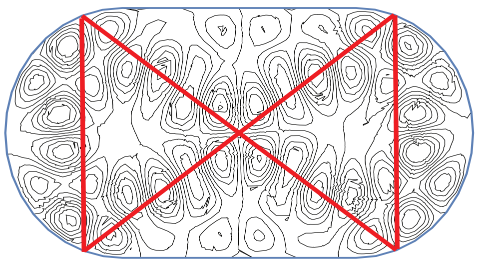
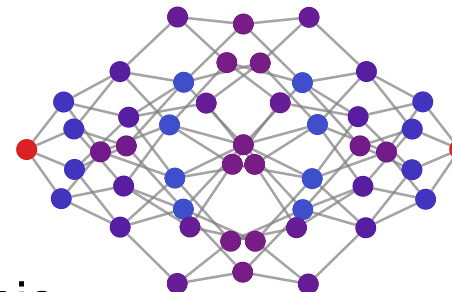
Weak ergodicity breaking from quantum many-body scars



Maksym Serbyn

In collaboration with
C. Turner, A. Michailidis, D. Abanin, Z. Papic



Nature Physics, 2018
arXiv:1806.10933

IST AUSTRIA

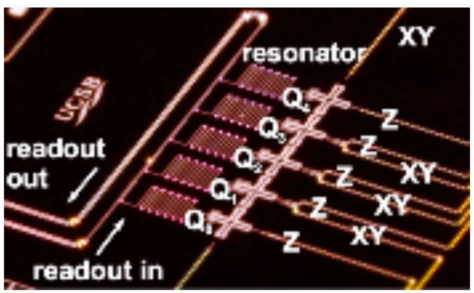
Mainz, 2018

SPICE

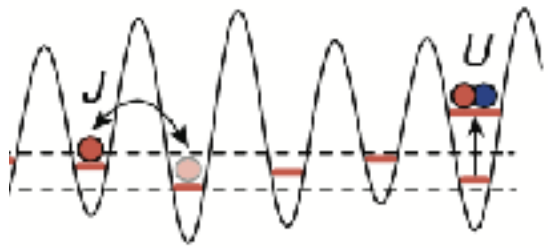
Universality in quantum dynamics

Isolated quantum systems out-of-equilibrium?

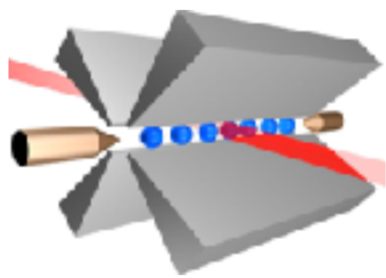
mutli-qubit systems



cold atoms



trapped ions



NV centers in diamond,

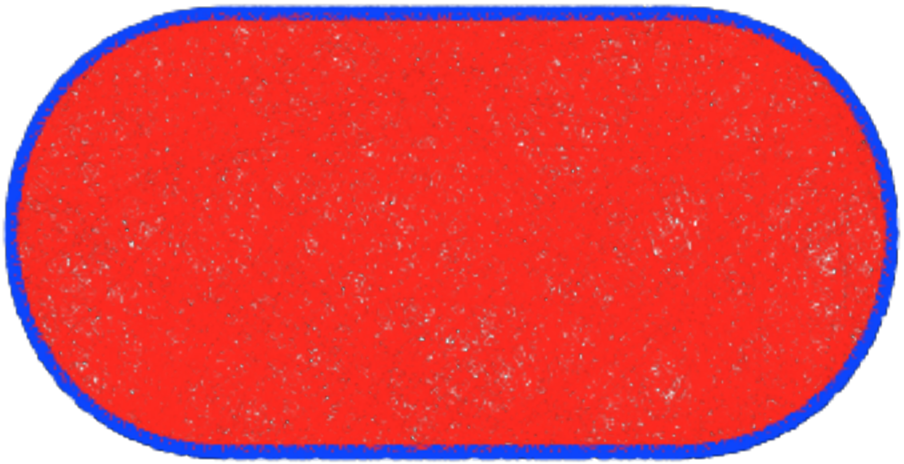
polar molecules,

....

Coarse classification: does system reach **equilibrium?**

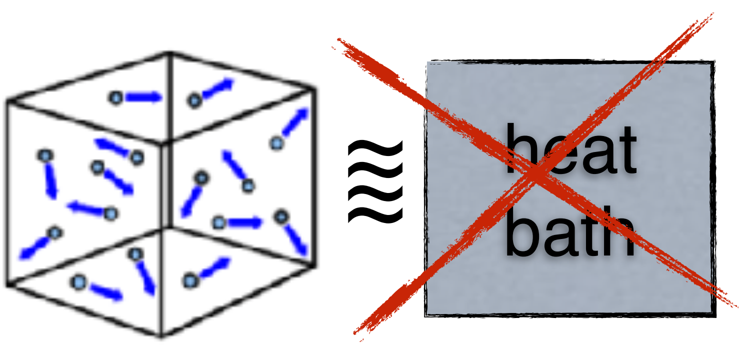
Chaos as a route to statistical equilibrium

- Systems "forget" initial conditions; explore all configurations:



- Generic systems are chaotic

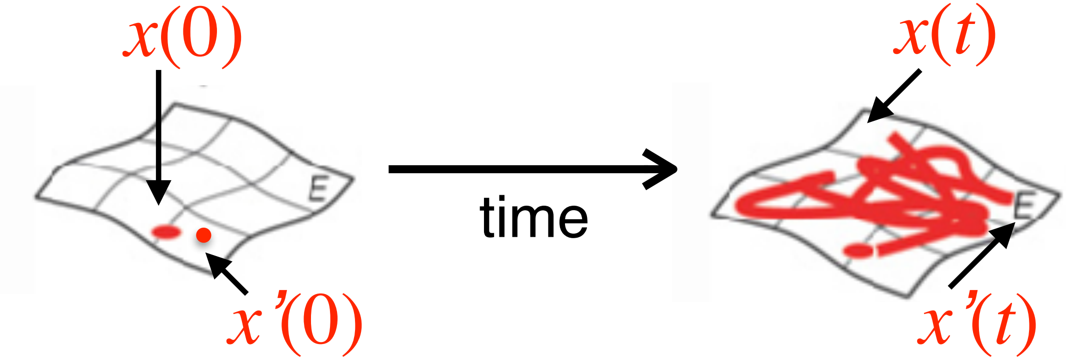
  Even isolated classical systems establish temperature



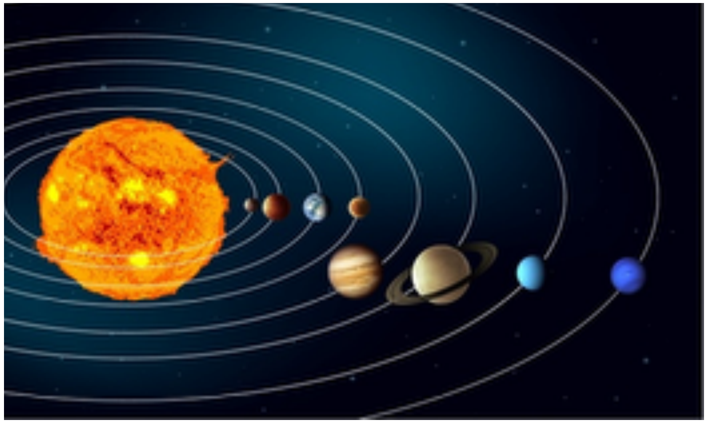
Not all systems are equally chaotic

Alexandr Lyapunov (1857-1918)

$$\delta x(t) \propto e^{\lambda t}\delta x(0)$$



Solar system



Lyapunov time $t=1/\lambda \sim 10^6$ years

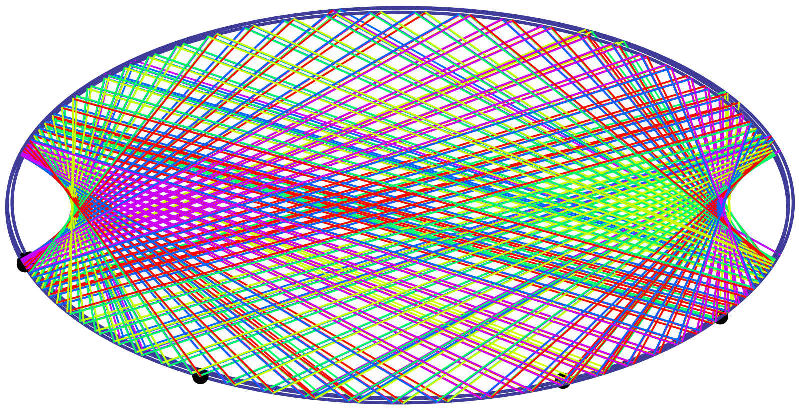
$\delta x(0)=10^{-42}$ m →

δx(500 mil. years)= 150 m

Small λ → longer transient

Escaping chaos: integrability

- Additional conservation laws/symmetries:



- Dynamics is constrained to tori; full phase space is not explored
- KAM theorem: non-resonant tori survive perturbations

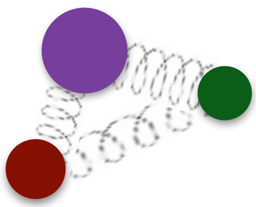
[Kolmogorov 1954; Arnold 1963; Moser 1962]

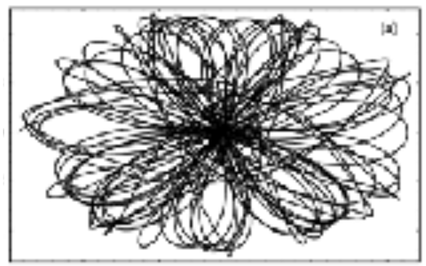
Ergodicity and integrability

Classical

Ergodic systems

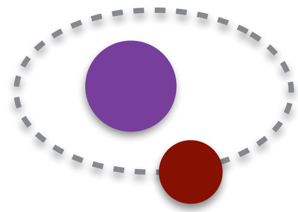
chaos → ergodicity

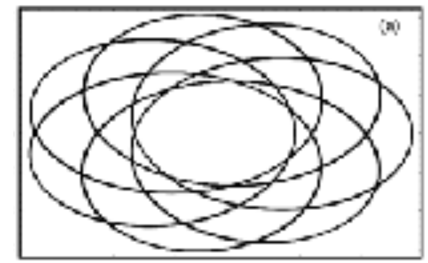




Integrable systems

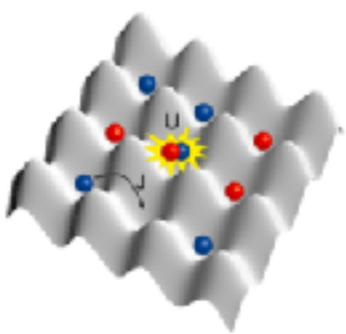
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

