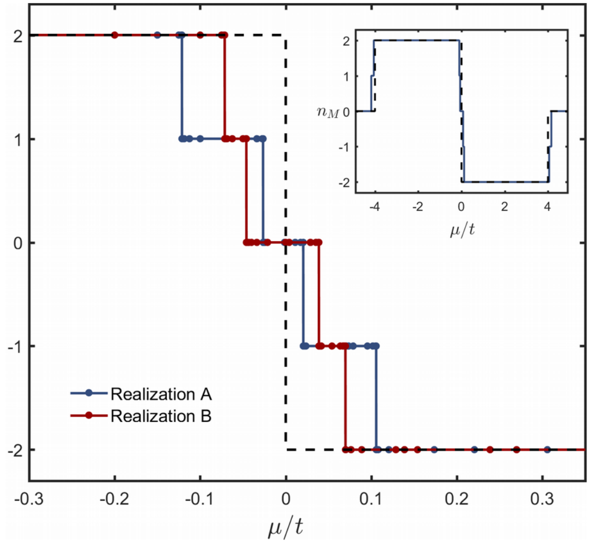
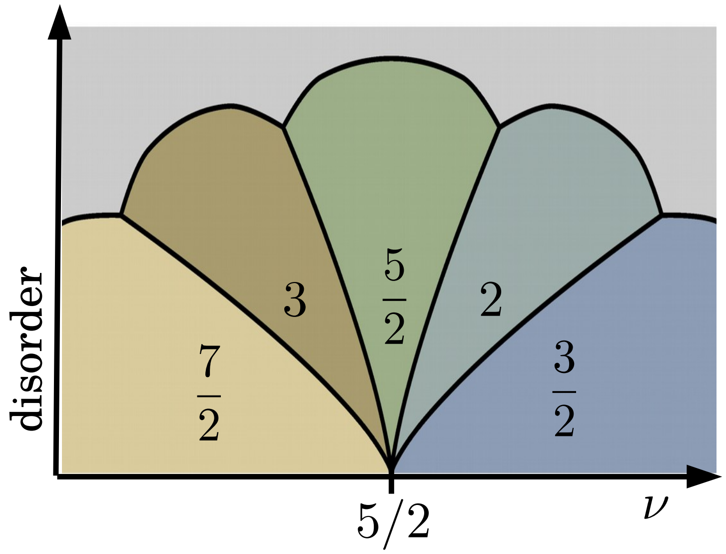
Theory of Disorder-Induced Half-Integer Thermal Hall Conductance

David F. Mross
Weizmann Institute of Science





DFM, Y. Oreg, A. Stern, G. Margalit, M. Heiblum, PRL 121, 026801 (2018)

Physics

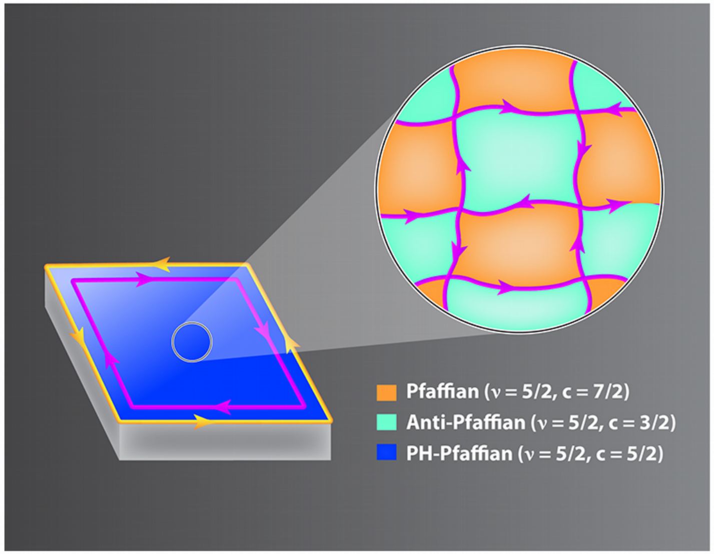
VIEWPOINT

A Hot Topic in the Quantum Hall Effect

Heat transport studies of fractional quantum Hall systems provide evidence for a new phase of matter with potential applications in fault-tolerant quantum computation.

Experiment:
M. Banerjee, M. Heiblum et al., Nature (2018)

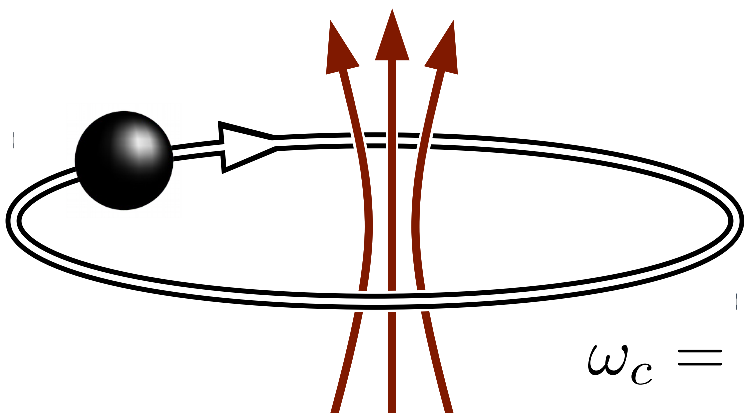
Theory:
DFM, Y. Oreg, A. Stern, G. Margalit M. Heiblum, PRL 121, 026801 (2018)
C. Wang, A. Vishwanath, B. Halperin, PRB 98, 045112 (2018)



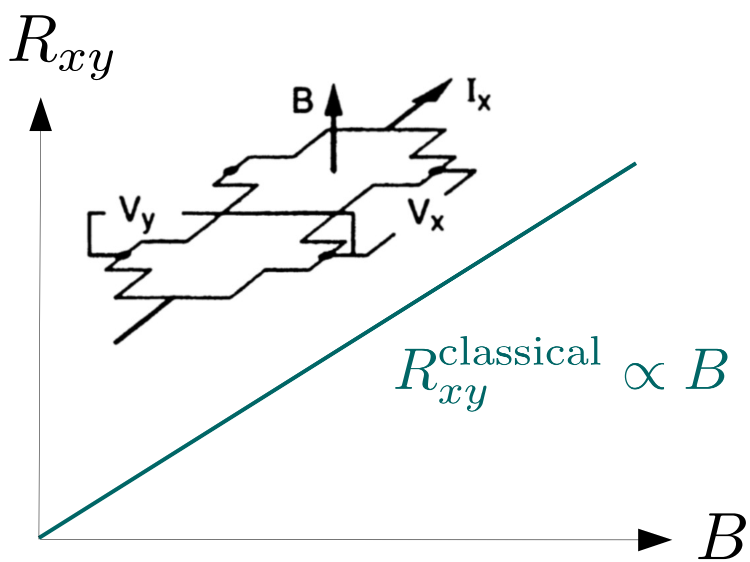
[Also related: B. Lian and J. Wang, PRB 97 165124 (2018)]

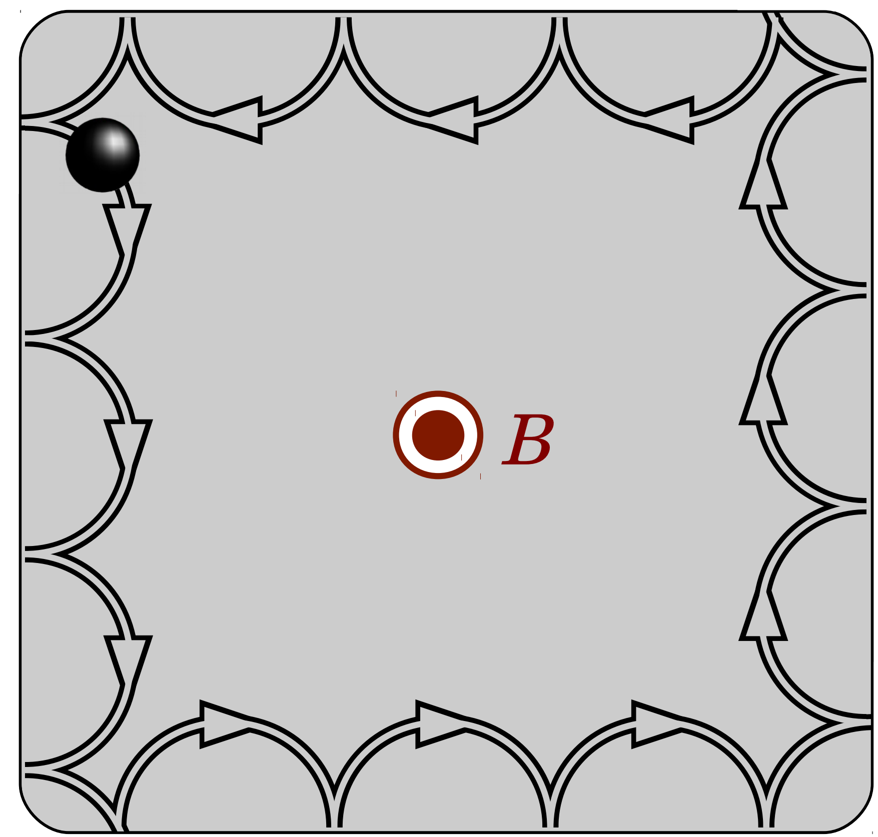
Quantum Hall effect in a nutshell

Classical: Cyclotron orbits



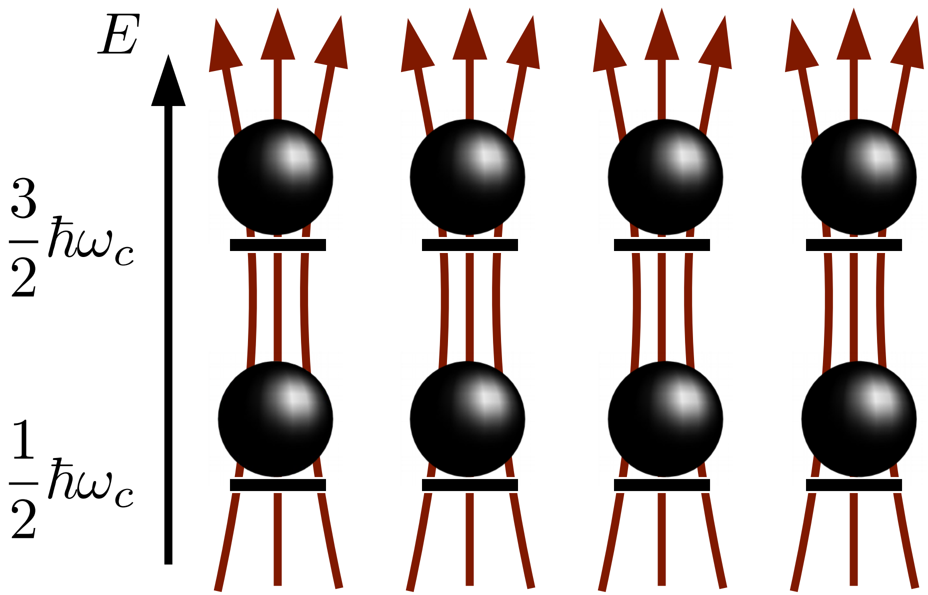
$$\omega_c = \frac{qB}{mc}$$

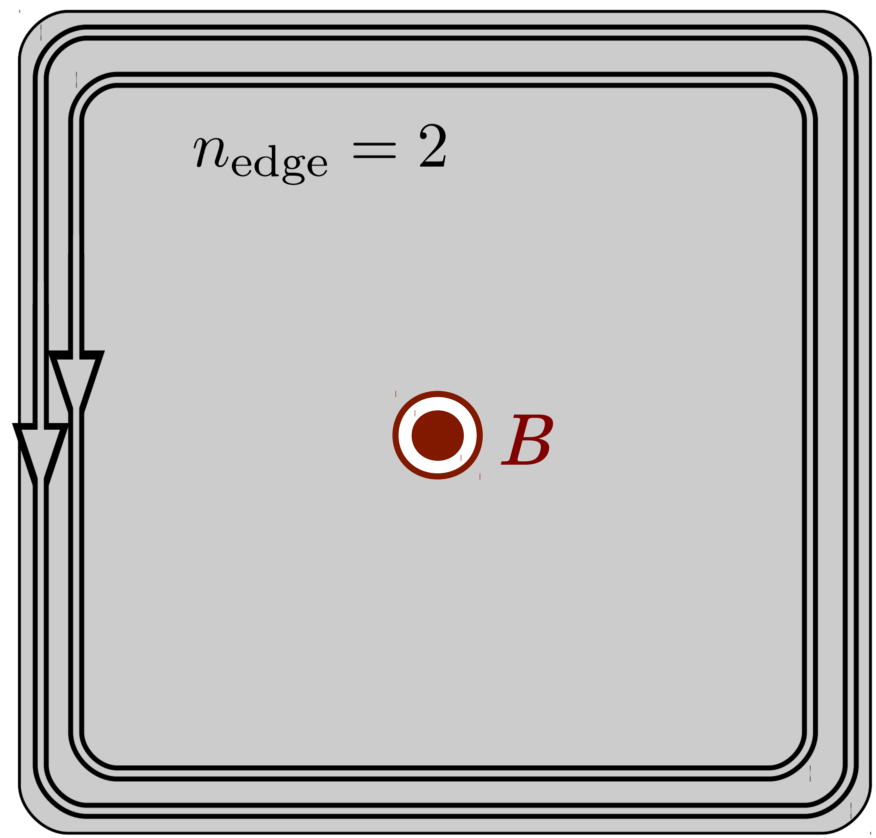




Quantum Hall effect in a nutshell

Quantum mechanical: Energy levels





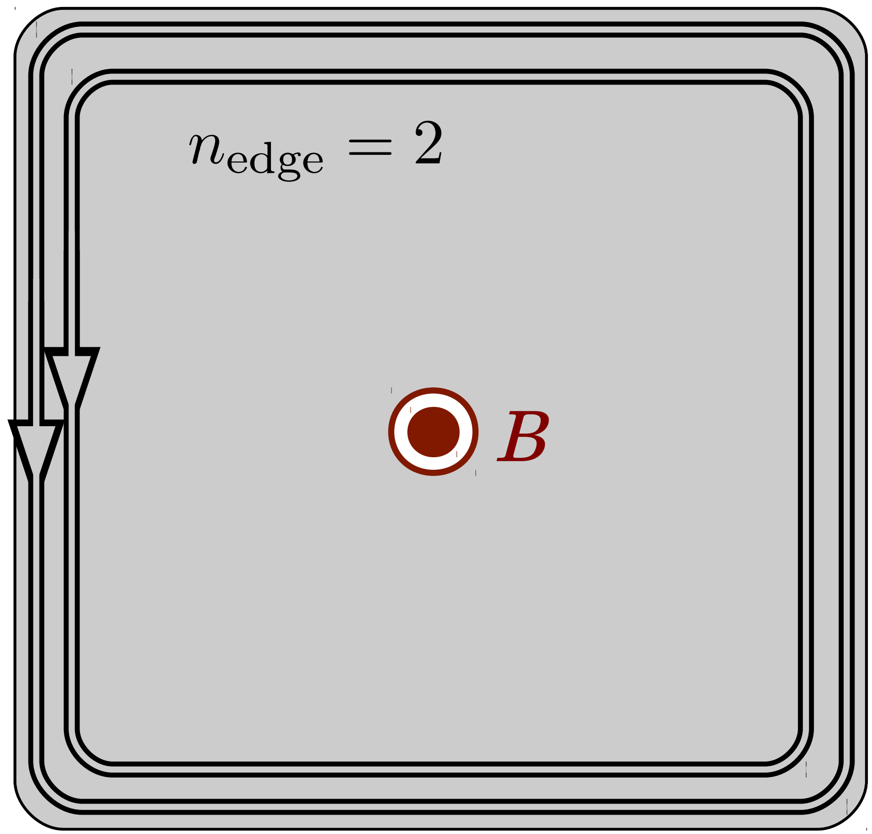
N_{flux} states per energy level

filling factor $\nu = \frac{N_{\text{electron}}}{N_{\text{flux}}}$

$n_{\text{edge}} = \nu$ chiral electron modes carry quantized flow of charge and energy

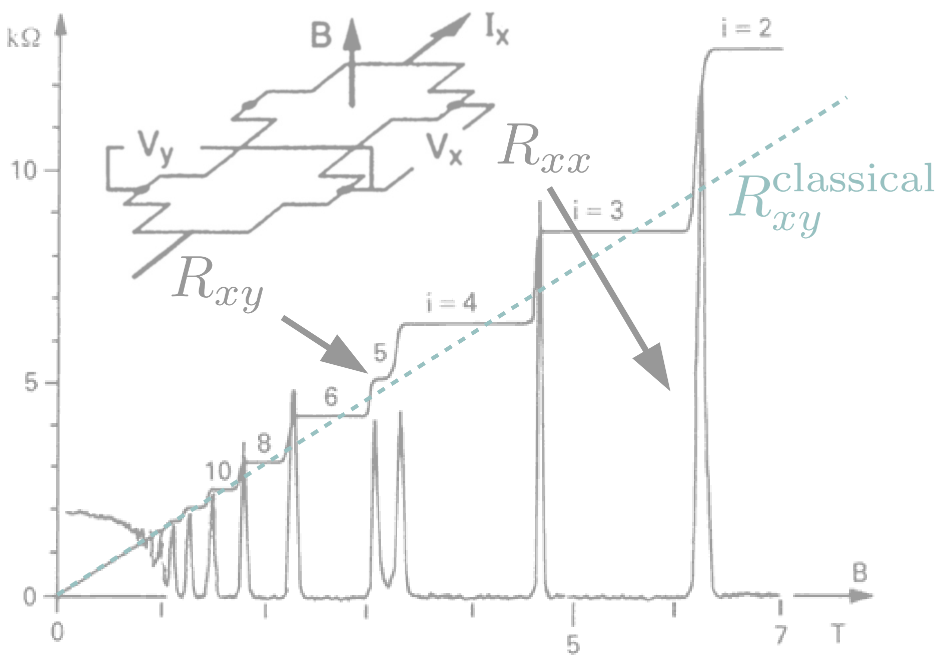
Quantum Hall effect in a nutshell

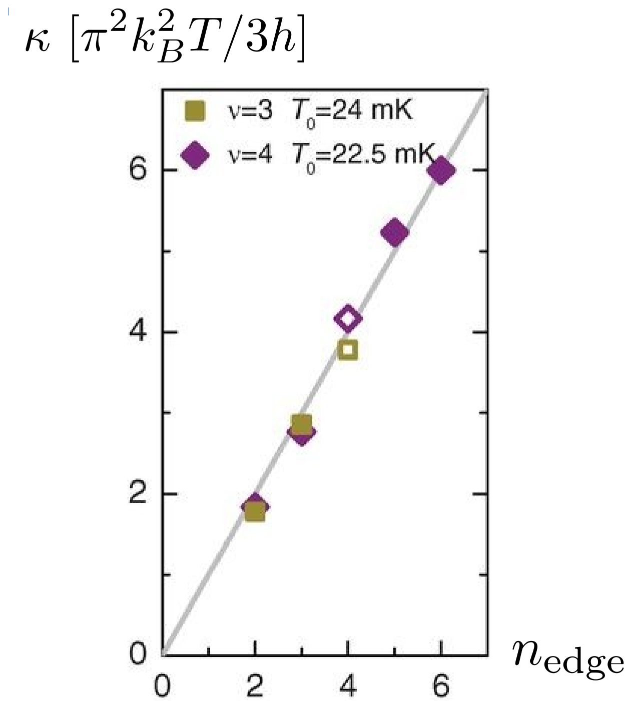
Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	−1	0	0	$\mathbb{Z}$	0
CI	1	−1	1	0	0	$\mathbb{Z}$



...ge $= \nu$ chiral electron modes carry
...antized flow of charge and energy

Quantum Hall effect in a nutshell





Jezouin *et al.* (2013)

