3D-2D equivalence of ordered states on harmonic honeycomb lattices

Lukas Janssen
(TU Dresden)

Matthias Vojta

Wilhelm G. F. Krüger

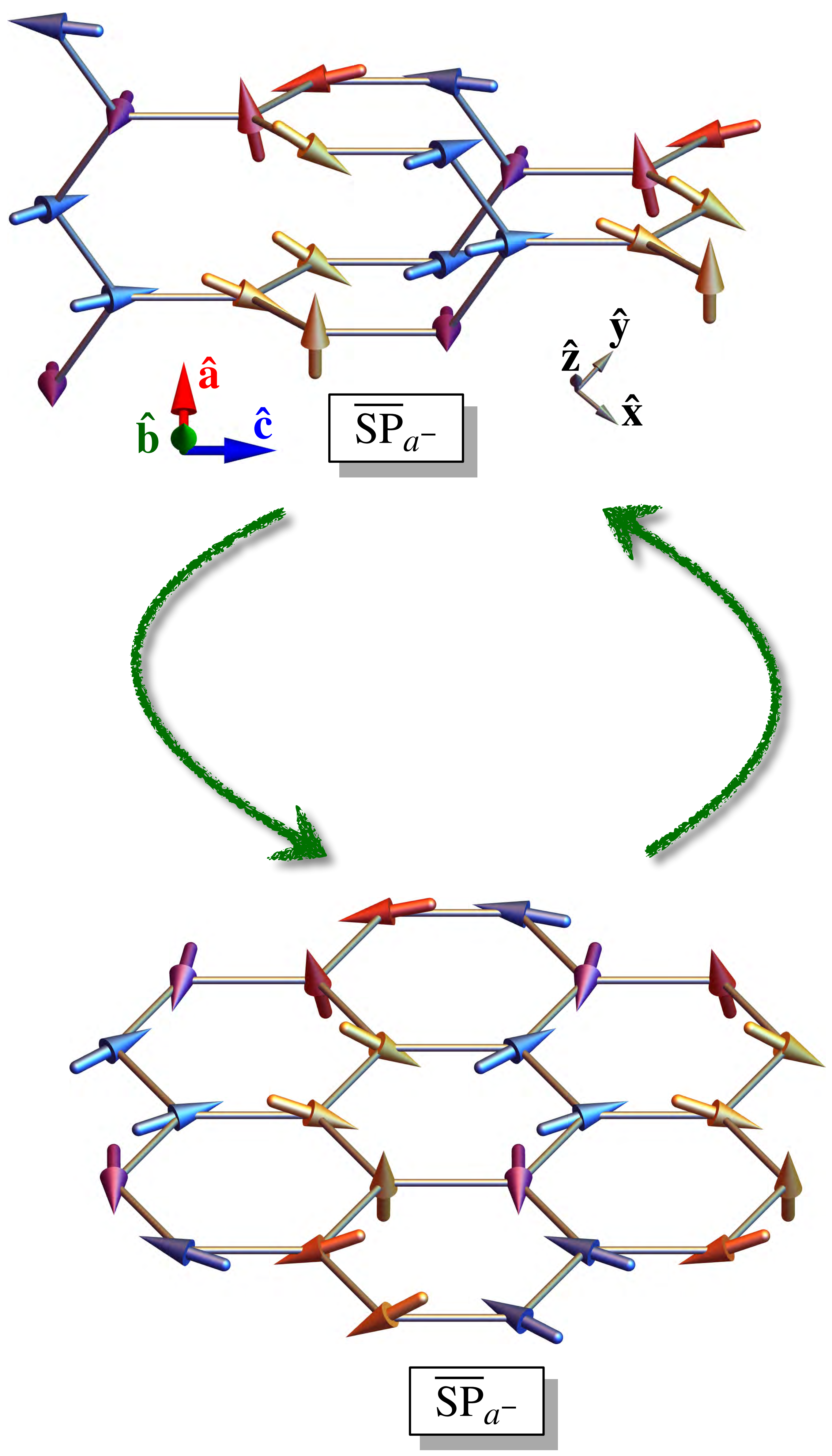
SFB 1143

ct.qmat
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

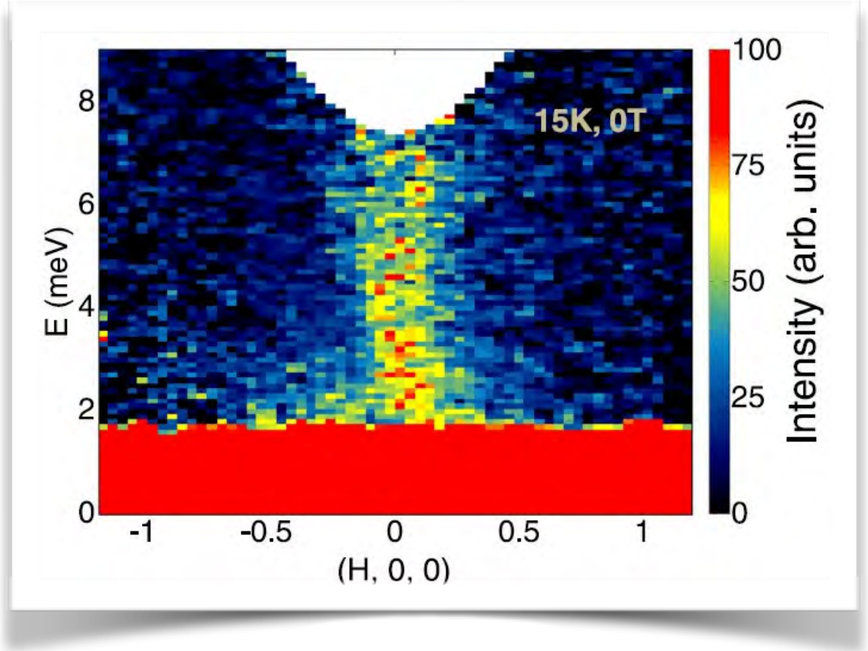
Quantum Critical Matter
JA 2306/4-1
Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Outline

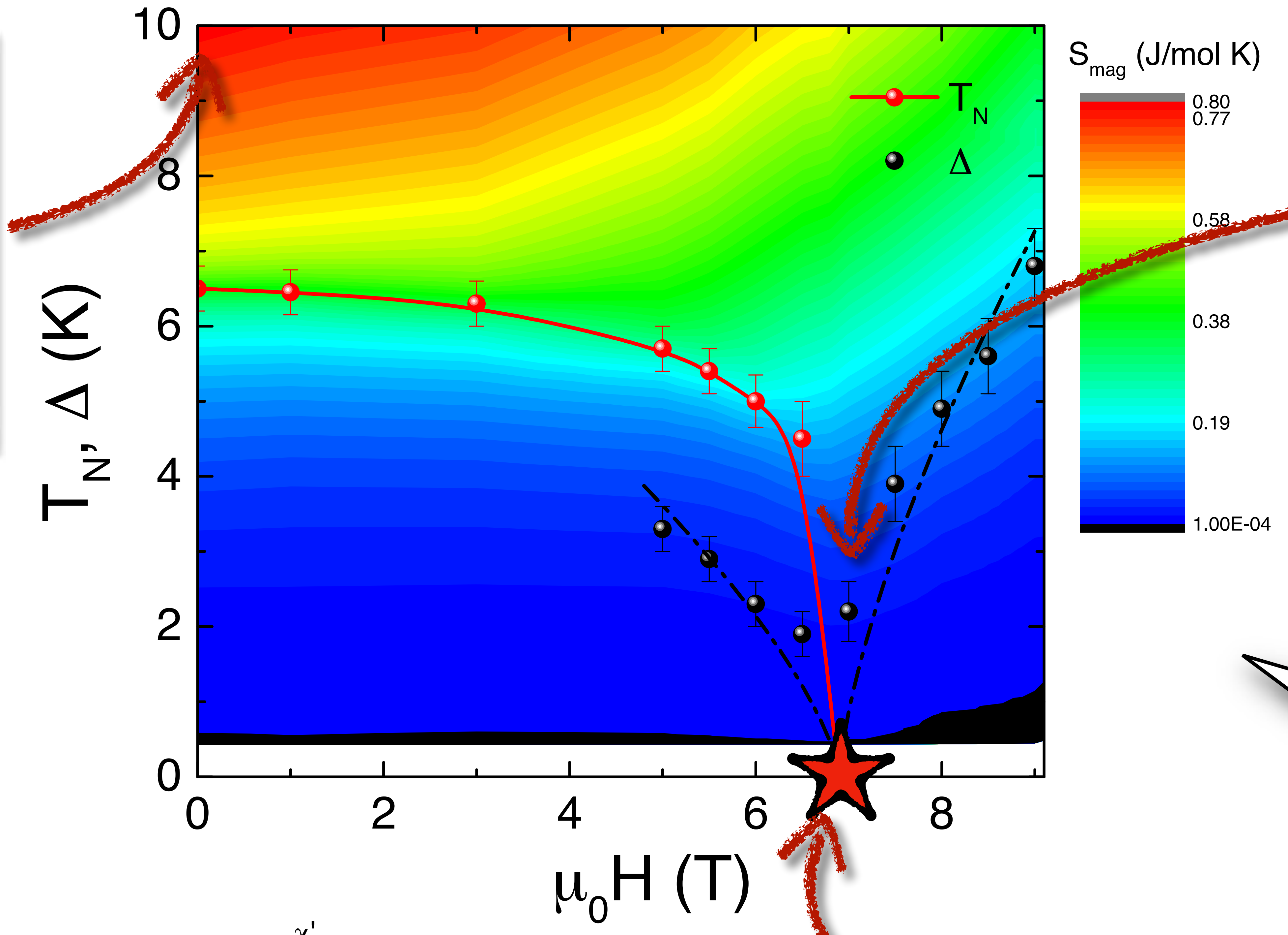
1. Introduction
2. 3D-2D mapping
3. Heisenberg-Kitaev-Γ models on the hyperhoneycomb lattice
4. Quantum effects
5. Conclusions



Honeycomb Kitaev materials

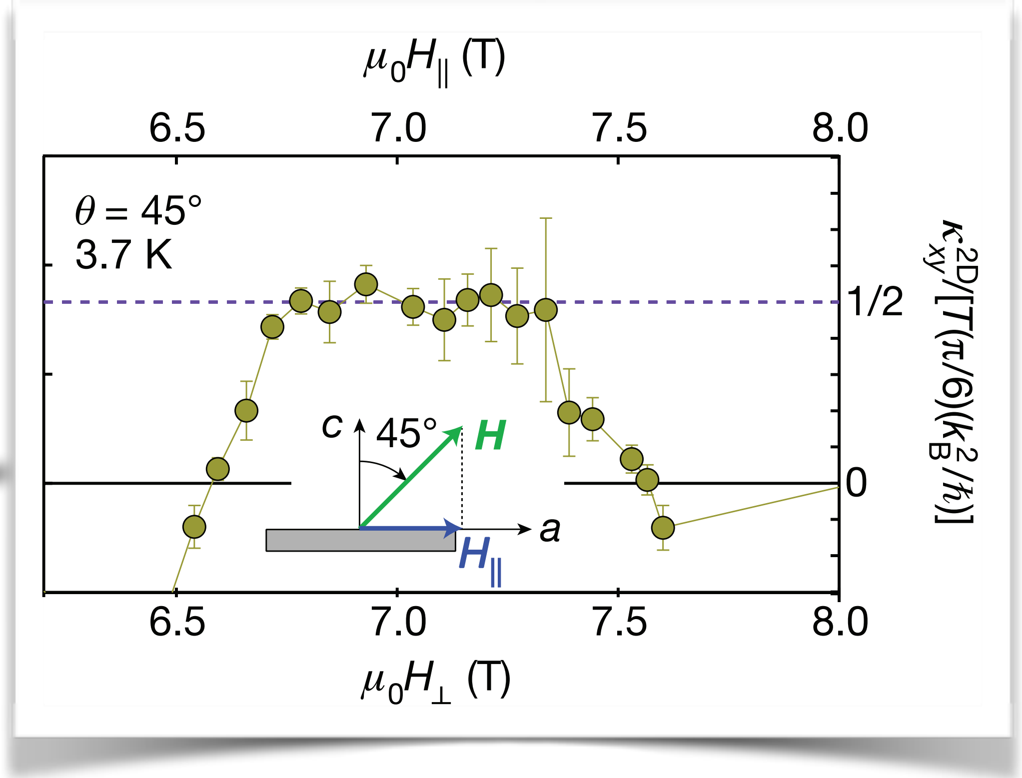


[Banerjee *et al.* '16; '17]

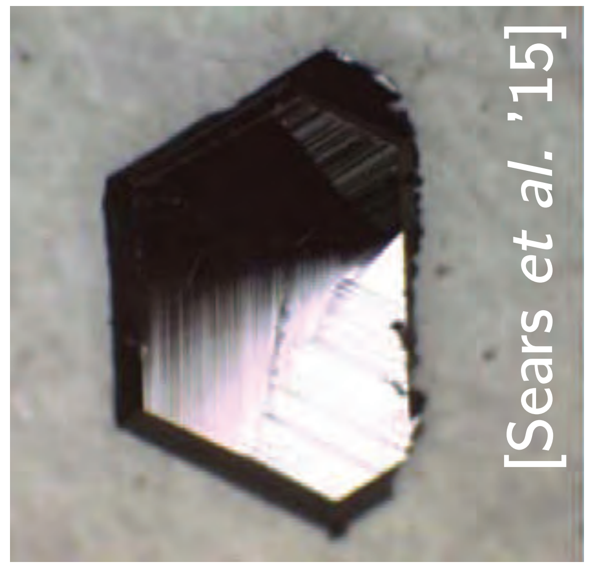


QCP

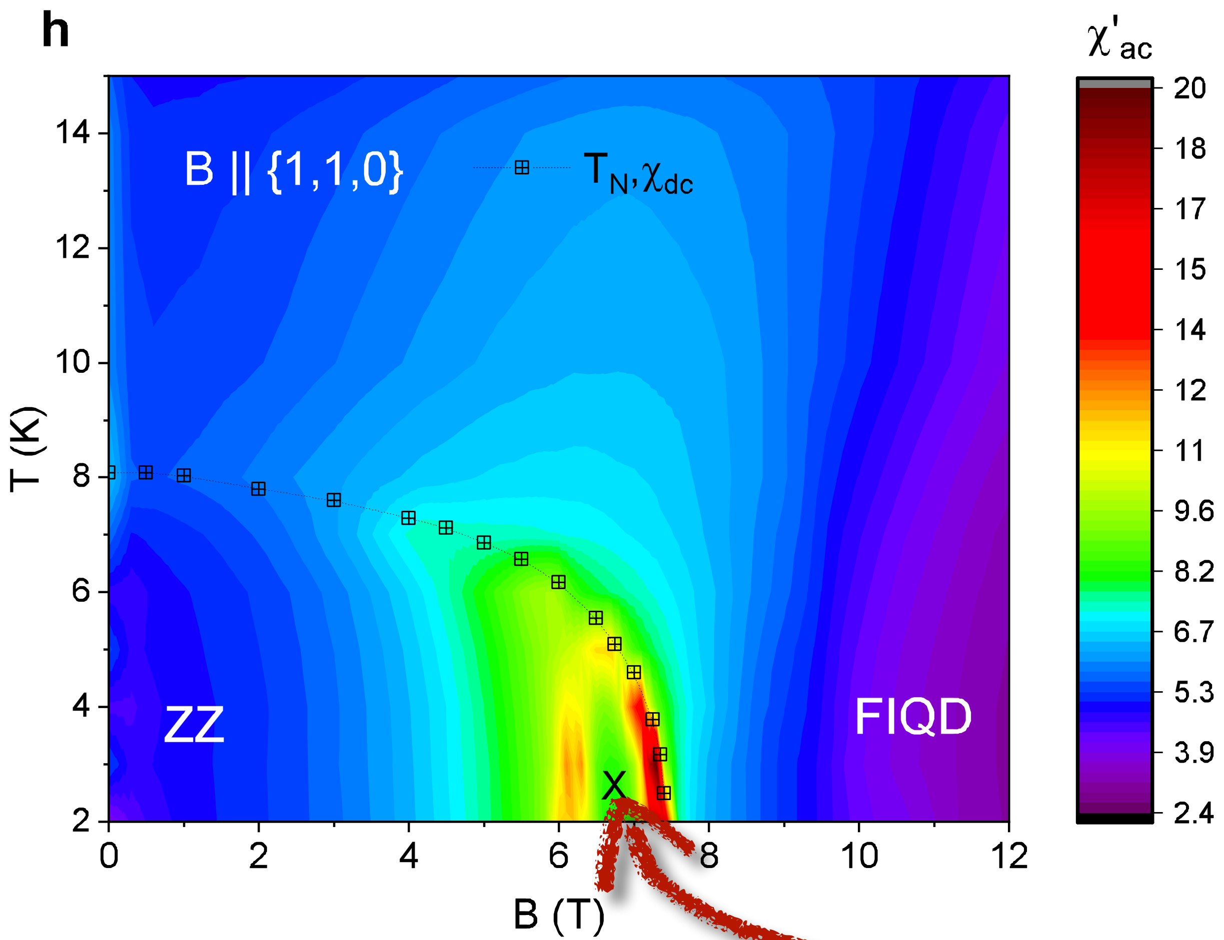
[Wolter, Corredor, LJ, *et al.* '17]



[Kasahara *et al.* '18]



α-$RuCl_3$



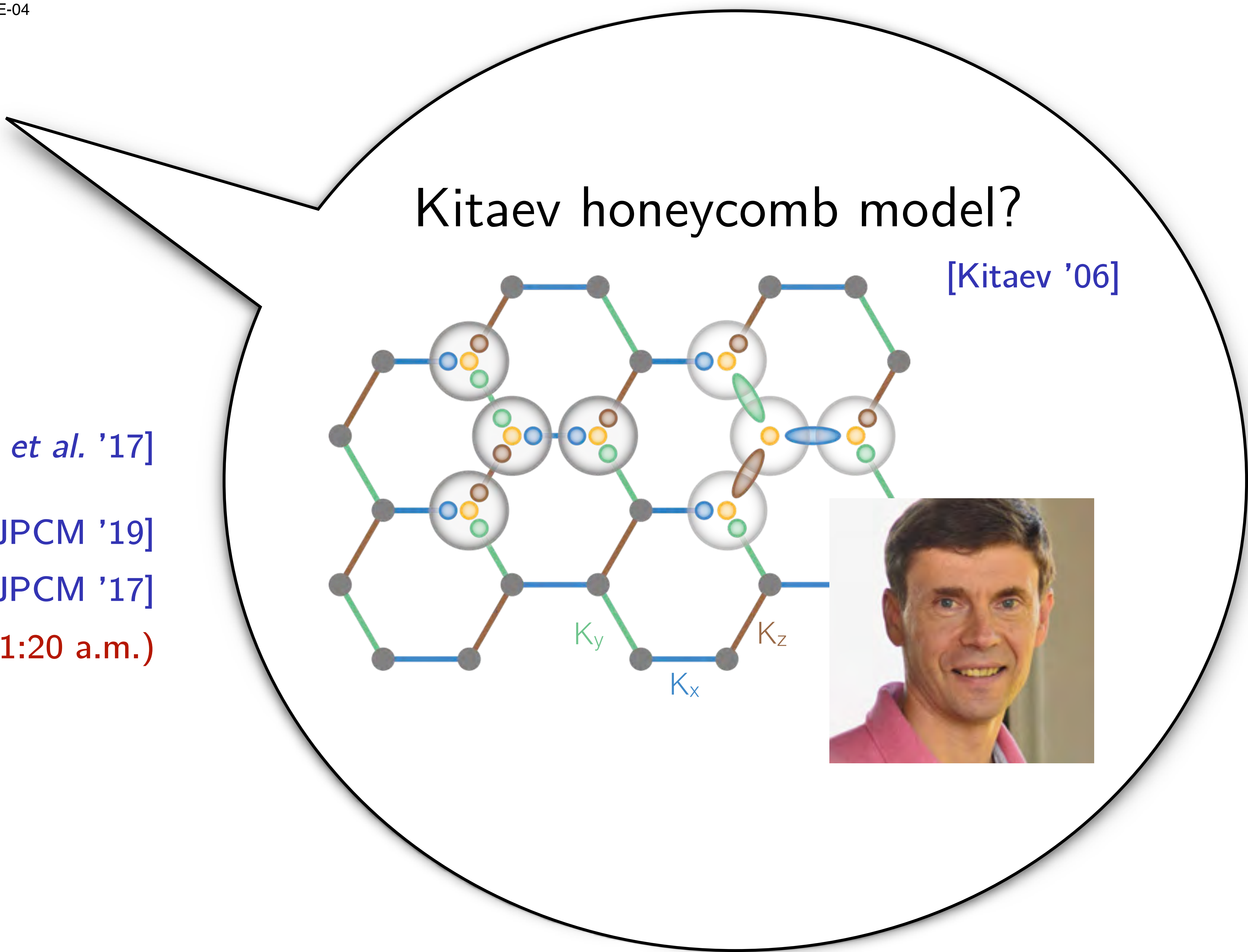
[Lampen-Kelley, LJ, *et al.* '18]

intermediate ordered phase

Topical Reviews: [LJ & Vojta, JPCM '19]

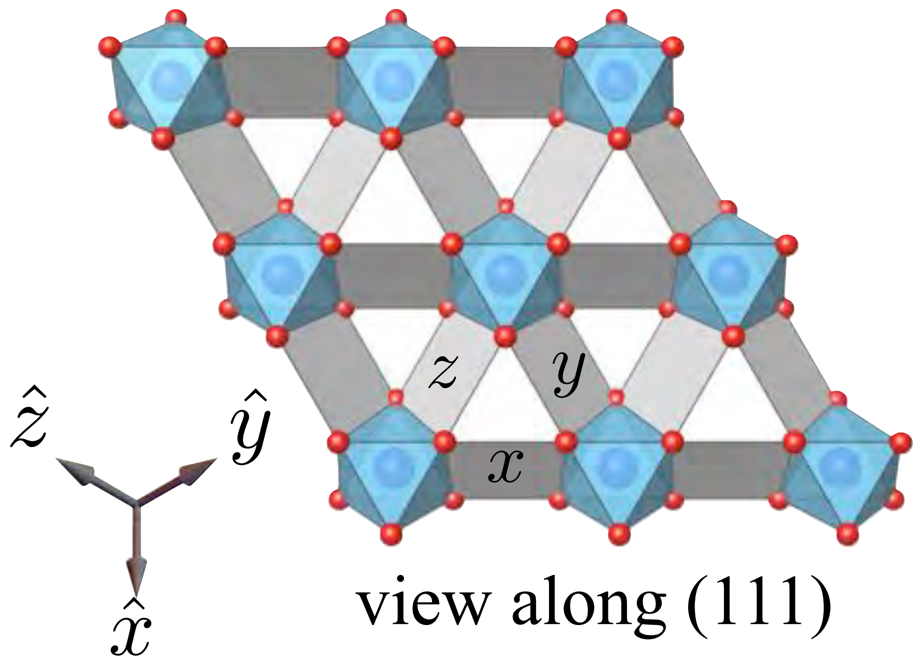
[Winter *et al.*, JPCM '17]

→ Talk by S. Winter (11:20 a.m.)

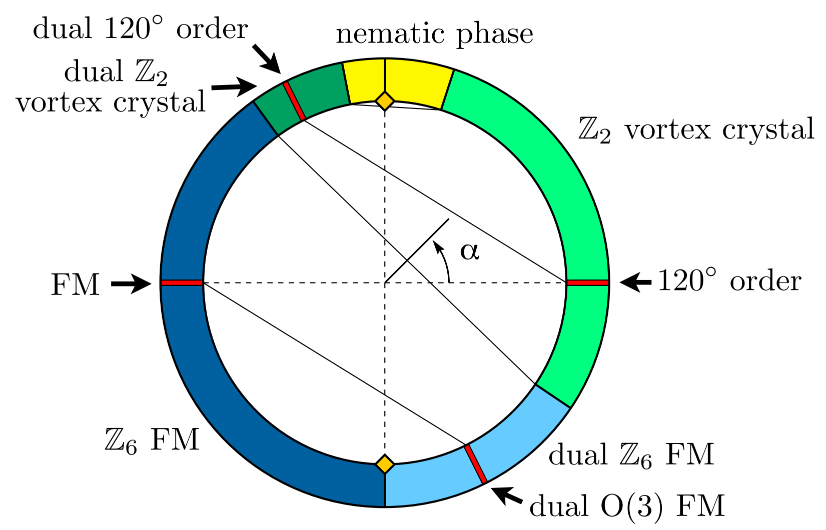


Other lattices?

2D: triangular ($Ba_3Ir_xTi_{3\text{-}x}O_9$, ...)



