

# **Intrinsic magnetic topological states in $\text{MnBi}_2\text{Te}_4$**

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**2D van der Waals spin systems, SPICE**

# Outline

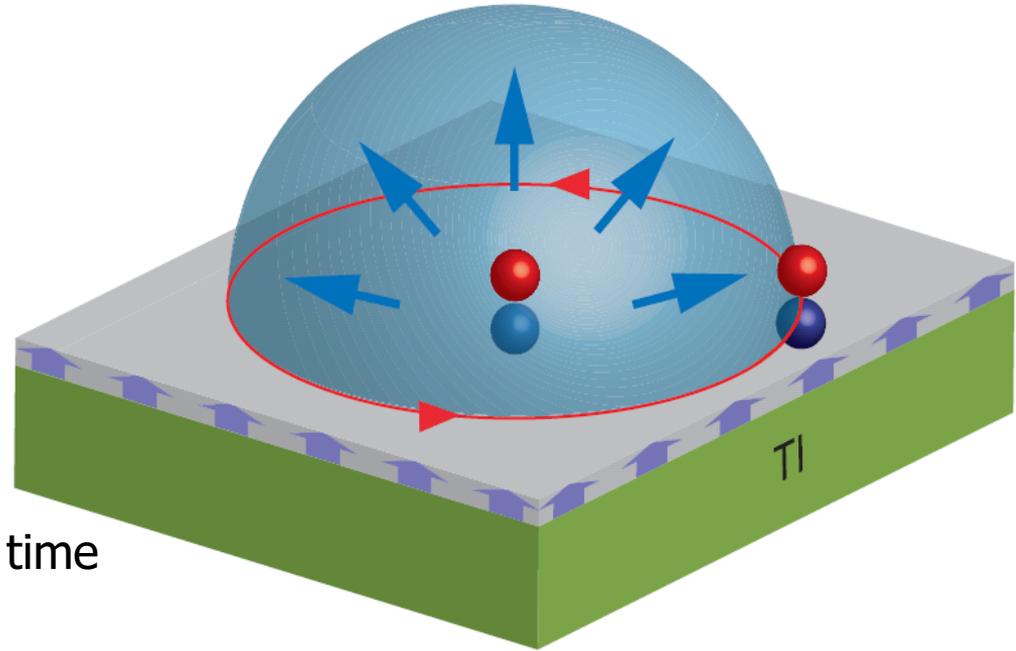
1. Axion response from topological insulators
2. Quantized anomalous Hall effect vs Topological magnetoelectric effect
3. Intrinsic magnetic topological state in  $\text{MnBi}_2\text{Te}_4$
4. Flat Chern band in twisted bilayer  $\text{MnBi}_2\text{Te}_4$

# General theory of topological insulators

- Topological field theory of topological insulators. Generally valid for interacting and disordered systems. Directly measurable physically. Quantized magneto-electric effect (Qi, Hughes and Zhang, Wilczek)

$$S_0 = \frac{1}{8\pi} \int d^3x dt \left( \epsilon \mathbf{E}^2 - \frac{1}{\mu} \mathbf{B}^2 \right)$$

- For a periodic system, the system is time reversal symmetric only when  $\theta=0 \Rightarrow$  trivial insulator  
 $\theta=\pi \Rightarrow$  non-trivial insulator



- Topological band theory based on  $\mathbb{Z}_2$  topological band invariant of single particle states.

(Fu, Kane and Mele, Moore and Balents, Roy)

$$S_\theta = \left( \frac{\theta}{2\pi} \right) \left( \frac{\alpha}{2\pi} \right) \int d^3x dt \mathbf{E} \cdot \mathbf{B}$$

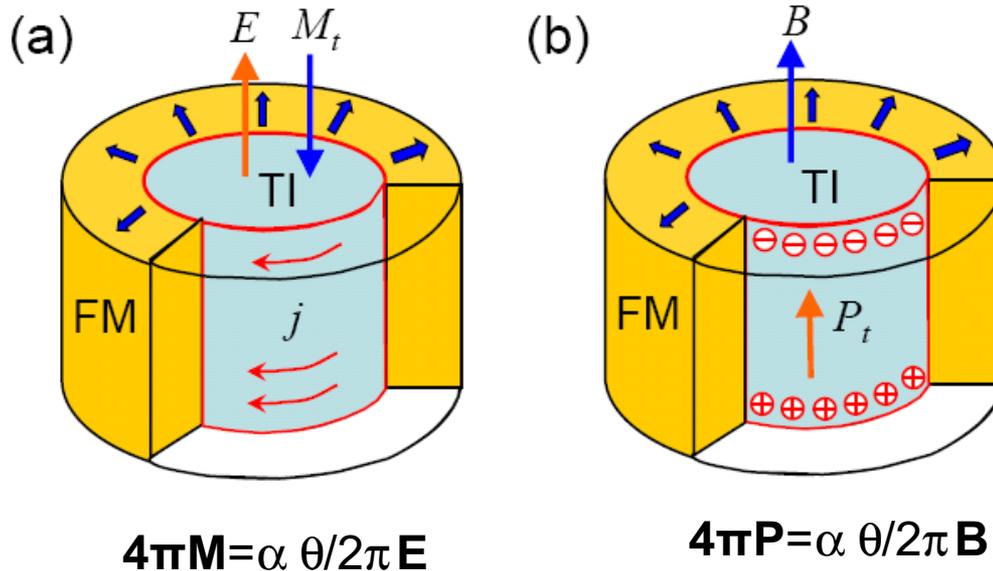
$$\alpha = \frac{e^2}{hc}$$

$$S_\theta = \frac{\theta}{2\pi} \frac{\alpha}{16\pi} \int d^3x dt \epsilon_{\mu\nu\rho\tau} F^{\mu\nu} F^{\rho\tau} = \frac{\theta}{2\pi} \frac{\alpha}{4\pi} \int d^3x dt \partial^\mu (\epsilon_{\mu\nu\rho\sigma} A^\nu \partial^\rho A^\sigma)$$

# The Topological Magneto-Electric (TME) effect

Slide 4

- Equations of axion electrodynamics predict the robust TME effect.



$$\begin{aligned}\nabla \cdot \mathbf{D} &= 4\pi\rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} \\ \mathbf{D} &= \mathbf{E} + 4\pi\mathbf{P} - 2P_3\alpha\mathbf{B} \\ \mathbf{H} &= \mathbf{B} - 4\pi\mathbf{M} + 2P_3\alpha\mathbf{E}\end{aligned}$$

Wilczek, axion electrodynamics

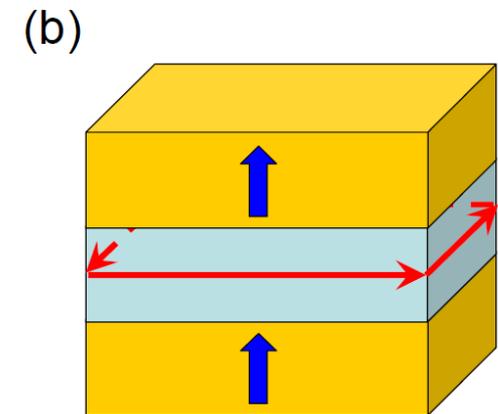
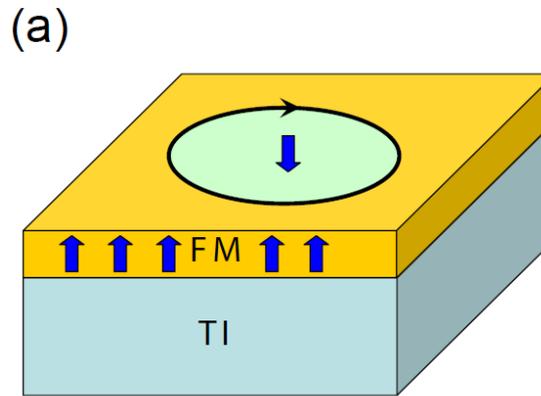
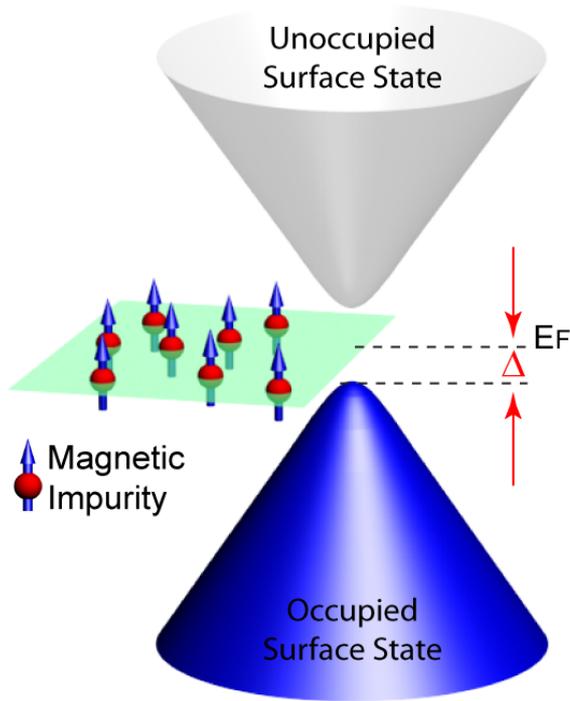
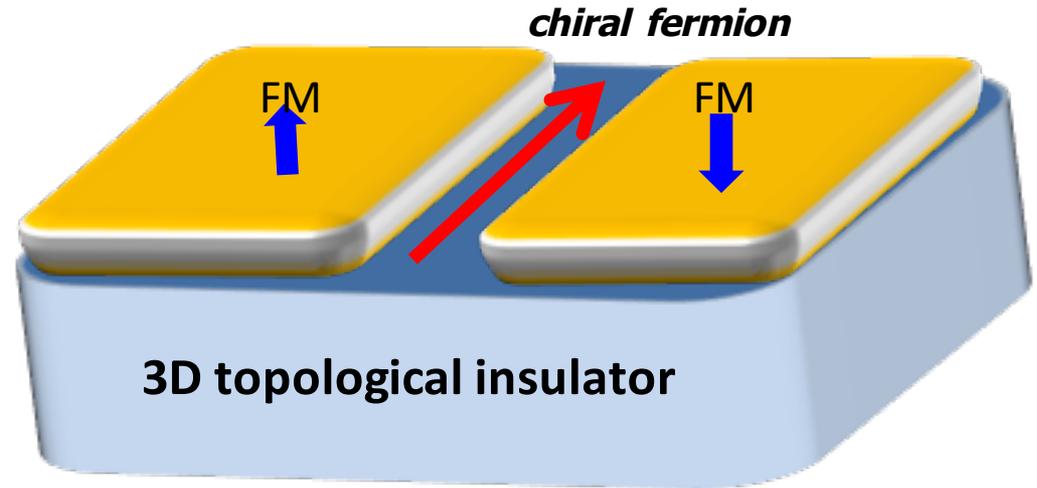
- $P_3 = \theta/2\pi$  is the electro-magnetic polarization, microscopically given by the CS term over the momentum space. Change of  $P_3 = 2^{\text{nd}}$  Chern number!

$$\begin{aligned}P_3(\theta_0) &= \int d^3k \mathcal{K}^\theta \\ &= \frac{1}{16\pi^2} \int d^3k \epsilon^{\theta_{ijk}} \text{Tr} \left[ \left( f_{ij} - \frac{1}{3} [a_i, a_j] \right) \cdot a_k \right]\end{aligned}$$

