Topological states on fractal lattices

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SPICE Workshop
30 September 2019

Acknowledgements

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PRB 98, 205116 (2018)
arXiv: 1807.00367

Outline

1. Motivation: beyond the ten-fold way
2. Setting up the problem
3. Methods and results
4. Conclusions and further directions

Classification of topological insulators and superconductors

		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	$+1$	0	0	-	-	-
	AII (symplectic)	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	$+1$	$+1$	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	$+1$	-1	1	-	-	$\mathbb{Z}$

Altland & Zirnbauer, PRB ’97; Kitaev, AIP Conf. Proc. ’09; Schnyder et al., PRB ’08

+ crystal symmetries

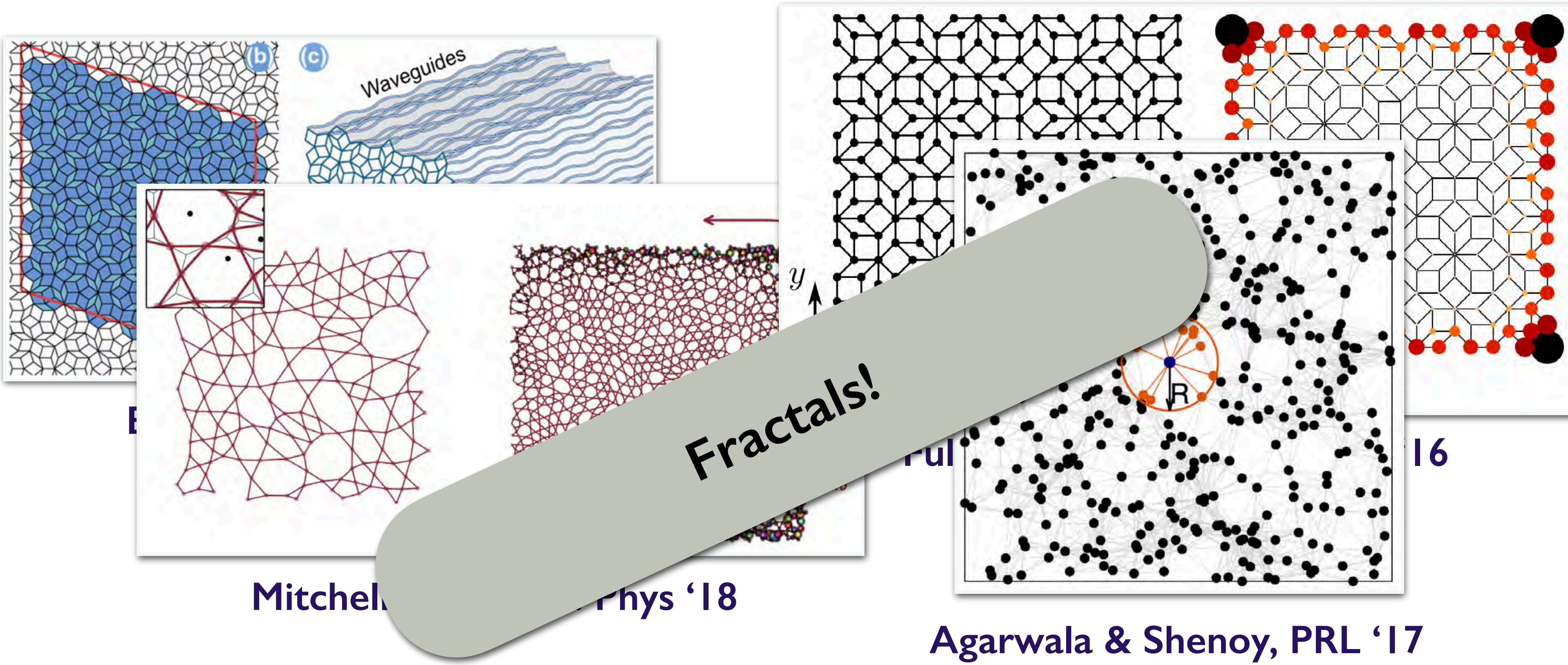
Slager et al., Nat. Phys ’12; Shiozaki & Sato, PRB ’14; Shiozaki, Sato & Gomi PRB ’15, ’16, ’17; Dong & Liu, PRB ’16; …

+ non-Hermiticity

Bernard & LeClair, 0110649 ’01; Shen, Zhen & Fu, PRL ’16; Gong et al. PRX ’18; Liu, Jiang & Chen, PRB ’19, …

Classification of topological insulators and superconductors

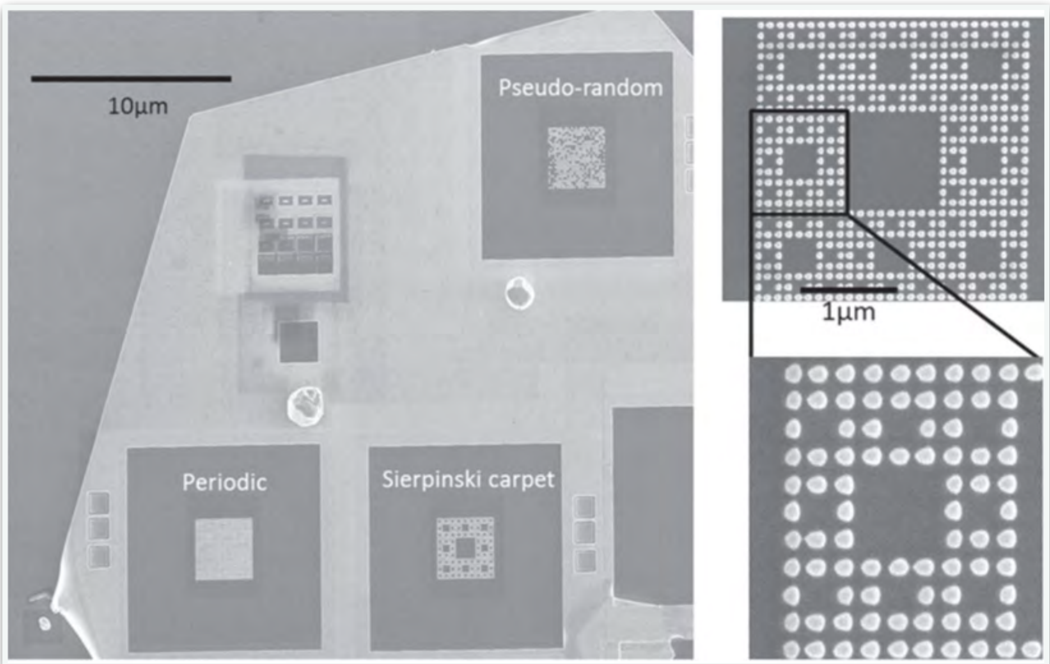
How to classify topological phases in **aperiodic** systems?



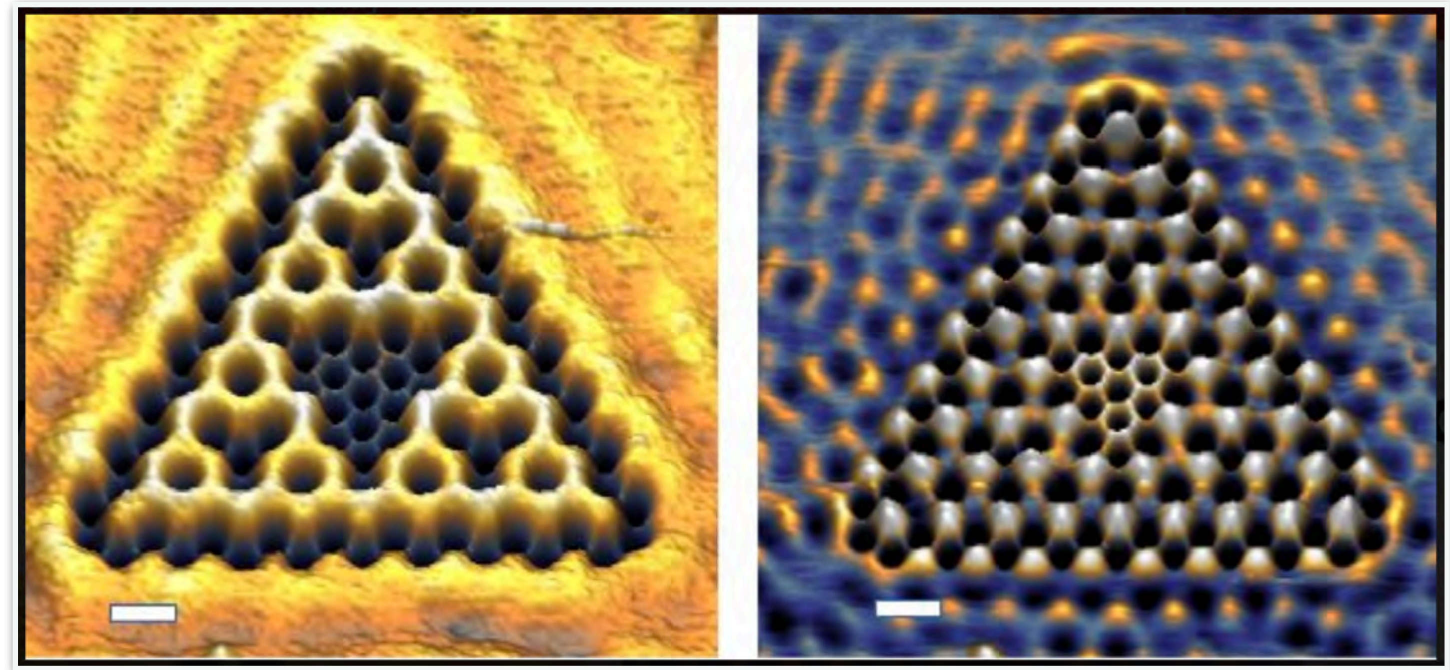
What if there is **no sharp distinction** between the edge and the bulk?

