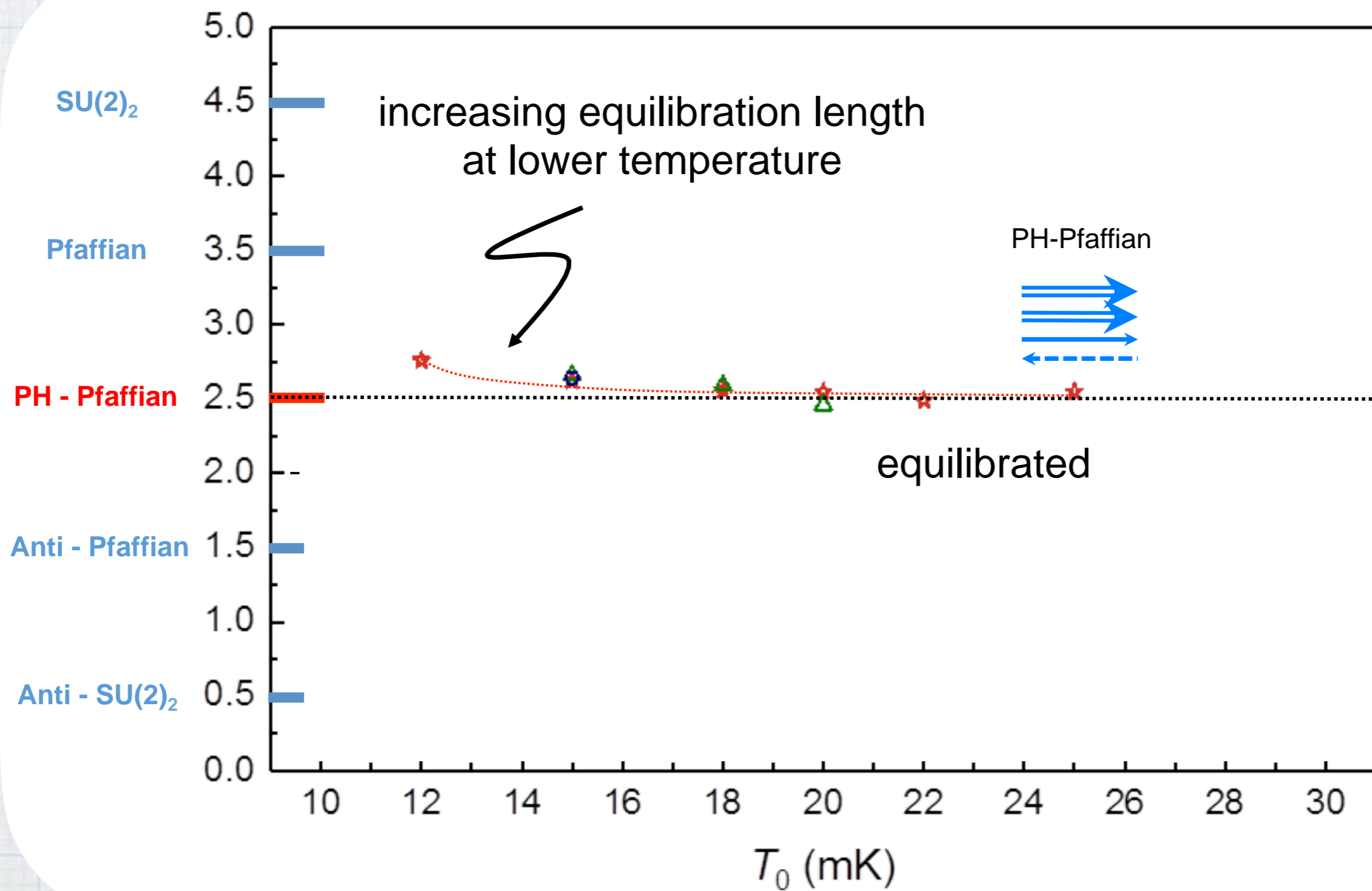


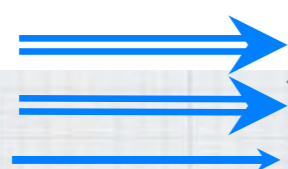
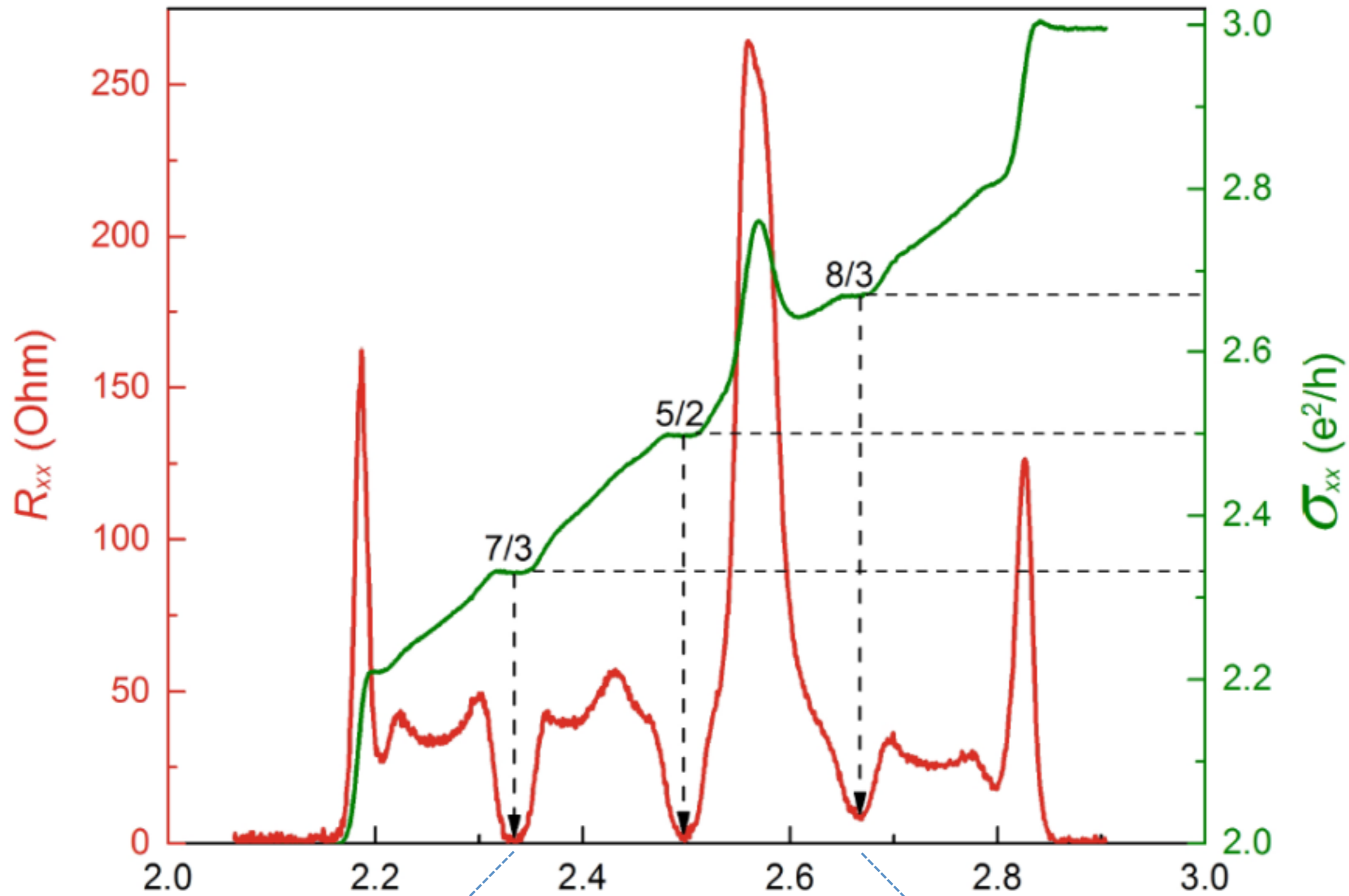




conductance determination *with*
phonons contribution



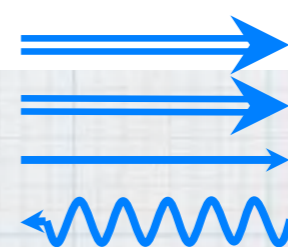




$3\kappa_0$



$2.5\kappa_0$



$2.15\kappa_0$

Theory of Disorder-Induced Half-Integer Thermal Hall Conductance

David F. Mross, Yuval Oreg, Ady Stern, Gilad Margalit, and Moty Heiblum
Braun Center for Submicron Research, Departments of Cond. Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Topological Order from Disorder and the Quantized Hall Thermal Metal: Possible Applications to the $\nu = 5/2$ State

Chong Wang,¹ Ashvin Vishwanath,¹ and Bertrand I. Halperin¹
¹*Department of Physics, Harvard University, Cambridge MA 02138, USA*

On the Interpretation of Thermal Conductance of the $\nu = 5/2$ Edge

Steven H. Simon¹

¹*Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford, OX1 3NP, UK*
(Dated: January 31, 2018)

Theory of

Disordered $\nu = 5/2$ Quantum Thermal Hall State: Emergent Symmetry and Phase Diagram

Biao Lian¹ and Juven Wang²

¹*Princeton Center for Theoretical Science, Princeton University, Princeton, NJ 08542, USA*

²*School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA*

$\nu = 5/2$likely non-abelian

measuring thermal conductance

reveals hidden information

ARTICLE

<https://doi.org/10.1038/s41586-018-0184-1>

Observation of half-integer thermal Hall conductance

Mitali Banerjee¹, Moty Heiblum^{1*}, Vladimir Umansky¹, Dima F. Feldman², Yuval Oreg¹ & Ady Stern¹

Thank you !!!

Future Directions

Starting with K

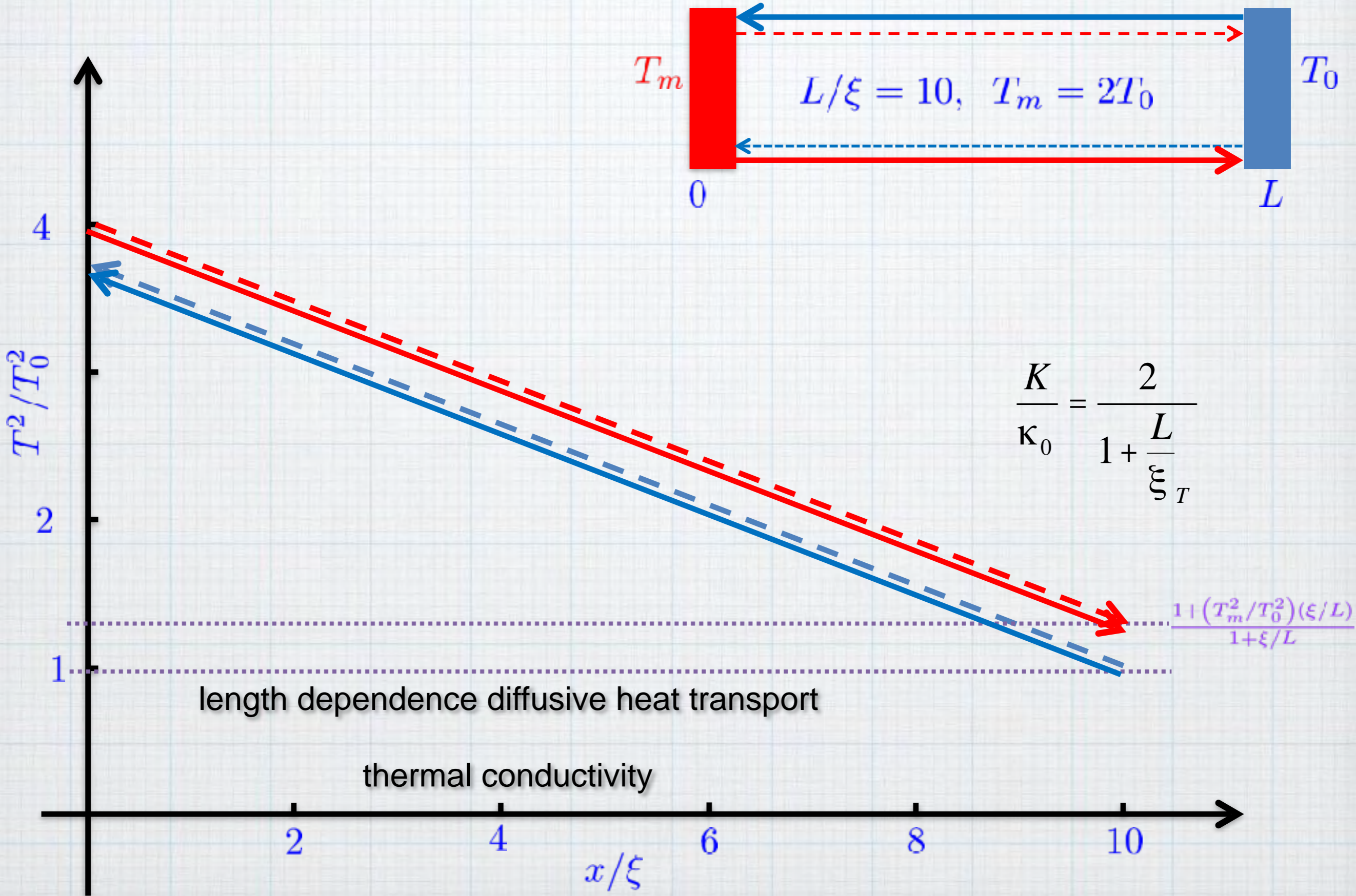
- Measuring K at $\nu=5/2$ at short distances – aided with noise measurements to check down and up neutral modes
- Doing the same in graphene (and bi-layer graphene) – as flakes are small
- Measuring K at $\nu=5/2$ at different B 's and different n 's – with aid of a back gate – testing if K is universal in this state or depends on parameters
- Studying K at $\nu=12/5$. Not easy as accuracy has to be better than $0.5k_0$.
- Studying $\nu=7/2$ – also predicted to be non-abelian (but our quick measurement didn't see it...)

Interference

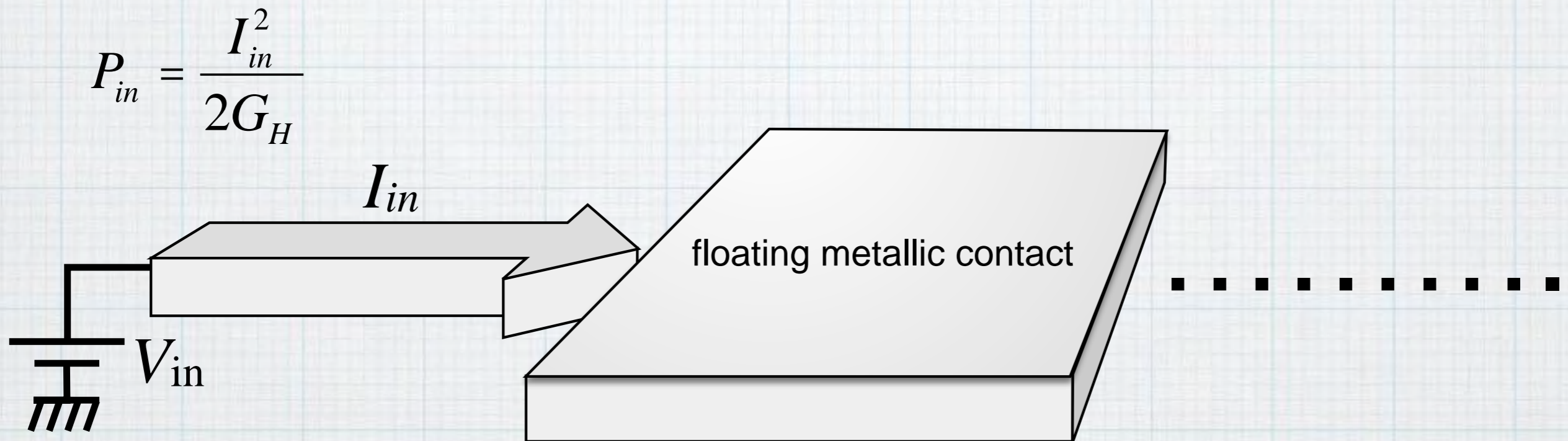
- Can we prevent neutral modes in GaAs 2DEG?
- If we can, looking for interference
- Looking for neutral modes in graphene (and others monolayer materials...)
- If no neutral modes, look for interference in graphene – first integer and fractions

Thank you !!!

