

# Crystal Hall Effects in Collinear Antiferromagnets



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7<sup>th</sup> of October 2019, Schloss Waldthausen  
3rd Antiferromagnetic Spintronics Workshop







**Jairo Sinova**



**Tomáš Jungwirth**

*Abundant collinear antiferromagnets may exhibit a new type of Hall effect:*  
***Crystal Hall effect.***

# Crystal Hall Effects in Collinear Antiferromagnets

## 1. Topological antiferromagnetic spintronics and spontaneous Hall effects intro

*LS, TJ, JS, Rapid.Res.Lett. (2017)*

*LS, Zelezny, JS, TJ, Phys.Rev.Lett. (2017)*

*LS, Y.Mokrousov, B.Yan, A.H.MacDonald, Nat.Phys.(2018)*

*LS & T.Jungwirth, Topology in magnetism, Springer (2018)*

## 2. Spontaneous Hall effect in collinear antiferromagnets?

*LS, R.H.Gonzales, JS, TJ, arXiv 1901.00445 (2019)*

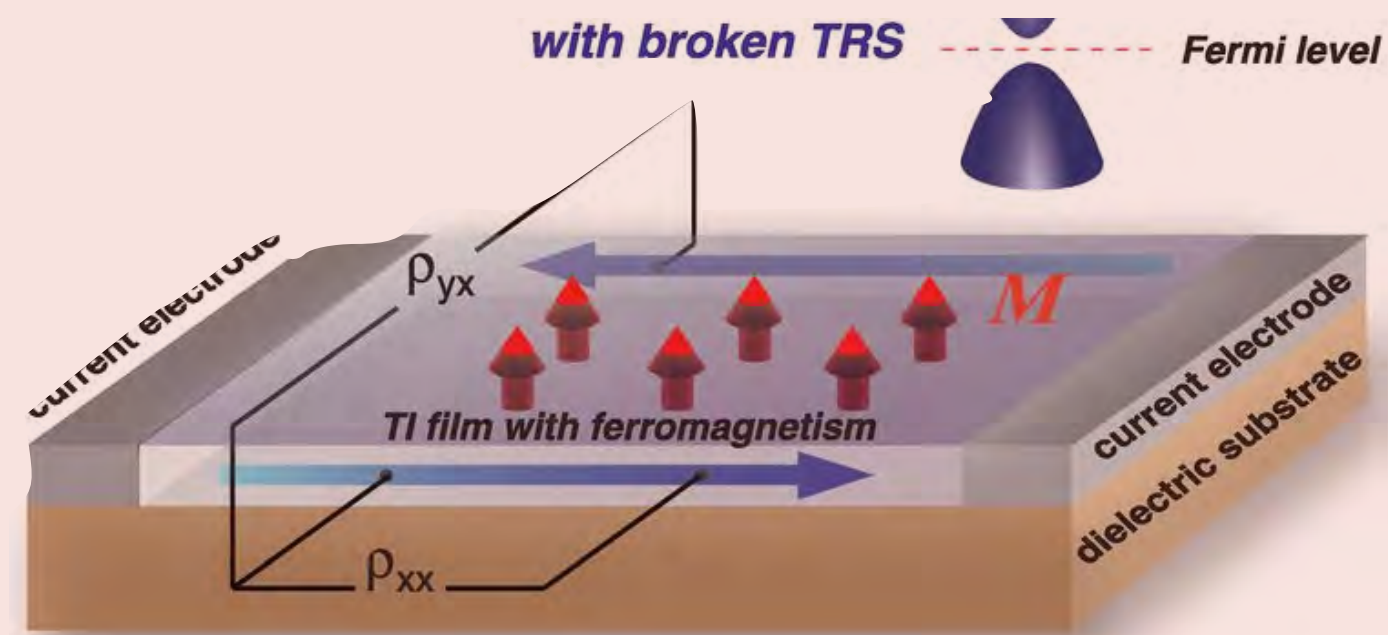
## 3. Classification, material candidates and applications of crystal Hall effect

*Gonzalez-Hernandez, Zelezny, LS et al. (2019)*





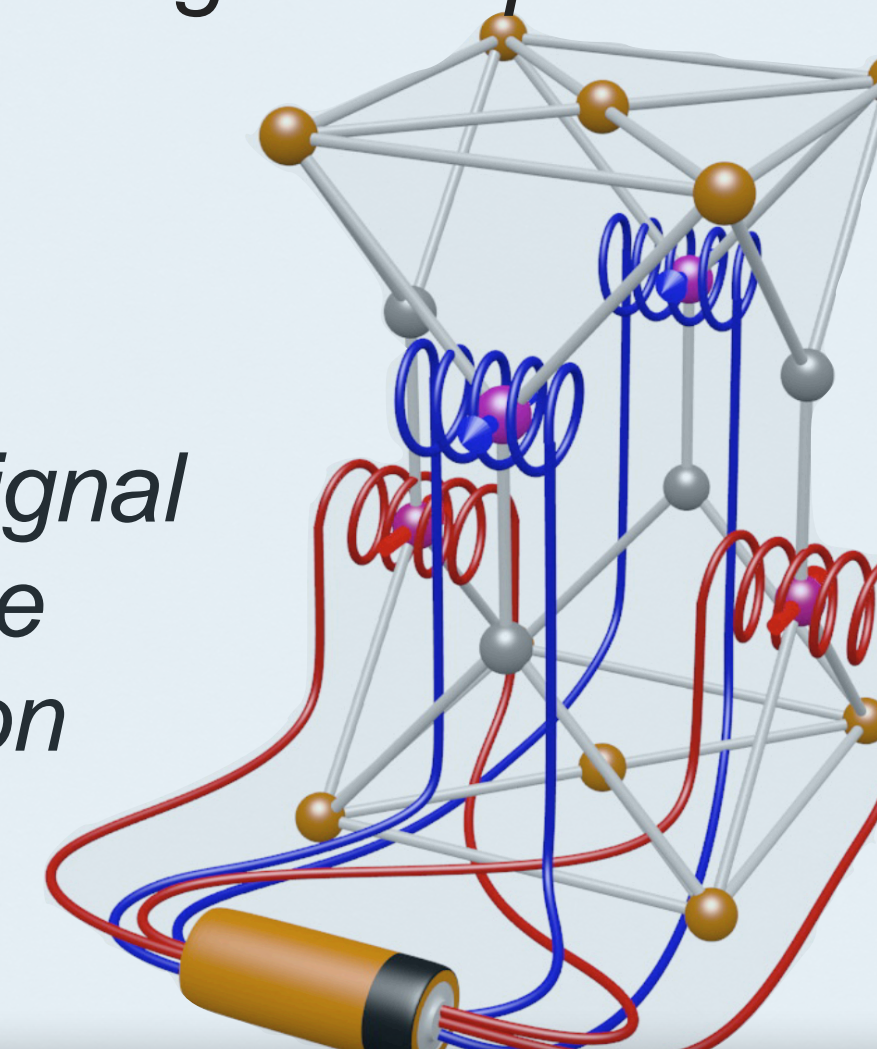
# Magnetically doped topological insulators



Fragile magnetism and critical temperatures

Unanticipated effects  
Large signal/noise  
Low dissipation

# Antiferromagnetic spintronics



Limited signal magnitude  
Dissipation

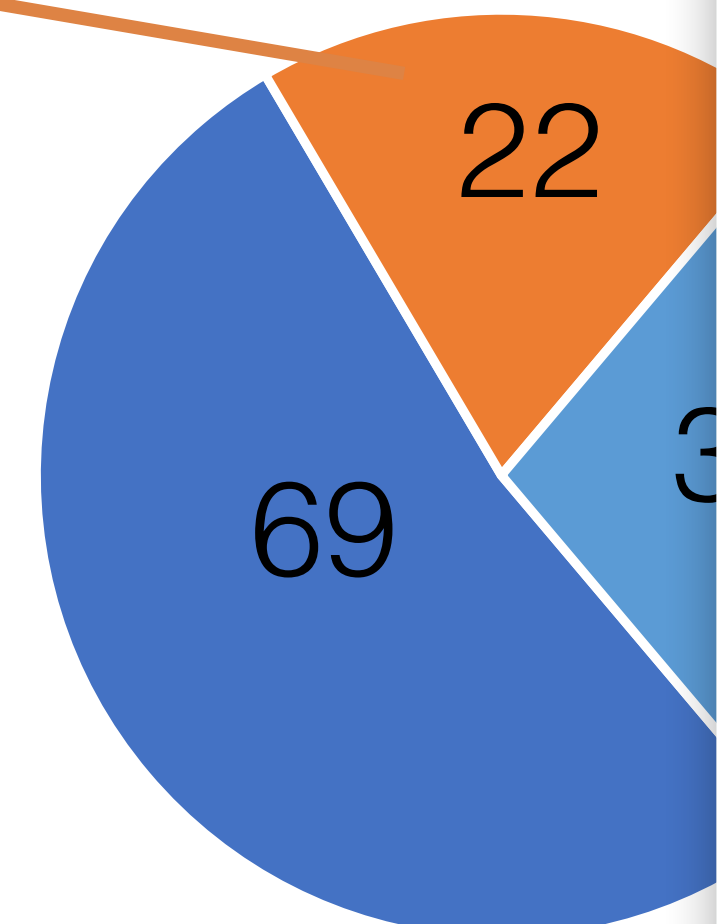
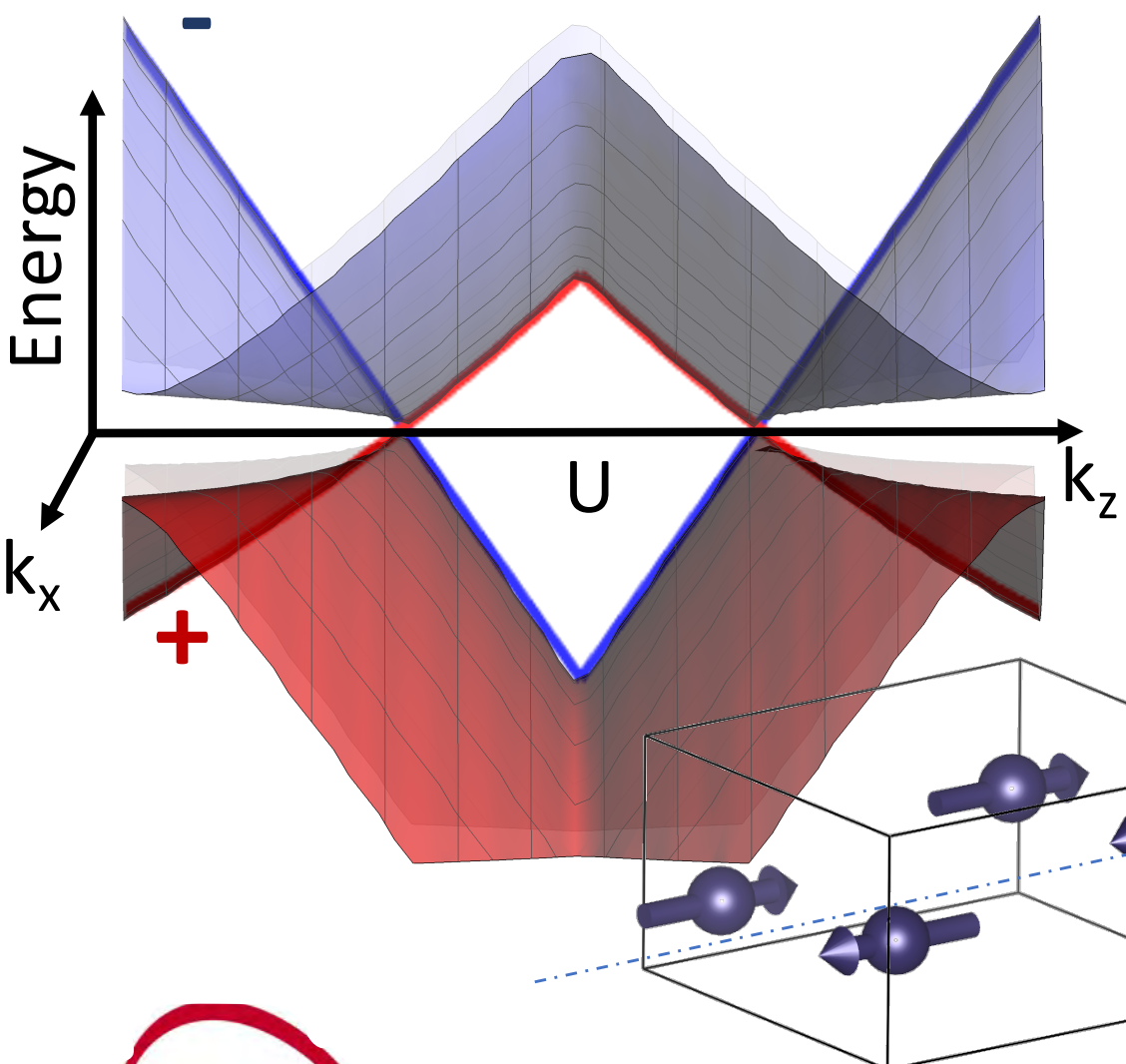
## Topological antiferromagnetic spintronics: n

### Dirac antiferromagnets

LŠ, Zelezny, JS, TJ,  
*Phys.Rev.Lett.* (2017)

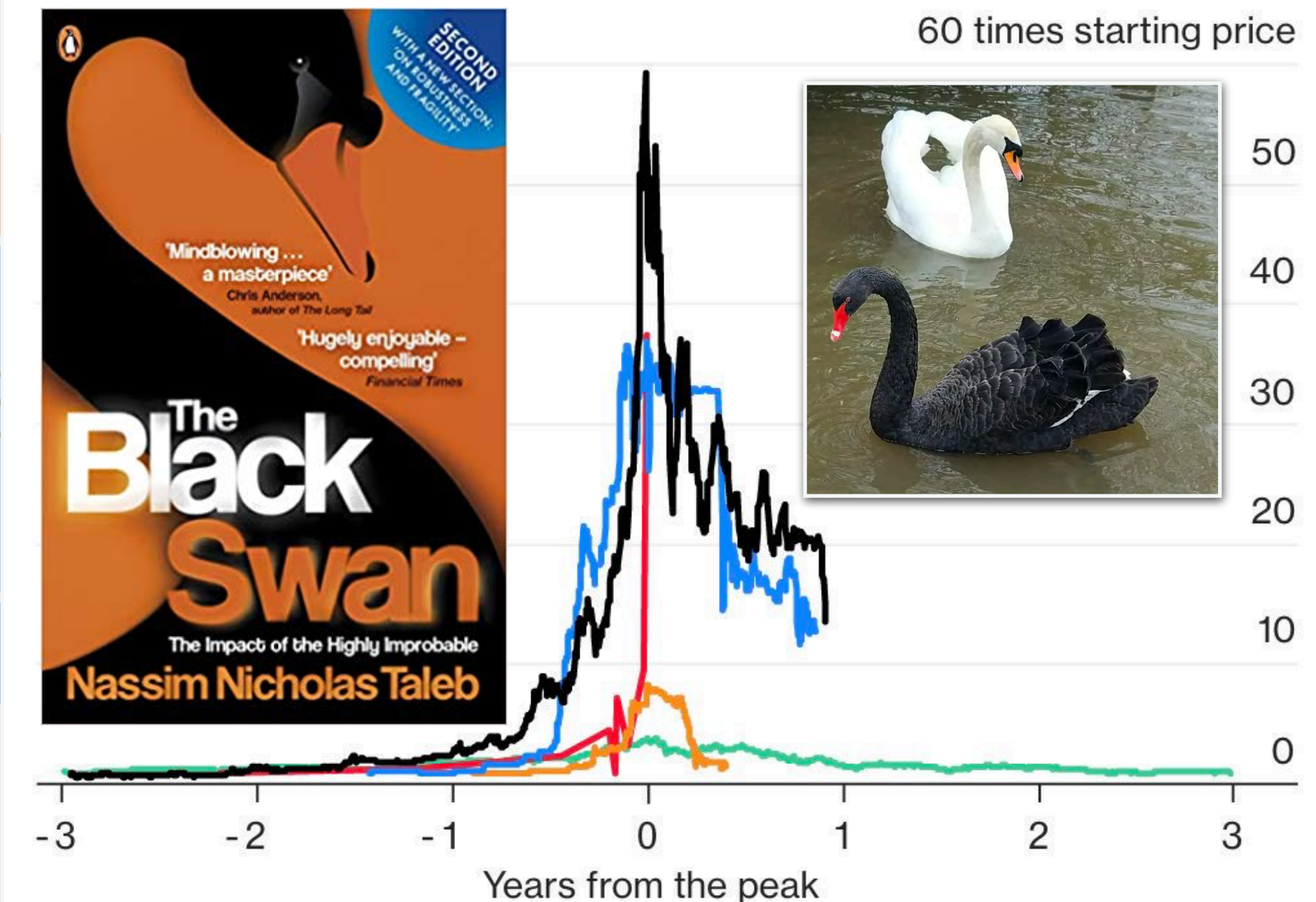
### Dynamical axions

D. J. E. Marsh, LŠ, and M. N. Al  
*Phys. Rev. Lett.* (2019)



LŠ & Tomáš Jungwirth, *Topology in Magnetism*, Springer (2018),  
LŠ, Y.Mokrousov, B.Yan, A.H.MacDonald, *Nat.Phys.*(2018)

Bitcoin Mississippi Tulips South Sea Nasdaq Composite



Sources: Bloomberg, International Center for Finance at Yale School of Management, Peter Garber



