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Nonequilibrium phenomena in superconductors in proximity to magnets

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FUNDING OPPORTUNITIES
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ACADEMY OF FINLAND
RESEARCH FUNDING AND EXPERTISE

Bergeret, Silaev, Virtanen, TTH, Rev. Mod. Phys. **90**, 041001 (2018); Progr. Surf. Sci. (2019)

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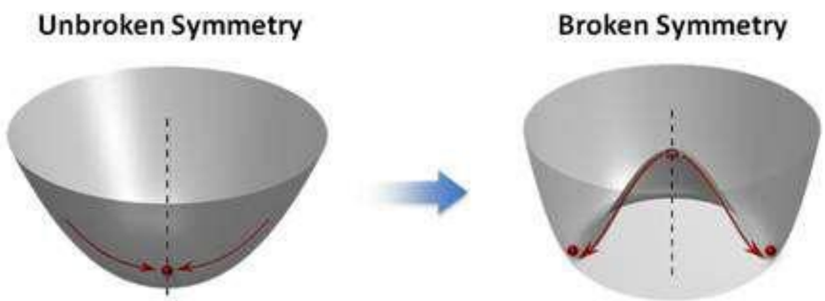
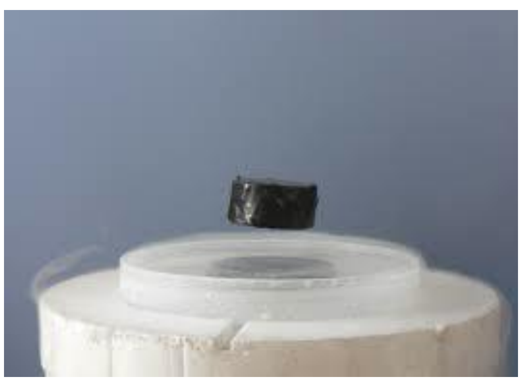
Celia Rogero, *San Sebastian*
Alessandro Monfardini, *Grenoble*

Superconductivity and magnetism

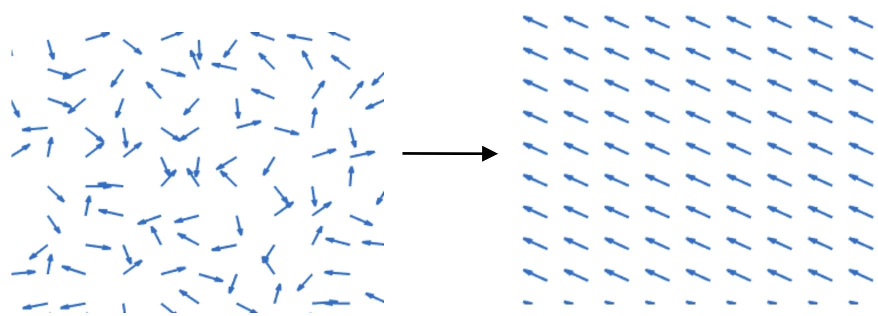
Ground state of most metals at T=0



Superconductivity



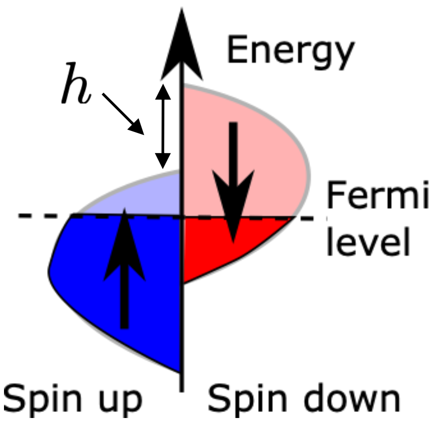
(Ferro)magnetism



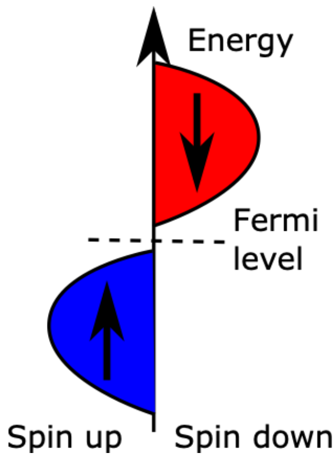
Picture sources: Phys. org, Wikimedia Commons

Magnetism

Basic properties: exchange field $\mathbf{h} \cdot \sigma = h\sigma_z$ and spin polarization



Ferromagnetic metal (FM)



Ferromagnetic insulator (FI)

Spin polarization P

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \in [-1, 1]$$

(Actual magnetic field not relevant for this talk)

This talk: (almost) no spin-orbit coupling, only spin-orbit relaxation

Superconductivity

- Order parameter: pairing amplitude, pair potential

$$F(\mathbf{r}) = \langle \psi_\uparrow(\mathbf{r}) \psi_\downarrow(\mathbf{r}) \rangle \qquad \Delta = \lambda F$$

Here: s-wave superconductivity

- Supercurrent: $F = |F| e^{i\phi} \Rightarrow \vec{j}_S \propto \nabla \phi$

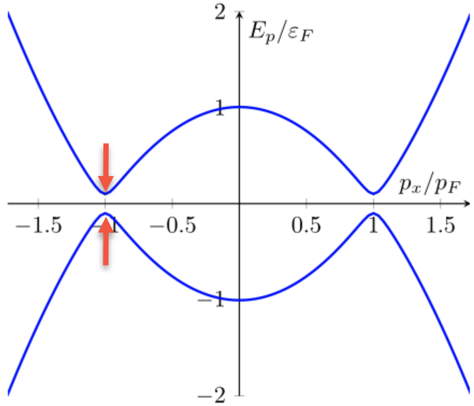
- Energy gap in the dispersion

Bardeen, Cooper, Schrieffer (1957);
Giaever & Megerle (1961)

$$E_p = \sqrt{\xi_p^2 + \Delta^2} \qquad |\Delta| \propto |F|$$

Non-linear system!

2Δ



Superconductivity

- Order parameter: pairing amplitude, pair potential

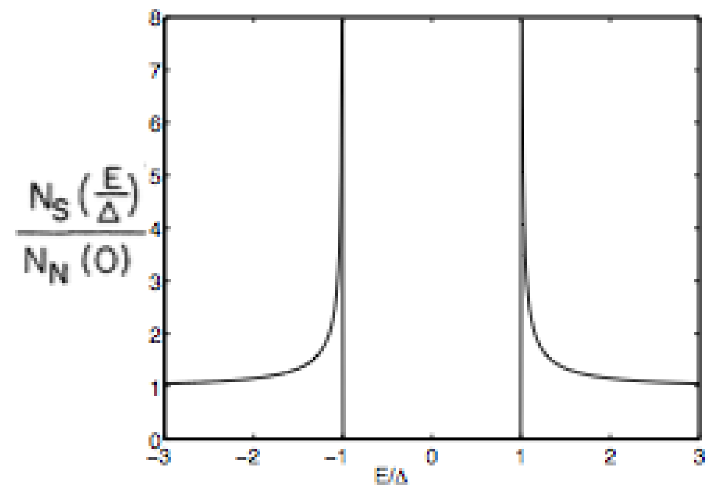
$$F(\mathbf{r}) = \langle \psi_\uparrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\rangle \qquad \Delta = \lambda F$$

Here: “singlet” s-wave superconductivity
(+ odd-frequency triplet)

- Supercurrent: $F = |F|e^{i\phi} \Rightarrow \vec{j}_S \propto \nabla\phi$

- Energy gap in the dispersion

BCS density of states



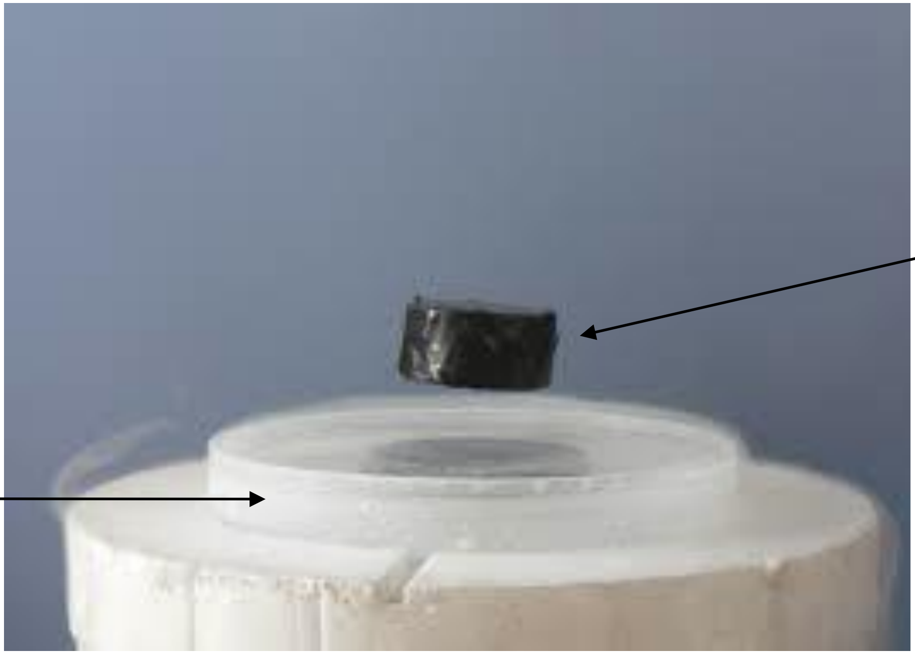
Bardeen, Cooper, Schrieffer (1957);
Giaever & Megerle (1961)

$$E_p = \sqrt{\xi_p^2 + \Delta^2} \qquad |\Delta| \propto |F|$$

Non-linear system!

Superconductivity and magnetism don’t usually like each other

They even physically repel each other!



Picture source: Wikimedia Commons

Contents

- Bring them together: spin-splitting a superconductor, how to study it via tunnelling
- Spin-resolved electron-hole asymmetry and thermoelectric effects
- Coupled charge, spin, energy and spin energy modes in bulk
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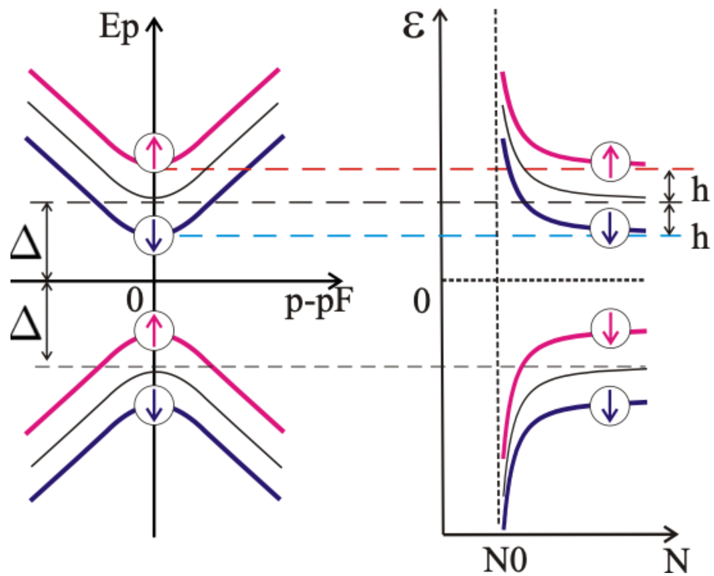
Spin splitting a superconductor

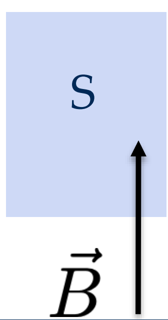
S-F bilayer (FI=F insulator)

Spectrum and DOS in superconductor
with spin splitting



FI: induced spin splitting (almost) without external field!

