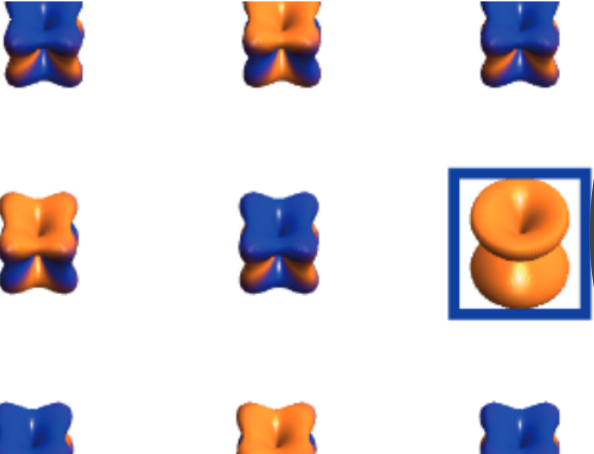
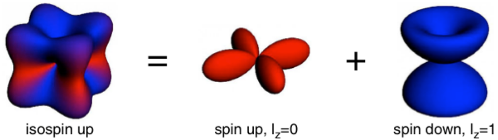
Universität Stuttgart



Maria Daghofer

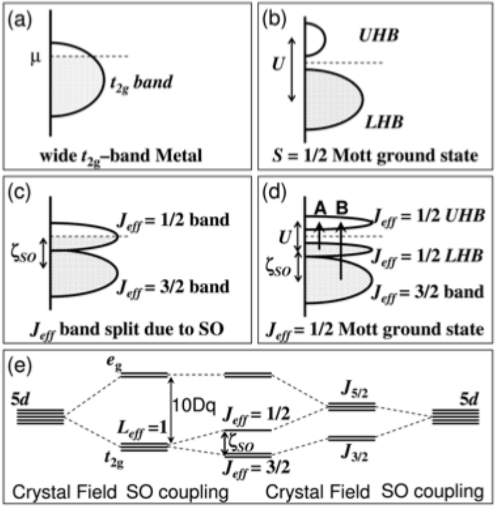
Excitonic magnetism in t_{2g}^4 systems

Spin-orbit coupling + correlations in iridates: 1 hole



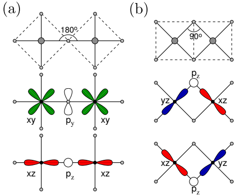
G. Jackeli and G. Khaliullin, PRL **102**, 017205 (2009)

- Three t_{2g} orbitals, two spins
- Spin-orbit coupling: $j = \frac{1}{2}$ and $j = \frac{3}{2}$ quartet
- Doublet has one hole

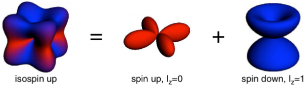


B. Kim *et al.*, PRL **101**, 076402 (2008)

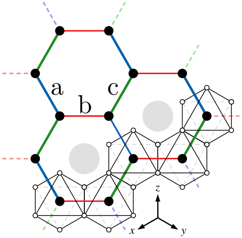
Spin-orbital Kitaev model



- $90°$ bond angle: Anisotropic couplings crucial
- honeycomb $\Rightarrow$ spin liquid
- variations thereon ...
- such bonds in A_2IrO_3, α-$RuCl_3$, $H_3LiIr_2O_6$



$$\mathcal{H} = J_K \sum_{\gamma} \sum_{\langle i,j \rangle \| \gamma} S_i^\gamma S_j^\gamma + J_H \sum_{\langle i,j \rangle} \mathbf{S}_i \mathbf{S}_j + \Gamma \sum_{\gamma} \sum_{\langle i,j \rangle \| \gamma} \left(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \right) + \ldots$$



G. Jackeli and G. Khaliullin, PRL **102**, 017205 (2009); J. Chaloupka, G. Jackeli, and G. Khaliullin, PRL **105**, 027204 (2010)

J.G. Rau *et al.*, PRL **112**, 077204 (2014)

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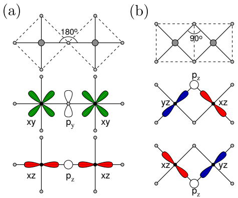

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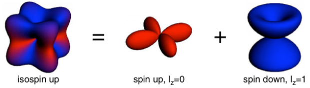

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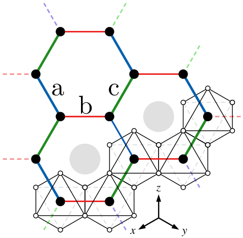

G. Jackeli and G. Khaliullin, PRL **102**, 017205 (2009); J. Chaloupka, G. Jackeli, and G. Khaliullin, PRL **105**, 027204 (2010)

J.G. Rau *et al.*, PRL **112**, 077204 (2014)

$j = \frac{1}{2}$ on many 3D and 2D lattices: well undestood and robust for square-lattice Sr_2IrO_4

Two holes with SOC

$\lambda \gg J_H$: j-j coupling

- two holes in $j = \frac{1}{2}$,
  $j = \frac{3}{2}$ filled
- band insulator
- J_H affects $j = \frac{3}{2}$

Two holes with SOC: jj vs. LS

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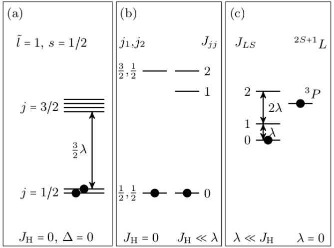

$J_H \gg \lambda$: L-S coupling

- two holes have $S_{tot} = 1$ and $L_{tot} = 1$
- λ acts on degenerate ^3P
- λ prefers $J = 0$

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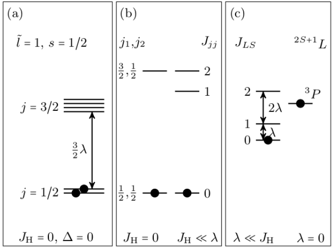

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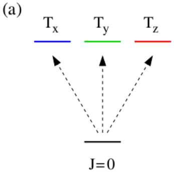

G. Khaliullin, PRL **111**, 197201 (2013)

$J_{LS} = 0$ scenario

- ionic ground state $J_{LS} = 0$
- superexchange can
  - induce triplet on both ions
  - let triplet and singlet exchange places
  - triplet $\hat{=}$ three flavors of hard-core bosons
- a bit like coupled singlet dimers

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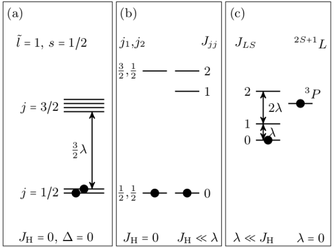

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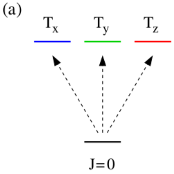

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Why would we want this?