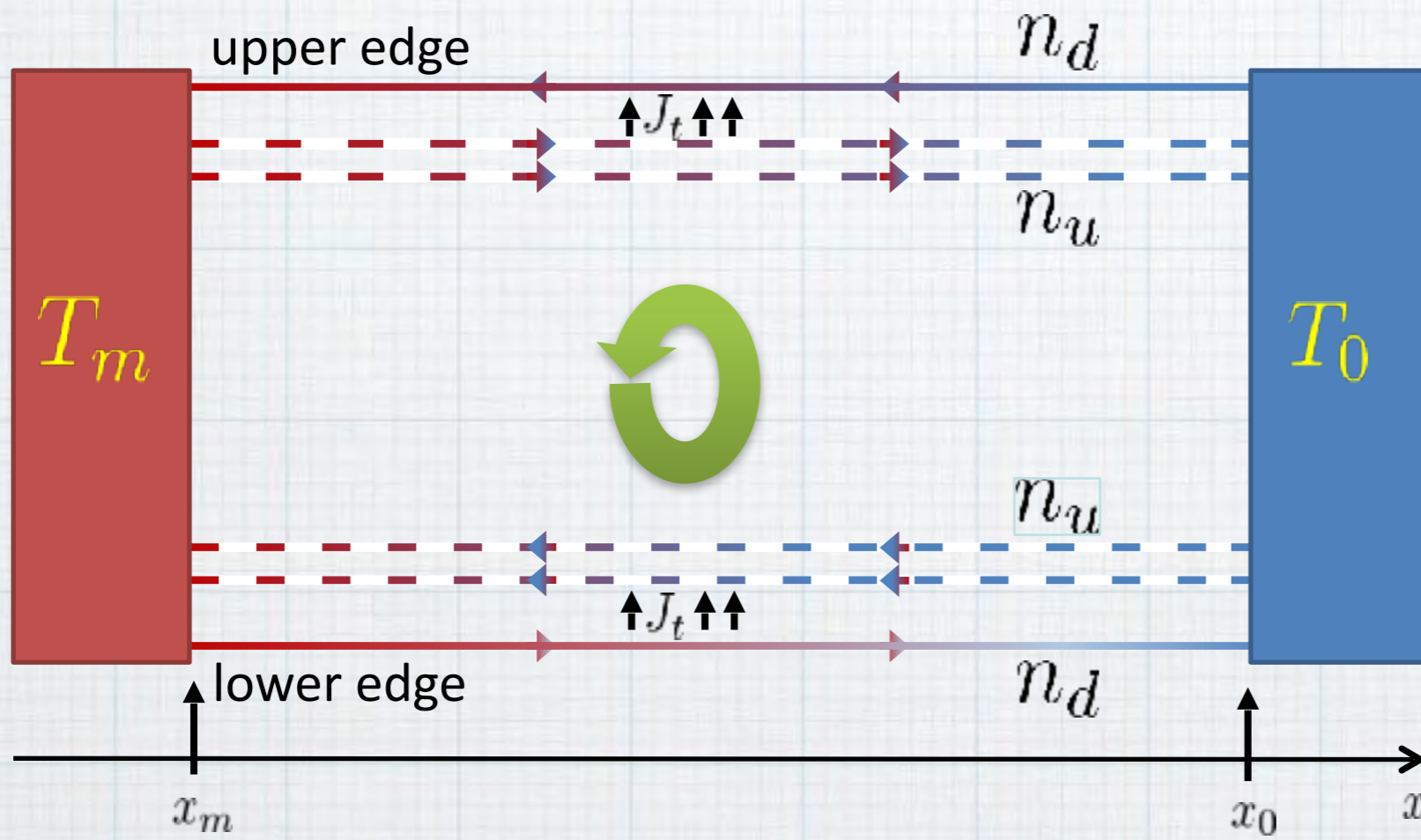
temperature profile, $\nu = 2/3$



heating the reservoir



calculating $T(x)$ & K $v = 3/5$



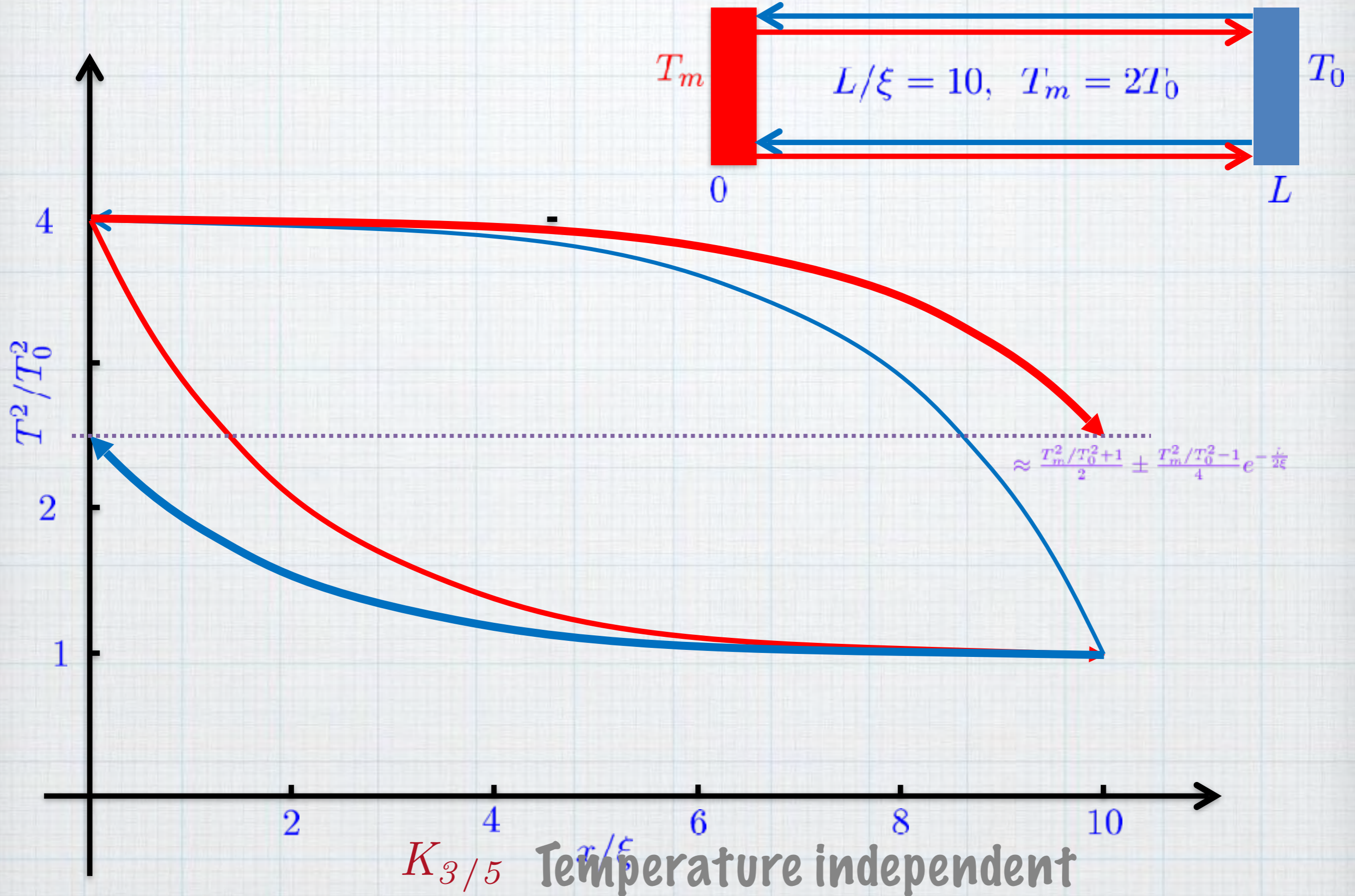
$$n_d = 1 \quad n_u = 2$$

$$J = KT^2$$

$$0.5n_u \kappa_0 \partial_x T_u^2(x) = -j_t(x)$$

$$0.5n_d \kappa_0 \partial_x T_d^2(x) = -j_t(x)$$

temperature profile, $\nu = 3/5$



difficulties due to structure:

- 'bulk heat conductance'free electrons in the donor layers
- poor contact of the floating reservoir – hence, reflections
- instability of QPC's

measured

$\nu = 7/3$ $\nu = 2 + 1/3$ particle like, downstream

$\nu = 8/3$ $\nu = 2 + 2/3$ hole-like, down + up

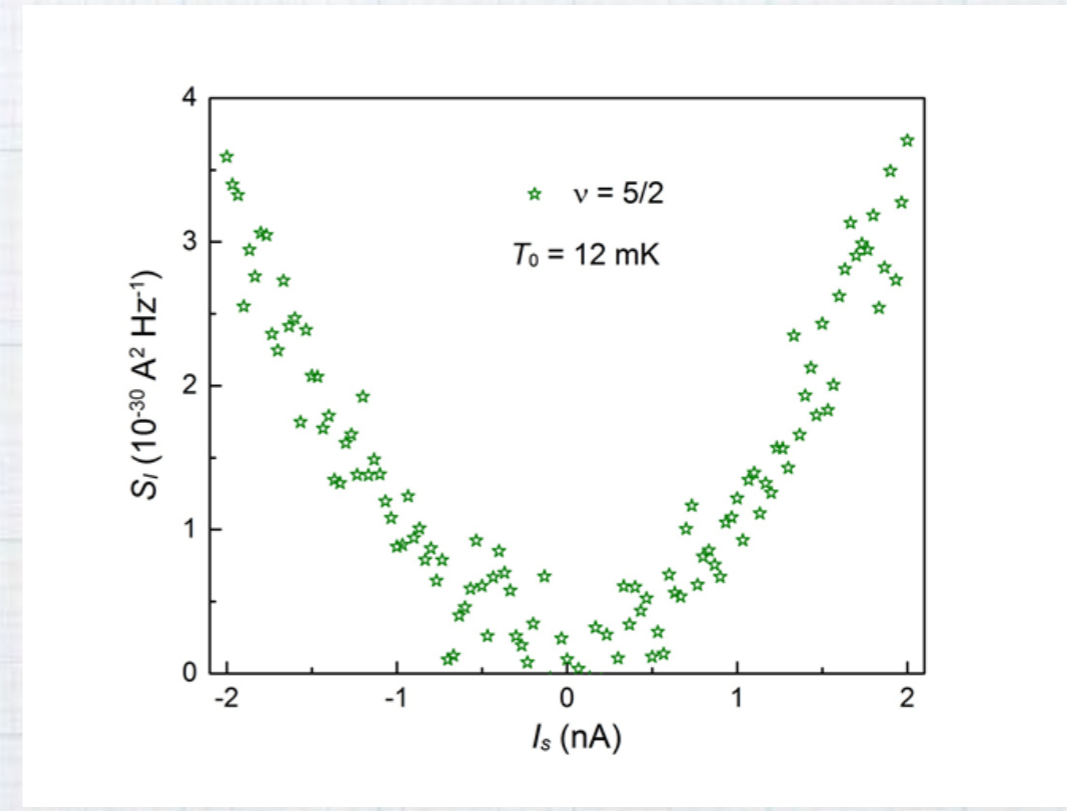
$K = 3\kappa_0$
 $K = (2 + \epsilon)\kappa_0$

• what do we know ?..... $\nu = 5/2$ $\nu = 2 + 1/2$

✓ quasiparticle charge $e^* = e / 4$

✓ upstream energy modes

✓ spin polarized



neutral fermionic

upstream Majorana

$K = ?$

Points of consideration:

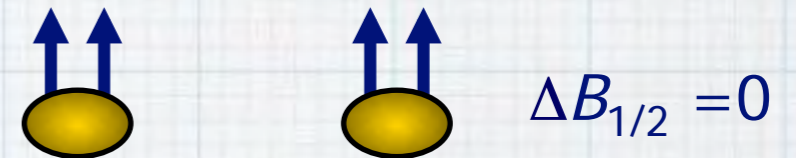
- * noiseless source current (**DC I_{in} \rightarrow in most cases**)
- * electrons fully equilibrate in the floating contact (**with T_m**)
- * outgoing currents only carry J-N noise (**low contact resistance**)
- * measurements at low temperature (**$J_{e-ph} \ll J_e$**)
- * no bulk energy modes exist (**may increase apparent conductance**)
- * required length of arms (**allowing temperature equilibration**)

$\nu = 5/2$ state if non-abelian $K/\kappa_0 = n + \frac{1}{2}$

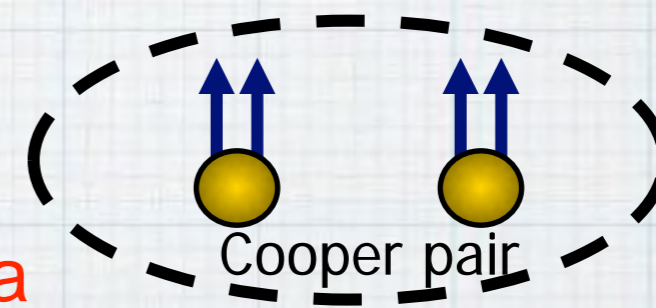
from electrons to non-abelian quasiparticles

* half - filled LL on top of two filled LL's..... $2 \frac{1}{2} = 2 + \frac{1}{2}$

* flux attachment.....spin polarized CFs
at zero average magnetic field



* CFs pair into Cooper pairs
p-wave superconductor



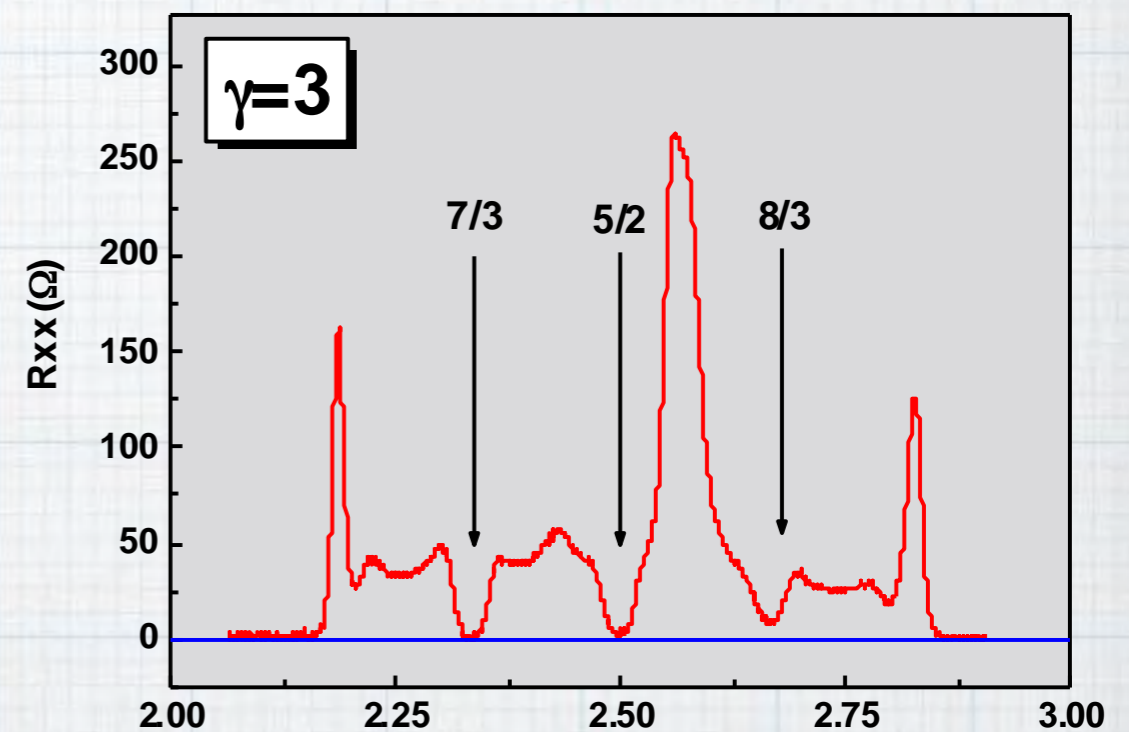
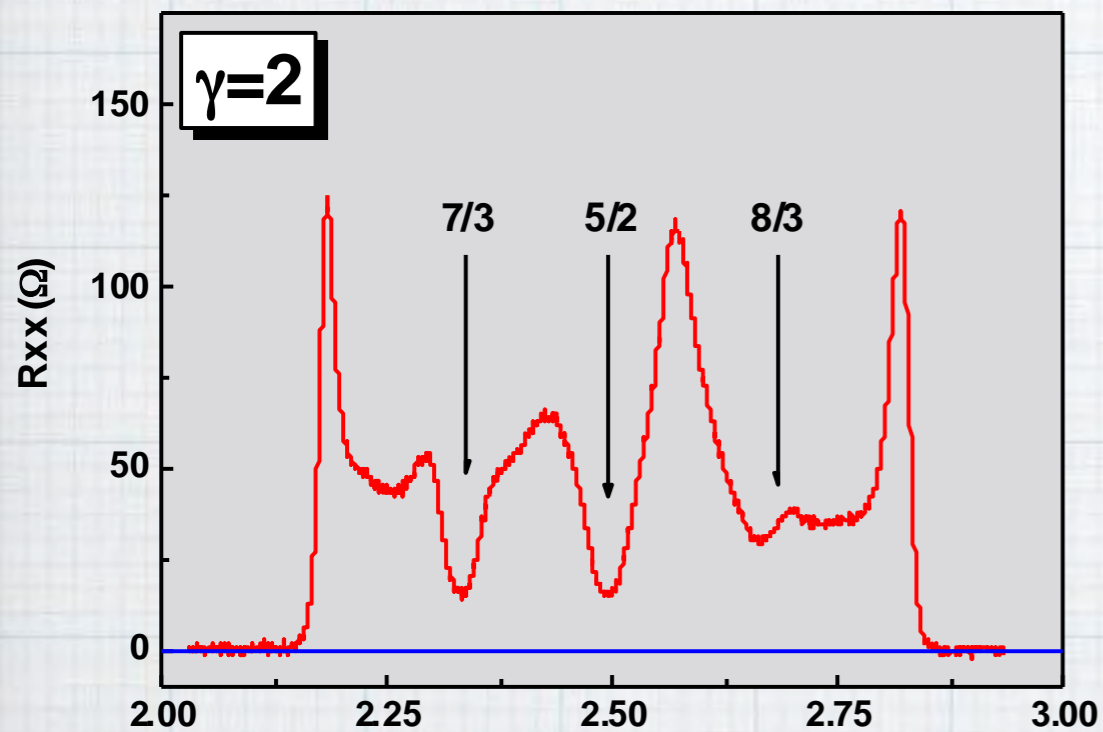
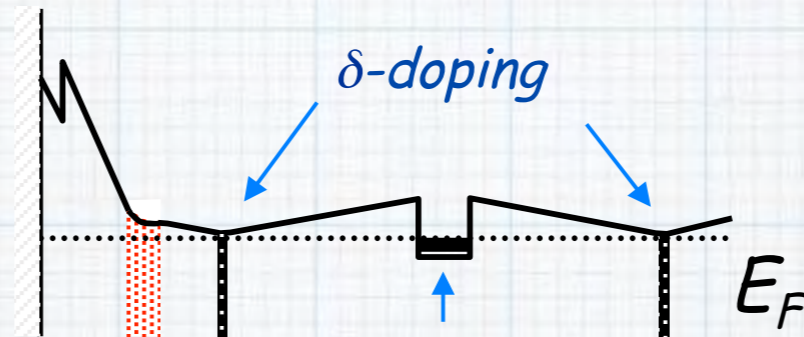
* vortices are charged..... $e^* = e/4 + \text{Majorana}$

