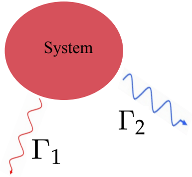
# Many-body sub radiant decay dynamics in 1D light-matter systems

Loïc Henriet

ICFO, in the group of Darrick Chang.
Ana Asenjo Garcia, Mariona Moreno Cardoner, Andreas Albrecht, James Douglas, Jeff Kimble, Paul Dieterle, Oskar Painter

# Characteristic decay timescale

- Finite quantum system.

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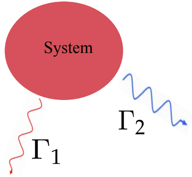


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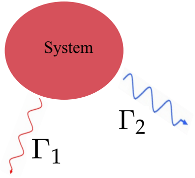


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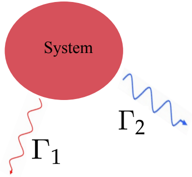


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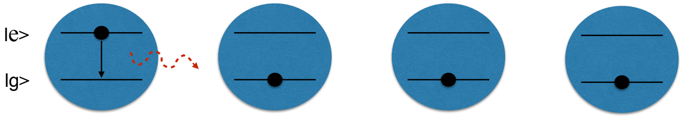
- Here, atomic ensembles with light-matter interaction.

# OUTLINE

- Atomic arrays, coupled to light modes.




[1] Asenjo-Garcia, Moreno-Cardoner, Albrecht, Kimble, and Chang, PRX 7, 031024 (2017). Albrecht*, Henriet*, Asenjo-Garcia, Dieterle, Painter and Chang, arxiv (2018)

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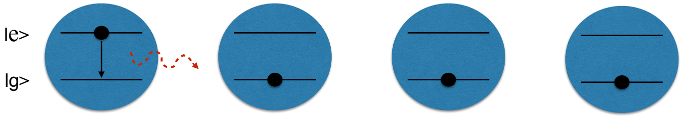


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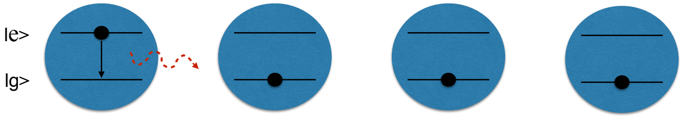


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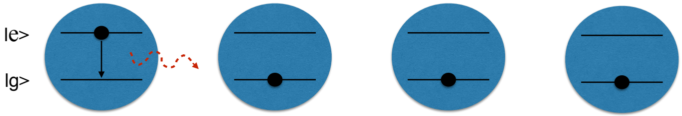


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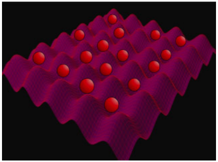

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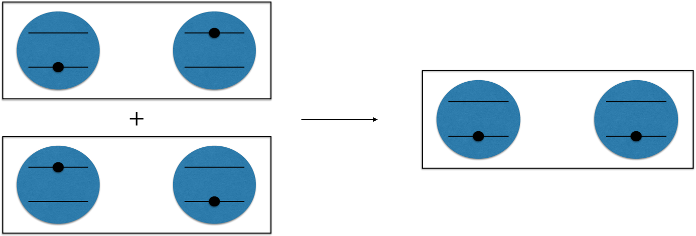

# Effects of interactions : spin model

We integrate the photonic degrees of freedom in the Born-Markov approximation[3],

$$\dot{\rho} = \underbrace{-(i/\hbar)\left(\mathcal{H}_{eff}\rho - \rho\mathcal{H}_{eff}^{\dagger}\right)}_{\mathcal{K}[\rho]} + \underbrace{\sum_{l,m=1}^{N} \Gamma_{m,n}\sigma_l^{-}\rho\sigma_m^{+}}_{\mathcal{J}[\rho]},$$

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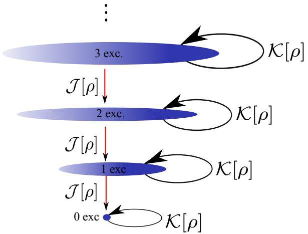


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# SINGLE EXCITATION MANIFOLD

$$\mathcal{H}_{eff} = \sum_{l,m=1}^{N} \hbar g_{l,m} \sigma_l^+ \sigma_m^- .$$

- Complex symmetric Hamiltonian $\mathcal{H}_{eff}^{(1)}$, with eigenvectors $|\psi_l^{(1)}\rangle = \sum_{j=1}^{N} \alpha_j^l \sigma_j^+ |g\rangle$ associated to eigenvalues $\lambda_l = \hbar(\omega_l - i\Gamma_l/2)$.

