Spintronics meets quantum spin liquids: a novel spectral probe of quantum magnets based on spin Hall phenomena

So Takei

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D. Joshi, A. P. Schnyder and S. Takei, Phys. Rev. B **98**, 064401 (2018)
J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

acknowledgments

- collaborators:



Joshua Aftergood (CUNY)

Andreas Schnyder (MPI Stuttgart)

Darshan Joshi (MPI Stuttgart → Harvard)

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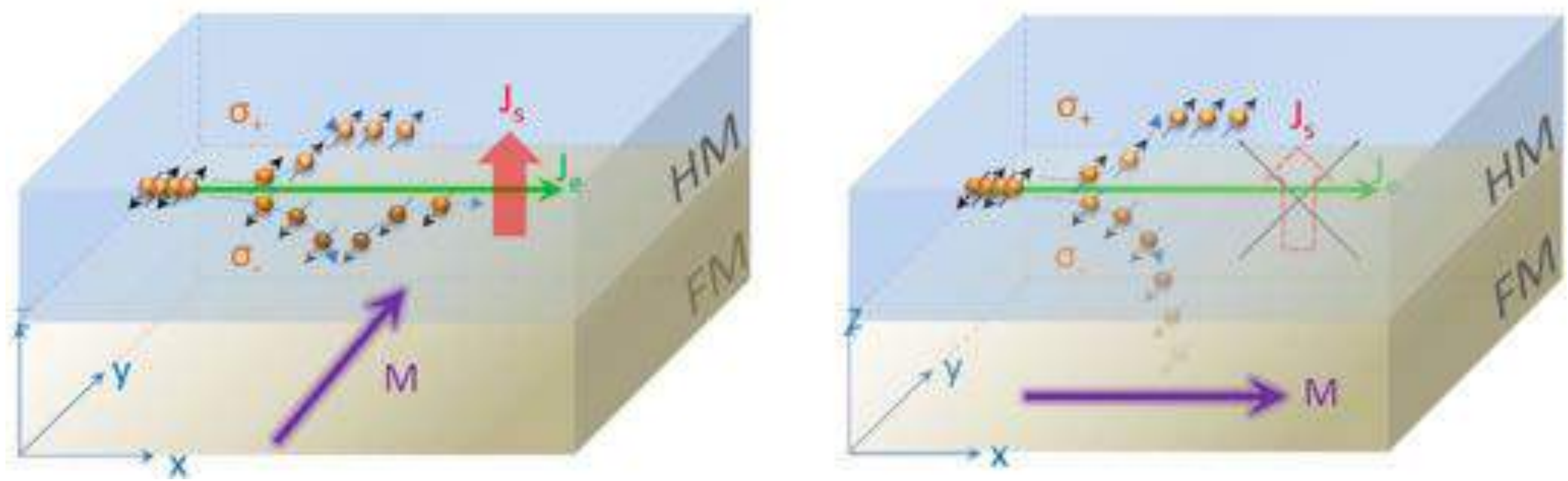
PSC CUNY

contents

- introduction to **spin Hall noise spectroscopy**
- application to a **quantum spin ladder**: detection of topological phase transitions
  - dimerized quantum antiferromagnet + Dzyaloshinskii-Moriya interaction + external magnetic field, e.g., $BiCu_2PO_6$.
- application to **quantum spin liquids**: detection of spin density of states
  - S=1/2 kagomé Heisenberg antiferromagnet, e.g., herbertsmithite $ZnCu_3(OH)_6Cl_2$.
  - the antiferromagnetic Kitaev honeycomb model, e.g., α-$RuCl_3$.
  - spinon Fermi surface coupled to gapless U(1) gauge fluctuations, e.g., organic salt compounds, $YbMgGaO_4$.
- summary & outlook

spin Hall magnetoresistance

- **spin Hall magnetoresistance (SMR)**: corrections to longitudinal and Hall resistivities of a strongly spin-orbit coupled metal in contact with a magnetic material.



H. Nakayama *et al.*, Phys. Rev. Lett. **110**, 206601 (2013)
Y.-T. Chen *et al.*, Phys. Rev. B **87**, 144411 (2013)

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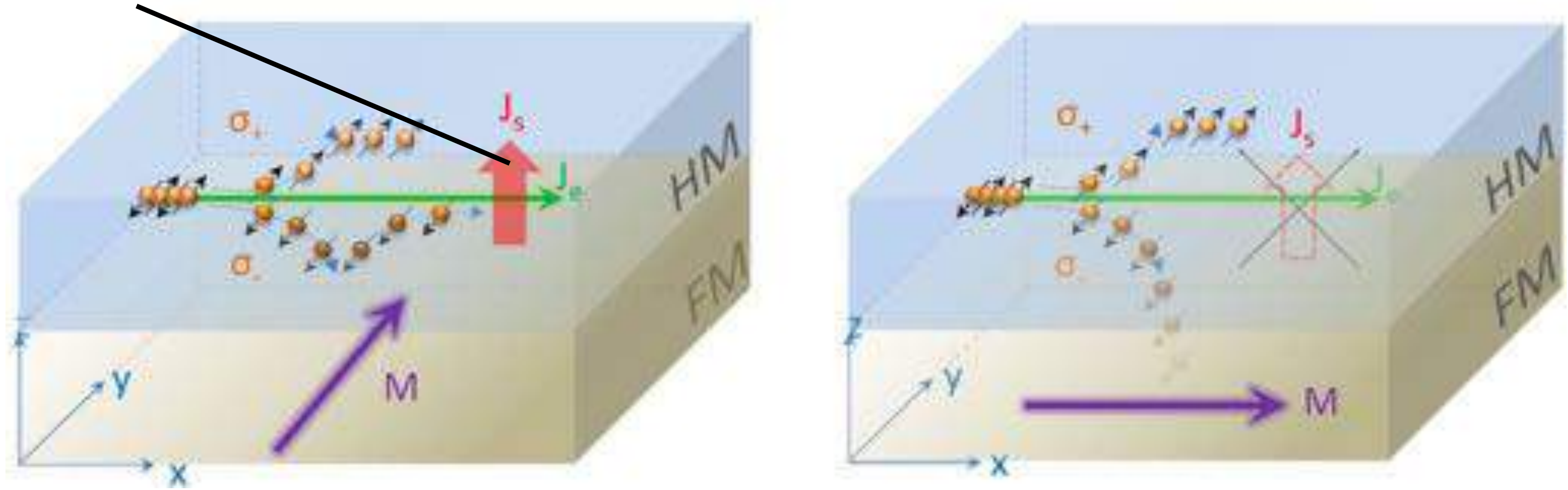

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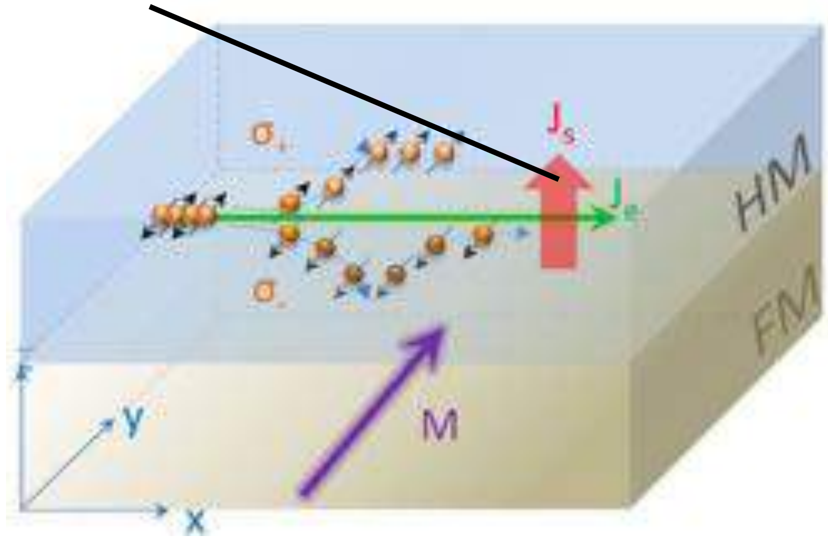

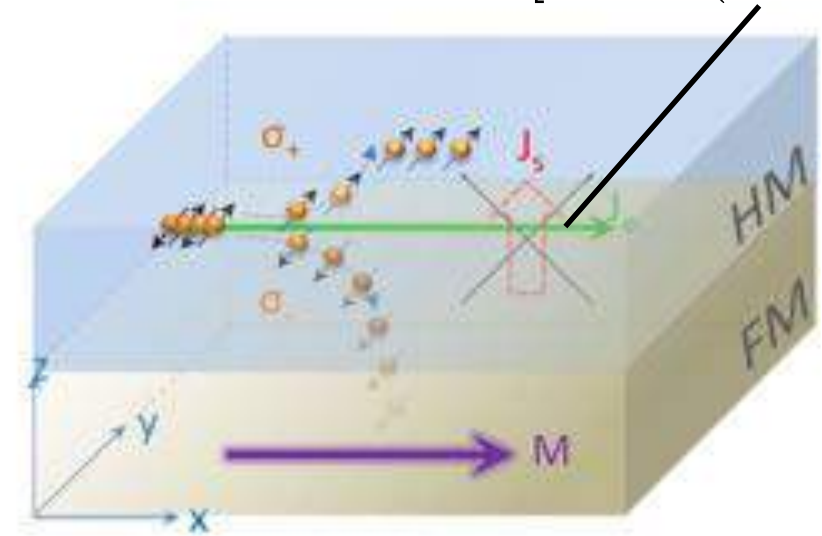

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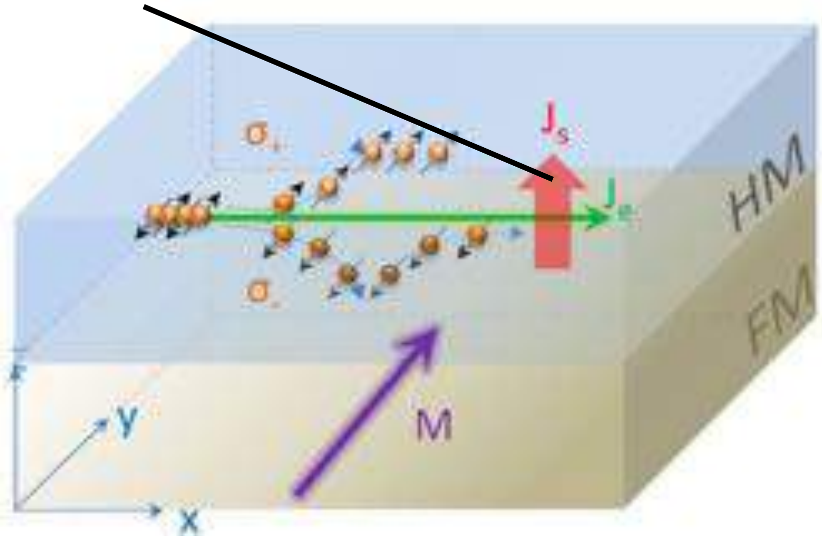
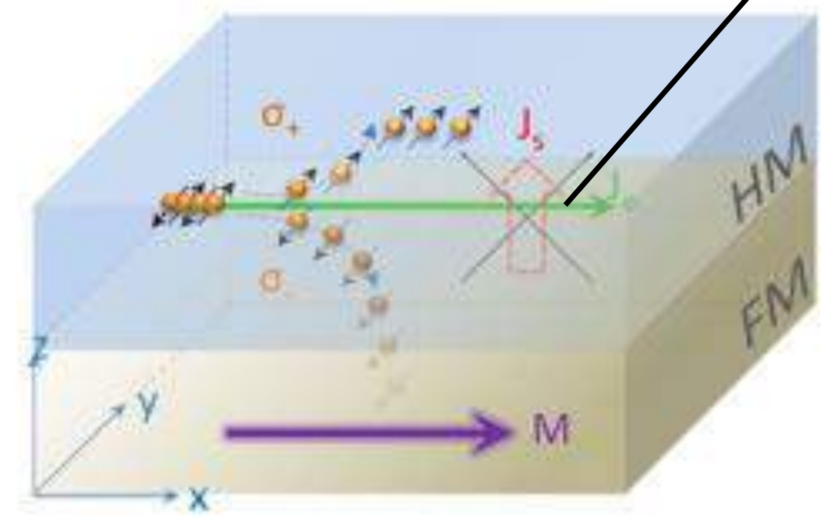
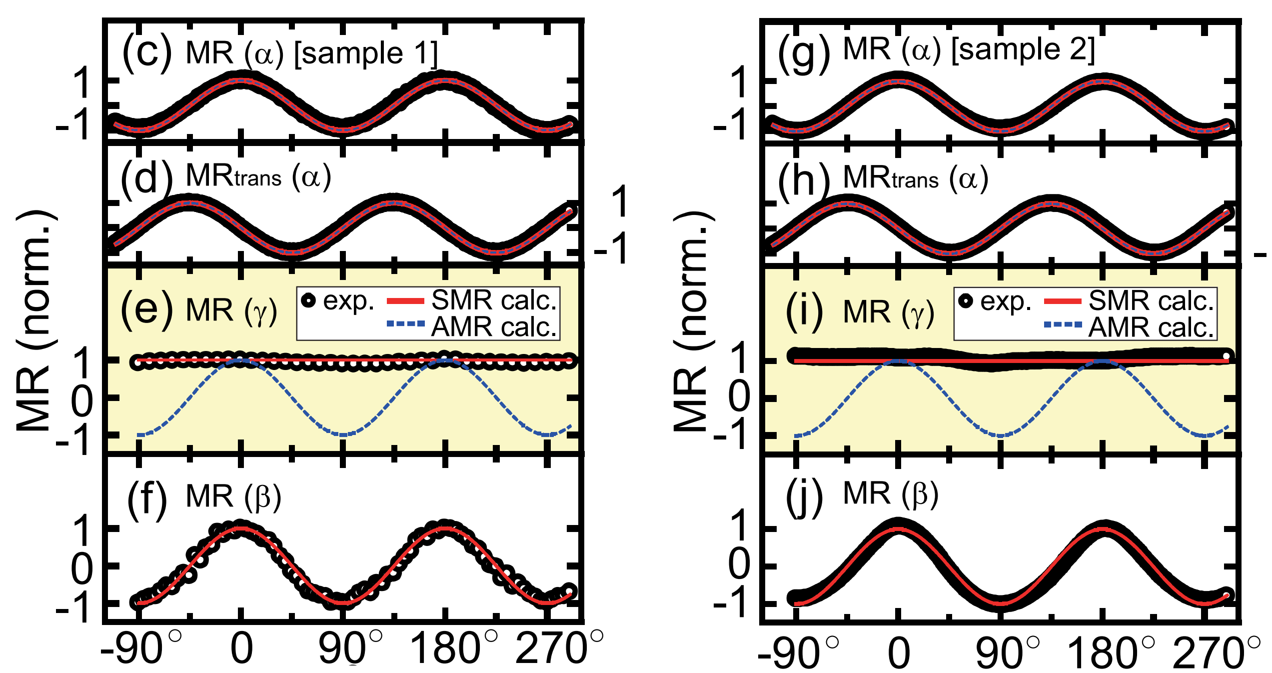
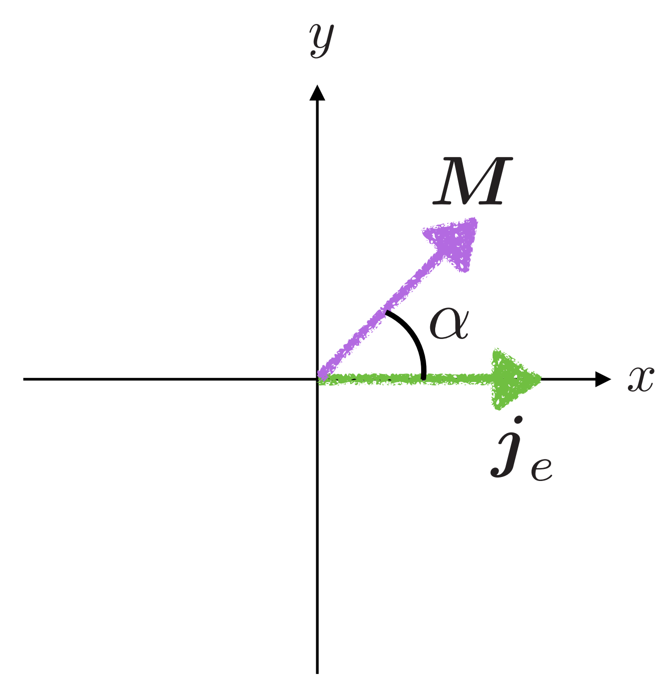




- longitudinal resistance for in-plane magnetization: $R = R_0 + \Delta R_0 + R_1 \cos^2 \alpha$





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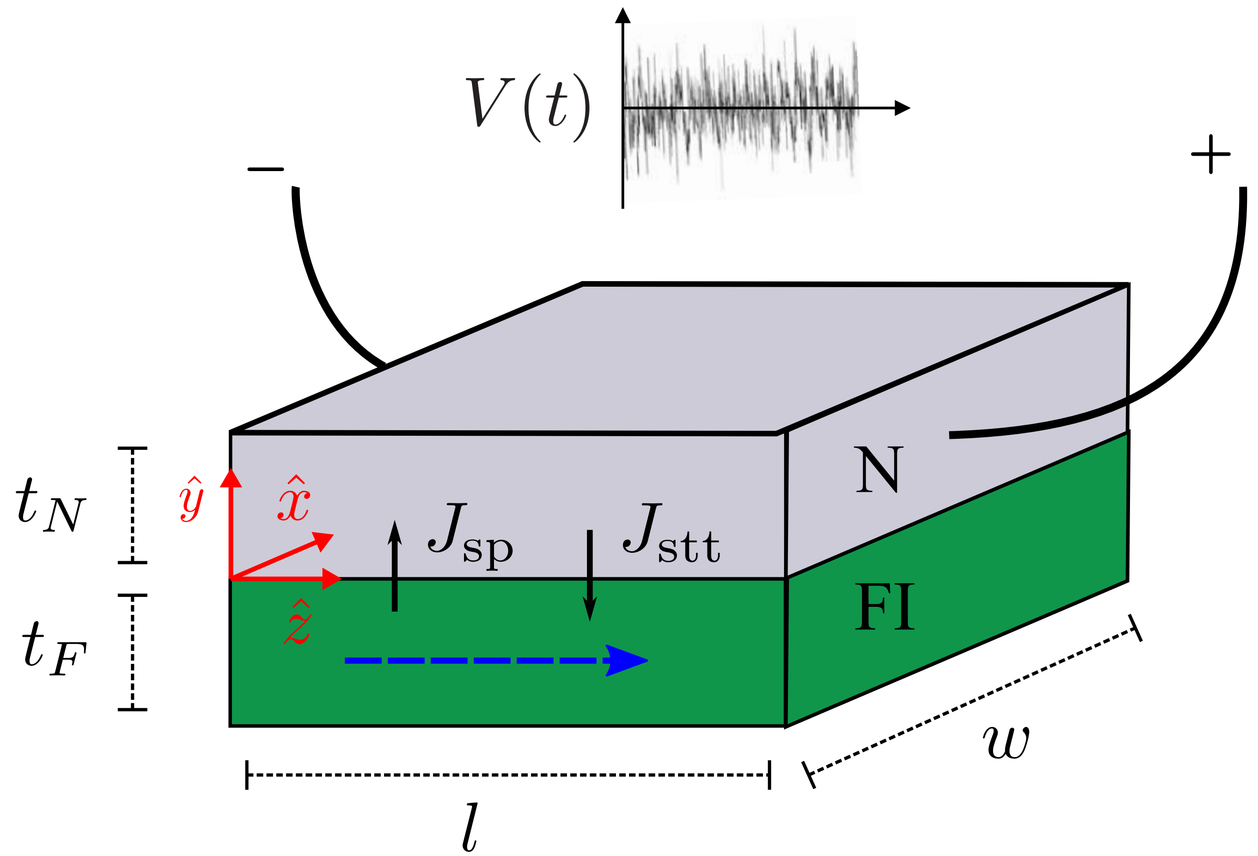
spin Hall noise

- Fluctuation-dissipation theorem: SMR gives additional contribution to the thermal voltage noise across the metal.