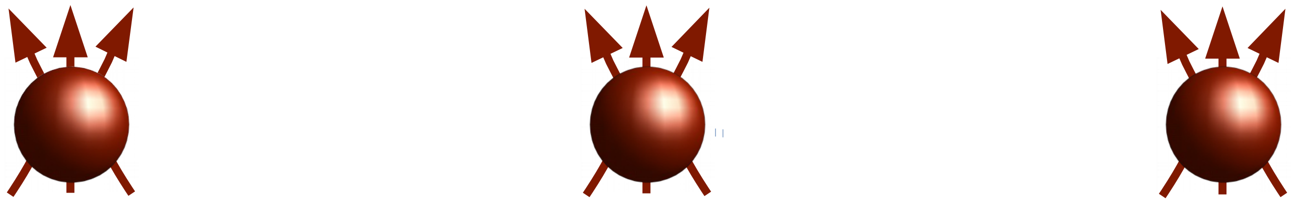
Hall conductance

$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

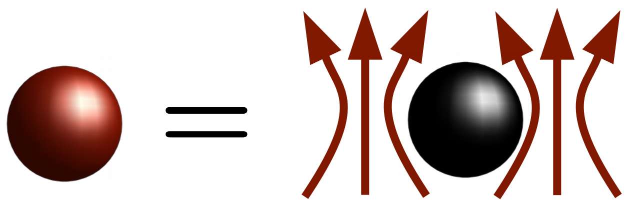
Thermal Hall conductance

$$\kappa_{xy} = n_{\text{edge}} \kappa_0$$

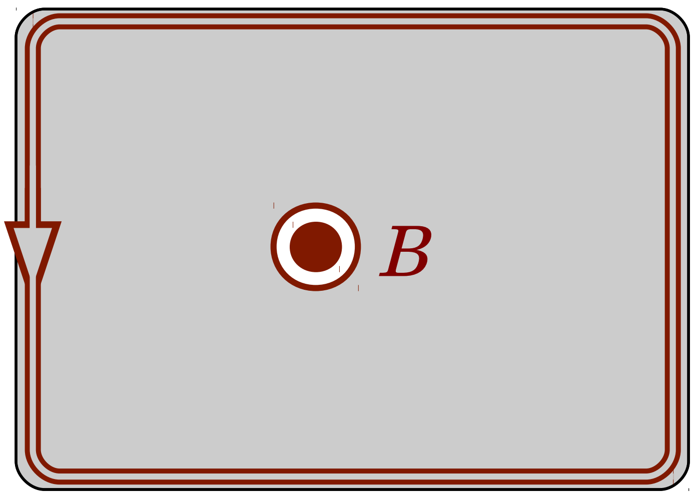
Fractional quantum Hall effect



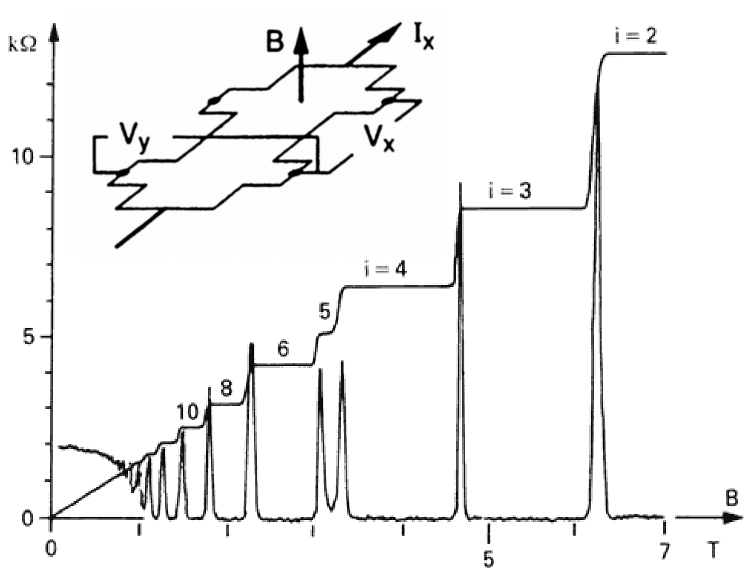
One composite fermions per flux quantum

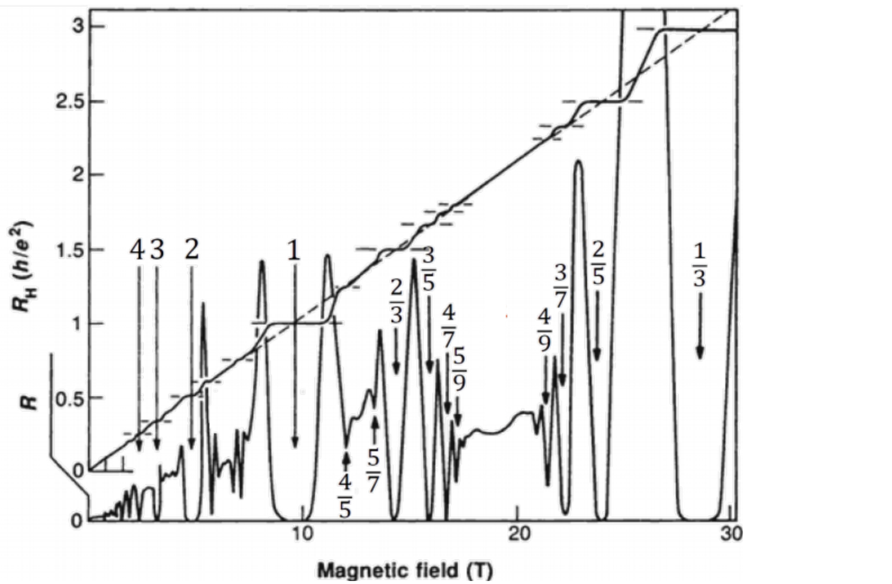


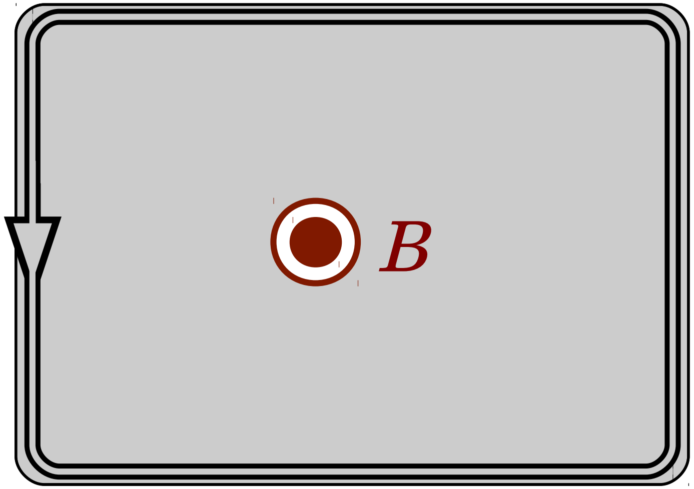
Composite fermions

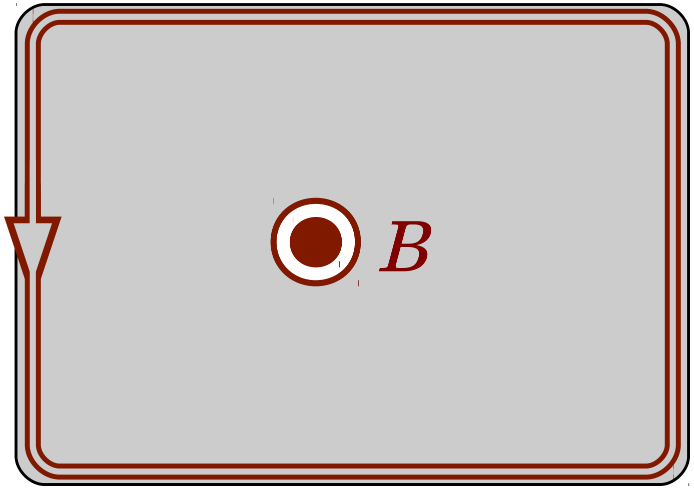


Fractional quantum Hall effect





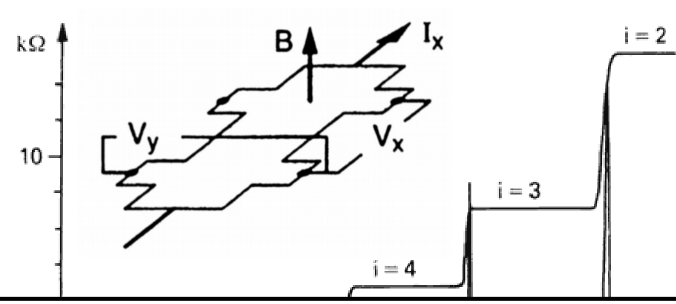


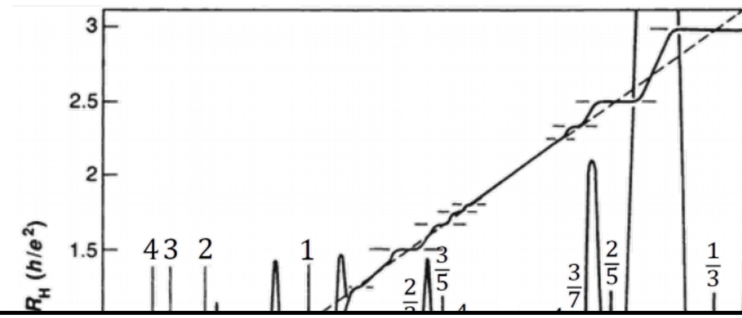


$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$

$$\sigma_{xy} = \frac{1}{2m+1} \frac{e^2}{h}$$

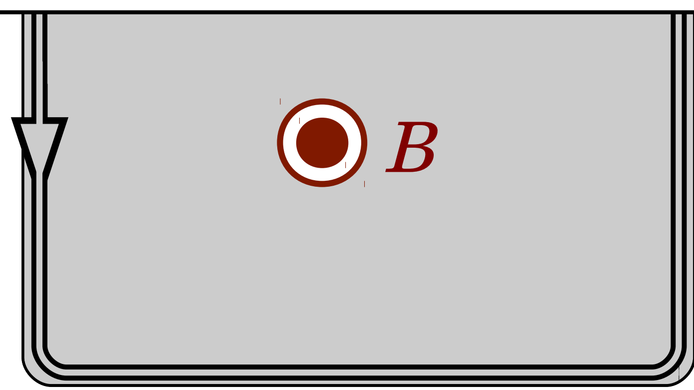
Fractional quantum Hall effect

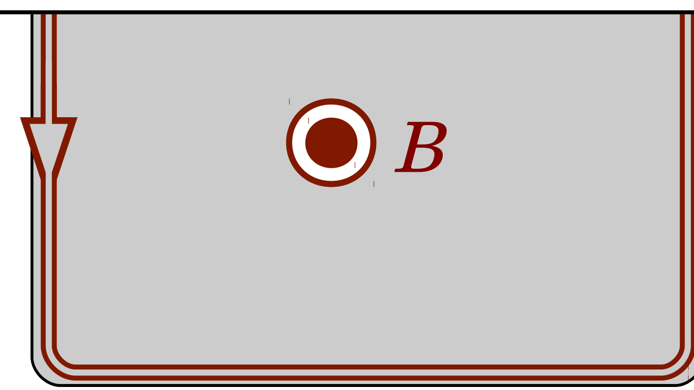




Any charge carrying edge state, fractional or integer, carries an integer thermal conductance κ_0

Theory: Kane and Fisher (1997)
Experiment: Banerjee *et al.* (2017)



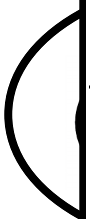


$$\sigma_{xy} = n_{\text{edge}} \frac{e^2}{h}$$
$$\kappa_{xy} = n_{\text{edge}} \kappa_0$$

$$\sigma_{xy} = \frac{1}{2m+1} \frac{e^2}{h}$$
$$\kappa_{xy} = \kappa_0$$

Topological superconductors

Pairing



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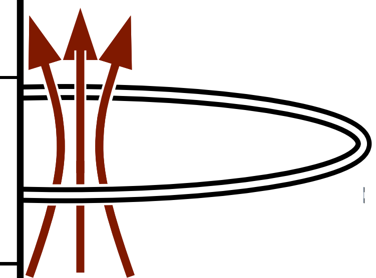
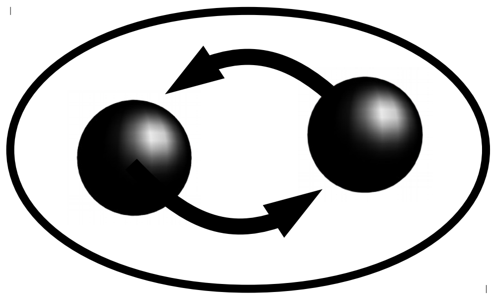
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

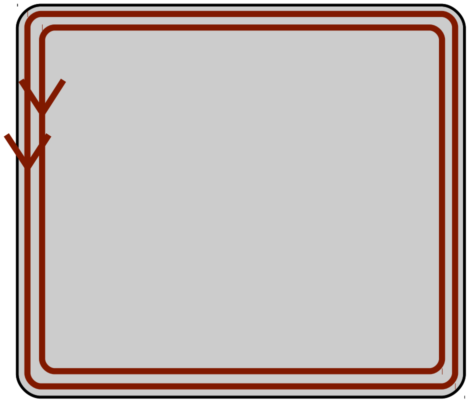
magnetic field



Topological superconductors

Pairing of spinless electrons

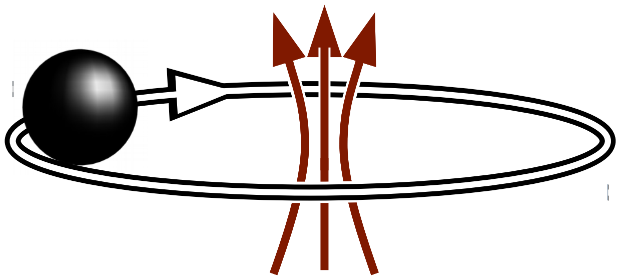


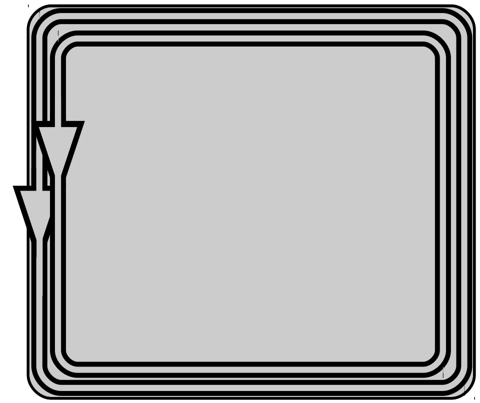


n_{Majorana} chiral Majoranas

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2}\kappa_0$$

Electrons in magnetic field





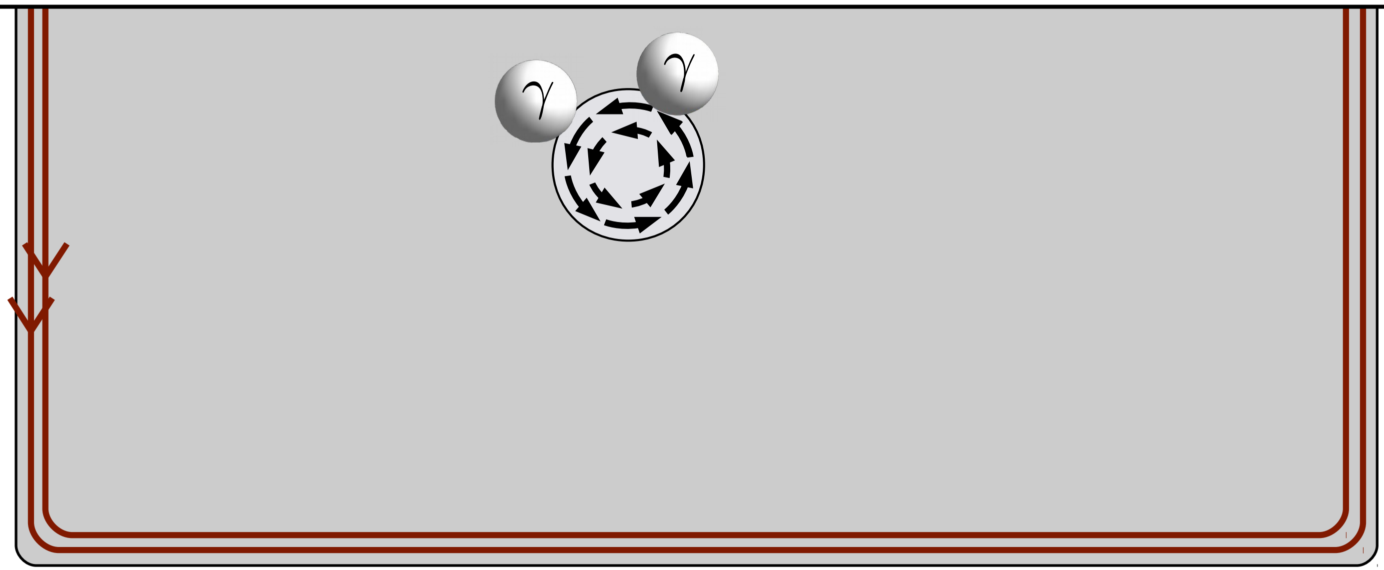
n_{edge} chiral electrons

$$\sigma_{xy} = n_{\text{edge}}\frac{e^2}{h}$$

$$\kappa_{xy} = n_{\text{edge}}\kappa_0$$

Topological superconductors

Half-odd integer κ_{xy} $\rightarrow$ Majorana zero modes

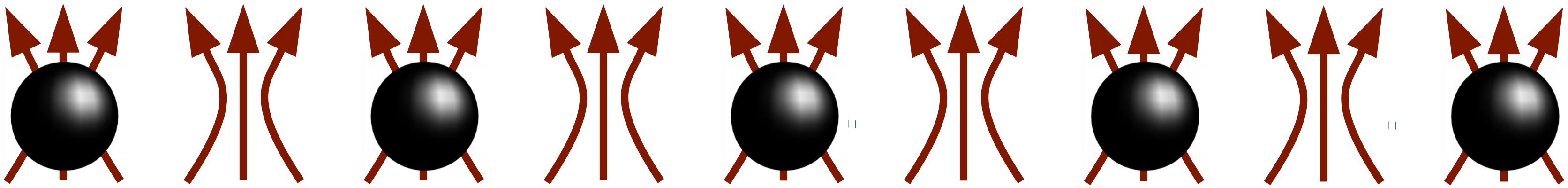


n_{Majorana} chiral Majoranas propagating at the edge (absolutely stable)

n_{Majorana} Majorana zero modes localized at a vortex (stable mod 2)

$$\kappa_{xy} = \frac{n_{\text{Majorana}}}{2} \kappa_0$$

Half-filled Landau level

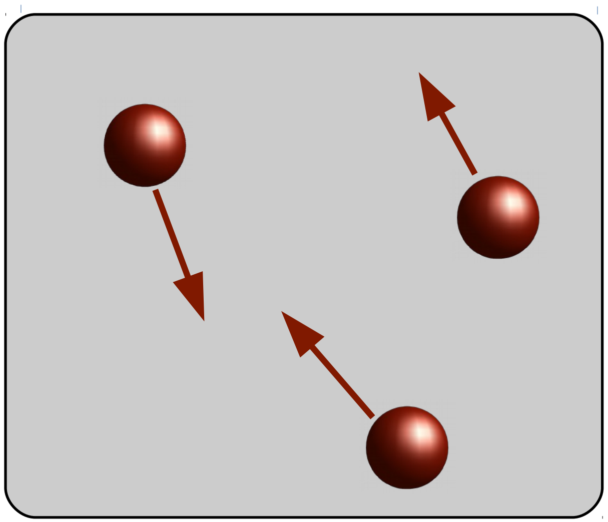


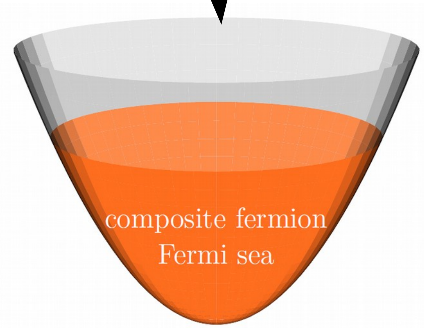
Electrons at $\nu = \frac{1}{2}$

Half-filled Landau level

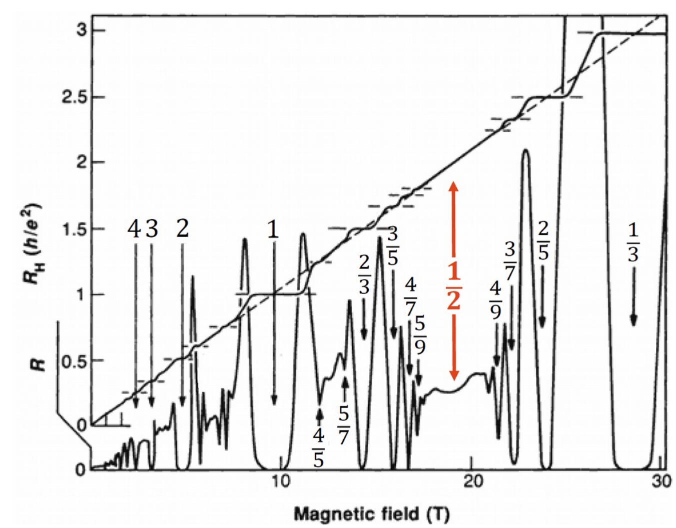
Recent developments: ‘Dirac composite fermions’

Son (2015); Wang, Senthil (2015); Metlitski, Vishwanath (2015); Kachru, Mulligan, Torroba, Wang (2015); **DFM**, Alicea, Motrunich (2015); Karch, Tong (2016), Seiberg, Senthil, Wang, Witten (2016); (and many more)



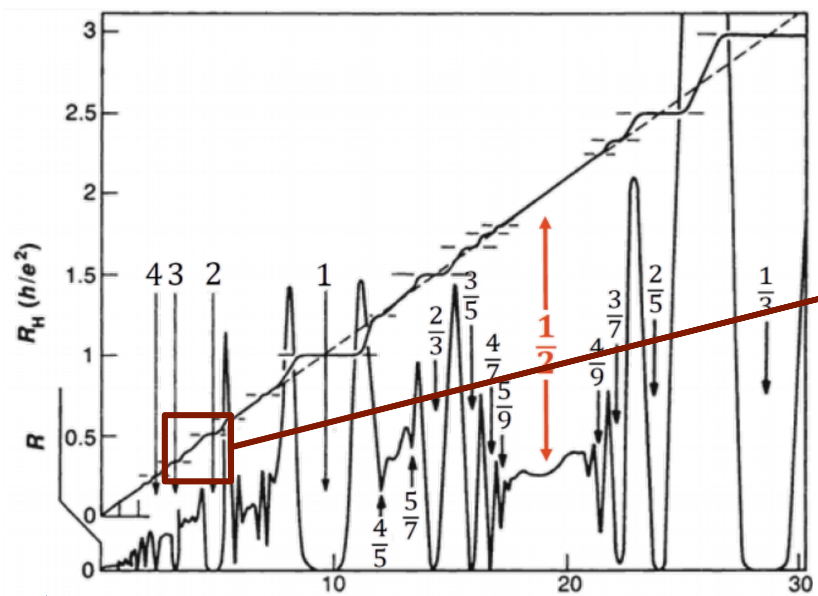


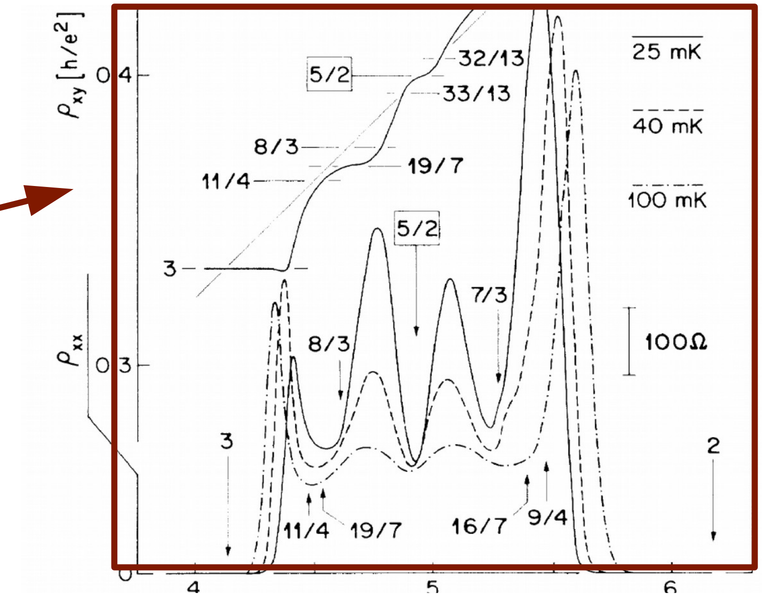
Halperin, Lee, and Read (1993)