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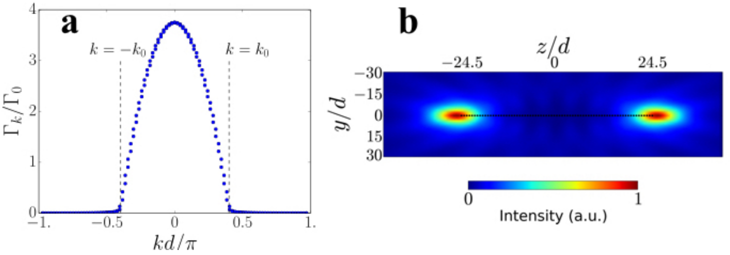

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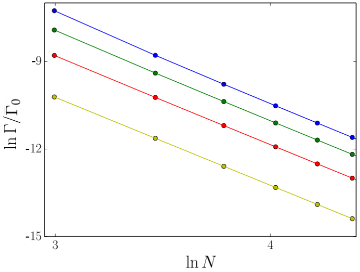
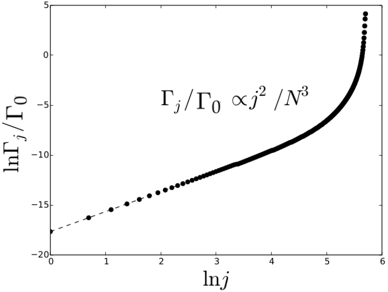
- $\Gamma_j \propto \Gamma_0 j^2/N^3$.



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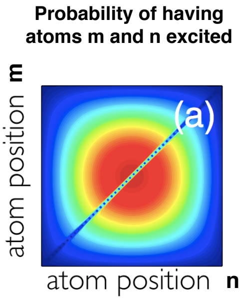

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- Complex symmetric Hamiltonian $\mathcal{H}_{eff}^{(2)}$, with eigenvectors $|\psi_l^{(2)}\rangle = \sum_{l=1}^{N-1} \sum_{m=l+1}^{N} \beta_{l,m}^{l} \sigma_m^+ \sigma_l^+ |g\rangle$.
- In the limit of large $N$, we have for the most subradiant states $\beta_{l,m}^{l} \underset{\sim}{\propto} [\alpha_l^{k_1} \alpha_m^{k_2} - \alpha_m^{k_1} \alpha_l^{k_2}]$, where $\alpha_j^{k_1}$ and $\alpha_j^{k_2}$ are coefficients of subradiant single-excitation states.

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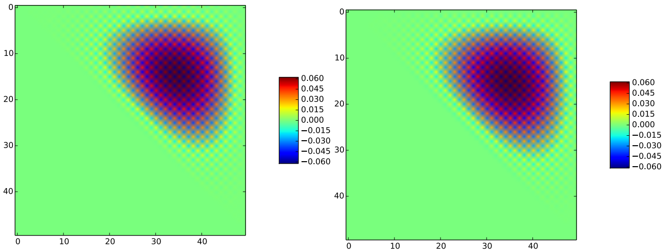
Plot of the real part of $\beta_{i,j}^l$.



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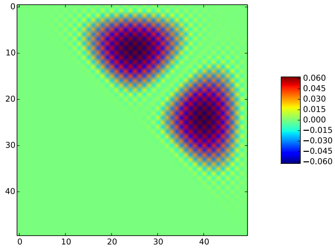


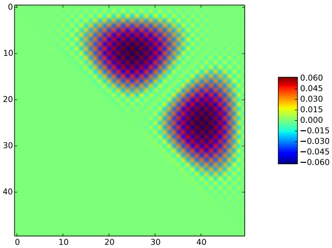

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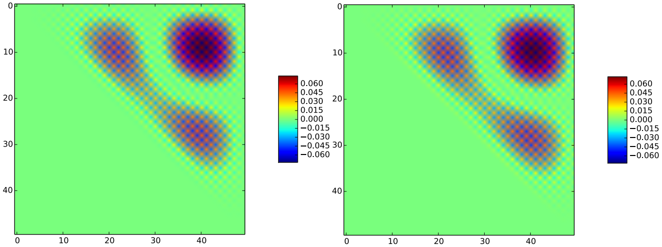