A. Kamra *et al.*, Phys. Rev. B **90**, 214419 (2014)

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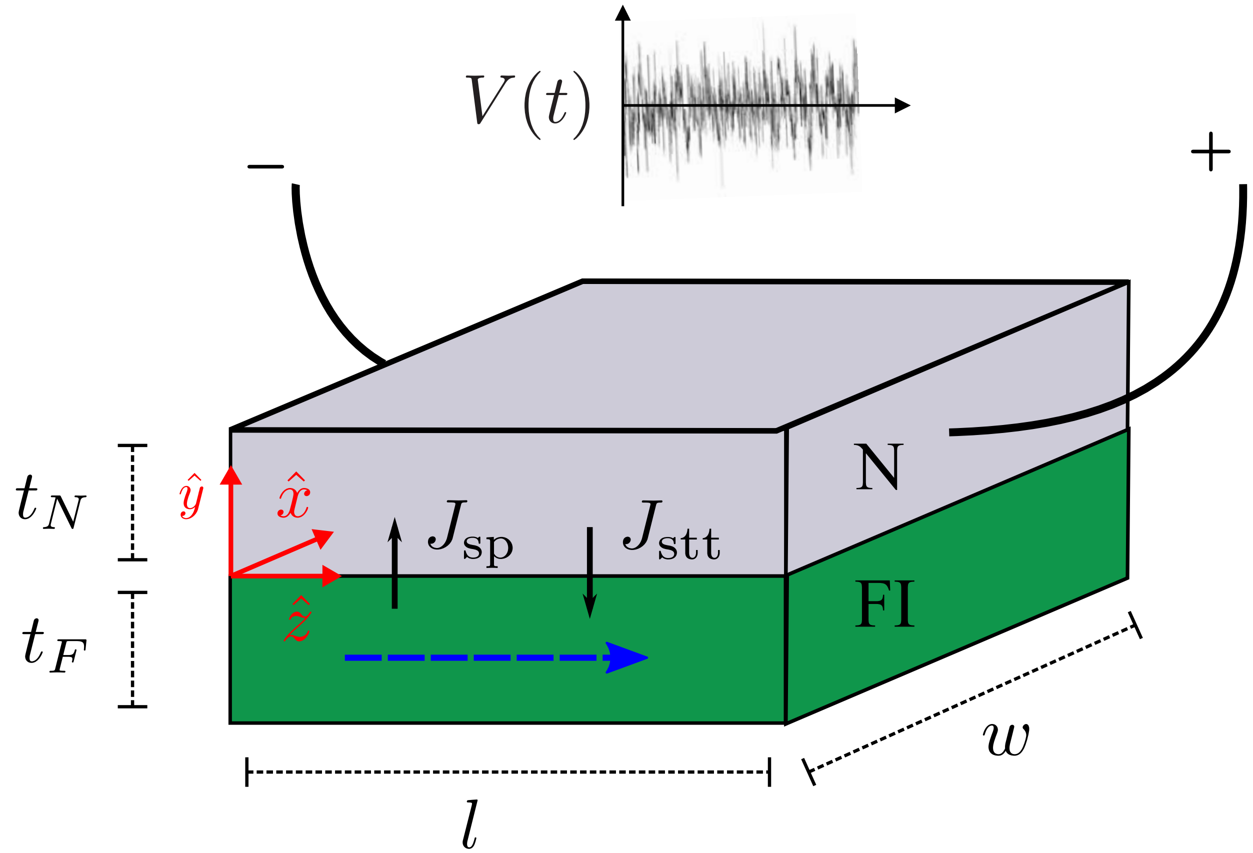
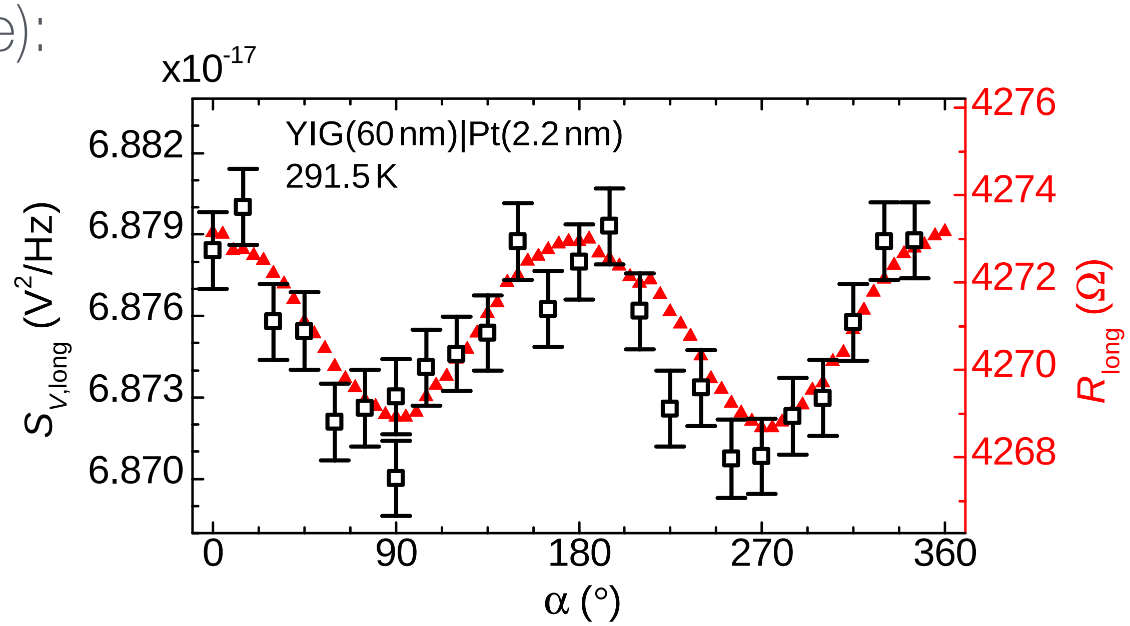


- DC thermal voltage noise (Johnson-Nyquist noise):

$$S_V(\Omega) = \int_{-\infty}^{\infty} \langle V(t)V(0)\rangle \, e^{-i\Omega t} dt$$

$$\overset{\Omega \to 0}{\Longrightarrow} 2k_B T(R_0 + \Delta R_0 + R_1 \cos^2 \alpha)$$



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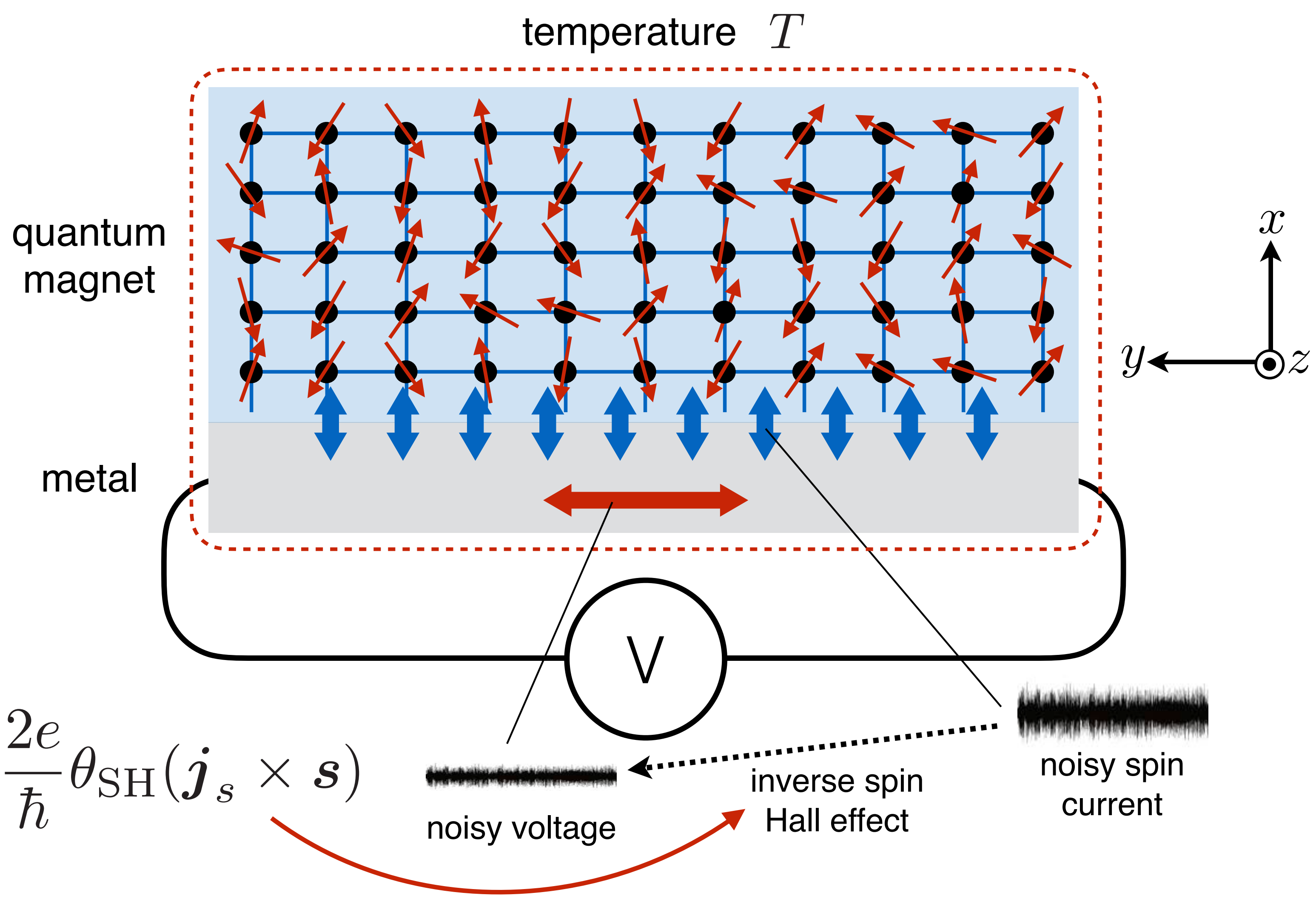
spin Hall noise spectroscopy (SHNS)

- heavy metal | quantum paramagnet (no long-range order) bilayer at temperature T.



$$\boldsymbol{j}_c = -\frac{2e}{\hbar}\theta_{\mathrm{SH}}(\boldsymbol{j}_s \times \boldsymbol{s})$$

- total (asymmetrized) voltage noise spectral density in the metal:

$$S_V(\Omega, T) \equiv \int_{-\infty}^{\infty} \langle V(t)V(0)\rangle\, e^{-i\Omega t} dt = S_V^{(0)}(\Omega, T) + \delta S_V(\Omega, T)$$

D. Joshi, A. P. Schnyder and ST, Phys. Rev. B **98**, 064401 (2018)
J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

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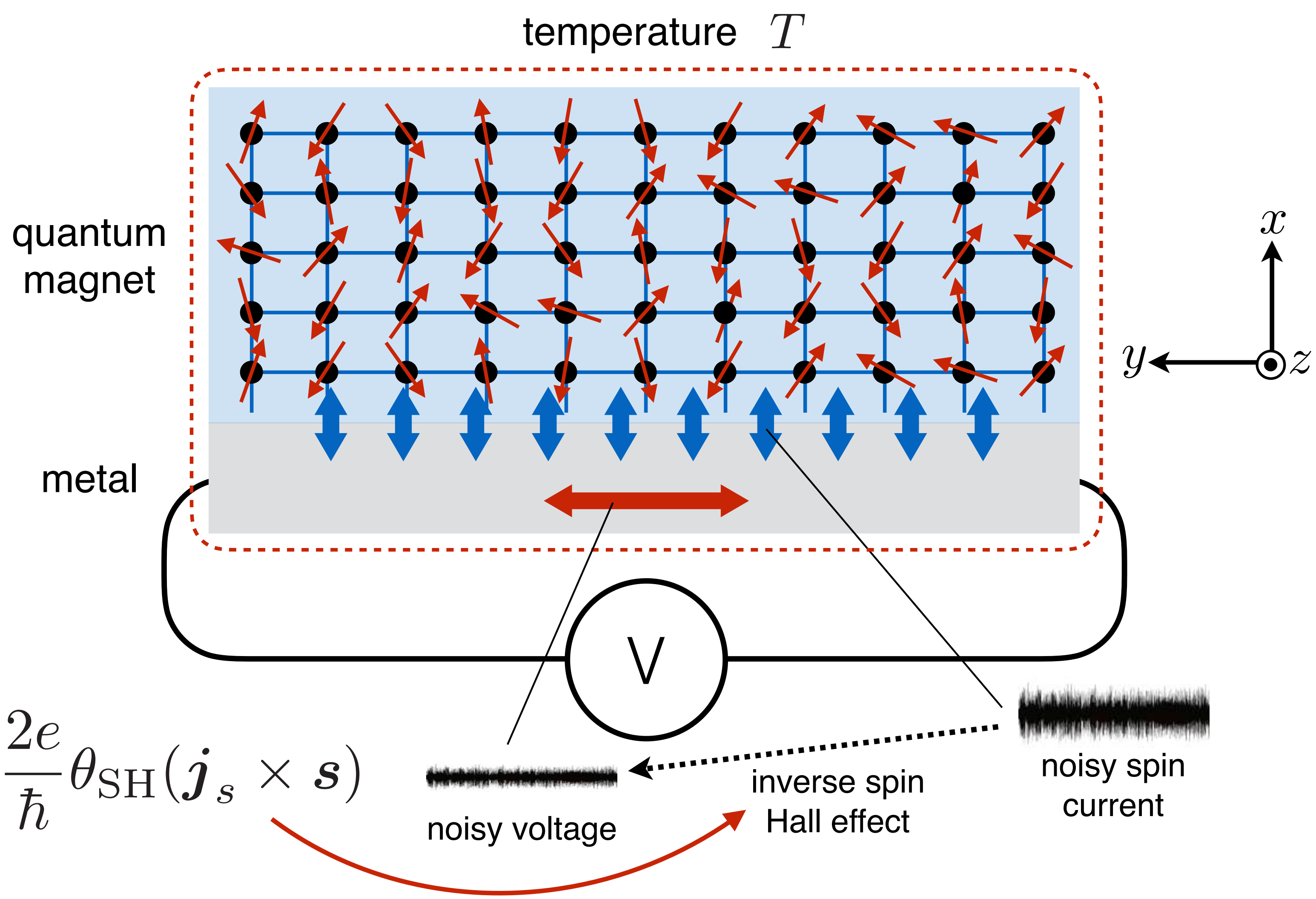

$$\boldsymbol{j}_c = -\frac{2e}{\hbar}\theta_{\mathrm{SH}}(\boldsymbol{j}_s \times \boldsymbol{s})$$

- interfacial z-polarized spin current fluctuations $S_s(\Omega, T)$ generates voltage fluctuations along the y axis in the metal:

$$\delta S_V(\Omega, T) = \Theta S_s(\Omega, T)$$

D. Joshi, A. P. Schnyder and ST, Phys. Rev. B **98**, 064401 (2018)
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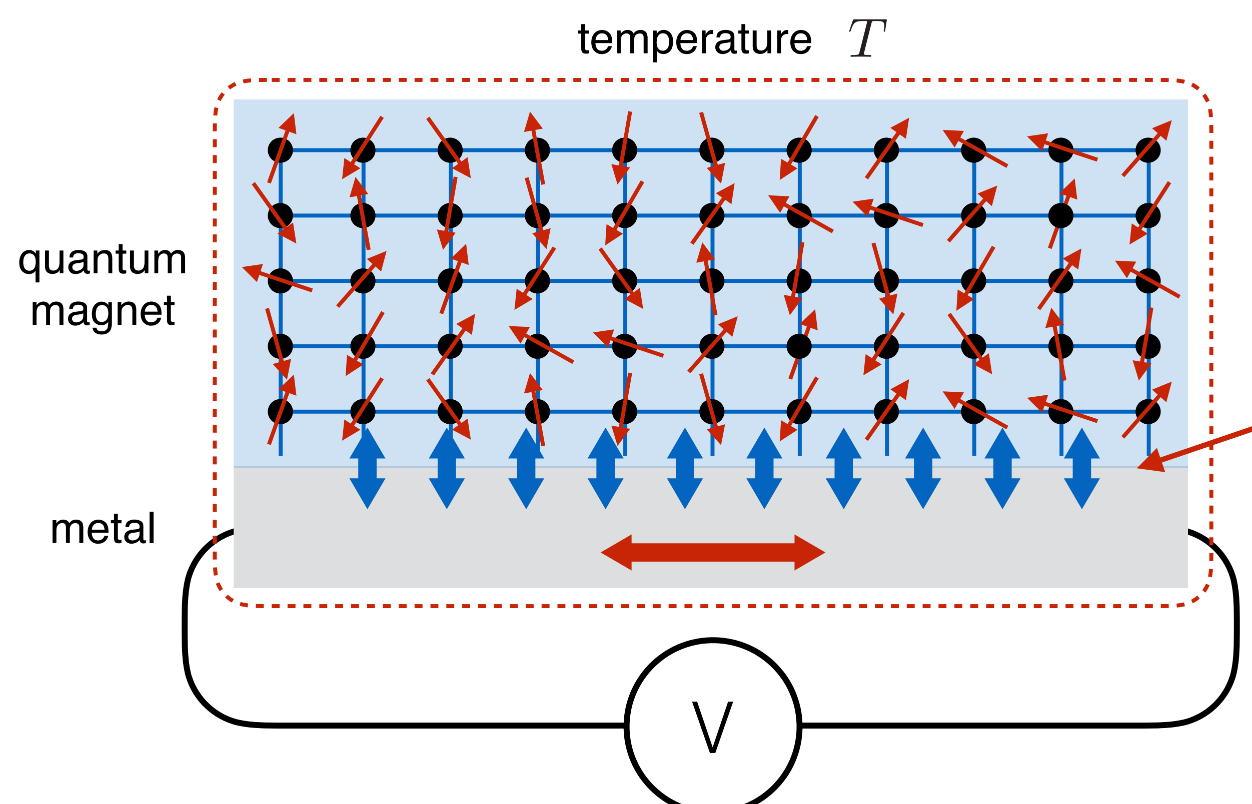
theory

- thermal spin current fluctuations at the metal | quantum paramagnet interface.
  - treat metal using the Sommerfeld model.
  - exchange coupling at the interface.
  - define spin current as total z-polarized spin entering the metal.



$$H_c = -\mathcal{J} v_0 \sum_{\boldsymbol{i}} \boldsymbol{s}(y=0, \boldsymbol{r_i}) \cdot \boldsymbol{S_i}$$

$$I_s(t) = \hbar \frac{\partial}{\partial t} \left[\int_{\text{metal}} d^3 \boldsymbol{r} s_z(\boldsymbol{r}, t) \right]$$

$$S_s(\Omega) = \int_{-\infty}^{\infty} dt \, \langle I_s(t) I_s(0) \rangle e^{i\Omega t} \qquad \chi_{\boldsymbol{ij}}^{\mp\pm}(\nu) \equiv -i \int dt \, \langle S_{\boldsymbol{i}}^{\mp}(t) S_{\boldsymbol{j}}^{\pm}(0) \rangle e^{i\nu t}$$

$$= 2i \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2 \hbar} \right)^2 \sum_{\boldsymbol{i}} \int_{-\infty}^{\infty} d\nu \, \frac{\nu - \Omega}{e^{\beta\hbar(\nu-\Omega)} - 1} \left[\chi_{\boldsymbol{ii}}^{+-}(\nu) + \chi_{\boldsymbol{ii}}^{-+}(\nu) \right]$$

Voltage noise correction is proportional to the imaginary part of the local dynamical spin structure factor of the adjacent quantum magnet.

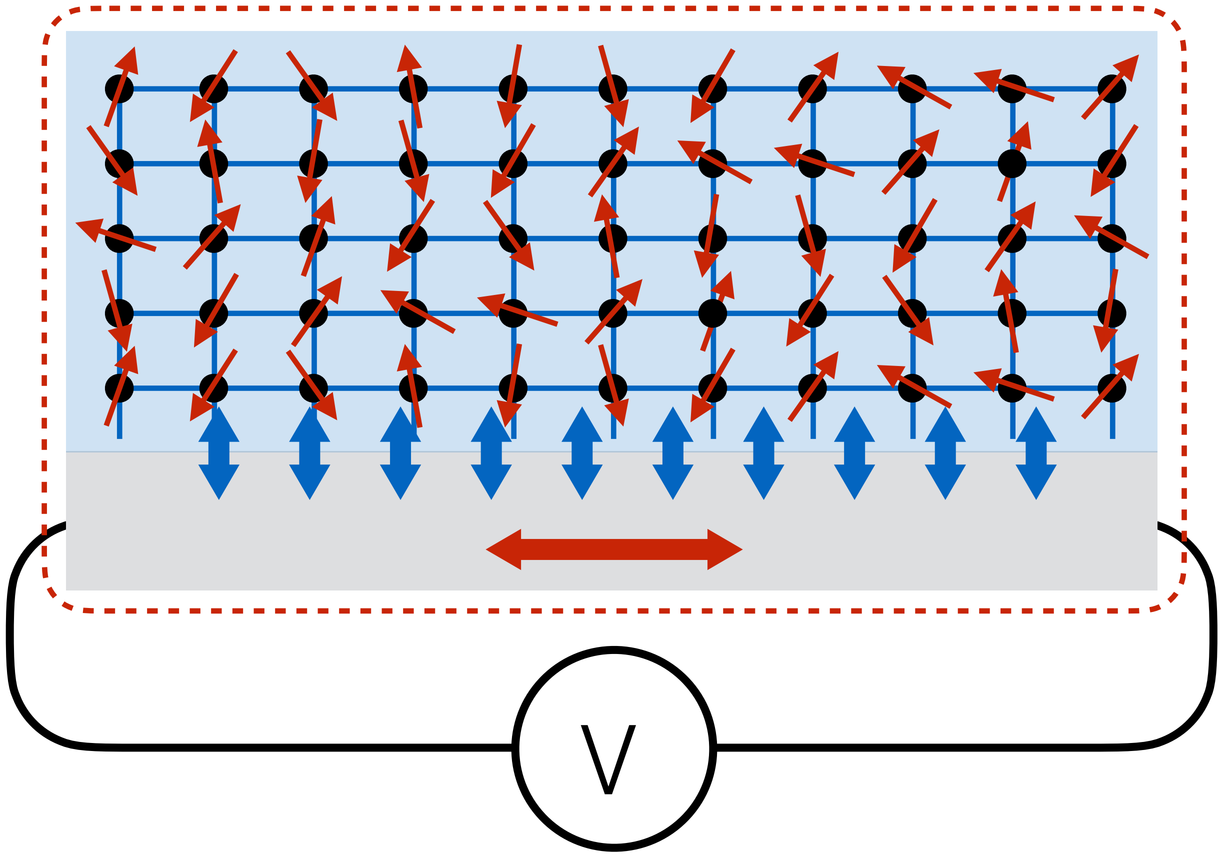
measurement

- noise due to proximate quantum magnet obtained via the difference between the total noise and the (known) background noise of the metal.

$$\underbrace{\delta S_V(\Omega, T)}_{\text{predicted}} = \underbrace{S_V(\Omega, T)}_{\text{measured}} - \underbrace{S_V^{(0)}(\Omega, T)}_{\text{known}}$$

- noise can also be determined through AC resistance measurements.