Similar effect from Zeeman field



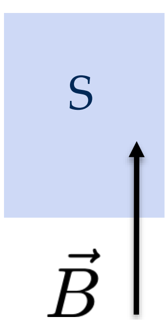
Exp: Meservey, Tedrow (1971)
Moodera, *et al.* (1990, 2013)
Beckmann, *et al.* (2014)
and many others

Alberto Hijano's poster yesterday

Spin splitting a superconductor

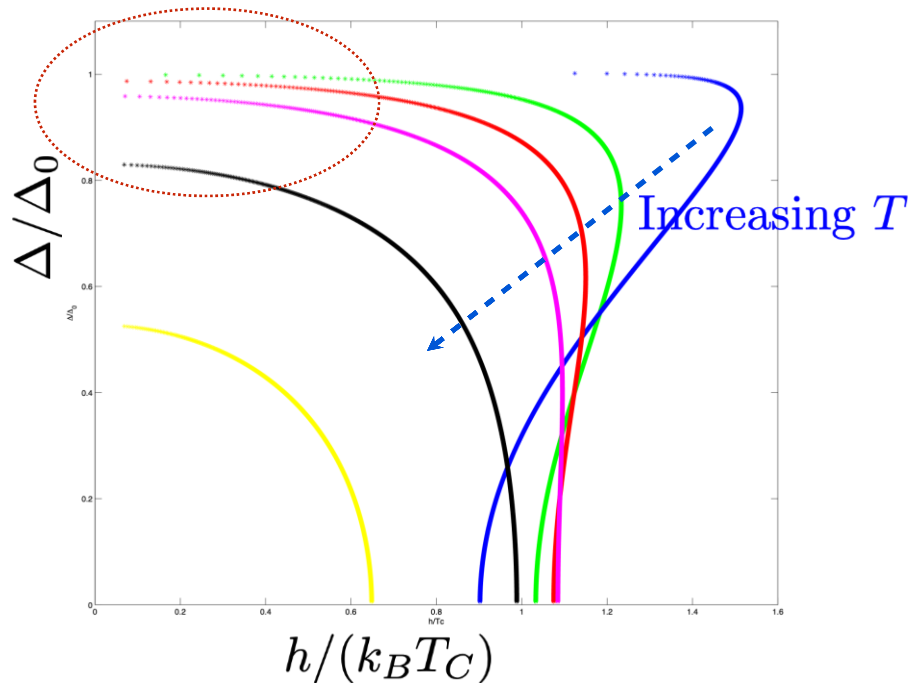
S-F bilayer (FI=F insulator)

Similar effect from Zeeman field



Exp: Meservey, Tedrow (1971)
Moodera, *et al.* (1990, 2013)
Beckmann, *et al.* (2014)
and many others

Spin-splitting field *h* in a superconductor: kills singlet superconductivity



Chandrasekhar (1962), Clogston (1962), Fulde & Ferrell (1964), Larkin & Ovchinnikov (1965), Maki & Tsuneto (1965)
Alexander, *et al.* (1985), Catelani, *et al.*, (2008), and many others

Theory description

Superconductor: Usadel equation, Keldysh technique

superconductivity

$$D\nabla \cdot (\check{g}\nabla\check{g}) + [i\varepsilon\tau_3 - i\boldsymbol{h}\cdot\boldsymbol{\sigma}\tau_3 - \check{\Delta} - \check{\Sigma}, \check{g}] = 0$$

diffusion   spin splitting   spin/energy relaxation

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}$$

spectrum

state of the system

Usadel 1970
Reviews: Belzig et al. 1999,
Bergeret, Efetov, Volkov 2005

Nambu space: $\hat{g}^R = \begin{pmatrix} g & f \\ \tilde{f} & -g \end{pmatrix}$ Spin matrices!

Interface between S and FM/FI:
Boundary condition -> either spin-polarised tunnelling or induced exchange field

Spin-split superconductor

$$\hat{g}^R = \begin{pmatrix} g & f \\ \tilde{f} & -g \end{pmatrix}$$

$$g = g_0\sigma_0 + g_z\sigma_z \quad f = f_0\sigma_0 + f_z\sigma_z$$

density of states — spin-resolved DOS — singlet pairing — (odd-frequency) triplet pairing (*“m=0”*)

spin-orbit coupling, non-collinear magnetism: $f = f_x\sigma_x + f_y\sigma_y + f_z\sigma_z$

(Next two talks) — “long-range” triplets

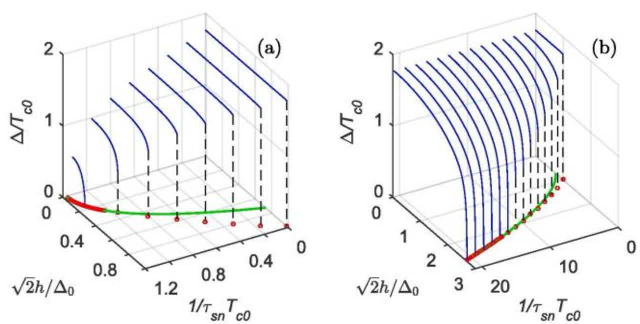
Here: interaction model that promotes only singlet pair potential:

$$\Delta = \frac{\lambda}{2}\int_{-\omega_D}^{\omega_D} d\epsilon \mathrm{Im}[f_0] \tanh\left(\frac{\epsilon}{2k_B T}\right)$$

Equilibrium!

(Singlet) pair potential: spin relaxation affects the critical fields

Spin flip scattering reduces critical field



Spin-orbit scattering increases critical field

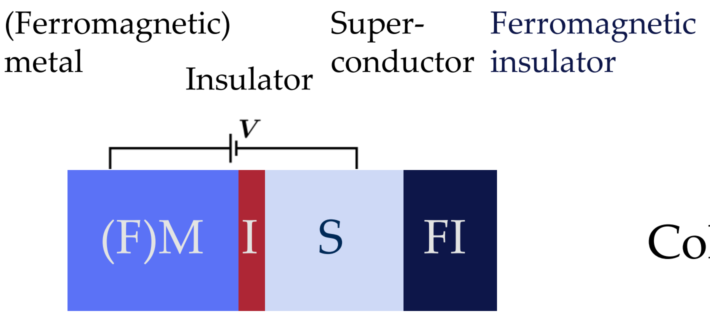
Details: TTH, et al. Progr. Surf. Sci. (2019)

Spin-polarized tunneling

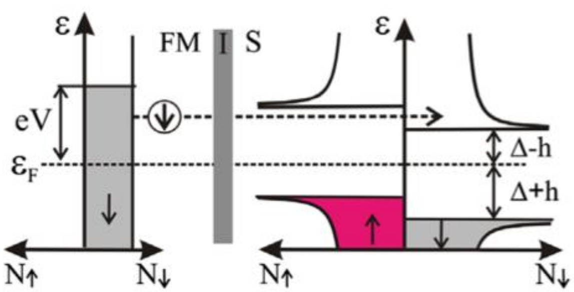
Meservey, Tedrow (1971)
Moodera, *et al.* (1990, 2013)
and many others
Rouco, et al. [TTH], PRB 2019

Probing the spin splitting: tunnelling experiments

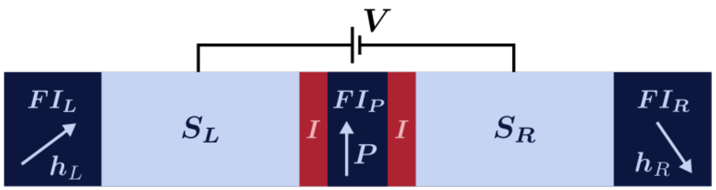
Simplest case:



(a) Spin-resolved tunneling



General case:



Collinear case:

$$I = \sum_\sigma \frac{G_{T\sigma}}{e} \int d\epsilon N_{L\sigma} N_{R\sigma}(\epsilon)(f_{L\sigma} - f_{R\sigma})$$

General non-collinear case: a bit more complicated…

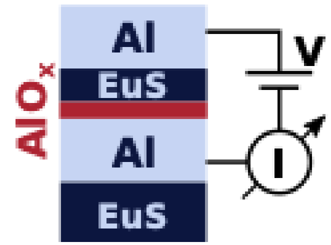
$$\begin{aligned} I = \frac{G_T}{2e} \int_{-\infty}^{\infty} & d\epsilon [f_0(\epsilon + eV, T_L) - f_0(\epsilon, T_R)] \\ & \times \{P[\mathcal{N}_{0L}(\epsilon + eV)\mathcal{N}_{3R}(\epsilon)\boldsymbol{n}_R \cdot \boldsymbol{n}_P \\ & + \mathcal{N}_{3L}(\epsilon + eV)\mathcal{N}_{0R}(\epsilon)\boldsymbol{n}_L \cdot \boldsymbol{n}_P] \\ & + \mathcal{N}_{0L}(\epsilon + eV)\mathcal{N}_{0R}(\epsilon) + \mathcal{N}_{3L}(\epsilon + eV)\mathcal{N}_{3R}(\epsilon) \\ & \times [\boldsymbol{n}_L^{\parallel} \cdot \boldsymbol{n}_R^{\parallel} + \sqrt{1 - P^2}\, \boldsymbol{n}_L^{\perp} \cdot \boldsymbol{n}_R^{\perp}]\}, \end{aligned}$$

Spin-polarized tunneling

Rouco, Chakraborty, Aikebaier, Golovach, Strambini, Moodera, Giazotto, TTH, Bergeret, Phys. Rev. B (2019)

Probing the spin splitting: tunnelling experiments

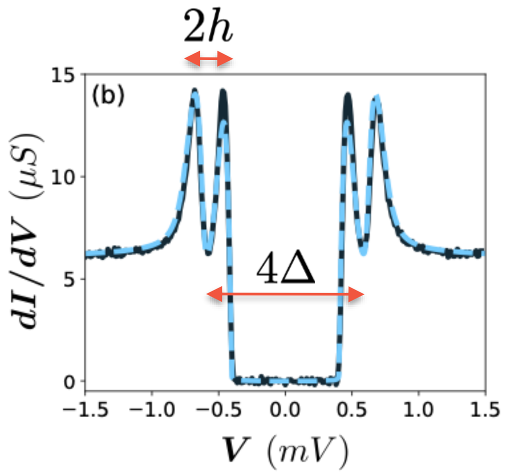
Here: examples from studying the case with two magnets

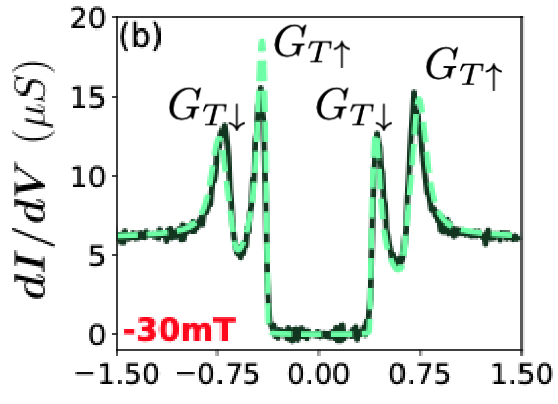


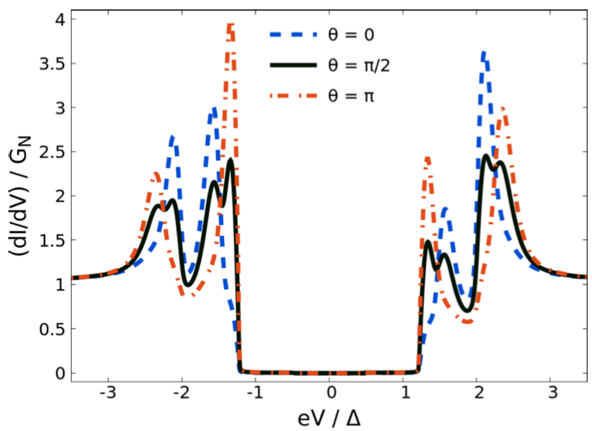
Non-polarized tunneling (random magnetic domains)