* chiral edge modes: charged + Majorana

* ground state degeneracy (braiding is non-abelian)

shallow DX centers over doping

delta doping in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ ($x=23\div 25\%$)



ν

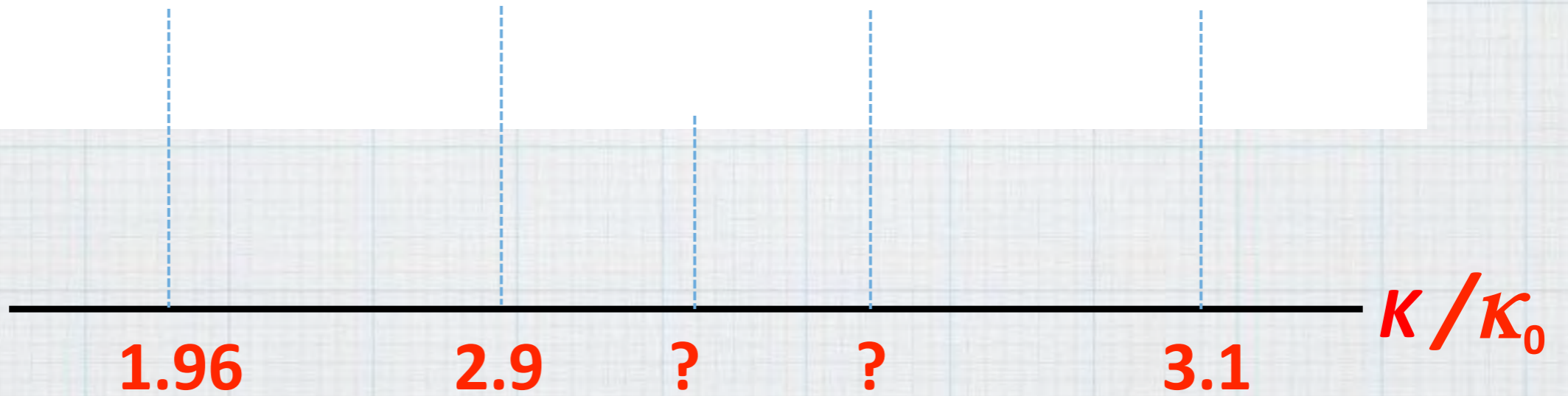
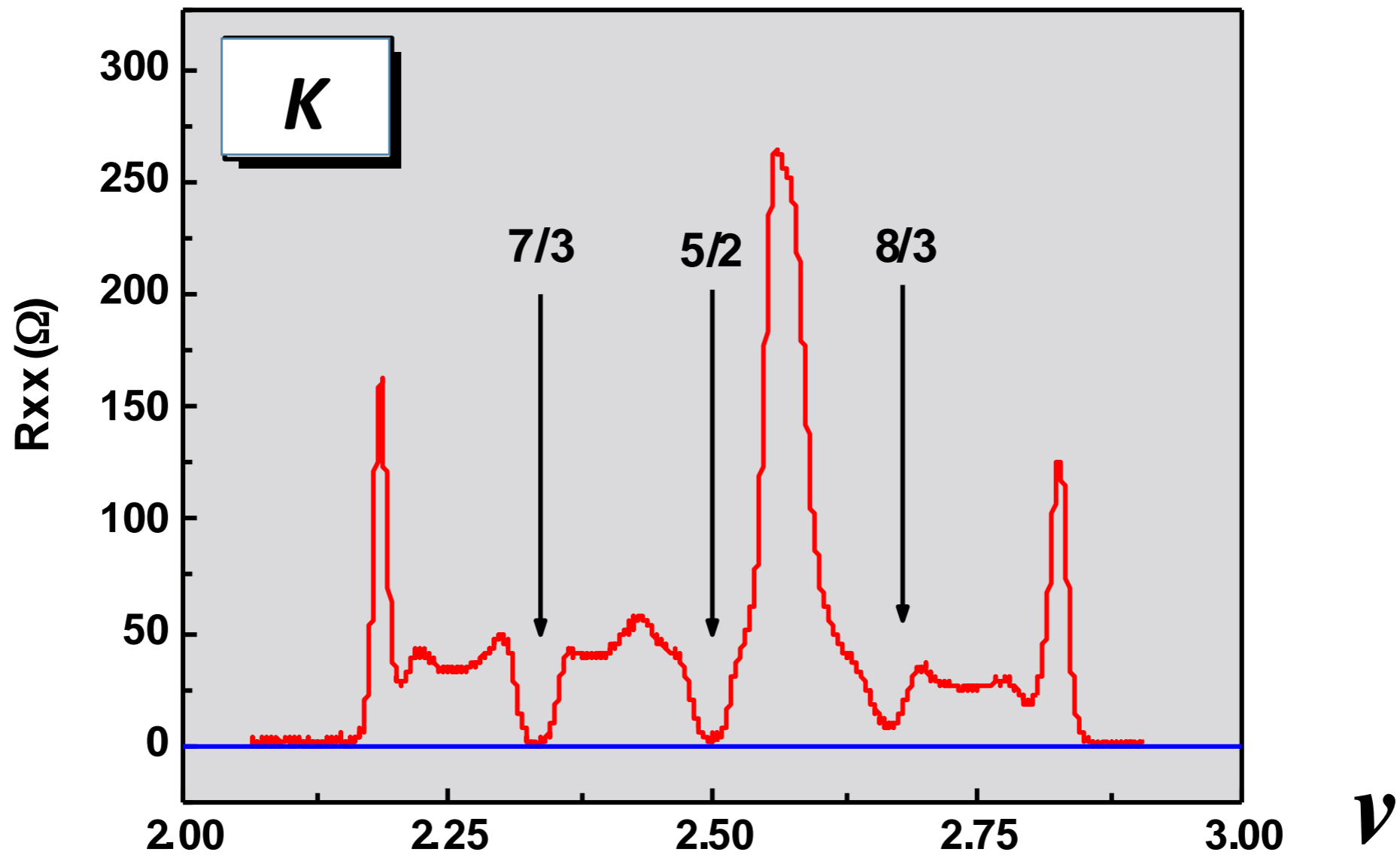
NO illumination

NO parallel conductance

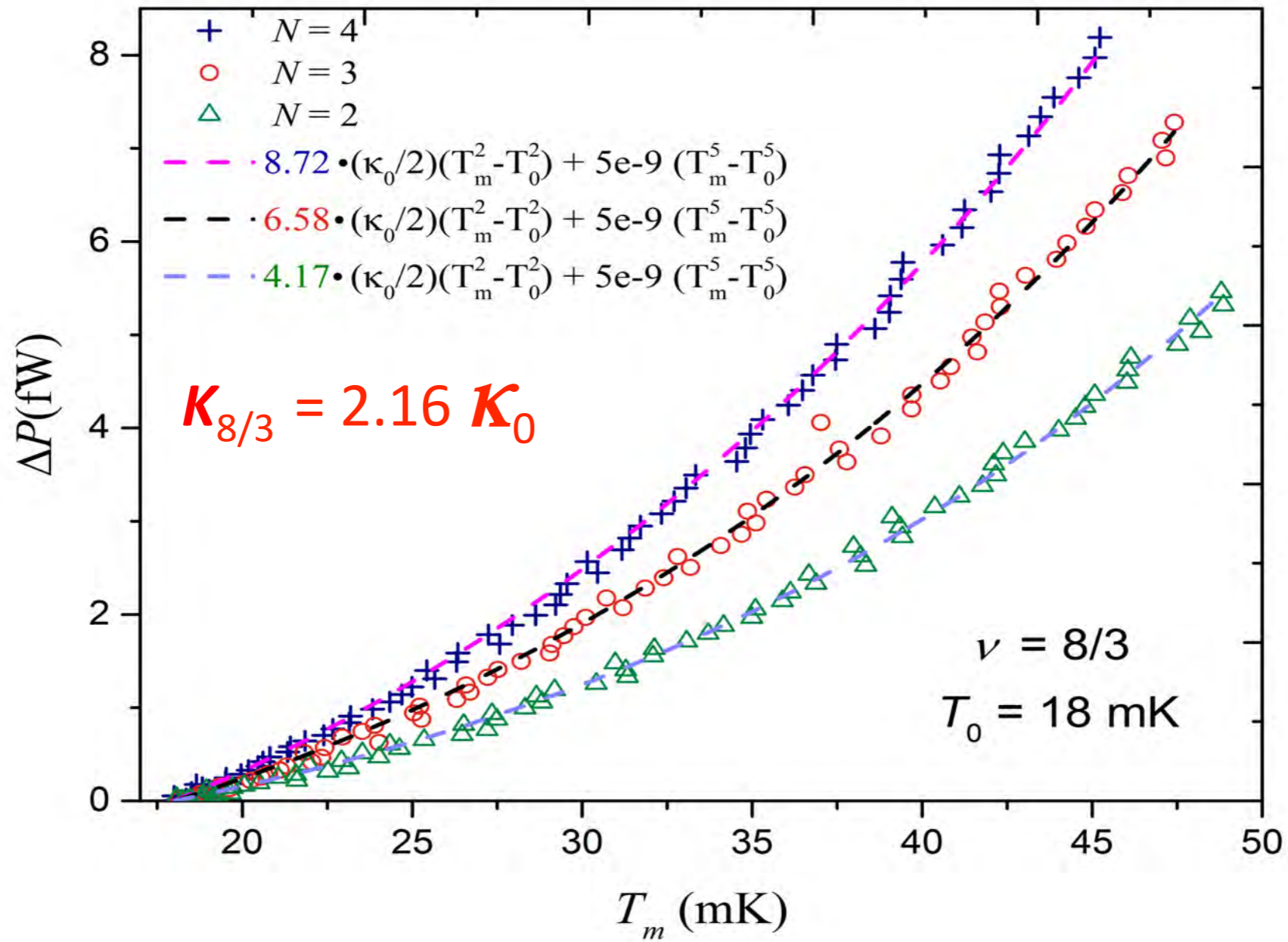
NO bulk thermal conductance

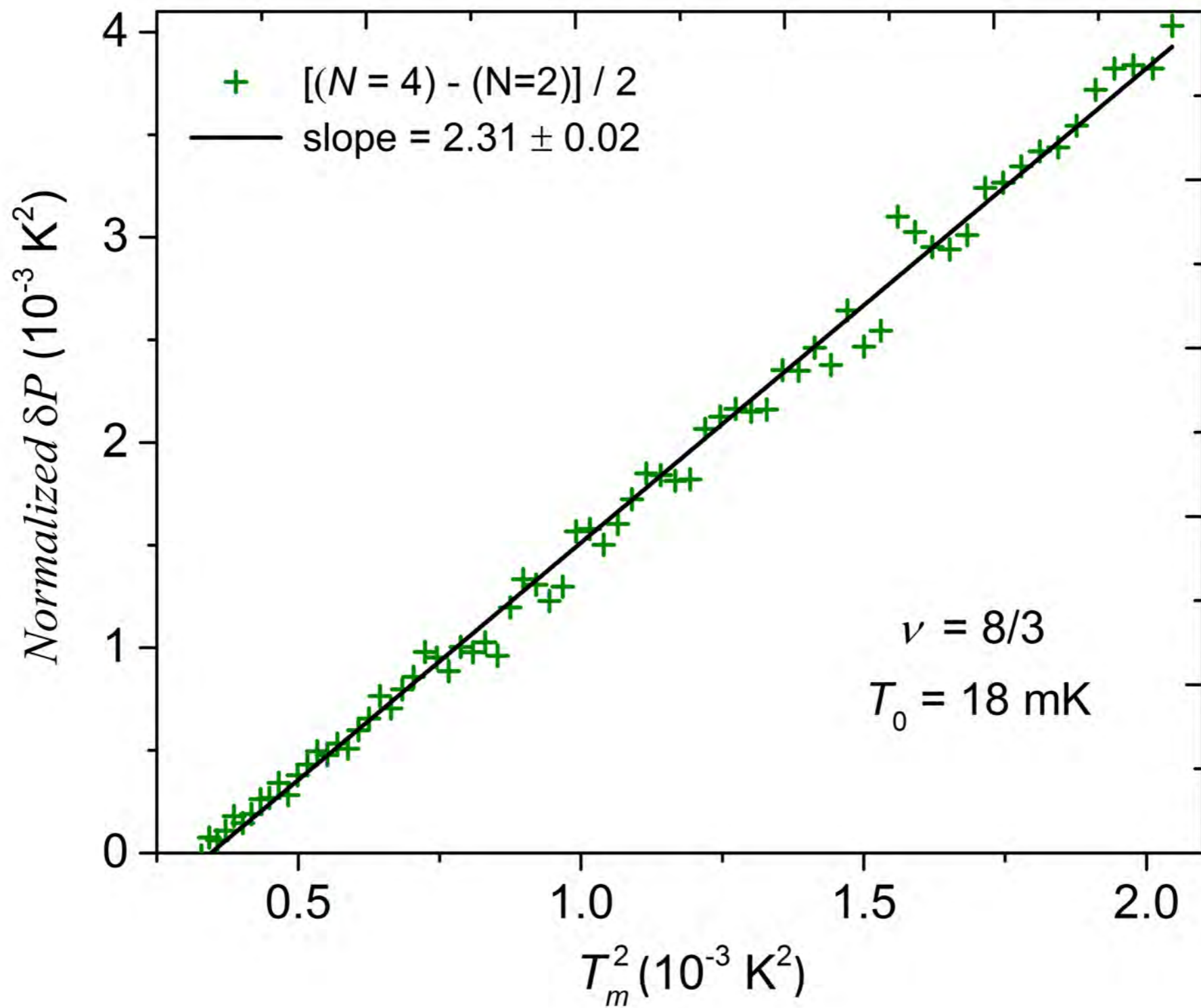
stable gates (QPC replaced by continuous gate)