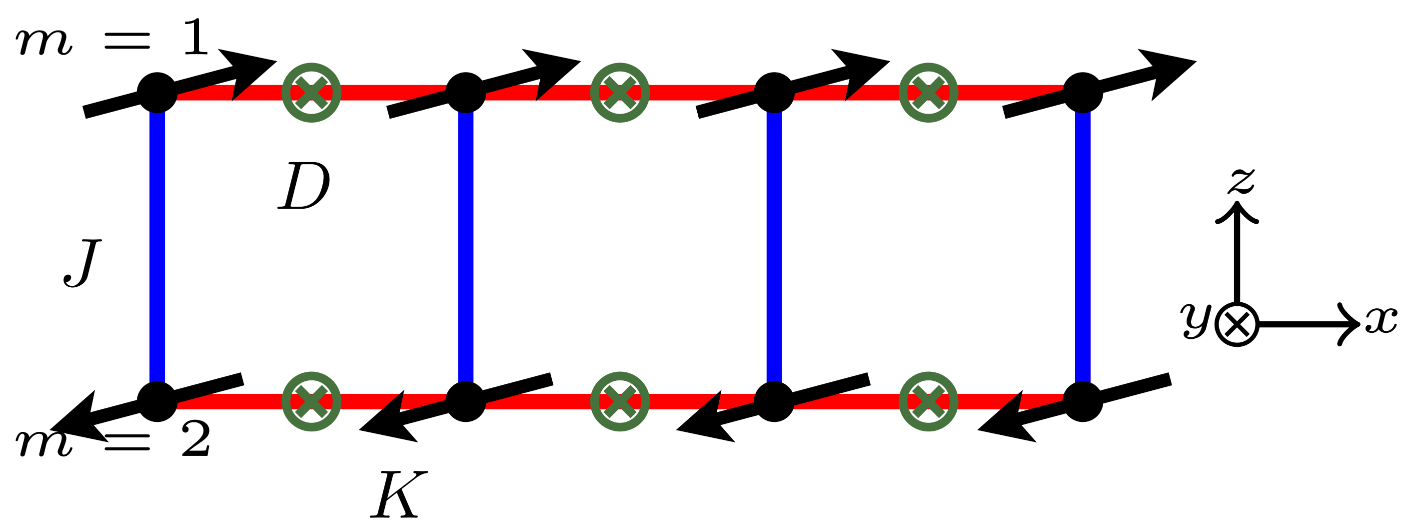
$$\underbrace{\delta S_V(\Omega, T)}_{\text{predicted}} = 4k_B T \frac{\hbar\Omega/k_B T}{e^{\hbar\Omega/k_B T} - 1}\left(\underbrace{R(\Omega, T)}_{\text{measured}} - \underbrace{R_0(\Omega, T)}_{\text{known}}\right)$$



application to quantum spin ladder

quantum spin ladder

- quantum spin ladder: intra-dimer exchange J, inter-dimer exchange K, odd-parity DM interaction D, even-parity spin-anisotropic interaction Γ, and external magnetic field h_y



$$\hat{H} = J\sum_i \hat{\boldsymbol{S}}_{1,i}\cdot\hat{\boldsymbol{S}}_{2,i} + K\sum_{m=1,2}\sum_i \hat{\boldsymbol{S}}_{m,i}\cdot\hat{\boldsymbol{S}}_{m,i+1} + h_y\sum_{m=1,2}\sum_i \hat{S}^y_{m,i}$$
$$+ D\sum_{m=1,2}\sum_i\left[\hat{S}^z_{m,i}\hat{S}^x_{m,i+1} - \hat{S}^x_{m,i}\hat{S}^z_{m,i+1}\right] + \Gamma\sum_{m=1,2}\sum_i\left[\hat{S}^z_{m,i}\hat{S}^x_{m,i+1} + \hat{S}^x_{m,i}\hat{S}^z_{m,i+1}\right]$$

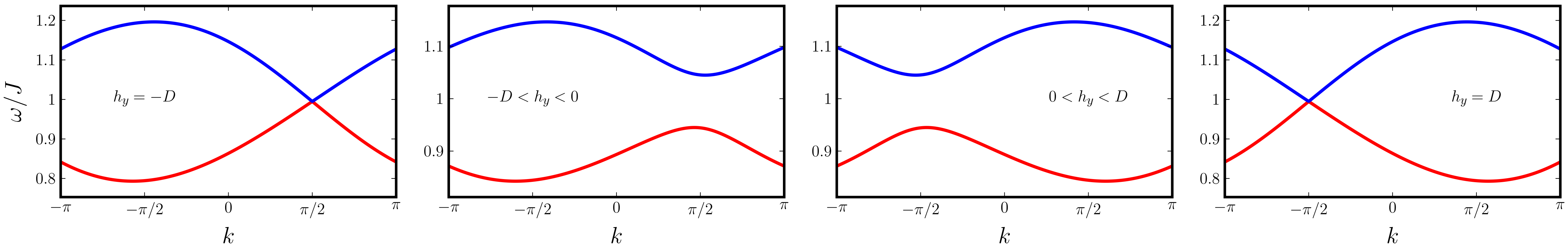
- dominant $J > 0 \rightarrow$ ground state: dimerized quantum antiferromagnet
- three gapped excitations (i.e., triplons): spin-1 triplet states on each dimer

$$|t_x\rangle = -\frac{|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle}{\sqrt{2}} \qquad |t_y\rangle = i\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}} \qquad |t_z\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

D. Joshi and A. P. Schnyder, Phys. Rev. B **96**, 220405 (2017)

topological phase transition

- bulk t_x and t_z triplon bands: topological quantum phase transition at $h_y = D$



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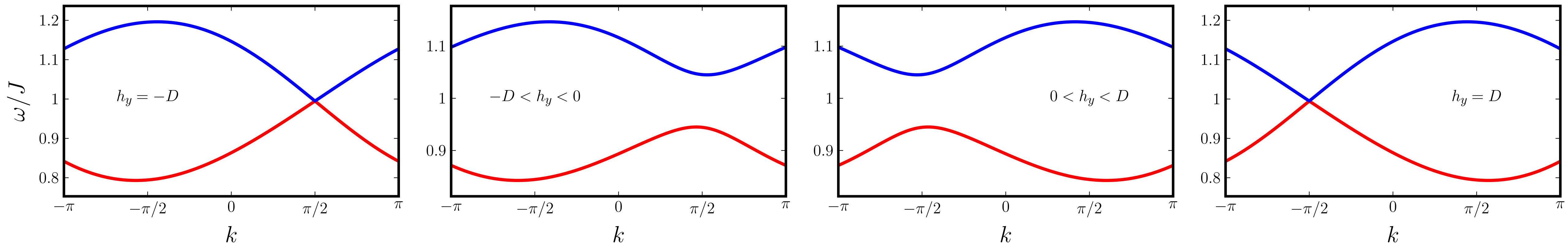
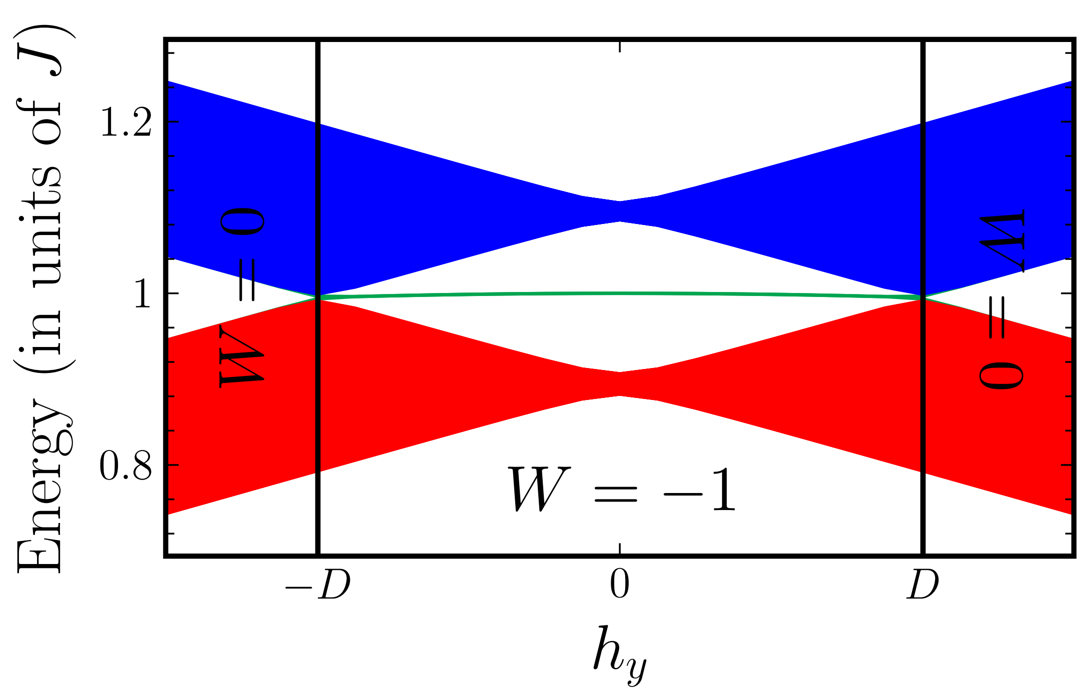
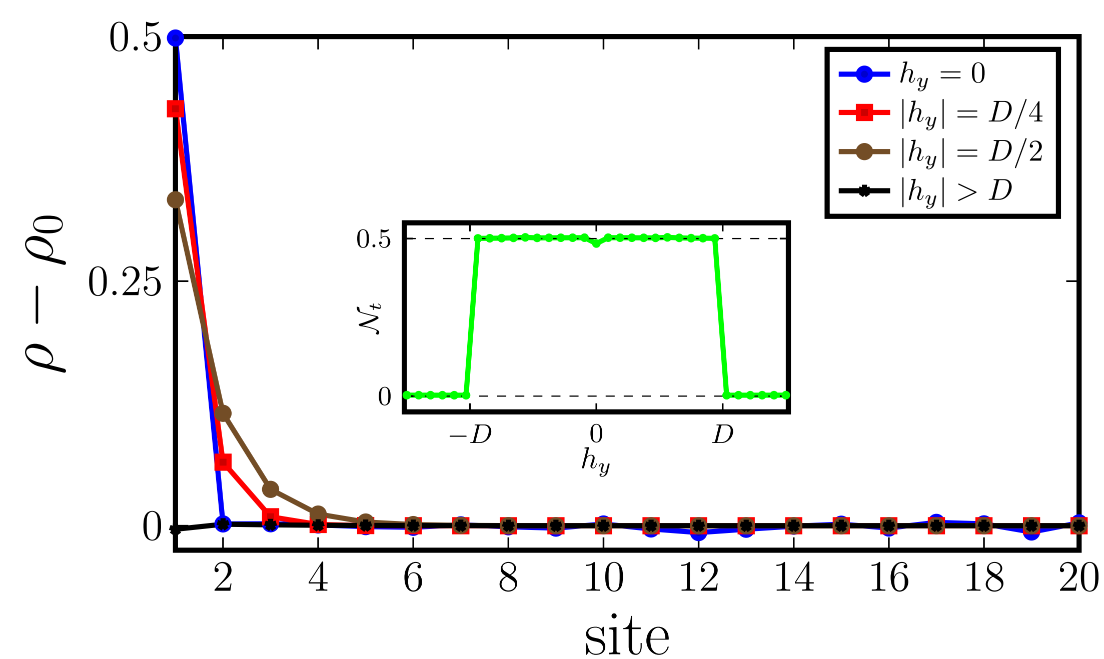


- t_x and t_z triplon bands with open ends.
- mid-gap states (energy J) are exponentially localized at the two ends of the ladder and have fractional particle number (i.e., $S = 1/2$)

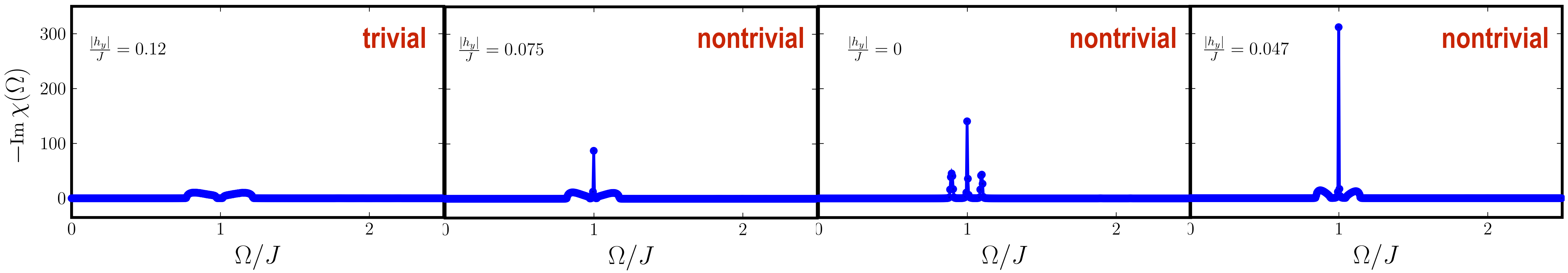
$$D/J = 0.1, \quad K/J = 0.01$$





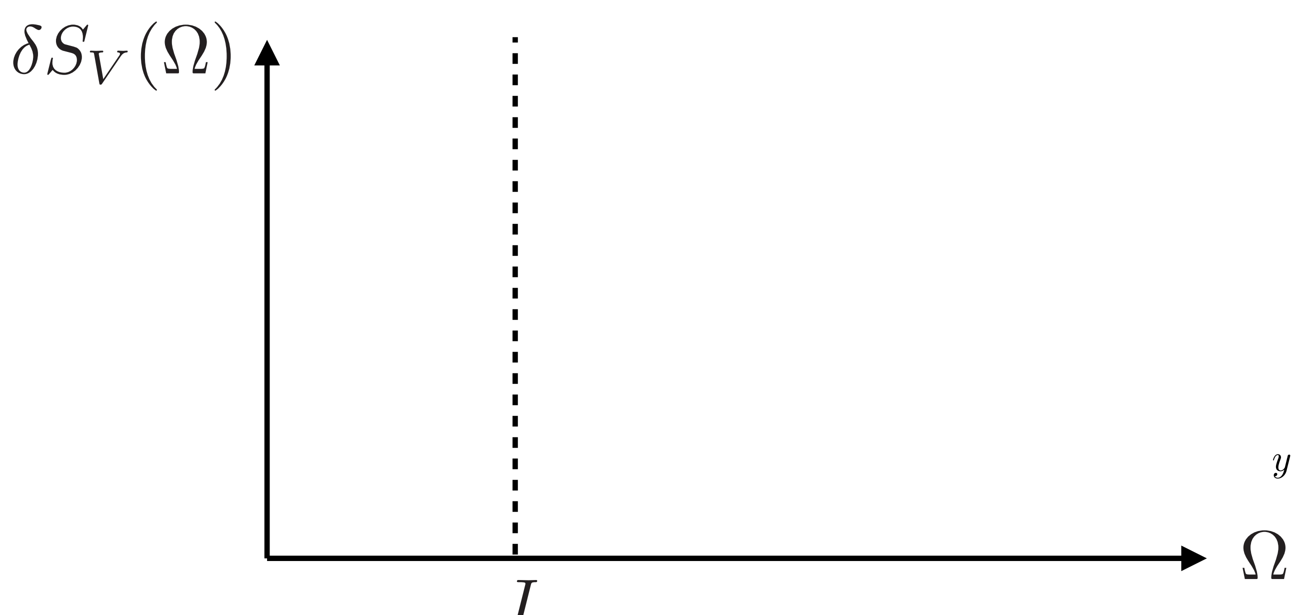
D. Joshi and A. P. Schnyder, Phys. Rev. B **96**, 220405 (2017)

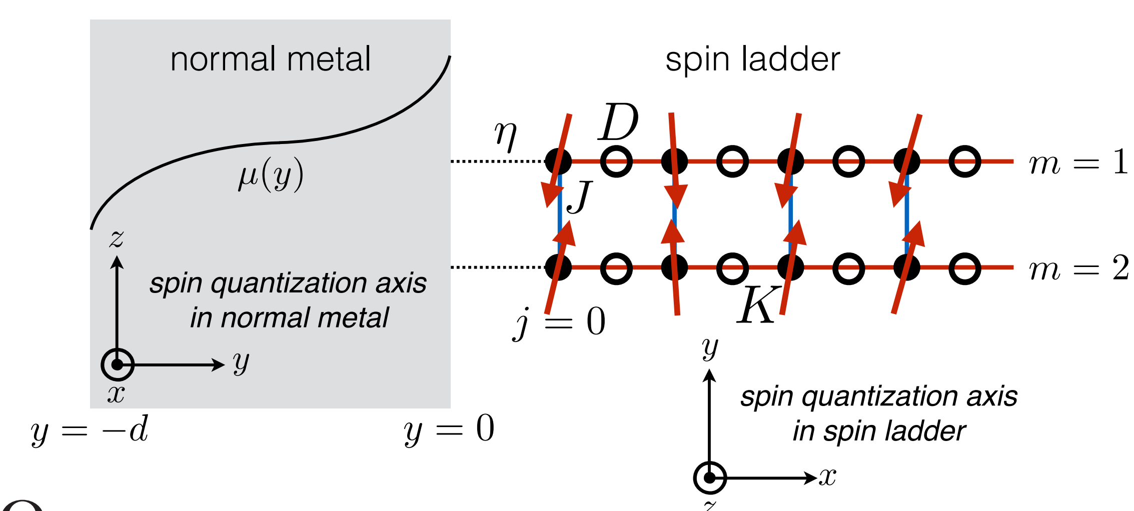
- sharp features in the (imaginary part of the) dynamical spin structure factor emerge in the topological phase at mid-gap energy $\hbar\Omega = J$.



- voltage noise correction at low temperatures:

$$S_s(\Omega, T \approx 0) \approx 2i \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2\hbar}\right)^2 \sum_{\boldsymbol{j}} \int_{-\infty}^{\infty} d\nu (\Omega - \nu) \left[\chi_{\boldsymbol{jj}}^{+-}(\nu) + \chi_{\boldsymbol{jj}}^{-+}(\nu)\right] \theta(\Omega - \nu)$$





D. Joshi, A. P. Schnyder and S. Takei, Phys. Rev. B **98**, 064401 (2018)

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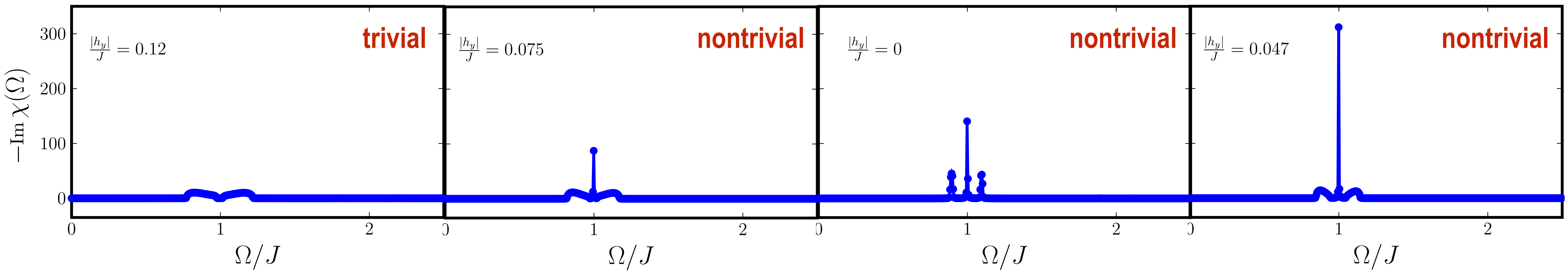

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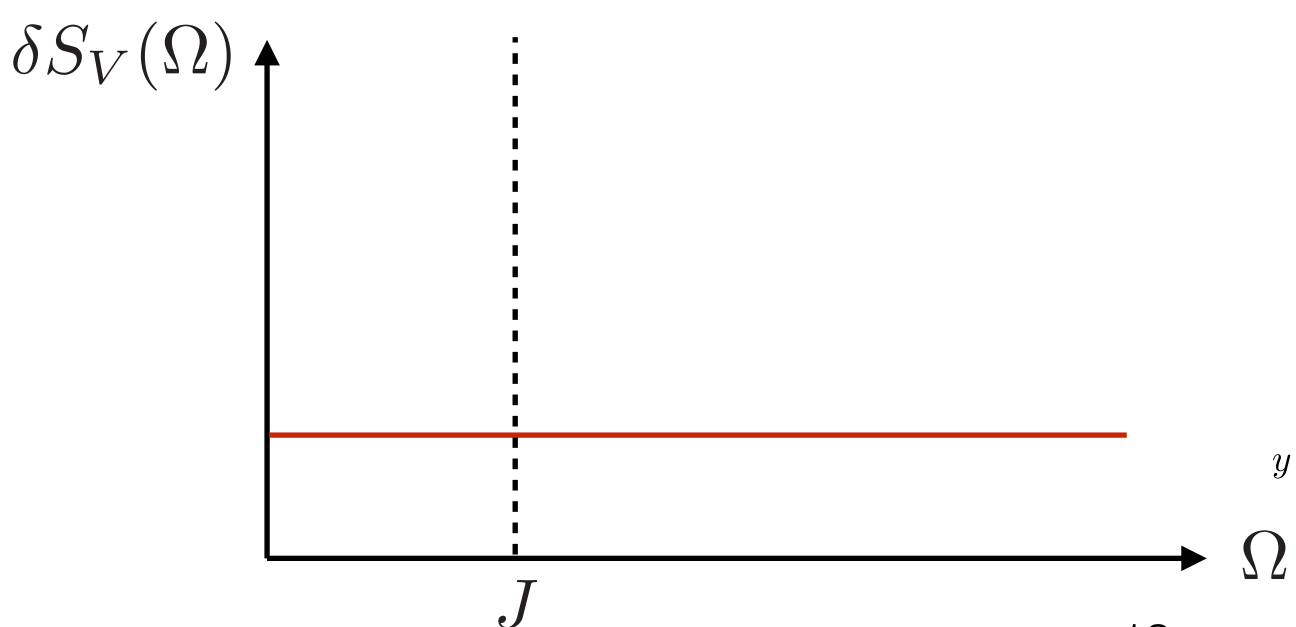