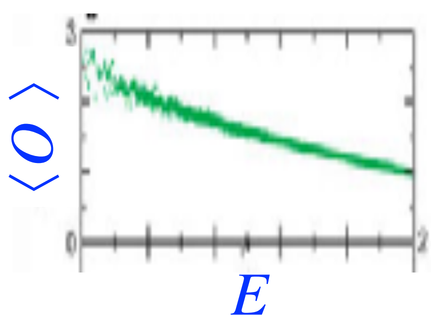




Quantum

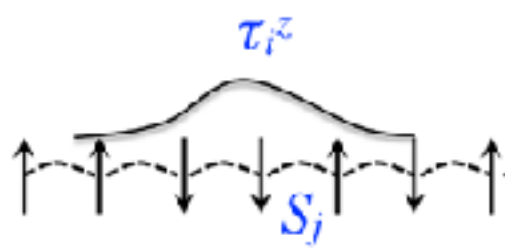
Thermalizing phases





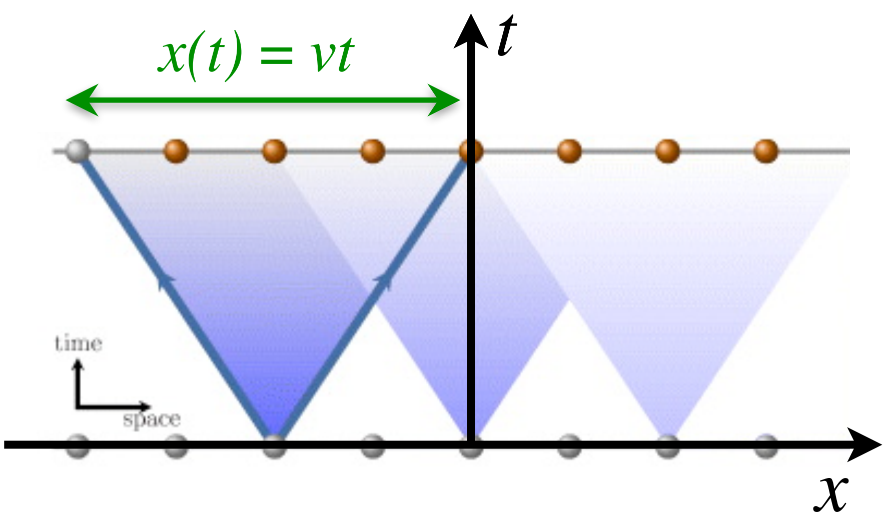
MBL phases

emergent integrability

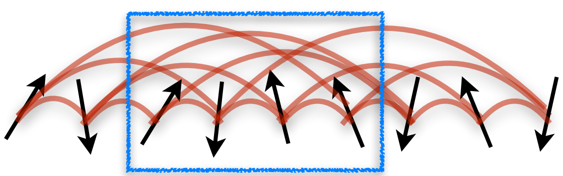


Ergodic phase vs MBL phase

$$S_{\rm ent} \propto vt$$



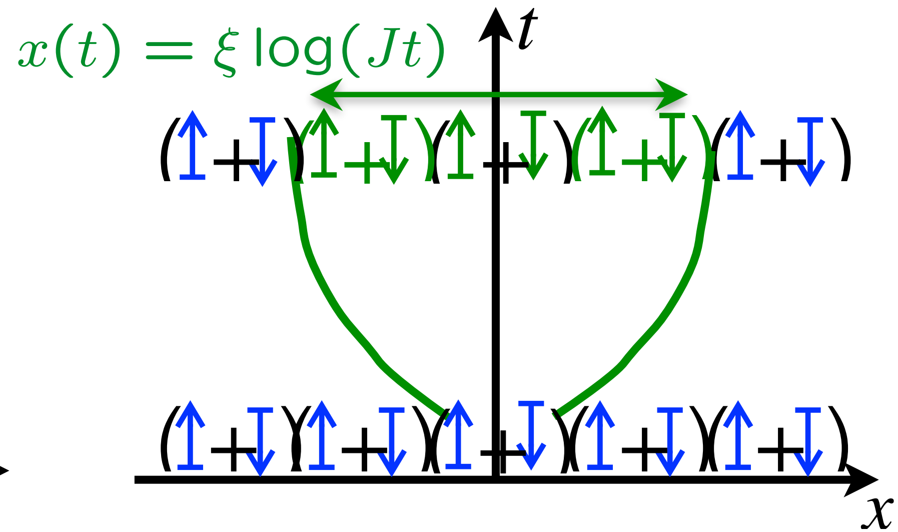
$$S_{\rm ent}(A) \propto \mathrm{vol}(A)$$



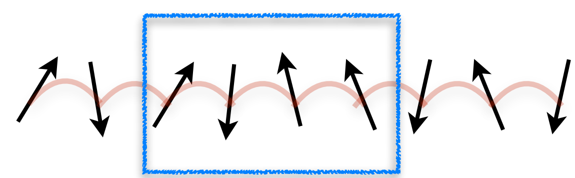
Described by ETH

[Srednicki'94]
[Rigol,Dunjko,Olshanii'08]

$$S_{\rm ent} \propto \xi \log Jt$$



$$S_{\rm ent}(A) \propto \mathrm{area}(A)$$

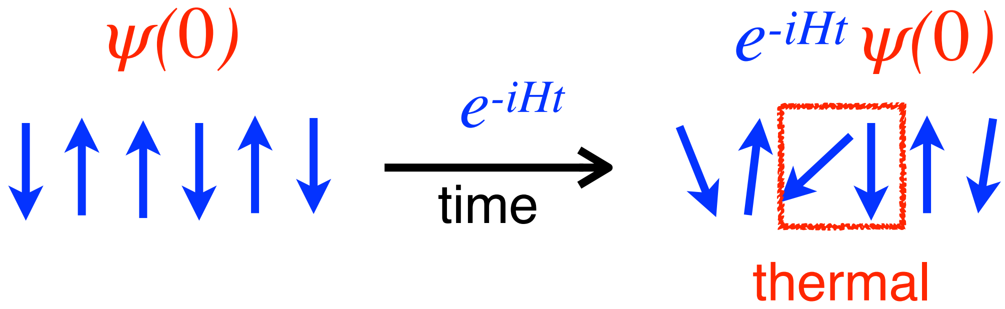


Described by quasi-local integrals of motion

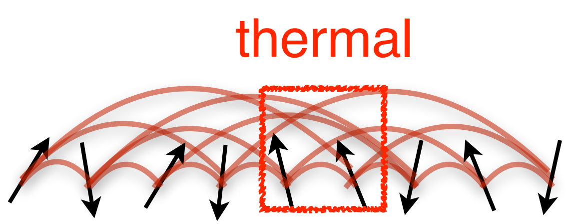
[see arXiv:1804.11065 for a review]

Eigenstate Thermalization Hypothesis

Quantum dynamics is linear:



ETH: eigenstates are thermal



→ volume-law entanglement

→ eigenstates ≈ random vectors

→ sensitivity to perturbations, level repulsion

→ matrix elements ansatz

$$O_{\alpha\beta} = \mathcal{O}(E)\delta_{\alpha\beta} + e^{-S(E)/2} f(E,\omega) R_{\alpha\beta}$$

local observables are smooth

off-diagonal matrix elements are small

[Srednicki'94] [Rigol,Dunjko,Olshanii'08]

Thermalizing vs MBL dynamics

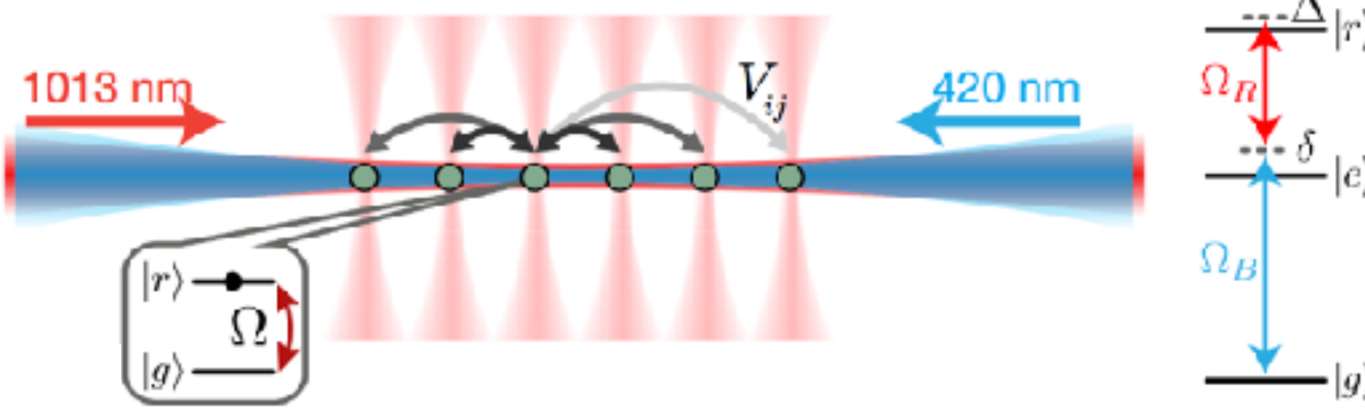
	Ergodic	Many-body localized
Entanglement growth	$S_{\text{ent}} \propto vt$	$S_{\text{ent}} \propto \xi \log Jt$
Eigenstates entanglement	$S_{\text{ent}}(A) \propto \text{vol}(A)$	$S_{\text{ent}}(A) \propto \text{area}(A)$
Integrals of motion	a few: energy, particle # $\downarrow$ Eigenstate Thermalization Hypothesis	extensive # of LIOMS $\downarrow$ Generalized Gibbs Ensemble

Is intermediate behavior possible?