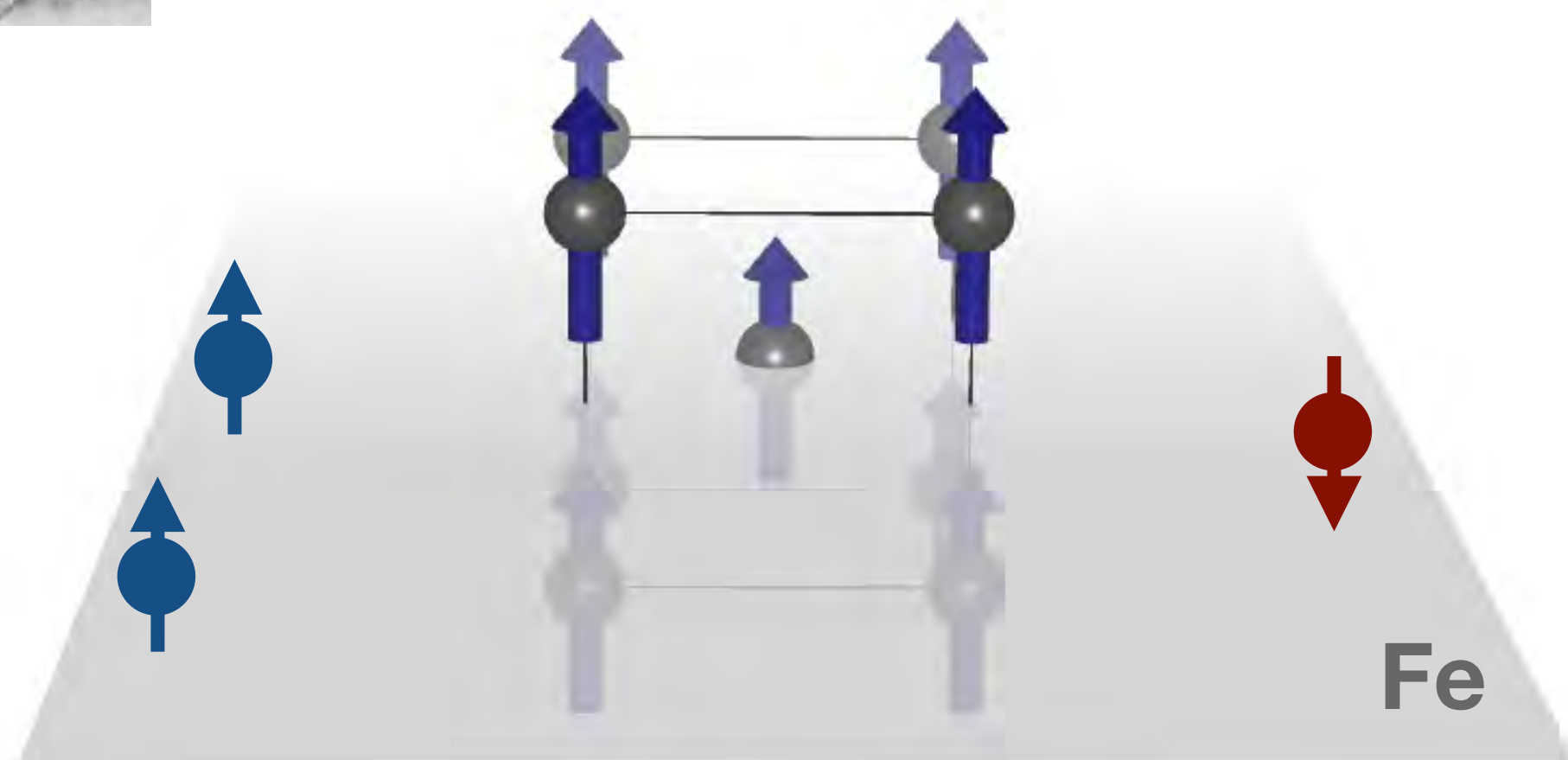
# Spontaneous Hall effect



$$\rho_{ab}(\mathbf{m}) = (E_a/j_{H,b} - E_b/j_{H,a})/2$$

Hall vector  $\sigma \sim \mathbf{E} \times \mathbf{j}_H$

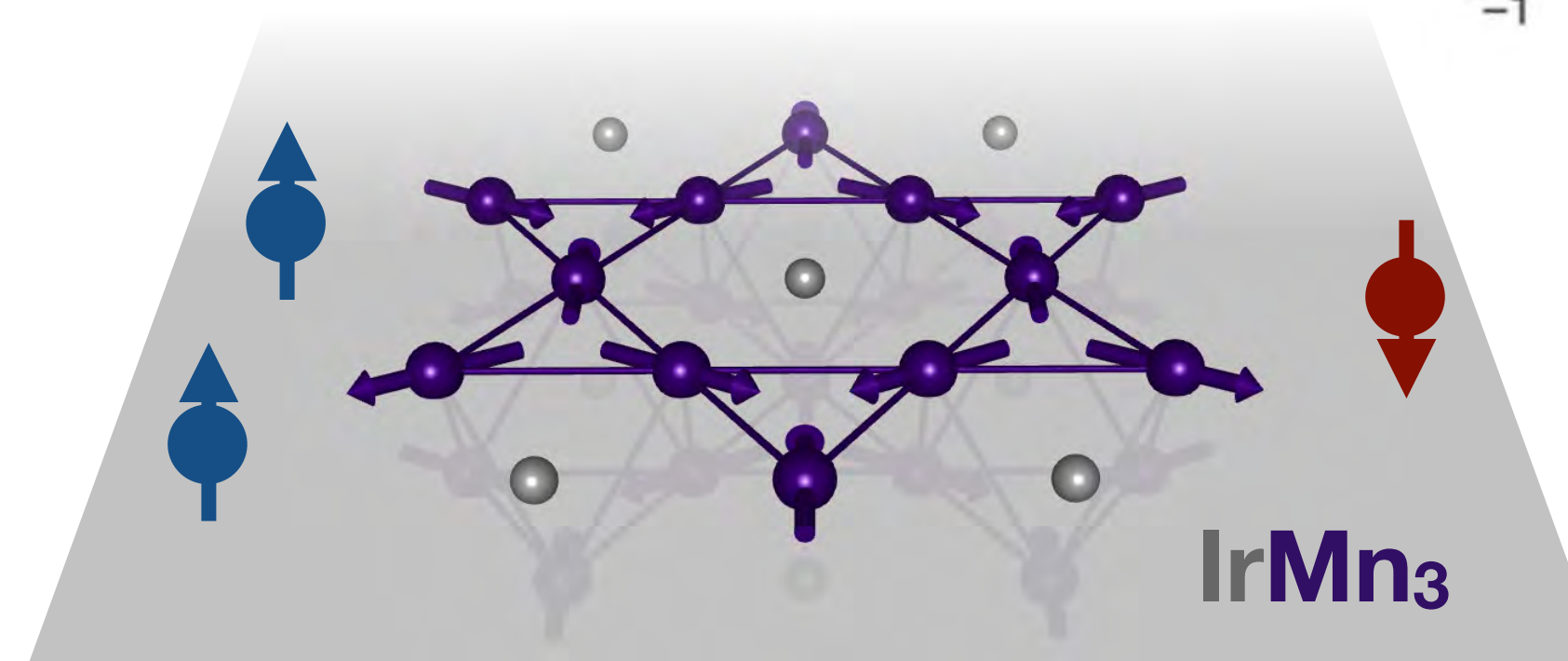
magnetisation



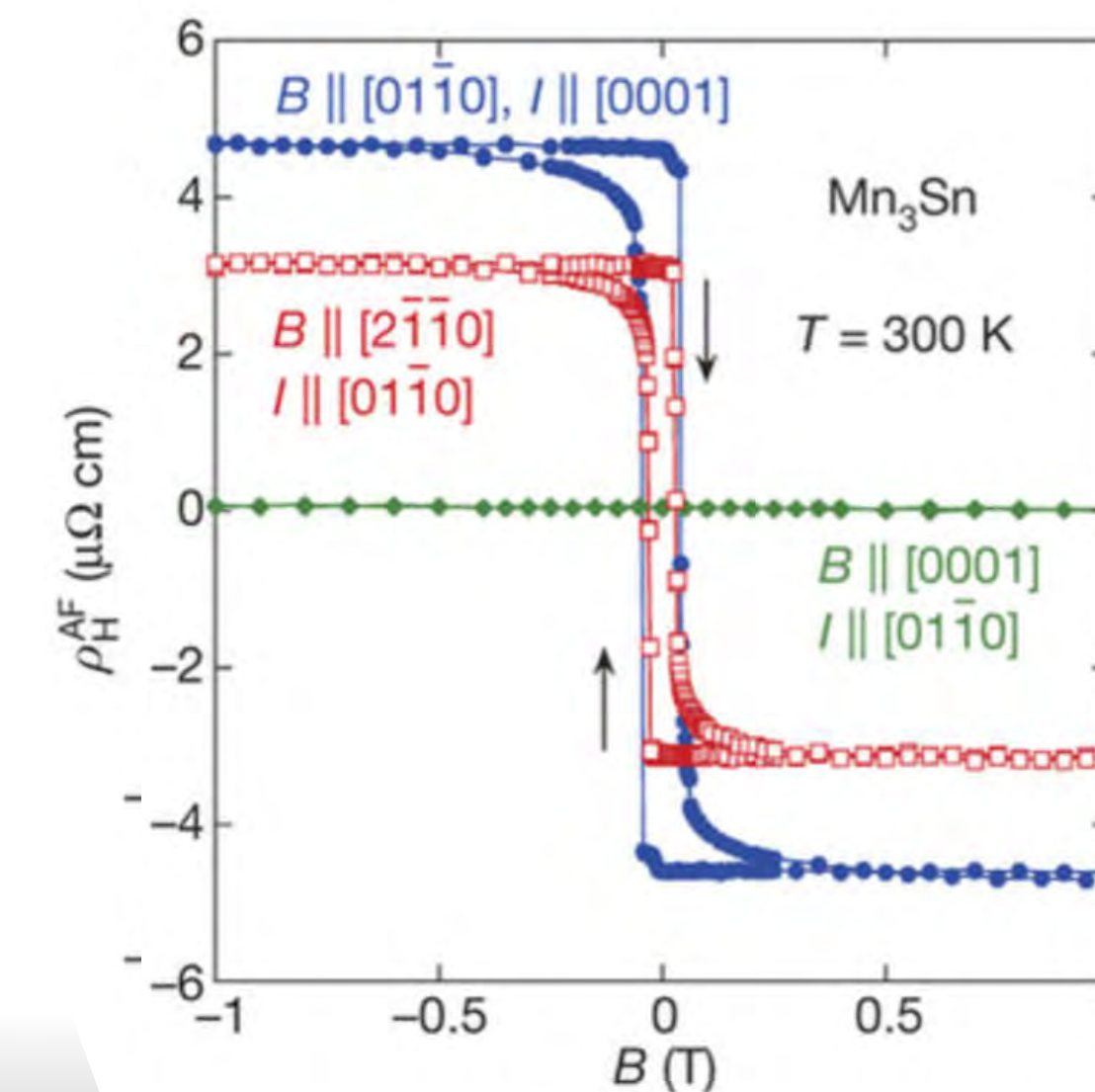
$$\rho_H = R_0 H + RM$$

magnetic chirality

$$\chi^{(M)} \sim \mathbf{S}_i \times \mathbf{S}_j$$



$$\rho_H = R_0 H + RM + \rho^{AF}$$



Hall, *Phil. Mag.* (1881)

Nagaosa et al, *RMP* (2010)

Hua Chen et al., *PRL* (2014), Falser & Kubler, *EPL* (2014)

Nakatsuji et al., *Nature* (2015), Nayak, Falser, Parkin et al., *Sci. Adv.* (2016)





## The spontaneous hall effect in ferromagnetics I

J. Smit

Show more

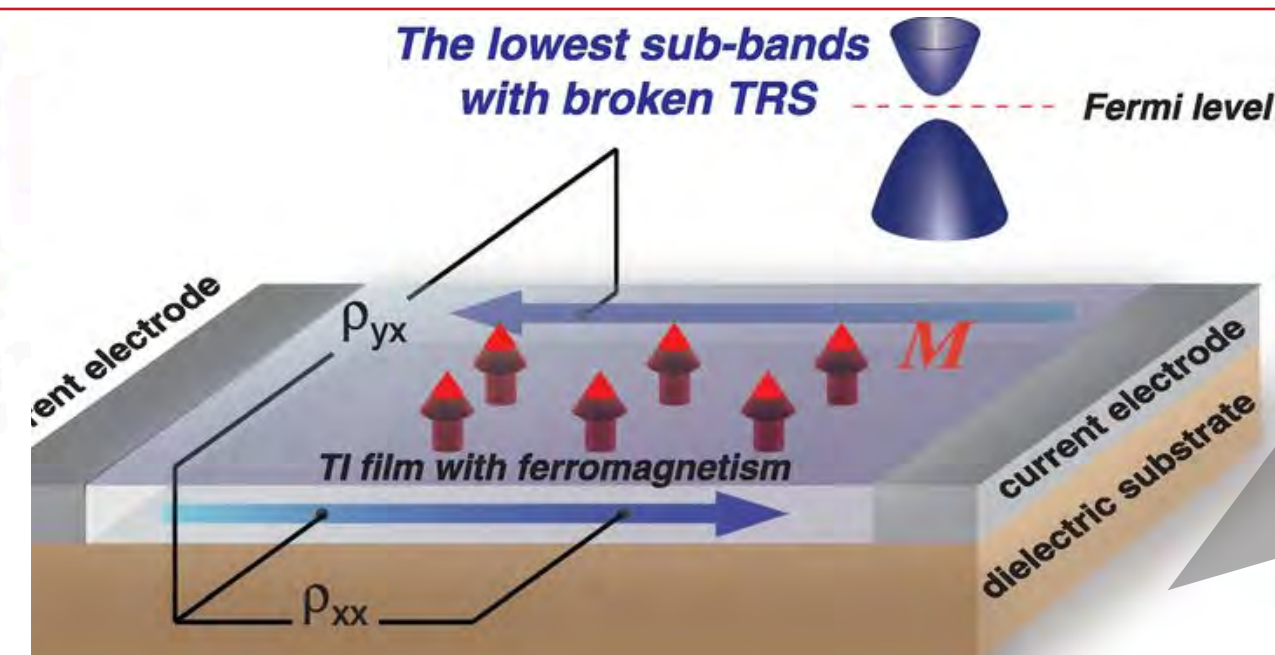
[https://doi.org/10.1016/S0031-8914\(55\)92596-9](https://doi.org/10.1016/S0031-8914(55)92596-9)

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### Synopsis

Apart from the normal Hall voltage a magnetized ferromagnetic material usually shows a relatively large extra voltage in the same direction, which can be found by linear extrapolation to  $B=0$ . It is shown that this spontaneous Hall effect cannot exist in a perfectly periodic lattice. Measurements at different temperatures suggest that the effect is closely related with the electrical-resistivity  $\pi$  of the material.

Existing theories on the origin of the effect are shown to be invalid, and it is shown that the explanation has to be based on the anisotropic scattering, caused by spinorbit interaction, of the conducting electrons against the imperfections of the lattice.



**2013** Quantum anomalous Hall in topological insulators

**1988** Haldane: quantum Hall effect **without Landau levels**

*Spontaneous Hall effect without complicated antiferromagnetism?*

**2015** Anomalous Hall effect in non collinear antiferromagnets **without magnetization**  
*Nakatsuji, Nature (2015)*

*Felser and Parkin, Sci.Adv.(2016)*

**2010** **without dipolar-magnetic order**  
*Machida, Nature (2010)*

**2002** Berry curvature - **topological properties of wave functions**  
*Jungwith et al. PRL (2002), Nagaosa et al., PRL (2002)*