G. Khaliullin, PRL **111**, 197201 (2013)

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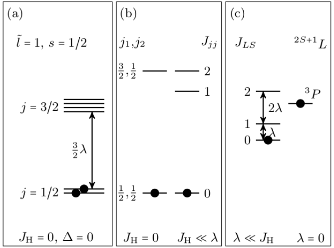

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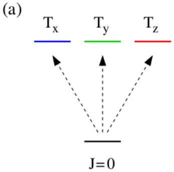

G. Khaliullin, PRL **111**, 197201 (2013)

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Why would we want this?

Where might this be relevant?

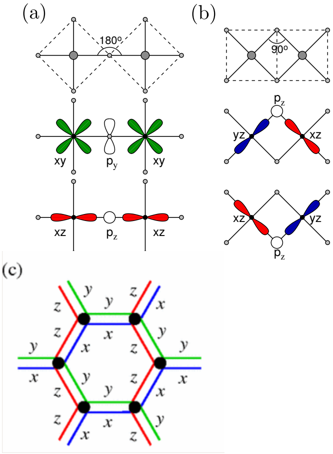
Boson Kitaev-Heisenberg model

Two-triplon terms: $c_J, c_K, c_\Gamma \approx 1$

$$H = \lambda \sum_{i,\alpha} n_{i,\alpha} + J \sum_{\langle i,j \rangle} \left(\vec{T}_i^\dagger \vec{T}_j - c_J \vec{T}_i^\dagger \vec{T}_j^\dagger + \text{H. c.} \right)$$
$$+ K \sum_{\alpha} \sum_{\langle i,j \rangle \| \alpha} \left(T_{i,\alpha}^\dagger T_{j,\alpha} - c_K T_{i,\alpha}^\dagger T_{j,\alpha}^\dagger + \text{H. c.} \right)$$
$$+ \Gamma \sum_{\substack{\alpha \neq \beta \neq \gamma \\ \alpha \neq \gamma}} \sum_{\langle i,j \rangle \| \alpha} \left(T_{i,\beta}^\dagger T_{j,\gamma} - c_\Gamma T_{i,\beta}^\dagger T_{j,\gamma}^\dagger + \text{H. c.} \right)$$

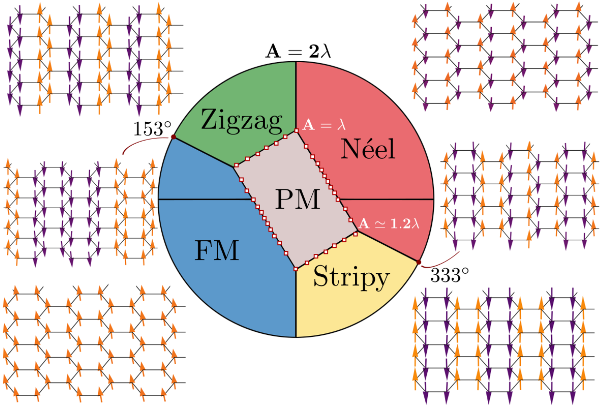
- Three- and four-triplon terms present
- Perturbation theory for $J_{\text{Hund}} = 0$:
  - only t via oxygen: $0 < J = -K$, $\Gamma = 0$
  - only direct t': $0 < J \ll K$, $\Gamma = 0$
  - $\Gamma \propto tt'$
- J_{Hund} promotes FM coupling $J < 0$

C. Svoboda, M. Randeria, N. Trivedi, PRB **95**, 014409 (2017)



G. Khaliullin, PRL **111**, 197201 (2013)

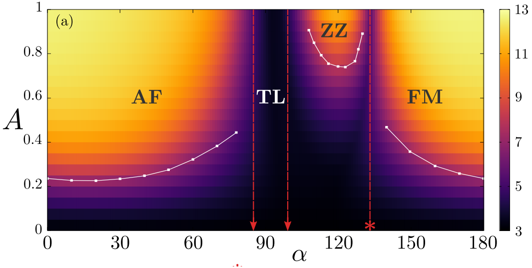
Classical Phase diagram on honeycomb



- center: large SOC suppresses triplons
- rim: strong superexchange overcomes SOC
- $J = A\cos\alpha$
- $K = 2A\sin\alpha$
- $\Gamma = 0$

- Ordered phases are those of classical Kitaev-Heisenberg(-Γ) model.
- Non-magnetic region for dominant λ
- Transition driven by $T^\dagger T^\dagger$ terms

Quantum Phase diagram from ED



- $J = A\cos\alpha,\ K = 2A\sin\alpha$
- Large $A \gg \lambda$: more classical than $j = \frac{1}{2}$ model

  D. Gotfryd *et al.*, PRB **95**, 024426 (2017)
- Keeps perfect symmetry $\alpha \to \alpha + 180$
- 1^{st}-order phase transitions at $K = -J$

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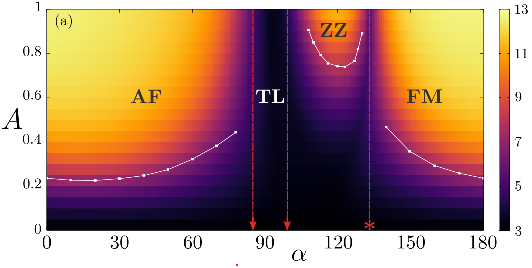

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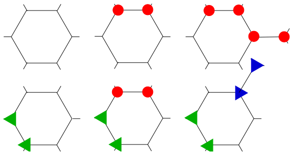
- 1^{st}-order phase transitions at $K = -J$

Regime without magnetic order at $J \approx 0$:

- magnetic correlations strictly short range at Kitaev point
- no other order found
- gapped: lower energy than dimer coverings $E < -K/2$
- non degenerate: order by disorder



P. S. Anisimov, F. Aust, G. Khaliullin, M. Daghofer, PRL **122**, 177201 (2019); J. Chaloupka and G. Khaliullin, arXiv:1910.00074