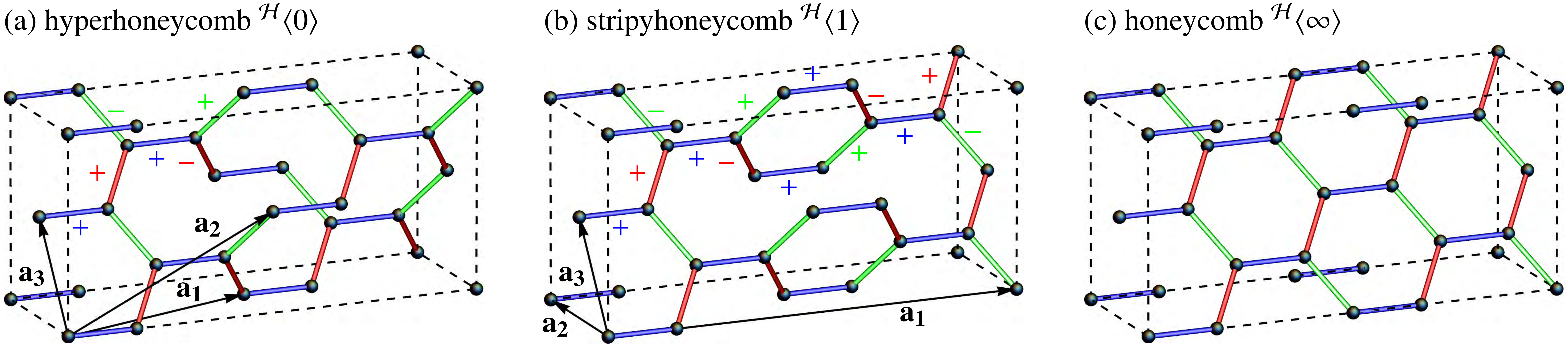
Heisenberg-Kitaev model:



... no (soluble) Kitaev limit

[Becker *et al.* '15]
[Rousochatzakis *et al.* '16]
Review: [Trebst '17]

3D:

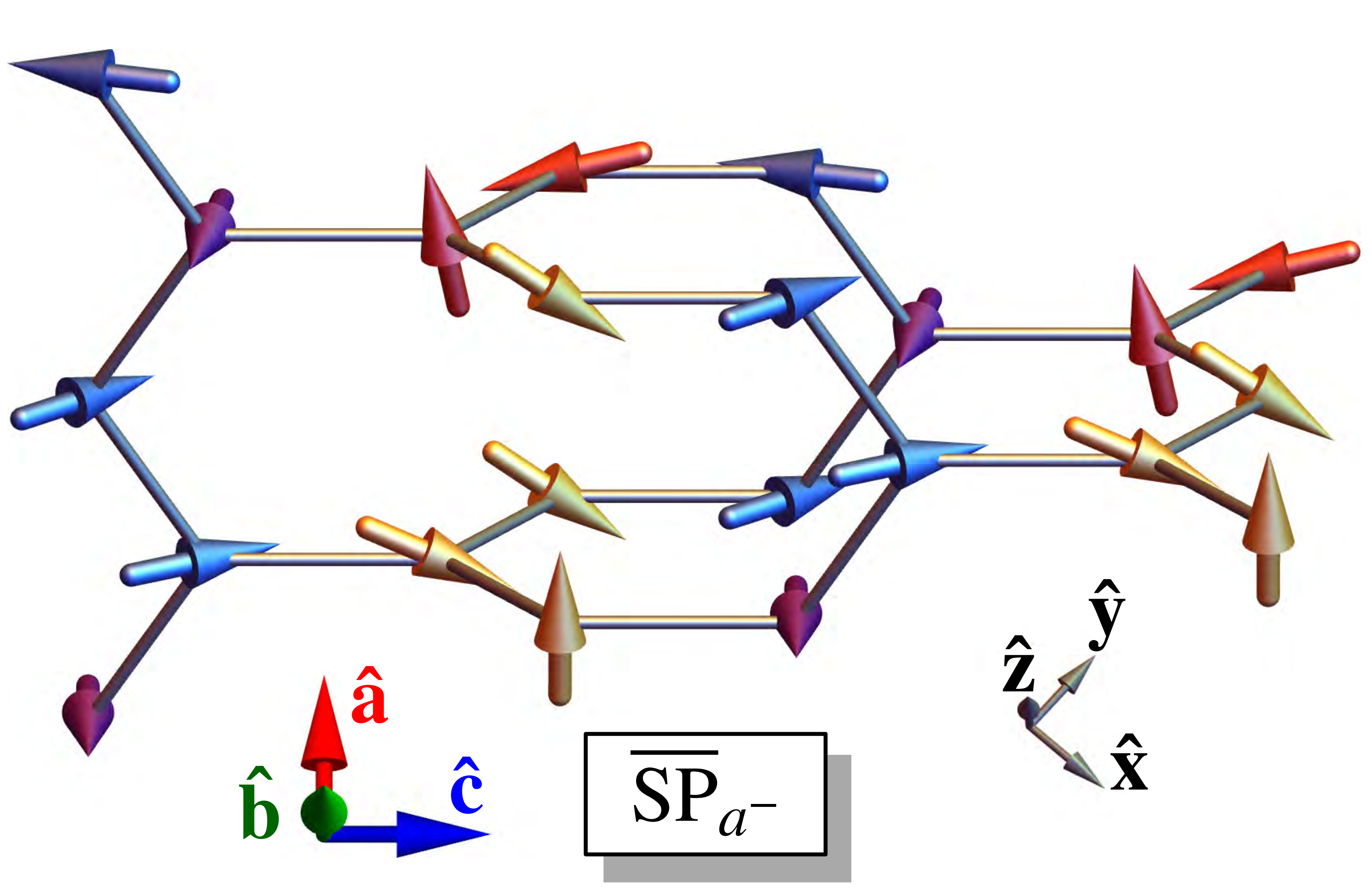


β-Li_2IrO_3, ... γ-Li_2IrO_3, ... α-Li_2IrO_3, α-$RuCl_3$, ...

[Modic *et al.*, '14]
[Kimchi *et al.* '14]

Li_2IrO_3: Magnetic order

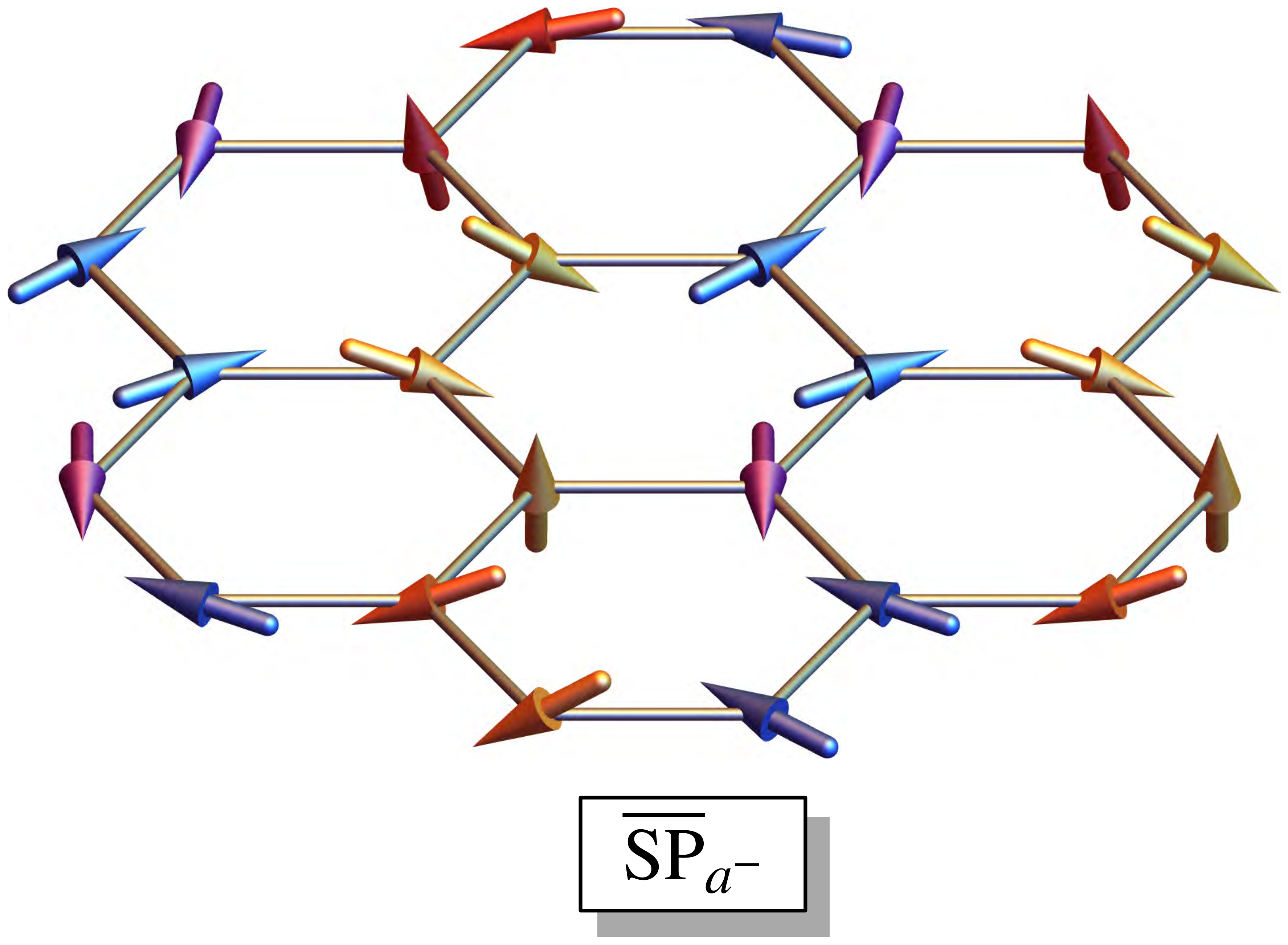
β-Li_2IrO_3



incommensurate spiral

[Biffin *et al.* '14]

α-Li_2IrO_3



incommensurate spiral

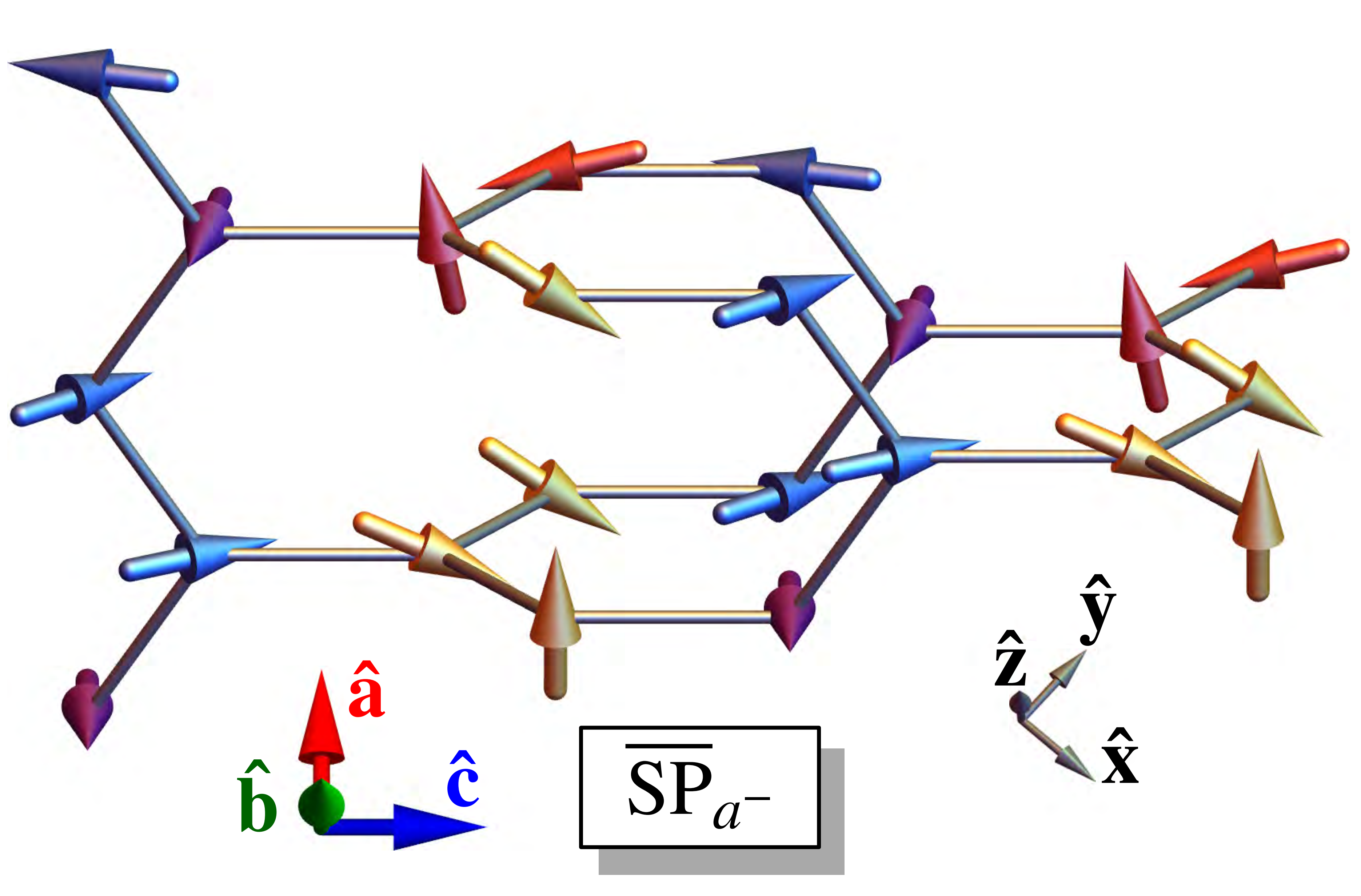
[Williams *et al.* '16]

... and γ-Li_2IrO_3 as well

[Modic *et al.* '14]

Li_2IrO_3: Magnetic order

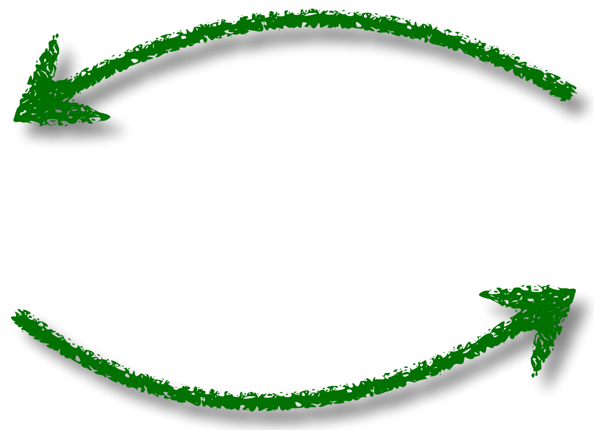
β-Li_2IrO_3



incommensurate spiral

[Biffin *et al.* '14]

3D-2D equivalence



Classical energies ✓

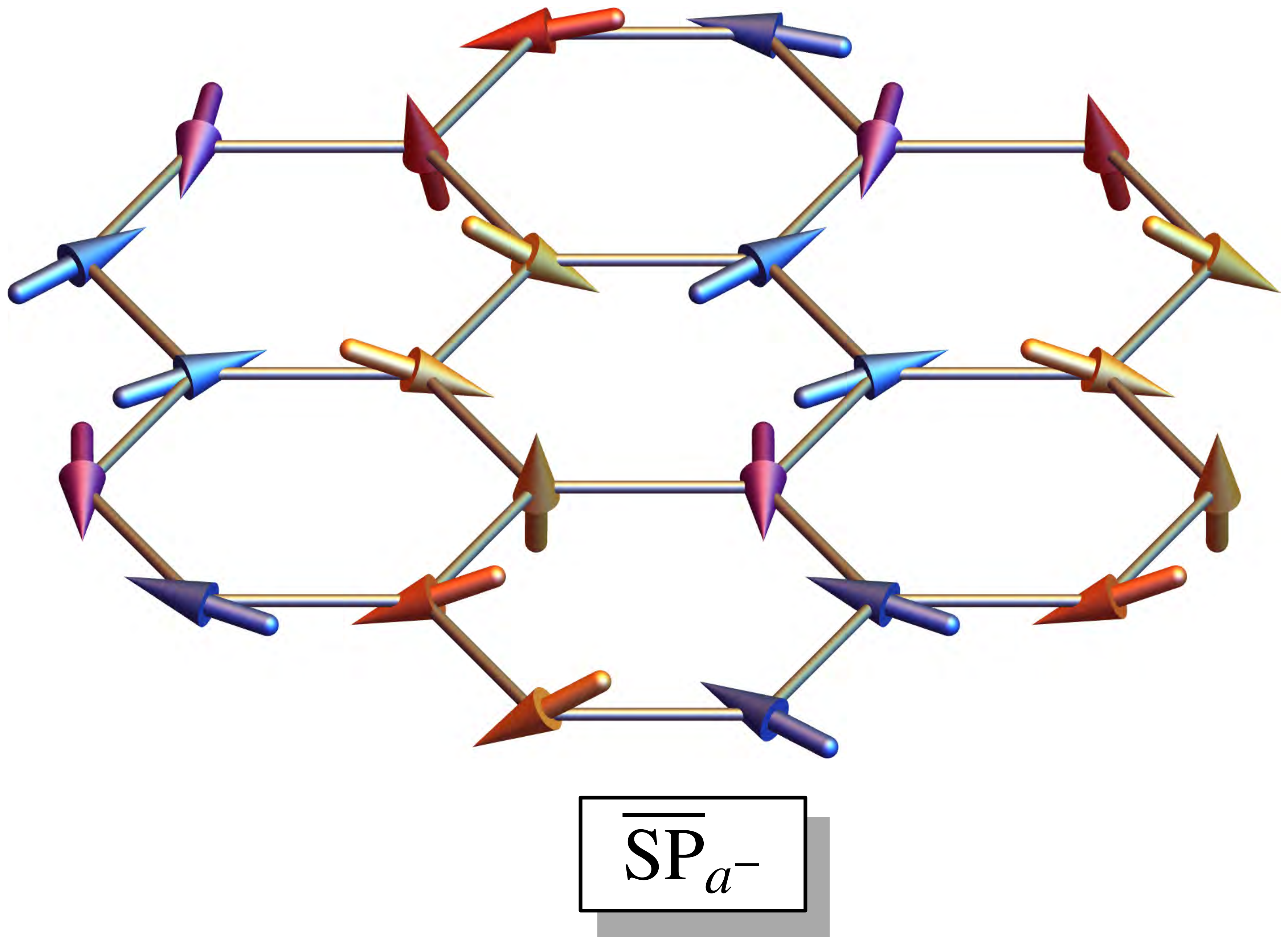
Phase diagrams ✓

Direction of moments ✓

Magnon bands (✓)

Quantum effects ✗

α-Li_2IrO_3



incommensurate spiral

[Williams *et al.* '16]

Coordination number: **3**

… and γ-Li_2IrO_3 as well

[Modic *et al.* '14]

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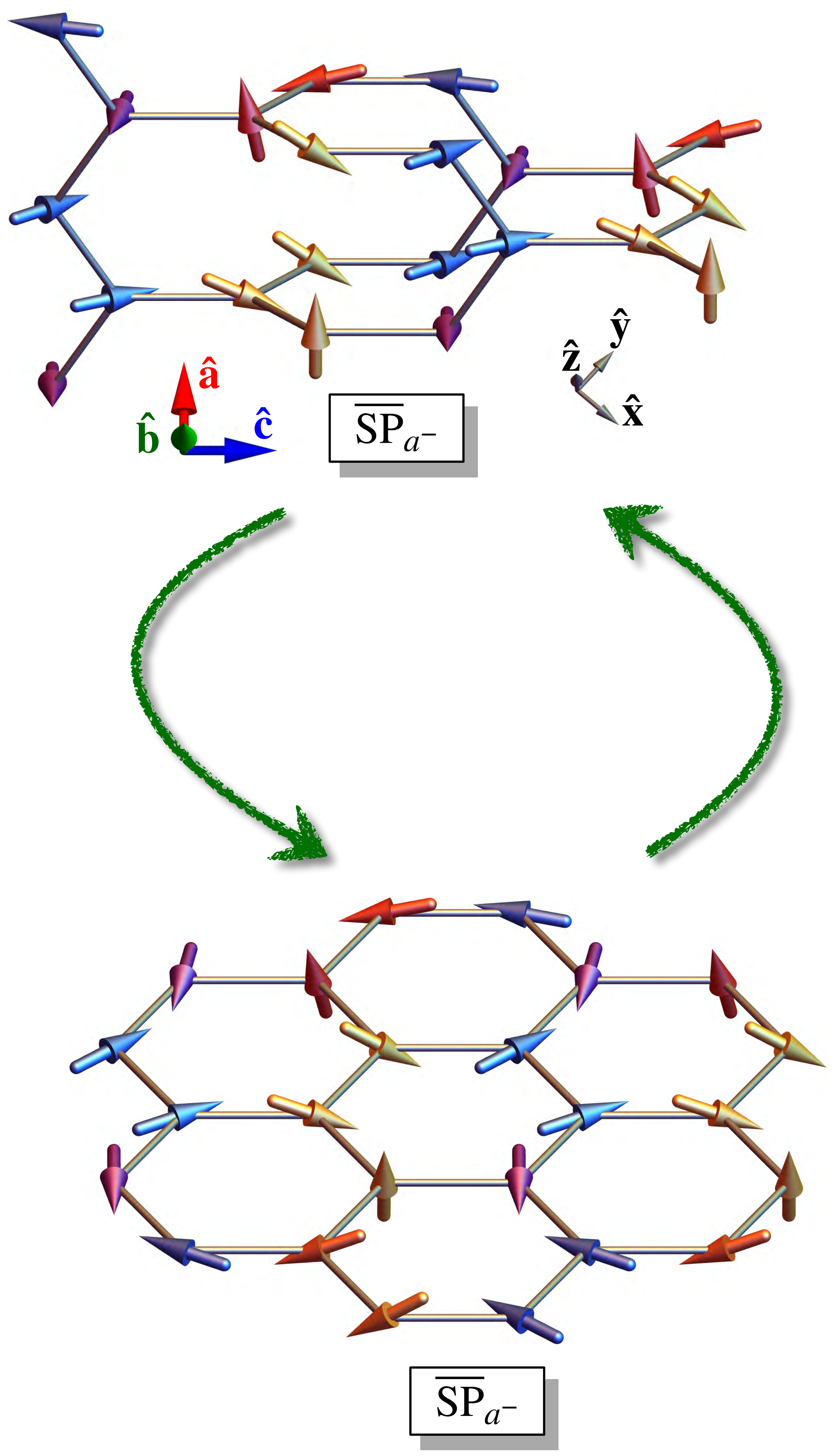
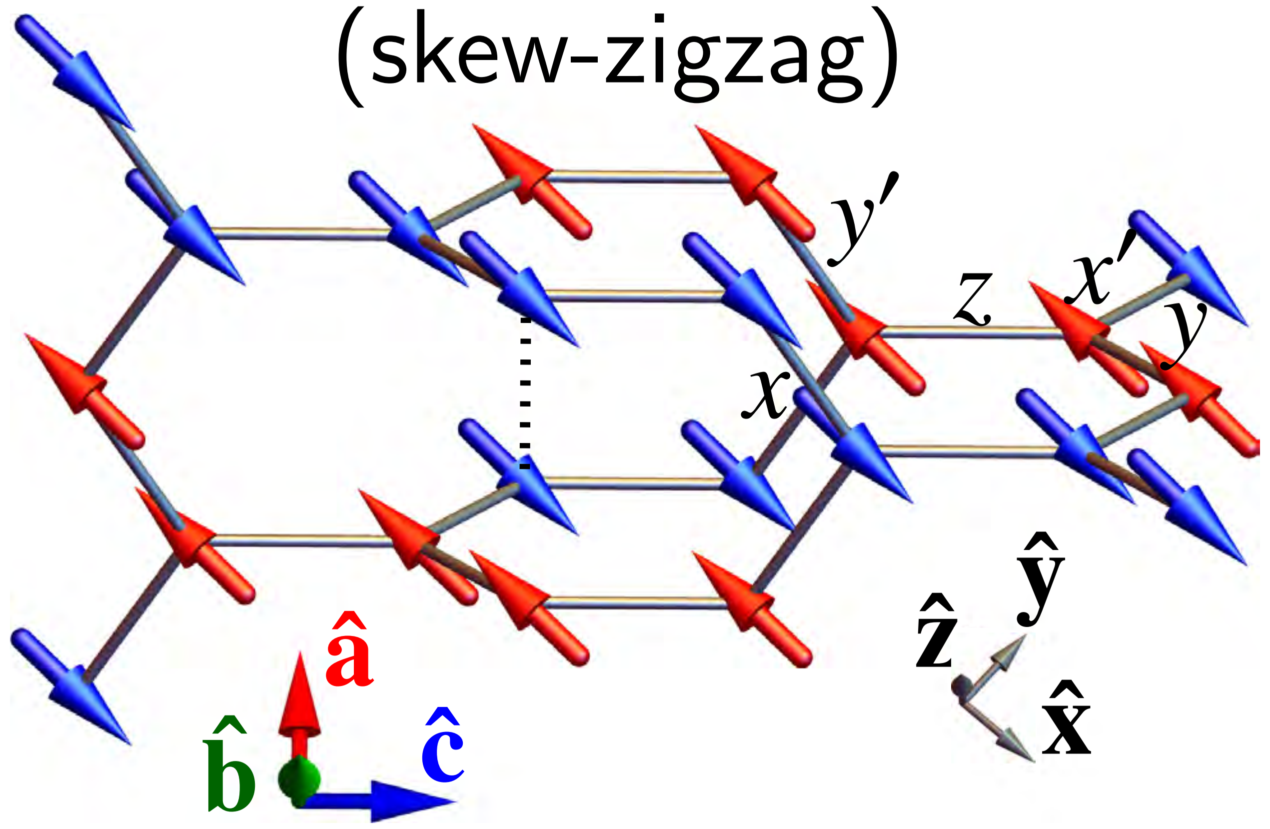


