Fractons: new pathway to topological order in 3D

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Outline and motivation

- Non-conventional magnetic order
    - Regular magnets
    - Non-conventional (but not yet topological) magnets: examples in 1D and 3D
    - Topological order
    - From 2D to 3D
    - 2D example: toric code

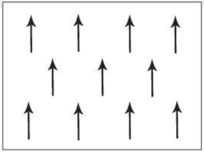
- Our fracton model
  OP, N. Regnault, Phys. Rev. B **96**, 224429 (2017)

Outline

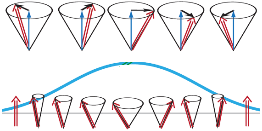
Regular magnets

- Spontaneous symmetry breaking
- Ground state: lower symmetry than Hamiltonian
  - Example: ferromagnet $H = -J \sum_n S_n \cdot S_{n+1}$
- Long range order

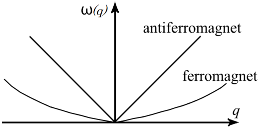
- Excitations: spin waves
- Magnons: quantized spin waves
  - Linear (AFM) or quadratic (FM) dispersion
- Inelastic neutron scattering: branches



Spin wave in a ferromagnet:



Magnons in FM, AFM:



Outline

Non-conventional magnetic order: spin liquids

- ~~Spontaneous symmetry breaking~~
- ~~Ground state: lower symmetry than Hamiltonian~~
- Ground state does not break any symmetries
- Disordered **but** local constraints
  Different from a paramagnet!

- ~~Excitations: spin waves~~
- ~~Magnons: quantized spin waves~~
- Fractionalized excitations
- Magnon fractionalizes into multiple quasiparticles $\Rightarrow$
  Momentum distributed in multiple ways $\Rightarrow$
  Inelastic neutron scattering: continuum

Heisenberg antiferromagnetic chain

XXZ antiferromagnetic spin 1/2 chain
($\Delta \gg 1$)

$$H = \sum_n \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z \right)$$

$\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow$

- "Magnon": flip a spin
- 2 upset bonds
  $\Delta \to \infty$: stays like this
- Magnon splits into two **spinons**
  $\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y$ give spinons dynamics
- Neutron scattering: continuum!

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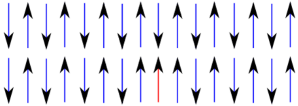

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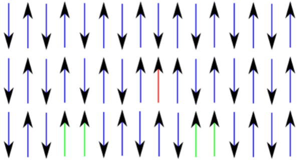

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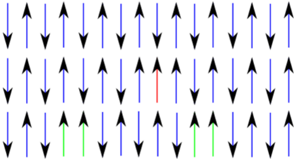
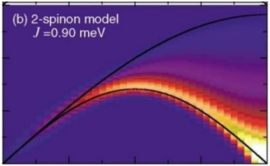


Stone *et al.*, PRL **91**, 037205 (2003)

model



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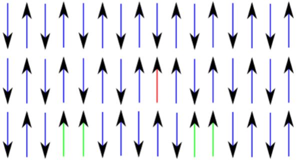

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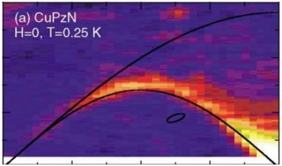
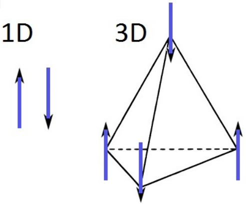
experimental data



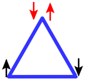
From 1D chain to a 3D model

$$H = \sum_n \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z\right), \quad \Delta \to \infty$$

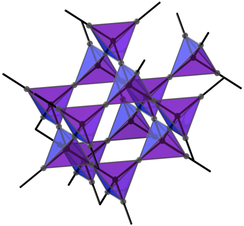


pyrochlore lattice

- Antiferromagnet in 1D: neighbors are misaligned
- Antiferromagnet on 3D pyrochlore lattice: neighbors are 4 spins on a tetrahedron
- Frustration : Ising spins on a triangle



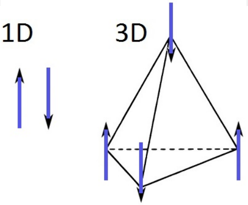
- Each tetrahedron: two up, two down
- Many degenerate configurations
  Each is disordered, but local constraints!
- Pauling residual entropy (like water ice!)