Excitations of the $J = 0$ regime

$$
\begin{aligned}
H =& \lambda \sum_{i,\alpha} n_{i,\alpha} + J \sum_{\langle i,j \rangle} \left(\vec{T}_i^\dagger \vec{T}_j - c_J \vec{T}_i^\dagger \vec{T}_j^\dagger + \text{H. c.} \right) \\
&+ K \sum_{\alpha} \sum_{\langle i,j \rangle \| \alpha} \left(T_{i,\alpha}^\dagger T_{j,\alpha} - c_K T_{i,\alpha}^\dagger T_{j,\alpha}^\dagger + \text{H. c.} \right) \\
&+ \Gamma \sum_{\substack{\alpha \neq \beta \neq \gamma \\ \alpha \neq \gamma}} \sum_{\langle i,j \rangle \| \alpha} \left(T_{i,\beta}^\dagger T_{j,\gamma} - c_\Gamma T_{i,\beta}^\dagger T_{j,\gamma}^\dagger + \text{H. c.} \right) \ ,
\end{aligned}
$$

stick to $\lambda \gg J, K, \Gamma \Rightarrow$ no magnetic order

Excitations of the $J = 0$ regime

$$H = \lambda \sum_{i,\alpha} n_{i,\alpha} + J \sum_{\langle i,j \rangle} \left(\vec{T}_i^\dagger \vec{T}_j - \cancel{c_J \vec{T}_i^\dagger \vec{T}_j^\dagger} + \mathrm{H.\ c.} \right)$$
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stick to $\lambda \gg J, K, \Gamma \Rightarrow$ no magnetic order

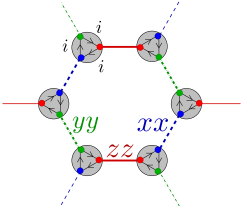
band structure for triplons:

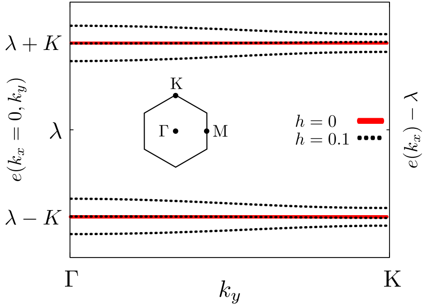
- once a triplon has been created, it can hop
- bands are here topologically trivial
- non-trivial intersite-dimer triplons known

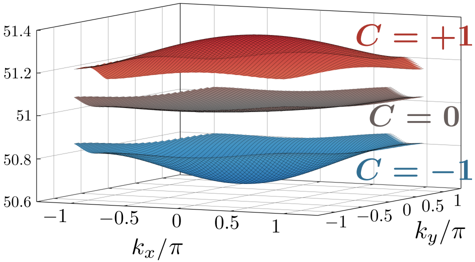
  J. Romhányi *et al.*, Nat. Commun. **6**, 6805 (2015); P. A. McClarty *et al.*, Nat. Phys. **13**, 736 (2017);
  D.G. Joshi and A.P.Schnyder, PRB **96**, 220405 (2017); **100**, 020407 (2019)

Triplons non-trivial in magnetic field

$$M_{j,x} = -i\sqrt{6}\left(T_{j,x} - T^\dagger_{j,x}\right) + ig_J\left(T^\dagger_{j,y}T_{j,z} - T^\dagger_{j,z}T_{j,y}\right)$$
$$\approx ig_J\left(T^\dagger_{j,y}T_{j,z} - T^\dagger_{j,z}T_{j,y}\right)$$



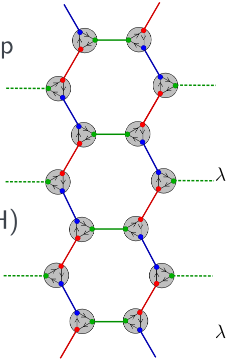




Two kinds of Edge States

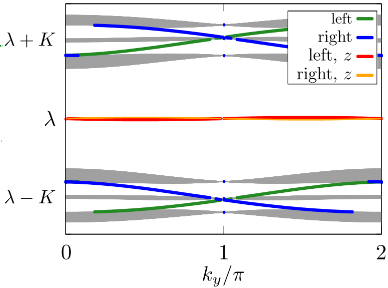
states at λ

- on edge sites, green flavor can't hop
- topological of a different sort
  G. van Miert *et al.*, 2D Mat. **4**, 015023 (2017); D. G. Joshi and A. P. Schnyder, PRB **100**, 020407 (2019)
- related to Su-Shrieffer-Heeger (SSH) chain; $Ba_2CuSi_2O_6Cl_2$?
  K. Nawa *et al.*, Nat. Comm., **10**, 2096 (2019)
- does not cross gap
- shifted with onsite energy

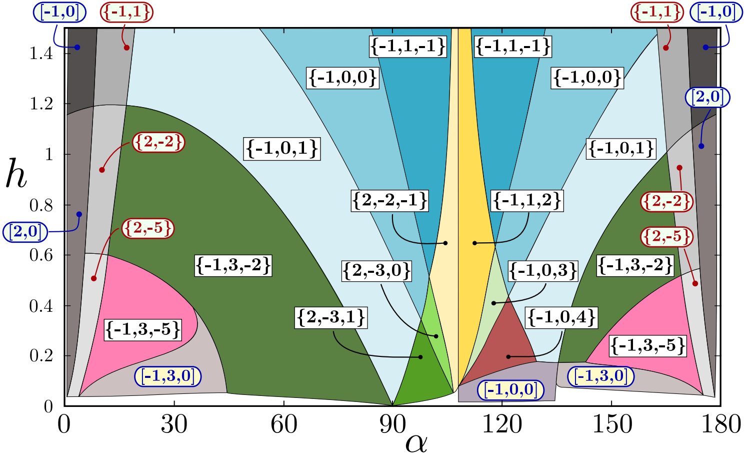


states at $\lambda \pm K$

- due to Chern number of bands
- crosses nontrivial band gap
- dissipationless transport?



Nontrivial triplon topology very robust



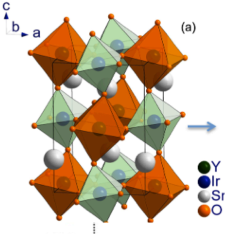
- $J = \cos\alpha$
- $K = \sin\alpha$
- $\vec{h} = (1,1,1)\cdot h$

P. S. Anisimov, F. Aust, G. Khaliullin, M. Daghofer, PRL **122**, 177201 (2019)

{-1,3,-2} $\rightarrow$ {-1,3,-2,2,-3,1}, {2,-5} $\rightarrow C_{1+2} = 2,\quad C_3 = -5,\quad C_4 = 5,\quad C_{5+6} = -2$

[2,0] $\rightarrow C_{1+2} = 2,\quad C_{3+4} = 0,$ [1,-1,0] $\rightarrow C_1 = 1,\quad C_2 = -1,\quad C_{3+4} = 0$

Where might $J = 0$ / $J = 1$ be relevant?