3D-2D mapping: real space

hyperhoneycomb

(skew-zigzag)

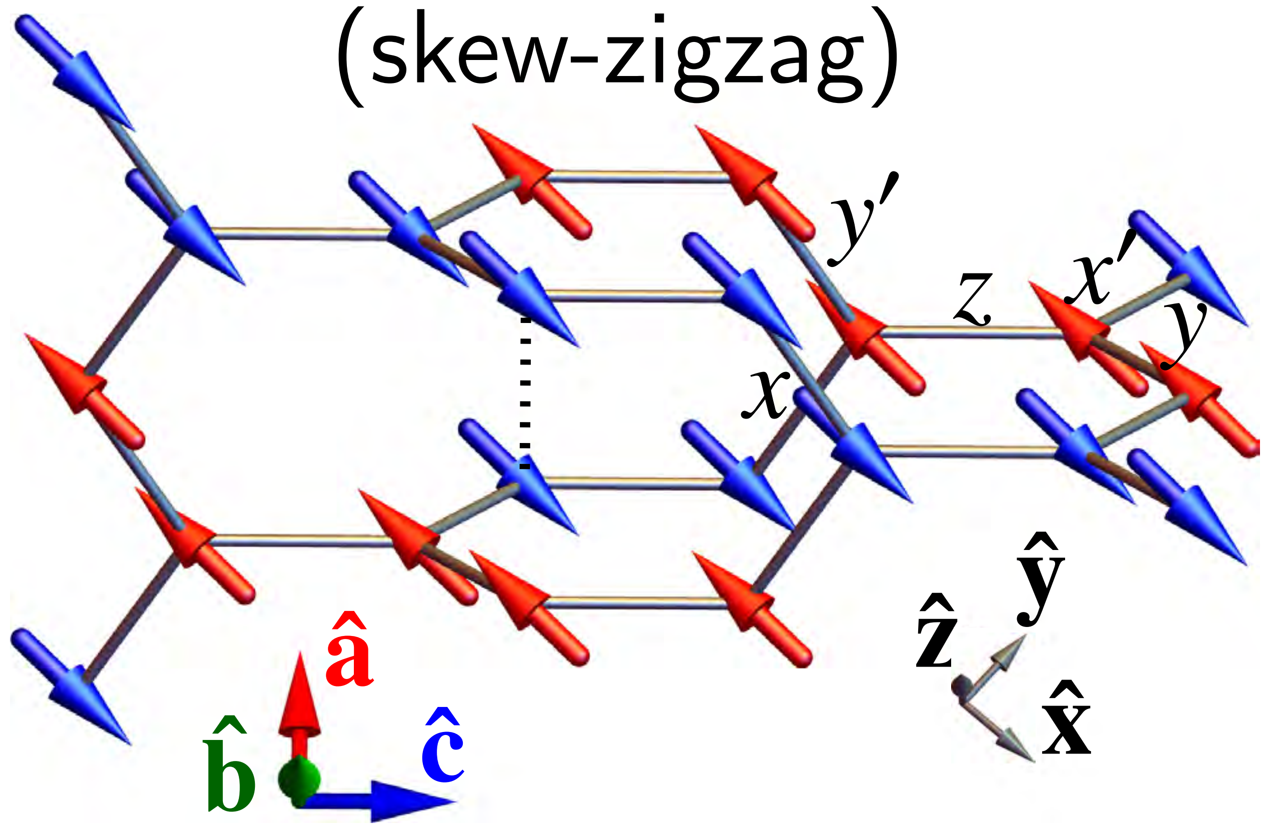


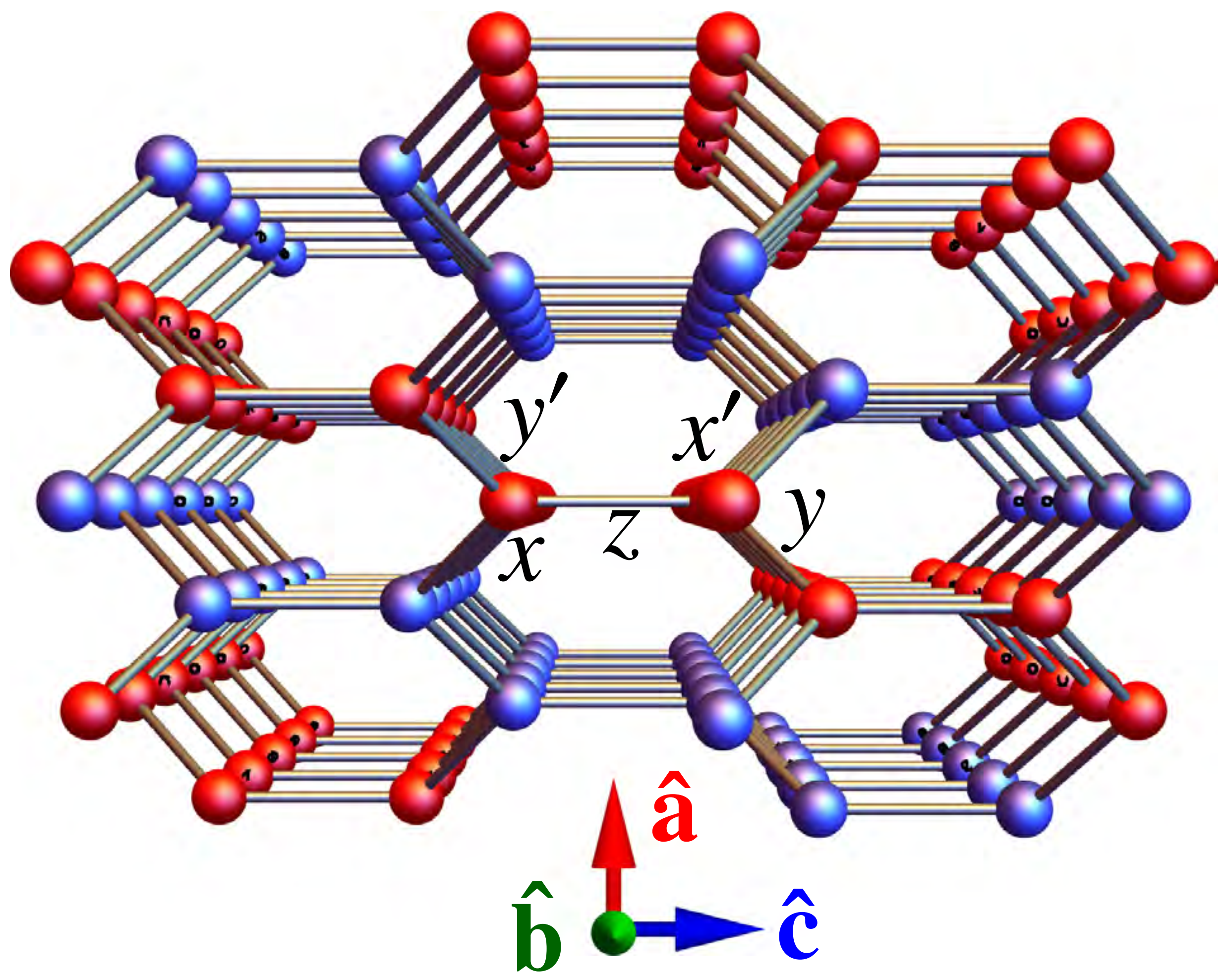
... can be induced in β-Li_2IrO_3 by magnetic field

[Ruiz *et al.* '17]

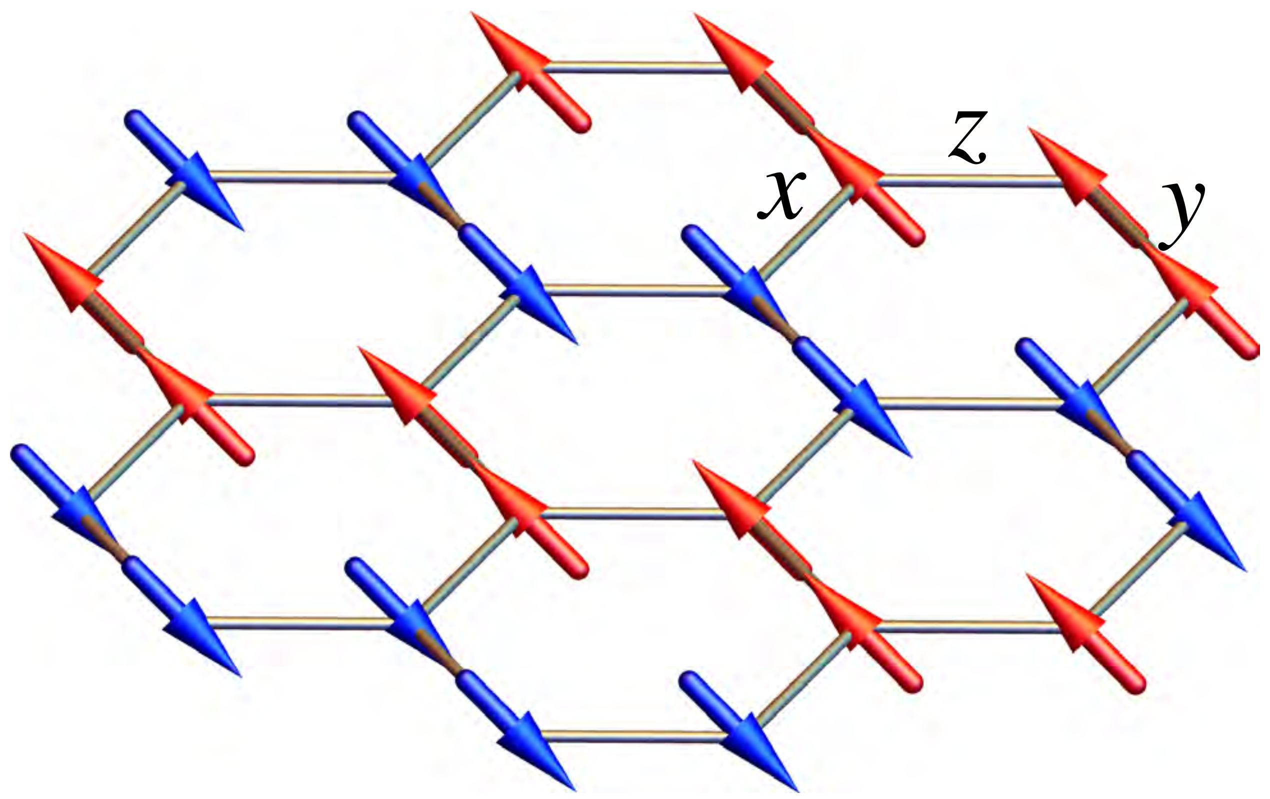
3D-2D mapping: real space

hyperhoneycomb
(skew-zigzag)



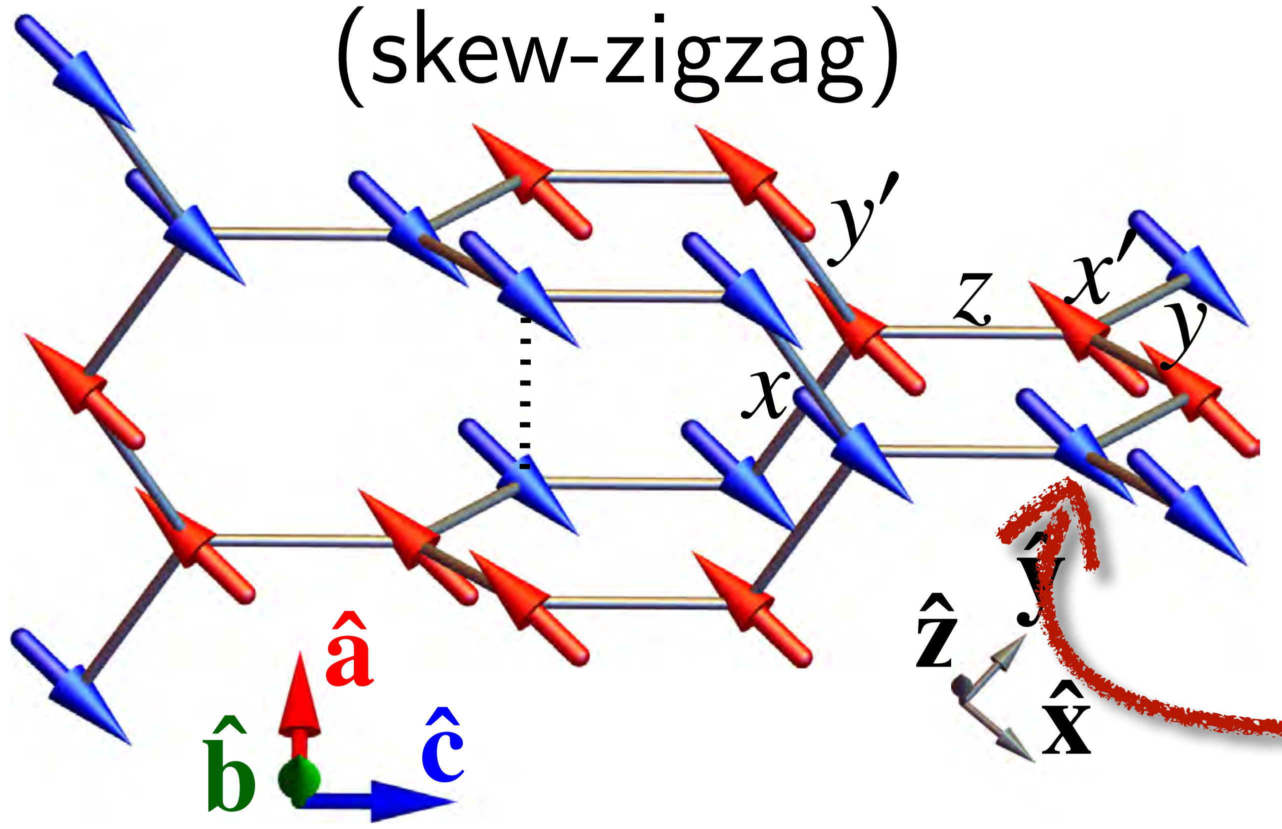


honeycomb
(zigzag)



3D-2D mapping: real space

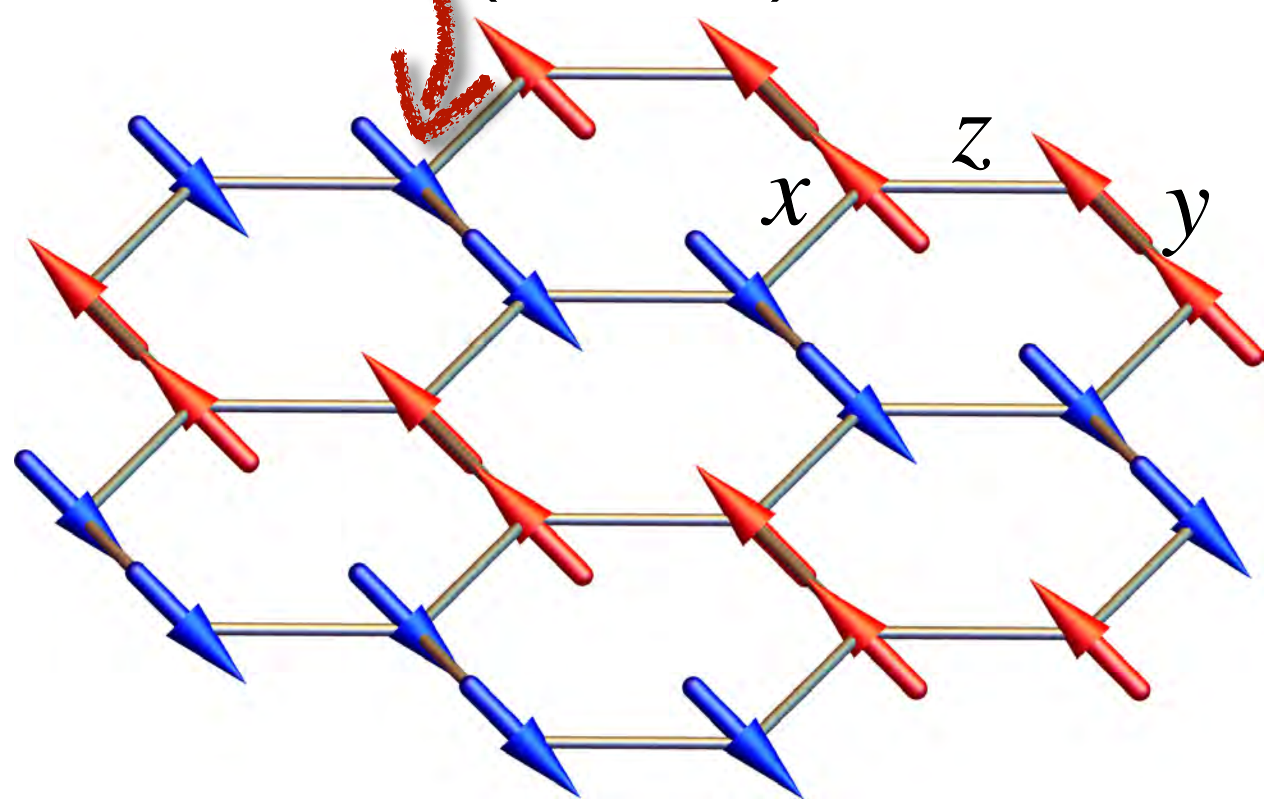
hyperhoneycomb
(skew-zigzag)

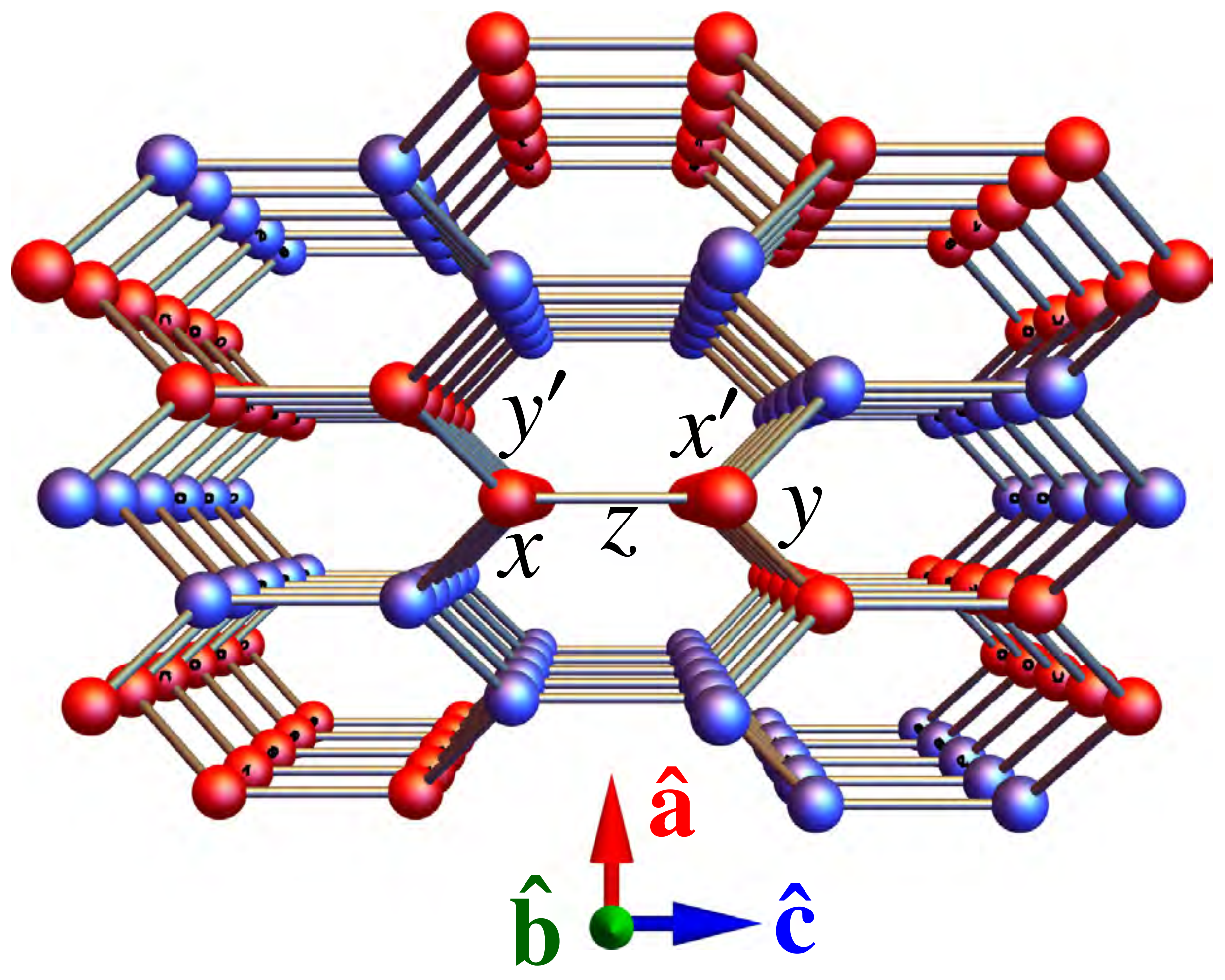


identical neighbors

$\Rightarrow$ identical classical energies

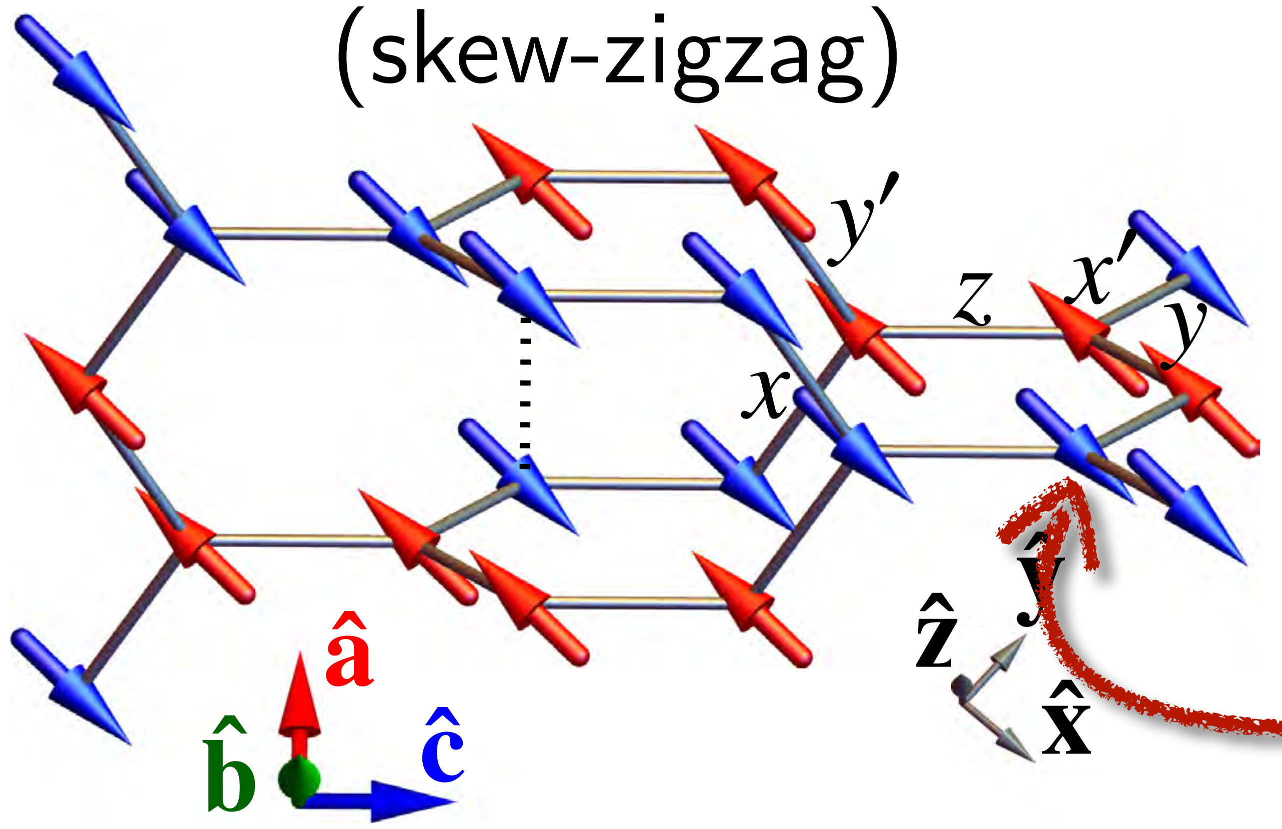
honeycomb
(zigzag)