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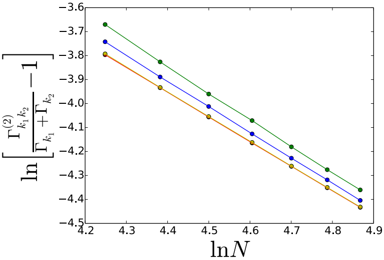

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- This structure extends to higher number of excitations $m \ll N$.

# "Single-excitation" eigenstates of $\mathcal{L}$

$$\mathcal{L}[\rho] = \mathcal{K}[\rho] + \mathcal{J}[\rho] = -(i/\hbar)\left(\mathcal{H}_{eff}\rho - \rho\mathcal{H}_{eff}^{\dagger}\right) + \sum_{l,m}\Gamma_{l,m}\sigma_l^{-}\rho\sigma_m^{+}$$

Effect of coherent and jump terms:

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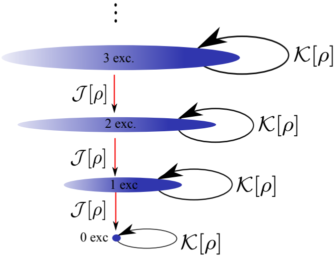

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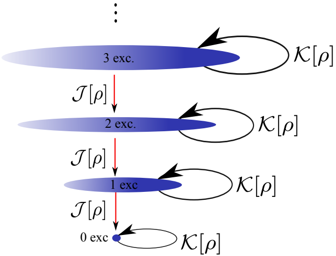

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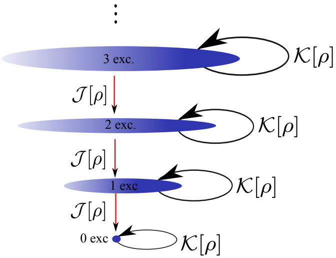


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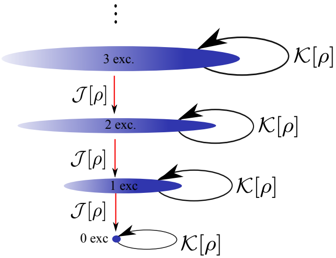


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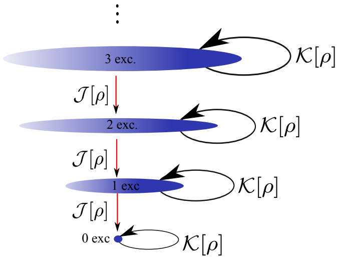


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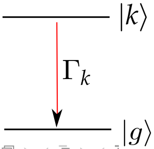


$$\rho_0 = |k\rangle\langle k| = Z_k + |g\rangle\langle g|$$
$$\rho(t) = Z_k e^{-\Gamma_k t} + |g\rangle\langle g| = e^{-\Gamma t}|k\rangle\langle k| + (1 - e^{-\Gamma t})|g\rangle\langle g|$$

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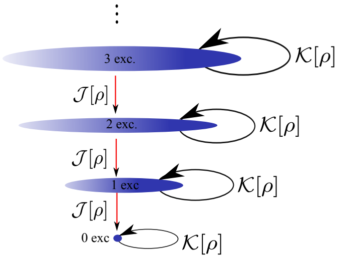

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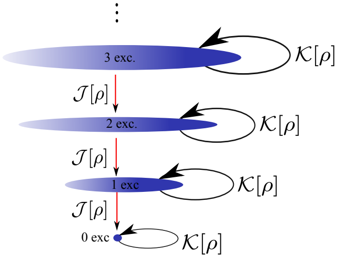

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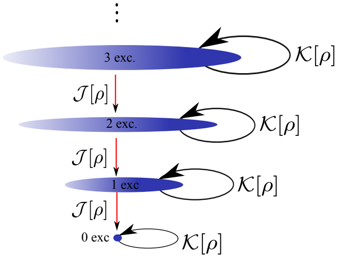


