

# Spontaneous out-of-equilibrium plasmonic magnetism

Justin Song

SPICE workshop on “Collective phenomena in driven quantum systems”

Mainz, 2018





## Berryogenesis: self-induced Berry flux and spontaneous out-of-equilibrium magnetism

MS Rudner, JS, arXiv 1807.01708 (2018)



Mark Rudner (KU)

## Hidden (emergent) plasmon internal structure and plasmon Berry phase

LK Shi, JS, PRX 8, 021020 (2018)



Li-kun Shi (Singapore)

Funding:

**NATIONAL  
RESEARCH  
FOUNDATION**



**NANYANG  
TECHNOLOGICAL  
UNIVERSITY  
SINGAPORE**

# Electronic matter close to equilibrium

determined by thermodynamics

## Topological Insulators

Surfaces of 3D TIs:

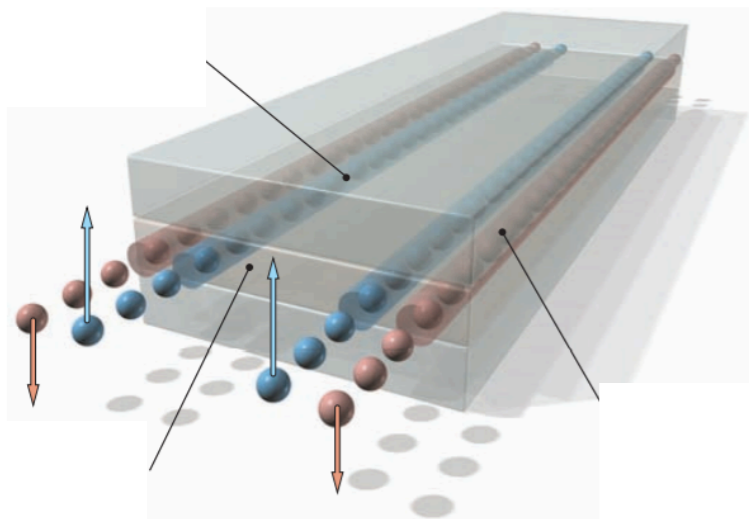
$\text{Bi}_2\text{Se}_3$ ,  $\text{Bi}_2\text{Te}_3$ ,  $\text{Bi}_x\text{Sb}_{1-x}$ , ...

Topological Crystalline Insulators:  
 $\text{SnTe}$ , ...

Magnetic Topological Insulators:  
Cr-doped  $\text{BiSbTe}$

$\text{Hg}_x\text{Cd}_{1-x}\text{Te}$  Quantum Wells,  
 $\text{InAs}/\text{GaSb}$  QWs

Monolayer  $\text{WTe}_2$



## 3D Dirac/Weyl

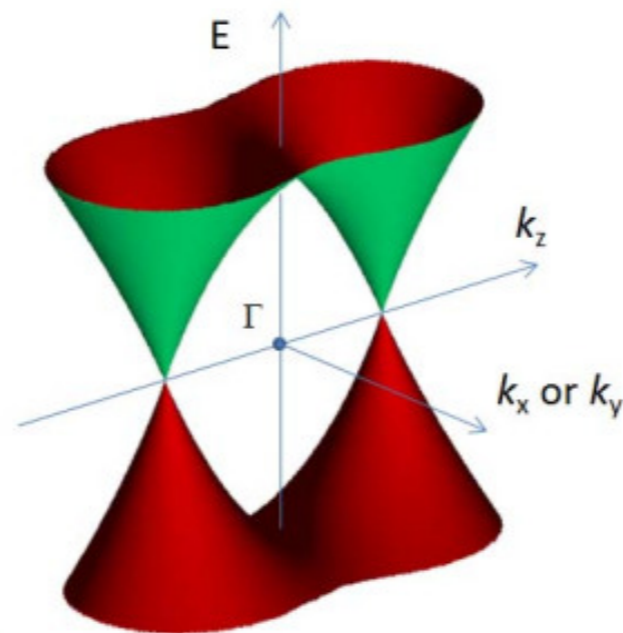
Experimentally Observed:

$\text{Cd}_3\text{As}_2$ ,  $\text{Na}_3\text{Bi}$ ,  $\text{TiBiSe}$   
 $\text{TaAs}_2$ , ...

Type II Weyl semimetals  
(candidates): (bulk)  $\text{WTe}_2$ ,  
 $\text{MoTe}_2$

Proposed in TI stacks;  
 $\text{HgCdTe}$  Stacks

Nodal-line semimetals



## 2D Dirac Materials

(materials that host Berry curvature)

Graphene heterostructures:  
 $\text{G}/\text{hBN}$ ,  
dual-gated Bilayer graphene, ...

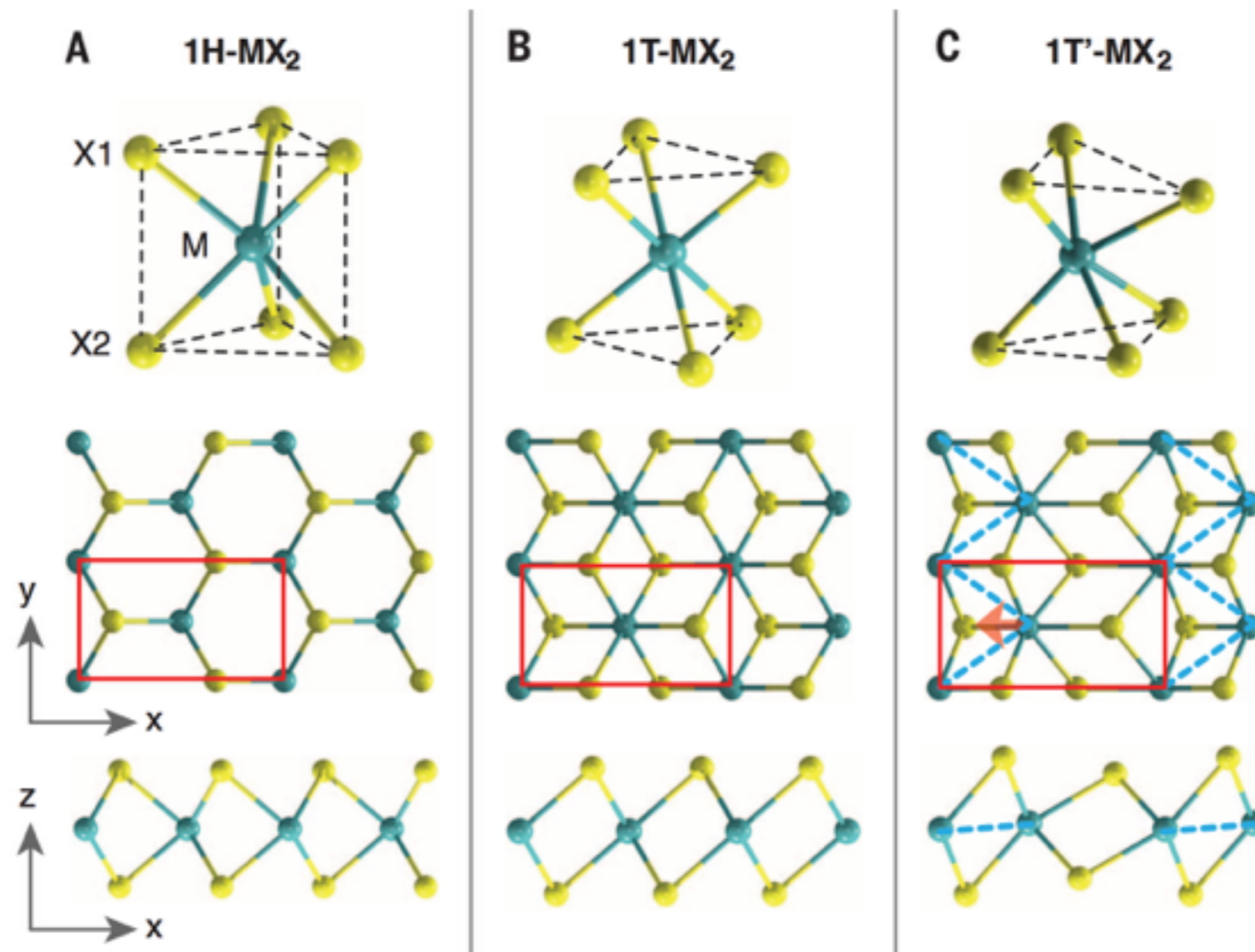
Transition metal dichalcogenides:  
 $\text{MoS}_2$ ,  $\text{WS}_2$ ,  $\text{WSe}_2$ ,  $\text{MoSe}_2$ ,  
 $\text{MoTe}_2$ , ...



# Electronic matter close to equilibrium

determined by thermodynamics

variety of possible structures for MX<sub>2</sub> monolayer



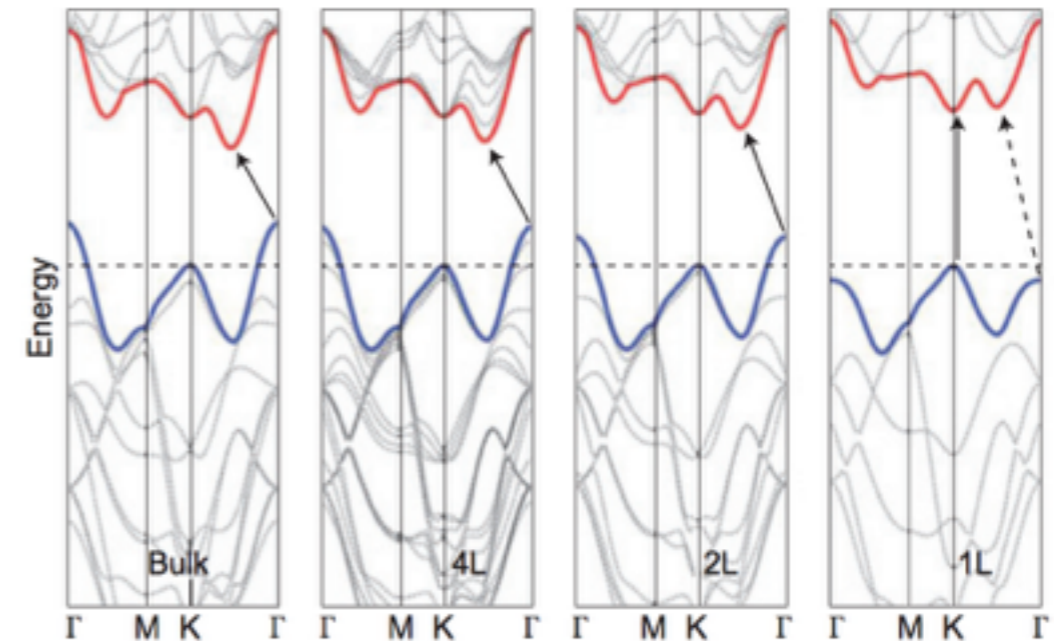
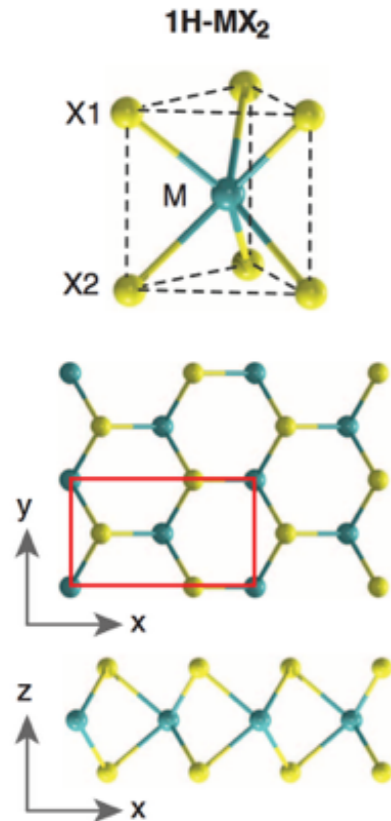
adapted from: Qian, Liu, Fu, Li Science (2014)

# Electronic matter close to equilibrium

determined by thermodynamics

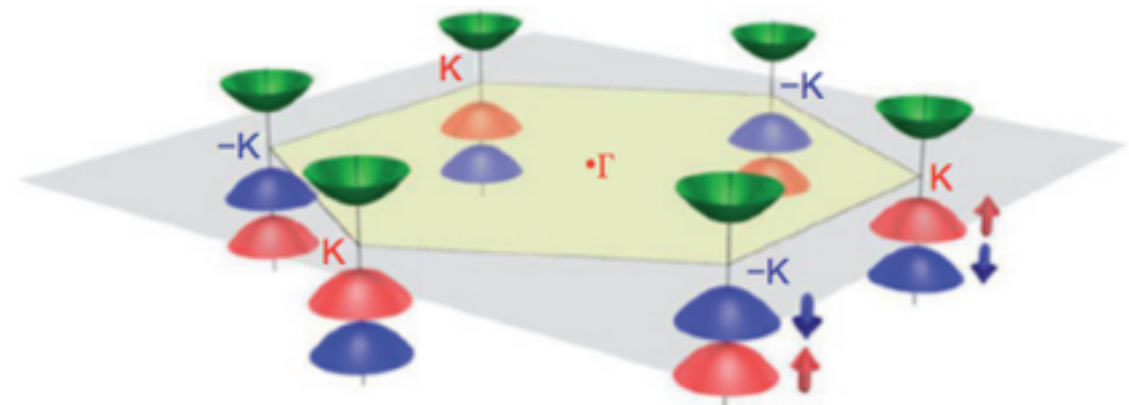
MoS<sub>2</sub>

bandstructure evolution from bulk to monolayer



adapted from Qian, Liu, Fu, Li Science (2014)

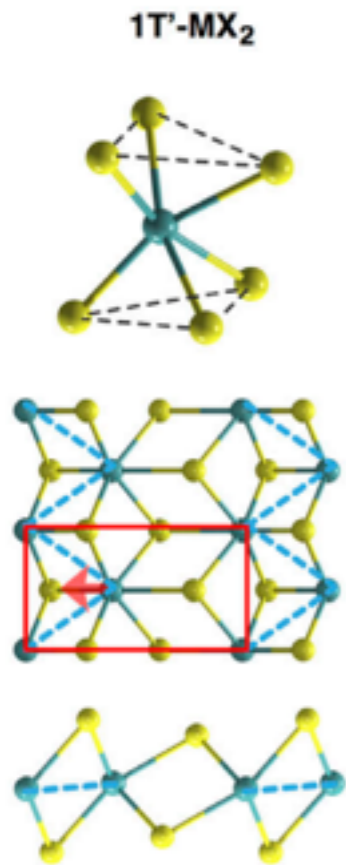
valley-locked spin structure



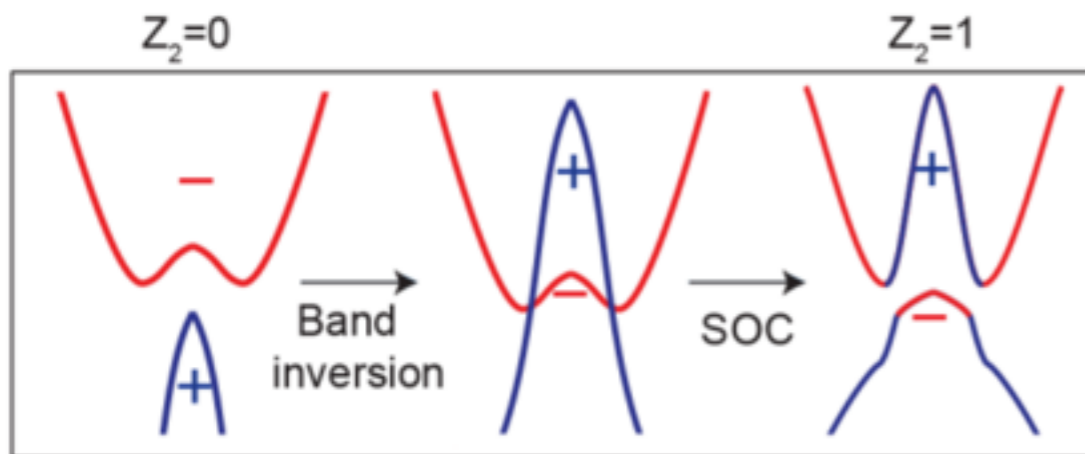
M Chhowalla et al, Nature Chemistry 2013

# Electronic matter close to equilibrium

determined by thermodynamics



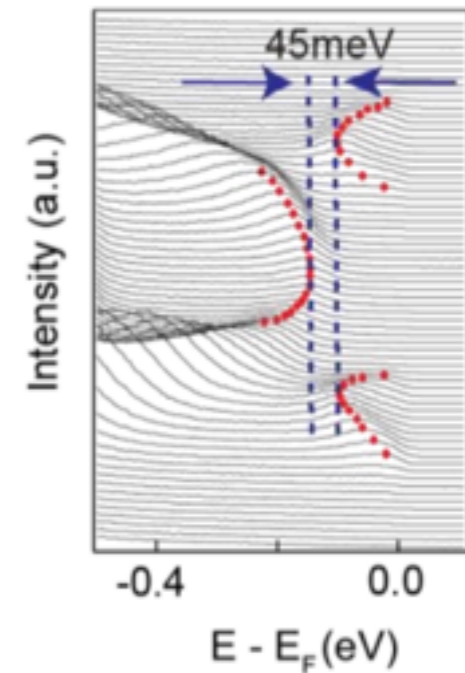
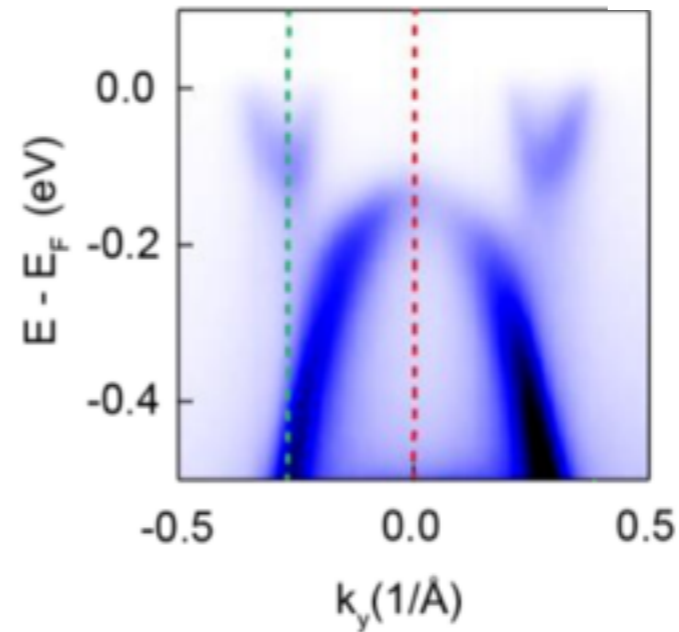
Qian, Liu, Fu, Li Science (2014)



adapted from Tang et al, Nature Physics (2017)

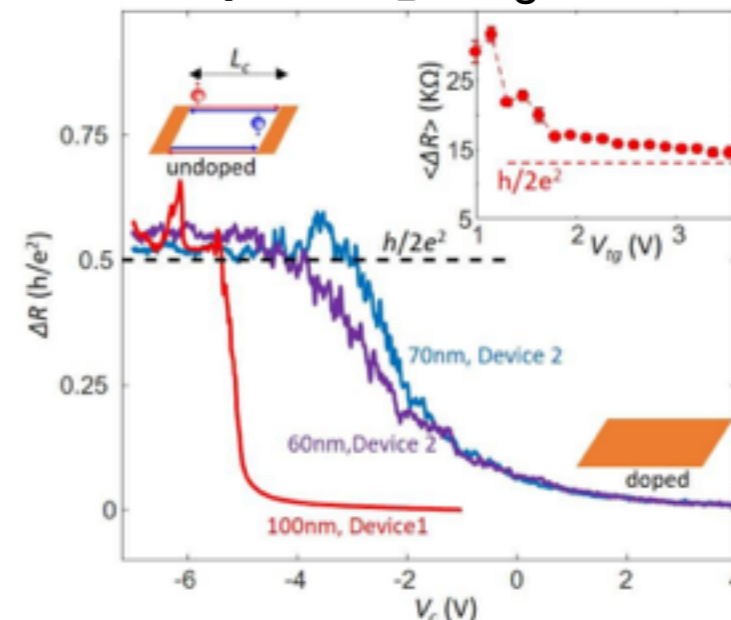
1T'-WTe<sub>2</sub>

[electronic bandstructure] ARPES data



Tang et al, Nature Physics (2017)

[electronic responses] Edge Conduction (“near quantized”)



Wu, Fatemi, et al, Science (2018)

# driven systems: out-of-equilibrium materials

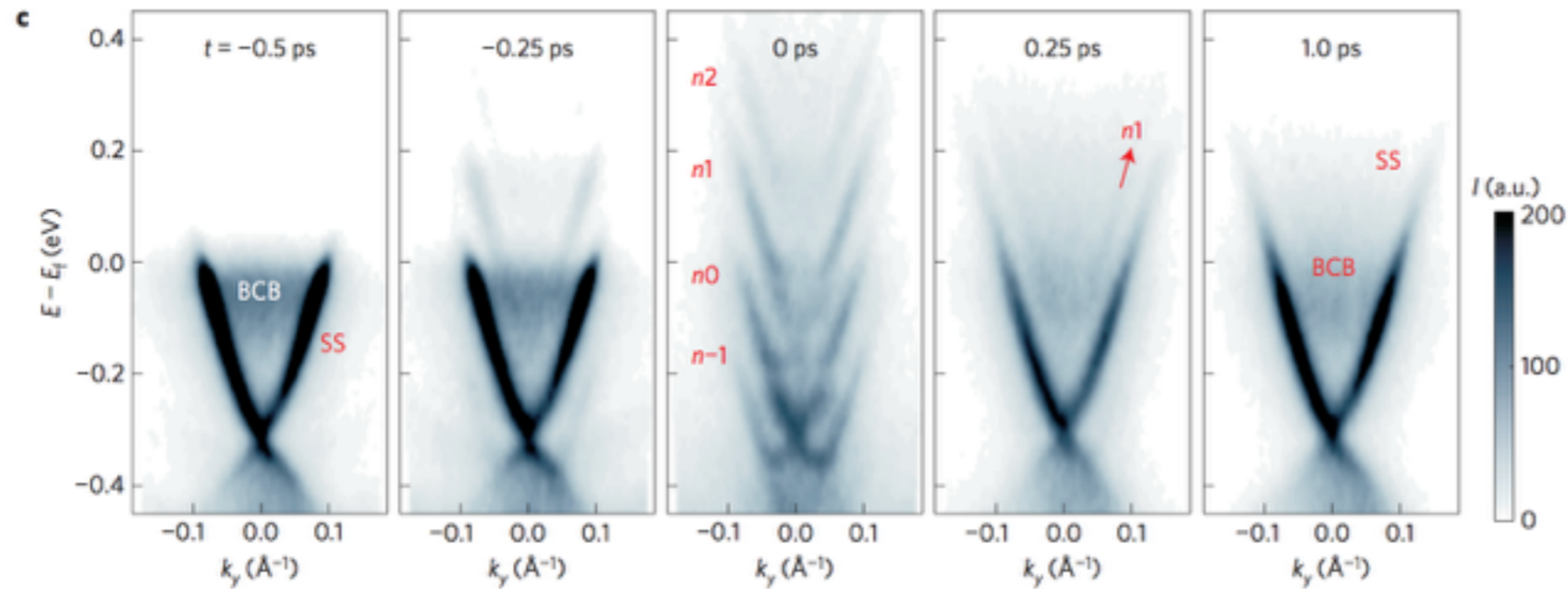
overcoming the tyranny of thermodynamics

some strategies:

- a. non-equilibrium driving for designer hamiltonians
- b. unconventional out-of-equilibrium responses
- c. exploiting collective modes for new phases/structure

# Non-equilibrium driving: tailored hamiltonians

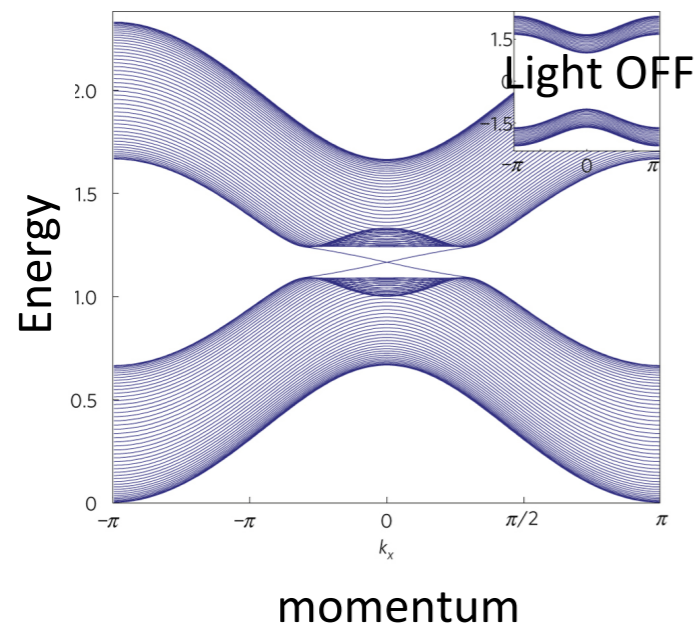
## Floquet engineering



see e.g., Fahad's talk on Wednesday

F Mahmood, et al, Nature Physics (2016)

## Floquet alchemy: transmuting trivial insulator into topological insulator



N Lindner, G Refael, V Galitski, Nature Physics (2011)

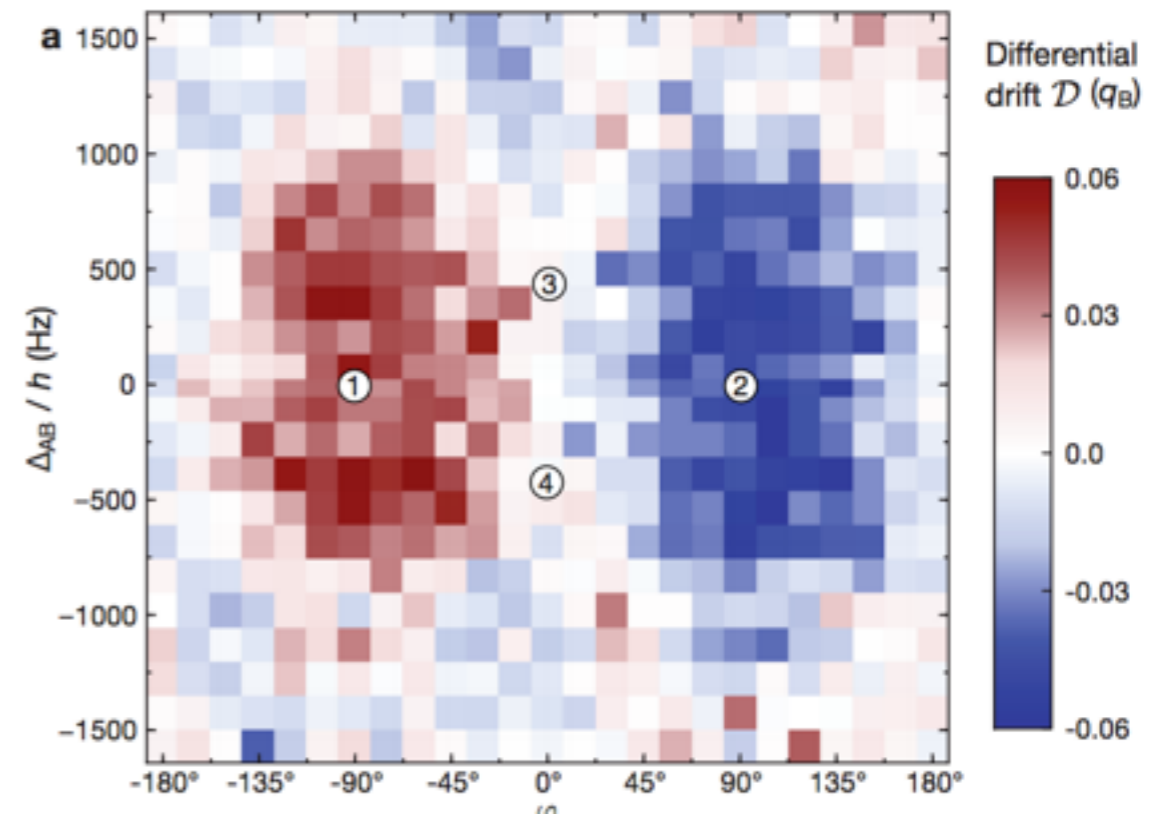
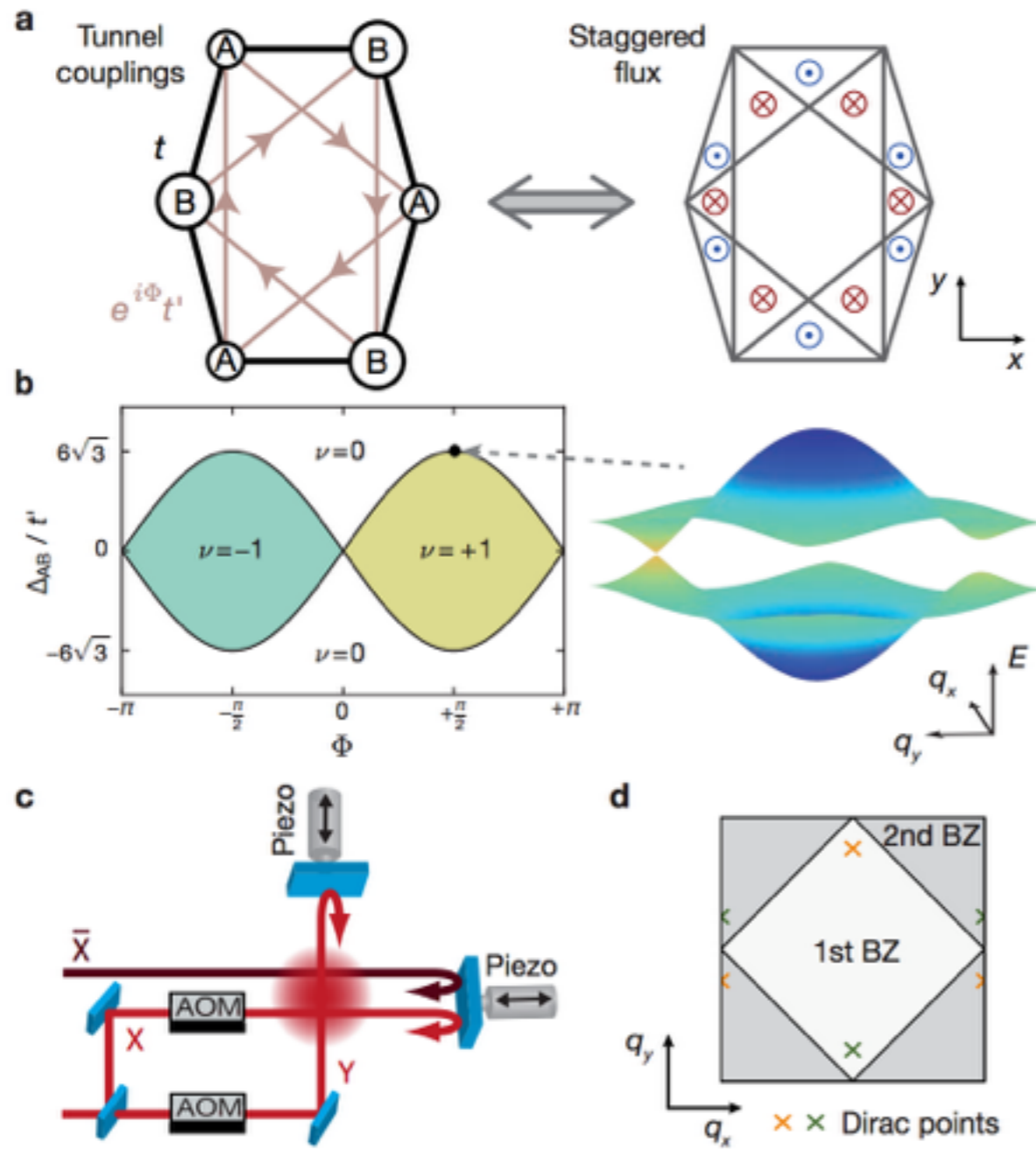
also large zoo of new Floquet effects:  
e.g., Rudner, Berg, Lindner, Levin PRX 2013



# Driven systems and Berry curvature

designer hamiltonians: engineering band structure

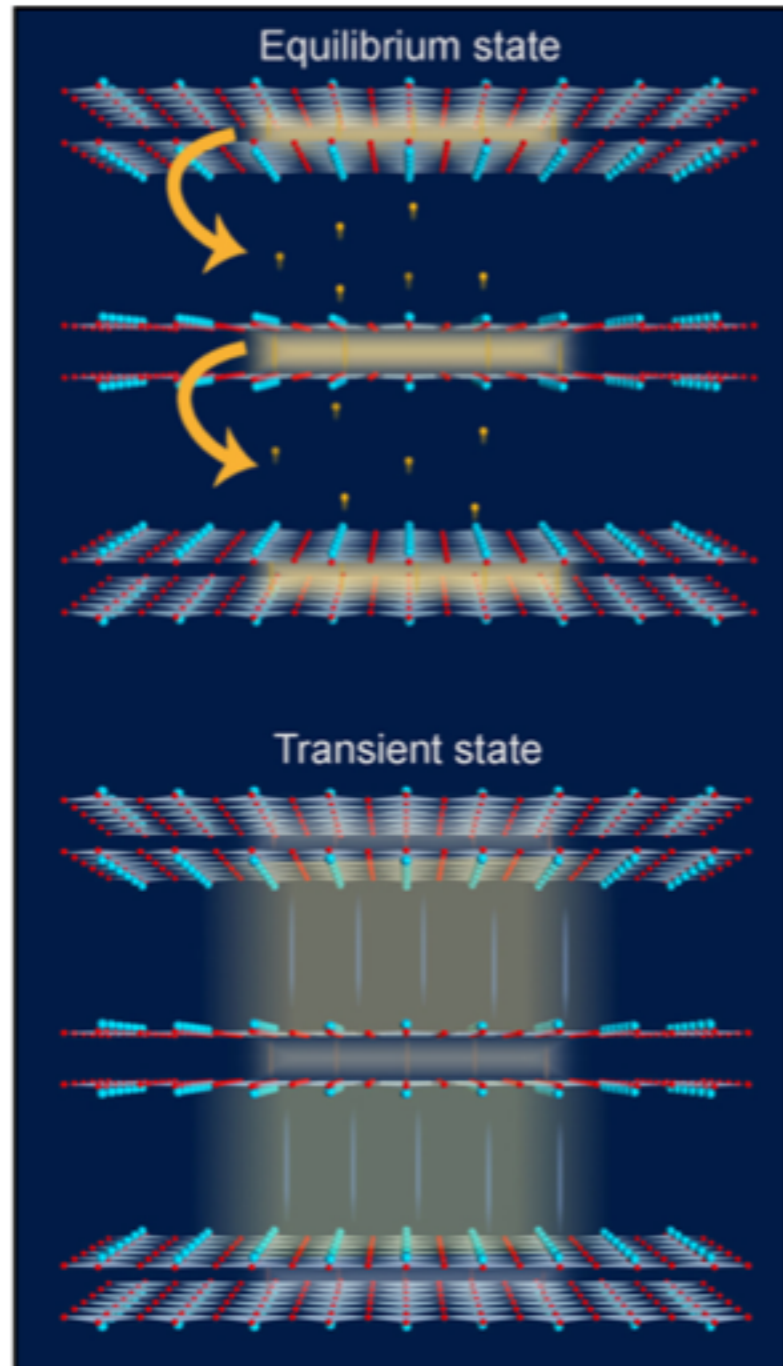
see e.g., Monika's talk on Tuesday



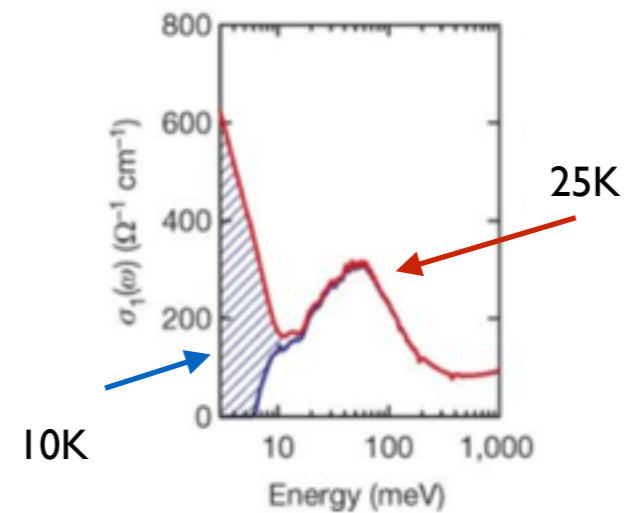
# Driven systems: Out-of-equilibrium phases

designer hamiltonians: engineering interactions

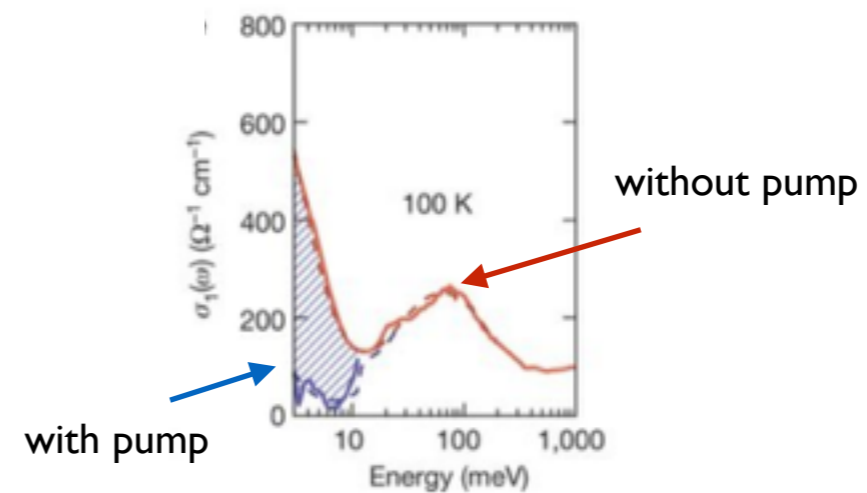
Light-induced superconductivity



equilibrium spectral weight transfer in K3C60



out-of-equilibrium spectral weight transfer in K3C60



M Mitrano, et al Nature (2016),

taken from cavilieri group website

see e.g., D Fausti, et al Science (2011),

# driven systems: out-of-equilibrium materials

overcoming the tyranny of thermodynamics

some strategies:

- a. non-equilibrium driving for designer hamiltonians
- b. unconventional out-of-equilibrium responses**
- c. exploiting collective modes for new phases/structure

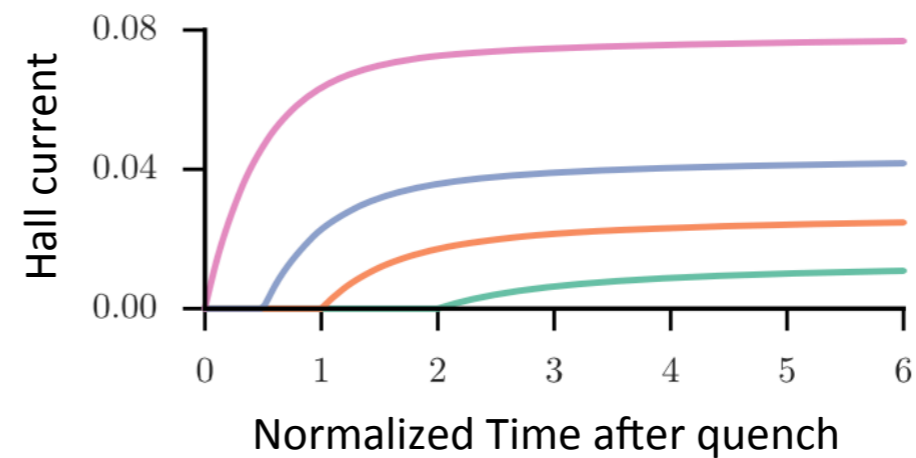
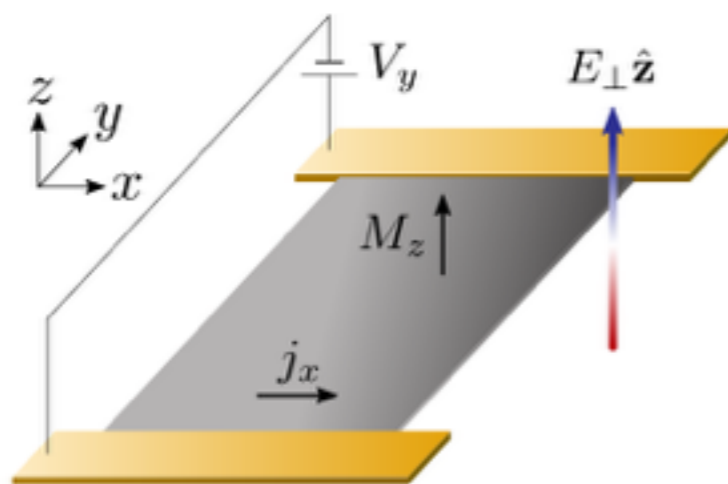
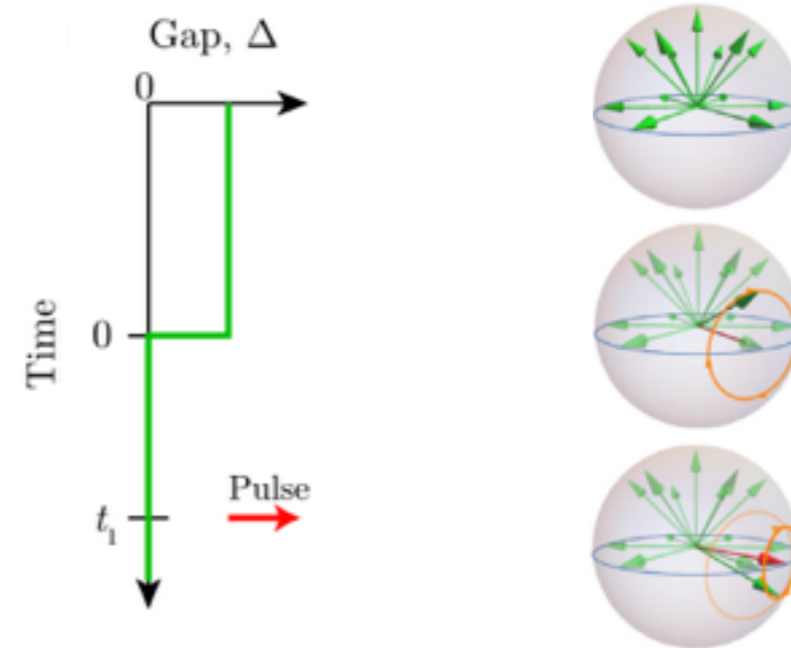
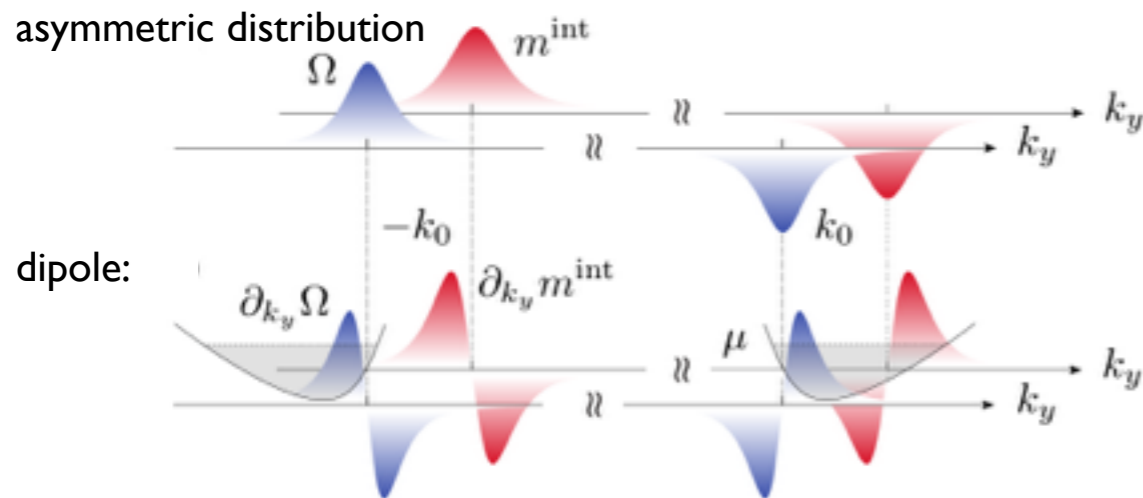
# Hall responses in a TRS preserving system

nonlinear Hall effect

quench induced responses

Berry curvature dipoles mediate transverse current

remnant geometrical Hall effect in a quantum quench





J Wilson, JS, G Refael, PRL (2016)

see also Hu, Zoller, Buddich PRL (2016)



# Anomalous Cyclotron motion without magnetic field

Slow center of mass/wavepacket dynamics

Group velocity  "Anomalous velocity" 

$$\dot{\mathbf{x}} = \frac{\partial \epsilon}{\partial \mathbf{p}} + \frac{1}{\hbar} \boldsymbol{\Omega}(\mathbf{p}) \times \dot{\mathbf{p}}$$
$$\dot{\mathbf{p}} = -\frac{\partial V}{\partial \mathbf{x}}$$

# Anomalous Cyclotron motion without magnetic field

Slow center of mass/wavepacket dynamics

Group velocity  $\rightarrow$  "Anomalous velocity"