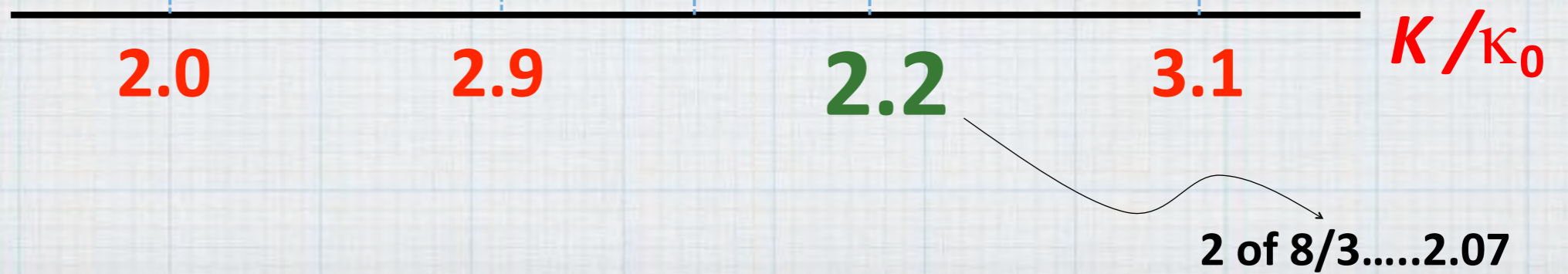
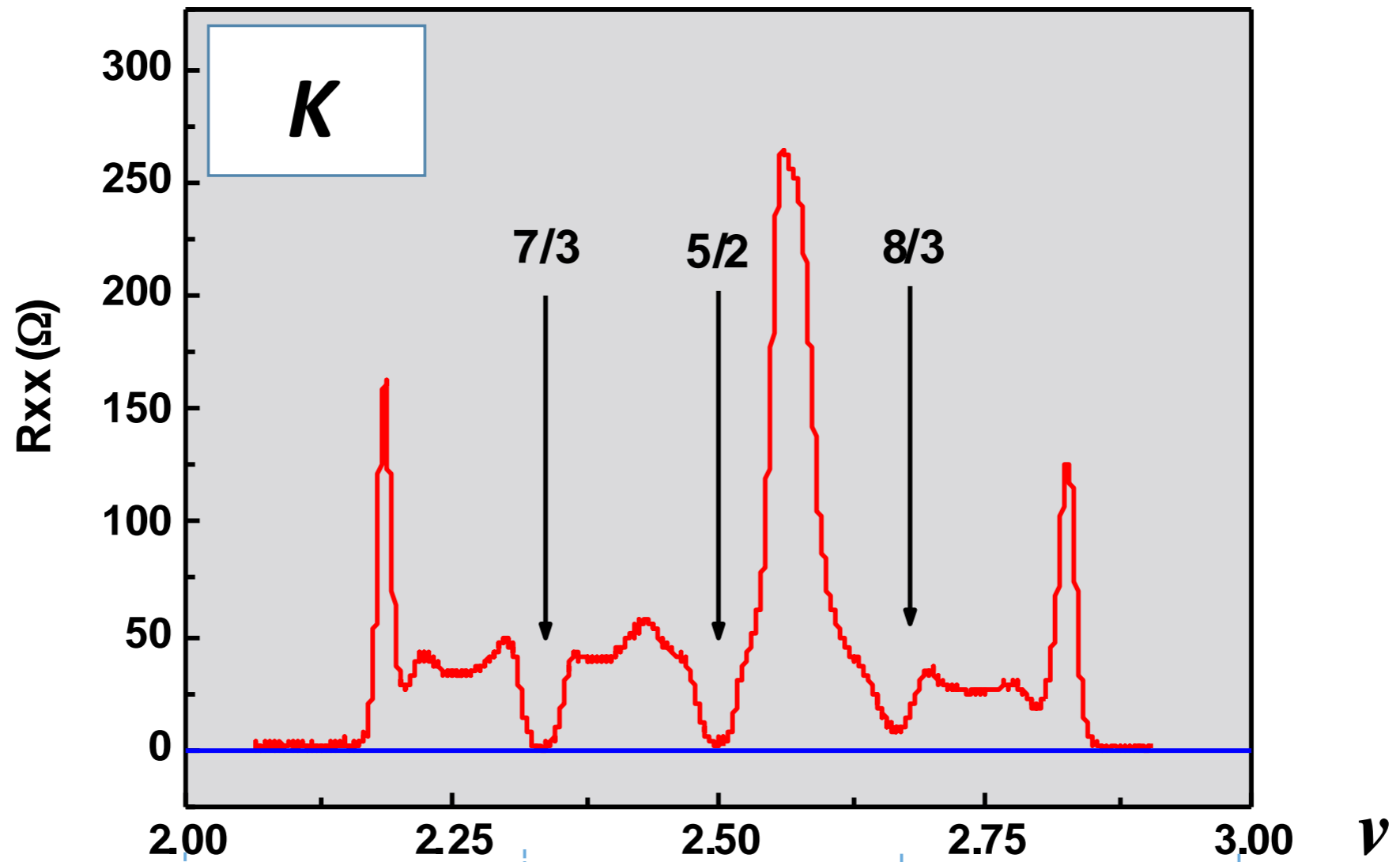
low resistance floating reservoir



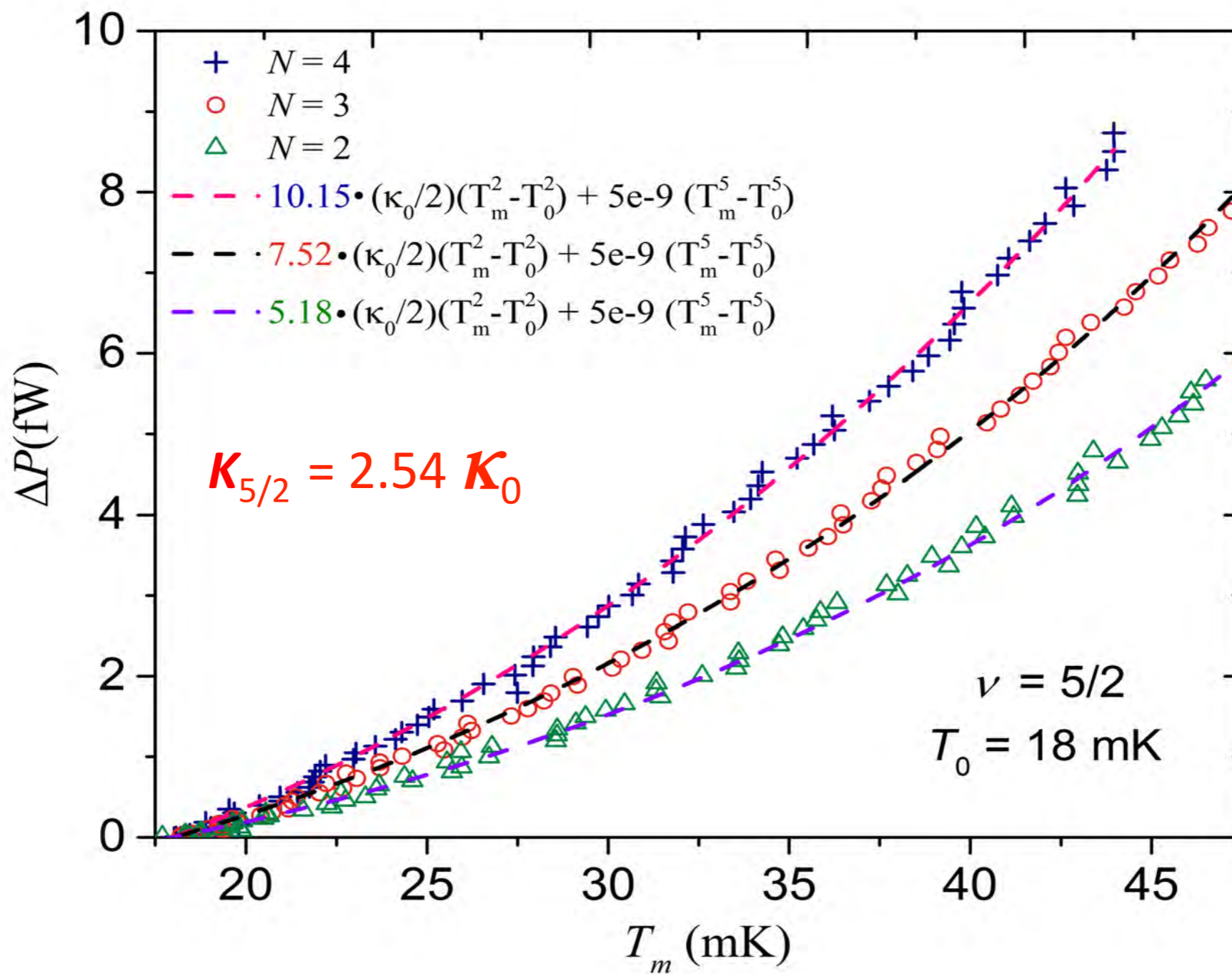
thermal conductance of $\nu = 8/3$ $R_{xx} > 0$



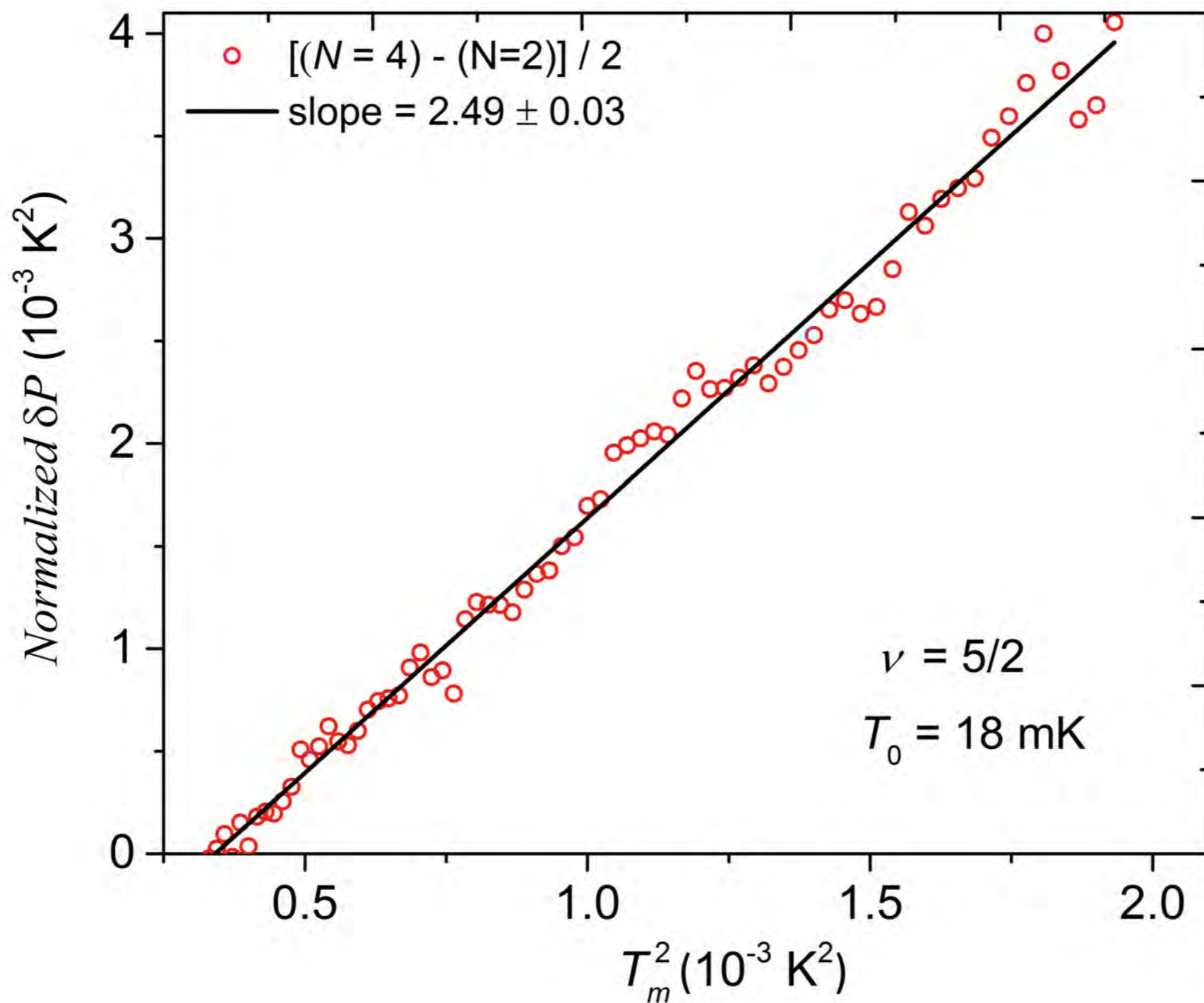


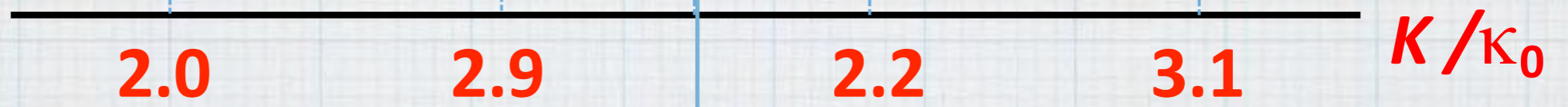
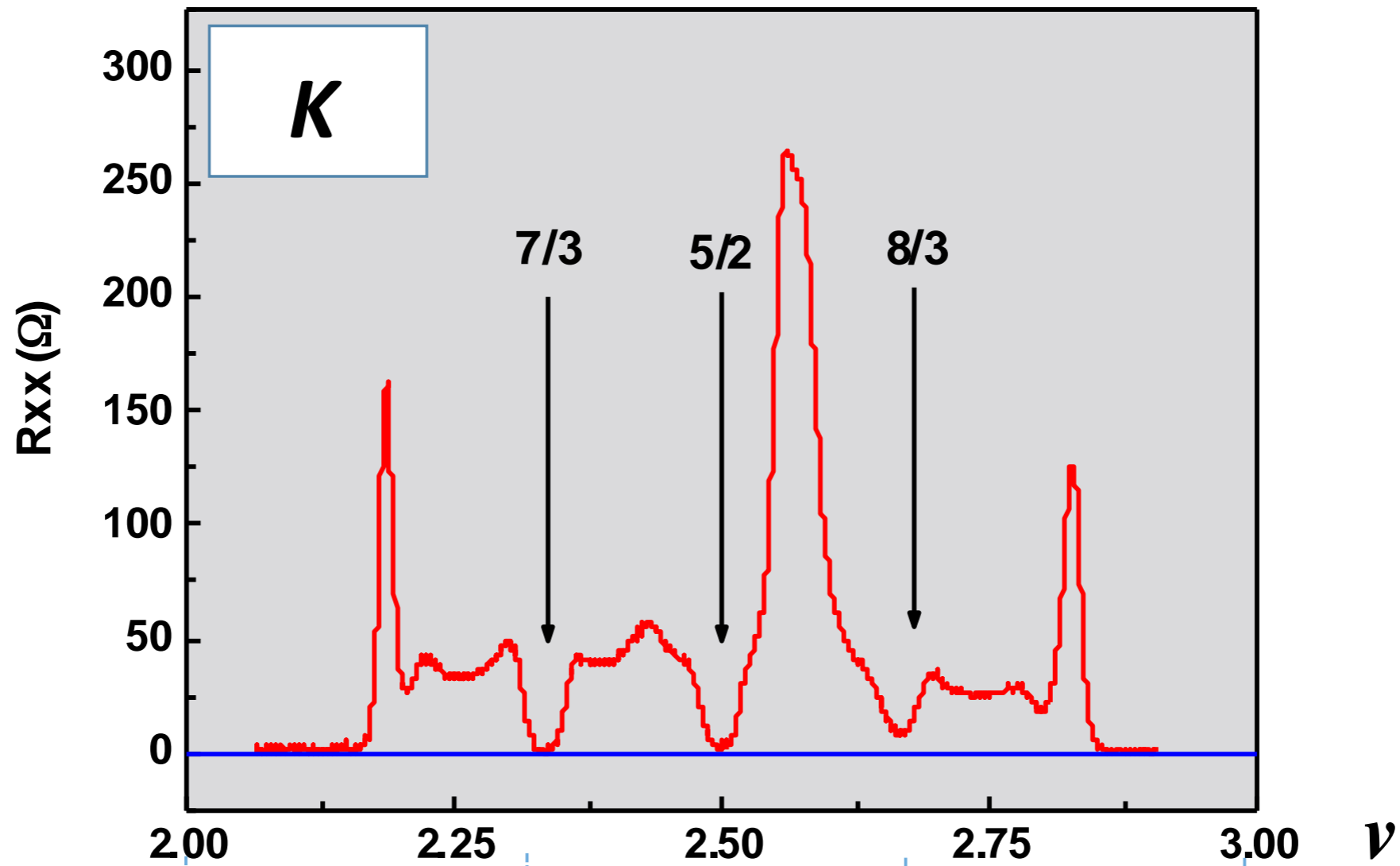


thermal conductance of $\nu = 5/2$




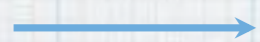


thermal conductance of $\nu = 5/2$






























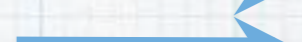





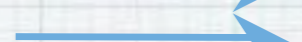
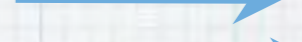