Spin polarised tunnelling: asymmetric

Non-collinear arrangement: eight peaks

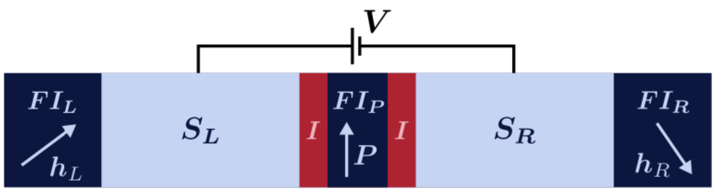






Requires a light (low Z) superconductor (Al, V…) to avoid spin relaxation due to SOC

Spin-polarized tunneling



Rouco, Chakraborty, Aikebaier, Golovach, Strambini, Moodera, Giazotto, TTH, Bergeret, Phys. Rev. B (2019)

Josephson current: "ϕ_0 effect"

$$I_s = J_1 \sin(\varphi) + J_2 \cos(\varphi)$$

Equilibrium case ($I_s = 0$): finite phase difference!

$$\phi_0 = -\arctan\left(\frac{J_2}{J_1}\right)$$

See also
Buzdin, PRL 2008
Konschelle, Tokatly, Bergeret, PRB 2015
Bergeret, Tokatly, EPL 2015
Tokayama, Eto, Nazarov, PRB 2014
Silaev, Tokatly, Bergeret, PRB 2017
Liu & Chan PRB 2010
Mal'shukov, Sadjina, Brataas, PRB 2010
Lu, TTH, PRB 2019
Margaris, Paltoglou, Flytzanis, JPCM 2010
Braude & Nazarov, PRL 2007
Moor, Volkov, Efetov, PRB 2015
Grein, Eschrig, Metalidis, Schön, PRL 2009
Mironov, Buzdin, PRB 2015

Exp:
Strambini, et al., Nature Nanotech (2020)

Regular contribution,
affected by spin polarization

$$J_1 = \frac{\pi G_T \Delta}{2e}[\sqrt{1-P^2}\eta + (\sqrt{1-P^2}\boldsymbol{n}_L^{\parallel}\cdot\boldsymbol{n}_R^{\parallel} + \boldsymbol{n}_L^{\perp}\cdot\boldsymbol{n}_R^{\perp})(\eta-1)]$$

Anomalous contribution,
requires non-coplanar magnetisations
(or SOC + exchange field)

$$J_2 = \frac{\pi G_T \Delta}{2e} P(\eta-1)\boldsymbol{n}_P\cdot(\boldsymbol{n}_L\times\boldsymbol{n}_R).$$

Here: $h_L = h_R = h$

$$\eta \equiv \frac{32\Delta^2(256\Delta^4 - 32\Delta^2 h^2 + 9h^4)}{(16\Delta^2 - h^2)^3} - 1.$$

Contents

- Spin-splitting a superconductor, how to study it via tunnelling
- Spin-resolved electron-hole asymmetry and thermoelectric effects
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Spin-resolved electron-hole asymmetry

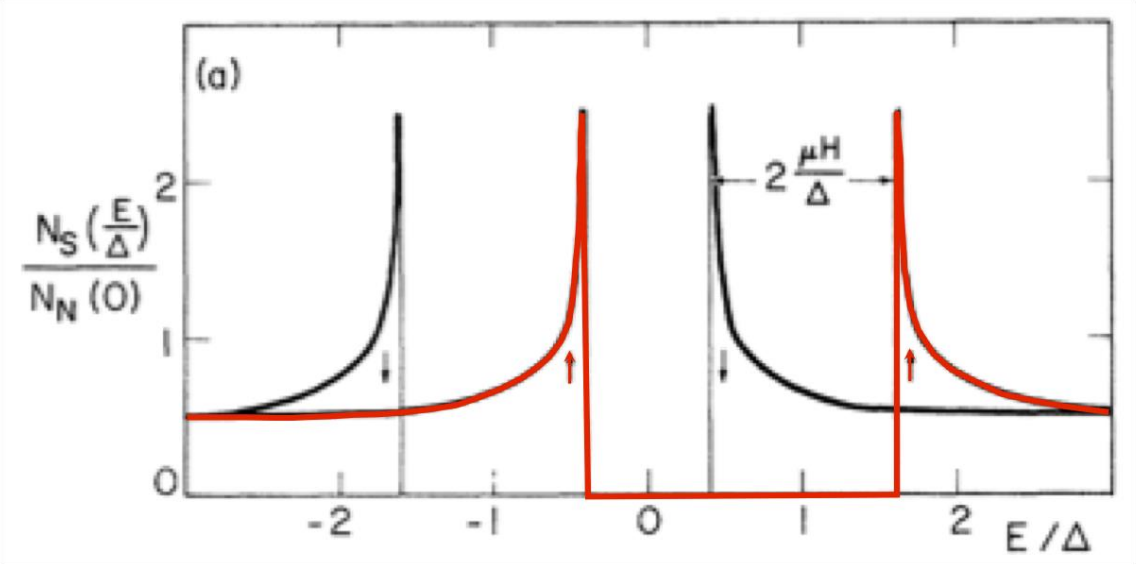
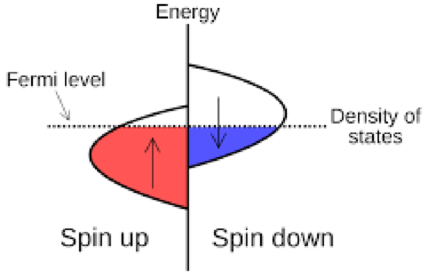


Figure: Meservey, Tedrow (1971)

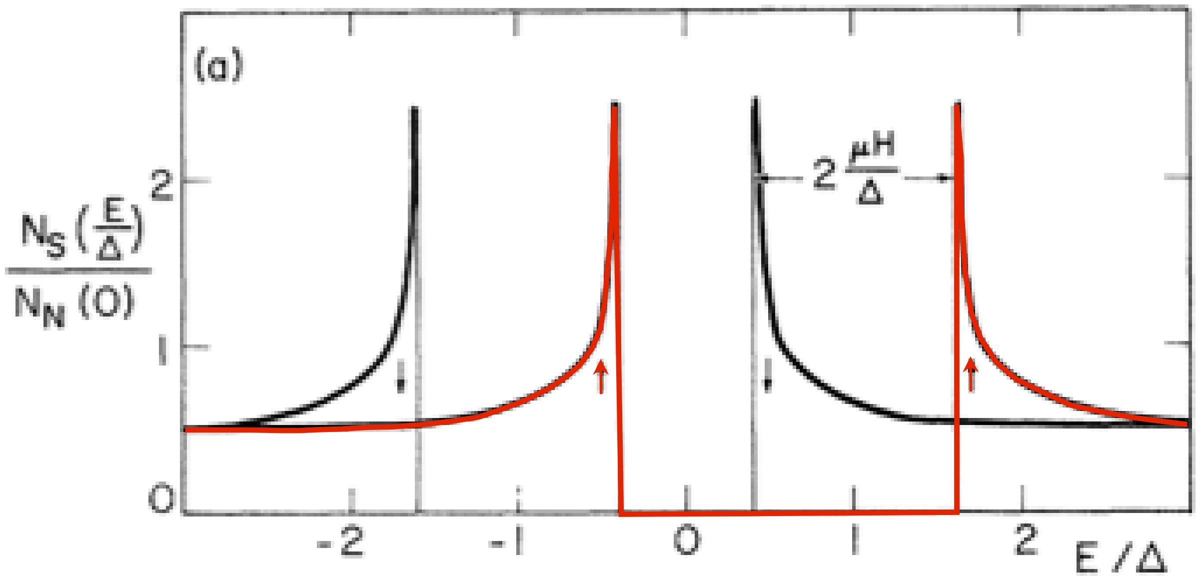
Combine this with the spin-polarised tunnelling:



$$G_{T\uparrow/\downarrow} = G_T(1 \pm P)/2$$

Spin polarization of tunnelling

Thermoelectric effect



1. Large e-h asymmetry *per spin*

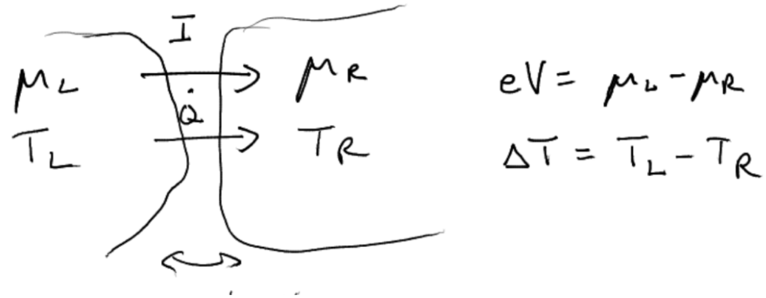
2. For $P \neq 0$, different spin contributions weighed differently

Ozaeta, Virtanen, Bergeret, TTH, PRL 2014
(See also Machon, et al. [Belzig], PRL 2013)

Thermoelectric effects

Linear response charge and heat currents across an interface:

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{\mathrm{th}} T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T / T \end{pmatrix}$$



Thermoelectric heat engine: efficiency

$$\max \eta = \eta_{\mathrm{Carnot}} \frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + 1} \xrightarrow{ZT \to \infty} 1$$

Thermoelectric figure of merit

$$ZT = \frac{\alpha^2}{G_{\mathrm{th}} G T - \alpha^2}$$

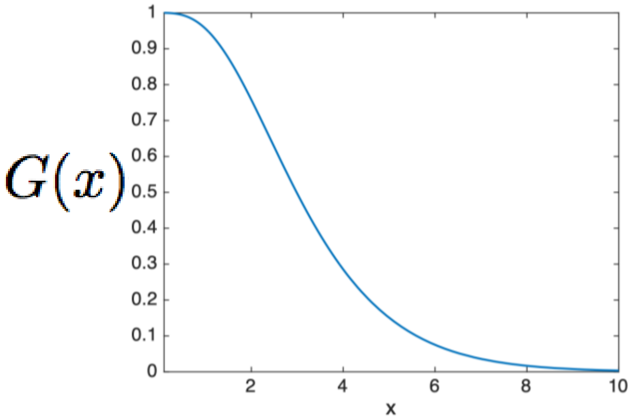
Thermoelectricity in superconductors

Many textbooks: no thermoelectric effects because they are cancelled by supercurrent (Meissner 1927).
Ginzburg (1944): possible to see in multiply connected structures, but very small

The size of thermoelectric coefficient in bulk superconductors:

$$\alpha = \alpha_N G(\Delta/T)$$

Small　　　Small



$$\alpha_N = \frac{\pi^2 G k_B}{6e} \frac{k_B T}{E_F}$$

Gal'perin, Gurevich & Kozub Sov. Phys. JETP (1974)

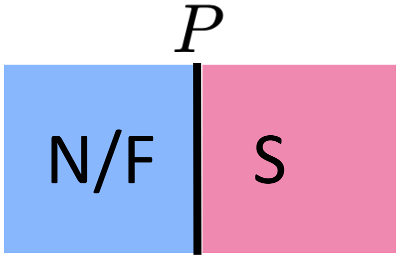
Main message (before 2014): superconductors are extremely poor thermoelectrics!

