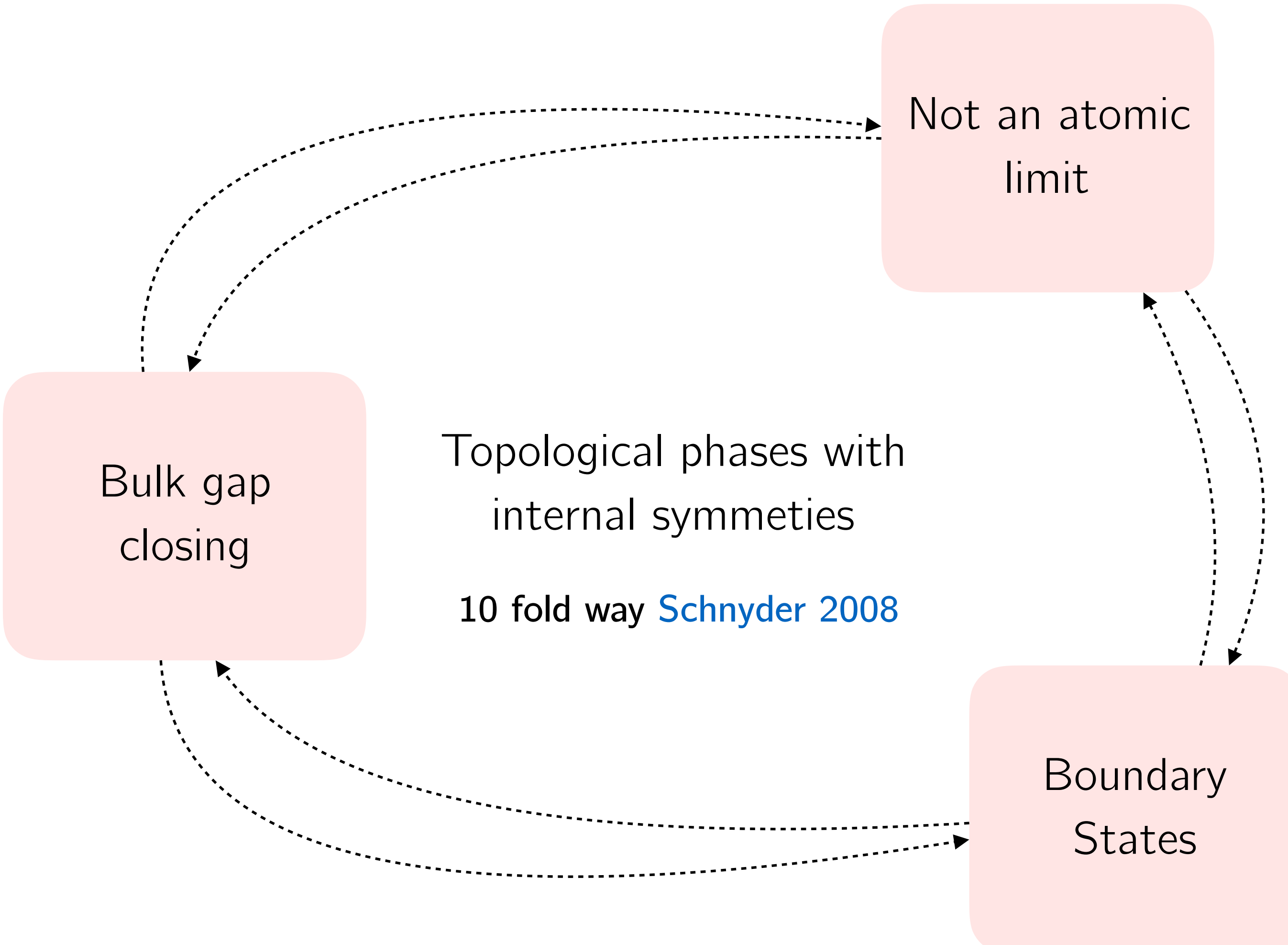
Boundary Obstructed Topological Phases

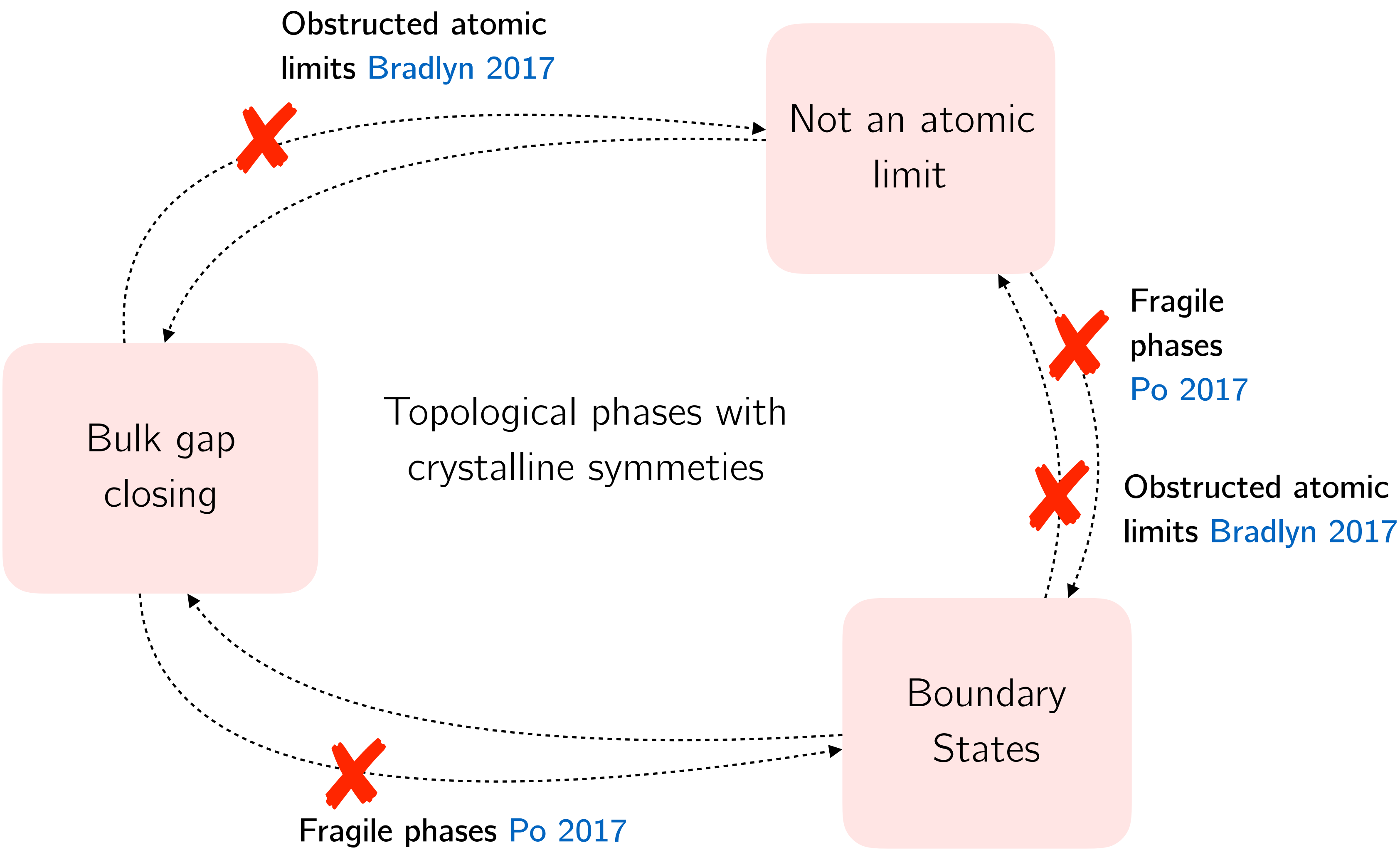
Raquel Queiroz

Boundary obstructed topological phases
Khalaf, Benalcazar, Hughes, ***RQ 1908.00011***
Partial lattice defects in higher order topological insulators
RQ*, Fulga, Avraham, Beidenkopf, Cano* ***1809.03518***