Hints from experiment!

Experiments on Rydberg atoms array

Atom-by-atom assembly of Rydberg chain



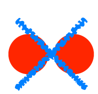
[Bernien et al, Nature 2017, arXiv:1707.04344]

Effective description: two states per atom:

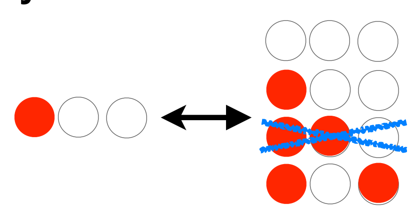
● excited (Rydberg) state

○ ground state

Rydberg blockade

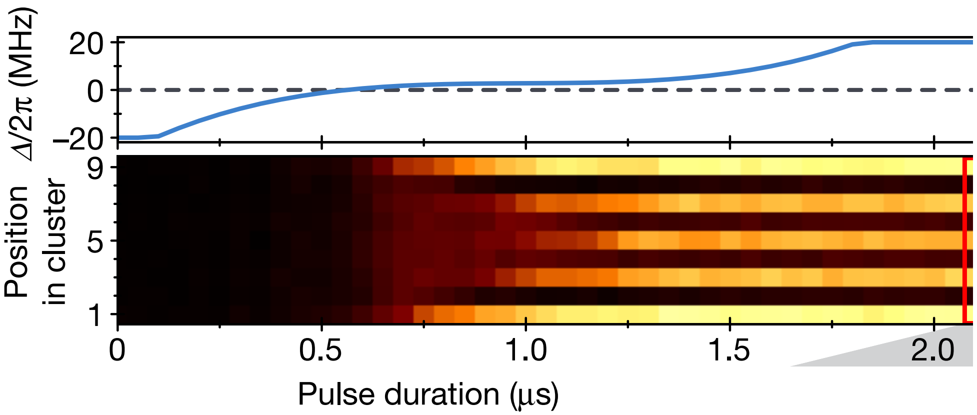


Dynamical constraint



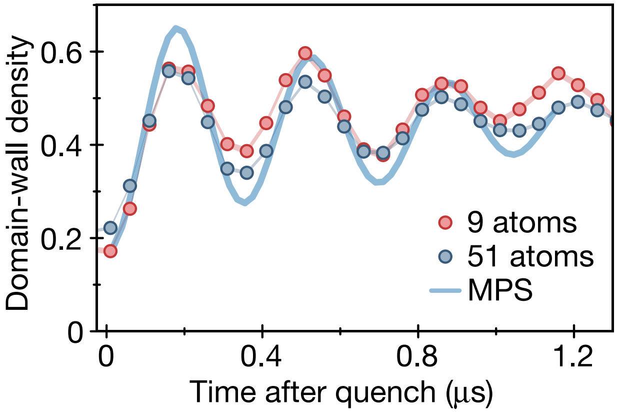
Experimental puzzle: long-time oscillations

Preparation of state



Observable:

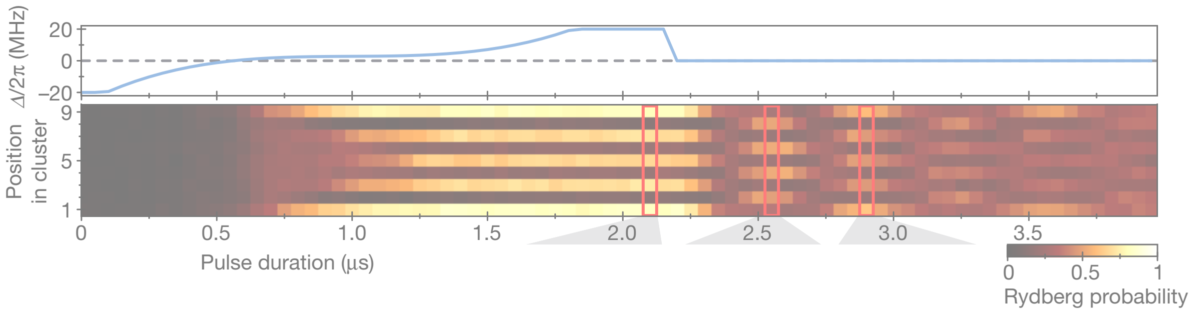
$$O = \frac{1}{L}\sum_i P_i^{\circ} P_{i+1}^{\circ}$$



Néel: long-time oscillations

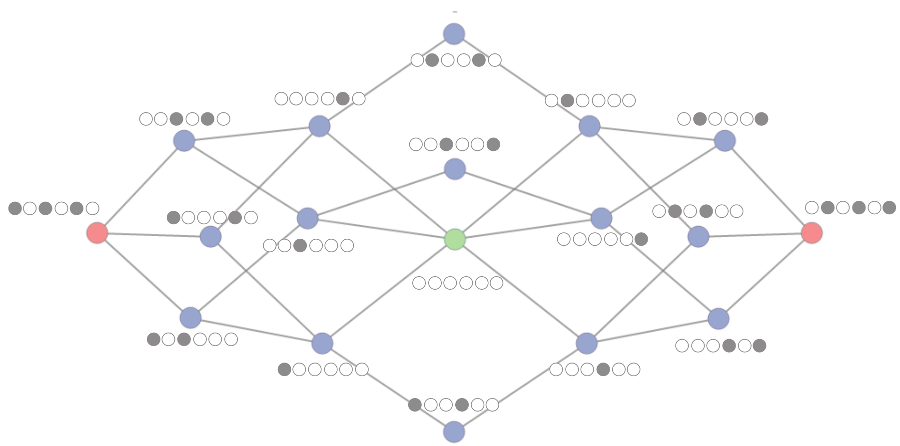
Other initial states: rapid relaxation

[Bernien et al, Nature 2017, arXiv:1707.04344]



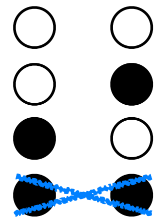
Eigenstates and dynamics of PXP model

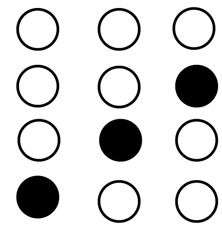
$$H = \sum_i P^{\circ}_{i-1} X_i P^{\circ}_{i+1}$$



PXP model as a graph

Hilbert space: $\mathcal{D}_2 = 3$ $\mathcal{D}_3 = 4$ $\mathcal{D}_L = F_{L-1} + F_{L+1}$



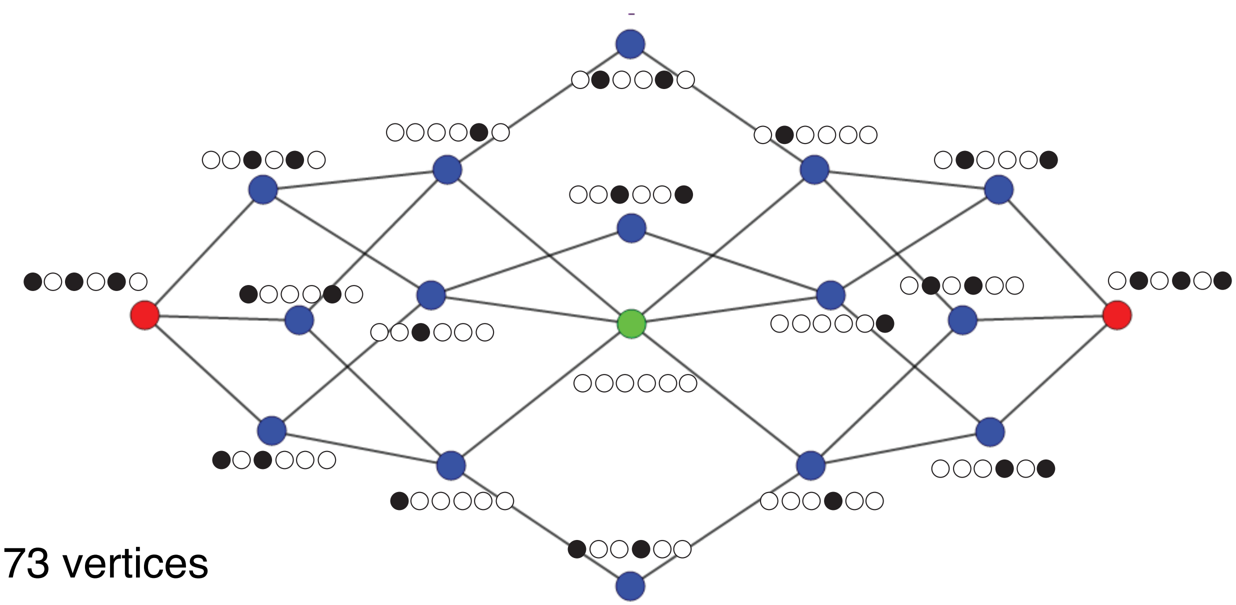


sum of Fibonacci #

no tensor product structure!