Spin-splitting field + polarization

Linear response: $\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & P\alpha \\ P\alpha & G_{th}T \end{pmatrix} \begin{pmatrix} V \\ \Delta T/T \end{pmatrix}$

Polarization!

$$\alpha = \frac{1}{2eR_T} \int_{-\infty}^{\infty} dE \, \frac{E[N_\uparrow(E) - N_\downarrow(E)]}{4k_B T \cosh^2\left(\frac{E}{2k_B T}\right)}$$



$$G \approx G_T \sqrt{2\pi\tilde{\Delta}} \cosh(\tilde{h}) e^{-\tilde{\Delta}} \, ,$$

$$G_{\rm th} \approx \frac{k_B G_T \Delta}{e^2} \sqrt{\frac{\pi}{2\tilde{\Delta}}} e^{-\tilde{\Delta}} \left[e^{\tilde{h}} (\tilde{\Delta} - \tilde{h})^2 + e^{-\tilde{h}} (\tilde{\Delta} + \tilde{h})^2 \right] \, ,$$

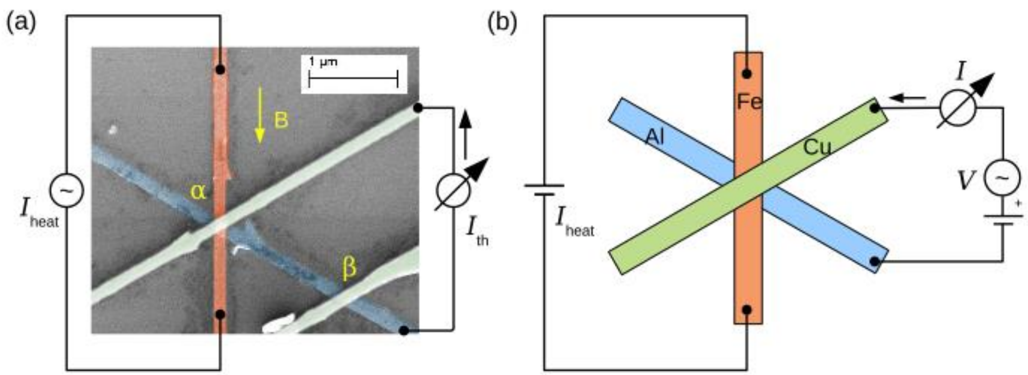
$$\alpha \approx \frac{G_T}{e} \sqrt{2\pi\tilde{\Delta}} e^{-\tilde{\Delta}} \left[\Delta \sinh(\tilde{h}) - h \cosh(\tilde{h}) \right]$$

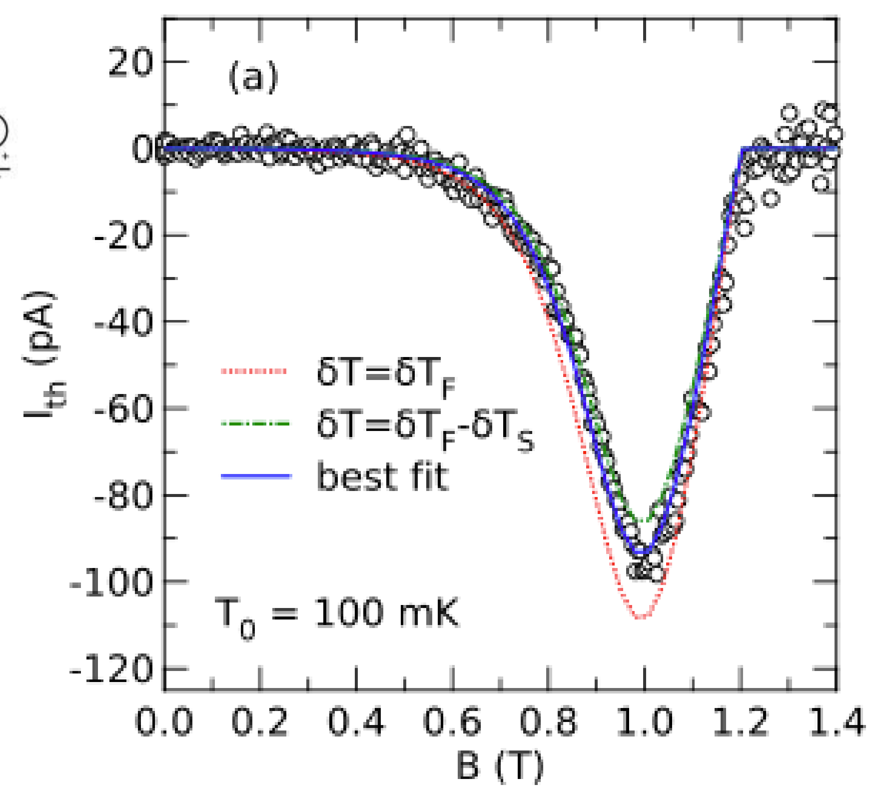
Ozaeta, Virtanen, Bergeret, TTH, PRL 2014

PRL **116,** 097001 (2016) PHYSICAL REVIEW LETTERS week ending 4 MARCH 2016

Observation of Thermoelectric Currents in High-Field Superconductor-Ferromagnet Tunnel Junctions

S. Kolenda, M. J. Wolf,* and D. Beckmann†
Institute of Nanotechnology, Karlsruhe Institute of Technology, Karlsruhe, Germany

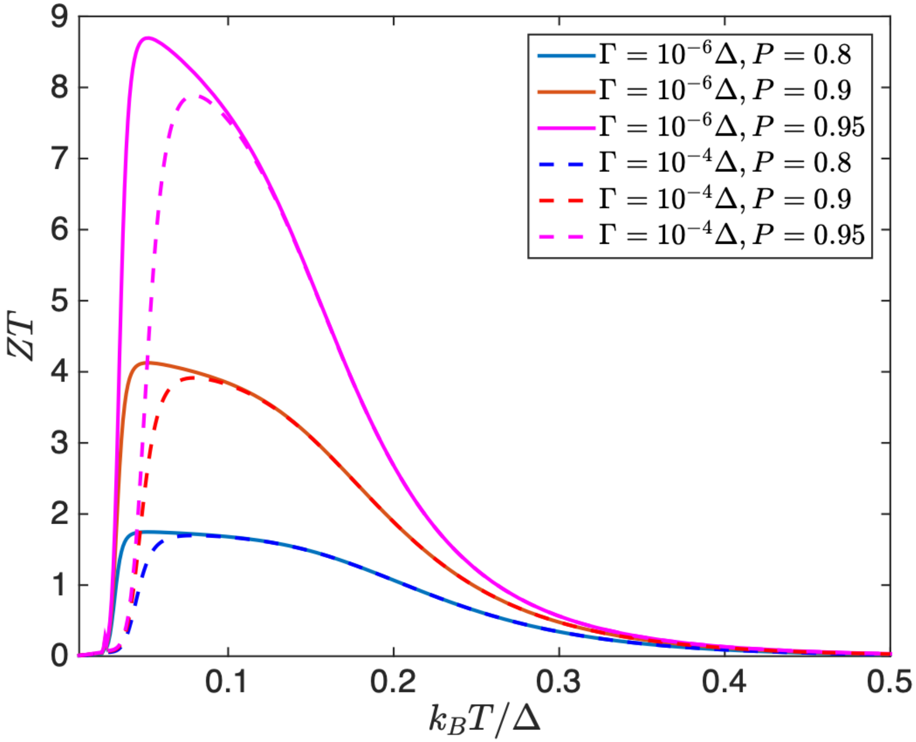


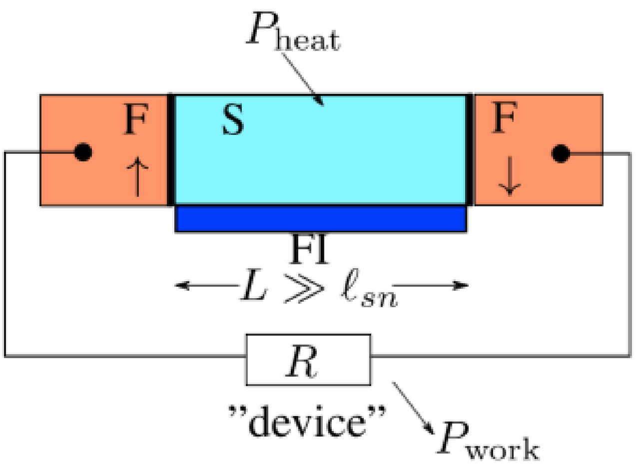


Heat engine

$$\begin{pmatrix} I \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} G & \alpha \\ \alpha & G_{\rm th} T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T / T \end{pmatrix}$$

$$ZT = \frac{\alpha^2}{G_{\rm th} G T - \alpha^2}$$

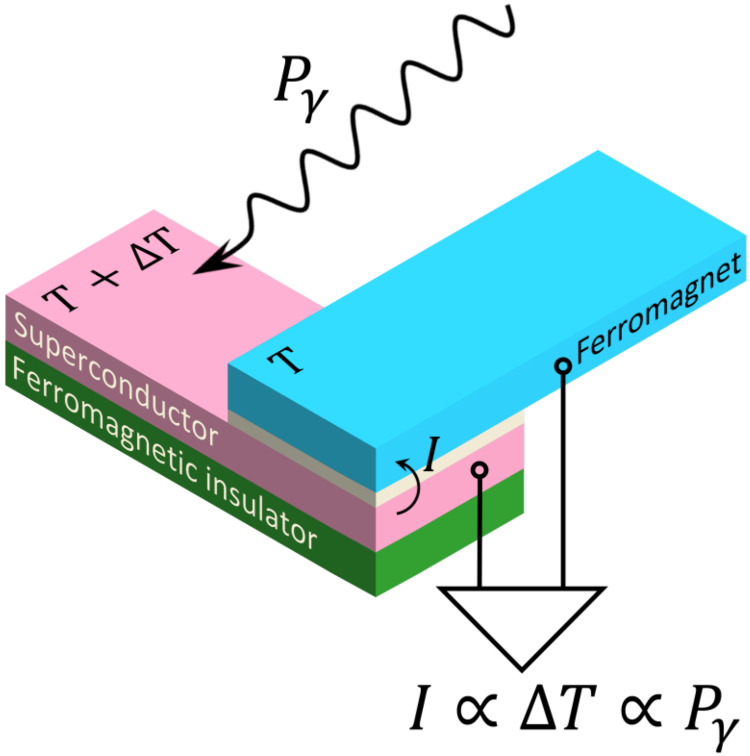




Bergeret, Silaev, Virtanen, TTH, Rev. Mod. Phys. **90**, 041001 (2018); Progr. Surf. Sci. (2019)

Where to use it?

Thermoelectric detector TED



SUPERTED

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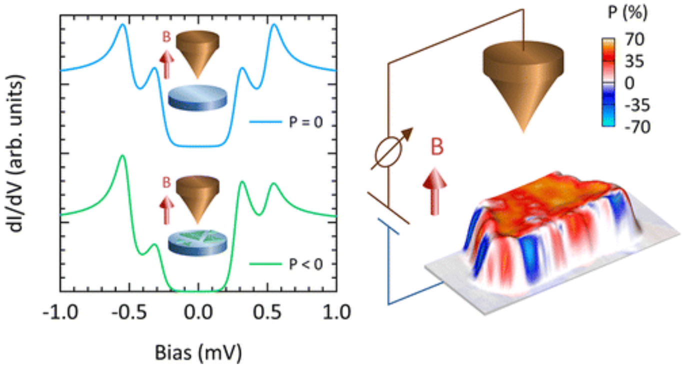
"Self-powered" by radiation:
no need for bias lines!