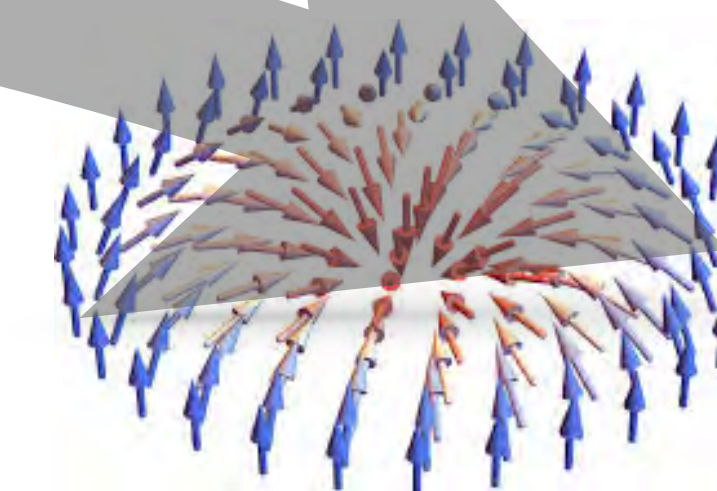
**1980** **Quantum** Hall effect

**1955+** Role of spin-orbit coupled impurities

**1953/58+** Kohn-Luttinger

**Microscopically: spin-polarisation and spin-orbit coupling**

**1996** "Topological" Hall effect **without spin-orbit coupling**



**1879** Hall effect from field

**1881** Hall effect from magnetisation





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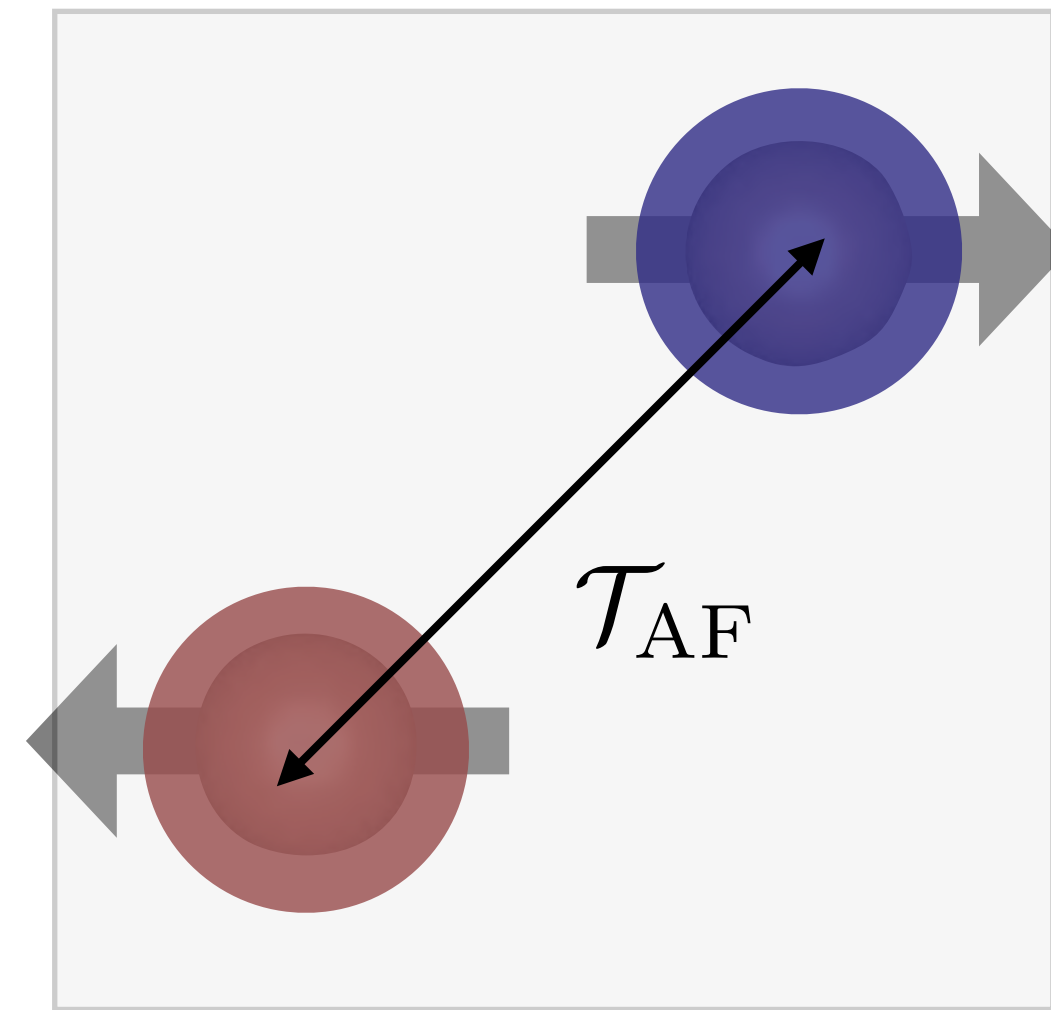
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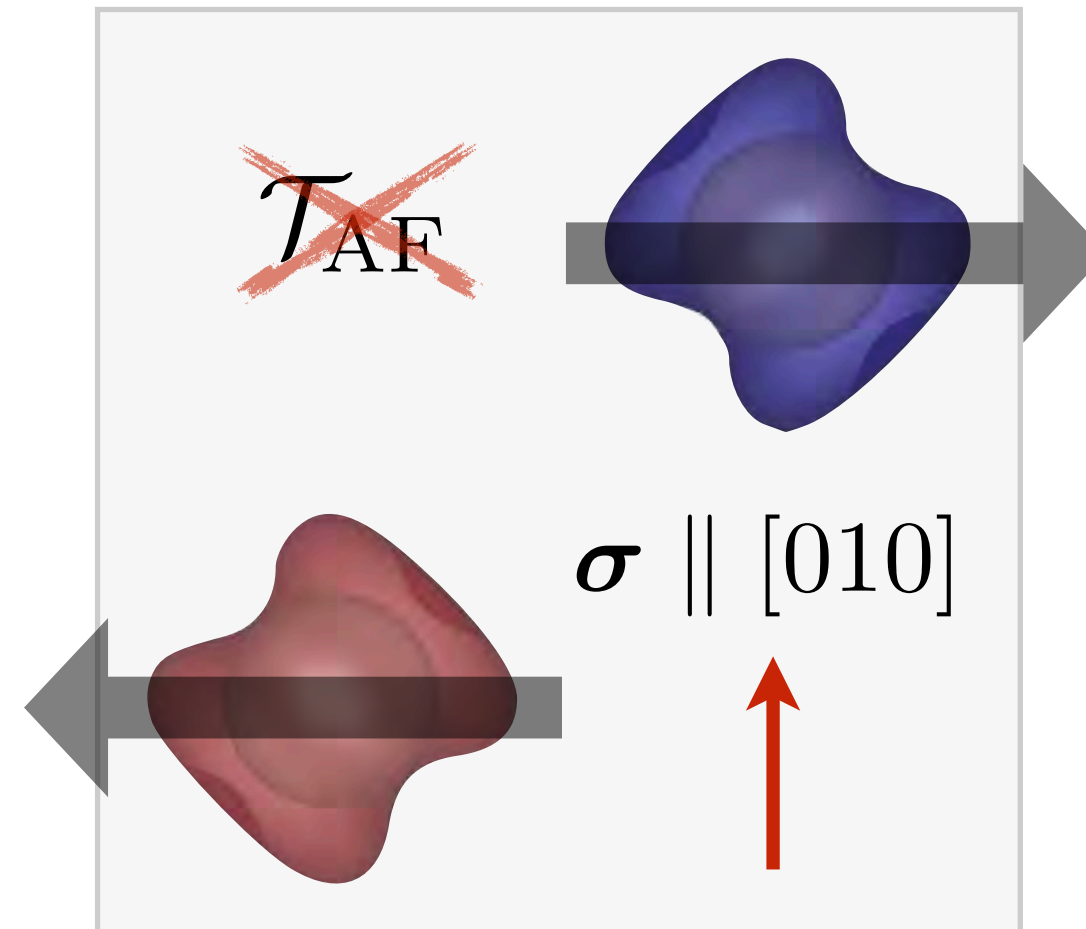


**spin degenerate bands**  
(IrMn, CuMnAs, Mn<sub>2</sub>Au, ...)



LŠ, J.Železny, JS, TJ,  
*Phys.Rev.Lett.* (2017)

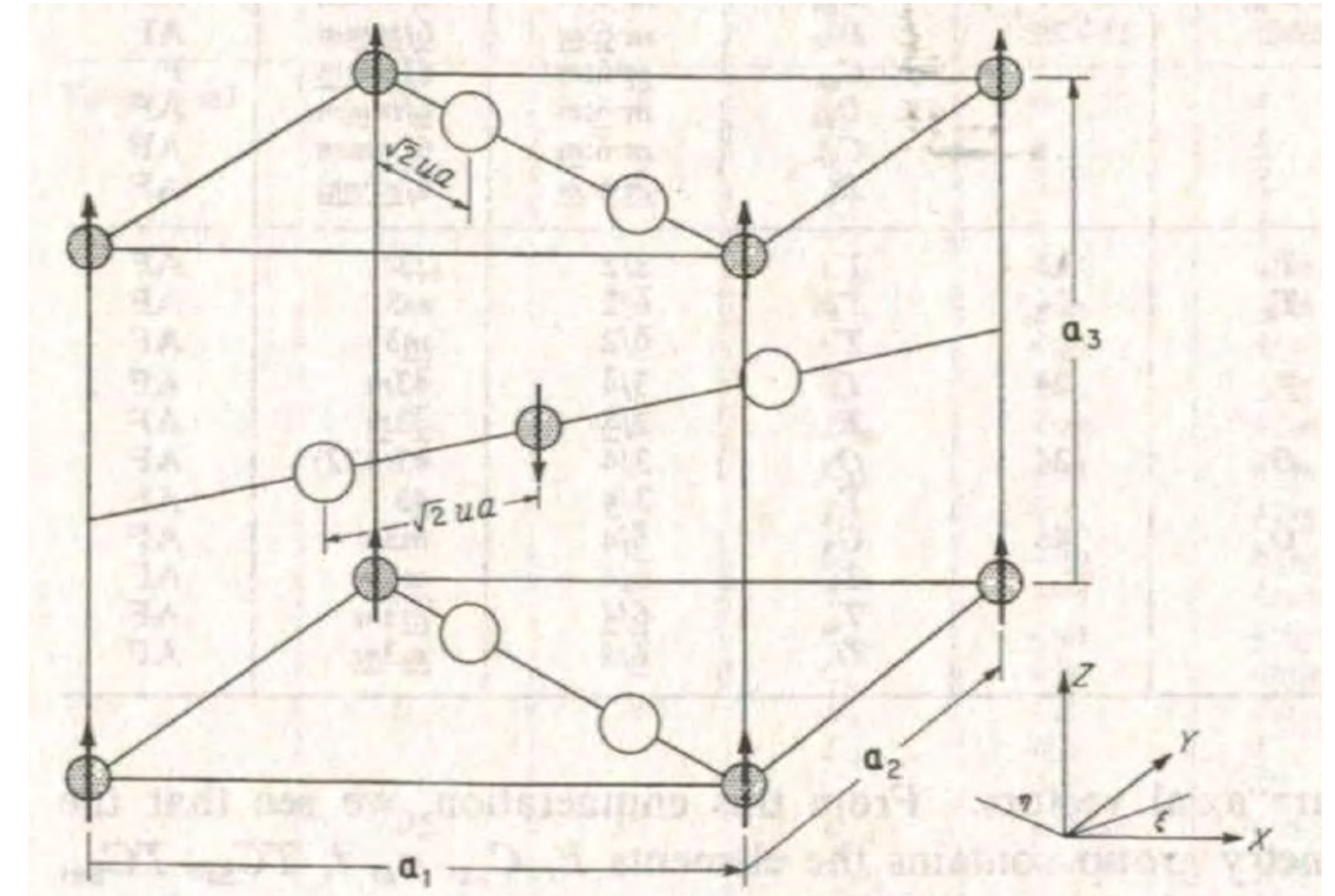
**Magnetisation density spontaneously breaks symmetries and allows Hall vector!** (RuO<sub>2</sub>, ...)



LŠ, R.H.Gonzales, JS, TJ,  
*arXiv 1901.00445* (2019)

**Analysing magnetisation projection vectors (black arrows) is incomplete!**

*Landau, Lifshitz, Electrodynamics of Continuous Media* (1972)



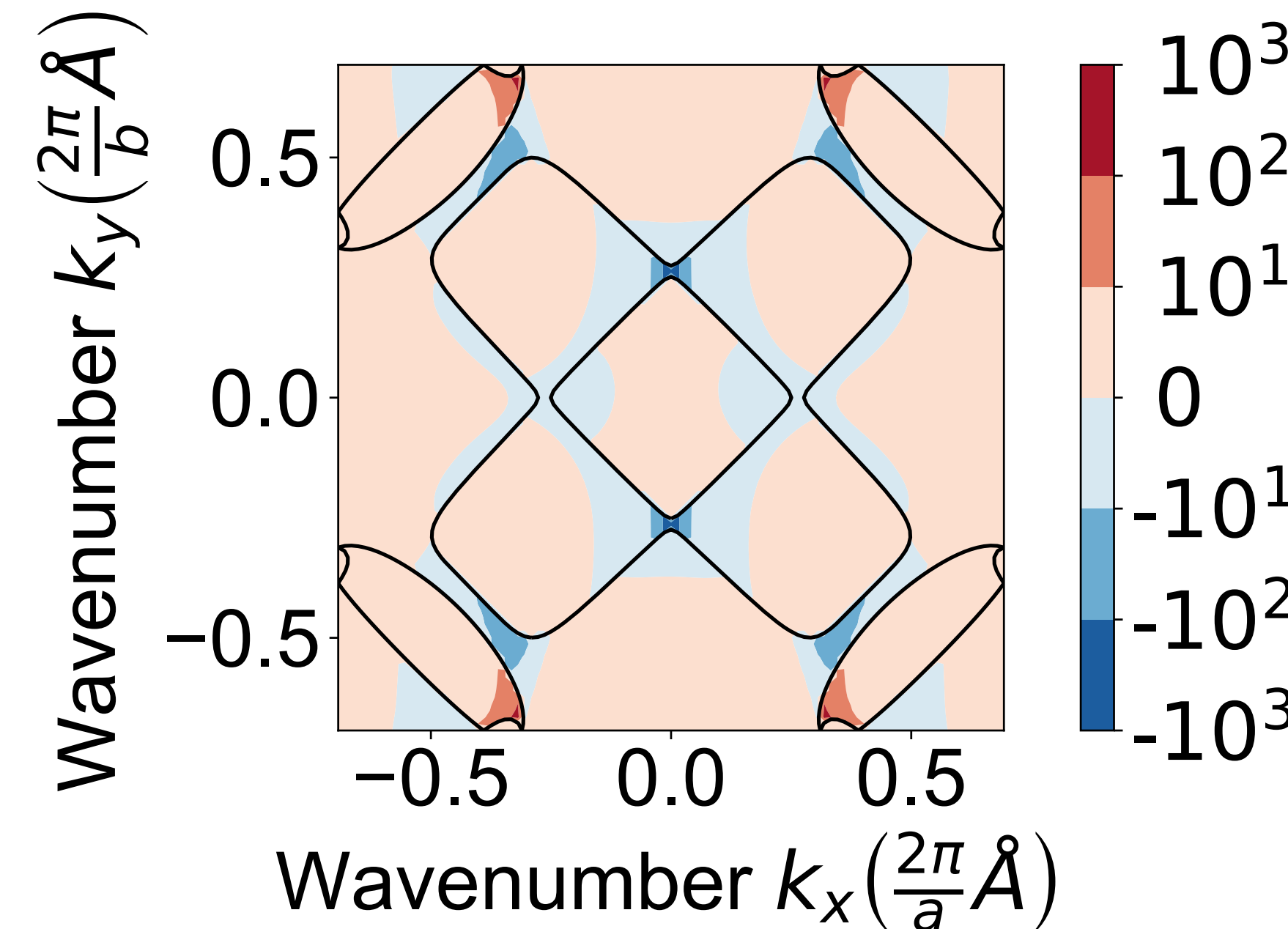
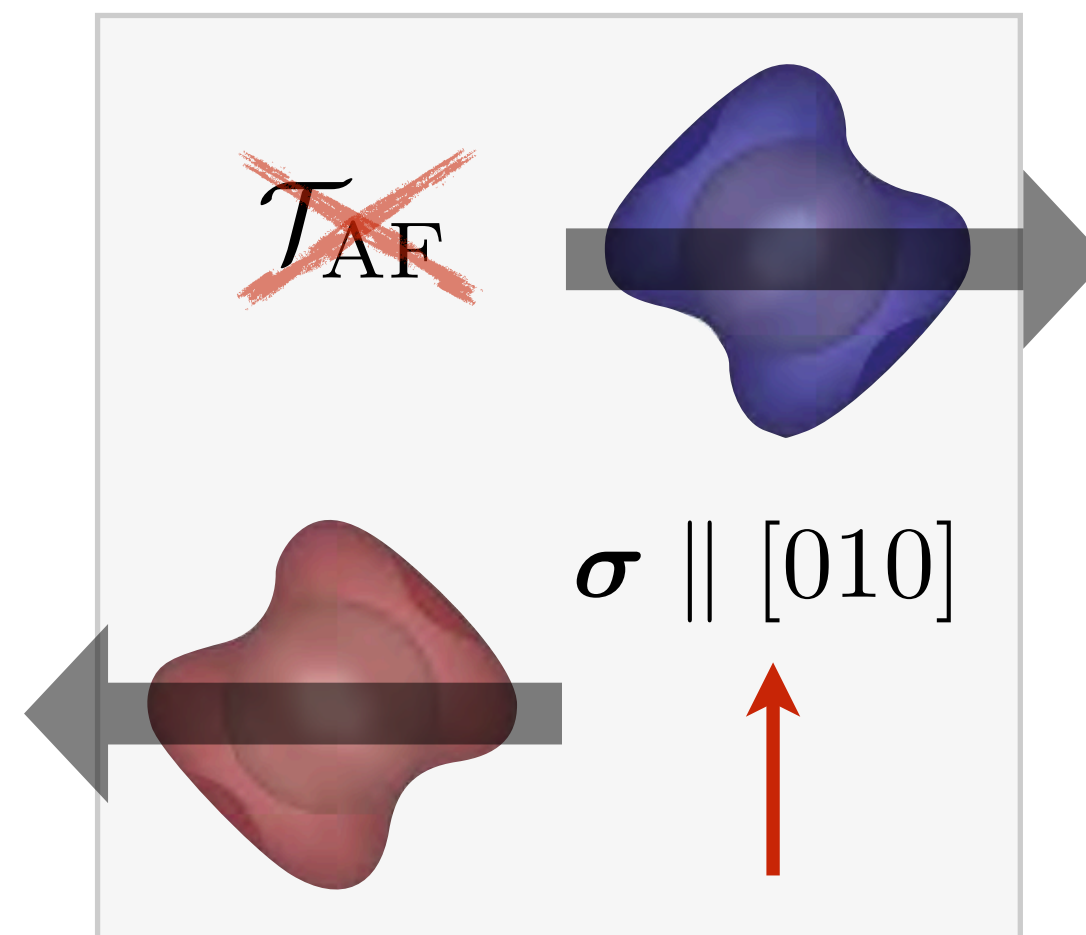
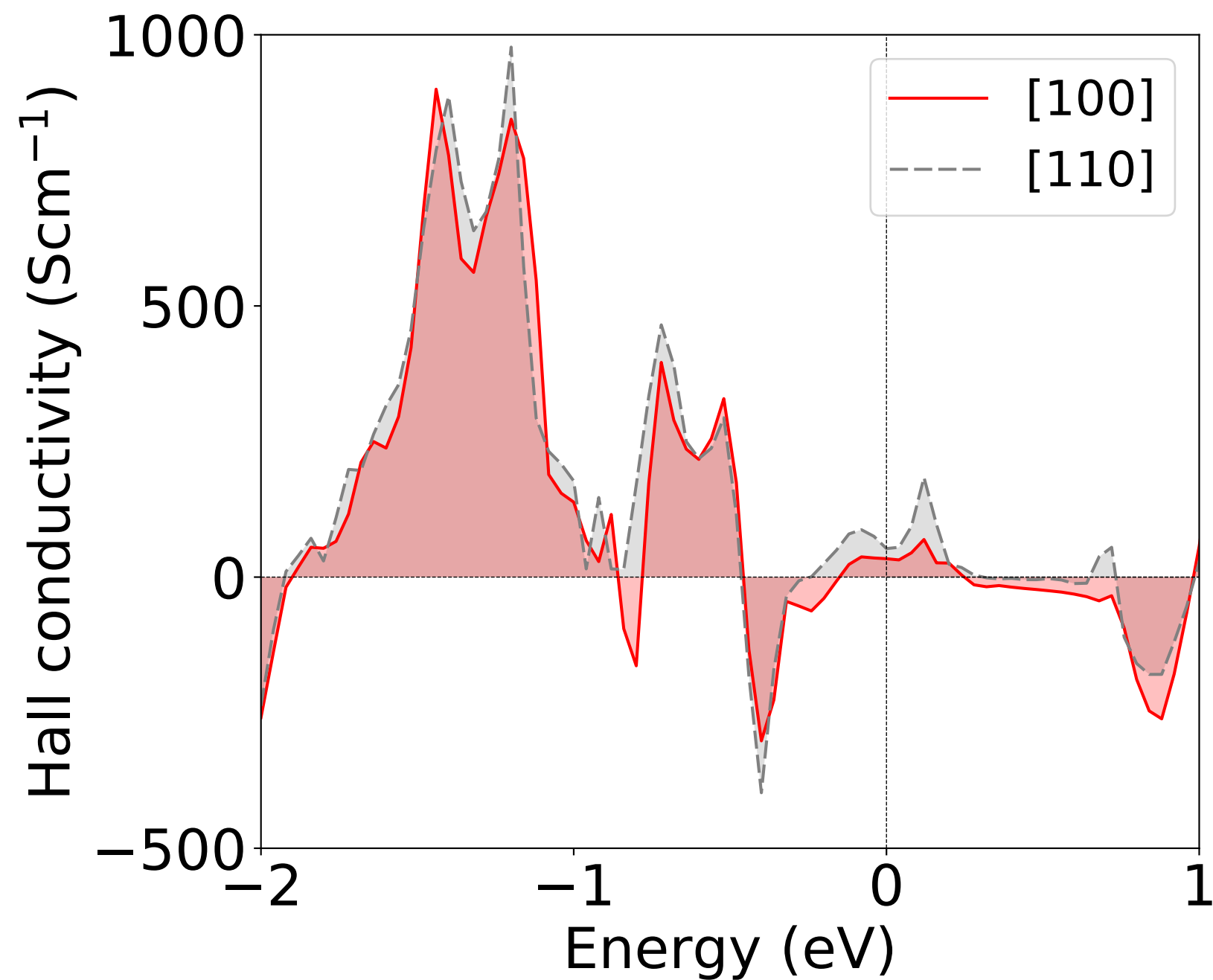
**Notorious antiferromagnets**

*Bradley & Cracknell, The Mathematical Theory of Symmetry in Solids* (1972)

*Tinkham, Group theory and quantum mechanics* (1964)