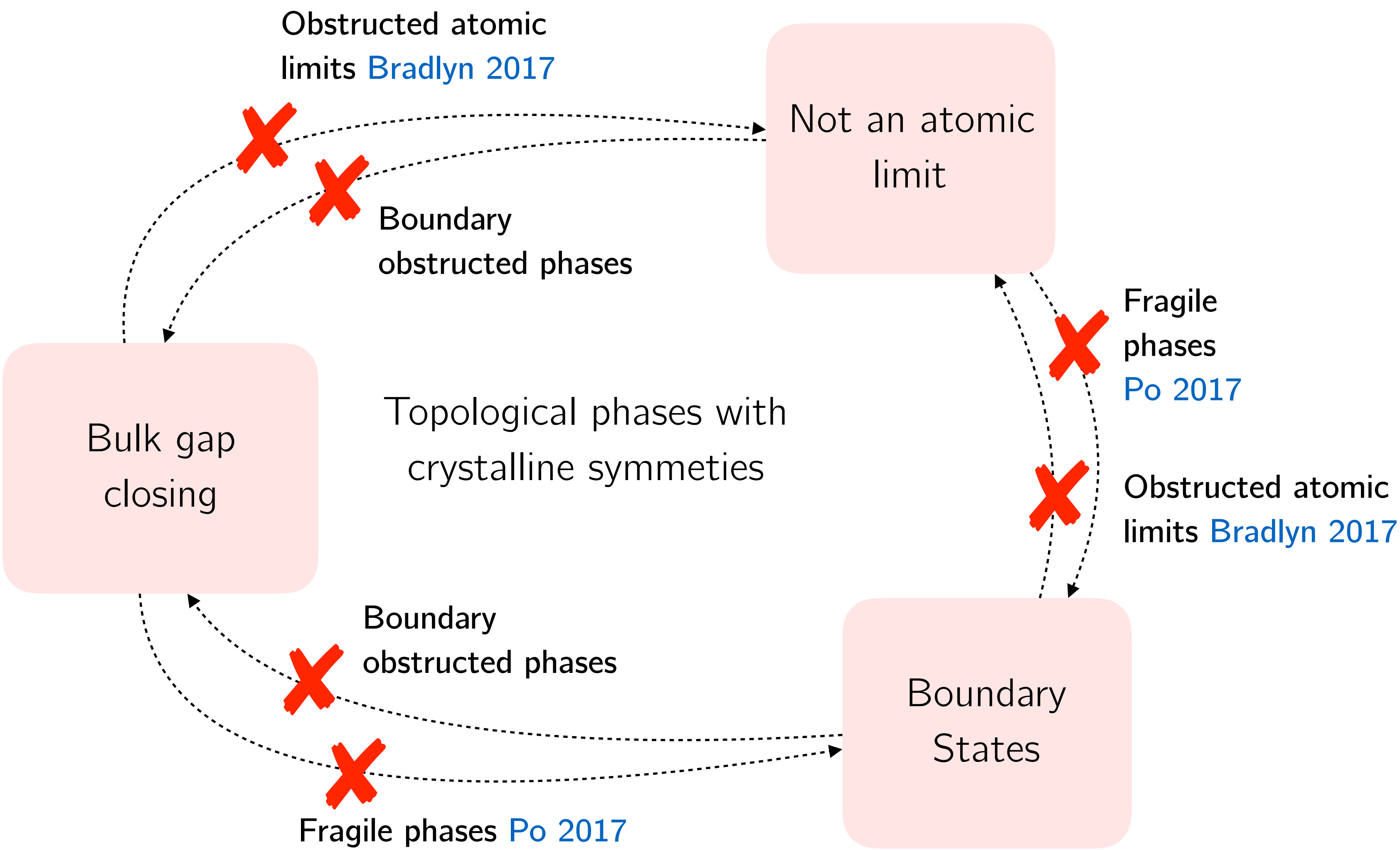
Different measures of topology



Different measures of topology



Different measures of topology



Khalaf, Benalcazar, Hughes and **RQ**, 1908.00011

Violates Bulk-boundary correspondence

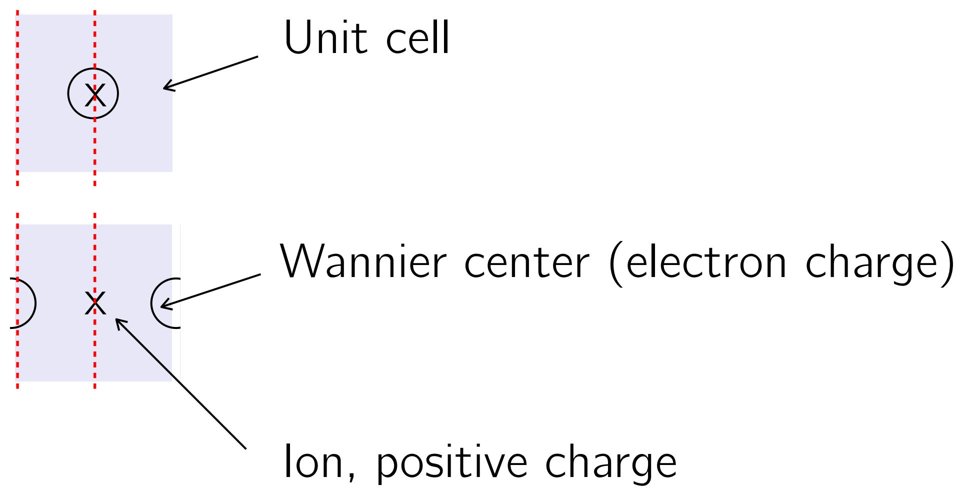
An example of a bulk obstructed atomic limit

SSH chain with inversion

Bulk obstruction: SSH chain with inversion

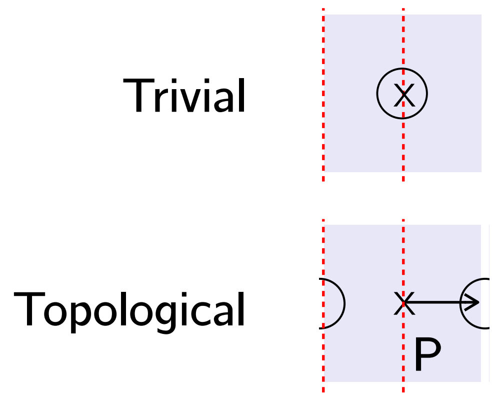
Obstructed: A transformation that can't be done while preserving the bulkgap

Simplest example: 1D chain with inversion symmetry. 1 electron per unit cell



Dashed red lines: inversion centers

Bulk obstruction: SSH chain with inversion

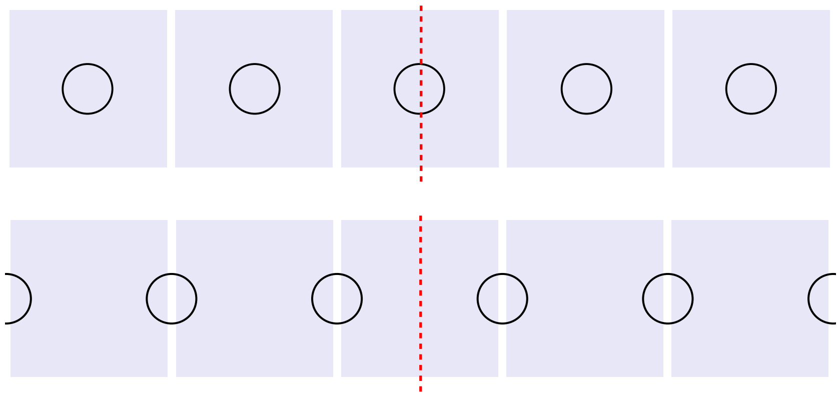


Symmetry: $I \times T^1$

Physical response:
quantized polarization
dipole moment

Topological invariant:
Distinct symmetry
representations ρ^k
protected by bulk gap closing
(Frank's talk!)

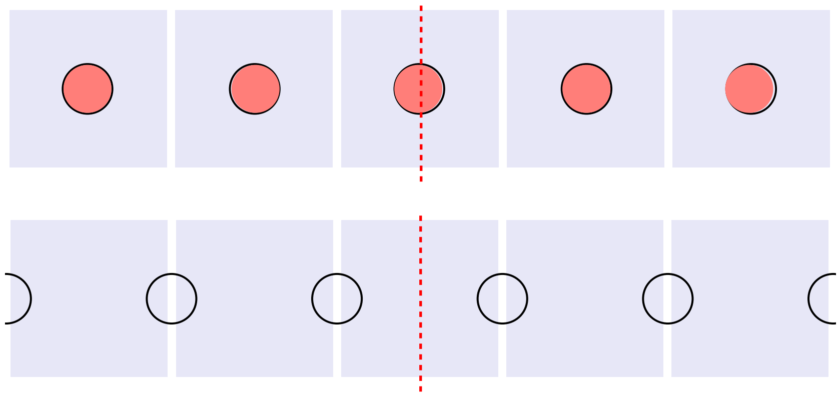
Bulk obstruction: SSH chain with inversion



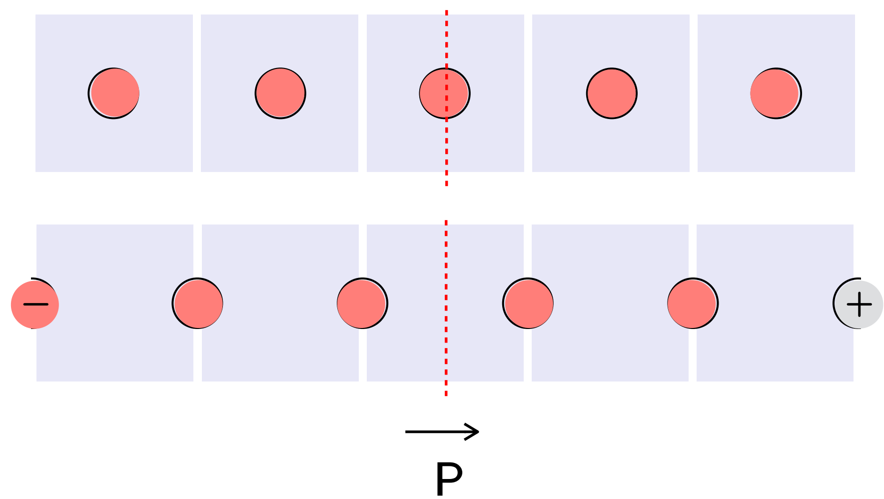
Open boundaries. Symmetry: I

Topological invariant?

Bulk obstruction: SSH chain with inversion



Bulk obstruction: SSH chain with inversion



Anomaly: can't fill the states and preserve inversion!

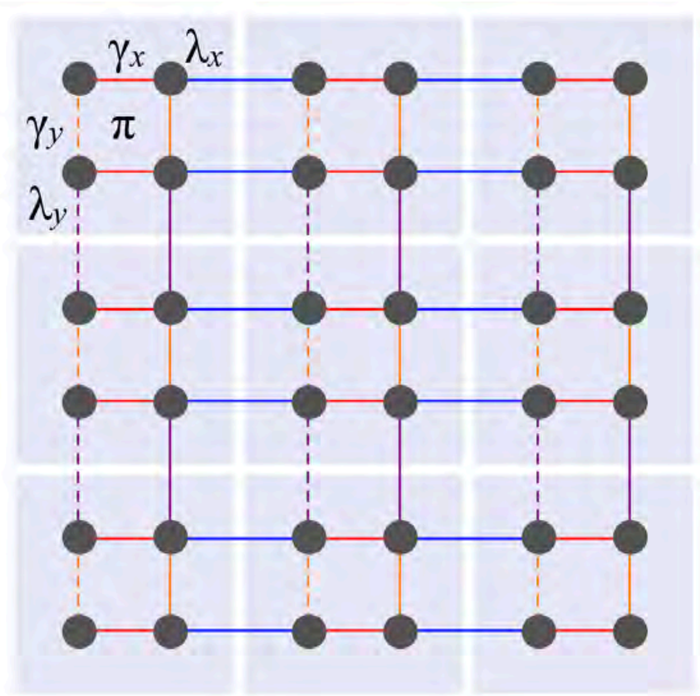
Topological invariant: is the inversion center filled?