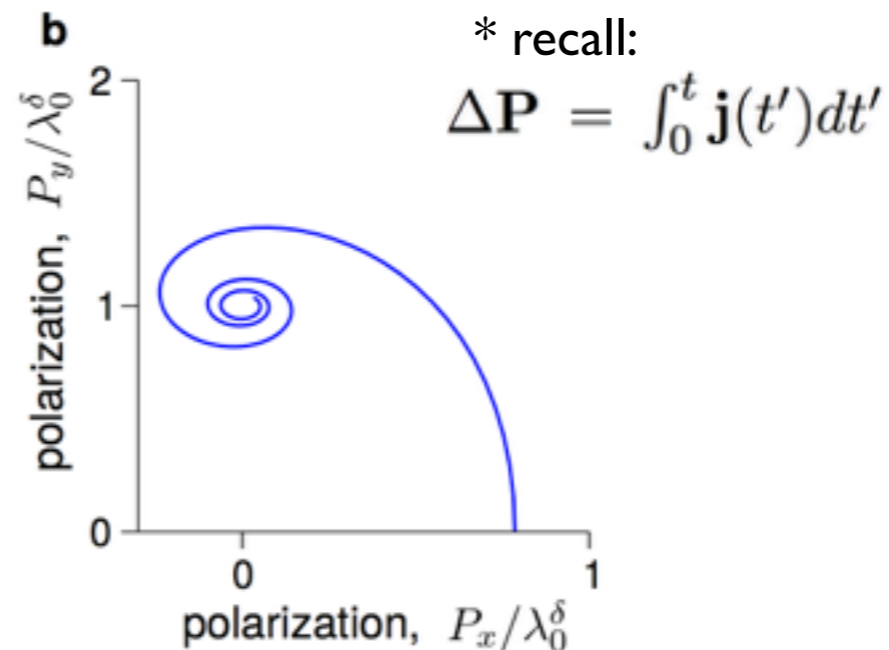
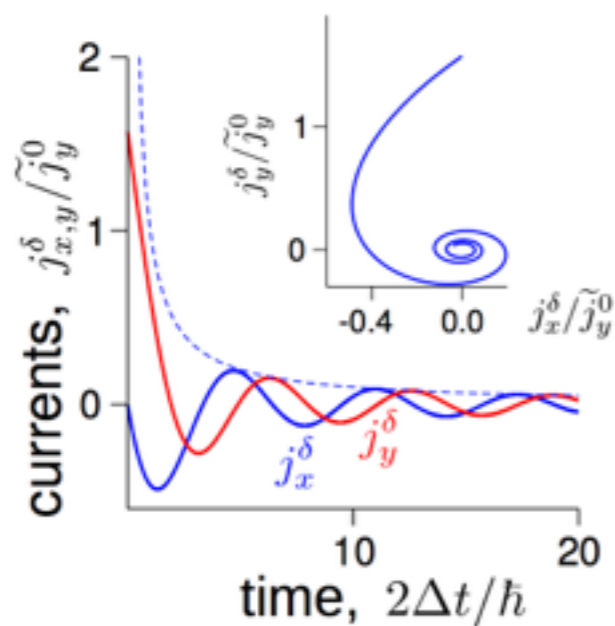
$$\dot{\mathbf{x}} = \frac{\partial \epsilon}{\partial \mathbf{p}} + \frac{1}{\hbar} \boldsymbol{\Omega}(\mathbf{p}) \times \mathbf{p}$$

$$\dot{\mathbf{p}} = -\frac{\partial V}{\partial \mathbf{x}}$$

## Intra-unit-cell dynamics

Pulse a (TRS broken) gapped Dirac system with  $\mathbf{E} = A_x \delta(t) \hat{\mathbf{x}} / c$

Track the dynamics of the current:



# driven systems: out-of-equilibrium materials

overcoming the tyranny of thermodynamics

some strategies:

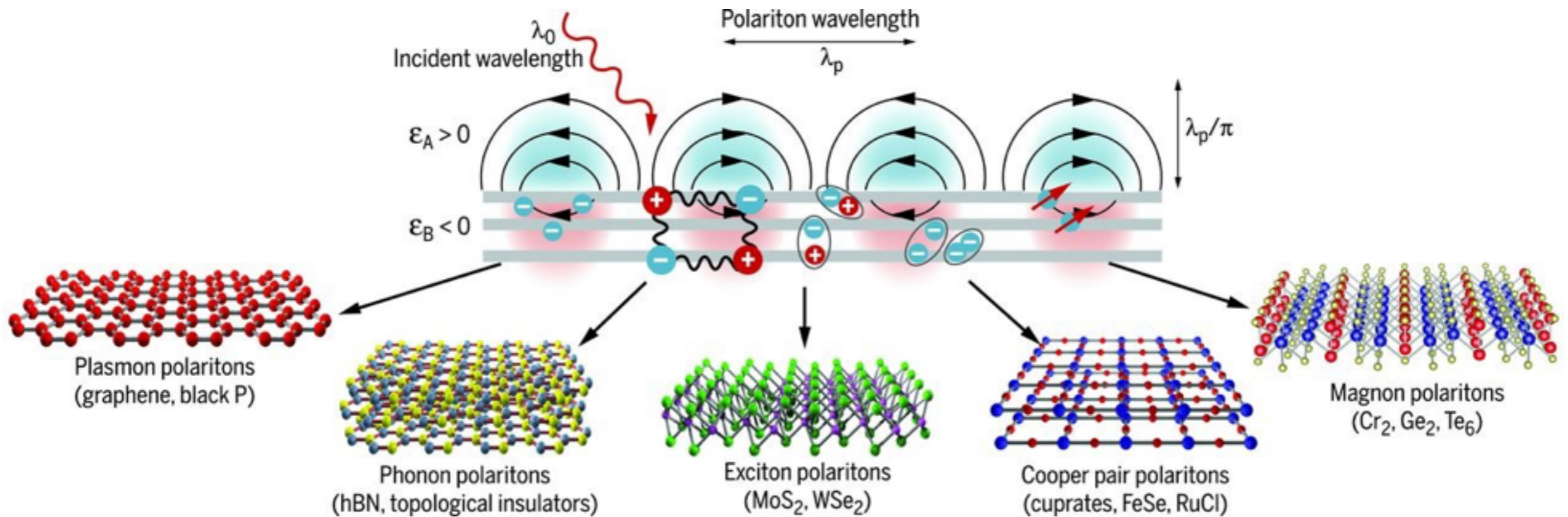
- a. non-equilibrium driving for designer hamiltonians
- b. unconventional out-of-equilibrium responses
- c. exploiting collective modes for new phases/structure

what we will focus on in this talk



# Rich tapestry of out-of-equilibrium excitations

large variety of excited states beyond nominal single-particle (bandstructure) excitations





# Plan

## Part I.

Exploiting out-of-equilibrium matter

## Part II.

Spontaneous symmetry breaking in a collective mode  
out-of-equilibrium plasmonic magnetism

MS Rudner JS, arXiv (2018)

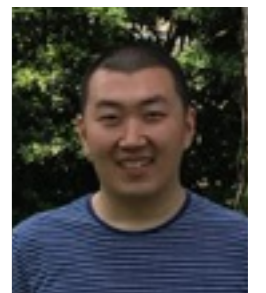


Mark Rudner (KU)

## Part III.

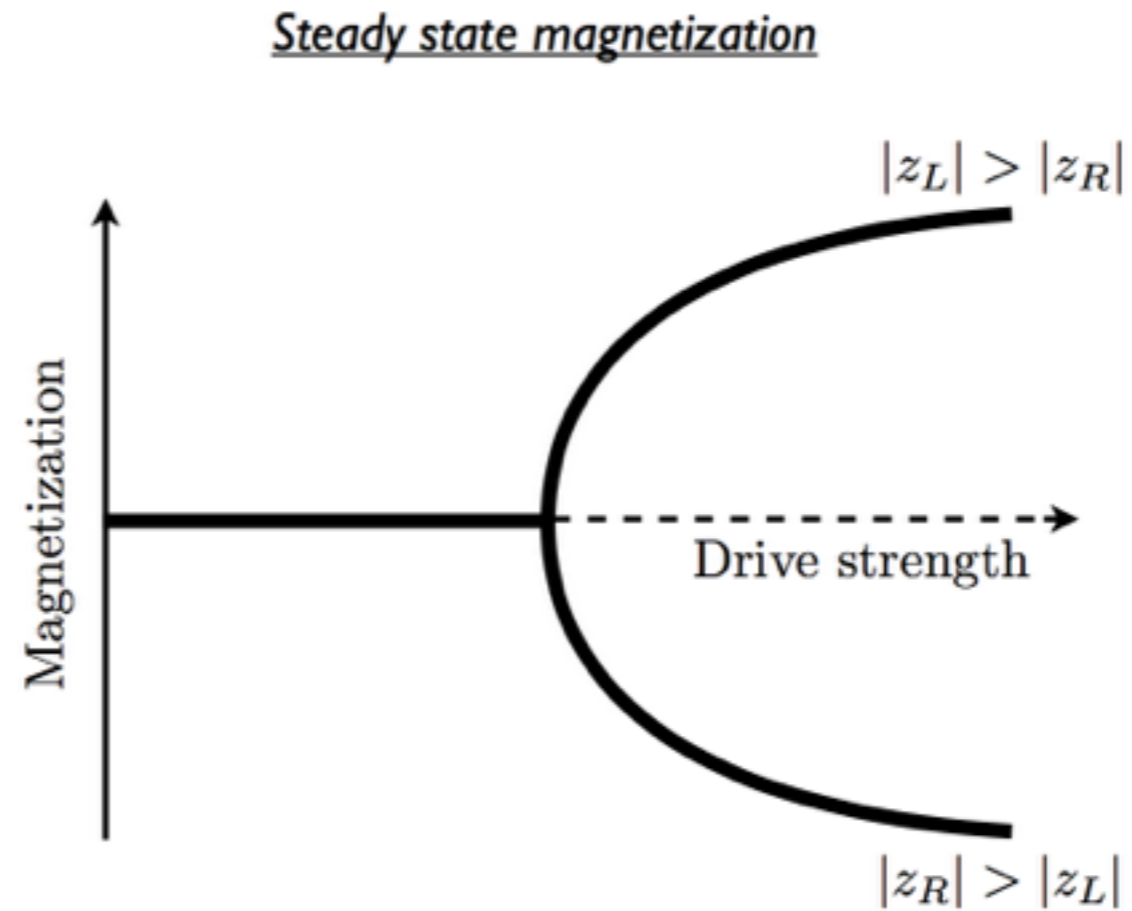
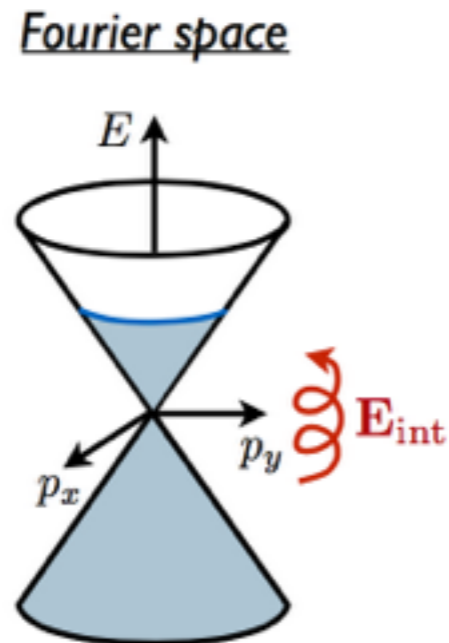
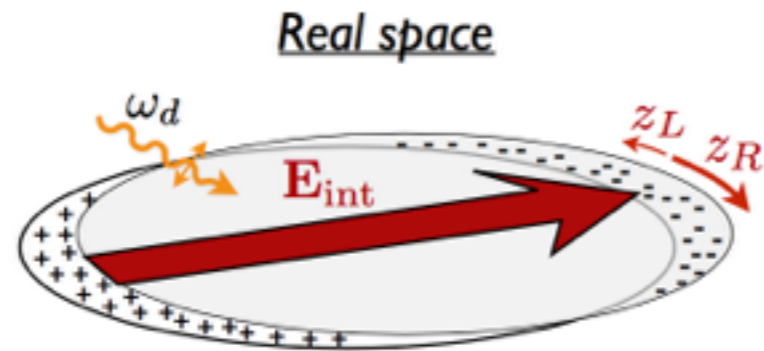
Emergent internal structure of plasmons and geometry

LK Shi, JS, PRX (2018)



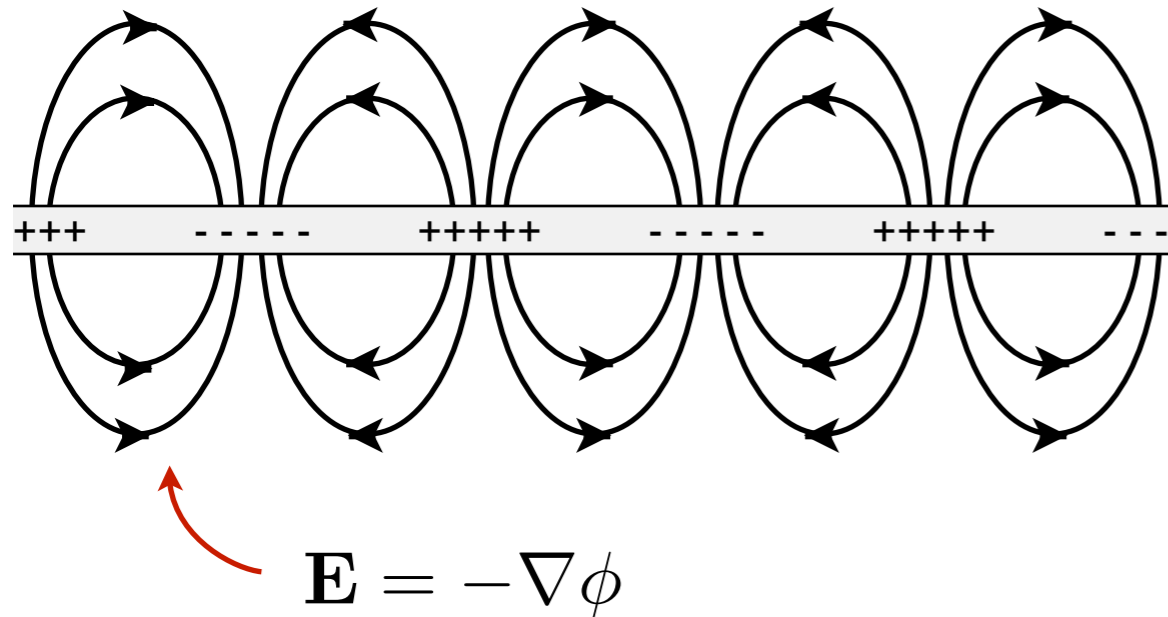
Li-kun Shi

# Claim: collective motion of plasmons gives rise to spontaneous TRS breaking



# Plasmons are collective density oscillations in metals

Plasmons



$$\phi(\mathbf{x}, t) = -e \int d^2\mathbf{x}' \frac{\delta n(\mathbf{x}', t)}{\kappa |\mathbf{x} - \mathbf{x}'|}$$

continuity equation:

$$\partial_t \delta n + \nabla \cdot \mathbf{v} = 0$$

force equation:

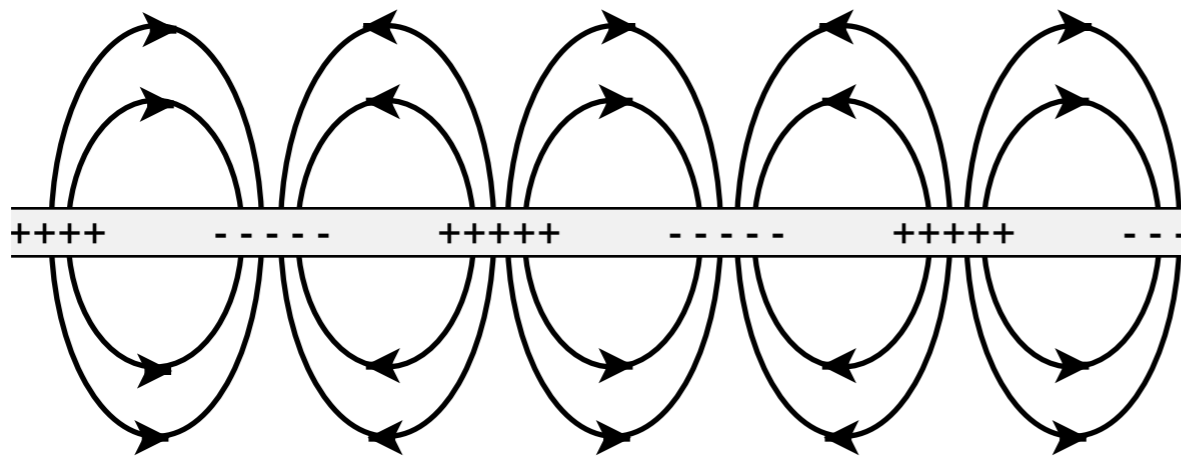
$$\partial_t \mathbf{p} + en_0 \mathbf{E} = 0$$

“constitutive” relation:

$$\mathbf{v} = \mathbf{p}/m$$

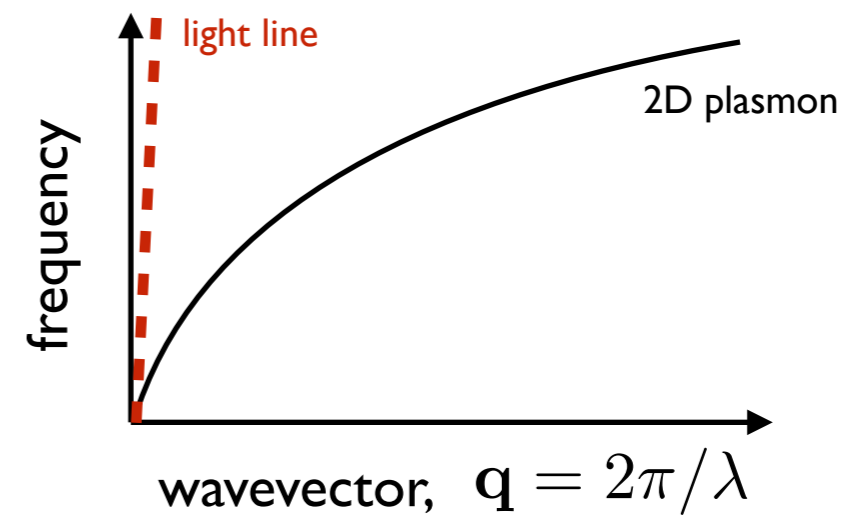
# Plasmons and strong light-matter interaction

Plasmons

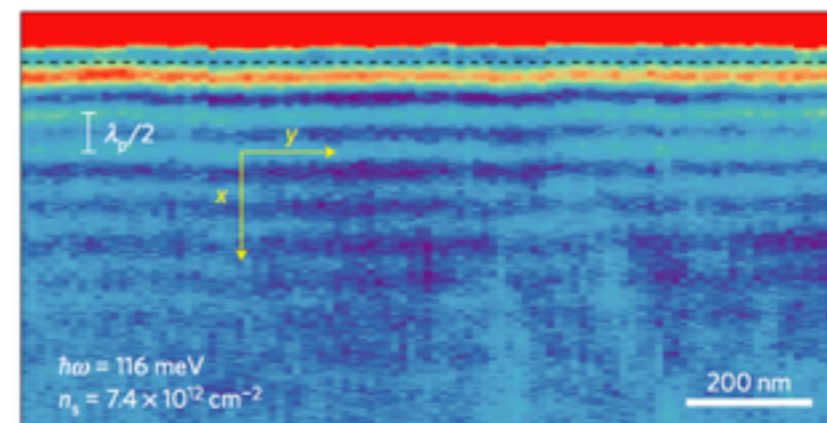
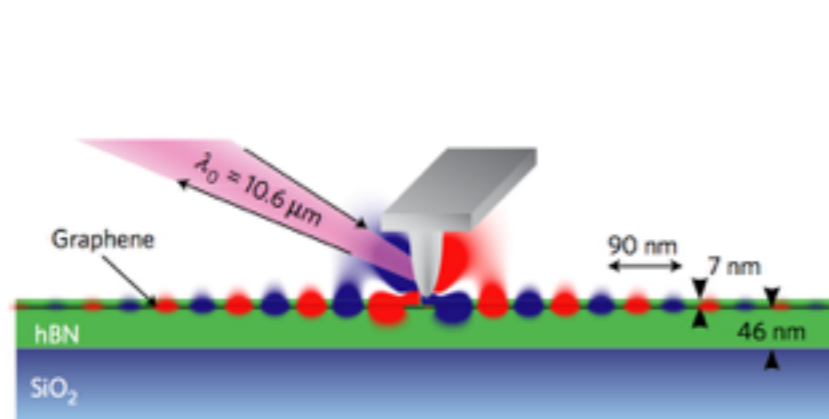


Large wavelength mismatch = high compression

$$\lambda_{\text{air}} \gg \lambda_{\text{plasmon}}$$



imaging/exciting plasmons in 2D materials using Scanning near-field optical microscope (SNOM)

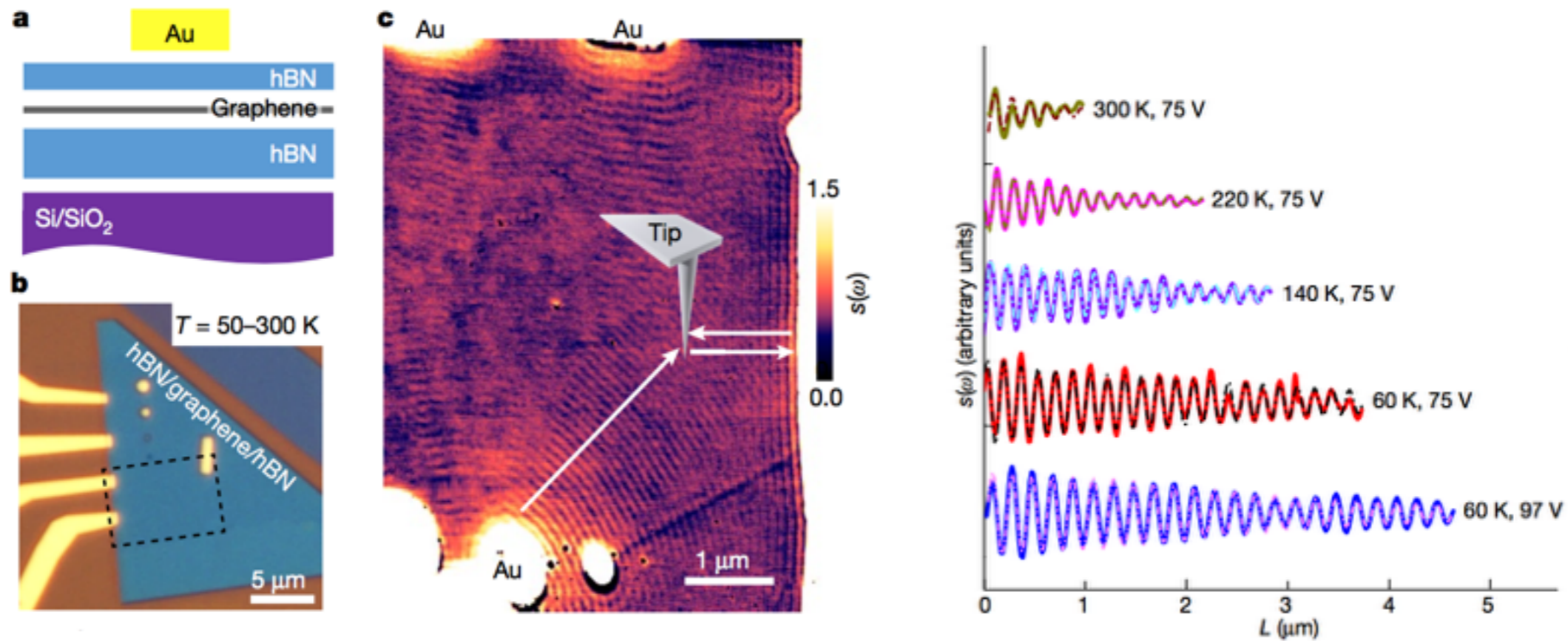


e.g., Woesnner, et al, Nature Materials (2015)

first achieved in Koppens group (Nature 2013), and Basov group (Nature 2013)