# Gapped Dirac fermions on the surface, chiral fermions on the domain wall

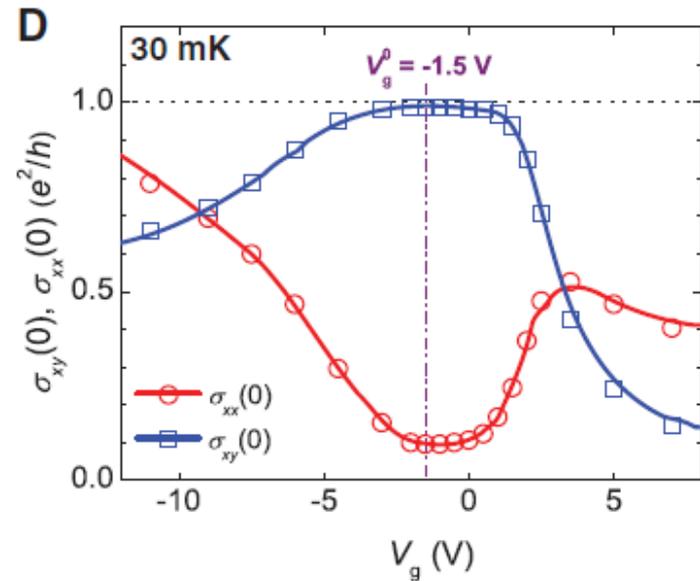
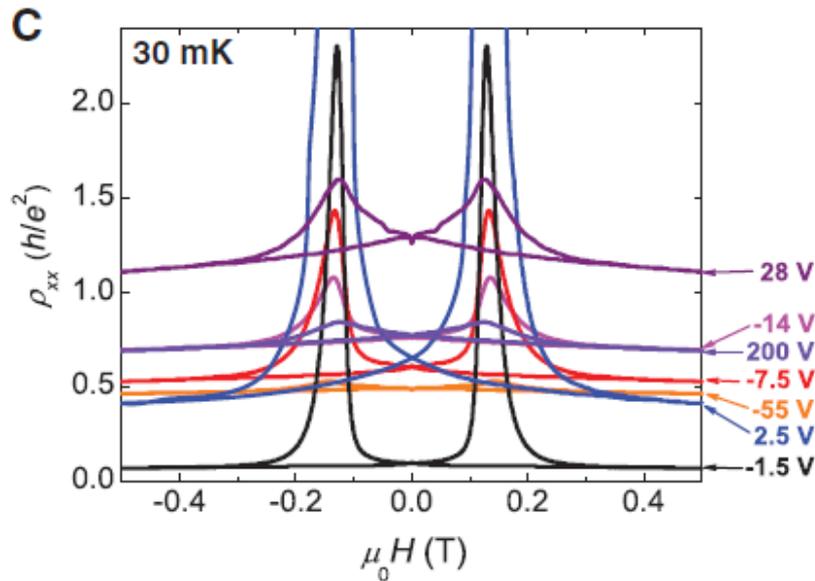
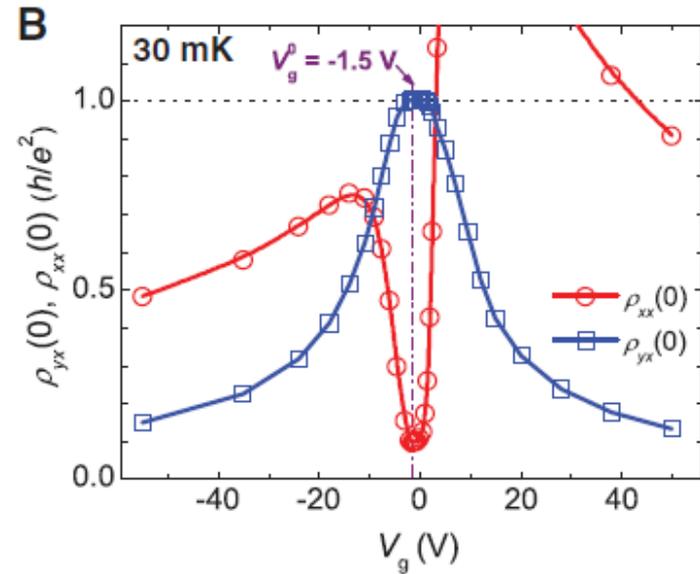
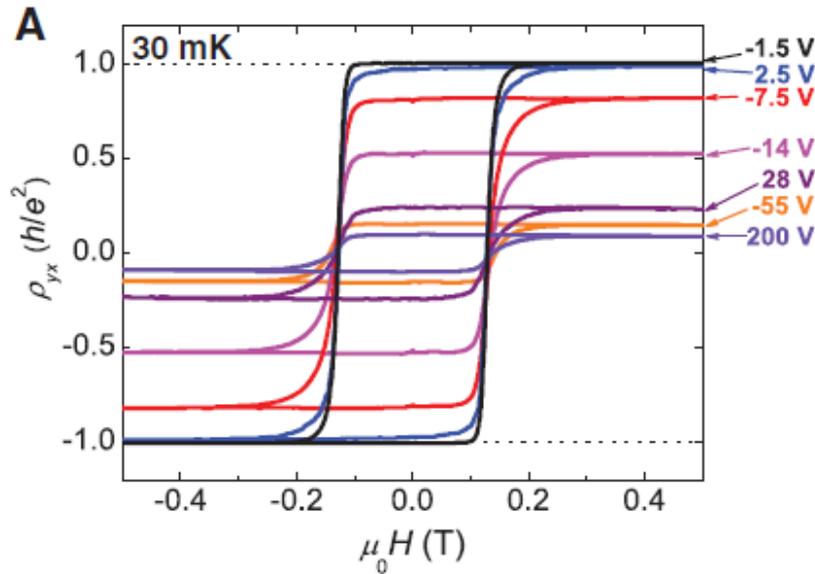
F-I $\uparrow$	F-I $\downarrow$
$\theta = 0$	$\theta = 0$
TI $\theta = \pi$	

(b)



QAH can be realized in ferromagnetic TI (Qi, Hughes, Zhang, PRB 2008)

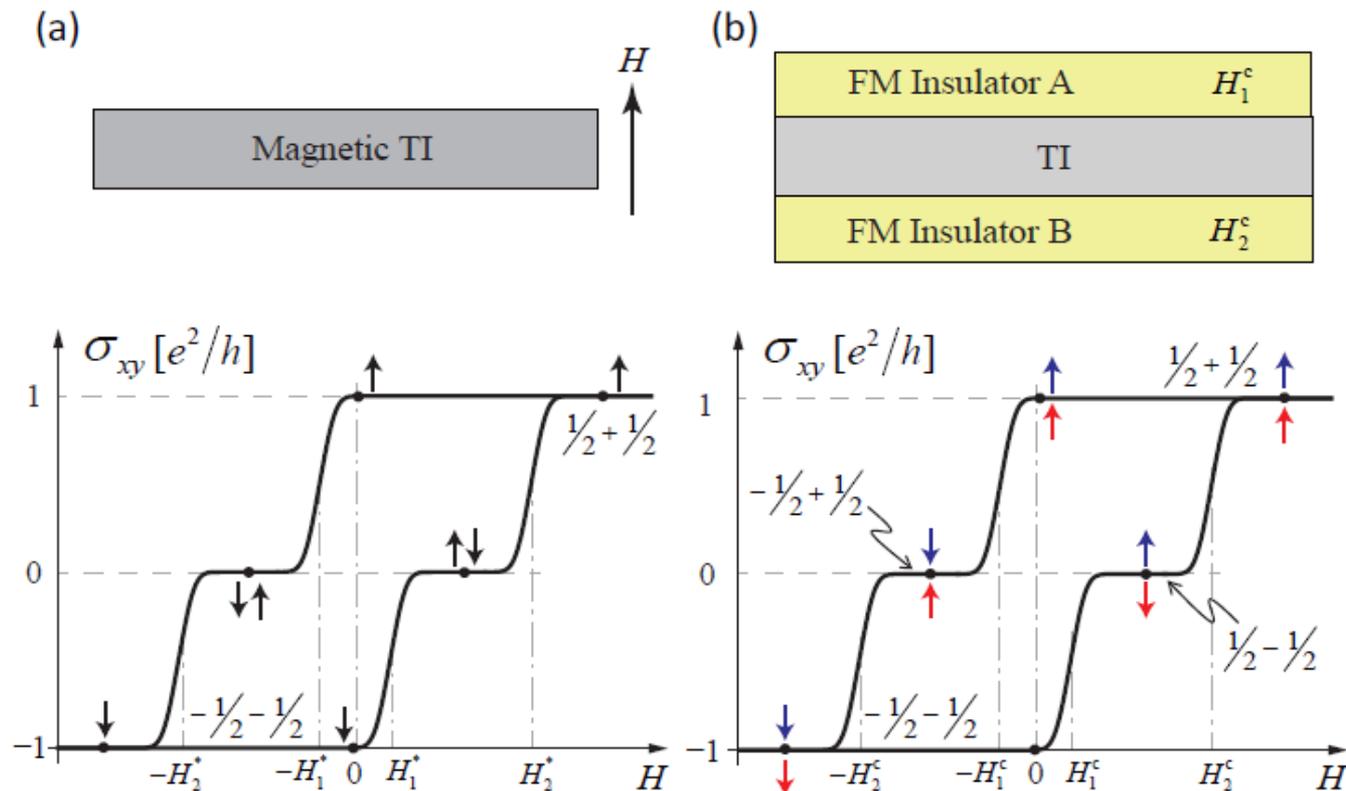
# Experimental observation of the QAH effect in 5 QL CrBiSbTe at 20 mK (Science 2013)



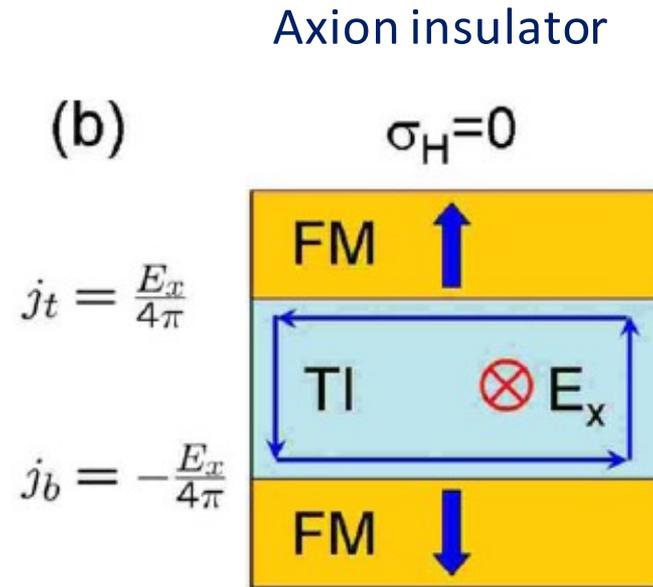
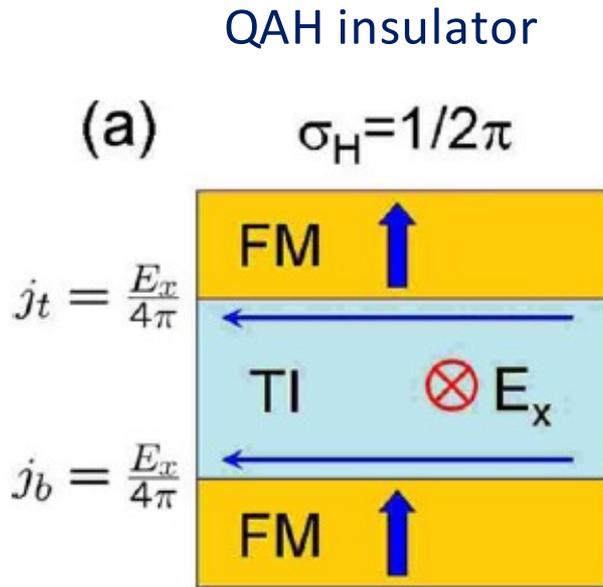
# Topological magnetoelectric effect in TIs and QAHE

$$S_\theta = \frac{\theta}{2\pi} \frac{e^2}{h} \int d^3x dt \mathbf{E} \cdot \mathbf{B},$$

1. T-breaking surface gap
2. Fully insulating
3. Finite size



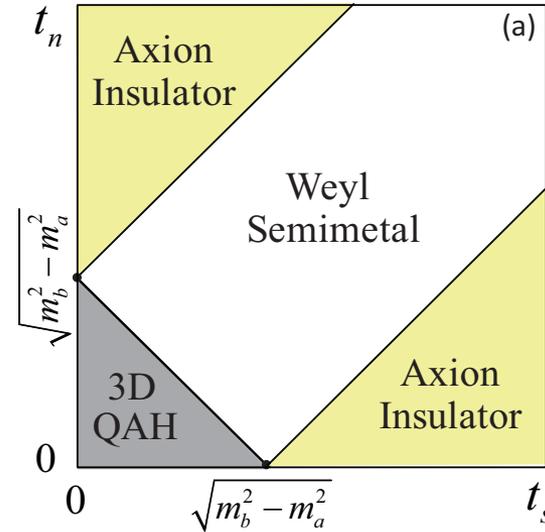
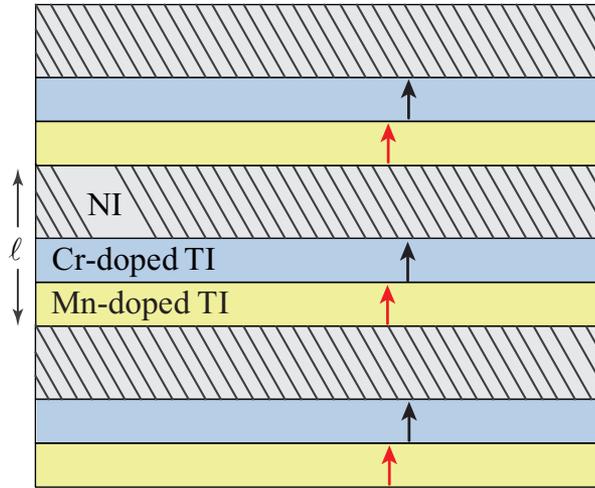
# QAH effect vs TME effect