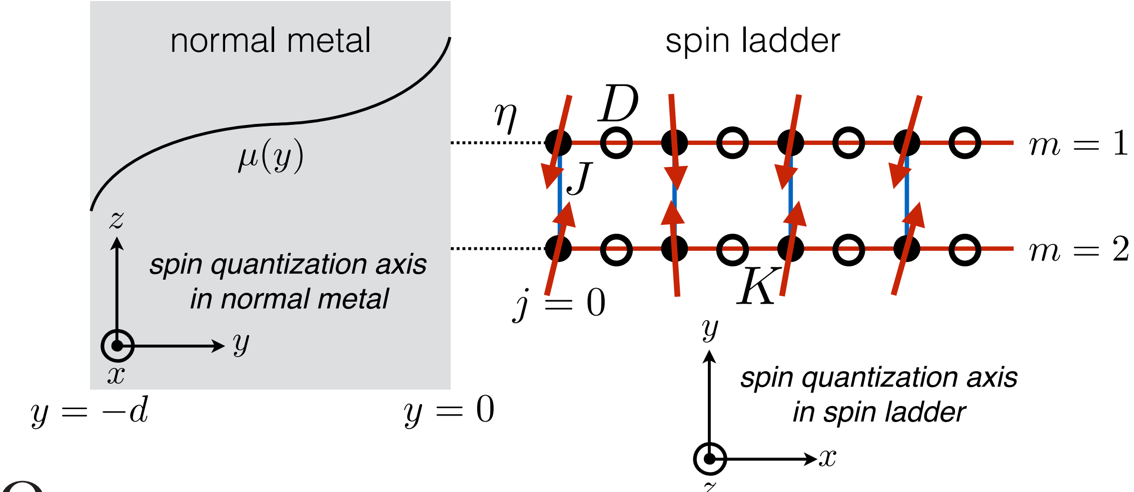

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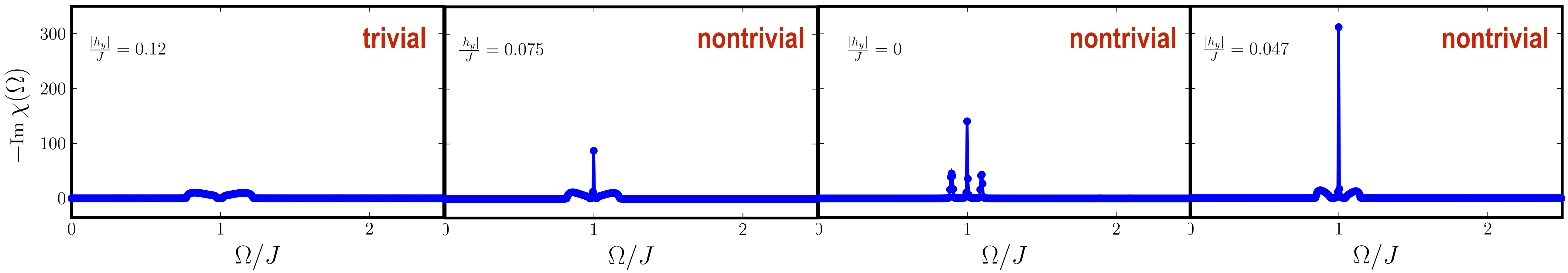

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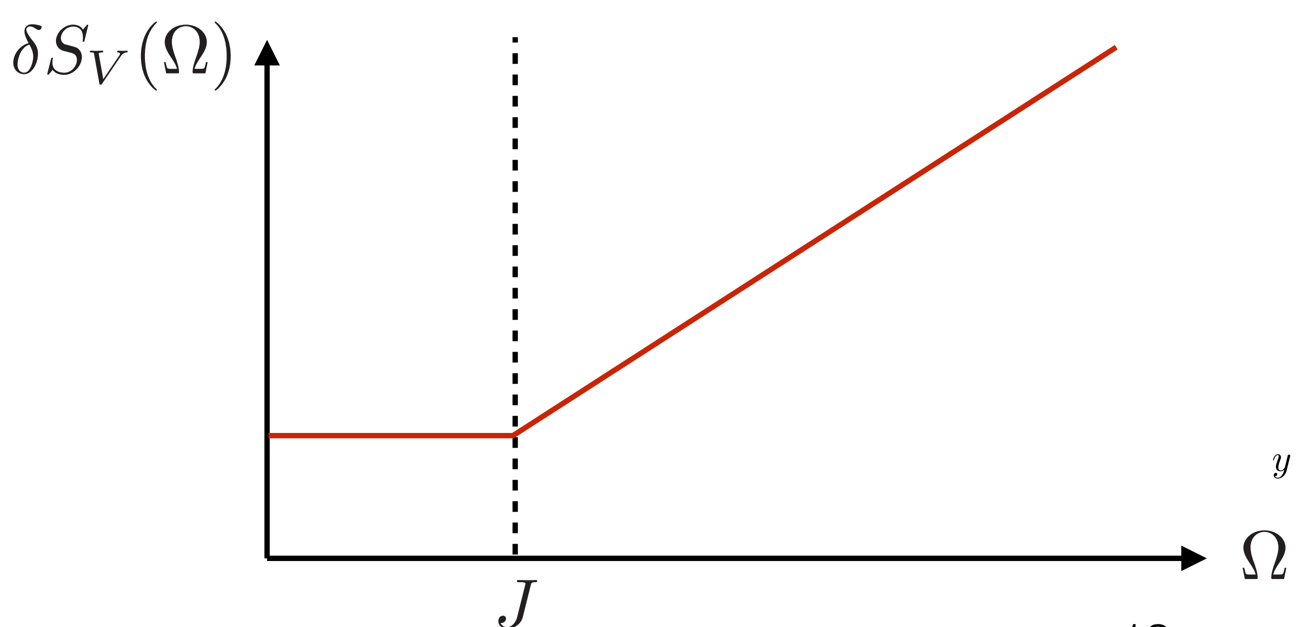
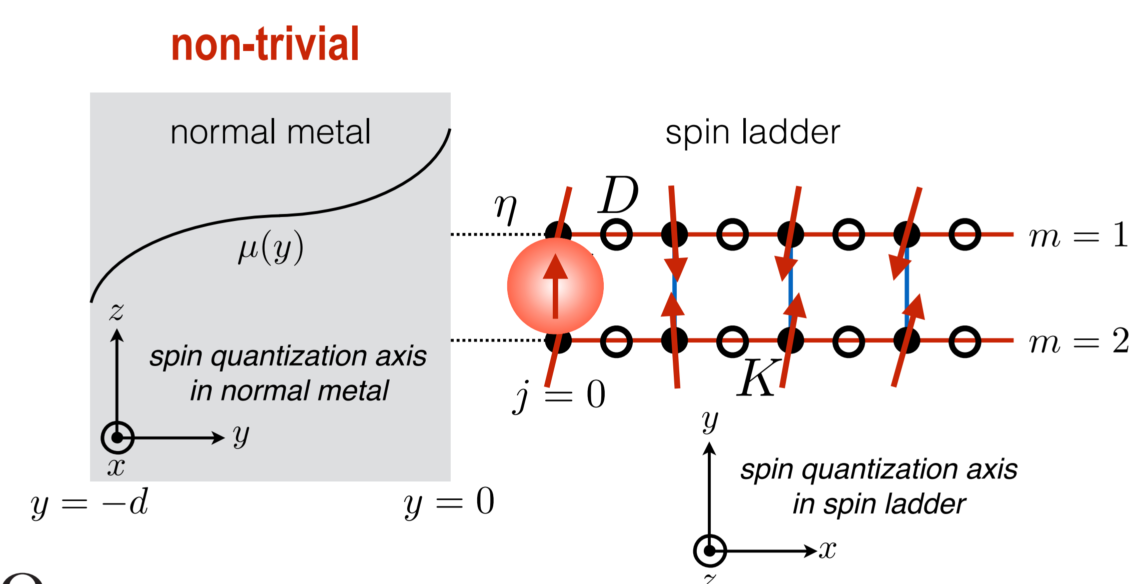
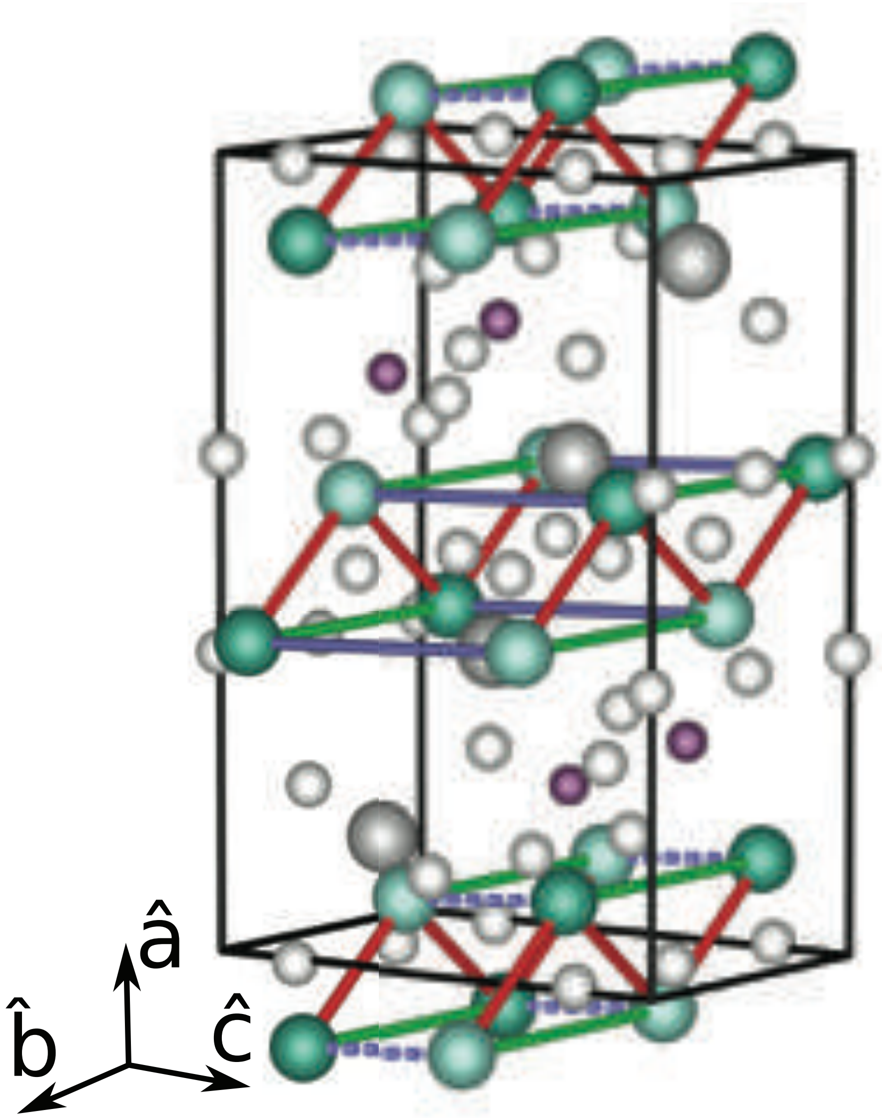




D. Joshi, A. P. Schnyder and S. Takei, Phys. Rev. B **98**, 064401 (2018)

candidate material

- e.g., $BiCu_2PO_6$

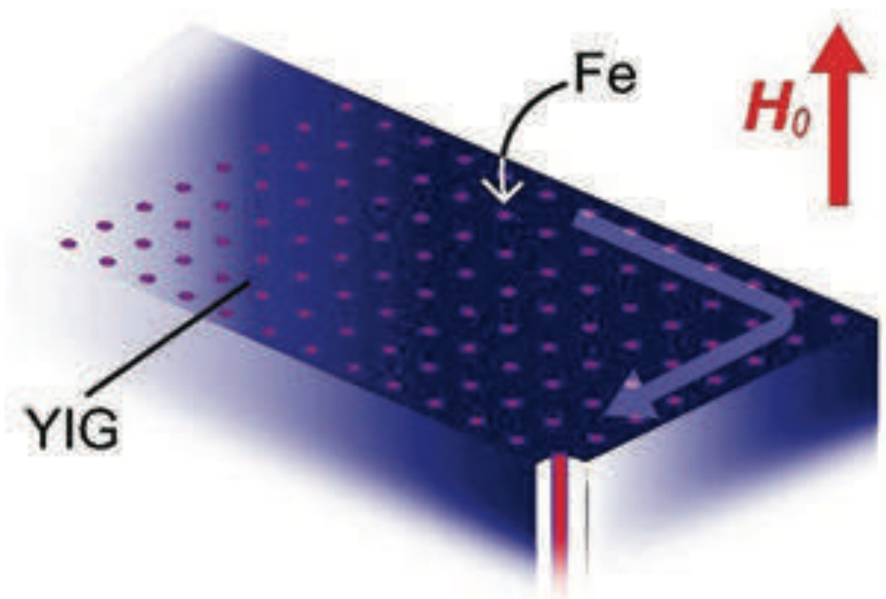


A. A. Tsirlin *et al.*, Phys. Rev. B **82**, 144426 (2010)
S. Wang *et al.*, J. Cryst. Growth **313**, 51 (2010)
Y. Kohama *et al.*, Phys. Rev. Lett. **109**, 167204 (2012)
K. W. Plumb *et al.*, Phys. Rev. B **88**, 024402 (2013)

- mid-gap energy scale ~100 GHz
- high-frequency noise spectral analyzer ~ few hundred GHz

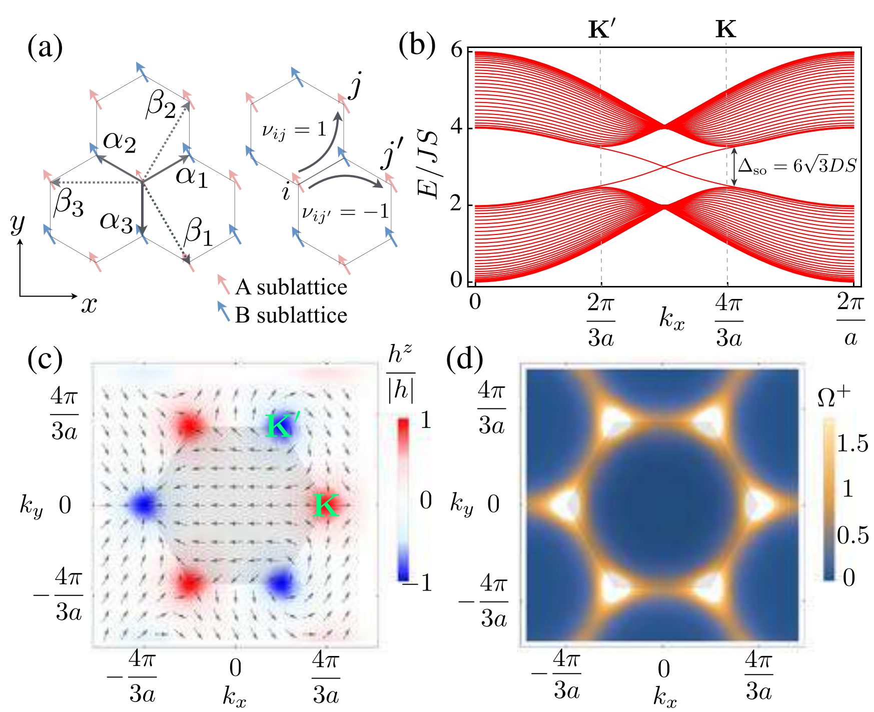
R. H. Koch *et al.*, Phys. Rev. Lett. **96**, 176601 (2006)

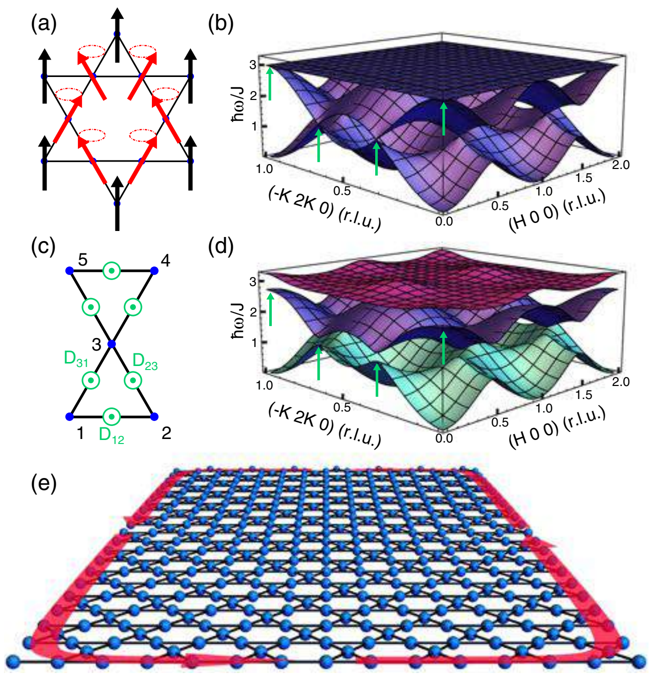
- SHNS is particularly sensitive to changes in local tunneling density of states at sample boundaries → useful probe of various topological edge-states in quantum magnets



Topological magnon bands in a magnonic crystal

Topological magnon bands in the honeycomb ferromagnet





Topological magnon bands in Kagome lattice ferromagnet

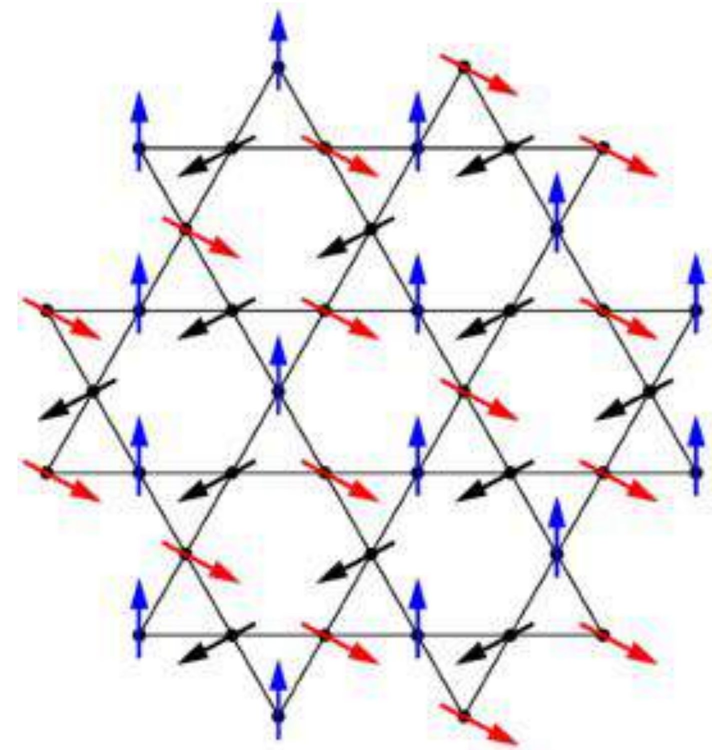
R. Shindou *et al.*, Phys. Rev. B **87**, 174427 (2013)
L. Zhang *et al*., Phys. Rev. B **87**, 144101 (2013)
R. Chisnell *et al*., Phys. Rev. Lett. **115**, 147201 (2015)
F.-Y. Li *et al*., Nature Comm. **7**, 12691 (2016)
S. K. Kim *et al*., Phys. Rev. Lett. **117**, 227201 (2016)
P. A. McClarty *et al*., Nature Phys. **13**, 736 (2017)

application to quantum spin liquids

kagomé quantum antiferromagnet

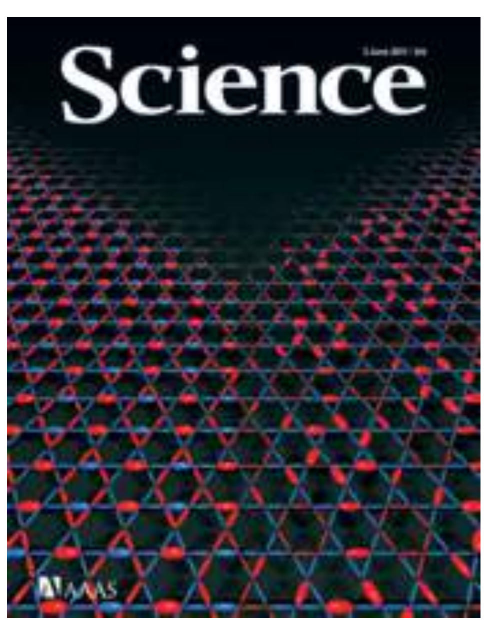
- nearest-neighbor $S = 1/2$ Heisenberg antiferromagnet on the kagomé lattice.

$$H = J \sum_{\langle jj' \rangle} \boldsymbol{S}_j \cdot \boldsymbol{S}_{j'}$$



- DMRG studies suggest a quantum spin liquid state with fully gapped excitations.