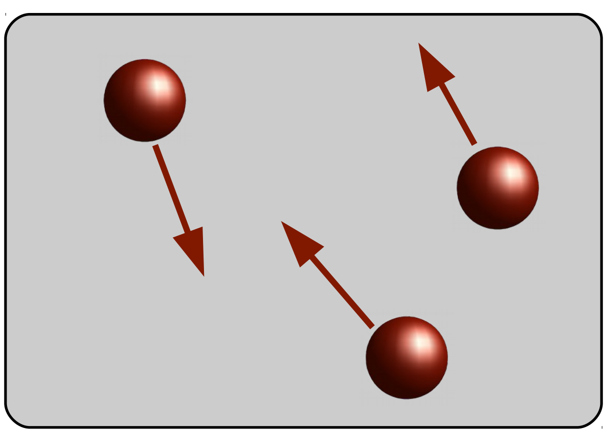
Composite fermions in zero magnetic field move in straight lines ⟶ form Fermi surface ⟶ metallic state at $\nu = \frac{1}{2}$

Half-filled Landau level

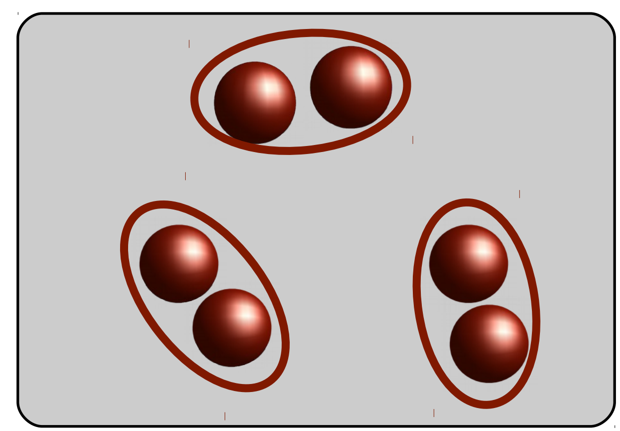




Willett *et al.* (1987)



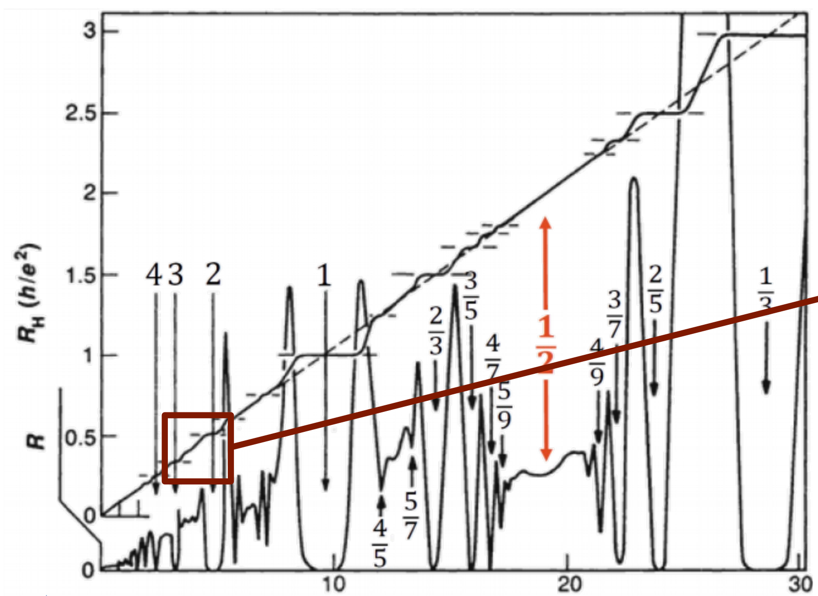
pairing

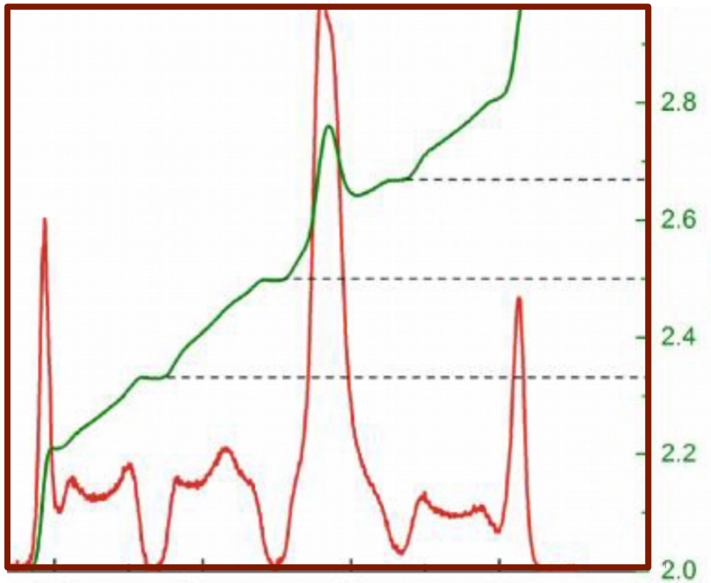


- Compressible state
- Hall conductance not quantized

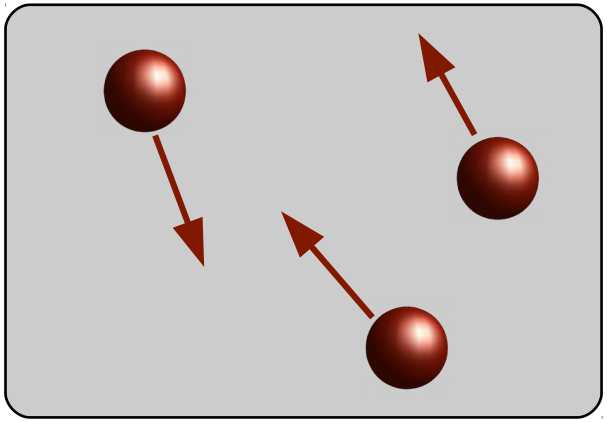
- Incompressible state
- Quantized Hall conductance

Half-filled Landau level

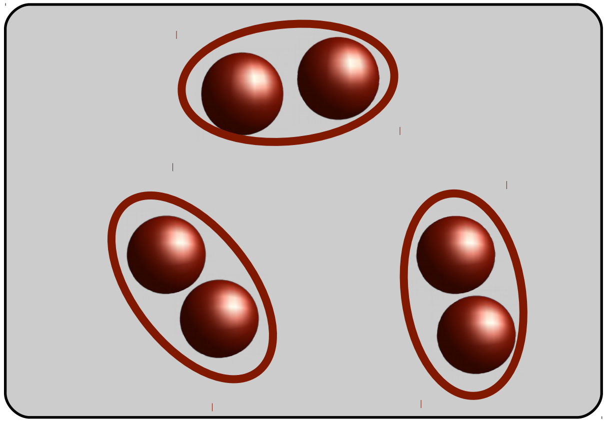




Banerjee *et al.* (2017)



pairing



- Compressible state
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- Incompressible state
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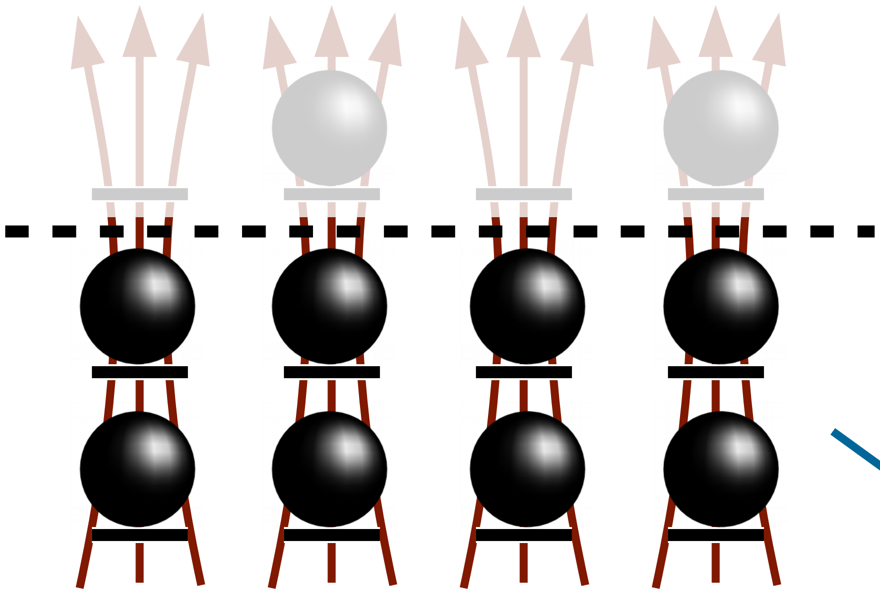
Electrons at ν=5/2



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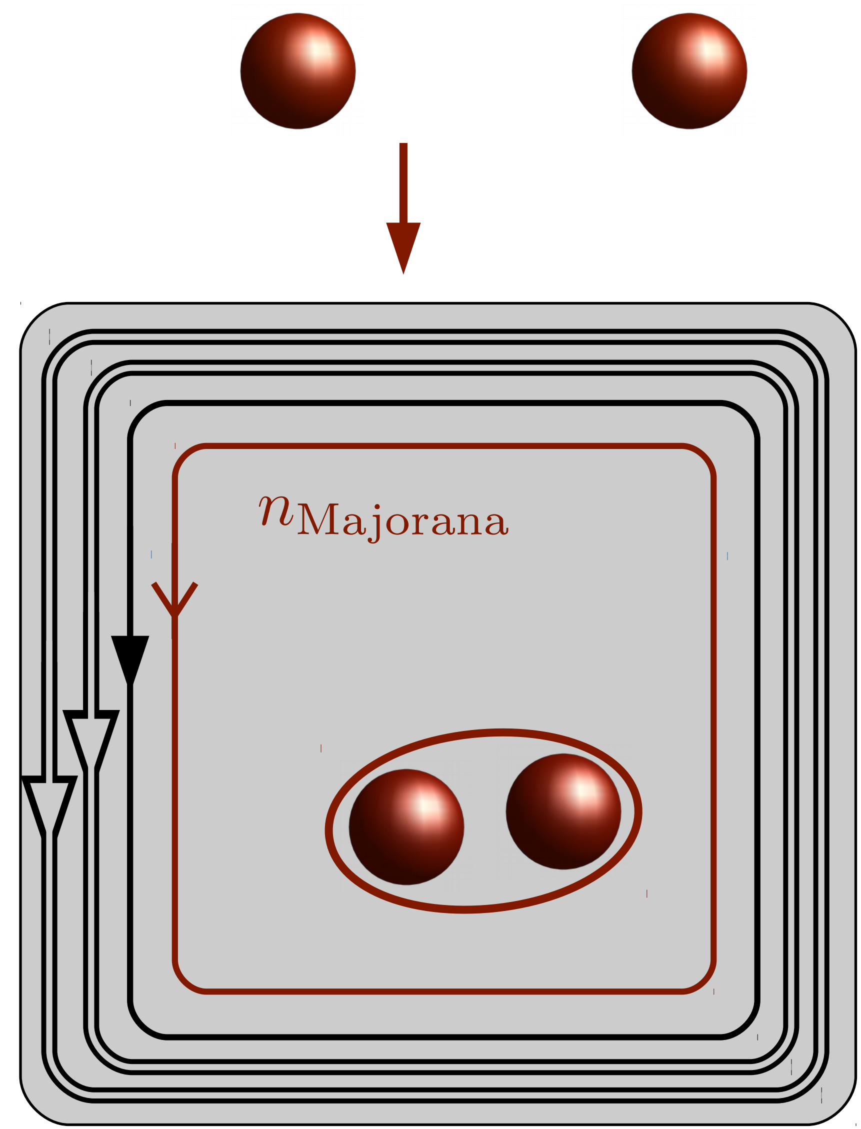


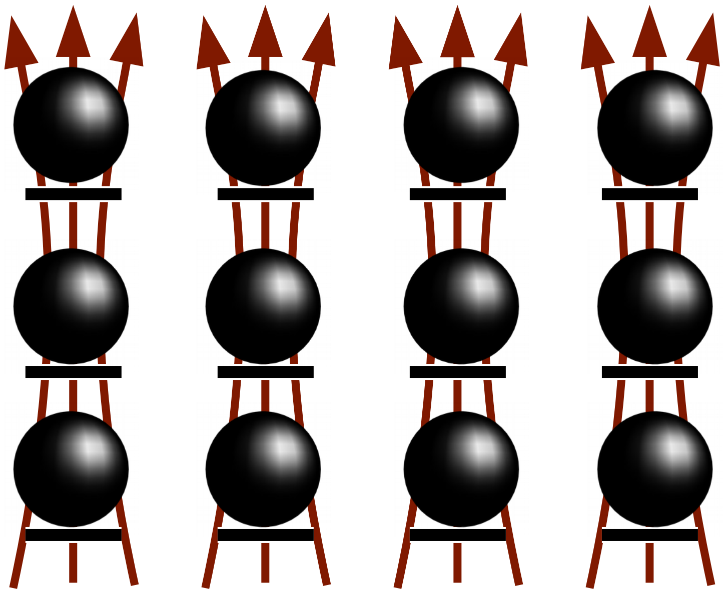
$$\sigma_{xy} = 2 * 1 + \frac{1}{2} = \frac{5}{2}$$

$$\kappa_{xy} = 2 * \kappa_0 + \kappa_0 + n_{\text{Majorana}} \frac{\kappa_0}{2}$$

$$= \left(3 + \frac{n_{\text{Majorana}}}{2}\right) \kappa_0$$

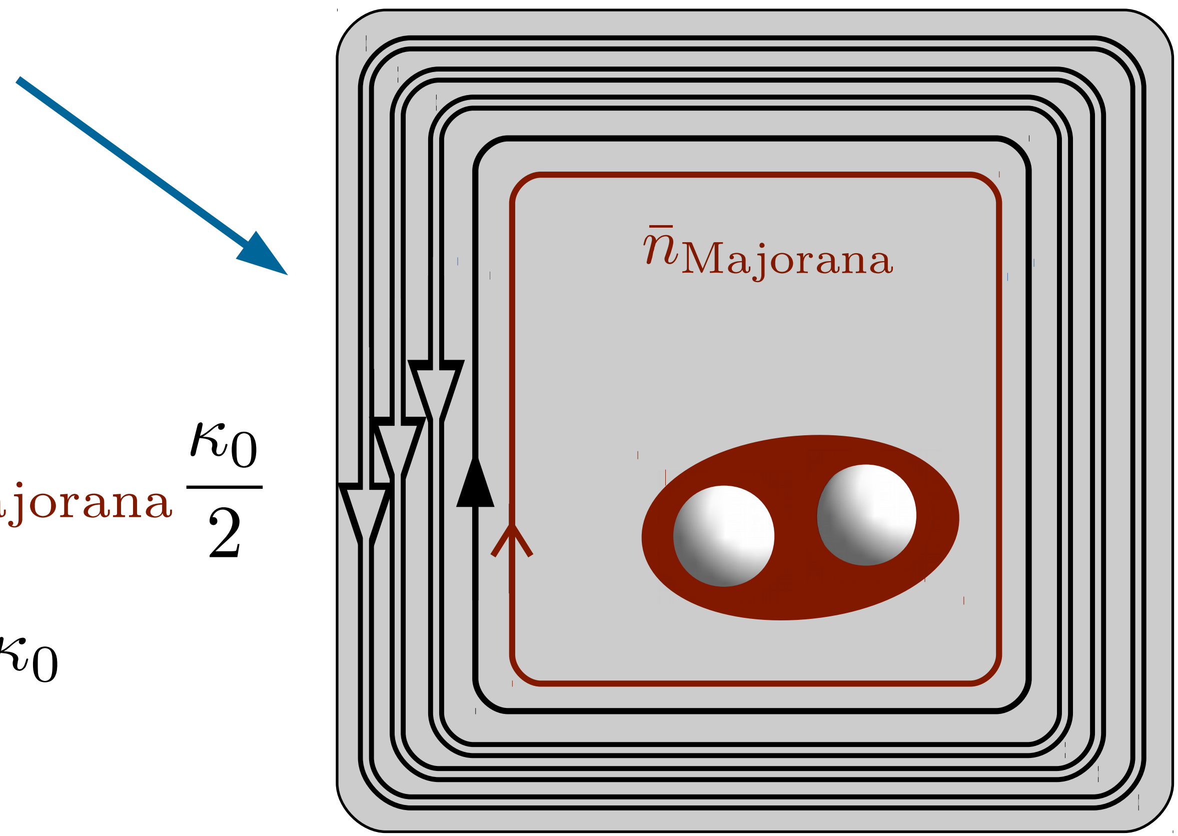
Many possible phases!

Particle-hole symmetry at ν=5/2



+

composite holes



$$\sigma_{xy} = 3 * 1 - \frac{1}{2} = \frac{5}{2}$$

$$\kappa_{xy} = 3 * \kappa_0 - \kappa_0 - \bar{n}_{\text{Majorana}} \frac{\kappa_0}{2}$$

$$= \left(2 - \frac{\bar{n}_{\text{Majorana}}}{2}\right) \kappa_0$$

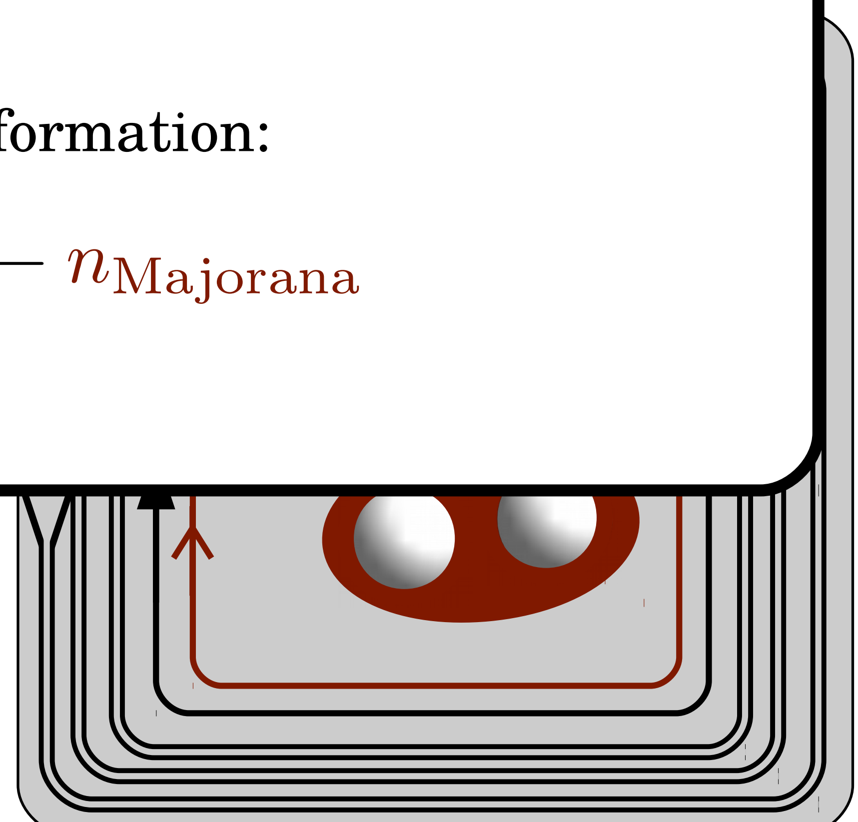
Particle-hole symmetry at ν=5/2

$$\left(3 + \frac{n_{\text{Majorana}}}{2}\right)\kappa_0 = \left(2 - \frac{\bar{n}_{\text{Majorana}}}{2}\right)\kappa_0$$