- T-breaking surfaces
- With magnetic domain
- Gapless edge mode at domain wall
- Fully insulating
- 2D system, need finite size

- T-breaking surfaces
- Without magnetic domain
- No gapless edge/hinge mode
- Fully insulating
- 3D system, finite size effect in 2D

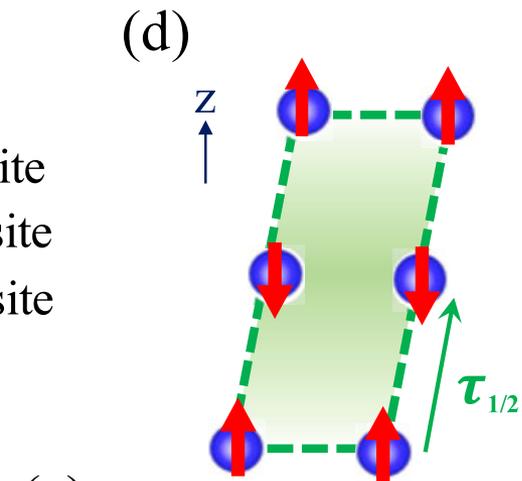
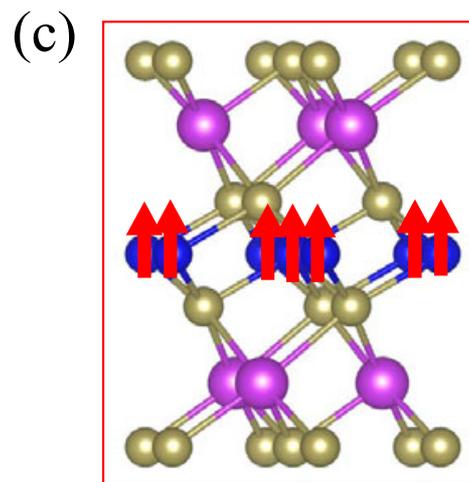
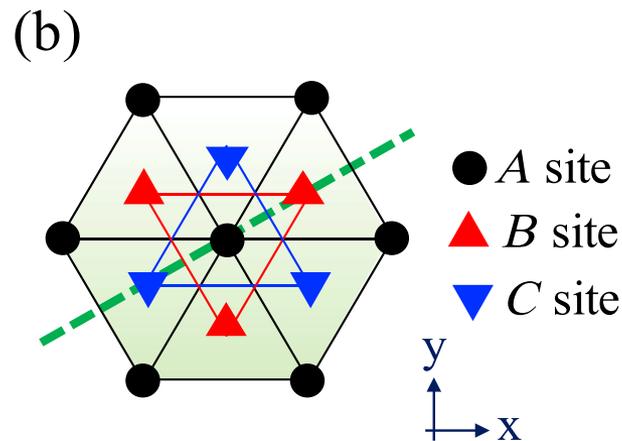
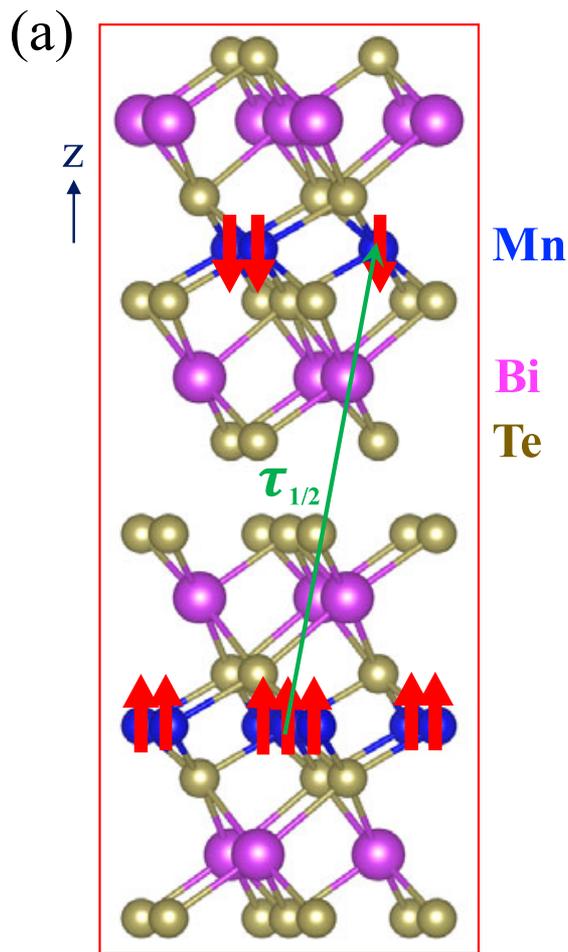
# Multiple types of topological states in superlattice



$$\mathcal{H} = \sum_{\mathbf{k}_{\parallel}, i, j} \left[ v_F \tau^z (\hat{\mathbf{z}} \times \boldsymbol{\sigma}) \cdot \mathbf{k}_{\parallel} \delta_{i, j} + m_a \tau^z \sigma^z \delta_{i, j} + m_b \sigma^z \delta_{i, j} \right. \\ \left. + t_s \tau^x \delta_{i, j} + \frac{t_n}{2} \tau^+ \delta_{i+1, j} + \frac{t_n}{2} \tau^- \delta_{i-1, j} \right] c_{\mathbf{k}_{\parallel} i}^{\dagger} c_{\mathbf{k}_{\parallel} j}, \quad (2)$$

# Hydrogen atom of magnetic TI: MnBi<sub>2</sub>Te<sub>4</sub>

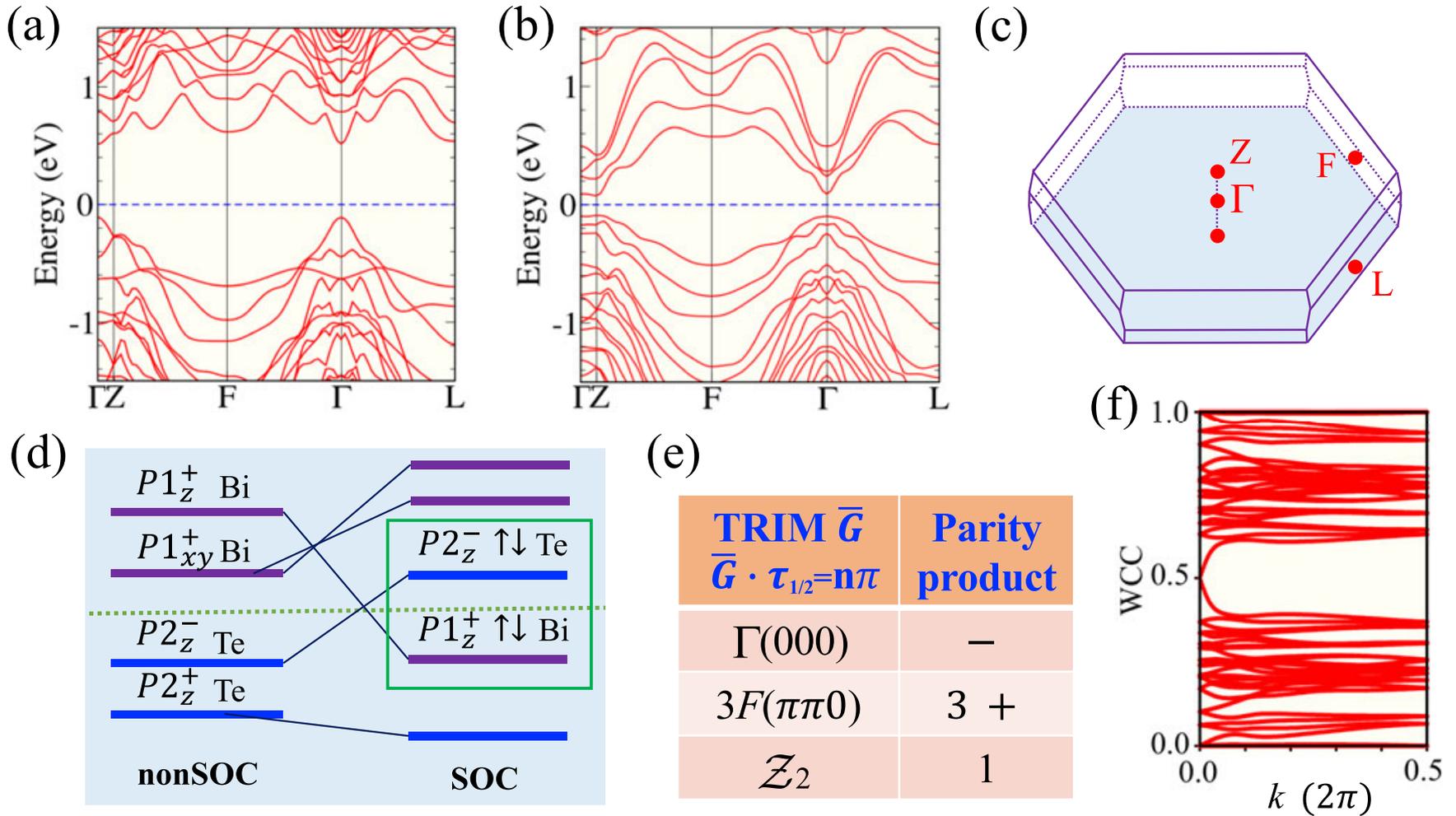
$$\mathcal{S} = \Theta\tau_{1/2}$$



(e)