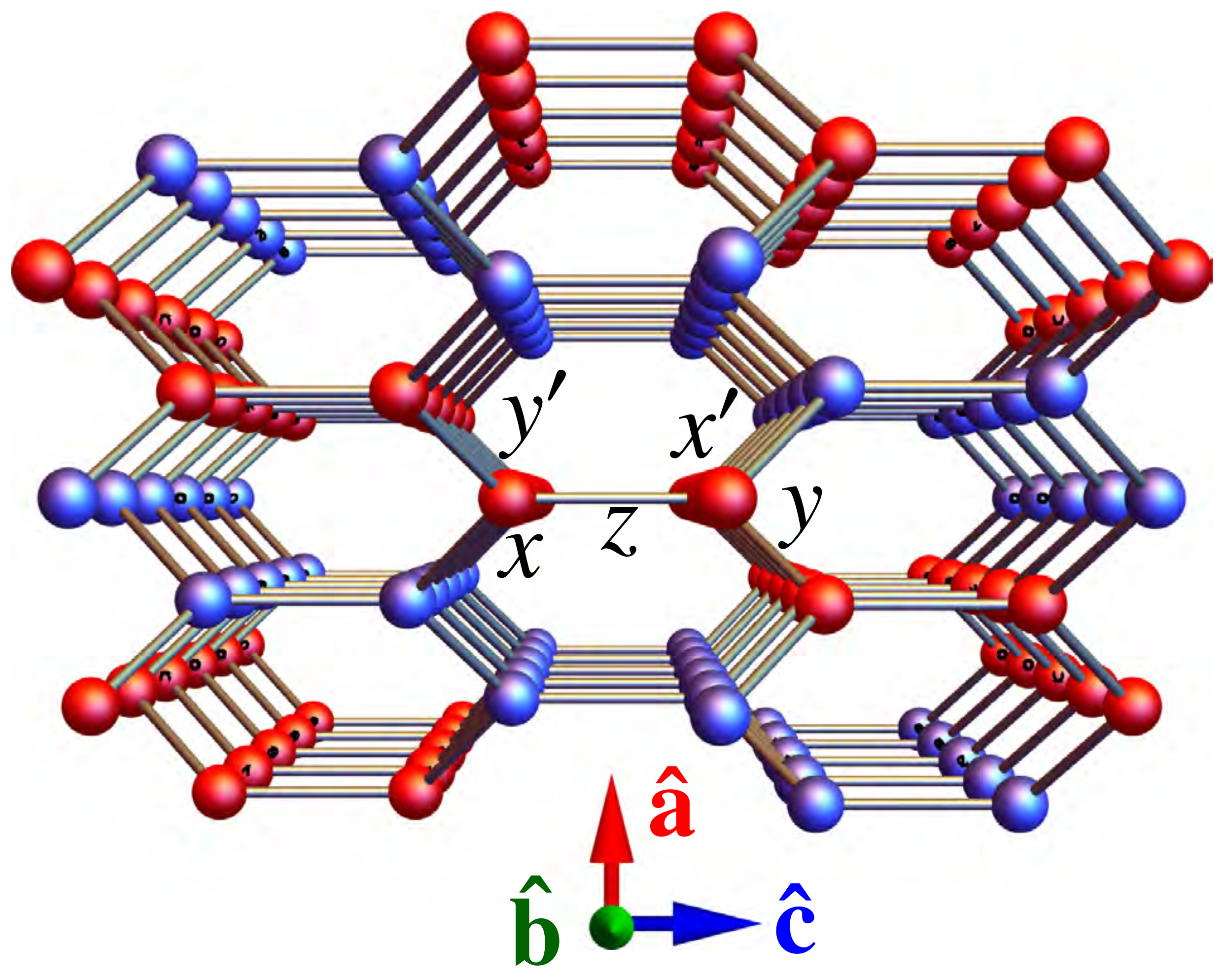




3D-2D mapping: real space

hyperhoneycomb
(skew-zigzag)

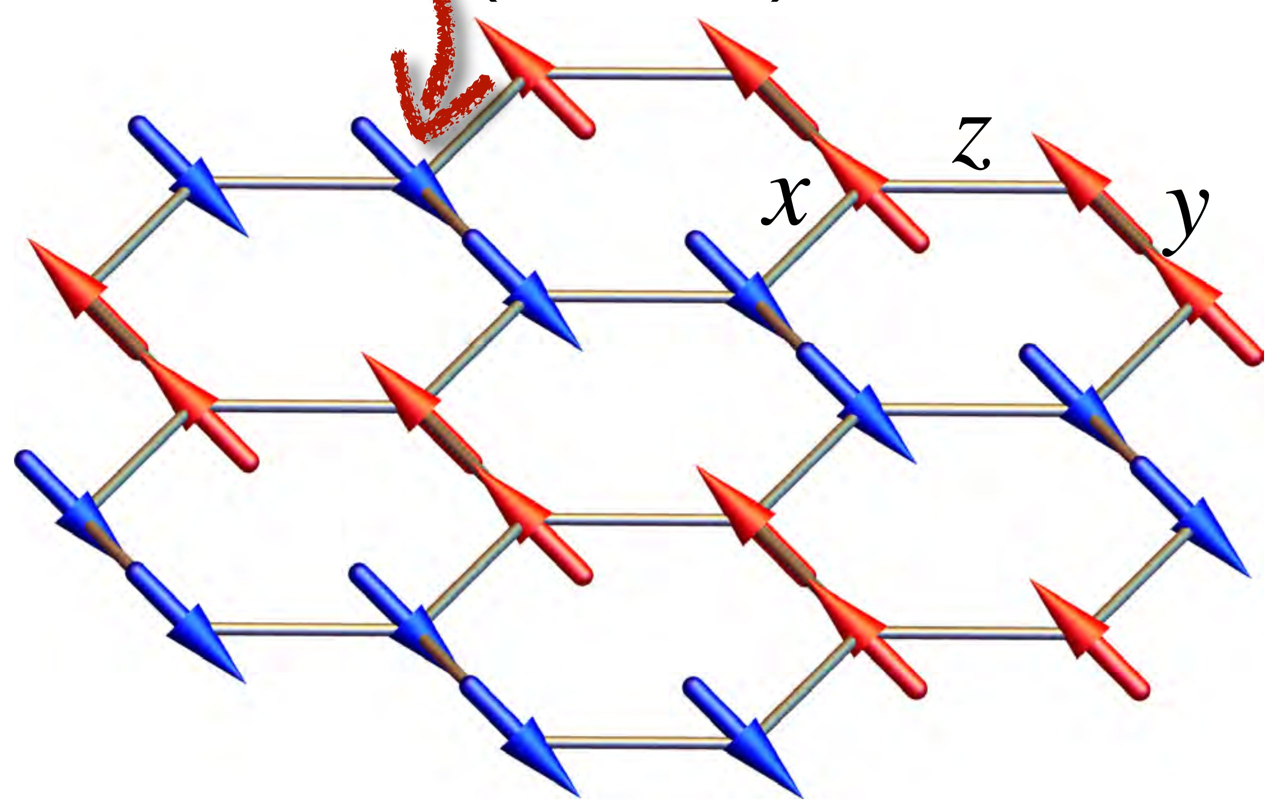




identical neighbors

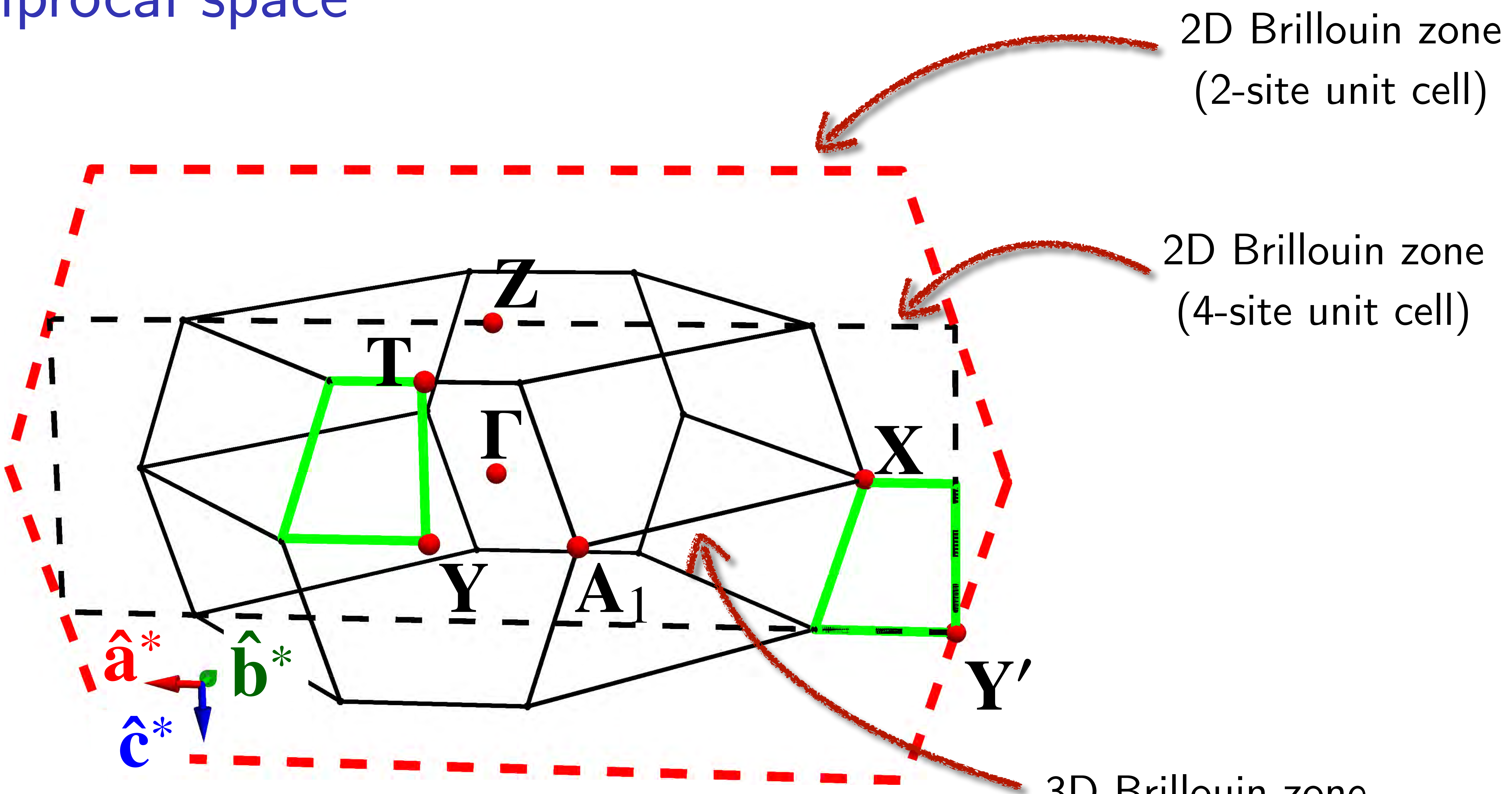
⇒ identical classical energies

honeycomb
(zigzag)



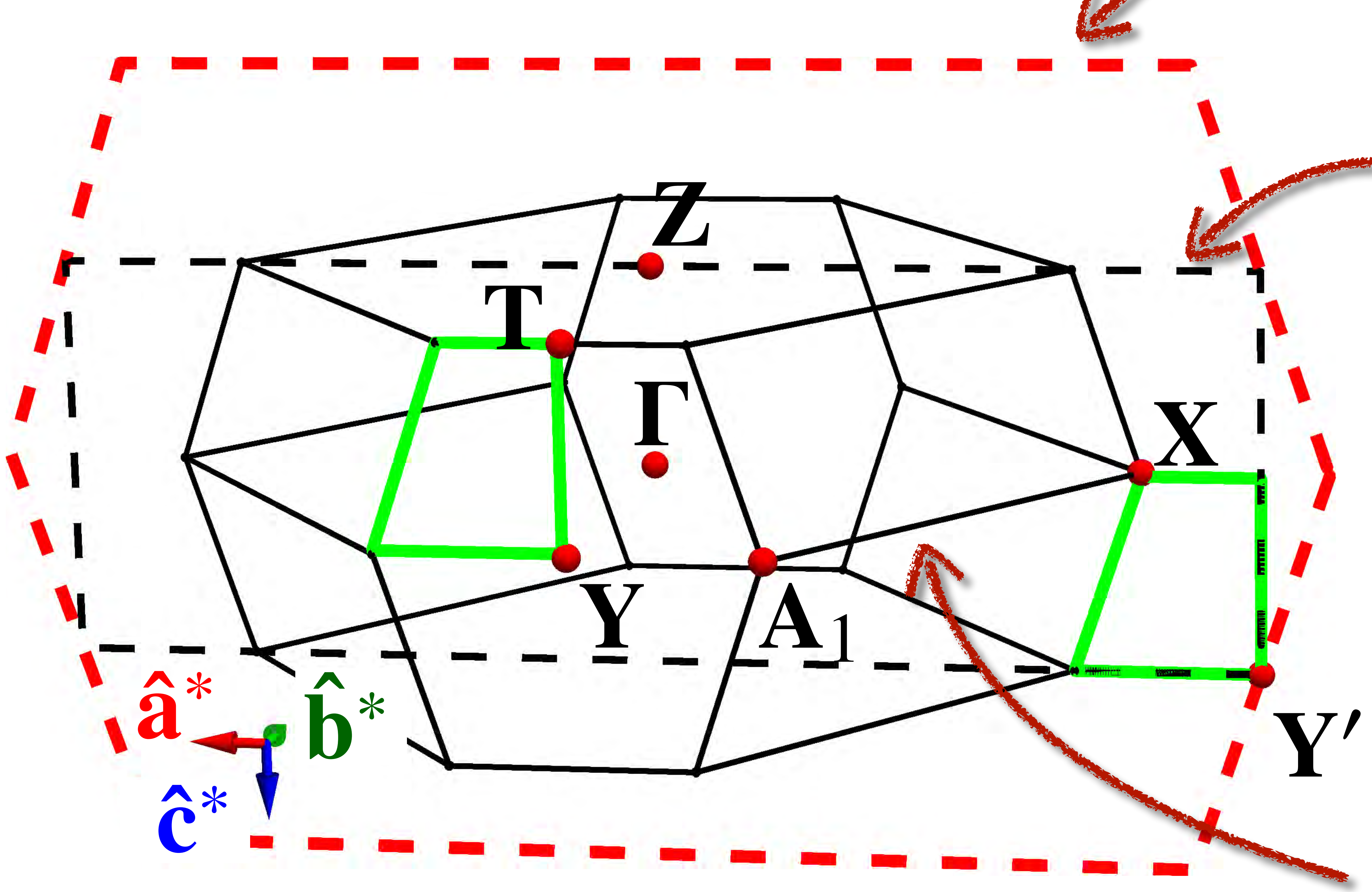
"quasi-2D" states $\Longleftrightarrow$ sites separated by **b** magnetically equivalent

3D-2D mapping: reciprocal space



... applies to *all* high-symmetry points

3D-2D mapping: reciprocal space



“quasi-2D” states $\Longleftrightarrow$ ordering wavevector in *ac* plane

(modulo reciprocal lattice translations)

... applies to *all* high-symmetry points

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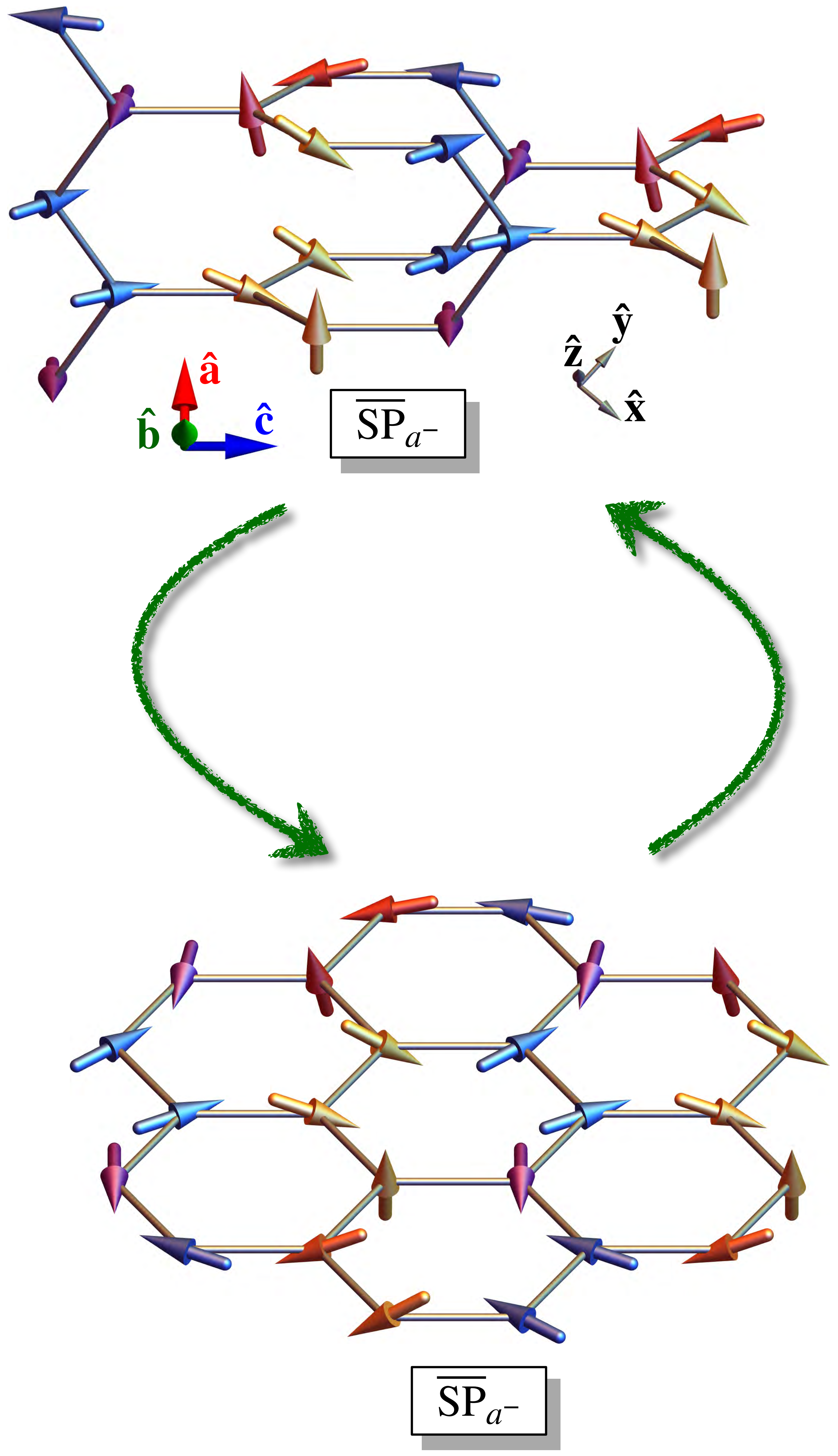
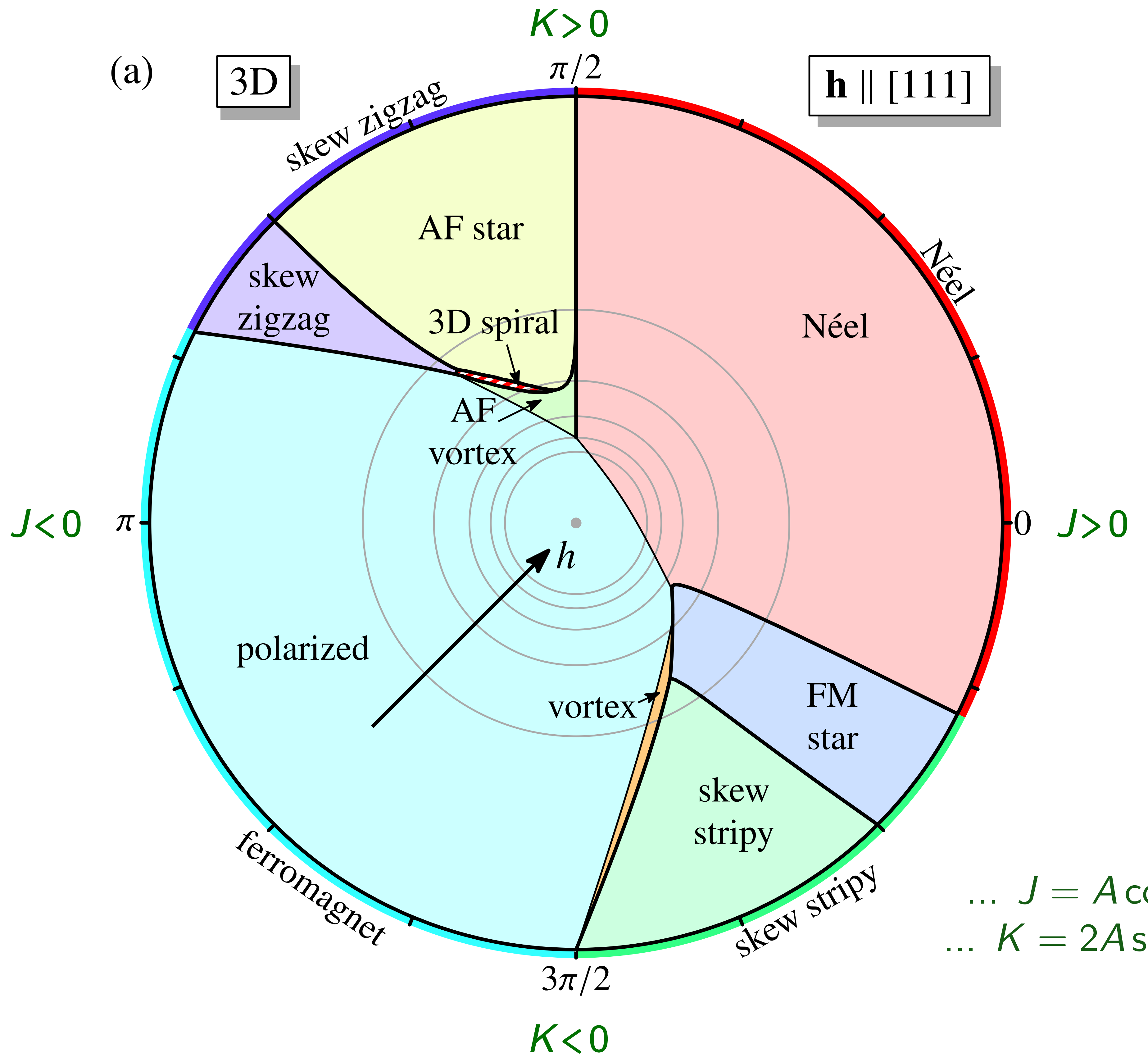


Example #1: Heisenberg-Kitaev model in a magnetic field

$$\mathcal{H}_{\mathrm{HK}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum_{\langle ij \rangle_\gamma} S_i^\gamma S_j^\gamma - \mathbf{h} \cdot \sum_i \mathbf{S}_i$$