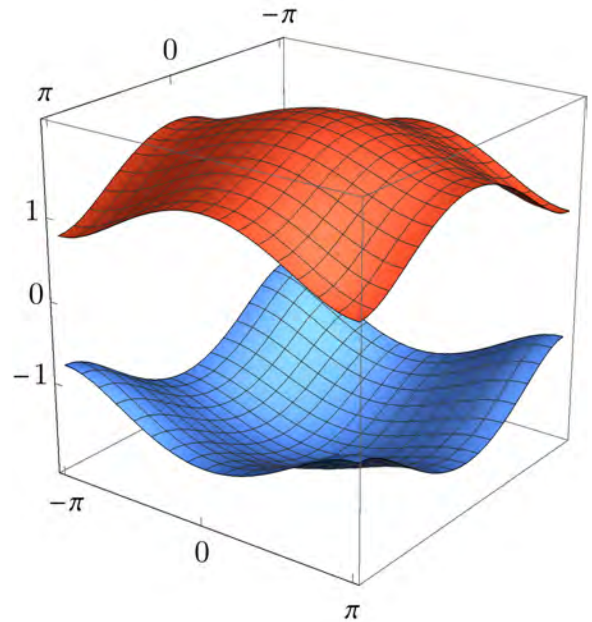
Correspondence: **bulk topology - boundary anomaly - physical response**

An example of a boundary obstructed atomic limit
Quadrupole insulator

Boundary obstruction: Quadrupole insulator

2D Quadrupole insulator (Double mirror) Benalcazar, Bernevig and Hughes, Science 2017



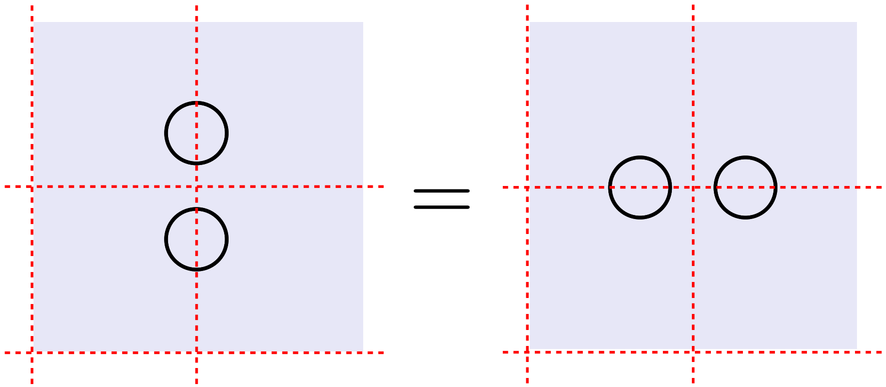


Corner modes and quantized quadrupole moment

no bulk gap closing transition? How can it be?

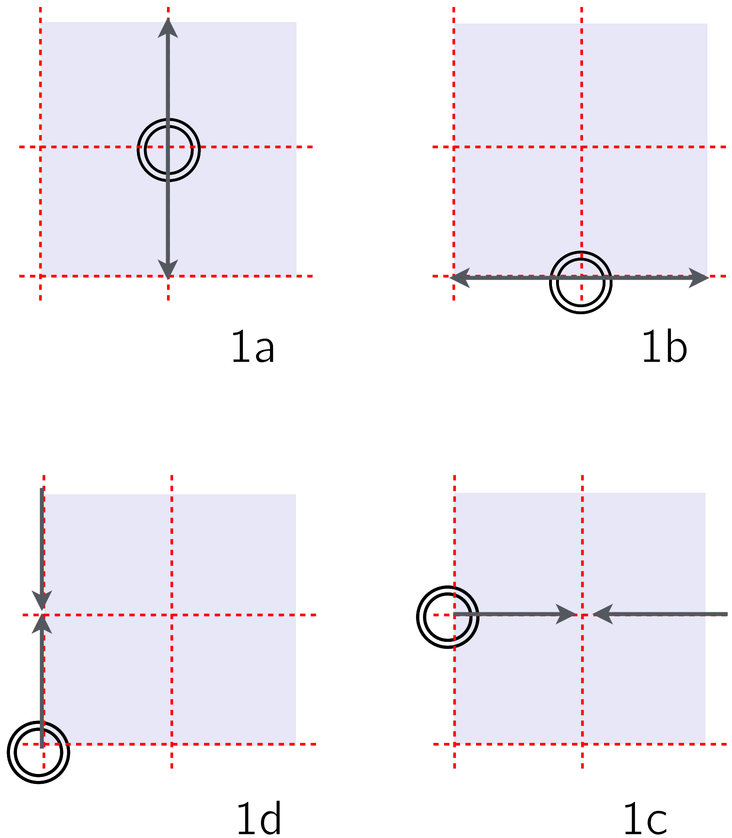
2D "Quadrupole insulator" Benalcazar, Bernevig and Hughes, Science 2017

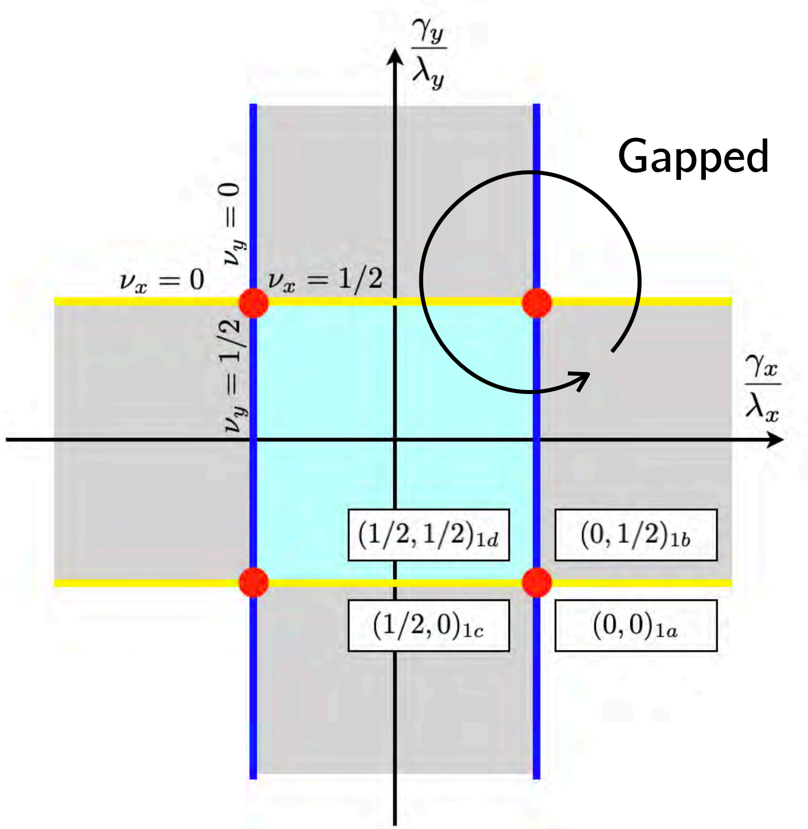
Two electrons per unit cell with M_x and M_y that form a **2D representation**



Charge centers have free movement along the mirror invariant lines

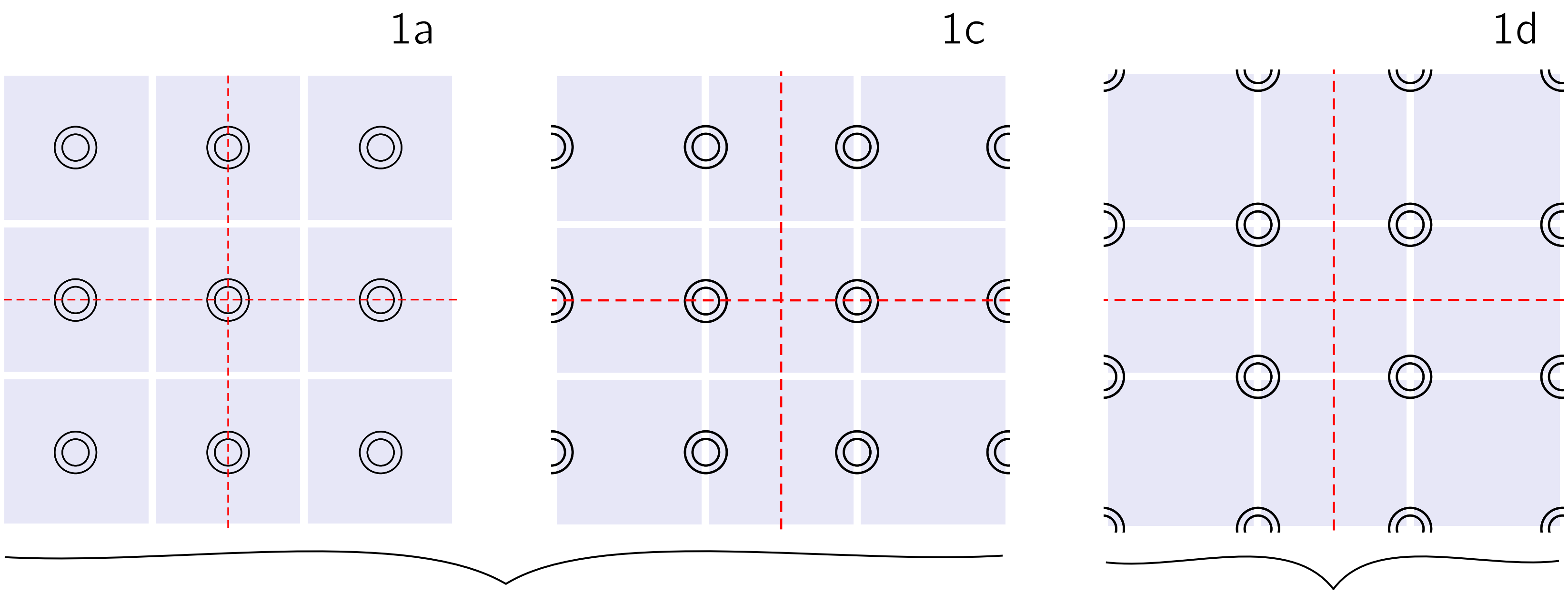
Boundary obstruction: Quadrupole insulator