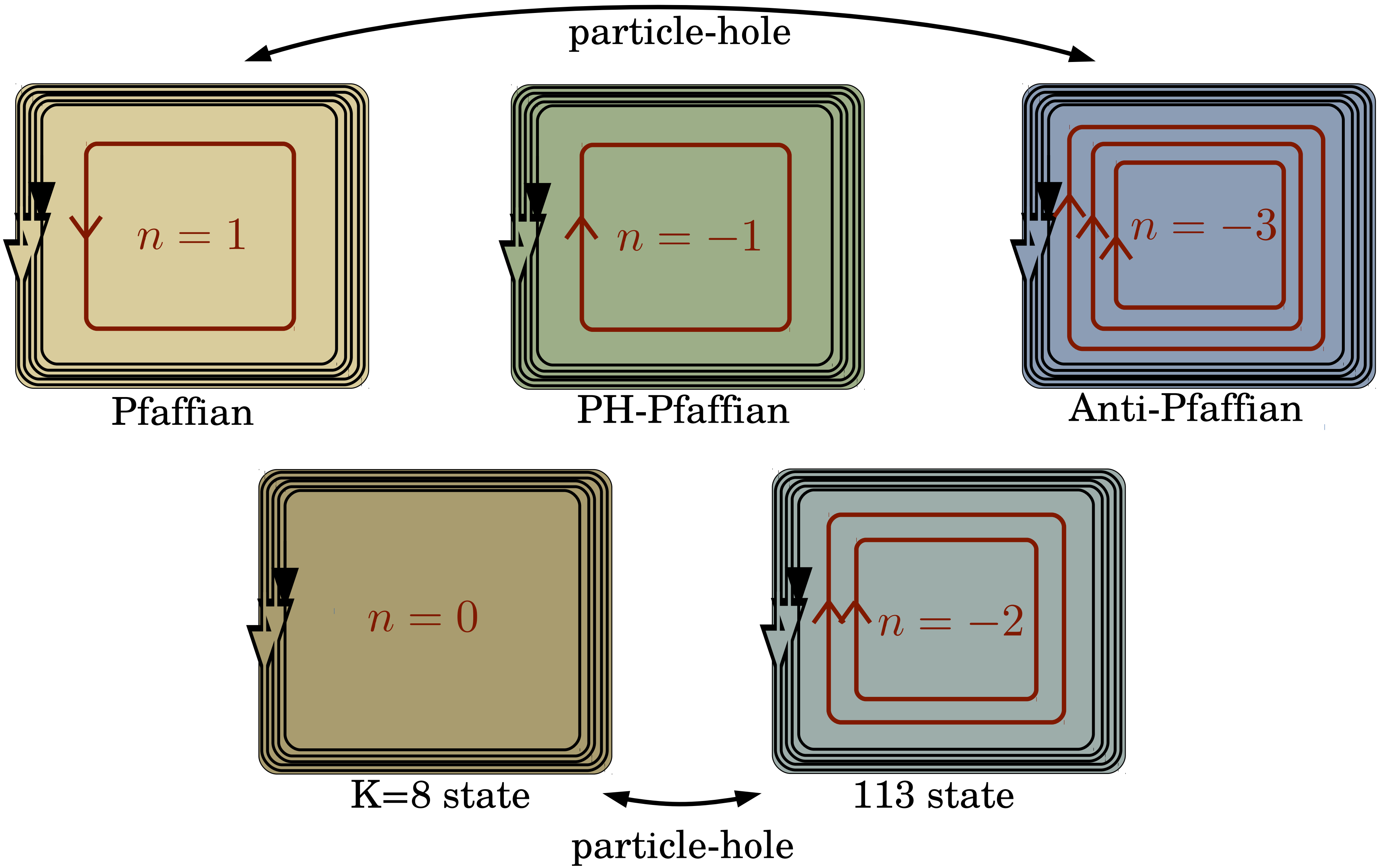
Particle-hole transformation:

$$P : n_{\text{Majorana}} \rightarrow -2 - n_{\text{Majorana}}$$

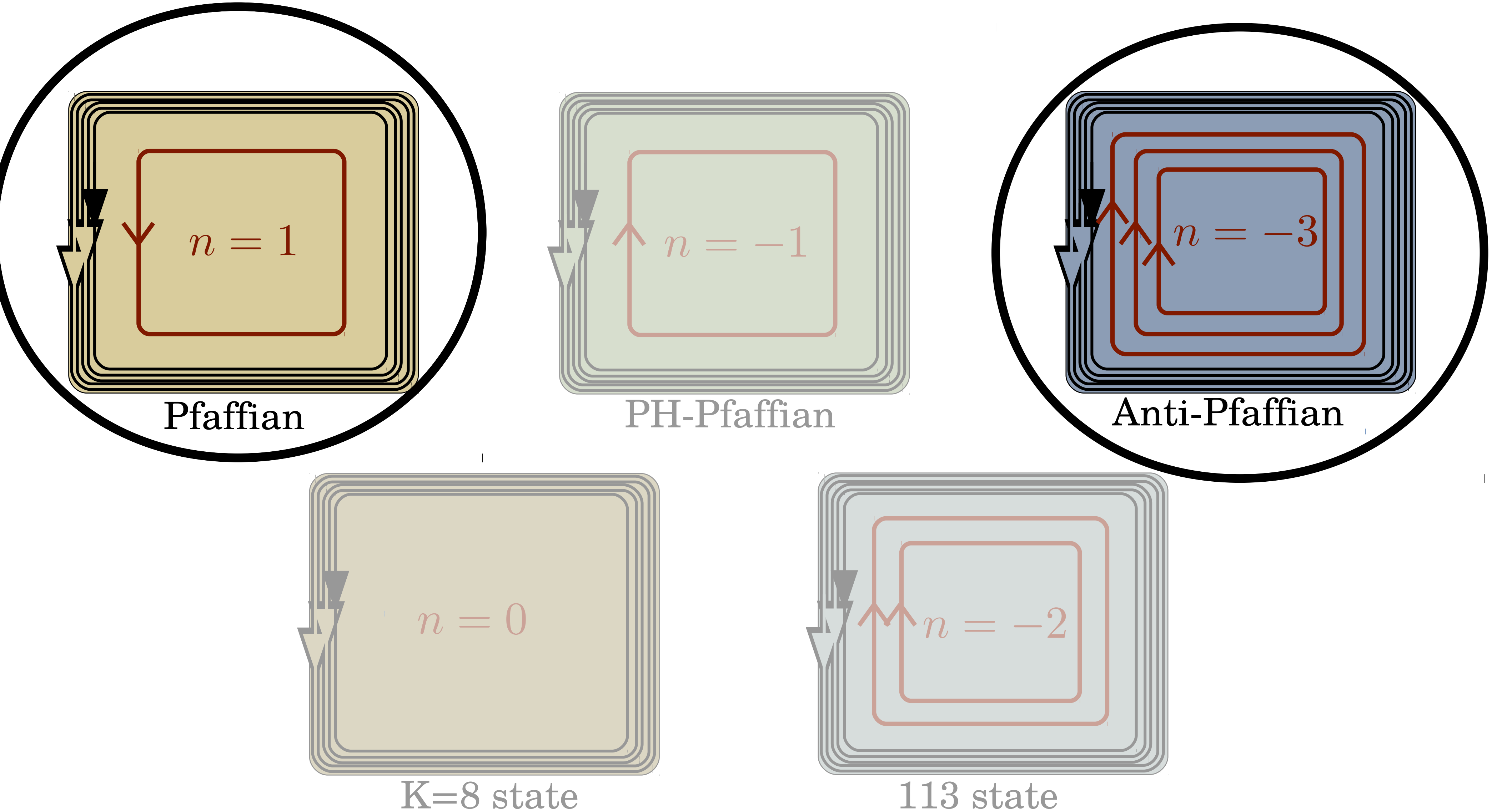
$$= \left(2 - \frac{\bar{n}_{\text{Majorana}}}{2}\right)\kappa_0$$



Electrons at ν=5/2

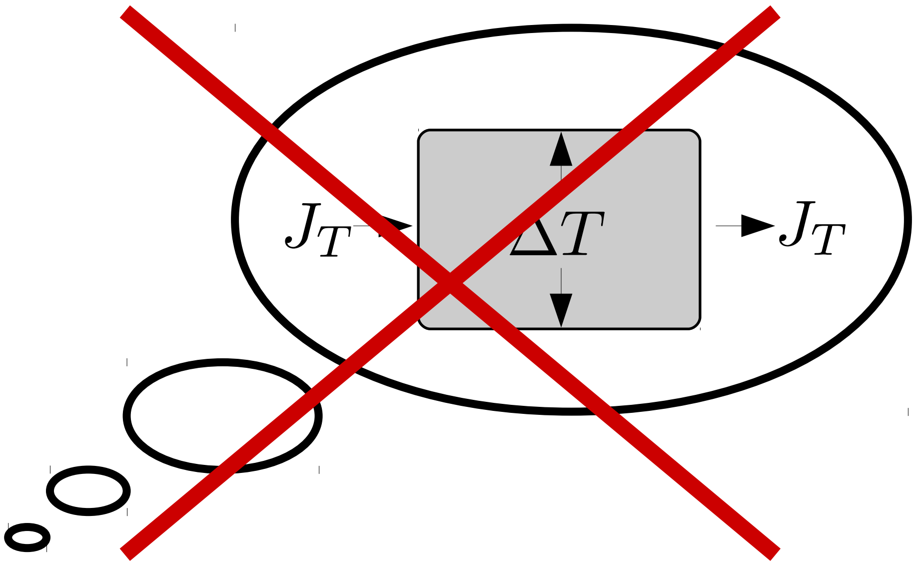


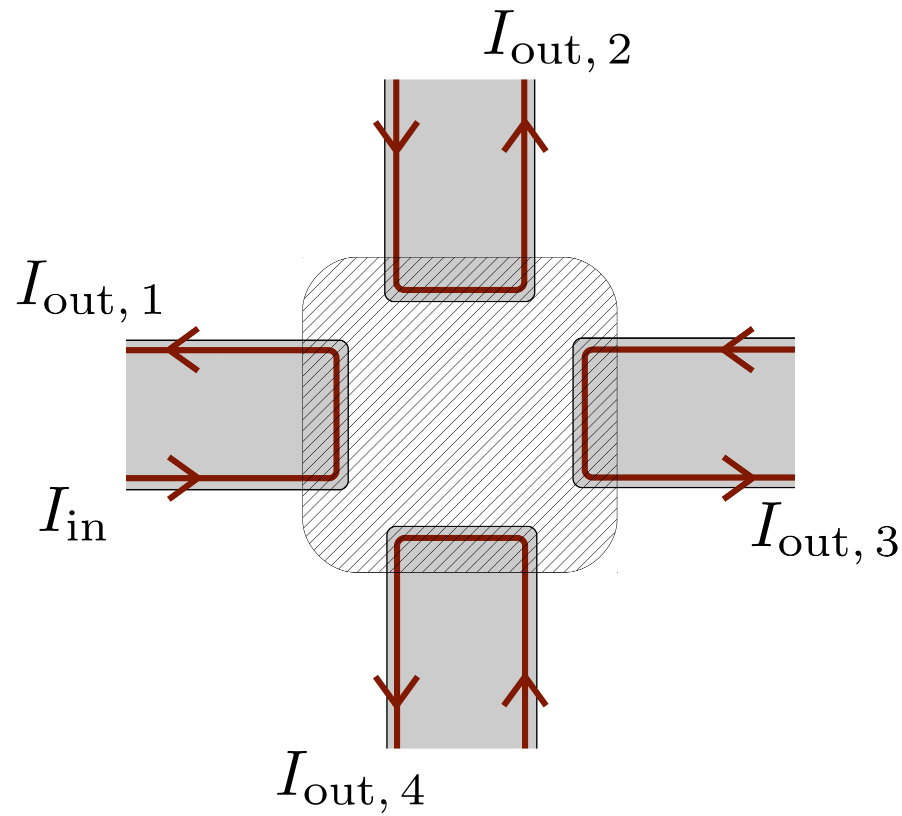
Electrons at ν=5/2



Numerics: $n_{\text{Majorana}} = 1$ or -3 Morf (1998), Rezayi and Haldane (2000)

Thermal Hall conductance at $\nu=5/2$





Thermal Hall conductance at $\nu=5/2$

Charge conservation

$$I_{\text{in}} = \sum_i I_{\text{out},\,i}$$

Joule's Law

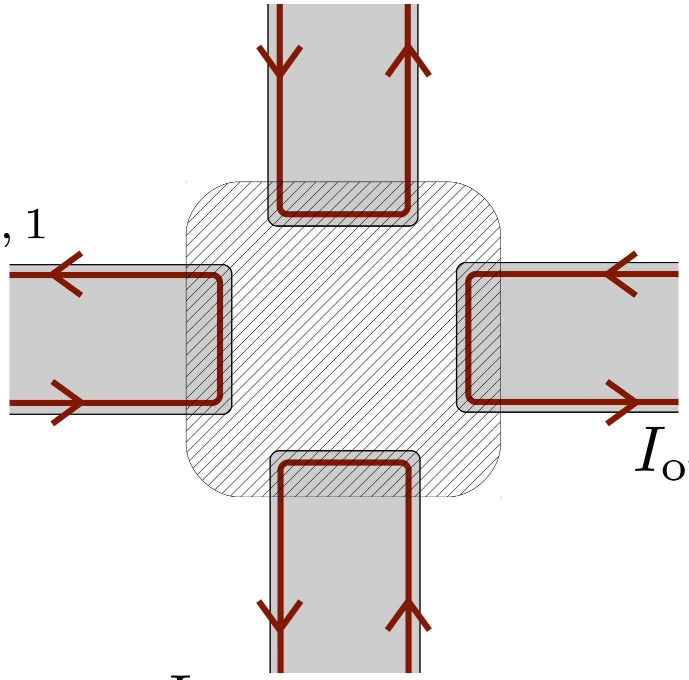
$$P_i = I_i^2/2G$$

Power balance

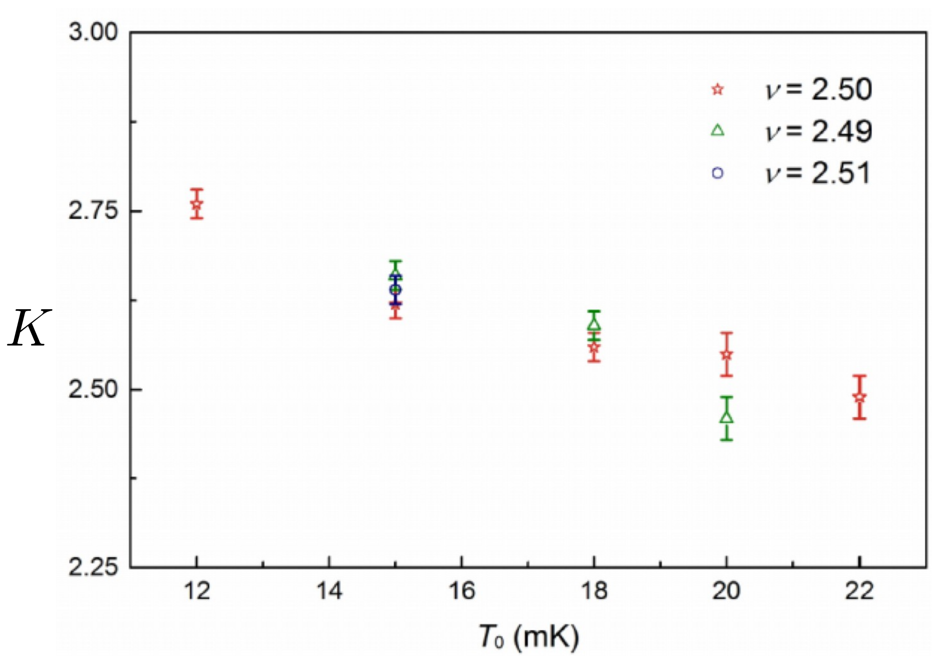
$$\Delta P = P_{\text{in}} - \sum_i P_{\text{out},\,i} = P_{\text{in}}(1 - N^{-1})$$

Heat flow out of metal

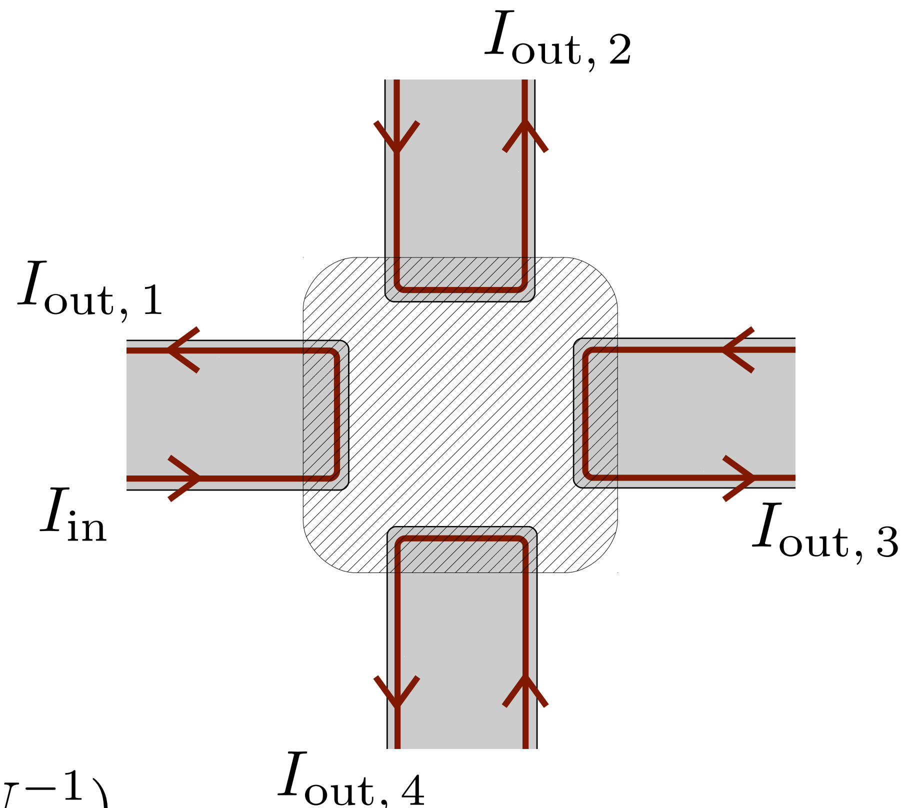
$$\Delta P = \frac{1}{2} K N T_{\text{metal}}^2$$



Thermal Hall conductance at $\nu=5/2$



Banerjee *et al.* (2017)

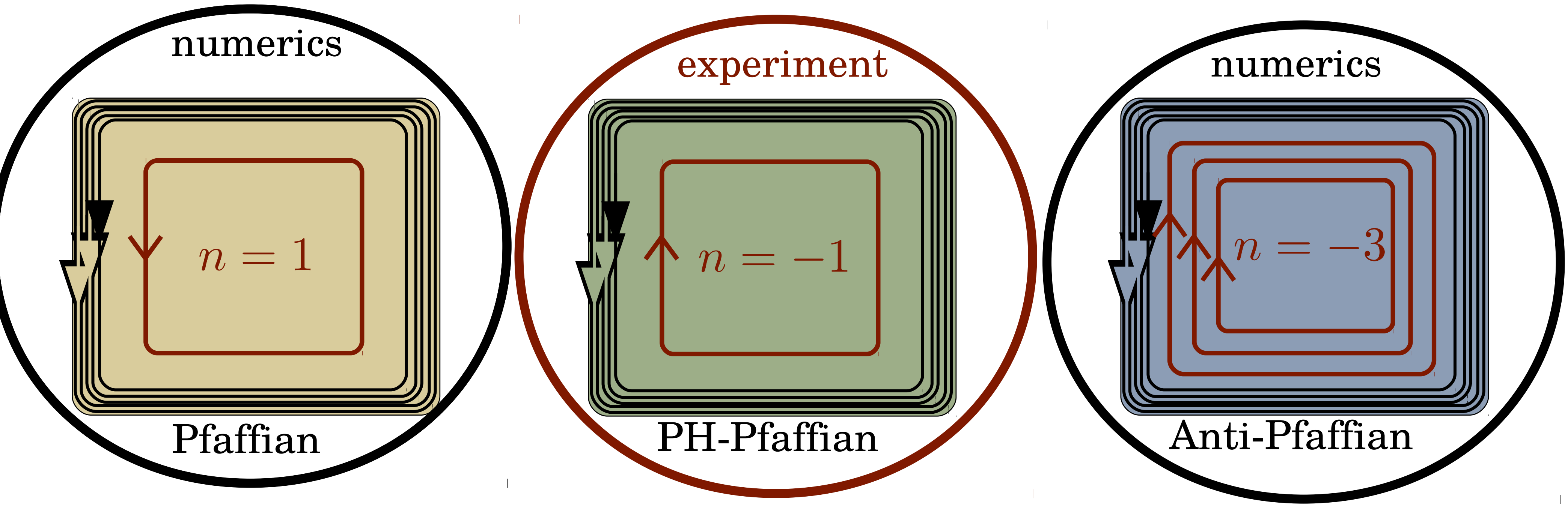


$N^{-1})$

Heat flow out of metal

$$\Delta P = \frac{1}{2} K N T_{\text{metal}}^2 \xrightarrow[\text{measure } T_{\text{metal}}]{} \text{find } K = \frac{5}{2} \quad \left(\kappa_{xy} = \frac{5}{2}\kappa_0\right)$$

Electrons at ν=5/2



Possible resolutions:

'numerics are wrong'

- Incorrect Hamiltonian
- Finite size not representative

'experiment is wrong'

- Alternative interpretation possible?

  Simon (2018), Feldman (2018)

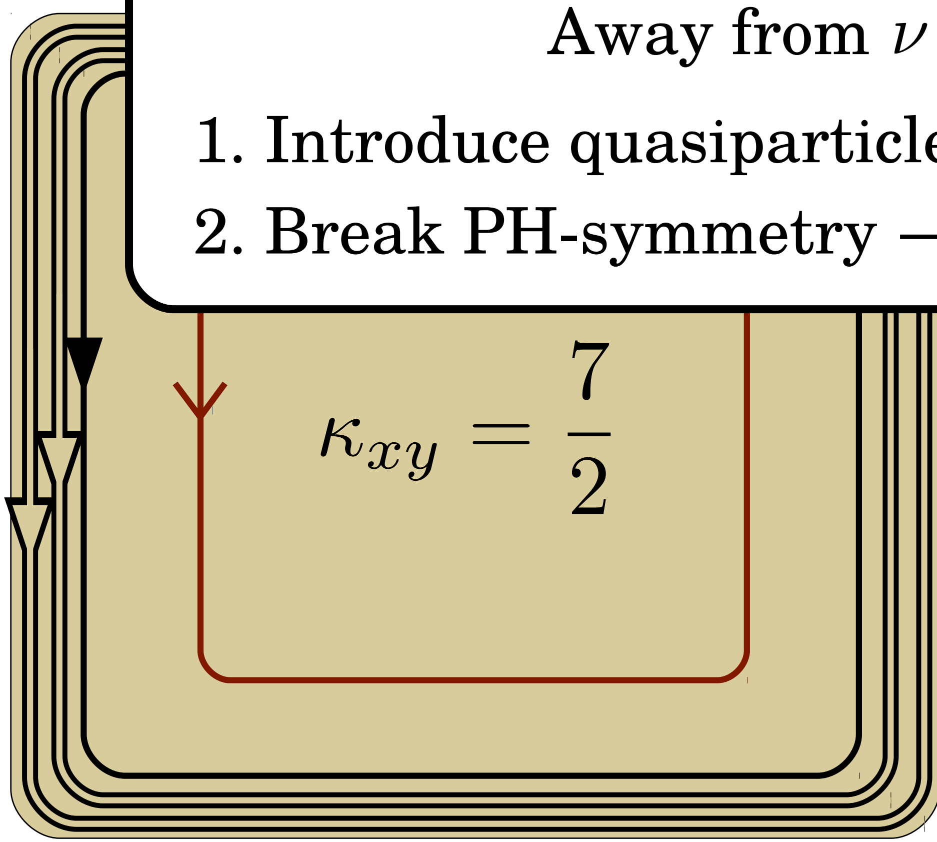
Can both be right?