## Spontaneous symmetry breaking is strong already without SOC!

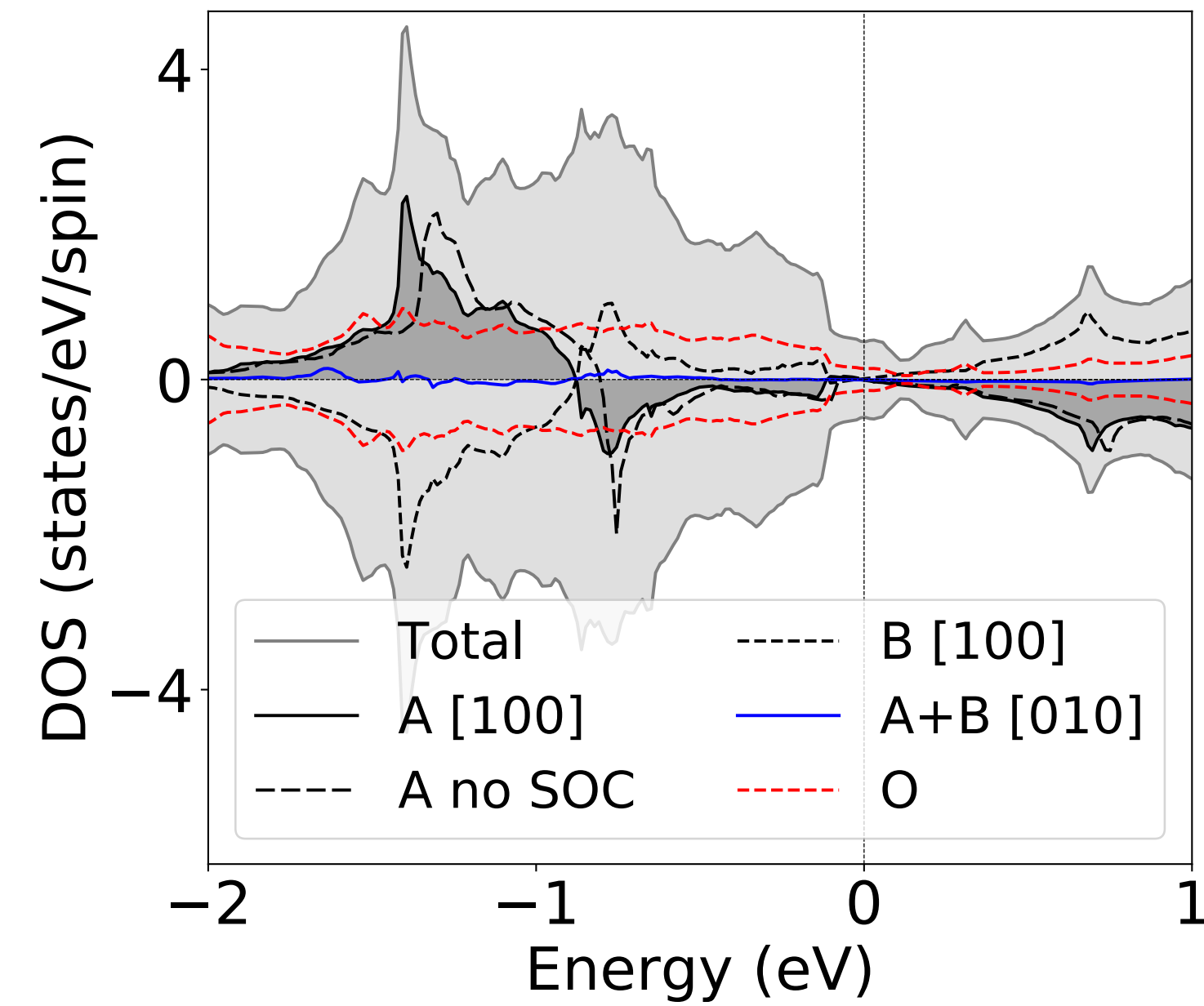


$$\sigma_{xy} = \frac{e^2}{h} \int_{\text{BZ}} \frac{dk}{(2\pi)^3} \sum_n f(\mathbf{k}) b_z^n(\mathbf{k})$$

LŠ, R.H.Gonzales, JS, TJ, arXiv 1901.00445 (2019)

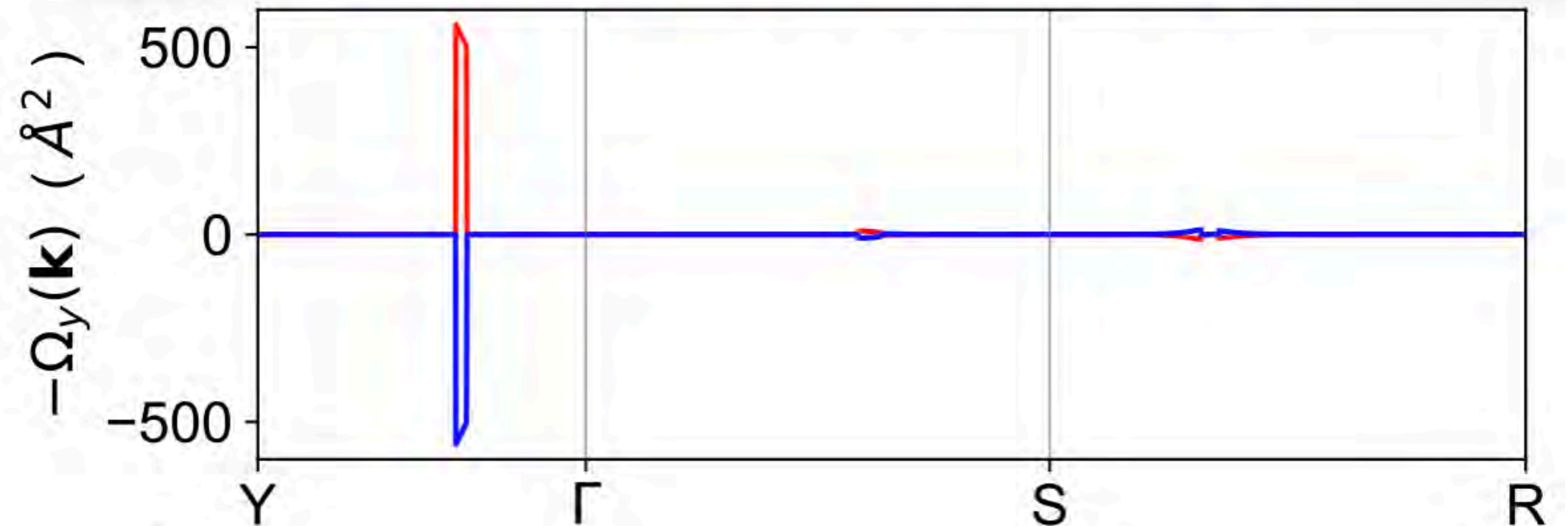
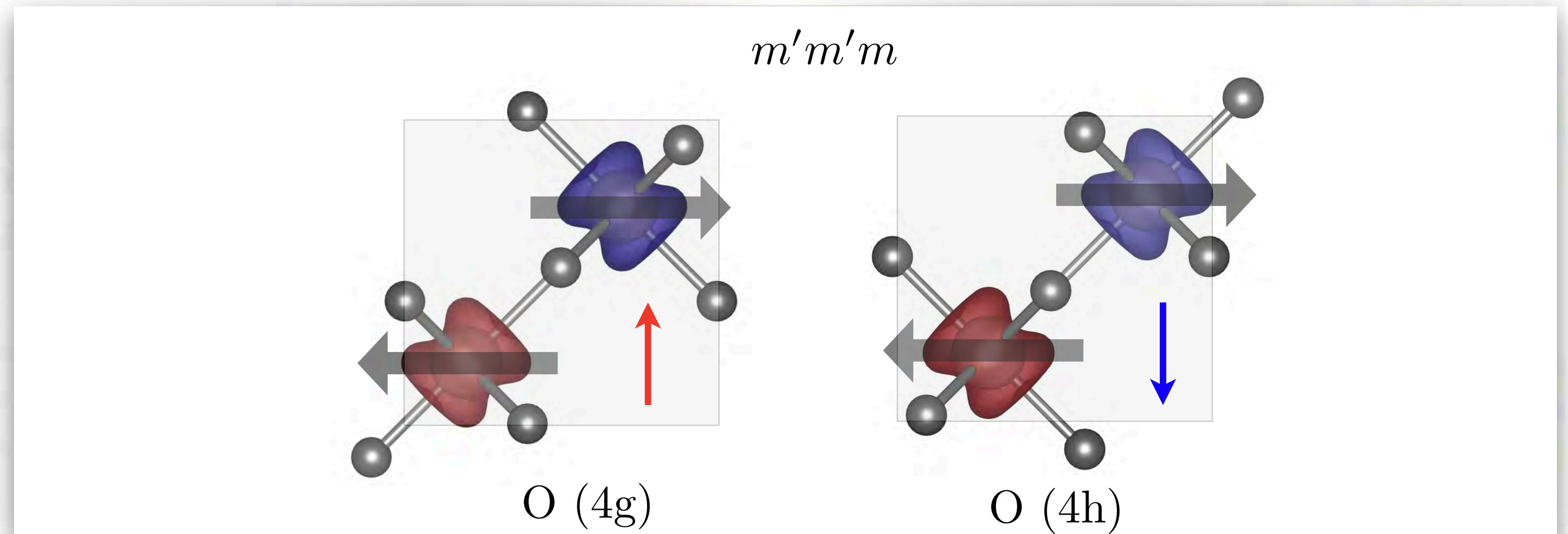
Magnitude can be as large as record values in ferromagnets





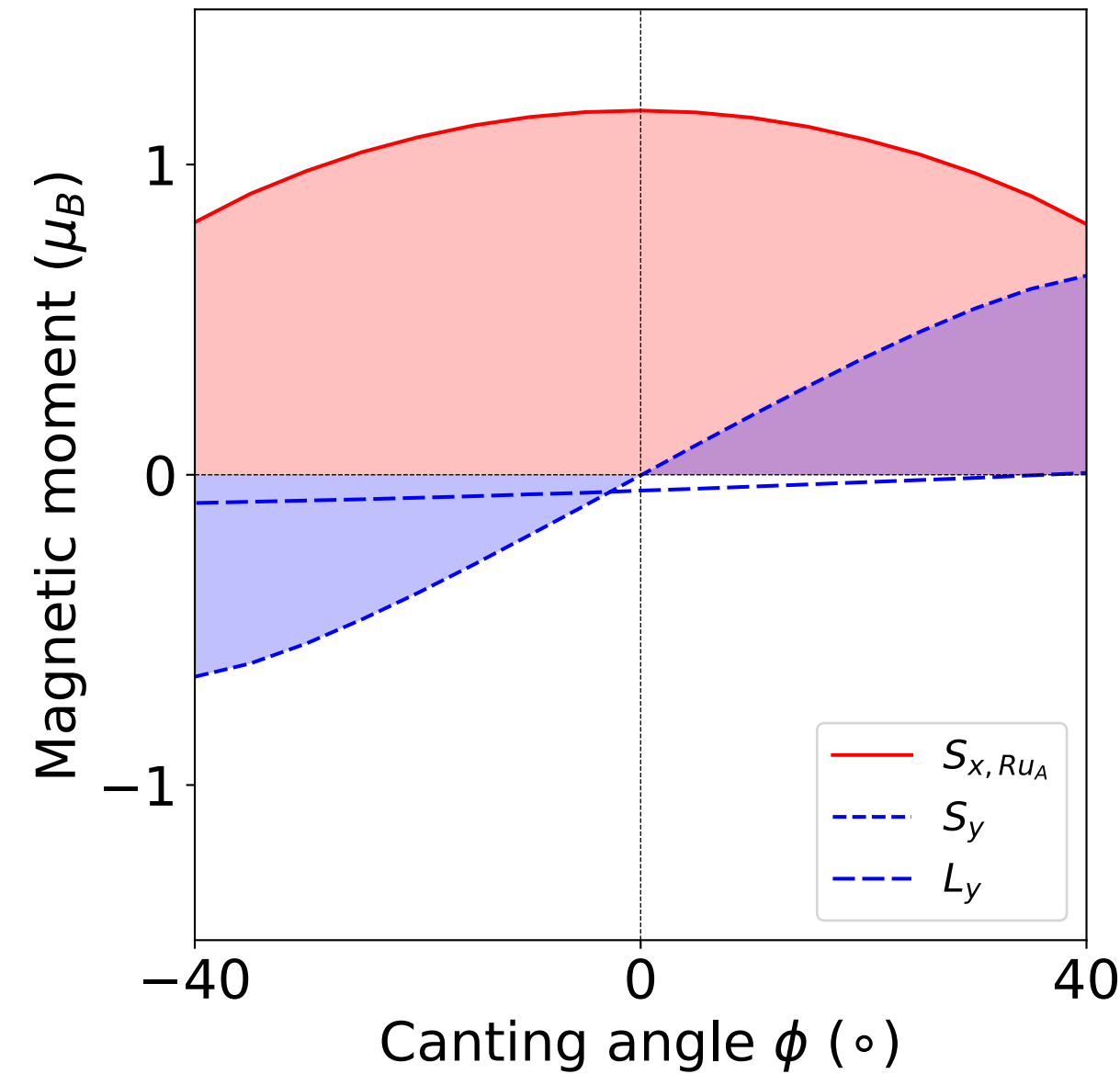
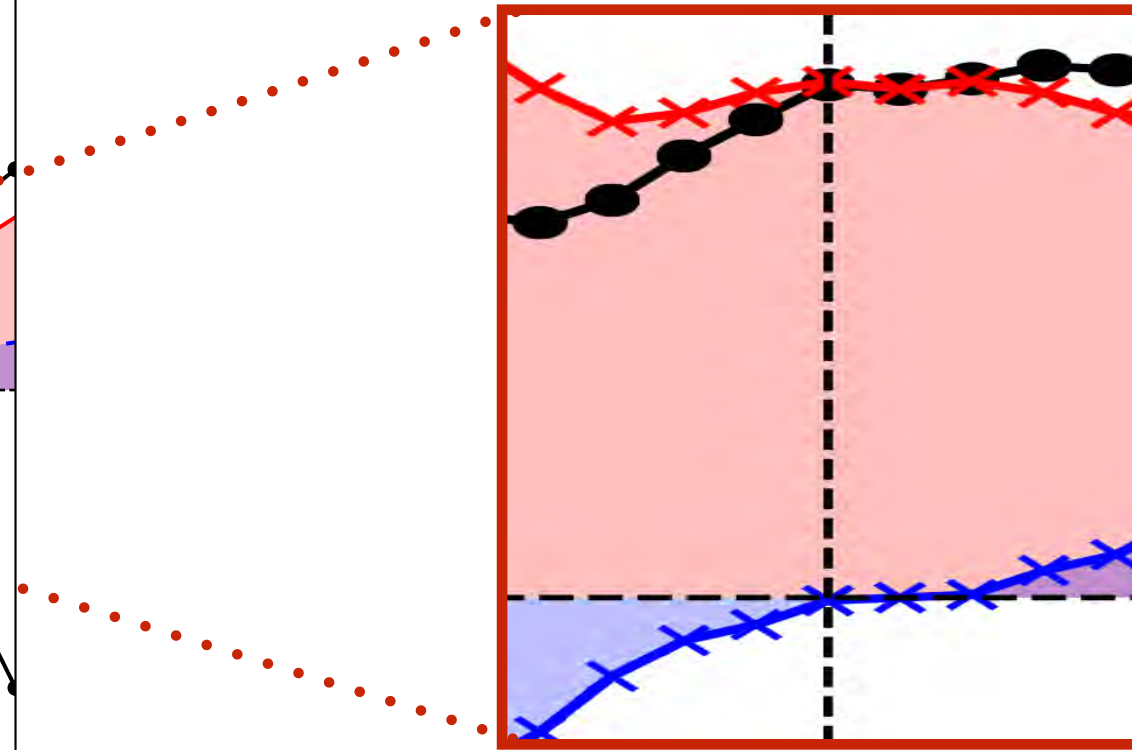
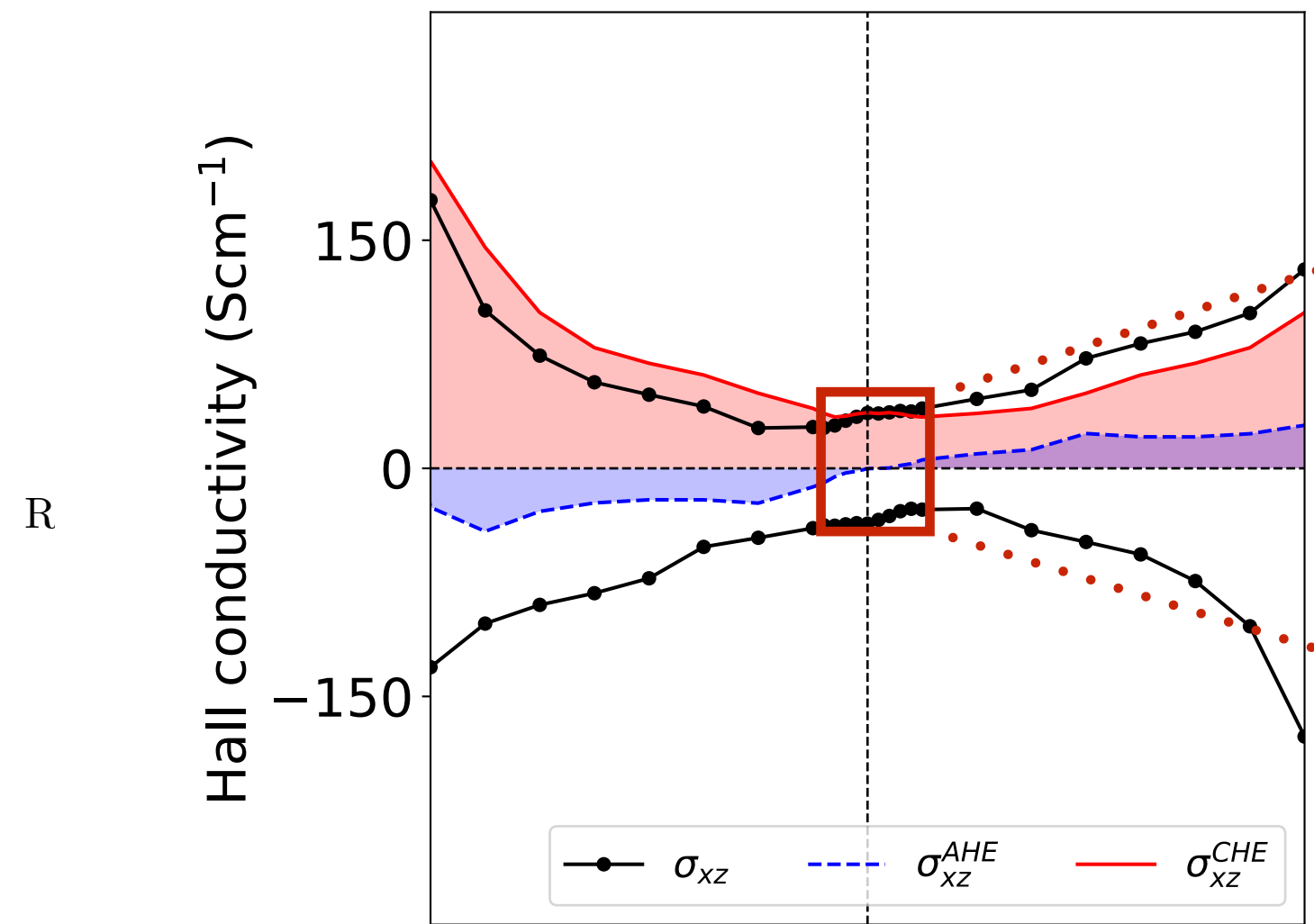
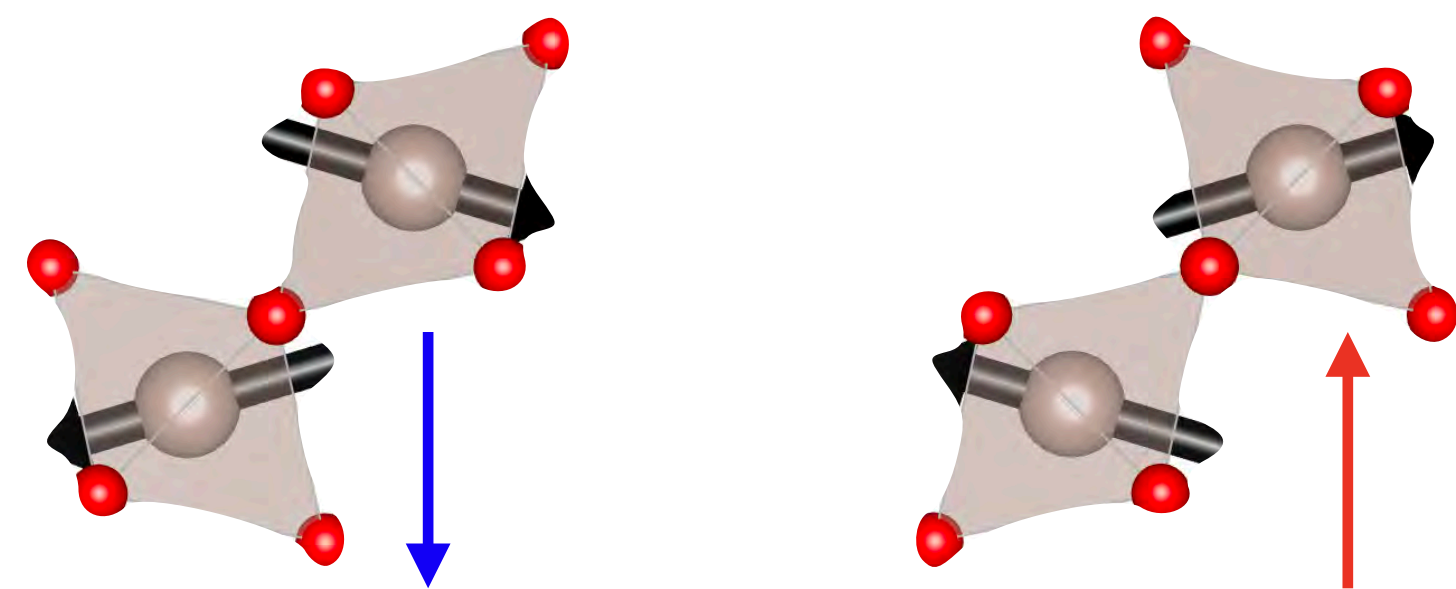
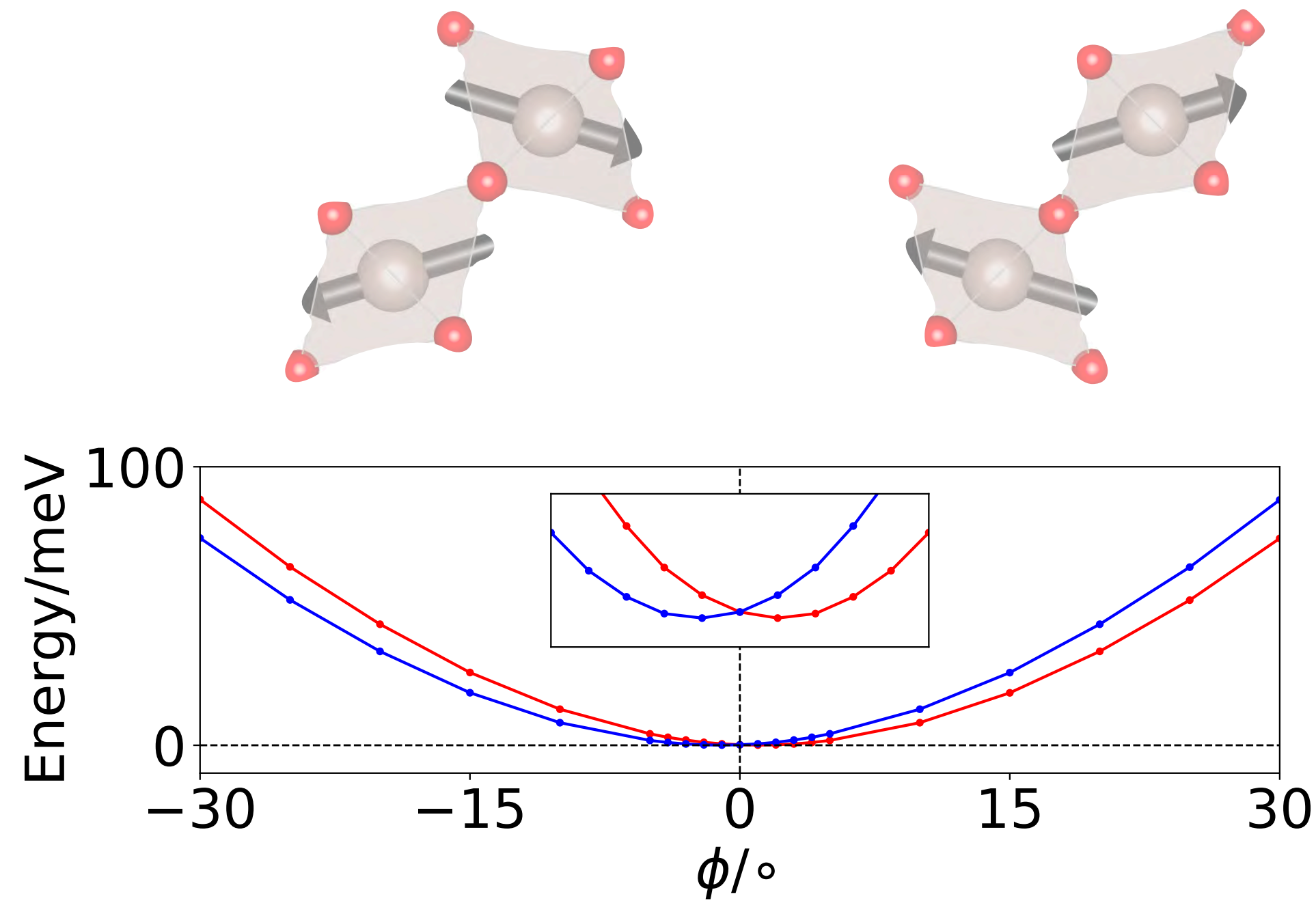
## Correlations stabilise antiferromagnetism