S. Yan *et al.*, Science **332** 1173 (2011)



Spin-Liquid Ground State of the $S = 1/2$ Kagome Heisenberg Antiferromagnet

Simeng Yan,[1] David A. Huse,[2,3] Steven R. White[1]*

We use the density matrix renormalization group to perform accurate calculations of the ground state of the nearest-neighbor quantum spin $S = 1/2$ Heisenberg antiferromagnet on the kagome lattice. We study this model on numerous long cylinders with circumferences up to 12 lattice spacings. Through a combination of very-low-energy and small finite-size effects, our results provide strong evidence that, for the infinite two-dimensional system, the ground state of this model is a fully gapped spin liquid.

Z_2 gapped quantum spin liquid

- analytical description via Schwinger-boson spin representation:

$$\boldsymbol{S_j} = \tfrac{1}{2} b^\dagger_{\boldsymbol{j}\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}\sigma'} \quad \Rightarrow \quad H = \tfrac{J}{4} \sum_{\langle \boldsymbol{j}\boldsymbol{j}'\rangle} (b^\dagger_{\boldsymbol{j}\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}\sigma'}) \cdot (b^\dagger_{\boldsymbol{j}'\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}'\sigma'})$$

$$\sum_{\sigma=\uparrow,\downarrow} b^\dagger_{\boldsymbol{j}\sigma} b_{\boldsymbol{j}\sigma} = 1$$

N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991)
S. Sachdev, Phys. Rev. B **45** 12377 (1992)

- mean-field theory for Z_2 QSL, i.e., $\langle b_{\boldsymbol{j}\sigma} \rangle = 0$ (justified via a large-N model):

$$H_{\mathrm{MF}} = -\frac{JQ}{2} \sum_{\langle \boldsymbol{j},\boldsymbol{j}'\rangle} \sum_{\sigma\sigma'} \varepsilon_{\sigma\sigma'} b^\dagger_{\boldsymbol{j}\sigma} b^\dagger_{\boldsymbol{j}'\sigma'} + h.c. + \lambda \sum_{\boldsymbol{j},\sigma} b^\dagger_{\boldsymbol{j}\sigma} b_{\boldsymbol{j}\sigma}$$

Z_2 gapped quantum spin liquid

- analytical description via Schwinger-boson spin representation:

$$\boldsymbol{S_j} = \frac{1}{2} b^{\dagger}_{\boldsymbol{j}\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}\sigma'} \quad \Rightarrow \quad H = \frac{J}{4} \sum_{\langle \boldsymbol{j}\boldsymbol{j}' \rangle} (b^{\dagger}_{\boldsymbol{j}\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}\sigma'}) \cdot (b^{\dagger}_{\boldsymbol{j}'\sigma} \boldsymbol{\tau}_{\sigma\sigma'} b_{\boldsymbol{j}'\sigma'})$$

$$\sum_{\sigma=\uparrow,\downarrow} b^{\dagger}_{\boldsymbol{j}\sigma} b_{\boldsymbol{j}\sigma} = 1$$

N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991)
S. Sachdev, Phys. Rev. B **45** 12377 (1992)

- mean-field theory for Z_2 QSL, i.e., $\langle b_{j\sigma} \rangle = 0$ (justified via a large-N model):

$$H_{\mathrm{MF}} = -\frac{JQ}{2} \sum_{\langle \boldsymbol{j},\boldsymbol{j}' \rangle} \sum_{\sigma\sigma'} \varepsilon_{\sigma\sigma'} b^{\dagger}_{\boldsymbol{j}\sigma} b^{\dagger}_{\boldsymbol{j}'\sigma'} + h.c. + \lambda \sum_{\boldsymbol{j},\sigma} b^{\dagger}_{\boldsymbol{j}\sigma} b_{\boldsymbol{j}\sigma}$$

- Z_2 model gives dynamical spin structure predictions in good agreement with inelastic neutron scattering measurements.

T.-H. Han *et al.*, Nature **492**, 406 (2012)
M. Punk *et al.*, Nature Phys. **10**, 289 (2016)

LETTER

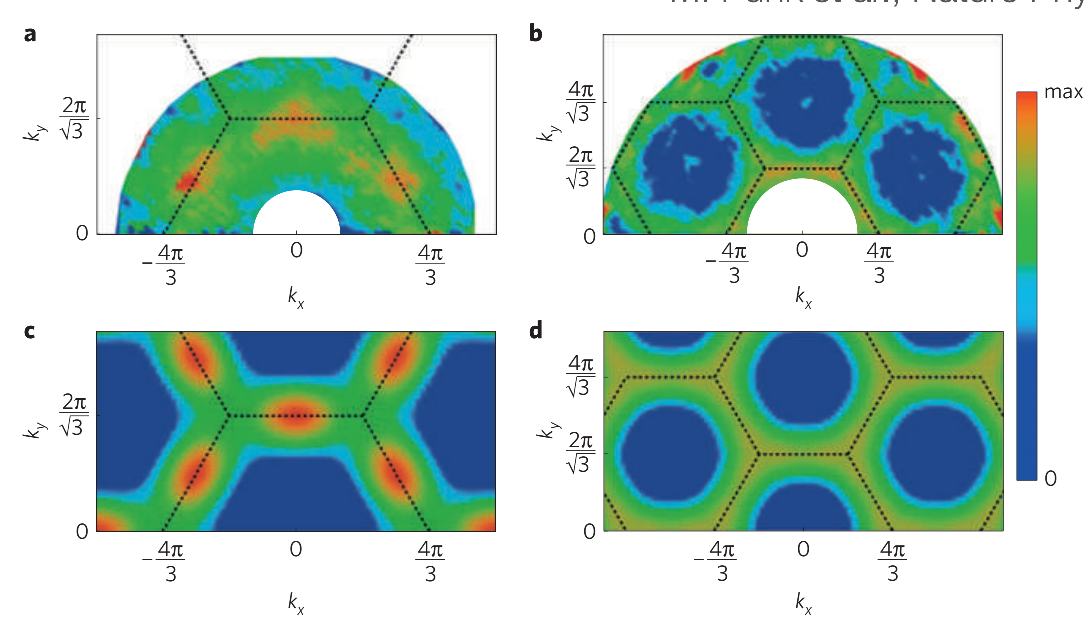
doi:10.1038/nature11659

Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han[1], Joel S. Helton[2], Shaoyan Chu[1], Daniel G. Nocera[4], Jose A. Rodriguez-Rivera[2,5], Collin Broholm[2,6] & Young S. Lee[1]

Topological excitations and the dynamic structure factor of spin liquids on the kagome lattice

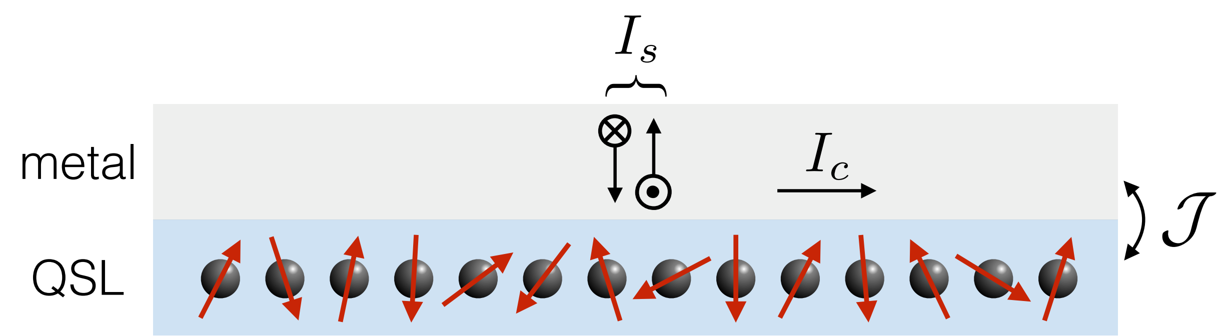
Matthias Punk[1,2], Debanjan Chowdhury[1] and Subir Sachdev[1]*



use mean-field parameters Q and λ obtained by Punk *et al.*

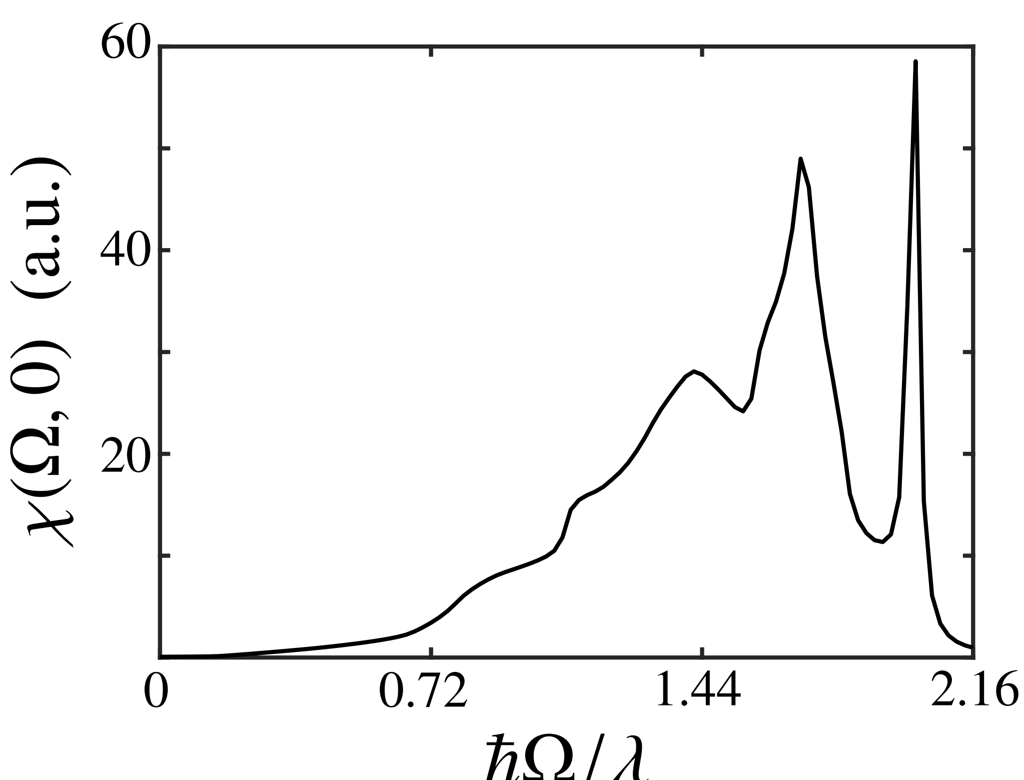
- correction to zero-temperature AC voltage noise across adjacent metal:

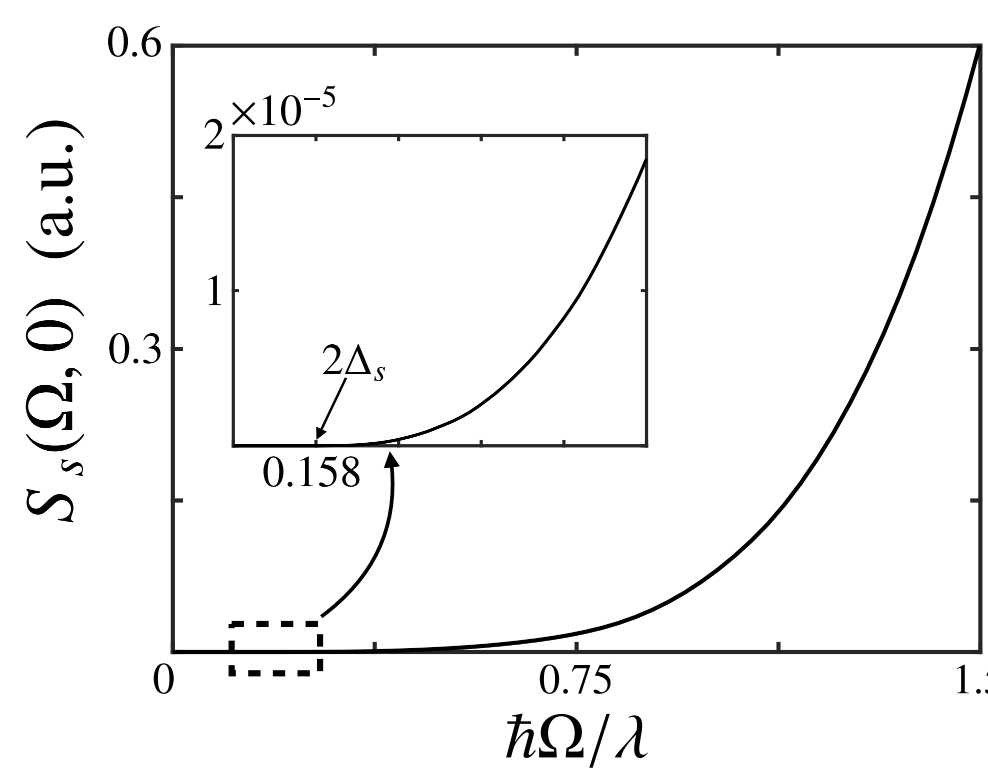
$$\delta S_V(\Omega) = \Theta S_s(\Omega)$$



$$S_s(\Omega, T) = 2i\left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2\hbar}\right)^2 \sum_{\boldsymbol{j}} \int_{-\infty}^{\infty} d\nu \frac{\nu - \Omega}{e^{\beta(\nu-\Omega)} - 1}\underbrace{\left[\chi_{\boldsymbol{jj}}^{+-}(\nu) + \chi_{\boldsymbol{jj}}^{-+}(\nu)\right]}_{\chi(\nu,0)}$$

$$\chi_{\boldsymbol{ij}}^{\mp\pm}(\nu) \equiv -\iota \int dt\, \langle S_{\boldsymbol{i}}^{\mp}(t) S_{\boldsymbol{j}}^{\pm}(0)\rangle_{H_{\mathrm{MF}}} e^{i\nu t}$$





J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

spinon Fermi surface + U(1) gauge field

- spinon metal coupled to gapless U(1) gauge field (photons):
  - slave-rotor representation of the half-filled Hubbard model on the triangular lattice.
  - fluctuations around a mean-field QSL state leads to a model of fermionic spinons with a Fermi surface and coupled to gapless gauge fluctuations.

S.-S. Lee and P. A. Lee, Phys. Rev. Lett. **95**, 036403 (2005)
P. A. Lee and N. Nagaosa, Phys. Rev. B **46**, 5621 (1992)
J. Polchinski, Nuclear Physics B **422**, 617 (1994)

$$S = \sum_\sigma \int dt d\boldsymbol{x} \left\{ \underbrace{\bar{c}_\sigma(t,\boldsymbol{x})(i\hbar\partial_t + \mu)c_\sigma(t,\boldsymbol{x})}_{S_0} - \frac{1}{2m_s}\bar{c}_\sigma(t,\boldsymbol{x}) \underbrace{[-i\hbar\boldsymbol{\nabla} + \boldsymbol{a}(t,\boldsymbol{x})]^2 c_\sigma(t,\boldsymbol{x})}_{S_{\mathrm{int}}} \right\}$$

- maybe relevant to quantum magnets on the triangular lattice.
  - linear-T low temperature specific heat and metal-like thermal conductivity even though electrically insulating, e.g., $EtMe_3Sb[Pd(dmit)_2]_2$

M. Yamashita *et al.*, Science **328**, 1246 (2010)
S. Yamashita *et al.*, Nature Comm. **2**, 275 (2011)

Highly Mobile Gapless Excitations in a Two-Dimensional Candidate Quantum Spin Liquid

Minoru Yamashita,[1*] Norihito Nakata,[1] Yoshinori Senshu,[1] Masaki Nagata,[1] Hiroshi M. Yamamoto,[2,3] Reizo Kato,[2] Takasada Shibauchi,[1] Yuji Matsuda[1*]

ARTICLE

Received 31 Aug 2010 | Accepted 14 Mar 2011 | Published 12 Apr 2011 DOI: 10.1038/ncomms1274

Gapless spin liquid of an organic triangular compound evidenced by thermodynamic measurements

Satoshi Yamashita[1,2], Takashi Yamamoto[1], Yasuhiro Nakazawa[1,3], Masafumi Tamura[4] & Reizo Kato[2]

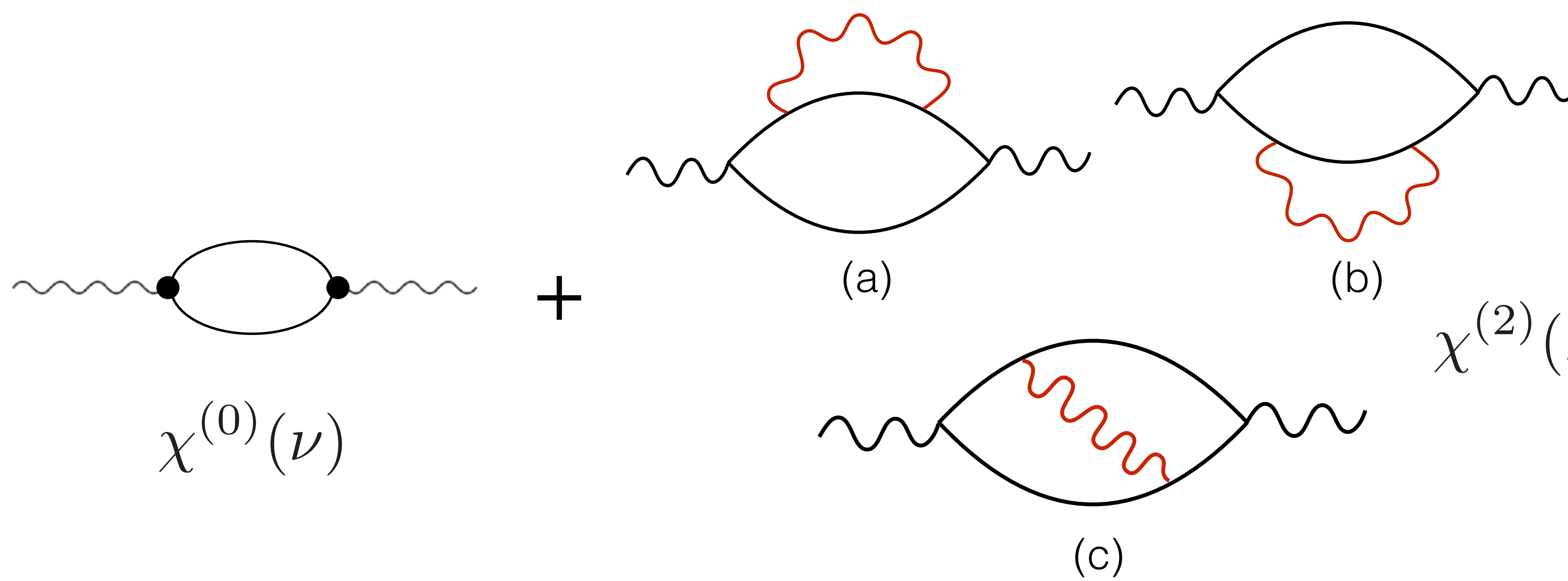
- local dynamic spin structure factor:

$$\chi(\nu) = \sum_{\boldsymbol{j}} \left[\chi^{+-}_{\boldsymbol{jj}}(\nu) + \chi^{-+}_{\boldsymbol{jj}}(\nu)\right]$$

$$= -2i \sum_{\boldsymbol{j}} \int dt e^{i\nu t} \left\langle T_K \bar{c}_{\boldsymbol{j}\downarrow}(t) c_{\boldsymbol{j}\uparrow}(t) \bar{c}_{\boldsymbol{j}\uparrow}(0) c_{\boldsymbol{j}\downarrow}(0) e^{iS_{\text{int}}} \right\rangle$$

- to two-loop order

$\chi(\nu) \sim$



$\chi^{(0)}(\nu)$

$\chi^{(2)}(\nu)$

J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

spinon Fermi surface + U(1) gauge field

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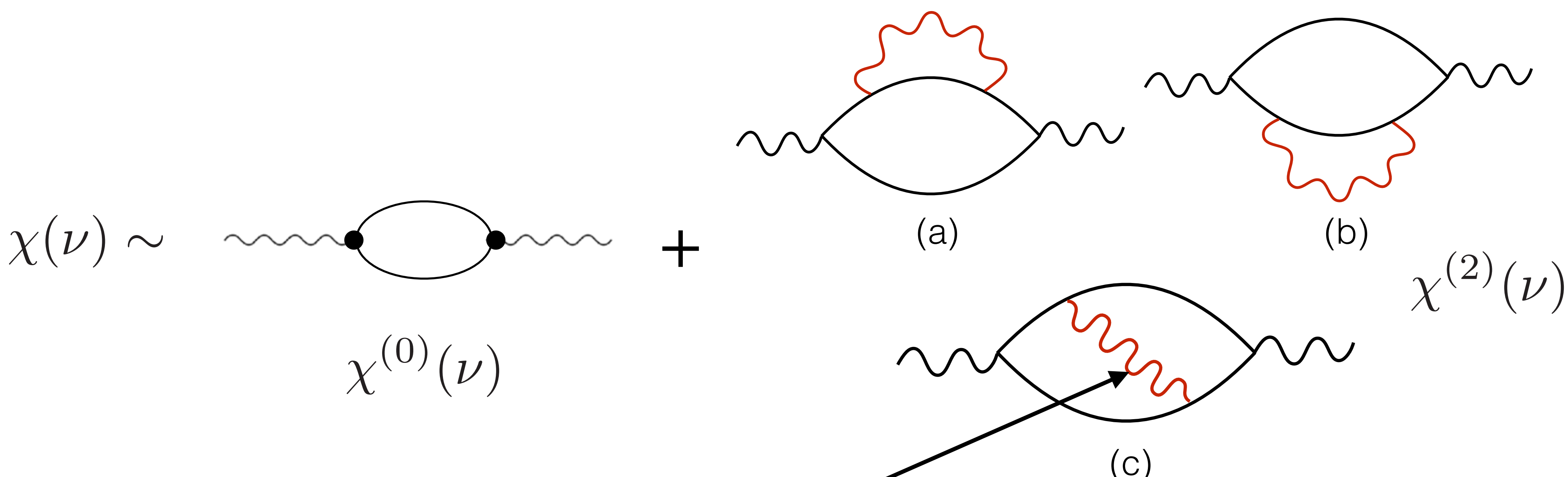
$$\chi(\nu) = \sum_{\boldsymbol{j}} \left[\chi^{+-}_{\boldsymbol{jj}}(\nu) + \chi^{-+}_{\boldsymbol{jj}}(\nu) \right]$$

$$= -2i \sum_{\boldsymbol{j}} \int dt e^{i\nu t} \langle T_K \bar{c}_{\boldsymbol{j}\downarrow}(t) c_{\boldsymbol{j}\uparrow}(t) \bar{c}_{\boldsymbol{j}\uparrow}(0) c_{\boldsymbol{j}\downarrow}(0) e^{iS_{\rm int}} \rangle$$