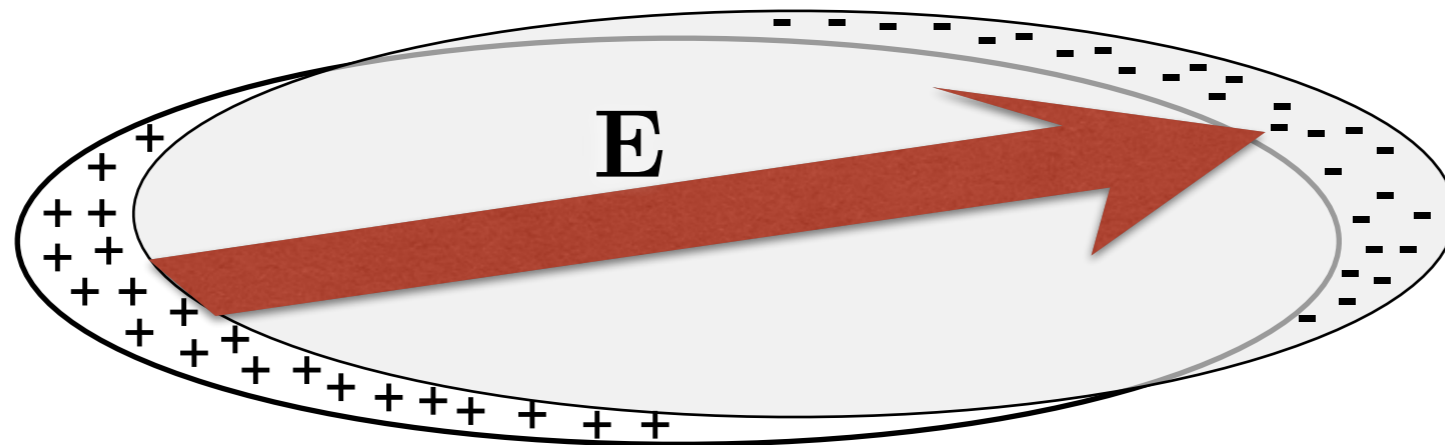
# High quality plasmons in graphene



	Confinement ratio	Quality factor	Lifetime (fs)
Definition	$\lambda_{IR}/\lambda_p$	$Q_p = q'_p/q''_p$	$\tau = 2Q_p/\omega$
Graphene (experimental, $T=60$ K)	66	130	1,600
Graphene (intrinsic, $T=60$ K)	66	970	12,000
$Ag^{a,b}$ ( $T=10$ K)	$\sim 1$	36	14
$n-InSb^c, n-CdO^a$ ( $T=300$ K)	$< 10$	37	270

# Equations of motion in a disk

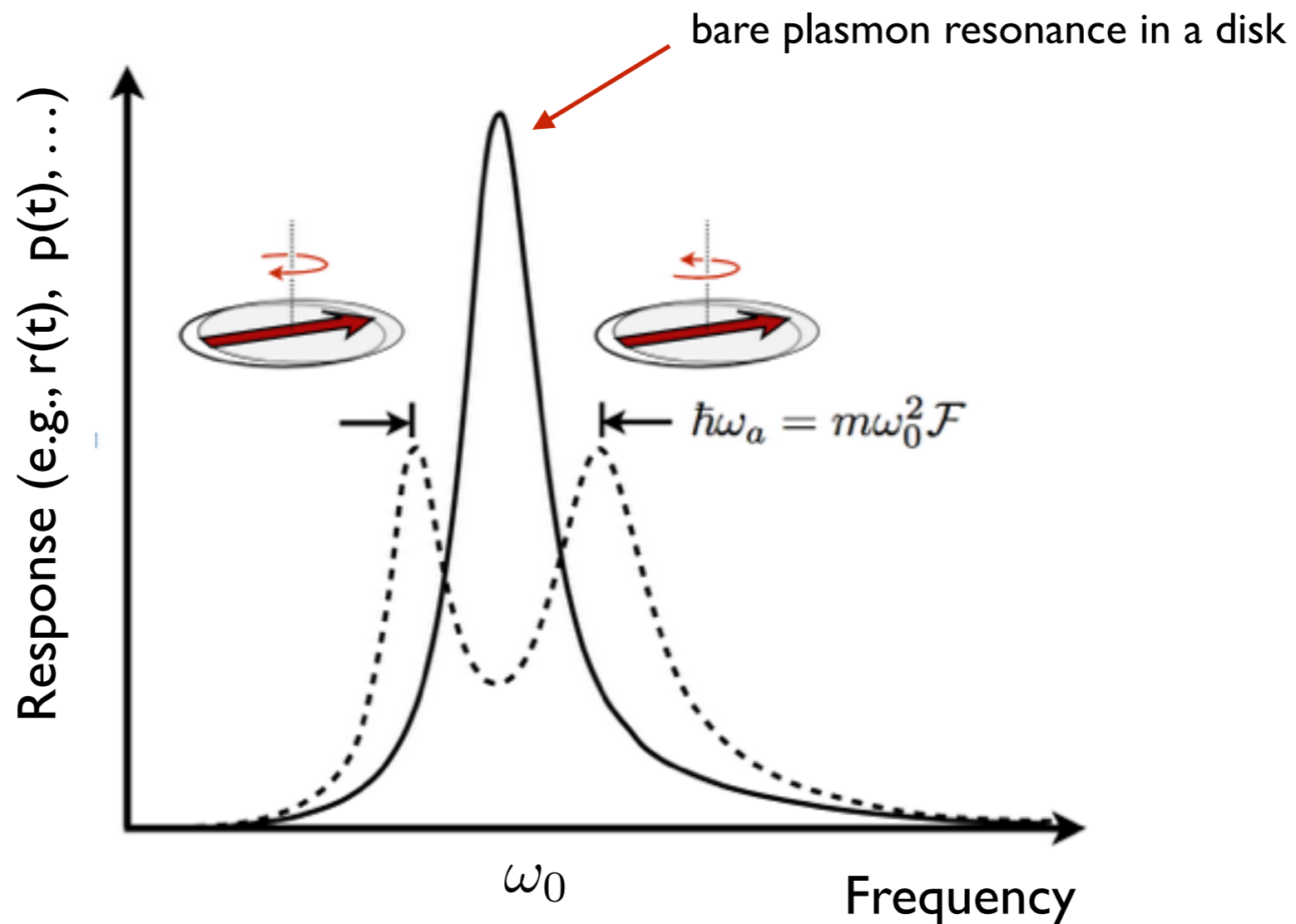
$$\frac{d\{\mathbf{r}\}}{dt} = \frac{\{\mathbf{p}\}}{m} - \frac{\mathcal{F}[\mathbf{E}_{\text{tot}}(t)]}{\hbar n_0} \hat{\mathbf{z}} \times e\mathbf{E}_{\text{tot}}(t),$$
$$\frac{d\{\mathbf{p}\}}{dt} = -m\omega_0^2\{\mathbf{r}\} - \gamma\{\mathbf{p}\} - e\mathbf{E}_{\text{drive}}(t),$$



# Equations of motion in a disk

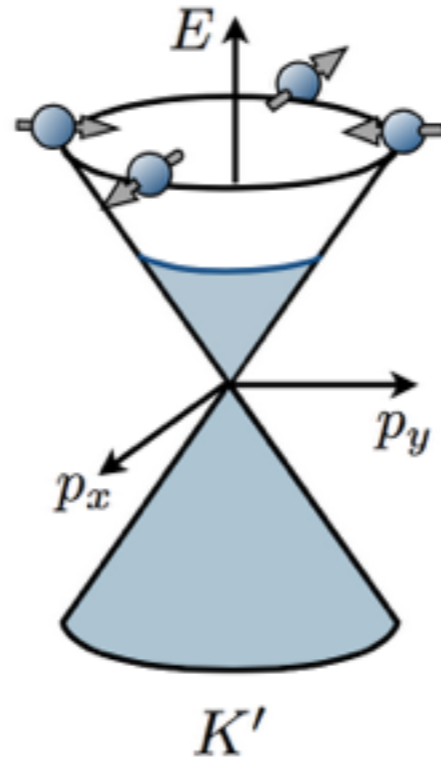
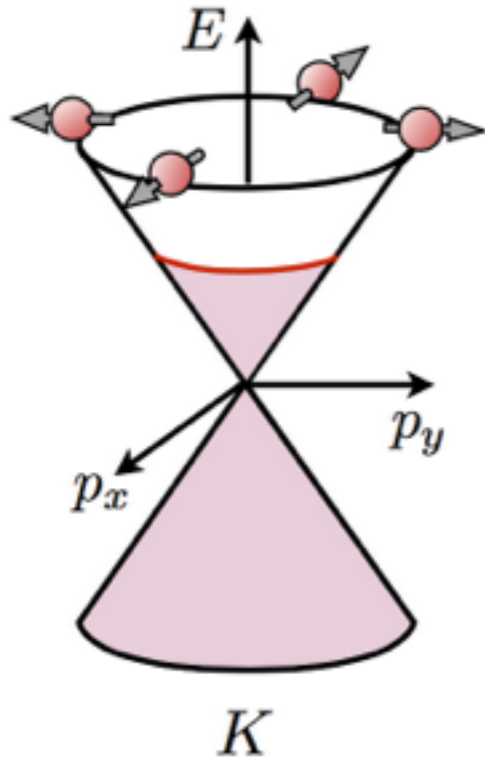
$$\frac{d\{\mathbf{r}\}}{dt} = \frac{\{\mathbf{p}\}}{m} - \frac{\mathcal{F}[\mathbf{E}_{\text{tot}}(t)]}{\hbar n_0} \hat{\mathbf{z}} \times e\mathbf{E}_{\text{tot}}(t),$$

$$\frac{d\{\mathbf{p}\}}{dt} = -m\omega_0^2\{\mathbf{r}\} - \gamma\{\mathbf{p}\} - e\mathbf{E}_{\text{drive}}(t),$$



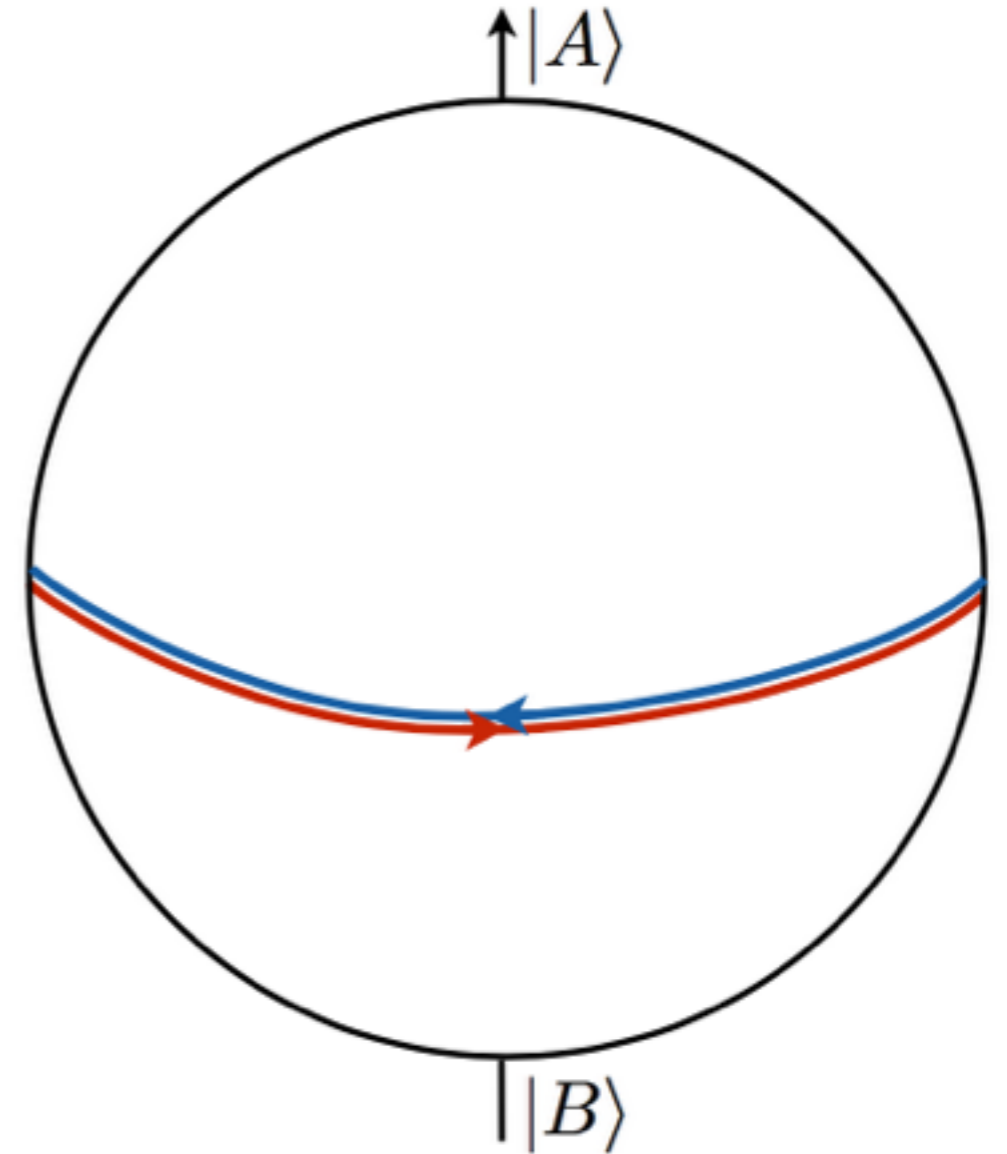
# Graphene and Berry flux

$$\mathcal{H}_K = E_F \tilde{\mathbf{k}} \cdot \boldsymbol{\sigma} \quad \mathcal{H}_{K'} = E_F \tilde{\mathbf{k}} \cdot [\boldsymbol{\sigma}]^*$$



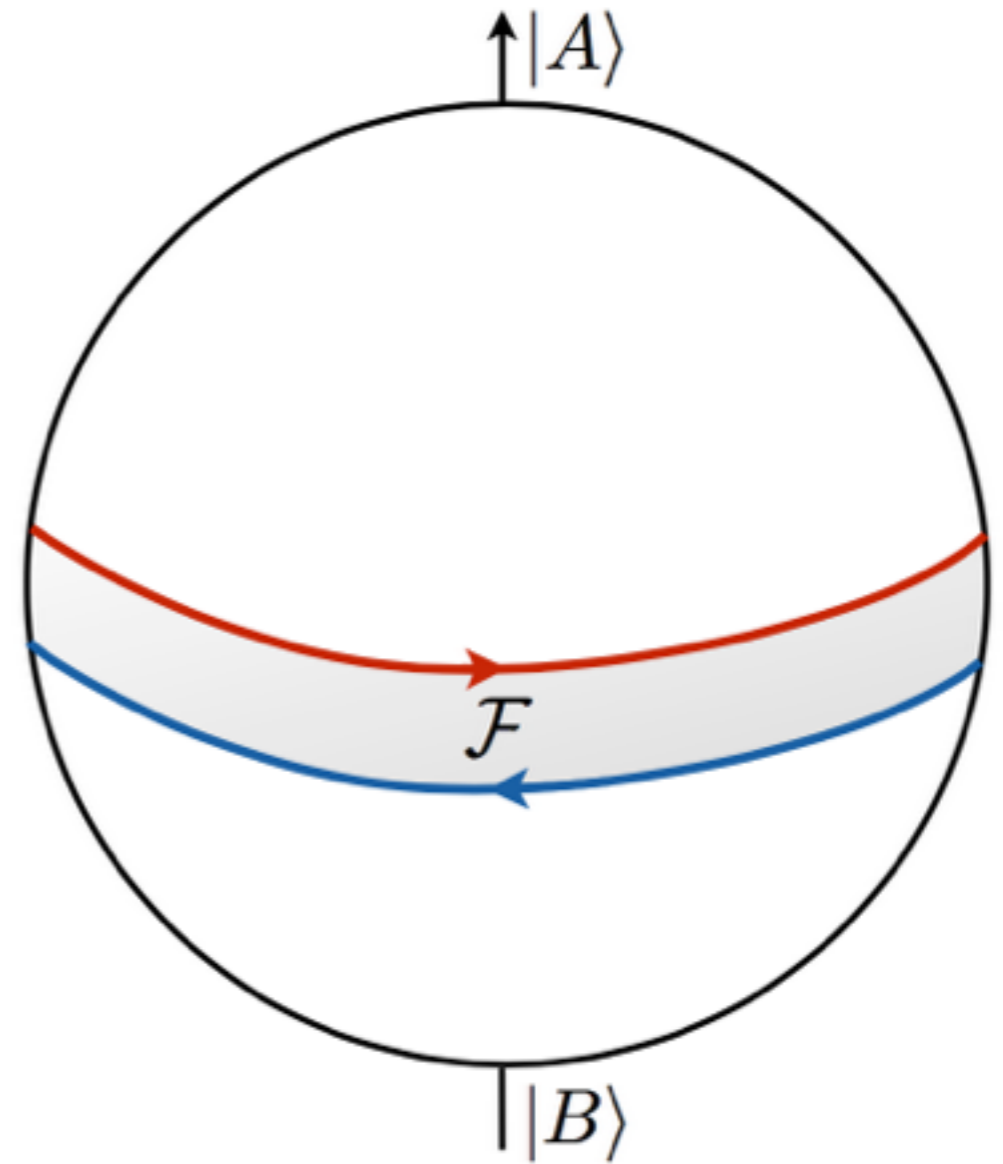
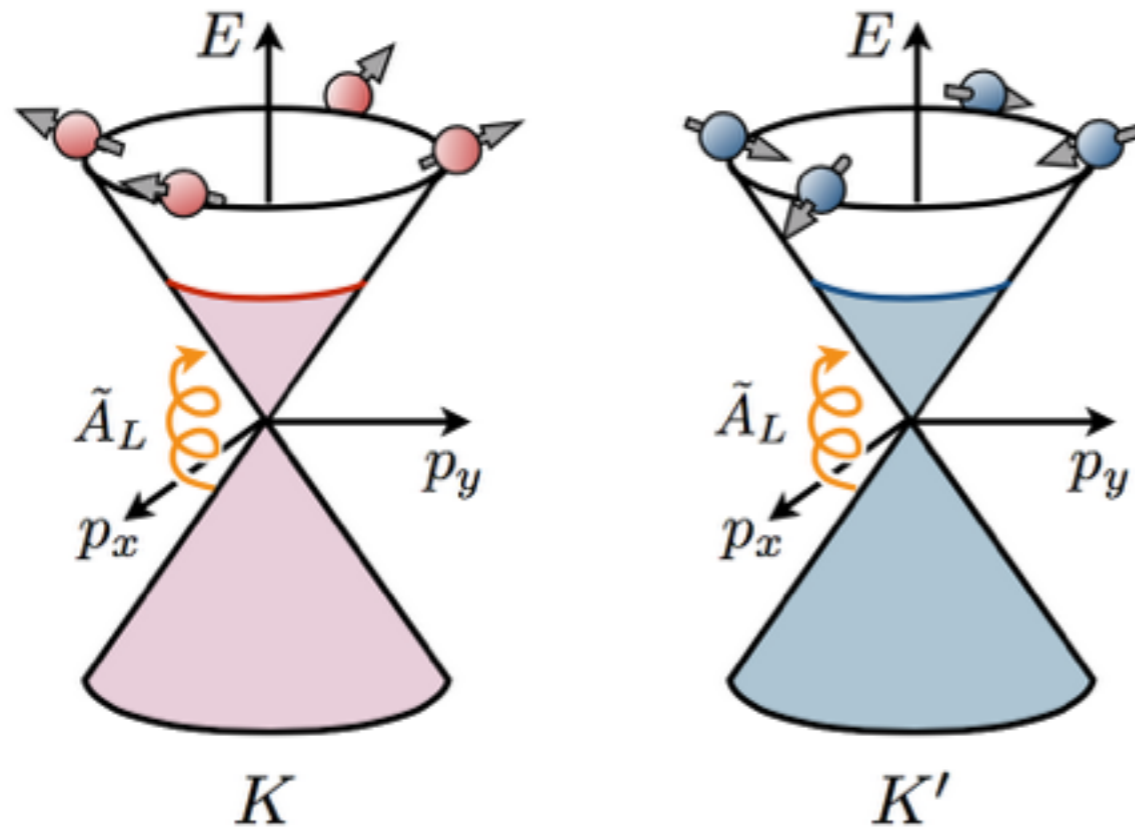
Berry flux as Berry phase  
across Fermi surface:

$$\mathcal{F} = \int d^2\mathbf{k} \Omega(\mathbf{k}) = \oint \mathbf{A}(\mathbf{k}) \cdot d\mathbf{k}$$



# Generation of Berry flux

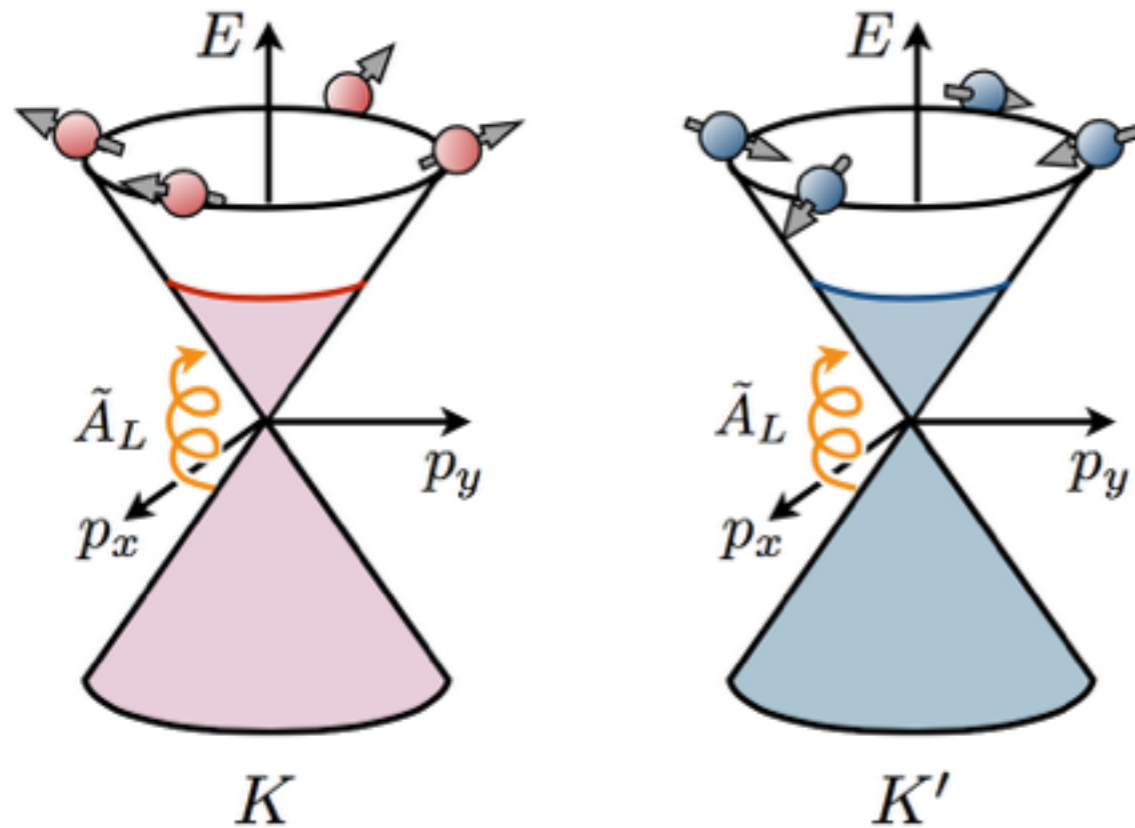
$$\mathcal{H}_K = E_F [\tilde{\mathbf{k}} - \tilde{\mathbf{A}}(t)] \cdot \boldsymbol{\sigma}, \quad \mathcal{H}_{K'} = E_F [\tilde{\mathbf{k}} - \tilde{\mathbf{A}}(t)] \cdot \boldsymbol{\sigma}^* \quad \tilde{\mathbf{A}}(t) = \frac{ev}{cE_F} \mathbf{A}(t),$$