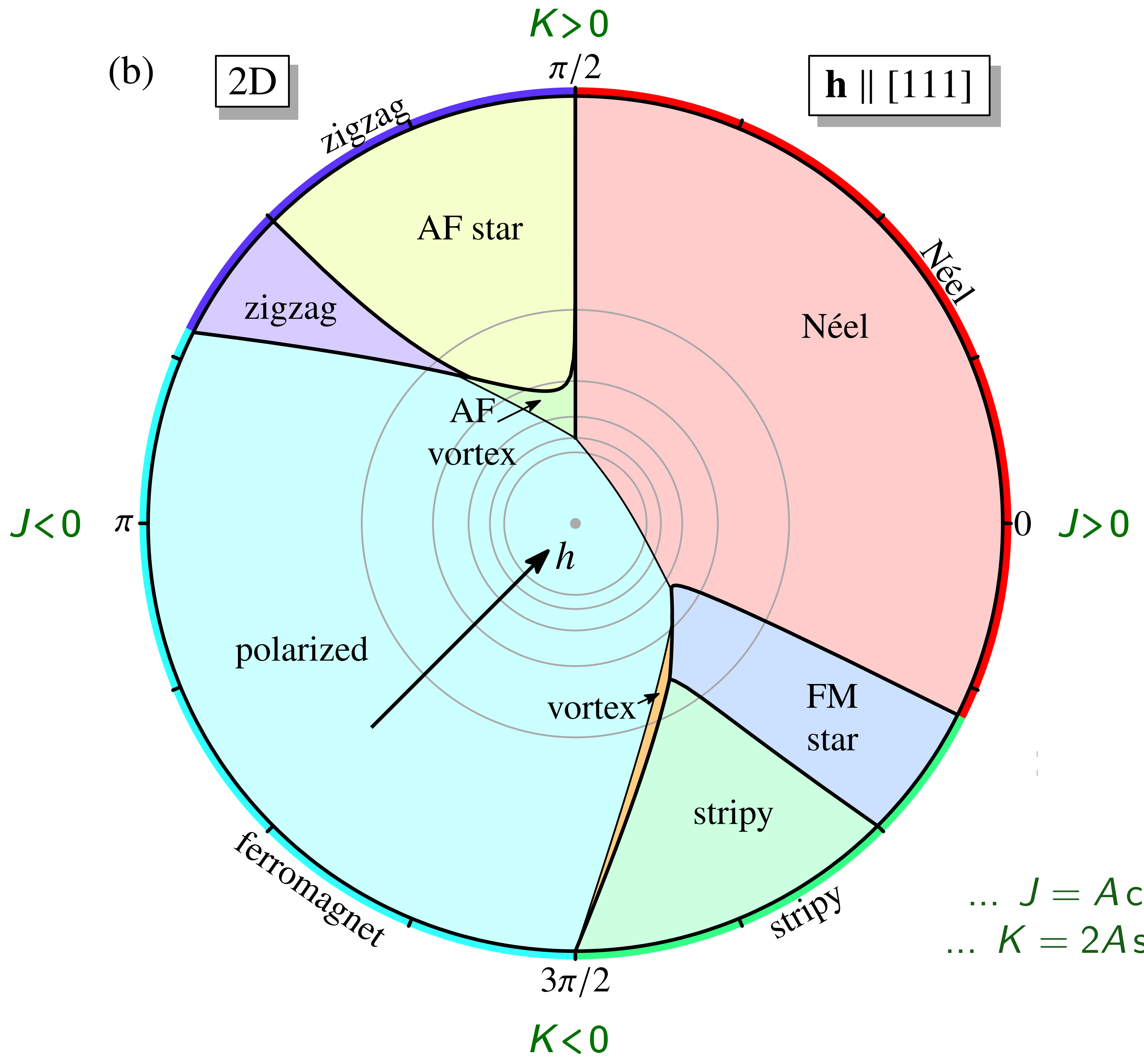
hyperhoneycomb



$\ldots\ J = A\cos\varphi$
$\ldots\ K = 2A\sin\varphi$

[Krüger, Vojta, LJ, arXiv:1907.05423]

honeycomb



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[LJ, Andrade, Vojta, PRL '16]

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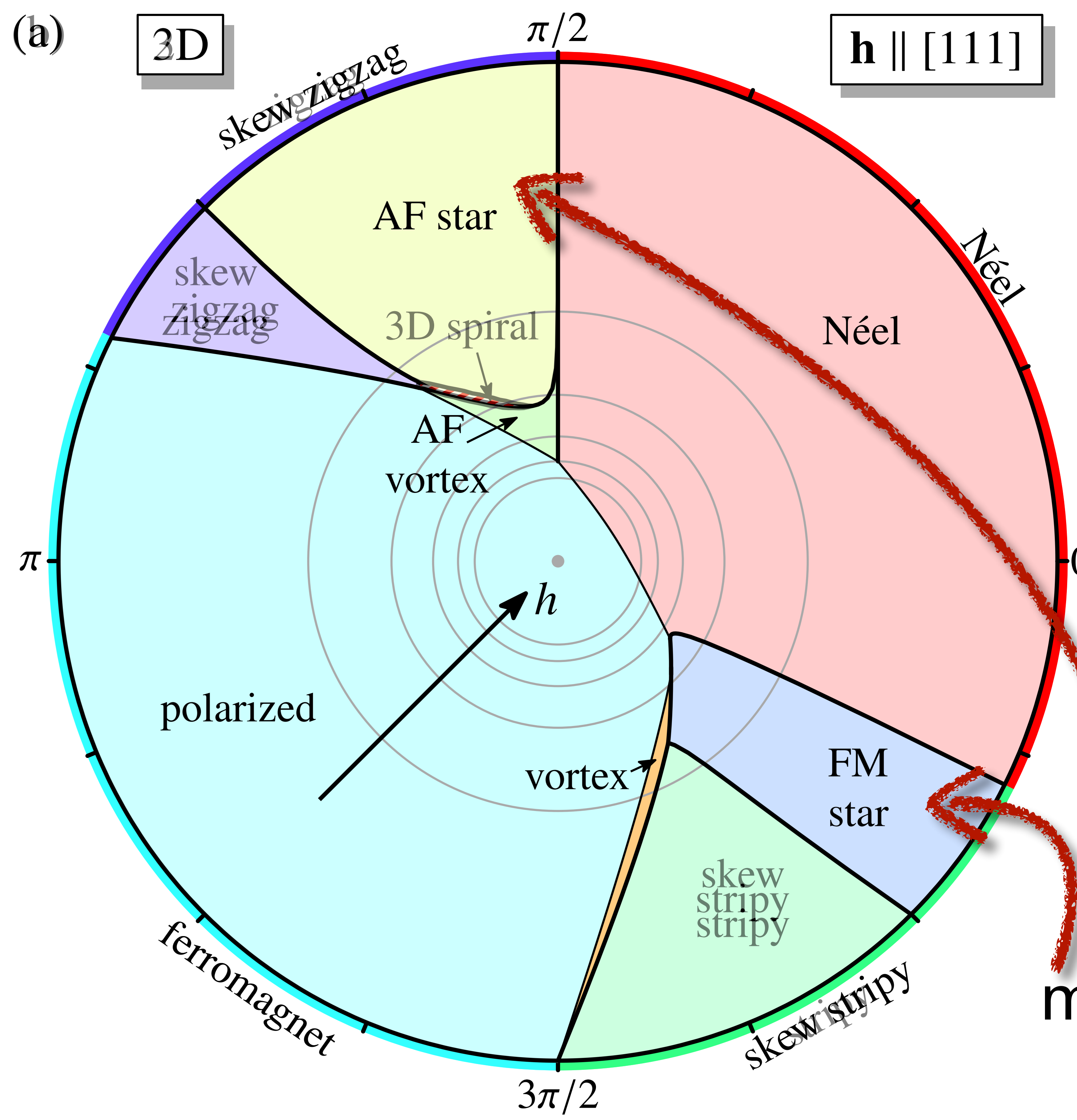
hyperhoneycomb+honeycomb



multi-**Q**

Example #1: Heisenberg-Kitaev model in a magnetic field

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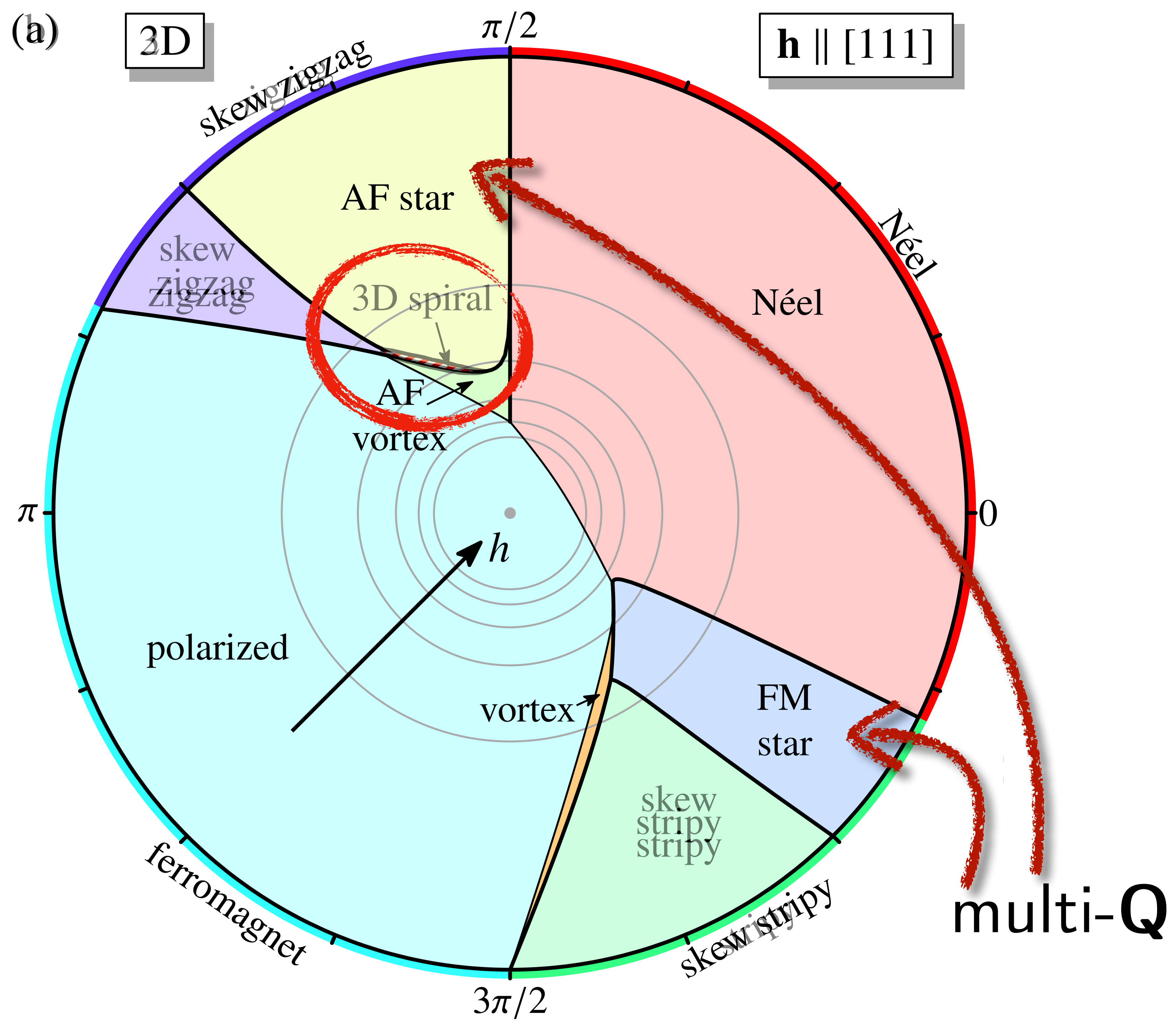
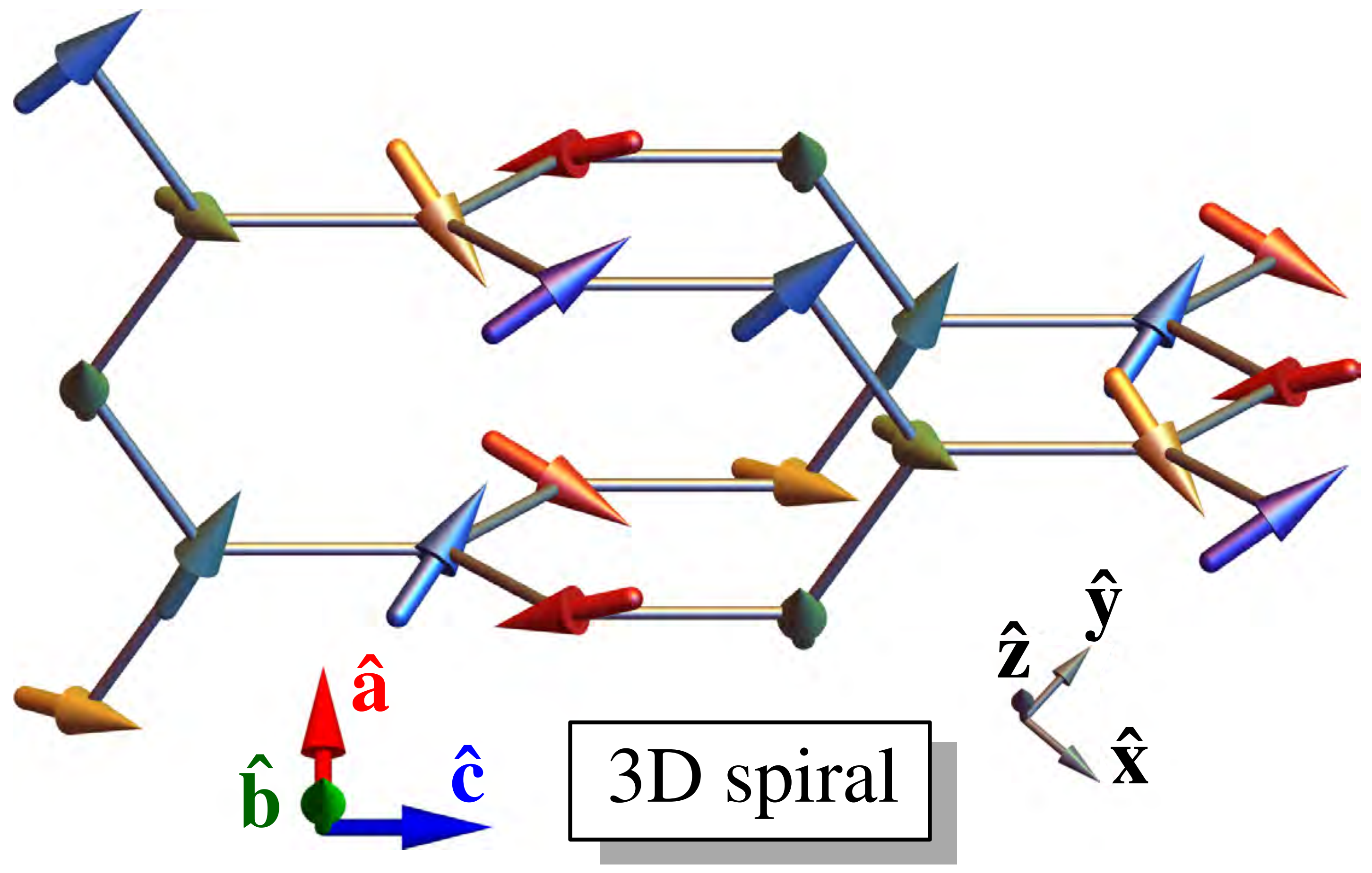
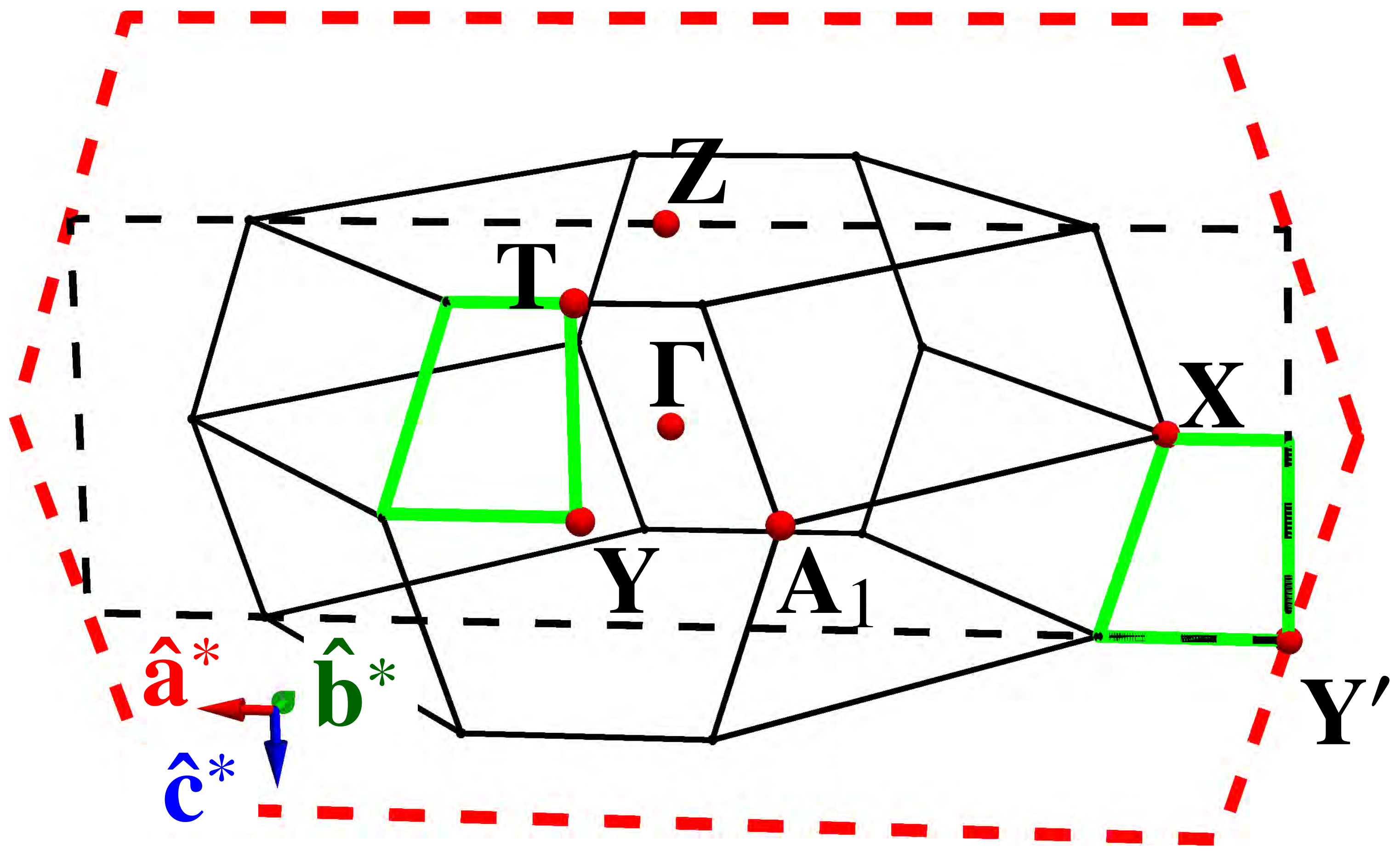
hyperhoneycomb+honeycomb



3D spiral state



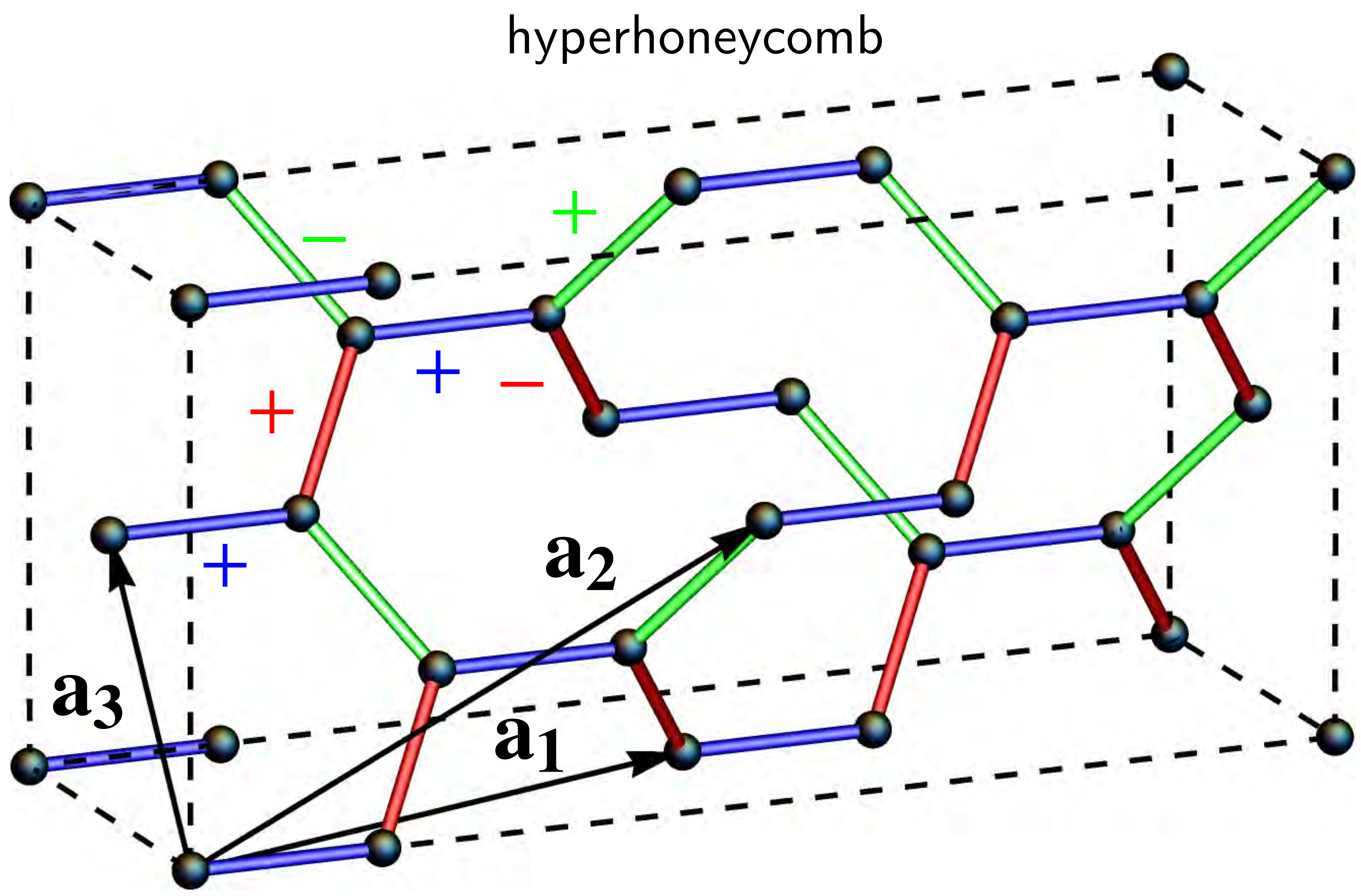
magnetically inequivalent sites along **b**!



$\mathbf{Q} = \frac{2}{3}\mathbf{Y} \notin$ *ac* plane!

