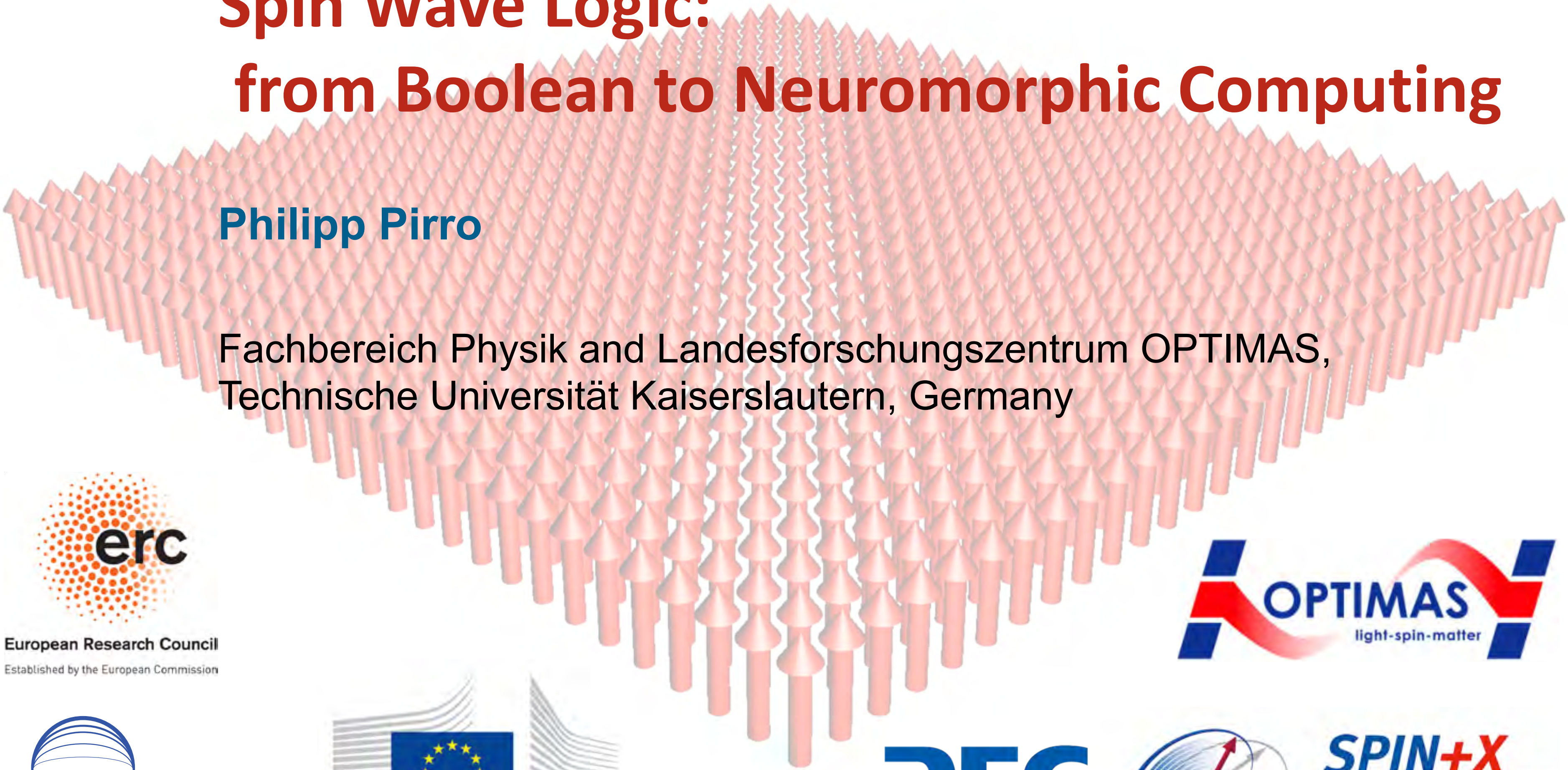


Spin Wave Logic: from Boolean to Neuromorphic Computing

Philipp Pirro

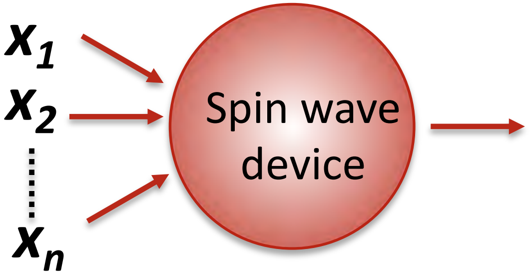
Fachbereich Physik and Landesforschungszentrum OPTIMAS,
Technische Universität Kaiserslautern, Germany

Preface

Up to now with spin waves:

- No feedback loops, no large neuronal networks
- Only first concepts for learning / how to change the w_i's

In this presentation:

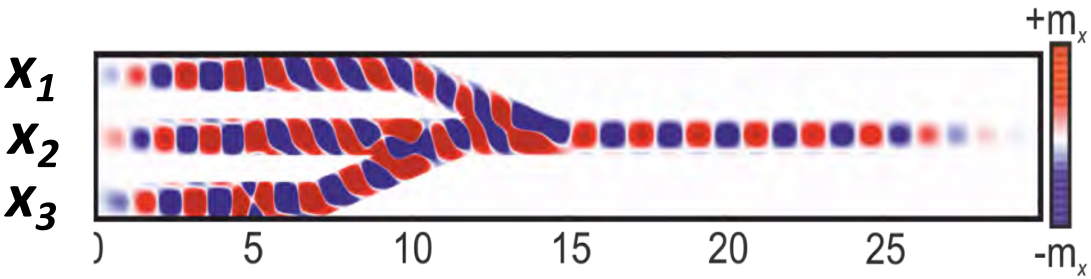


$X_{out}=f(\sum_i^n w_i x_i)$

Input
spin waves

Output
spin waves

Main question:
How to shape **complex transfer/activation functions** ***f***
using only spin waves?



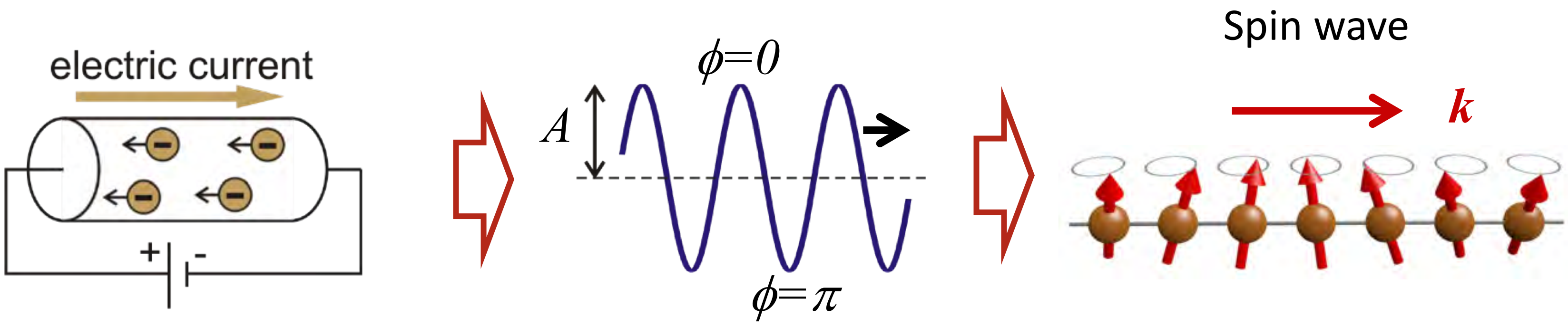
$X_{out}=\sum_i^n w_i x_i$ w_i is mainly in the wave's phase

Linear wave superposition

Outline

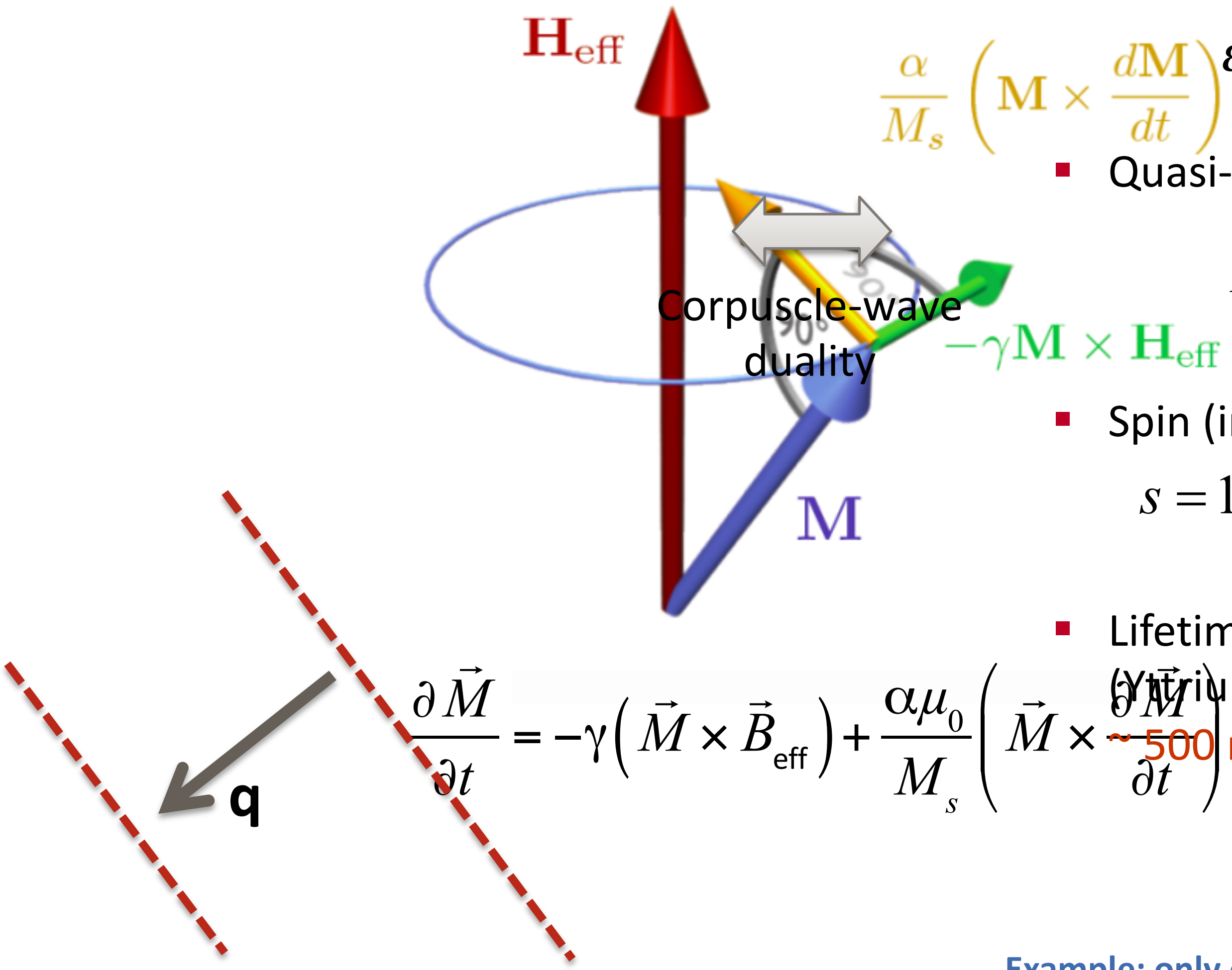
- **Wave-based** logic and particular advantages of **spin waves**
- Prototype of **linear spin-wave logic** element**: Majority Gate**
- **Frequency multiplexer and spectrum analyzer** based on **caustic beams**
- **Towards integrated magnonic circuits:**
  - Magnonic Half adder (quasi binary)
  - Analog magnon adder

Wave-based computing



Variable	**Scalar**	**Vector**	$(A, \phi, f, \mathbf{k})$
Transport	Driven by **potential gradient**	Driven by **wave propagation**	Novel circuit design
Logic	Semiconductor based **transistors**	**Interference** phenomena	Simple realization of logic functionalities

Spin waves and magnons



Corpuscle-wave duality

$$\frac{\partial\vec{M}}{\partial t} = -\gamma\left(\vec{M}\times\vec{B}_{\text{eff}}\right) + \frac{\alpha\mu_0}{M_s}\left(\vec{M}\times\frac{\partial\vec{M}}{\partial t}\right)$$

MAGNON quasi-particle :

- Energy

$$\varepsilon = \hbar\omega = \frac{\eta}{\hbar}p^2$$

- Quasi-momentum

$$\vec{p} = \hbar\vec{q}$$

- Spin (in Planck's constant)

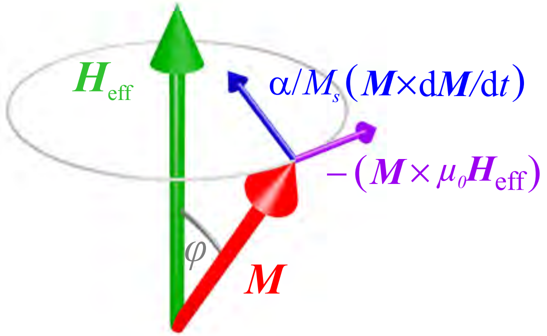
$$s = 1$$

- Lifetime
(Yttrium Iron Garnet, $Y_3Fe_5O_{12}$)
~ 500 ns !

Example: only exchange interaction considered

Effective field $B_{\mathrm{eff}}=\mu_0 H_{\mathrm{eff}}$

$$\frac{1}{|\gamma|}\frac{d\vec{M}(\vec{r},t)}{dt} = -\vec{M}(\vec{r},t)\times\boxed{\vec{B}_{eff}(\vec{r},t)} + \frac{\alpha}{M_s}\vec{M}(\vec{r},t)\times\frac{d\vec{M}(\vec{r},t)}{dt}$$



$$\vec{B}_{\mathrm{eff}} = \mu_0\vec{H}_{\mathrm{eff}} = -\nabla_{\vec{M}}\varepsilon\left(\vec{M}\right)$$

ε: Magnetic energy density

$$= \vec{B}_0 + \vec{b}_0(t) + \int_V \tilde{G}(\vec{r},\vec{r}')\cdot\vec{M}(\vec{r})\,\mathrm{d}\vec{r}'^3 + \frac{\eta}{\gamma M_s}\Delta\vec{M} + \frac{D}{M_s}\left(\hat{z}\times\frac{\partial M}{\partial\hat{x}}\right)$$

Dipolar interaction described by Green's function *G*

Symmetric Heisenberg exchange interaction

Antisymmetric Dzyaloshinskii–Moriya exchange interaction

Contribution to the spin-wave dispersion relation (ferromagnetic ground state):

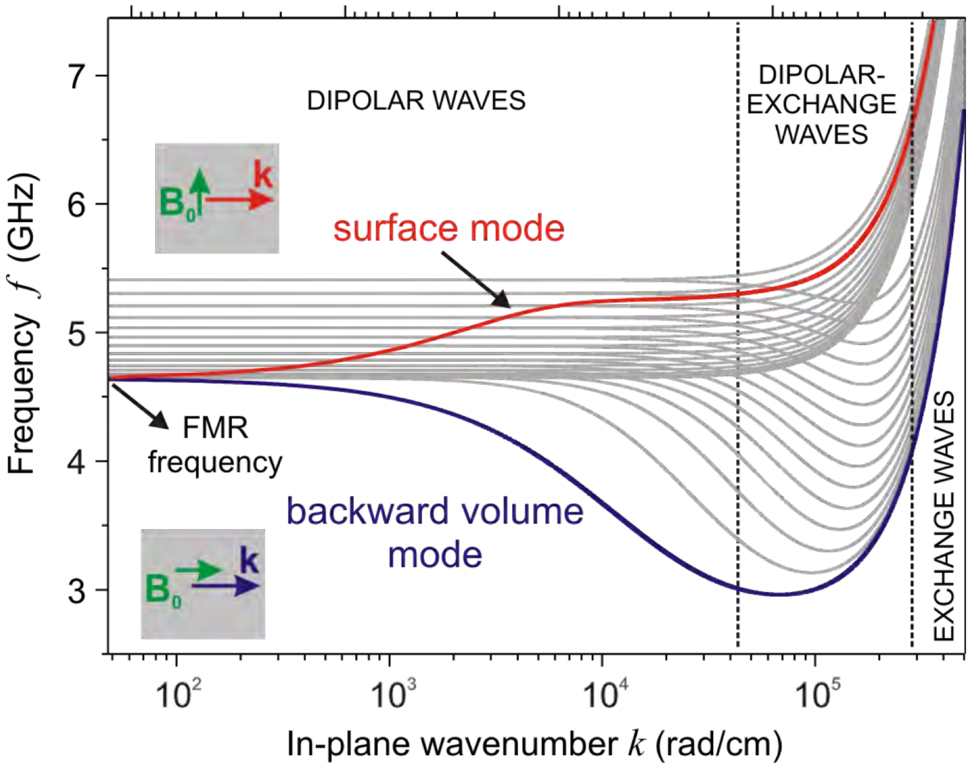
Geometry dependent, complex contribution

$f \sim k^2$

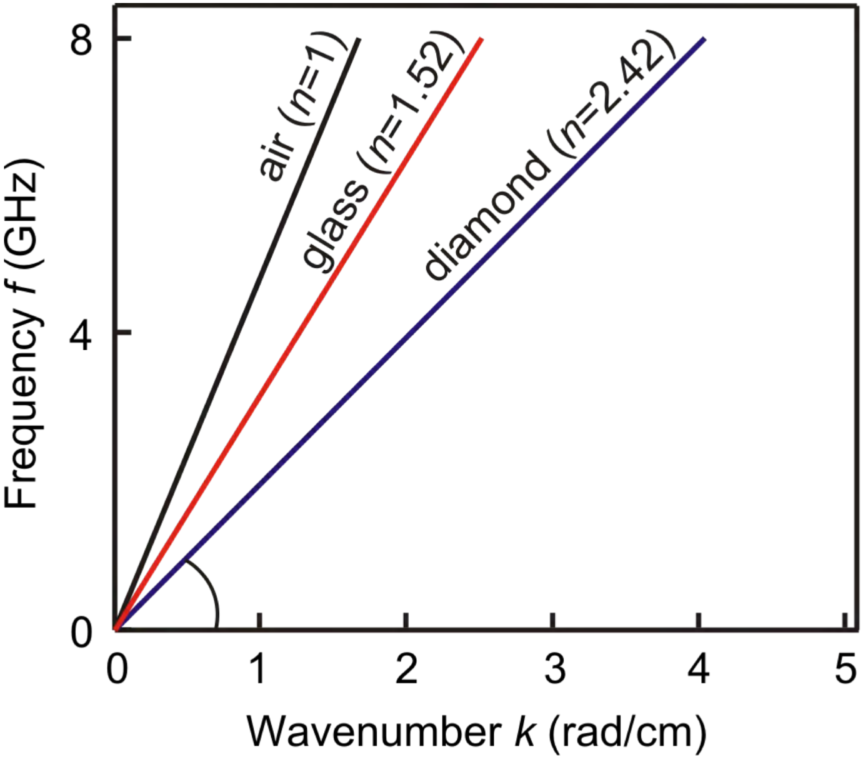
$f \sim k$

Magnon- and photon dispersion relations

Magnons in a YIG film



Photons in different media



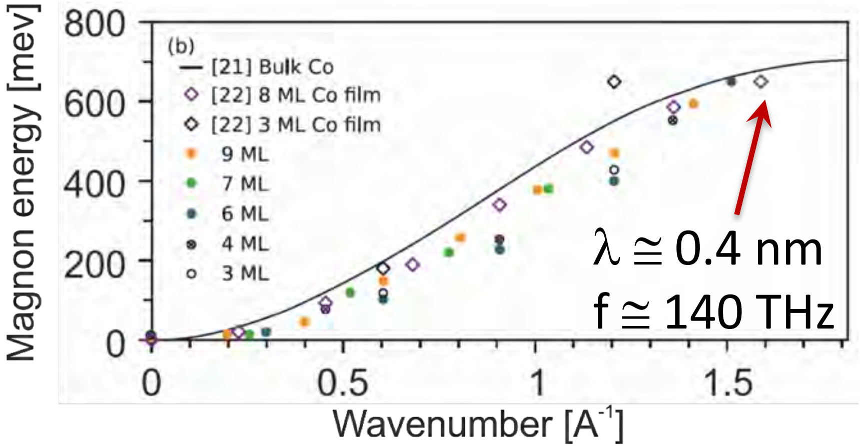
Magnons: about 10^3-10^5 higher wavenumbers for the same frequency range:

- **Miniaturization** is facilitated
- Magnons travel much **slower** than photons: direct **time resolved investigation** possible

Why SPIN waves?

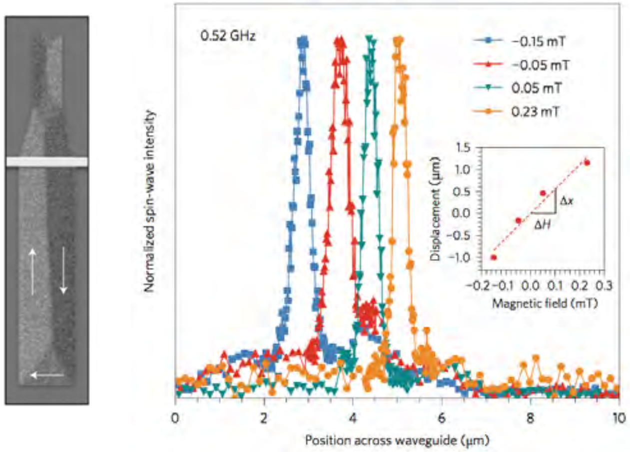
- Scaling: frequency up to several THz, wavelength down to nanometers
- Efficient nonlinear effects
- Non-reciprocal effects (DMI, dipole-dipole interaction)
- Design of bandstructure, reconfigurable propagation
- Room temperature
- CMOS compatibility (frequency, converters, nano-technology)

Magnon Spintronics, Nature Phys. **11**, 453 (2015)



$\lambda \cong 0.4$ nm
f $\cong$ 140 THz

Balashov, et al., J. Phys.: Condens. Matter **26**, 394007 (2014):
Inelastic tunneling spectroscopy (ITS)



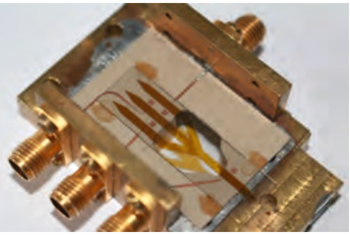
Wagner et al., Nature Nanotech 11, 432 (2016).