SPIN ICE

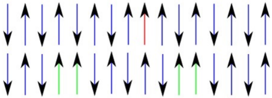
Fractionalization in spin ice

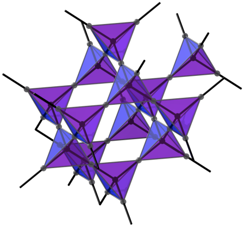
$$H = \sum_n \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z \right), \quad \Delta \to \infty$$



pyrochlore lattice

- Antiferromagnet in 1D: magnon → two spinons





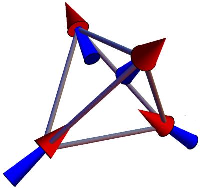
SPIN ICE

- Spin flip: two excited tetrahedra
- Fractionalizes into two quasiparticles
- Dipolar spin terms in H: ≈ Coulomb interaction
- Magnetic monopoles

Spin ice: classical and quantum

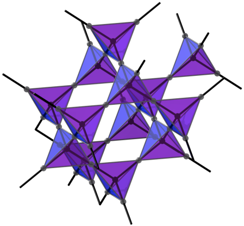
- Classical spin ices: $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$

$$H = J_{||} \sum_n \sigma_n^z \sigma_{n+1}^z$$ Spin flips from thermal fluctuations



- **Quantum** spin ice (many candidate materials!)

$$H = J_{\perp} \sum_n \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) + J_{||} \sum_n \sigma_n^z \sigma_{n+1}^z$$

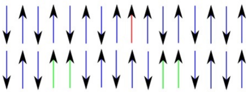


- Lifts the extensive ground state degeneracy
- Ground state is featureless, but not the excitations!

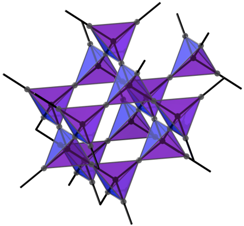
''Hydrogenic states of monopoles in diluted quantum spin ice'', **OP**, R. Moessner, S. L. Sondhi PRB **92**, 100401 (2015)

''Mesonic excited states of magnetic monopoles in quantum spin ice'', **OP**, arXiv:1807.02193

What changes when we go 1D $\rightarrow$ higher dimensions?



- Examples we discussed:
- Richer physics, but $\approx$ same phenomena (Frustration is not specific to 3D)
- Topological order:
- 2D vs. 3D: qualitatively different

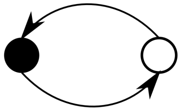


Outline

Topological order: anyons

Will discuss what is **topological** on the next slide!

- Exchange two particles: pick up a phase $e^{i\phi}$
- Double exchange $\approx$ closed loop: $e^{i2\phi}$
- 3D: can lift the loop and shrink it to a point
  $\Rightarrow e^{i2\phi} = 1 \Rightarrow e^{i\phi} = \pm 1$, bosons and fermions
- 2D: cannot shrink the loop
  $\Rightarrow e^{i\phi}$ can in principle take any value $\Rightarrow$ anyons
- Anyons are a sign of **topological order**
- Pointlike anyons: only in 2D
- Non-identical particles: **mutual exchange statistics**



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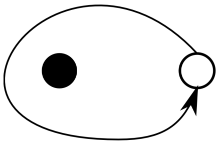

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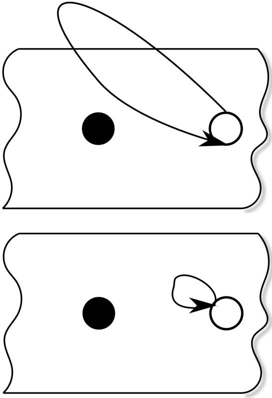

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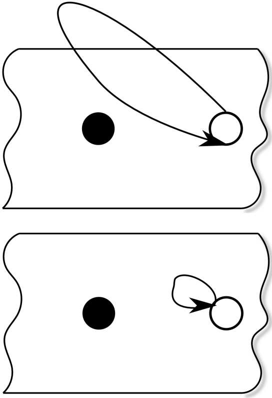


Topological degeneracy

- What is topological?
- Ground state degeneracy depends on topology
  - Example: non-degenerate on a sphere, but 4-fold on a torus
- Closely related to the existence of **anyon** excitations
- Can tell what kind of order we have from topological degeneracy!