⇒ "true-3D" state

Example #2: Γ interactions



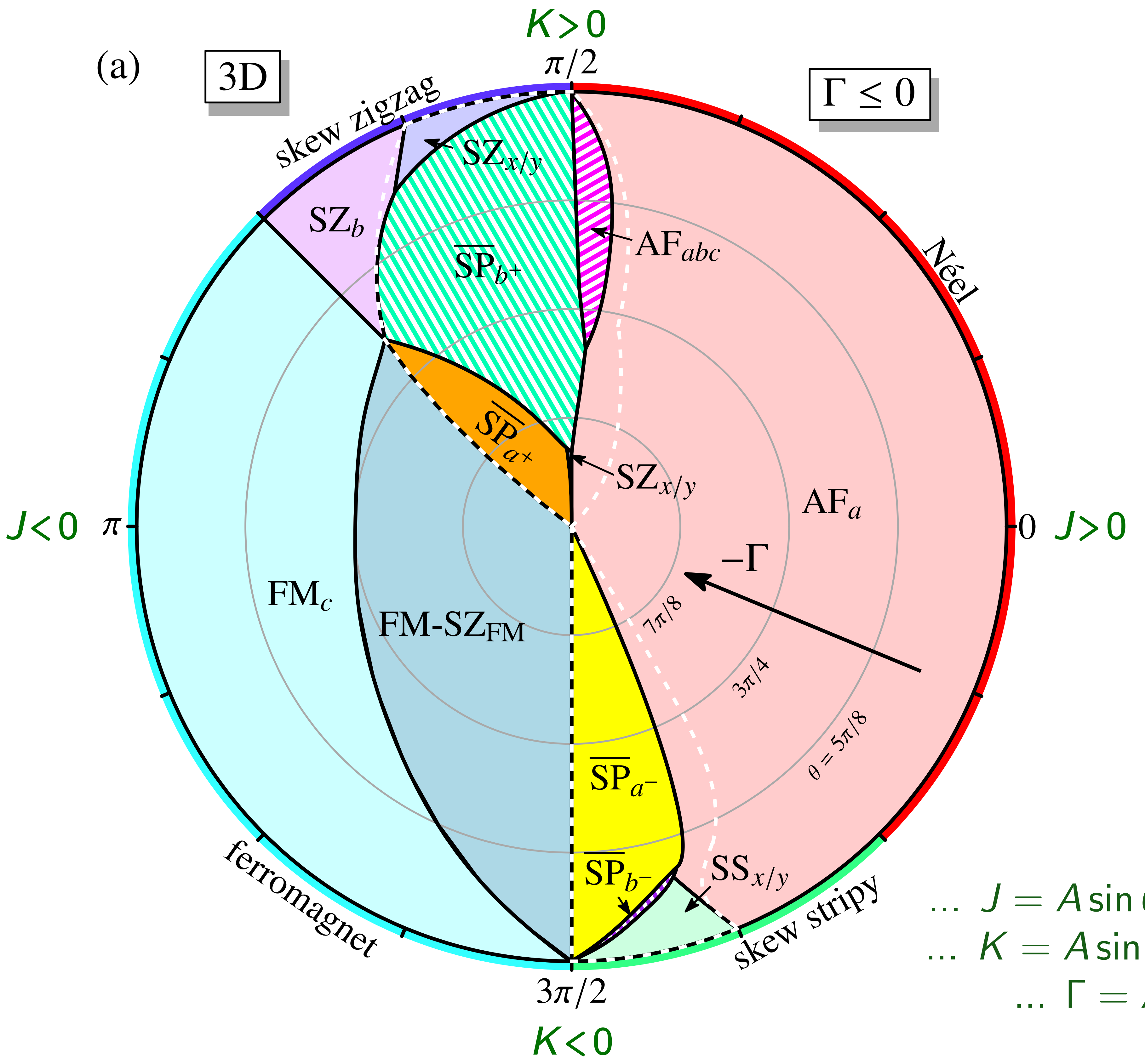
2 different types of x and y bonds

... but same local environment
... choose interactions accordingly
[Lee & Kim, PRB '15]

Example #2: HK±Γ model

$$\mathcal{H}_{\mathrm{HK\Gamma}} = \sum_{\langle ij\rangle_\gamma}\left[J\mathbf{S}_i\cdot\mathbf{S}_j + KS_i^\gamma S_j^\gamma \pm \Gamma\left(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha\right)\right]$$

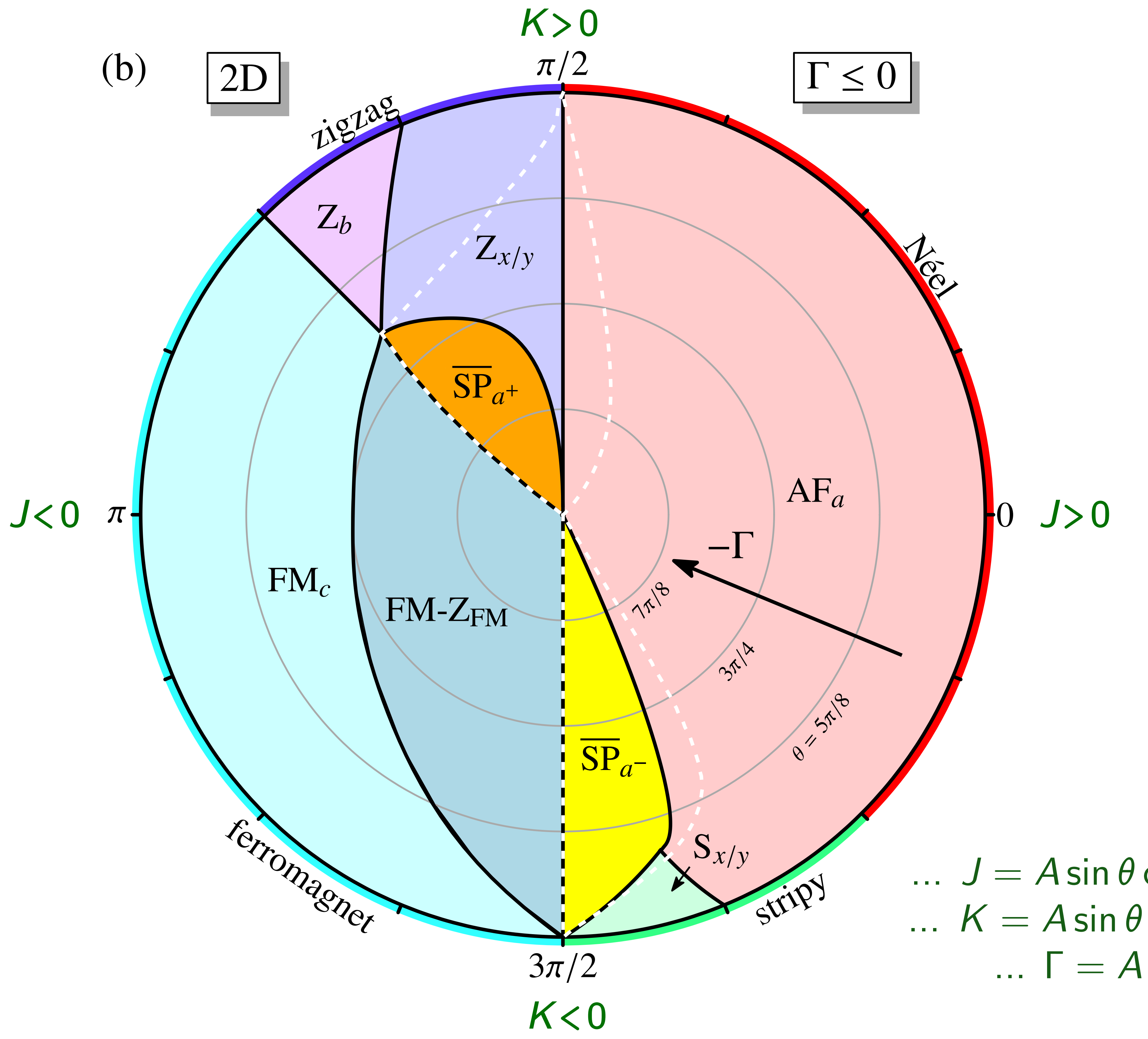
hyperhoneycomb



... $J = A\sin\theta\cos\varphi$
... $K = A\sin\theta\sin\varphi$
... $\Gamma = A\cos\theta$

[Lee & Kim, PRB ’15]

honeycomb



... $J = A\sin\theta\cos\varphi$
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[Krüger, Vojta, LJ, arXiv:1907.05423]

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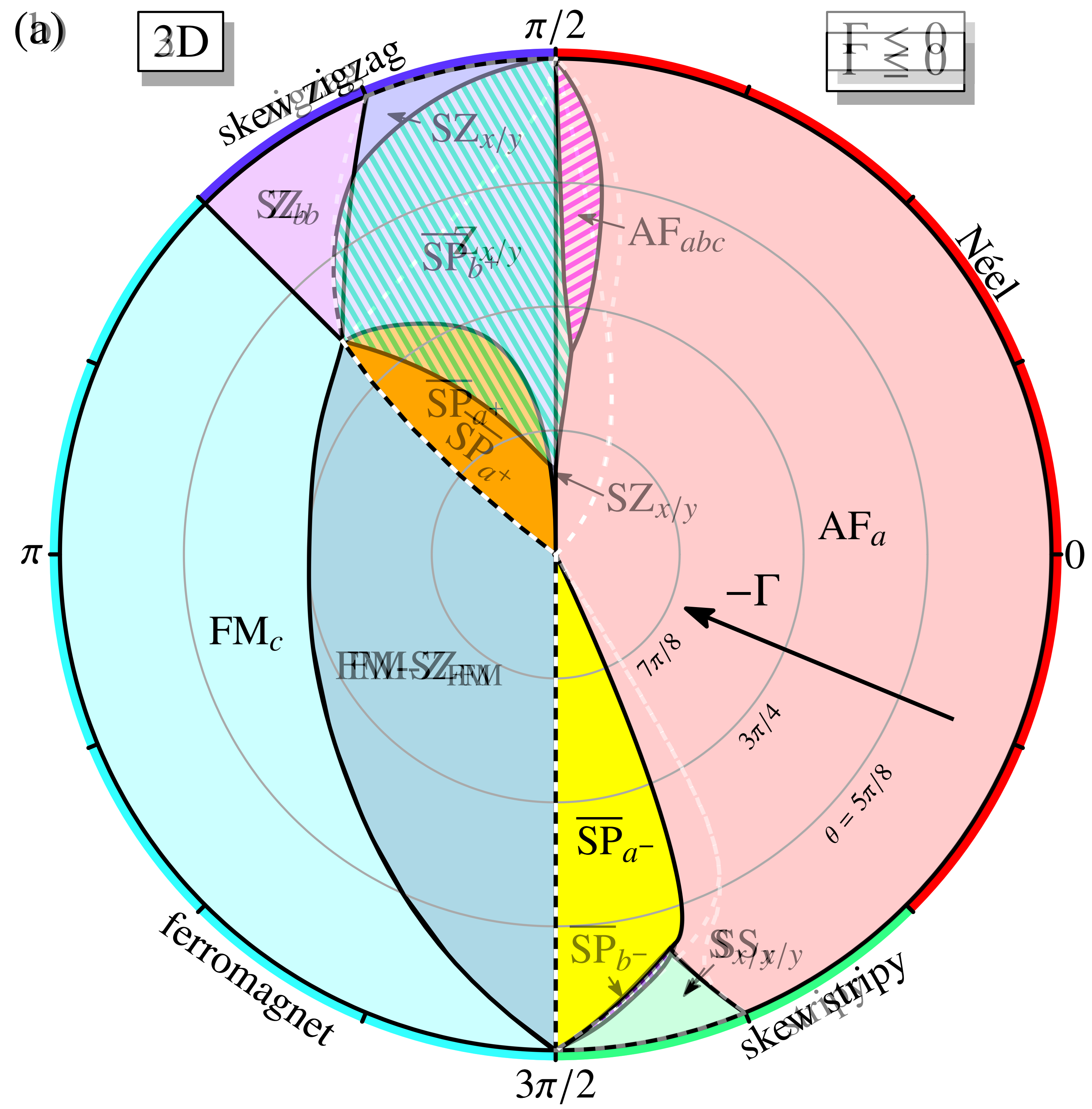
hyperhoneycomb+honeycomb



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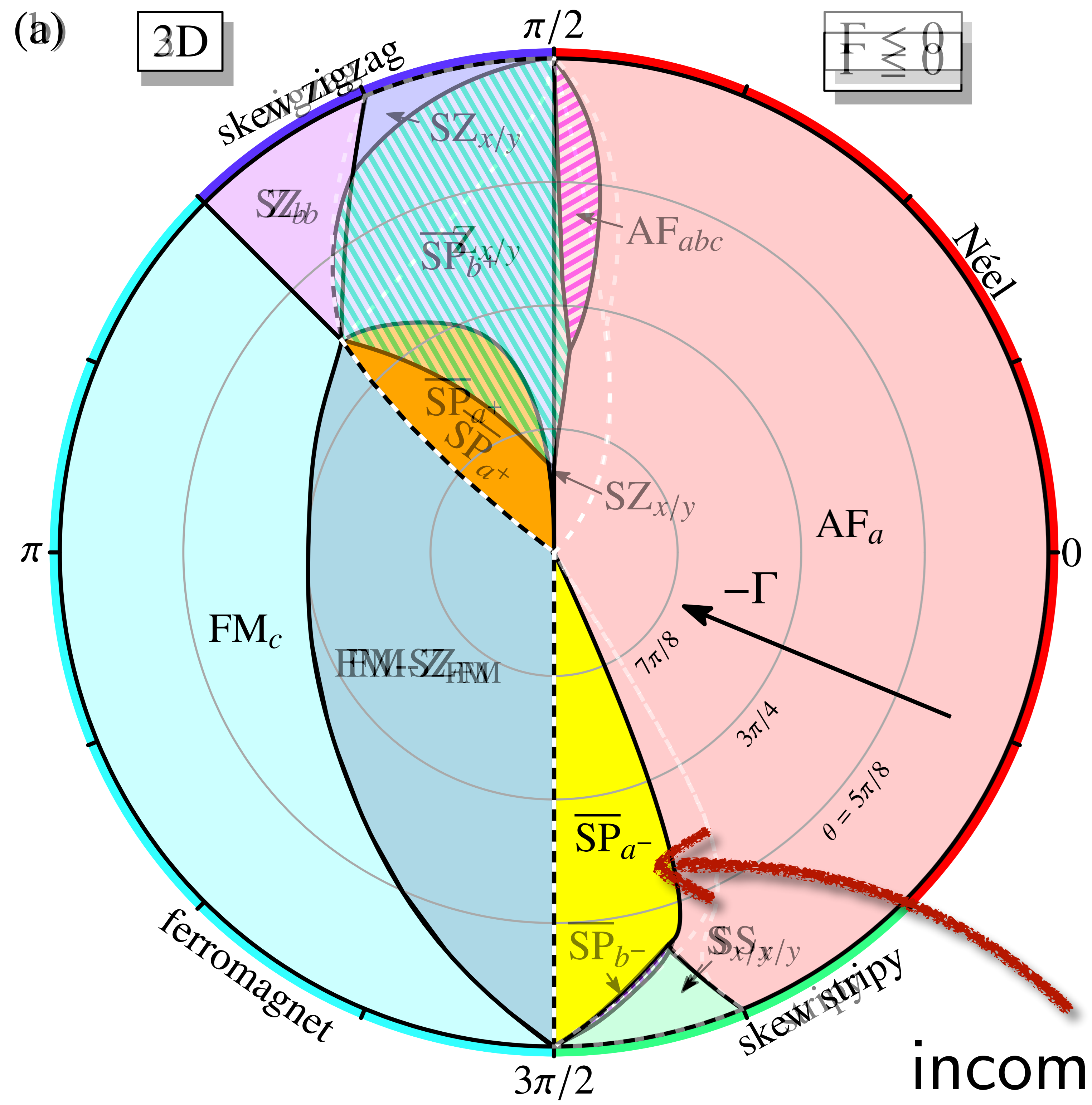
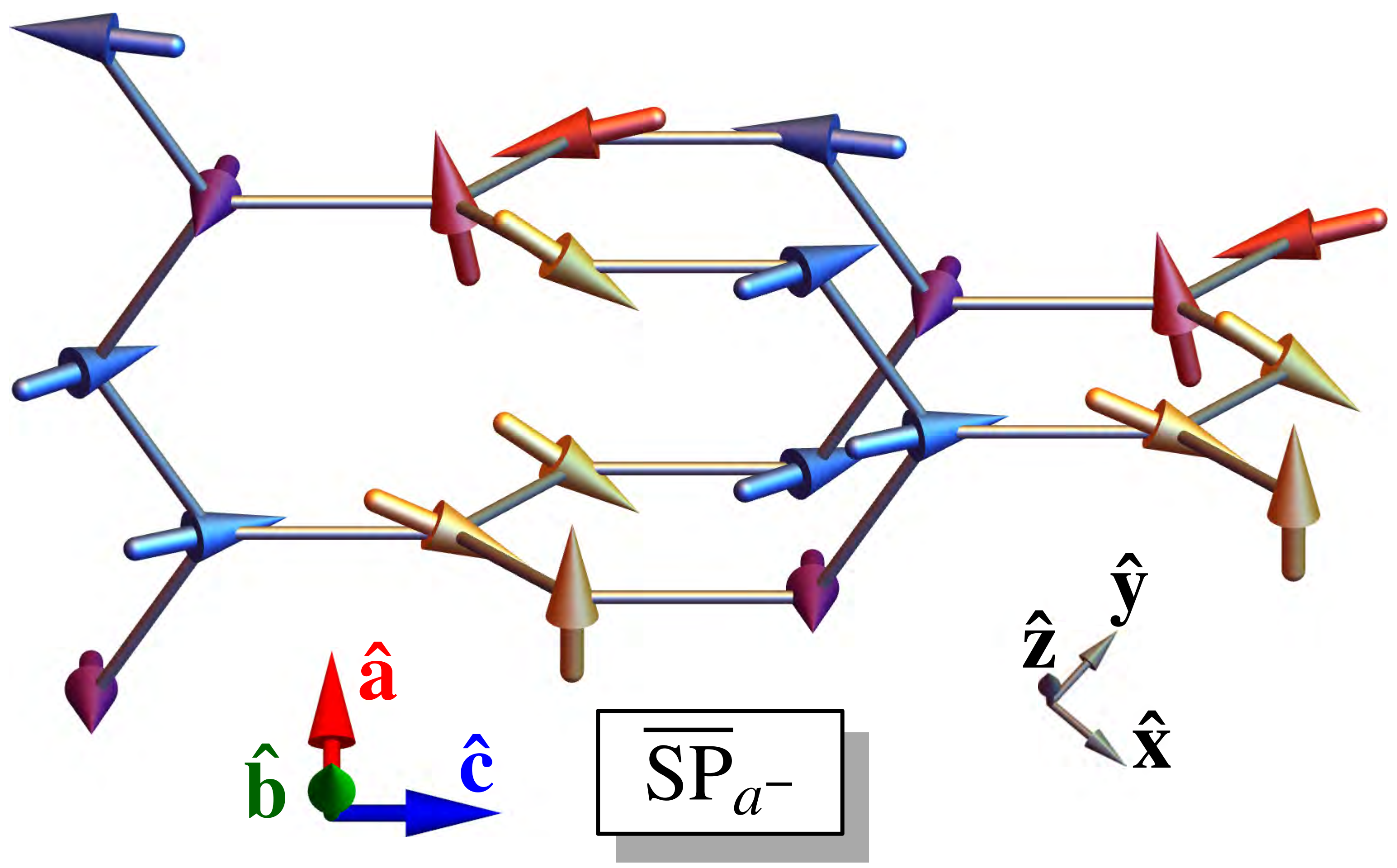
hyperhoneycomb+honeycomb

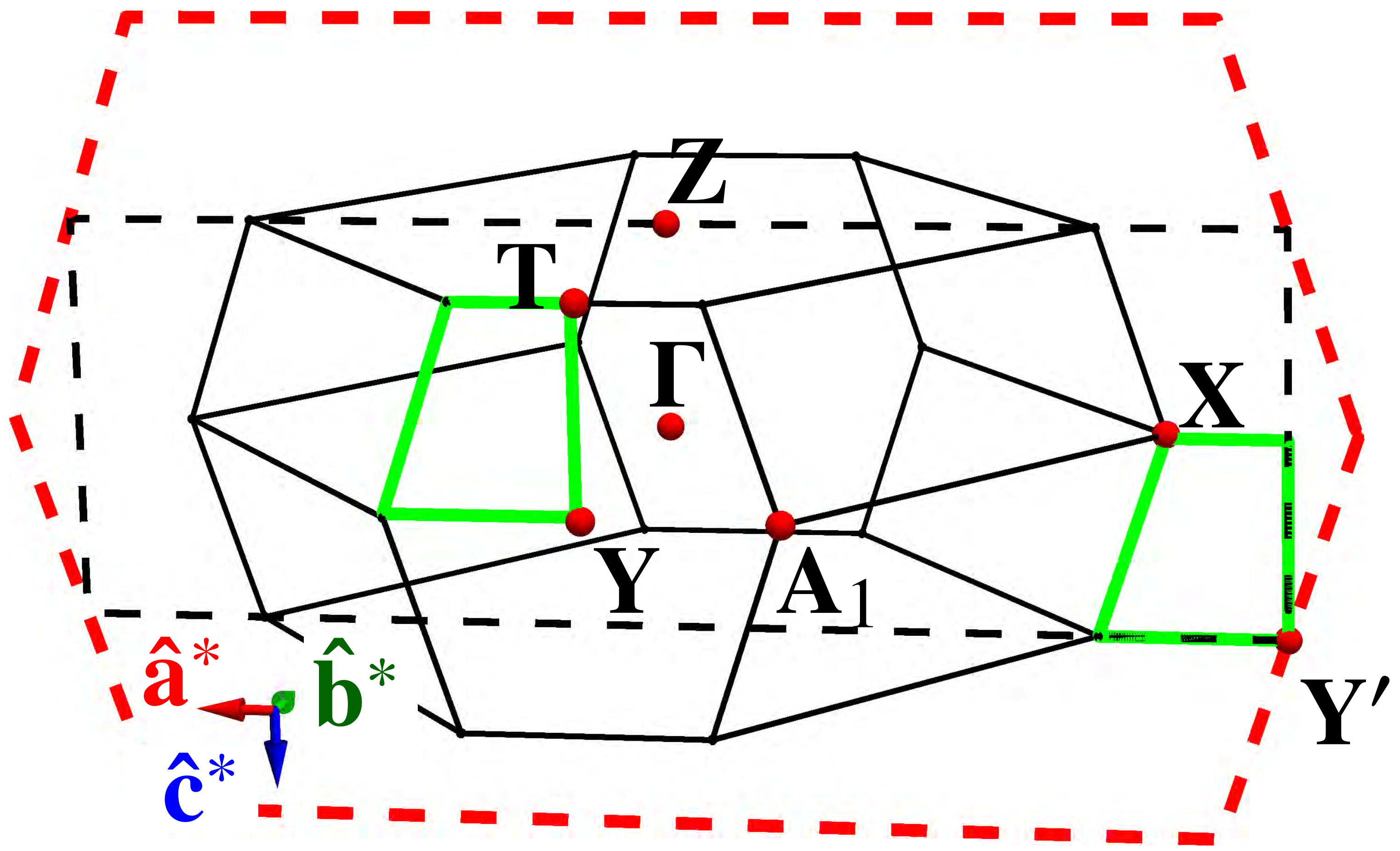


incommensurate
spiral (Li_2IrO_3)

Incommensurate spiral



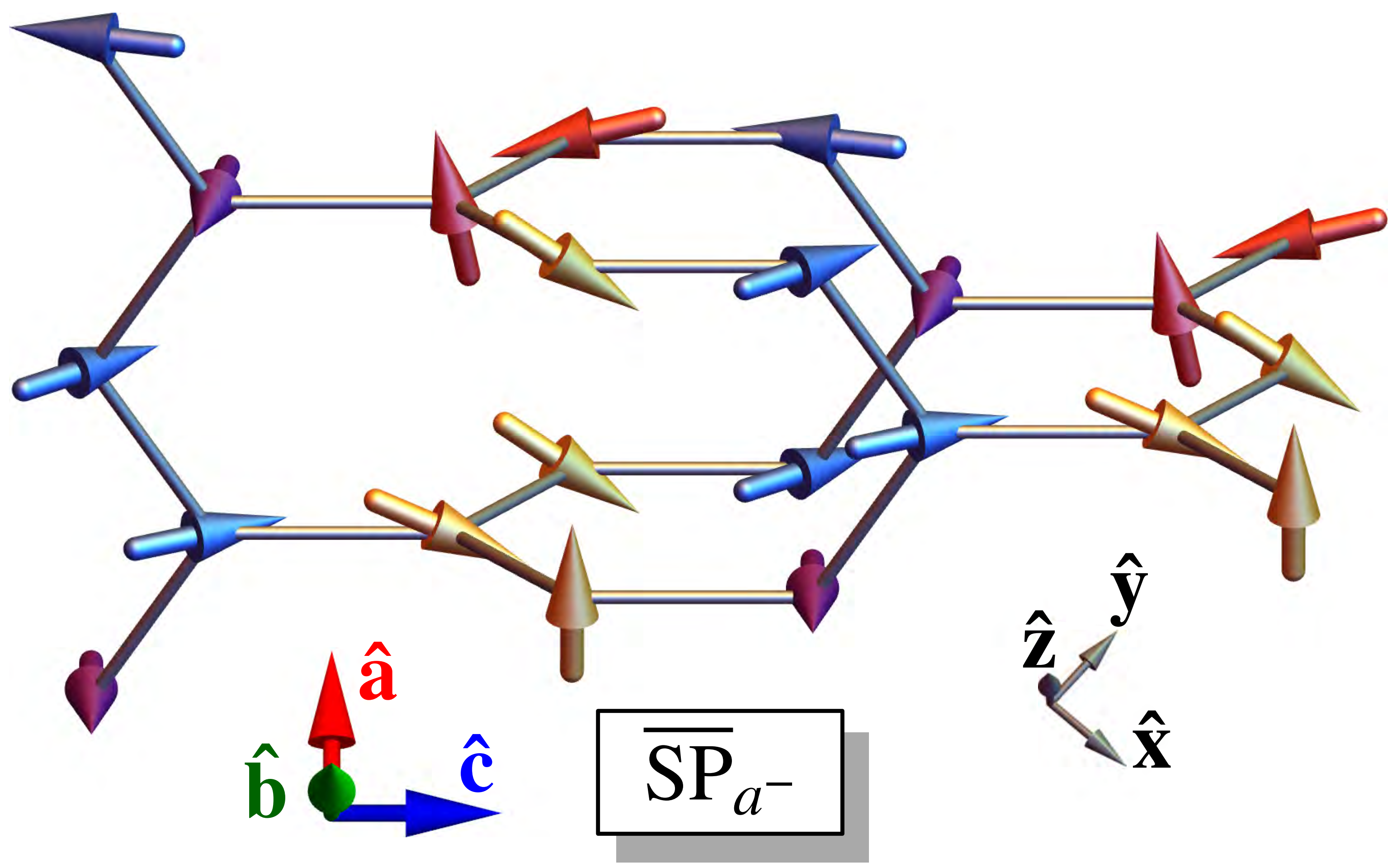
magnetically equivalent sites along **b**!