*Berlijn et al., Phys. Rev. Lett. (2017)*





# Crystal Hall .VS. anomalous Hall from canting



$$\sigma_{xz}^{\text{CH}} = \frac{\sigma_{xz}(\mathbf{n}, \mathbf{m}) + \sigma_{xz}(\mathbf{n}, -\mathbf{m})}{2}$$

$$\sigma_{xz}^{\text{AH}} = \frac{\sigma_{xz}(\mathbf{n}, \mathbf{m}) - \sigma_{xz}(\mathbf{n}, -\mathbf{m})}{2}$$

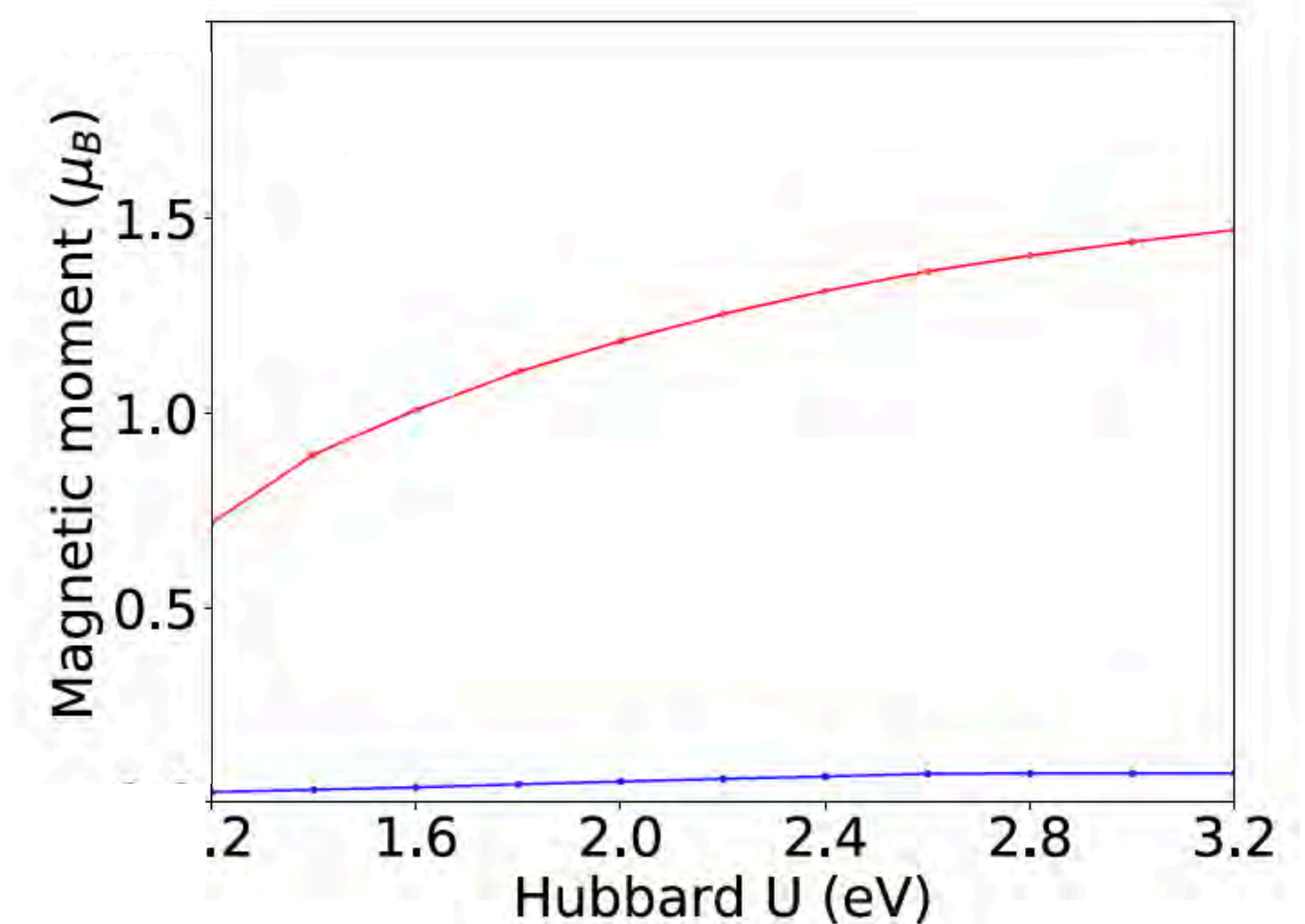
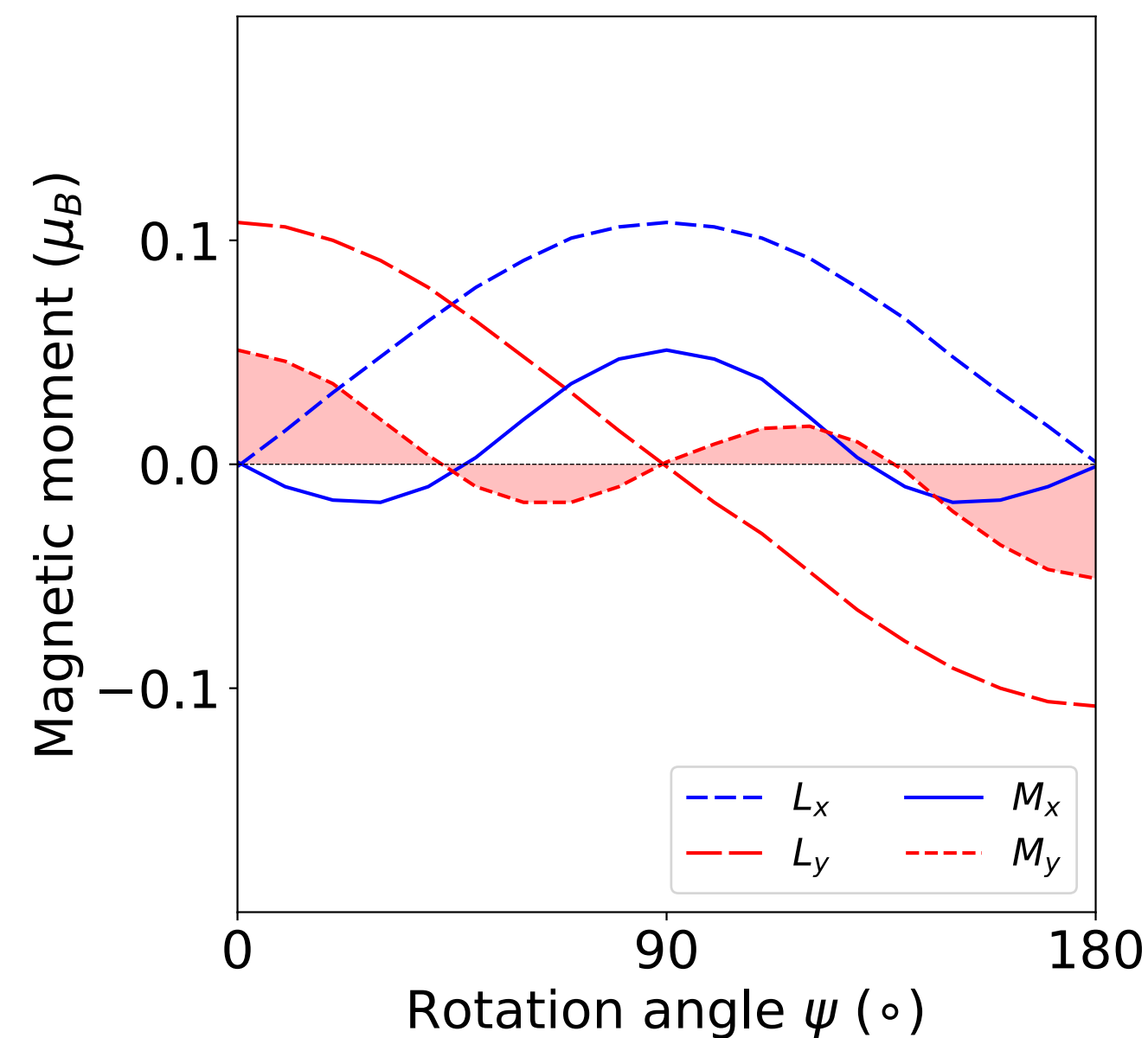
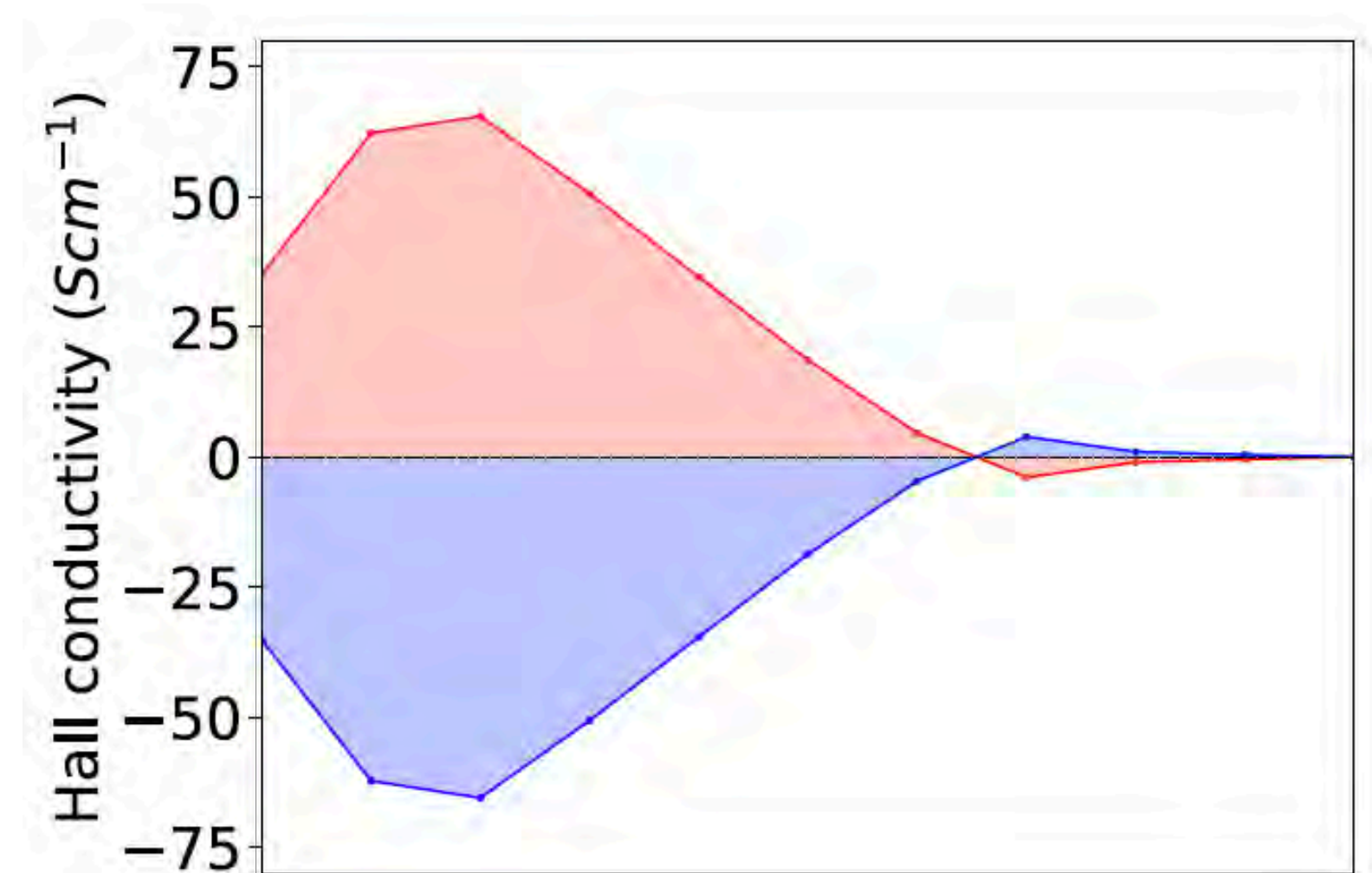
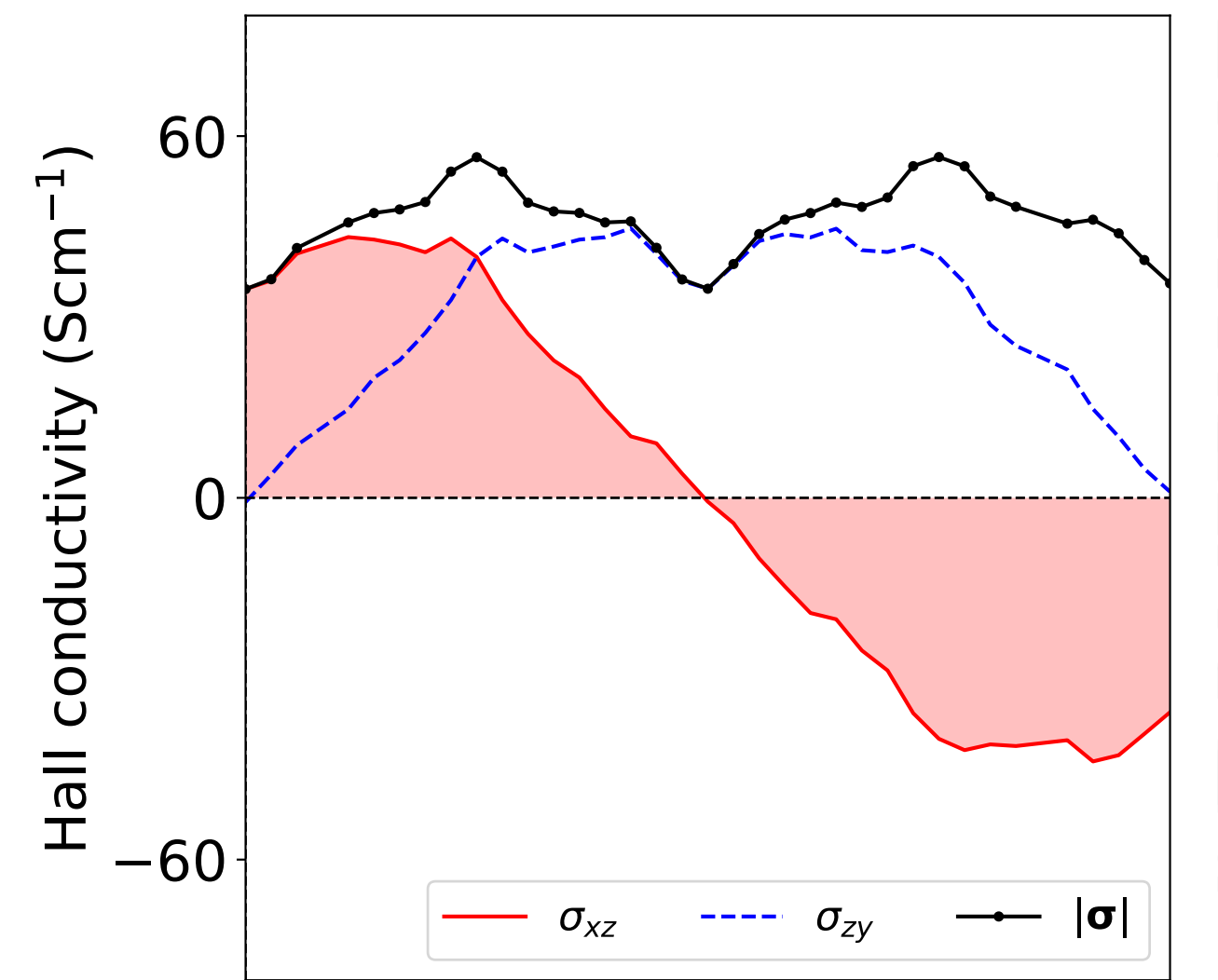
$$\sigma_{xz}(\mathbf{n}, \mathbf{m}) = -\sigma_{xz}(-\mathbf{n}, -\mathbf{m})$$