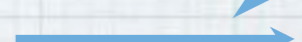







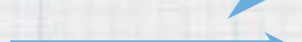


2.45 - 2.65


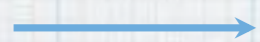


2 of 5/2.....1.9 ←

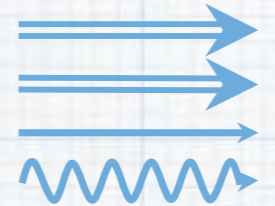

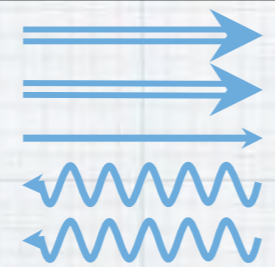
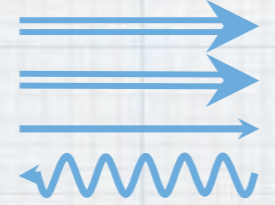
integer, e , $\kappa = 1$

 fraction, $e/4$, $\kappa = 1$

 neutral, 0 , $\kappa = 1$

 Majorana, 0 , $\kappa = 0.5$


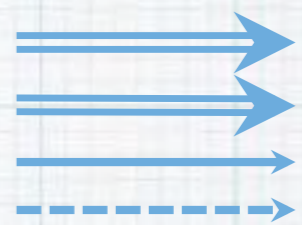
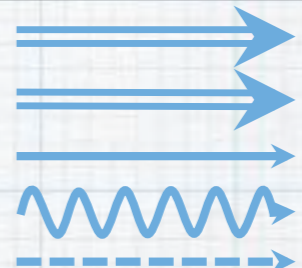
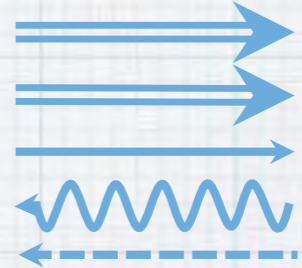

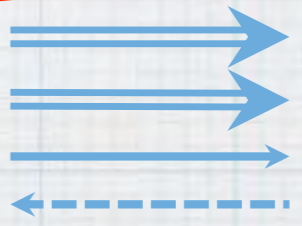
upstream


331    	$\kappa = 4$
$\kappa = 8$    	$\kappa = 3$
Anti-331      	$\kappa = 1$ 
113     	$\kappa = 2$

Pfaffian    	$\kappa = 3.5$
$SU(2)_2$     	$\kappa = 4.5$
Anti - Pfaffian      	$\kappa = 1.5$ 
Anti - $SU(2)_2$       	$\kappa = 0.5$ 
PH - Pfaffian     	$\kappa = 2.5$ 

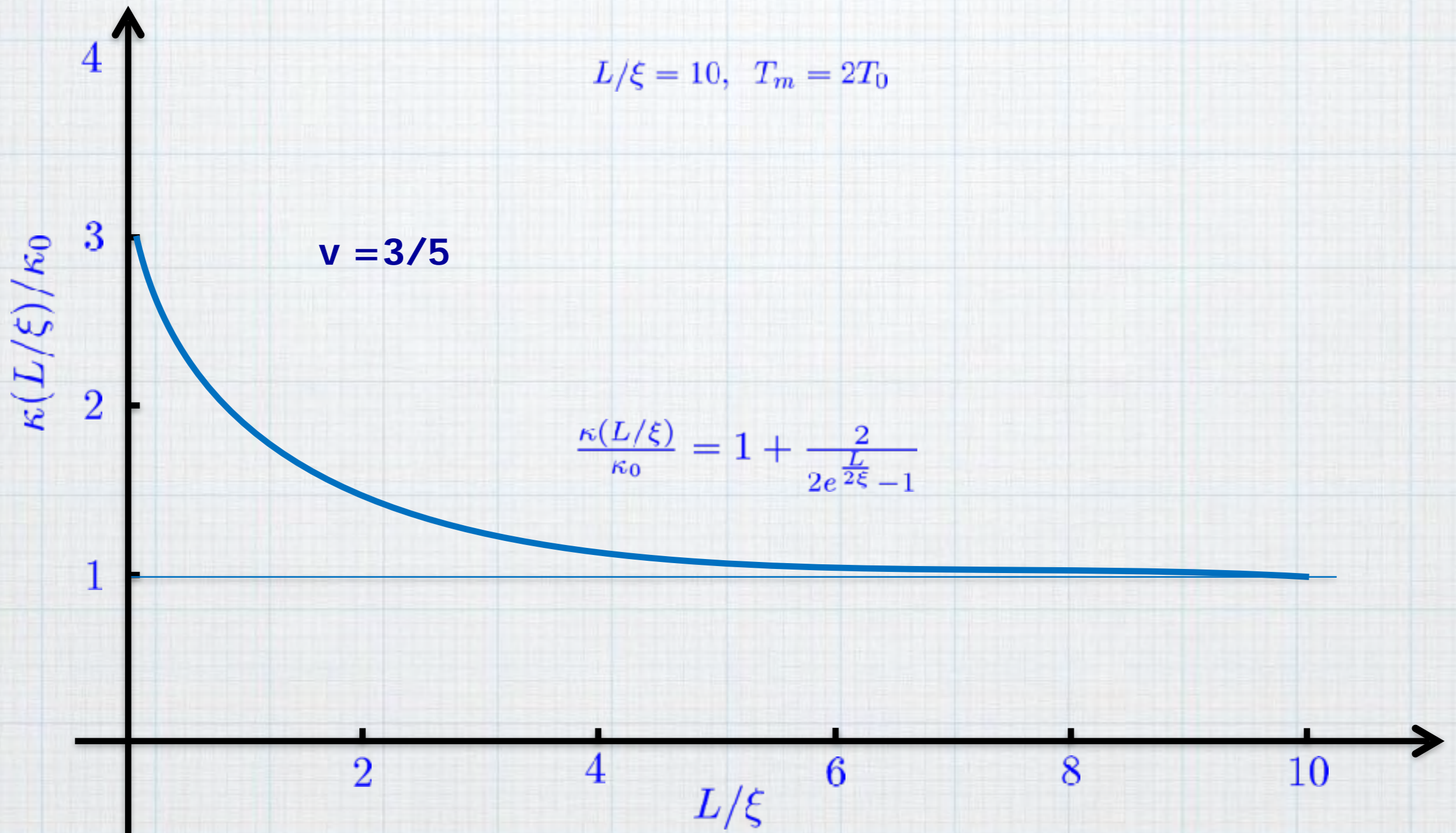
integer, e , $\kappa = 1$

 fraction, $e/4$, $\kappa = 1$

 neutral, 0 , $\kappa = 1$

 Majorana, 0 , $\kappa = 0.5$


331		$\kappa = 4$
$\kappa = 8$		$\kappa = 3$
Anti-331		$\kappa = 1$
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Pfaffian		$\kappa = 3.5$
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Anti - $SU(2)_2$		$\kappa = 0.5$
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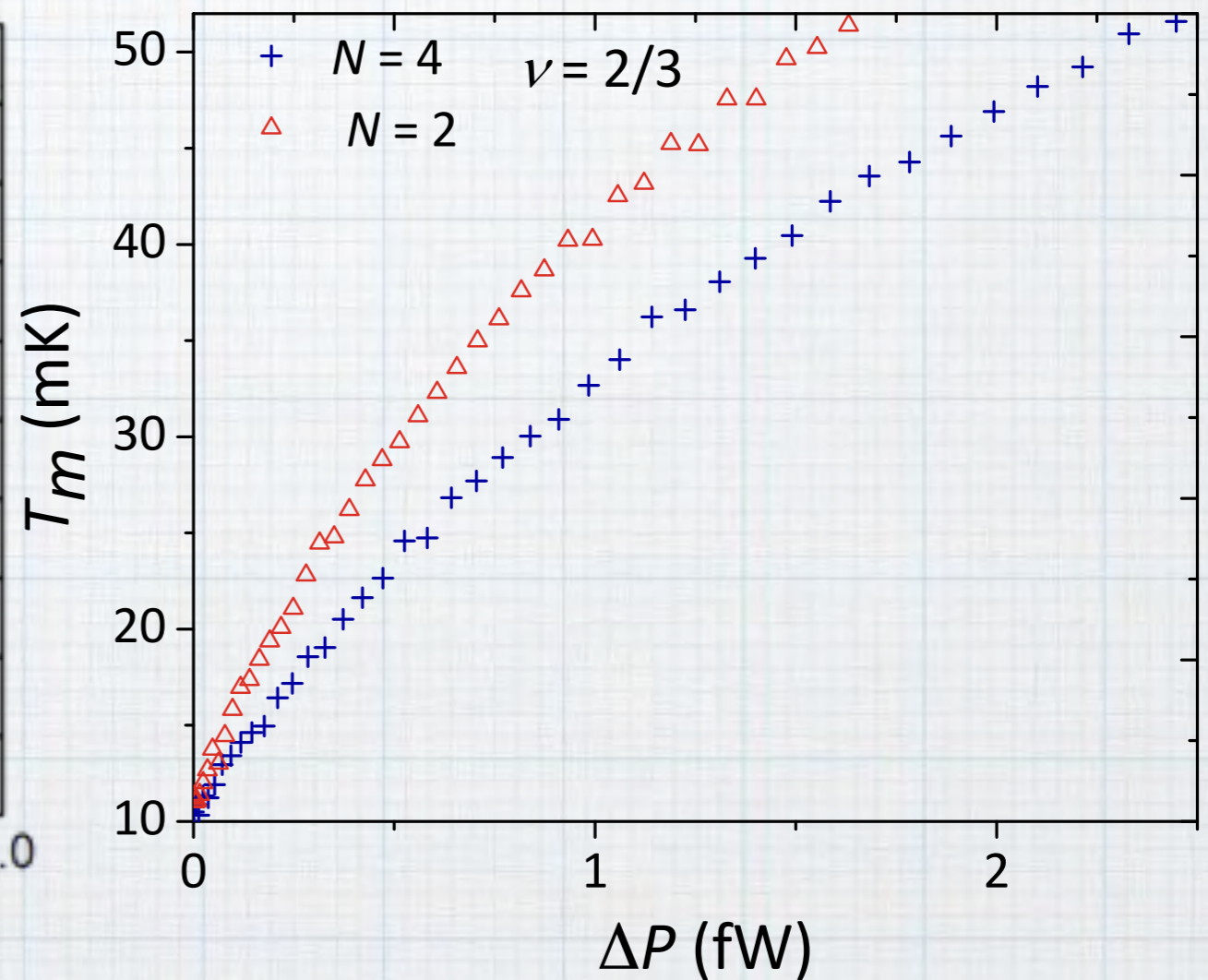
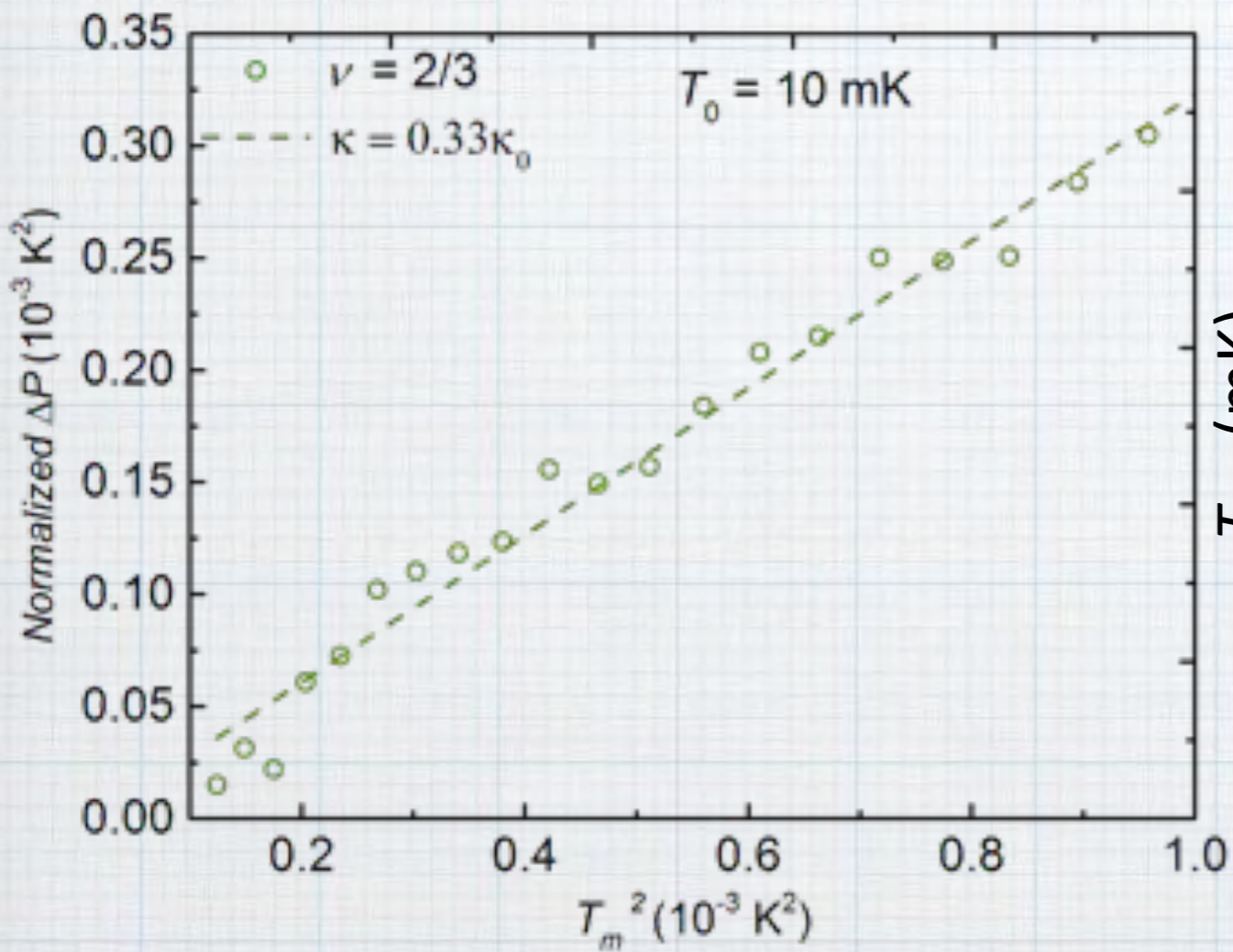