Can topological states be realized in systems defined in **non-integer** spatial dimensions?

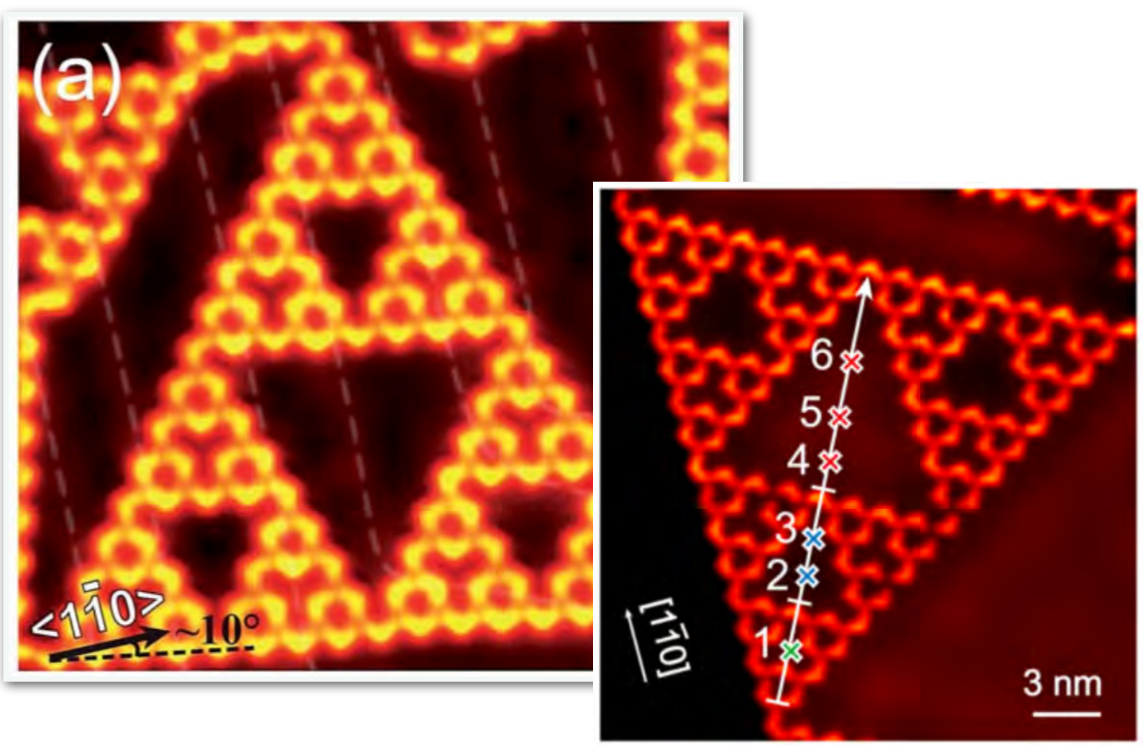
Fractals in a lab: experimental developments