Examples of spin-wave logic

Non exhaustive list, many more approaches

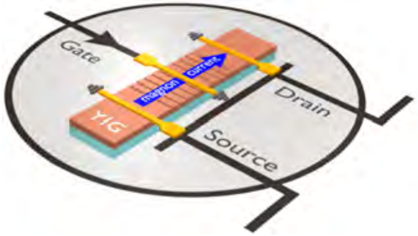
Focusing on spin-wave physics

Majority Gate



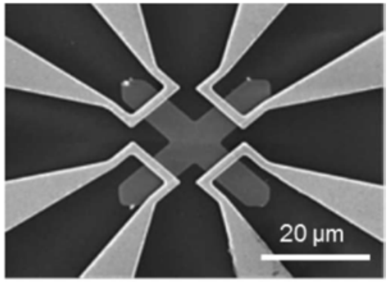
Fischer et al., APL **110**, 152401 (2017)

Magnon Transistor



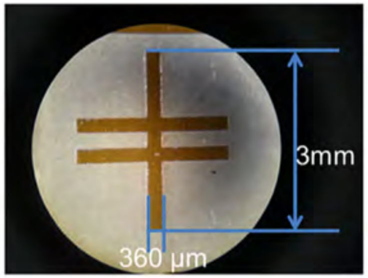
Chumak et al., Nat. Commun. **5**, 4700 (2014)

Holographic Memory



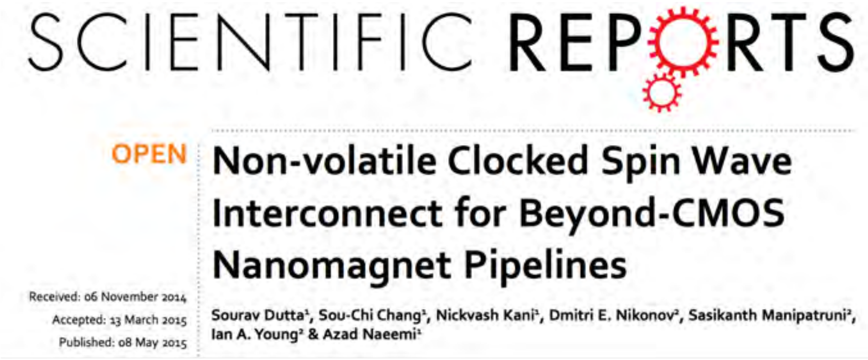
F. Gertz et a., IEEE Magn. Lett. **7**, 3200204 (2016)

Factorization Problem



Y. Khivintsev et a., JAP **120**, 123901 (2016)

Focusing on technology



SCIENTIFIC REPORTS

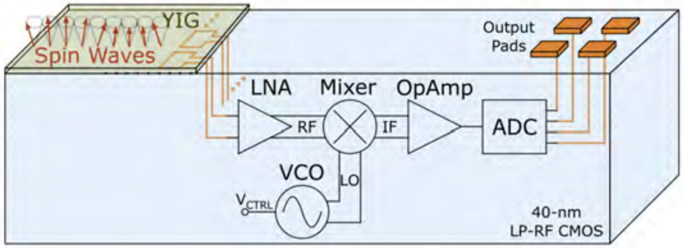
OPEN

Non-volatile Clocked Spin Wave Interconnect for Beyond-CMOS Nanomagnet Pipelines

Received: 06 November 2014
Accepted: 13 March 2015
Published: 08 May 2015

Sourav Dutta[1], Sou-Chi Chang[1], Nickvash Kani[1], Dmitri E. Nikonov[2], Sasikanth Manipatruni[2], Ian A. Young[2] & Azad Naeemi[1]

Broadband on-chip detection



E. Egel, G. Csaba, et al ,
AIP Advances 8, 056001 (2018).

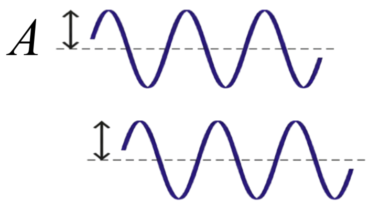
Benchmarks: Spin wave circuits interesting for low power applications (mobile devices, IOT..)

Outline

- **Wave-based** logic and particular advantages of **spin waves**
- Prototype of **linear spin-wave logic** element**: Majority Gate**
- **Frequency multiplexer and spectrum analyzer** based on **caustic beams**
- **Towards integrated magnonic circuits:**
  - Magnonic Half adder (quasi binary)
  - Analog magnon adder

Magnon majority gates: General idea

Data is coded into spin-wave **phase**



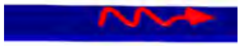
0: ϕ=0

1: ϕ=π

Majority gate

Inverter

λ/2 =>Δϕ=π



Input A	Input B	Input C	Output
0	0	0	0
1	0	0	0
0	1	0	0
1	1	0	1
0	0	1	0
1	0	1	1
0	1	1	1
1	1	1	1

0: ϕ=0 **1: ϕ=π**

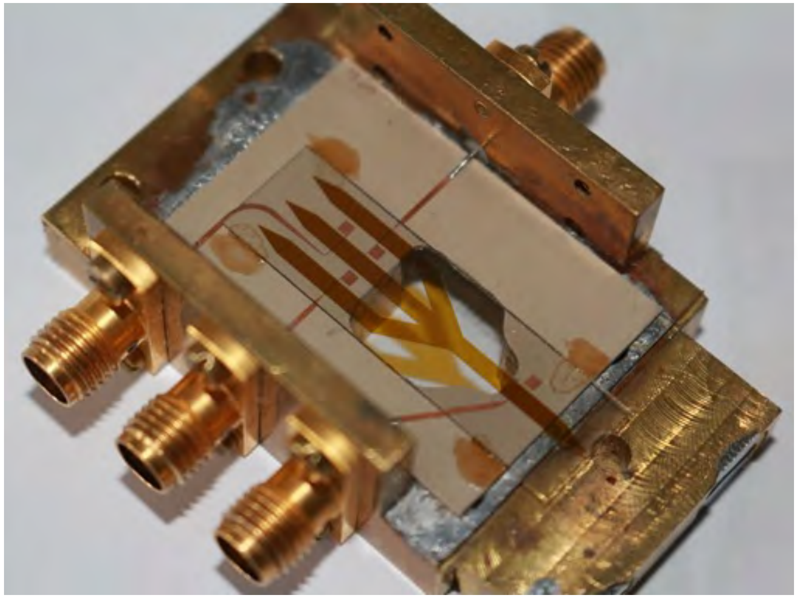
A. Khitun, et al., J. Phys. D. **43**, 264005 (2010)

- Simple realization (spin-wave combiner), all magnon, trivial inverter
- Majority gate + inverter are building blocks for full logic functionality: access to AND, OR, NAND, NOR and Full Adder gates with reduced complexity

Klingler, et al. APL **105**, 152410 (2014) Klingler, et al. APL **106**, 212406 (2015)

Experimental realization

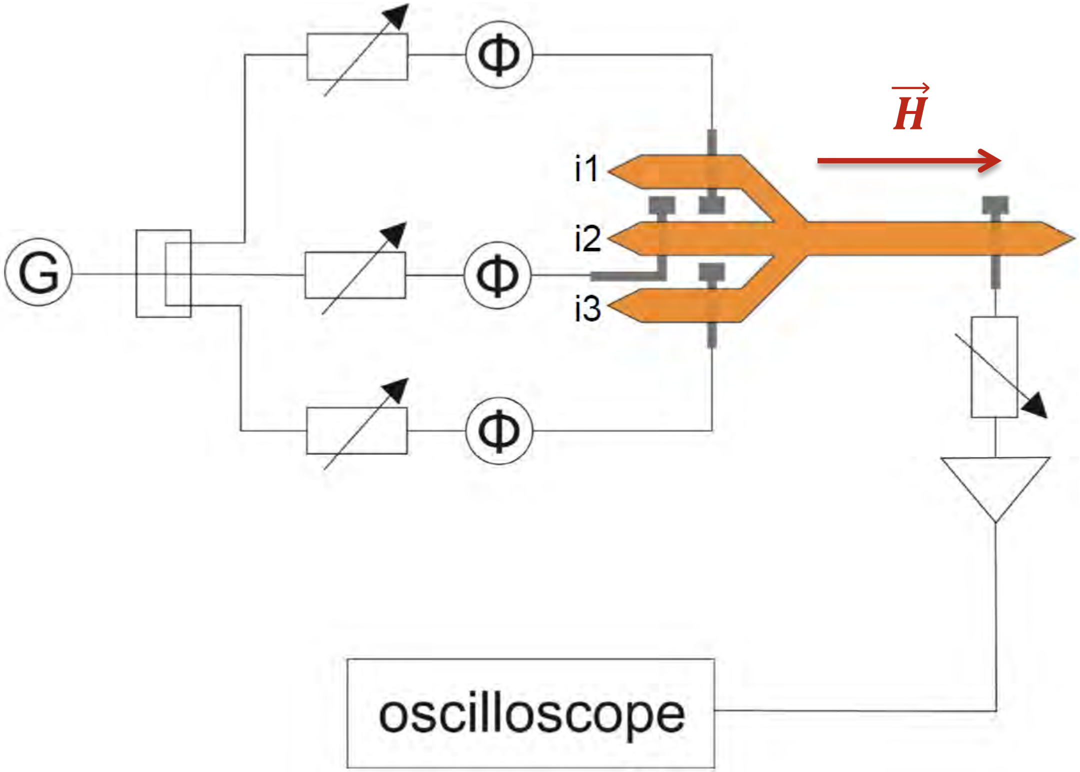
Macroscopic majority gate



YIG sample:
- - thickness 5.4 µm
- - waveguide width 1.5 mm

Produced by Carat, Lviv, Ukraine

Experimental setup

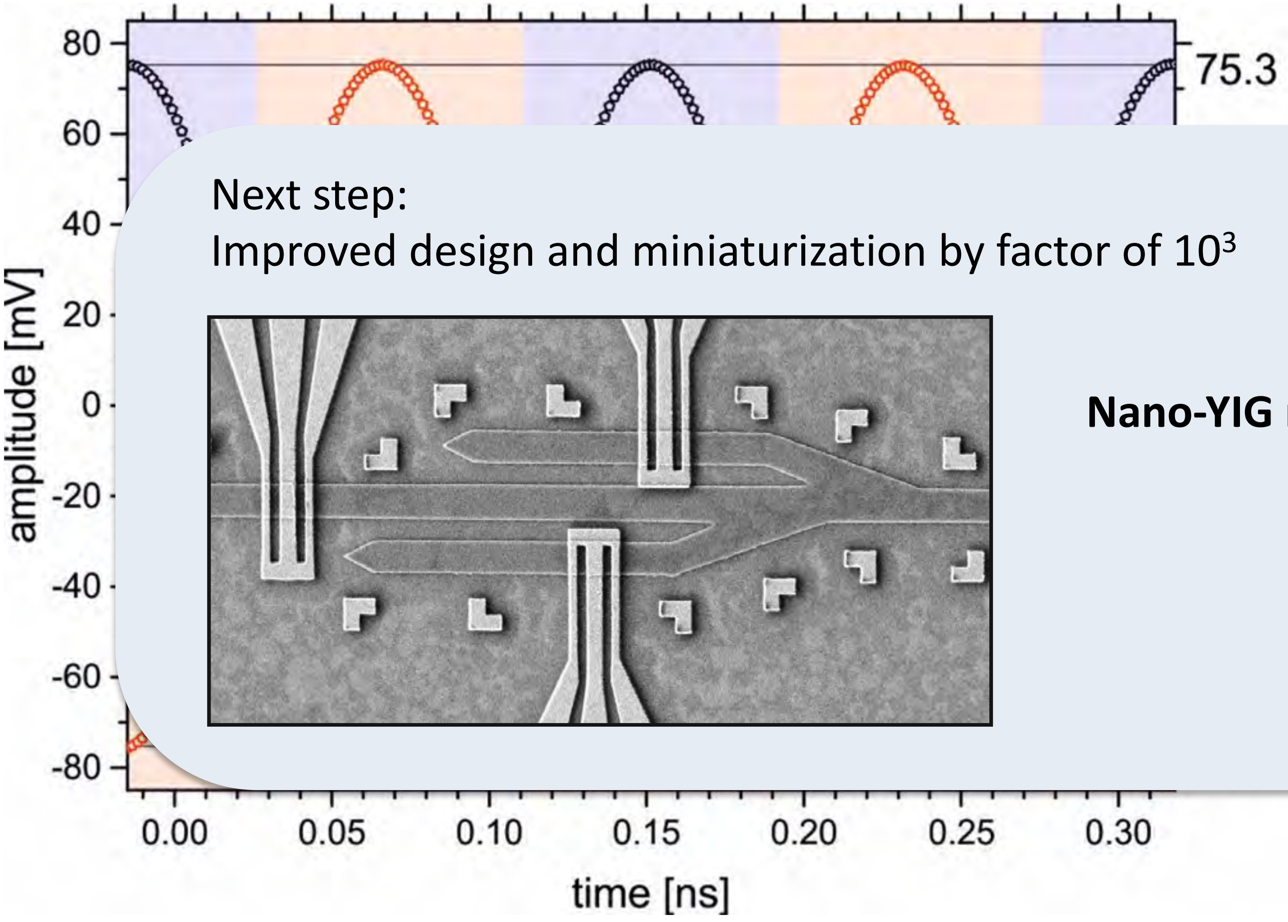


Geometry	BVMSW
Frequency	6.035 GHz
Magnetic field	142.9 mT

T. Fischer et al., APL **110**, 152401 (2017)

Superposition of all channels

Real time observation of the resulting spin-wave interference via fast oscilloscope:



Next step:
Improved design and miniaturization by factor of 10^3

Nano-YIG majority gate

e output
s defined
by the majority of the
input phase

amplitude ratio: 75.3/24.7=**3.05,** expected: 3

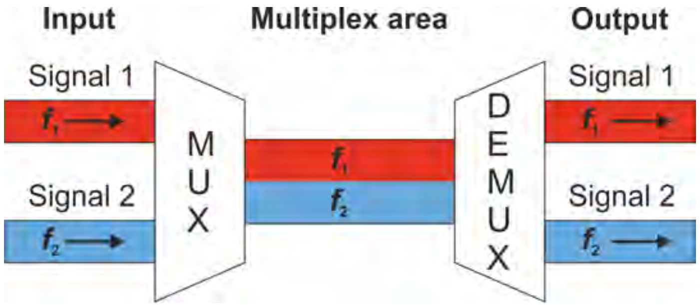
T. Fischer et al., APL **110**, 152401 (2017)

Outline

- **Wave-based** logic and particular advantages of **spin waves**
- Prototype of **linear spin-wave logic** element: **Majority Gate**
- **Frequency multiplexer and spectrum analyzer** based on **caustic beams**
- **Towards integrated magnonic circuits:**
  - Magnonic Half adder (quasi binary)
  - Analog magnon adder

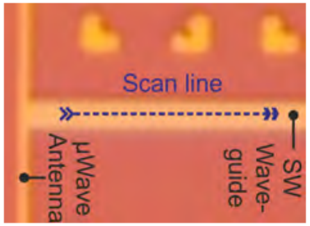
Frequency-division multiplexing (FDM)

Concept

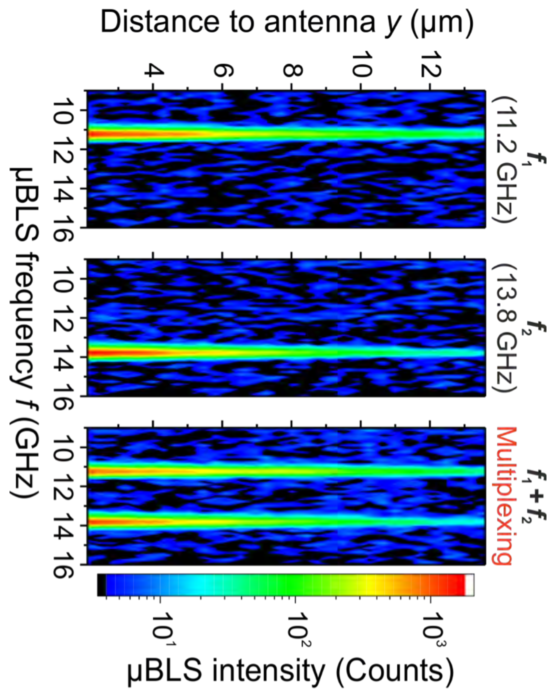


Spin-wave realisation

sample



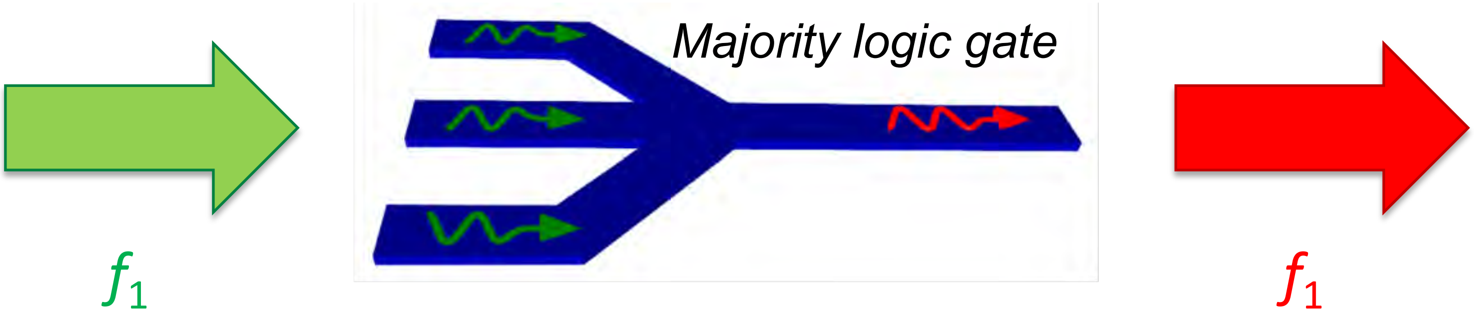
BLS measurement



- Signals are **separated by frequency**
- Simultaneous transport of data through **one transmission line**

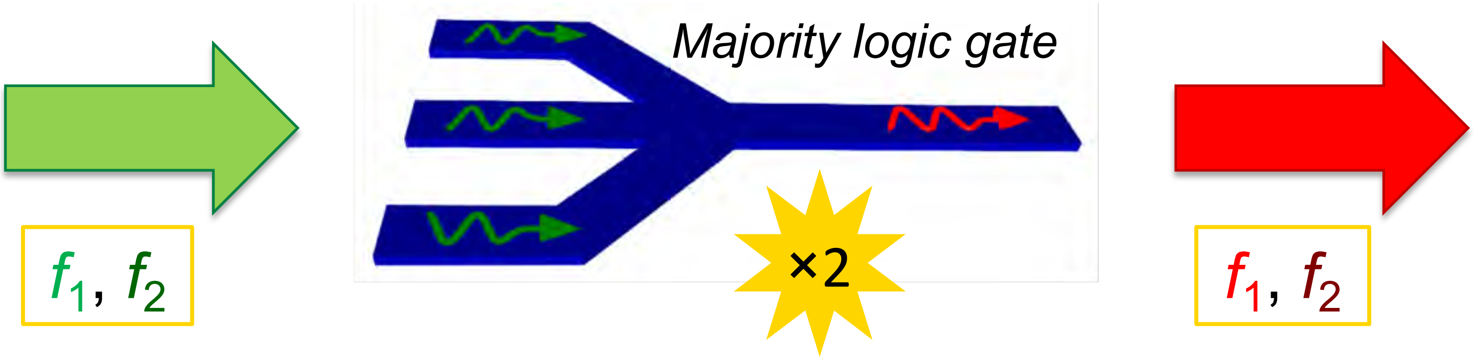
Multiplexing

Multi-frequency magnonic logic circuits



Multiplexing

Multi-frequency magnonic logic circuits



- Simultaneous logic operations in a single device

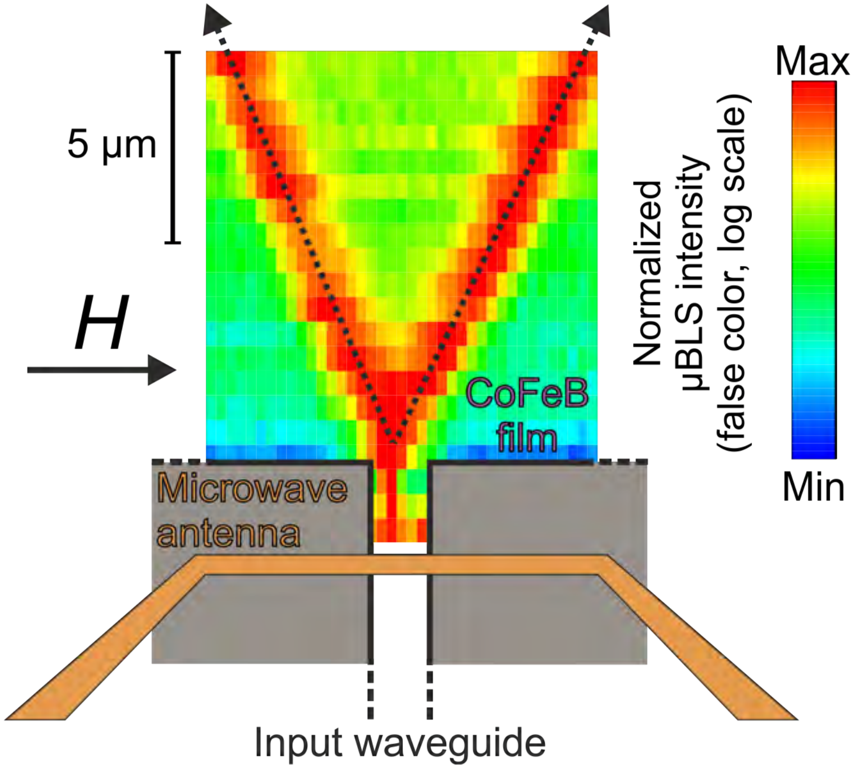
  → Multiplication of the processing rate