Different regions characterized by ***nested polarization***

Boundary obstruction: Quadrupole insulator



no anomaly

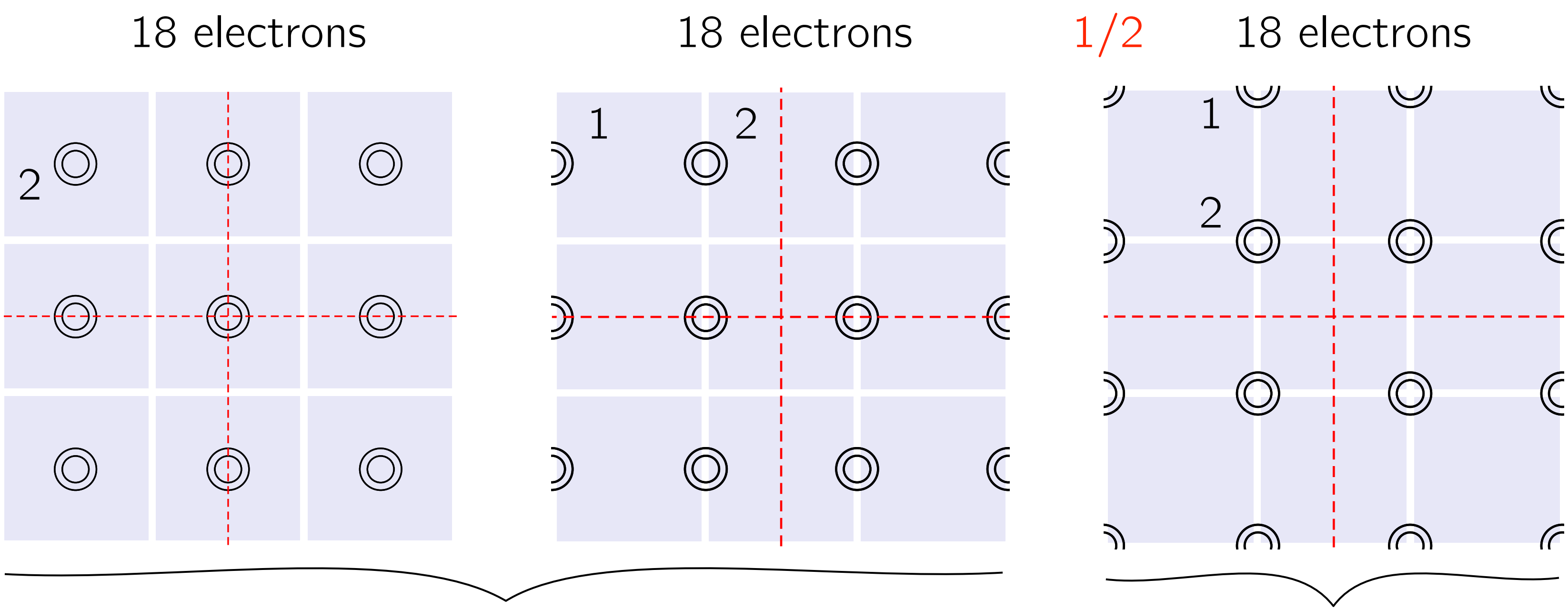
can be deformed into each other

filling anomaly:

2 states shared by **4 sites**

Corner modes

Boundary obstruction: Quadrupole insulator



no anomaly

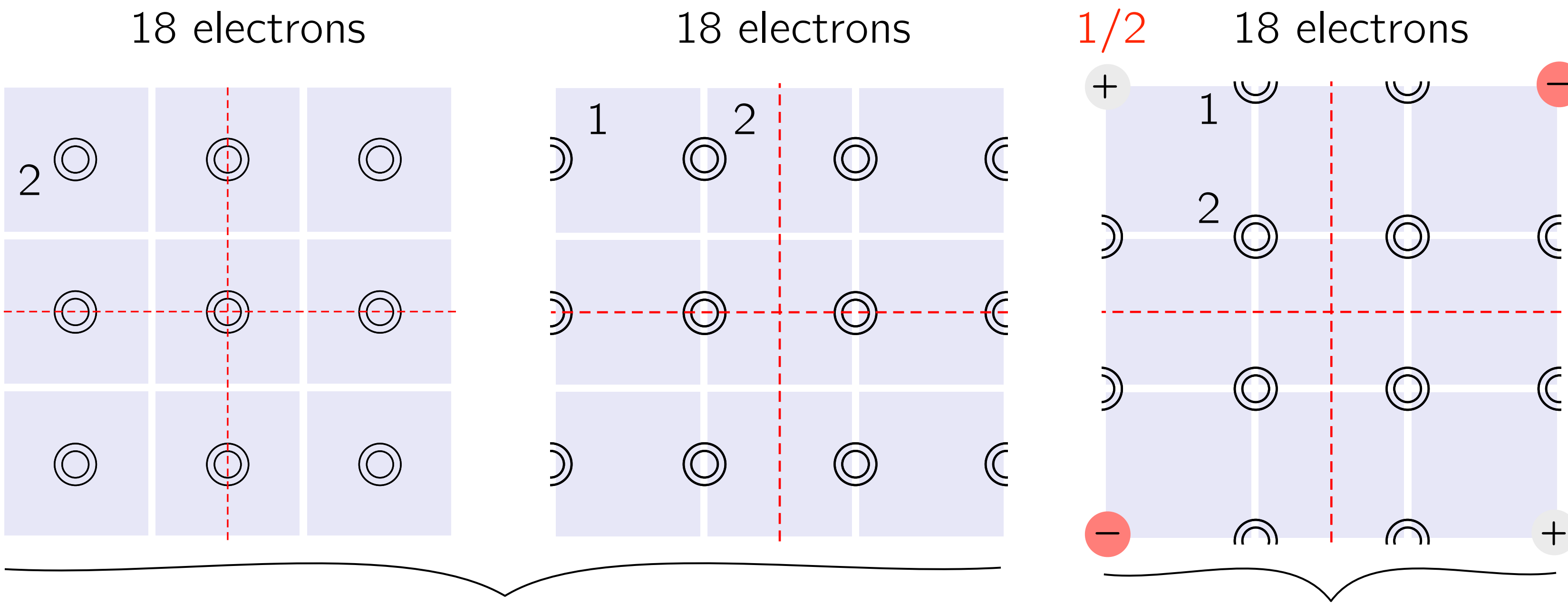
can be deformed into each other

filling anomaly:

2 states shared by **4 sites**

Corner modes

Boundary obstruction: Quadrupole insulator



no anomaly

can be deformed into each other

filling anomaly:

2 states shared by **4 sites**

Corner modes

Quantized Quadrupole

Boundary ambiguity:

Phase with corner charge depends on where the corner sits in the unit cell

Definition of BOTPs

1. Bulk phase
2. **Adiabatically equivalent** with periodic boundary conditions.
3. We can **choose a boundary** that makes them **adiabatically inequivalent**
4. **Distinguished by globally robust boundary modes**

Fails to be captured by known classification schemes with PBC:

→ *Symmetry indicators (Watanabe et.al.)*

→ *Topological Quantum Chemistry (Bradlyn et.al.)*

Captured by physical quantities defined in lower dimensions:

→ *Surface spectrum*

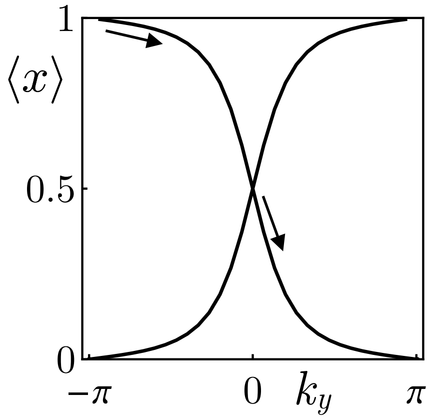
→ *Wannier spectrum*

→ *Entanglement spectrum*

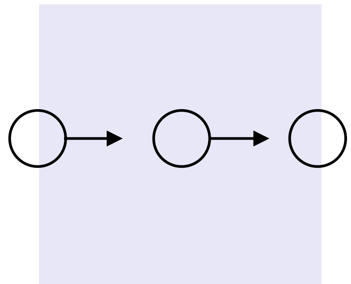
How to diagnose this topology?

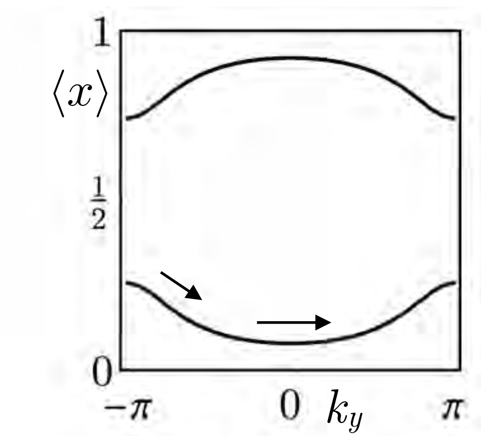
Use the Wannier spectrum

Band Representation theory of BOTPs

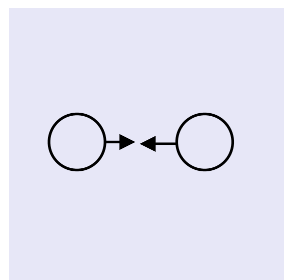


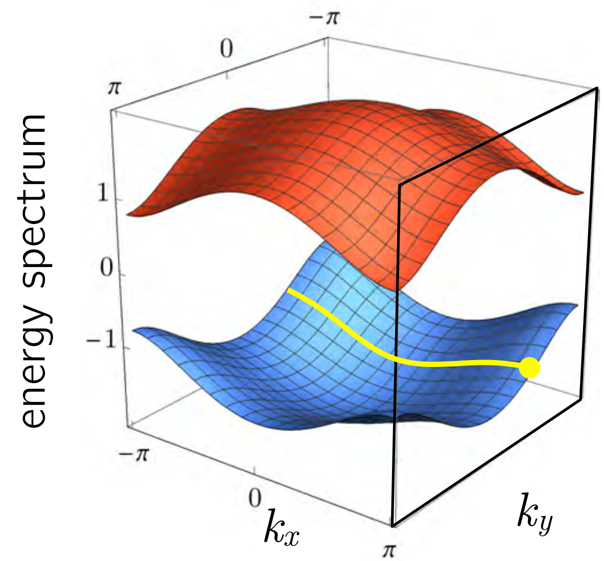
unit cell:





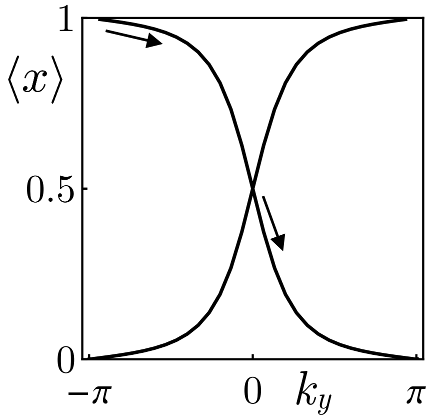
unit cell:



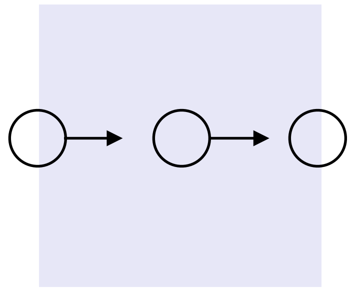


Wannier spectrum $\sim$ surface spectrum. Fidkowski 2011

Band Representation theory of BOTPs



unit cell:

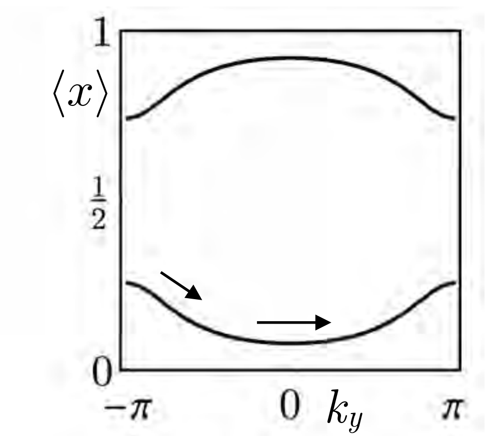


Spectral flow

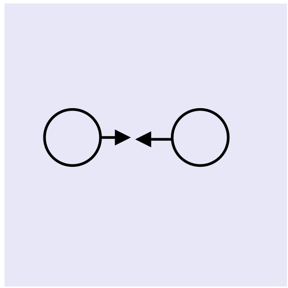
Not Wannier representable

No band representation

Absolute Topology ✓



unit cell:



No spectal flow

Wannier representable

Band representation $\rho^{\mathbf{k}}$

Relative Topology ?

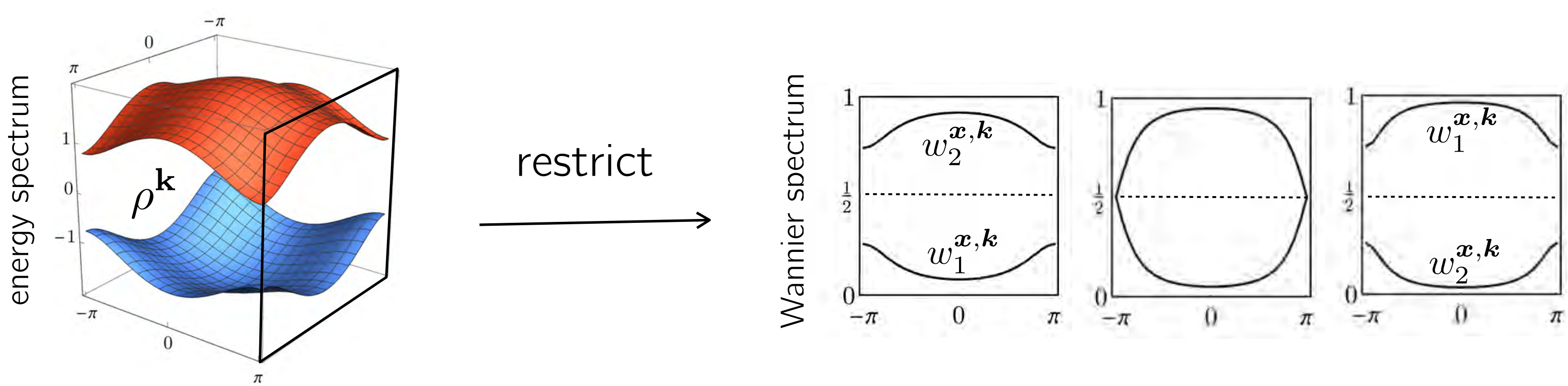
Band Representation theory of BOTPs

1) Described by a band representation $\rho^{\mathbf{k}}$, is Wannier localisable in $G \times T^d$

2) Breaking some translation symmetry along $\boldsymbol{b}$, $\rho^{\mathbf{k}}$ is decomposed into Wannier bands, described by Wannier band representations

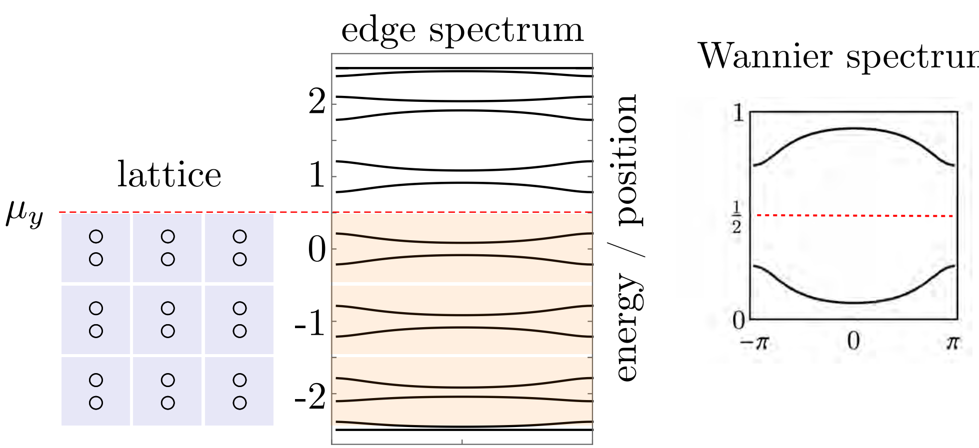
$$w^{\boldsymbol{b},\boldsymbol{k}_\perp} = \rho^{\boldsymbol{k}} \downarrow (G^{\boldsymbol{b}} \times T^{d-1}). \quad \text{with} \quad w^{\boldsymbol{b},\boldsymbol{k}} = \bigoplus_a w_a^{\boldsymbol{b},\boldsymbol{k}}$$

3) The decompositions are **topologically distinct** $\rightarrow$ separated by gap closing



Band Representation theory of BOTPs

4) Lattice termination defines a **Wannier Chemical potential μ_b**



Close the edge gap

Mix with bulk DOFs

Wannier spectrum $\sim$ surface spectrum. Fidkowski 2011

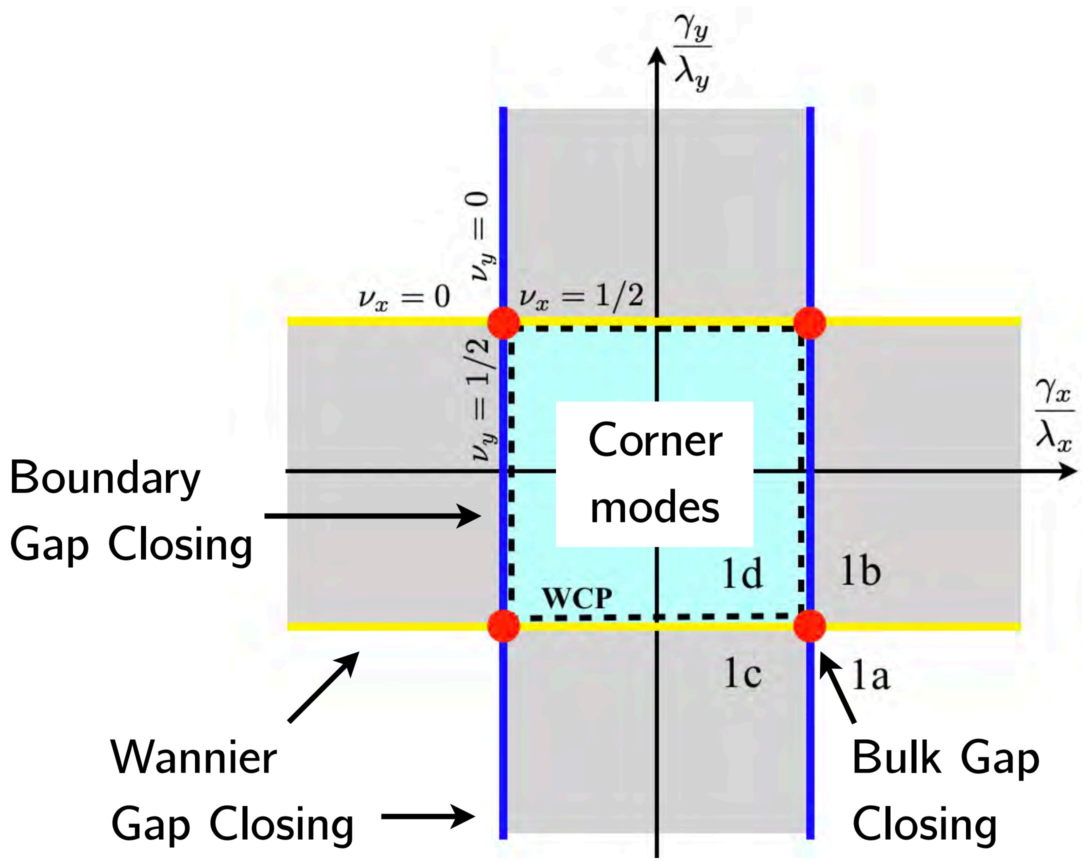
∂ Gap Closing $\Rightarrow$ Wannier Gap Closing

Wannier Gap Closing $\not\Rightarrow$ ∂ Gap Closing

Boundary ambiguity

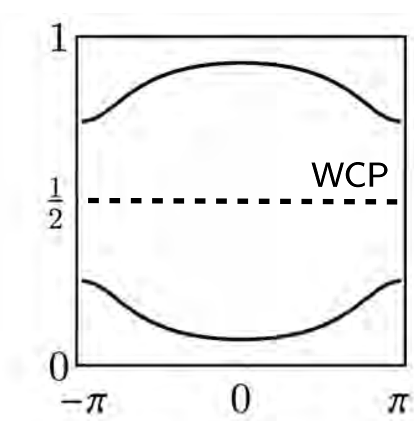
Band Representation theory of BOTPs

5) Boundary introduces a topological distiction between regions characterized by distinct boundary modes: **Wannier chemical potential**



Real invariant accounts for ∂. Nested polarization not enough:

$Z_2 \times Z_2 \to Z_2$



Types of boundary modes

Boundary States

Globally robust
Protected by bulk **symmetry**
TIs, Higher-order or Crystalline TIs

Locally robust
Not protected **by symmetry**
Trivial insulators (extrinsic HOTIs)

New!