Chen et al., NJP '14

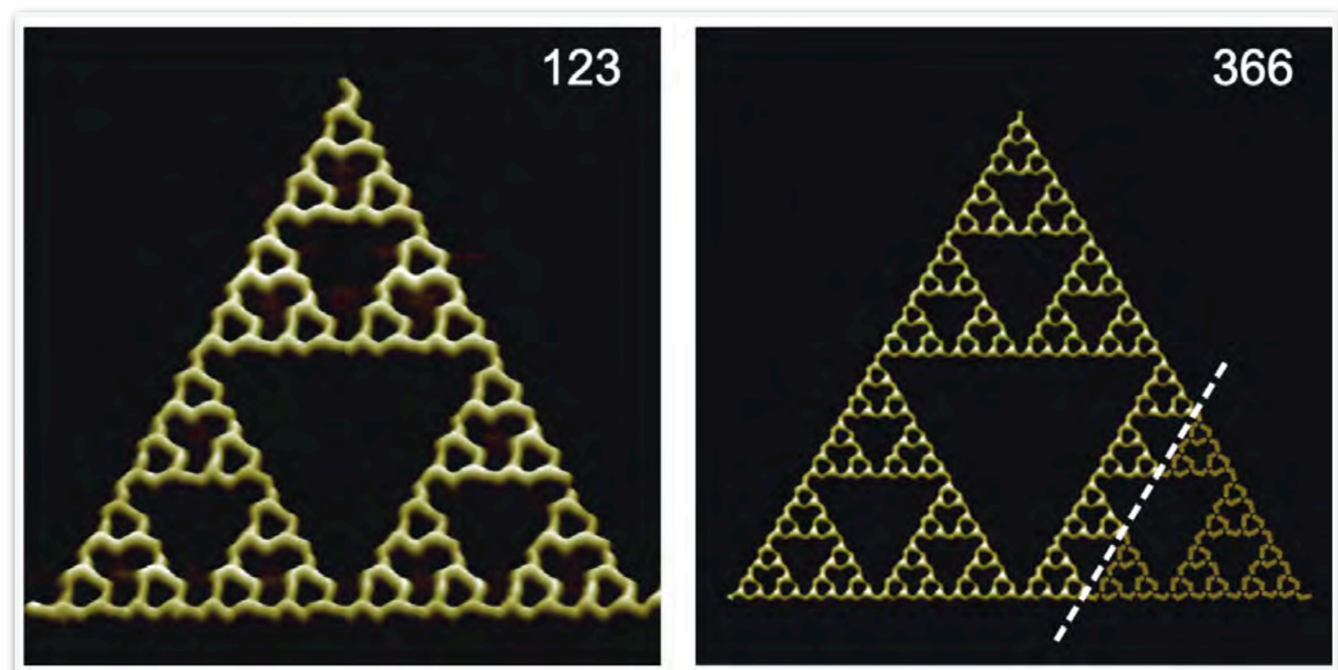


Kempkes et al., Nat. Phys '18



Zhang et al., RSC Adv. '18

Wang et al., PRB '18



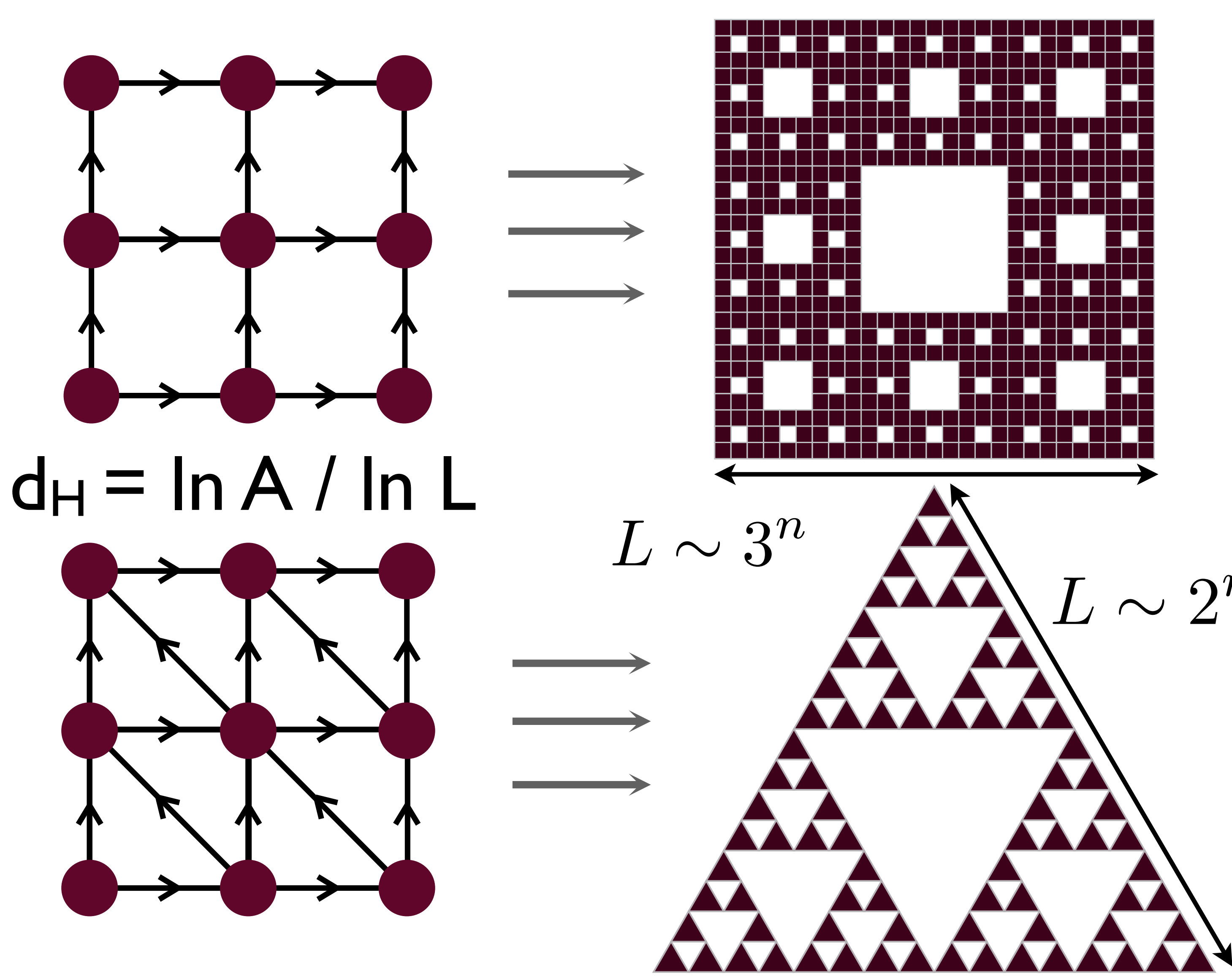
Shang et al., Nat. Chem '15

Model

Goal: investigate IQHE in fractal dimensions

Consider spinless fermions in a magnetic field on **Sierpiński carpet** and **gasket**

$$H = -t \sum_{\langle i,j \rangle} e^{\mathrm{i}A_{ij}} c_i^\dagger c_j + H.c.$$



$d_H = 1.892\ldots$

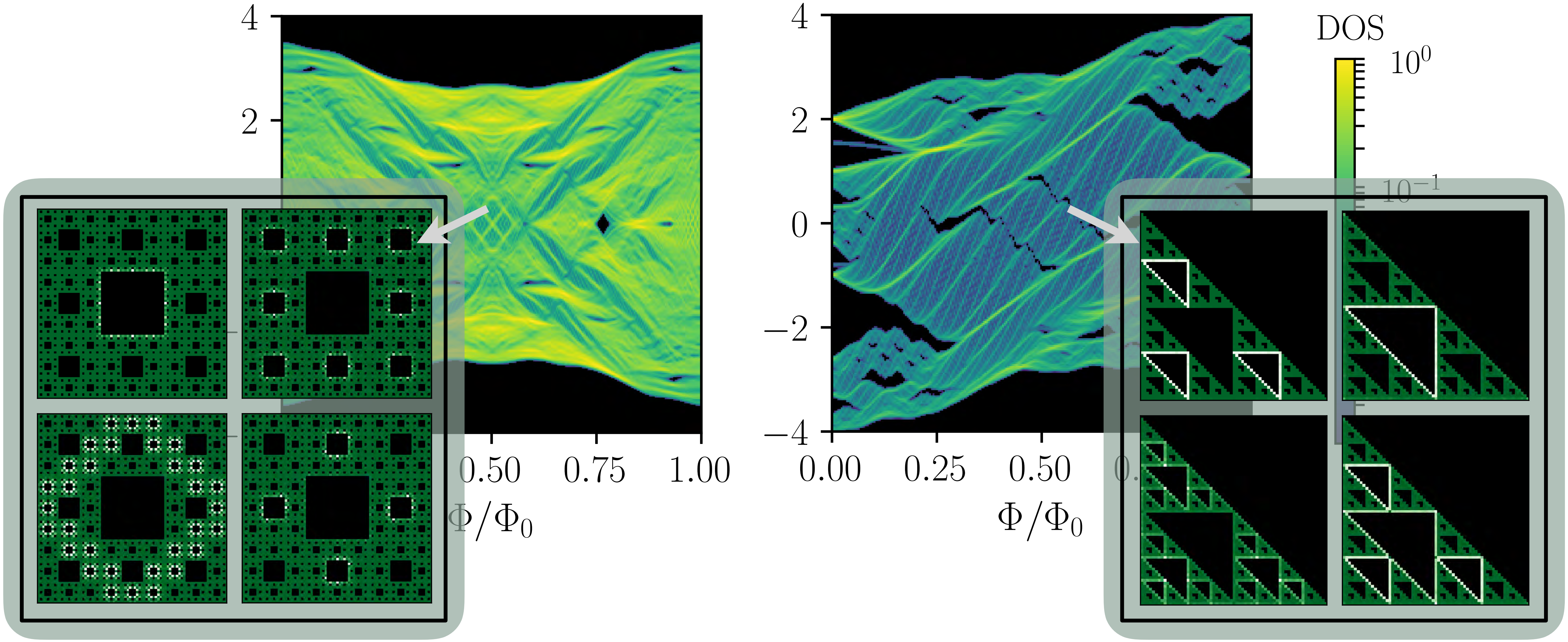
infinite ramification #

$d_H = 1.585\ldots$

finite ramification #

Density of states

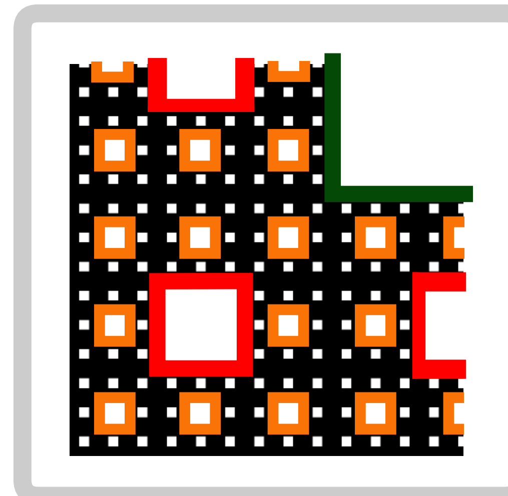
Magnetic field gives rise to **Hofstadter's butterfly** also on fractal lattices