- Efficient signal transport

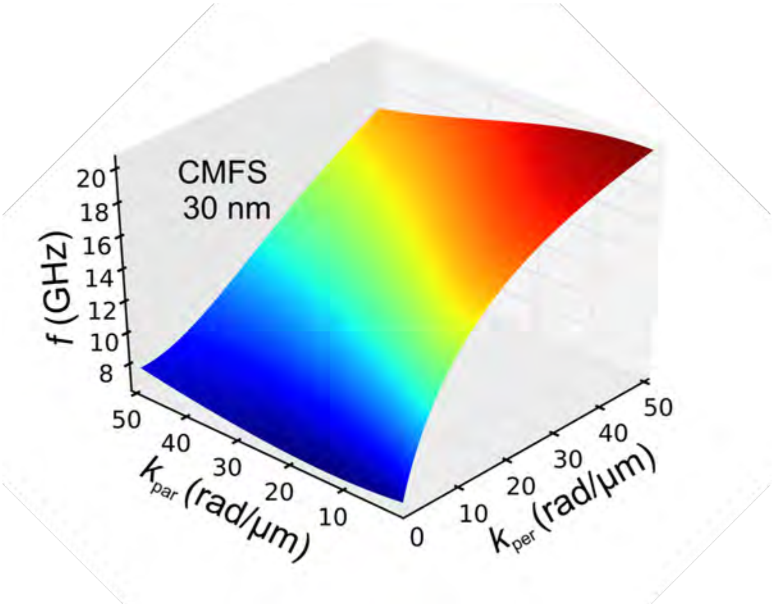
Requirements:

Frequency-division multiplexer for magnonic networks:
Combining and separating multi-frequency spin-wave signals

Caustic-like spin-wave beams



Microfocused Brillouin Light Scattering measurement



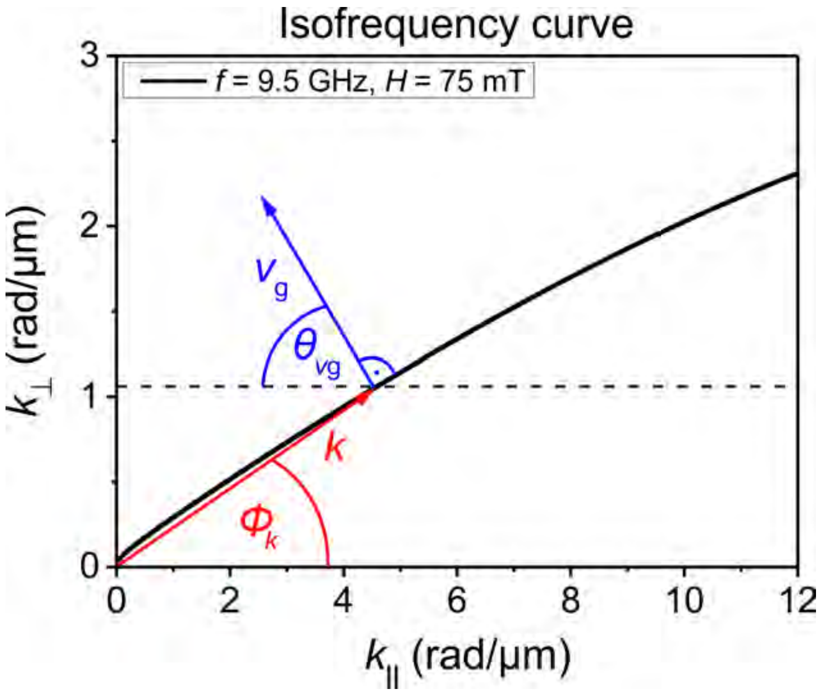
Spin-wave dispersion in a thin film

- Low-diffractive spin-wave beams in 2D magnetic films (in-plane magnetized)
- Formation due to the anisotropy of the spin-wave dispersion (dipole-dipole)

Origin of caustic-like SW beams

Anisotropic spin-wave dispersion

Material parameters of the Heusler compound $Co_2Mn_{0.6}Fe_{0.4}Si$ (CMFS)



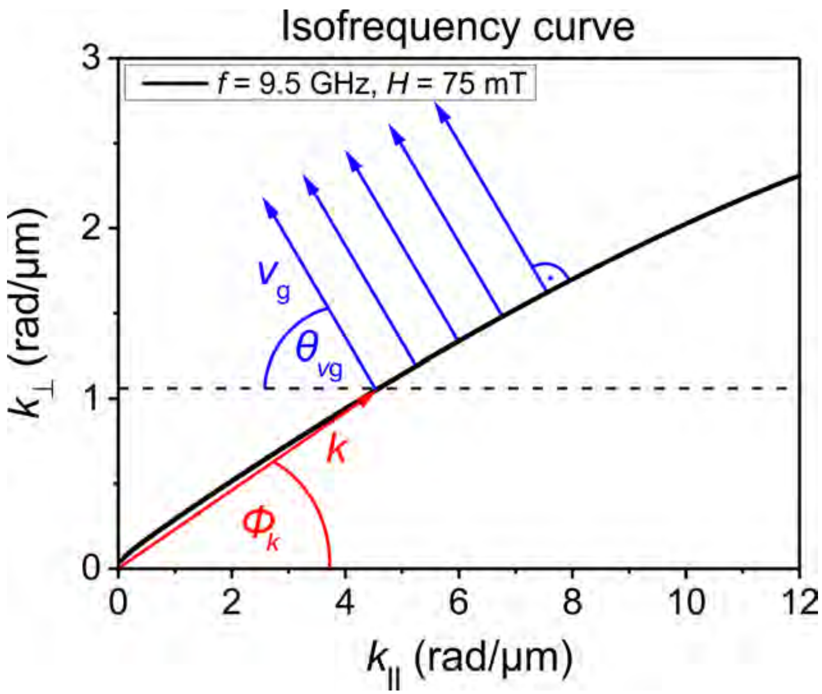
$$\boxed{\vec{v}_g \nparallel \vec{k}}$$

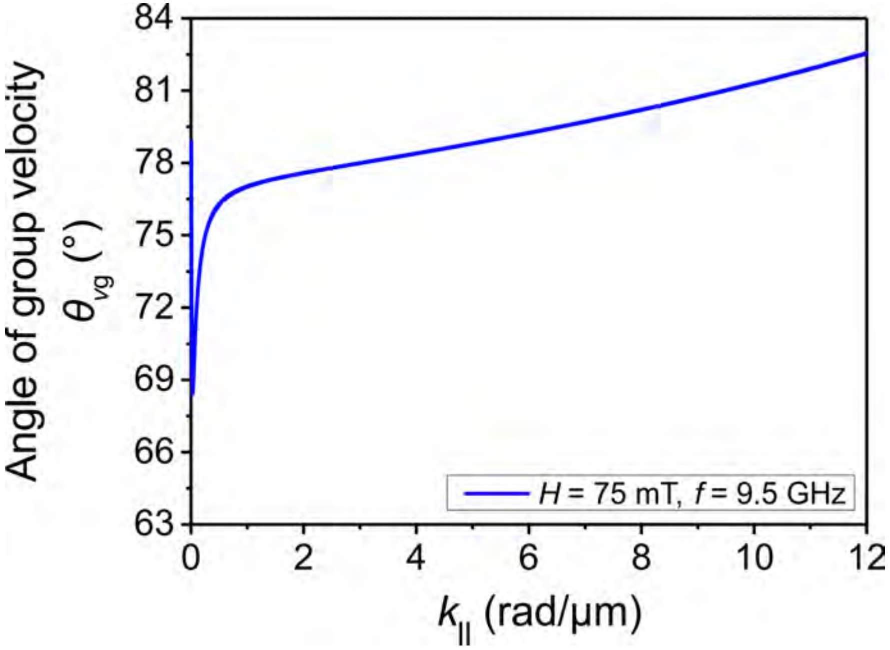
T. Sebastian et al., PRL **110**, 067201 (2013)

Origin of caustic-like SW beams

Anisotropic spin-wave dispersion

Material parameters of the Heusler compound $Co_2Mn_{0.6}Fe_{0.4}Si$ (CMFS)





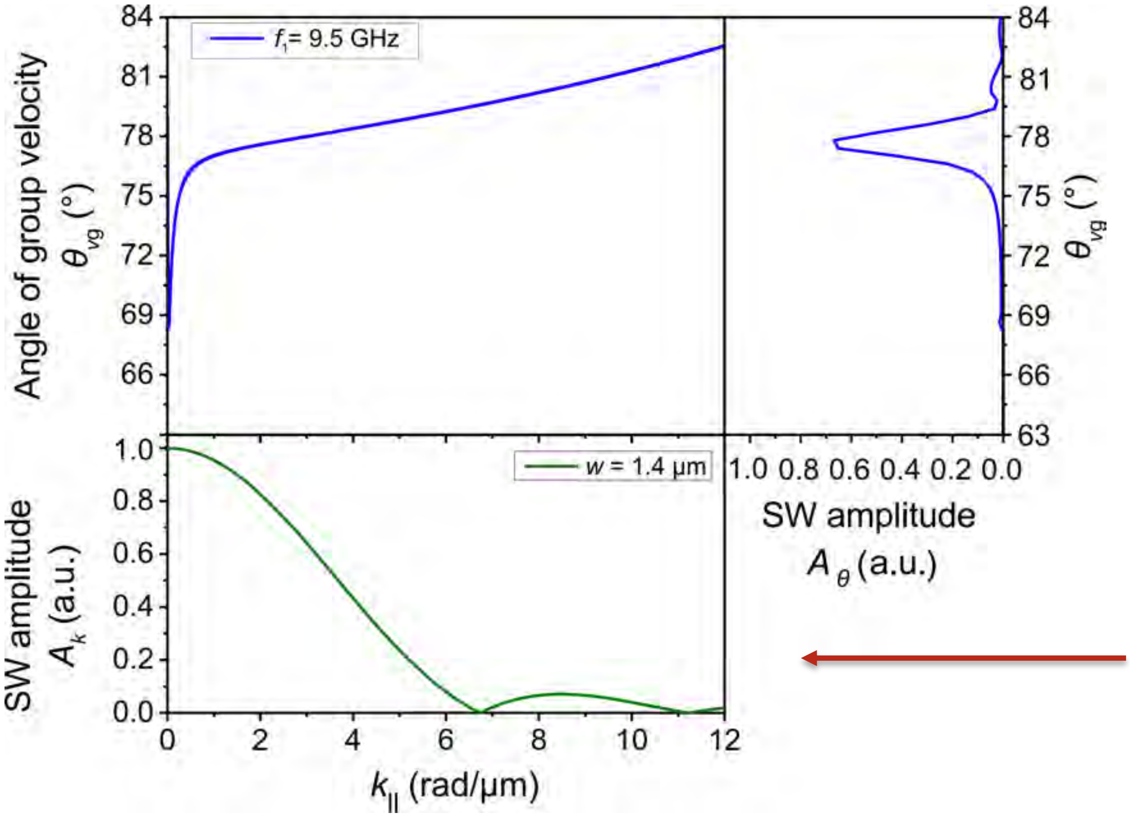
$\vec{v}_g \nparallel \vec{k}$

Large k-ranges with similar angle θ_{vg}

T. Sebastian et al., PRL **110**, 067201 (2013)

Origin of caustic-like SW beams

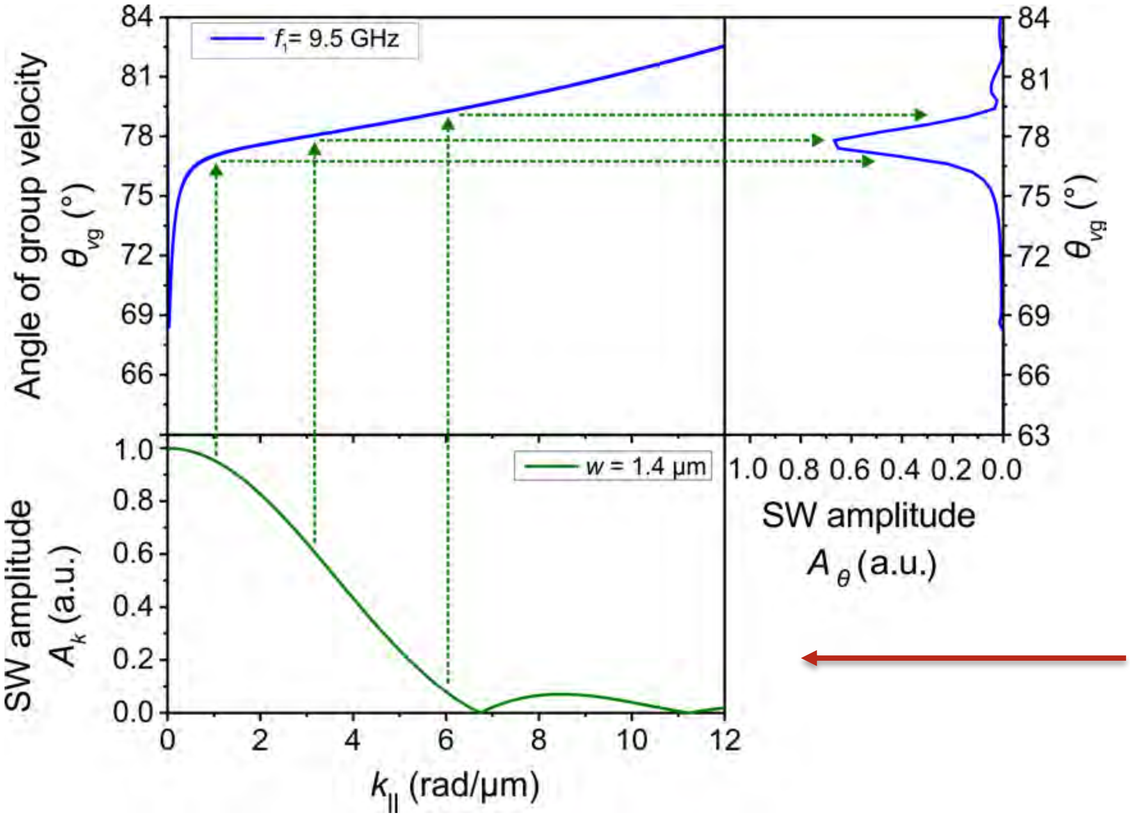
Excitation scheme of spin-wave caustics



Broad *k*-spectrum due to narrow input waveguide

Origin of caustic-like SW beams

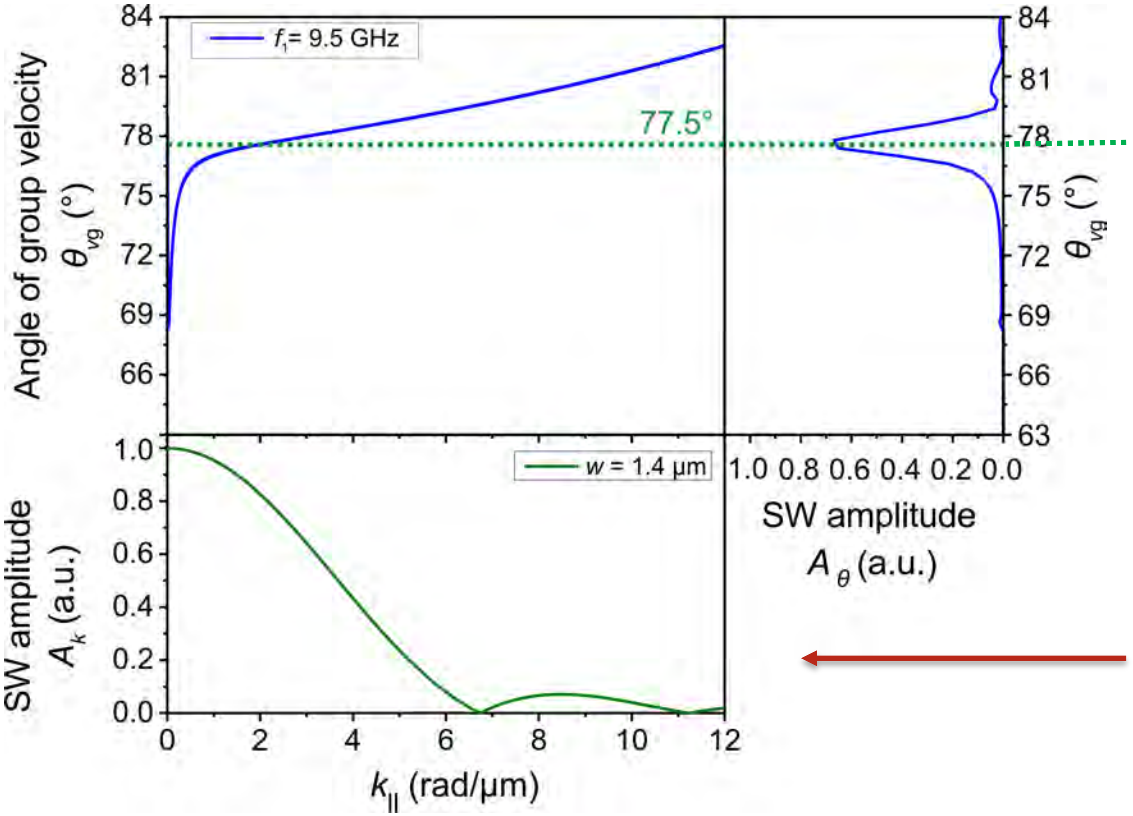
Excitation scheme of spin-wave caustics



Broad *k*-spectrum due to narrow input waveguide

Origin of caustic-like SW beams

Excitation scheme of spin-wave caustics



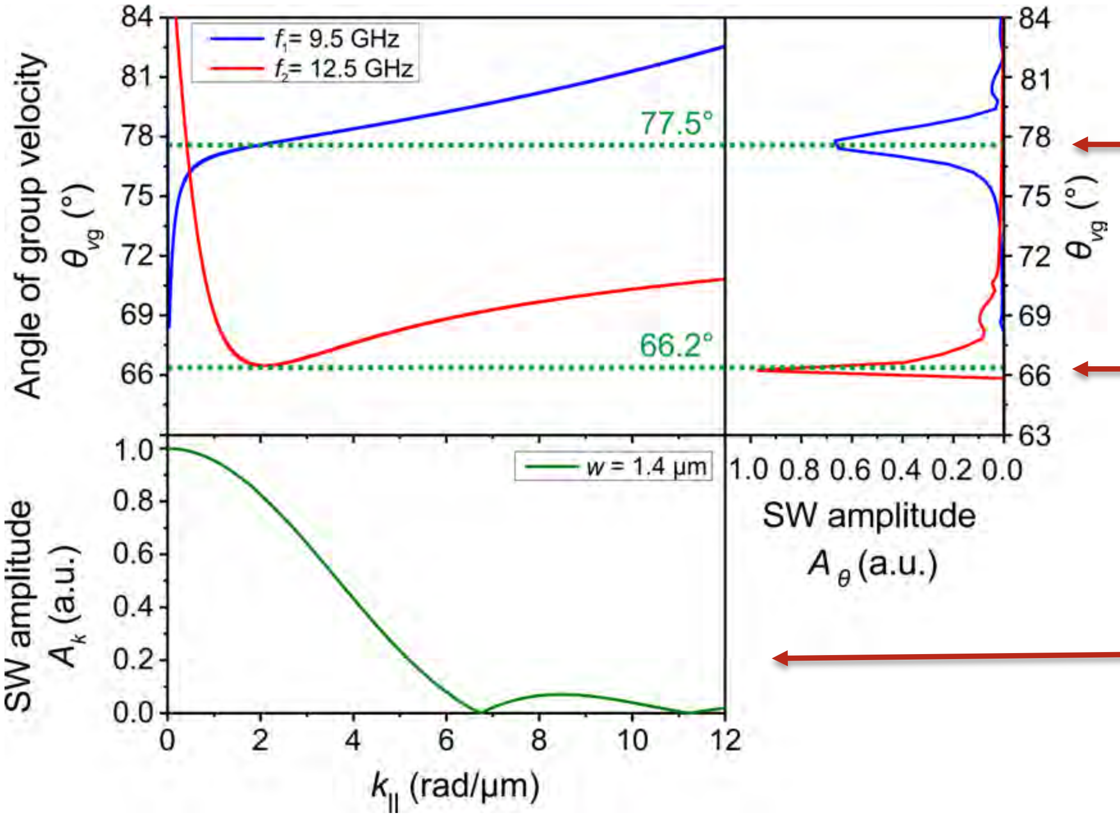
Direction of energy flow

Broad k-spectrum due to narrow input waveguide

Concentration of the spin-wave amplitude at angle θ_B → **Beam formation**

Origin of caustic-like SW beams

Frequency dependency of propagation direction



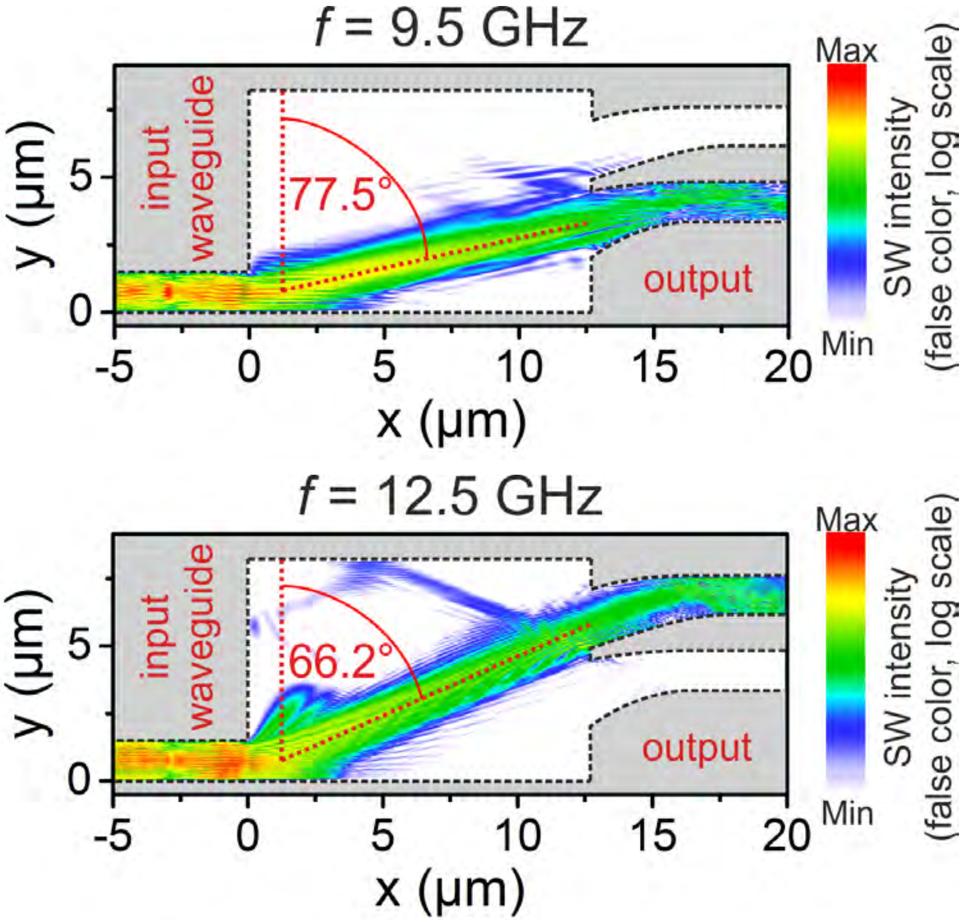
Beam angle depends strongly on SW frequency