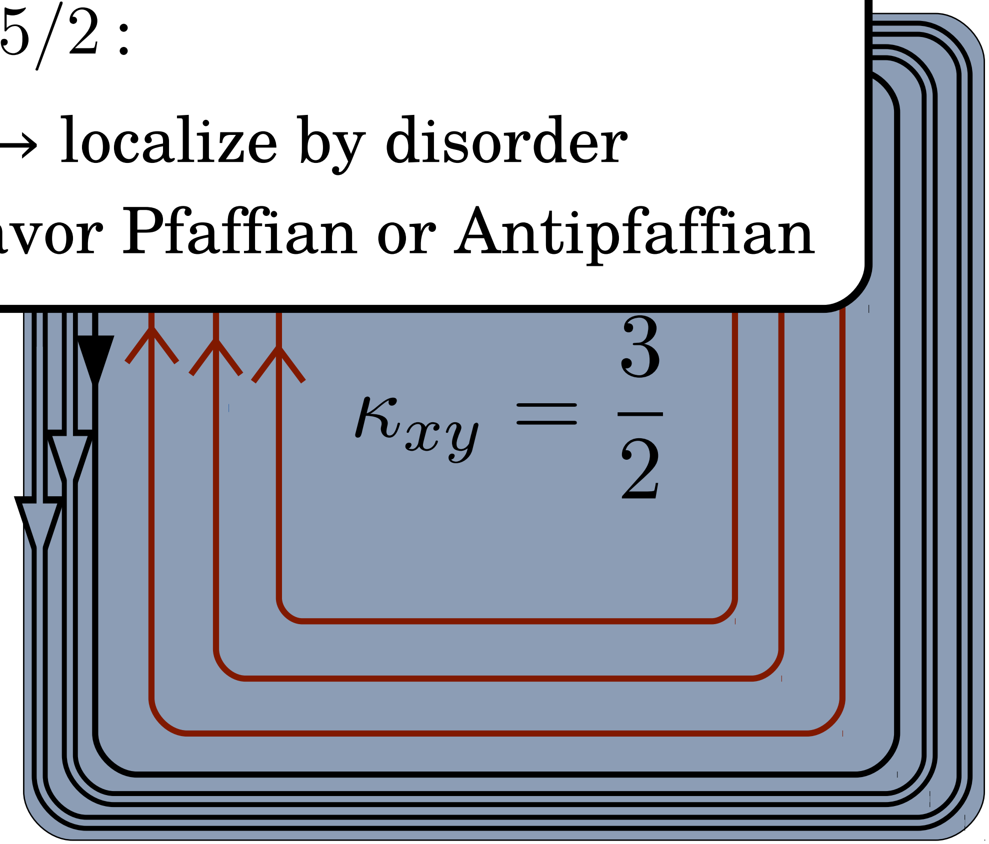
Electrons at ν=5/2

Numerics: In **clean** system, Pfaffian or Antipfaffian

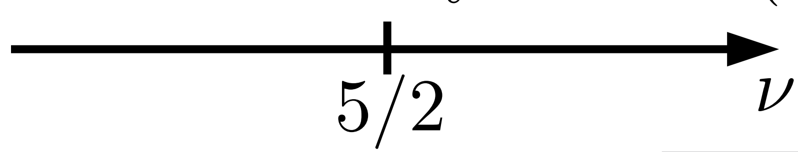
Away from $\nu = 5/2$:

1. Introduce quasiparticles → localize by disorder
2. Break PH-symmetry → favor Pfaffian or Antipfaffian



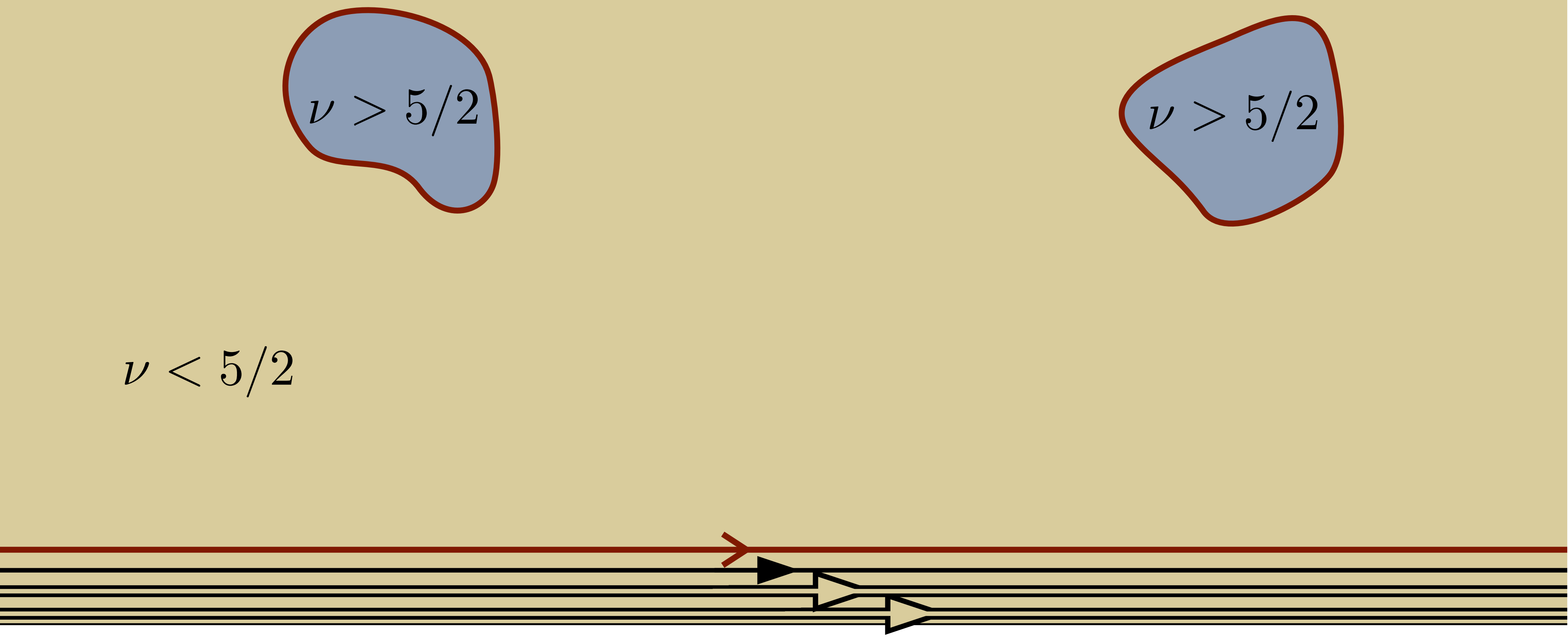


degenerate when PH-symmetric ($\nu \approx 5/2$)