Phase	Energy
AFM1	0.0000eV
AFM2	0.0017eV
FM1	0.0460eV
FM2	0.0464eV

# Electronic band structure of AFM MnBi<sub>2</sub>Te<sub>4</sub>

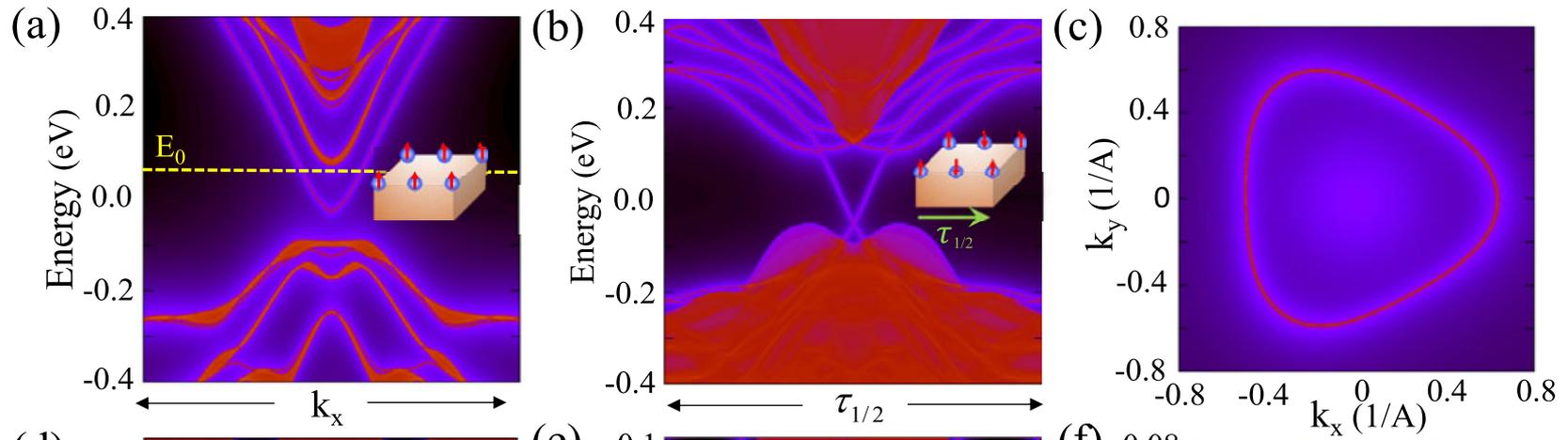


# Model for AFM topological insulator MnBi2Te4

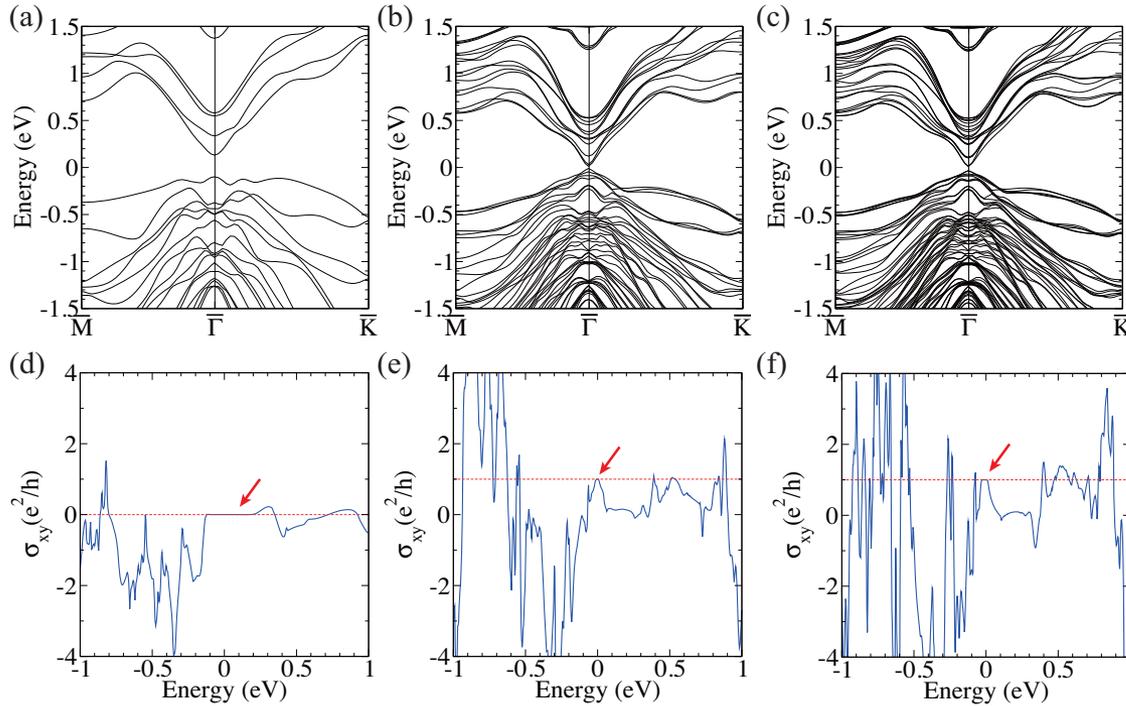
$$H(\mathbf{k}) = \epsilon_0(\mathbf{k})\mathbf{I}_{4\times 4} + \begin{pmatrix} \mathcal{M}(\mathbf{k}) & A_1k_z & 0 & A_2k_- \\ A_1k_z & -\mathcal{M}(\mathbf{k}) & A_2k_- & 0 \\ 0 & A_2k_+ & \mathcal{M}(\mathbf{k}) & -A_1k_z \\ A_2k_+ & 0 & -A_1k_z & -\mathcal{M}(\mathbf{k}) \end{pmatrix} + o(\mathbf{k}^2)$$

Pz+, up, Pz-, up, Pz+, down, Pz-, down

## Single Gapped Dirac cone on the surface of MnBi2Te4



# QAH and Zero plateau in MnBi2Te4 odd and even layers



Even layer, conserved  $\mathcal{I}\Theta$

$$\sigma_{xy} = 0$$

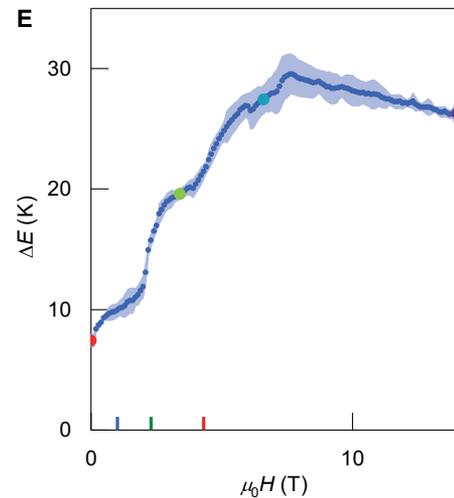
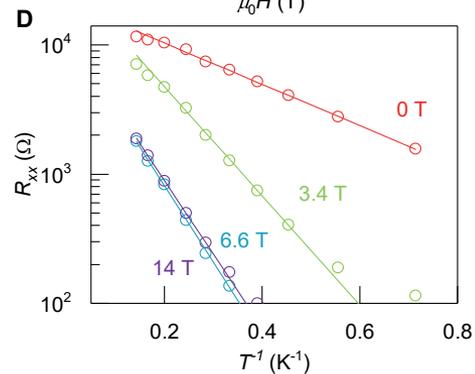
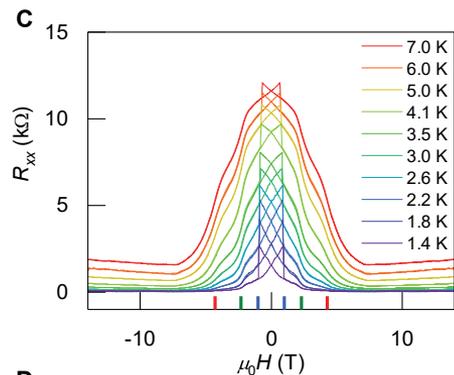
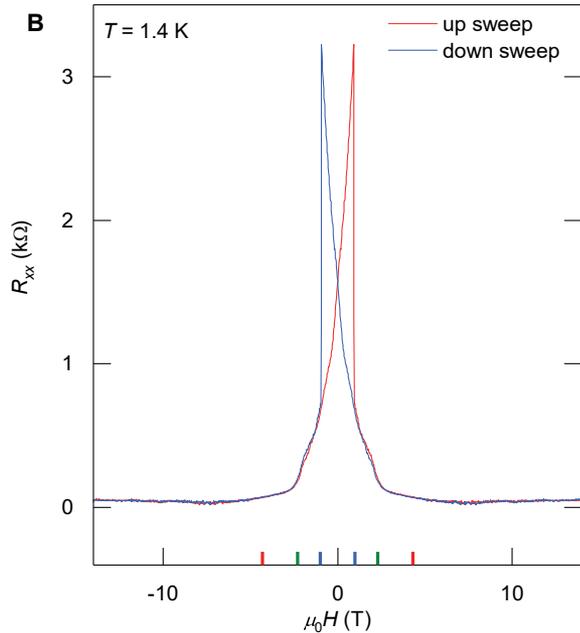
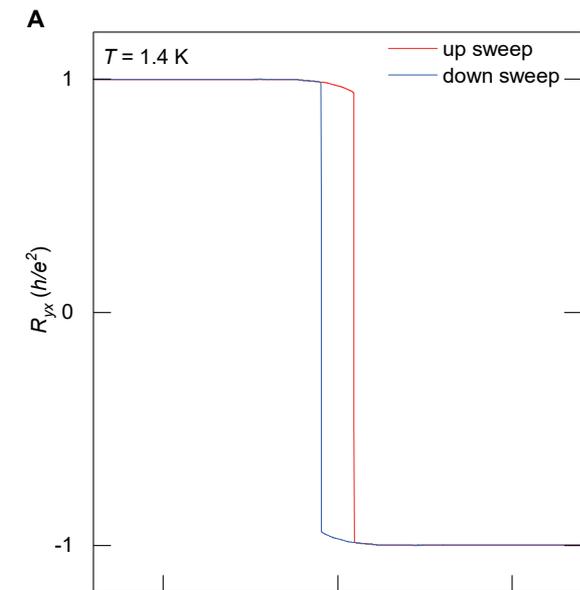
Odd layer, broken  $\mathcal{I}\Theta$ , with band inversion

$$\sigma_{xy} = \pm e^2/h$$

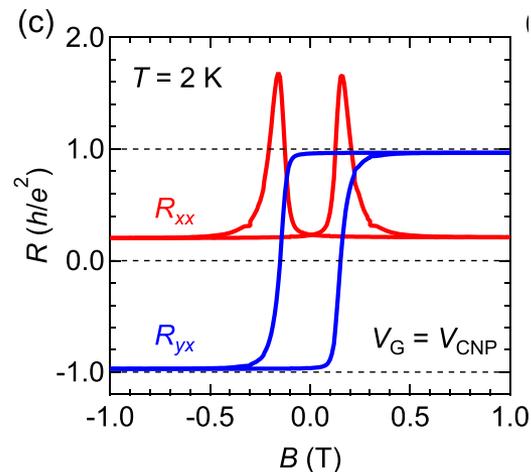
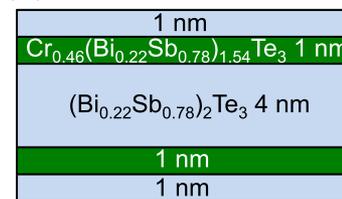
Thickness (SL)	Topology	Bandgap (meV)
1	Trivial	321
2	ZPQAH	107
3	QAH	66
4	ZPQAH	97
5	QAH	77
6	ZPQAH	87
7	QAH	85
$\infty$ (bulk)	3D AFM TI	225

# Observation of zero-field QAH in 5 SL layer (Yuanbo Zhang Group)

Deng *et al*, Science 367, 895 (2020)