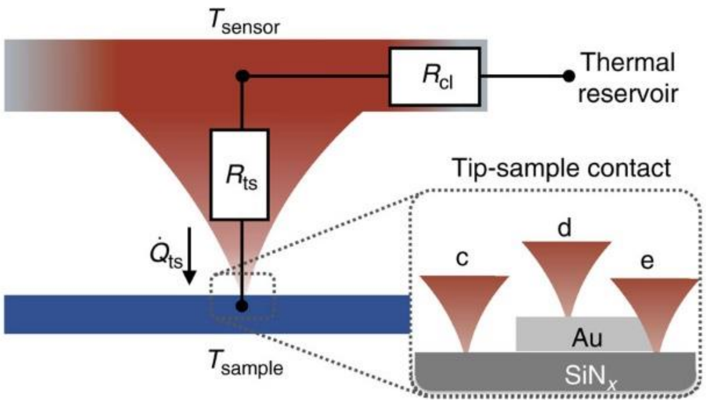
TTH, Ojajärvi, Maasilta, Strambini, Giazotto, Bergeret, [Phys. Rev. Applied **10**, 034053 (2018).]
Chakraborty, TTH, [J. Appl. Phys. **124**, 123902 (2018).]
Geng, Helenius, TTH, Maasilta, JLTP **199**, 585 (2020)

Scanning thermometry



Spin-split superconductor tip for probing spatially resolved spin polarization

Eltschka, et al., Nano Lett. 2014; APL 2015



Scanning probe thermometry at RT

Menges, et al., Nature Commun. 2016

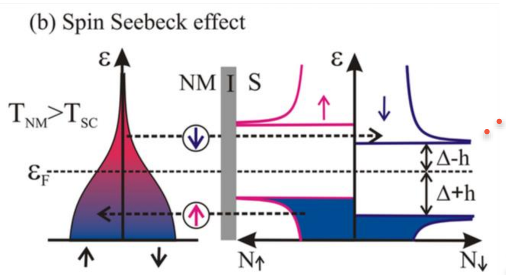
Combination: spatially resolved temperatures (I=0) at low T!

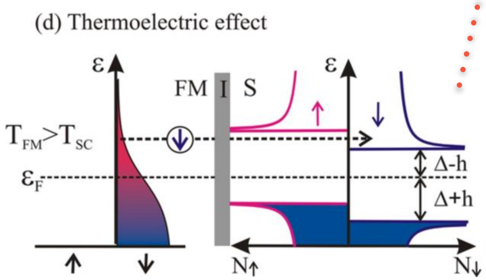
F. Giazotto, P. Solinas, A. Braggio, F.S. Bergeret, PR Appl. (2015)

Full thermoelectric matrix

Currents: Onsager! *Biases*

Currents			Biases
Charge Heat Spin Spin heat	$$\begin{pmatrix} I \\ \dot{Q} \\ I_s \\ \dot{Q}_s \end{pmatrix} = \begin{pmatrix} G & P\alpha & PG & \alpha \\ P\alpha & G_{\rm th}T & \alpha & PG_{\rm th}T \\ PG & \alpha & G & P\alpha \\ \alpha & PG_{\rm th}T & P\alpha & G_{\rm th}T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T/T \\ V_s/2 \\ -\Delta T_s/2T \end{pmatrix},$$		Voltage T-difference Spin voltage Spin T-difference





$$\alpha = \frac{1}{2eR_T}\int_{-\infty}^{\infty} dE\, \frac{E[N_\uparrow(E) - N_\downarrow(E)]}{4k_BT\cosh^2\left(\frac{E}{2k_BT}\right)}$$

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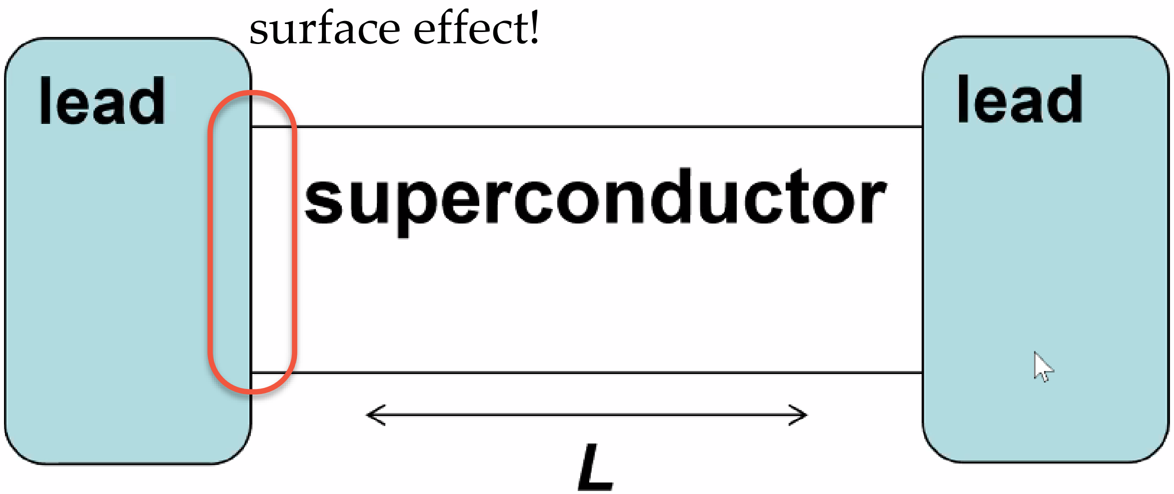
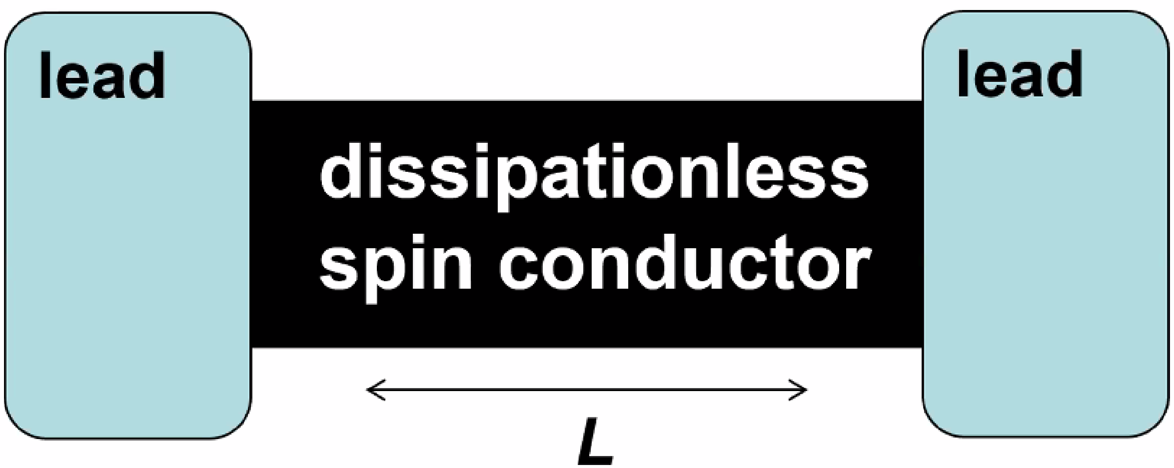
Long-term motivation



I=V/R with
R independent of *L*

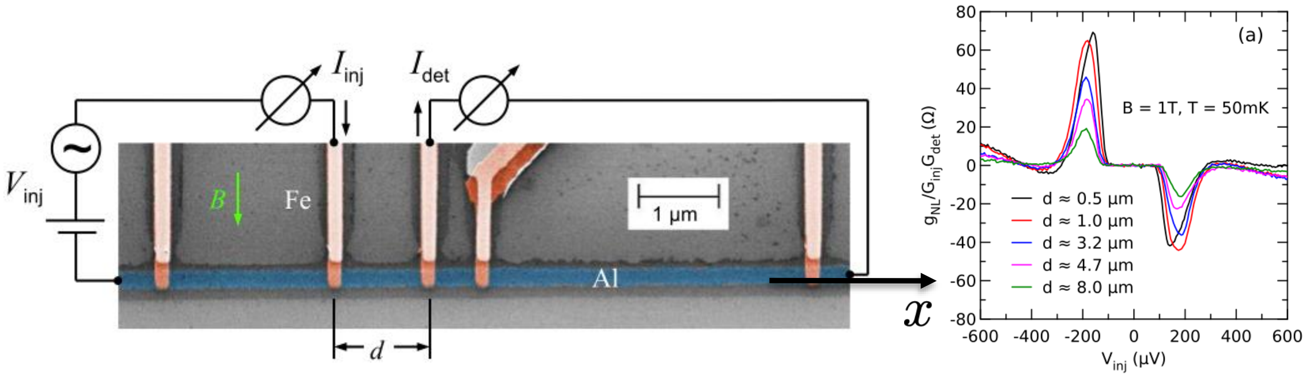
below room *T*



?

above room *T*?

Non-local transport and nonequilibrium modes



Hübler, Wolf, Beckmann, von Löhneysen, PRL **109,** 207001 (2012)

$$g_{NL} = \frac{dI_{\text{det}}}{dV_{\text{inj}}}$$

Normal, nonmagnetic metal: $I_{\text{det}} = 0$ due to charge conservation

Clarke and Tinkham (1972): charge imbalance in superconductors

$$\mu_{\text{qp}}(x) \neq \mu_{\text{condensate}} = 0$$ can decay (usually via inelastic collisions)

Johnson & Silsbee (1985), Jedema, Filip & van Wees (2001): spin imbalance

$$\mu_s(x) = \mu_\uparrow(x) - \mu_\downarrow(x) \neq 0$$ can decay via spin relaxation

Several probes: way to measure the relaxation length

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)

Theory: modes of nonequilibrium

$$\hat{g}^K = \hat{g}^R \hat{f} - \hat{f} \hat{g}^A$$

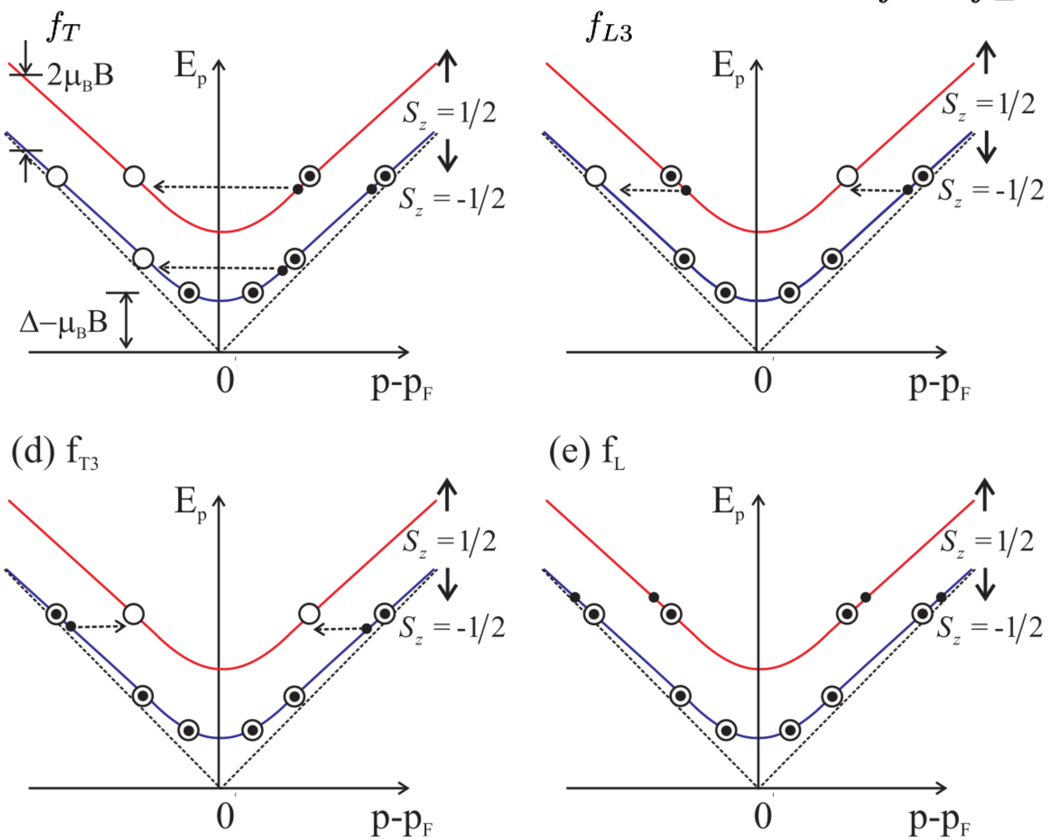
$$\hat{f} = f_L + f_T \tau_3 + f_{T3} \sigma_3 + f_{L3} \tau_3 \sigma_3$$