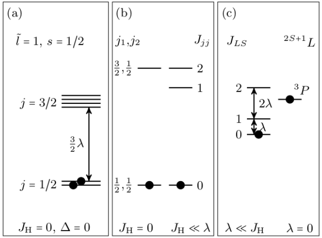
from G. Cau *et al.*, PRL **112**, 056402 (2014)

Ir based double perovskites

- spin-orbit coupling (SOC) large for Ir
- Ir far apart
- likely nonmagnetic

K. Pajskr *et al.*, PRB **93**, 035129 (2016), S. Fuchs *et al.*, PRL **120**, 237204 (2018)

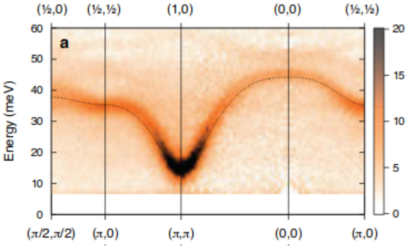
- very strong onsite singlet
- superexchange too weak to mix in triplet
- Picture applicable, but maybe a bit too robust



Where might this be relevant?

Ca_2RuO_4

- AFM order
- excitations well explained with $J = 0/J = 1$ scenario
- Esp. maximum at $(0, 0)$ rater than minimum



from A. Jain *et al.*, Nat. Phys. **13**, 633 (2017)

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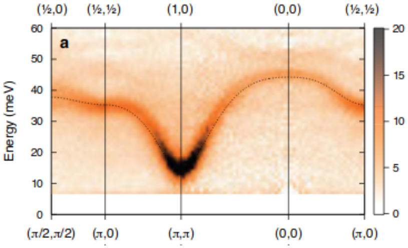
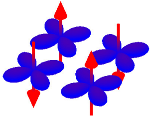


from A. Jain *et al.*, Nat. Phys. **13**, 633 (2017)

Spin-and-Orbital vs. Spinorbital

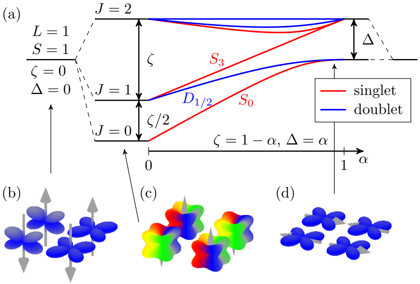
- OTOH: spin and orbital also decent description
  T. Mizokawa *et al.*, PRL **87**, 077202 (2001); M. Cuoco *et al.*, PRB **74**, 195124 (2006)
- *ab-initio* treatment: strong orbital polarization → spin $S = 1$ good description
  G. Zhang and E. Pavarini PRB **95**, 075145 (2017); D. Sutter *et al.*, Nat. Comm. **8**, 15176 (2017)

- correlations enhance SOC → shows up in superexchange
  G. Zhang and E. Pavarini PRB **95**, 075145 (2017)



Strongly correlated t_{2g} model with SOC

- Model studies: investigation of excitonic vs. 'normal spin' magnetism
  - Variational cluster approach applied to model for electrons
  - Exact-diagonalization spectra of Kugel-Khomskii-type model for spins and orbitals
- Ca_2RuO_4:
  - parameters from DFT (hopping, crystal field) and experiment (interactions, SOC)
  - one-particle spectra do not really show SOC, magnetic spectra do
  - excitonic despite orbital polarization



t_{2g} model on 2D square lattice

- Hopping (nearest neighbors or DFT derived)

$$H_{\text{kin}} = -t \sum_{\langle i,j\rangle,\sigma} c^\dagger_{i,xy,\sigma} c_{j,xy,\sigma} - t \left(\sum_{\langle i,j\rangle \| x,\sigma} c^\dagger_{i,xz,\sigma} c_{j,xz,\sigma} + \sum_{\langle i,j\rangle \| y,\sigma} c^\dagger_{i,yz,\sigma} c_{j,yz,\sigma} \right) ,$$

- tetragonal crystal field $\qquad H_\Delta = -\Delta \sum_{i,\sigma} n_{i,xy,\sigma}$
- spin-orbit coupling

$$H_{\text{SOC}} = \zeta \sum_i \vec{l}_i \cdot \vec{s}_i = \frac{i\zeta}{2} \sum_i \sum_{\substack{\alpha,\beta,\gamma \\ \sigma,\sigma'}} \varepsilon_{\alpha\beta\gamma} \tau^\alpha_{\sigma\sigma'} c^\dagger_{i,\beta,\sigma} c_{i,\gamma,\sigma'}$$

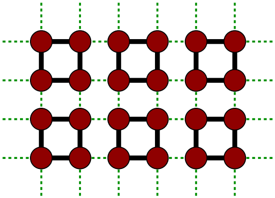
- onsite Coulomb and Hund's coupling

$$H_{\text{int}} = U \sum_{i,\alpha} n_{i\alpha\uparrow} n_{i\alpha\downarrow} + \frac{U'}{2} \sum_{i,\sigma} \sum_{\alpha\neq\beta} n_{i\alpha\sigma} n_{i\beta\bar{\sigma}}$$

$$+ (U' - J_H) \sum_{i,\sigma} \sum_{\alpha>\beta} n_{i\alpha\sigma} n_{i\beta\sigma} - J_H \sum_{i,\alpha\neq\beta} (c^\dagger_{i\alpha\uparrow} c_{i\alpha\downarrow} c^\dagger_{i,\beta\downarrow} c_{i\beta\uparrow} - c^\dagger_{i\alpha\uparrow} c^\dagger_{i\alpha\downarrow} c_{i\beta\downarrow} c_{i\beta\uparrow})$$

Variational Cluster Approximation

- Get free energy and one-particle Green function (GF) for small cluster → self energy Σ
- Plug Σ into GF of big system → GF and grand potential of big system
- Find optimal cluster self-energy ('self-energy approach')



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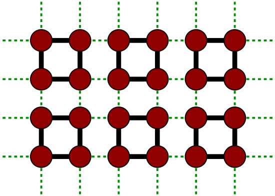

Symmetry-breaking field: $h' \sum_i \Lambda_i \mathrm{e}^{i\vec{Q}\vec{R}_i}$

- one-particle operator Λ_i, e.g. S^z_i, L^x_i, $n_{i,xy}$
- $\vec{Q} = (0,0)$, (π,π), $(\pi,0)$, $(0,\pi)$ strongly restricted by cluster