Hamiltonian: $$H = \sum_i P^{\circ}_{i-1} X_i P^{\circ}_{i+1}$$

Hilbert space + Hamiltonian = graph + adjacency matrix



Experiment: L=51

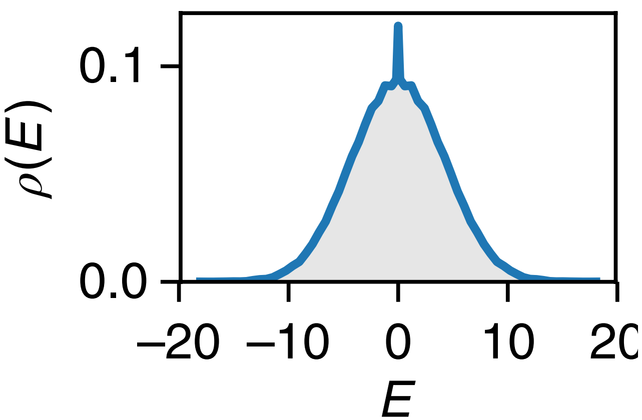
F_{53} =53,316,291,173 vertices

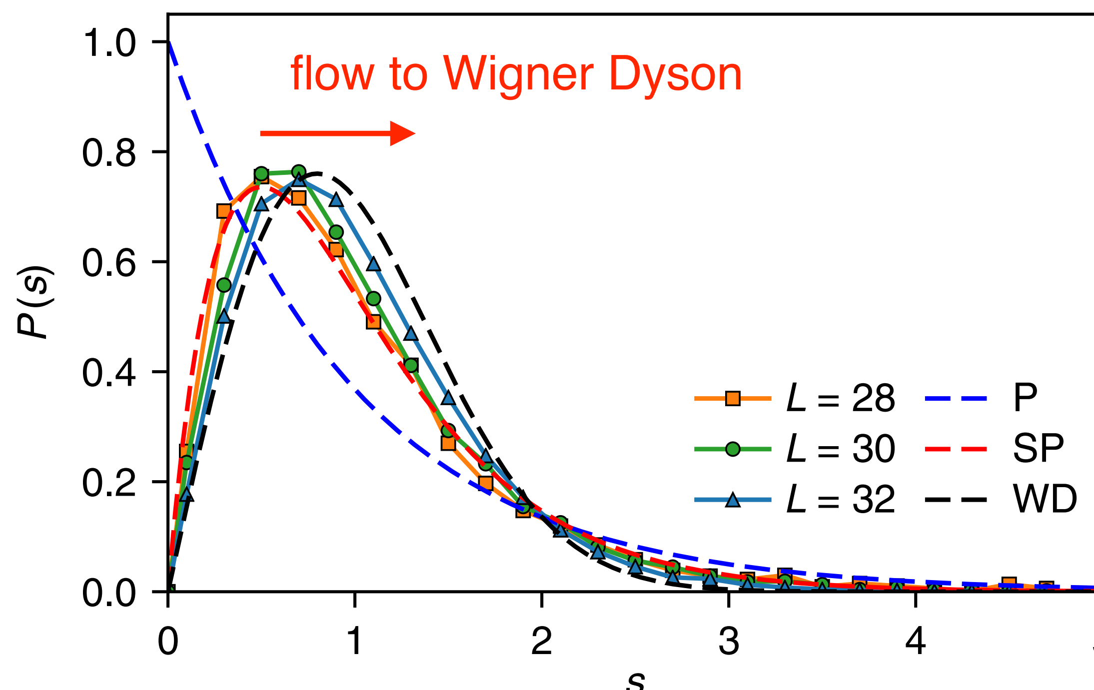
PXP model is non-integrable

Statistics of level spacings

Chaotic/ergodic	Wigner-Dyson	$P(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}$
Intermediate/critical	Semi-Poisson	$P(s) = 4 s e^{-2s}$
Integrable/MBL	Poisson	$P(s) = e^{-s}$

Gaussian DOS



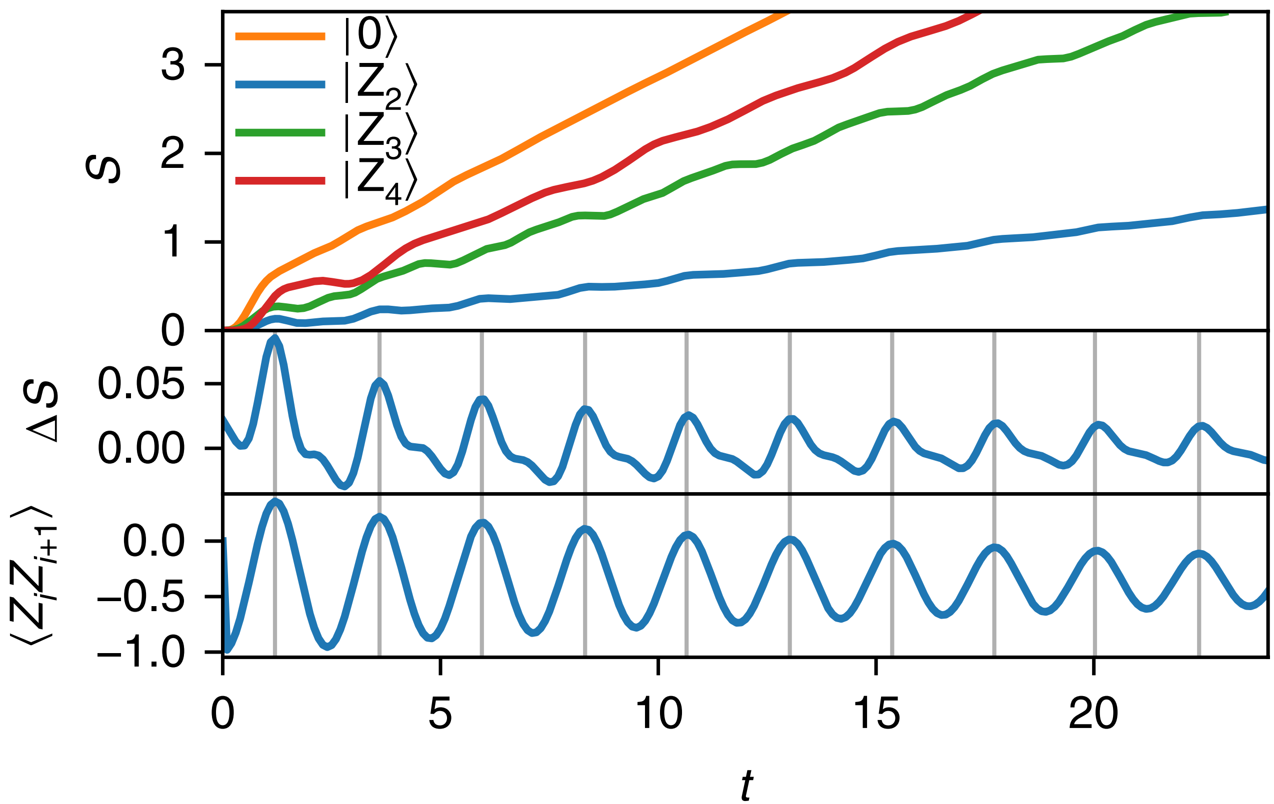


[Turner et al., Nature Physics 2018, arXiv:1711.03528]

Dynamics: ballistic growth of entanglement

Entanglement spreading depends on initial state

$|\mathbb{Z}_2\rangle = \bullet\circ\bullet\circ\bullet\circ$ $\quad |\mathbb{Z}_3\rangle = \bullet\circ\circ\bullet\circ\circ$ $\quad |0\rangle = \circ\circ\circ\circ\circ\circ$



Long-time oscillations are observed!

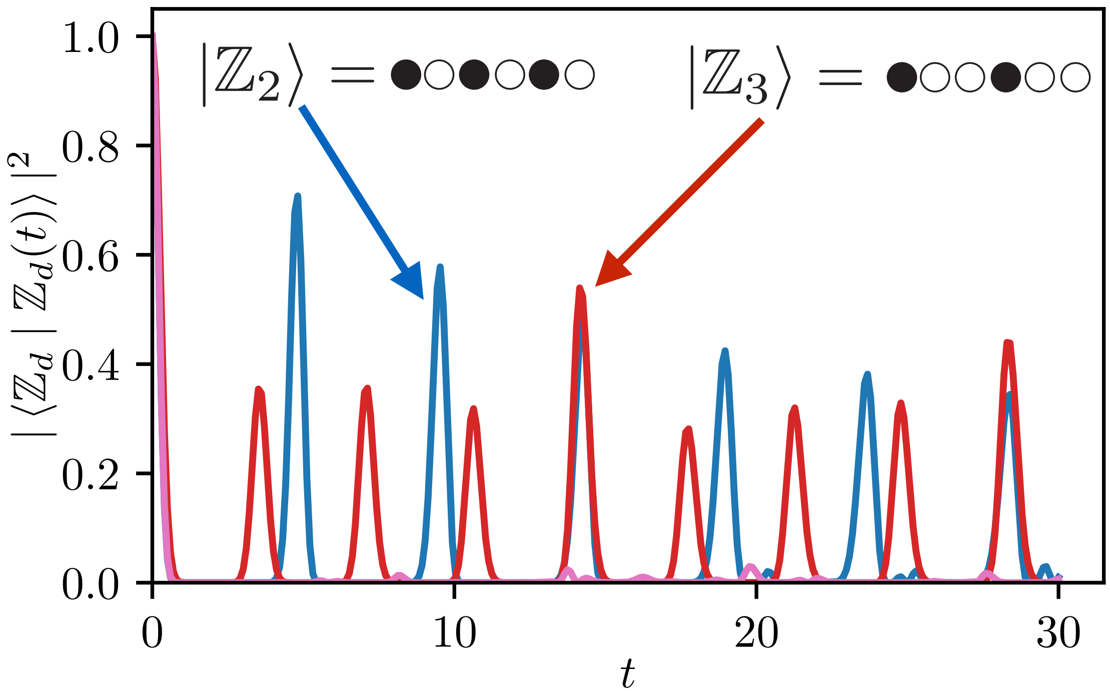
[Turner et al., Nature Physics 2018, arXiv:1711.03528]

Dynamics: revivals of many-body fidelity

Probability to return to Néel/Z_3 state:

$$\mathcal{F} = |\langle Z_2 | e^{-iHt} | Z_2 \rangle|^2$$

L=24 atoms; full Hilbert space:103,682

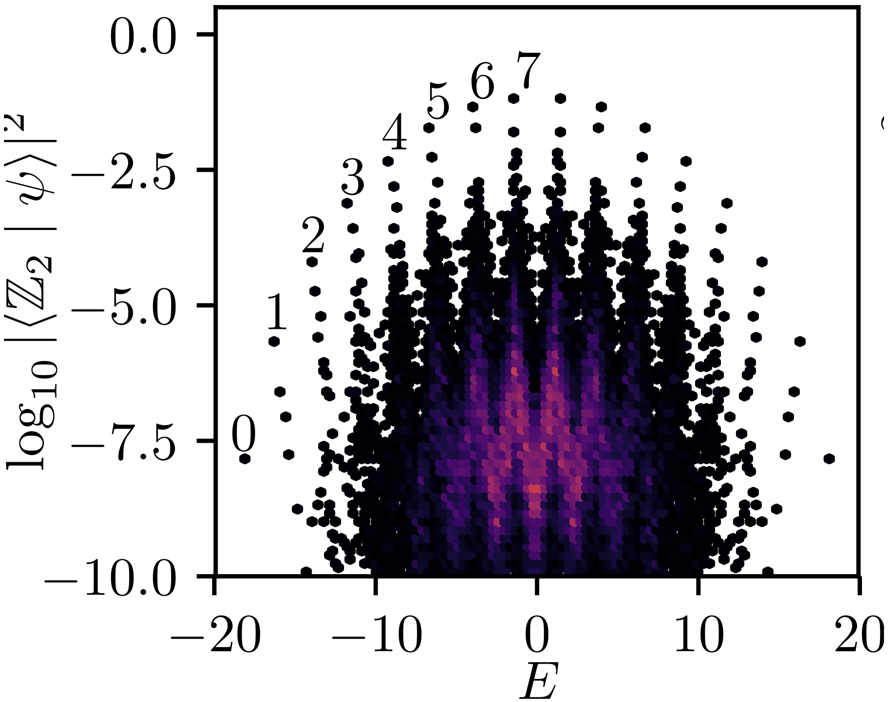


Origin of periodic revivals?