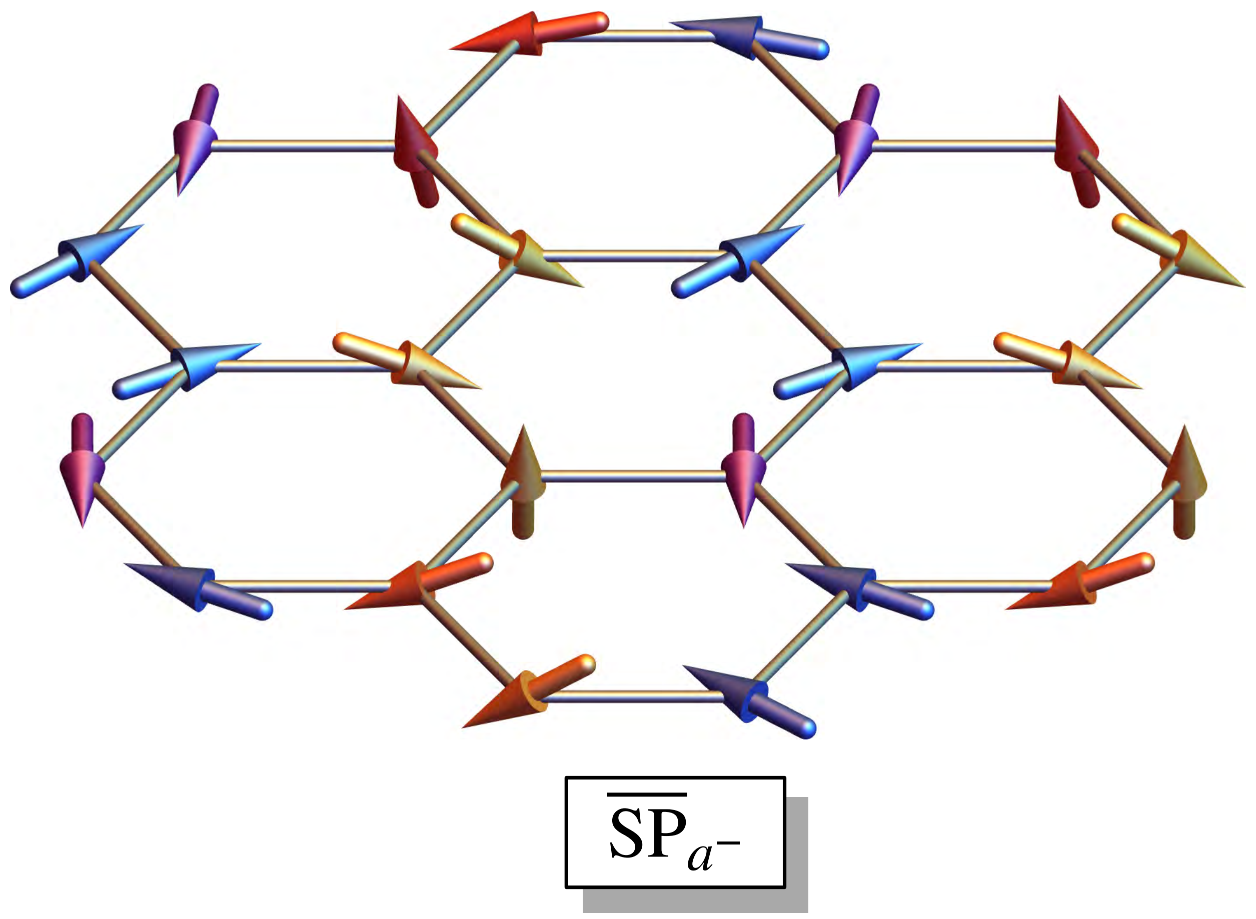
$\mathbf{Q} \parallel \mathbf{a}^* \in$ *ac* plane!

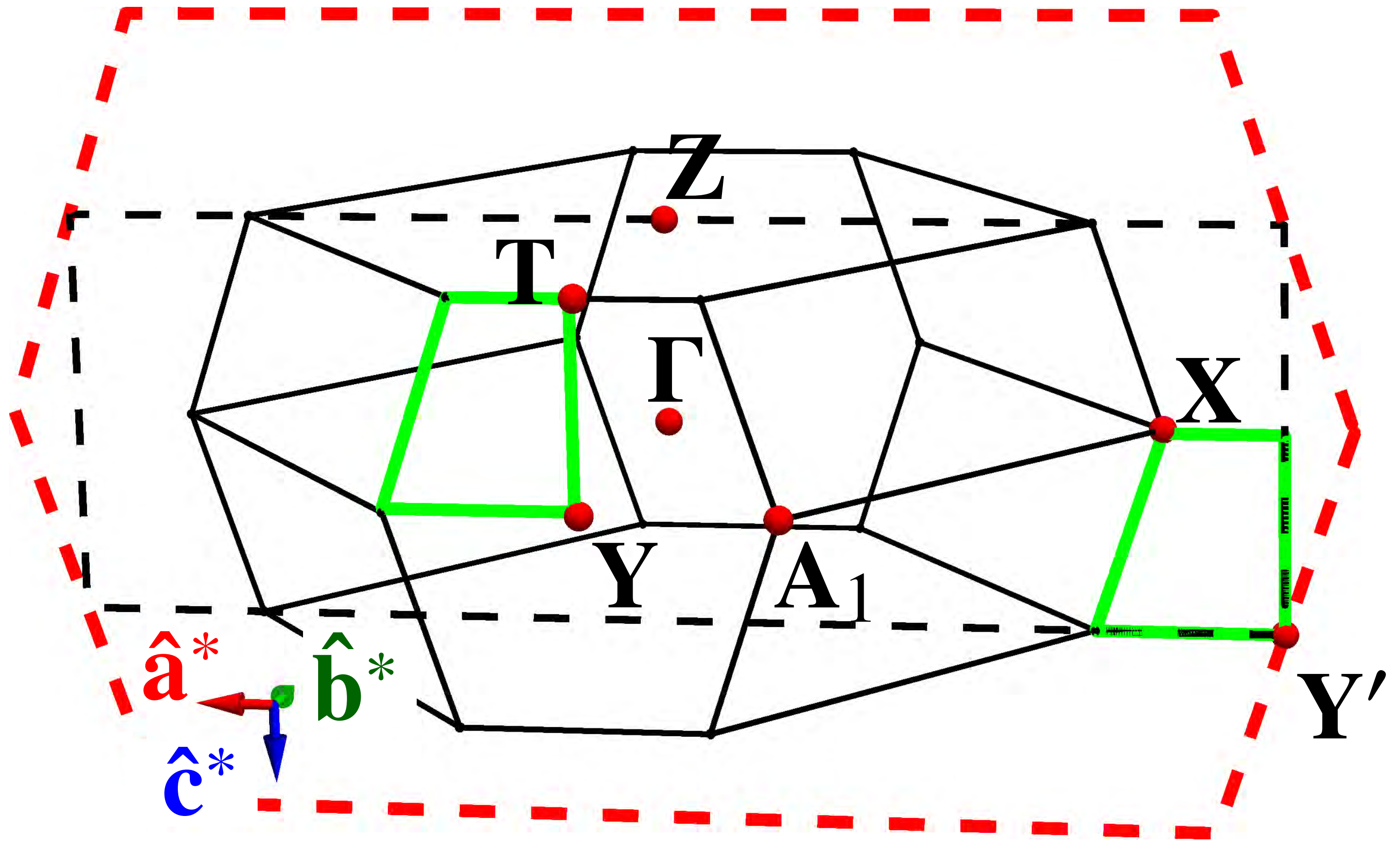
⇒ "quasi-2D" state

Incommensurate spiral



3D-2D equivalence

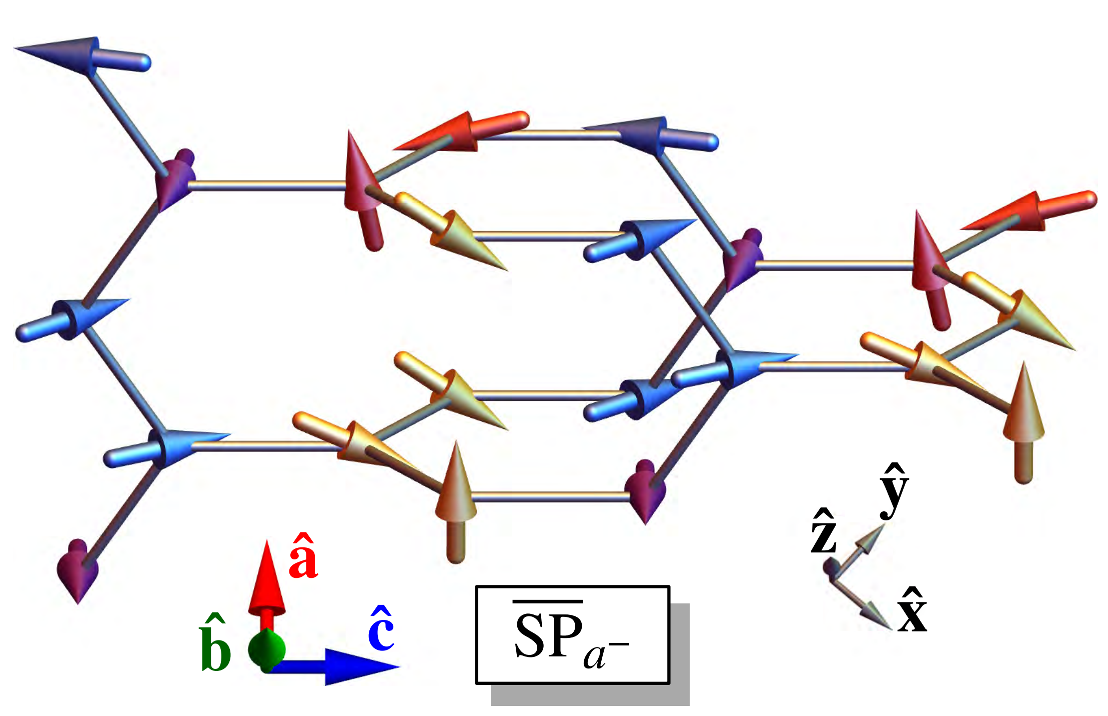




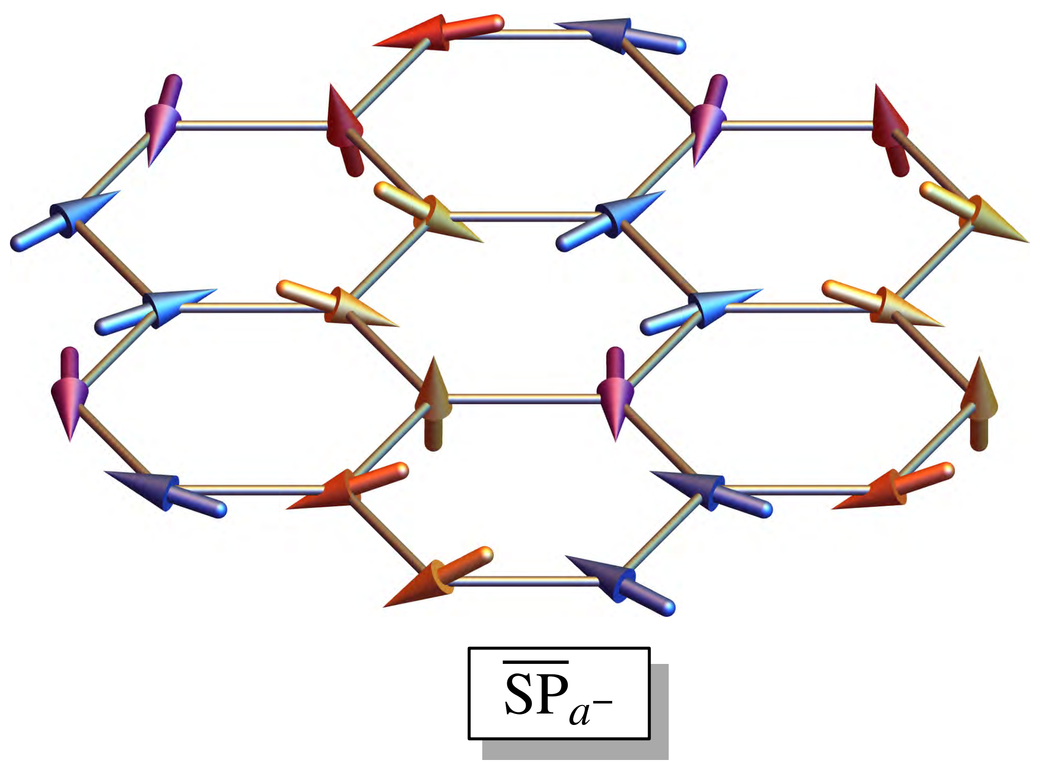
$\mathbf{Q} \parallel \mathbf{a}^* \in$ *ac* plane!

$\Rightarrow$ "quasi-2D" state

Commensurate period-3 state ($J \ll |K|, |\Gamma|$)

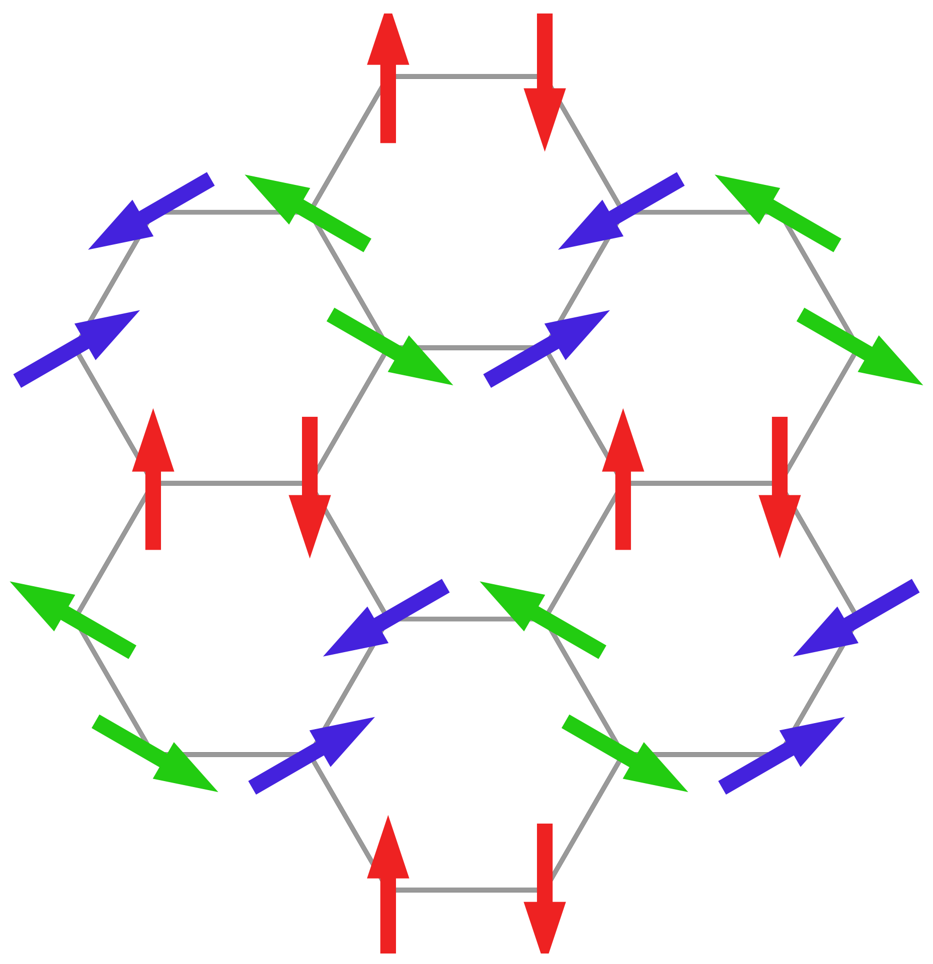


3D-2D equivalence



Duality transformation

... that rotates spins by $\pi/2$ ($3\pi/2$) about z axis

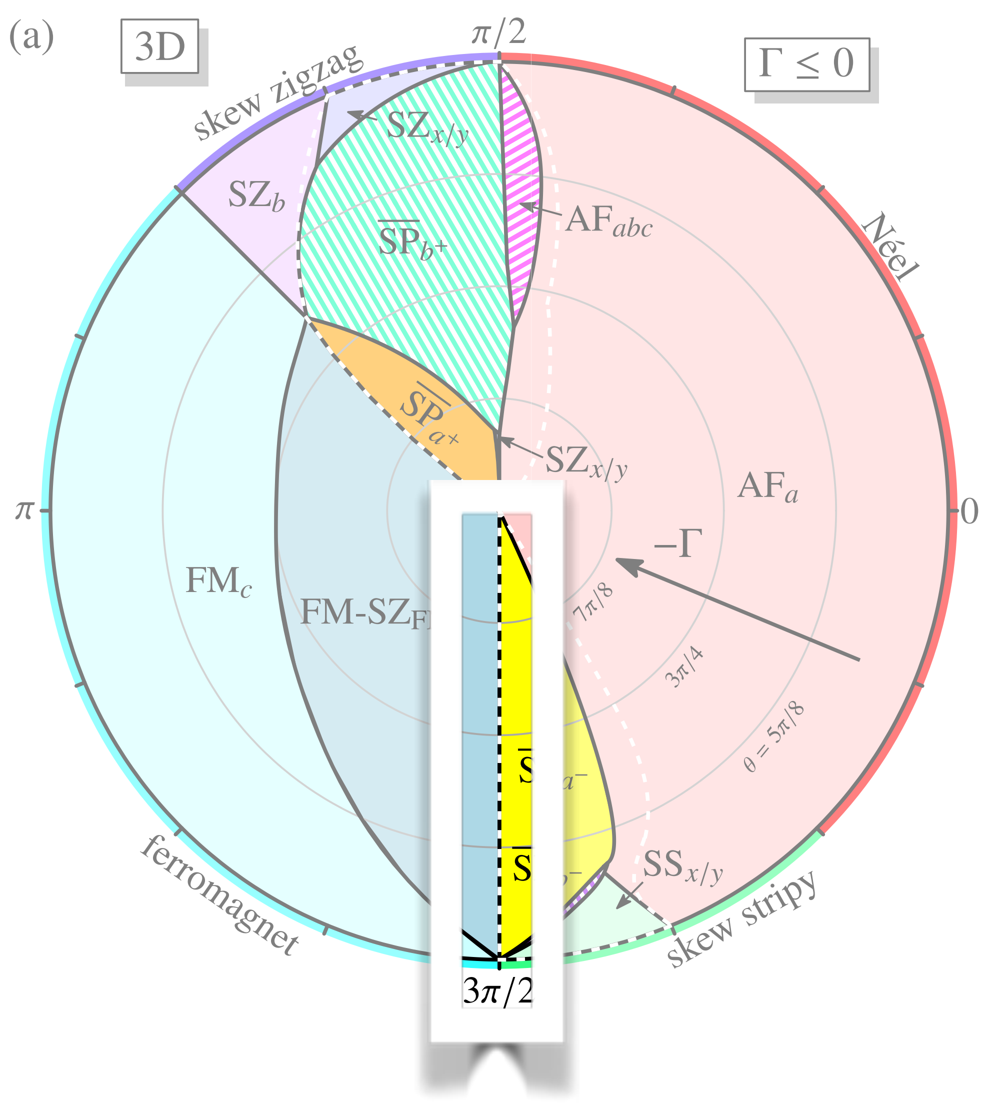


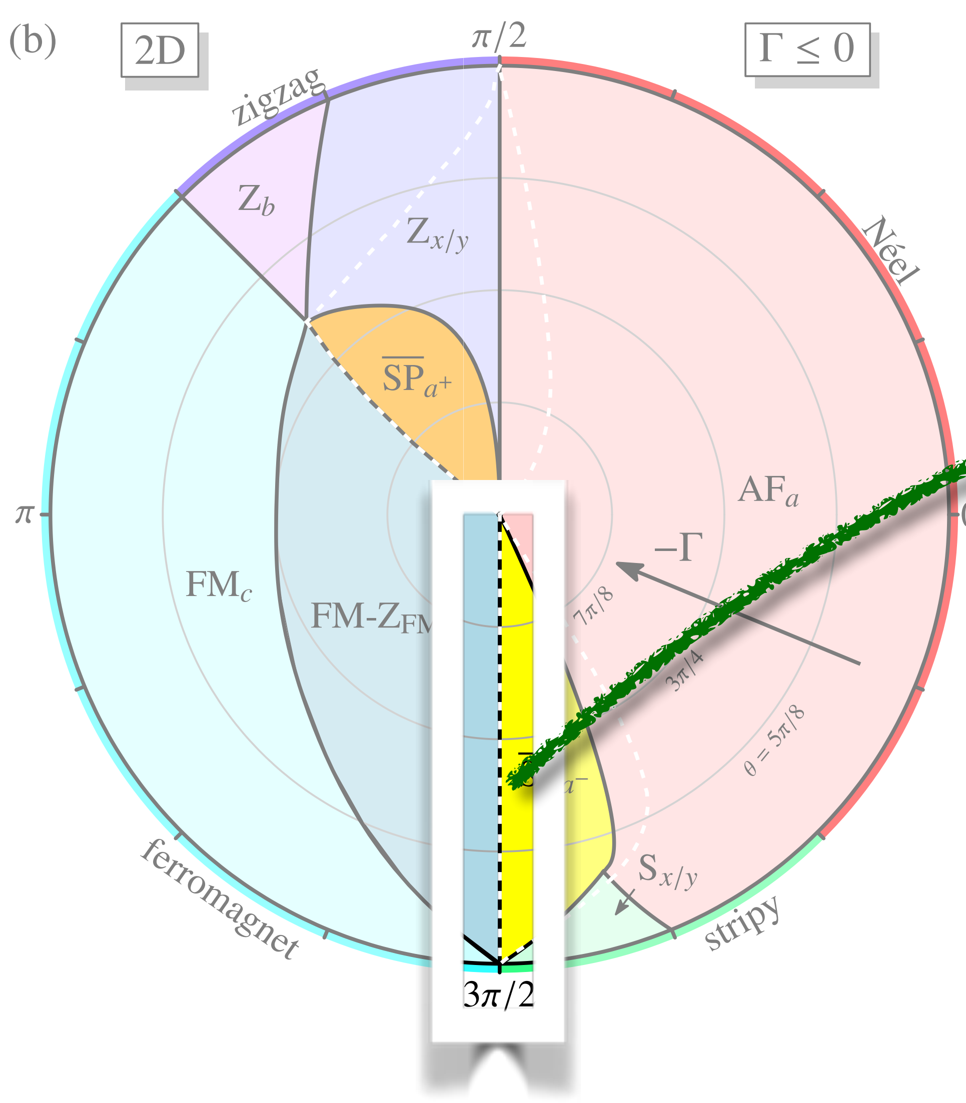
Period-3 state
3D HK$\pm\Gamma$

"K state"
[Ducatman *et al.*, PRB '18]

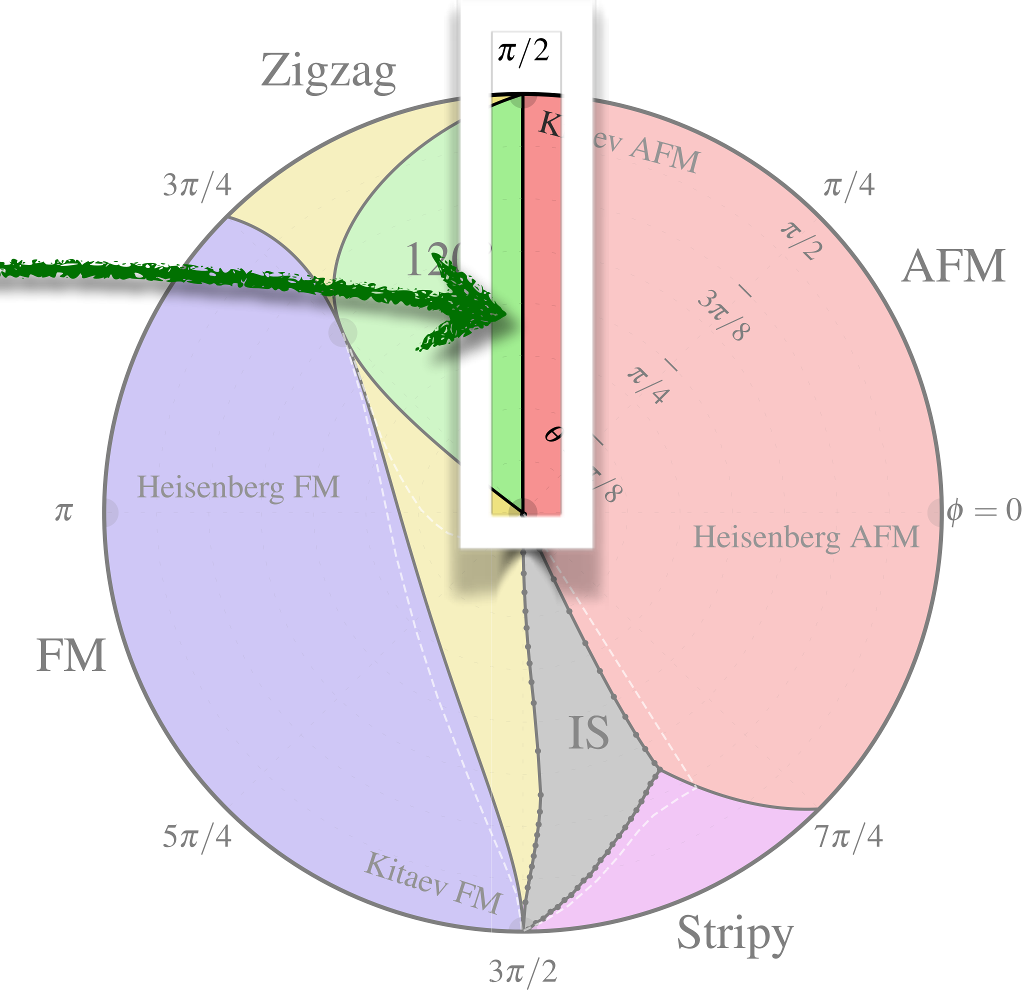
Period-3 state
2D HK$\pm\Gamma$

120° state
2D HKΓ





K → − K
Γ → − Γ



→ Talk by N. Perkins (2:40 p.m.)