Charge

$\mu_{qp} \neq \mu_S$

Spin energy

$T^*_\uparrow \neq T^*_\downarrow$



Spin

$\mu_\uparrow \neq \mu_\downarrow$

Energy

$T^* \neq T$

Self-consistency equation:

$$\Delta = \frac{\lambda}{2} \int_{-\omega_D}^{\omega_D} d\epsilon \, \{\mathrm{Im}(f_0) f_L + \mathrm{Im}(f_3) f_{T3} + i[\mathrm{Re}(f_0) f_T + \mathrm{Re}(f_3) f_{L3}]\}$$

Diffusion equations

We start from Usadel equation $\check{\Lambda} = i\varepsilon\tau_3 - i(\boldsymbol{h}\cdot\boldsymbol{S})\tau_3 - \check{\Delta},$

$$D\nabla\cdot(\check{g}\nabla\check{g}) + [\check{\Lambda} - \check{\Sigma}_{so} - \check{\Sigma}_{sf} - \check{\Sigma}_{orb}, \check{g}] = 0.$$

Spin-orbit

Spin flips

Orbital effect of a field

Kinetic equation (ignoring the "Hanle" spin precession):

Energy
Spin
Charge
Spin energy

$$\nabla\cdot\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{T3} & 0 & 0 \\ 0 & 0 & R_T & R_{L3} \\ 0 & 0 & R_{L3} & R_T + S_{L3} \end{pmatrix}\begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

+ inelastic scattering

currents

noneq modes

$R_{T/L3}$: coupling to superconducting condensate
$S_{T3/L3}$: spin relaxation

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)
F. Aikebaier, M. Silaev, and TTH, PRB **98**, 024516 (2018)

Kinetics

Energy
Spin
Charge
Spin energy

$$\nabla \cdot \begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{T3} & 0 & 0 \\ 0 & 0 & R_T & R_{L3} \\ 0 & 0 & R_{L3} & R_T + S_{L3} \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

currents noneq modes

Normal metal: no mixing between the modes

$$\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = D \begin{pmatrix} \nabla & 0 & 0 & 0 \\ 0 & \nabla & 0 & 0 \\ 0 & 0 & \nabla & 0 \\ 0 & 0 & 0 & \nabla \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

Integrate over energy: for example spin diffusion equation

$$D\nabla^2 \mu_s = -\frac{\mu_s}{\tau_{sr}}$$

$$\mu_s = \nu_F \int d\epsilon f_T(\epsilon)$$

Kinetics

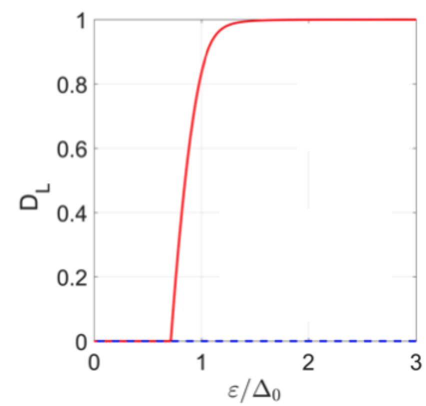
Energy
Spin
Charge
Spin energy

$$\nabla \cdot \begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{T3} & 0 & 0 \\ 0 & 0 & R_T & R_{L3} \\ 0 & 0 & R_{L3} & R_T + S_{L3} \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

currents noneq modes

Superconductor: energy dependent spectral coefficients

$$\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = D \begin{pmatrix} D_L \nabla & 0 & 0 & 0 \\ 0 & D_L \nabla & 0 & 0 \\ 0 & 0 & D_T \nabla & 0 \\ 0 & 0 & 0 & D_T \nabla \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$



Explains charge imbalance, poor energy transport in superconductors

Kinetics

Energy
Spin
Charge
Spin energy

$$\nabla \cdot \begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{T3} & 0 & 0 \\ 0 & 0 & R_T & R_{L3} \\ 0 & 0 & R_{L3} & R_T + S_{L3} \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

currents

noneq modes

Superconductor: energy dependent spectral coefficients, supercurrent effect

$$\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se} \end{pmatrix} = D \begin{pmatrix} D_L \nabla & 0 & j_E \nabla \phi & 0 \\ 0 & D_L \nabla & 0 & j_E \nabla \phi \\ j_E \nabla \phi & 0 & D_T \nabla & 0 \\ 0 & j_E \nabla \phi & 0 & D_T \nabla \end{pmatrix} \begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3} \end{pmatrix}$$

Supercurrent-driven thermoelectric conversion between temperature gradient and charge imbalance

Kinetics

Energy
Spin
Charge
Spin energy

$$\nabla\cdot\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se}\end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{T3} & 0 & 0 \\ 0 & 0 & R_T & R_{L3} \\ 0 & 0 & R_{L3} & R_T + S_{L3}\end{pmatrix}\begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3}\end{pmatrix}$$

currents | noneq modes

Spin-split superconductor: full matrix

spectral spin supercurrent

$$\begin{pmatrix} j_e \\ j_s \\ j_c \\ j_{se}\end{pmatrix} = D\begin{pmatrix} D_L\nabla & D_{T3} & j_E\nabla\phi & j_{Es}\nabla\phi \\ D_{T3} & D_L\nabla & j_{Es}\nabla\phi & j_E\nabla\phi \\ j_E\nabla\phi & j_{Es}\nabla\phi & D_T\nabla & D_{L3} \\ j_{Es}\nabla\phi & j_E\nabla\phi & D_{L3} & D_T\nabla\end{pmatrix}\begin{pmatrix} f_L \\ f_{T3} \\ f_T \\ f_{L3}\end{pmatrix}$$

Without supercurrent, energy and spin modes couple $\Rightarrow$ long-range spin transport in superconductors

M. Silaev, P. Virtanen, F.S. Bergeret, and TTH, PRL **114**, 167002 (2015)
F. Aikebaier, M. Silaev, and TTH, PRB **98**, 024516 (2018)

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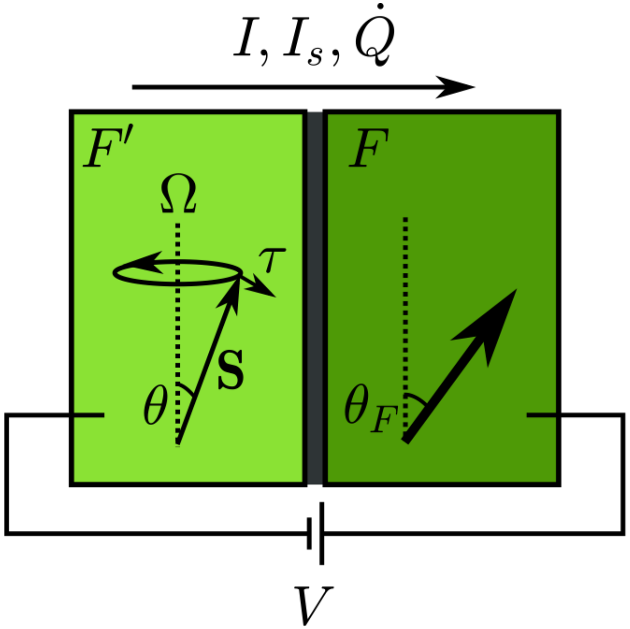
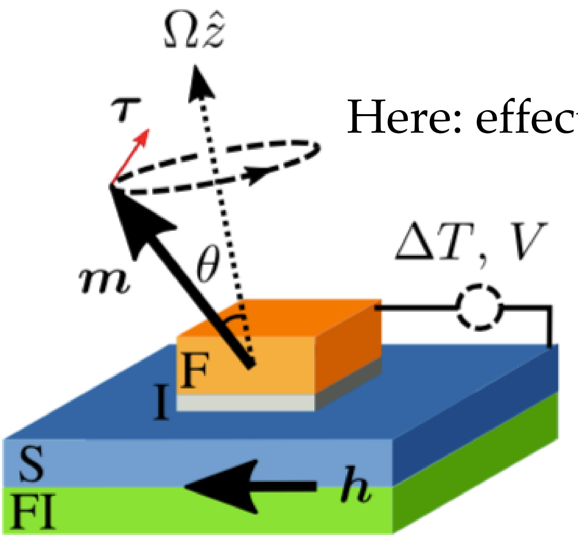
Dynamic magnetization

Two coupled metallic magnets, one with “free” and one with “fixed” magnetisation.



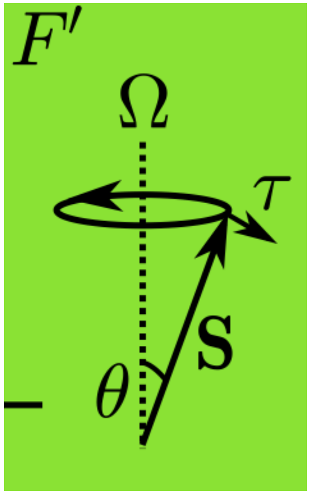
Non-collinear magnetisations
⇒ spin transfer torque
⇒ spin pumping from precessing magnetisation

Onsager!



Here: effect of superconductivity

Semiclassical magnetization dynamics



Single-domain magnet:

Magnetization $\mathbf{M} = \gamma \mathbf{S}/\mathcal{V}$

Gyromagnetic ratio

Large spin

Volume

Assume that $M = |\mathbf{M}|$ stays constant,
$\hat{m} \equiv \mathbf{M}/M$ can change

$$\partial_t \hat{m} = -\gamma \hat{m} \times \mathbf{H}_{\text{eff}} - \alpha \hat{m} \times \hat{m} \times \mathbf{H}_{\text{eff}} + \frac{\gamma}{M\mathcal{V}} \hat{m} \times \hat{I}_s \times \hat{m}$$

LLG

Precession

Damping

(Effective) magnetic field

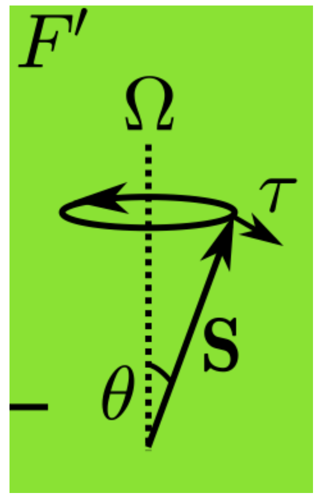
spin transfer torque
(damping or anti-damping)

Spin torque and pumping

Non-collinear spin current

$$\left.\frac{\partial \mathbf{m}}{\partial t}\right|_{\text{torque}} = \frac{\gamma}{M_s V}\mathbf{m} \times \mathbf{I}_s \times \mathbf{m}$$

Slonczewski (1996)



Pumped spin current

$$\mathbf{I}_s^{\text{pump}} = \frac{\hbar}{4\pi}\left(A_r \mathbf{m} \times \frac{d\mathbf{m}}{dt} - A_i \frac{d\mathbf{m}}{dt}\right)$$

Tserkovnyak, Brataas, Bauer (2002)

Torque “conductance”

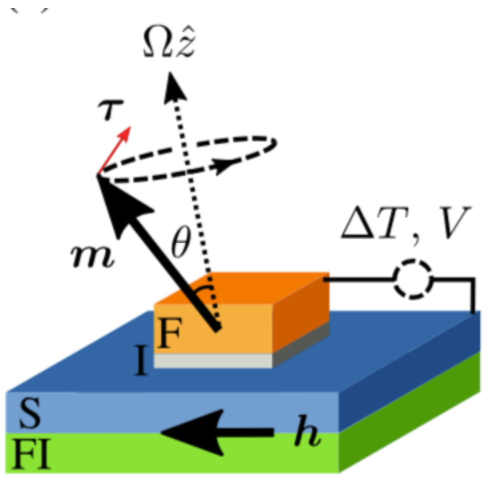
$$\begin{pmatrix} M_s \mathcal{V} \dot{\mathbf{m}} \\ \mathbf{I}_s \end{pmatrix} = \begin{pmatrix} \tilde{L}^{(mm)} & \tilde{L}^{ms} \\ \tilde{L}^{(sm)} & \tilde{L}^{(ss)} \end{pmatrix} \begin{pmatrix} \mathbf{H}_{\text{eff}} \\ \Delta\mu_s \end{pmatrix}$$