- to two-loop order



$$D^R(\boldsymbol{q}, \Omega) = -\frac{1}{2} \frac{1}{\chi_D q^2 - i\frac{\Omega}{q}}$$

RPA propagator for gauge fluctuations

J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

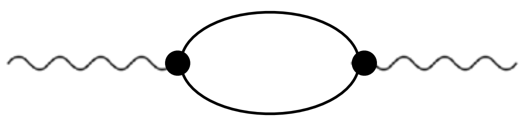
SHNS

- interfacial spin current noise

$$S_s(\Omega, T \approx 0) \approx 2i \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2 \hbar} \right)^2 \int_{-\infty}^{\infty} d\nu (\Omega - \nu) \chi(\nu) \theta(\Omega - \nu)$$

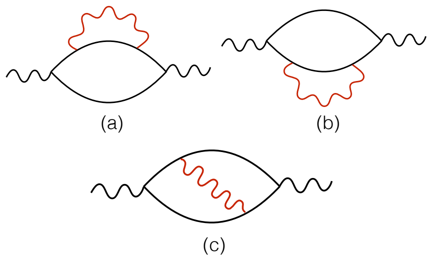
- zeroth-order correction:

$$S_s^{(0)}(\Omega, 0) = \frac{\mathcal{N}}{3\pi} \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2 \hbar} \right)^2 \left(\frac{m_s a_s}{\hbar} \right)^2 \Omega^3$$



- two-loop correction:

$$\delta S_s^{(2)}(\Omega, 0) = 0.157 \mathcal{N} \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2 \hbar} \right)^2 \left(\frac{m_s a_s}{2\pi \hbar} \right)^2 \Omega^3 \left(\frac{\Omega}{\Omega_{Fs}} \right)^{4/3}$$



- total noise correction:

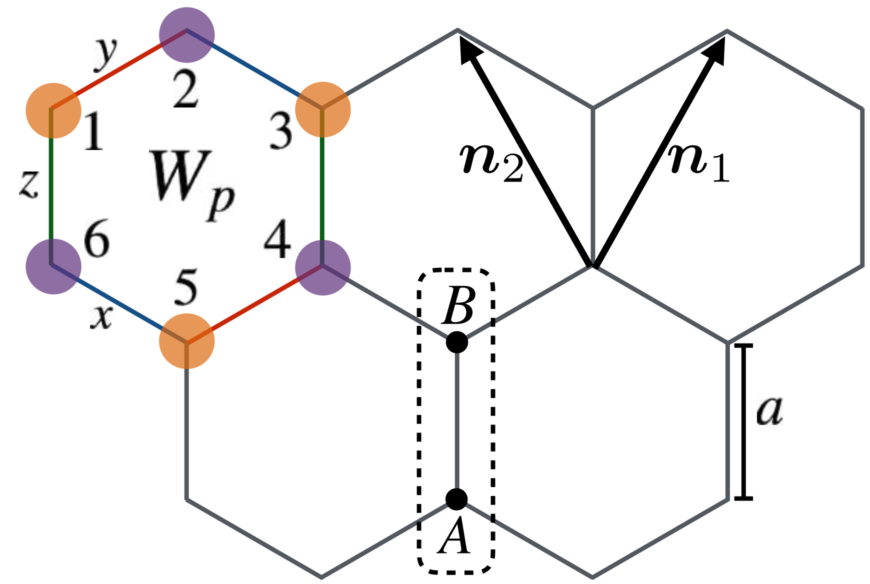
$$S_s(\Omega, 0) = \frac{\mathcal{N}}{3\pi} \left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2 \hbar} \right)^2 \left(\frac{m_s a_s}{\hbar} \right)^2 \Omega^3 \left[1 + 0.037 \left(\frac{\Omega}{\Omega_{Fs}} \right)^{4/3} \right]$$

J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

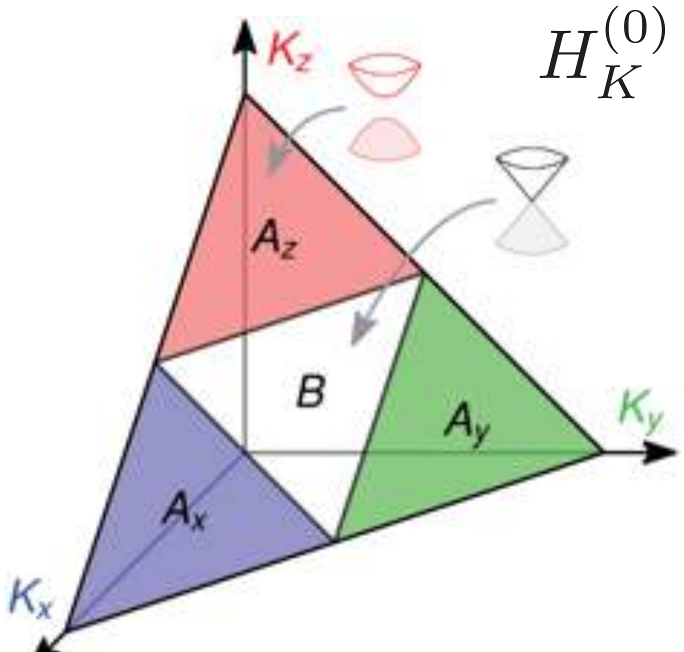
Kitaev honeycomb model

- exactly solvable model of a quantum spin liquid:

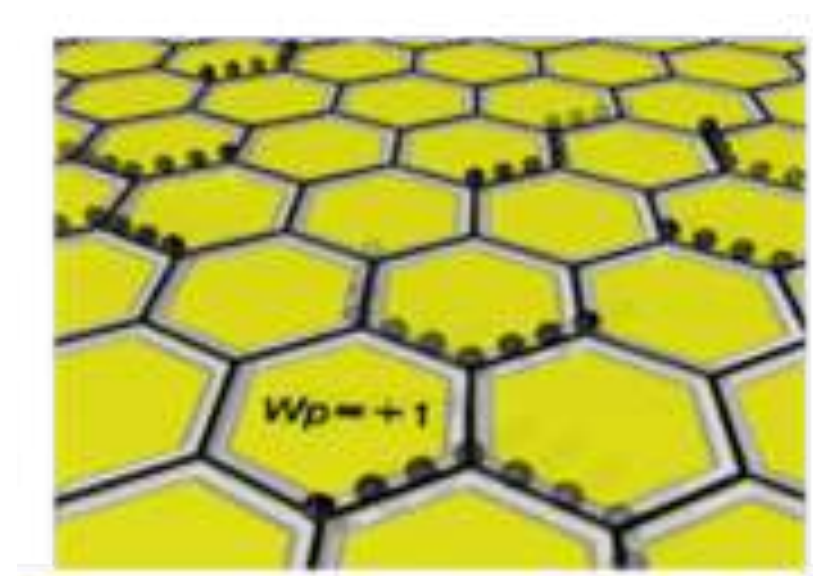
$$H_K = \sum_{\gamma,\langle \boldsymbol{i},\boldsymbol{j}\rangle_\gamma} K_\gamma S_{\boldsymbol{i}}^\gamma S_{\boldsymbol{j}}^\gamma \ ,$$



- gapless quantum spin liquid ground state at the isotropic point $K_x = K_y = K_z \equiv K$: can be mapped to a gas of gapless Majorana-Dirac fermions (spinons) hopping on the honeycomb lattice.



$$H_K^{(0)} = iK \sum_{\langle \boldsymbol{i},\boldsymbol{j}\rangle} c_{\boldsymbol{i}} c_{\boldsymbol{j}}$$

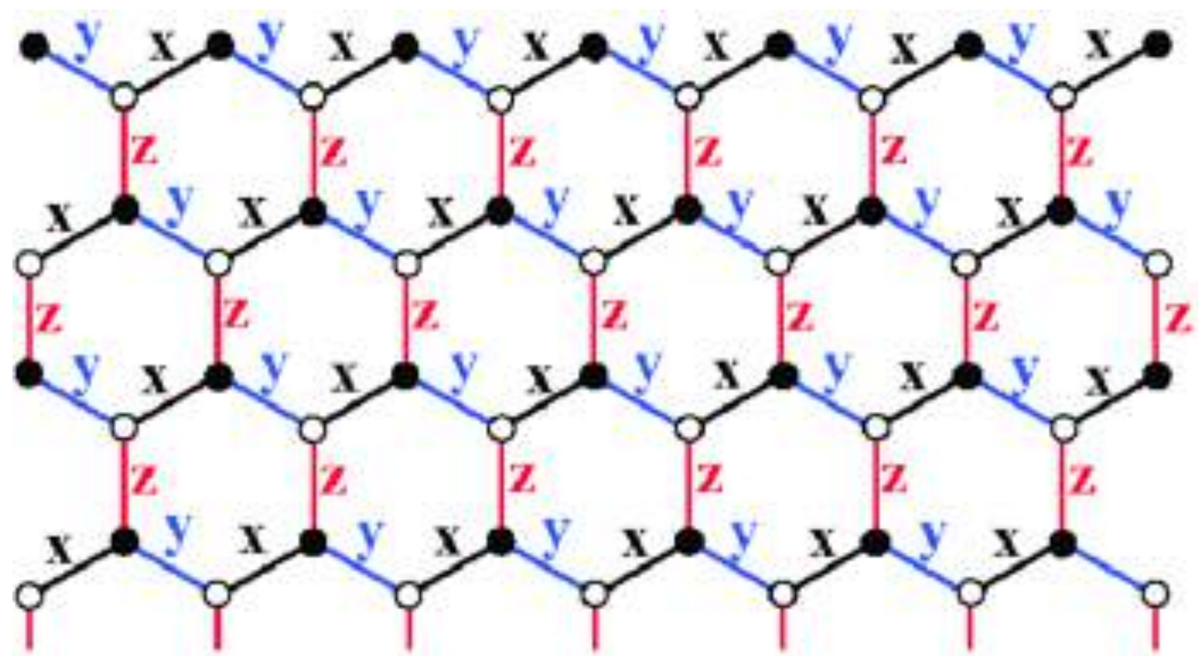


A. Yu. Kitaev, Ann. Phys. (Amsterdam) **321**, 2 (2006)

local dynamic spin structure factor

- dynamic local spin structure factor can be viewed as a quantum quench problem: re-arrangement of Majorana fermion gas following sudden appearance of magnetic fluxes

$$-i\left\langle S^x_{\boldsymbol{j}}(t_2) S^x_{\boldsymbol{j}}(t_1)\right\rangle \quad \Rightarrow$$



G. Baskaran *et al.*, Phys. Rev. Lett. **98,** 247201 (2007)
J. Knolle *et al.*, Phys. Rev. Lett. **112**, 207203 (2014)

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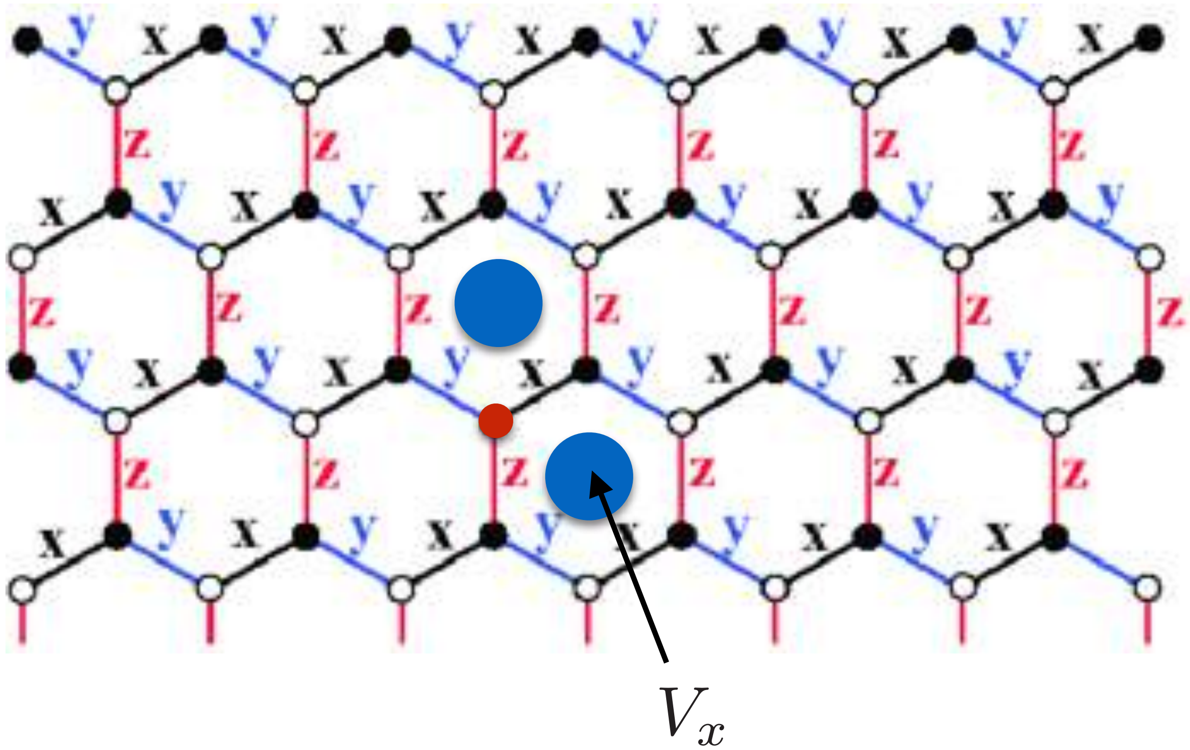
$t = t_1$



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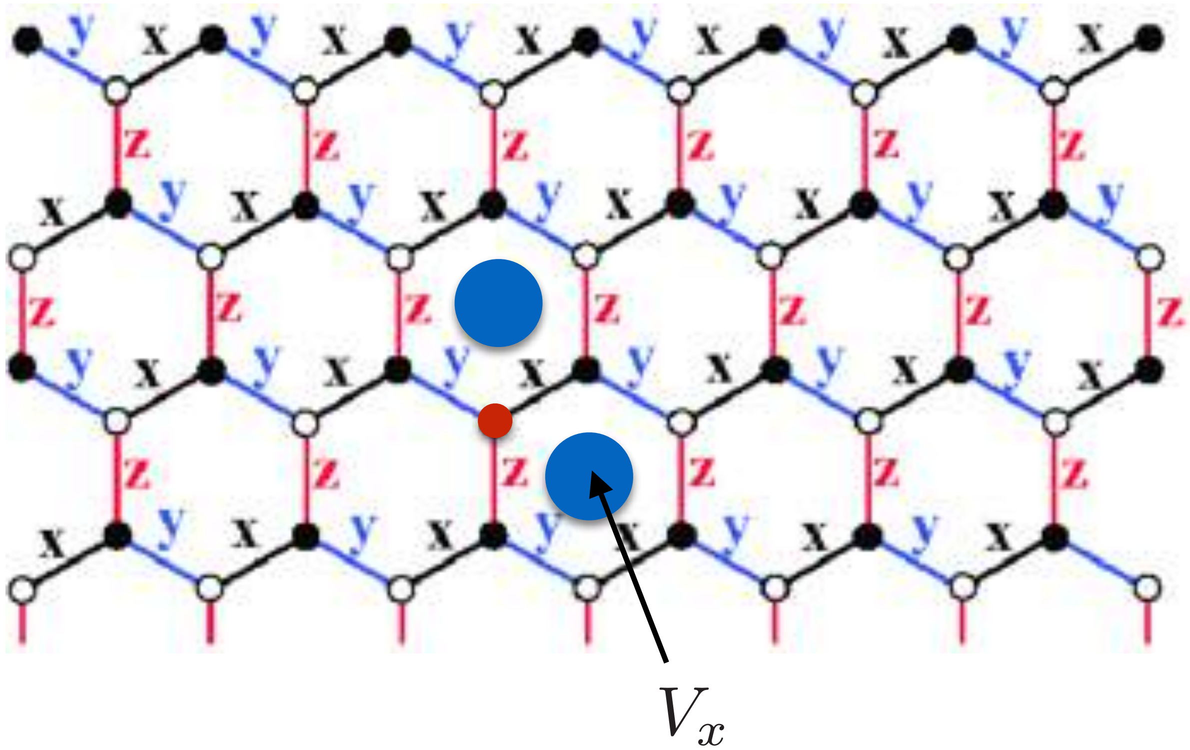
$t_1 < t < t_2$



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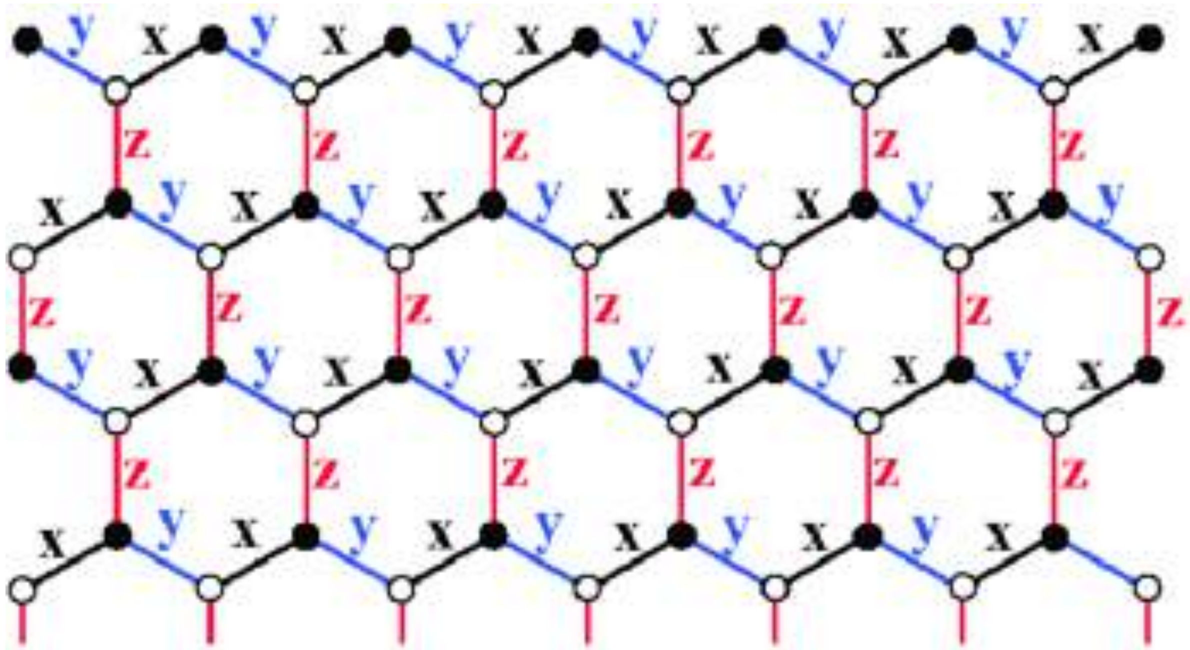

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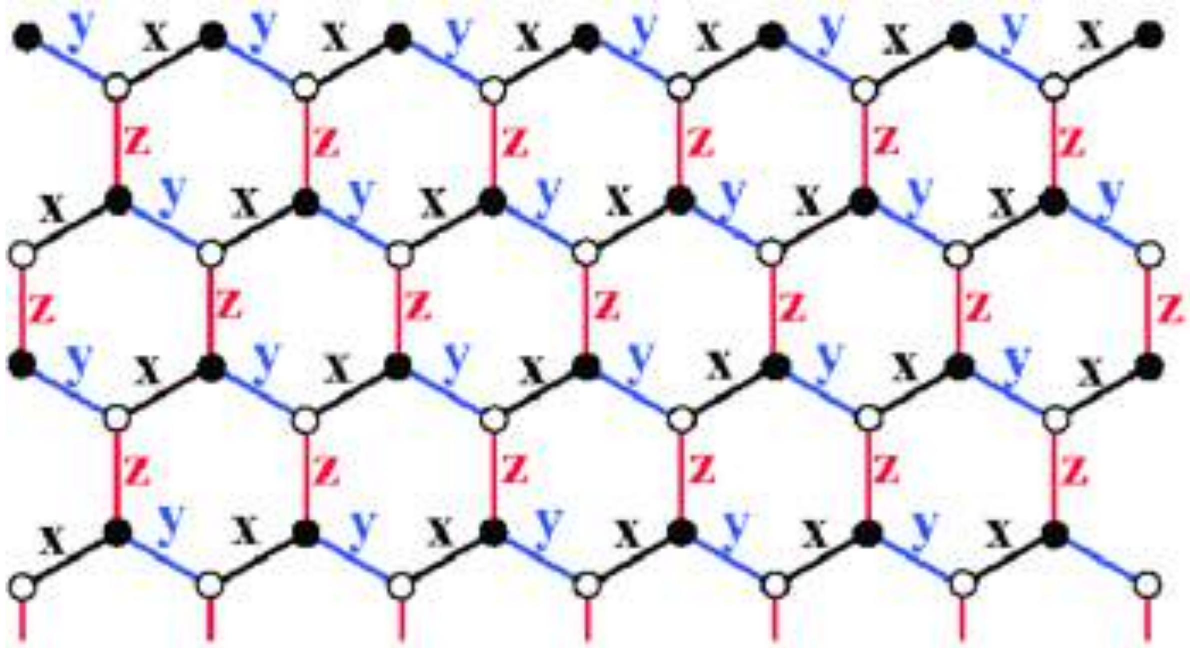