Independent of frequency

SW caustic direction is tunable by frequency

Frequency Demultiplexer (FDM-DEMUX)

Micromagnetic simulation (mumax3)



- Frequency dependent channeling of caustic beams into output waveguides
- Passive element, only proper structuring necessary
- Very compact device

Separation of SW signals of different frequencies

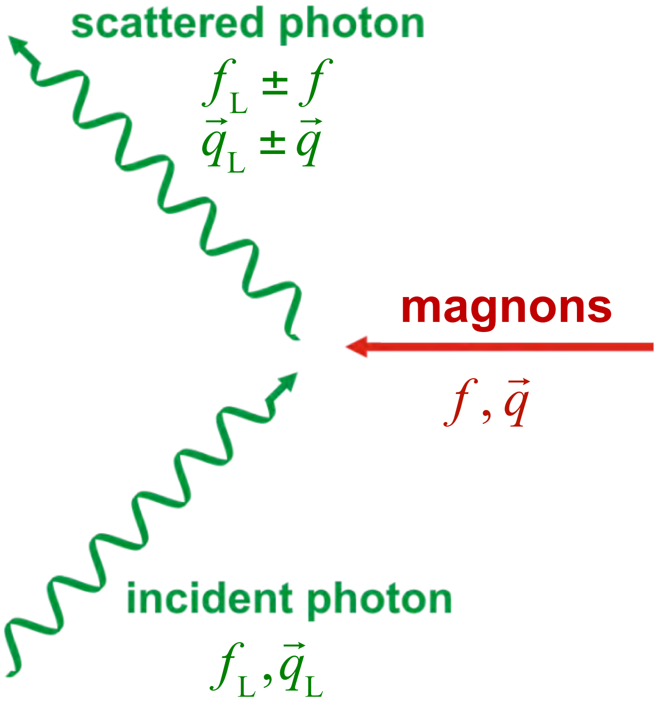
Heusner et al.,
Phys. Status Solidi RRL 2018,
1800409, (in print)

Experimental realization

Measurement technique:

Brillouin light scattering (BLS) process

= inelastic scattering of photons from spin waves



scattered photon

$f_L \pm f$

$\vec{q}_L \pm \vec{q}$

magnons

$f, \vec{q}$

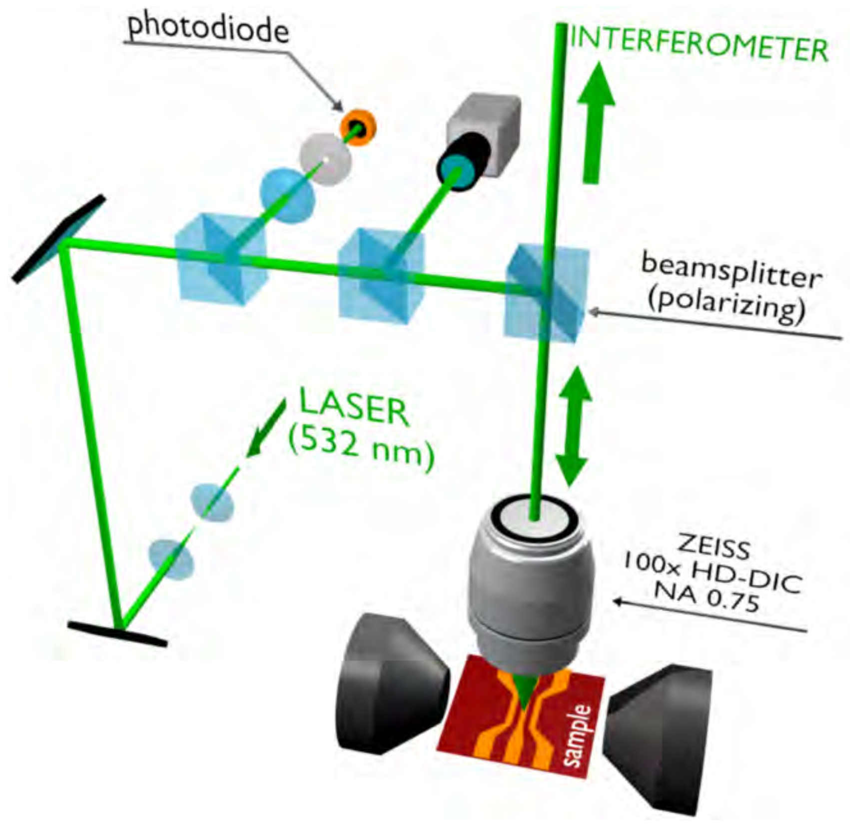
incident photon

$f_L, \vec{q}_L$

$$f_{\text{scattered L}} = f_L \pm f$$

$$\vec{q}_{\text{scattered L}} = \vec{q}_L \pm \vec{q}$$

Microfocused BLS microscopy



Features:

- Spectral resolution:

$$\Delta f \approx 100 \text{ MHz}$$

- Spatial resolution:

≈ 300 nm

- Wave vector range:

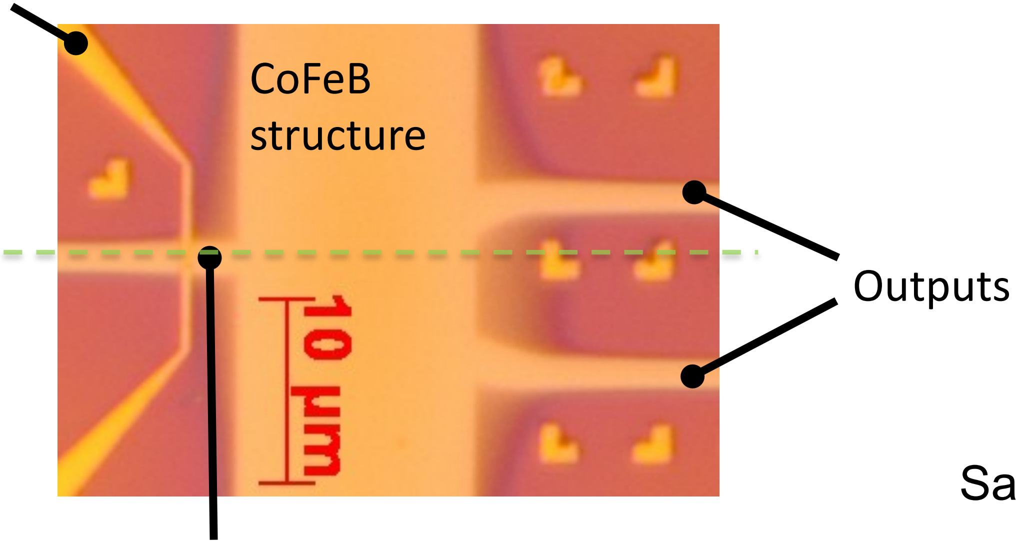
$$q \in [0;17] \text{ rad/µm}$$

- Loss of wave vector selectivity

→ Study of *microstructured* spin-wave samples

Review: T. Sebastian *et al.*,“*Micro-focused Brillouin light scattering: imaging spin waves at the nanoscale*” Front. Phys. 3:35 (2015)

Microstructured sample

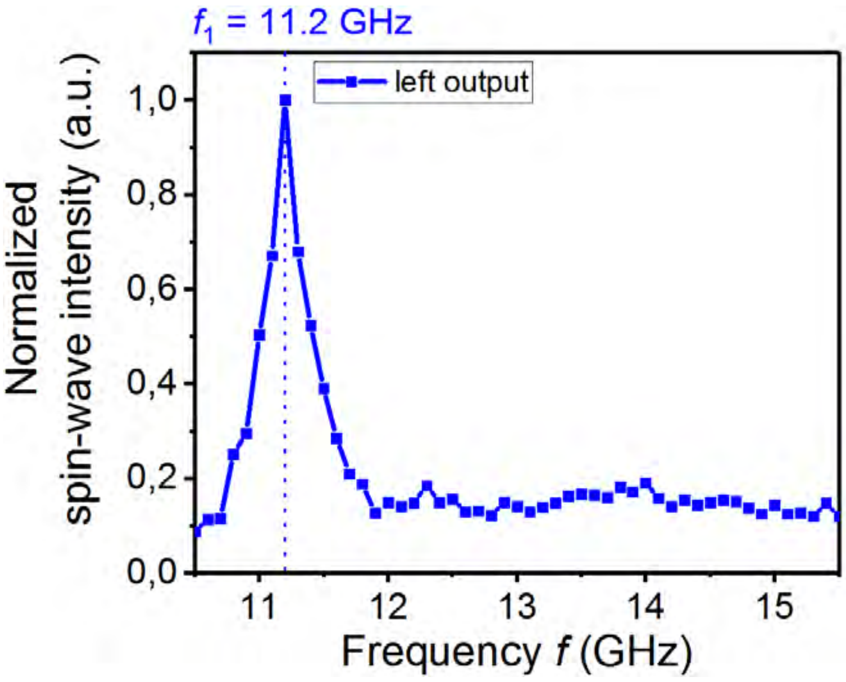


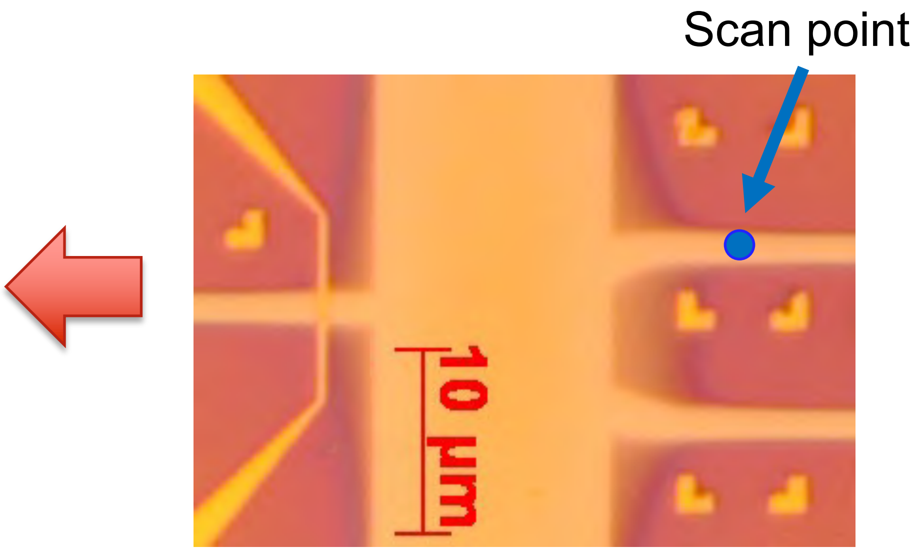
Sample fabricated by imec

- Made of CoFeB: M_S = 1550 kA/m, A_{ex} = 17.6 pJ/m, α = 0.0043, t = 30 nm
- Microstrip antenna (Au): spin-wave excitation by microwave magnetic field

Demultiplexer: experimental results

Frequency dependence of output signals:

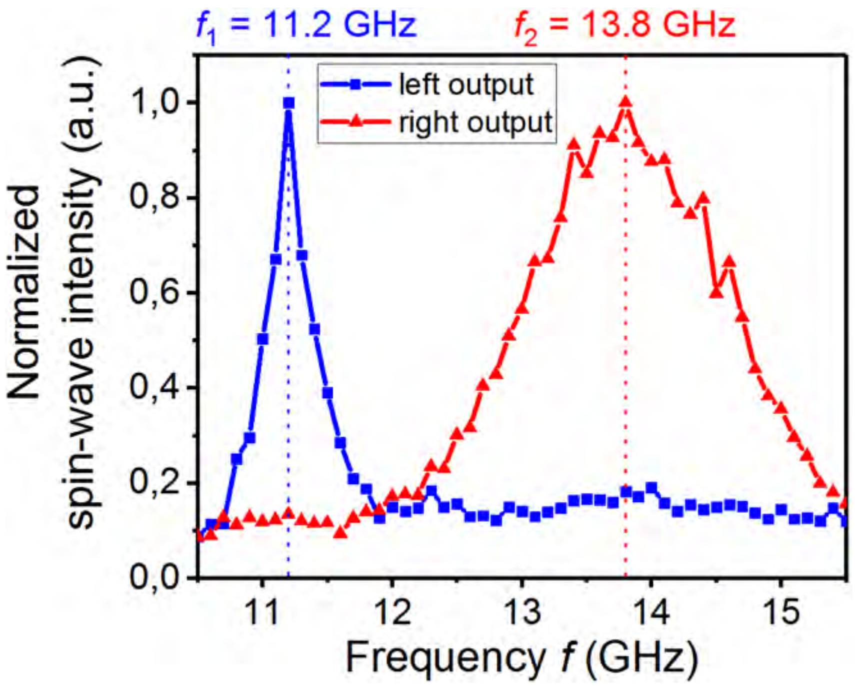


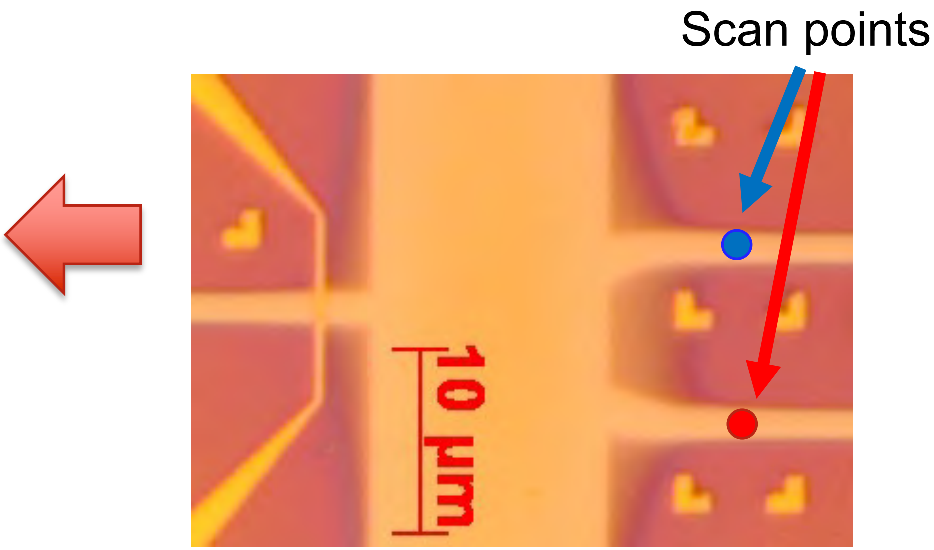


- Efficient caustic creation and channeling into output waveguides

Demultiplexer: experimental results

Frequency dependence of output signals:

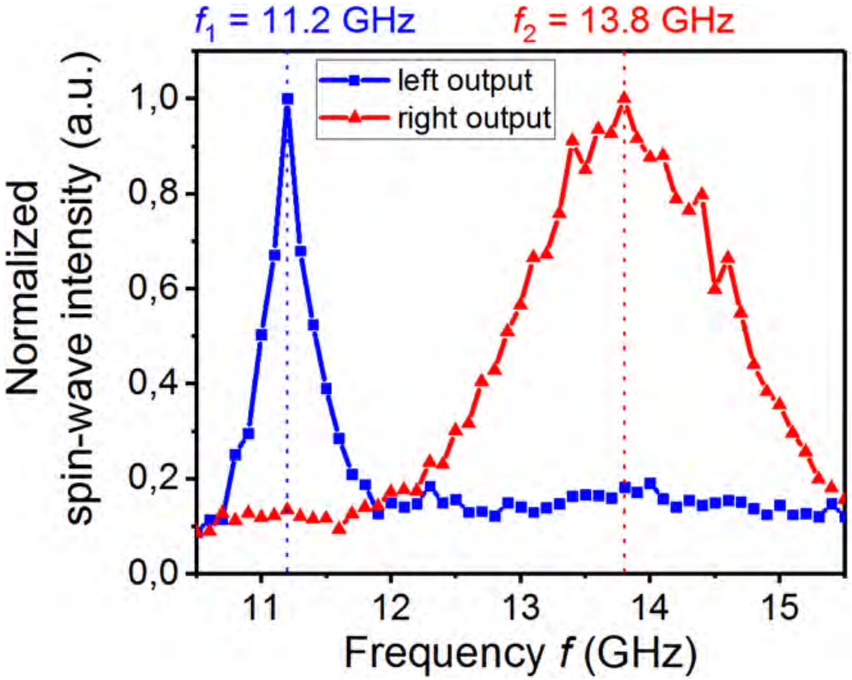


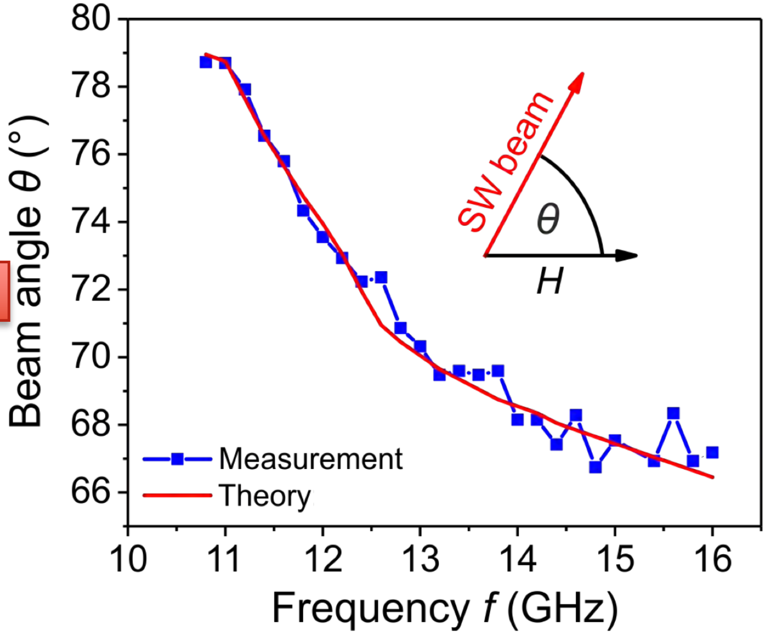


- Efficient caustic creation and channeling into output waveguides
- Signals in different outputs are clearly separated in frequency

Demultiplexer: experimental results

Frequency dependence of output signals:

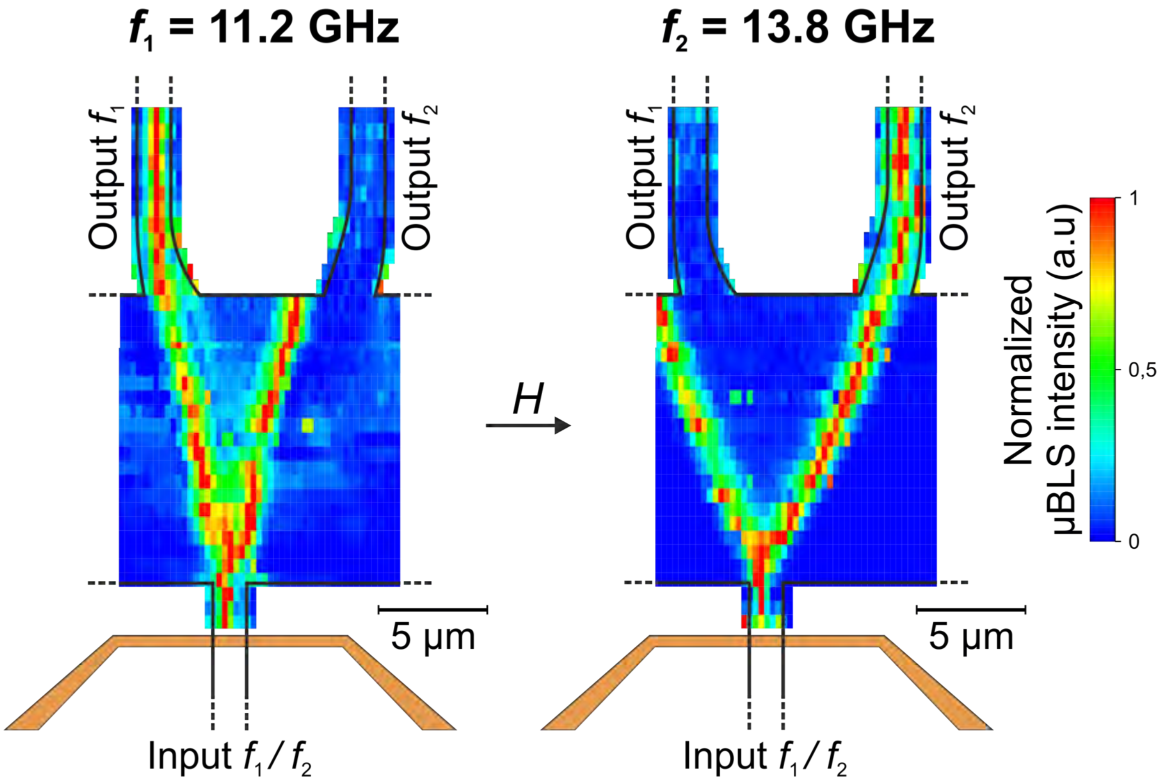




- Efficient caustic creation and channeling into output waveguides
- Signals in different outputs are clearly separated in frequency

Experimental results

Two-dimensional spin-wave intensity maps:

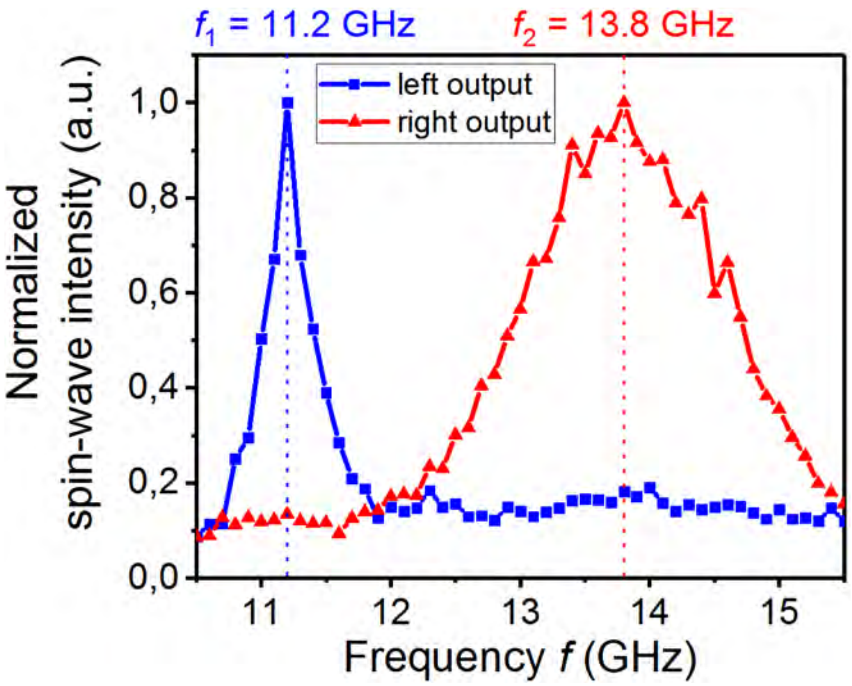


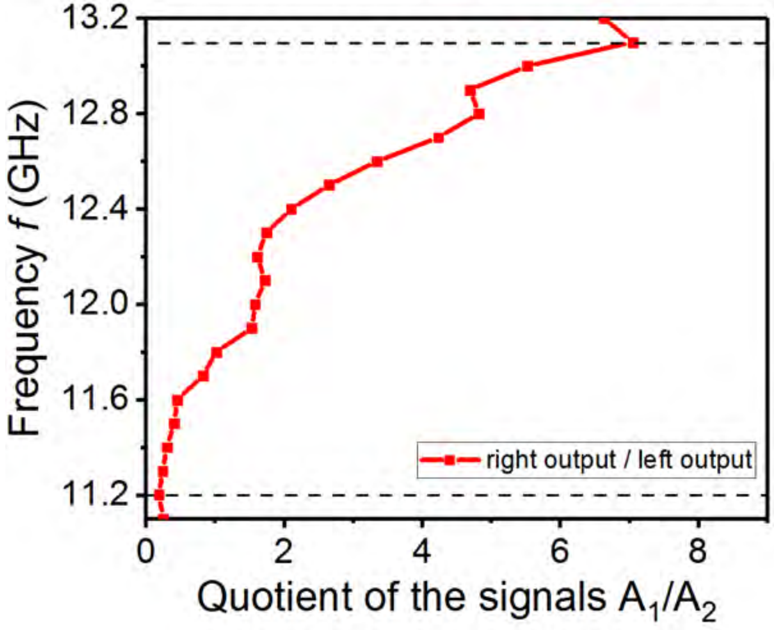
- Input signals of different frequencies channeled into different outputs

→ Experimental realization of the spin-wave frequency demultiplexer

Caustics for microscaled GHz spectrum analyzer

Frequency dependence of output signals:





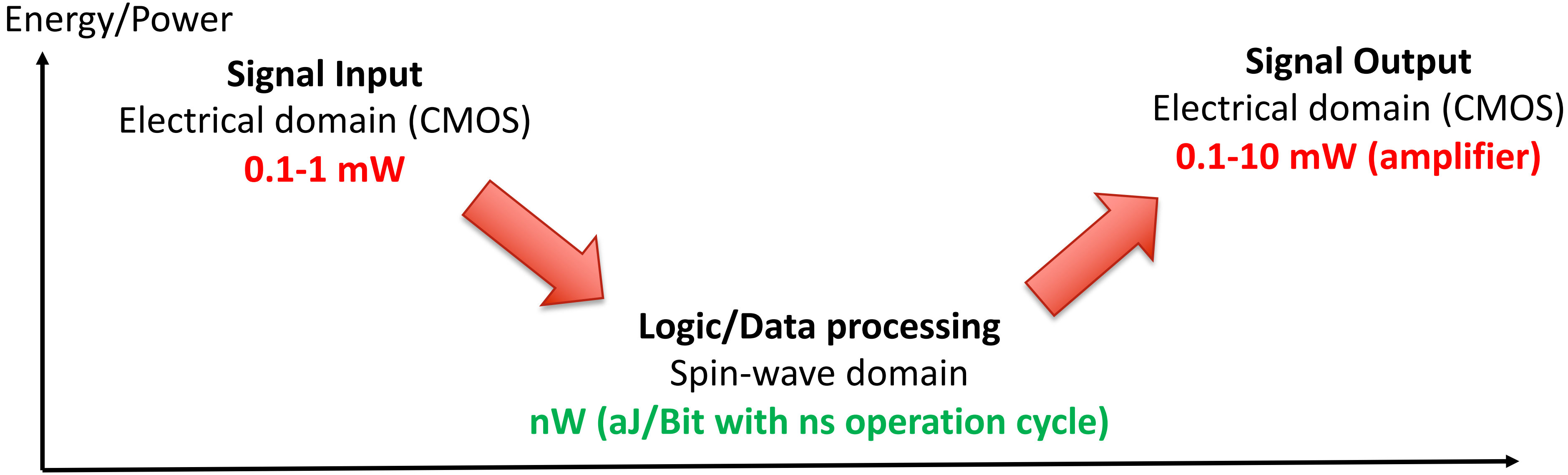
- Strong frequency dependence of the channeling direction can be used to **determine the frequency based on a intensity measurement**

Outline

- **Wave-based** logic and particular advantages of **spin waves**
- Prototype of **linear spin-wave logic** element: **Majority Gate**
- **Frequency multiplexer and spectrum analyzer** based on **caustic beams**
- **Towards integrated magnonic circuits:**
    - Magnonic Half adder (quasi binary)
    - Analog magnon adder

Why integrated magnon circuits ?

Energy consumption in a prototype magnonic device:



- Most energy is consumed for moving the signal between the electrical and the spin-wave domain

→ **“All-magnonic logic” reduces conversion losses** and gives access to low power computation

- Improve conversion efficiency in nanostructures (Spin-orbit torques, Voltage controlled anisotropy, magneto-electric and –acoustic coupling…)

Acknowledgements: IMEC team, Leuven and G. Csaba, Budapest

Integrated circuit: Half adder

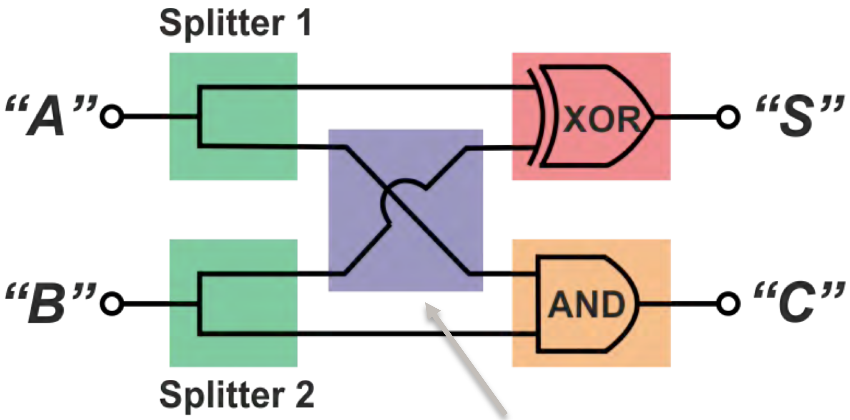
Standard element of binary data processing

Truth table of Half adder

Input A	Input B	„S“	„C“
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

“Sum” “Carry”

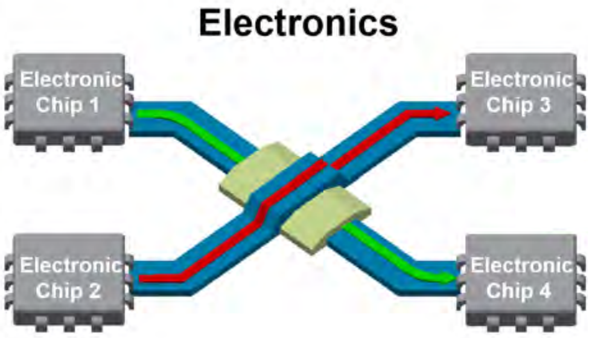
Realization:

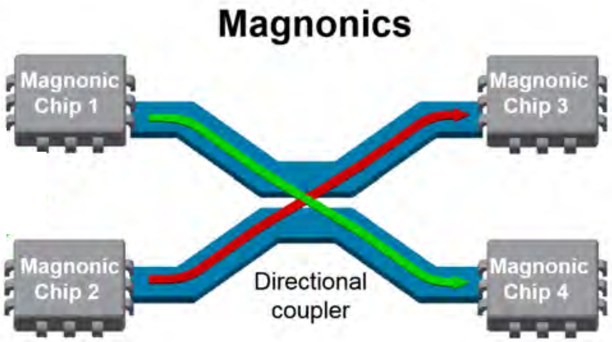


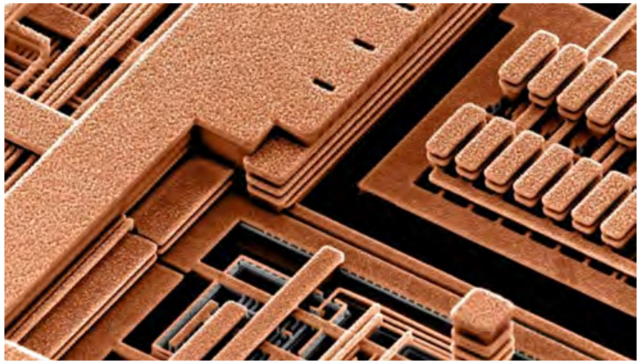
Crossed interconnect

- Integrated circuits require **crossed interconnects**

Crossed interconnects

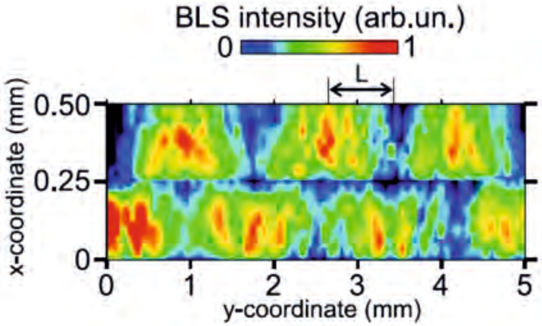






Copper interconnects for semiconductor circuits (Intel)

Waveguide coupling



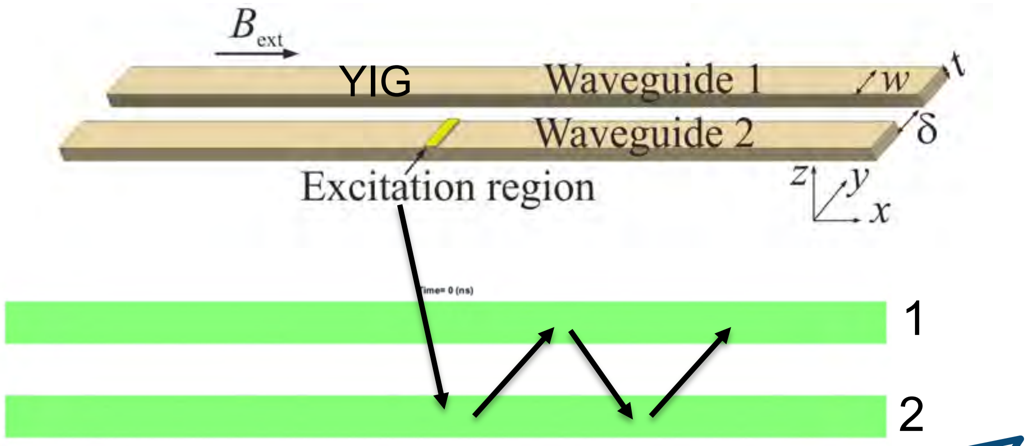
A.V. Sadovnikov et al., Appl. Phys. Lett. 107, 202405 (2015) and Appl. Phys. Lett. 109, 042407 (2016)

Complex and expensive 3D interconnects

2D interconnects made of single material with one lithography step

Dipolar coupled waveguides

Micromagnetic simulation:

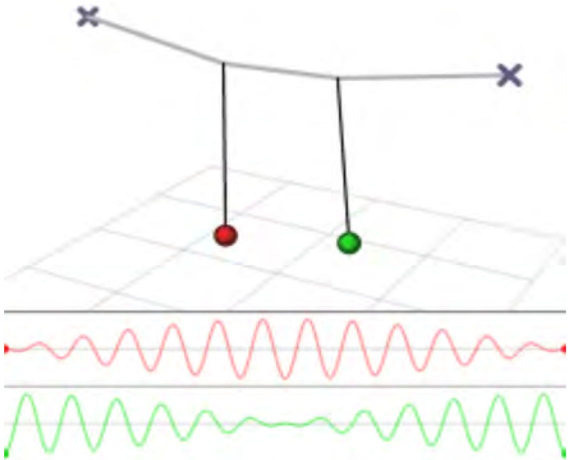


Yttrium Iron Garnet (YIG):

$M_s = 1.4 \times 10^5$ A/m
A = 3.5 pJ
$\alpha = 2 \times 10^{-4}$

t = 50 nm
w = 100 nm
δ = 100 nm

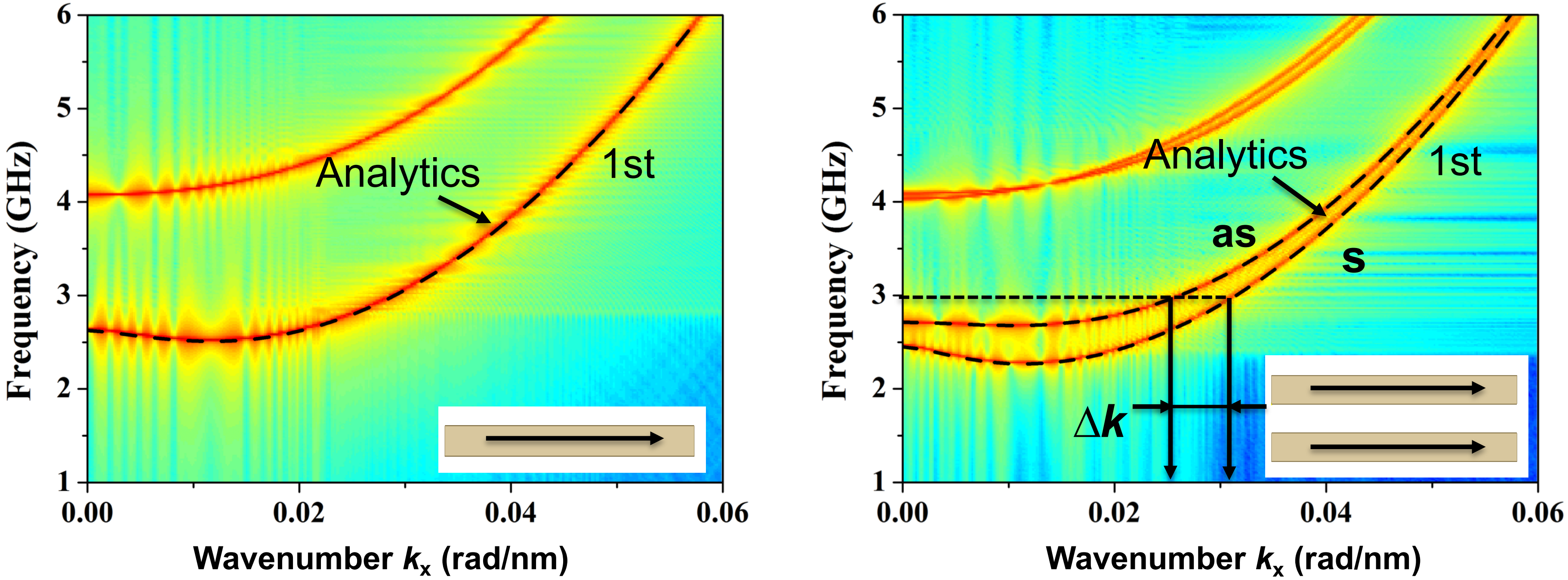
Coupled oscillators

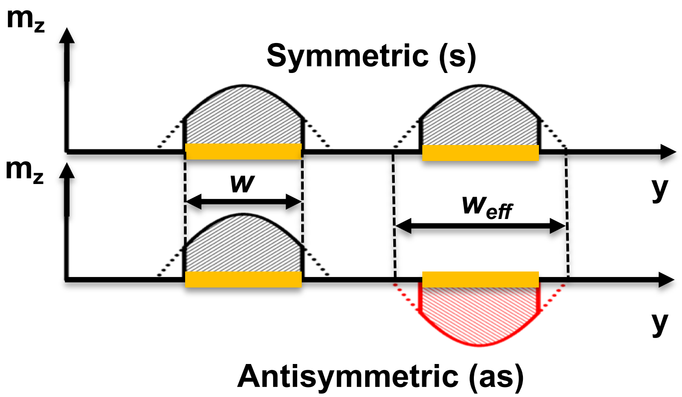


From: Wikipedia

Using Mumax3: A. Vansteenkiste, et al., *AIP Advance* **4**, 107113 (2014)

Spin-wave dispersions





Coupling length:

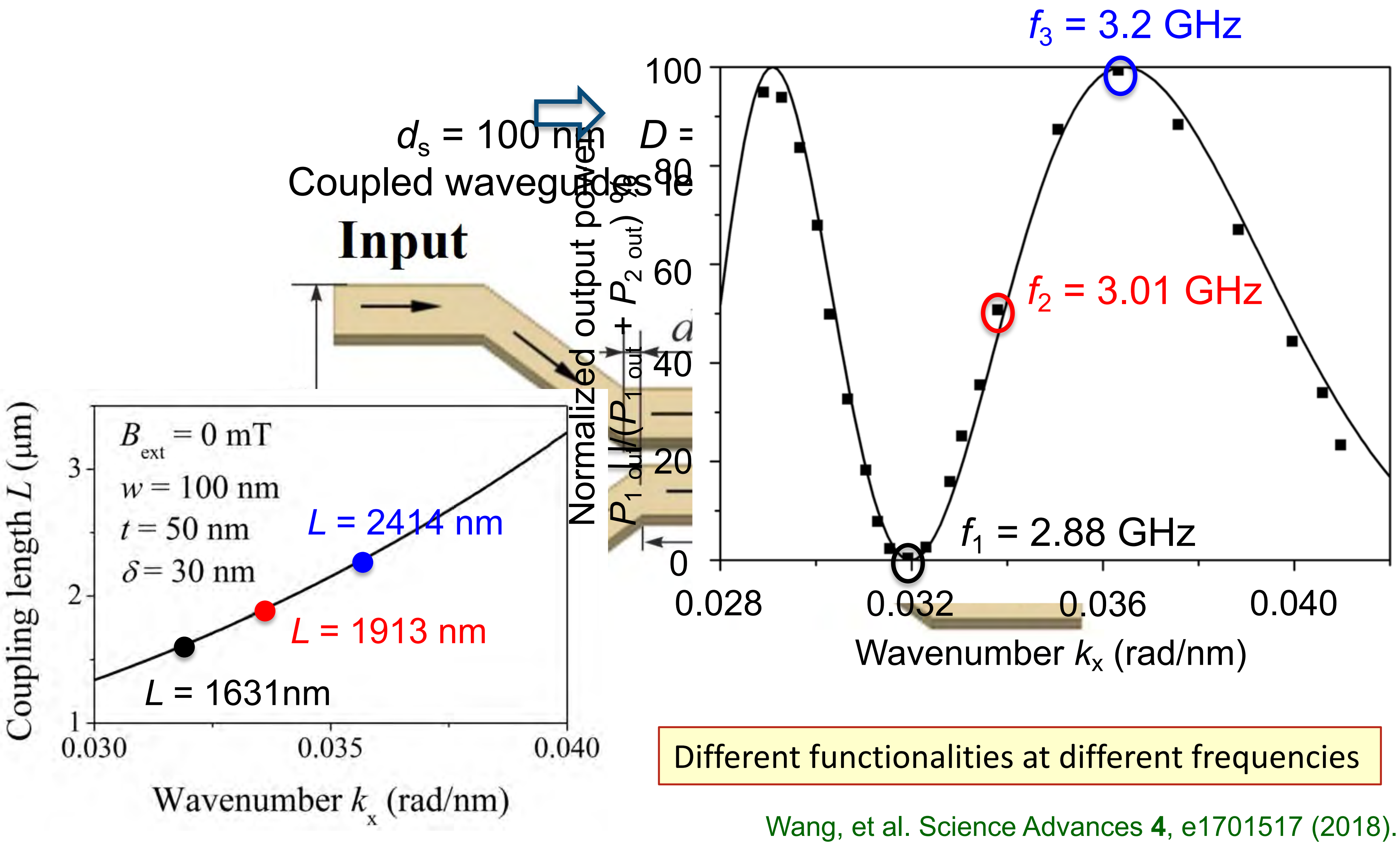
$$\boxed{L = \frac{\pi}{\Delta k}}$$

Analytics: R. Verba, A. Slavin

R. Verba et al., Phys. Rev. B **85**, 014427 (2012)

R. Verba, Ukrainian journal of physics **58**, 758 (2013)

Directional coupler design

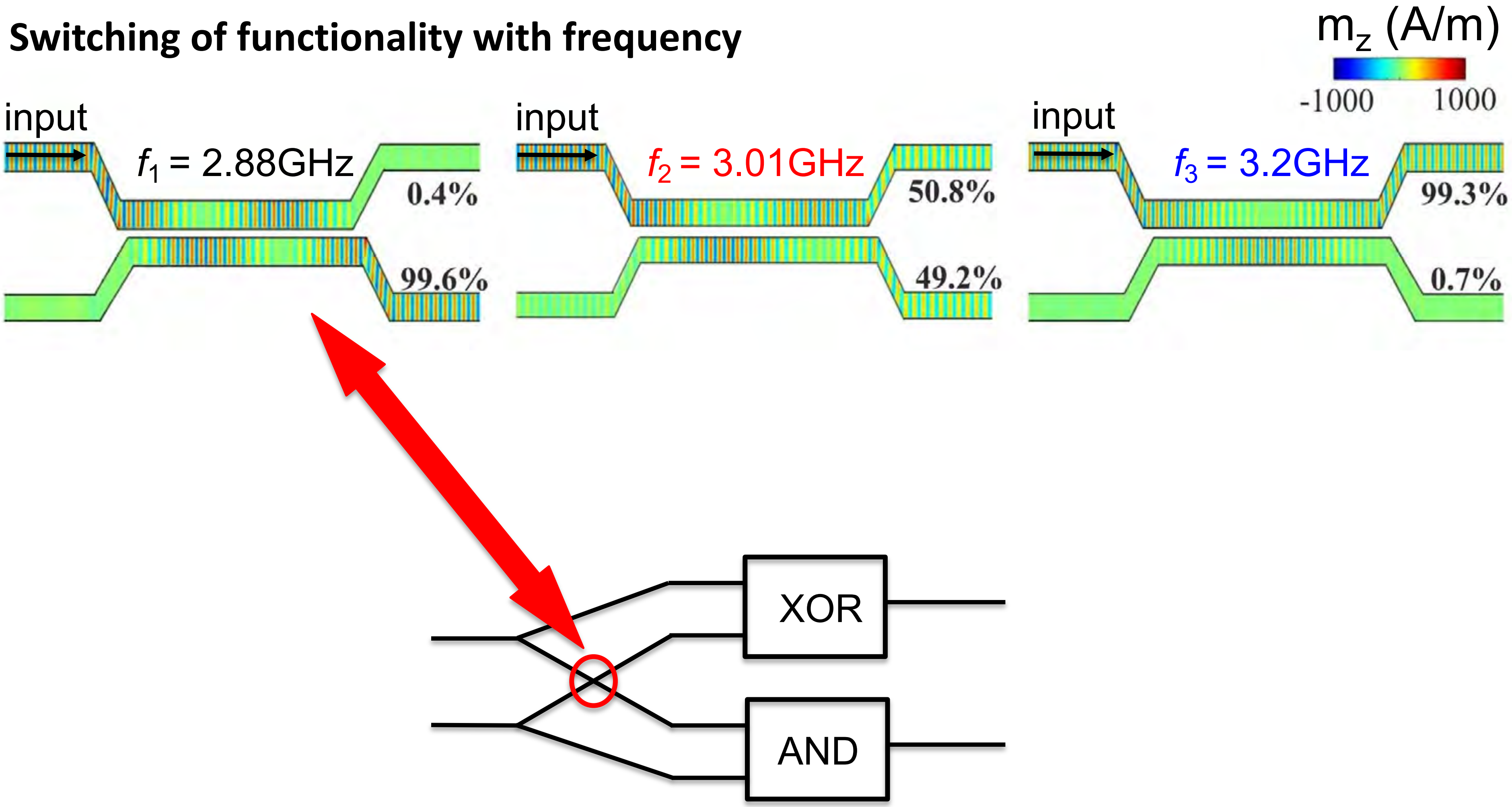


Different functionalities at different frequencies

Wang, et al. Science Advances **4**, e1701517 (2018).