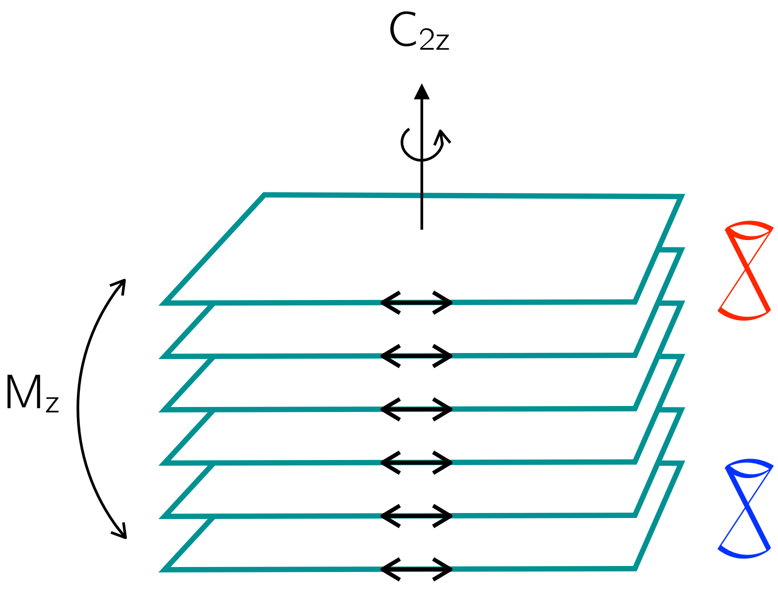
Globally robust
Protected by boundary **symmetry**
Boundary obstructed TIs

Khalaf, Benalcazar, Hughes and **RQ**, 1908.00011

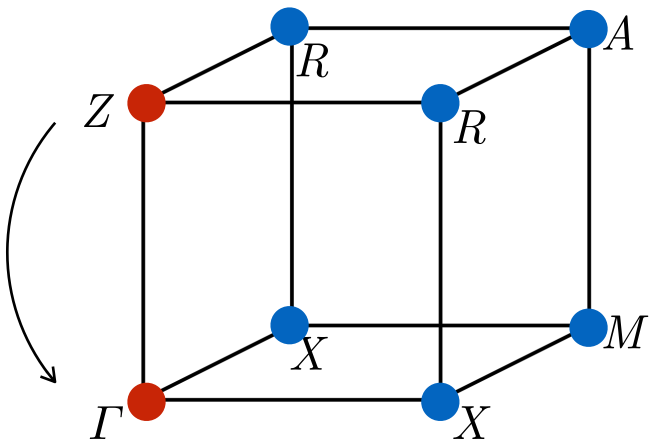
An example with helical modes

Dimerized weak topological insulator

Weak topological insulator



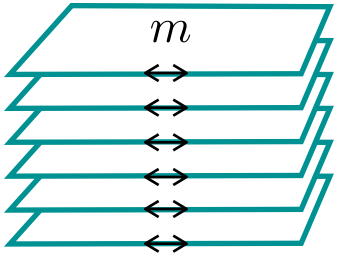
Stack of 2D TI

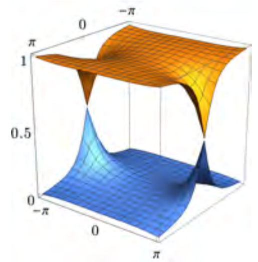


Protected by translation

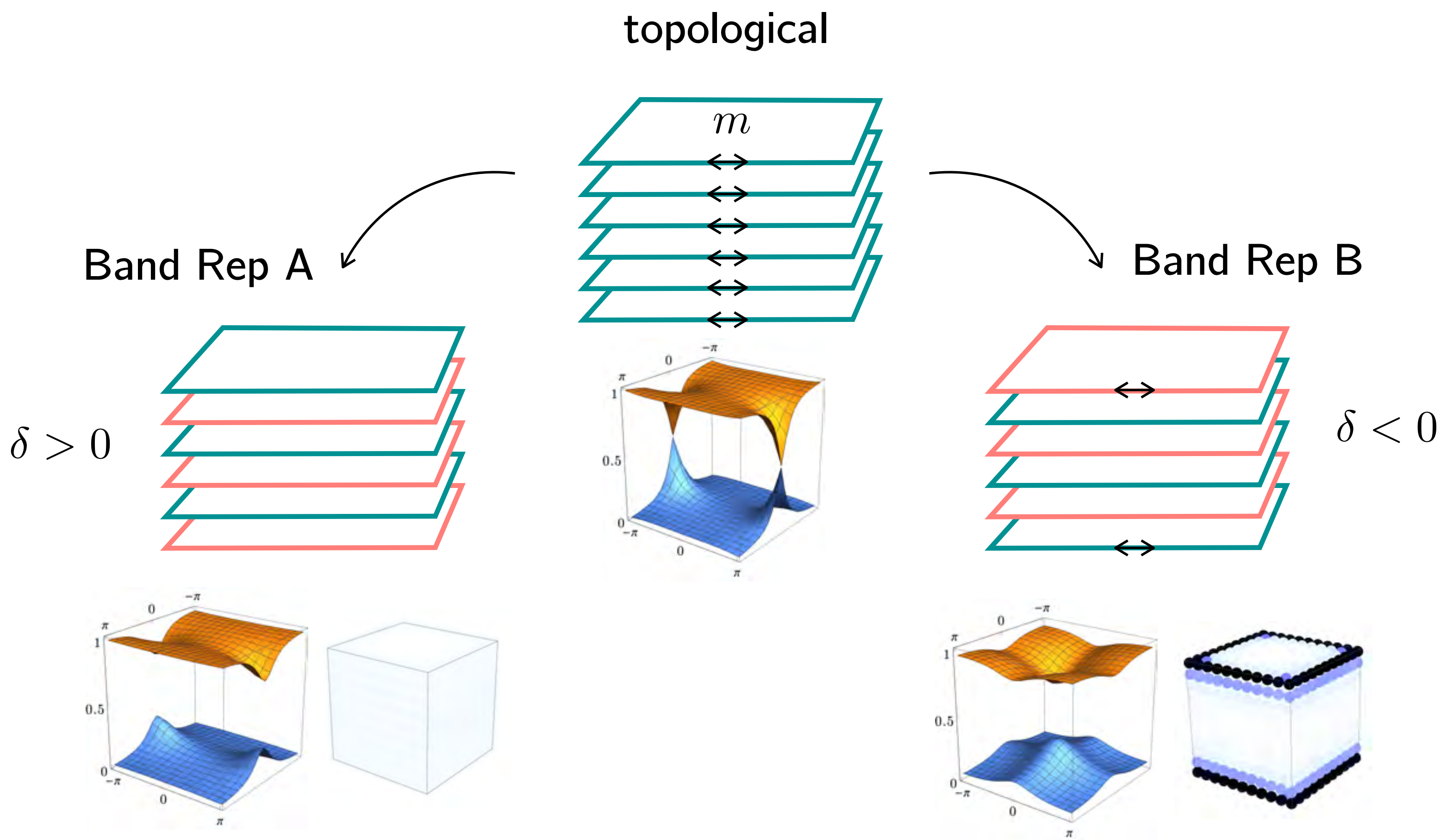
Dimerized weak topological insulator

topological



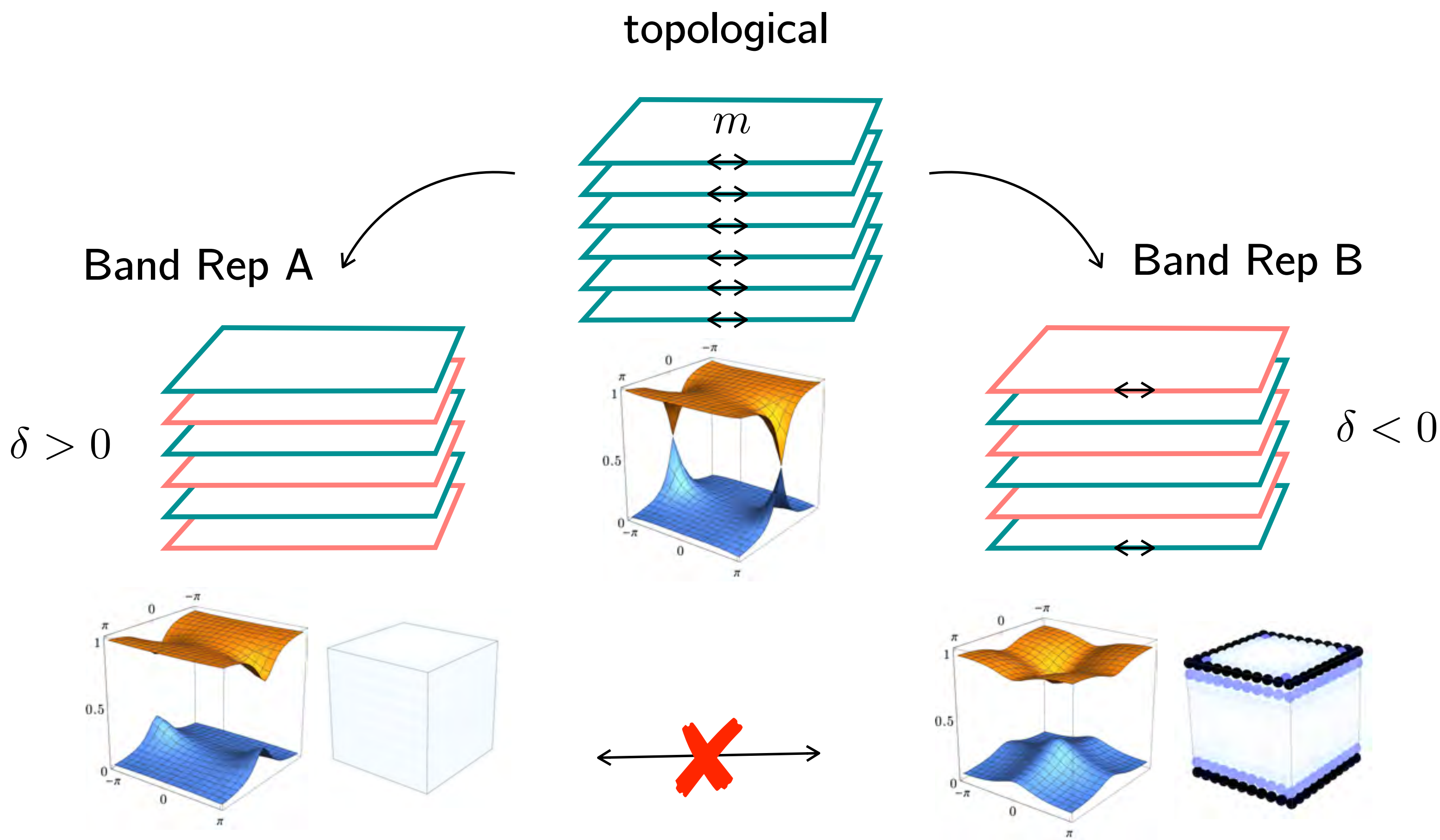


Dimerized weak topological insulator



Bulk remains gapped! Band Rep A = Band Rep B $\quad \Delta \sim \sqrt{m^2 + \delta^2}$