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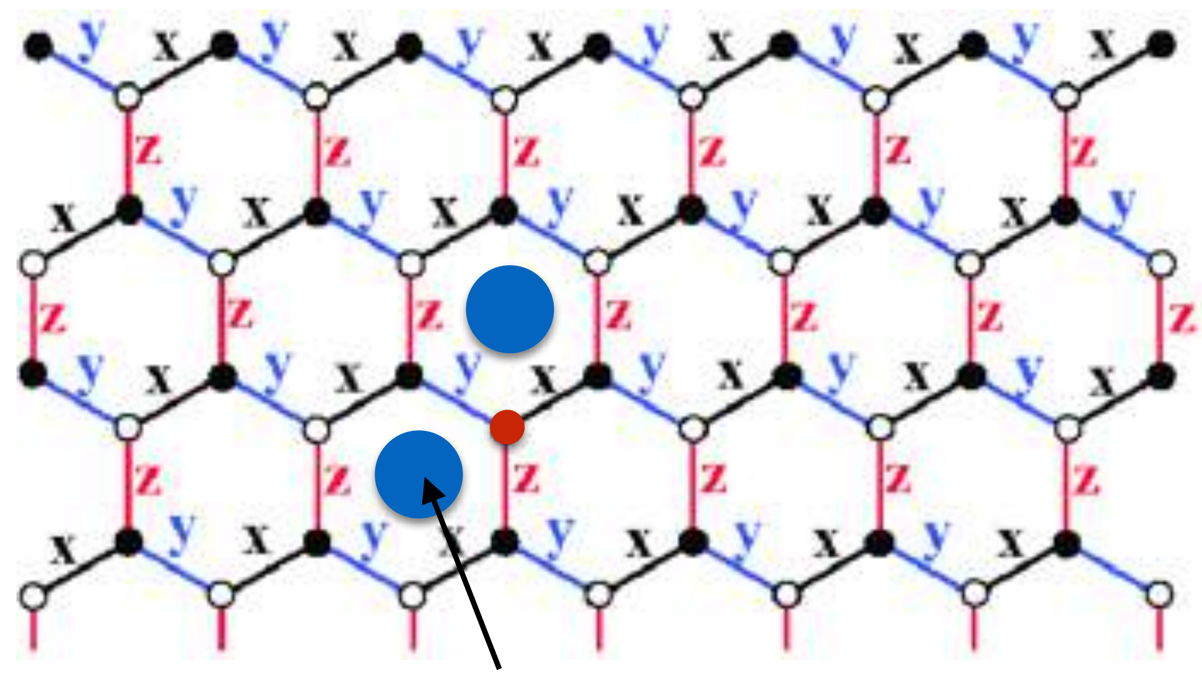

V_y

G. Baskaran *et al.*, Phys. Rev. Lett. **98,** 247201 (2007)
J. Knolle *et al.*, Phys. Rev. Lett. **112**, 207203 (2014)

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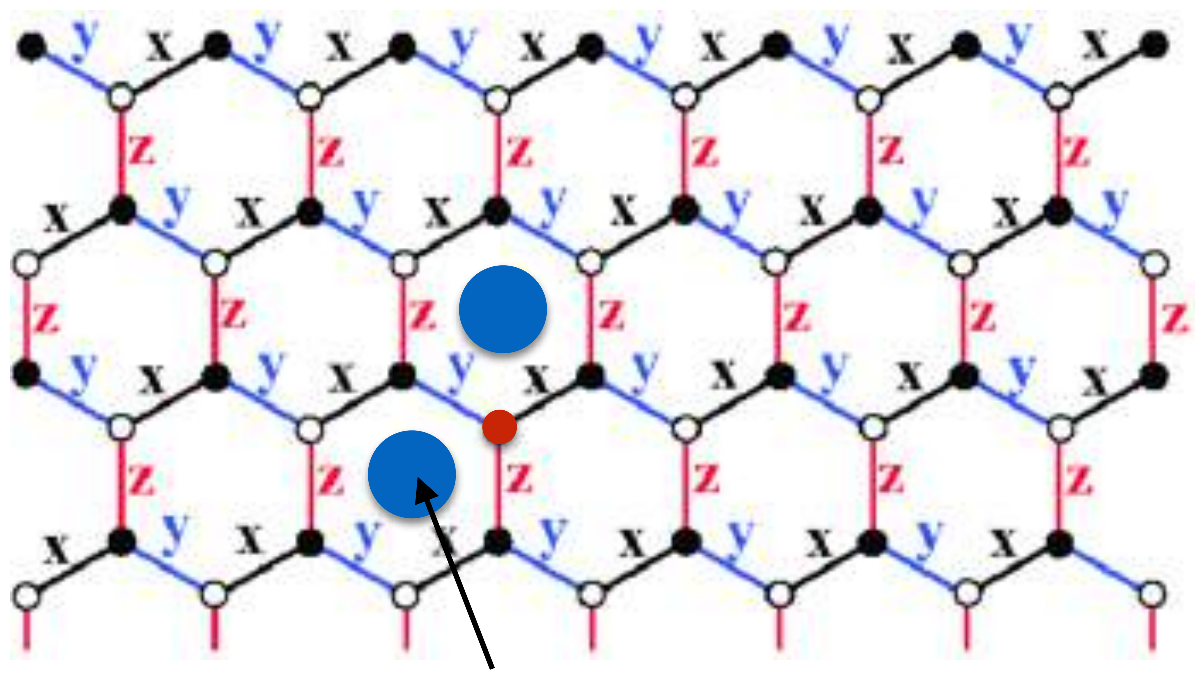
$$-i \left\langle S_{\boldsymbol{j}}^{y}(t_2) S_{\boldsymbol{j}}^{y}(t_1) \right\rangle \quad \Rightarrow$$



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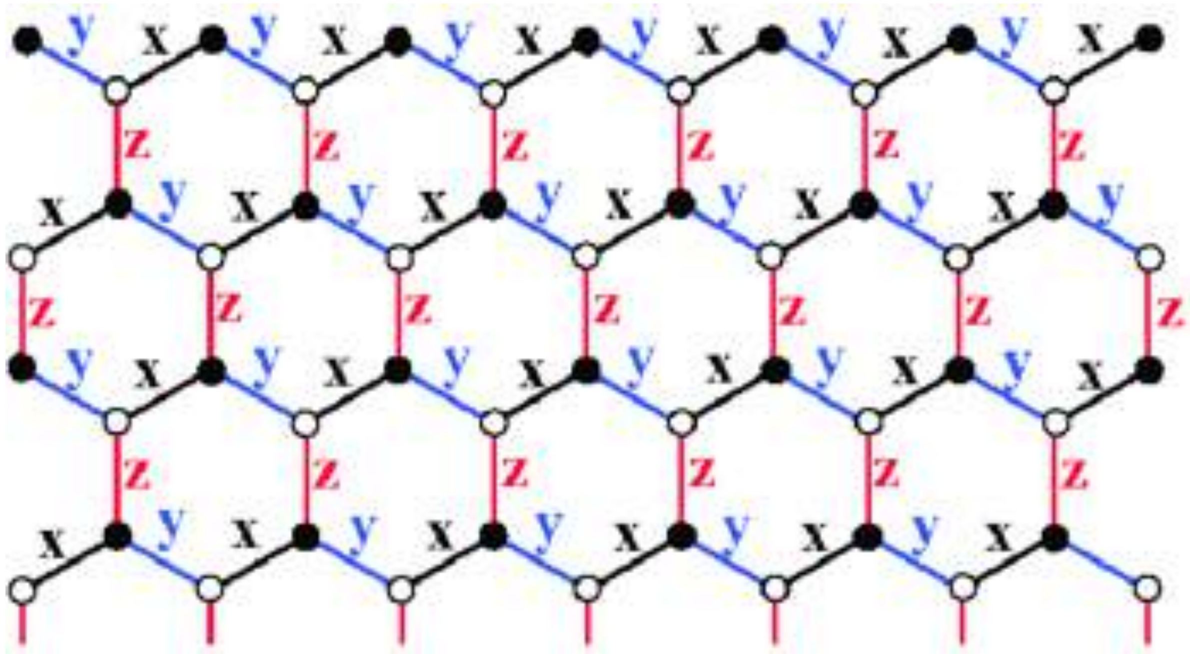

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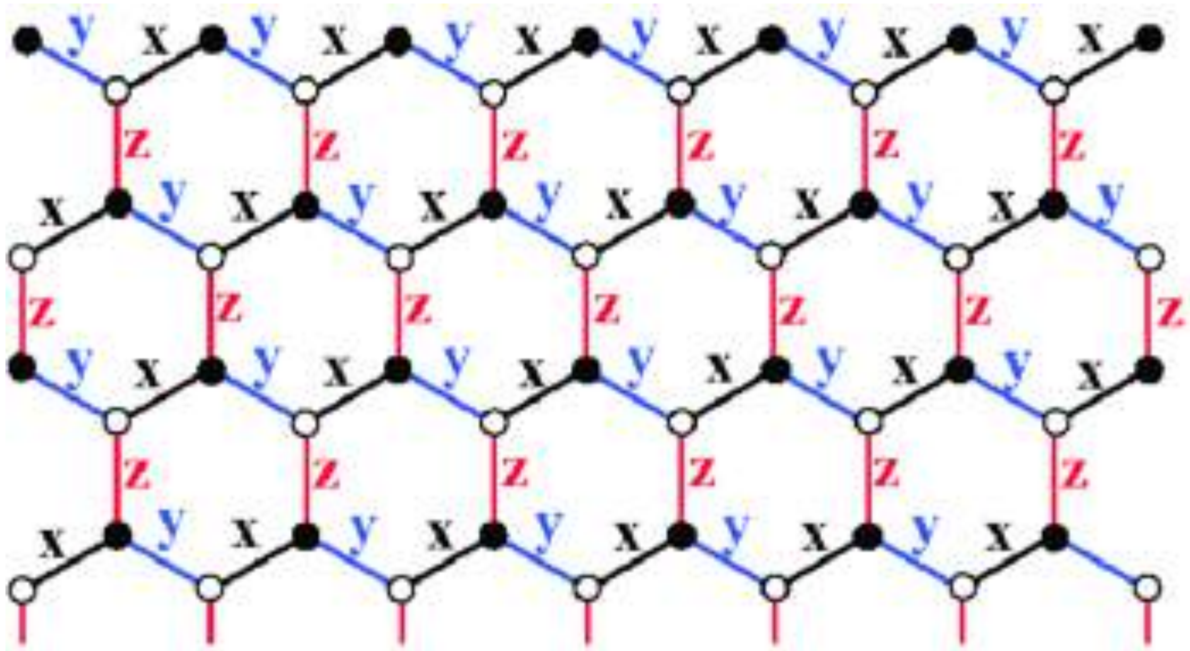

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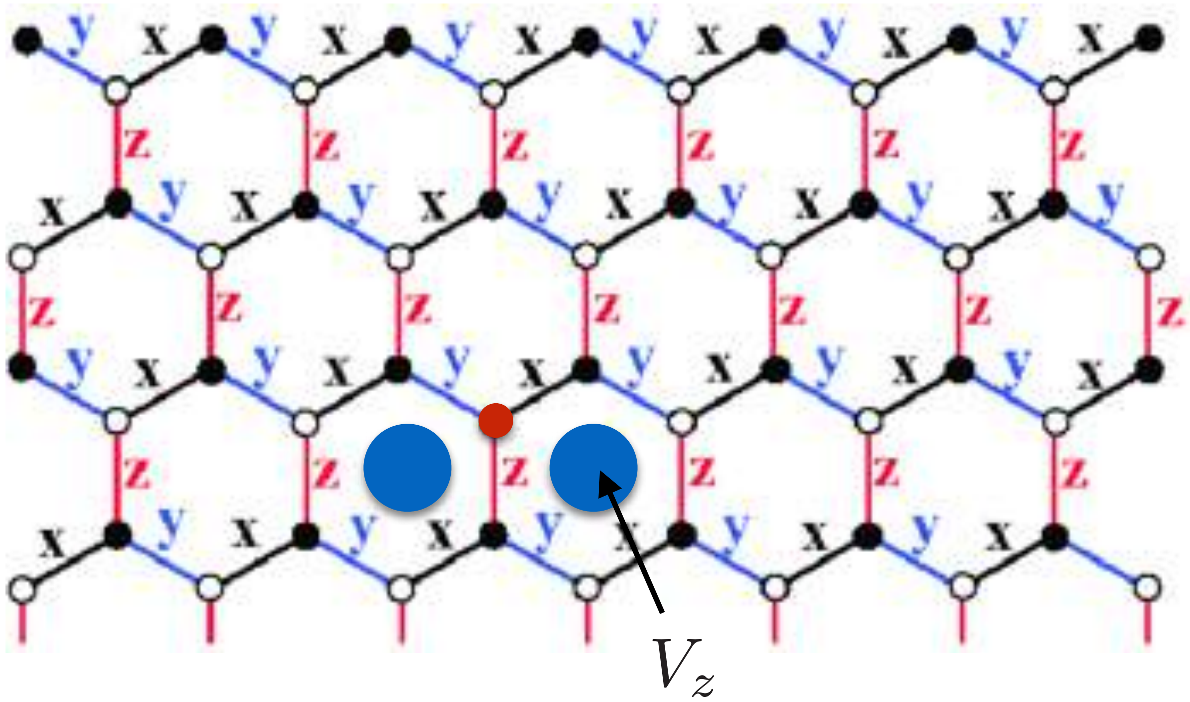

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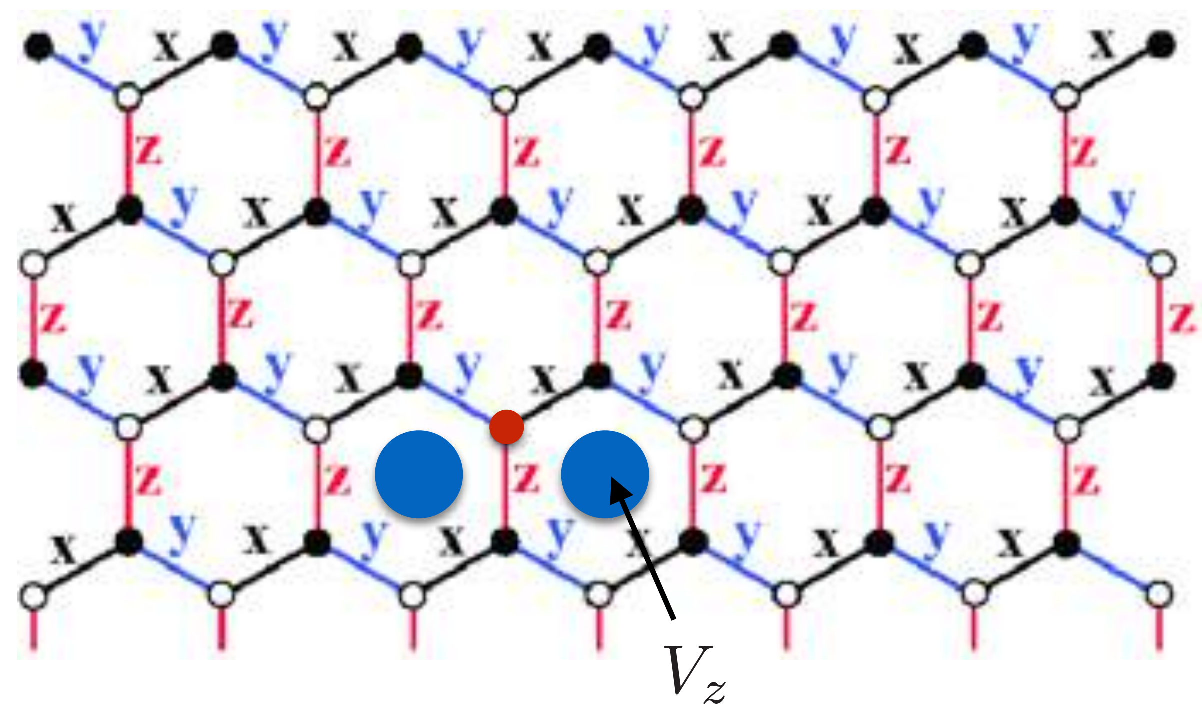

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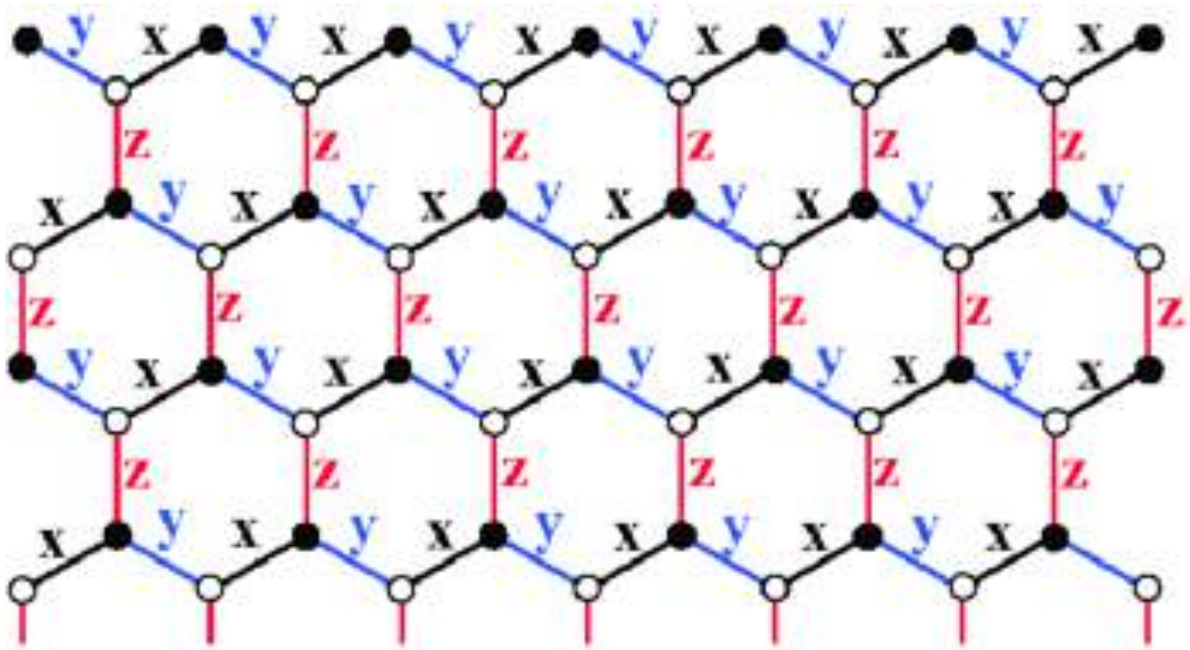

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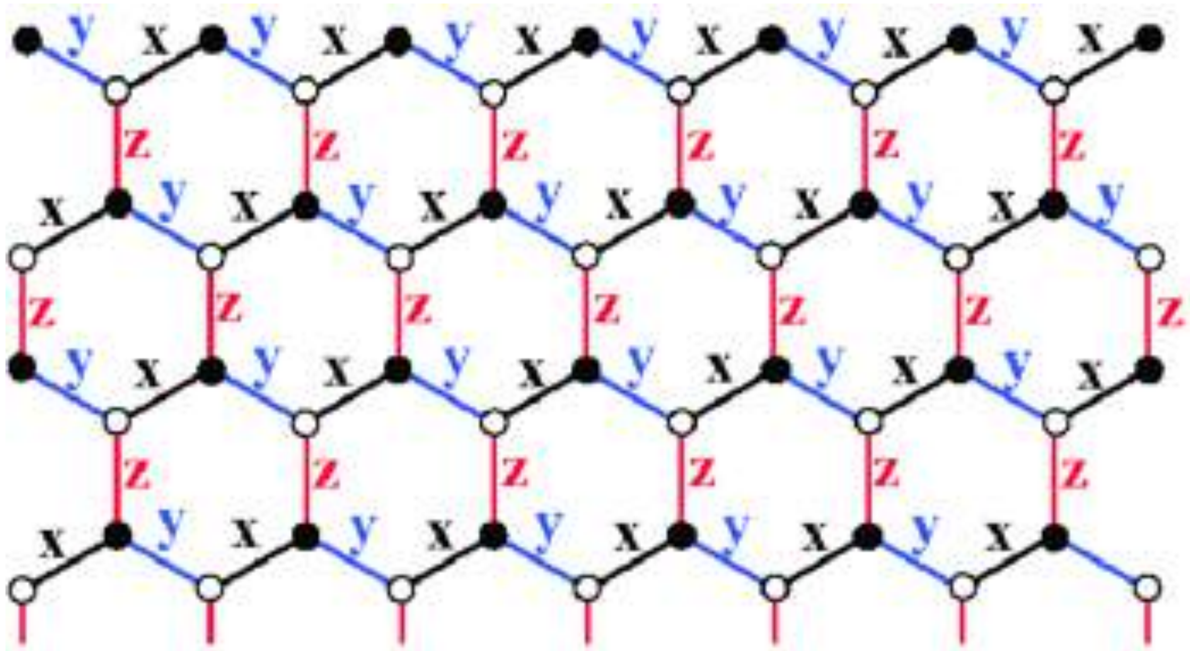