Functionalities of a directional coupler

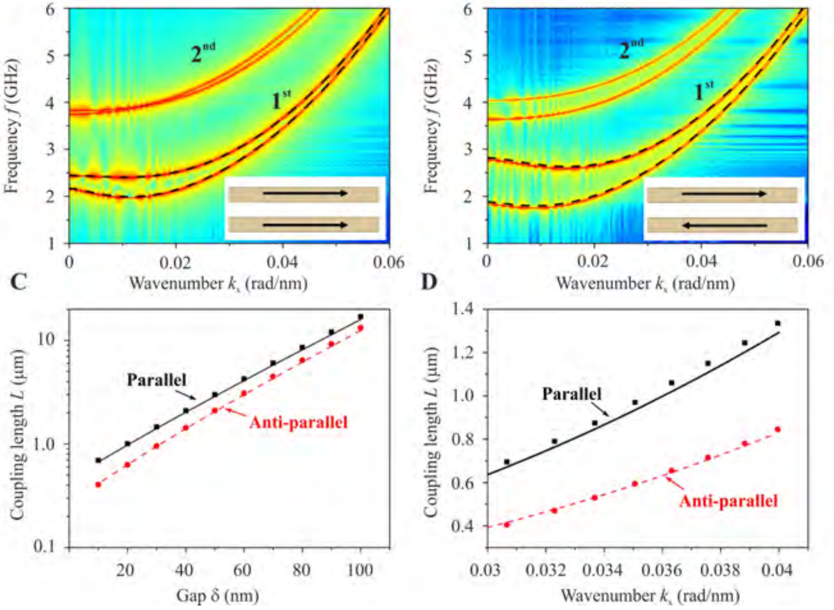
Switching of functionality with frequency



Wang, et al. Science Advances **4**, e1701517 (2018).

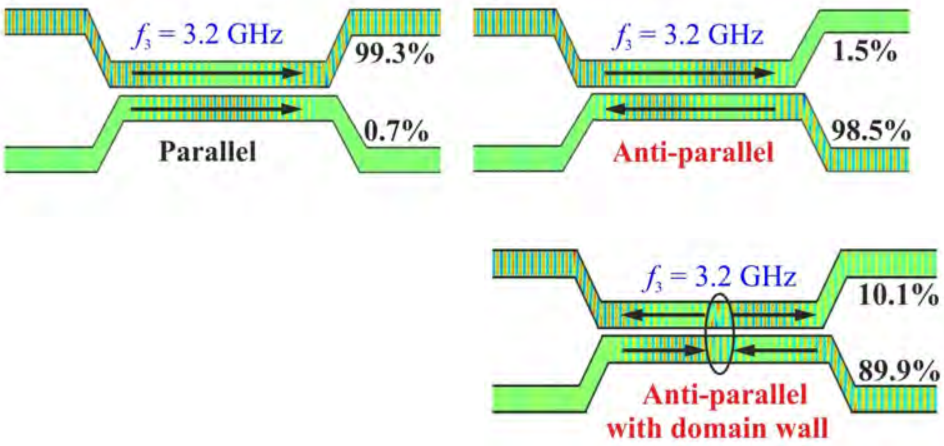
Reconfiguring the directional coupler

Without magnetic bias field: parallel and antiparallel magnetization configurations are stable

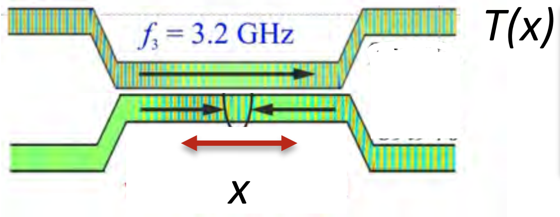


Antiparallel state has stronger coupling / lower coupling length

Simple waveguide **Waveguide crossing**



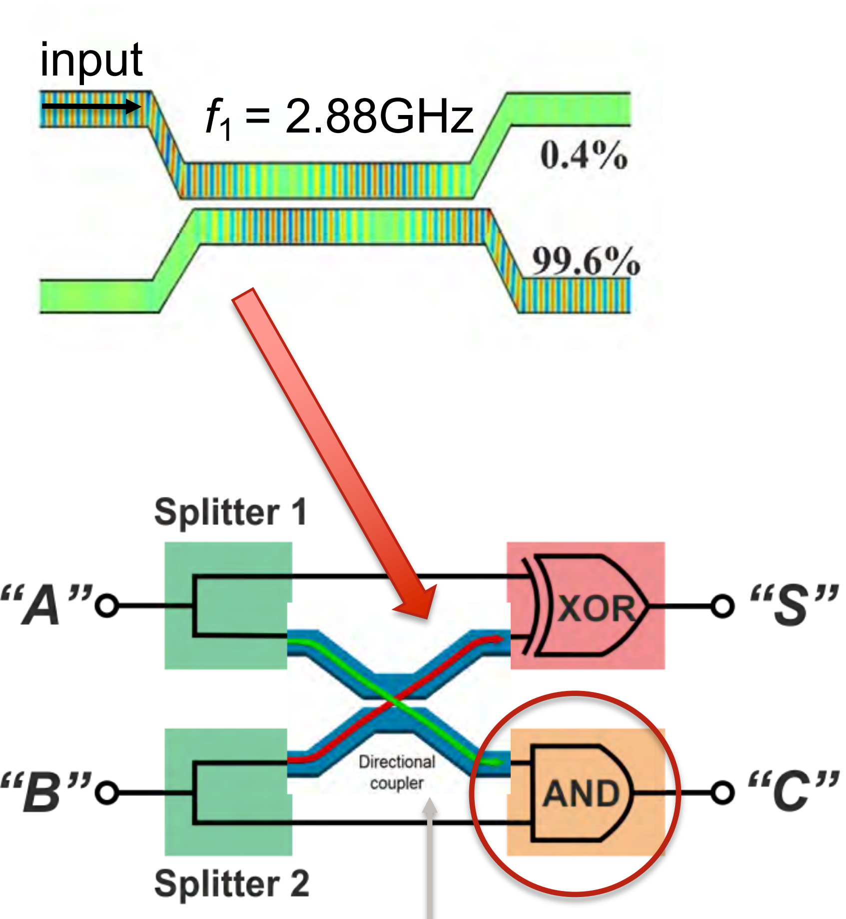
If the domain wall is present in only one waveguide, the transmission *T* could be continuously tuned by the DW positon.



“Reconfigurable magnonic synapse”

Wang, et al. Science Advances **4**, e1701517 (2018).

Directional coupler as spin-wave crossing



✓ **Crossed interconnect: Directional coupler**

- Half adder requires **AND gate**

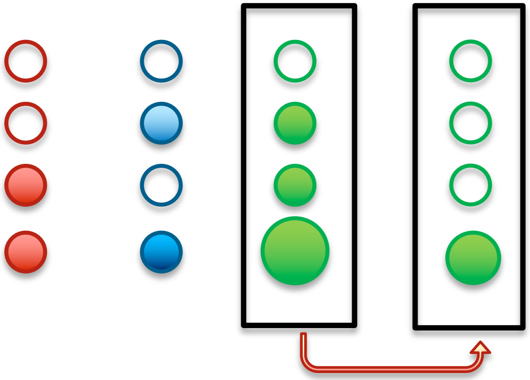
Wang, et al. Science Advances **4**, e1701517 (2018).

Magnonic AND-gate

Encoding in amplitude:

Input 1	Input 2	Output
0	0	0
0	1	1
1	0	1
1	1	1

Linear superposition:



Truth table of AND gate:

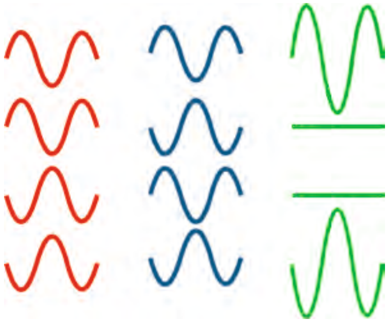
Input 1	Input 2	Output
0	0	0
0	1	0
1	0	0
1	1	1

Nonlinear signal transformation by **nonlinear transmission**

Cavities to enhance nonlinear effects

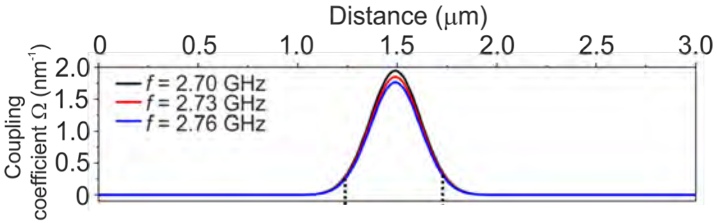
Encoding in phase:

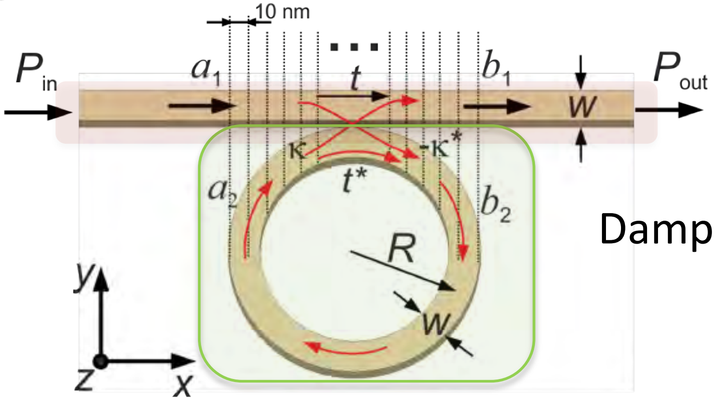
Input 1	Input 2	Output
0	0	0
0	π	–
π	0	–
π	π	π



No simple **linear realization** of wave-based **AND gate** which can be easily **implemented in a magnonic network**

Magnonic nano-cavity: analytics





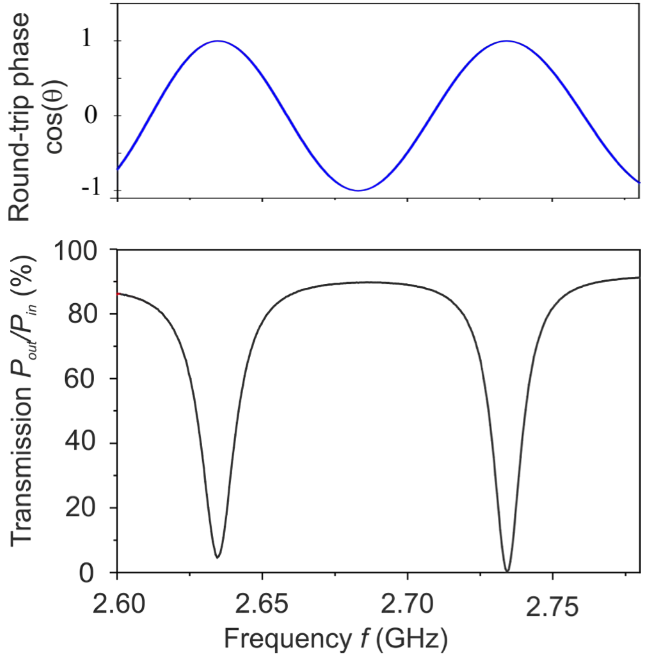
Describe coupled system (waveguide-ring) using matrix approach from optics:

Transmission parameter

Coupling parameter

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} t & \kappa \\ -\kappa^* & t^* \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad |\kappa|^2 + |t|^2 = 1$$

Damping coefficient for one round trip: $\beta = \exp(-C / L_d)$



Transmitted power at resonance :

$$P_{out} = (\beta - |t|)^2 / (1 - \beta|t|)^2$$

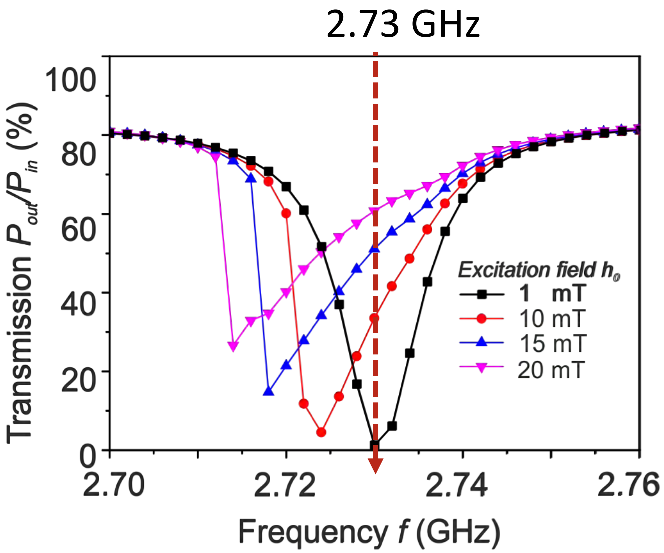
Resonance condition:

$$2\pi R\, k_{SW} = 2\pi n \qquad P_{\mathrm{out}} = 0 \text{ if } \beta = |t| = \sqrt{1 - |\kappa|^2}$$

“Critical coupling condition”:
complete destructive interference

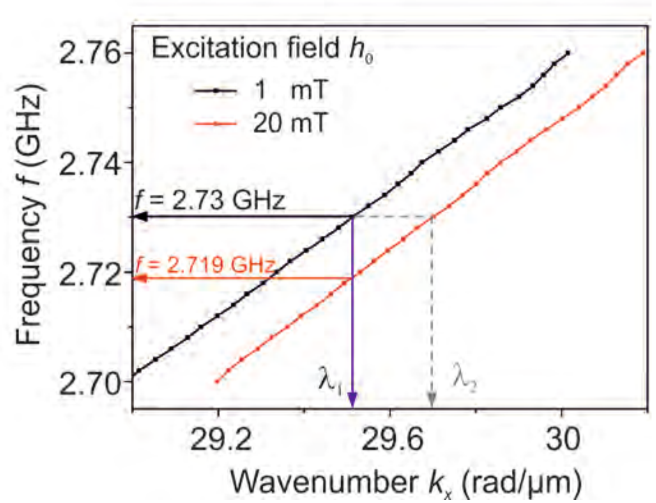
$$|a_1 t| = |a_2 \kappa|$$

Nonlinear magnonic nano-cavity



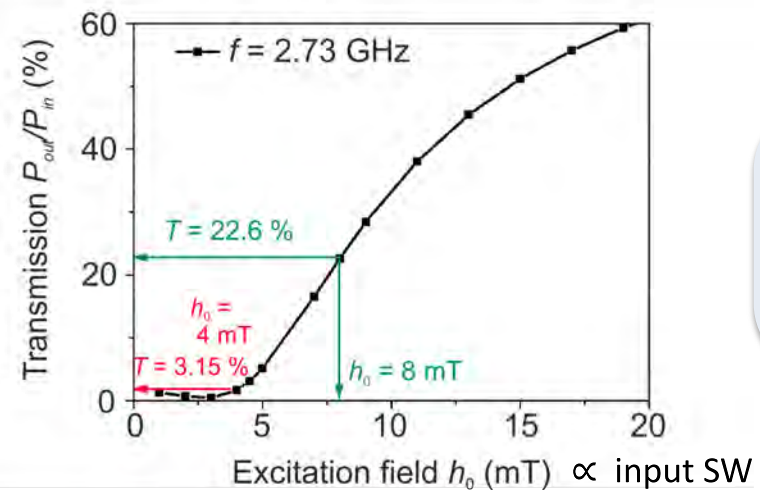
$$2\pi R\, k_{SW}(f, |u|^2) = 2\pi n$$

Nonlinear shift of dispersion relation:



$$k_{SW} = k_{SW}(f, |u|^2),\ \ u \propto h_0$$

Spin wave amplitude

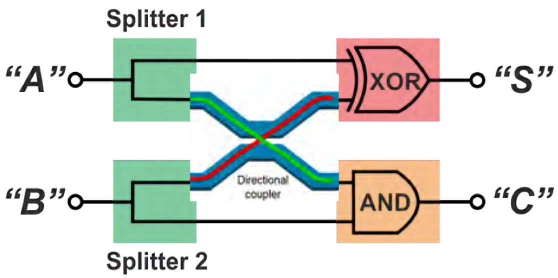


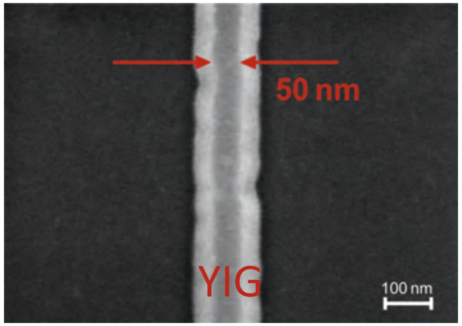
∝ input SW amplitude

Nonlinear shift of dispersion relation changes resonance frequency of nano-cavity
→ Nonlinear signal transformation: Increase of transmission for fixed frequency

Micromagnetic simulation: mumax3

“CMOS inspired” design of magnonic half adder



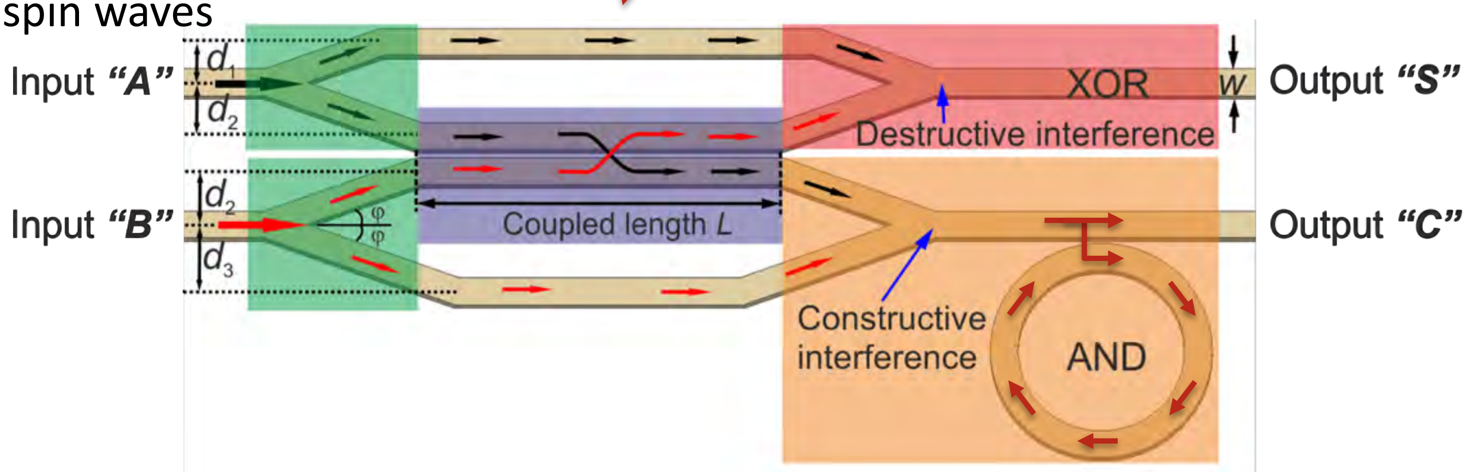


Length of active structure: 5.5 µm
Total width: 2 µm
Coupled length=L_c=1.2 µm

Experimentally accessible sizes

Wang, Heinz et al., arXiv: 1807.01358
Brächer et al., Nano Lett. **17**, 7234 (2017)