- Locally indistinguishable ground states
- Transform via global operations

- Stable to local noise
- Applications in quantum computing

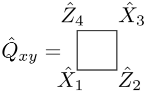
Example: the toric code

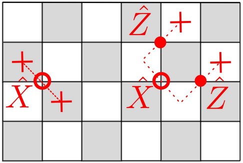
Wen plaquette model: X.-G. Wen, PRL **90**, 016803 (2003)
Kitaev toric code: A. Kitaev, Ann. Phys. **303**, 2 (2003)

$$\sigma^x = \hat{X};\ \sigma^z = \hat{Z}$$

- Spins 1/2 on the sites of a square lattice

$$H = -\sum_p \hat{Q}_{xy}, \quad \hat{Q}_{xy} = \sigma_1^x \sigma_2^z \sigma_3^x \sigma_4^z$$

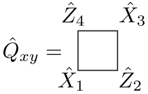


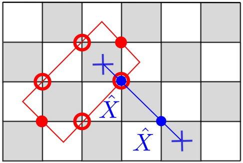


- All $\hat{Q}_{xy}$ commute $\Rightarrow$ exactly solvable
- $Q_{xy} = \pm 1$; ground state: all $Q_{xy} = +1$
- Excitations $Q_{xy} = -1$: spin flip $\rightarrow$ excitation pair
  Act on ground state with σ^x or σ^z
- $Q_{xy} = -1$ at the ends of string operators
- Two flavors of plaquettes: e and m

Anyons in the toric code

- e and m are bosons
- **Mutual** statistics: take e around m in a loop
- e and m strings anticommute when they cross
- e and m are mutual anyons:
  e goes around $m \Rightarrow$ phase $= -1$
- GS degeneracy on a torus: 4-fold
  - N spins; N plaquettes; N Q_{xy} values
  - GS: all $Q_{xy} = +1$, but they are not all linearly independent
  - 4 locally indistinguishable ground states





Fracton topological order

2D topological order

- Finite topological GS degeneracy
- Excitations: pointlike anyons
- Anyons at the endpoints of strings

3D topological order

- Finite topological GS degeneracy
- Excitations: loop-like anyons
- Anyons at the 1D boundaries of 2D open membranes

Fracton topological order in 3D

- (Sub)extensive topological degeneracy, $2^{\alpha L}$
- Excitations: immobile fractons
- Immobile = energy penalty to move
- Composite particles mobile in 2D/1D
- Can construct 3D fracton phases from layered 2D topological models

H. Ma *et al.*, PRB **95**, 245126 (2017)

S. Vijay, arXiv:1701.00762

OP, N. Regnault, PRB **96**, 224429 (2017)

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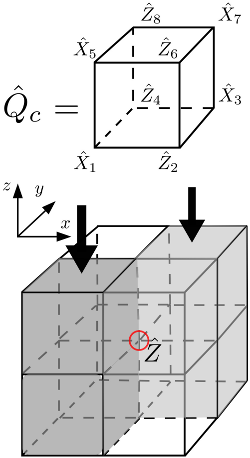
The fracton model

- Spins 1/2 on the sites of a cubic lattice

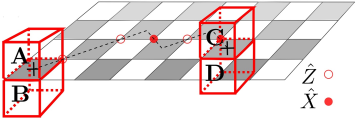
$$H = -\sum_c \hat{Q}_c$$

- $\hat{Q}_c = \hat{Q}_{xy}(z) \times \hat{Q}_{xy}(z+1)$
- All $\hat{Q}_c$ commute; $Q_c = \pm 1$
- Ground state: $Q_c = +1$
  - In a column: all $Q_{xy} = +1$ or all $Q_{xy} = -1$
    spontaneous symmetry breaking
- Flip a spin: create four $Q_c = -1$
- Two flavors of columns: e and m



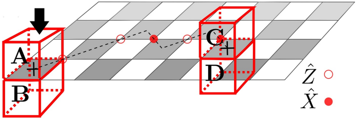
Can we take our excitations apart as we did in the toric code?

Immobile fractons and mobile composites



- Four excited cubes:
  A, B, C, and D
- Toric code-like string in xy plane:
  separates AB and CD
- Can we move all four apart?
- Flip the upper Q_{xy} of cube A
- Where to put another flipped Q_{xy}?
- It has to be the top of cube C
- The 4 cubes are **fractons**

Immobile fractons and mobile composites



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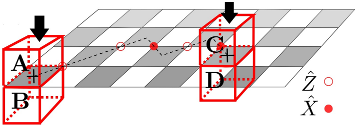

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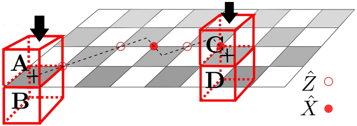