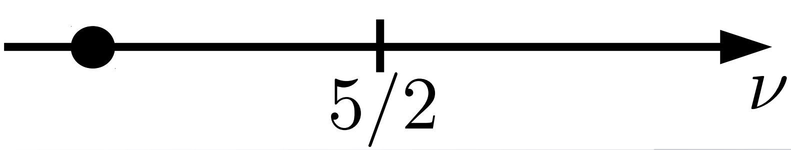


Electrons at ν=5/2

With disorder: Regions of Pfaffian and Antipfaffian

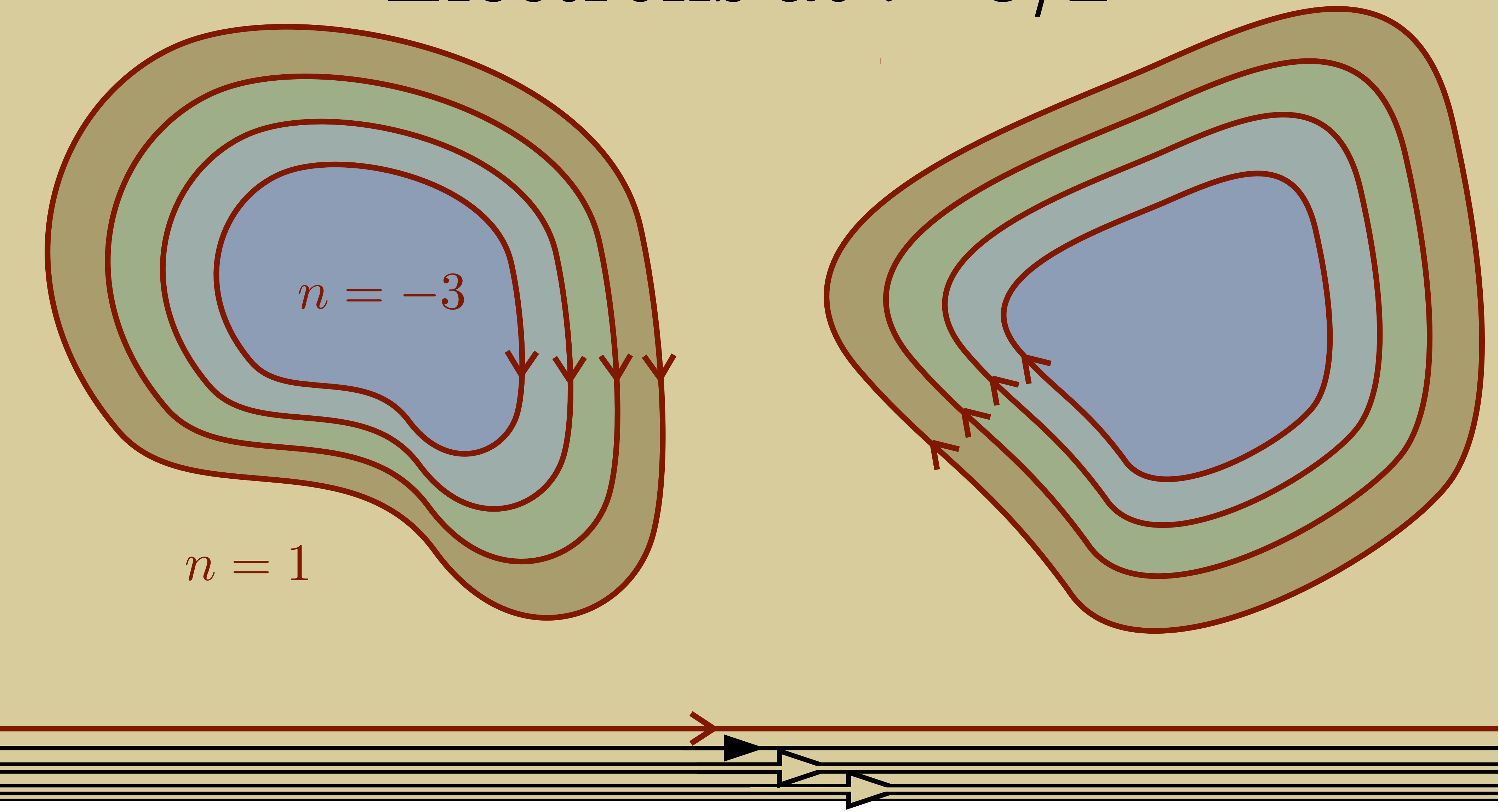


$\kappa_{xy} = \frac{7}{2}$

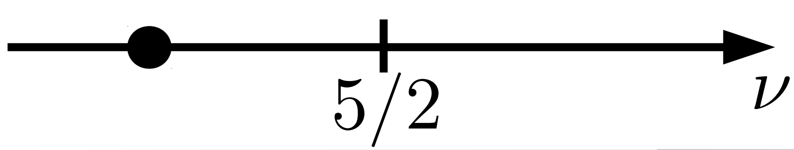


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

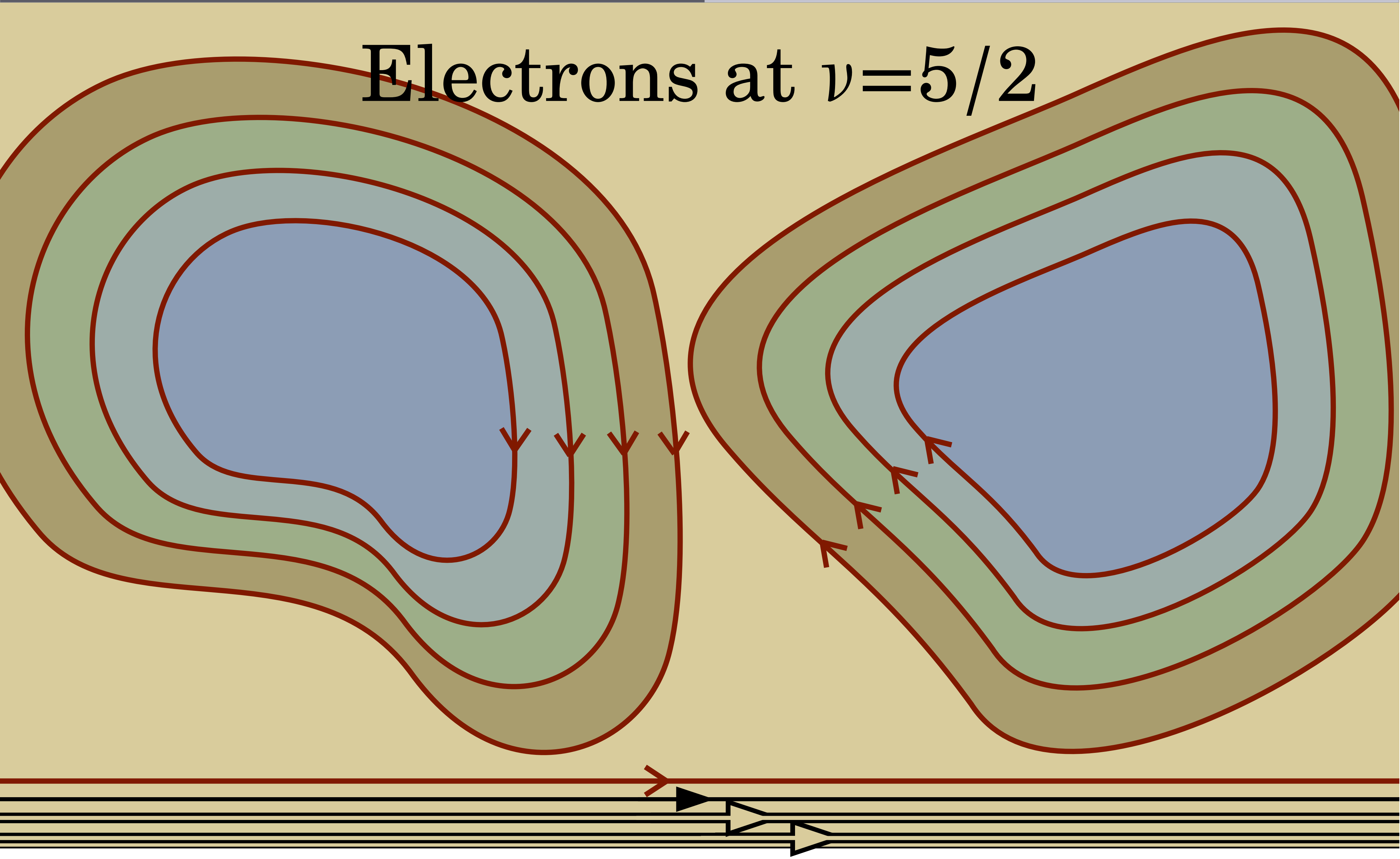


$\kappa_{xy} = \frac{7}{2}$

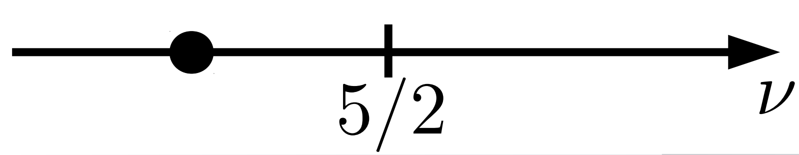


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

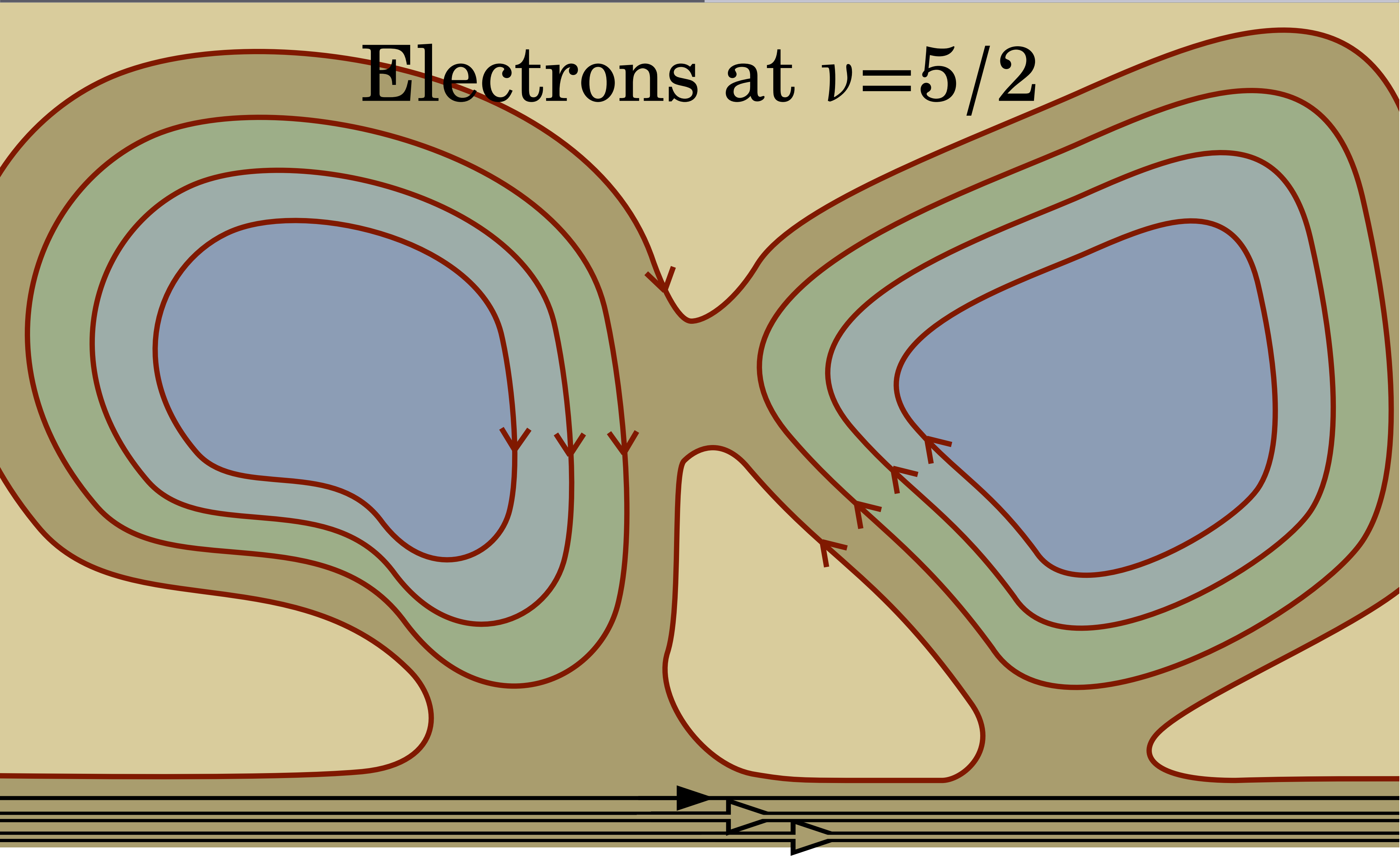


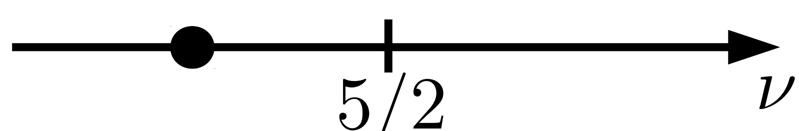
$\kappa_{xy} = \frac{7}{2}$



$\sigma_{xy} = \frac{5}{2}$

Electrons at $\nu=5/2$

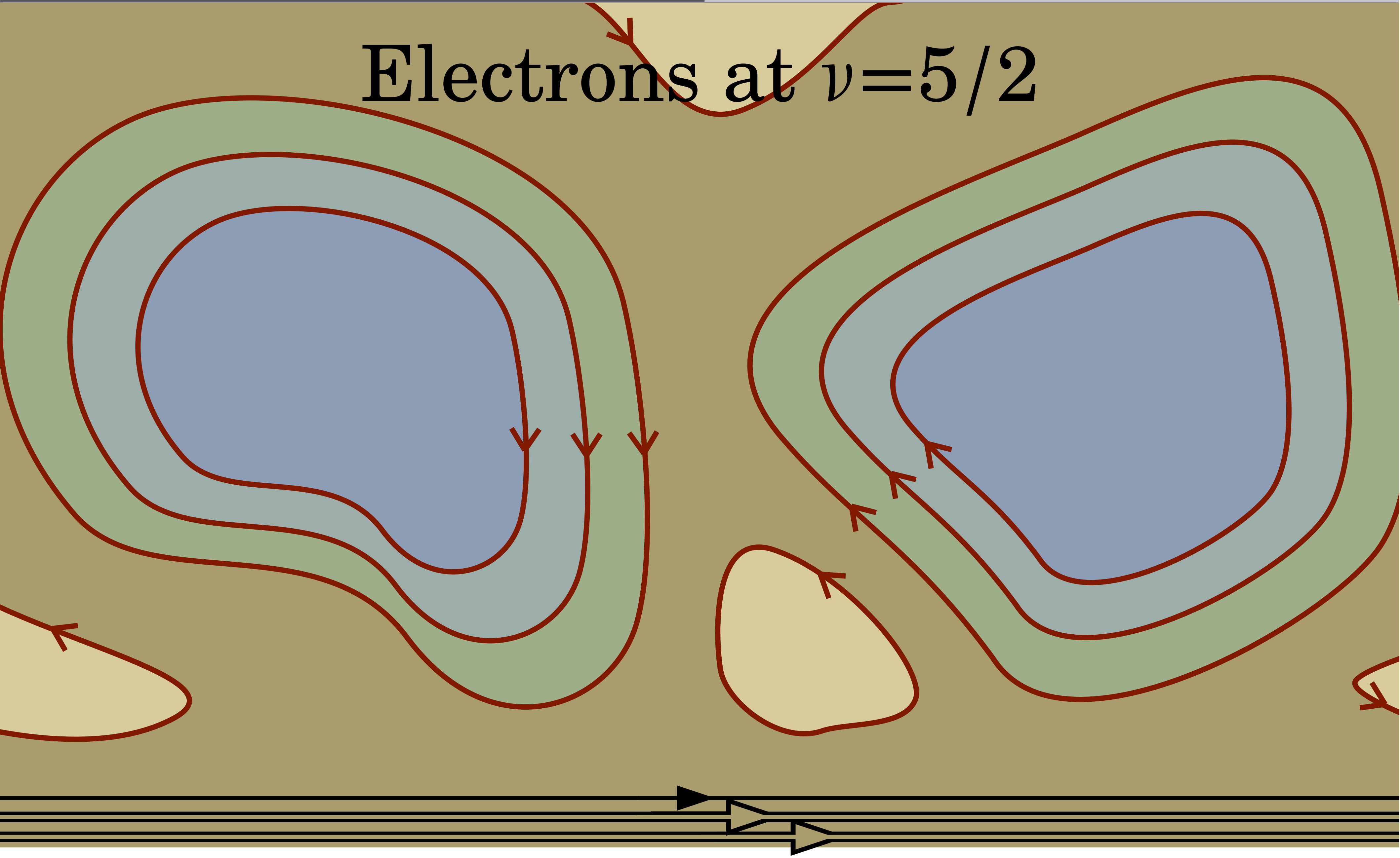




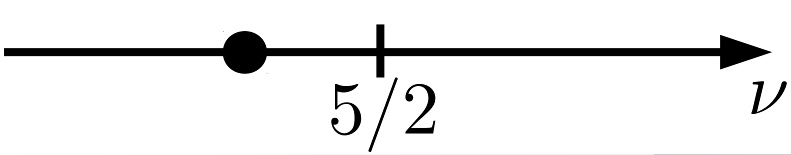
κ_{xy}
not quantized

$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

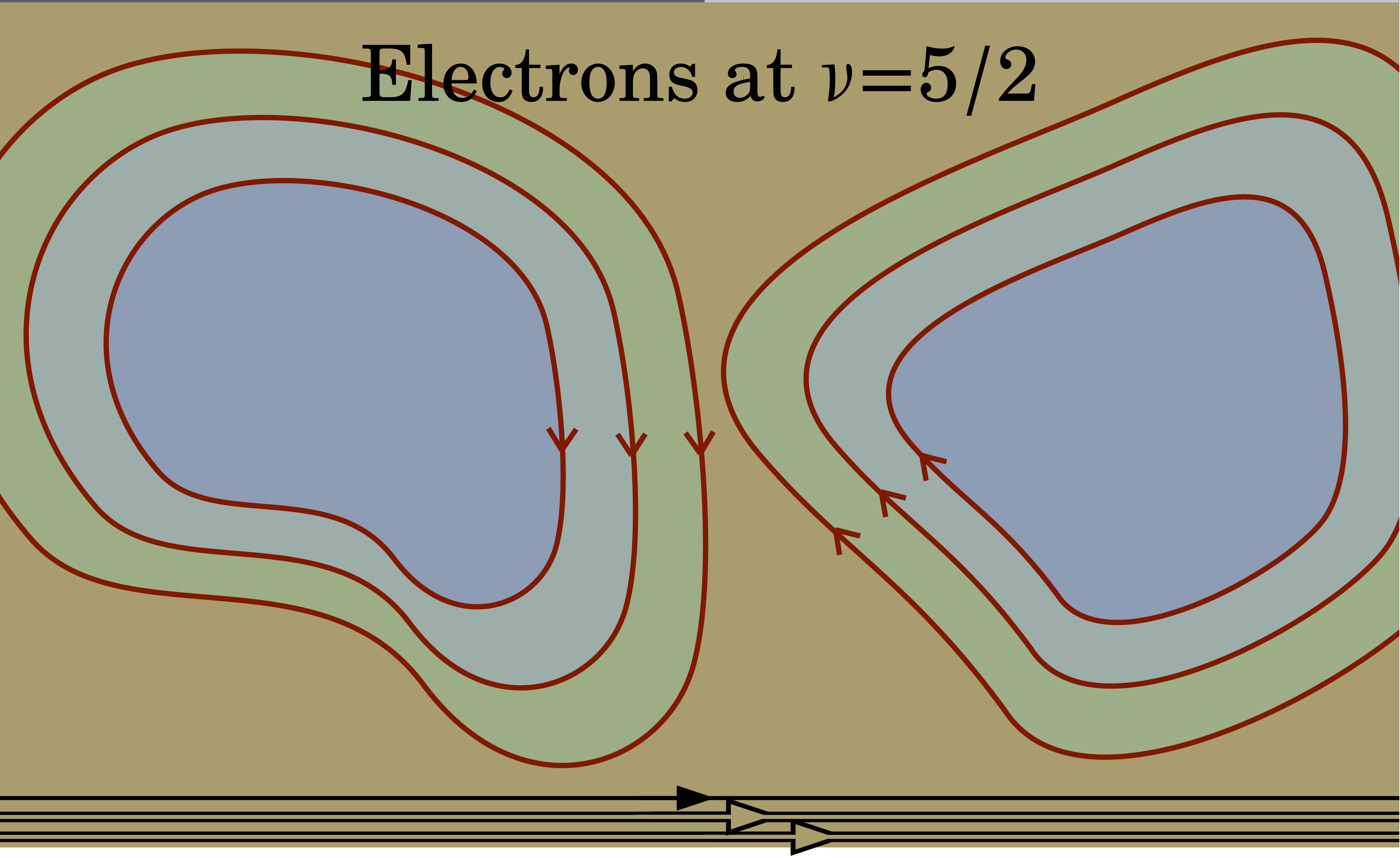


$\kappa_{xy} = 3$

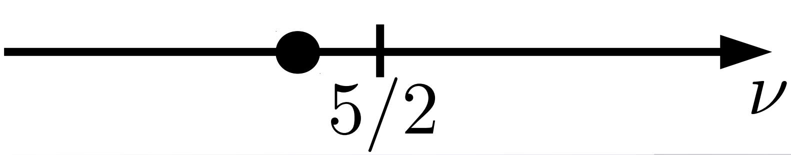


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

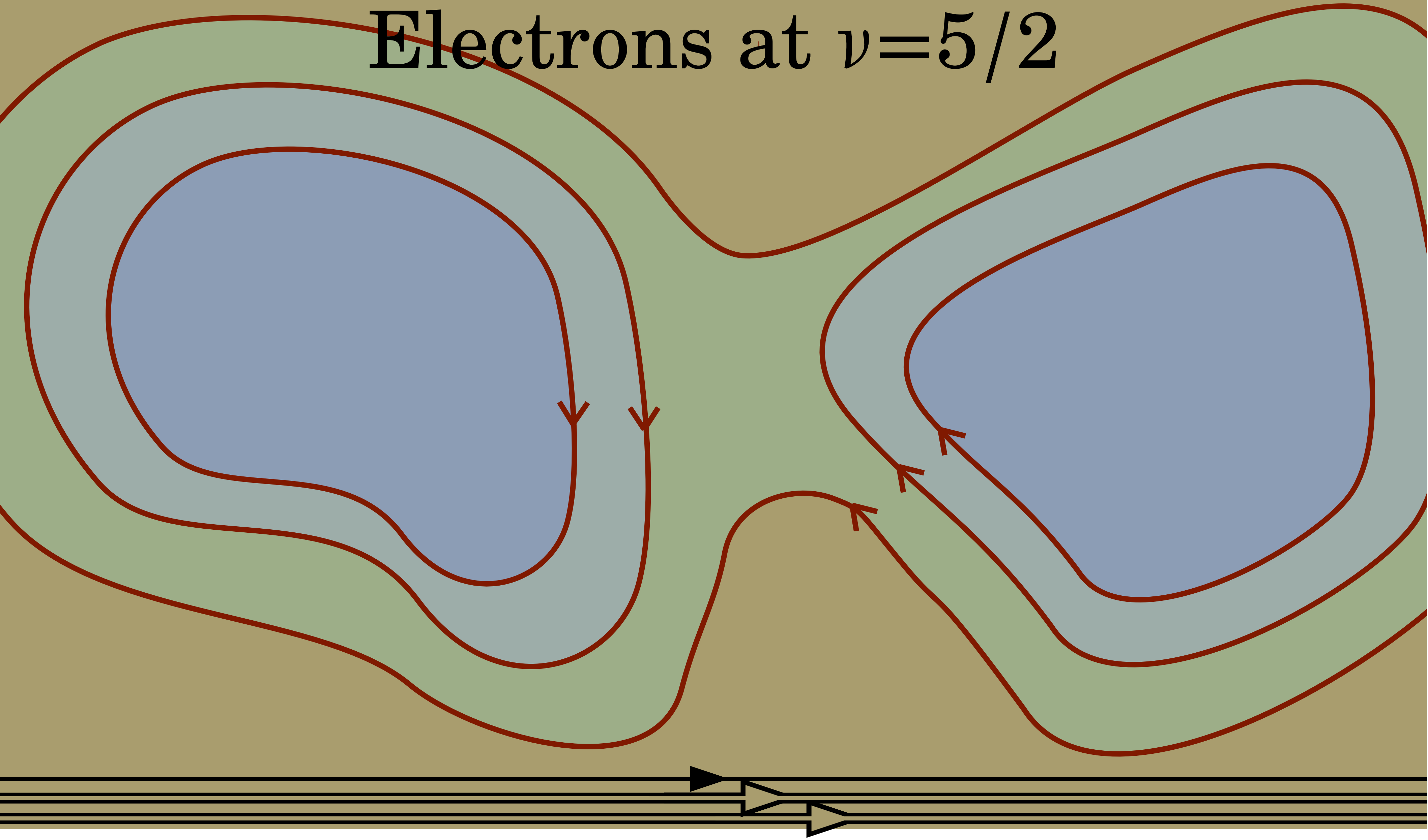


$\kappa_{xy} = 3$

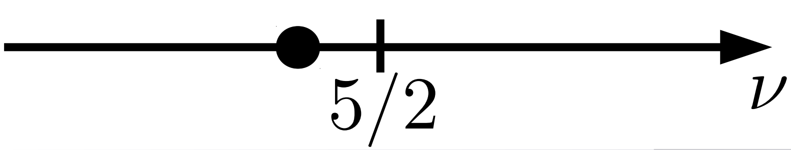


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

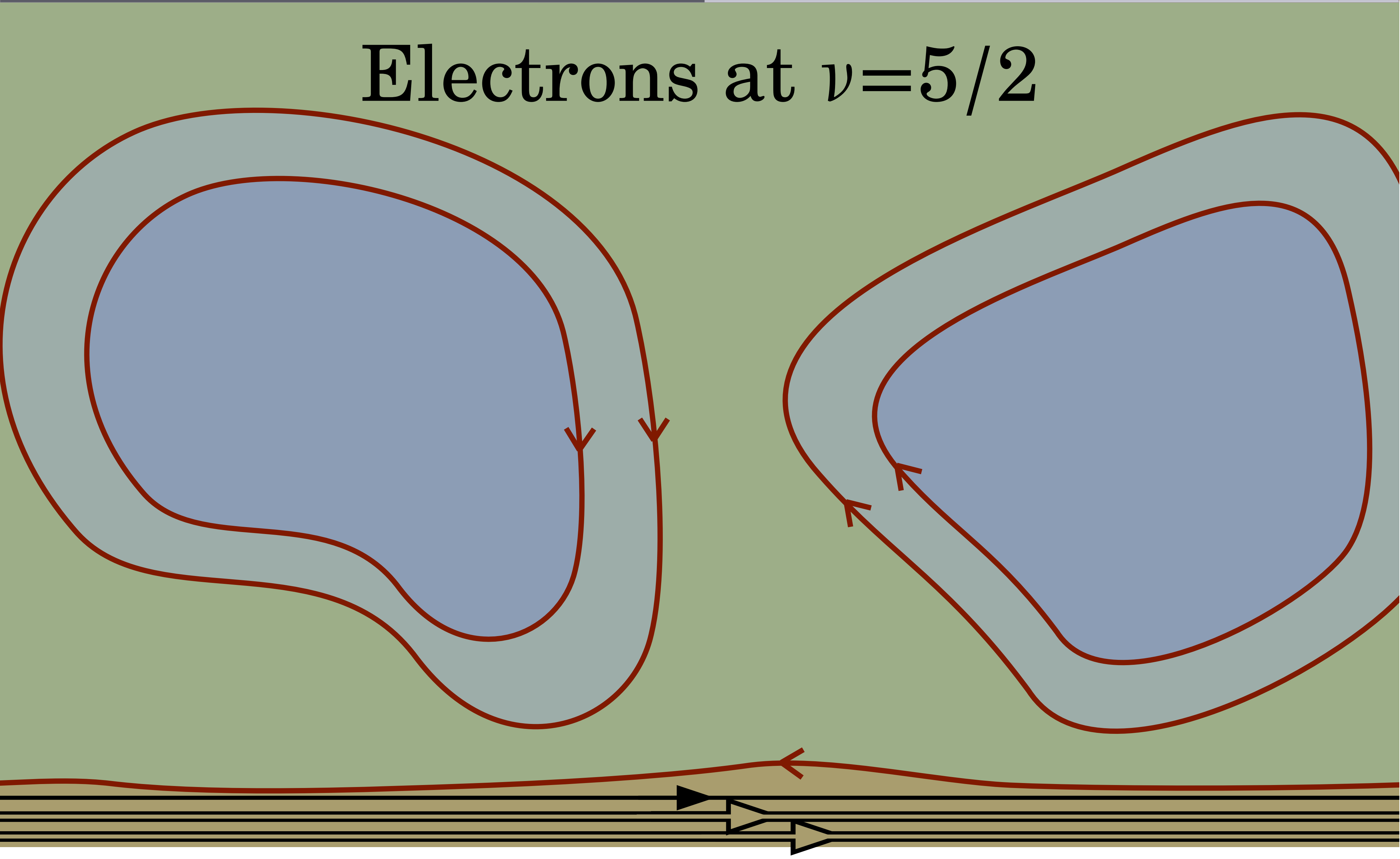


$\kappa_{xy} = 3$

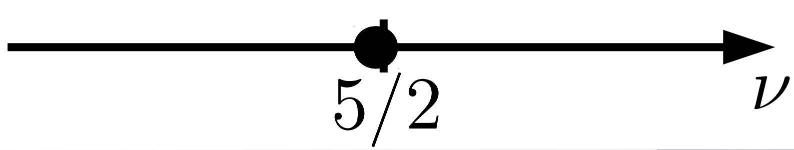


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

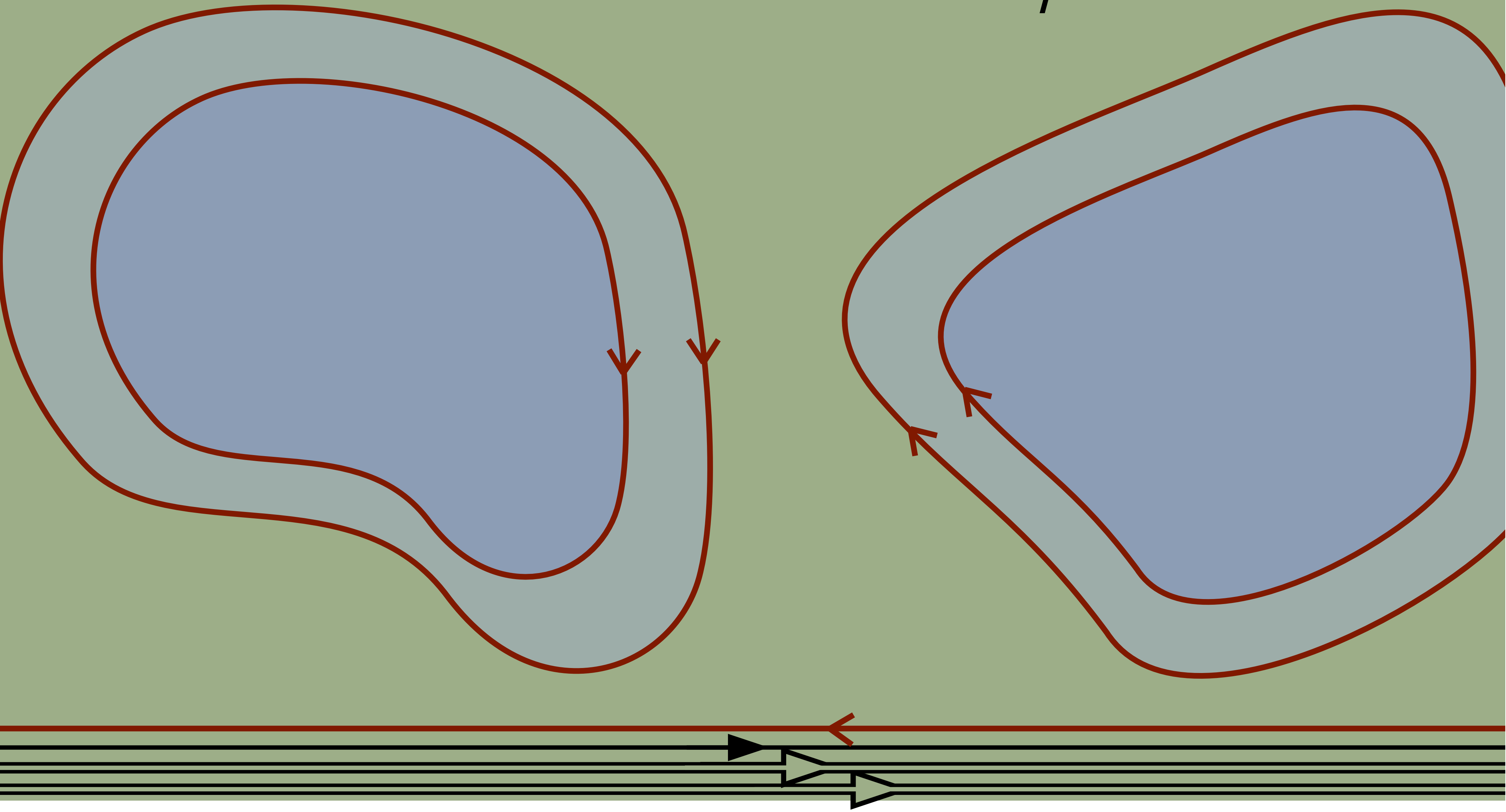


$$\kappa_{xy} = \frac{5}{2}$$

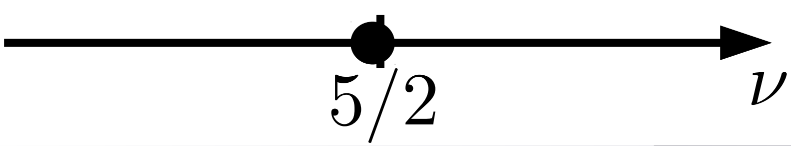


$$\sigma_{xy} = \frac{5}{2}$$

Electrons at ν=5/2

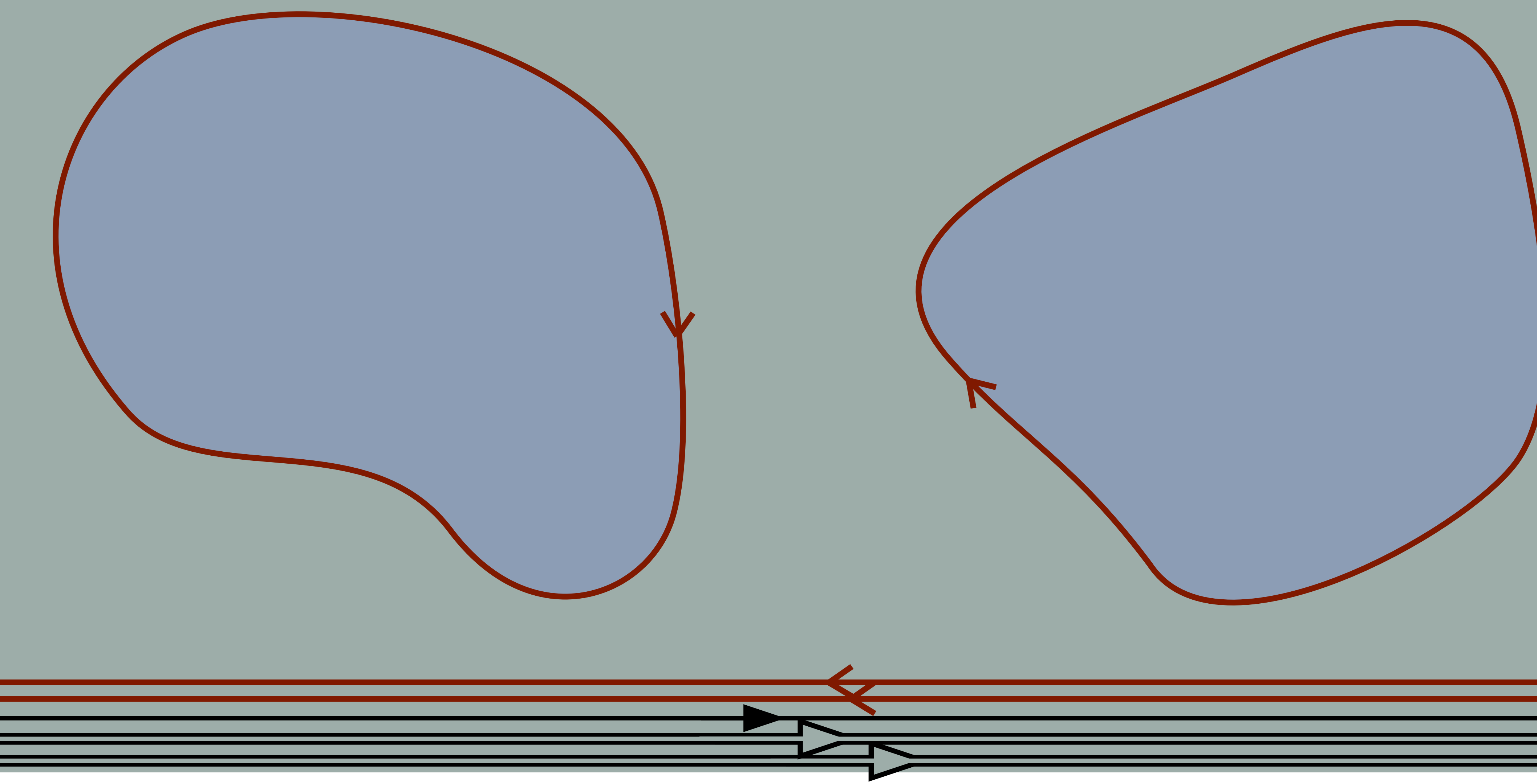


$$\kappa_{xy} = \frac{5}{2}$$

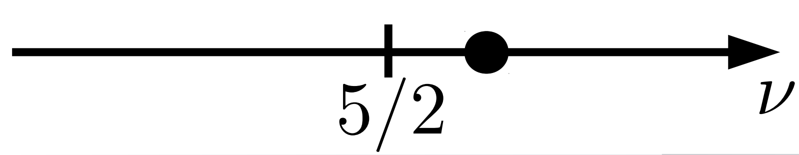


$$\sigma_{xy} = \frac{5}{2}$$

Electrons at ν=5/2

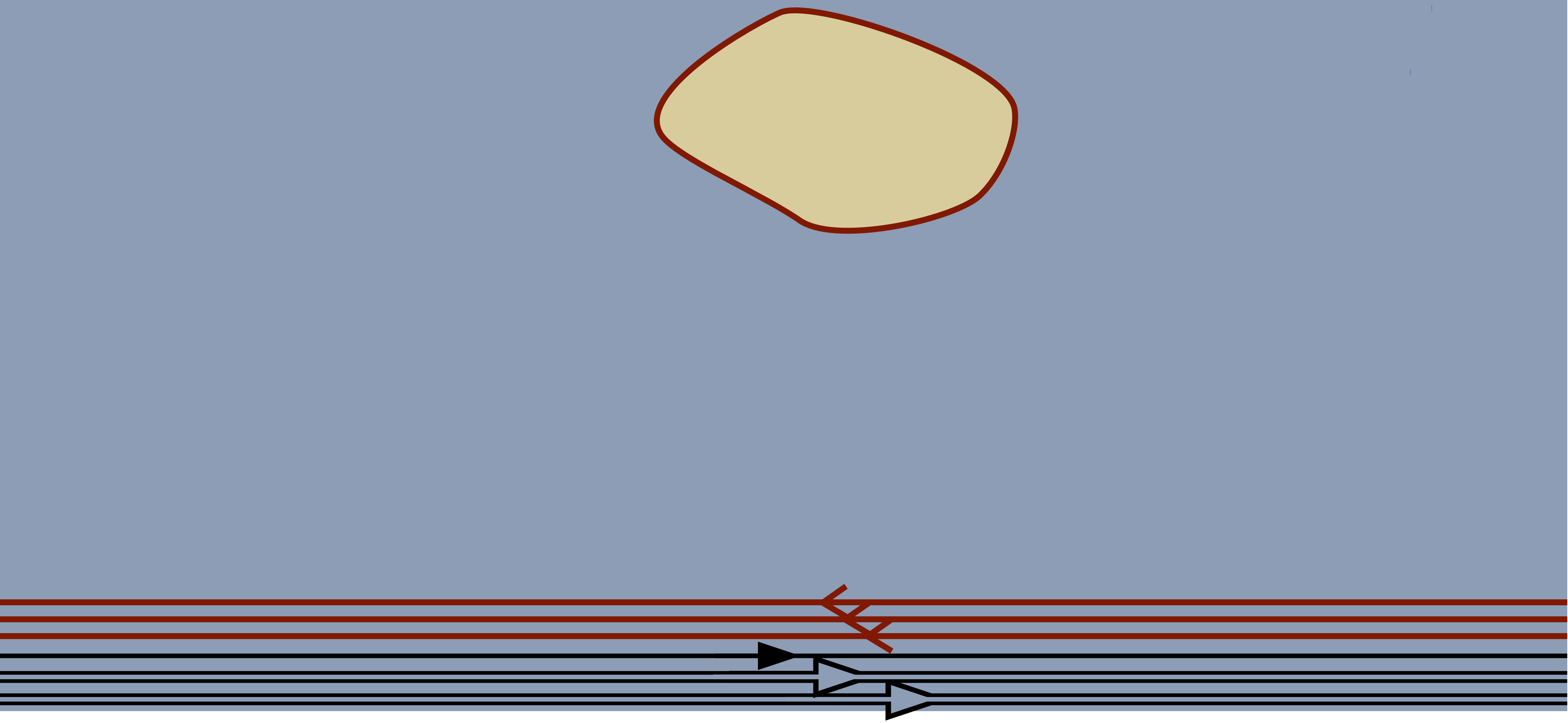


$\kappa_{xy} = 2$

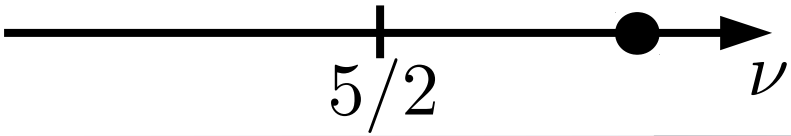


$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

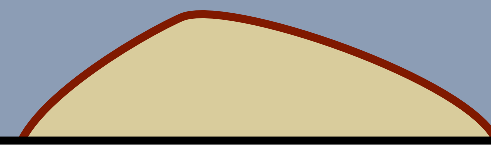


$\kappa_{xy} = \frac{3}{2}$



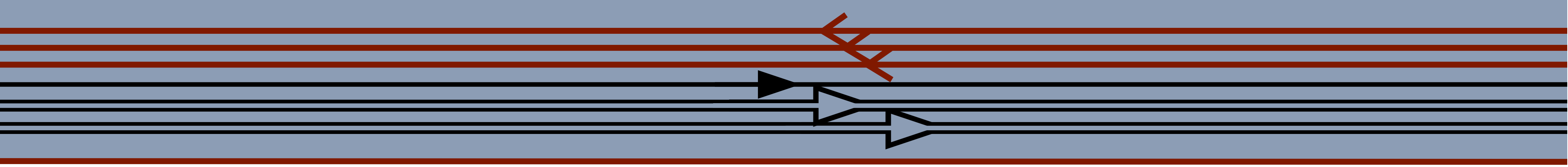