Input of spin waves



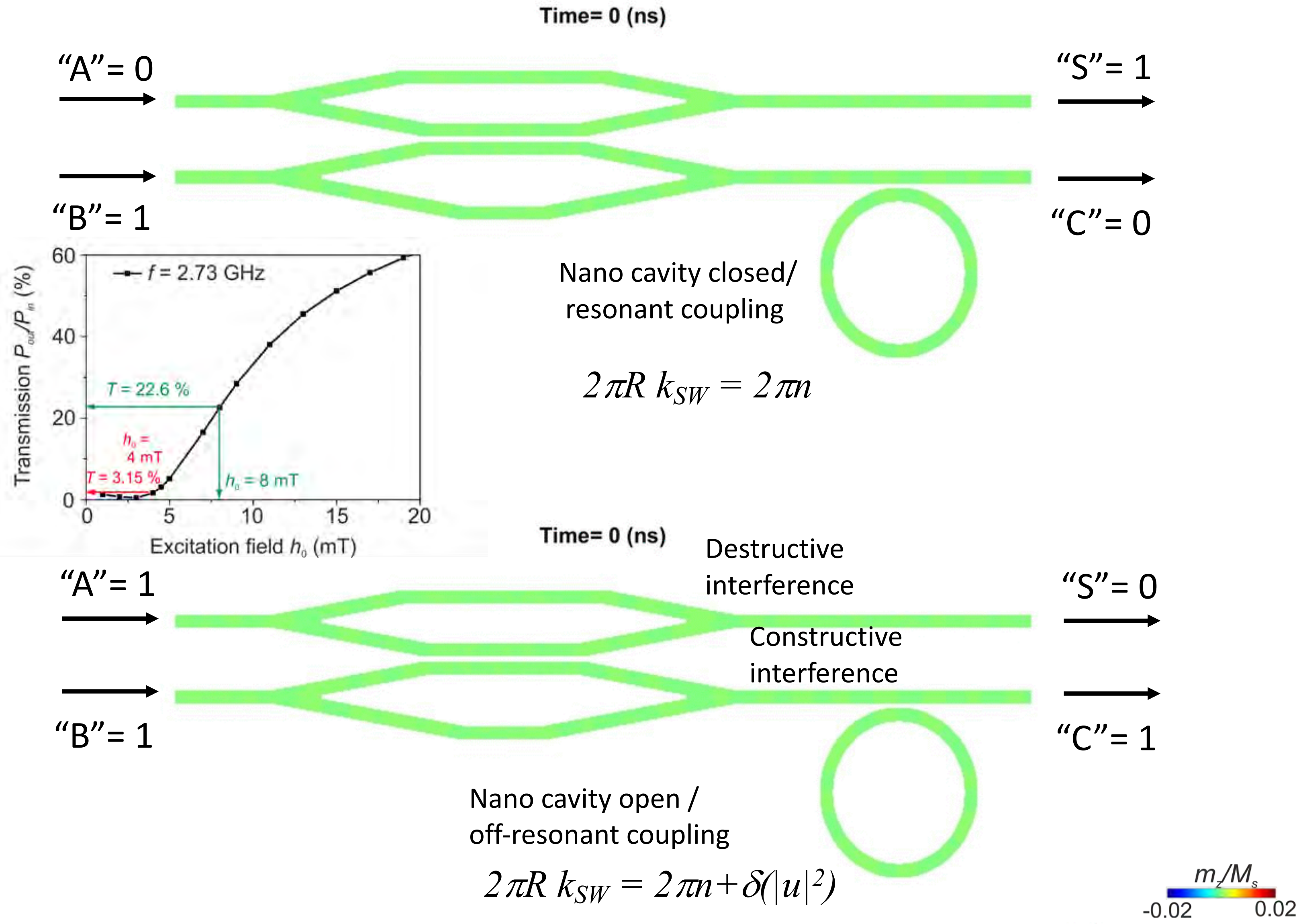
Nonlinear nano cavity:
inspired by integrated optics

Choice of length d_1,d_2,d_3 allows to match phases

- destructive interference for XOR
- constructive interference for AND

Using Mumax3: A. Vansteenkiste, et al., AIP Advance **4**, 107113 (2014)

Half adder: simulation



Nano cavity closed/
resonant coupling

$$2\pi R\, k_{SW} = 2\pi n$$

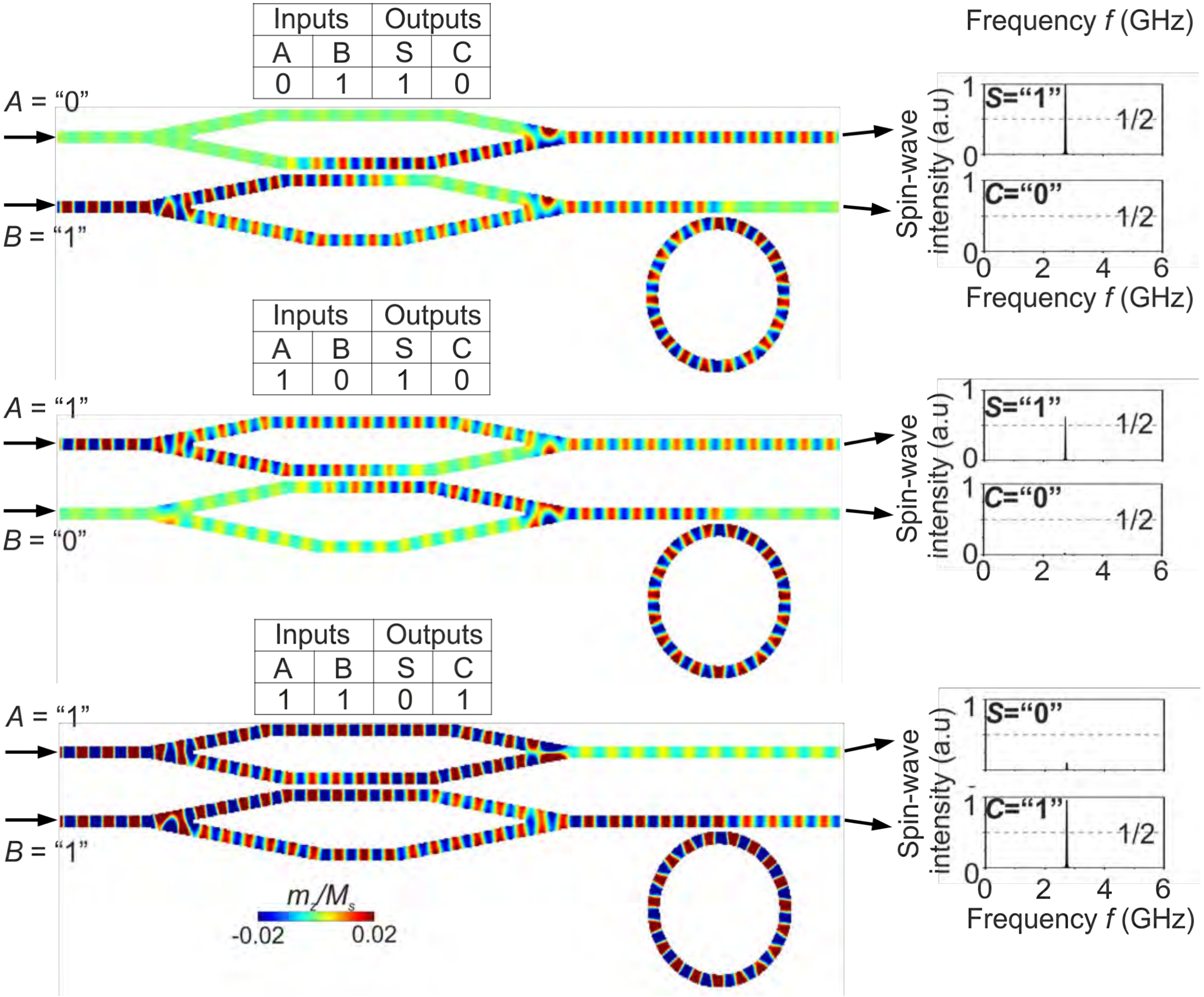
Destructive
interference

Constructive
interference

Nano cavity open /
off-resonant coupling

$$2\pi R\, k_{SW} = 2\pi n + \delta(|u|^2)$$

CMOS inspired Half adder: simulation



Inputs		Outputs	
A	B	S	C
0	1	1	0

Inputs		Outputs	
A	B	S	C
1	0	1	0

Inputs		Outputs	
A	B	S	C
1	1	0	1

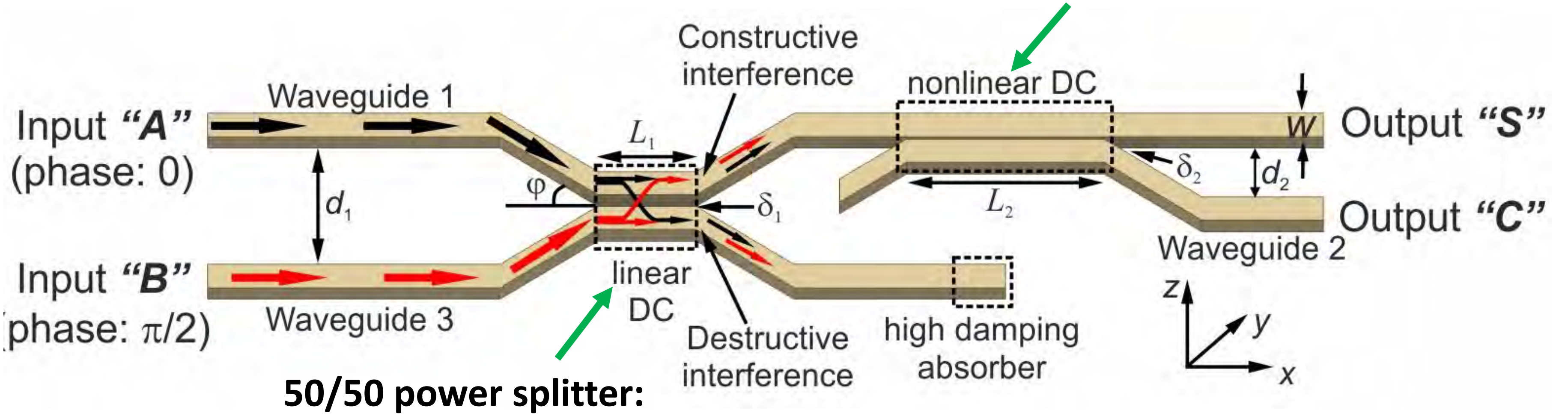
First proof-of concept-study of an **integrated all magnon circuit**

Energy consumption per operation ca 1.5 aJ

Problem:
Phase of (A=0;B=1) is shifted by 90° compared to (A=1;B=0)

→No **network** of half-adders is possible

New design: Half-adder 2.0

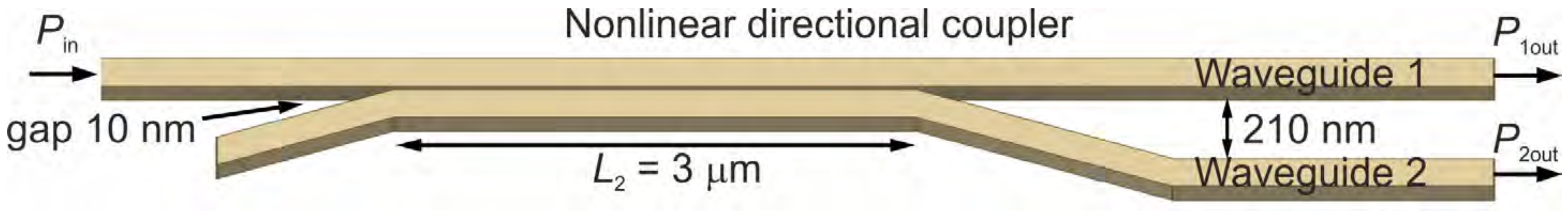


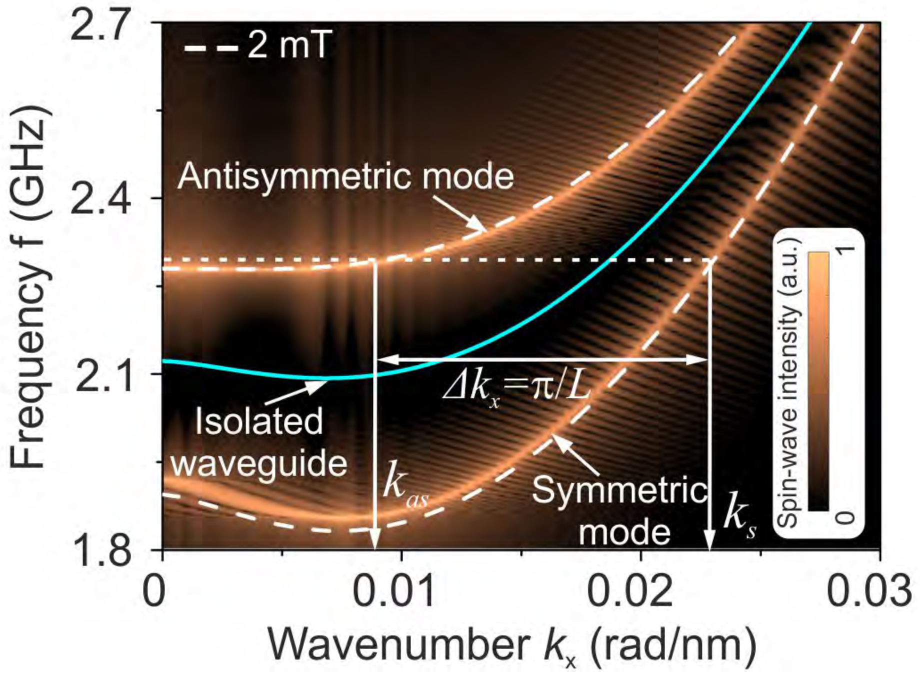
Directional coupler in nonlinear regime (small gap, long length)

50/50 power splitter: Linear Directional coupler (large gap, short length)

- No ring cavity anymore, only **two directional couplers ('DC's)** with different coupling strength
- Device design allows to build an **extended network**: amplitudes AND phases are matched

Nonlinear directional coupler





Nonlinear shift in directional couplers

Nonlinear dispersion curves in coupled waveguides:

$$\omega_{s,as\_nonliner}\left(k_x\right)=\omega_{s,as}\left(k_x\right)+T_{k_x}\left|a_{k_x}\right|^2$$

Nonlinear shift coefficient:

$$T_{k_x}=-A_{k_x}+\frac{\omega_M B_{k_x}^2}{2\omega_0^2}\left(4\lambda^2k_x^2-F_0^{xx}\left(0\right)+F_{2k_x}^{xx}\left(0\right)\right)$$

where:

$$A_{k_x}=\gamma B_{ext}+\frac{\omega_M}{2}\left(2\lambda^2k_x^2+F_{k_x}^{yy}\left(0\right)+F_{k_x}^{zz}\left(0\right)\right) \qquad B_{k_x}=\frac{\omega_M}{2}\left(F_{k_x}^{yy}\left(0\right)-F_{k_x}^{zz}\left(0\right)\right)$$

Canonical variable a_{k_x}

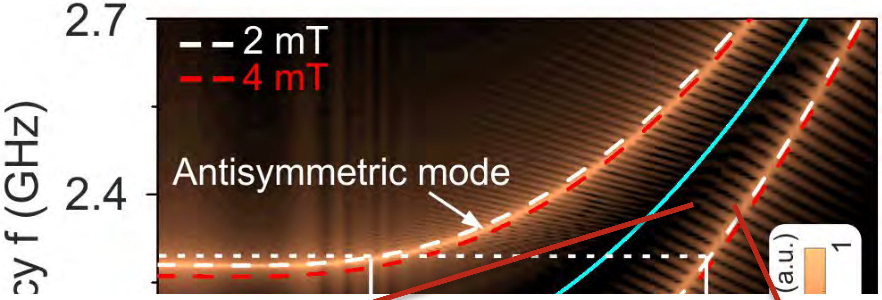
$$m_z=M_s a_{k_x}\sqrt{2-\left|a_{k_x}\right|^2}\left(u_{k_x}-v_{k_x}\right)$$

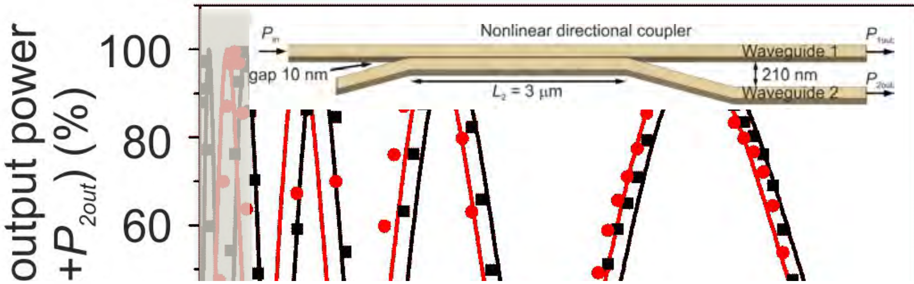
$$u_{k_x}=\sqrt{\frac{A_{k_x}+\omega_0}{2\omega_0}} \qquad v_{k_x}=-\mathrm{sign}\left[B_{k_x}\right]\sqrt{\frac{A_{k_x}-\omega_0}{2\omega_0}}$$

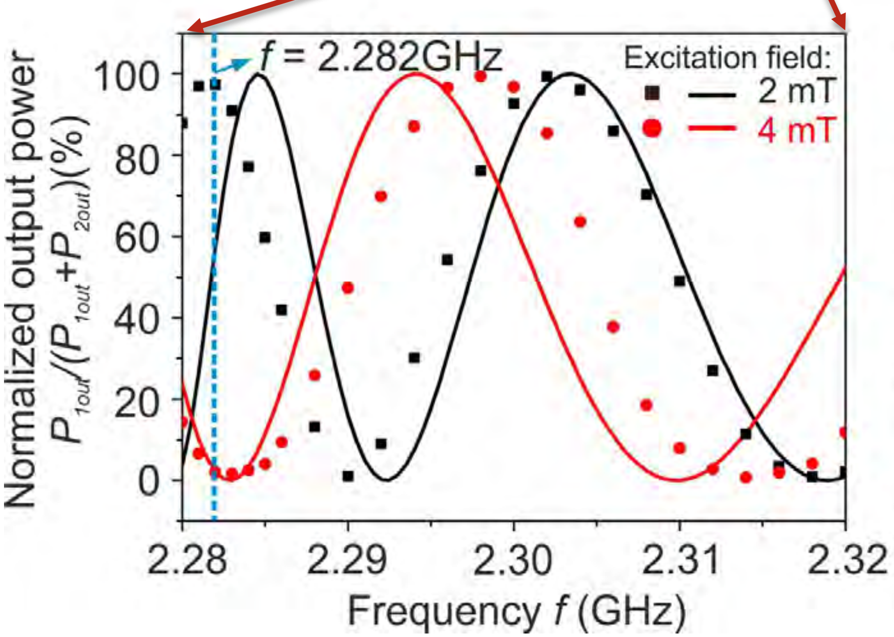
Roman Verba,
Kiew

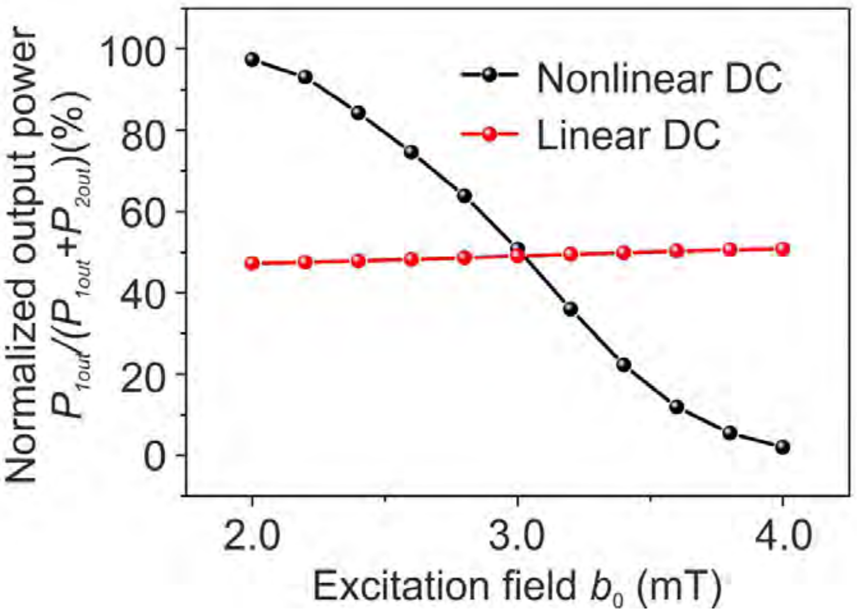
P. Krivosik and C. E. Patton, Phys. Rev. B 82, 184428 (2010)
R. Verba, *et al*, Sci. Rep. 6, 25018 (2016)