$\mathbf{n}$	MSG	$\mathbf{m}$ ( $\mu_B$ )	$\sigma_H$ ( $\text{Scm}^{-1}$ )
[001]	$P4'_2/mnm'$	0	0
[100]	$Pnn'm'$	$\parallel [010]$ 0.05	36
[110]	$Cmm'm'$	$\parallel [\bar{1}10]$ 0.0075	58

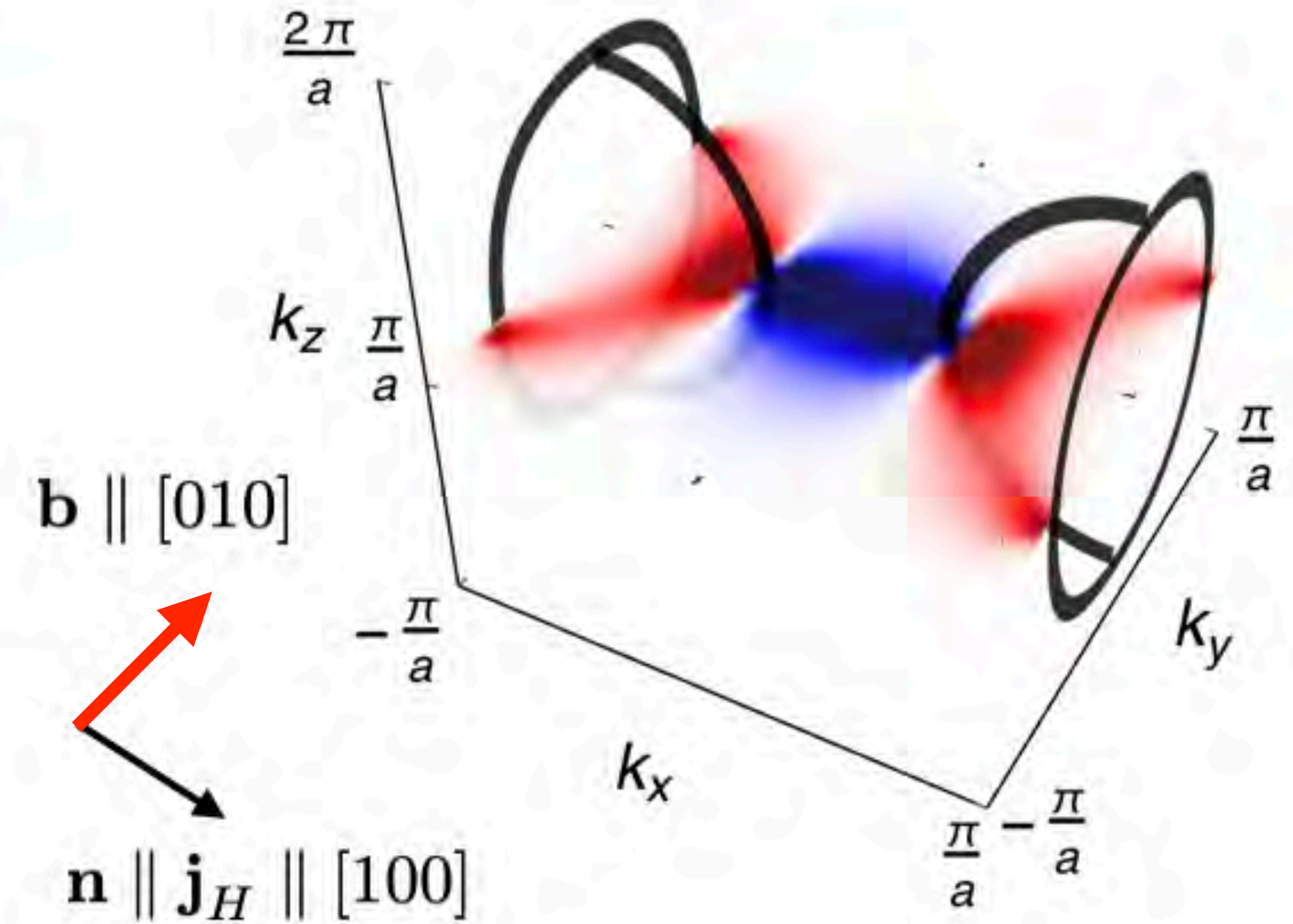
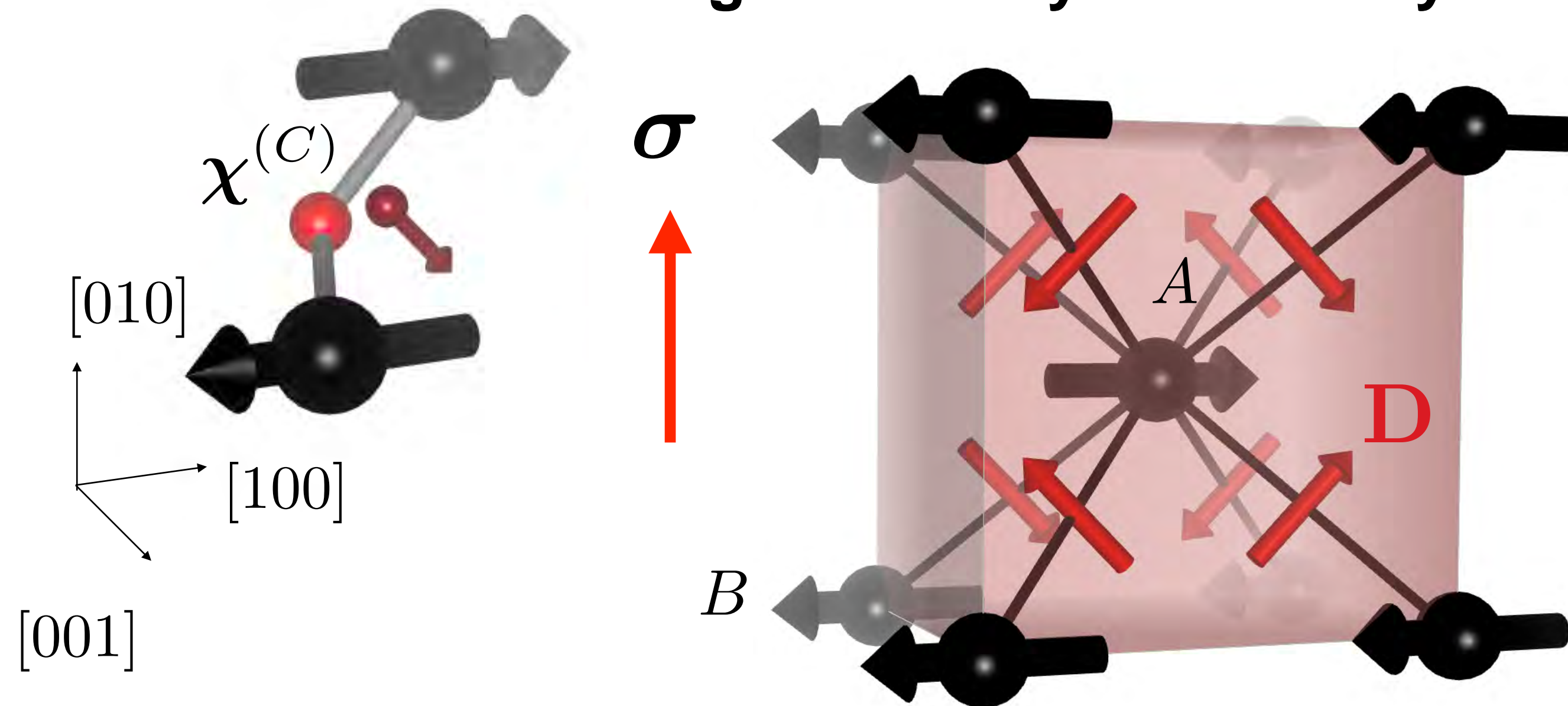
**Hall vector can be switched on-off**  
**Ultimate anisotropy of Hall effect**

**Hall vector is not proportional to**  
**Sublattice magnetisation**  
**Net magnetisation**





## Nonmagnetic atoms at noncentrosymmetric positions generate crystal chirality!



$$H = t \sum_{ij} c_i^\dagger c_j + J_n \sum_i \mathbf{u}_i \cdot \mathbf{s} c_i^\dagger c_i + \lambda \sum_{ij,k} \chi^{(C)}_{ij,k} \cdot \mathbf{s} c_i^\dagger c_j$$

$$\chi^{(C)} = \text{sign}(\mathbf{d}_{k,A} \times \mathbf{d}_{k,B})$$

**Multiple mirrors stabilise nodal chains.**

*Bzdusek et al., Nature (2016), Yan et al., Nat. Phys. (2018)*

Asymmetric spin-orbit coupling:  
swap  $\leftrightarrow$  swap Hall vector



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*LS & T.Jungwirth, Topology in magnetism, Springer (2018)*

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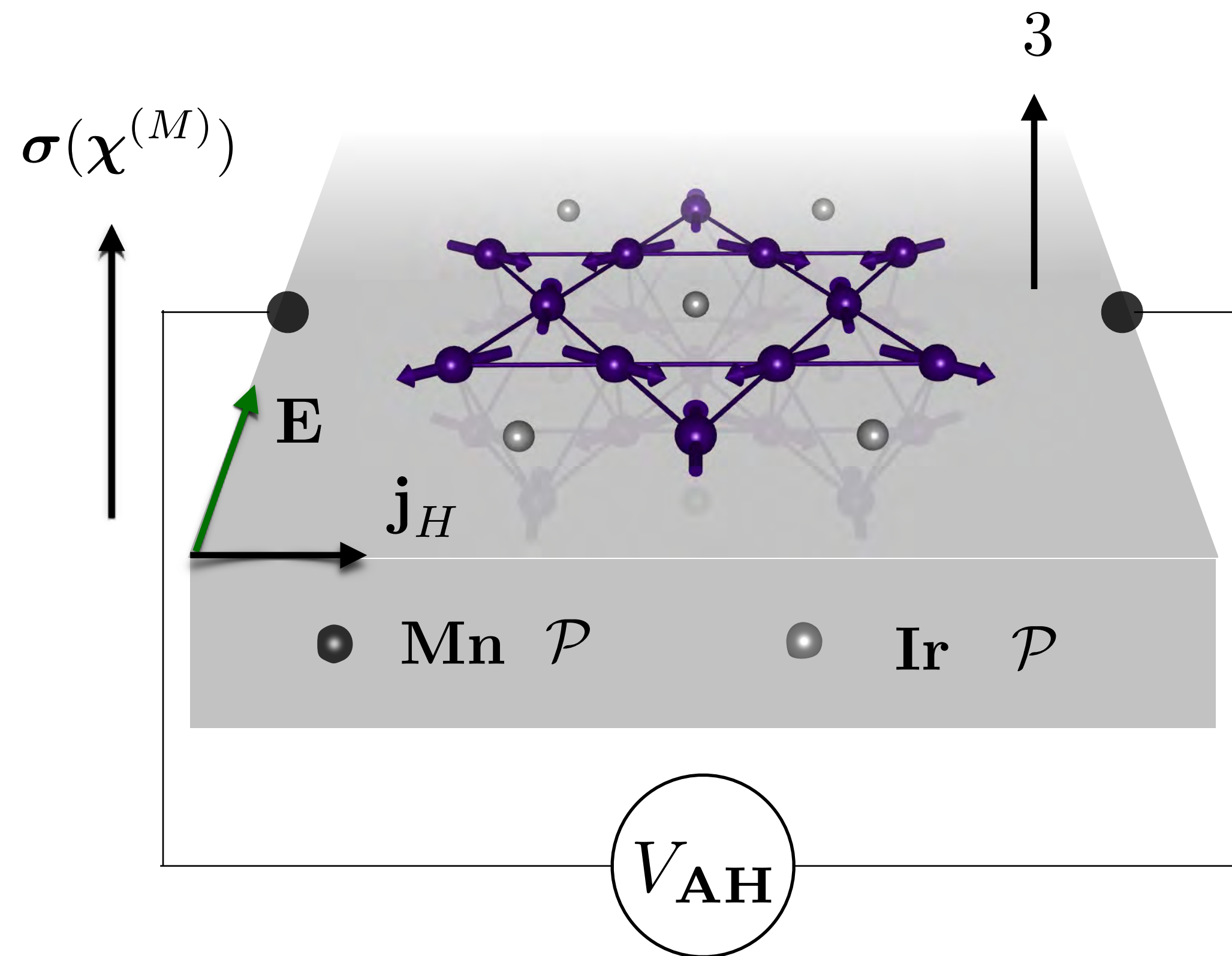
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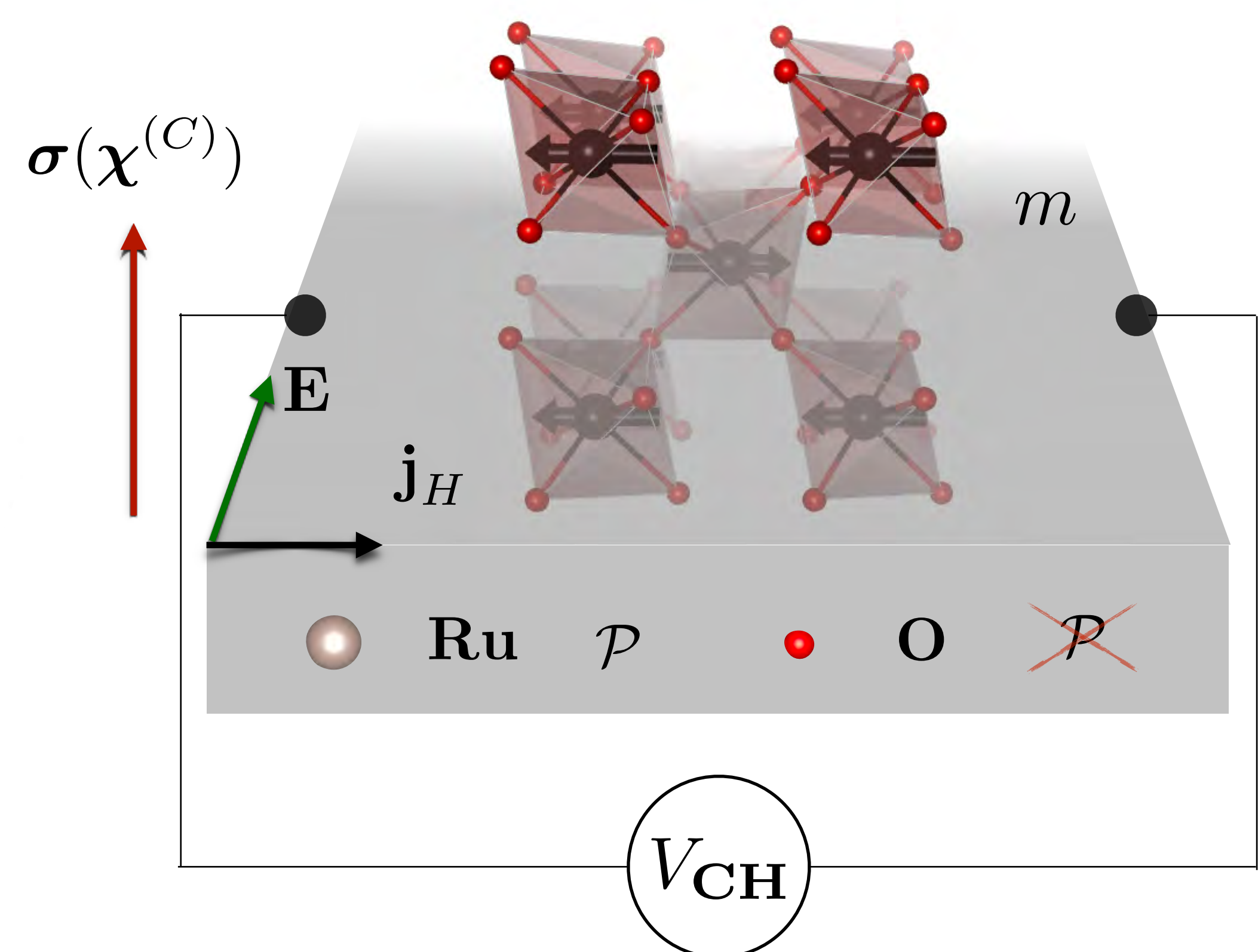