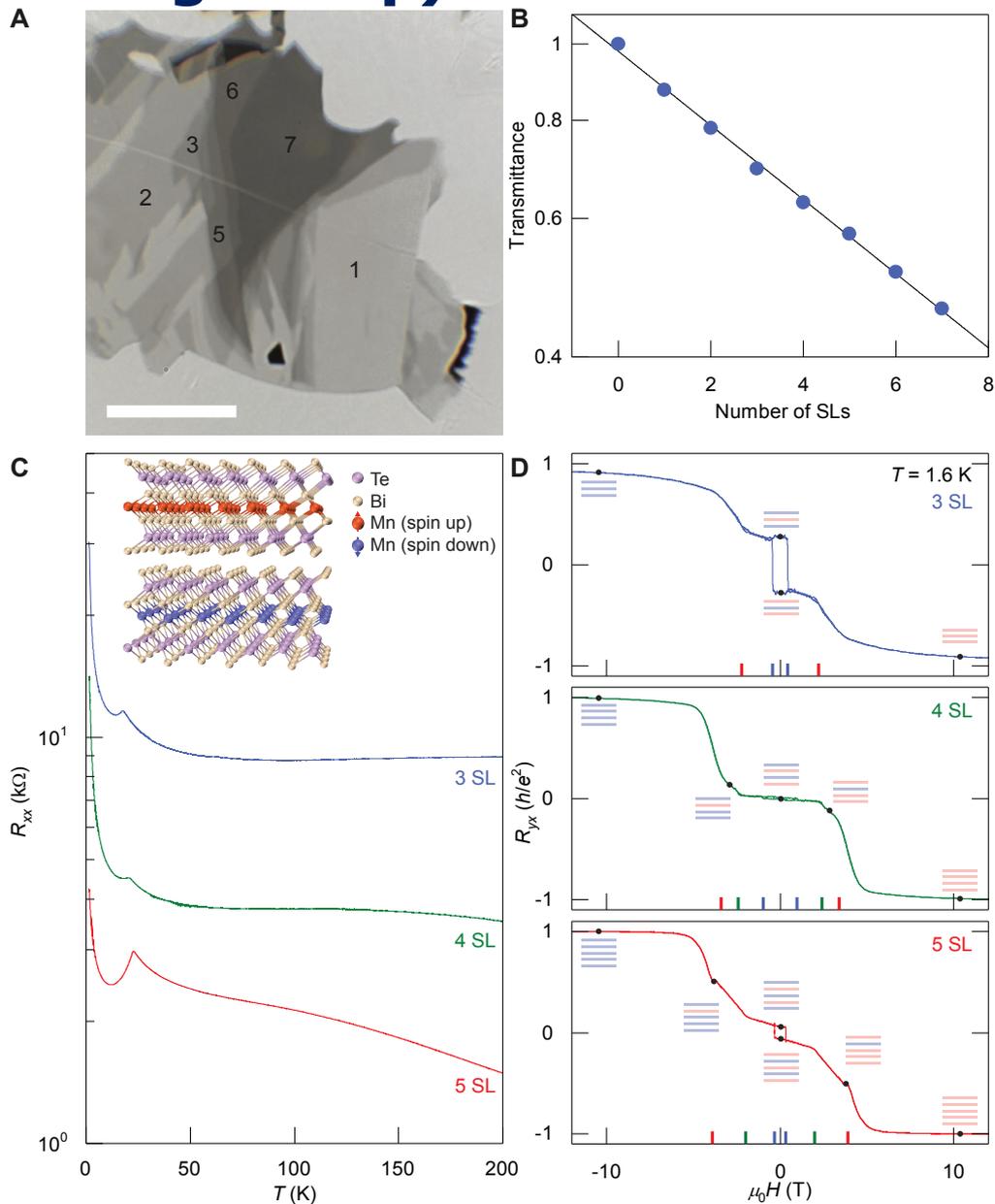
(c) penta-layer



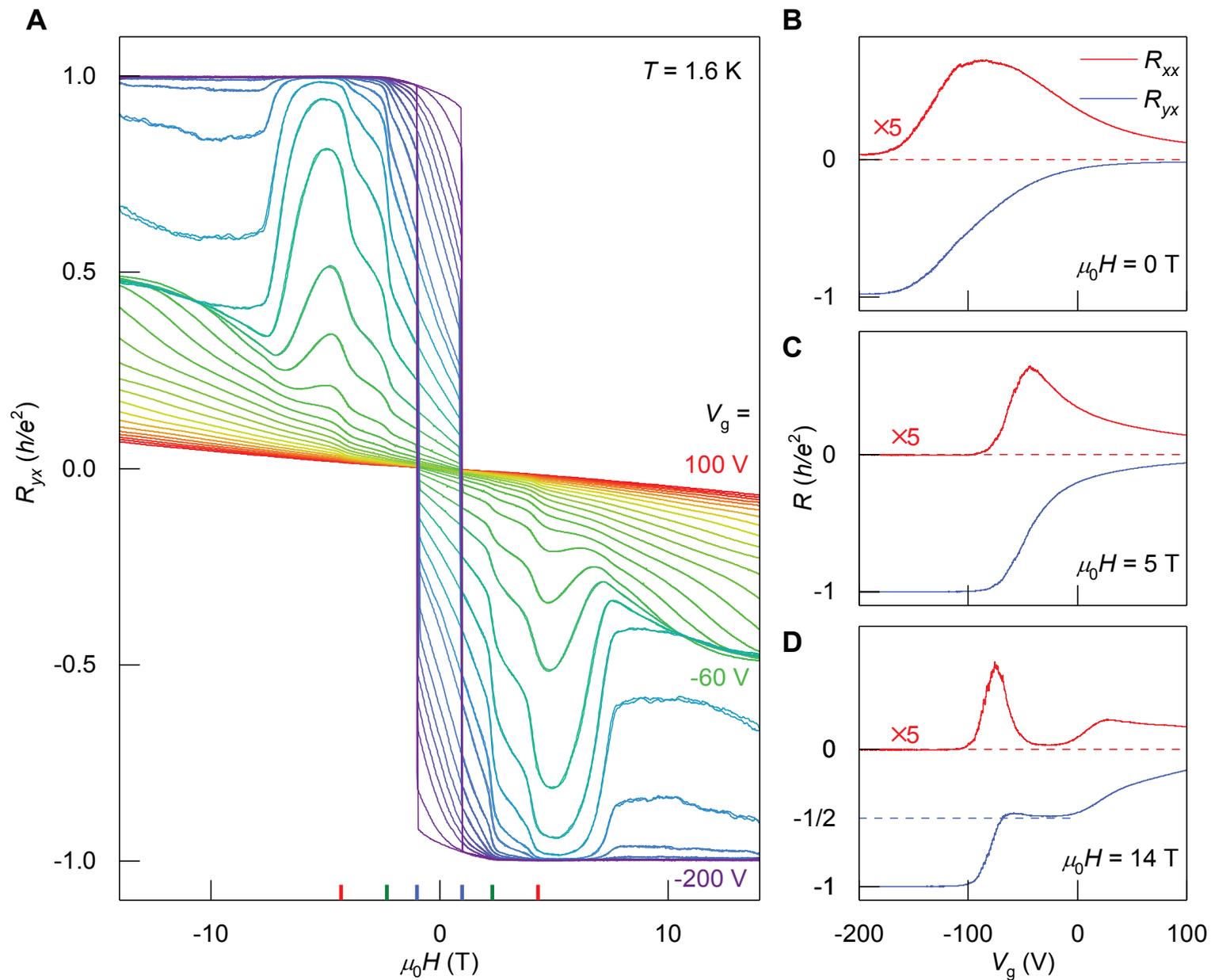
Tokura group 2015

# Observation of QAH in 5 SL and possible AI in 4 SL (Yuanbo Zhang Group)

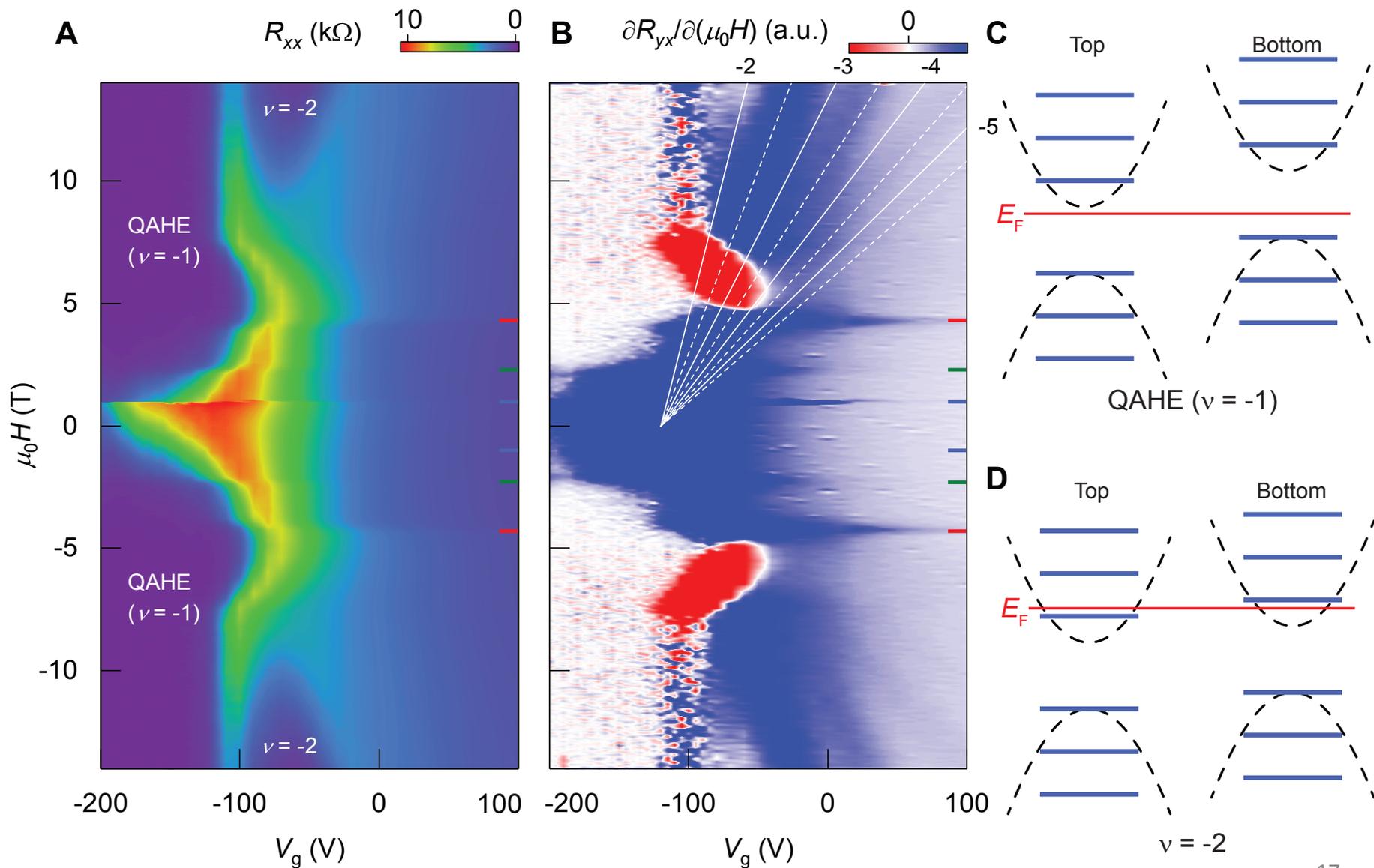
Deng *et al*, Science 367, 895 (2020)



# Coexistence of QAH and QHE in odd layer



# Phase diagrams of H and gate voltage



# Higher plateau QAH: multi-channel chiral edge states

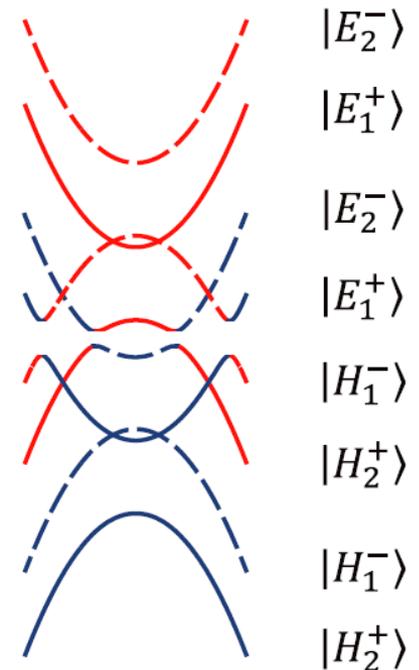
**Basic physics** ( realization of  $\sigma_{xy} = n \frac{e^2}{h}$  ):

**System:** thin film of 3D TI with magnetic dopping.

1) Dopping induced exchange field  $\Delta$  splits the 3D bulk bands with spin  $\uparrow$  and  $\downarrow$ , and only one pair of bulk bands ( $s \uparrow, p \downarrow$ ) are inverted, while the other are not ( $s \downarrow, p \uparrow$ ).

2) Make 3D TI into a thin film (compactify the  $z$  direction), so that

3D bulk bands ( $s \uparrow, p \downarrow$ )  $\xrightarrow{k_z \text{ discrete}}$  2D sub-bands.



QAH with  $n = 2$

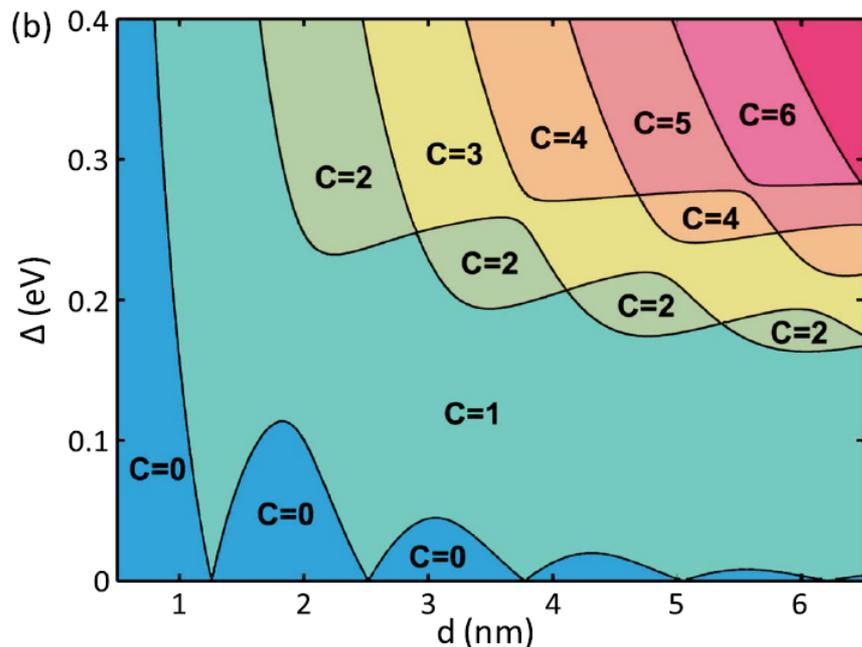
If the lowest  $n$  2D sub-bands are inverted, we get a QAH with Chern number  $n$ .