Z_2 special band of eigenstates

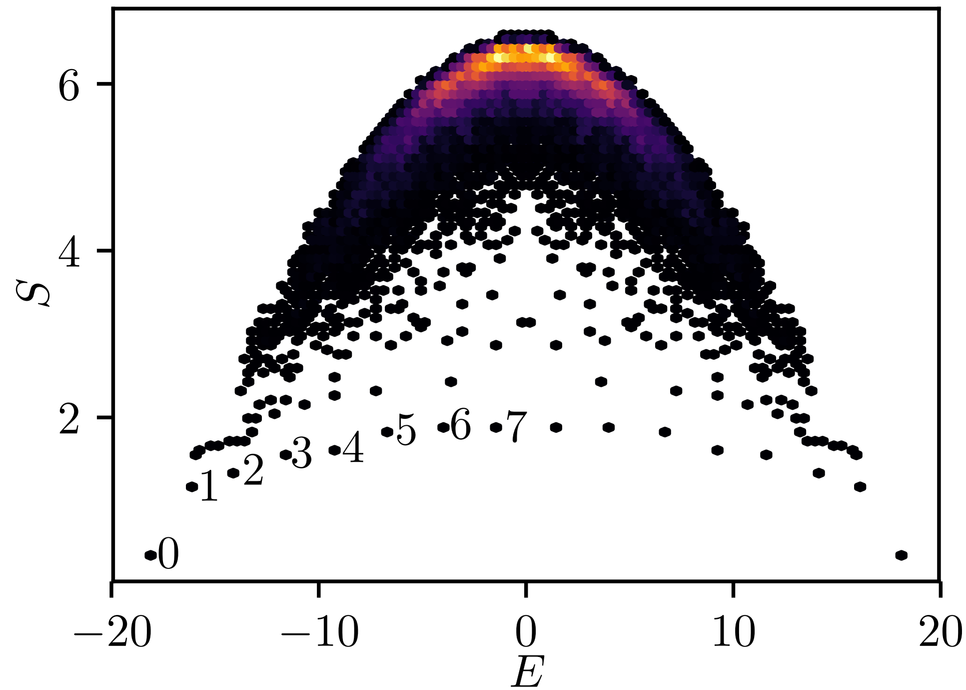
enhanced overlaps with

$|\mathbb{Z}_2\rangle = $ ●○●○●○



Vs. random eigenstates in ergodic systems

anomalously low entanglement entropy

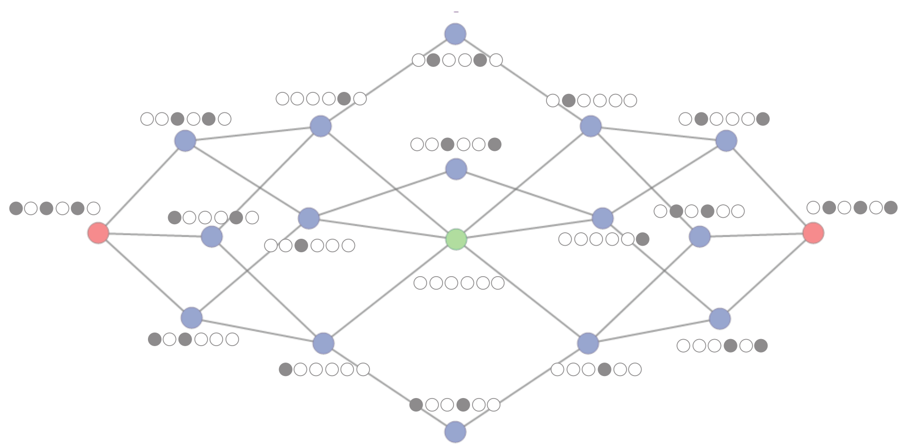


Vs. volume law S_{ent} in ergodic systems area-law in MBL systems

How to understand these special states?

$$H^{+} = \sum_{i \in \text{ even}} P_{i-1}\sigma_i^{+}P_{i+1} + \sum_{i \in \text{ odd}} P_{i-1}\sigma_i^{-}P_{i+1}$$

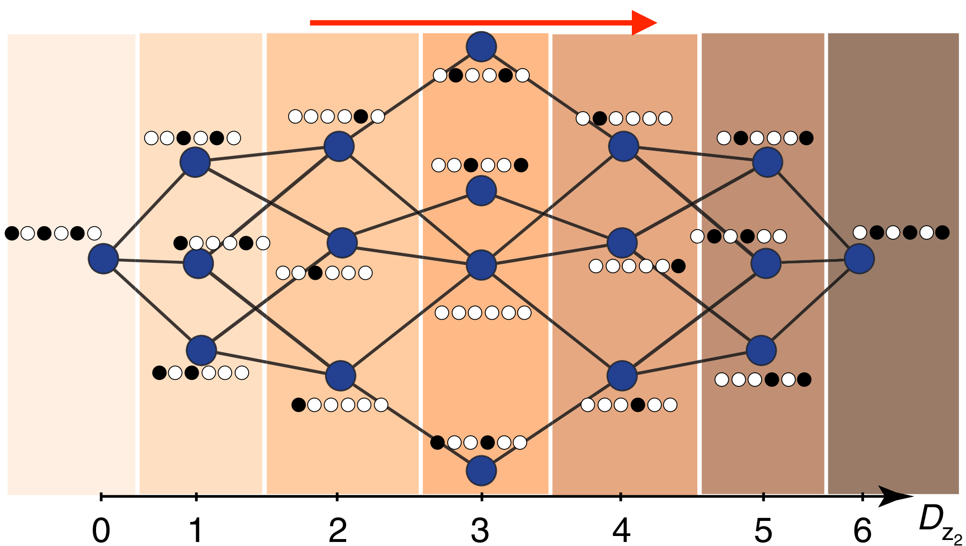
Forward scattering approximation



Constructing the special band of eigenstates

$$H = H_+ + H_- \quad = \text{forward} + \text{backward}$$

$$H^+ = \sum_{i \in \text{ even}} P_{i-1}\sigma_i^+ P_{i+1} + \sum_{i \in \text{ odd}} P_{i-1}\sigma_i^- P_{i+1}$$



$$H^z = \sum_{i \in \text{ even}} P_{i-1} Z_i P_{i+1} - \sum_{i \in \text{odd}} P_{i-1} Z_i P_{i+1}$$

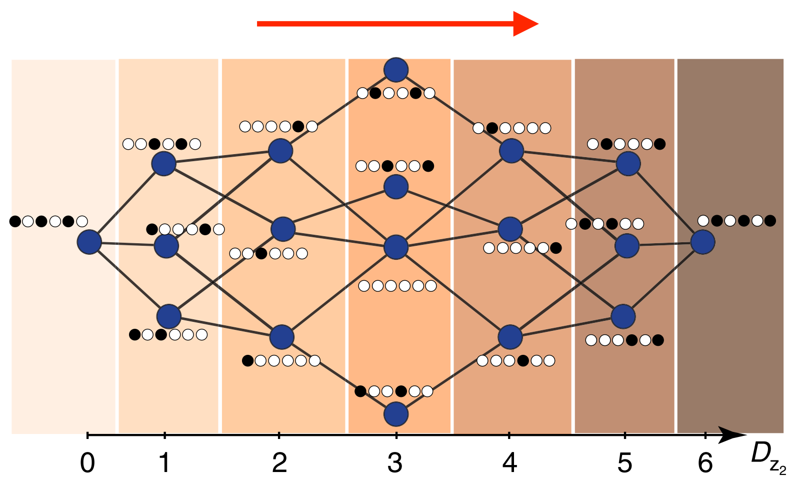
Incomplete spin algebra: $\left[H^+, H^-\right] = H^z + \sum_i P_{i-1}\sigma_i^+ P_{i+1} P_{i+2} + \ldots$

corrections

Forward scattering approximation

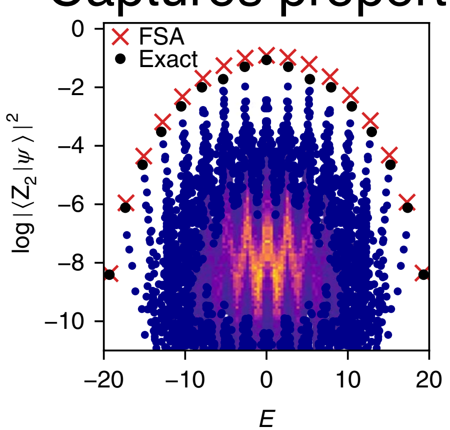
Forward scattering H^+ to construct basis:

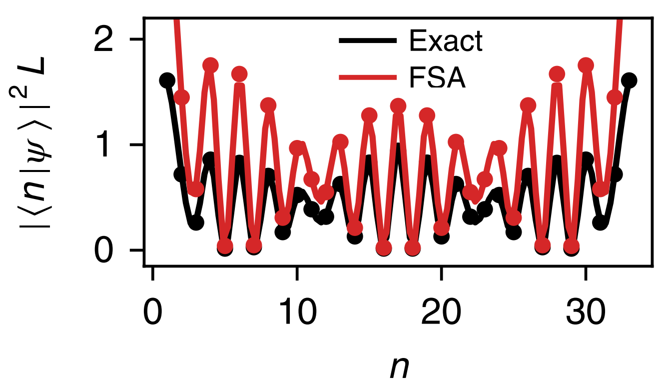
$$|\mathbb{Z}_2\rangle,\;\; H^+|\mathbb{Z}_2\rangle,\;\; [H^+]^2|\mathbb{Z}_2\rangle,\;\; \ldots,\;\; [H^+]^n|\mathbb{Z}_2\rangle$$



Projected Hamiltonian = tridiagonal matrix

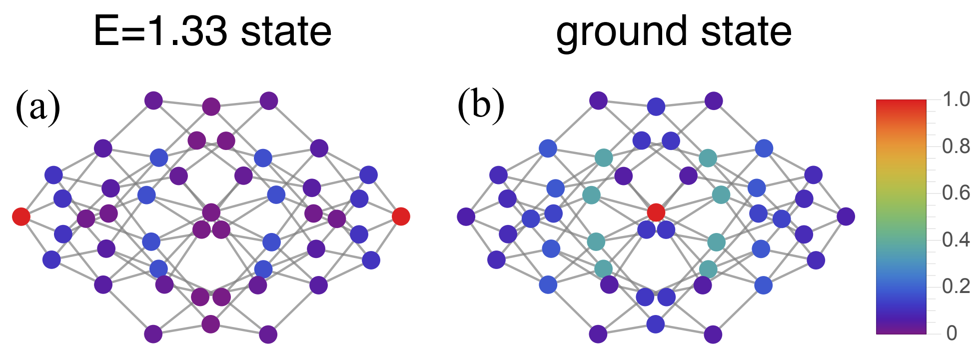
Captures properties of highly excited special states





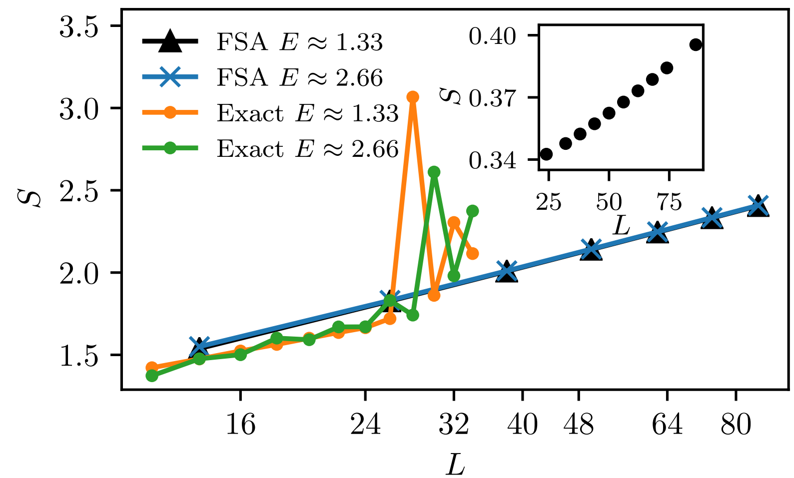
Structure of special eigenstates

Concentration on parts of Hilbert space

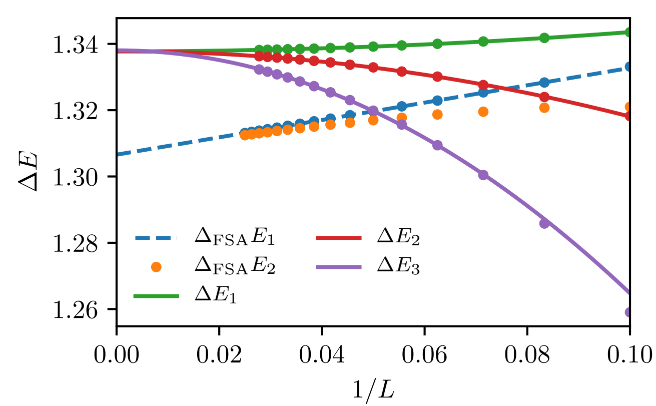


Low entanglement

$S_{\mathrm{ent}}(A) \propto \log L_A$

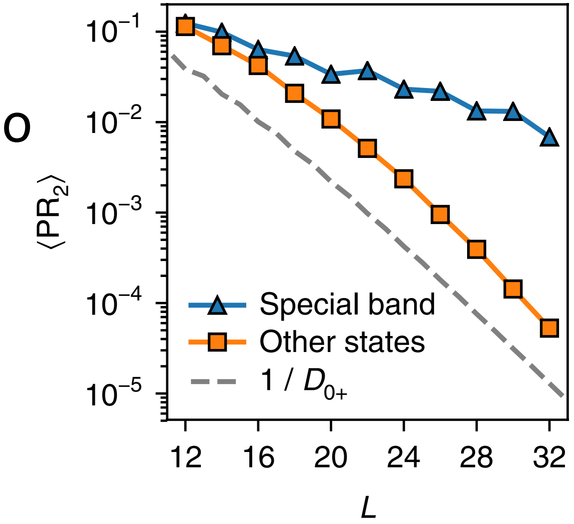


Constant ΔE
+ O(1/L^2) corrections



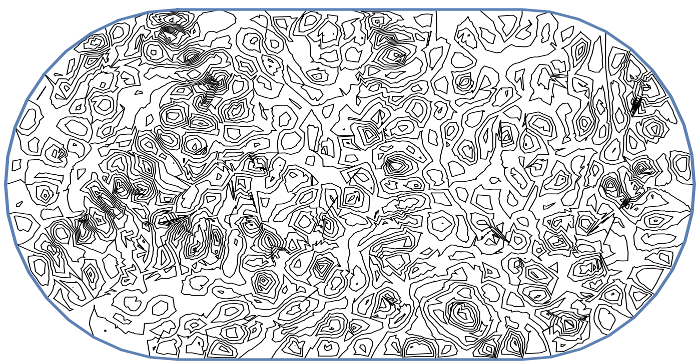
Special eigenstates as quantum many-body scars

- Concentration, low entanglement, participation ratio
- Constant energy separation
- Remaining eigenstates are “conventional”

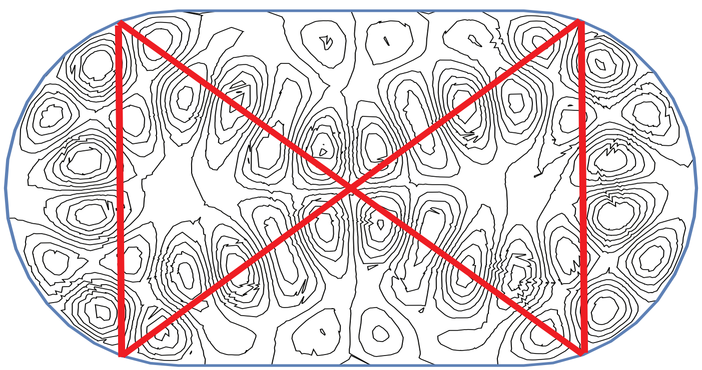


Quantum scars in single-particle chaos

typical eigenstates

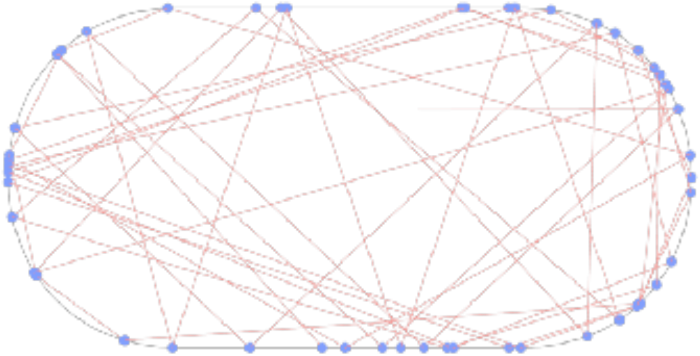


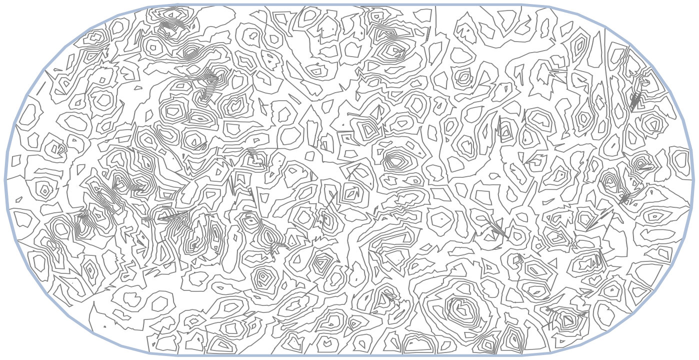
quantum scarred eigenstates



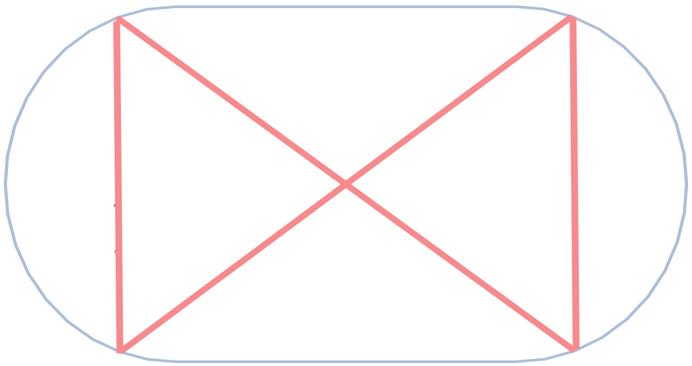
[Heller, PRL’84]