## Centrosymmetric Wyckoff Magnetic chirality



## Non-centrosymmetric Wyckoff Crystal chirality



**Simple collinear antiferromagnetism not compatible with 3, 4, 6 symmetries!**



# Classification of crystal Hall antiferromagnets

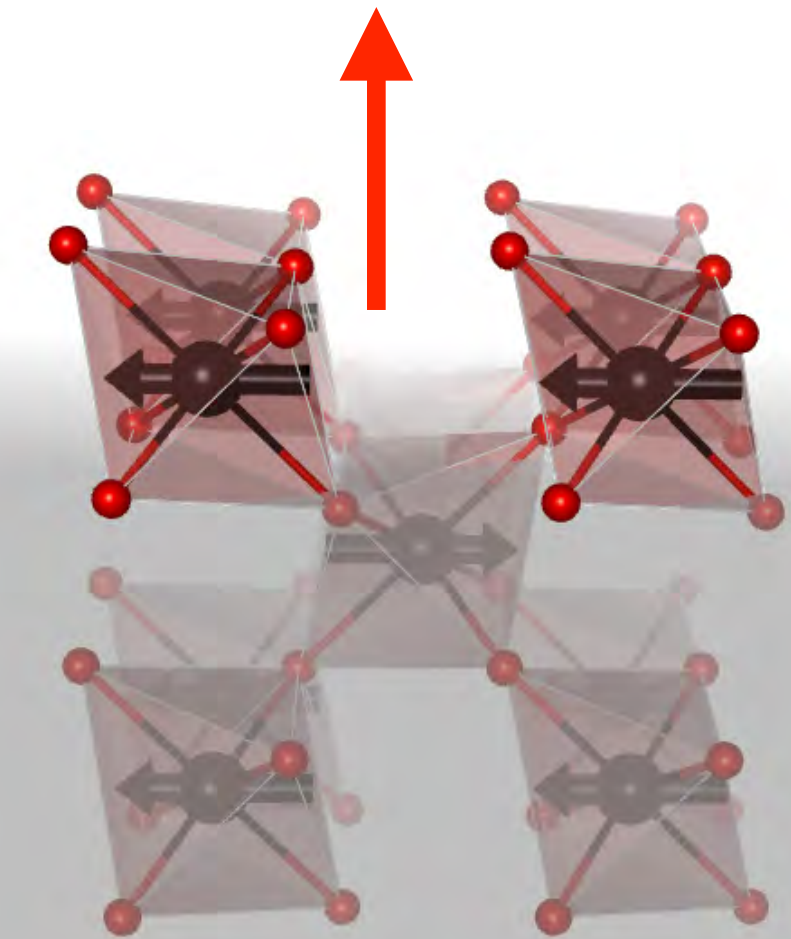
Centrosymmetric MPG	$\sigma$	Noncentrosymmetric MPG	$\sigma$	Tensor
$\bar{1}$	arb.	1	arb.	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$
$2/m$	$\parallel \mathbf{a}_{c_2}$ $\perp \mathcal{M}$	$\frac{2}{m}$ $\parallel \mathbf{a}_{c_2}$ $\perp \mathcal{M}$		$\begin{pmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{xz} & 0 & \sigma_{zz} \end{pmatrix}$
$\bar{3}$ $4/m$ $6/m$	$\parallel \mathbf{a}_{c_2}$	3 4 6 $\bar{4}$ $\bar{6}$	$\parallel \mathbf{a}_{c_2}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$2'/m'$	$\perp \mathbf{a}_{\mathcal{T}c_2}$ $\in \mathcal{T}\mathcal{M}$	$\frac{2'}{m'}$ $\perp \mathbf{a}_{\mathcal{T}c_2}$ $\in \mathcal{T}\mathcal{M}$		$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ -\sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ -\sigma_{xz} & -\sigma_{yz} & \sigma_{zz} \end{pmatrix}$
$m'm'm$	$\parallel \mathbf{a}_{c_2}$ $\perp \mathcal{M}_z$	$\frac{2'2'2m'm'2}{m'm2'}$ $\parallel \mathbf{a}_{c_2}$ $\perp \mathcal{M}_y$		$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$
$\bar{3}m'$ $4/m'm'm$ $6/m'm'm$	$\parallel \mathbf{a}_{c_n}$	$42'2' 4m'm' \bar{4}2'm'$ $32' 3m' 62'2'$ $6m'm' \bar{6}m'2'$	$\parallel \mathbf{a}_{c_n}$	$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$

$\mathcal{T}$  time inversion  
 $\bar{1} = \mathcal{P}$  space inversion  
 $m = \bar{2}$  mirror  
 $\bar{1}' = \mathcal{P}\mathcal{T}$

RuO<sub>2</sub>  
 CoNb<sub>3</sub>S<sub>6</sub>  
 CaMnO<sub>3</sub>\*

Mn<sub>3</sub>Pt

$$\sigma \sim \mathbf{E} \times \mathbf{j}_H$$



~5/10 ferromagnetic  
 Laue point groups  
 ~1/10 of magnetic  
 material database  
**are** crystal Hall  
 antiferromagnets!

**Hall plane does not have to be given by symmetry!**



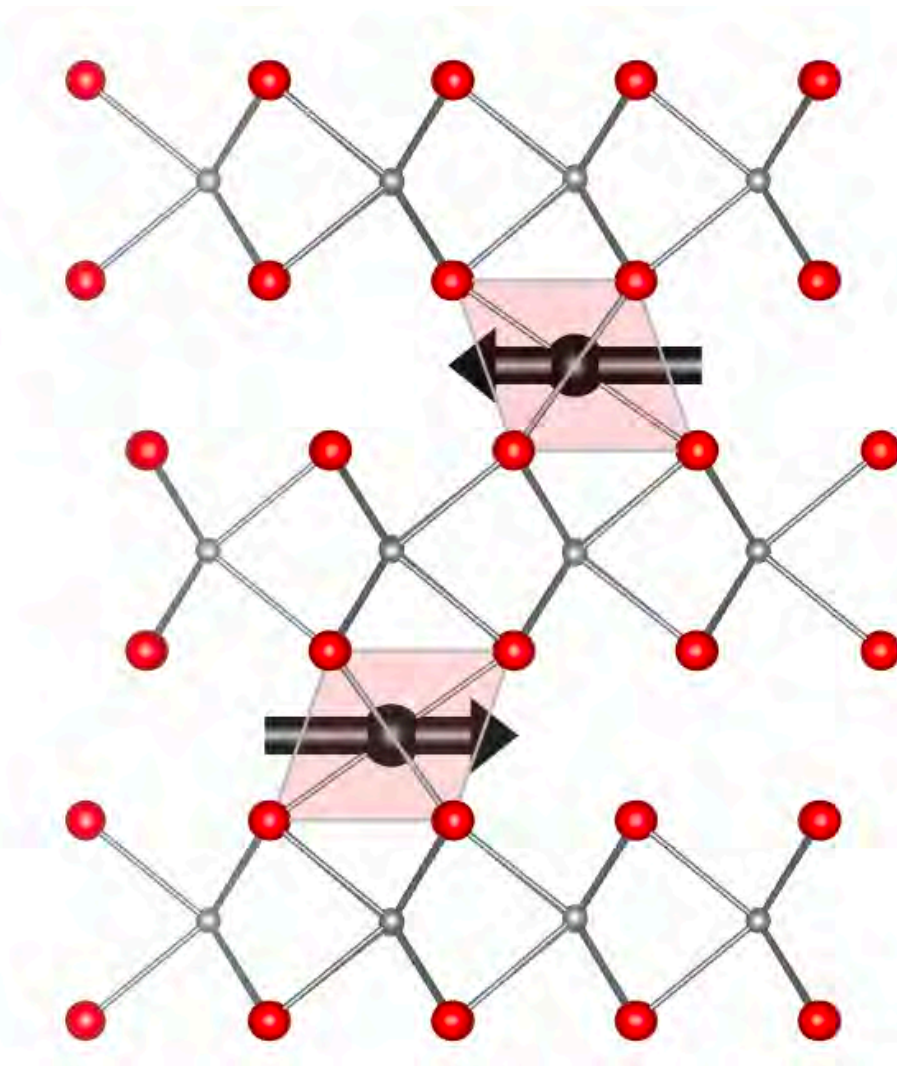
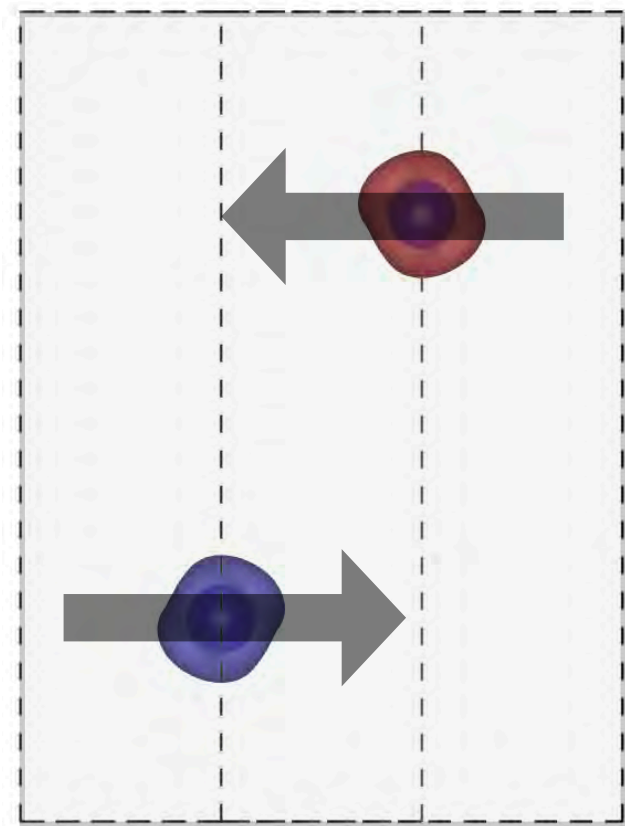
# Global chirality in $\text{CoNb}_3\text{S}_6$ antiferromagnet



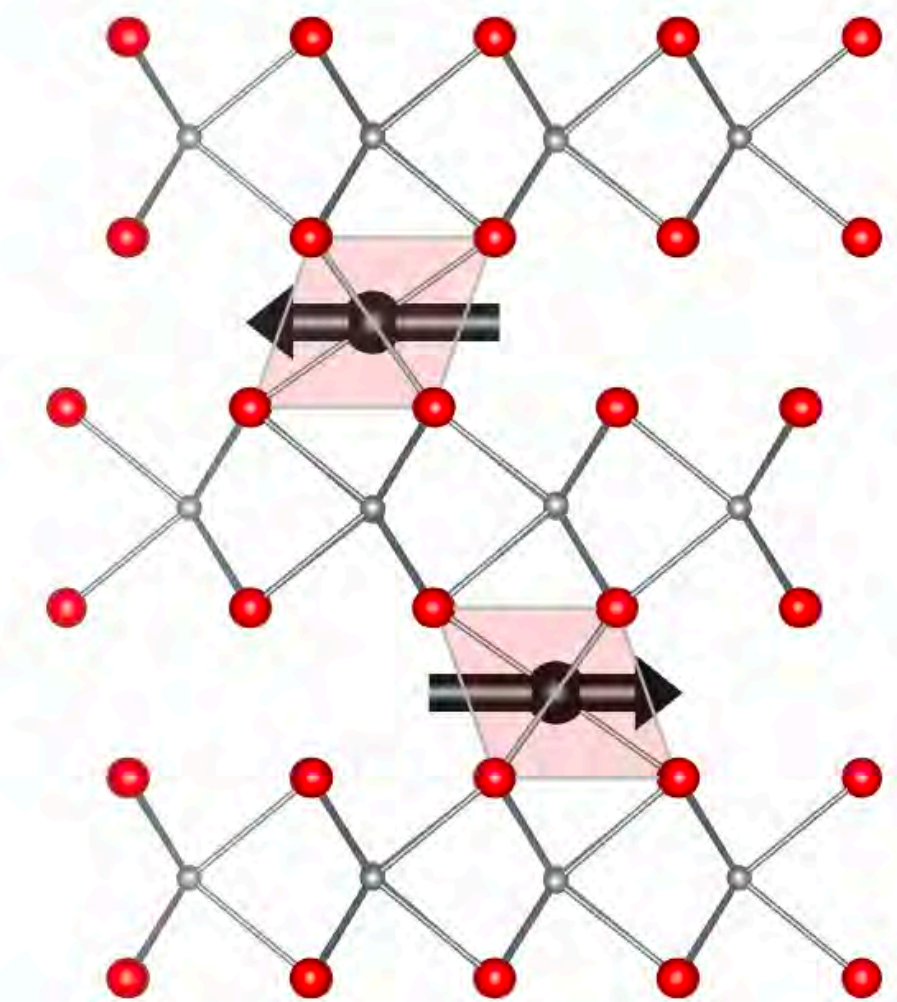
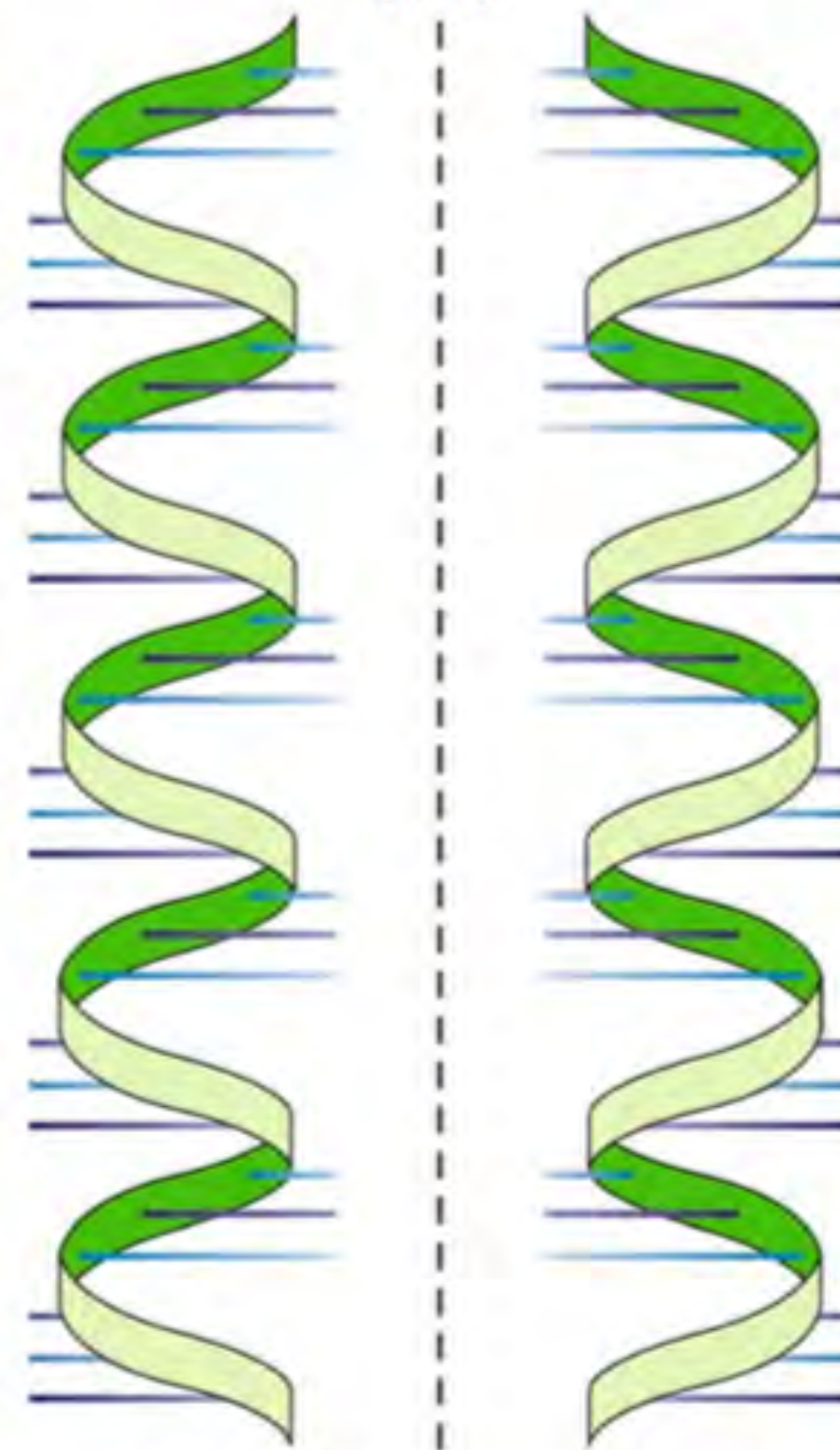
left-handed