# Higher plateau QAH: multi-channel chiral edge states

Theoretical phase diagram of QAH ( thickness  $d$  , exchange field  $\Delta$  )

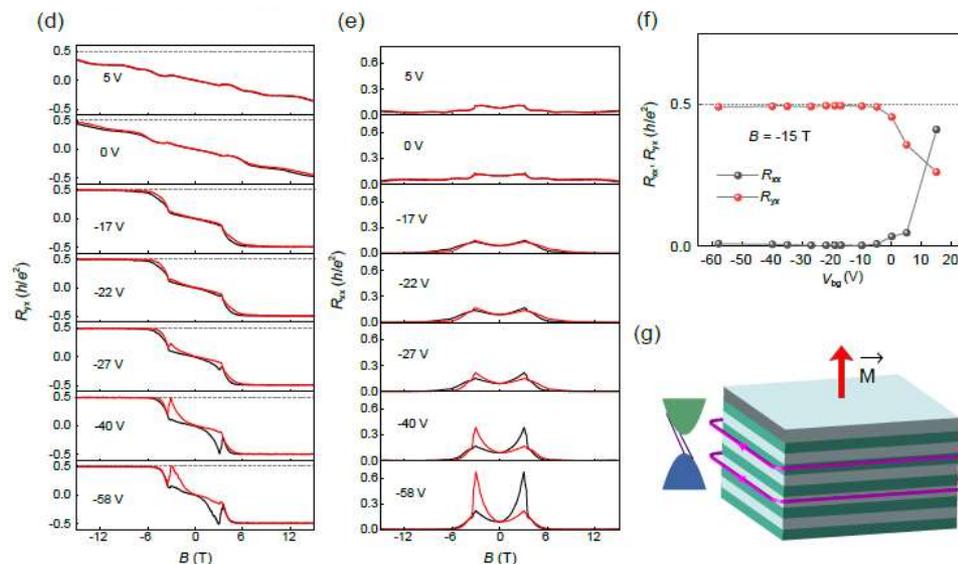
Promising to realize in FM MnBi<sub>2</sub>Te<sub>4</sub>

10 SL FM MnBi<sub>2</sub>Te<sub>4</sub>, C=2



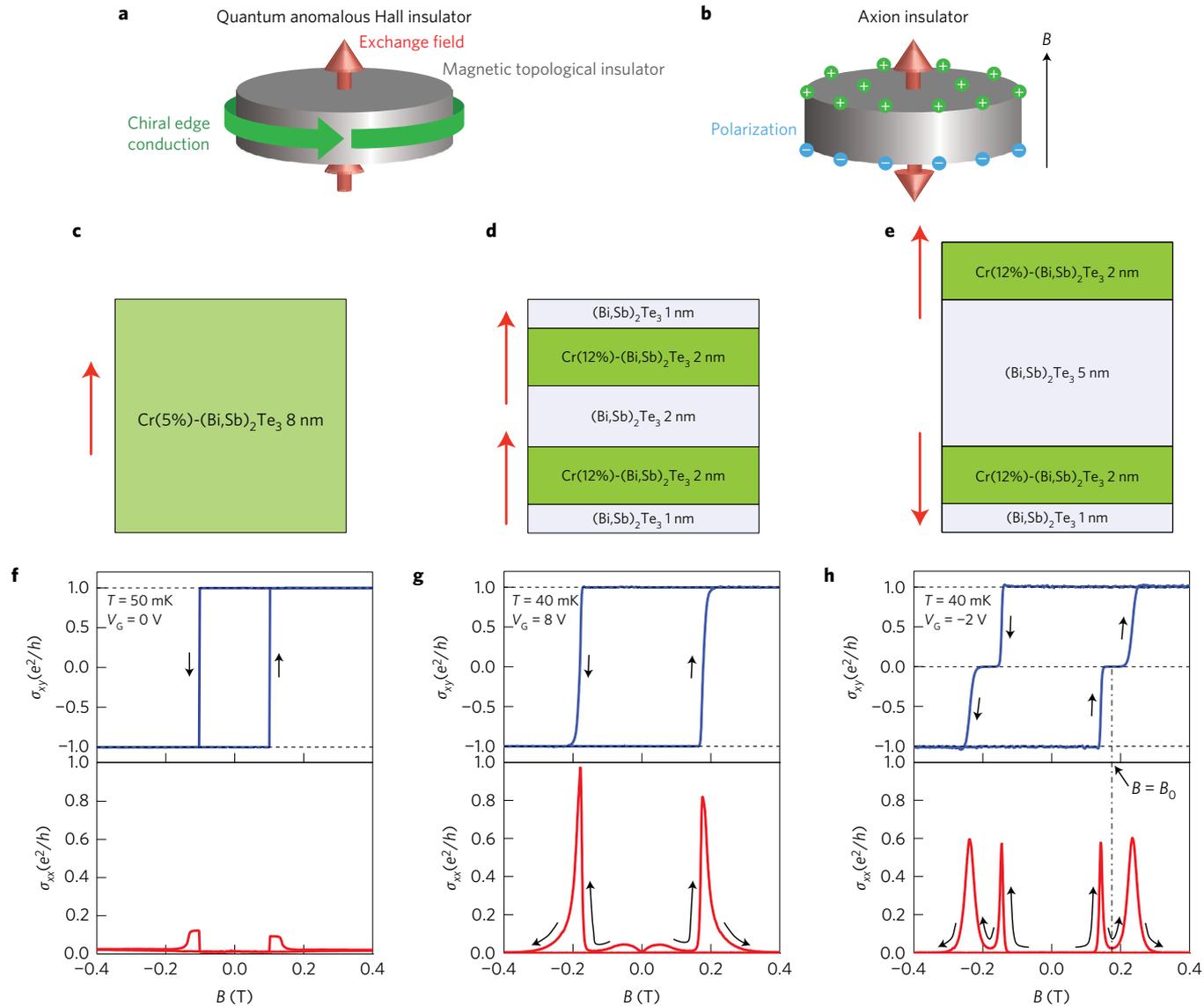
With z direction SOC

$$A_1 \neq 0$$



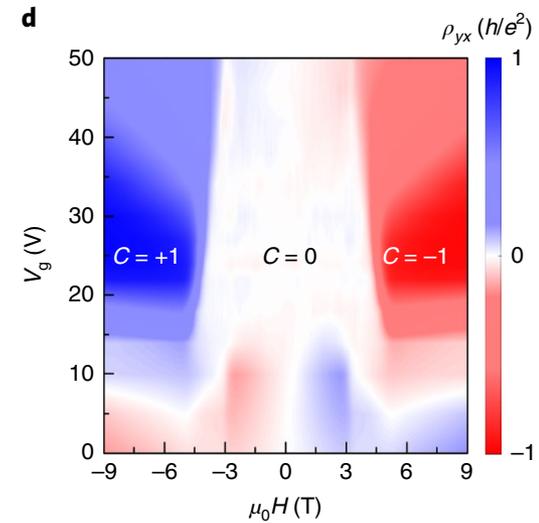
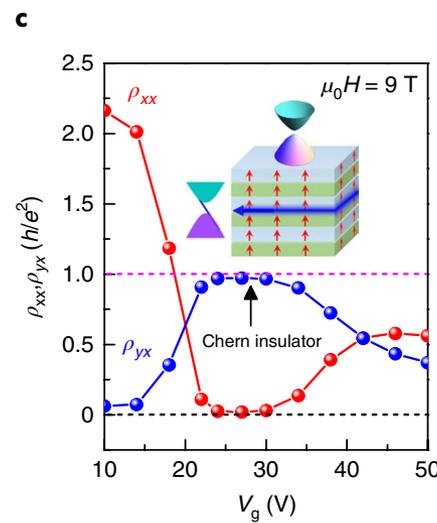
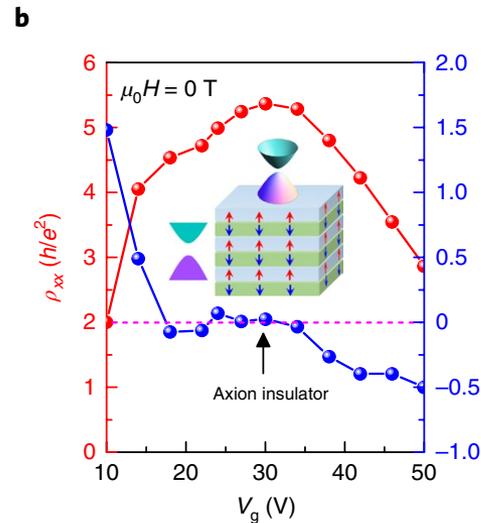
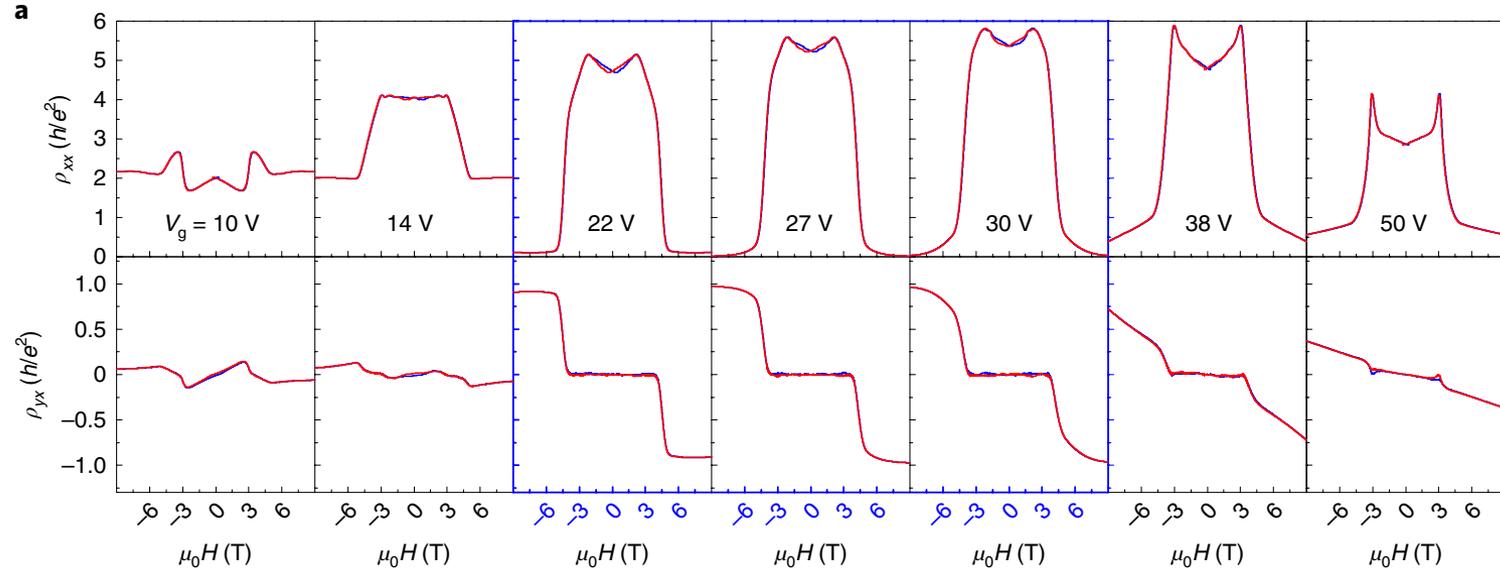
Jian Wang group. National Science Review, nwa089 (2020)

# Axion state in magnetic TI heterostructure



# Observation of possible AI in 6 SL MnBi<sub>2</sub>Te<sub>4</sub> (Yayu Wang Group)

Liu *et al*, Nature Mat. 19, 522 (2020)