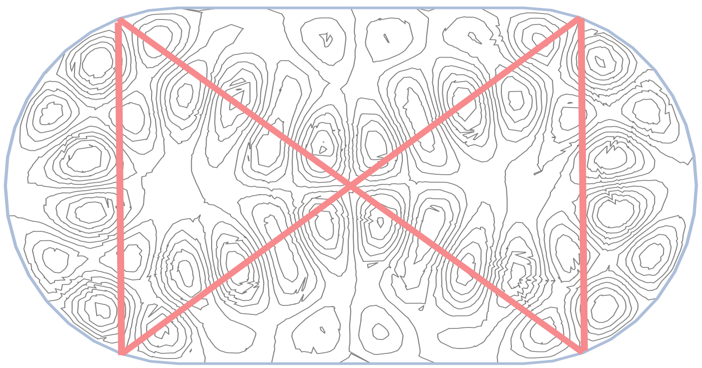
Unstable **classical** orbits influence **quantum** eigenstates





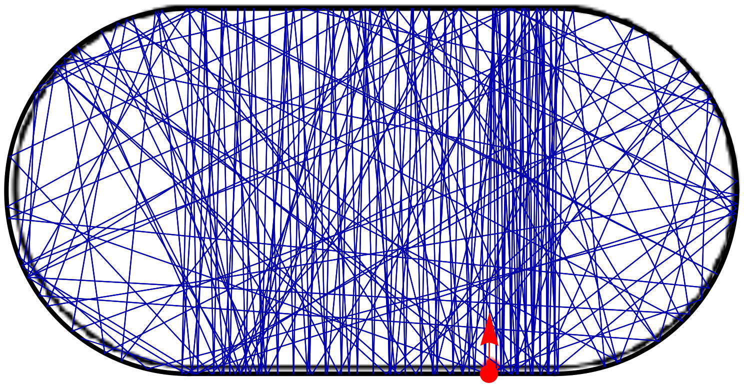
Quantum many-body scars

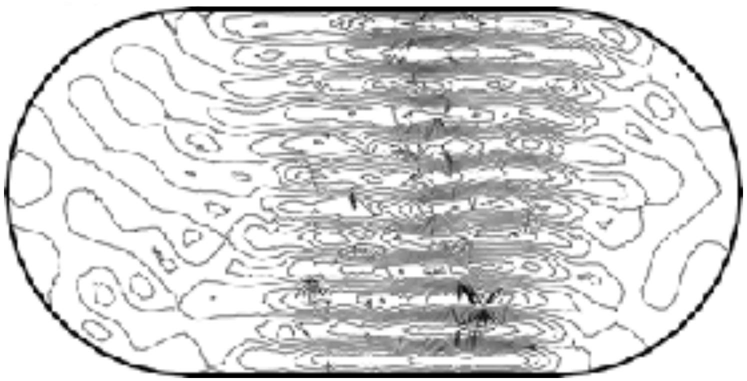




Properties of single-particle quantum scars

I. Different trajectories with small Lyapunov exponent $\lambda_L T \ll 1$





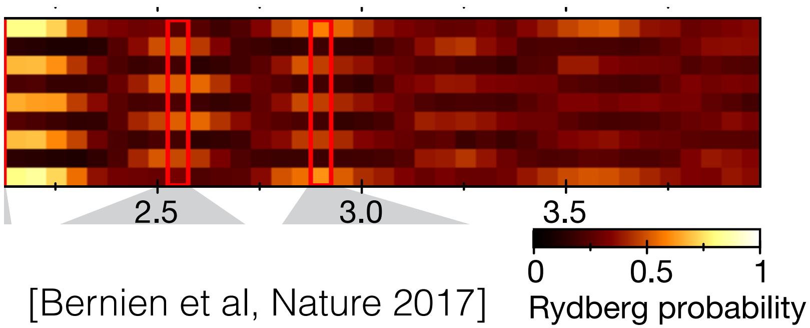
Small Lyapunov exponent $\rightarrow$ stronger scarring

II. Stability to perturbations when periodic orbit is not destroyed

Do these properties hold in many-body system?

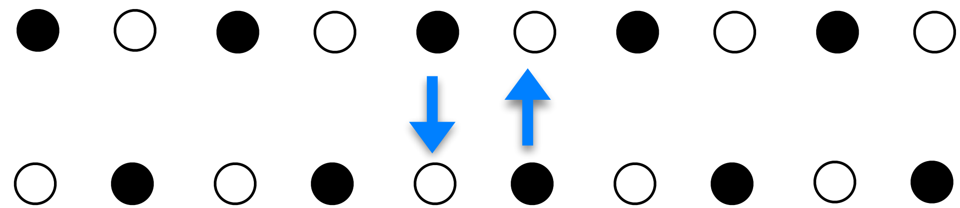
Trajectories and special bands: Z_2 and Z_3

Z_2 special eigenstates

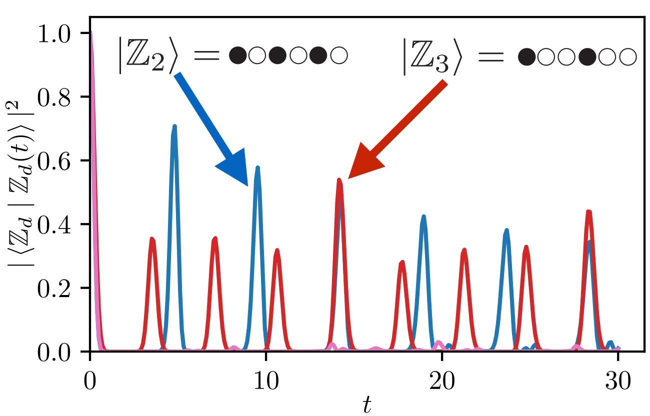


[Bernien et al, Nature 2017]

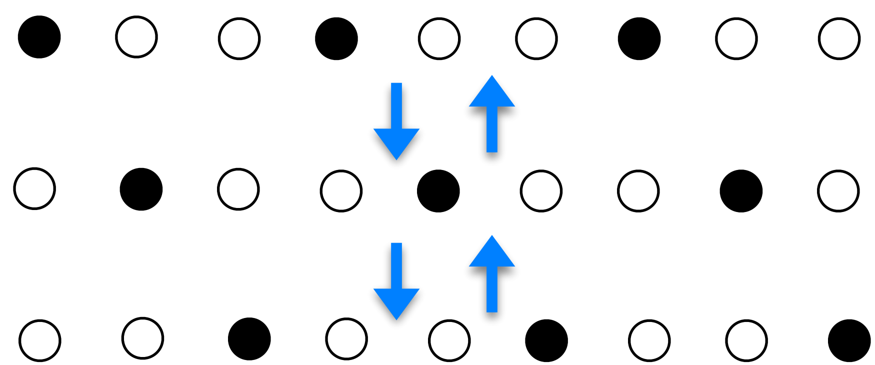
trajectory connecting 2 Néel states



Z_3 special eigenstates

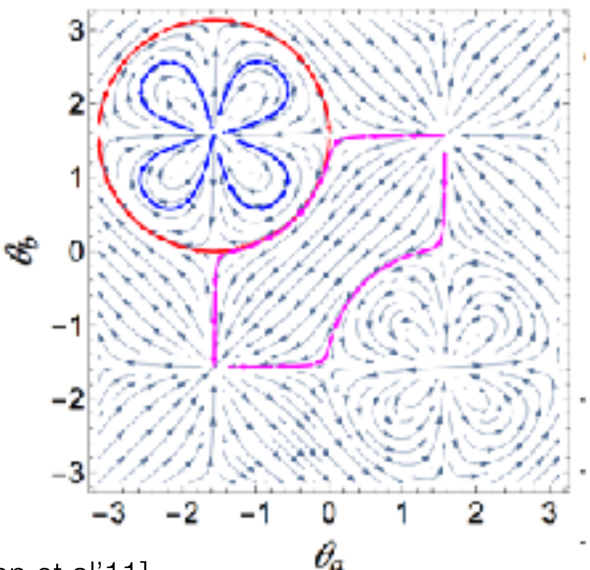


trajectory connecting period-3 CDW



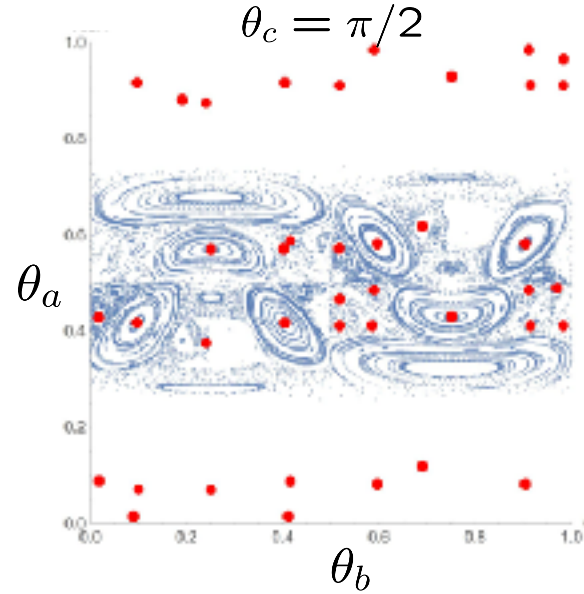
Time dependent variational principle

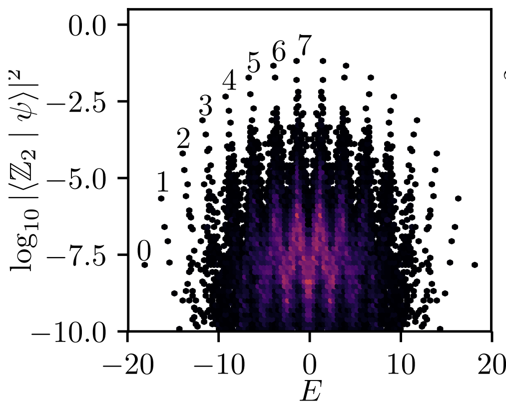
Z_2 band trajectory with TDVP

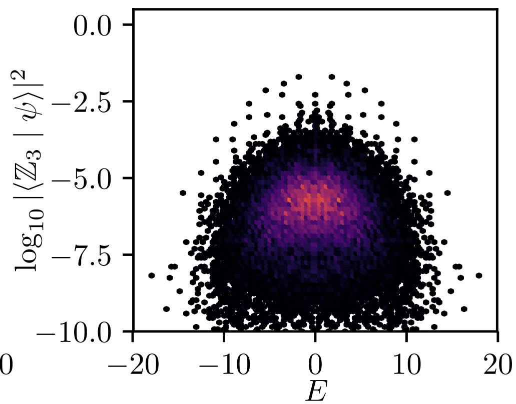


[Haegeman et al'11]
[Bernien et al, Nature'17]
[WW Ho et al, arXiv:1807.01815]