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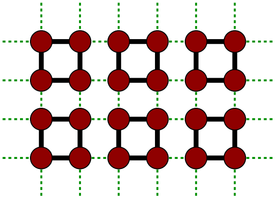

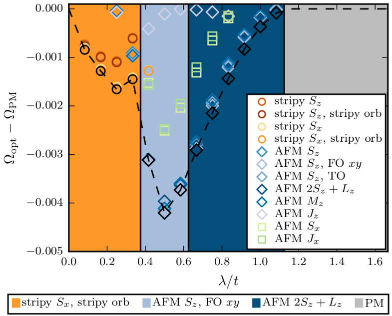

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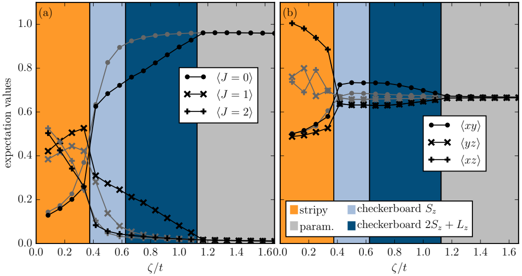
- important: $\vec{Q}$ and in-plane (x) vs. out-of-plane (z)
- less important: α in $S_i^{x/z} + \alpha L_i^{x/z}$

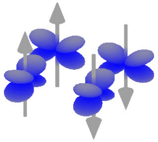
SOC ζ reduces onsite degeneracy: $J_{LS} = 0$ / $J_{LS} = 1$ order



complex stripy pattern:

out of nine-fold $L = 1$, $S = 1$ degeneracy

fits M. Cuoco *et al.*, PRB **74**, 195124 (2006)



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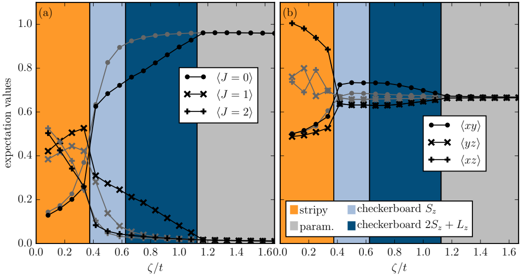

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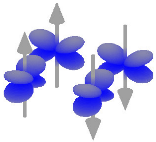

(excitonic) checkerboard AFM:

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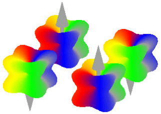

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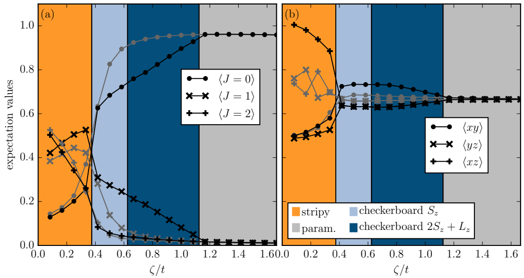

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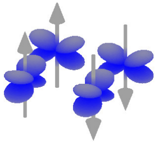

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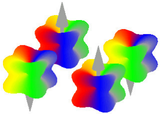

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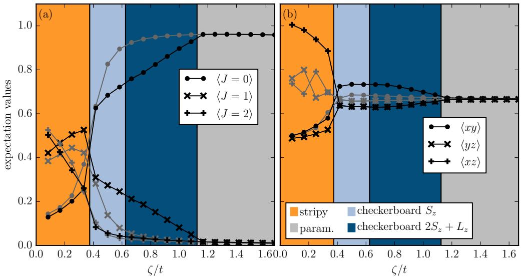

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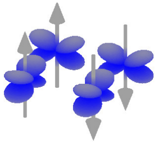

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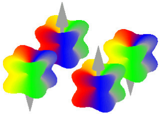
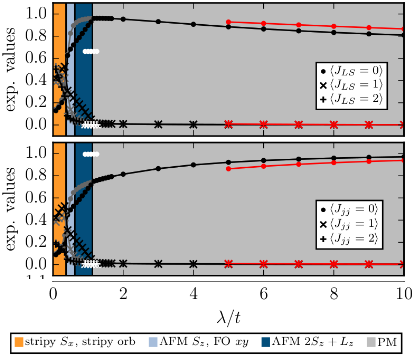


PM: ζ wins, $J = 0$ state

- grey symbols: no symmetry breaking
  $J = 0$ stabilized by ζ, soon dominates
- black symbols: optimized order
  weight shifted into $J = 1$
- $J = 2$ unimportant

L-S to j-j: correlated to uncorrelated



SOC $\lambda \gg J_H$ Hund coupling: j-j

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white: $J_H = 0$, red: $J_H = U/3$

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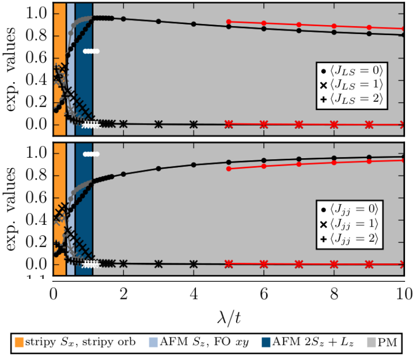

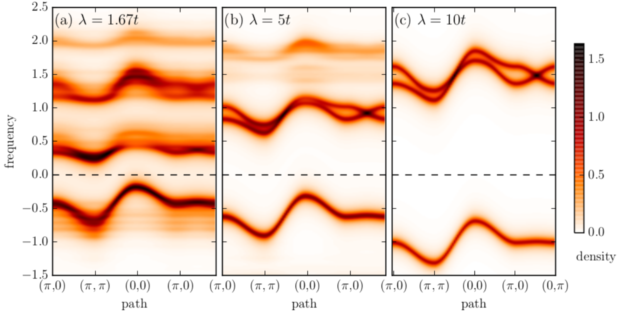

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correlated bands split by J_H

j-j: like non-interacting bands; both holes in $j = \frac{1}{2}$

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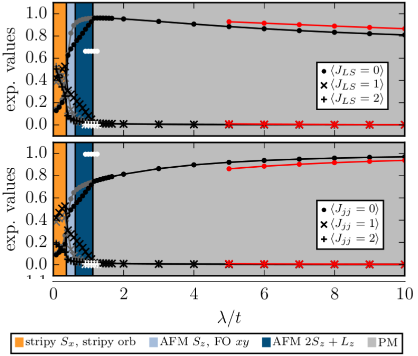