heat conductance w/length



$$\nu = 2/3$$

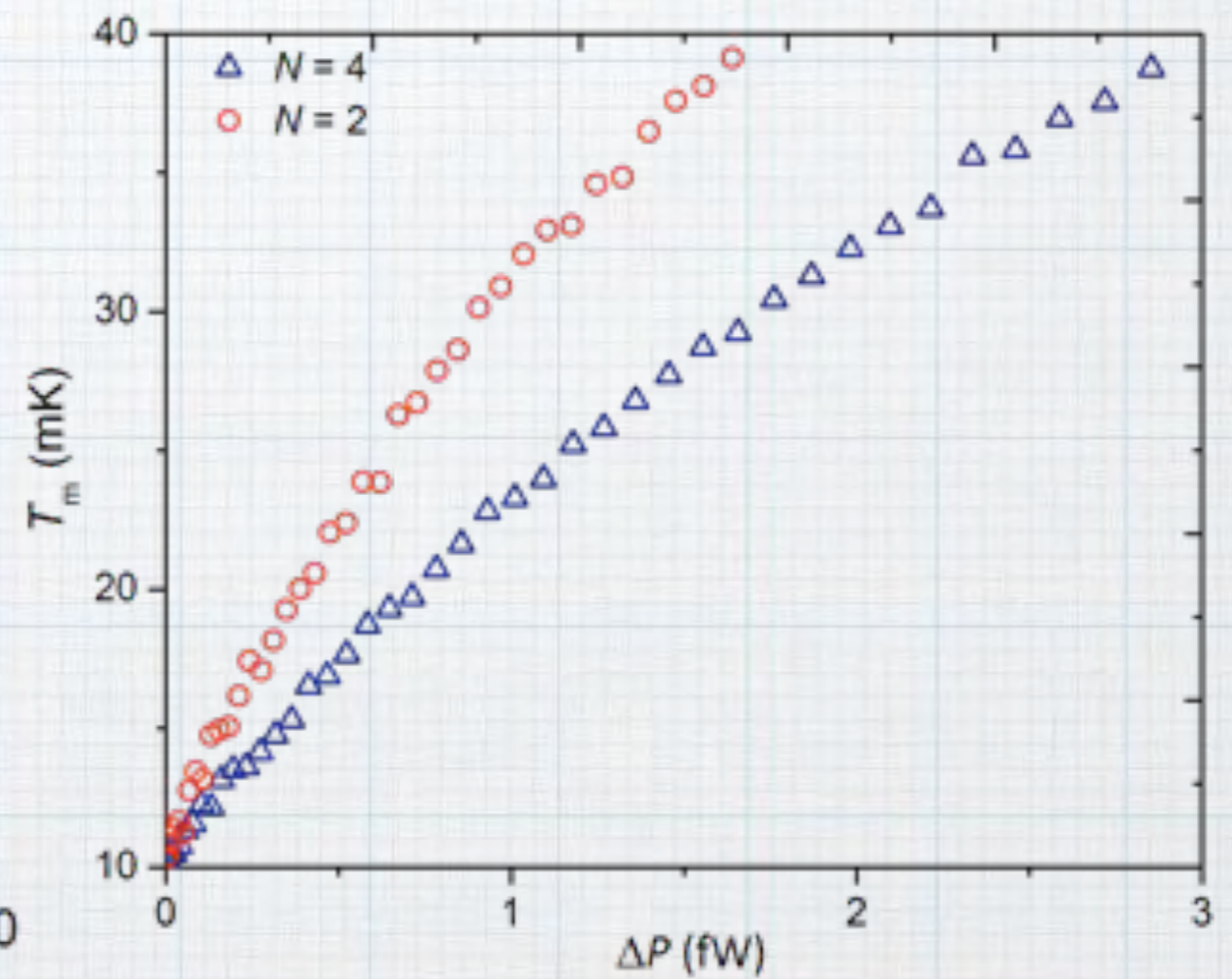
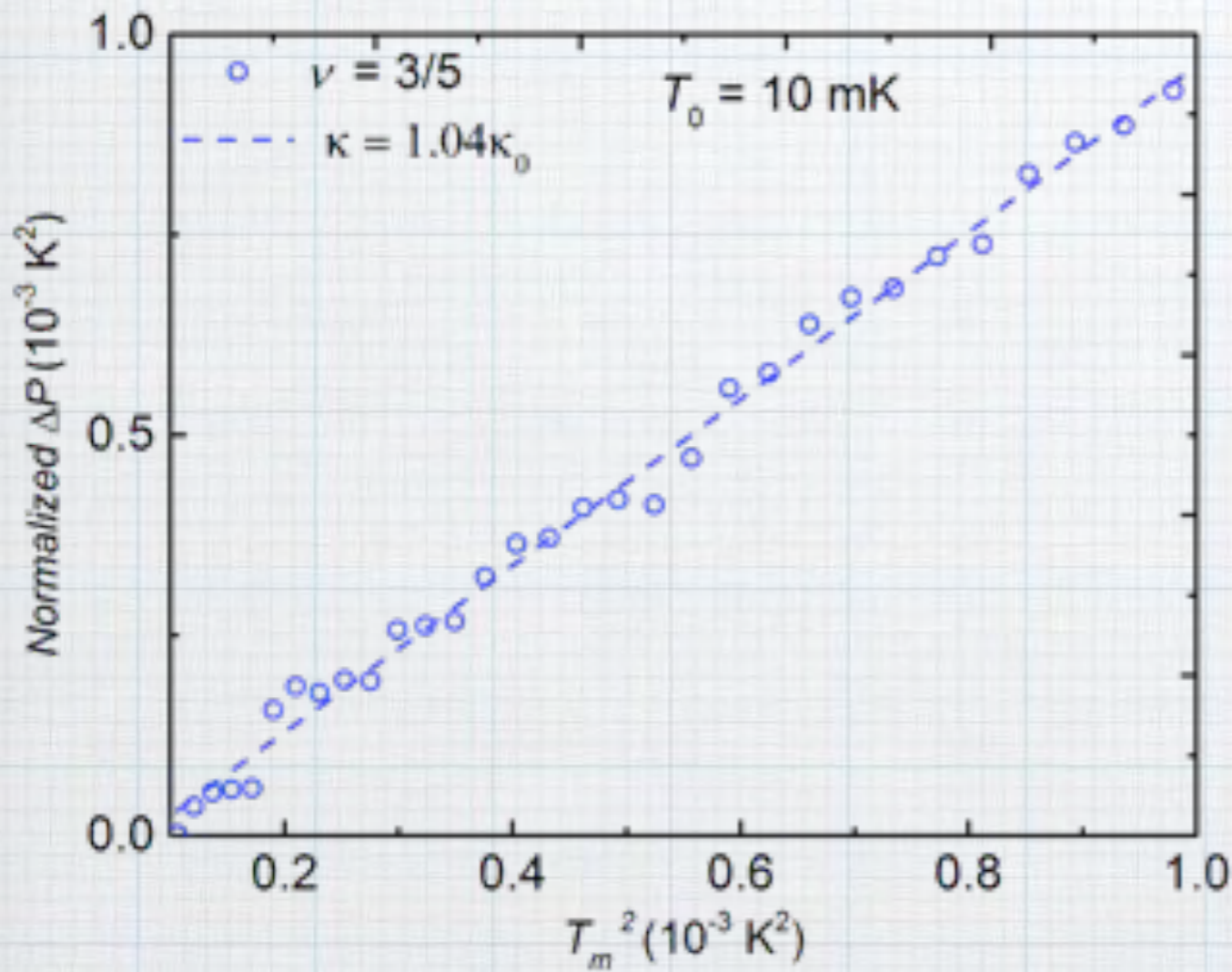
$$J_e \approx 0.33 \cdot 0.5\kappa_0 (T_m^2 - T_0^2) \quad T_0 = 10\text{mK}$$



$\kappa > 0$symmetric up and down of arms, hence.... actual $\kappa / 2$

$$\nu = 3/5$$

$$J_e \approx 1.04 \cdot 0.5\kappa_0 (T_m^2 - T_0^2) \quad T_0 = 10\text{mK}$$



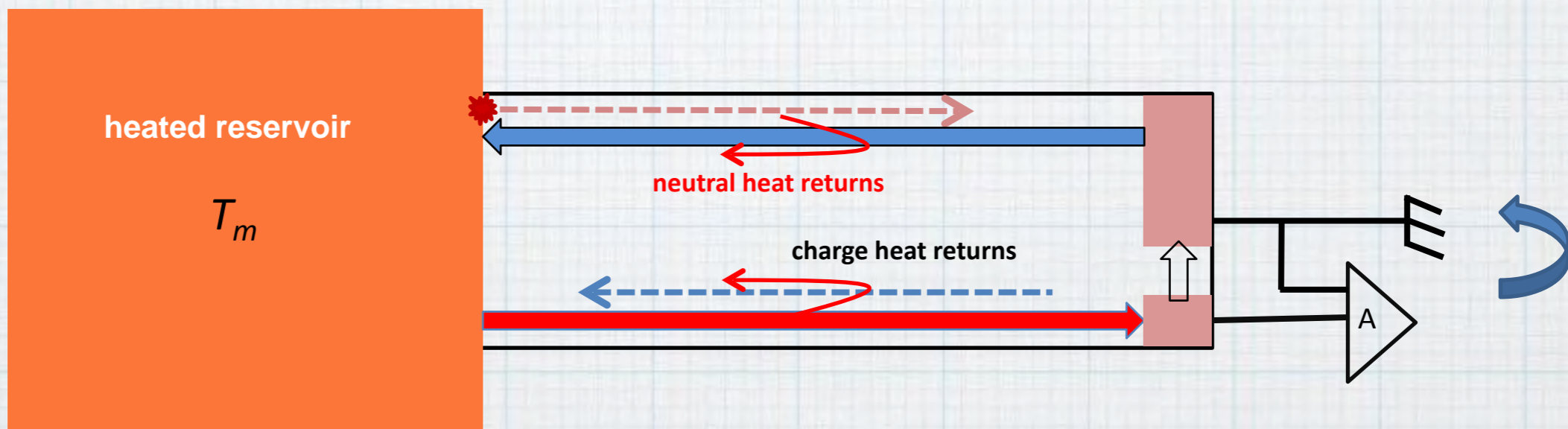
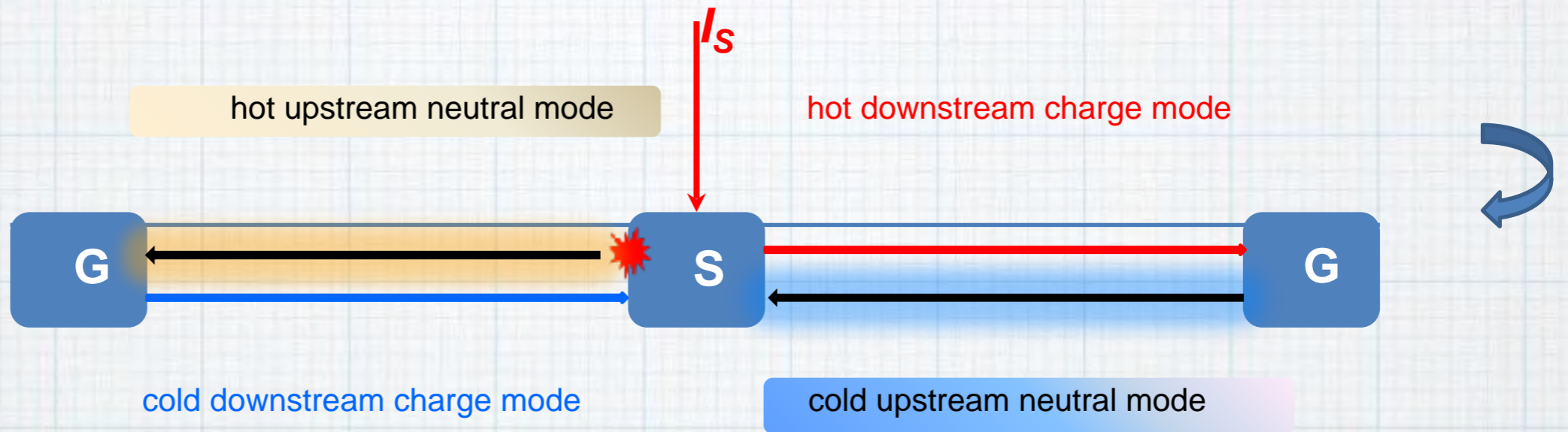
$$v = 2/3$$

$$\frac{K}{\kappa_0} = \frac{2}{1 + \frac{L}{\xi_T}} \quad L \sim 150 \mu m$$

$$J_e \simeq 0.33 \cdot 0.5 \kappa_0 (T_m^2 - T_0^2) \quad T_m^{ava} = 20 mK \quad \Rightarrow \xi_T = 30 \mu m$$

$$J_e \simeq 0.25 \cdot 0.5 \kappa_0 (T_m^2 - T_0^2) \quad T_m^{ava} = 45 mK \quad \Rightarrow \xi_T = 20 \mu m$$

example..... $v = 2/3$why $K=0$?

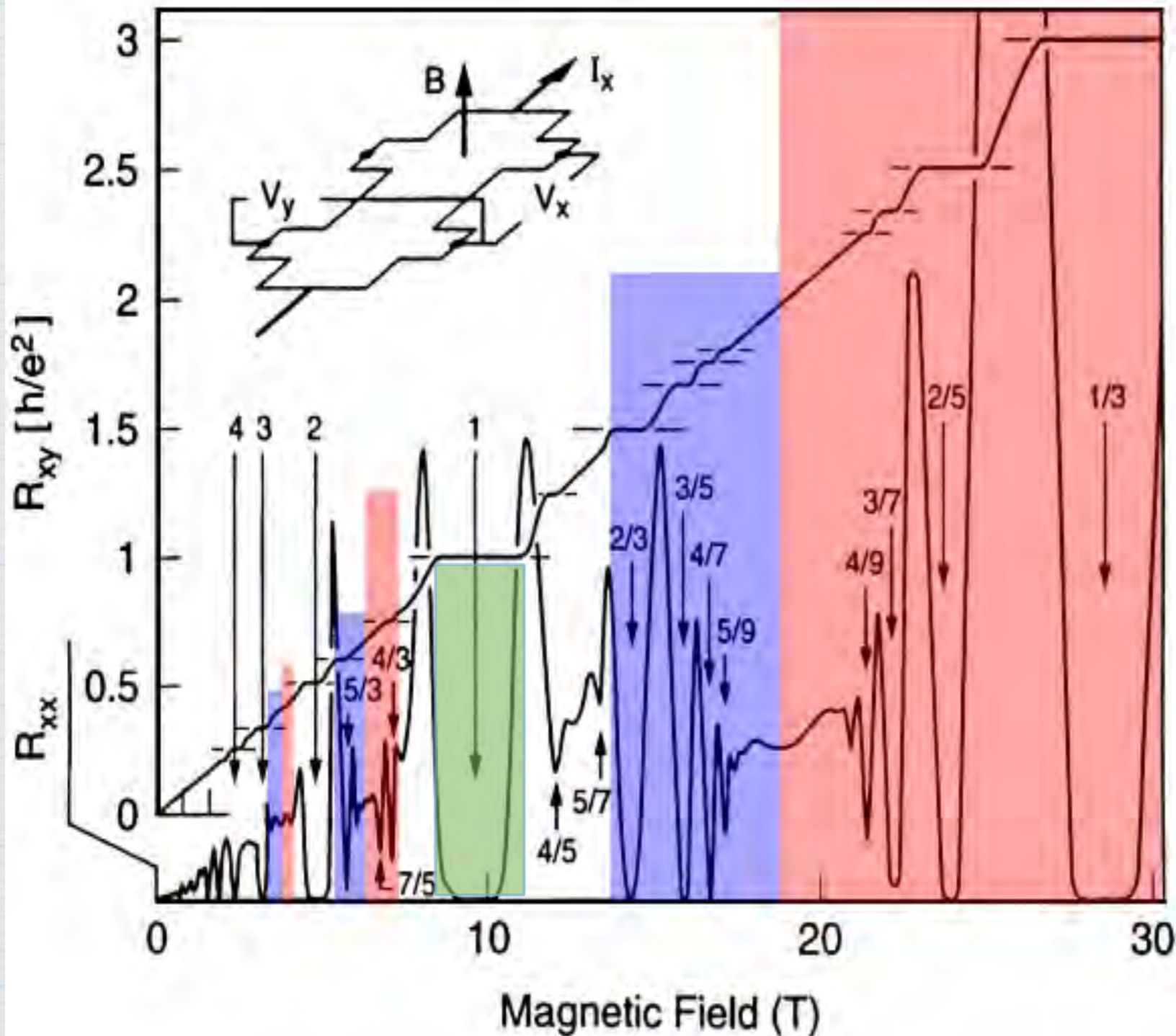


heat diffuses

thermal conductivity vs thermal conductance

length dependence thermal conductance

observation of upstream neutral edge modes



shot noise

$\nu = 2/3, 3/5, 1+2/3, 2+2/3$ & $5/2$
Bid, Nature (2010), Dolev, PRL (2011)

QD thermometry

$\nu = 2/3$ edge at $\nu = 1 +$ bulk heat transport
Venkatachalam, Nature Physics (2012)

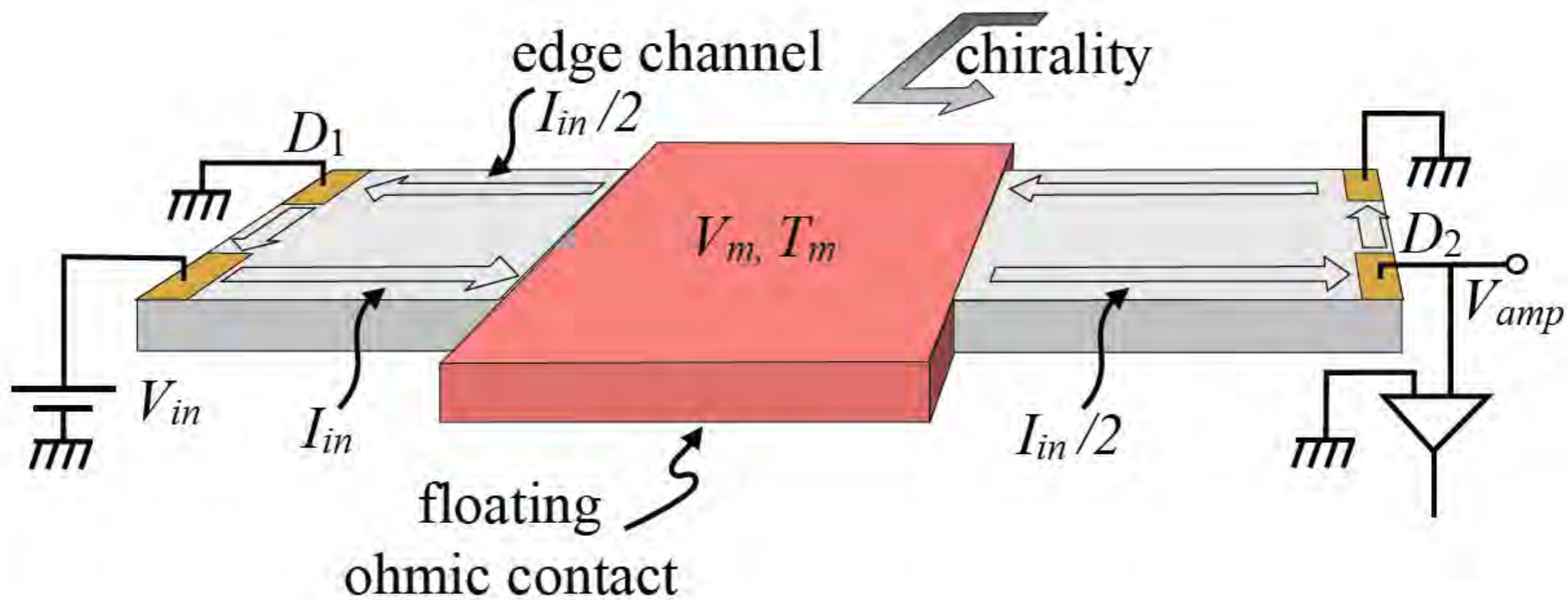
QD thermoelectric current

$\nu = 2/3$
Gurman, Nature Comm. (2012)