Eigenelements:

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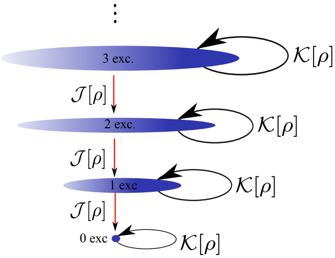


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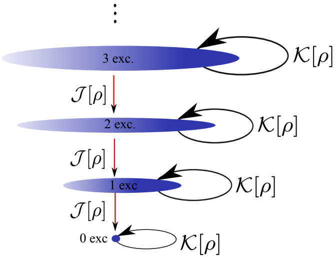


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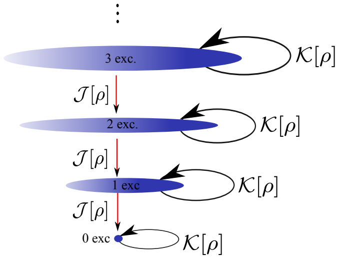


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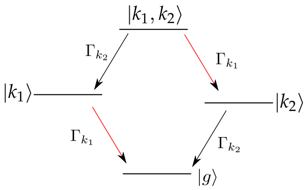

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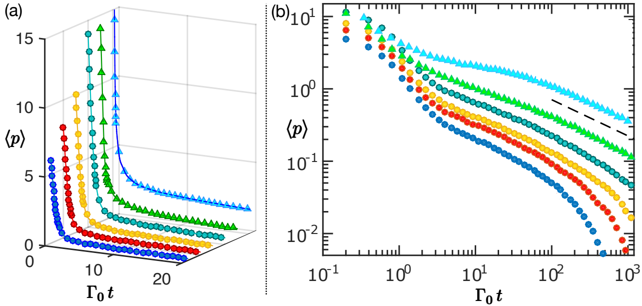

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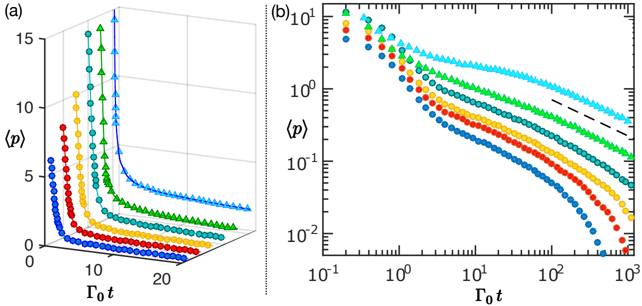

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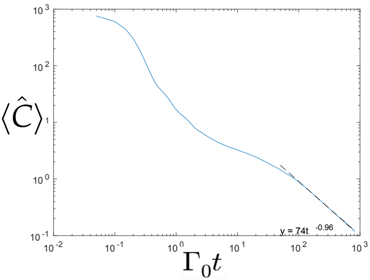

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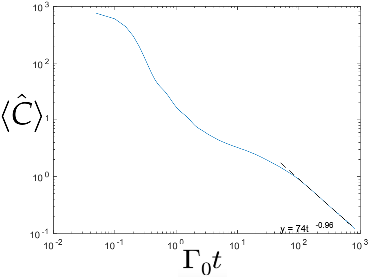
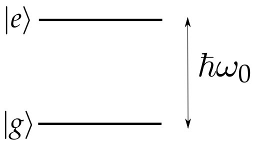


# ATOMIC CLOCK PROTOCOL

Consider an atom with two well isolated energy levels $|g\rangle$ and $|e\rangle$.




Clock protocol : determination of $\omega_0$.

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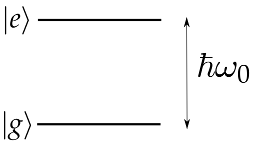


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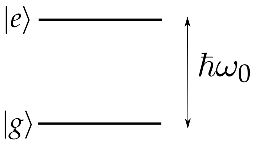


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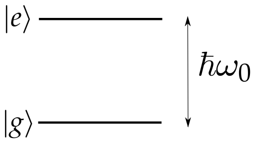


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$$\frac{|g\rangle + e^{i\delta t}|e\rangle}{\sqrt{2}} \to \frac{(1 + e^{i\delta t})|g\rangle + (-1 + e^{i\delta t})|e\rangle}{2}$$

Measurement of $\langle\sigma^z\rangle = -\cos(\delta t)$.