$\sigma_{xy} = \frac{5}{2}$

Electrons at ν=5/2

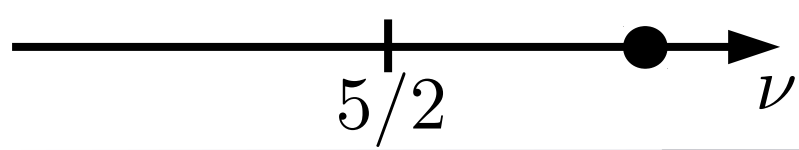


tuning the filling factor **within the $\boldsymbol{\sigma_{xy} = 5/2}$ plateau**,

plateaus with $\boldsymbol{\kappa_{xy} = \frac{7}{2}, 3, \frac{5}{2}, 2, \frac{3}{2}}$

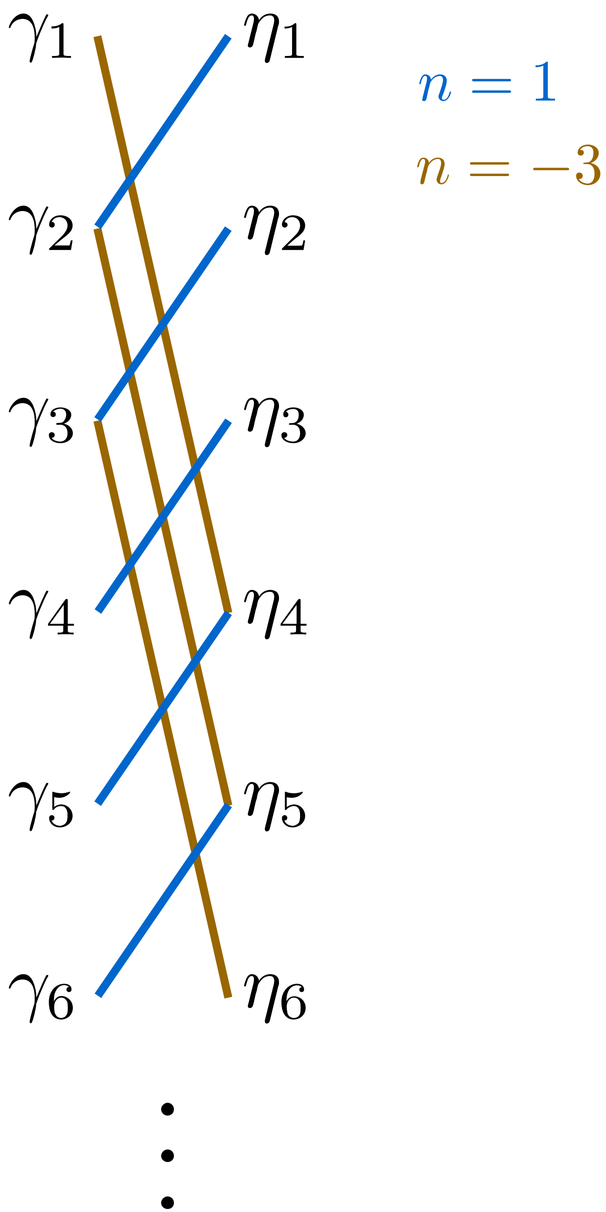


$\kappa_{xy} = \frac{3}{2}$



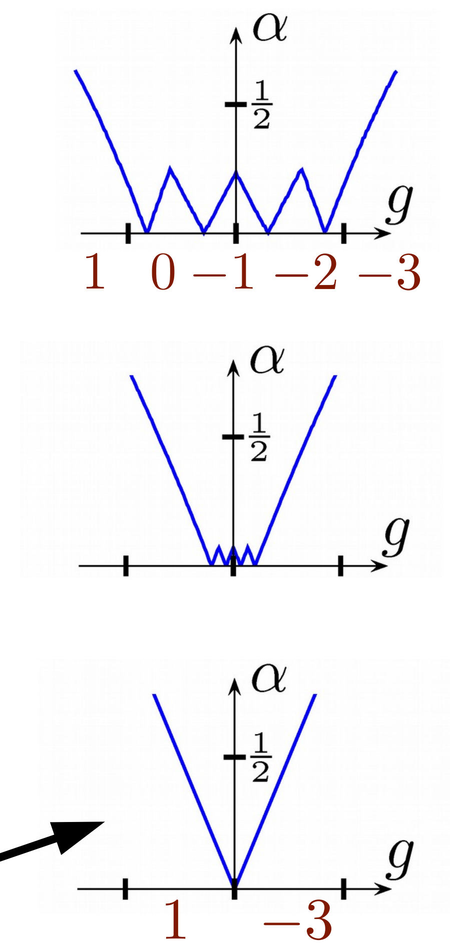
$\sigma_{xy} = \frac{5}{2}$

Numerics at weak disorder



Symmetry				d		
AZ	Θ	Ξ	Π	1	2	3
A	0	0	0	0	$\mathbb{Z}$	0
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	1	0	0	0	0	0
BDI	1	1	1	$\mathbb{Z}$	0	0
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$
C	0	-1	0	0	$\mathbb{Z}$	0
CI	1	-1	1	0	0	$\mathbb{Z}$

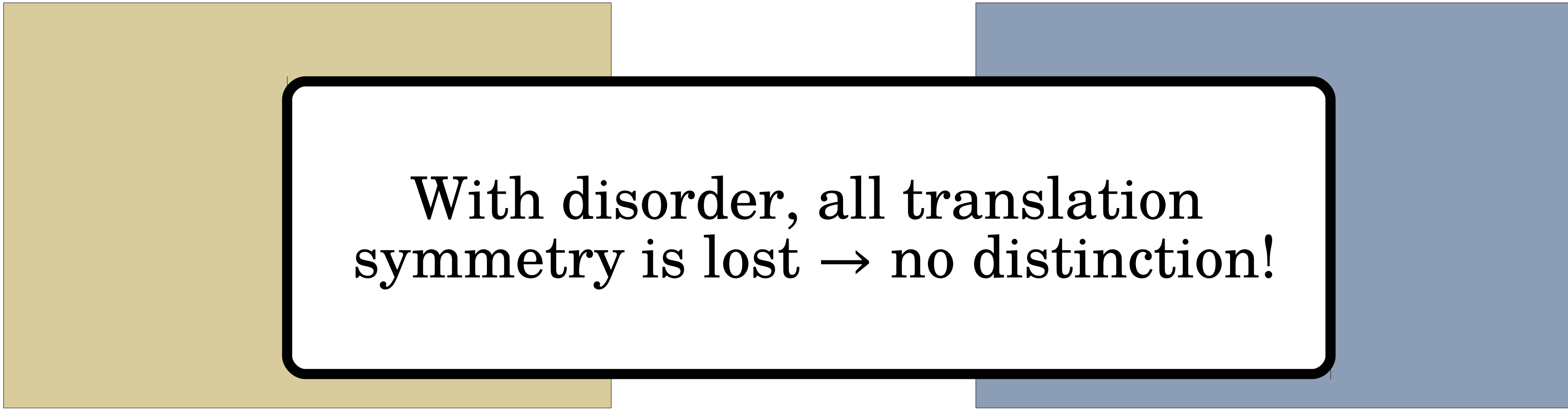
Numerics at weak disorder



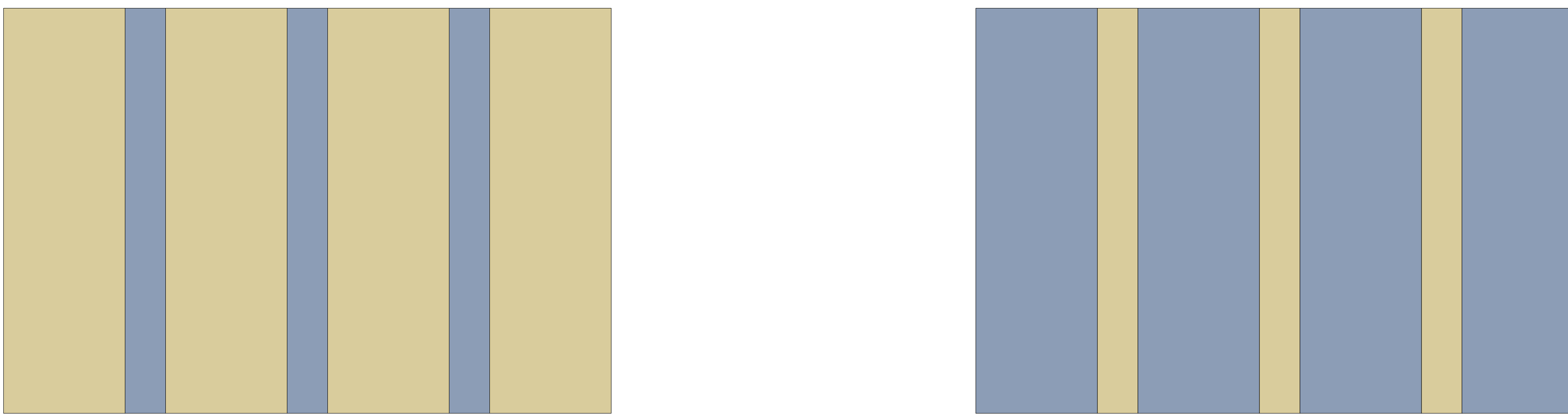
$$H = \sum_{i=1}^{4} \xi_i^T \left[\tau_x(-i\partial_x) + m\tau_y \right] \xi_i$$

Continuous phase transition in clean system

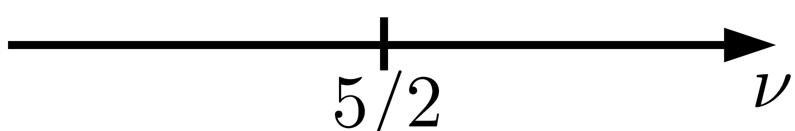
Numerics at weak disorder



Continuous translation symmetry



Discrete translation symmetry

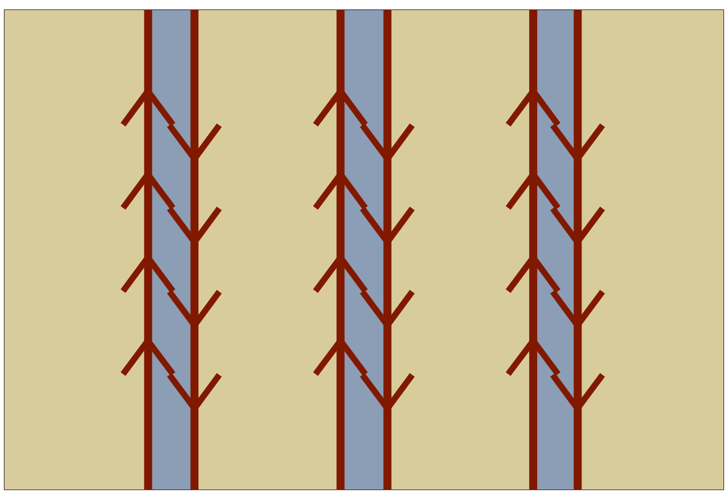


Numerics at weak disorder

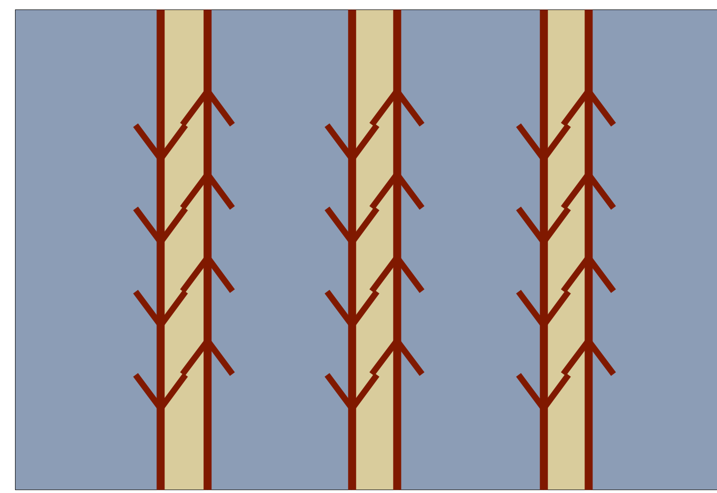
$$H = \sum_{i=1}^{4} \xi_i^T \underbrace{\left[\tau_z(-i\partial_z)\right.}_{\text{motion along domain walls}} + \underbrace{\left.\tau_x(-i\partial_x) + m\tau_y\right]}_{\text{tunneling across domains}} \xi_i$$

motion along domain walls

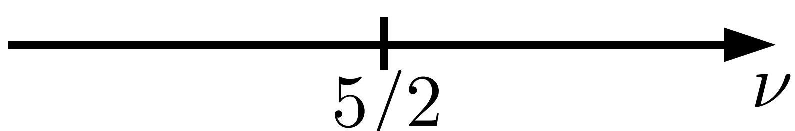
tunneling across domains



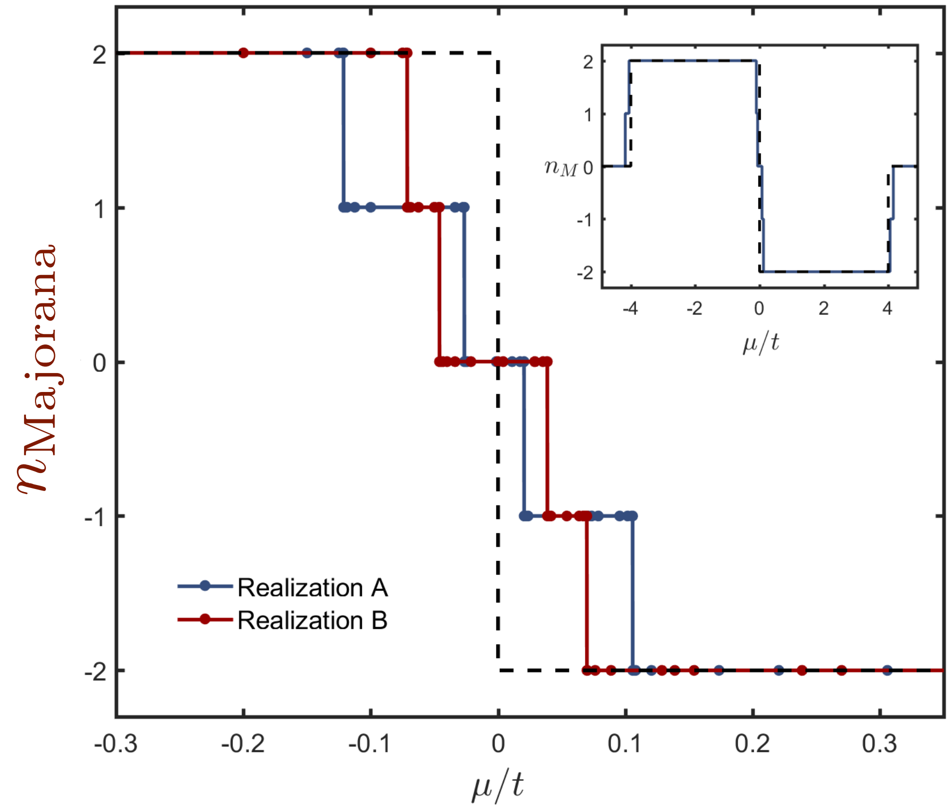
2nd order transition



Discrete translation symmetry



Numerics at weak disorder

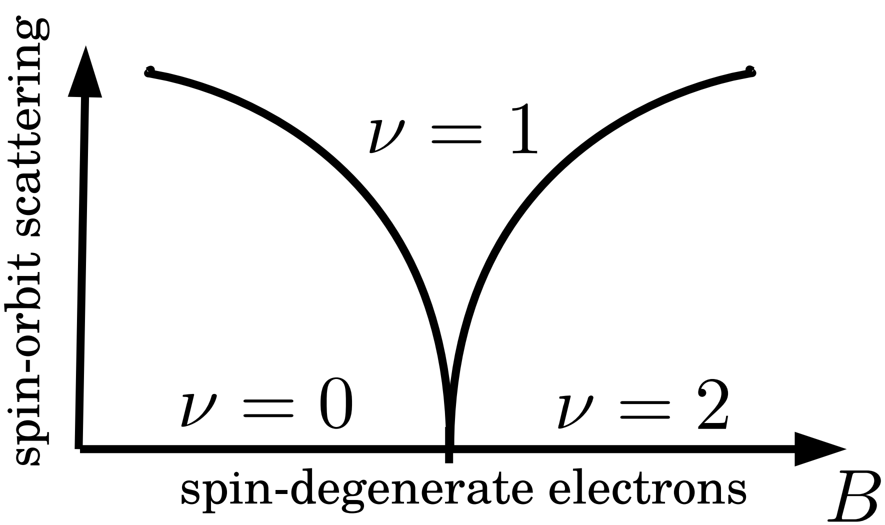


Two-dimensional superconductor

A useful analogy

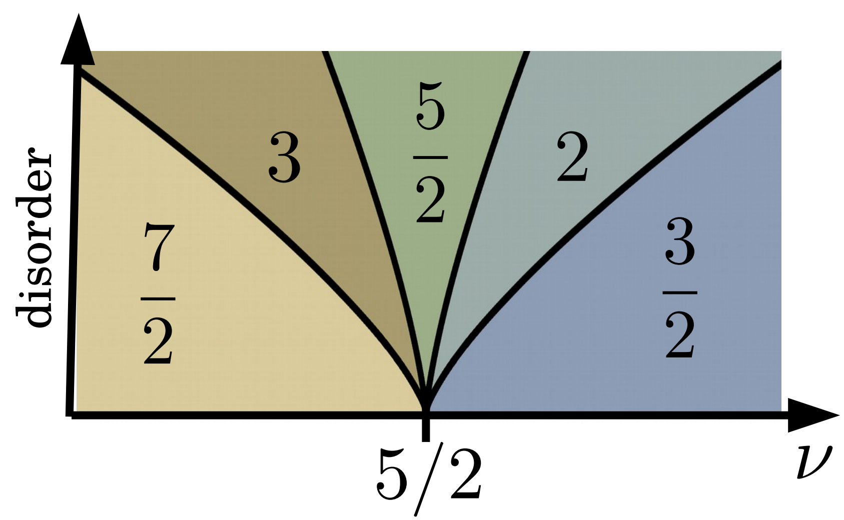
Integer quantum Hall

- Class A for electrons
- Integer classification ($n = \#$ of edge electrons)
- Generic transition: $\Delta n = 1$