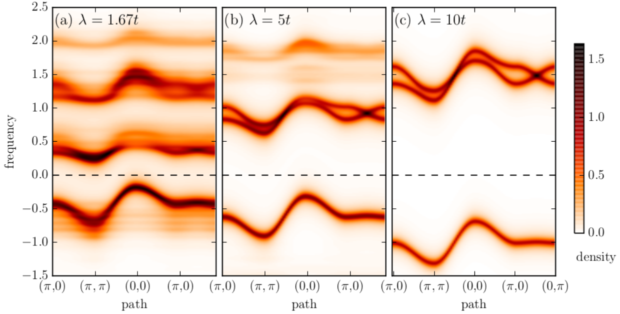

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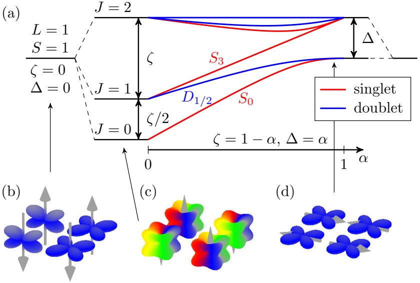
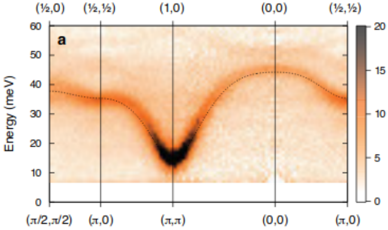
Sticking to correlated L-S regime here.

Crystal field vs. spin-orbit coupling



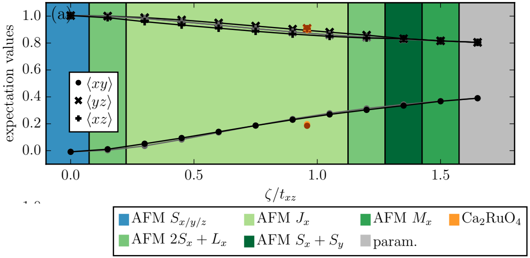
- $\Delta \approx 0$: excitonic limit with $J=0/J=1$
- $\zeta \approx 0$: Spin-1 with doubly occ. xy
- Ca_2RuO_4 somewhere in between
  - structural transition: xy almost doubly occupied (DFT+DMFT, ARPES)
  - Neutrons clearly show spin-orbit coupling

- Where in between is Ca_2RuO_4?
- Orbitally polarized $S=1$ with some spin-orbit corrections?
- Excitonic with some crystal-field corrections?



A. Jain *et al.*, Nat. Phys. **13**, 633 (2017)

SOC in the presence of a Strong Crystal Field



- SOC $\zeta = 0$: one hole on xz, one in $yz \Rightarrow$ polarized $S = 1$ with $n^h_{xy} = 0$, $n^h_{xz/yz} = 1$
- $\zeta \approx t$: pretty polarized with $n^h_{xy} \approx 0.2$, $n^h_{xz/yz} \approx 0.9$

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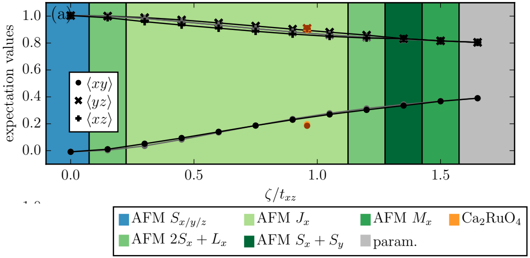

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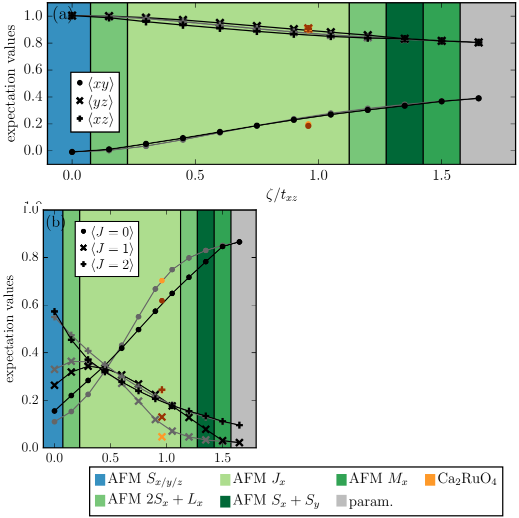

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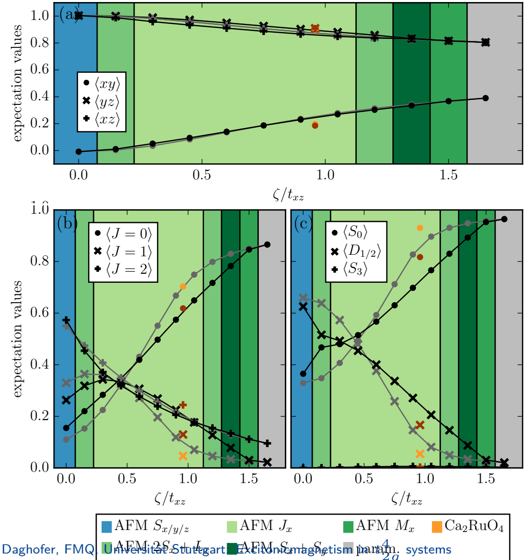

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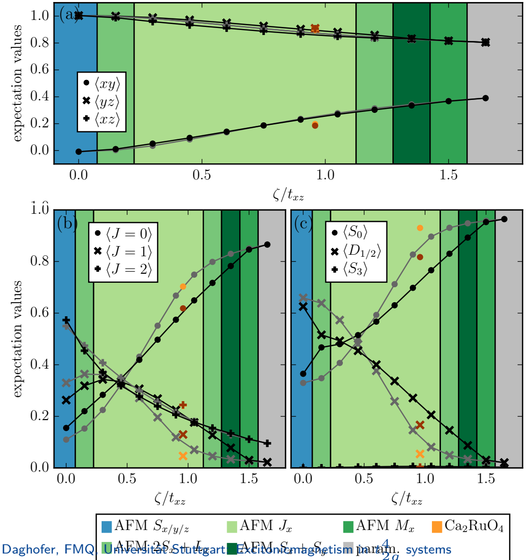