Mirror

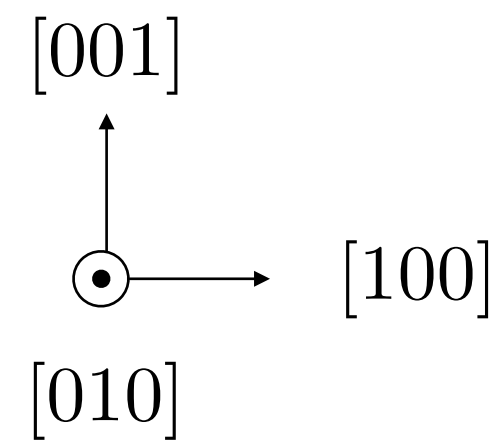
right-handed



$\sigma$



$\sigma$



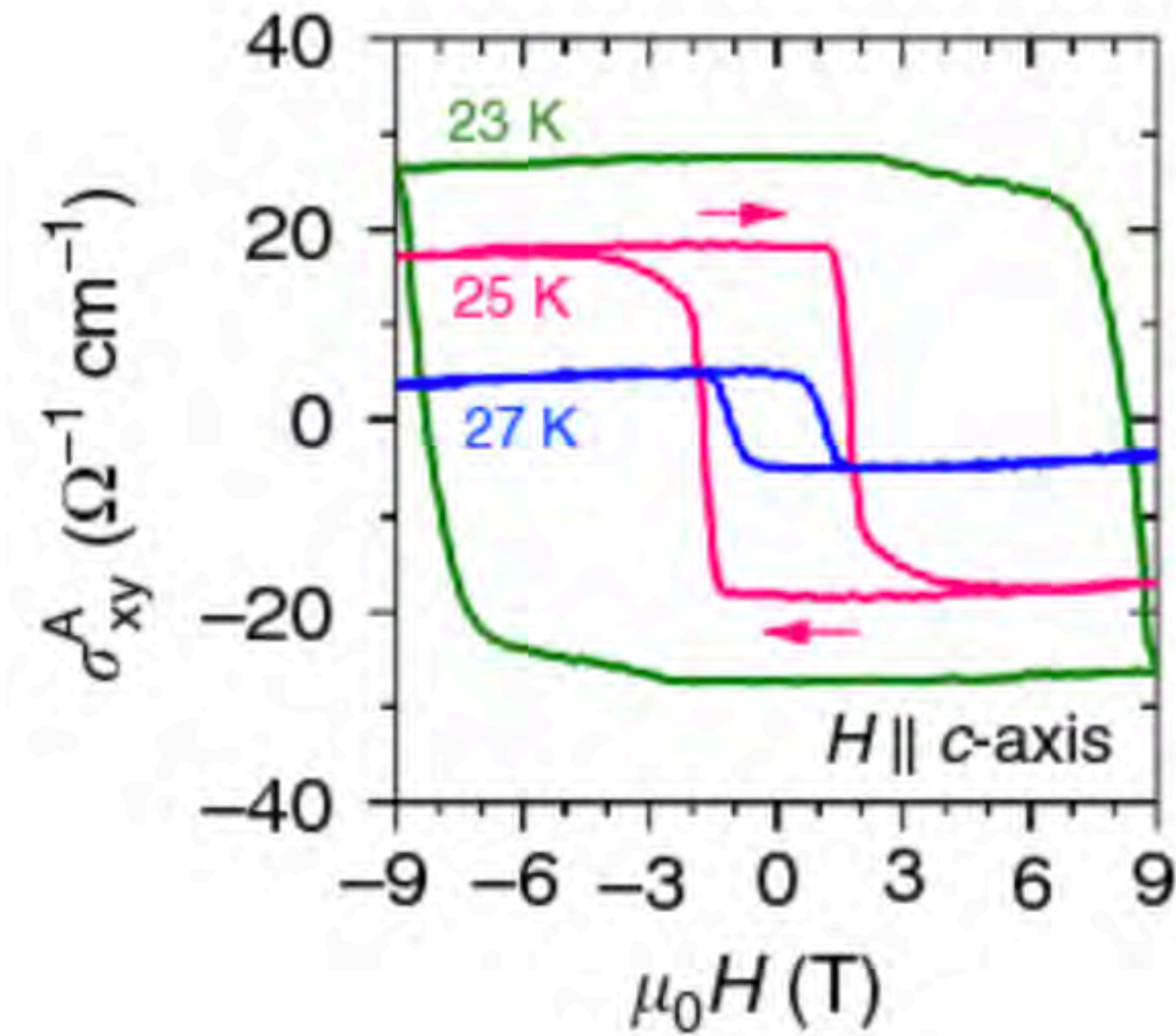
● Co    ● Nb    ● S

Experiment:  $\sim 27$  S/cm  
*Ghimire et al. Nat. Comm. (2018)*  
Our theory:  $\sim 80$  S/cm

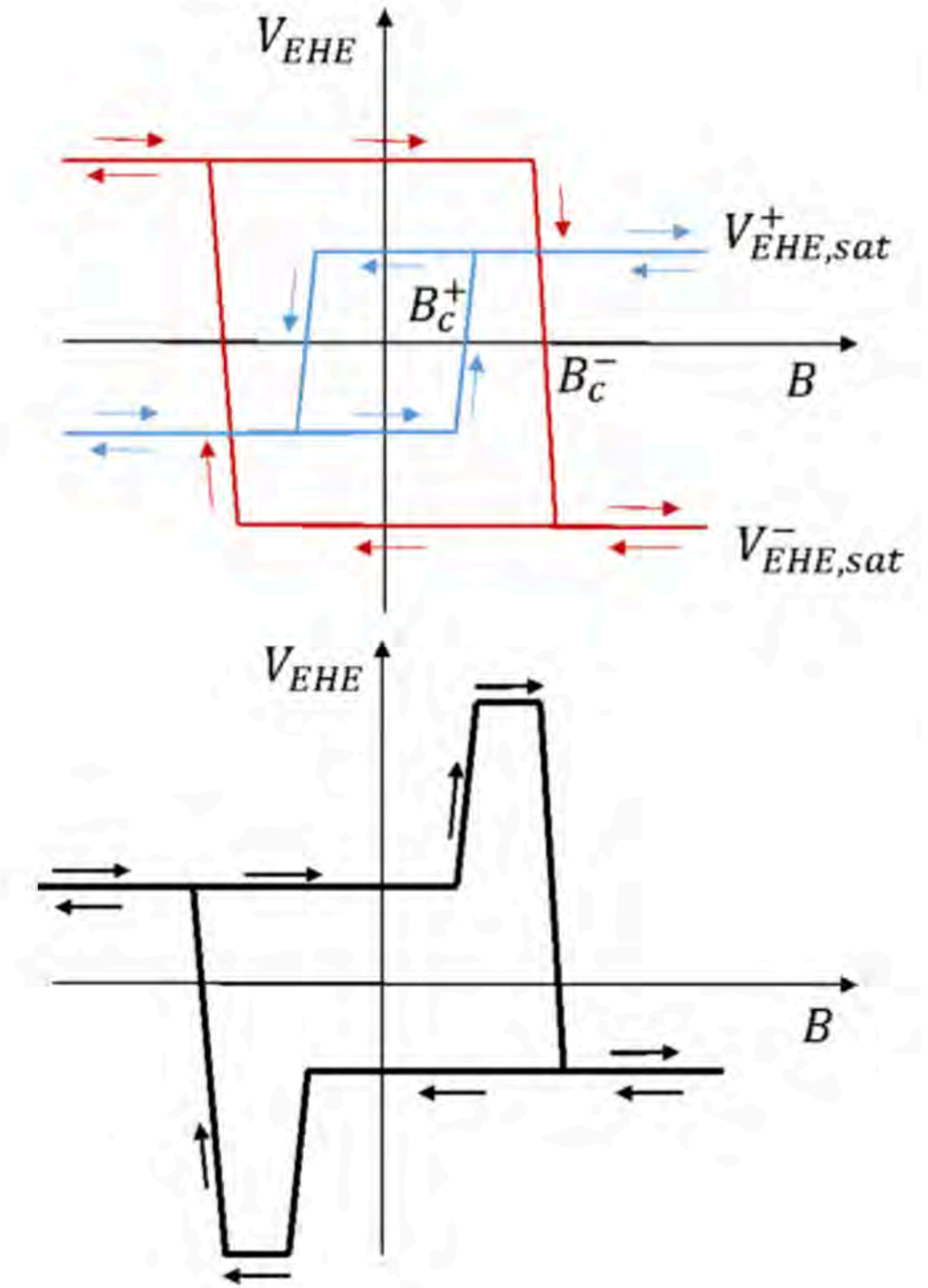
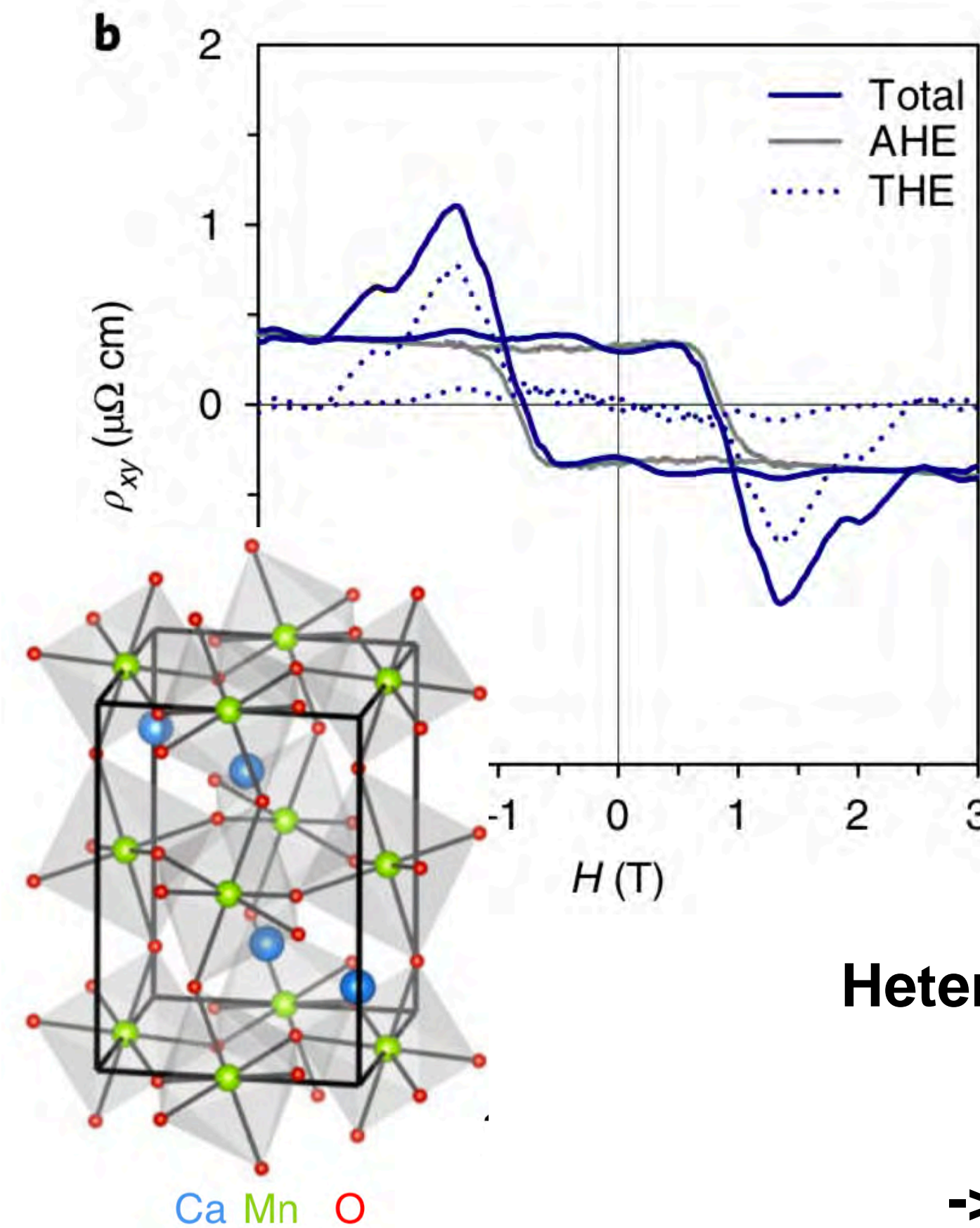
**Global crystal chirality controls Hall sign!**



Experiment: ~27 S/cm  
*Ghimire et al. Nat. Comm. (2018)*  
Our theory: ~80 S/cm



Experiment  
*Bibes et al. Nat. Phys. (2018)*  
Topological Hall effect?



Heterogenous ferromagnets with anomalous Hall with opposite polarities

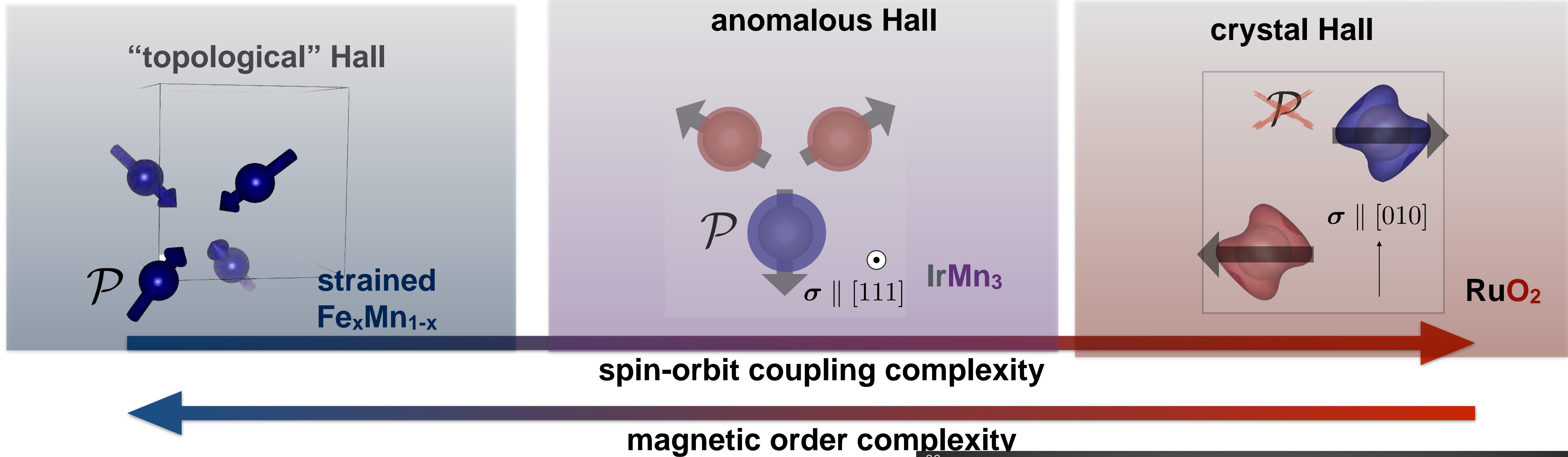
*Gerber, PRB (2018)*

-> crystal chiral domains -> Crystal Hall effect

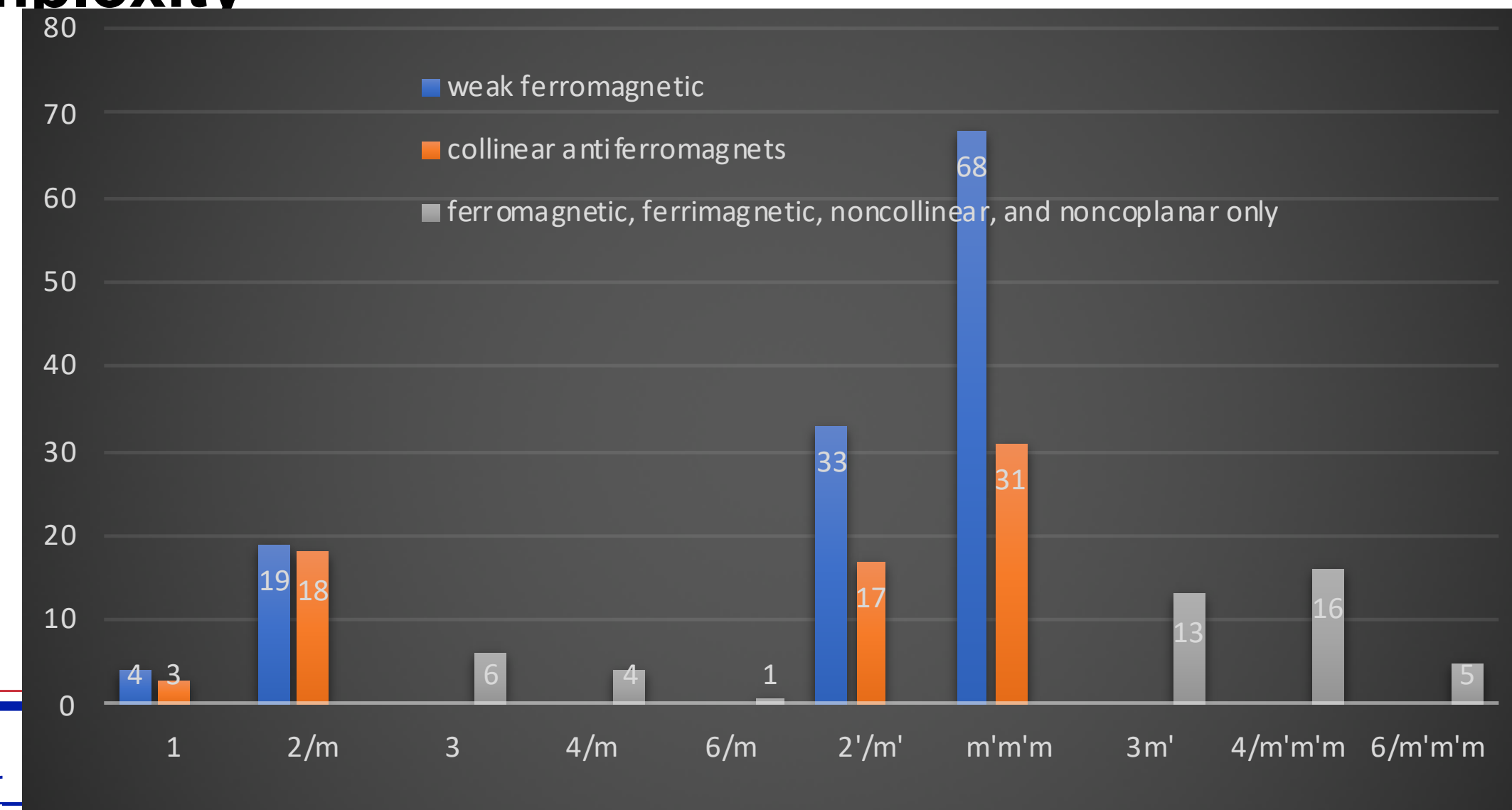


# Spontaneous Hall antiferromagnets

$$\rho_H = \rho_{THE} + \rho_{AHE} + \rho_{CHE}$$



- A. Allows Hall effect in abundant collinear antiferromagnets  $\sim 1/10!$
- B. Fundamental understanding (anisotropy, orbital, spin-orbit)
- C. Many more variants to come.



LS, R.Hernandez-Gonzales, TJ, JS, arXiv  
1901.00445 (2019)

<https://www.researchgate.net/profile/Libor>



*Thank you for your attention and looking forward for your questions!*

Ilja Turek, Karel Carva, Jakub Železný (Prague)  
Rafael Hernandez-Gonzalez, Kei Yamamoto,  
Olena Gomonay, Martin Jourdan, Stanislav Yu  
Bodnar, Mathias Kläui (Mainz)

Yuriy Mokrousov (Julich)  
Helena Reichlova, Dominik Kriegner (Dresden)  
Laura Mihalceanu (Kaiserslautern)  
Mazhar Ali (Halle), David Marsch (Gottingen)

$R^G$



**@LiborPhysics**