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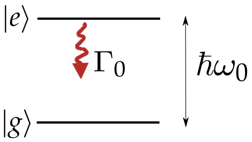


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Measurement of $S = -\cos(\delta t)e^{-\Gamma_0 t/2}$.

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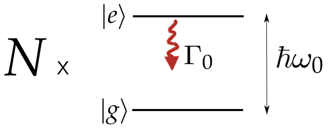


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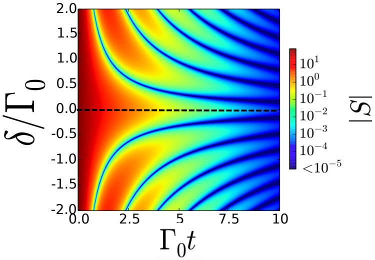
Measurement of $S = -N\cos(\delta t)e^{-\Gamma_0 t/2}$.

- Many atoms with spontaneous emission, $S = -N\cos(\delta t)e^{-\Gamma_0 t/2}$

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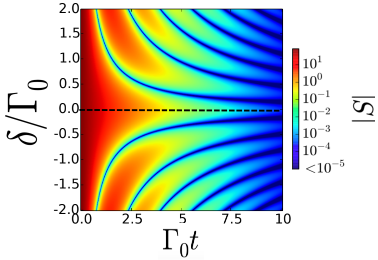


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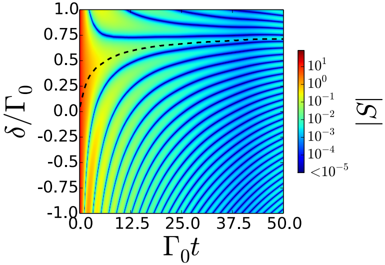
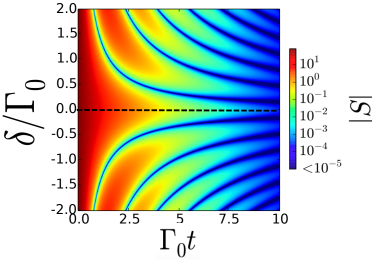
- 1D Atomic lattice clock with subradiant modes, $S \sim \cos(\delta_m t)(\Gamma_0 t)^{-0.5}$



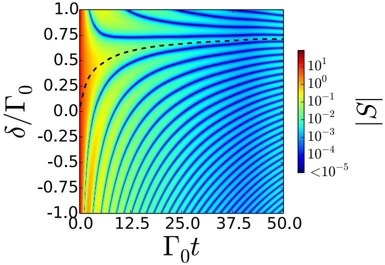

- Central fringe → reference the laser frequency toward the frequency of the most subradiant modes $\delta \sim \omega^{(1)}_{j=1}$ .

- Many atoms with spontaneous emission, $S = -N\cos(\delta t)e^{-\Gamma_0 t/2}$




- Central fringe → reference the laser frequency to the atomic frequency $\delta = 0$ .
- Sensitivity, for $N$ independent atoms, $\Delta\omega \sim \sqrt{\Gamma_0/(NT_{\text{avg}})}$

- 1D Atomic lattice clock with subradiant modes, $S \sim \cos(\delta_m t)(\Gamma_0 t)^{-0.5}$




- Central fringe → reference the laser frequency toward the frequency of the most subradiant modes $\delta \sim \omega^{(1)}_{j=1}$ .
- Sensitivity independent of $\Gamma_0$

# CONCLUSIONS / PERSPECTIVES

Spontaneous emission represents a fundamental barrier for atom-light interactions.
But, with subradiant collective excitations

[4] Asenjo-Garcia, Moreno-Cardoner, Albrecht, Kimble, and Chang, PRX 7, 031024 (2017). Albrecht*, Henriet*, Asenjo-Garcia, Dieterle, Painter and Chang, arxiv (2018)

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- Exponential improvements in photon storage fidelity when increasing the system size[4].

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- Evade the limit of spontaneous emission for atomic lattice clocks.

What can be done for other applications ? Photon gates, nonlinear optics, ...,

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