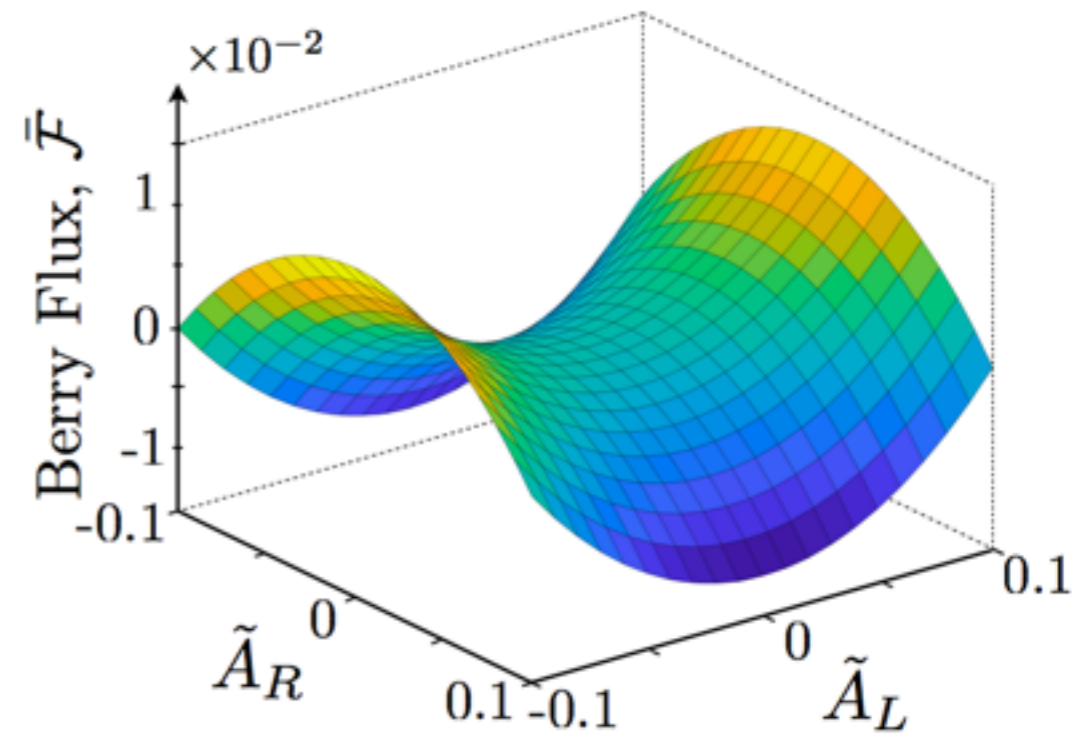


# Generation of Berry flux

$$\mathcal{H}_K = E_F [\tilde{\mathbf{k}} - \tilde{\mathbf{A}}(t)] \cdot \boldsymbol{\sigma}, \quad \mathcal{H}_{K'} = E_F [\tilde{\mathbf{k}} - \tilde{\mathbf{A}}(t)] \cdot \boldsymbol{\sigma}^* \quad \tilde{\mathbf{A}}(t) = \frac{ev}{cE_F} \mathbf{A}(t),$$



Steady state Berry flux generated



$$\bar{\mathcal{F}} = \beta(|\tilde{A}_L|^2 - |\tilde{A}_R|^2)$$

order unity dimensionless number

\* For realistic graphene parameters ( $E_F = 160$  meV, drive = 100 meV), we find  $\beta = 2.3$

M Rudner, JS, arXiv (2018)

# Feedback: Flux induced plasmon non-linearity

$$\frac{d\{\mathbf{r}\}}{dt} = \frac{\{\mathbf{p}\}}{m} - \frac{\mathcal{F}[\mathbf{E}_{\text{tot}}(t)]}{\hbar n_0} \hat{\mathbf{z}} \times e\mathbf{E}_{\text{tot}}(t),$$

$$\frac{d\{\mathbf{p}\}}{dt} = -m\omega_0^2\{\mathbf{r}\} - \gamma\{\mathbf{p}\} - e\mathbf{E}_{\text{drive}}(t),$$

Generated by rotating electric fields (drive + internal)

$$\mathcal{F} = 0 + \mathcal{F}[\mathbf{E}_{\text{tot}}(t)]$$

internally plasmonic enhanced electric fields  
internal electric fields up to Q times larger

graphene has zero flux

$$e\mathbf{E}_{\text{tot}}(t) = e\mathbf{E}_{\text{drive}}(t) + m\omega_0^2\{\mathbf{r}\}.$$

Flux depends on plasmon motion/displacement: **plasmon non-linear**

$$\bar{\mathcal{F}} = f(|\mathcal{Z}_-^{(0)}|^2/l^2, |\mathcal{Z}_+^{(0)}|^2/l^2), \quad l^{-1} = \frac{vm\omega_0^2}{E_F\omega_d},$$

complex representation for circular basis

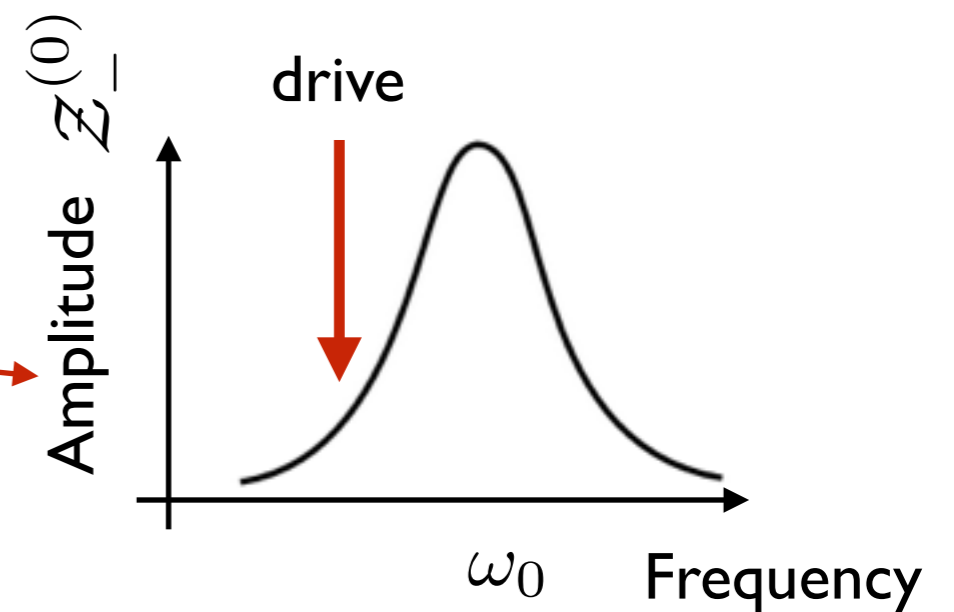
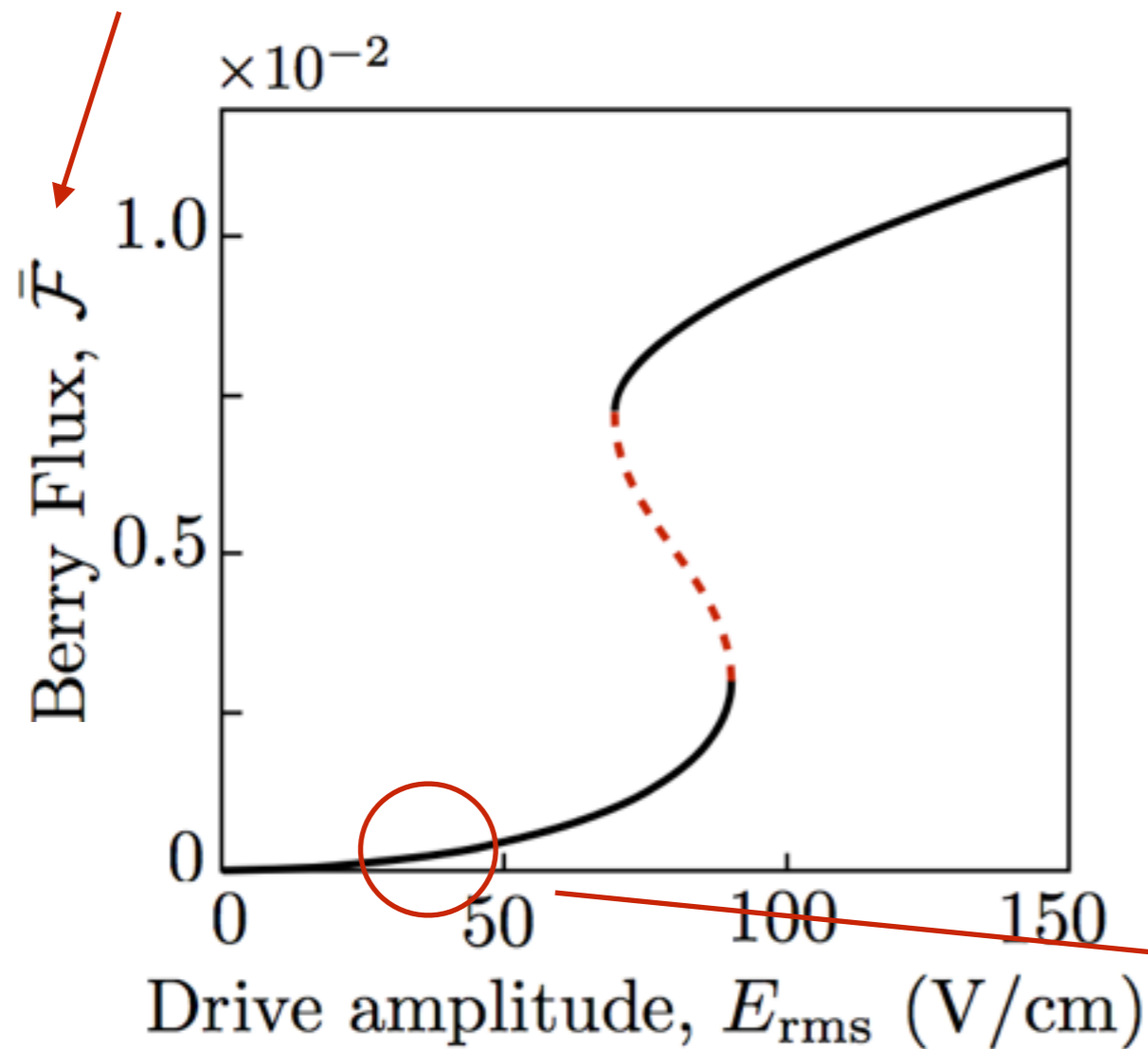
$$\mathcal{Z}_\pm^{(0)} = \frac{1}{\sqrt{2}}[x^{(0)} \pm iy^{(0)}]$$

# Feedback: nonlinearity and bistability

## case I: circularly polarized driving

Captures the amplitude of the circular motion of the plasmons

$$z_{\pm}^{(0)} = \frac{1}{\sqrt{2}} [x^{(0)} \pm iy^{(0)}]$$

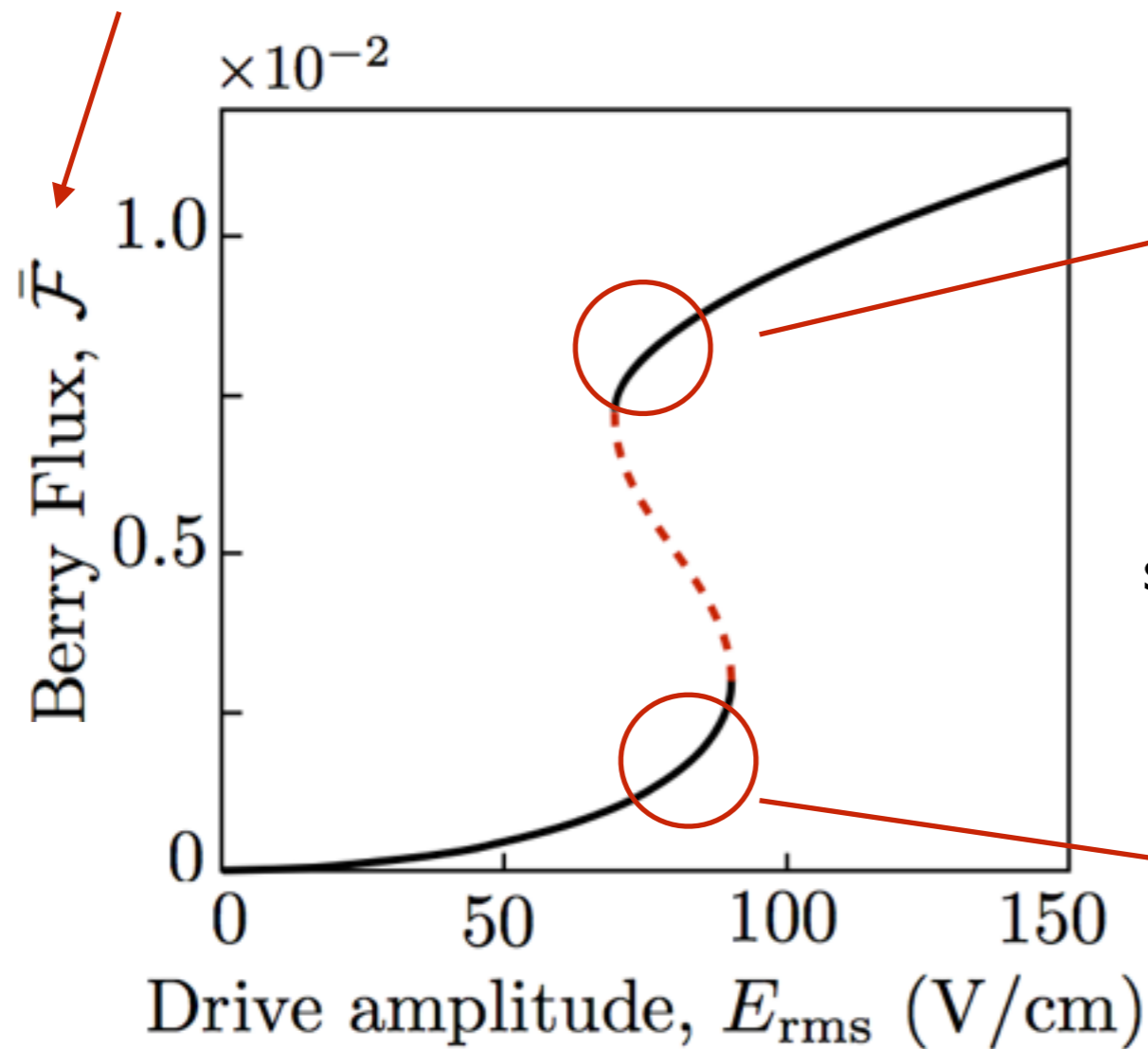


# Feedback: nonlinearity and bistability

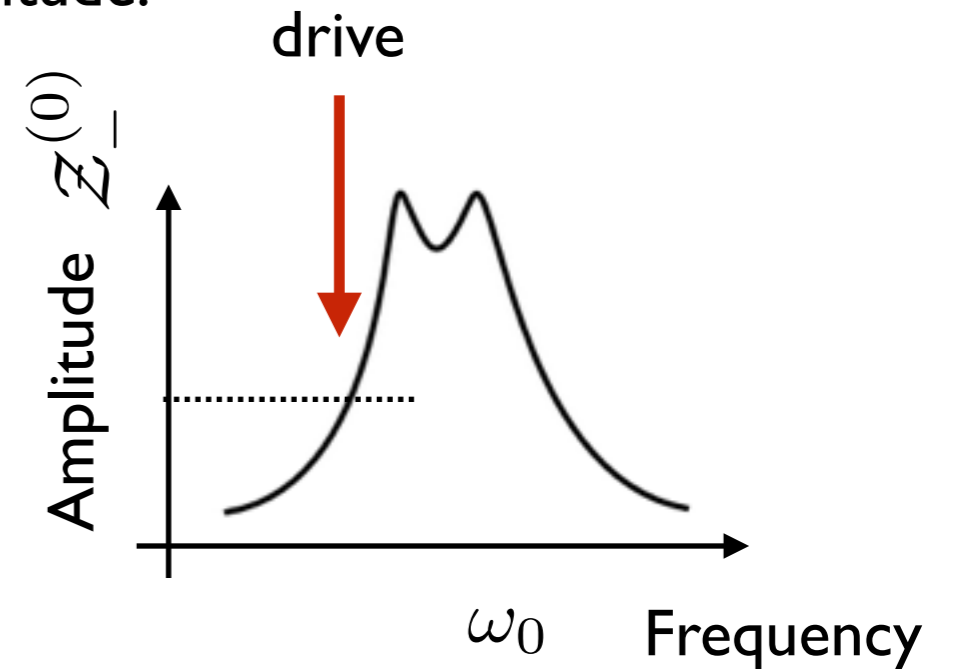
## case I: circularly polarized driving

Captures the amplitude of the circular motion of the plasmons

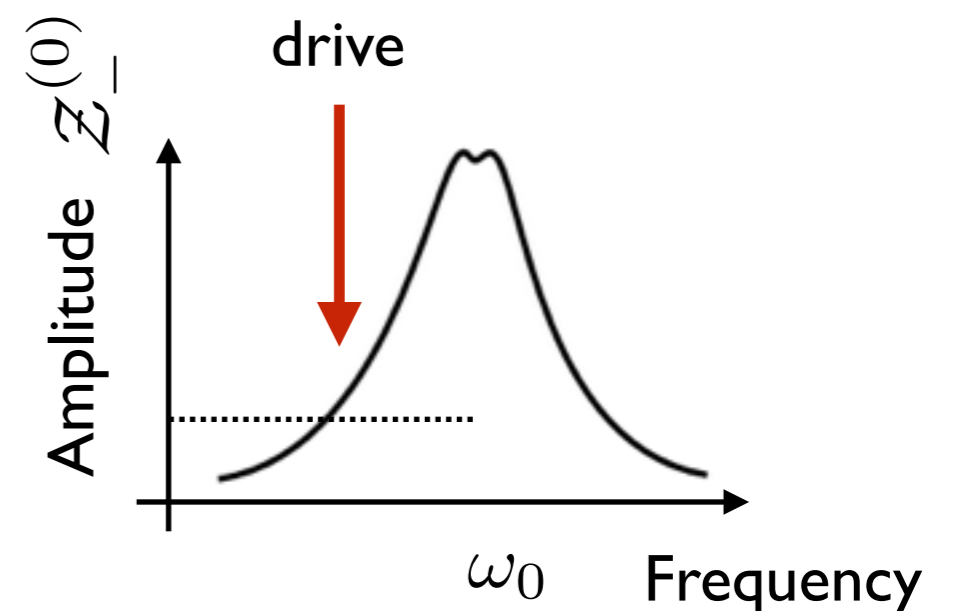
$$Z_{\pm}^{(0)} = \frac{1}{\sqrt{2}} [x^{(0)} \pm iy^{(0)}]$$



large amplitude:



small amplitude:



TRS preserving drives (e.g., linear polarization)?



# Self-Floquet: spontaneous collective mode *magnetism*

## case II: linearly polarized driving

$$\frac{d\{\mathbf{r}\}}{dt} = \frac{\{\mathbf{p}\}}{m} - \frac{\mathcal{F}[\mathbf{E}_{\text{tot}}(t)]}{\hbar n_0} \hat{\mathbf{z}} \times e\mathbf{E}_{\text{tot}}(t),$$

$$\frac{d\{\mathbf{p}\}}{dt} = -m\omega_0^2\{\mathbf{r}\} - \gamma\{\mathbf{p}\} - e\mathbf{E}_{\text{drive}}(t),$$

self-consistent equation for symmetry breaking  $\eta \equiv |\mathcal{Z}_+^{(0)}|^2 - |\mathcal{Z}_-^{(0)}|^2$ .

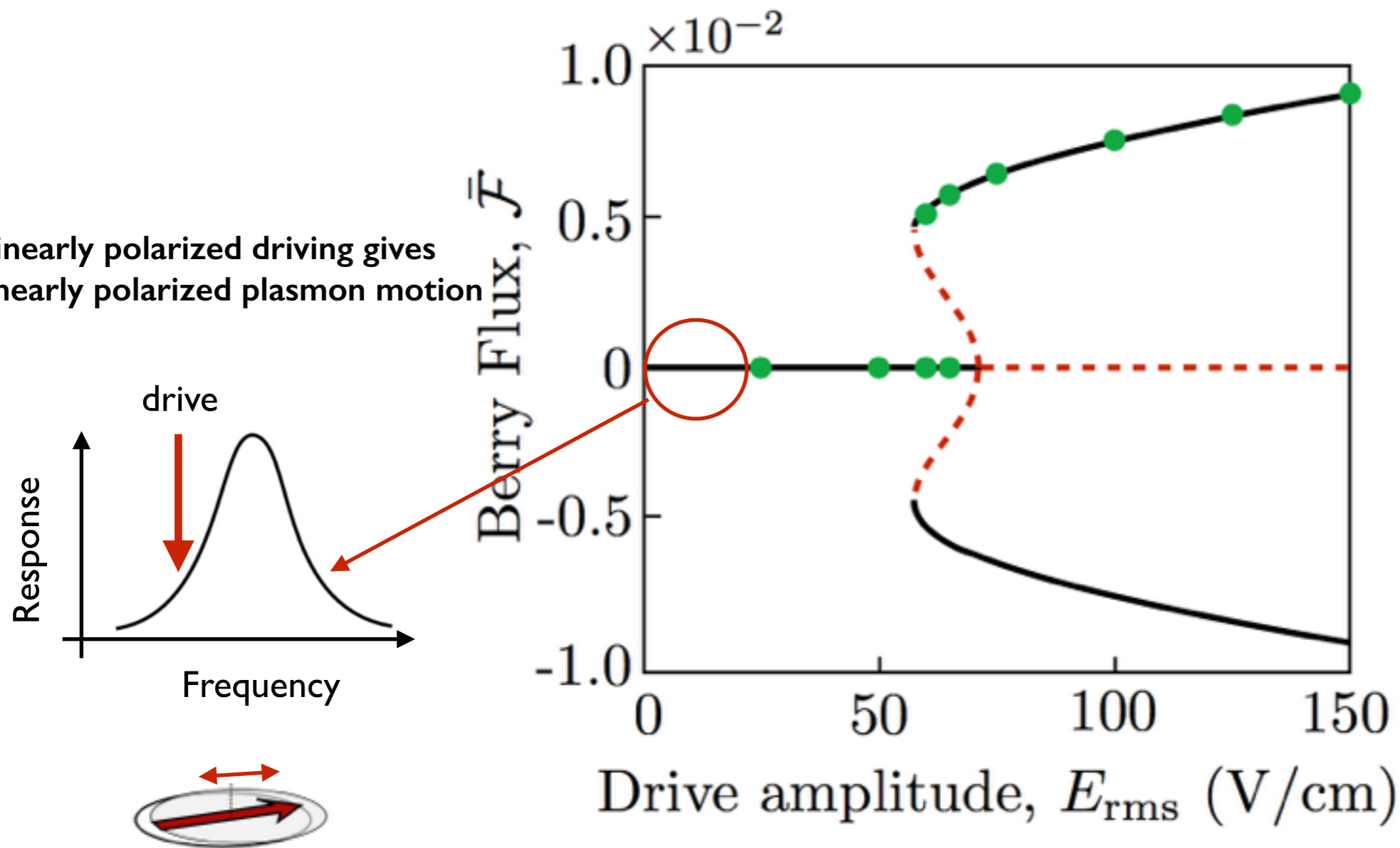
$$\eta \left[ 1 + 4\nu\omega_d(\omega_d^2 + \gamma^2 - \omega_0^2) \frac{|eE_{\text{rms}}/m|^2}{D_+D_-} \right] = 0,$$

where

$$D_{\pm} = [\omega_0^2 - \omega_d^2 \mp \nu\omega_d\eta]^2 + \gamma^2(\omega_d \pm \nu\eta)^2$$