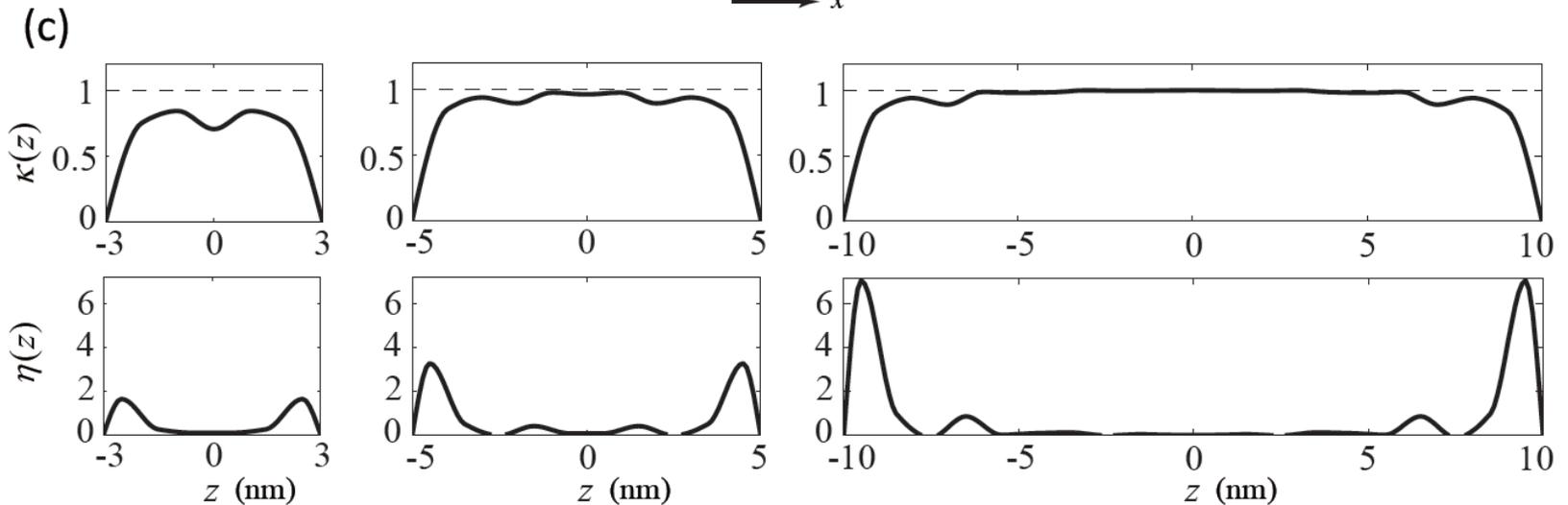
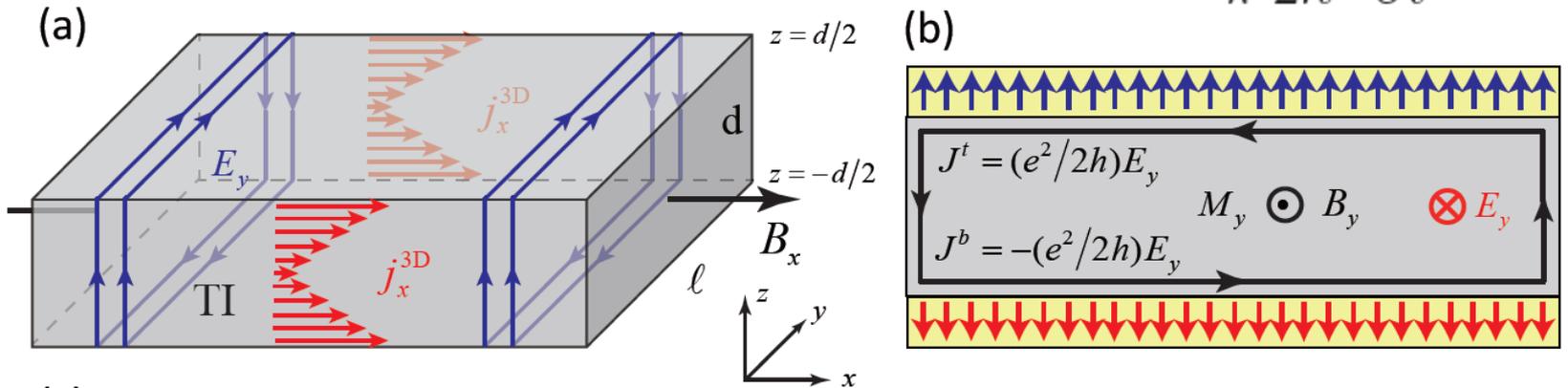
# TME: ac B field induces ac current in the same direction

$$\mathbf{E}^t = -\mathbf{E}^b = (\partial B_x / \partial t)(d/2)\hat{y}.$$

$$\mathcal{J} = \mathbf{j}^{2D} \ell = \mathcal{J} \hat{x},$$

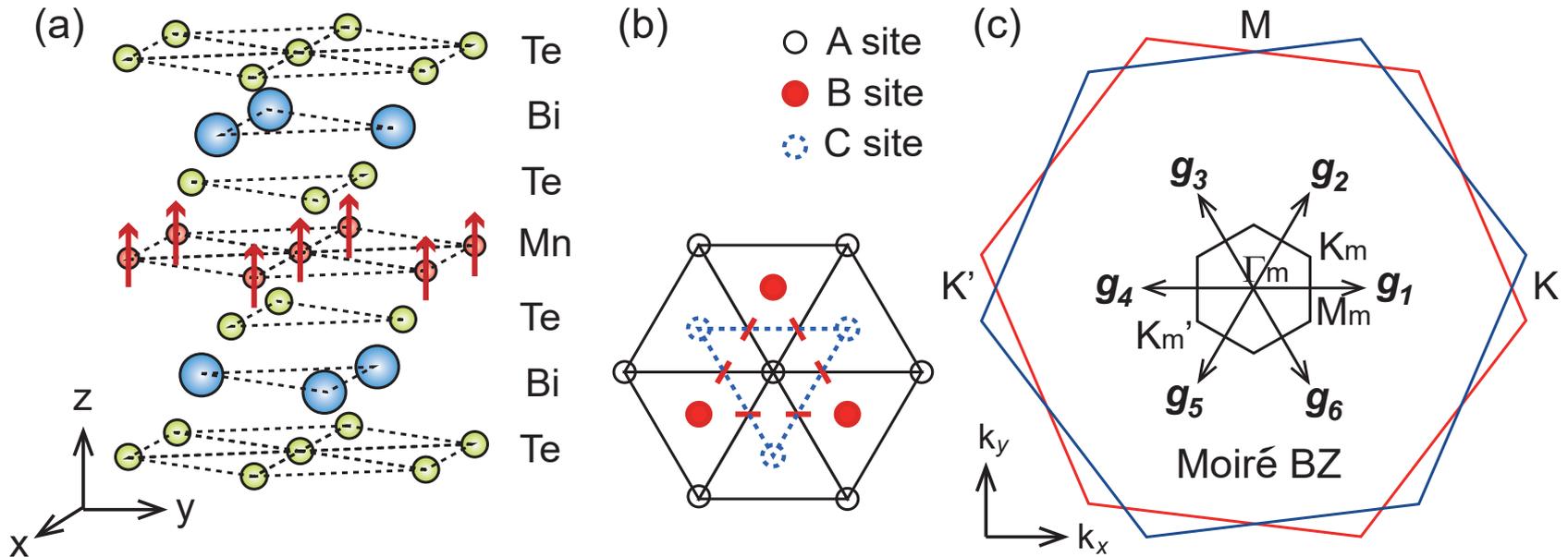
$$\mathbf{j}_t^{2D} = \sigma_{xy}^t \hat{z} \times \mathbf{E}^t \quad \mathbf{j}_b^{2D} = \sigma_{xy}^b \hat{z} \times \mathbf{E}^b.$$

$$\mathcal{J} = \frac{\theta}{\pi} \frac{e^2}{2h} \frac{\partial B_x}{\partial t} \ell d.$$



# Flat band from twisted 2D bilayer-SL MnBi2Te4

Motivation: to get a time-reversal breaking flat Chern band in the single particle level.



$$H = \begin{pmatrix} h_{1,\theta/2}(-i\nabla) + U_d & T(\mathbf{r}) \\ T^\dagger(\mathbf{r}) & h_{2,-\theta/2}(-i\nabla) - U_d \end{pmatrix},$$

$$T(\mathbf{r}) = T_0 + \sum_{j=1}^6 T_j e^{i\mathbf{g}_j \cdot \mathbf{r}}$$

# Flat Chern band from twisted bilayer MnBi2Te4: FM

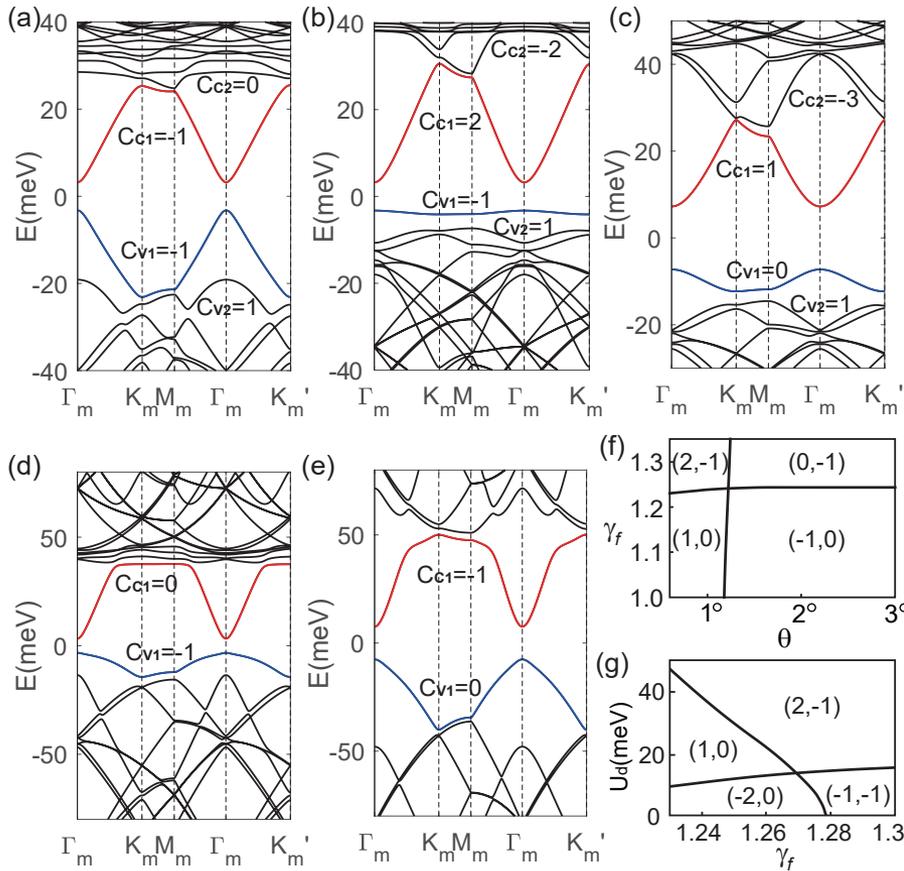
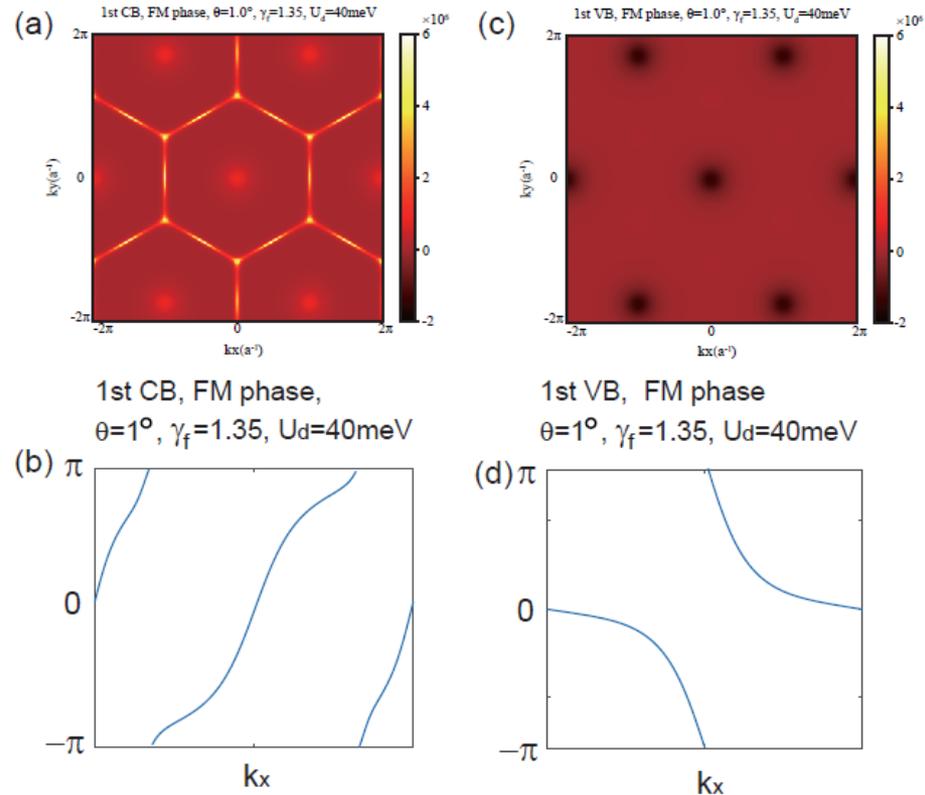


FIG. 2. The band structure of the FM tBMBT for (a)  $\theta = 1^\circ$ ,  $U_d = 10$  meV,  $\gamma_f = 1.35$ , (b)  $\theta = 1^\circ$ ,  $U_d = 40$  meV,  $\gamma_f = 1.35$ , (c)  $\theta = 1^\circ$ ,  $U_d = 40$  meV,  $\gamma_f = 1$ , (d)  $\theta = 2^\circ$ ,  $U_d = 40$  meV,  $\gamma_f = 1.35$  and (e)  $\theta = 3^\circ$ ,  $U_d = 40$  meV,  $\gamma_f = 1$ . (f) Chern numbers of the lowest conduction and valence bands ( $C_{C1}, C_{V1}$ ) as a function of angle  $\theta$  and exchange field strength  $\gamma_f$ , where  $U_d = 40$  meV is set. (g) ( $C_{C1}, C_{V1}$ ) for  $\theta = 1^\circ$  as a function of  $\gamma_f$  and staggered layer potential  $U_d$ .