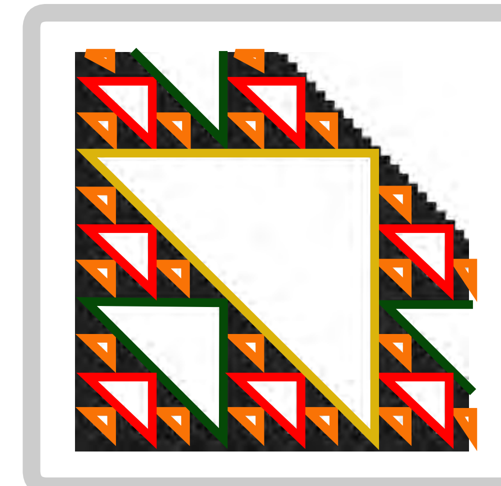


And leads to **localized states at the internal edges** of fractals

Eigenstates localization properties

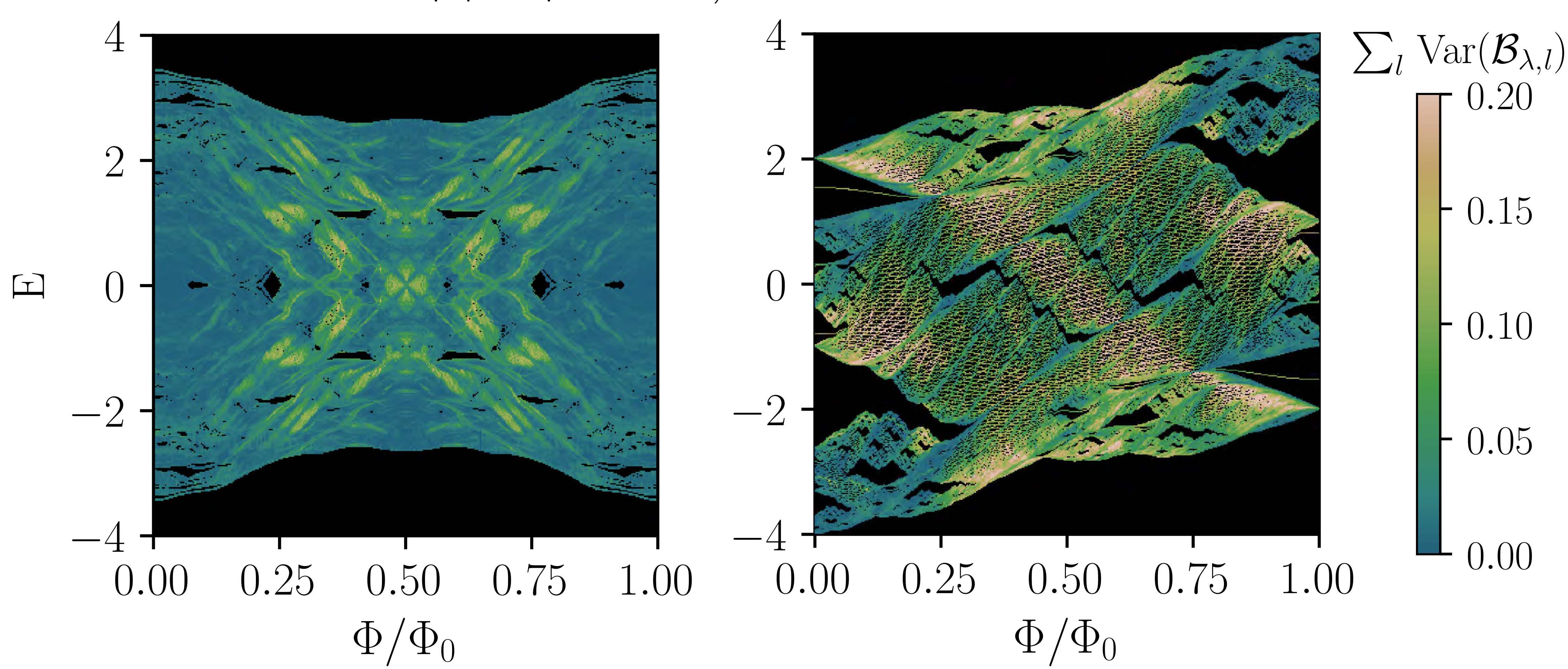
We use the **edge-locality marker** to quantify the degree of localization of the eigenstates





$$\mathcal{B}_{\lambda,l} = \sum_{i \in \varepsilon_l} |\psi_{\lambda,i}|^2$$

$$\langle i|\psi_\lambda\rangle = \psi_{\lambda,i}$$



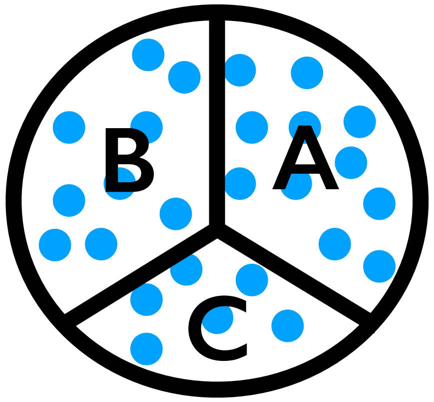
Low DOS regions exhibit rapidly varying **hierarchy of edge-localized states**