# Spontaneous collective mode *magnetism*

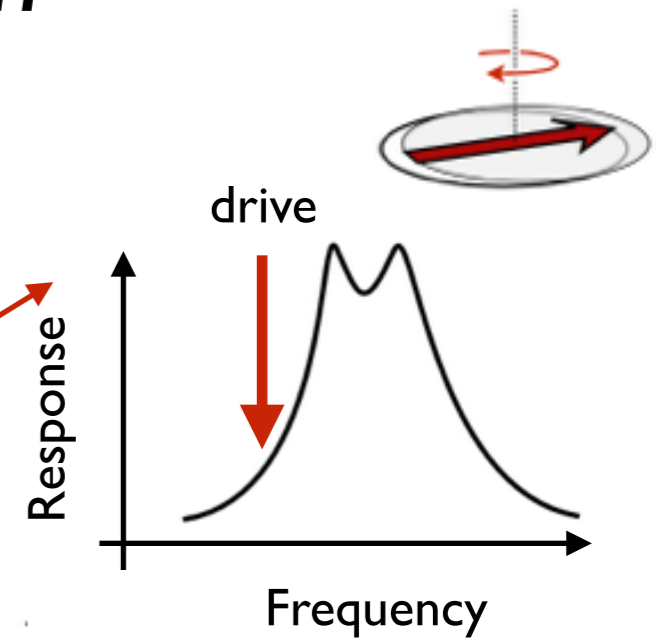
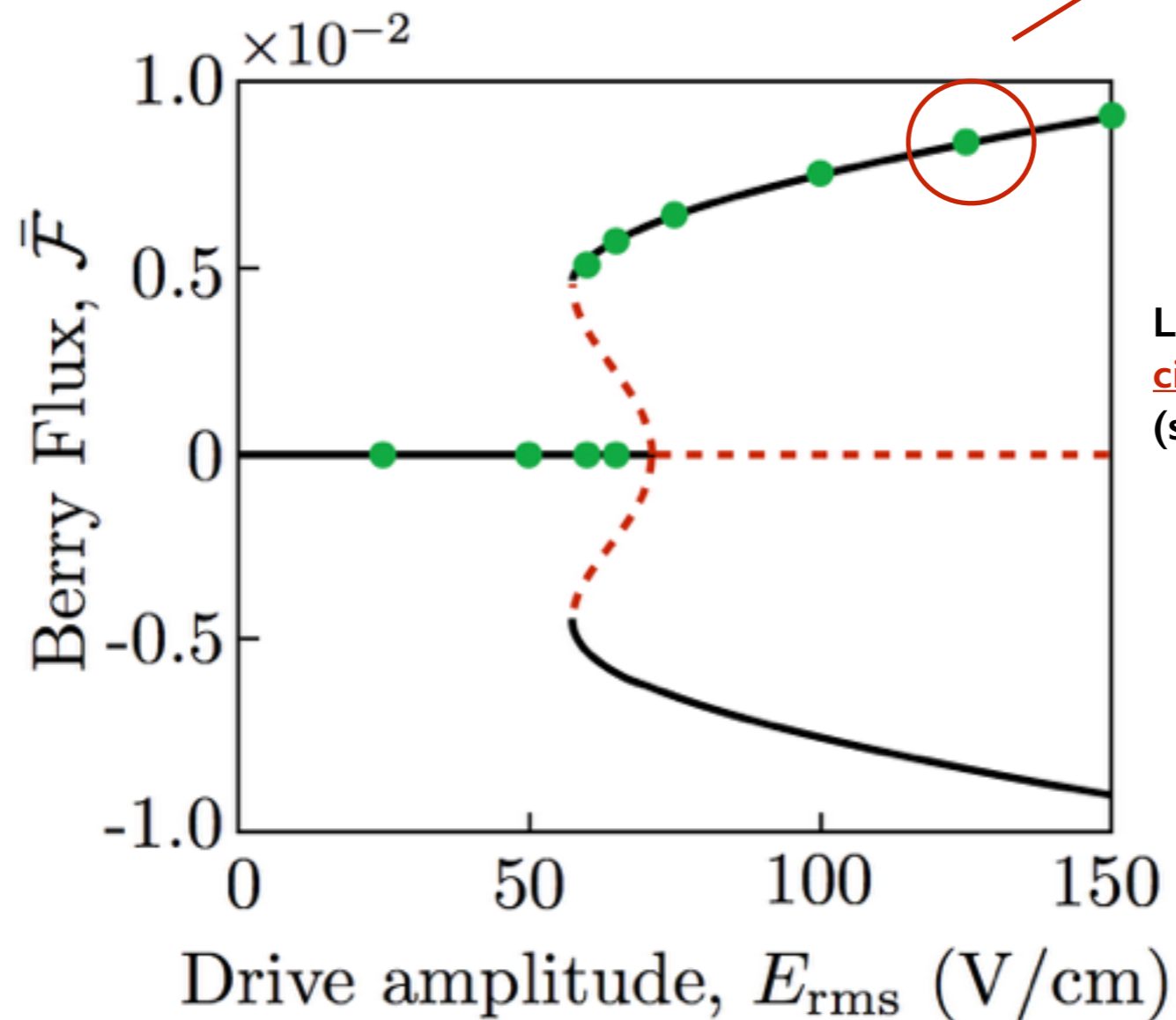
case II: linearly polarized driving



\*solid lines from analytic expression of steady states, green dots from self-consistent full numerical simulation

# Spontaneous collective mode *magnetism*

case II: linearly polarized driving

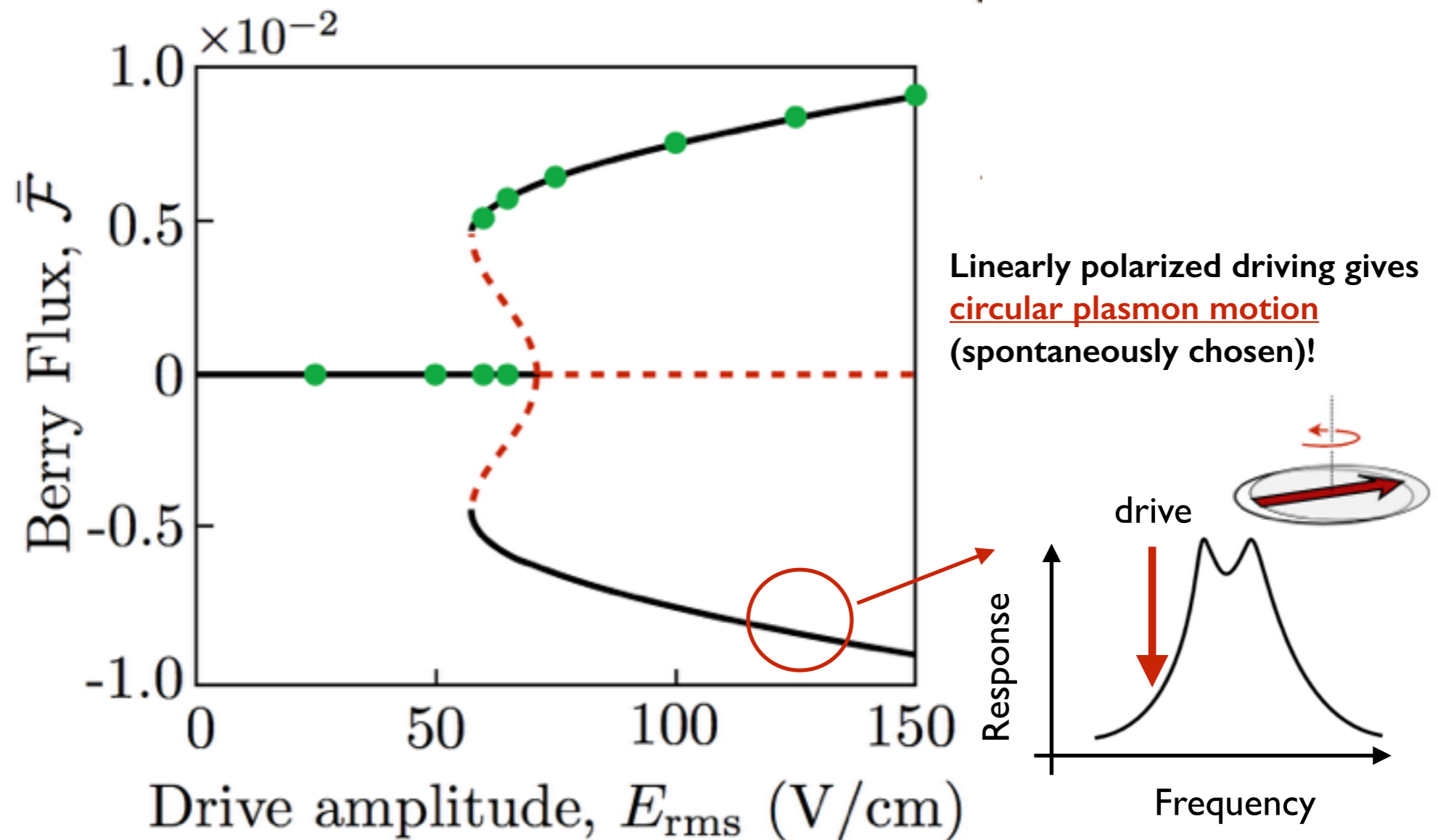


Linearly polarized driving gives circular plasmon motion (spontaneously chosen)!

\*solid lines from analytic expression of steady states, green dots from self-consistent full numerical simulation

# Spontaneous collective mode *magnetism*

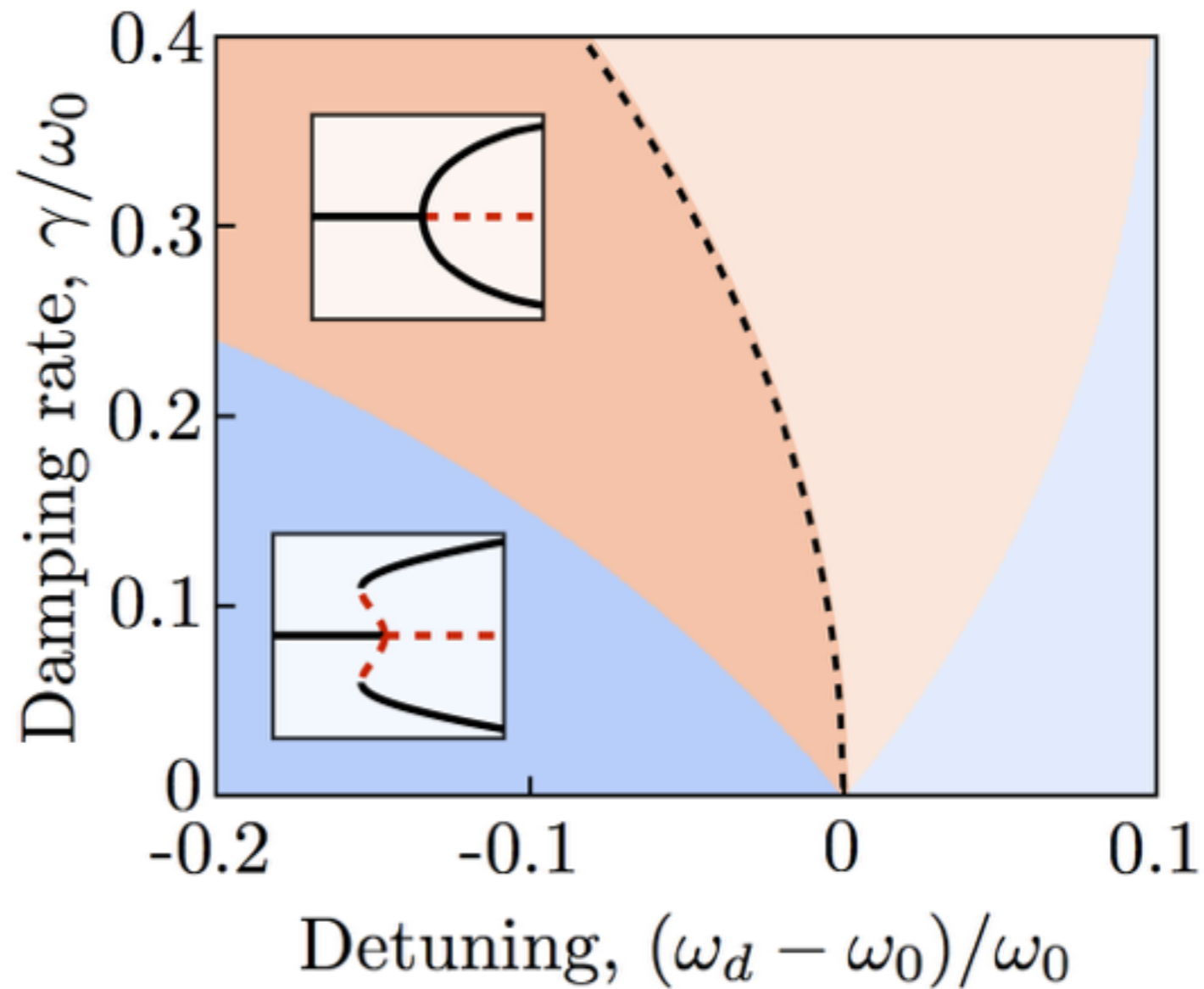
case II: linearly polarized driving



\*solid lines from analytic expression of steady states, green dots from self-consistent full numerical simulation

# Tuning the type of phase transition

case II: linearly polarized driving





# Drives are outside particle-hole continuum

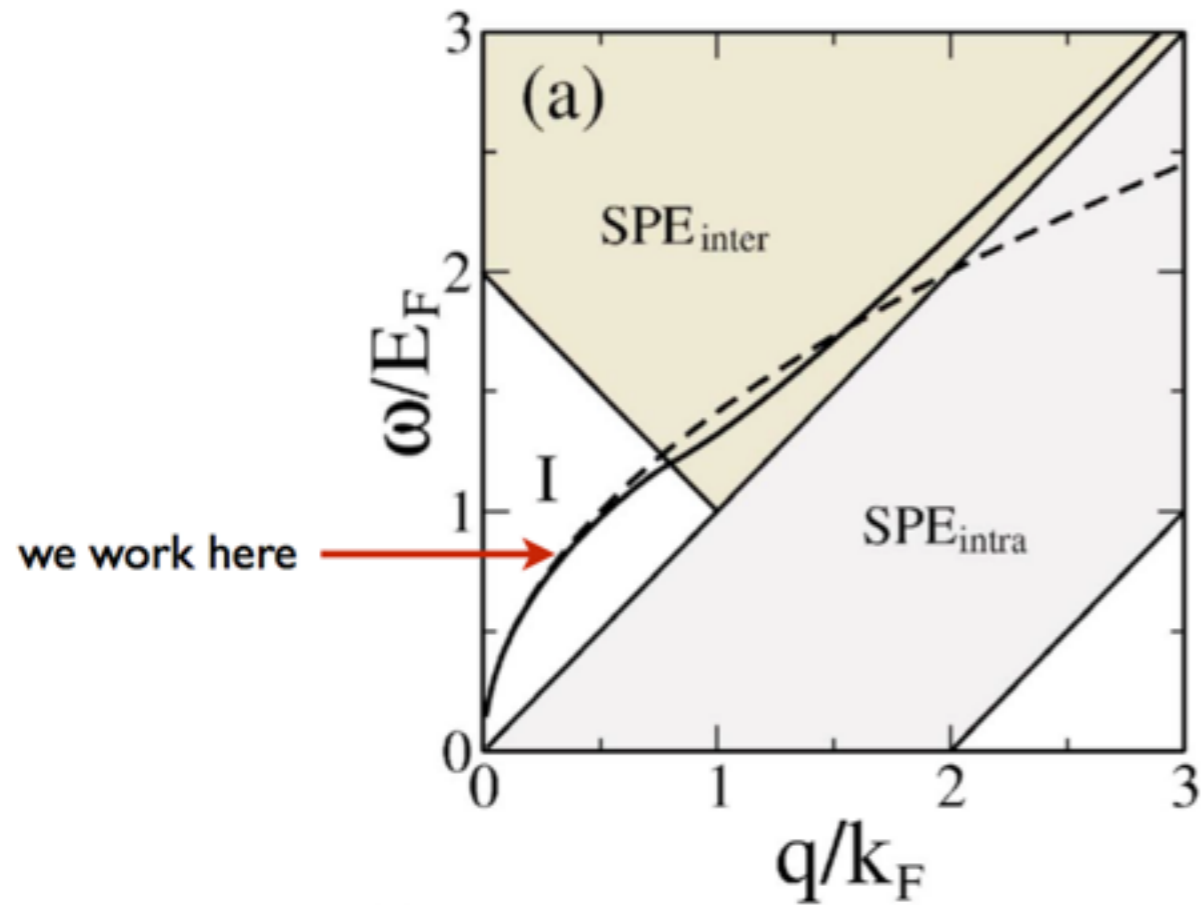
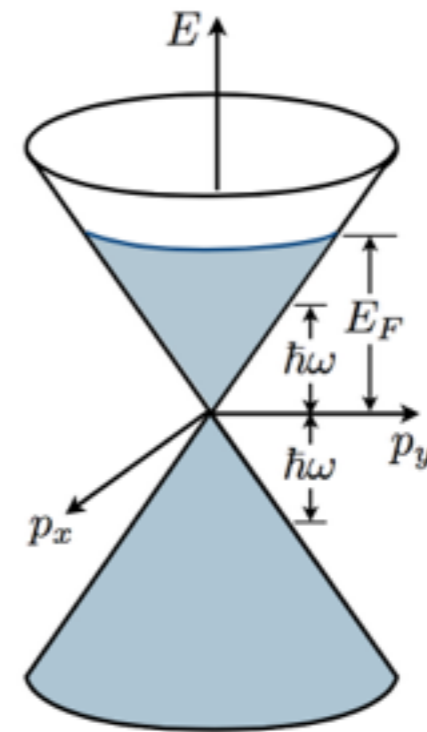


Figure from: Hwang and Das Sarma, PRB (2007)

Direct inter-band transitions are Pauli Blocked:



# Plan

## Part I.

Exploiting out-of-equilibrium matter

## Part II.

Spontaneous symmetry breaking in a collective mode  
out-of-equilibrium plasmonic magnetism

MS Rudner JS, arXiv (2018)

## Part III.

Emergent internal structure of plasmons and geometry

LK Shi, JS, PRX (2018)

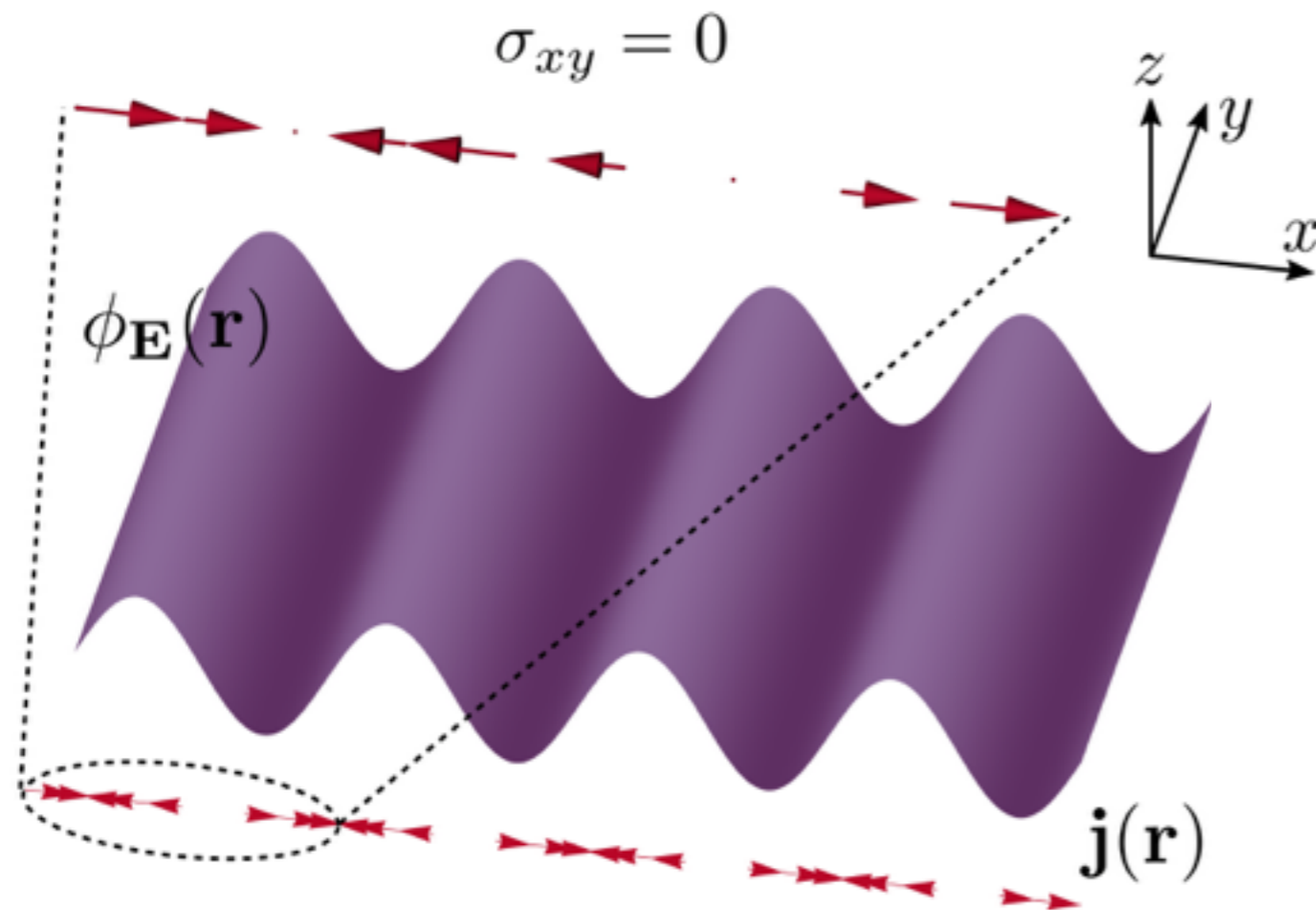


Li-kun Shi

# Plasmons: electric fields, density, and current density

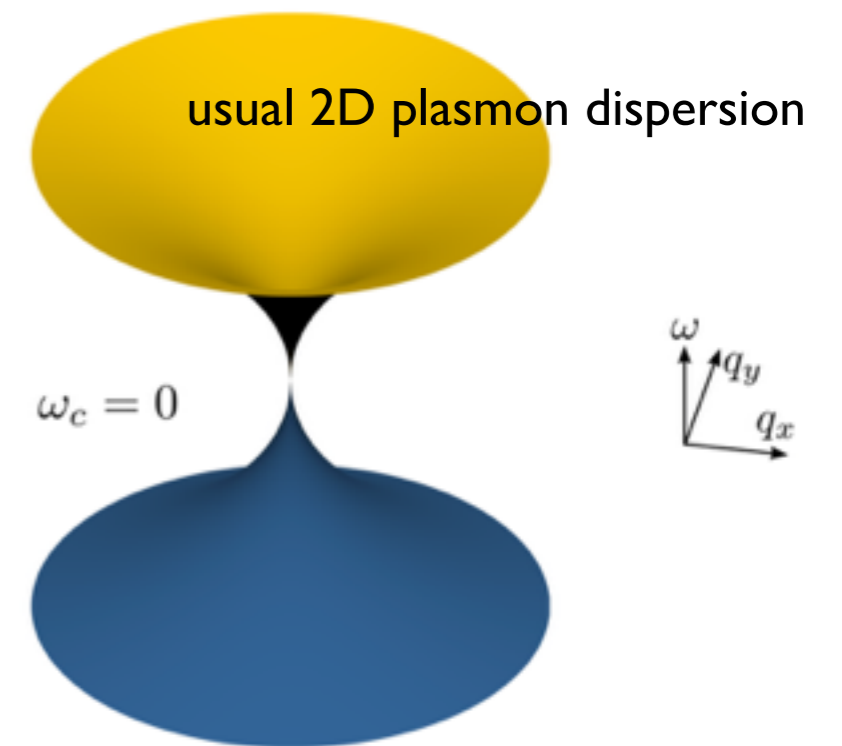
## Longitudinal electric mode

locked orientations:  $(\mathbf{q}, \mathbf{E}, \mathbf{j})$



charge dynamics:

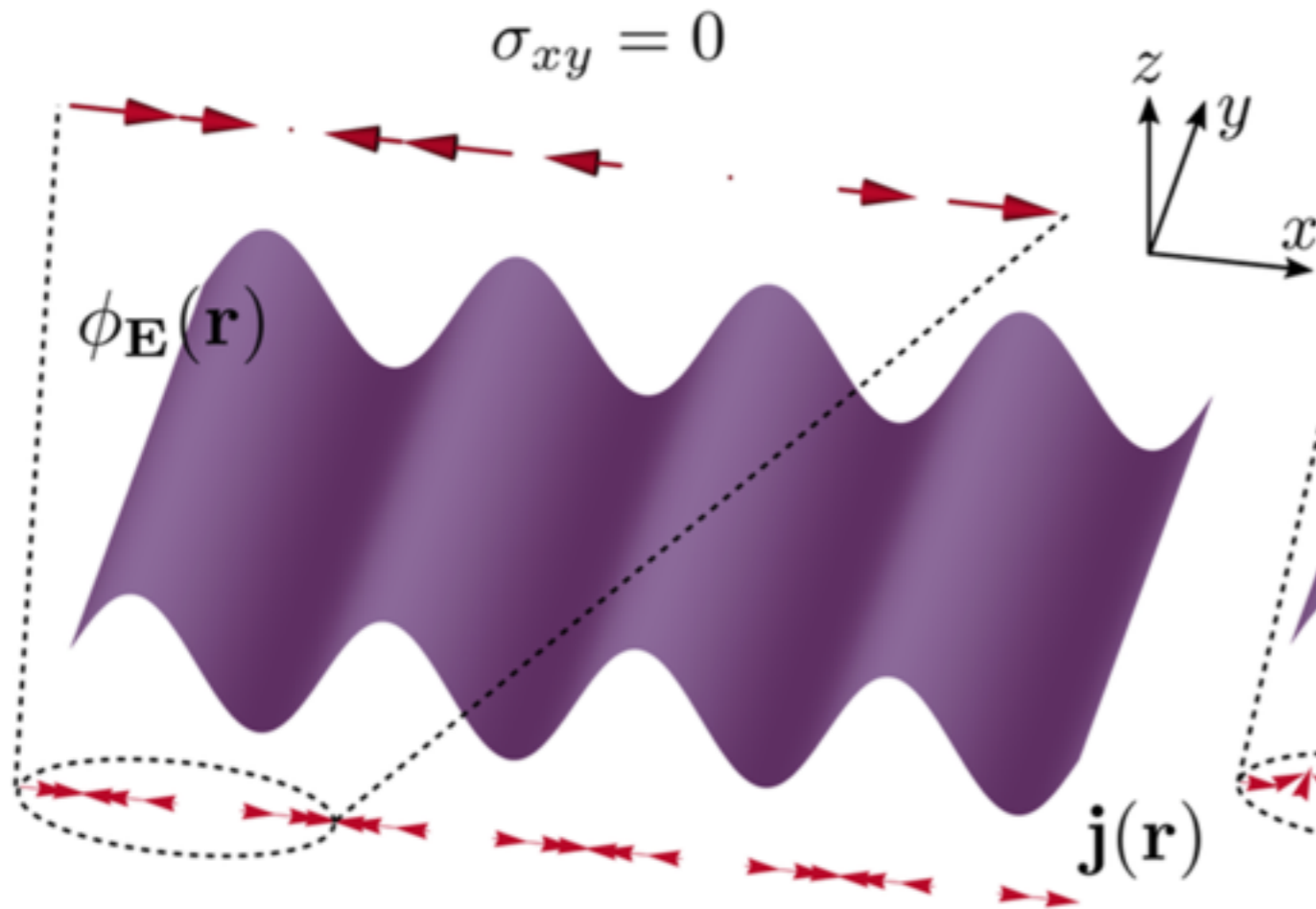
$$\partial_t \delta n(\mathbf{r}, t) + \nabla \cdot [\boldsymbol{\sigma} \mathbf{E}(\mathbf{r}, t)] = 0$$



# Plasmon internal current density structure

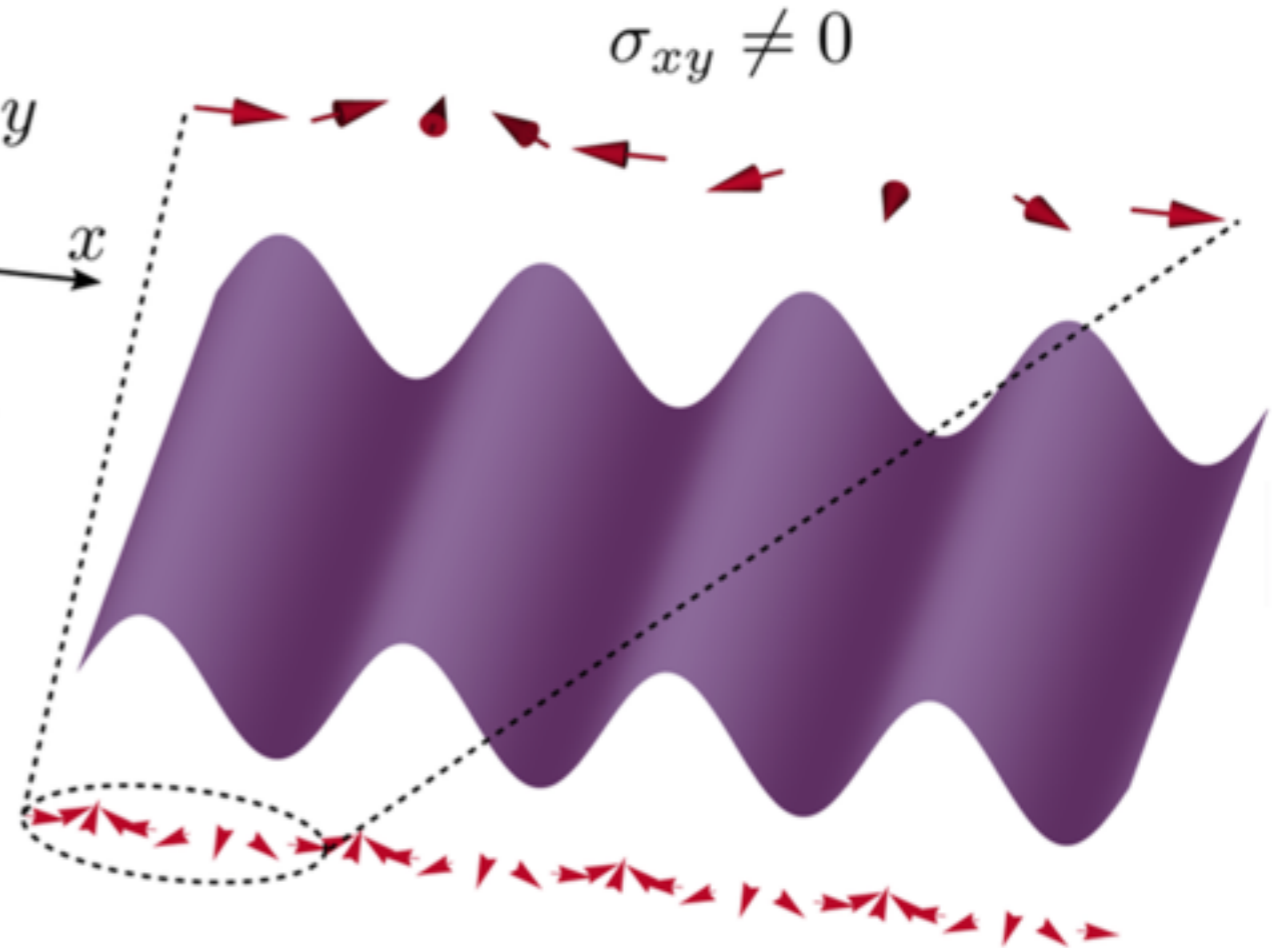
## Longitudinal electric mode

locked orientations:  $(\mathbf{q}, \mathbf{E}, \mathbf{j})$



## Longitudinal electric mode

canted orientation:  $(\mathbf{q}, \mathbf{E}) \nparallel \mathbf{j}$



# Internal current density structure

Formal treatment

Electric potential of plasmon determined by (full 3D) Maxwell's equation

$$\nabla \times [\nabla \times \mathcal{E}(\mathbf{r}, z, t)] = \frac{\omega^2}{c^2} \mathcal{E}(\mathbf{r}, z, t) - i \frac{4\pi\omega}{c^2} \mathcal{J}(\mathbf{r}, z, t),$$

As a result, Maxwell demands that electric field inside the metallic (2D) plane is related to the current density

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \mathcal{F} \begin{pmatrix} j_x \\ j_y \end{pmatrix}, \quad \mathcal{F} = \frac{2\pi i}{\omega\beta} \begin{pmatrix} \beta^2 - q_y^2 & q_x q_y \\ q_x q_y & \beta^2 - q_x^2 \end{pmatrix}. \quad \text{in non-retarded limit}$$

$$\beta = \sqrt{\mathbf{q}^2 - \omega^2/c^2} \approx |\mathbf{q}|$$

Supplemented with the conductivity (constitutive relation of the metal) plasmons can be obtained as zero modes of

$$\mathcal{M}\mathbf{j} = 0, \quad \mathcal{M} = \mathcal{F} - \sigma^{-1}, \quad \text{where} \quad \sigma_{xx} = \frac{(1 + i\omega\tau) \sigma_0}{(1 + i\omega\tau)^2 + (\omega_c\tau)^2}, \quad \sigma_{xy} = \frac{-\omega_c\tau \sigma_0}{(1 + i\omega\tau)^2 + (\omega_c\tau)^2}$$

yielding magneto-plasmon solutions as

$$\omega = \sqrt{2\pi D_0 |\mathbf{q}| + \omega_c^2} \quad \mathbf{u}(\mathbf{q}) = \begin{pmatrix} j_x(\mathbf{q}) \\ j_y(\mathbf{q}) \end{pmatrix} = \frac{\mathcal{N}}{q} \begin{pmatrix} -iq_x + \eta_q q_y \\ +iq_y - \eta_q q_x \end{pmatrix}.$$

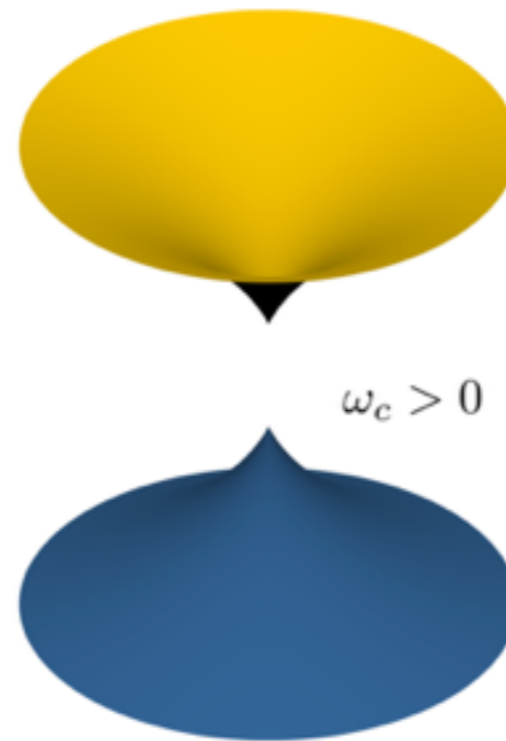
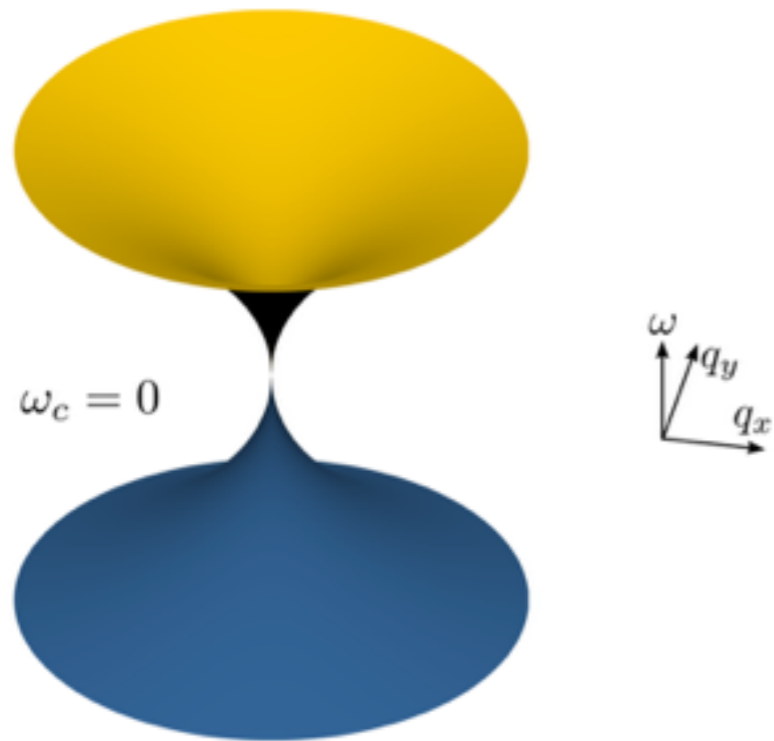
looks like a "spinor wavefunction"!

$\omega_c/\omega_q$



# Plasmon *emergent* pseudo-spin

plasmon “pseudo-spin” tracks canted orientation



$$\langle \mathbf{s}_{1,2,3} \rangle = \langle \mathbf{j} | \boldsymbol{\tau}_{1,2,3} | \mathbf{j} \rangle$$

Pauli matrices

Zero magnetic field  $\omega_c = 0$

Finite magnetic field  $\omega_c \neq 0$

LK Shi, JS, PRX (2018)

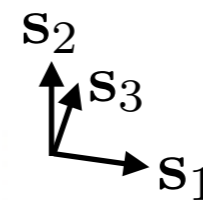
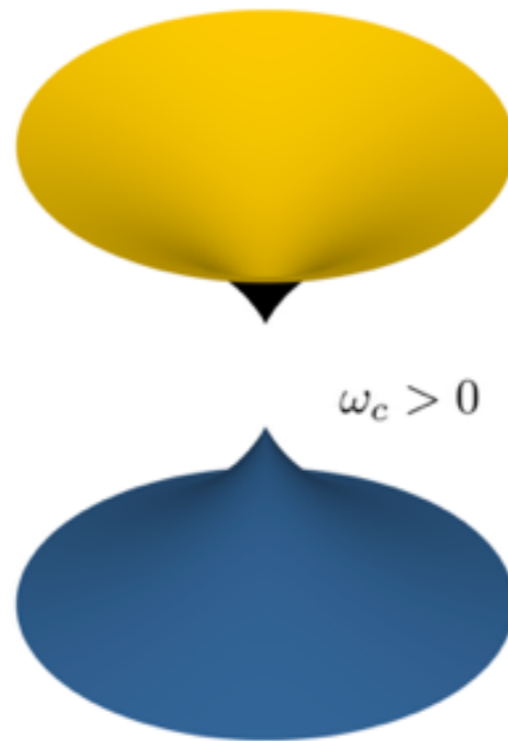
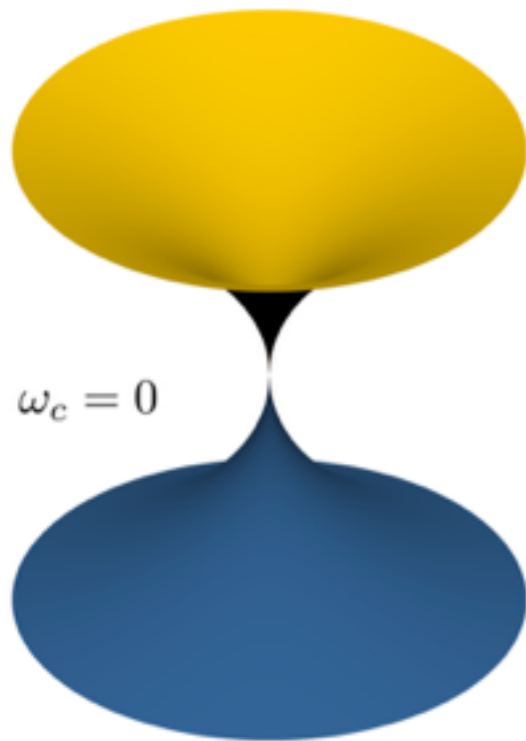
# Plasmon emergent pseudo-spin

plasmon “pseudo-spin” tracks canted orientation

$$s_1 = \frac{1 - \eta_q^2}{1 + \eta_q^2} \sin 2\phi_s, \quad s_2 = \frac{-2\eta_q}{1 + \eta_q^2}, \quad s_3 = \frac{1 - \eta_q^2}{1 + \eta_q^2} \cos 2\phi_s$$

$$\langle \mathbf{s}_{1,2,3} \rangle = \langle \mathbf{j} | \boldsymbol{\tau}_{1,2,3} | \mathbf{j} \rangle$$

Pauli matrices



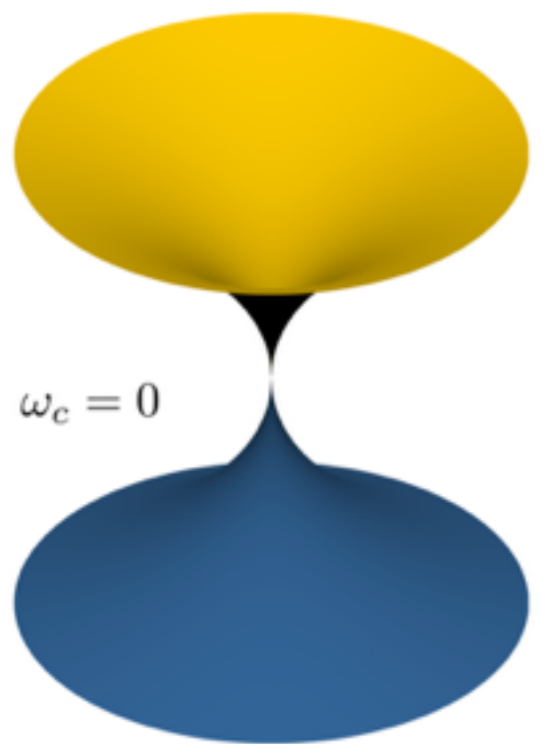
Zero magnetic field  $\omega_c = 0$

Finite magnetic field  $\omega_c \neq 0$

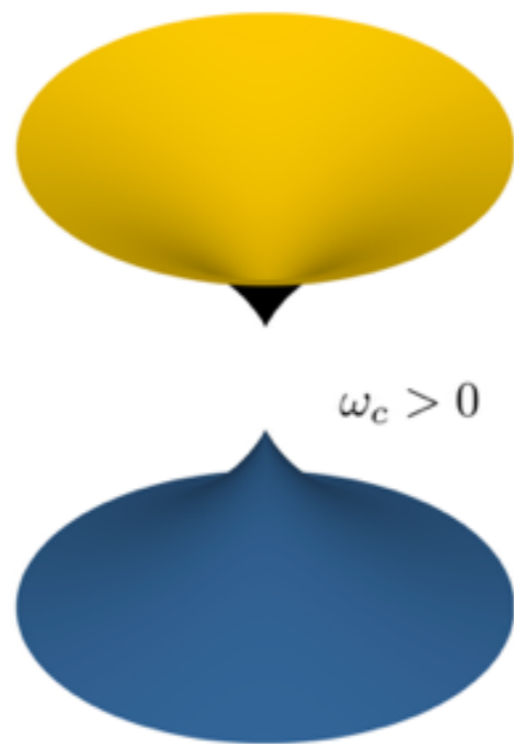
LK Shi, JS, PRX (2018)

# Plasmon *emergent* pseudo-spin

plasmon “pseudo-spin” tracks canted orientation



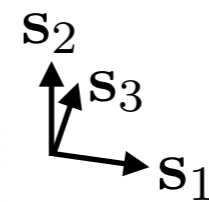
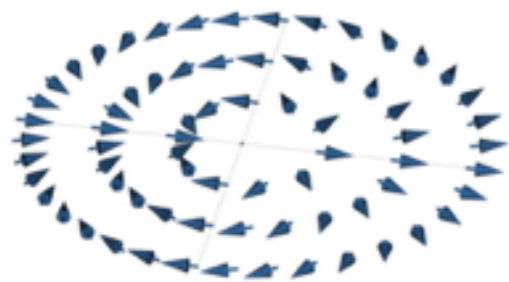
$\omega_c = 0$



$\omega_c > 0$

$$\langle \mathbf{s}_{1,2,3} \rangle = \langle \mathbf{j} | \boldsymbol{\tau}_{1,2,3} | \mathbf{j} \rangle$$

Pauli matrices



Zero magnetic field  $\omega_c = 0$

Finite magnetic field  $\omega_c \neq 0$

LK Shi, JS, PRX (2018)