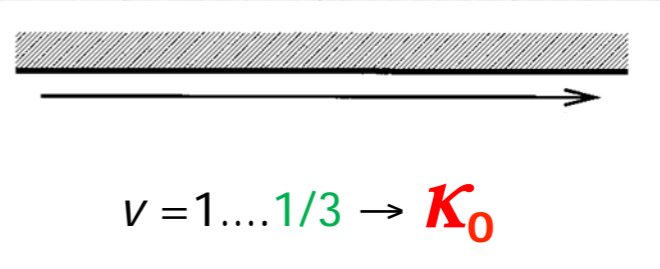
QD thermometry

$\nu = 1+1/3 +$ bulk heat transport
Altimiras, PRL (2012)

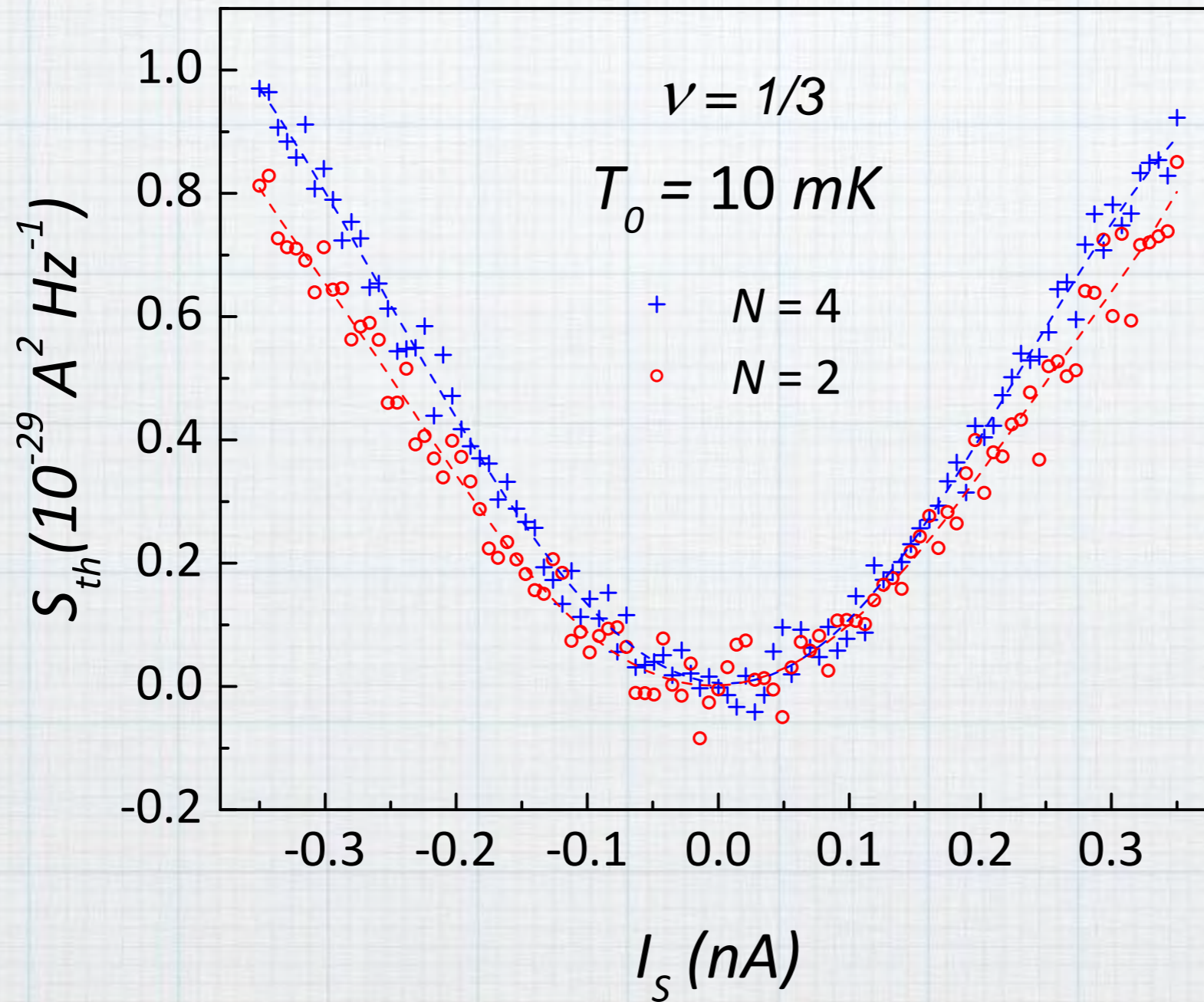
two-arm device



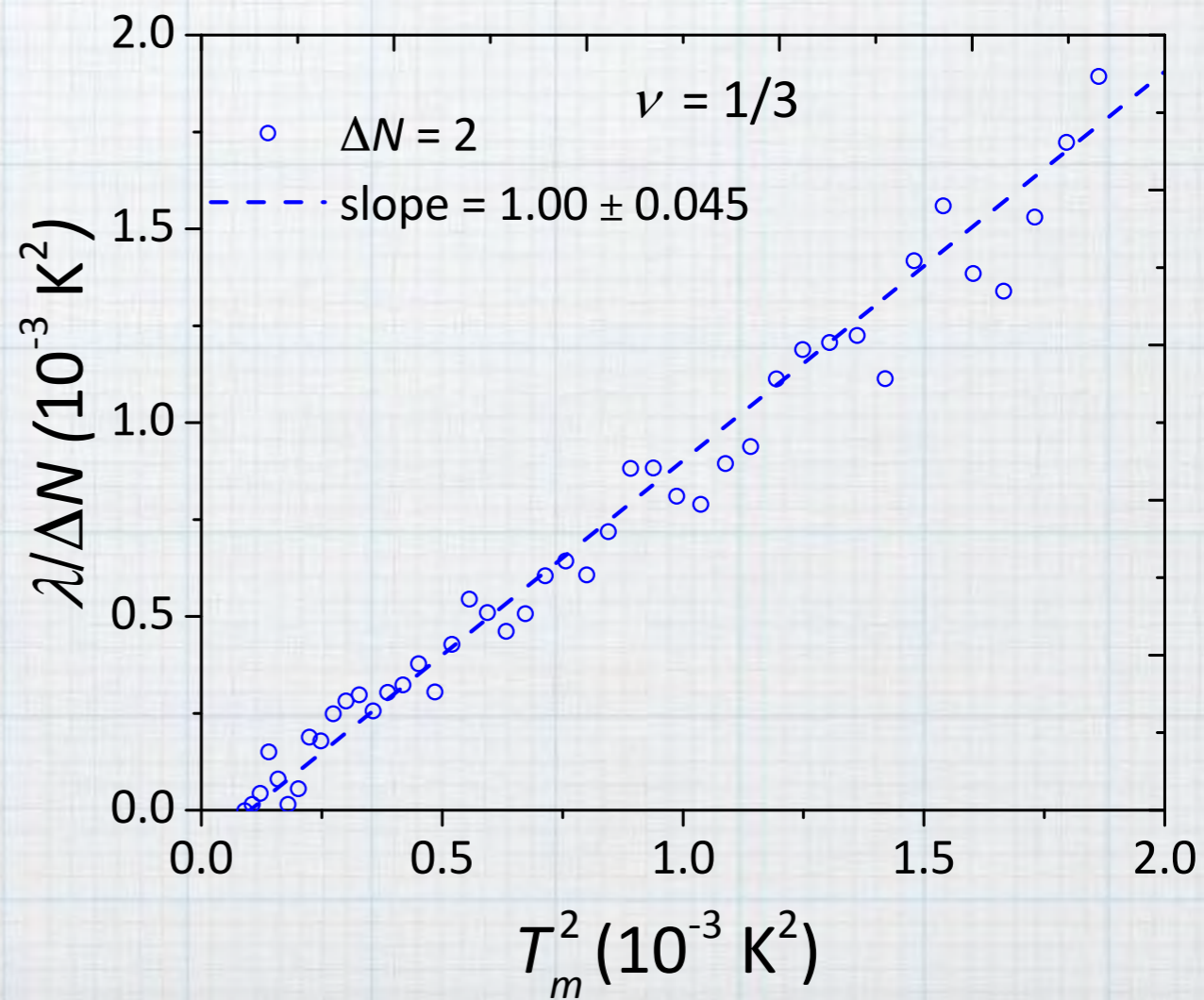
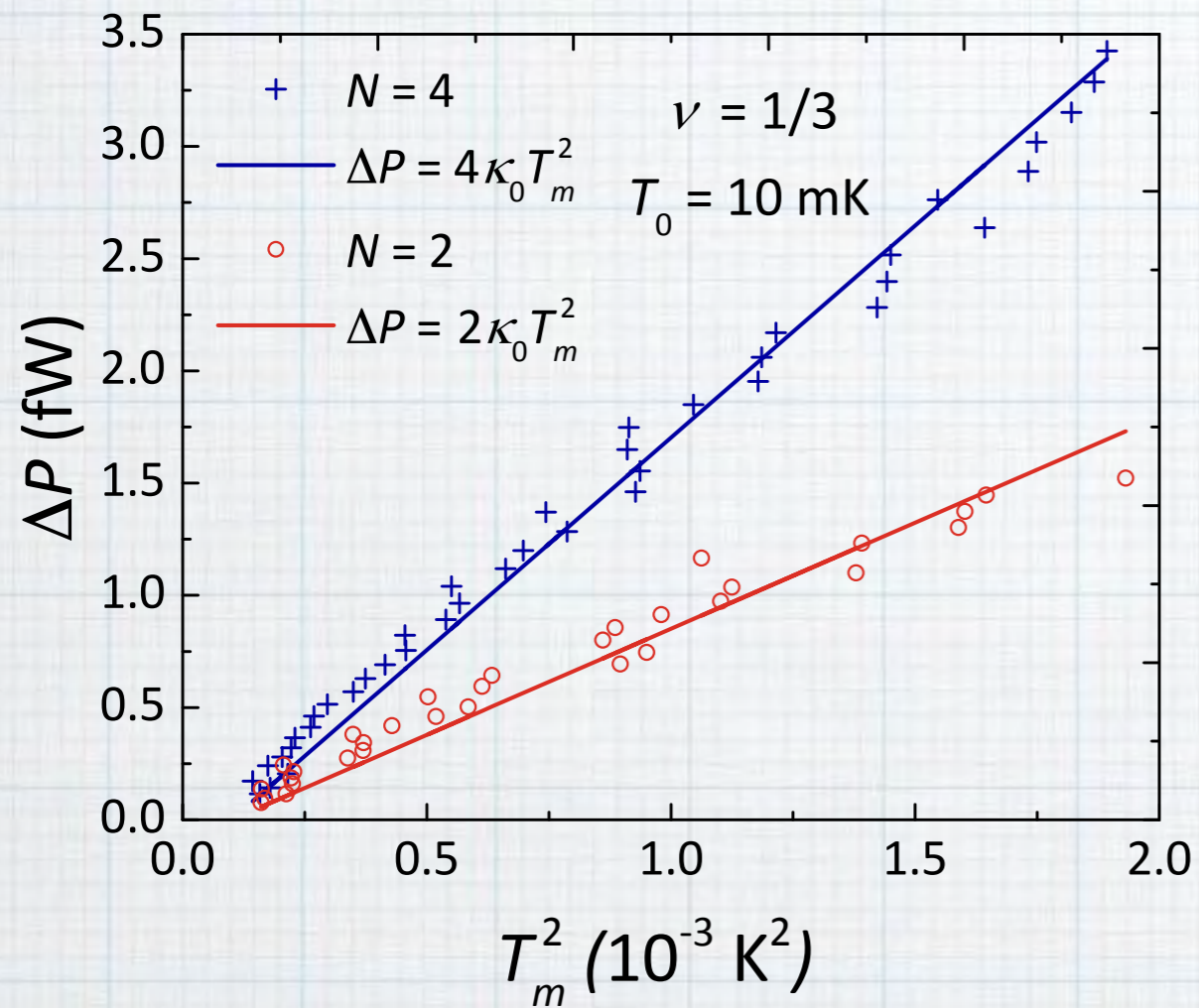
Results :



1 electron mode... 1 composite fermion mode



Results :



What sets the limit on heat flow

$E.t \approx \hbar$ sets a lower bound to the energy flow

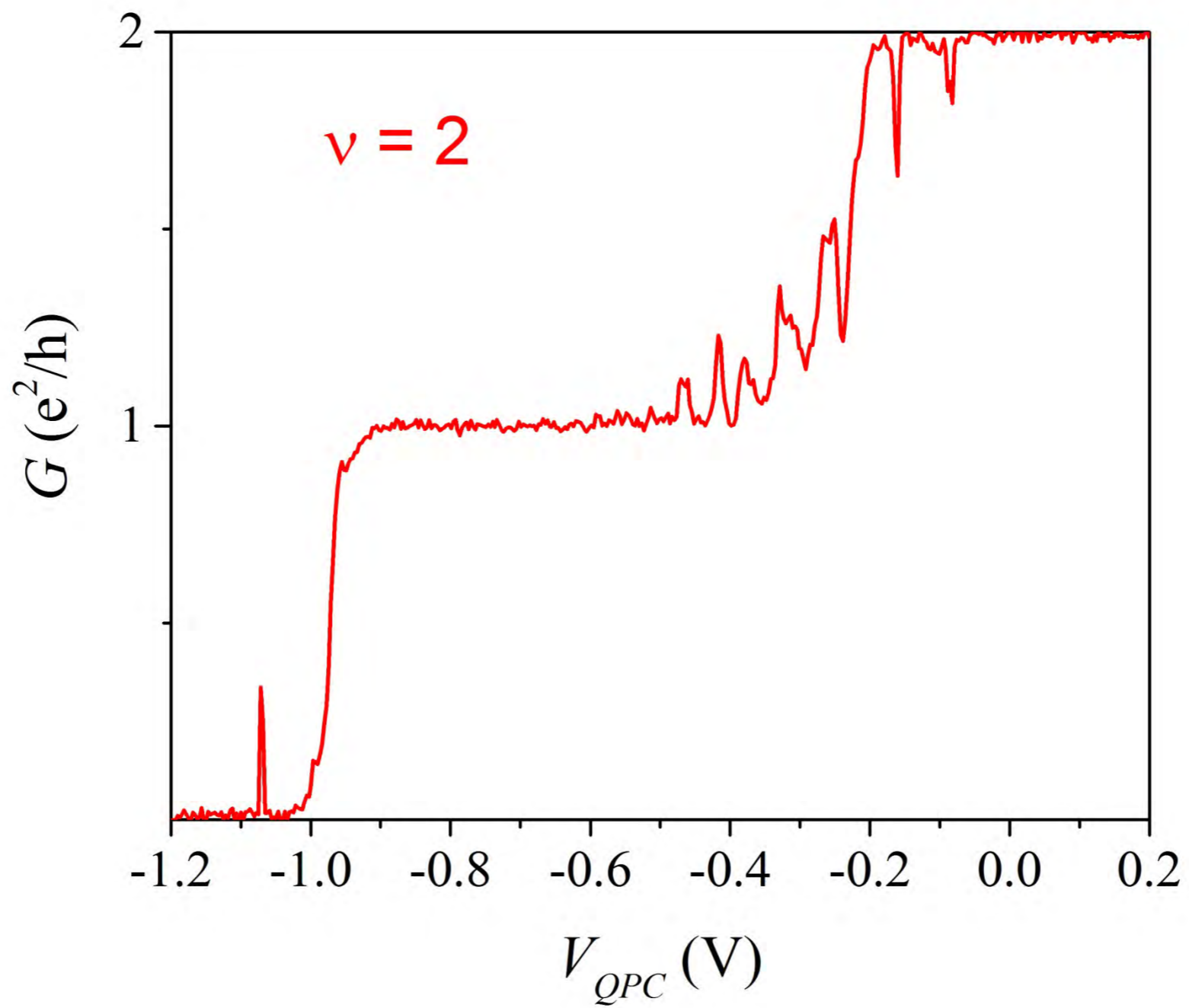
In steady state, $\dot{E} = \frac{\pi k_B^2}{12\hbar} T^2$ & $\dot{S} = \frac{\pi k_B^2}{6\hbar} T$