Nonlinear directional coupler



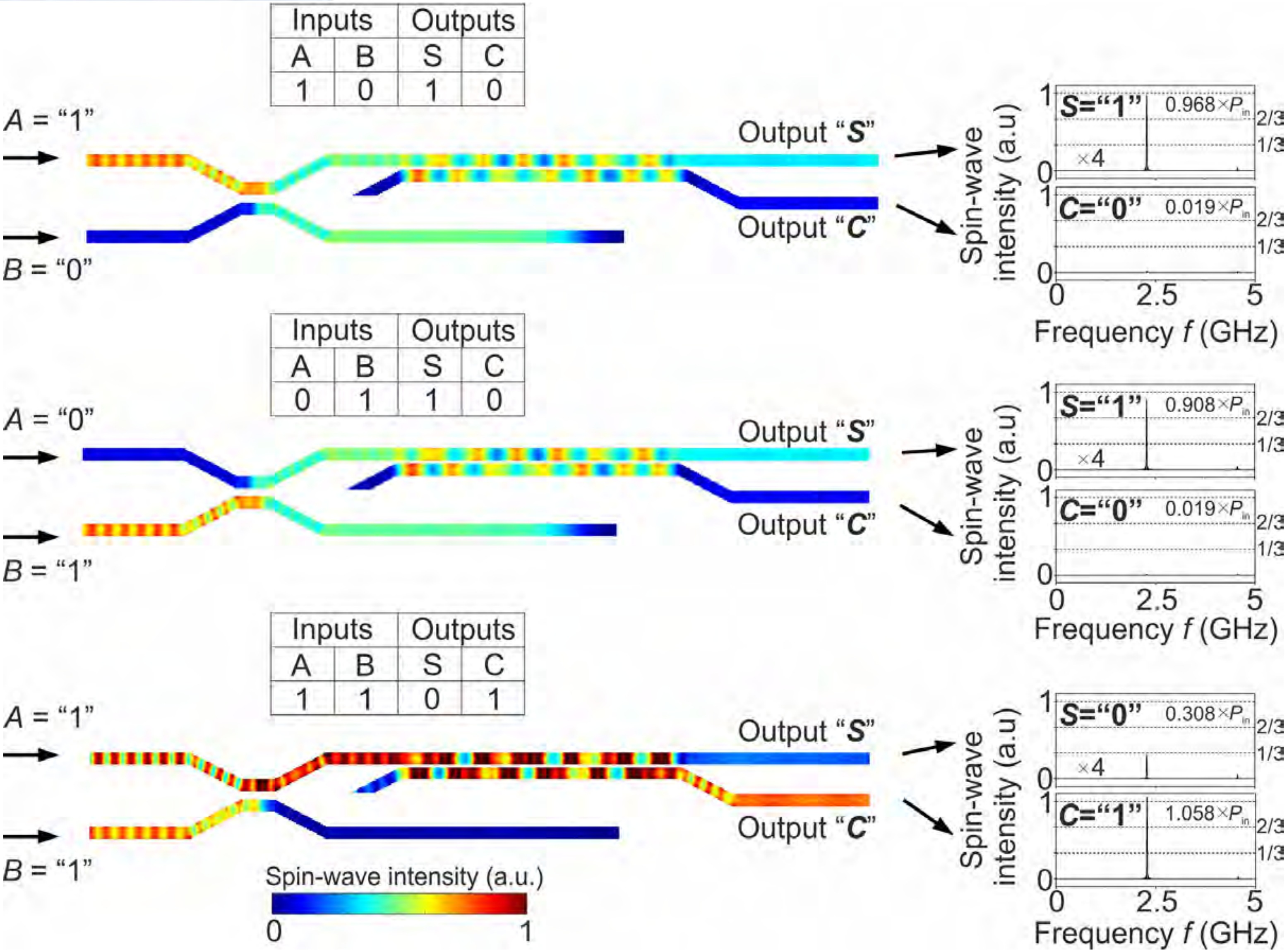






∝ input SW amplitude

Working principle



Inputs		Outputs	
A	B	S	C
1	0	1	0

Inputs		Outputs	
A	B	S	C
0	1	1	0

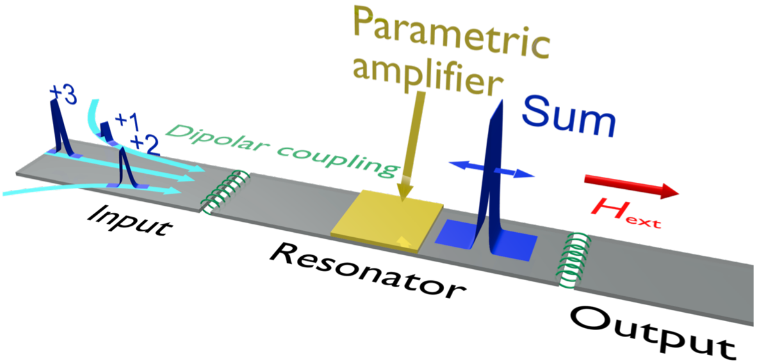
Inputs		Outputs	
A	B	S	C
1	1	0	1

Outline

- **Wave-based** logic and particular advantages of **spin waves**
- Prototype of **linear spin-wave logic** element: **Majority Gate**
- **Frequency multiplexer and spectrum analyzer** based on **caustic beams**
- **Towards integrated magnonic circuits:**
  - Magnonic Half adder (quasi binary)
  - Analog magnon adder

Concept of analog magnon adder

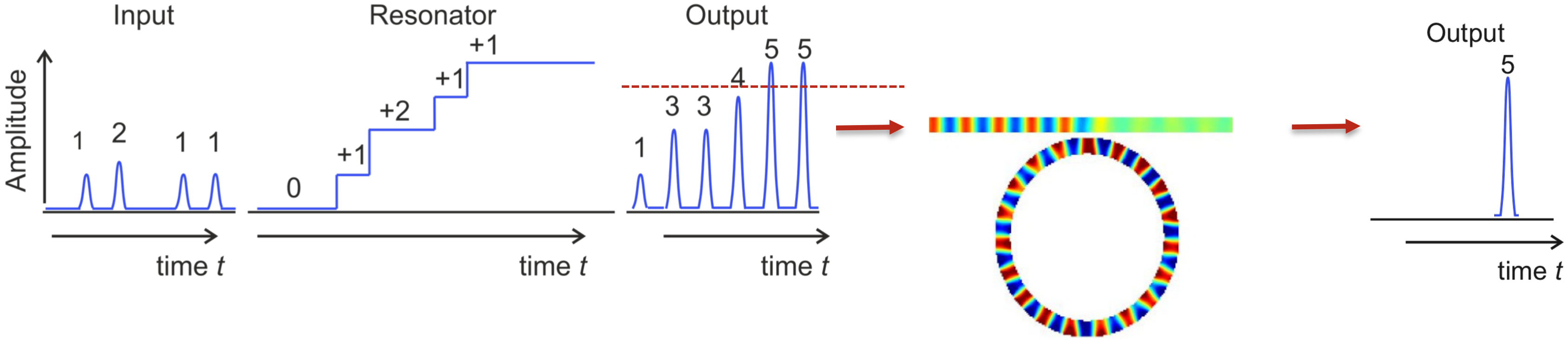
Basic layout:



Idea:
use a (damping compensated) resonator to add up spin-wave pulses which arrive with a delay equal to the resonator round trip time.

Next step (to be done):
Connect nonlinear magnonic element for threshold functionality

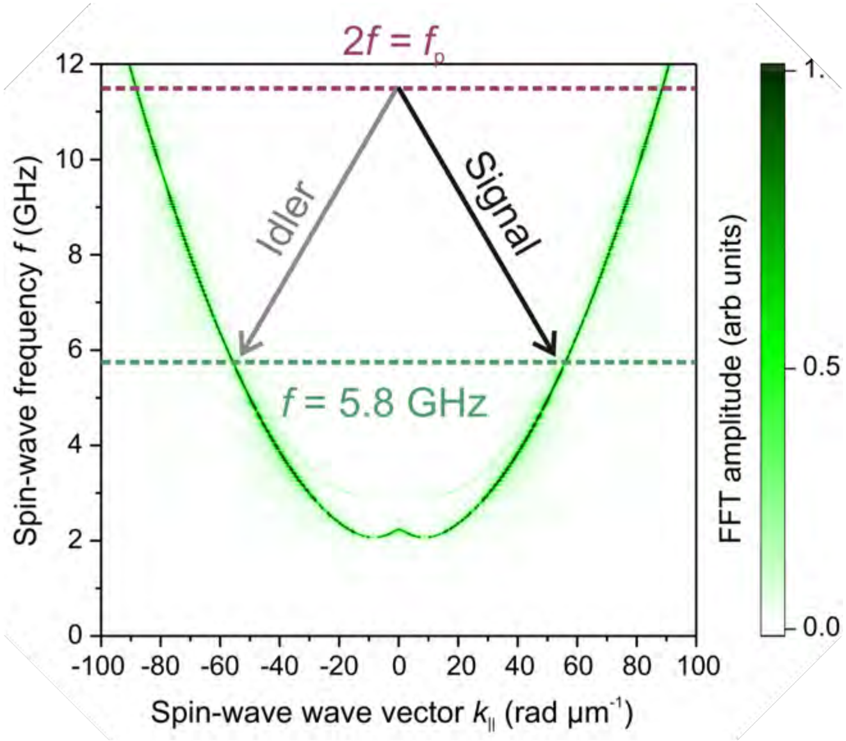
Working principle:



Negative values: phase difference of π

T. Brächer and P. Pirro, J Appl Phys **124**, 152119 (2018).

Parallel parametric amplification



Micromagnetic simulation (mumax3)

- Microwave field in **parallel to static field**: "Pumping field" with $f_p = 2 f_{sw}$
- Pumping field **acts only on dynamic magnetization**
- Requirement: Elliptical trajectory of magnetization (by, e.g., shape anisotropy)

Quasi-Particle picture:

A microwave photon (with f_p) is split up into a signal (f_s) and an idler magnon (f_i) with phase relation:

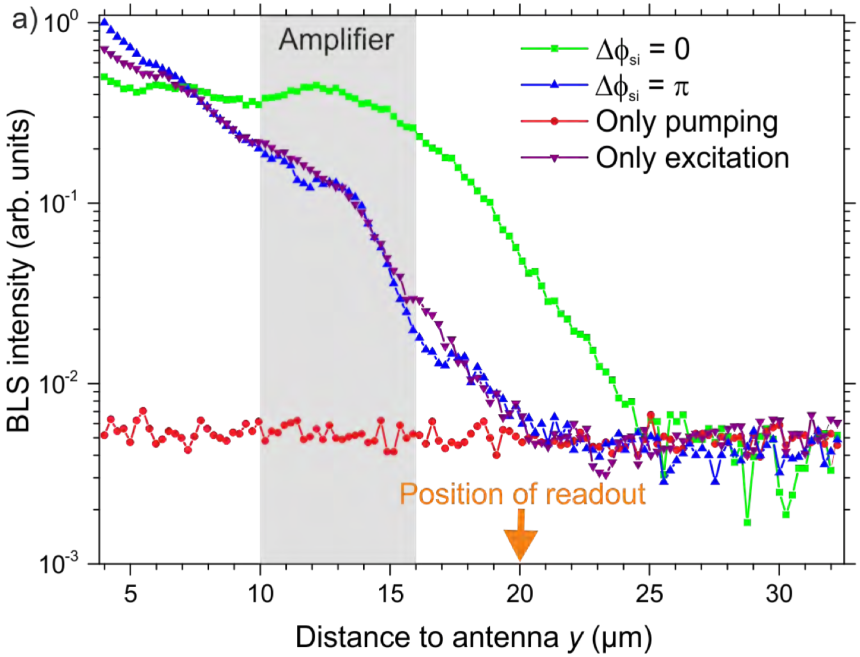
$$\phi_s + \phi_i = \phi_p + \pi / 2$$

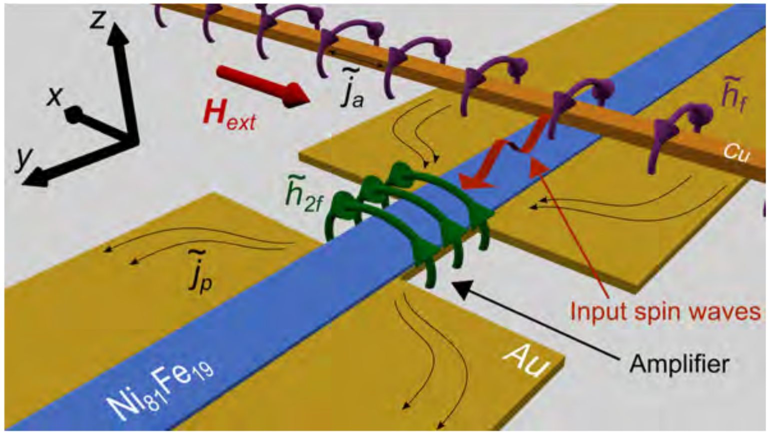
Signal phase Idler phase Pumping phase

Animation by D. Bozhko

T. Brächer, P. Pirro, and B. Hillebrands, Physics Reports **699**, 1 (2017).

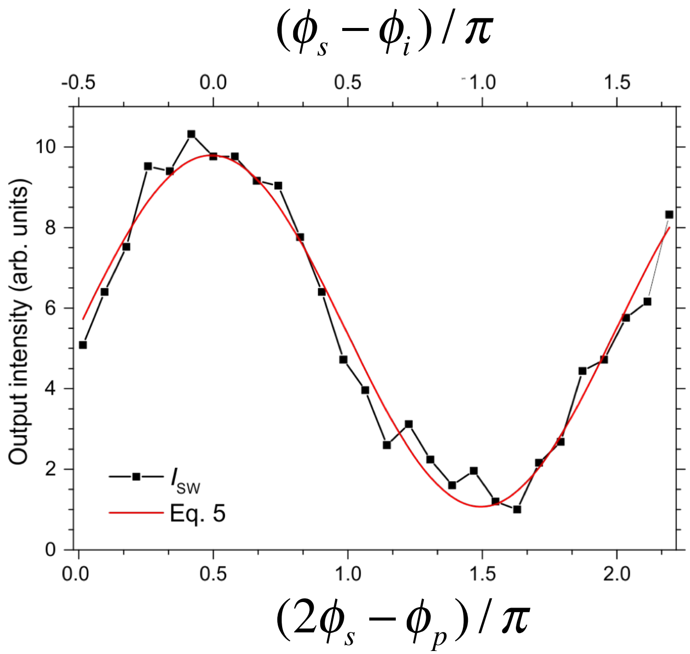
Parallel parametric amplification





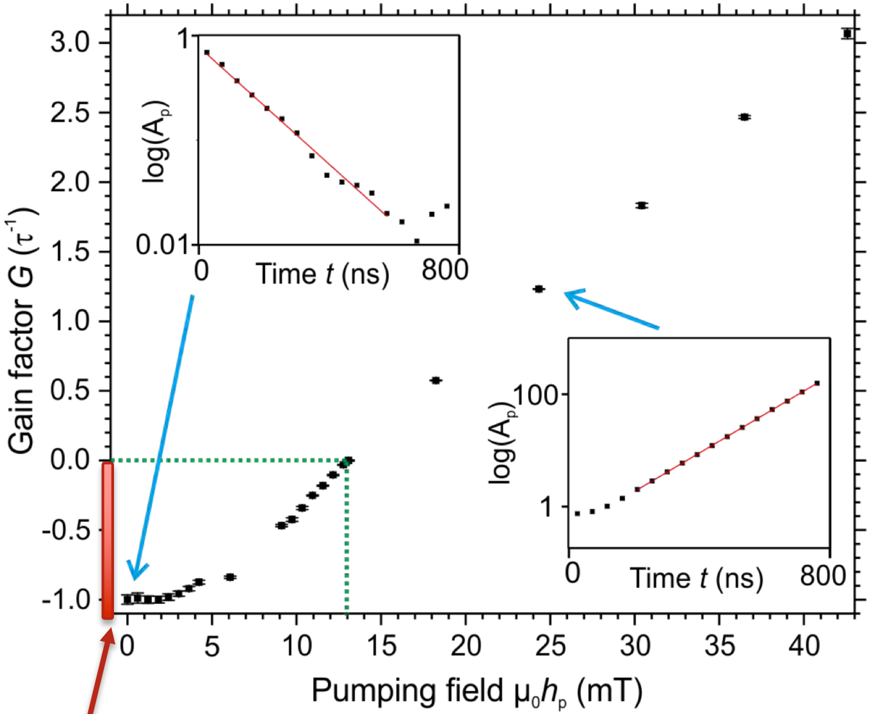
- Strong spin-wave amplification if proper phase relation is chosen

Continuous phase-to-intensity conversion for spin waves

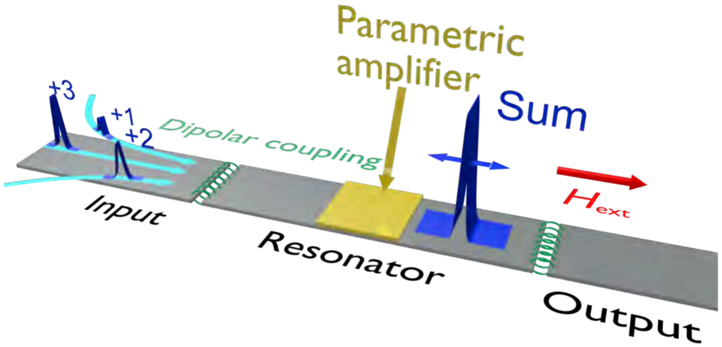


T. Brächer, et al., Sci. Rep. **6**, 38235 (2016).

Pumped resonator: gain factor G



Range of *G* where the memory of a pulse is decaying (exponentially) with time.



Amplitude after one round trip:

$$A_p(\Delta t_{\mathrm{rt}}) = A_p(0)\cdot\exp(G^*(h_p))$$

Exponential gain factor per round trip:

$$G^*(h_p) = 2V\mu_0 h_p \Delta t_{\mathrm{pump}} - \Delta t_{\mathrm{rt}}/\tau$$

Gain factor normalized to intrinsic losses:

$$G(h_p) = \frac{G^*(h_p)}{G^*(0)}$$

G=0 : spin waves amplitude is "stationary"
G=-1 : intrinsic decay of the system

Micromagnetic simulation (mumax3)

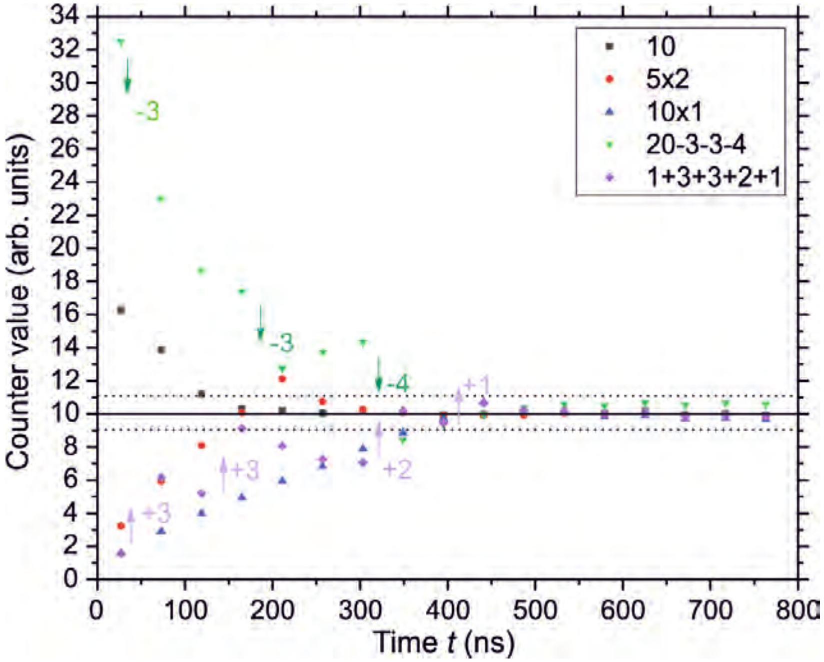
T. Brächer and P. Pirro, J Appl Phys **124**, 152119 (2018).

Analog magnon adder: gain factor G=0

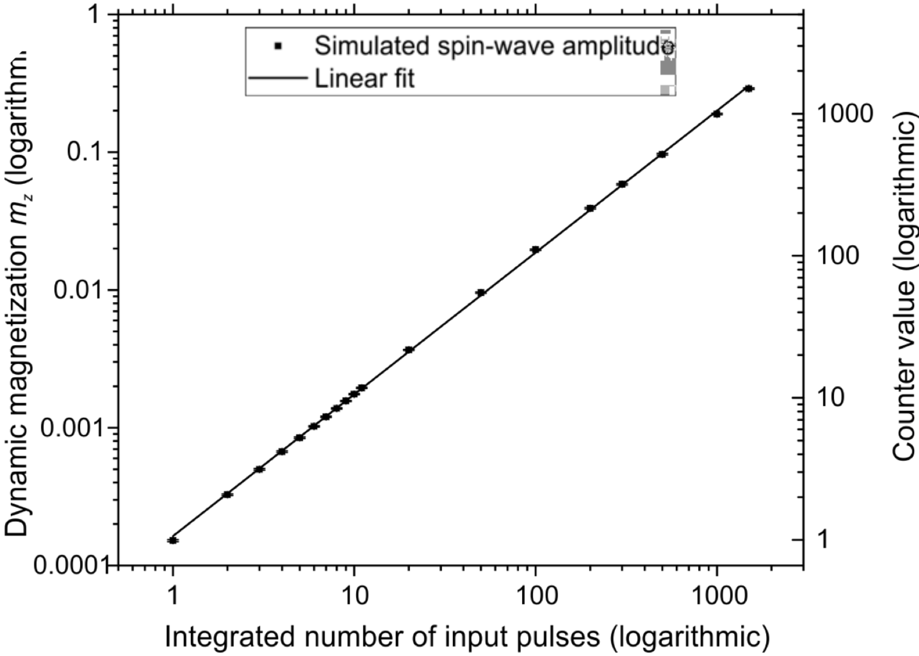
$$A_p(\Delta t_{\mathrm{rt}}) = A_p(0) \cdot \exp(G^*(h_p))$$

$$G(h_p) = \frac{G^*(h_p)}{G^*(0)}$$

Example: different ways to sum to “10”



Linear summation up to “1000”



Different transitional dynamics until a steady state is reached, but without impact on the adding function.

Micromagnetic simulation (mumax3)

T. Brächer and P. Pirro, J Appl Phys **124**, 152119 (2018).

TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Acknowledgements

Kaiserslautern team:

Prof. Burkard Hillebrands
Jun. Prof. Andrii Chumak

Dr. Thomas Brächer

Frank Heussner
Qi Wang
Martin Kewenig
Tobias Fischer
Moritz Geilen

IMEC team in Leuven:

Dr. Christoph Adelmann
Dr. Florin Ciubotarou

Giacomo Talmelli

Theory support:

Dr. Roman Verba

Inst. of Magnetism
Kyiv, Ukraine

Thank you for your attention!

Conclusion

- Spin-waves can use **phase** and **amplitude** to carry information in logic circuits based on **interference phenomena**
- Multiplexing in frequency opens novel ways to improve performance
- **Magnonic Half adder** as first proof-of-concept of integrated magnon circuits with **nonlinear functionality**
- **Analog magnon adder** with tunable gain to mimic pulse accumulation in neurons