# “hidden” internal current density

Hall current does not contribute to dispersion relation in bulk

$$\mathcal{M}\mathbf{j} = 0, \quad \mathcal{M} = \mathcal{F} - \sigma^{-1},$$

yielding solutions as

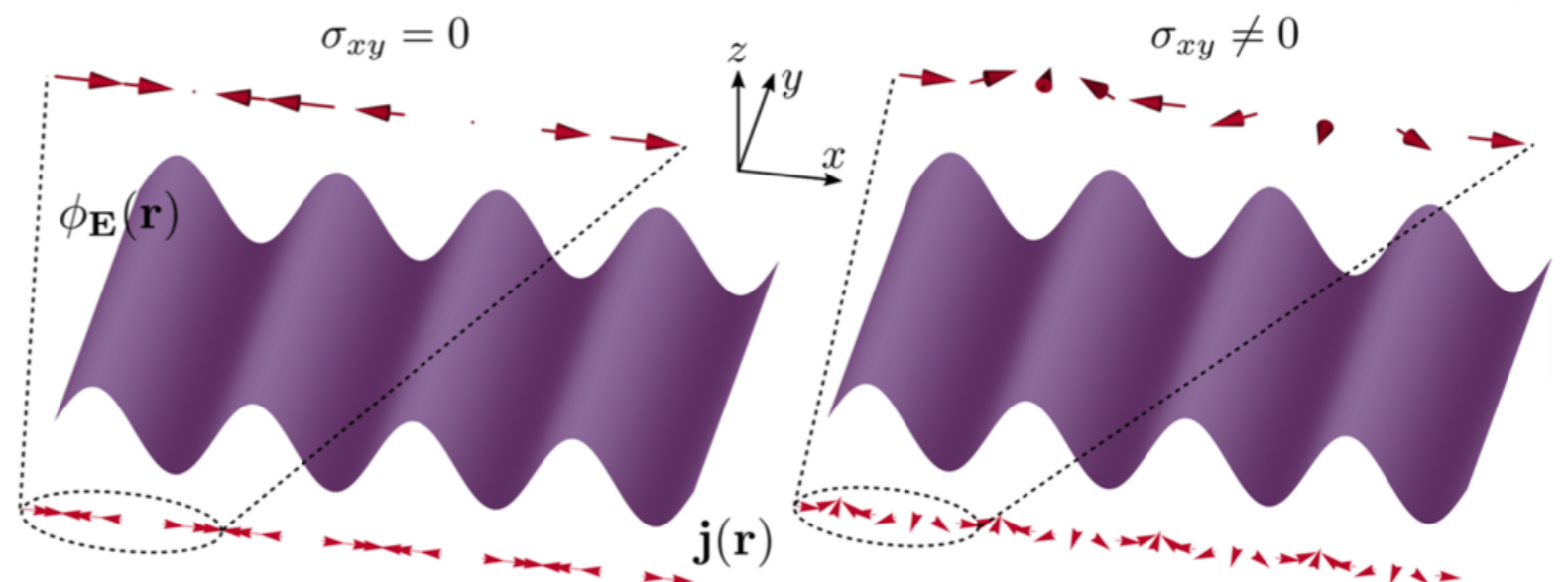
$$\mathbf{u}(\mathbf{q}) = \begin{pmatrix} j_x(\mathbf{q}) \\ j_y(\mathbf{q}) \end{pmatrix} = \frac{\mathcal{N}}{q} \begin{pmatrix} -iq_x + \eta_q q_y \\ +iq_y - \eta_q q_x \end{pmatrix}.$$

current density pattern possesses hedge-hog like texture



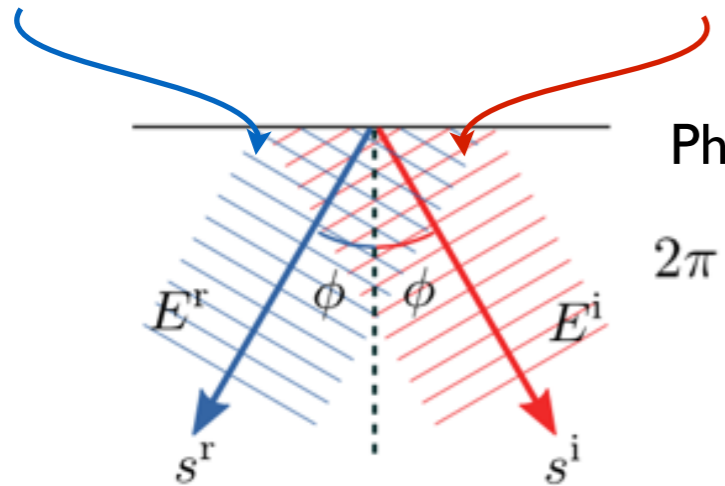
\* note that even though current density cants in the presence of B field, electric field remains Longitudinal.  
Longitudinal electric modes (as required for deep sub wavelength plasmons)

$$\epsilon(\mathbf{q}) = \mathcal{F}(\mathbf{q})\mathbf{u}(\mathbf{q}) = (2\pi\mathcal{N}/\omega_q)\mathbf{q},$$



# Plasmon geometrical phases

wave reflected off boundary      wave incident on boundary



Phase shift  $\rho$

$2\pi$

(zero B field case) typically reflected waves acquire  $\pi$  phase shift

-1

$\pi$

0

complex number

eigen vector (spinor) for current density

$$\mathbf{J}^{i,r}(\mathbf{r}) = \sum_n g_n^{i,r} \mathbf{u}(\mathbf{q}_n^{i,r}) \exp(-i\mathbf{q}_n^{i,r} \cdot \mathbf{r})$$

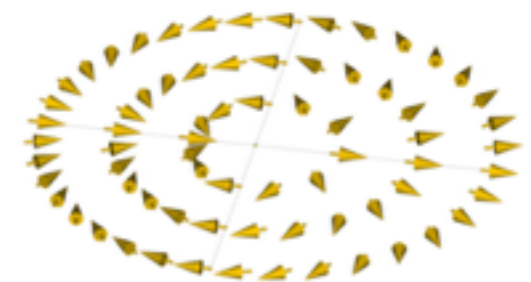
boundary condition

$$\hat{\mathbf{x}} \cdot [\mathbf{J}^i(x=0, y) + \mathbf{J}^r(x=0, y)] = 0.$$

phase shift:

$$\frac{g_n^r}{g_n^i} = -\frac{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^i)}{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^r)} = -\exp(i\rho_n)$$

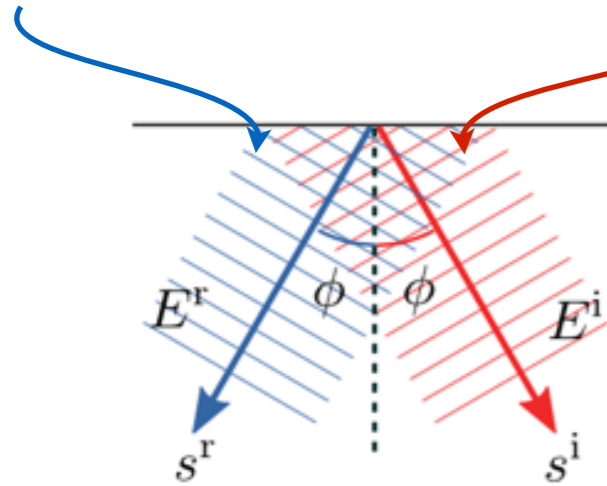
$\omega_c/\omega_q$



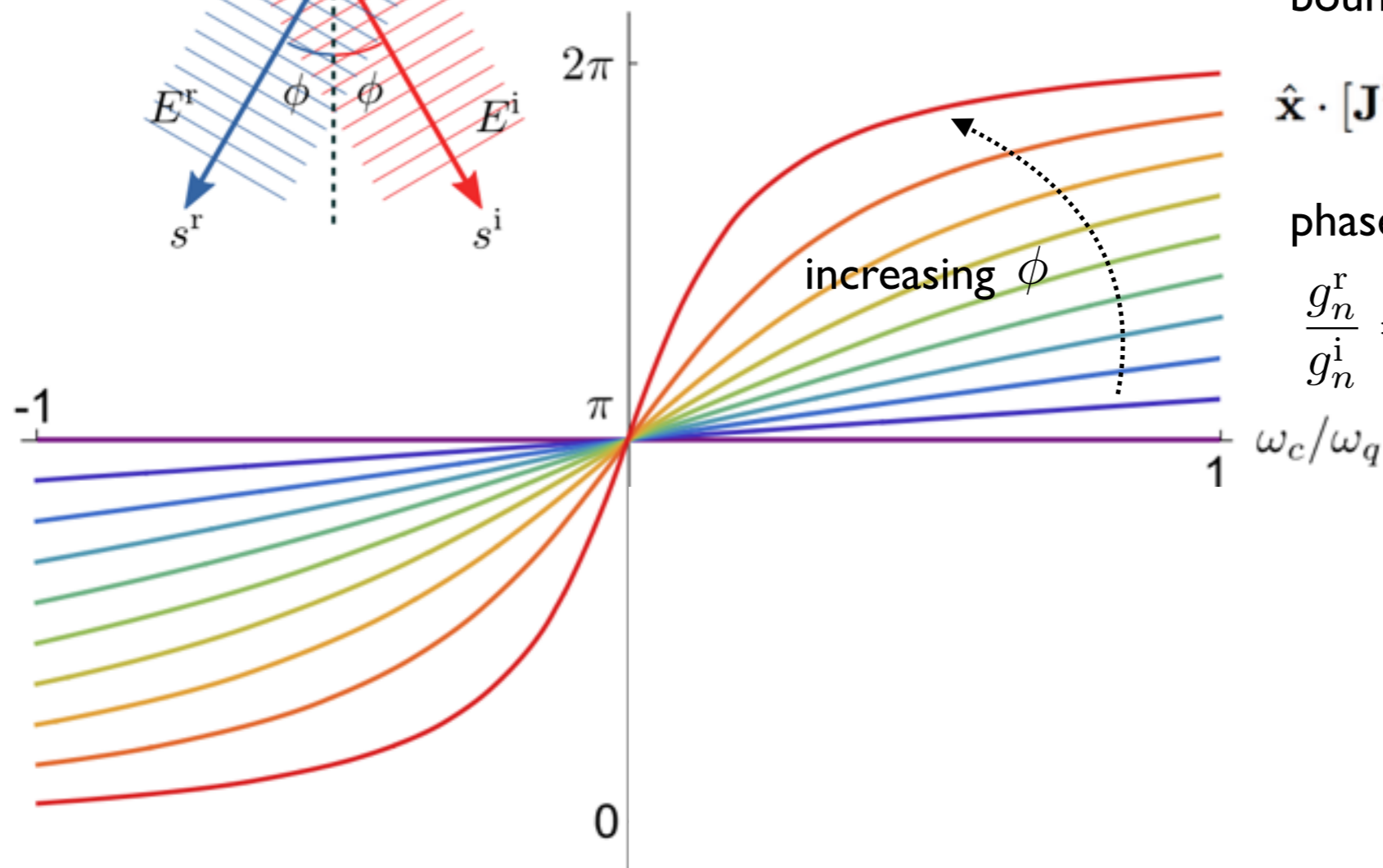
plasmon current density spinor

# Plasmon geometrical phases

wave reflected off boundary      wave incident on boundary



Phase shift  $\rho$



complex number

eigen vector (spinor) for  
current density

$$\mathbf{J}^{i,r}(\mathbf{r}) = \sum_n g_n^{i,r} \mathbf{u}(\mathbf{q}_n^{i,r}) \exp(-i\mathbf{q}_n^{i,r} \cdot \mathbf{r})$$

boundary condition

$$\hat{\mathbf{x}} \cdot [\mathbf{J}^i(x=0, y) + \mathbf{J}^r(x=0, y)] = 0.$$

phase shift:

$$\frac{g_n^r}{g_n^i} = -\frac{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^i)}{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^r)} = -\exp(i\rho_n)$$



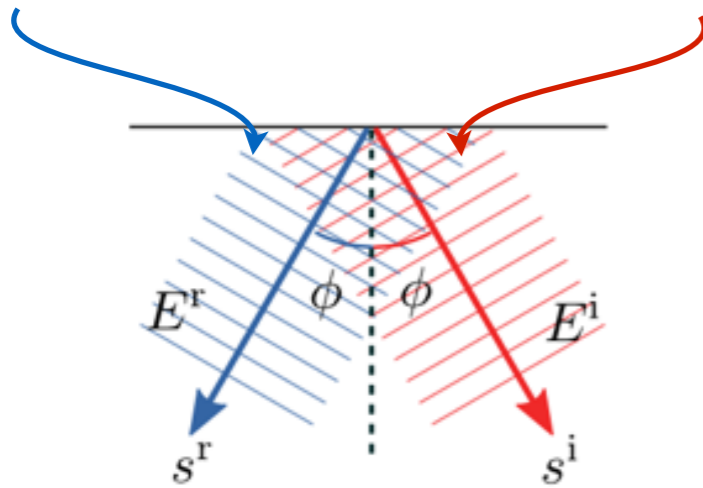
plasmon current density spinor



# Plasmon geometrical phases

wave reflected off boundary

wave incident on boundary



$$\frac{g_n^r}{g_n^i} = -\frac{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^i)}{\hat{\mathbf{x}} \cdot \mathbf{u}(\mathbf{q}_n^r)} = -\exp(i\rho_n)$$

written more suggestively (take  $\phi$  continuous determined by  $\mathbf{q}$ )

$$\nabla_{\mathbf{q}}\rho(\mathbf{q}) = \mathcal{A}(\mathbf{q}_0^r, \hat{\mathbf{n}}) - \mathcal{A}(\mathbf{q}_0^i, \hat{\mathbf{n}})$$

phase shift depends on a geometrical connection

$$\mathcal{A}(\mathbf{q}, \hat{\mathbf{n}}) = \langle u_{\hat{\mathbf{n}}}(\mathbf{q}) | i\nabla_{\mathbf{q}} | u_{\hat{\mathbf{n}}}(\mathbf{q}) \rangle$$

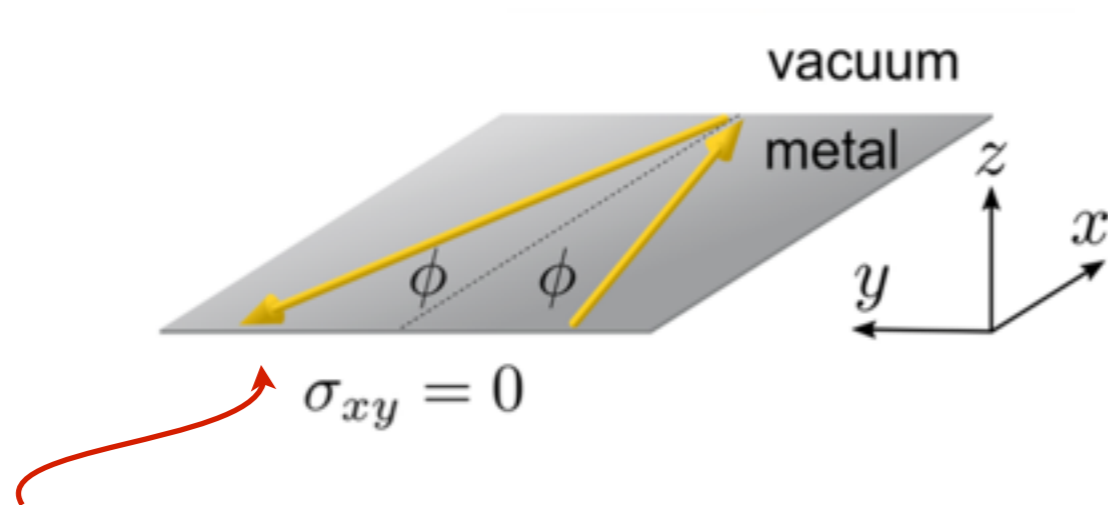


plasmon current density spinor

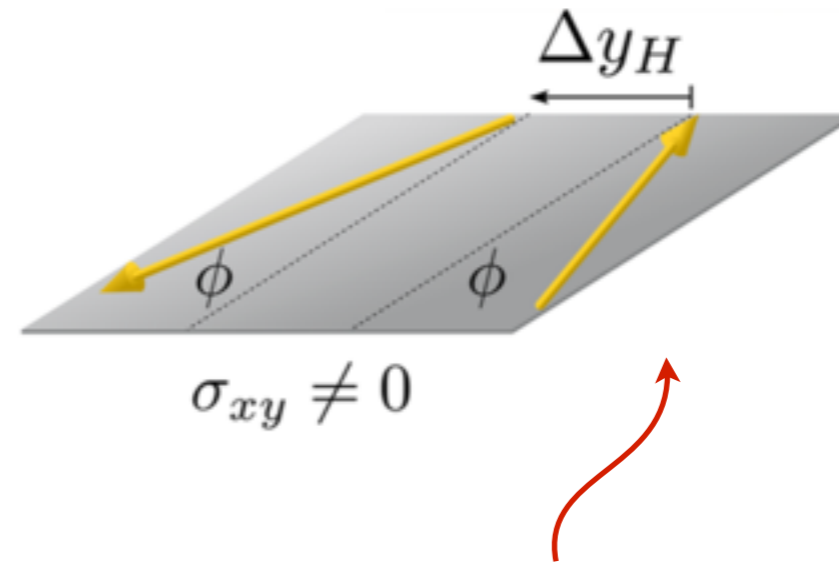
\* not quite the berry connection since only normal component of  $\mathbf{u}$  matters, nevertheless it captures geometry of pseudo-spinor texture

# Geometrical phases: Plasmon Hall effect

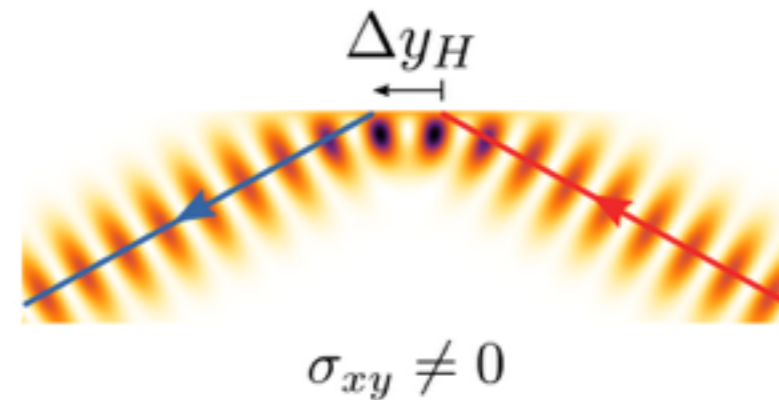
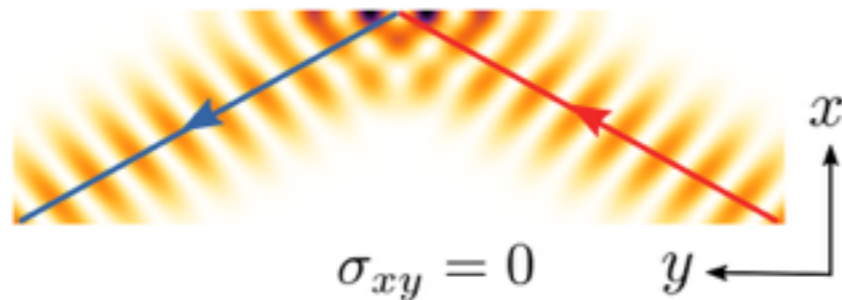
Geometric phase for multiple waves in a wave packet accumulate shifting the reflection trajectories



Conventional ray optics

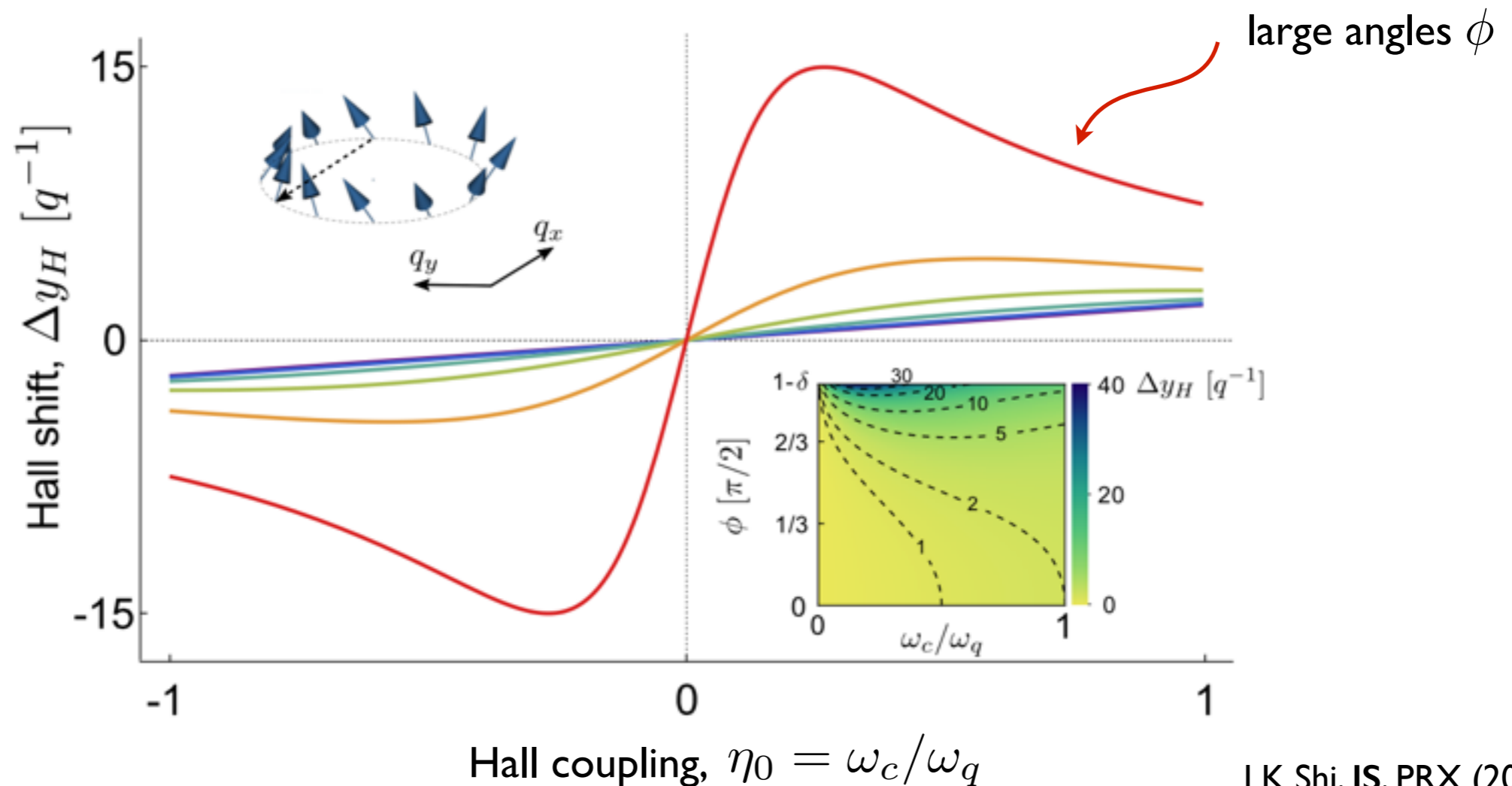
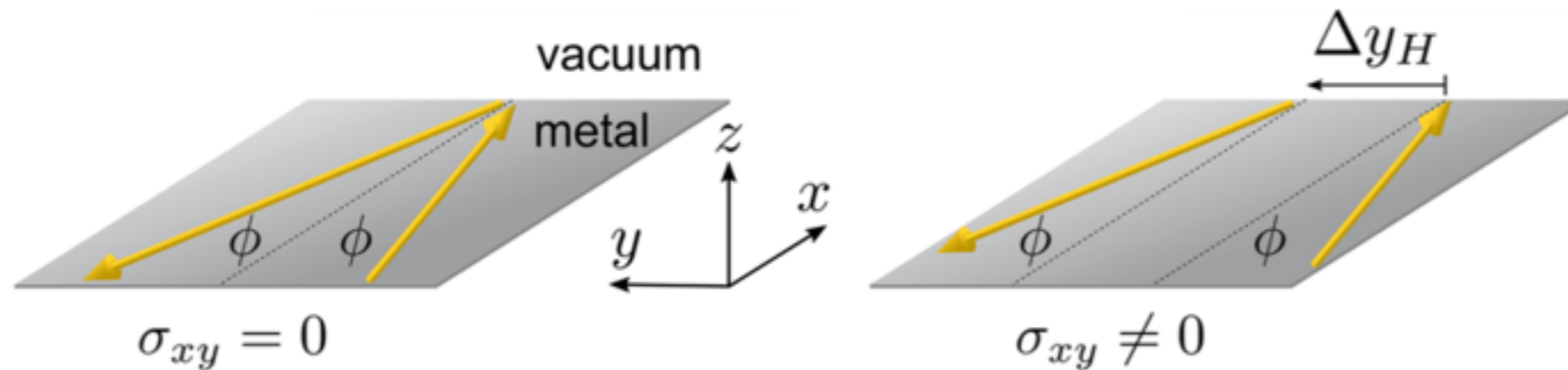


Unconventional shift



# Geometrical phases: Plasmon Hall effect

Plasmon wavepackets acquire geometrical phases, shifting their reflection trajectories



# Collective modes are platform for new out-of-equilibrium phenomena

- \* Collective modes can be a platform to realize new spontaneously broken phases
- \* Collective modes can possess an emergent structure distinct from that of the underlying crystal

## References:

Berryogenesis: spontaneous out-of-equilibrium magnetism

MS Rudner, JS, arXiv (2018)

Plasmon internal structure and geometric phase

LK Shi, JS, PRX (2018)

We gratefully acknowledge our Funding sources:

NATIONAL  
RESEARCH  
FOUNDATION