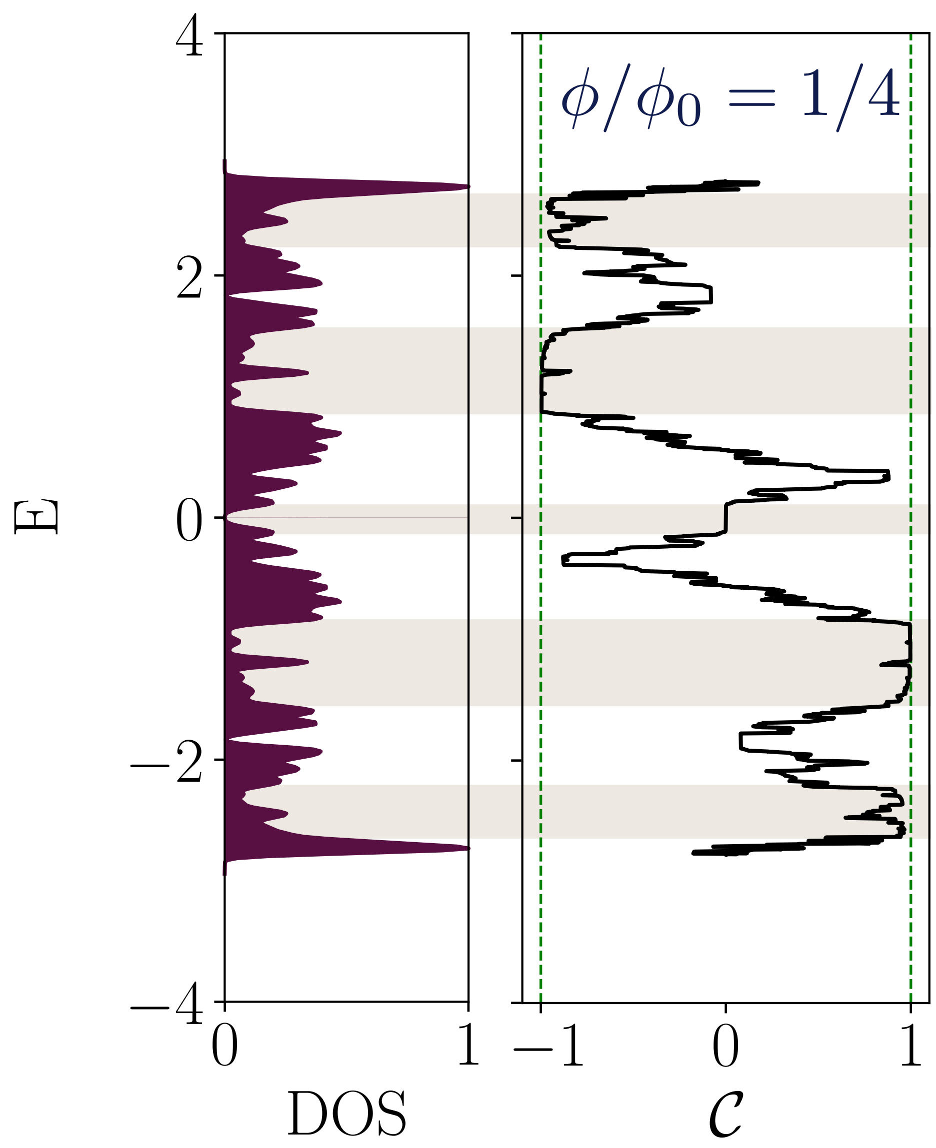
Real-space Chern number calculations

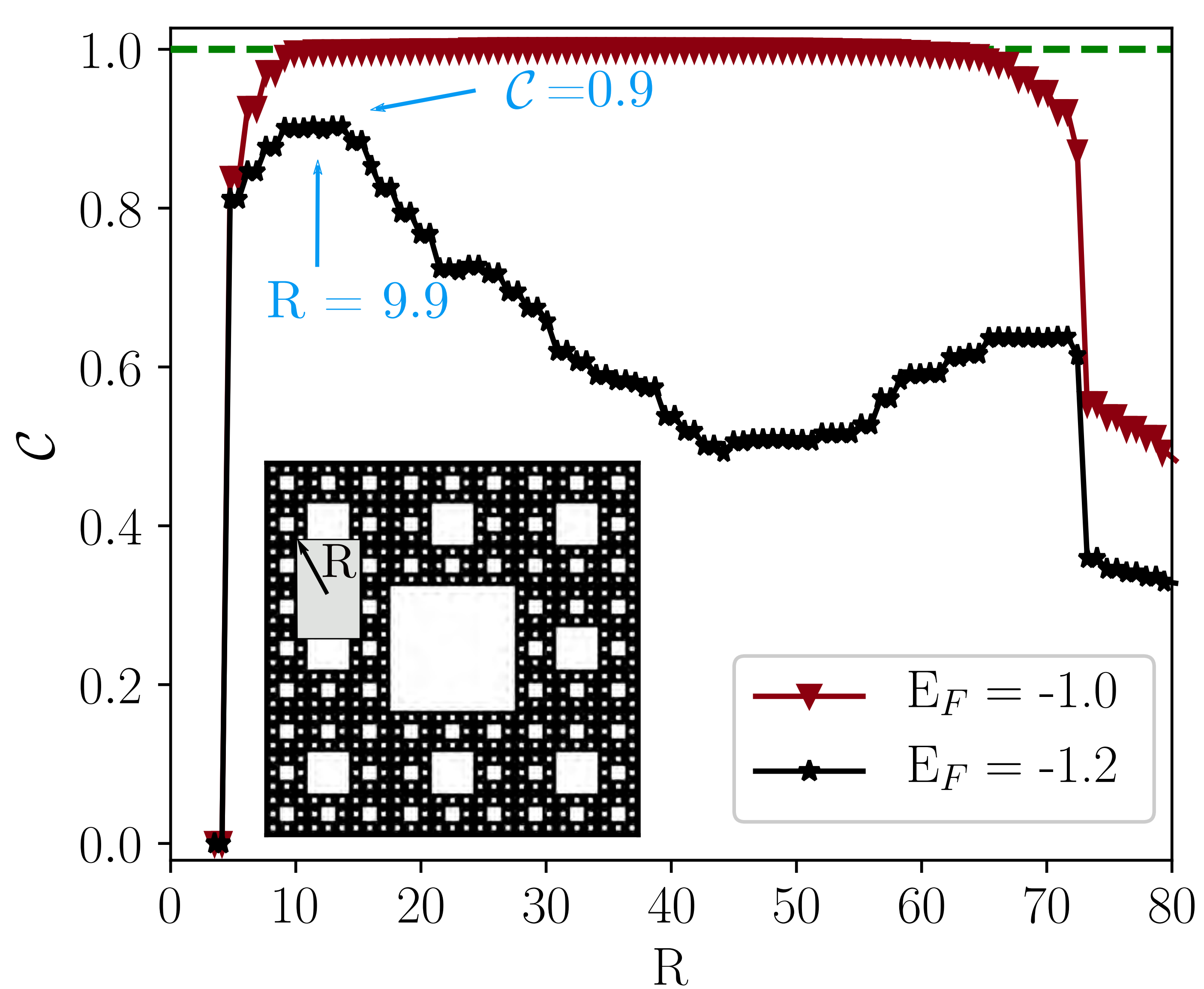
For systems without translational invariance, the Chern number can be computed through the projection operator P

$$\mathcal{C} = 12\pi \mathrm{i} \sum_{j\in A} \sum_{k\in B} \sum_{l\in C} (P_{jk}P_{kl}P_{lj} - P_{jl}P_{lk}P_{kj})$$

Kitaev, Ann. Phys. '06

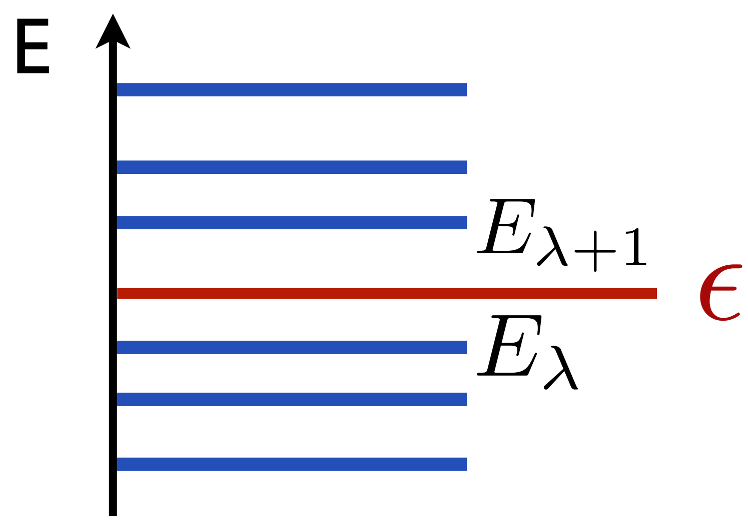






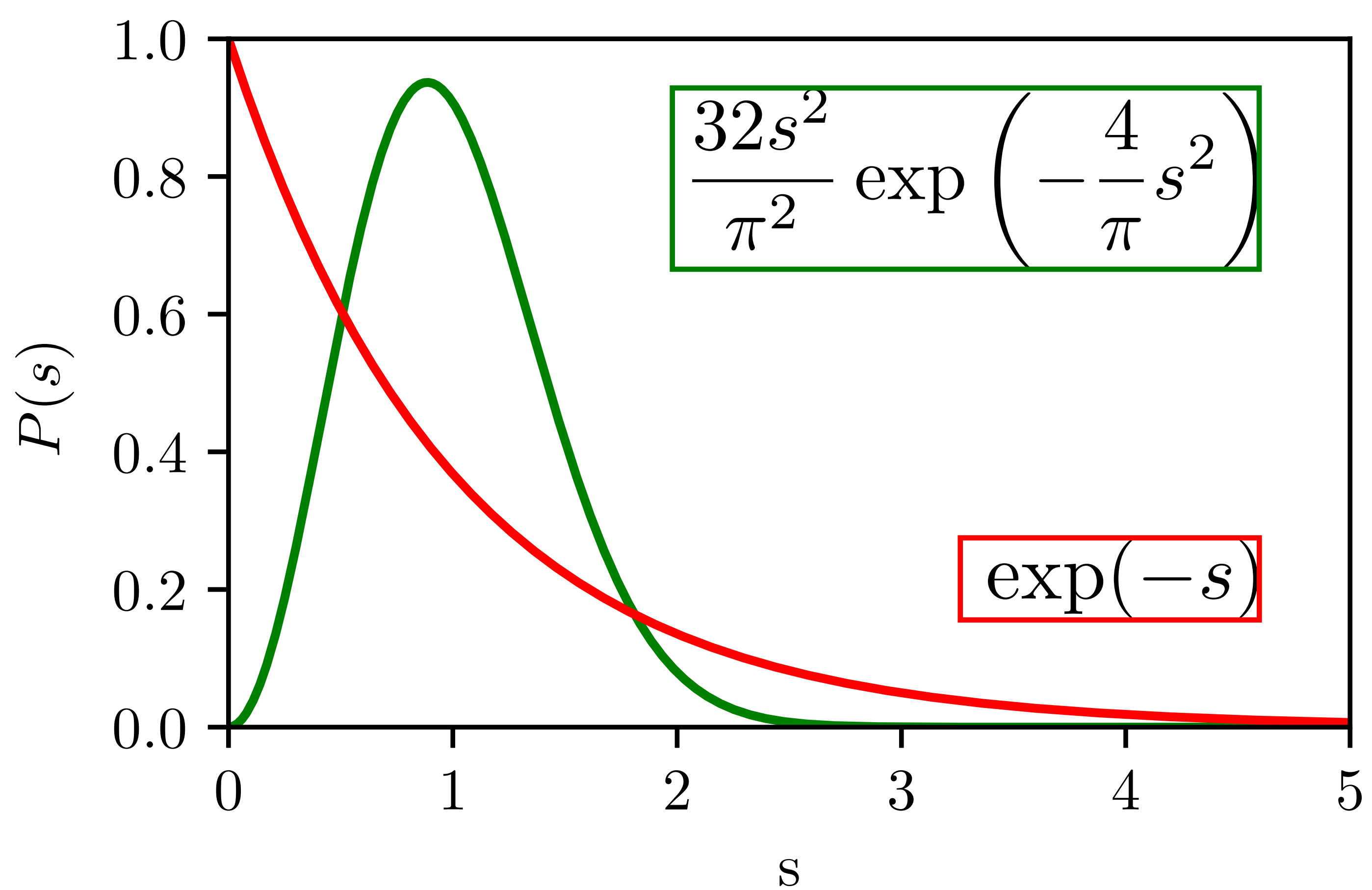
Level spacings analysis

To study disorder-induced phase transition, we compute **level spacings** as a function of disorder strength



$$s_{\epsilon,m} = E_{\lambda+1+m} - E_{\lambda+m}$$

$$Var(s_\epsilon) = \langle s_\epsilon^2 \rangle - \langle s_\epsilon \rangle^2$$