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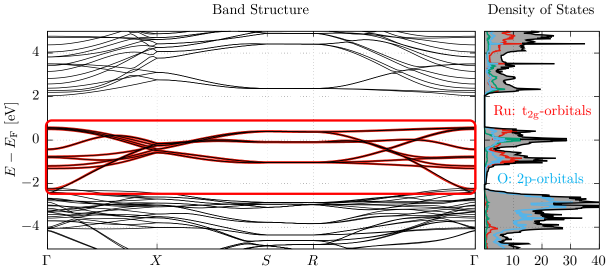
One-particle parameters for Ca_2RuO_4



- Wannier projection $\Rightarrow$ get t's and Δ

  suppl. mat. to J. Bertinshaw *et al.*, PRL **123**, 137204 (2019)
- get SOC $\zeta \approx 0.13$ eV, $U \approx 2$ eV and $J_H \approx 0.34$ eV from

  H. Gretarsson *et al.*, PRB **100**, 045123 (2019)
- plug into VCA $\Rightarrow$ red/orange symbols

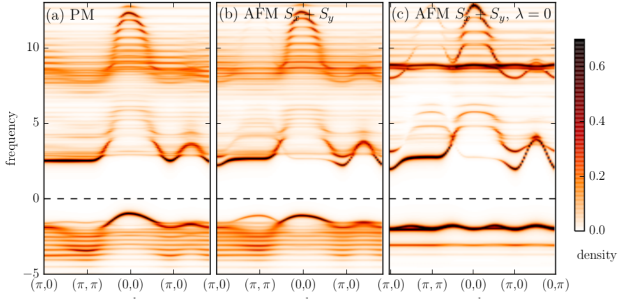
$$\epsilon_{xy,xy}(\vec{k}) = -2t_{xy}^{\mathrm{NN}}(\cos k_x + \cos k_y) - 4t_{xy}^{\mathrm{NNN}} \cos k_x \cos k_y - \Delta \ ,$$

$$\epsilon_{xz,xz}(\vec{k}) = -2t_{xz}^{\mathrm{NN}} \cos k_x, \quad \epsilon_{yz,yz}(\vec{k}) = -2t_{yz}^{\mathrm{NN}} \cos k_y,$$

$$\text{and} \quad \epsilon_{xz,xy}(\vec{k}) = -2t^{o} \cos k_x \ .$$

with $t_{xy}^{\mathrm{NN}} \approx 0.2$ eV, $t_{xz}^{\mathrm{NN}} = t_{yz}^{\mathrm{NN}} \approx 0.14$ eV, $t_{xy}^{\mathrm{NNN}} \approx 0.09$ eV, $t^{o} \approx 0.09$ eV

SOC small correction to $n_{xy/yz/yz}$ and ARPES



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  D. Sutter *et al.*, Nat. Comm. **8**, 15176 (2017); A. Kłosiński *et al.*, arXiv:1910.01605

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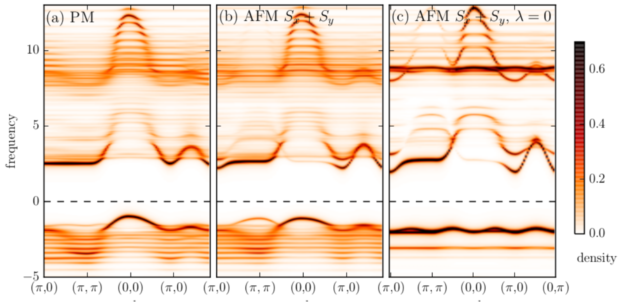

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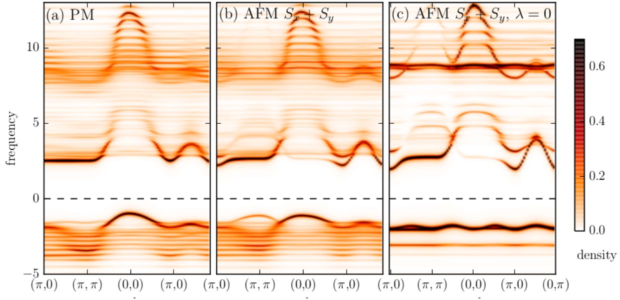

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What about magnetism?

Magnetic Hamiltonian

- VCA: $\vec{L} = 1$, $\vec{S} = 1$ valid description (9 states/site)
- 2nd order perturbation theory ⇒ Kugel-Khomskii-type model $H_{i,j} = H_{\vec{S}\cdot\vec{S}} + H_p + H_{\vec{T}\vec{T}}$

$$H_{\vec{S}\cdot\vec{S}} = J_{i,j}\left(\vec{S}_i\vec{S}_j - 1\right) \otimes |T_i;T_j\rangle\langle T_i;T_j| \,, \qquad J_{i,j} = \begin{cases} (t_\alpha^2 + t_\beta^2)\frac{U+J_H}{U(U+2J_H)} & \text{for } T_i = T_j = \gamma \\ \frac{t_\gamma^2(U+J_H)}{U(U+2J_H)} - \frac{J_H(t_\alpha^2+t_\beta^2)}{U(U-3J_H)} & \text{for } T_i = \alpha \neq T_j = \beta \text{ and } \alpha,\beta \neq \gamma \end{cases} \quad (1)$$

$$H_p = \left(\vec{S}_i\vec{S}_j - 1\right)\left[\sum_{\alpha\neq\beta}\frac{-t_\alpha t_\beta J_H}{U(U+2J_H)}|\alpha;\alpha\rangle\langle\beta;\beta|\right] . \quad (2)$$

$$H_{\vec{T}\vec{T}} = -\sum_{\beta\neq\gamma}\frac{t_\beta^2 + t_\gamma^2}{U-3J_H}|\beta;\gamma\rangle\langle\beta;\gamma| + \sum_{\beta\neq\gamma}\frac{t_\beta t_\gamma}{U(U-3J_H)}\left[2J_H + (U-J_H)\left(\vec{S}_i\vec{S}_j + 1\right)\right] \otimes |\beta;\gamma\rangle\langle\gamma;\beta| \, . \quad (3)$$

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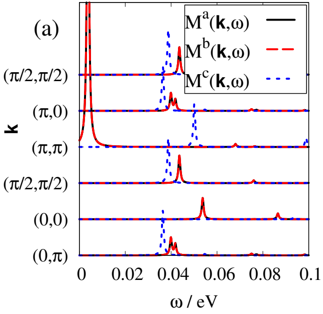
- onsite part: $H_{\text{ion}} = \frac{\zeta}{2}\vec{S}\vec{L} + \Delta(L^z)^2$