- adiabatic approximation: turn on local flux potential in the infinite past and turn it off in the infinite future.

$$-i \left\langle S_{\boldsymbol{j}}^{\alpha}(t_2) S_{\boldsymbol{j}}^{\alpha}(t_1) \right\rangle = \langle 0 | \, T c_{\boldsymbol{j}}(t_2) c_{\boldsymbol{j}}(t_1) e^{\frac{i}{\hbar} \int_{-\infty}^{\infty} [H_K^{(0)}(t') + V_\alpha(t')] dt'} \, | 0 \rangle$$

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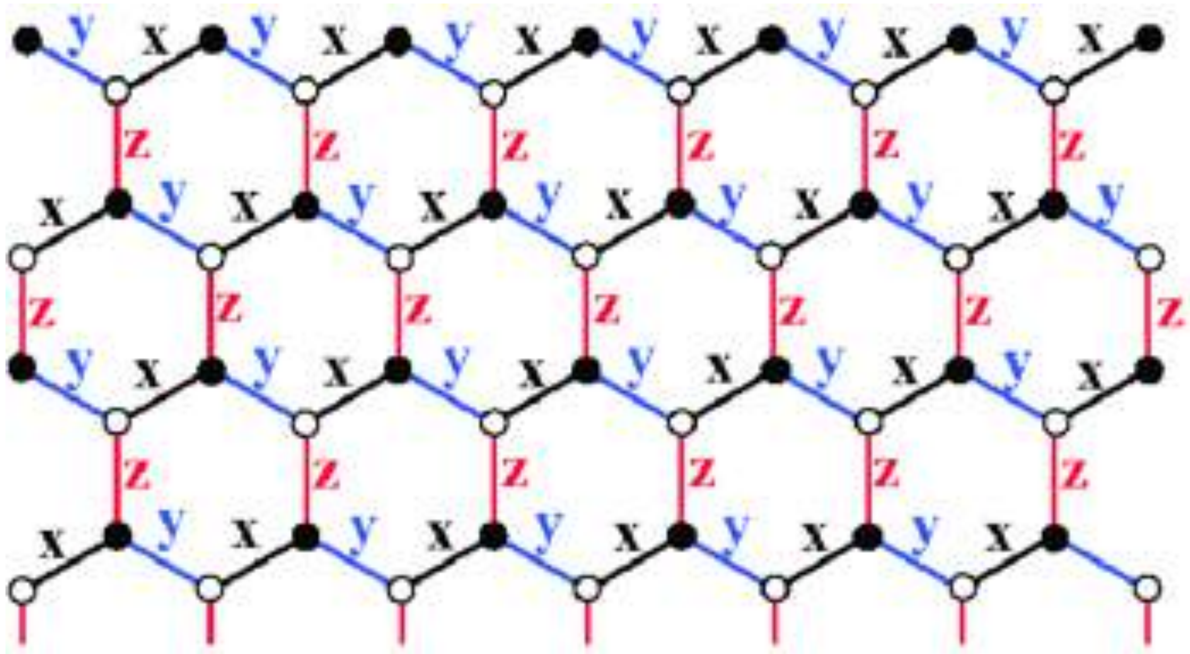

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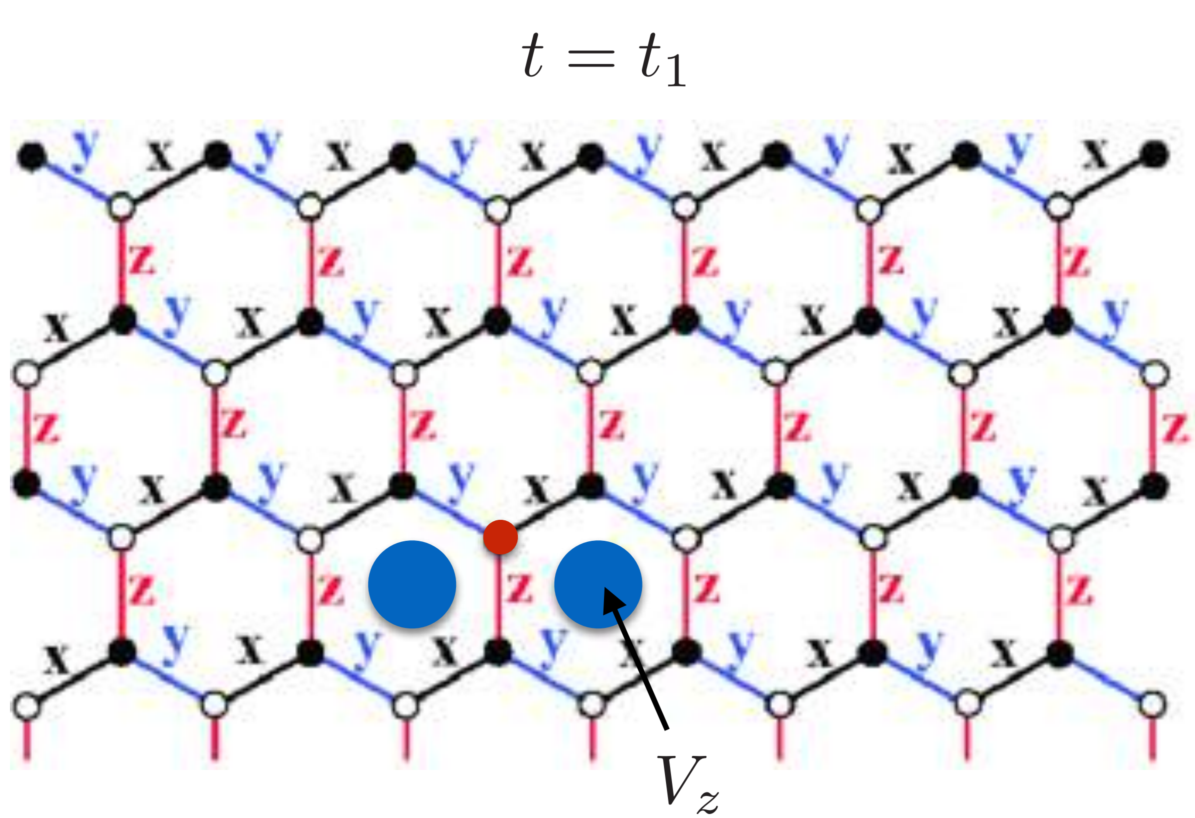

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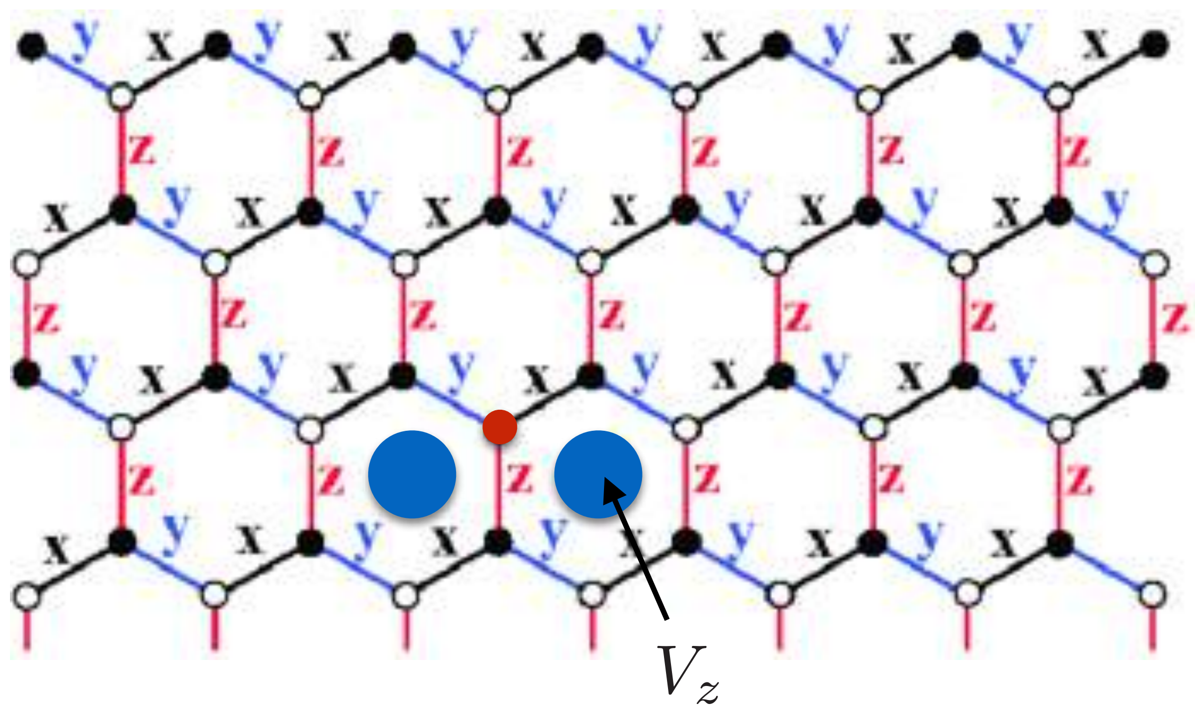

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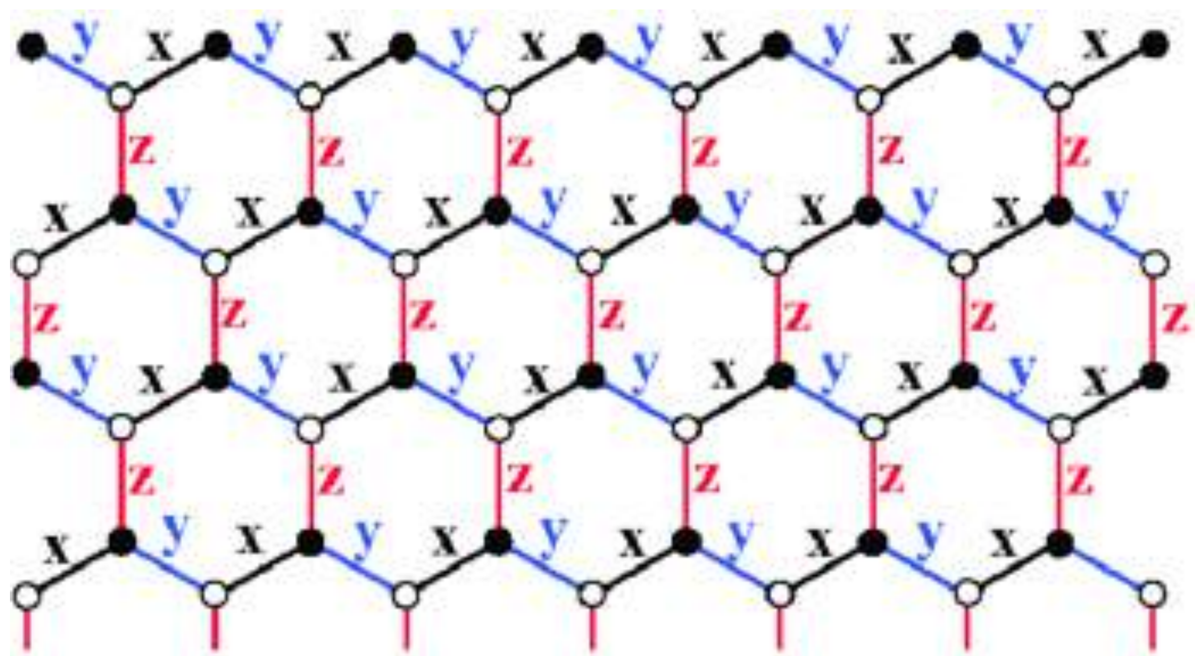

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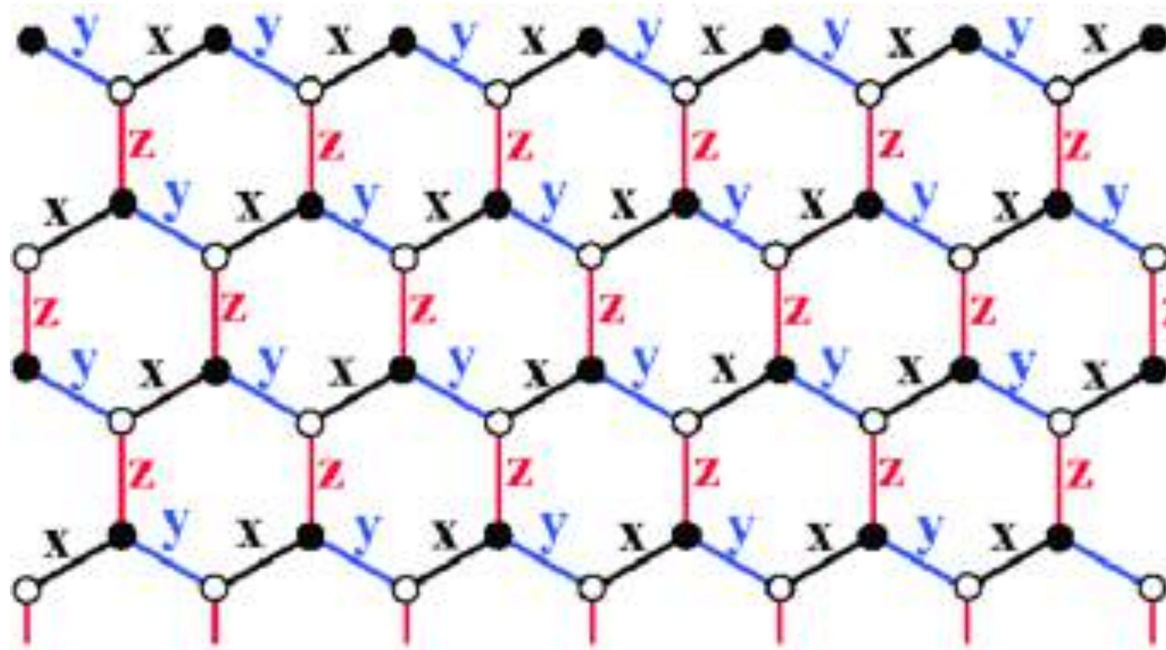

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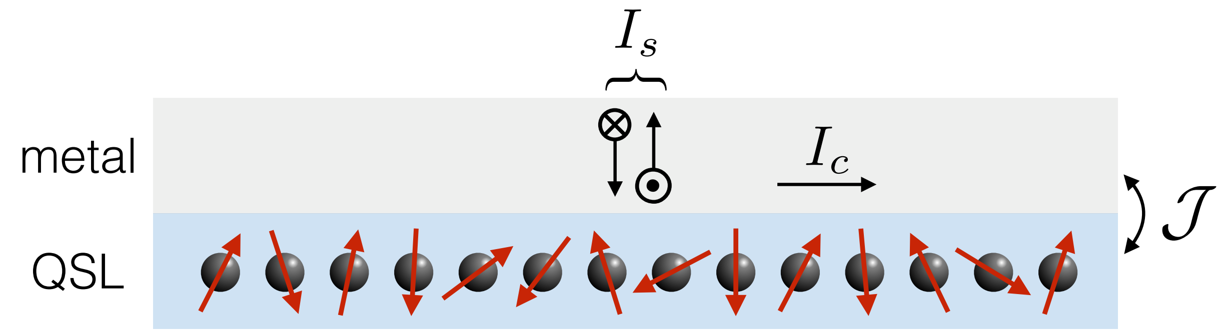
J. Knolle *et al.*, Phys. Rev. Lett. **112**, 207203 (2014)

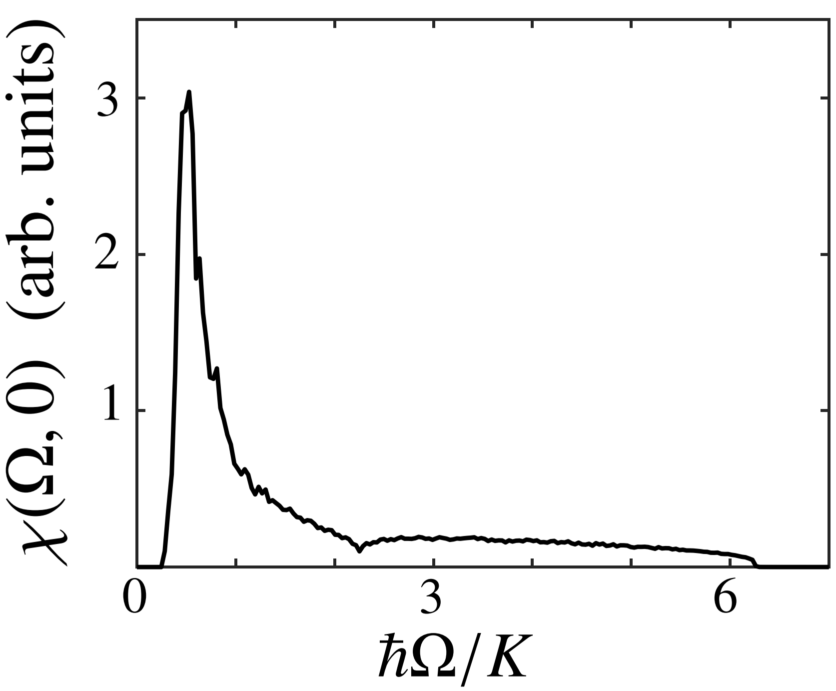
- correction to zero-temperature AC voltage noise across adjacent metal:

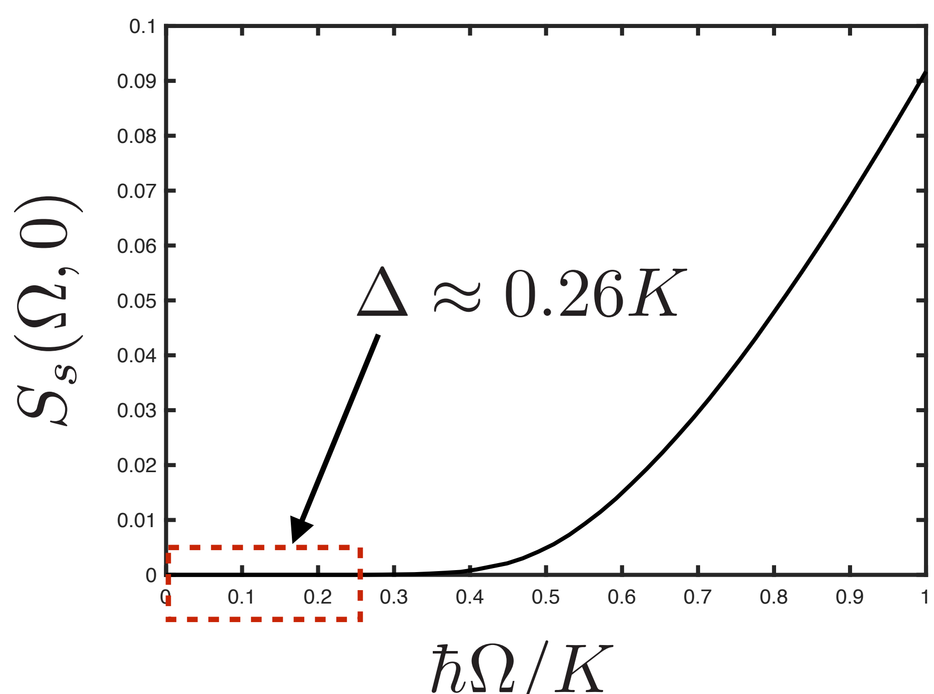
$$\delta S_V(\Omega) = \Theta S_s(\Omega)$$



$$S_s(\Omega, T) = 2i\left(\frac{\mathcal{J} v_0 m k_F}{2\pi^2\hbar}\right)^2 \sum_{\boldsymbol{j}} \int_{-\infty}^{\infty} d\nu \, \frac{\nu - \Omega}{e^{\beta(\nu-\Omega)} - 1} \underbrace{\left[\chi_{\boldsymbol{jj}}^{+-}(\nu) + \chi_{\boldsymbol{jj}}^{-+}(\nu)\right]}_{\chi(\nu,0)}$$

$$\chi(\nu, 0) \equiv -4i \int dt \; e^{i\nu t} \langle S_{\boldsymbol{i}}^x(t) S_{\boldsymbol{j}}^x(0)\rangle_{H_K^{(0)}+V_x}$$





J. Aftergood and S. Takei, Phys. Rev. Research **2**, 033439 (2020)

summary & outlook

- **spin Hall noise spectroscopy**: probes local spin density of states of a quantum magnet via voltage fluctuations using inverse spin Hall effect.
    - useful for probing topological edge states in quantum paramagnets.
    - useful for probing spin density of states of quantum spin liquids: test of QSL models against candidate materials.

- effect of visons in the Z_2 model?
- incoherent magnon scenario for α-$RuCl_3$?
- microscopic model for how thermal spin current noise at the interface converts into measurable voltage noise via ISHE?

$$\delta S_V(\Omega, T) = \Theta S_s(\Omega, T)$$