Spin pumping coefficient

Onsager reciprocity: $L_{ij}^{(sm)}(\mathbf{m}) = L_{ji}^{(ms)}(-\mathbf{m})$

Brataas, Tserkovnyak, et al., arXiv:1108.0385

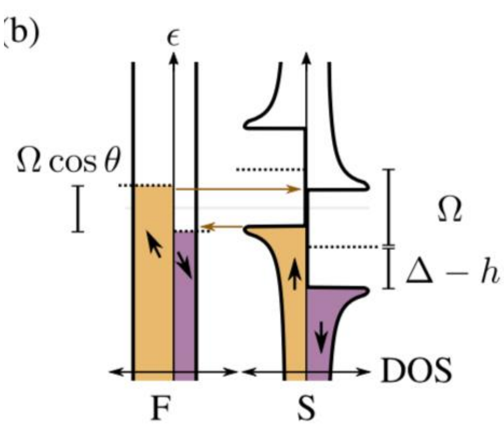
Including superconductivity



The Onsager matrix for collinear transport:

$$\begin{pmatrix} I \\ \dot{Q} \\ I_s \\ \dot{Q}_s \end{pmatrix} = \begin{pmatrix} G & P\alpha & PG & \alpha \\ P\alpha & G_{\rm th}T & \alpha & PG_{\rm th}T \\ PG & \alpha & G & P\alpha \\ \alpha & PG_{\rm th}T & P\alpha & G_{\rm th}T \end{pmatrix} \begin{pmatrix} V \\ -\Delta T/T \\ V_s/2 \\ -\Delta T_s/2T \end{pmatrix},$$

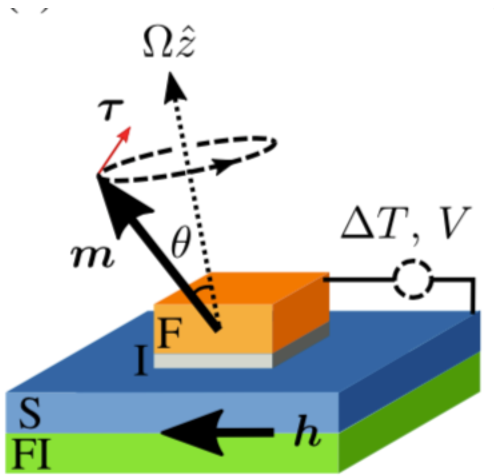
What about non-collinear magnetizations and magnetization precession?

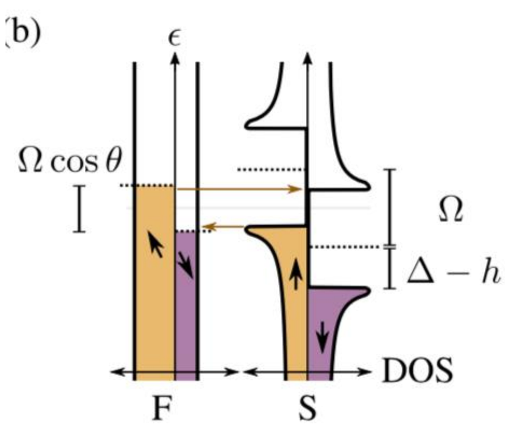


FMR: switch to the frame precessing with the magnetization
Precession rate becomes a spin bias driving currents into S

R. Ojajärvi, J. Manninen, TTH, and P. Virtanen, PRB 2020

Including superconductivity



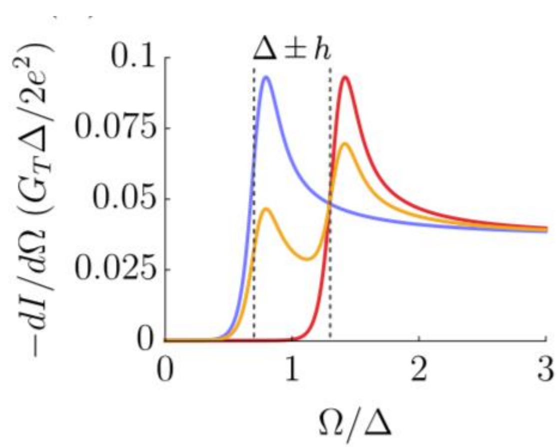


Linear response:

$$\begin{pmatrix} \overline{I_c} \\ \overline{\dot{E}_S} \\ \overline{\tau_z} \end{pmatrix} = \begin{pmatrix} G & P\alpha\cos\theta & 0 \\ P\alpha\cos\theta & G_{\mathrm{th}}T & \frac{\alpha}{2}\sin^2\theta \\ 0 & -\frac{\alpha}{2}\sin^2\theta & -\frac{G}{4}\sin^2\theta \end{pmatrix} \begin{pmatrix} V \\ -\Delta T/T \\ \Omega \end{pmatrix}$$

Thermospin torque! Spin-pumping cooling

Nonlinear: also spin-pumped current



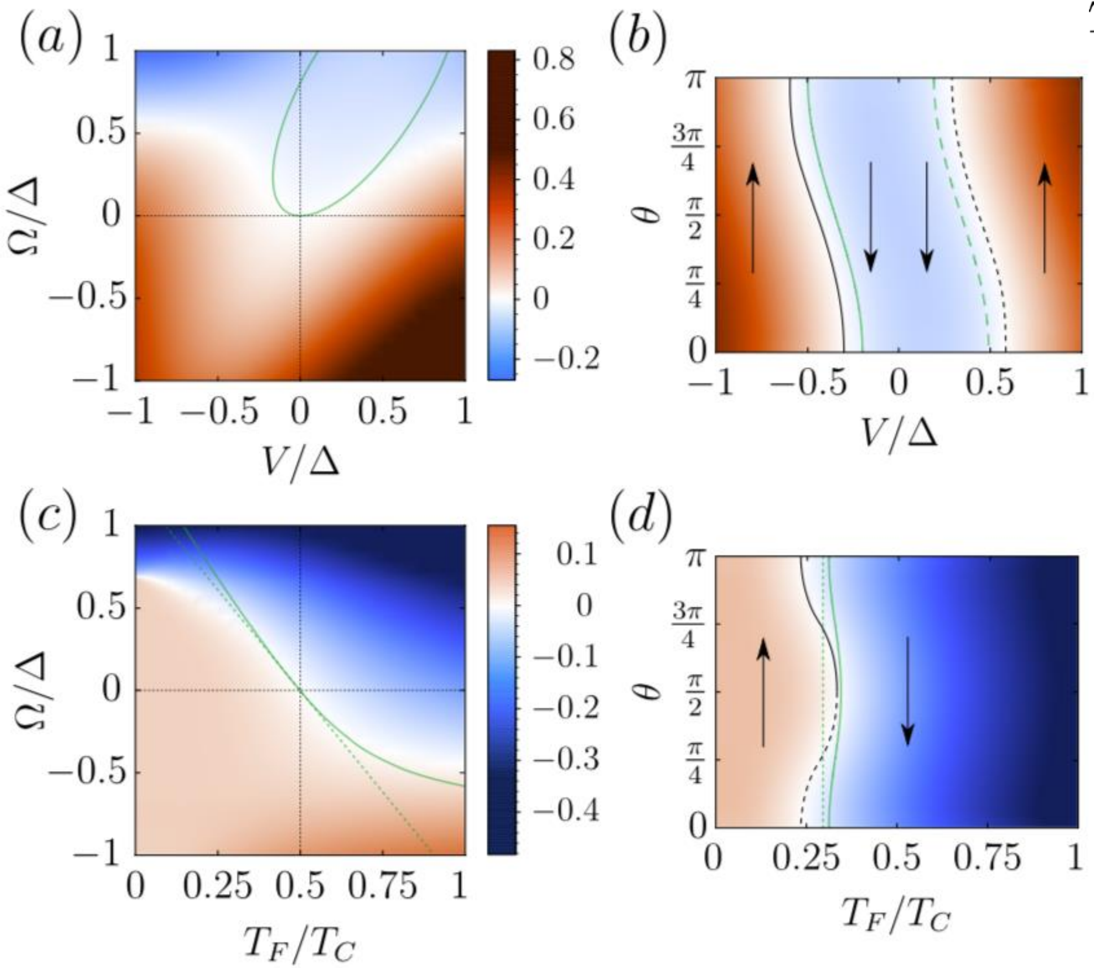
(Without spin splitting:
Trif & Tserkovnyak PRL 2013)

R. Ojajärvi, J. Manninen, TTH, and P. Virtanen, PRB 2020

STT and auto-precession

$\Omega = 0.3\Delta$

$T_S = 0.5T_c$



Spin transfer torque ($\theta \approx 0$)

Stable precession angle

Assumes weak intrinsic damping (hard, but not impossible)

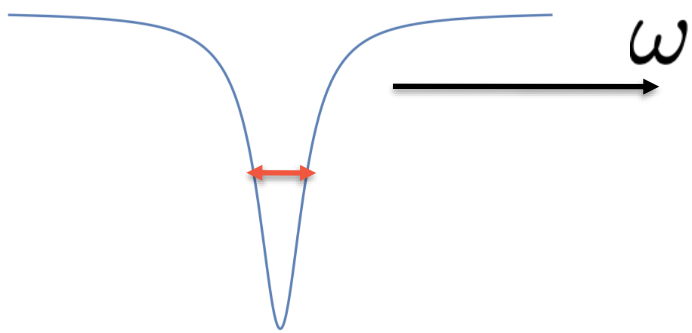
R. Ojajärvi, J. Manninen, TTH, and P. Virtanen, PRB 2020

Previous: dc biases
Next: ac bias

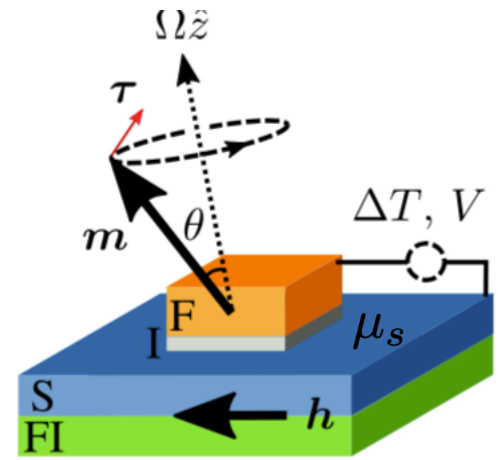
Idea in the measurement (and more in the talk by Chiara Ciccarelli):

Input signal, frequency ω

Reflected signal

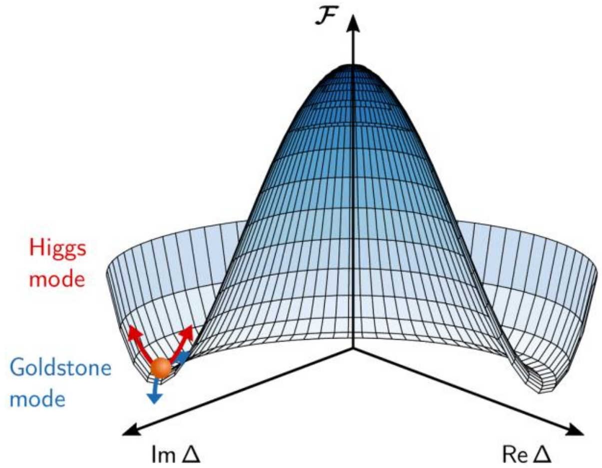


$$\omega_0 = \omega_{\mathrm{FMR}} \text{ or } \omega_0 = \Omega_{\mathrm{Higgs}}$$



Linewidth: damping
by spin pumping into S (Ciccarelli)
or Higgs damping (here)

Higgs mode detection



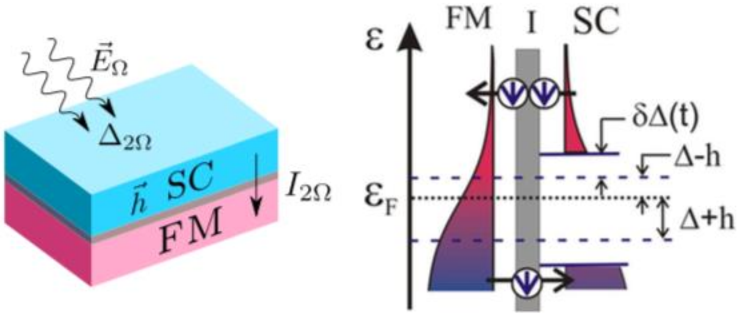
Picture stolen from Dirk Manske

See also Risto Ojajärvi's poster!