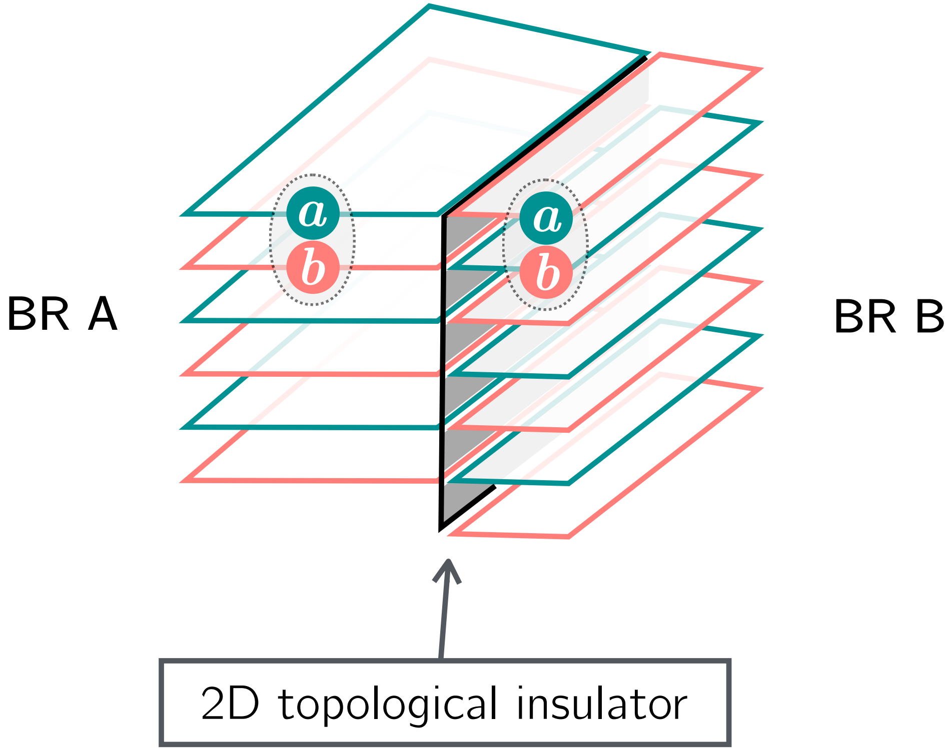
Dimerized weak topological insulator



Boundary gap closing at C_{2z} or M_z invariant points

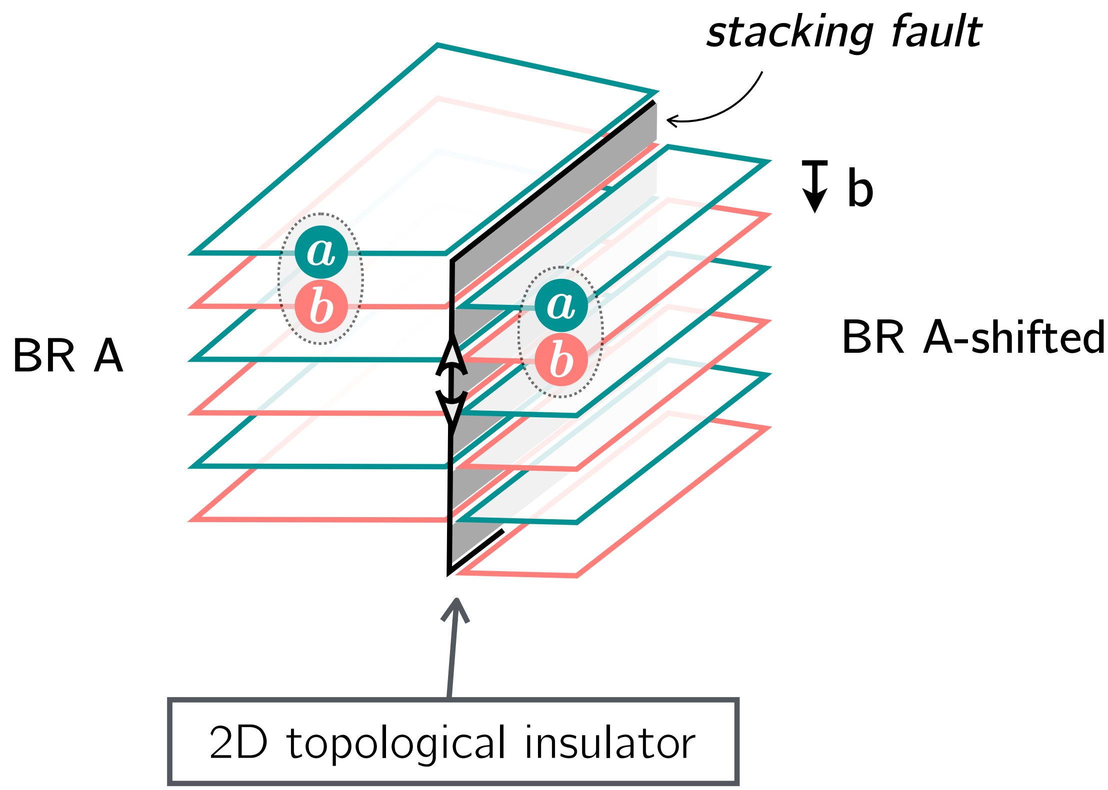
distinct with open boundaries

Topology is well defined once a boundary is introduced

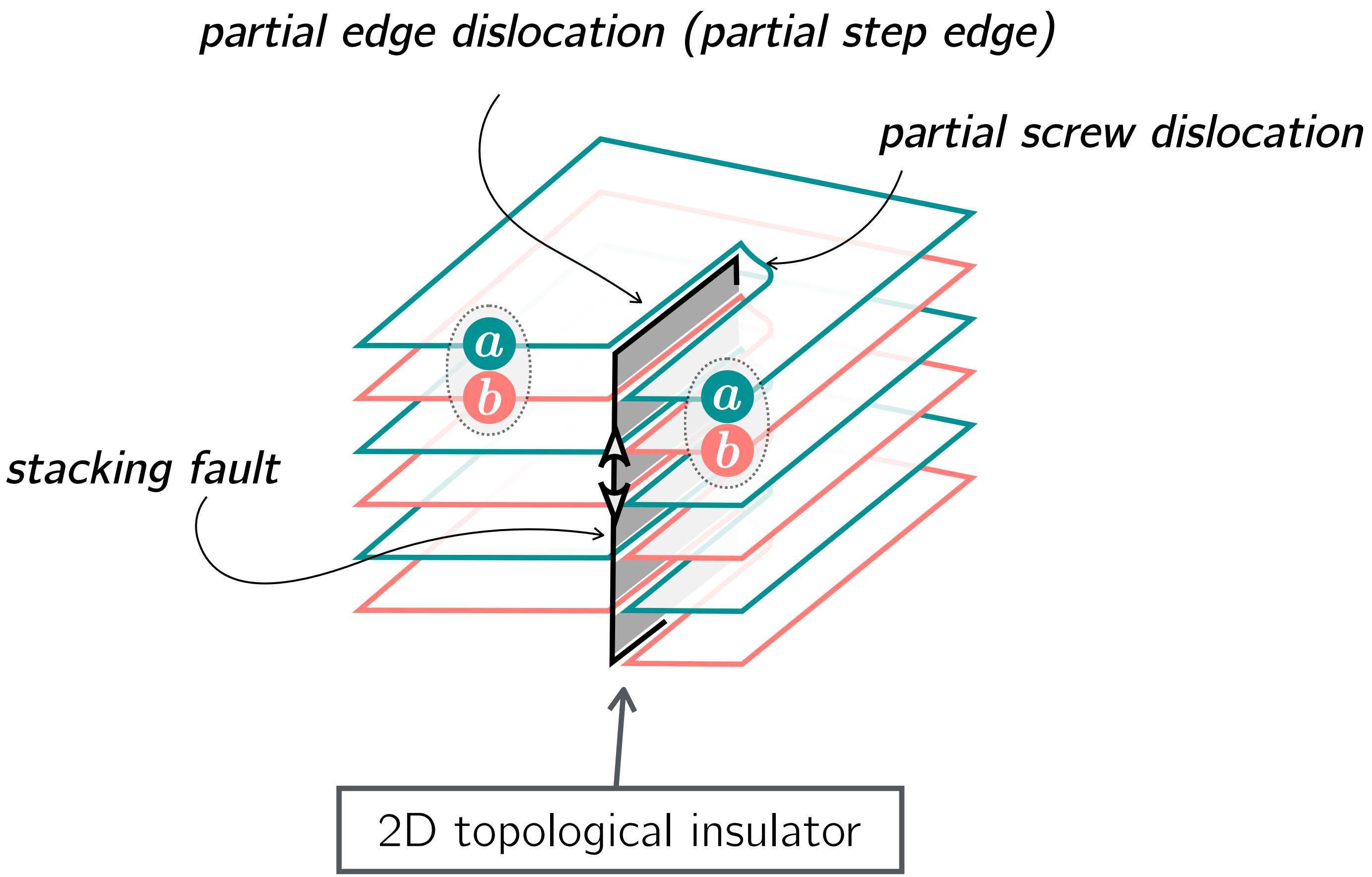


Partial lattice defects

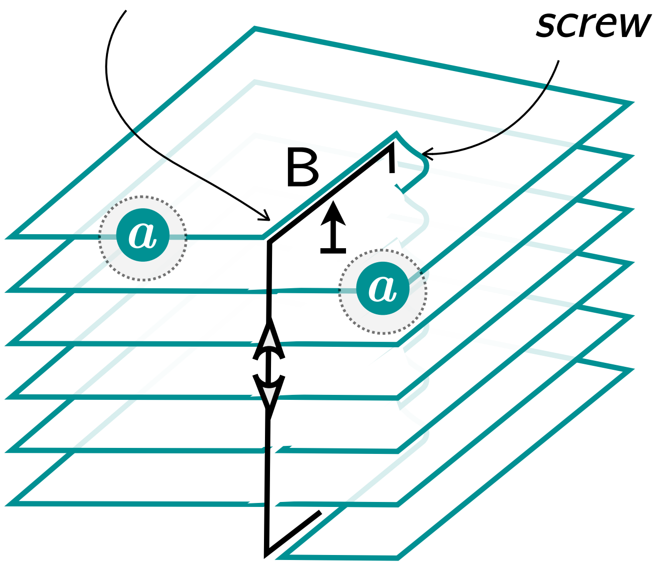
translation defect in a boundary obstructed phase