FM: need  $C_{2x}\mathcal{T}$  symmetry breaking

Staggered layer potential

Platform for fractionalized QAH

# Flat Chern band from twisted bilayer MnBi2Te4: AFM

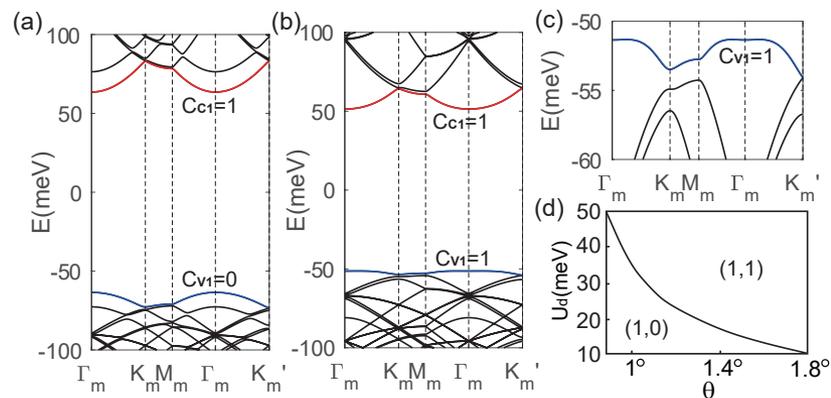
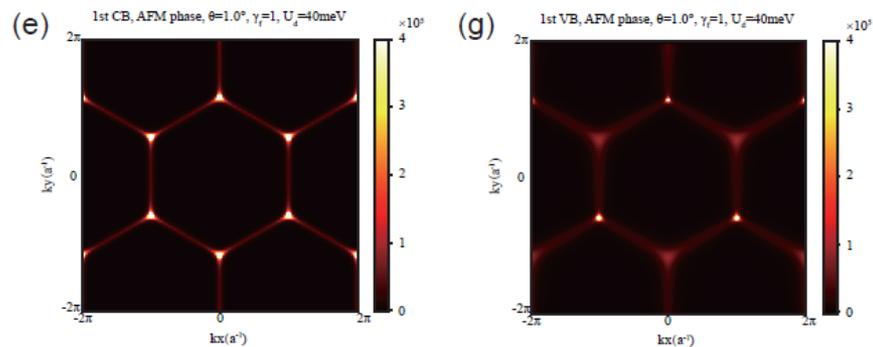
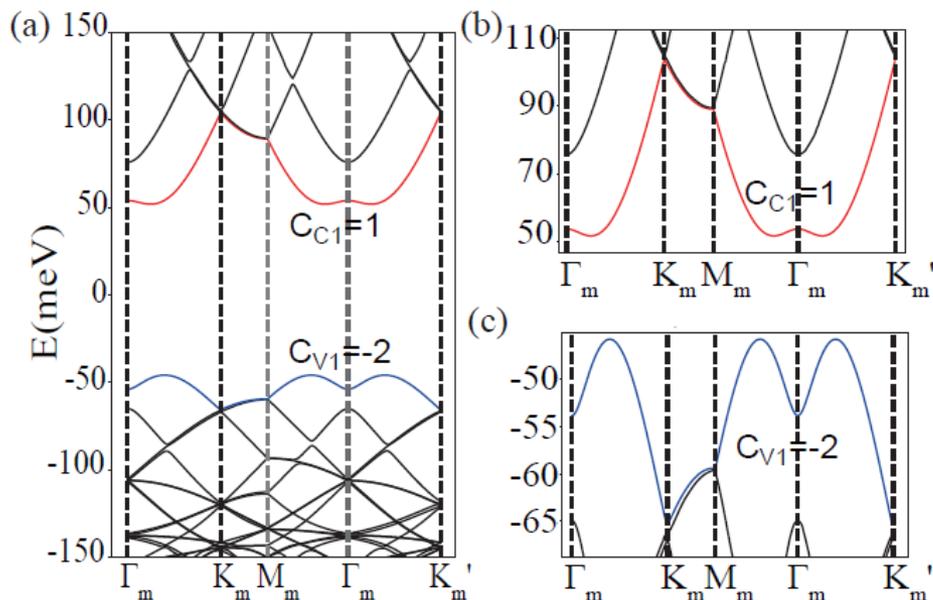
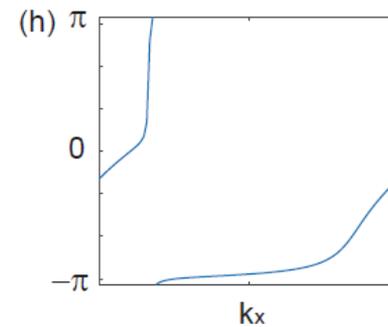
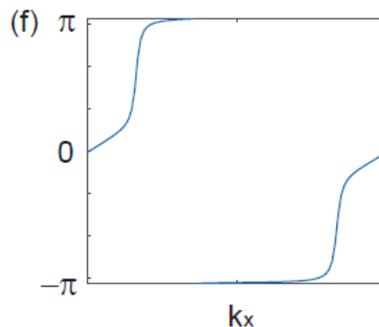


FIG. 3. The band structure of the AFM tBMBT for (a)  $\theta = 1^\circ$ ,  $\gamma_{af} = 1$ ,  $U_d = 10$  meV, and (b)  $\theta = 1^\circ$ ,  $\gamma_{af} = 1$ ,  $U_d = 40$  meV. (c) Zoom-in plot of the valence band structure in (b), showing the bandwidth of the first valence band. (d)  $(C_{C1}, C_{V1})$  as a function of the twist angle  $\theta$  and the staggered layer potential  $U_d$ .



1st CB, AFM phase  
 $\theta=1^\circ$ ,  $\gamma_f=1$ ,  $U_d=40\text{meV}$

1st VB, AFM phase  
 $\theta=1^\circ$ ,  $\gamma_f=1$ ,  $U_d=40\text{meV}$

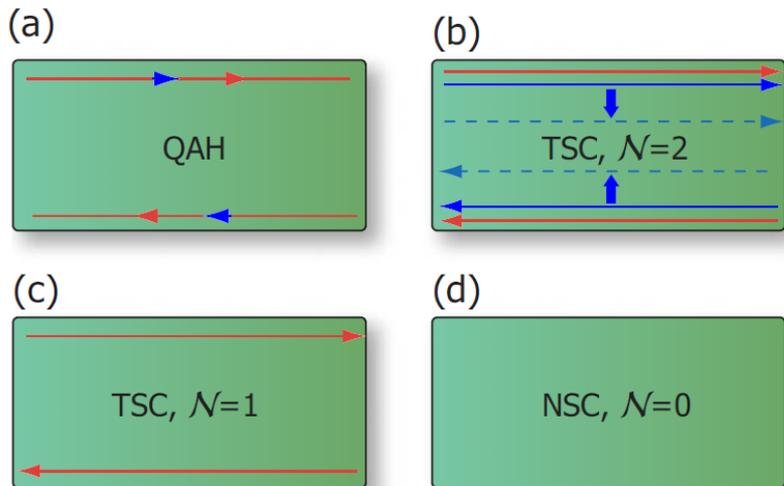
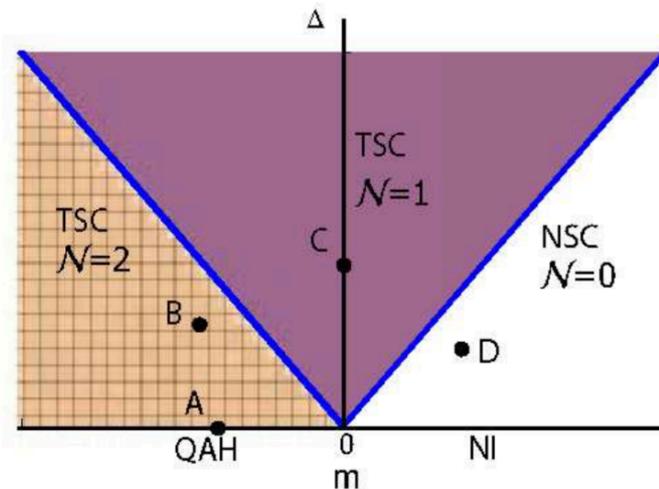
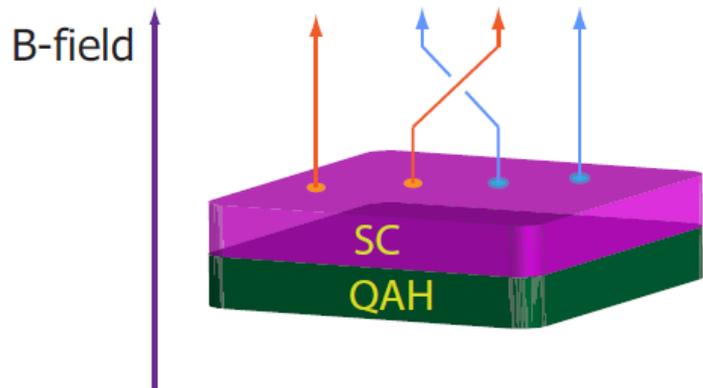


AFM: need  $C_{2x}$  symmetry breaking

Staggered layer potential

# Chiral topological superconductivity from QAH

X L Qi *et al*, PRB **82**, 184516 (2010); J Wang *et al*, PRB **92**, 064520 (2015)



$$h_{\text{QAH}}(\mathbf{p}) = \begin{pmatrix} m(p) & A(p_x - ip_y) \\ A(p_x + ip_y) & -m(p) \end{pmatrix}$$

$$H_{\text{BdG}} = \frac{1}{2} \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^\dagger \begin{pmatrix} h_{\text{QAH}}(\mathbf{p}) - \mu & i\Delta\sigma^y \\ -i\Delta^*\sigma^y & -h_{\text{QAH}}^*(-\mathbf{p}) + \mu \end{pmatrix} \Psi_{\mathbf{p}}$$

As one sweep the magnetic field, there is **NECESSARY** an intermediate phase with chiral topological superconductivity!

# Outlook

1. Zero-field QAH at higher temperature.
2. TME and axion electrodynamic from even SL MnBi<sub>2</sub>Te<sub>4</sub>.
3. Flat Chern bands in twisted bilayer MnBi<sub>2</sub>Te<sub>4</sub>, promising for fractionalized QAH.
4. QAH/SC heterostructure for chiral Majorana fermion.

Bulk (AFM)	Bulk (FM)	Odd layer film	Even layer film
AFM TI	Minimal Weyl semimetal	QAH effect	axion insulator, zero Hall plateau
gapped Dirac SS (T dependent)	Fermi arc	chiral edge states	TME, image monopole

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**Thank you for your attention!**