Usually measuring Higgs: 3rd harmonic measurements

Silaev, PRB 2019: impurities increase the Higgs excitation

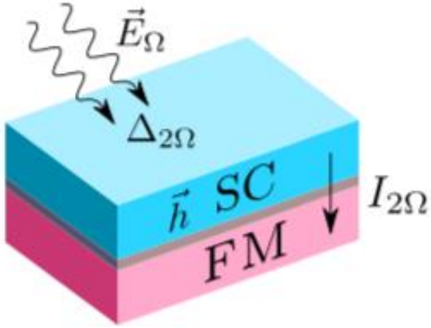
Here:



Second harmonic!
Higgs: ac spin current
Spin polarised barrier: charge

Silaev, Ojajärvi, TTH, PRRes (2020)

Higgs mode detection: second harmonic generation



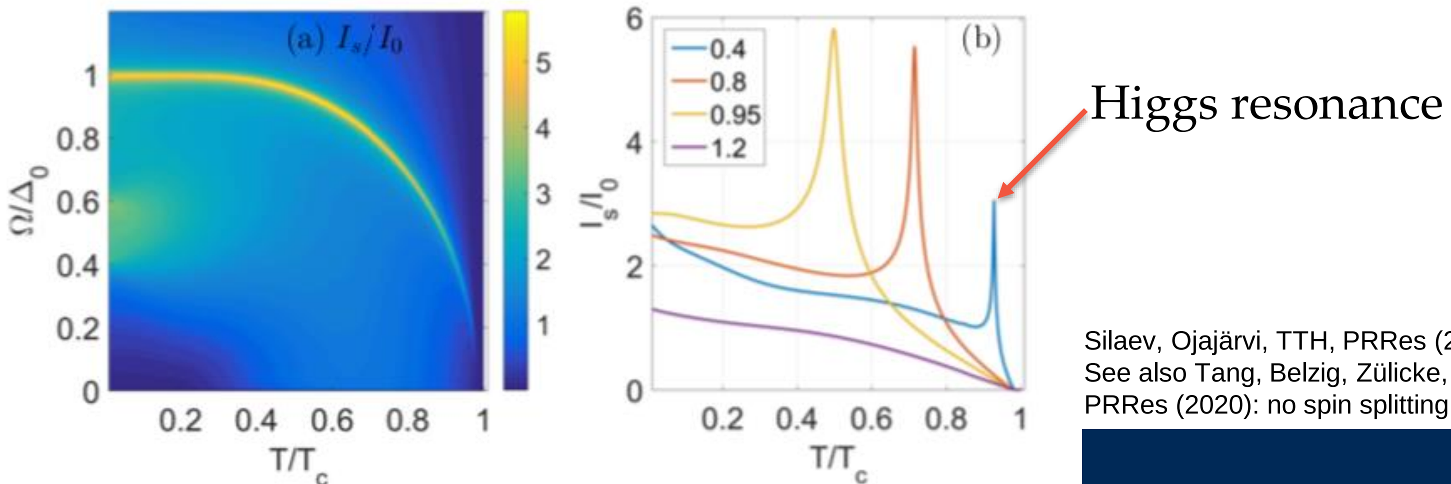
Order parameter modification due to ac irradiation

$$F_\Delta(2\Omega) = -\lambda T \sum_\omega \mathrm{Tr}[\hat{\tau}_2 \hat{g}_{AA}] \qquad \Delta_{2\Omega} = F_\Delta / [1 - \Pi(2\Omega)]$$

Higgs resonance: $\Pi(2\Omega = 2\Delta) - 1 \approx \Gamma \to 0$

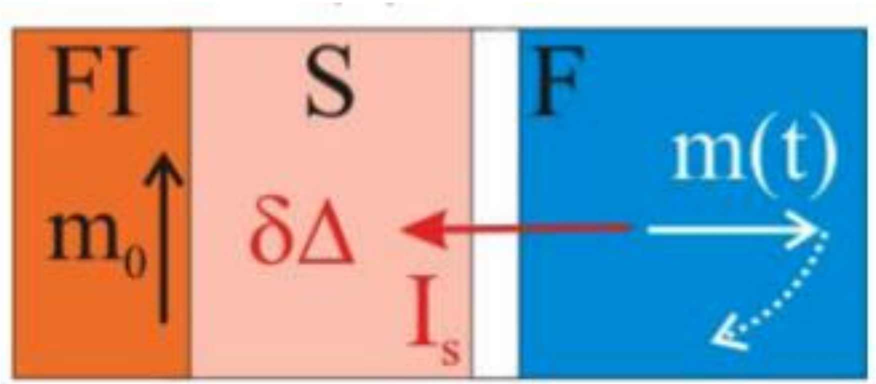
Spin-split superconductor: $\Delta_{2\Omega}$ drives an ac spin current
$\Rightarrow$ can be converted into charge current by spin filtering

Amplitude of double-frequency spin current



Silaev, Ojajärvi, TTH, PRRes (2020)
See also Tang, Belzig, Zülicke, Bruder, PRRes (2020): no spin splitting, finite dc voltage

Higgs mode and FMR



Magnetization precession pumps spin current into S, affecting its dynamics; affects back to the Gilbert damping (spin battery effect)

Order parameter modification:

spin accumulation

$$\delta\Delta = \frac{\lambda\Delta}{1-\Pi}\mu_s \partial_\Delta (N_+ - N_-)$$

Silaev, Ojajärvi, TTH, PRRes 2020

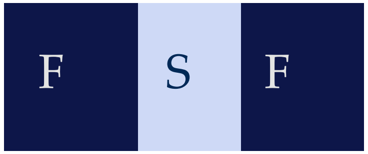
Some other exciting topics…

Spin-orbit coupling:

- Anomalous phase
- “Long-range” (m=1) triplet correlations
- Spin Hall effect

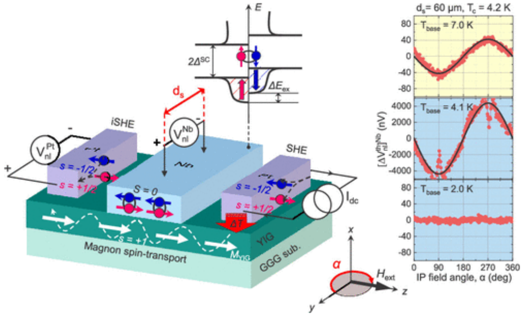
Tokatly & Bergeret 2014-2020

Superconductor-mediated coupling between ferromagnets



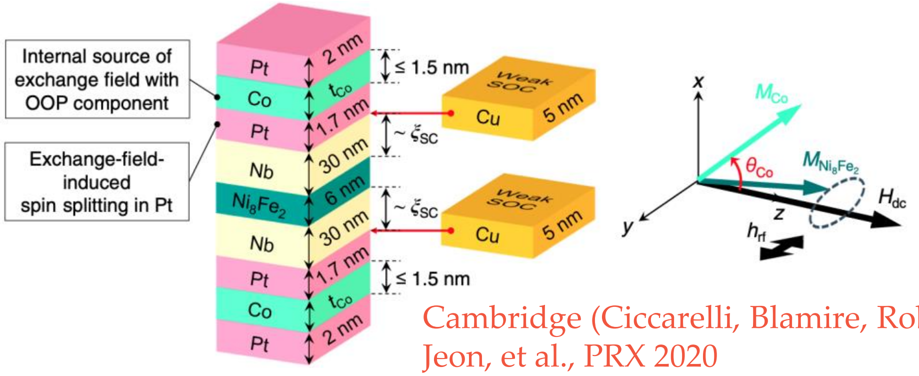
de Gennes 1966
Zhu, et al., [Robinson, Blamire], Nature Mat 2017

ISHE into a superconductor



Jeon, et al. [Parkin]
ACS Nano 2020

Ferromagnetic resonance and spin supercurrents?



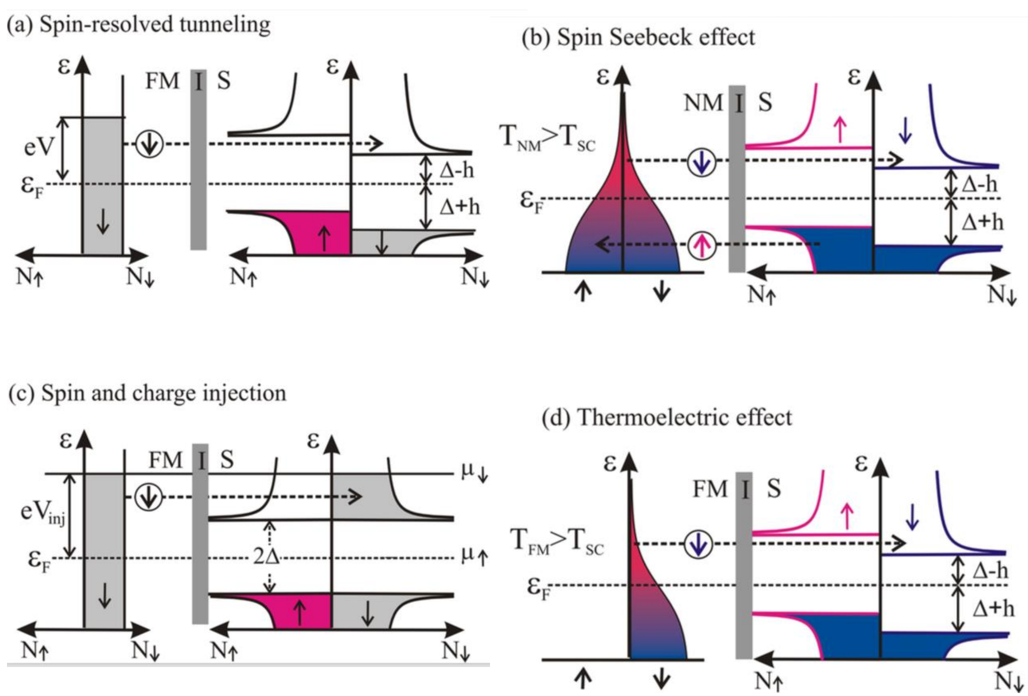
Cambridge (Ciccarelli, Blamire, Robinson)
Jeon, et al., PRX 2020

Measurement of a spin temperature

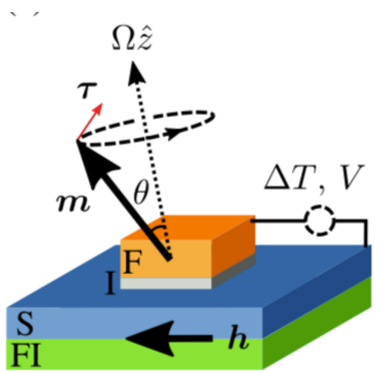
Kuzmanovic, et al. [Aprili], arXiv:2001.04422

Summary

Spin-split superconductor: coupling between spin, charge and heat



Bergeret, Silaev, Virtanen, TTH, Rev. Mod. Phys. 90, 041001 (2018)



Towards non-linear spin torque/pumping into spin-split superconductors, using other collective modes (Higgs)