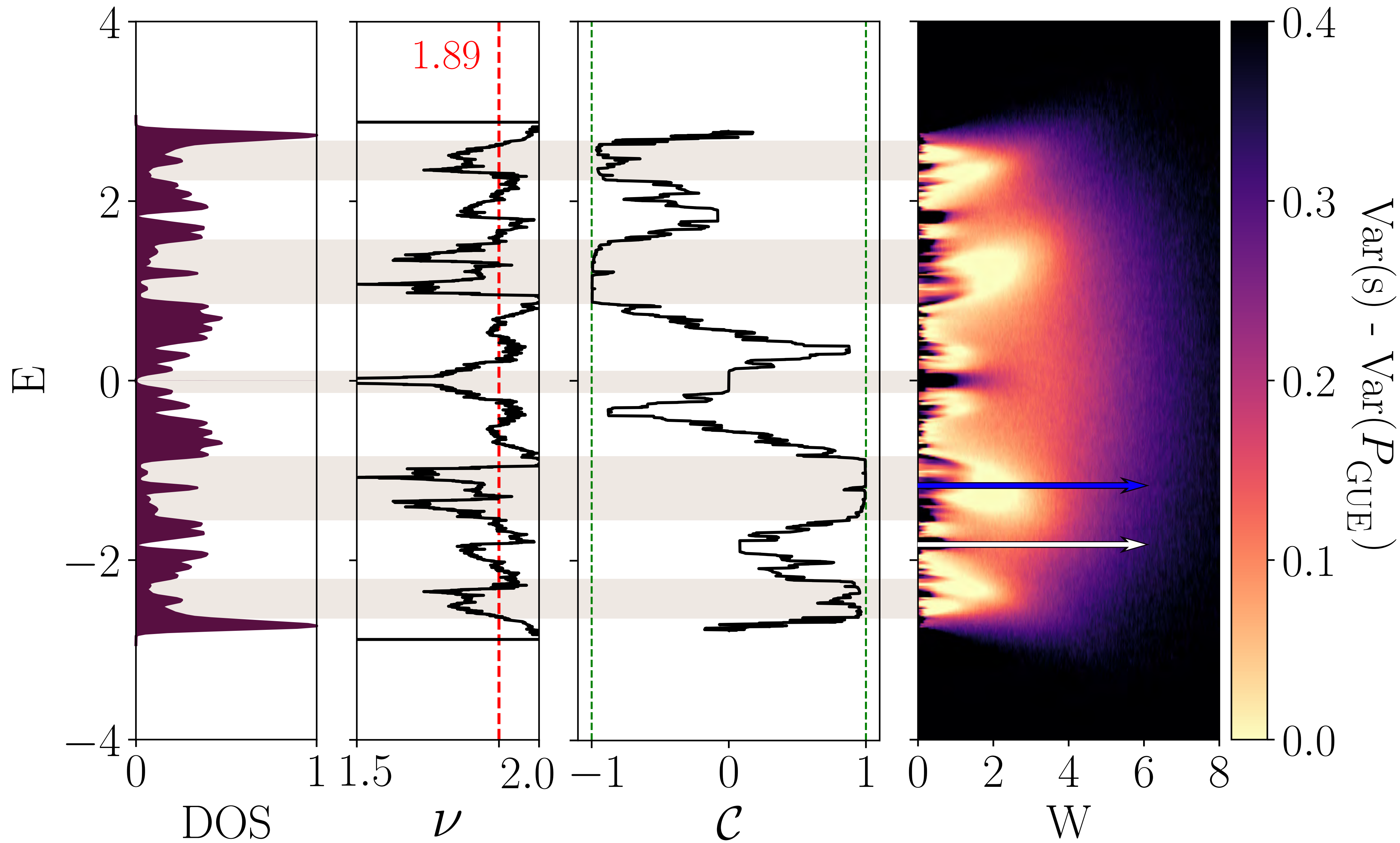
Delocalized case:
Wigner-Dyson distribution

$$Var(s) = 0.174$$

Localized case:
Poisson distribution

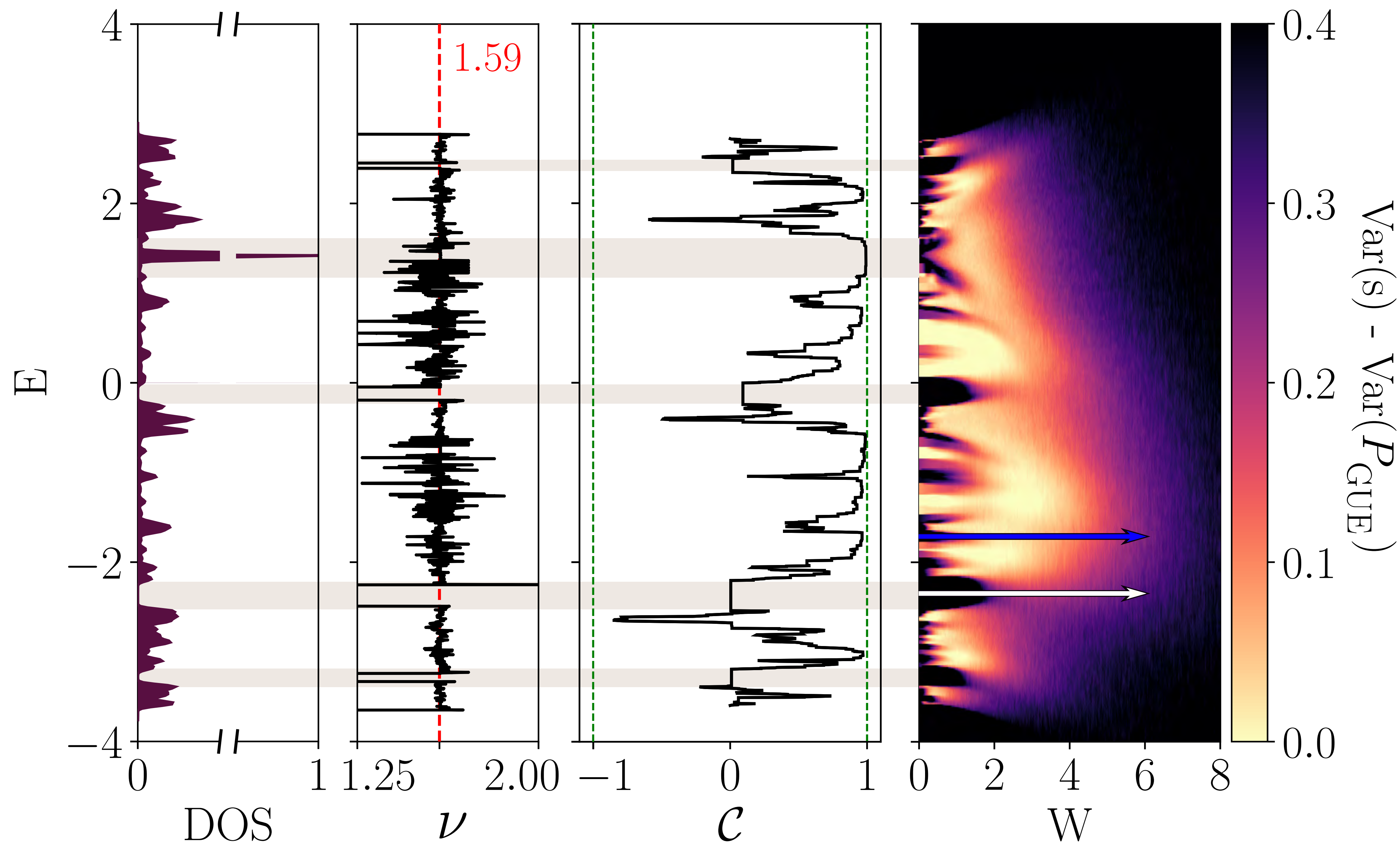
$$Var(s) \sim \mathcal{O}(1)$$

Topological properties of Sierpiński carpet



Plateaus with non-trivial Chern number coincide with **localization-delocalization transition**

Topological properties of Sierpiński gasket



Results for the gasket are less clearly identifiable - **finite size effects** or **dimensionality**?