[Rau, Lee, Kee, PRL '14]

β-Li_2IrO_3 order: Dual of 120°-state

Duality explains key features of β-Li_2IrO_3:

(i) Zigzag chains of coplanar spins

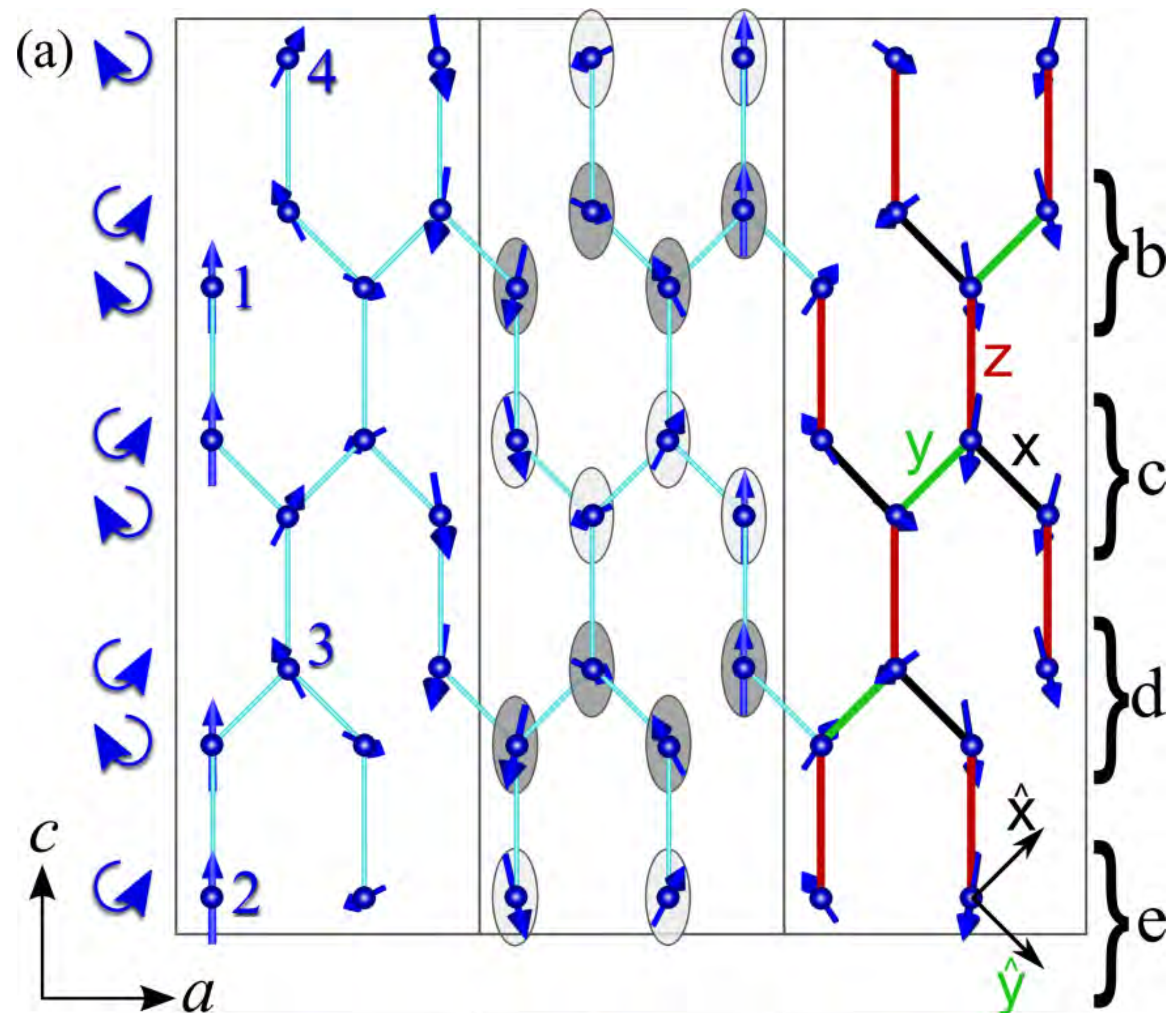
... as duality transformation preserves coplanarity along zigzag chains

(ii) Counterrotating spirals

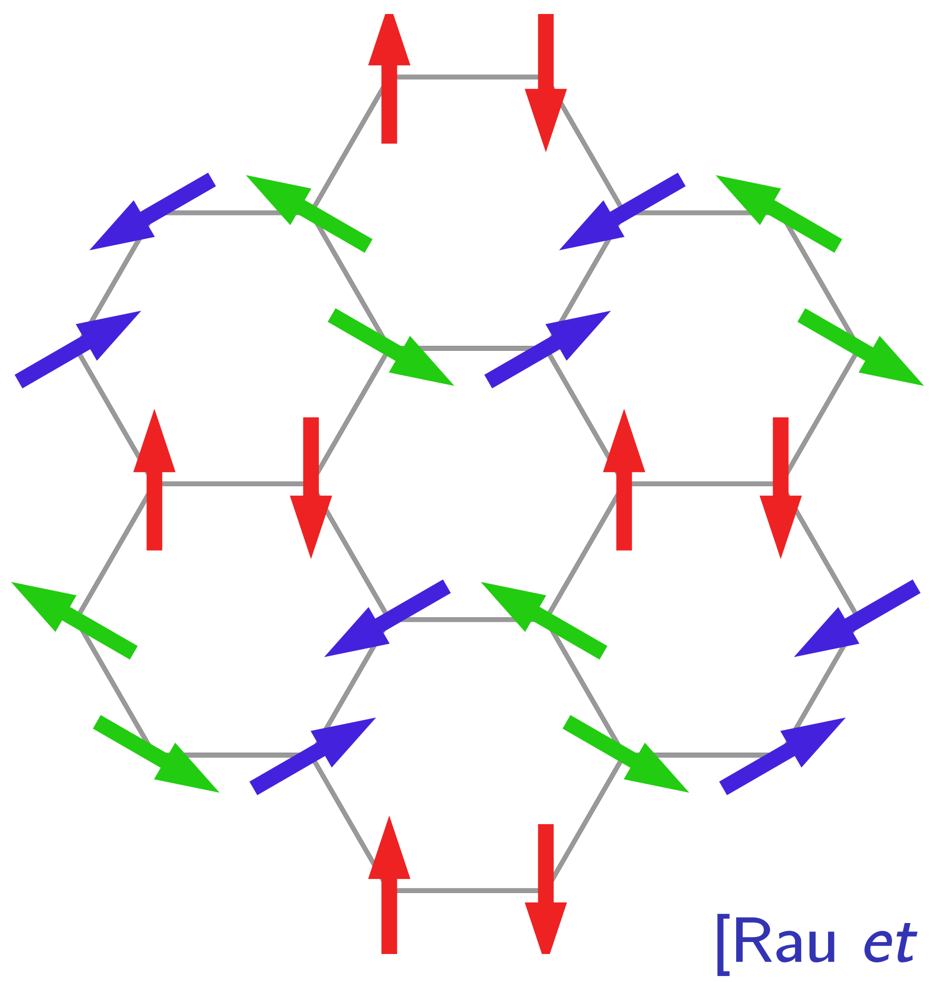
... spins on two sublattices rotate in opposite directions

(iii) Angle between next-nearest neighbors ≈ 120°

... with ordering wavevector $\mathbf{q} = 0.57(1)\mathbf{a}^* \approx 2/3\mathbf{a}^*$



[Biffin *et al.* '14]



[Rau *et al.*, '14]

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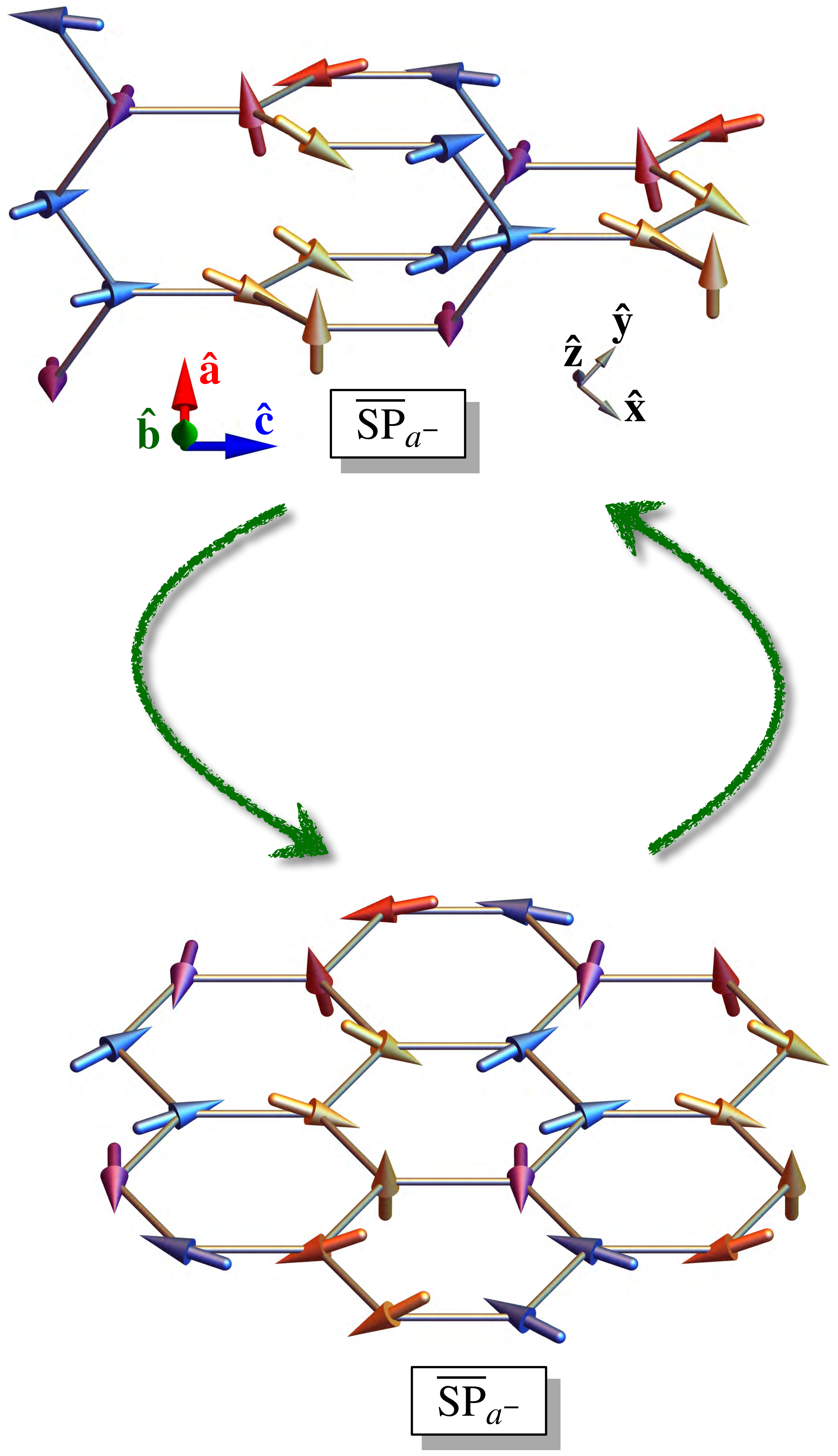
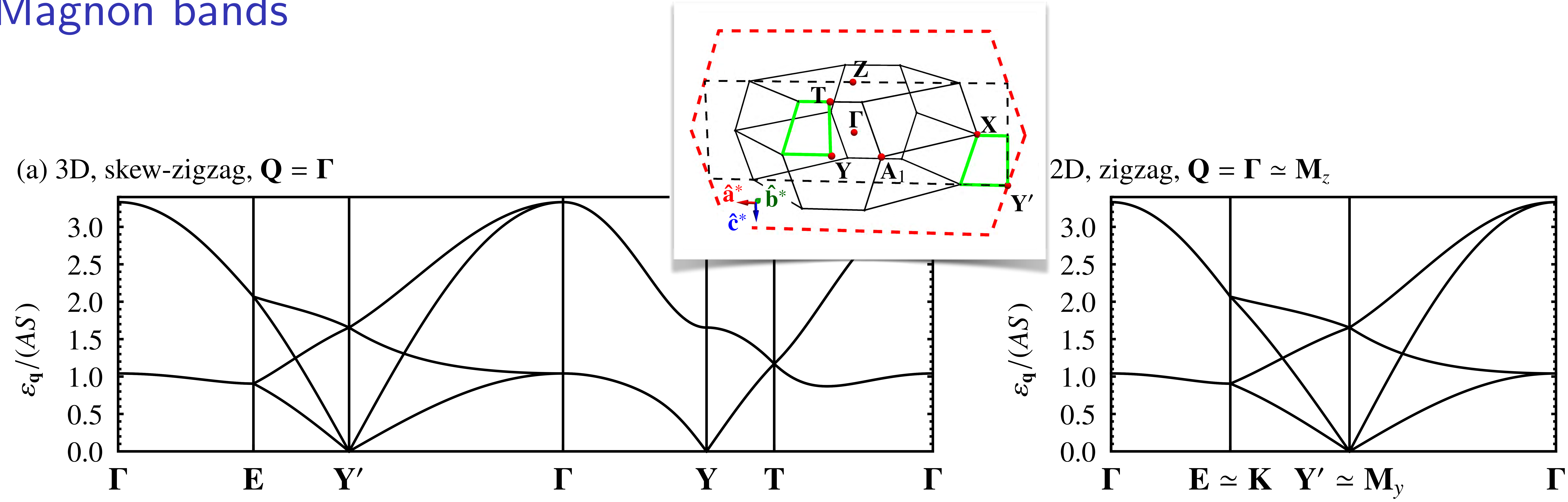
4. Quantum effects
5. Conclusions



Magnon bands

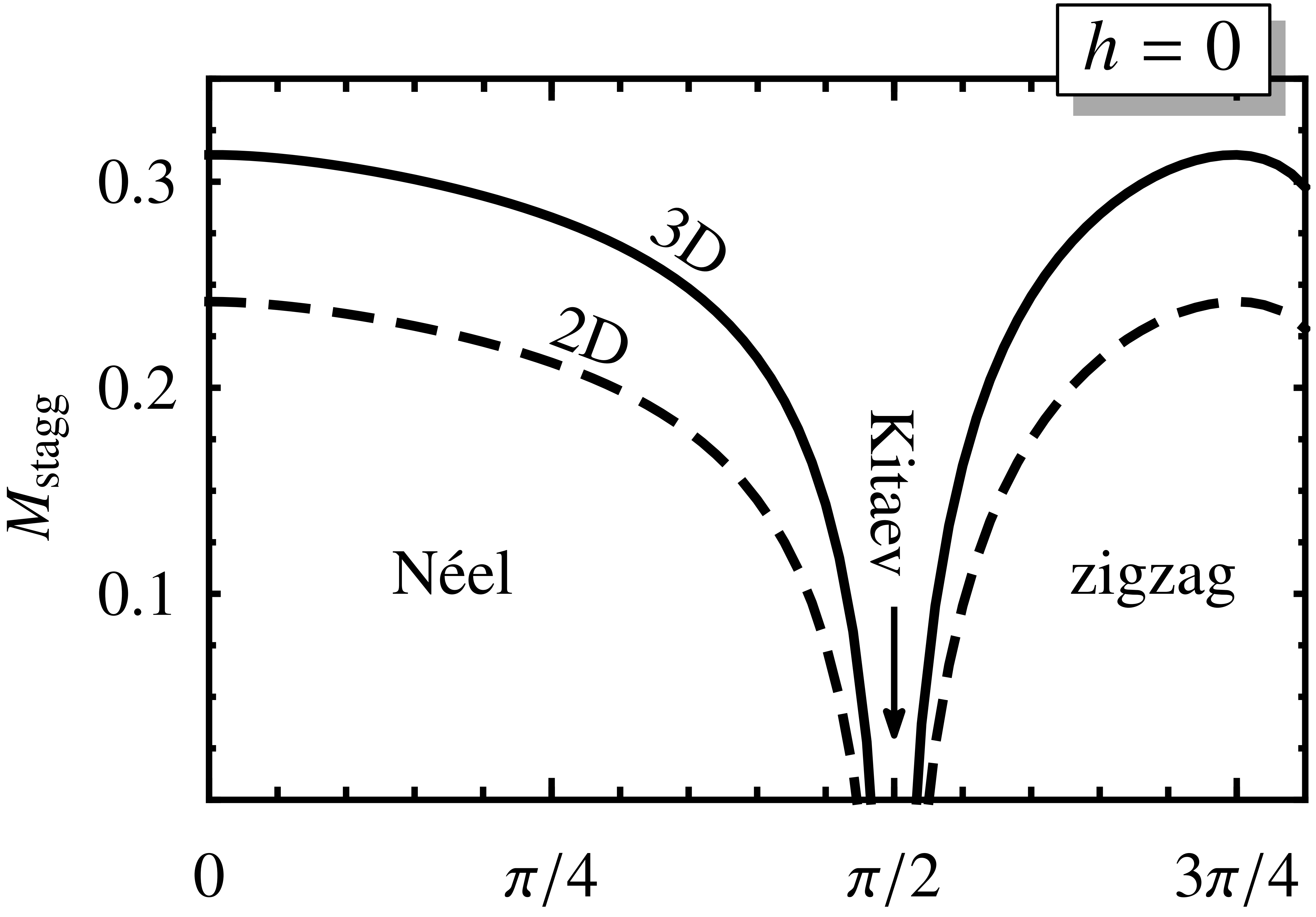


hyperhoneycomb

honeycomb

$$\boxed{\varepsilon_{\mathbf{q}}(\text{hyperhoneycomb}) = \varepsilon_{\mathbf{q}}(\text{honeycomb}) \text{ for all } \mathbf{q} \in ac \text{ plane}}$$

Staggered magnetization near Kitaev limit



$\varphi = \arg(2J + \mathrm{i}K)$

... for S = 1/2 Heisenberg-Kitaev model

Conclusions

3D-2D equivalence of ordered states ...

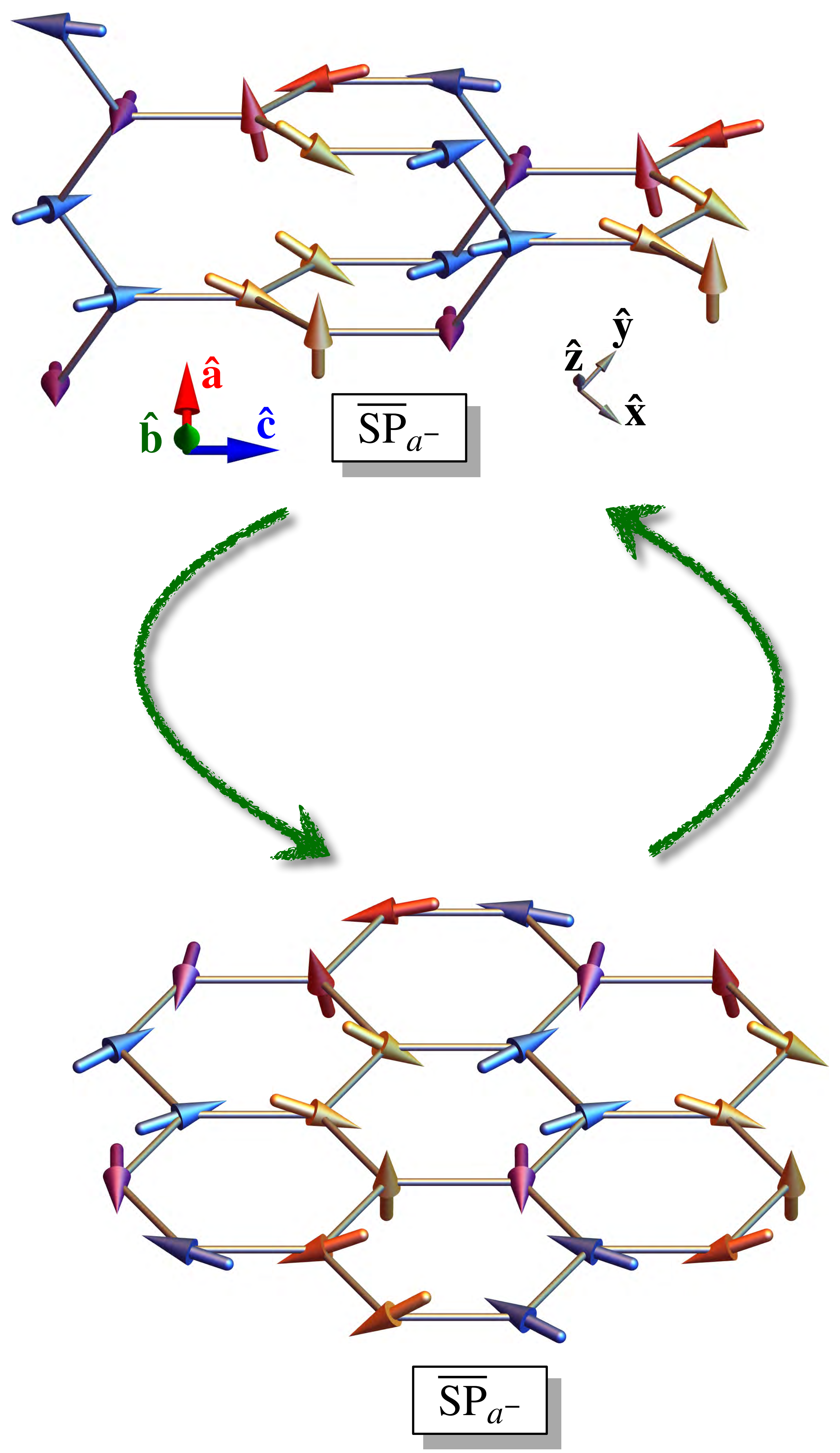
... applies to all ordered states with $\mathbf{Q} \in ac$ plane

... leads to (largely) identical phase diagrams

... can be extended to full harmonic series $^{\mathcal{H}}\langle n \rangle$

... independent of model

... establishes equivalence of magnetic order
in α-Li_2IrO_3, β-Li_2IrO_3, and γ-Li_2IrO_3



[Krüger, Vojta, LJ, arXiv:1907.05423]