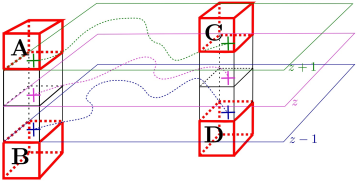

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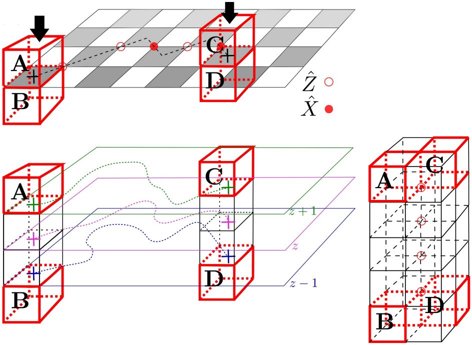
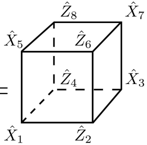


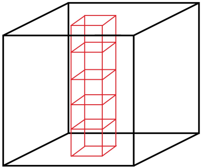
- Fractons ⇒ mobile particles:
  - AB and CD are mobile in 2D
  - AC and BD are mobile in 1D

(Sub)extensive degeneracy

- $L_x \times L_y \times L_z$ with PBCs
- 2D toric code: 4-fold degenerate
- Topological degeneracy: $4^{L_z} = 2^{2L_z}$
- Fracton order: subextensive GS degeneracy!
- Additional **symmetry-breaking** degeneracy: $Q_{xy} = \pm 1$ per column
- $L_x \times L_y - 2$ linearly independent columns
- Total GS degeneracy

$$2^{2L_z + L_x \times L_y - 2}$$

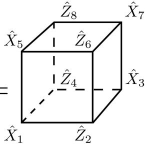
$\hat{Q}_c =$ 

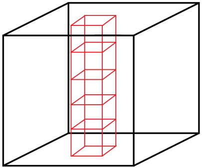


(Sub)extensive degeneracy

- $L_x \times L_y \times L_z$ with PBCs
- 2D toric code: 4-fold degenerate
- Topological degeneracy: $4^{L_z} = 2^{2L_z}$
- Fracton order: subextensive GS degeneracy!
- Additional **symmetry-breaking** degeneracy: $Q_{xy} = \pm 1$ per column
- $L_x \times L_y - 2$ linearly independent columns
- Total GS degeneracy

$$2^{2L_z + L_x \times L_y - 2}$$

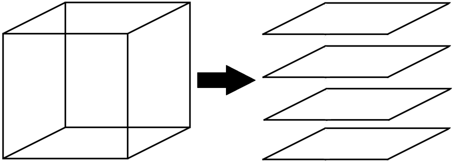
$\hat{Q}_c =$ 



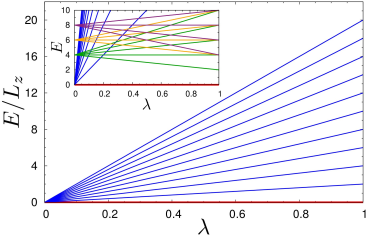
Fracton order vs. layered toric code

$$H(\lambda) = -\frac{(1-\lambda)}{2}\sum_c \hat{Q}_c - \frac{\lambda}{2}\sum_z \sum_p \hat{Q}_{xy}(z)$$

- $\lambda = 0$: 3D fracton order
- $\lambda = 1$: decoupled 2D toric codes
- Gap closes at $\lambda = 0$
- $\lambda \neq 0$: toric code phase



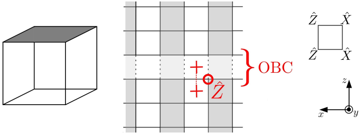
partial lifting of GS degeneracy

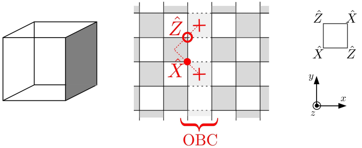


Open boundary conditions

Ground state is featureless in the bulk
but the **surface modes** are not!
Signature of topological order

- OBC = PBC with "missing" layers
- OBC in $\hat{z}$:
  no boundary charge w/o bulk charge
- OBC in $\hat{x}$ or $\hat{y}$:
  free charge at the boundary $\Rightarrow$
  zero energy surface modes
- Surface modes vs. 1D and 2D mobile
  composite particles





Conclusions

- Rich topological physics in 3D
- Much harder to tackle than 2D
- Layered construction: use 2D insights
- Exactly solvable model

- Anisotropic 3D fracton topological model
- Immobile fractons at corners of membranes
- Composite particles mobile in 1D and 2D
- (Sub)extensive topological degeneracy
- Symmetry breaking + topological order