Summary and further remarks

We identified states exhibiting similar features to the quantum Hall effect which occur on lattices with **non-integer** Hausdorff dimension

Open questions:

- general classification scheme for fractals
- interactions & topological order: what dimensional properties a graph must have in order to support long-range entangled ground states of local Hamiltonians?

Thank you for attention!

Slide: IPR

$$I_\psi = \frac{\sum_i |\psi_i|^4}{\left(\sum_i |\psi_i|^2\right)^2}$$

$\sim \frac{1}{N}$, if state is **delocalized**

large IPR, if state is **localized**

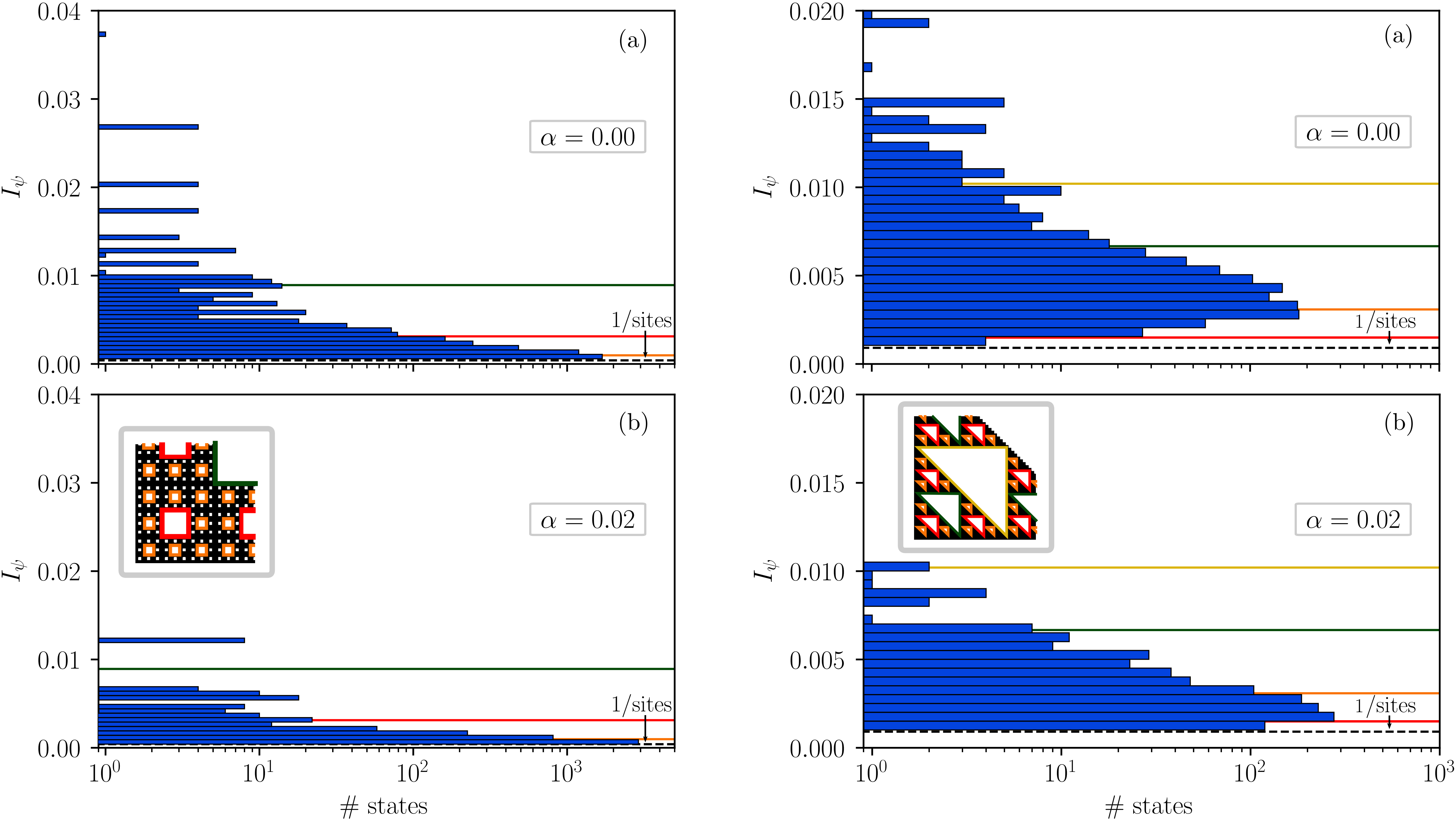


Figure: IPR at zero and finite magnetic flux

Slide: comparison with regular lattices

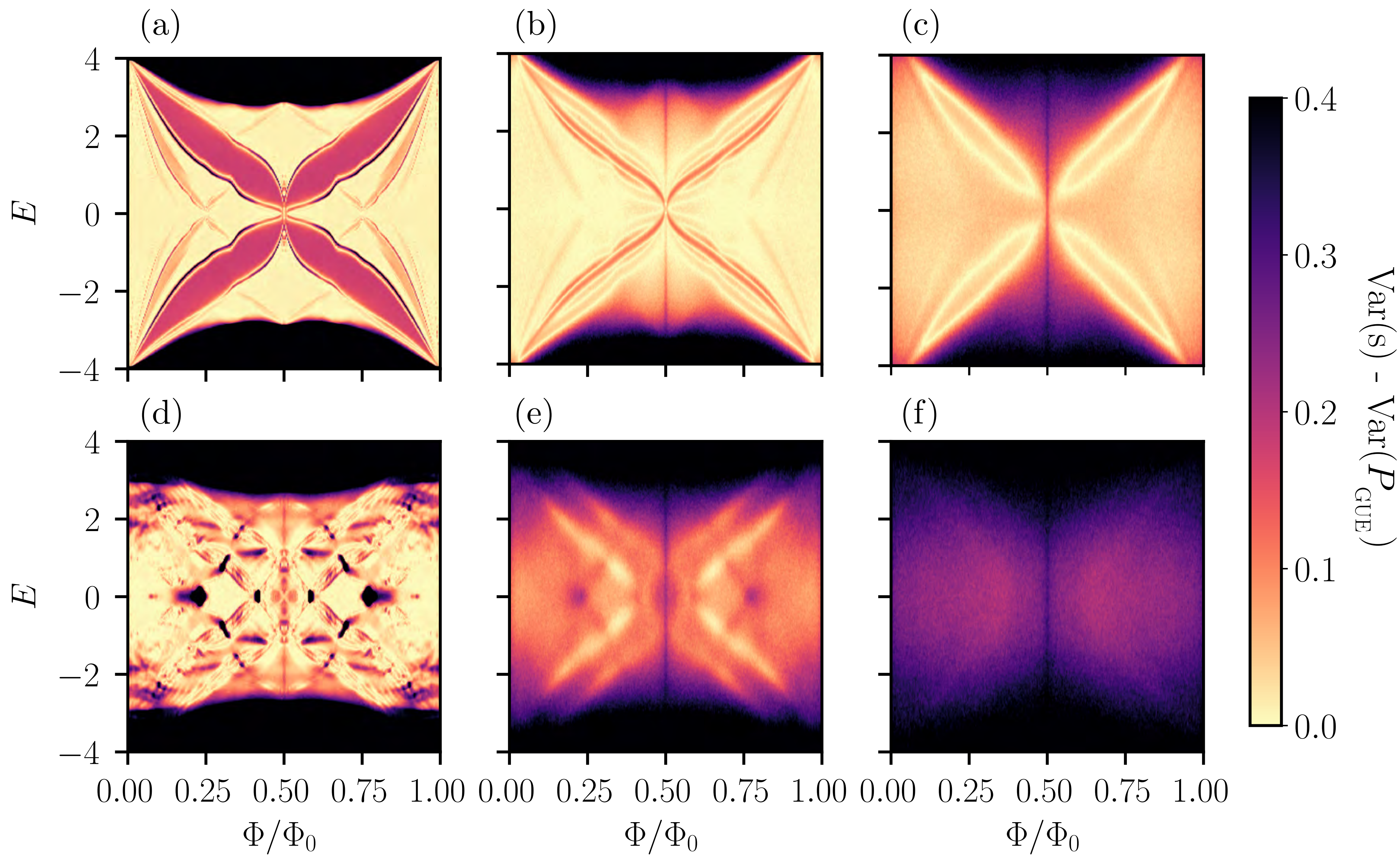
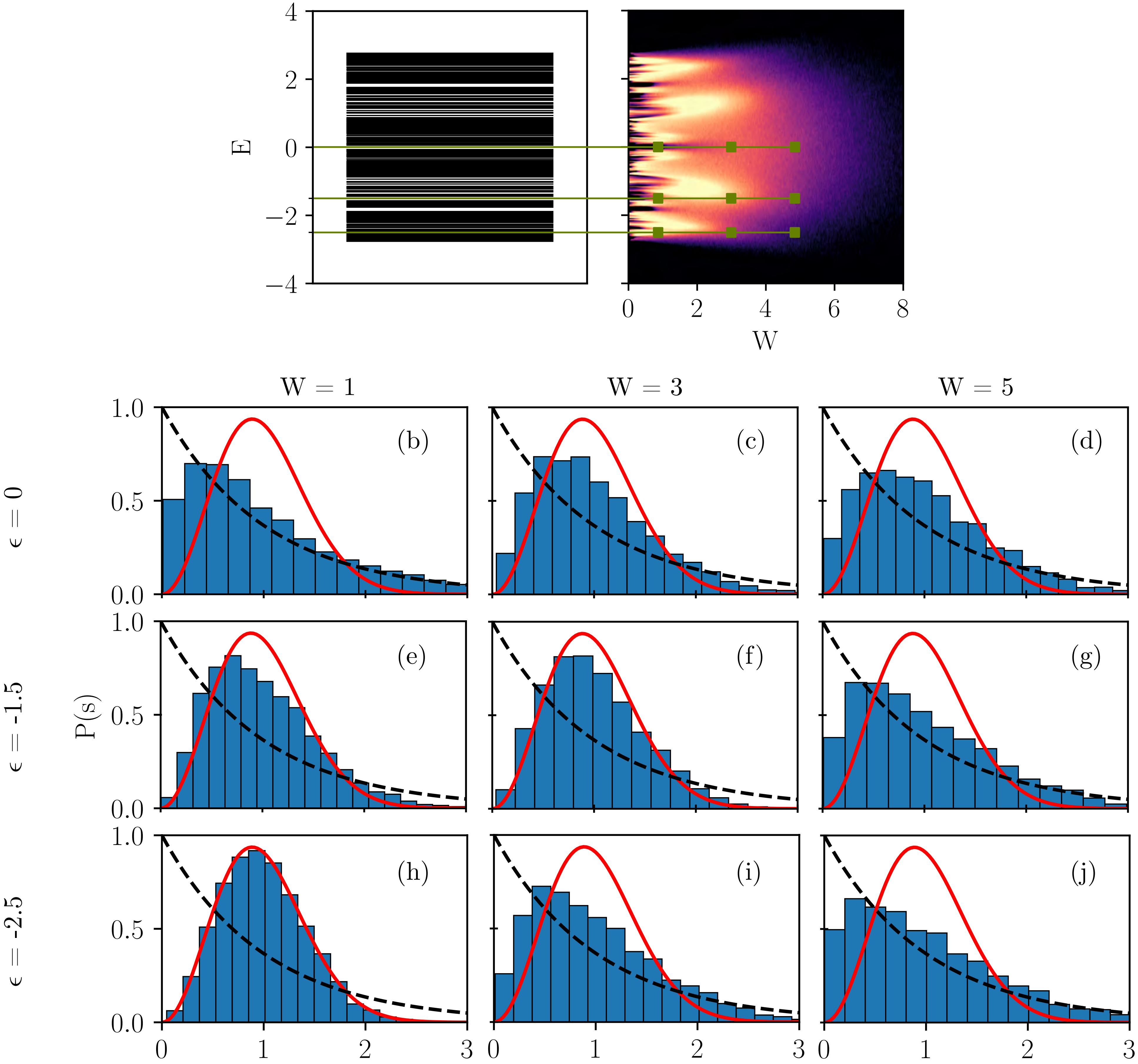


Figure: Variance of level spacings at W = 1, W = 3 and W = 5 for a square lattice and SC

Slide: level spacings for SC



Slide: comparison with regular lattices

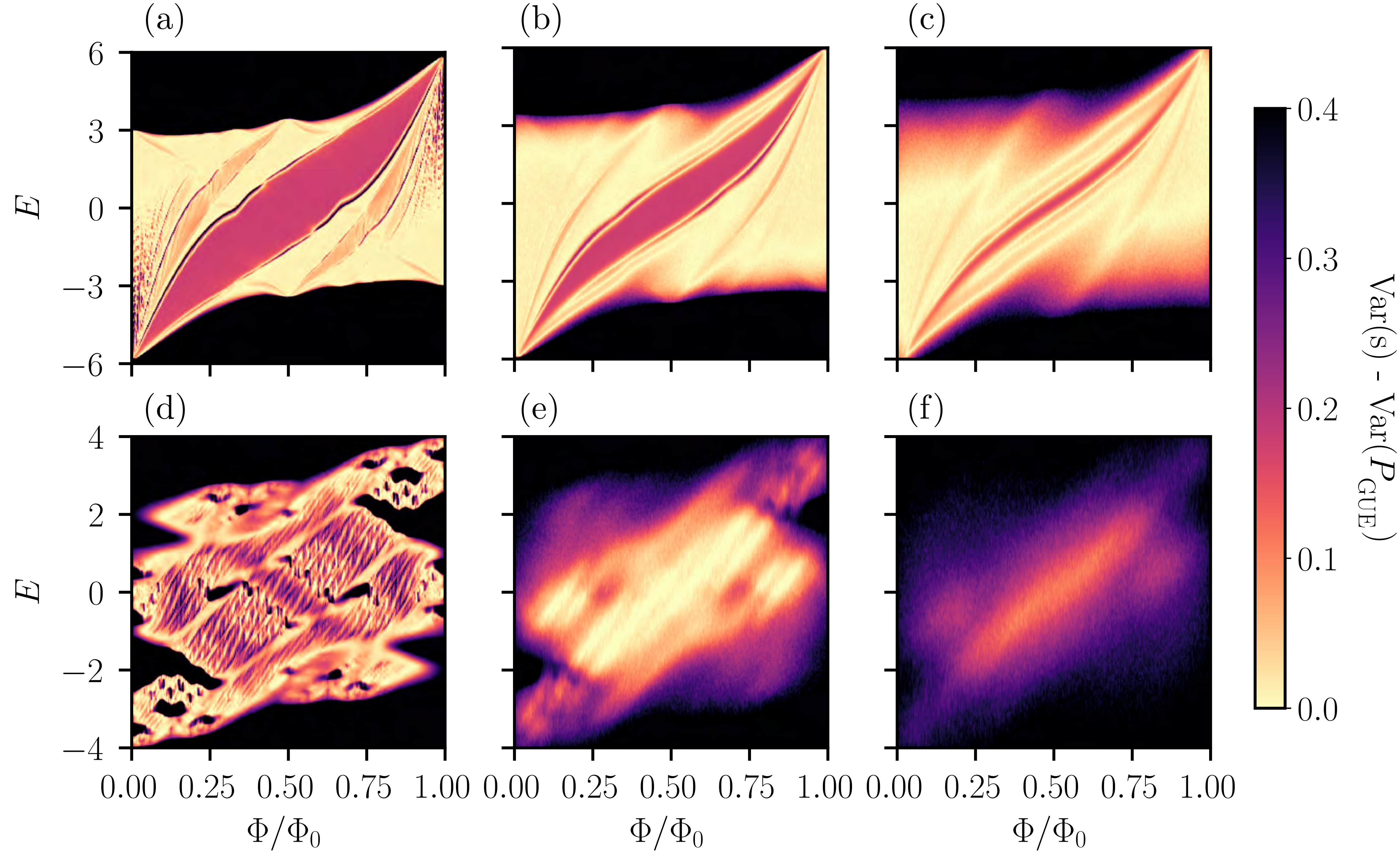


Figure: Variance of level spacings at W = 1, W = 3 and W = 5 for a triangular lattice and SG

Slide: level spacings for SG