Partial lattice defects



RQ, Fulga, Avraham, Beidenkopf, Cano, 1809.03518



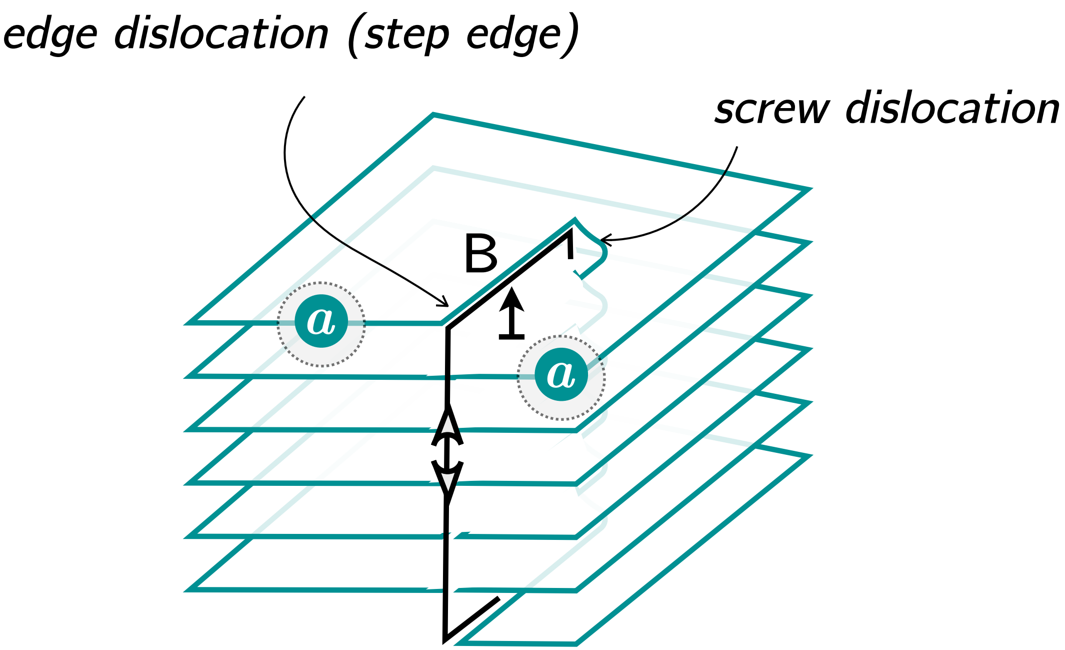
Ran and Vishwanath criteria:

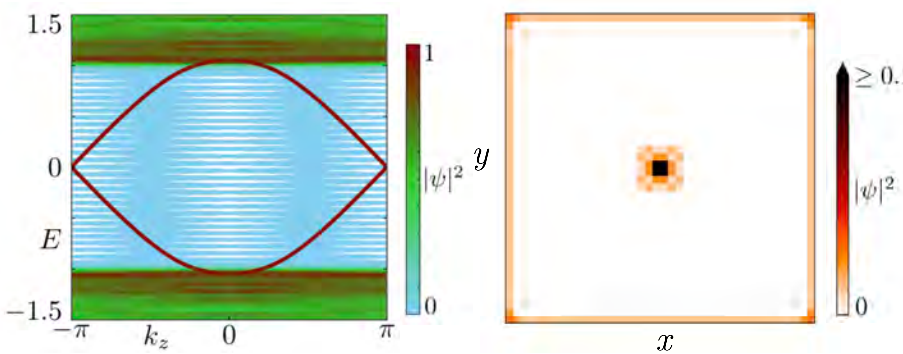
WTI implies states at dislocations if $\mathsf{B} \cdot \mathsf{M}_\nu = \pi$ **with**

$$\mathsf{M}_\nu = \frac{1}{2}(\nu_1, \nu_2, \nu_3) \cdot \mathsf{G}$$

RQ, Fulga, Avraham, Beidenkopf, Cano, 1809.03518

Partial lattice defects

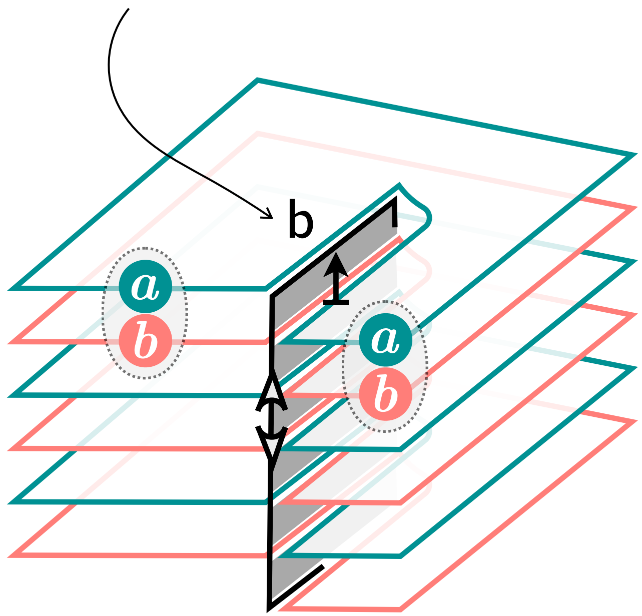




RQ, Fulga, Avraham, Beidenkopf, Cano, 1809.03518

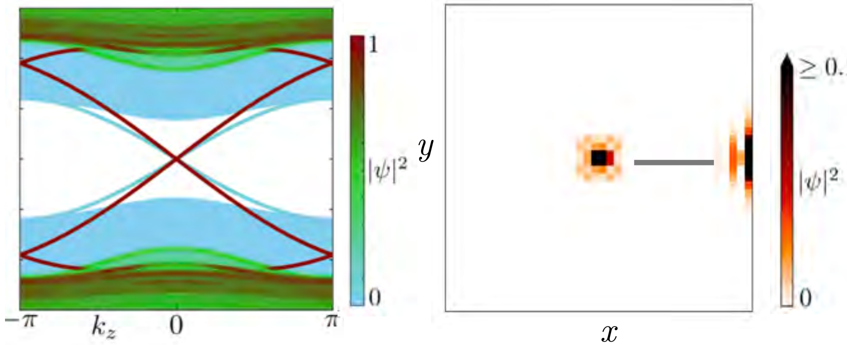
Partial lattice defects

fractional Burgers vector



TI at stacking fault is the signature that the two dimerizations *are topologically distinct*

Robust bulk signature!



Partial dislocations and stacking faults

- → Stacking faults can host robust topological insulators
- → Common in crystallography
- → Can pick up on boundary obstructions in seemingly trivial systems
- → Relevant for other higher order topological insulators
- → Just the beginning

Hoffmann 2018

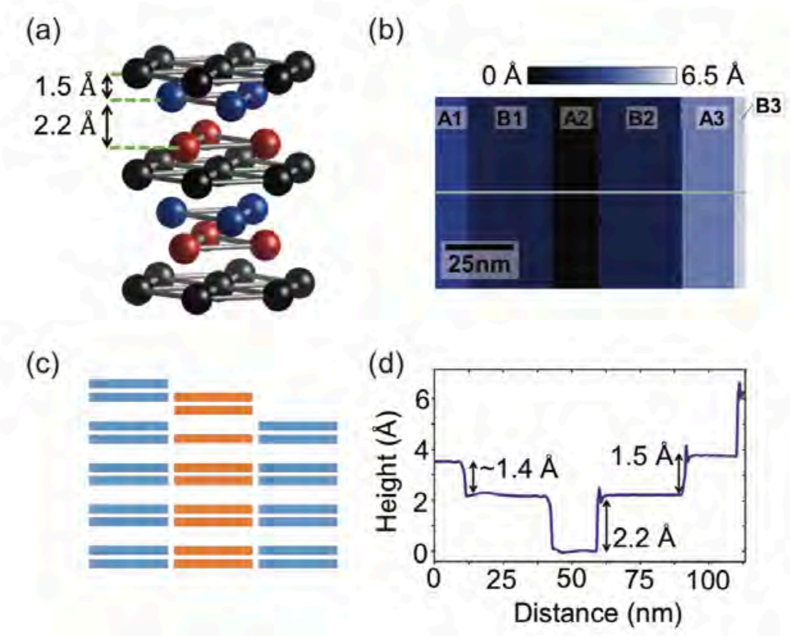


FIG. 1. **Terrace Structure of Antimony.** **(a)** Bilayer crys-

RQ, Fulga, Avraham, Beidenkopf, Cano, 1809.03518

Overview

→ Introduced boundary obstructed topological phases: Topology of bulk revealed given a certain boundary

→ Violates bulk-boundary correspondence, generalize adiabatic theorem

→ Introduced Wannier band representations and Wannier chemical potential

→ Seen that the BBH model and the dimerized WTI are BOTPs

→ Introduced topological stacking faults and partial dislocations

Thank you!