Expression relating single channel entropy and energy flow is

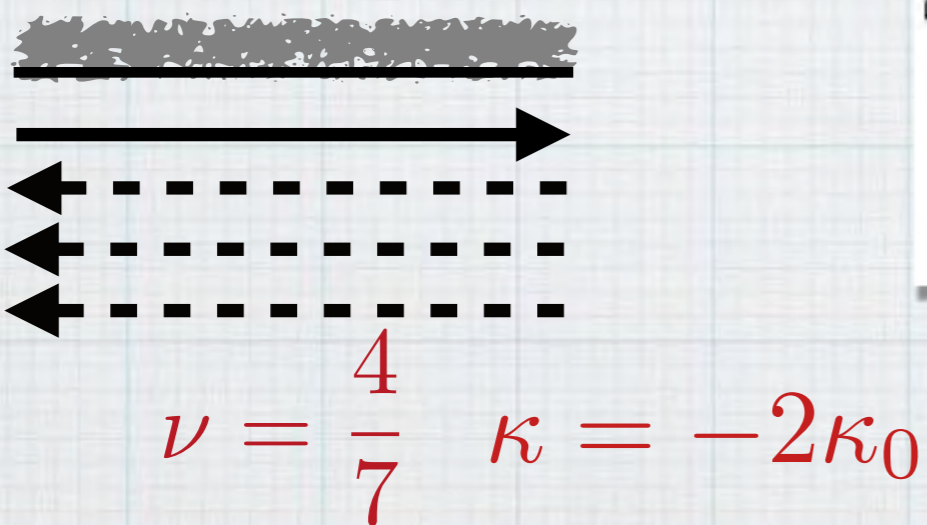
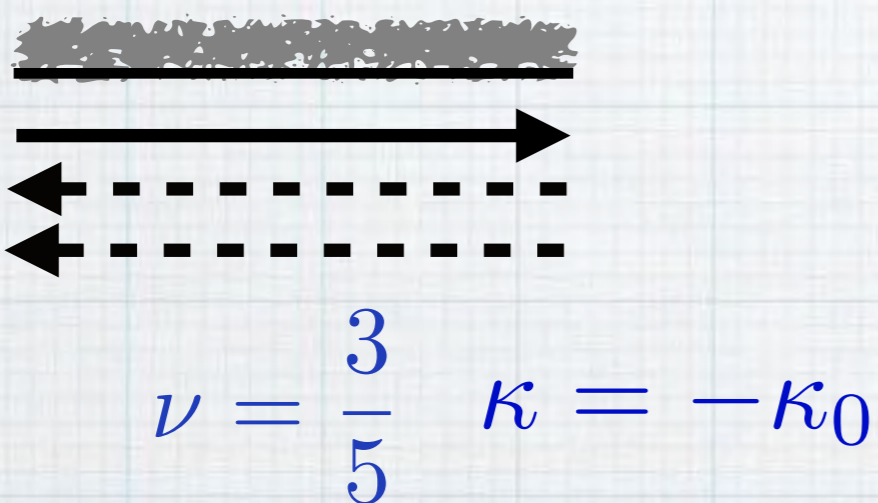
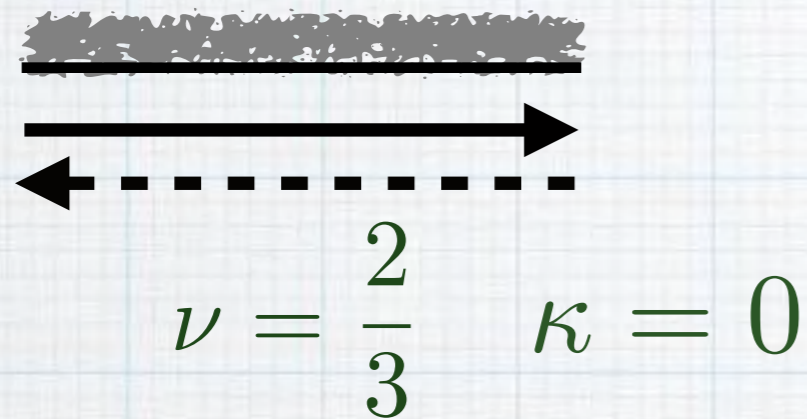
$$\dot{S}^2 \leq \frac{\pi k_B^2}{3\hbar} \dot{E}$$

using, $\dot{Q} = \dot{E}$ & $\frac{\dot{Q}}{T} \leq \dot{S}$

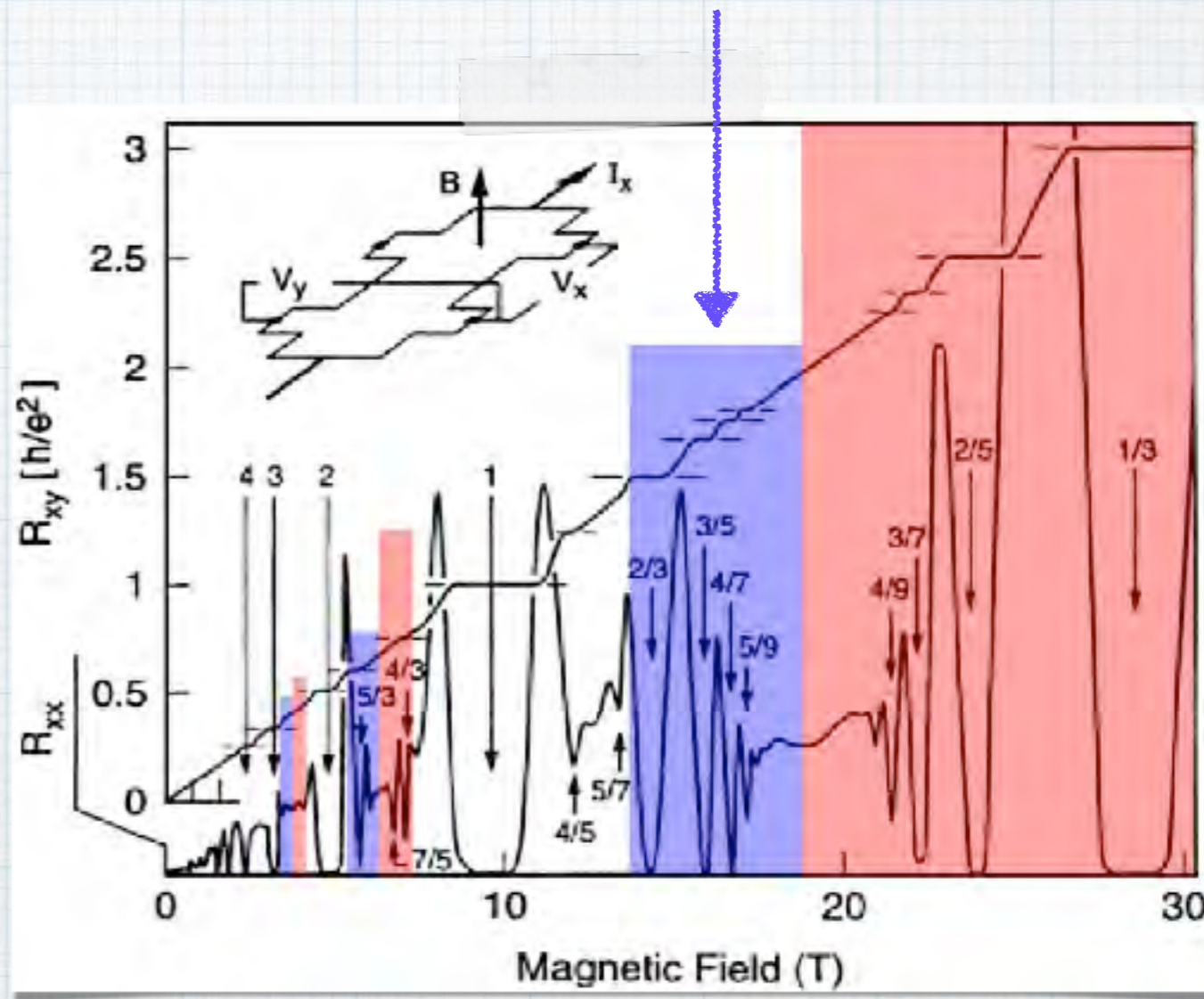
$$\dot{Q} \leq \frac{\pi k_B^2}{3\hbar} T^2 \quad \text{or} \quad J \leq \frac{\pi^2 k_B^2}{6\hbar} T^2$$



hole-like states :



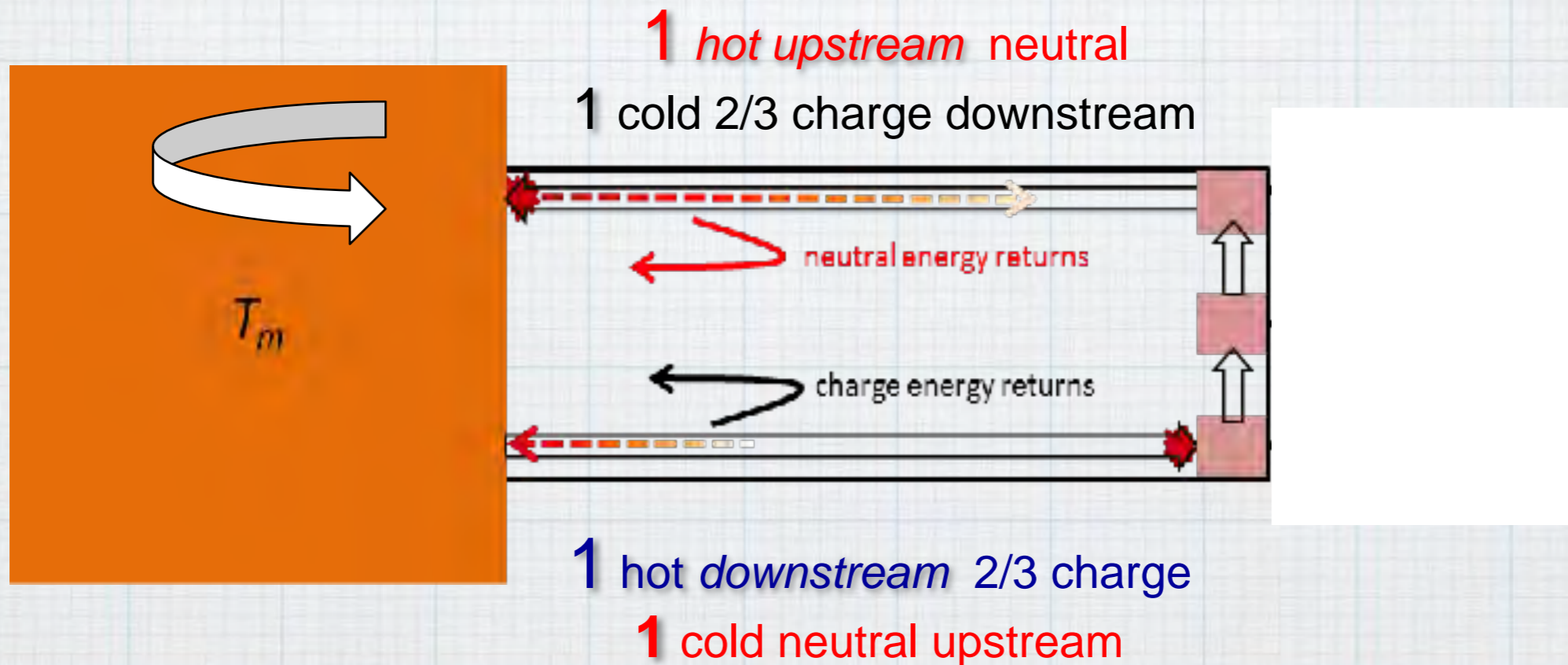
hole-like states
more interesting states



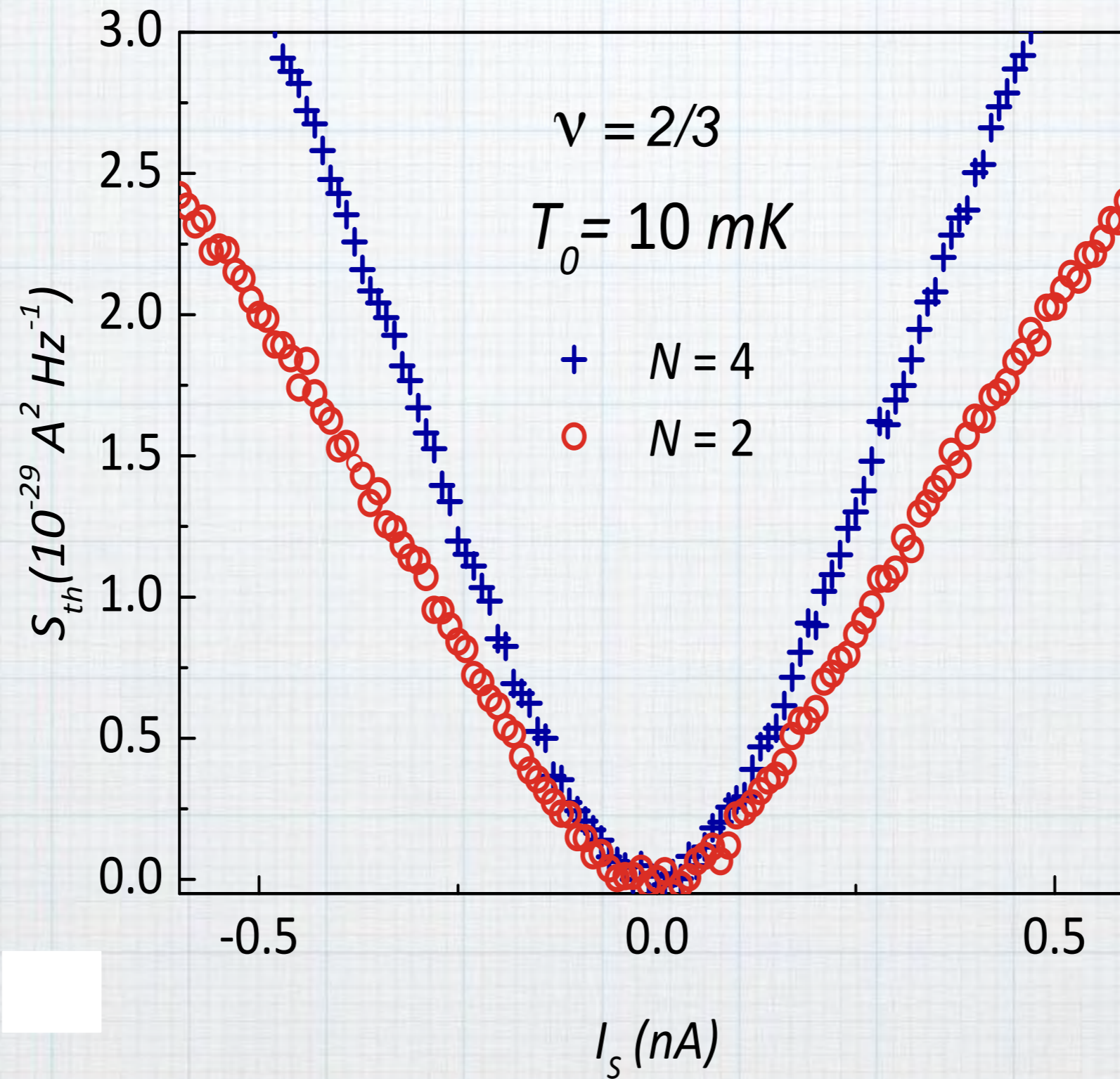
hole-like states + neutral modes..... $\nu = 2/3$

expected $\kappa = 0$ all electrical heat returns

distance $\sim 150\mu\text{m}$, $T_0 \sim 10\text{mK}$



thermal noise – spectral density



Generalization....

G.C Rego and G Kirczenow
Fractional exclusion statistics and the universal thermal conductance: A unifying approach
Phys. Rev. B **59**, 13080-13086 (1999)

$$J_q = \frac{q}{h} \int_{\varepsilon} d\varepsilon (\eta_R - \eta_L) \dots \dots \dots \text{electric current}$$

$$J_{th} = \frac{1}{h} \int_{\varepsilon} d\varepsilon \cdot \varepsilon (\eta_R - \eta_L) \dots \dots \dots \text{heat current}$$

$$\eta_g = \frac{1}{Z(x, g) + g} \quad x = \frac{\varepsilon - \mu}{k_B T}$$

$g = 0$	bosonic
$g = 1$	fermionic
$g = 3$	$\nu = 1/3$

$$G_q = \frac{1}{g} \cdot \frac{e}{h} \cdot e \dots \dots \dots g \text{ dependent}$$

$$G_{th} = 1 \cdot \frac{\pi^2 k_B^2}{3h} \cdot T \dots \dots \dots g \text{ independent}$$