Z_3 band trajectories







Z_3 trajectory
is more unstable
→ weaker scarring

Stability to perturbations

Oscillations can persists with O(1) perturbation

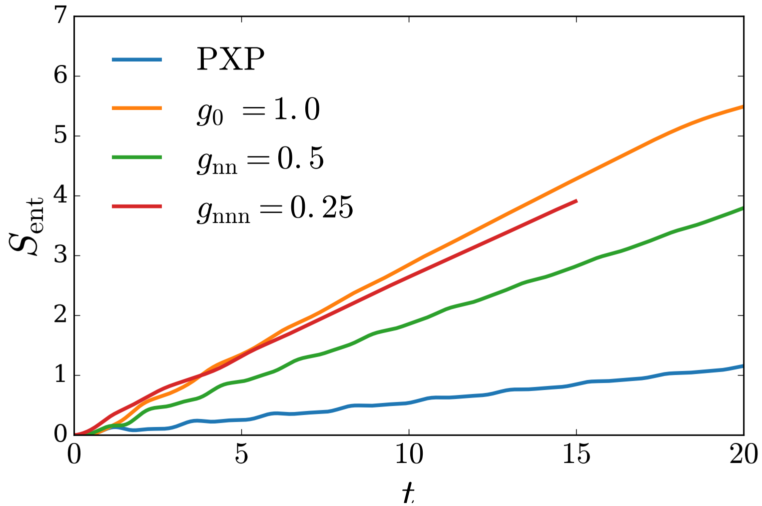
[Turner et al., arXiv:1806.10933]

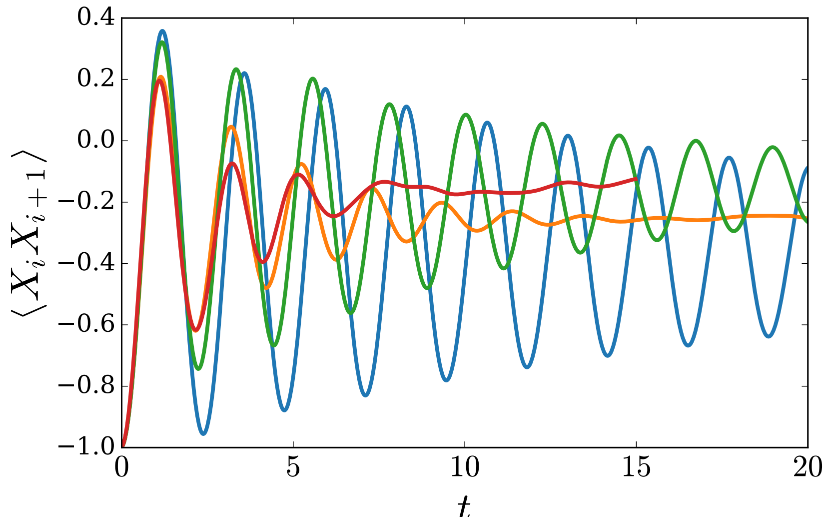
$$\delta H_0 = g_0 \sum_j Q_j,$$

$$\delta H_{\mathrm{nn}} = g_{\mathrm{nn}} \sum_j P_{j-1}(\sigma_j^+ \sigma_{j+1}^- + \sigma_j^- \sigma_{j+1}^+) P_{j+2}$$

$$\delta H_{\mathrm{nnn}} = g_{\mathrm{nnn}} \sum_j P_{j-1} X_j P_{j+1} X_{j+2} P_{j+3}.$$

Enhancing entanglement growth ≠ destroying orbit





Perturbations that remove scars restore “canonical” ETH

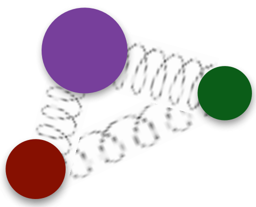
Summary and outlook

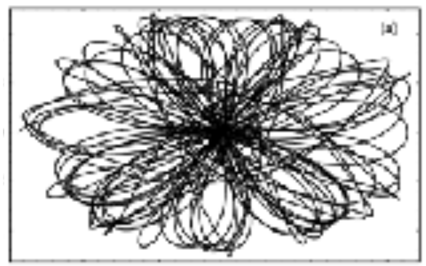
Ergodicity and integrability

Classical

<u>Ergodic systems</u>

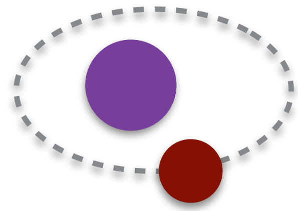
chaos → ergodicity

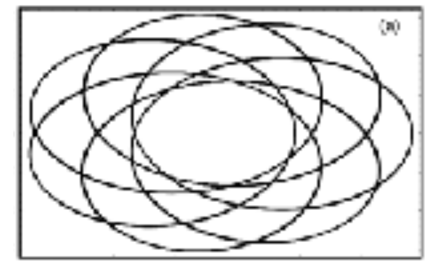




<u>Integrable systems</u>

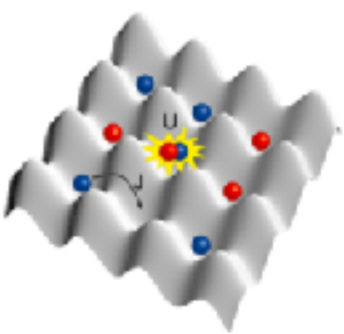
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

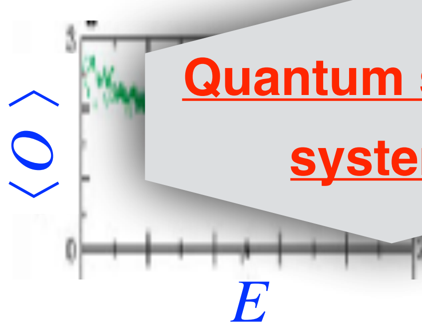




Quantum

Thermalizing phases

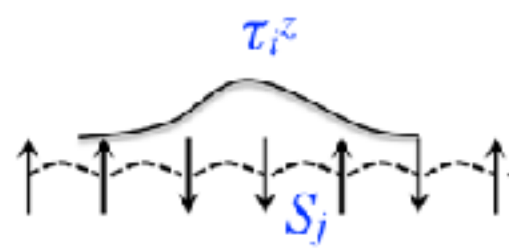




<u>Quantum scarred systems</u>

MBL phases

emergent integrability

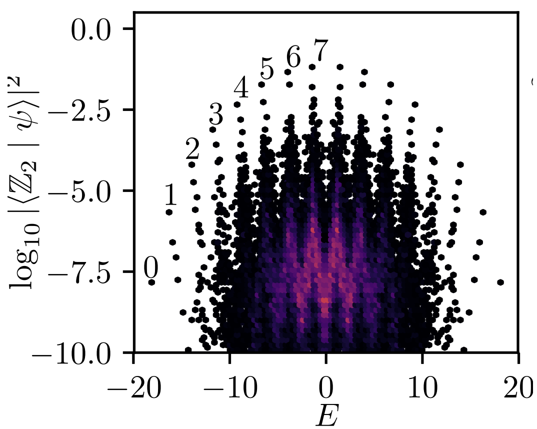


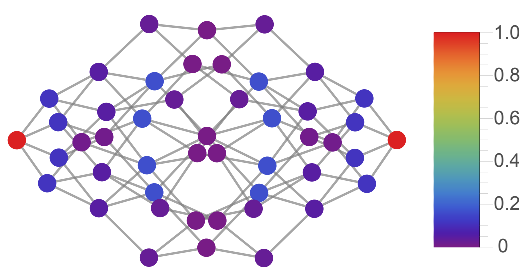
Quantum many-body scars

- Weak ergodicity breaking:
  - ✳ special eigenstates: low entanglement, large $Z_{2,3}$ overlaps, no ETH
  - ✳ explains recent experiments; stable to perturbations

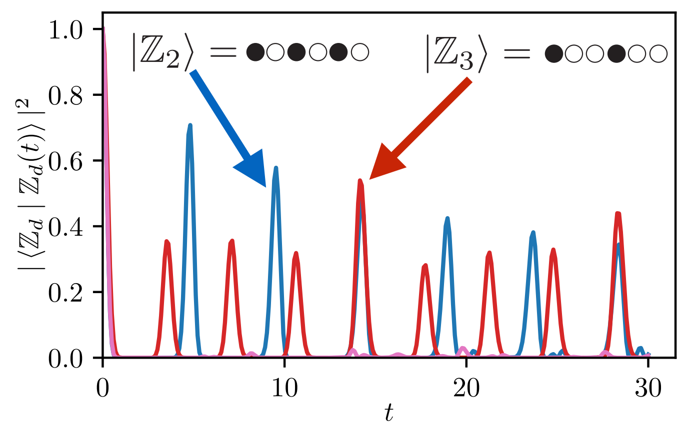
- Open questions:
  - ✳ classes of models with quantum scars?
  - ✳ quasiclassics, meaning of Lyapunov exponent?
  - ✳ use scars and/or zero modes to protect quantum information?

[Turner et al., Nature Physics 2018]



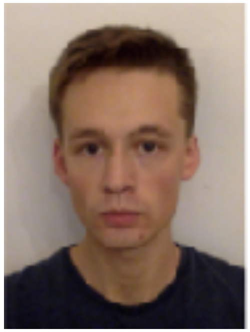


[Turner et al., arXiv:1806.10933]



Acknowledgments

Christopher Turner



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