Electrons at ν=5/2

- Class D for comp. fermions
- Integer classification ($n = \#$ of edge Majoranas)
- Generic transition: $\Delta n = 1$

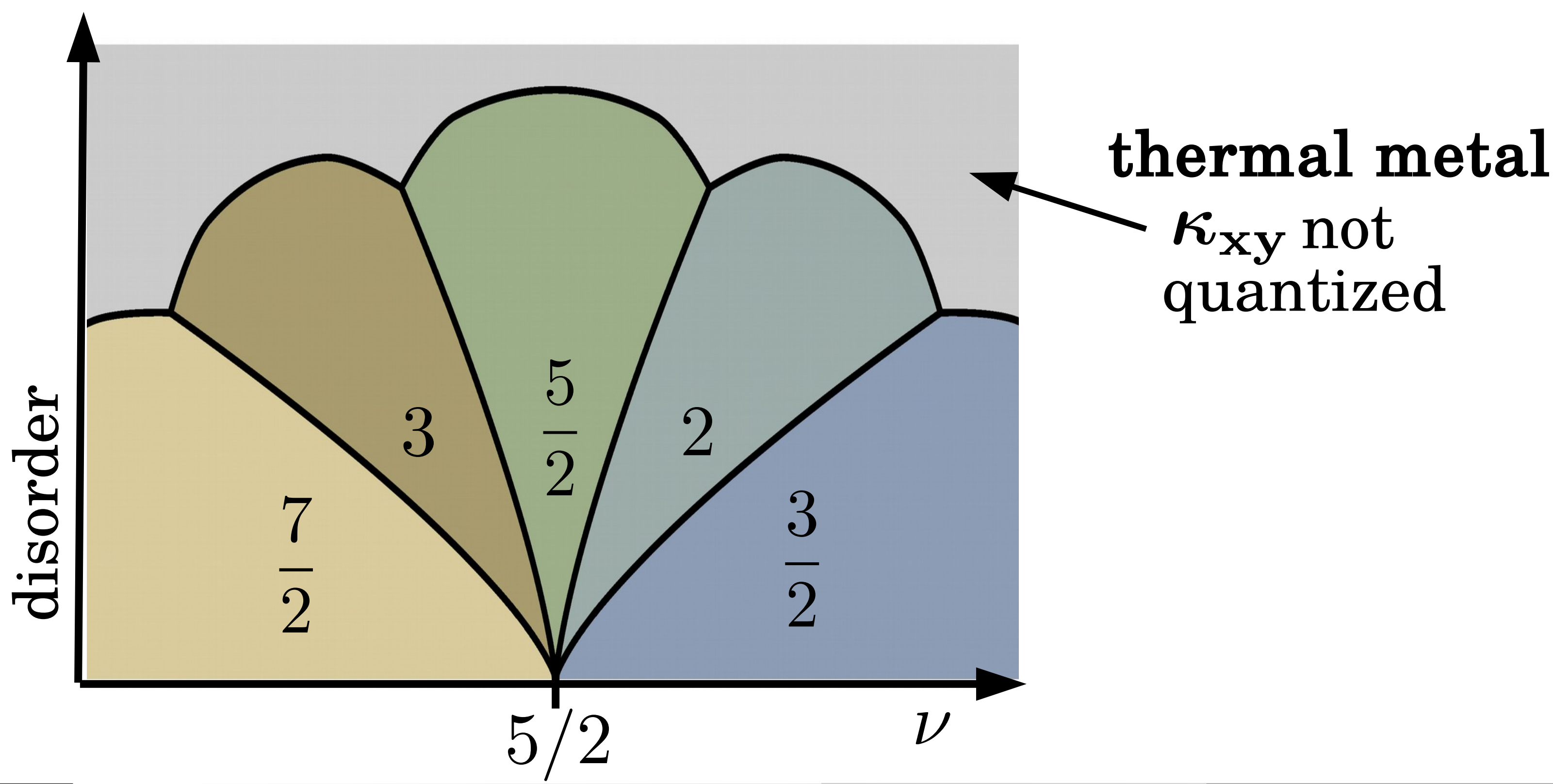


Strong disorder

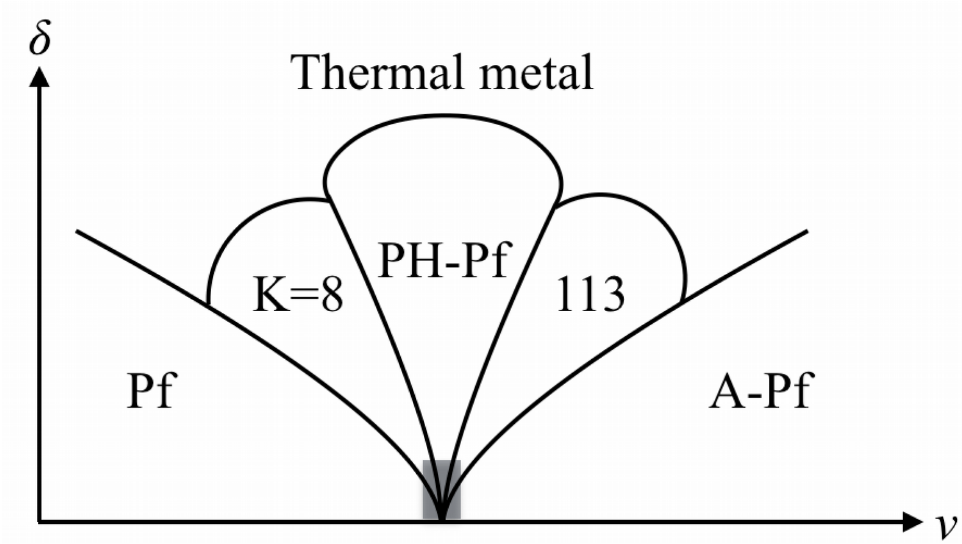
- a localized phase (well defined n_{Majorana}) not guaranteed

Cho and Fisher (1997), Senthil and Fisher (2000), Bocquet, Serban and Zirnbauer (2000)
Read and Ludwig (2000), Chalker *et al.* (2001)

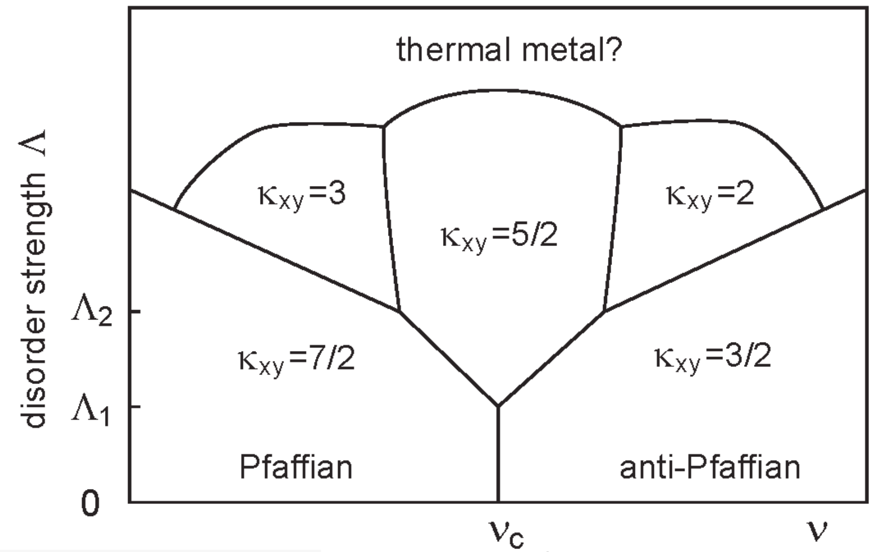
- depends on details of the disorder potential



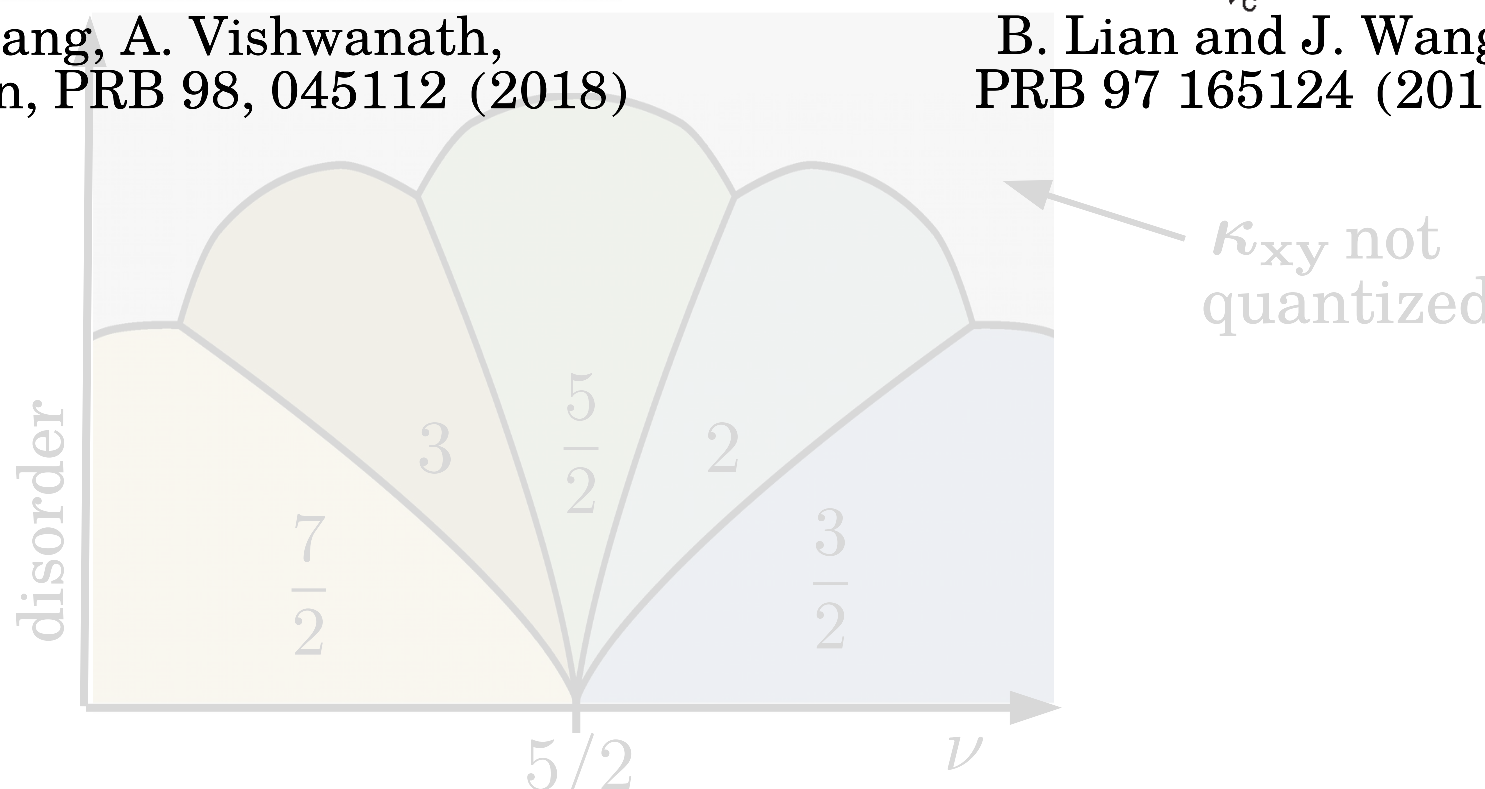
Related work



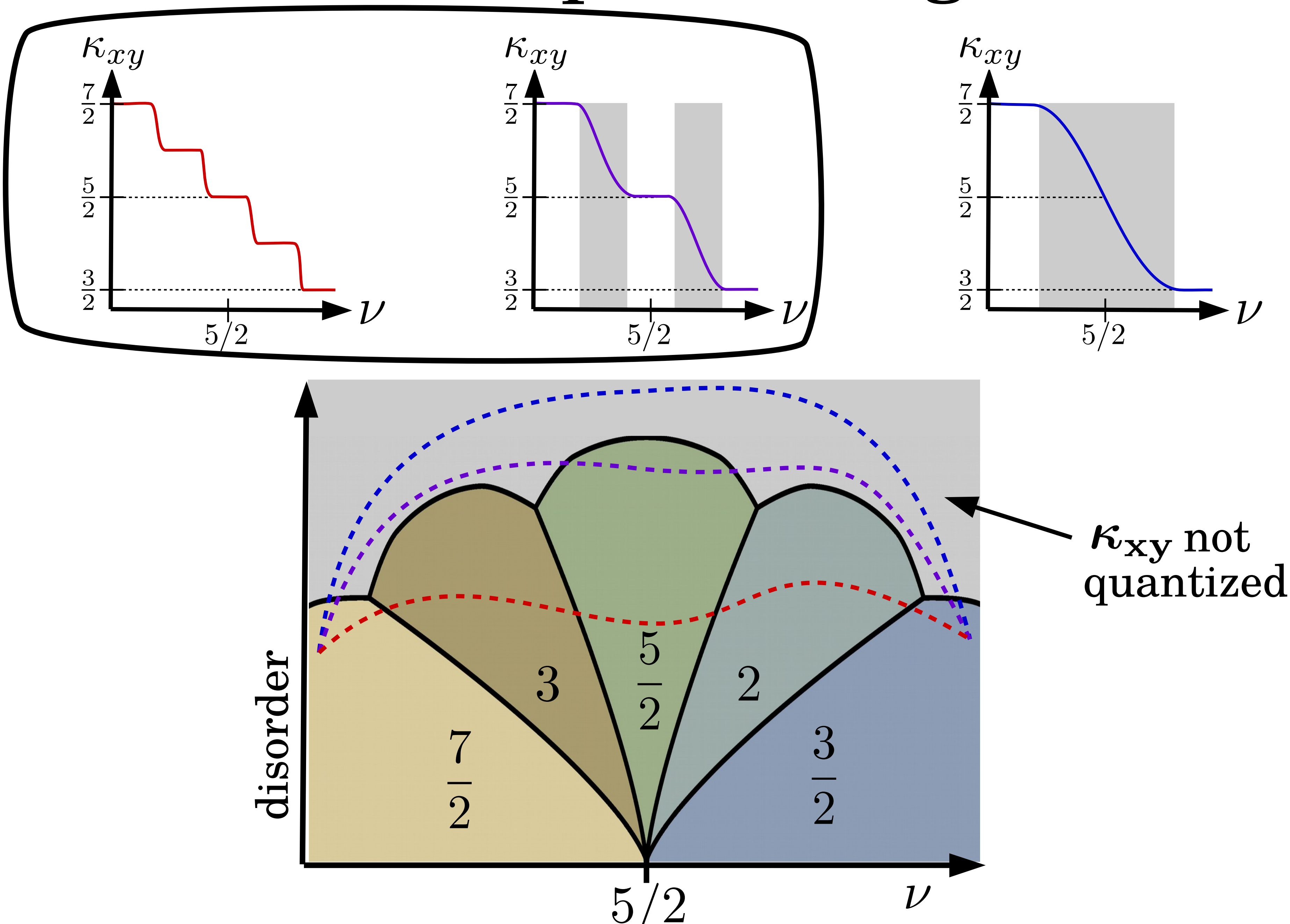
C. Wang, A. Vishwanath,
B. Halperin, PRB 98, 045112 (2018)



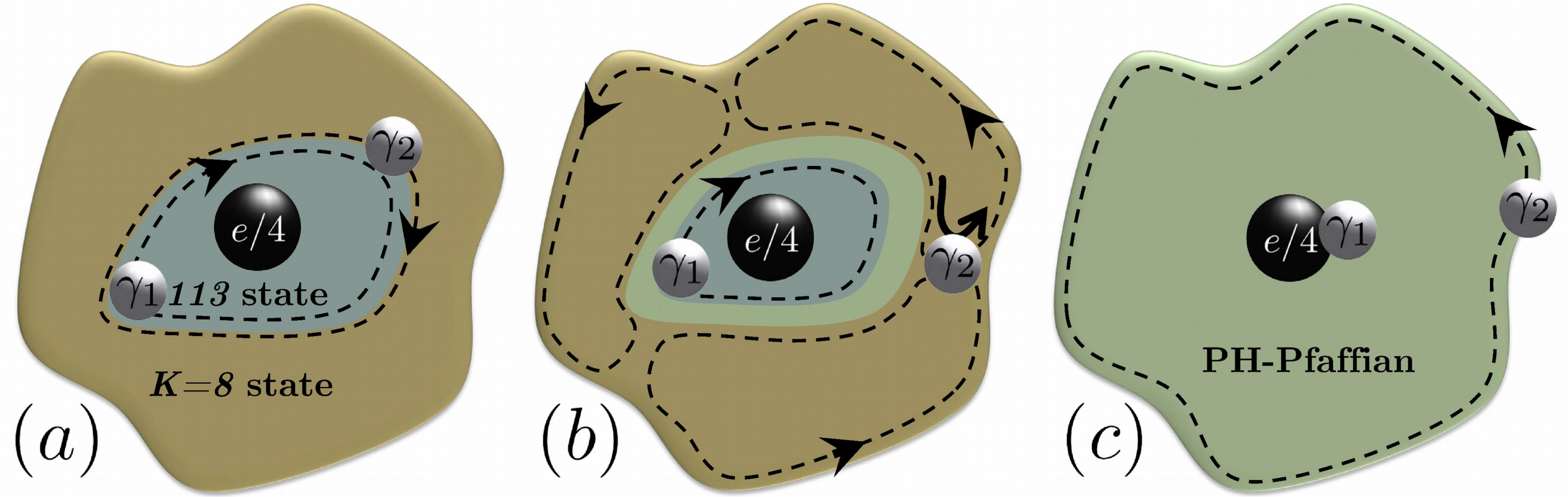
B. Lian and J. Wang
PRB 97 165124 (2018)



General phase diagram



From Abelian to non-Abelian



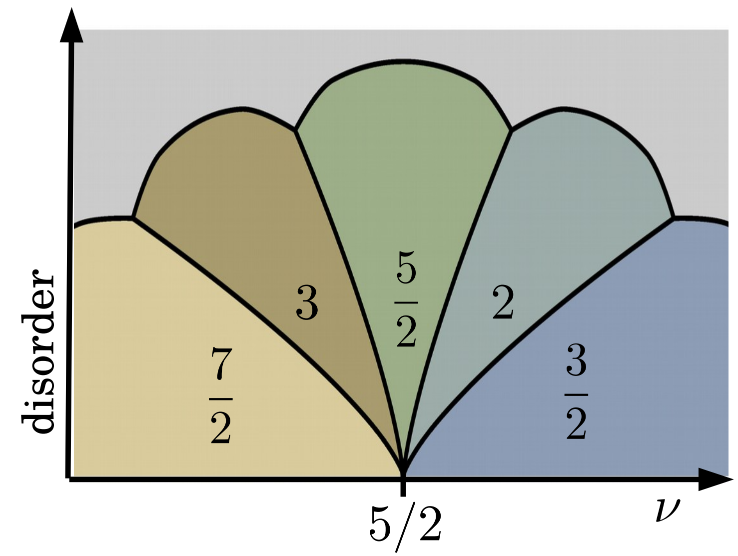
(a) No isolated Majorana modes in Abelian phase

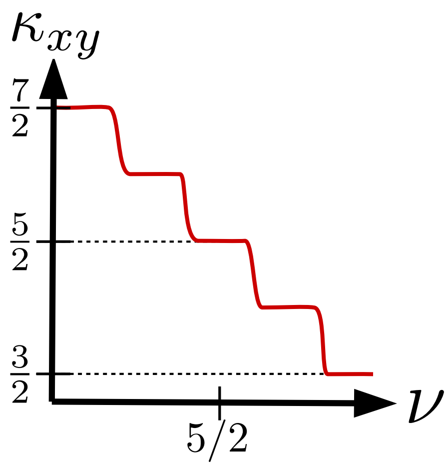
(b) Transfer of Majorana mode at transition

(c) Isolated Majorana mode, i.e., non-Abelian phase

Conclusions

- Weak disorder resolves discrepancy between numerics and experiment





- Predict additional plateaus in thermal Hall conductance

- Disorder can induce non-Abelian statistics