Magnetic excitations spectra

$$M^{\alpha}(\vec{k},\omega) = -\frac{1}{\pi}\Im\langle\phi_0|M^{\alpha}(-\vec{k})\frac{1}{\omega - H + ß0^+}M^{\alpha}(\vec{k})|\phi_0\rangle\ , \qquad (4)$$

from ED for 8-site cluster, tetragonal '*ab-initio*' parameters ($t^o = 0$)



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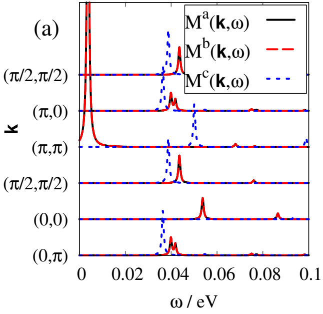

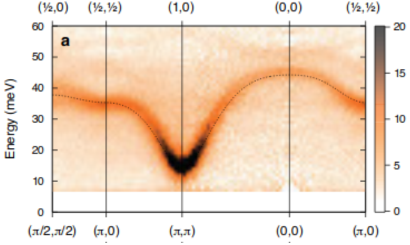

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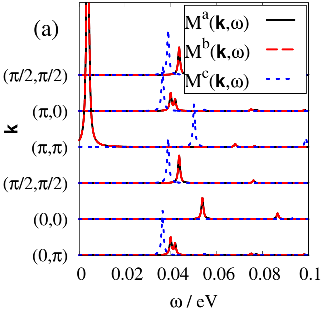

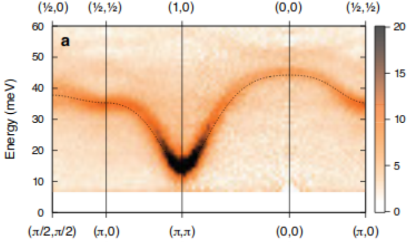

from A. Jain *et al.*, Nat. Phys. **13**, 633 (2017)

good agreement, especially max at $(0,0)$, no fitting parameters

Magnetic excitations spectra with in-plane anisotropy

- before: $t_{xy}^{\mathrm{NN}} \approx 0.2$ eV, $t_{xz}^{\mathrm{NN}} = t_{yz}^{\mathrm{NN}} \approx 0.14$ eV, $t_{xy}^{\mathrm{NNN}} \approx 0.09$ eV, $\Delta \approx 0.25$ eV, $t^o = 0$; finite $t^o = 0.09$ made spin prefer $b = (x - y)$ direction
- now: add $\delta \frac{1}{2}(S^x - S^y)^2$ to mimic t^o and gap at (π, π)

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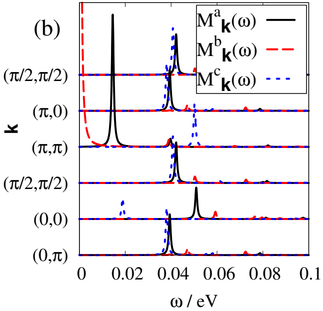
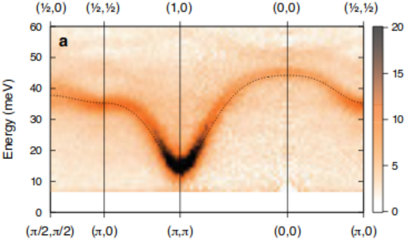
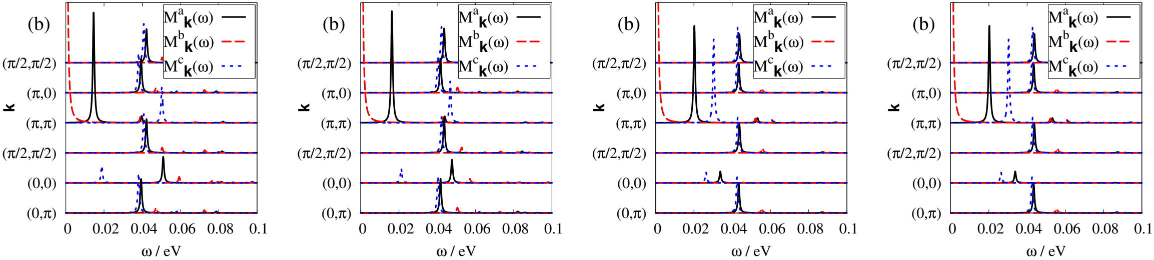


transverse modes + amplitude mode M^b



from A. Jain *et al.*, Nat. Phys. **13**, 633 (2017)

Max at $(0,0)$ needs some (small) n_{xy}^h



$\Delta = 0.25$ eV, $n_{xy}^h \approx 0.25$

$\Delta = 0.3$ eV, $n_{xy}^h \approx 0.18$

$\Delta = 0.3$ eV, $n_{xy}^h \approx 0.05$

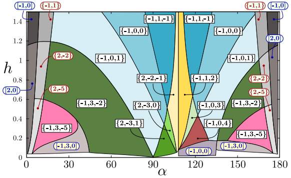
$\Delta = 0.75$ eV, $n_{xy}^h \approx 0.02$

- strong orbital polarization ($n_{xz,yz}^h \approx 0.88$ vs. $n_{xy}^h \approx 0.25$)
- quite sensitive to n_{xy}^h
- Second-order superexchange model + SOC + Δ works rather well

Thanks

- Pavel Anisimov, Friedemann Aust
- Teresa Feldmaier, Pascal Strobel
- Michael Schmid, Philipp Hansmann
- (many) discussions with Giniyat Khaliullin, George Jackeli, Hide Takagi

Excitonic antiferromagnetism in d^4 systems



Triplons of honeycomb $J = 0$ phase

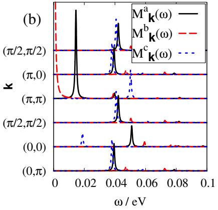
- nontrivial triplons all over parameter space
- same ordered magnetic phases as $j = \frac{1}{2}$; triplon liquid

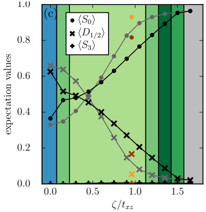
P. S. Anisimov, F. Aust, G. Khaliullin, M. Daghofer, PRL **122**, 177201 (2019)

Square-lattice models, Ca_2RuO_4

- magnetic ordering via weight transfer $J = 0 \to J = 1$
- valid for magnetic transition even at strong crystal fields/orbital polarization
- Kugel-Khomskii–like model + SOC + Δ give neutron-scattering spectra

T. Feldmaier, P. Strobel, M. Schmid, P. Hansmann, M. Daghofer, arXiv:1910.13977





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